A Market-Based Term Structure of Expected Inflation for Mexico

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Disclaimer:
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Goals

1 To obtain a market-based term structure of inflation expectations, i.e., \( \mathbb{E}_t [\pi_{t+k}] \) for \( k = 1, \ldots, 10 \) years. In particular, we do not use inflation explicitly.

2 Case of study: Mexico.

3 To compare our estimates with Banco de Mexico’s inflation surveys, for short-, medium- and long-term horizons.
   - Informational content
   - Inflation forecasters
Motivation

1. The monetary authority has a keen interest in understanding how its policies affect inflation expectations, as changes in its policy affect the economy with a time lag.

2. Also, it is interested in the anchorage of long-term inflation expectations.

3. Portfolio managers commonly assess expected inflation, given its relevance as an economic indicator.

4. The bulk of financial contracts that agents celebrate are in nominal terms.

5. Break-even inflation (nominal minus real interest rates) contains liquidity (see e.g., for the US, Gürkaynak et al., 2010; Andreasen et al., 2018), default and other risk premiums. Such premiums might notably grow during financial or economic stress episodes.
Literature Review

1. Affine Interest Rate Models (Piazzesi, 2010)

2. Estimating Affine Interest Rate Models (Singleton, 2006)
   - Interest Rates’ Errors / Excess Returns-Holding Period Return Errors (Ang and Piazzesi, 2003 / Adrian and Wu, 2009)
   - Max Likelihood (Ang and Piazzesi, 2003) / Max Likelihood + Kalman Filter (Grishchenko et al., 2017) / OLS (Adrian et al., 2013)

3. Inflation Expectations
   - Econometric/Macroeconomic Approach (Stock and Watson, 1999)
   - Survey-based approach (Aruoba, 2014)
   - Market-based approach (inflation swaps: Fleckenstein et al, 2016; nominal and real interest rates: Abrahams et al., 2016; our paper)
   - Any combination of the above (Adrian and Wu, 2006)
   - Which one performs better? (Grothe and Meyler, 2015; Ang et al., 2016)
Main Approaches to Obtaining (Inflation) Expectations

1. Econometric / Macroeconomic Based
   - Popular
   - Direct implementation
   - Frequency is limited to that of the time series (temporal disaggregation possible)

2. Survey-Based
   - Limited frequency (for Mexico, Citi’s is fortnightly, Banco de México’s monthly)
   - Limited forecasting horizons
   - Some uncertainty on the exact date stamp for the information set of the surveyees

3. Market-Based
   - Higher frequency, in particular, if based on financial time series
   - More flexible forecasting horizons
   - Explicit information set
   - Model (and sample) dependent
Data

1. Zero-coupon nominal interest rates, with maturities 1-month, 1-, 5-, 10- and 20-year (Valmer)

2. Zero-coupon real (index-linked) interest rates, with maturities 5-, 10- and 20-year (Valmer)

3. We estimate the following zero-coupon real (annualized) interest rates:

\[
1 \text{ year: } (1 + i_t^{(12m)}) = (1 + r_t^{(12m)}) E_t [1 + \pi_t^{(12m)}]
\]

\[
1 \text{ month: } (1 + i_t^{(1m)}) = (1 + r_t^{(1m)}) E_t [1 + \pi_t^{(12m)}],
\]

assuming (see e.g., Magud and Tsounta, 2012):

\[
E_t [1 + \pi_t^{(1m)}] \approx E_t [1 + \pi_t^{(12m)}]
\]

4. To obtain the rest of needed interest rate nodes, we use cubic interpolation.

5. We use \( E_t [\pi_t^{(12)}], E_t [\pi_t^{(12-48)}] \) and \( E_t [\pi_t^{(60-98)}] \) from Banco de Mexico’s surveys.

