A Framework for Debt-Maturity Management

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How should a country manage its debt?

- **Large-stake** but **highly complex** problem
  - **Large dimensional state-space:** vector of bonds of different maturity
  - **Several risks:** income, interest-rate, default

- **Current literature**
  - use standard bonds $\implies$ curse of dimensionality
  - forces use of consols
  - typically a maximum of two consols
What we do...

- Small open economy with a **continuum of bonds**,  
  - Exogenous paths for income and (risk-free) interest rates

- **Solution using infinite dimensional calculus**  
  - **Realistic debt structure**  
  - Analytic expressions, rich dynamics

- **Highlight liquidity cost**  
  - Price impact of issuances  
  - Microfounded from OTC frictions
A simple formula for issuances

Solution: in each maturity

\[
\text{issuance} = (\text{price impact})^{-1} \times \text{value gap},
\]

where

\[
\text{value gap} = \frac{\text{market price} - \text{domestic valuation}}{\text{market price}}
\]

and domestic valuation is the price of a bond using the aggregate stochastic discount factor (domestic discount).

This formula holds even as the price impact \( \to 0 \) (no indeterminacy)
(Some) related literature

- **Preferred Habit Models**: Duffie, Garleanu, Pedersen (2005), Vayanos Vila (2010)


Maturity management with liquidity costs
Environment: shocks, preference and state

- Exogenous paths: income $y(t)$, short rate $\tilde{r}(t)$
- Preferences:

  \begin{align*}
  V_0 &= \int_0^{\infty} e^{-\rho t} u(c(t))dt \quad \text{and} \quad u(x) \equiv \frac{c^{1-\sigma} - 1}{1-\sigma}
  \end{align*}

- State: debt $f(\tau, t)$, expiration $\tau \in [0, T]$
Environment: constraint set

- **Budget Constraint:**

\[
c(t) = y(t) + \int_0^T q(\tau, t, \iota) \iota(\tau, t)d\tau - f(0, t) - \delta \int_0^T f(\tau, t)d\tau.
\]

- **Bond price:**

\[
q(\tau, t, \iota) = \psi(\tau, t) - \left(1 - \frac{1}{2}\lambda \iota\right) \psi(\tau, t)
\]

- **PDE constraint:**

\[
\frac{\partial f(\tau, t)}{\partial t} = \iota(\tau, t) + \frac{\partial f(\tau, t)}{\partial \tau}; \quad f(\tau, 0) = f_0(\tau)
\]
Bond Price $\psi(\tau, t)$:

- Short rate $\bar{r}(t)$ path and no-arbitrage:

\[
\psi(\tau, t) = e^{-\int_0^\tau \bar{r}(t+s)ds} + \delta \int_0^\tau e^{-\int_0^s \bar{r}(t+z)dz} ds
\]

- In PDE form:

\[
\bar{r}(t)\psi(\tau, t) = \delta + \frac{\partial \psi}{\partial t} - \frac{\partial \psi}{\partial \tau}; \quad \psi(0, t) = 1
\]
Environment: liquidity cost

- Primary / secondary bond markets
  - Primary: walrasian auction of \( \iota(\tau, t) \)
    - Primary dealers only
    - Cost of capital \( \bar{r}(t) + \eta \)
    - Auction price \( q(\tau, t) \)
  - Secondary: OTC market
    - \( \mu \cdot y_{ss} \) order flow
    - Client valuation \( \psi(\tau, t) \)

- Auction price (small issuances):

\[
q(\tau, t, \iota) \simeq \psi(\tau, t) - \frac{1}{2} \underbrace{\bar{\lambda} \cdot \psi(\tau, t)}_{\lambda(\iota)}
\]

where

\[
\bar{\lambda} \equiv \frac{\eta}{\mu y_{ss}}.
\]
Government problem

Perfect Foresight

$$V[f(\cdot,0)] := \max_{\{\iota(\tau,t)\}_{t \in [0,\infty), \tau \in [0,T]}} \int_t^\infty e^{-\rho(s-t)} u(c(s)) \, ds$$

$$s.t. \ c(t) = y(t) - f(0,t) + \int_0^T [q(\tau,t,\iota) \iota(\tau,t) - \delta f(\tau,t)] \, d\tau$$

$$\frac{\partial f}{\partial t} = \iota(\tau,t) + \frac{\partial f}{\partial \tau}; f(\tau,0) = f_0(\tau)$$
Solution

