Multivariate Filter Estimation of Potential Output for the United States

An Extension with Labor Market Hysteresis

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Presentation Outline

• Model of the paper before hysteresis.
• Application of paper’s model: Mexico’s monetary stance.
• Back to the paper: U.S.
• Why add hysteresis.
• How we add hysteresis to the model?
• U.S. results with hysteresis.
Model before Hysteresis

Multivariate filter with the following ingredients

Alichi (2015); Alichi and others (2015, 2017):

• Philips curve.
• Unemployment and capacity utilization gap blocks.
• Equation relating output gap to unemployment and capacity utilization gaps.

Alichi, Torosyan, and many others (2018), added:

• A Taylor rule.

Advertisement: Ongoing Latin America 5 work adds:

• An exchange rate block.
Application of the paper’s model: Mexico’s monetary stance
Mexico Potential Growth

Mexico: Output growth and potential growth

Annual

- Growth
- Potential Growth
- Empirical potential
- Steady state

(%)
Mexico Output Gap
Mexico Neutral Rate

Mexico: Real Interest Rates

Annual

- Short-term real interest rate
- Short-term real interest rate equilibrium
- Empirical average
- Steady state

[Graph showing the annual trend of Mexico's real interest rates from 2005 to 2025, with significant fluctuations and a shaded area indicating a specific period.]
Back to the paper: U.S.
A Tale of Two Recessions

**Unemployment rate**

Faster decline of unemployment

Source: Federal Reserve Bank of St. Louis
A Tale of Two Recessions (continued)

Employment Dynamics in Two Recessions

Sources: Federal Reserve Bank of St. Louis, D. Plotnikov “Hysteresis in unemployment and jobless recoveries”
A Tale of Two Recessions (continued)

CPI Inflation

Inflation declined as a result of significant increase of policy rate: Volcker disinflation

Inflation did not fall significantly below the target despite persistently high unemployment

Source: Federal Reserve Bank of St. Louis

CPI Inflation target 2.3% which is consistent with 2% PCE Inflation target

Percent change y-o-y


-7 0 3 6 9 12 15 18


-2 0 2 4 6 8 10 12 14 16 18


-7 0 3 6 9 12 15 18


-2 0 2 4 6 8 10 12 14 16 18


-7 0 3 6 9 12 15 18


-7 0 3 6 9 12 15 18


-7 0 3 6 9 12 15 18

NBER: US recession

NBER: US recession
Downward revisions of US GDP and potential output.

Source: IMF WEO database: April 2007 (before the crisis), April 2011 (after the first wave of the crisis), and April 2017
Incorporating Hysteresis into the Model

**Hysteresis in NAIRU**

\[
\bar{u}_t = (1 - \rho_{\bar{u}}) \bar{u}_{t-1} + \rho_{\bar{u}} u_{ss}^{\bar{u}} + g_{u,t} + \epsilon_{\bar{u},t} - \xi \times \text{movavg}(\hat{y}_{t+3},-6)
\]

\[
g_{\bar{u},t} = \rho_{g\bar{u}} g_{\bar{u},t-1} + \epsilon_{g\bar{u},t}
\]

where

- \(\bar{u}_t\) – the equilibrium value of the unemployment rate (NAIRU)
- \(\epsilon_{\bar{u},t}\) - shock on NAIRU
- \(g_{\bar{u},t}\) - growth rate of NAIRU
- \(\epsilon_{g\bar{u},t}\) - shock on the growth rate of NAIRU
- \(\hat{y}_t\) - output gap
Incorporating Hysteresis into the Model: NAIRU

Length of “ordinary” business cycle – *6 years*

- Moving average term in NAIRU doesn’t alter model behavior during “ordinary” business cycles
- Captures the effects deep recessions and prolonged recoveries on NAIRU

* http://www.nber.org/cycles.html
Incorporating Hysteresis into the Model: Potential GDP and NAIRU

**NAIRU effects on potential output**

\[
\bar{y}_t = \bar{y}_{t-1} + g_{\bar{y},t} + \epsilon_{\bar{y},t} - \eta_1(\bar{u}_t - \bar{u}_{t-1}) - (1 - \eta_1)(\bar{u}_{t-1} - \bar{u}_{t-5})/4 + \eta_2(\bar{c}_t - \bar{c}_{t-1})
\]

\[
g_{\bar{y},t} = (1 - \rho_{g_{\bar{y}}})g_{\bar{y},t-1} + \rho_{g_{\bar{y}}}g^{ss}_{\bar{y}} + \epsilon_{g_{\bar{y}},t}
\]

where

- \(\bar{y}_t\) - potential output
- \(g_{\bar{y},t}\) - growth rate of potential output
- \(\epsilon_{\bar{y},t}\) - shock on potential output
- \(\bar{c}_t\) - equilibrium value of capacity utilization
- \(\epsilon_{g_{\bar{y}},t}\) - shock on the growth rate of potential output
Comparison of Results

Hysteresis NAIRU vs No hysteresis NAIRU

Much of the persistence of unemployment rate can be explained by the persistence of NAIRU

NAIRU peaks at 7% lagging the peak in unemployment

CBO’s estimate of NAIRU incorporates structural factors that are temporarily boosting the natural rate beginning in 2008

Source: Authors’ estimates
Potential growth declines more due to structural damage in labor market.
Comparison of Results (continued)

Lower output gap due to the deterioration of potential output

Source: Authors’ estimates
Better explanation of post-crisis period in the augmented model with hysteresis

Source: Authors’ estimates
A Tale of Two Recessions

Source: Ali, Torosyan and many others (2018)
Conclusions

• Multivariate filter is a flexible framework to incorporate channels not considered in standard models.