6. Sample: January, 2004 – September, 2018
Assume the dynamics of \( K \) state variables (risk factors) \( X_t \) follow a VAR(1):

\[
X_{t+1} = \mu + \Phi X_t + v_{t+1}; \quad v_{t+1}|\{X_s\}_{s=0}^t \sim N(0, \Sigma)
\]

and that the (log) excess holding period return are

\[
rx_{t+1}^{(n-1)} = \beta^{(n-1)'} [\gamma_0 + \gamma_1 X_t] + \beta^{(n-1)'} v_{t+1} + e_{t+1}^{(n-1)}
\]

\[
rx_{t+1}^{(n-1)} = \ln P_{t+1}^{(n-1)} - \ln P_{t}^{(n)} + \ln P_{t}^{(1)}
\]

Next

\[
rx_{t+1}^{(n-1)} = a^{(n-1)} + \beta^{(n-1)'} v_{t+1} + c^{(n-1)'} X_t + e_{t+1}^{(n-1)} \quad (\text{OLS estimation})
\]

\[
\hat{a} = (\hat{a}^{(1)}, \ldots, \hat{a}^{(N)}); \quad \hat{\beta} = (\hat{\beta}^{(1)}', \ldots, \hat{\beta}^{(N)'}) \quad ; \quad \hat{c} = (\hat{c}^{(1)}', \ldots, \hat{c}^{(N)'})
\]

\[
\text{Var} \left( e_{t+1}^{(n-1)} \right) = \sigma^2
\]

Therefore, \( \hat{\gamma}_0 = (\hat{\beta}' \hat{\beta})^{-1} \hat{\beta}' \hat{a}, \) and \( \hat{\gamma}_1 = (\hat{\beta}' \hat{\beta})^{-1} \hat{\beta}' \hat{c}. \)
Arbitrage Free Affine Interest Model

\[ P_t^{(n)} = \mathbb{E}_t \left[ M_{t+1} P_{t+1}^{(n-1)} \right] \]

(Pricing equation)

\[ M_{t+1} = \exp \left( -y_t^{(1)} - \frac{\lambda'_t \lambda_t}{2} - \lambda'_t \Sigma^{-1/2} v_{t+1} \right) \]

(Pricing Kernel as in Duffee, 2002)

\[ \lambda_t = \Sigma^{-1/2} (\lambda_0 + \lambda_1 X_t) \]

(Risk Pricing)
Recursive Relations amongst Coefficients

Assume log prices are affine in $X_t$ plus an error term

$$\ln P_t^{(n)} = A_n + B'_nX_t + u_t^{(n)}$$

$$A_n = A_{n-1} + B'_{n-1}(\mu - \lambda_0) + \frac12 (B'_{n-1}\Sigma B_{n-1} + \sigma^2) - \delta_0$$

$$A_0 = 0$$

$$B'_n = B'_{n-1}(\Phi - \lambda_1) - \delta_1$$

$$B_0 = 0$$, for $n = 1, ..., N$

$$A^*_n = A^*_{n-1} + (B^*_{n-1})'\mu + \frac12 [(B^*_{n-1})'\Sigma B^*_{n-1} + \sigma^2] - \delta_0$$

$$A^*_0 = 0.$$  

$$(B^*_n)' = (B^*_{n-1})'\Phi - \delta_1;$$

$$B^*_0 = 0$$, for $n = 1, ..., N$. 


Recursive Relations amongst Coefficients

\[ A_n = A_{n-1} + B'_{n-1}(\mu - \lambda_0) + \frac{1}{2} (B'_{n-1} \Sigma B_{n-1} + \sigma^2) - \delta_0 \]
\[ A_0 = 0 \]
\[ B'_n = B'_{n-1}(\Phi - \lambda_1) - \delta_1 \]
\[ B_0 = 0, \text{ for } n = 1, \ldots, N \]

\[ A^*_n = A^*_{n-1} + (B^*_{n-1})' \mu + \frac{1}{2} [(B^*_{n-1})' \Sigma B^*_{n-1} + \sigma^2] - \delta_0 \]
\[ A^*_0 = 0 \]
\[ (B^*_n)' = (B^*_n - 1)' \Phi - \delta_1 \]
\[ B^*_0 = 0, \text{ for } n = 1, \ldots, N \]
Estimation Procedure in a Nutshell