- **Optimal issuances** $\nu(\tau, t)$:

  $$\nu = \frac{1}{\bar{\lambda}} \left( \frac{\psi(\tau, t) - v(\tau, t)}{\psi(\tau, t)} \right)$$

  (Price impact)$^{-1}$ Value gap

- **Domestic valuation** $v(\tau, t)$ solves the "individual trader" price-PDE:

  $$r(t) v(\tau, t) = \delta + \frac{\partial v}{\partial t} - \frac{\partial v}{\partial \tau}, \text{ if } \tau \in (0, T]$$

  $$v(0, t) = 1$$
Frictionless benchmark ($\bar{\lambda} = 0$)

- If $\psi$ arbitrage free:
  
  $$r(t) = \bar{r}(t),$$
  $$\psi(\tau, t) = v(\tau, t)$$

- Discount Factor pins consumption:
  
  $$\frac{\dot{c}(t)}{c(t)} = -\frac{\rho - \bar{r}(t)}{\sigma}$$

- Indeterminate maturity!
Calibration

- **Liquidity cost** $\bar{\lambda} \equiv \frac{\eta}{\mu y_{ss}} = 7.08$
  - Arrival rate $\eta = 0.0011$: US primary dealers exhaust 60 percent of their inventory at the end of one week (Fleming and Rosenberg, 2009)
  - Spread $\eta = 0.015$: AA-, A- and BBB-option-adjusted-spreads of bonds from the 5 largest US banks

- **Spanish data**
  - Discount factor $\rho = 0.0416$: average Spanish Government Net Debt of 46 percent (1985-2016).
  - Maximum bond maturity $T = 20$

- **Other**
  - Rest of the parameters taken from literature (Aguiar and Gopinath, 2006)
  - Steady state risk-free interest rates and coupons $\bar{r}_{ss} = \delta = 0.04$
  - Risk aversion $\sigma = 2$
  - Steady state output $y_{ss} = 1$
Asymptotic values - as function of $\bar{\lambda}$

(a) Consumption, $c_\infty$
(b) Discount, $r_\infty$
(c) Issuances, $\iota_\infty$
(d) Distribution, $f_\infty$
Steady State: model versus Spanish data (July 2018)
Transitional Dynamics: overview

▶ Input: $c(t)$

▶ Discount Factor:

$$r(t) = \rho + \sigma \frac{\dot{c}(t)}{c(t)}$$

▶ Valuations:

$$r(t) \left\{ \begin{array}{c} \text{Value of Debt} \\ \v(\tau, t) \end{array} \right\} = \delta \left\{ \begin{array}{c} \text{Coupon} \\ \frac{d\v(\tau, t)}{dt} \end{array} \right\} + \frac{\partial \v}{\partial t} - \frac{\partial \v}{\partial \tau}$$

▶ Optimal Issuance:

$$\iota(\tau, t) = \frac{1}{\lambda} \left( \frac{\psi(\tau, t) - \v(\tau, t)}{\psi(\tau, t)} \right)$$

▶ Output: $c(t)$ (follows from KFE+BC)
Income shocks: consumption smoothing tilts duration upwards

Impulse response to an unanticipated 5 percent decline in income
Interest rate shocks: a race between bond-price reaction and consumption smoothing

Impulse response to an unanticipated increase from 4 to 5 percent in rates
Conclusion

- **Main contribution:** new approach to study maturity
  - Liquidity costs
  - Continuum of bonds of any class

- **Forces at play**
  - Consumption smoothing vs. bond-price reaction
  - Self-insurance and hedging
  - Revenue-echo effect

- **Future:** lack of commitment
Appendix
Solving it

Step 1. Lagrangian

\[ \mathcal{L}(\nu, f) = \int_0^\infty e^{-\rho t} u(c(t)) \, dt \]

\[ + \int_0^\infty \int_0^T e^{-\rho t} j(\tau, t) \left( -\frac{\partial f}{\partial t} + \nu(\tau, t) + \frac{\partial f}{\partial \tau} \right) \, d\tau \, dt, \]

\[ c(t) = y(t) - f(0, t) + \int_0^T [q(t, \tau, \nu) \nu(\tau, t) - \delta f(\tau, t)] \, d\tau. \]
Solving it