• A model with potential output and neutral rate is estimated. Appropriate for assessing the monetary stance.

• Illustration of labor market hysteresis hypothesis revitalized in the literature after the GFC.

• Incorporation of hysteresis helps explain the persistence of high unemployment after the GFC and lack of sizeable inflation deviations from the target.
Thank You
Extra Slides
Earlier Output Gap Working Papers


Model Specification: Output

• Output defined as the deviation of real GDP from its potential level.
  \[ \hat{y}_t = y_t - \bar{y}_t \]

• Stochastic process for output has three types of shocks.

\[
\bar{y}_t = \bar{y}_{t-1} + g_{\bar{y},t} + \epsilon_{\bar{y},t} \\
g_{\bar{y},t} = (1 - \rho_{g_{\bar{y}}})g_{\bar{y},t-1} + \rho_{g_{\bar{y}}}g_{\bar{y}}^{ss} + \epsilon_{g_{\bar{y}},t} \\
\hat{y}_t = \phi_1 \hat{y}_{t-1} - \phi_2 \hat{r}_{t}^{1Y} - \phi_3 \hat{r}_{t-1}^{1Y} + \phi_4 \epsilon_{g_{\bar{y}},t} + \epsilon_{\hat{y},t}
\]
Model Specification: Output

Source: Alichiet al 2017.
Model Specification: Inflation

- To help identify the shocks, add a Phillips curve.

\[ \pi_t = \lambda_1 E_t \pi_{t+1} + (1 - \lambda_1)\pi_{t-1} + \lambda_3 \hat{y}_t + \epsilon_{\pi,t} - \lambda_4 \epsilon_{\bar{y},t} \]

- The last term in the equation allows the model to mimic the effects in a fully-structured model where level shocks to productivity can lower the marginal cost and therefore reduce inflation.

- Has the Phillips curve flattened over the past several decades?
Model Specification: Unemployment

- Unemployment block.

\[ \hat{u}_t = \bar{u}_t - u_t \]

\[ \bar{u}_t = (1 - \rho_{\bar{u}}) \bar{u}_{t-1} + \rho_{\bar{u}} u^{ss} + g_{\bar{u},t} + \epsilon_{\bar{u},t} \]

\[ g_{\bar{u},t} = \rho g_{\bar{u}} g_{\bar{u},t-1} + \epsilon_{g_{\bar{u},t}} \]

\[ \hat{u}_t = \rho_{\hat{u}} \hat{u}_{t-1} + \tau \hat{y}_t + \epsilon_{\hat{u},t} \]
Model Specification: Capacity Utilization

- Use information from measures of capacity utilization rates in the manufacturing sector to help identify the overall slack in the economy.

\[
\hat{c}_t = c_t - \bar{c}_t
\]

\[
\bar{c}_t = (1 - \delta_2) \bar{c}_{t-1} + \delta_2 \bar{c}^{SS} + g\bar{c},t + \epsilon_{\bar{c},t}
\]

\[
g\bar{c},t = (1 - \delta_1)g\bar{c},t-1 + \epsilon_{g\bar{c},t}
\]

\[
\hat{c}_t = \kappa \hat{y}_t + \epsilon_{\hat{c},t}
\]
Model Specification: Interest Rates

• To close the model, a policy interest rate reaction function is introduced, where the one-year nominal interest rate responds to the deviation of inflation from target and the output gap.

\[ rs_{t}^{1Y} = \alpha_1 rs_{t-1}^{1Y} + (1 - \alpha_1)[rr_{t}^{1Y} + \pi_t^e + \alpha_2 (\pi_t - \pi_t^{Tar}) + \alpha_3 \hat{y}_t] + \epsilon_{rs^{1Y},t} - \alpha_4 \epsilon_{\pi^{Tar},t} \]

\[ rr_{t}^{1Y} = \rho rr_{t-1}^{1Y} + (1 - \rho rr_{t}^{1Y}) rr^{SS} + \epsilon_{rr^{1Y},t} \]

\[ rs_{t}^{10Y} = \frac{\sum_{i=t}^{t+9} rs_{i}^{1Y}}{10} + \sigma_t^{Term} + \epsilon_{rs^{10Y}} \]

\[ \sigma_t^{Term} = \rho \sigma_{t-1}^{Term} + (1 - \rho \sigma_{t}^{Term}) \sigma^{Term,SS} + \epsilon_{\sigma^{term},t} \]
Model Specification: Adding Expectations