1. Use PCA on nominal interest rates to obtain nominal risk factors ($K_n = 5$ in our case).

2. Use PCA on real interest rates to obtain real risk factors ($K_r = 4$ in our case).

3. Estimate Nominal Affine Model with 3-step OLS (Adrian et al., 2013).

4. Estimate Real Affine Model with 3-step OLS (Adrian et al., 2013).

5. Extract risk-neutral and term premium components of model implied nominal (real) interest rates.
Models’ Fit: Nominal Interest Rates

Note: Simple Interest Rates. Source: Own estimations with data from Valor de Mercado and Banco de México.
Models’ Fit: **Real** (Index-linked) Rates

*Note: Simple Interest Rates. Source: Own estimations with data from Valor de Mercado and Banco de México.*
# Models’ Fit: Errors

## Interest Rates Pricing Errors

<table>
<thead>
<tr>
<th>Horizon (months)</th>
<th>1</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>1.84</td>
<td>-0.22</td>
<td>-0.31</td>
<td>1.29</td>
<td>0.85</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.68</td>
<td>3.86</td>
<td>2.95</td>
<td>2.47</td>
<td>1.26</td>
<td>1.56</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.45</td>
<td>0.54</td>
<td>-0.04</td>
<td>-0.06</td>
<td>0.99</td>
<td>0.37</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>0.82</td>
<td>-0.09</td>
<td>-0.54</td>
<td>-0.49</td>
<td>0.56</td>
<td>-0.04</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>2.27</td>
<td>6.80</td>
<td>2.80</td>
<td>3.22</td>
<td>3.01</td>
<td>1.36</td>
</tr>
<tr>
<td>$\rho(6)$</td>
<td>2.27</td>
<td>6.80</td>
<td>2.80</td>
<td>3.22</td>
<td>3.01</td>
<td>1.36</td>
</tr>
</tbody>
</table>

## Real Rates Pricing Errors

<table>
<thead>
<tr>
<th>Horizon (months)</th>
<th>1</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>5.76</td>
<td>0.66</td>
<td>0.35</td>
<td>0.92</td>
<td>0.87</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>3.17</td>
<td>6.40</td>
<td>3.80</td>
<td>4.71</td>
<td>3.86</td>
<td>1.57</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.69</td>
<td>0.03</td>
<td>0.65</td>
<td>1.34</td>
<td>-0.14</td>
<td>0.74</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>2.59</td>
<td>1.19</td>
<td>2.02</td>
<td>3.44</td>
<td>0.41</td>
<td>1.49</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>2.27</td>
<td>6.80</td>
<td>2.80</td>
<td>3.22</td>
<td>3.01</td>
<td>1.36</td>
</tr>
<tr>
<td>$\rho(6)$</td>
<td>2.27</td>
<td>6.80</td>
<td>2.80</td>
<td>3.22</td>
<td>3.01</td>
<td>1.36</td>
</tr>
</tbody>
</table>

## Excess Return Pricing Errors

<table>
<thead>
<tr>
<th>Horizon (months)</th>
<th>1</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Rates</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.00</td>
<td>0.00</td>
<td>0.01</td>
<td>0.00</td>
<td>0.02</td>
<td>0.11</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.65</td>
<td>-0.62</td>
<td>0.55</td>
<td>0.82</td>
<td>-0.52</td>
<td>-0.62</td>
</tr>
<tr>
<td>Excess Kurtosis</td>
<td>0.89</td>
<td>1.22</td>
<td>1.40</td>
<td>2.66</td>
<td>0.39</td>
<td>1.04</td>
</tr>
<tr>
<td>$\rho(1)$</td>
<td>0.37</td>
<td>0.34</td>
<td>-0.05</td>
<td>-0.04</td>
<td>0.09</td>
<td>-1.00</td>
</tr>
<tr>
<td>$\rho(6)$</td>
<td>0.62</td>
<td>0.83</td>
<td>0.77</td>
<td>0.82</td>
<td>0.82</td>
<td>0.52</td>
</tr>
</tbody>
</table>

## Notes:
Mean and Std. Dev. in basis points. $\rho(n)$ stands for the lag-$n$ autocorrelation coefficient.
Extracting Inflation Expectations