Step 2. First-Order Condition and Envelope

\[ u'(c) \left( q(\tau, t, \nu) + \frac{\partial q}{\partial \nu} \nu(\tau, t) \right) = -j(\tau, t) \]

\[ \rho j(\tau, t) = -\delta u'(c(t)) + \frac{\partial j}{\partial t} - \frac{\partial j}{\partial \tau}, \text{ if } \tau \in (0, T] \]

and \( j(0, t) = -u'(c(t)) \).
Introducing risk
The challenge

▶ What about Risk?
▶ Complication: fixed point in sets of functions (akin to heterogeneous-agent models with aggregate shocks)
Introducing risk

- Only one shock

- Shock arrival $\sim \text{Exp}(\phi)$

- After shock:
  - Draw: $\{y(0), \bar{r}(0)\} \sim F$
  - Transition deterministic: $\{y(t), \bar{r}(t)\} \rightarrow \{y_{ss}, \bar{r}_{ss}\}$

- Risky Steady State (RSS):
  - Steady state of the model before shock arrival
Pre-shock - Risky Steady State (RSS)

▶ Risk Adjusted Valuation:

\[ \rho v_{rss}(\tau) = \delta + \frac{\partial v_{rss}(\tau)}{\partial \tau} + \phi \left[ \mathbb{E} \left[ v(\tau,0) \frac{u'(c(0))}{u'(c_{rss})} \right] - v_{rss}(\tau) \right] \]

▶ Intuition:

▶ Jump in \( v(\tau,0) \) and \( c(0) \) given by \( f_{rss}(\tau) \)
▶ **Marginal-utility ratio**: "exchange-rate" between states
Income shock: a race between consumption smoothing and self-insurance

Impulse response to a 5 percent decline in income
Interest rate shock: hedging too costly, bond-price reaction dominates

Impulse response to an increase from 4 to 5 percent in rates

(a) Debt distribution, $f(\tau)$
(b) Consumption, $c(t)$
(c) Total debt, $b(t)$
(d) Average duration
(e) Bond prices, $\psi(\tau, t)$
(f) Domestic valuations, $v(\tau, t)$
The option to default
Default

- **Option of default** when a shock arrives
- **At date of default**
  - Draw value of autarky from distribution $\Theta(\cdot)$
- **Debt policy**
  - **Under commitment** (no debt dilution)
Default

- Domestic valuation:

\[
\begin{align*}
\hat{r}(t)\hat{v}(\tau, t) & = \\
& = \delta + \frac{\partial \hat{v}}{\partial t} - \frac{\partial \hat{v}}{\partial \tau} \\
& + \phi \left[ \mathbb{E}_s^X \left[ \left( \Theta(V(t)) + \Omega(t) \right) \frac{U'(c(t))}{U'(\hat{c}(t))} v(\tau, t) \right] - \hat{v}(\tau, t) \right]
\end{align*}
\]

- Bond prices:

\[
\begin{align*}
\bar{r}(t)\bar{\psi}(\tau, t) & = \\
& = \delta + \frac{\partial \bar{\psi}}{\partial t} - \frac{\partial \bar{\psi}}{\partial \tau} \\
& + \phi \left( \Theta(V(t)) + \psi(\tau, t, X_t) - \hat{\psi}(\tau, t) \right)
\end{align*}
\]
Revenue-echo effect

Main term:

\[ \Omega(t) = \theta(V(t)) U'(\hat{c}(t)) \ldots \]

\[ \int_0^T \int_{\max\{t+m-T,0\}}^t e^{-\int_z^t (\bar{r}(u) - \hat{r}(u))du} \ldots \]

\[ \frac{\psi(m, t)}{2\bar{\lambda}} \left[ 1 - \left( \frac{\hat{v}(m + t - z, z)}{\hat{\psi}(m + t - z, z)} \right)^2 \right] dzdm \]
Revenue-echo effect

Figure: Illustration of the revenue echo effect (axes inverted).
Income shock: (i) bond-price reaction through risk premium, (ii) echo effect, (iii) self-insurance and (iv) insurance through default

Impulse response to a 5 percent decline in income