- Add inflation and growth expectations data, from Consensus Economics, to help identify shocks, pin down potential growth, also helps improve estimates and at the end of sample.

\[
\pi^c_{t+j} = \pi_{t+j} + \epsilon_{\pi^c,t+j}, j = 5
\]

\[
g^c_{t+j} = g_{t+j} + \epsilon_{g^c,t}, j = 1,\ldots,5
\]

- At the end of sample, the model-consistent j-year ahead expectations for inflation and growth will be equal to j-year consensus projections of inflation and growth.
Incorporating Hysteresis into the Model

**Hysteresis in NAIRU**

\[ \bar{u}_t = (1 - \rho_{\bar{u}}) \bar{u}_{t-1} + \rho_{\bar{u}} u^{ss} + g_{\bar{u},t} + \epsilon_{\bar{u},t} - \xi \ast \text{movavg}(\hat{y}_{t+3},-6) \]

\[ g_{\bar{u},t} = \rho g_{\bar{u}} g_{\bar{u},t-1} + \epsilon_{g_{\bar{u},t}} \]

where

\( \bar{u}_t \) – the equilibrium value of the unemployment rate (NAIRU)

\( \epsilon_{\bar{u},t} \) - shock on NAIRU

\( g_{\bar{u},t} \) - growth rate of NAIRU

\( \epsilon_{g_{\bar{u},t}} \) - shock on the growth rate of NAIRU

\( \hat{y}_t \) - output gap
Incorporating Hysteresis into the Model: Potential GDP and NAIRU

**NAIRU effects on potential output**

\[
\bar{y}_t = \bar{y}_{t-1} + g_{\bar{y},t} + \varepsilon_{\bar{y},t} - \eta_1 (\bar{u}_t - \bar{u}_{t-1}) - (1 - \eta_1) \bar{u}_{t-1} - \bar{u}_{t-5})/4 + \eta_2 (\bar{c}_t - \bar{c}_{t-1})
\]

\[
g_{\bar{y},t} = (1 - \rho_{g_{\bar{y}}}) g_{\bar{y},t-1} + \rho_{g_{\bar{y}}} g_{\bar{y}}^{ss} + \varepsilon_{g_{\bar{y}},t}
\]

where
\[
\bar{y}_t - \text{potential output}
\]
\[
g_{\bar{y},t} - \text{growth rate of potential output}
\]
\[
\varepsilon_{\bar{y},t} - \text{shock on potential output}
\]
\[
\varepsilon_{g_{\bar{y}},t} - \text{shock on the growth rate of potential output}
\]
Explanation of Specification

Capital adjusts in almost 4 years after 1pp negative NAIRU shock

Source: Authors' simulations
Bayesian estimation:

• Incorporates theoretical insights to prevent counterintuitive empirical results.

• Splits weights between priors (economist's perception) and data (actual economic performance).

• Allows small sample sizes.

• Allows estimation of many coefficients and latent variables, even with small sample sizes.
Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$G^{ss}$</td>
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<tr>
<td>$\bar{U}^{ss}$</td>
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<tr>
<td>$\sigma(\varepsilon_{t+j}^{\pi^c}), j = 1$</td>
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<tr>
<td>$\sigma(\varepsilon_{t+j}^{\text{GROWTH}^c}), j = 1, \ldots, 5$</td>
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Note: $\sigma$ represents standard deviation.
### Parameter Estimates

<table>
<thead>
<tr>
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<th>Mode</th>
<th>Standard Error</th>
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<td>Prior</td>
<td>Posterior</td>
</tr>
<tr>
<td></td>
<td>Prior</td>
<td>Posterior</td>
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<td>$\rho_{y_{T-1}y}$</td>
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<td>0.880</td>
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<td>$\delta_1$</td>
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<td>$\delta_2$</td>
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<td>0.198</td>
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<tr>
<td>$\kappa$</td>
<td>2.167</td>
<td>2.148</td>
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</tbody>
</table>

Source: Authors’ estimates
Supply-side Damage in Labor Markets

Labor-market hysteresis channels:

• Reallocation of labor across different sectors.
• Deterioration of efficiency of the matching process between workers and vacancies.
• Deterioration of skills and employability of long-term unemployed.
Reallocation of Labor Across Different Sectors: Faster Job Recovery in Services

Source: Bureau of Economic Analysis
Outward shift in Beveridge curve is consistent with temporary increase in structural unemployment (NAIRU)

Source: Federal Reserve Bank of St. Louis
Long-term Damage of Skills and Employability of Long-term Unemployed

Source: Federal Reserve Bank of St. Louis
The Return of the Beveridge Curve: Partial Hysteresis

• The skill mismatch is a temporary process.
• Job matching becomes easier as unemployed people develop new skills.
• However, some people left the labor force - a scarring effect of high unemployment.
Illustrative Effects of Hysteresis

- In case of hysteresis, inflation deviates from the target less to generate large and persistent unemployment.
- Hysteresis generates the persistence in unemployment series observed after the GFC.
- In the baseline model the same unemployment is generated by bigger output gap.

Source: Authors’ estimates