Nominal and Real Interest Rates

\[
(1 + i_t^{(n)}) = (1 + R_t^{(n)}) \left( 1 + \mathbb{E}_t[\pi_{t,t+n}] \right) (1 + \rho_t^{(n)})
\]

\[
(1 + r_t^{(n)}) = (1 + R_t^{(n)}) \left( 1 + \tilde{\rho}_t^{(n)} \right)
\]

Risk-Neutral Nominal and Real Interest Rates

\[
(1 + i_t^{(n,*)}) = (1 + R_t^{(n)}) \left( 1 + \mathbb{E}_t[\pi_{t,t+n}] \right) \quad [\lambda_0, \lambda_1 = 0]
\]

\[
(1 + r_t^{(n,*)}) = (1 + R_t^{(n)}) \quad [\tilde{\lambda}_0, \tilde{\lambda}_1 = 0]
\]

As \( \rho_t^{(n)} = \tilde{\rho}_t^{(n)} = 0. \)

\[
\mathbb{E}_t[\pi_{t,t+n}] = \left( 1 + i_t^{(n,*)} \right) \left( 1 + r_t^{(n,*)} \right)^{-1}
\]
Extracting Inflation Expectations

\[
(1 + i_t^{(n)}) = (1 + R_t^{(n)}) (1 + \mathbb{E}_t[\pi_{t,t+n}]) (1 + \rho_t^{(n)})
\]

\[
(1 + r_t^{(n)}) = (1 + R_t^{(n)}) (1 + \tilde{\rho}_t^{(n)})
\]

\[
(1 + i_t^{(n,*)}) = (1 + R_t^{(n)}) (1 + \mathbb{E}_t[\pi_{t,t+n}]) \quad [\lambda_0, \lambda_1 = 0]
\]

\[
(1 + r_t^{(n,*)}) = (1 + R_t^{(n)}) \quad [\tilde{\lambda}_0, \tilde{\lambda}_1 = 0]
\]

\[
\mathbb{E}_t[\pi_{t,t+n}] = (1 + i_t^{(n,*)}) (1 + r_t^{(n,*)})^{-1}
\]
Results: Inflation Expectations

Source: Own estimations with data from Valor de Mercado and Banco de México.
Results: Inflation Expectations

Source: Own estimations with data from Valor de Mercado and Banco de México.
Results: Basic Statistics, Inflation Expectations

<table>
<thead>
<tr>
<th>Inflation Expectations Horizon</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Std. Dev.</td>
</tr>
<tr>
<td>Skewness</td>
</tr>
<tr>
<td>Kurtosis</td>
</tr>
</tbody>
</table>

Time Series, Mean

Time Series, Standard Deviation
Inflation Expectations Using a Subsample

Note: Expected inflations, 12-month horizon.
Source: Own estimations with data from Valor de Mercado and Banco de México.
Model vs Surveys – 12 months

Sources: Banco de Mexico’s surveys and own estimations with data from Valor de Mercado and Banco de México.
Model vs Surveys – 1 to 4 years

Sources: Banco de Mexico’s surveys and own estimations with data from Valor de Mercado and Banco de México.
Model vs Surveys – 4 to 8 years

Sources: Banco de Mexico’s surveys and own estimations with data from Valor de Mercado and Banco de México.
Granger Wald Causality Tests

### Short-term Expectations (12-months)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Excluded</th>
<th>$\chi^2$</th>
<th>DF</th>
<th>Pr &gt; $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>Survey</td>
<td>0.57</td>
<td>1</td>
<td>0.45</td>
</tr>
<tr>
<td>Survey</td>
<td>Market</td>
<td>23.03</td>
<td>1</td>
<td>&lt;0.00</td>
</tr>
<tr>
<td>Market</td>
<td>Survey</td>
<td>3.63</td>
<td>2</td>
<td>0.16</td>
</tr>
<tr>
<td>Survey</td>
<td>Market</td>
<td>18.25</td>
<td>2</td>
<td>&lt;0.00</td>
</tr>
</tbody>
</table>

### Medium-term Expectations (1-4 years)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Excluded</th>
<th>$\chi^2$</th>
<th>DF</th>
<th>Pr &gt; $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>Survey</td>
<td>&lt;0.00</td>
<td>2</td>
<td>0.98</td>
</tr>
<tr>
<td>Survey</td>
<td>Market</td>
<td>22.59</td>
<td>2</td>
<td>&lt;0.00</td>
</tr>
</tbody>
</table>

### Long-term Expectations (5-8 years)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Excluded</th>
<th>$\chi^2$</th>
<th>DF</th>
<th>Pr &gt; $\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>Survey</td>
<td>0.21</td>
<td>1</td>
<td>0.64</td>
</tr>
<tr>
<td>Survey</td>
<td>Market</td>
<td>5.32</td>
<td>1</td>
<td>0.02</td>
</tr>
<tr>
<td>Market</td>
<td>Survey</td>
<td>3.11</td>
<td>2</td>
<td>0.21</td>
</tr>
<tr>
<td>Survey</td>
<td>Market</td>
<td>5.29</td>
<td>2</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Null hypothesis: the “excluded” variable does not cause in the Granger sense the “equation” variable. Thus, rejecting the null is evidence of the “excluded” variable Granger-causing the “equation” variable.
Diebold – Mariano Tests

Market-based Model

<table>
<thead>
<tr>
<th>Horizon</th>
<th>DM statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-month</td>
<td>1.09</td>
<td>0.28</td>
</tr>
<tr>
<td>1 to 4-year</td>
<td>-0.80</td>
<td>0.42</td>
</tr>
<tr>
<td>5 to 8-year</td>
<td>-1.38</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Notes: Diebold-Mariano (DM) test on mean squared errors. A positive DM statistic means that surveys’ expected inflation are better forecaster for inflation, and vice versa.
One could include observable risk-factors. For instance, observed inflation.

This limits our estimation procedure to monthly (fortnightly) frequency.

When we include it, the Diebold-Mariano test suggest that inflation expectations from this alternative model are better predictors of future inflation than Banco de México’s surveys, for the 5 – 8 years period.
Estimation Using Inflation as a Risk Factor

Time Series, Mean

Time Series, Standard Deviation

Horizon (n)

Market-based
Market-based+Inflation as RF

Market-based
Market-based+Inflation as RF
Diebold – Mariano Tests (alternative model)

Market-based Model + **Inflation** as a (Nominal) Risk Factor

<table>
<thead>
<tr>
<th>Horizon</th>
<th>DM statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-month</td>
<td>0.05</td>
<td>0.96</td>
</tr>
<tr>
<td>1 to 4-year</td>
<td>-1.26</td>
<td>0.21</td>
</tr>
<tr>
<td>5 to 8-year</td>
<td>-2.59</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Notes: Diebold-Mariano (DM) test on mean squared errors. A **positive** DM statistic means that surveys’ expected inflation are better forecaster for inflation, and vice versa.
Final Remarks

1. We obtained a market-based term structure of inflation expectations in Mexico with daily availability (it can be done with high-frequency data).

2. We interpret the difference of nominal and real interest rates, assuming risk-neutral investors, as a measure of expected inflation; accounting for liquidity and inflation risk premiums.

3. We found that market-based expectations Granger-cause survey-based means, but not conversely.

4. Market-based- and survey-measures are on comparable performance levels accordingly to Diebold–Mariano tests.

5. The term structure of expected inflation appears to contain information that the survey-based measures do not have. While hard to distinguish statistically, market-based measures could improve or complement more traditional approaches.
Appendices
Models’ Fit, Errors B’s vs Betas – Nominal

-1.00 -0.75 -0.50 -0.25 0.00 0

B(1)

b(1)

-1.2 -1.0 -0.8 -0.6 -0.4 -0.2 0.0 0.2 0.4

B(2)

b(2)

-2.0 -1.5 -1.0 -0.5 0.0 0.5 1.0 1.5

B(3)

b(3)

33
Models’ Fit, Errors B’s vs Betas – Real