A Market-Based Term Structure of Expected Inflation

Manuel Ramos-Francia2 Santiago García-Verdú3 Manuel Sánchez-Martínez4

October 2019

Preliminary. Please do not quote or cite without the author’s permission.

Abstract

We use the difference of nominal and real (i.e., inflation-linked) interest rates, assuming risk-neutral investors, as a measure of expected inflation. We contend that such an approach accounts for liquidity and inflation risk premiums, both of which can be sizeable. As a case study, we implement this approach for Mexican government bonds. We estimate a daily market-based term structure of expected inflation, associated with 1- to 120-month horizons. We document that such expectations Granger-cause survey-based means for short-, medium- and long-horizons, but not conversely. This suggests that the former could have more informational content than the latter. Finally, we assess its forecasting performance by implementing Diebold-Mariano tests, to conclude that our market-based measures are comparable with survey-based ones.

JEL Codes: C58, E43, G12, G17.
Key words: Inflation, Inflation Expectations, Market-Based Expectations, Affine Interest Rate Models.

1 The opinions in this paper are those of the authors and should not be attributed to the Banco de México. We would like to thank the Central Bank Operations Division for providing us with some of the data, and to Raúl Álvarez-del-Castillo, Julio Cacho-Díaz, Julio Carrillo-Abrego, Pólux Díaz-Ruiz, Alexandros Fakos, Ricardo Gimeno, Renata Herreras-Franco, Serafín Martínez-Jaramillo, Felix H.A. Matthys, Lautaro Silva-Ibarguren, and the participants of the ITAM Business School Seminar for their comments and suggestions. Any errors are our own.
2 CEMLA
3 CEMLA and Banco de México (Contacting Author). Email: sgarciaav@banxico.org.mx.
4 CEMLA and Banco de México.
1. Introduction
We interpret the difference of nominal and real interest rates, assuming risk-neutral investors, as a market-based measure of inflation expectations. We argue that such an approach accounts for inflation and liquidity risk premiums, both of which can be sizeable, particularly so, in emerging market economies (EMEs) and in the case of financial stress episodes. As a case study, we apply such an approach to Mexican government bond data.

To obtain a daily market-based term structure of expected inflation, we estimate affine interest models for nominal interest rates and, separately, for real (i.e., inflation-linked) interest rates, following Adrian et al. [2]. We do not use inflation as an input to our main model. This makes our estimation *bona fide* market-based and allows us to obtain it on a daily basis. Other common approaches to extract inflation expectations from market information are generally unavailable. For instance, there are few economies in which inflation swaps are available, which are also used to obtain market-based inflation expectations, e.g., in the U.S. (Fleckenstein et al. [17]).

Having an understanding of inflation and its expectations is paramount to market participants (Adrian and Wu [3]). For instance, monetary authorities have a keen interest in understanding how policy changes affect inflation expectations. This is not only because these changes impact the economy with a time lag but also because the control of long-term inflation expectations is central to their policies (Bernanke et al. [11]). The bulk of financial contracts that agents celebrate are in nominal terms, but ultimately care about their payments in real terms. Altogether, inflation expectations are central to decision-making processes of market participants.

There are, at least, three common approaches to obtain inflation expectations. In the most common of such approaches, one adjusts an econometric model that includes inflation and, commonly, other time series. Once the model's suitability has been evaluated, one proceeds to elaborate a forecast. The second one uses survey-based measurements, usually considering the median of the individual surveyees' expectations. The third one uses market-based expectations, extracting information from financial assets that depend on inflation. The main model implemented in this paper falls under the latter approach.

On their relative advantages, the implementation of an econometric approach is usually more direct. Its frequency though could be limited to that of the variables of interest. For instance, inflation is typically published on a monthly basis. Survey-based methods have limited frequency and forecasting horizons. Their output is largely circumscribed to design of the survey. For instance, there is some ambiguity on the time stamp of each datum, in effect, it might have taken place before the publication of the survey. In contrast, market-based expectations are typically available at a higher frequency. A market-based approach to obtain inflation expectations, widely used by practitioners and policy makers alike, considers the difference between the nominal and real interest rates, the so-called break-even inflation. Nonetheless, such an approach does not account for the presence of risk premiums, which can be quantitatively important. This
is particularly so, for long-term horizons, during episodes of financial stress and in emerging markets (Andreasen et al. [4]), an issue we address.

Our approach is close to that of Abrahams et al. [1]. Still, ours differs from theirs in, at least, the following aspects. First, we use the IR- and risk-neutral measures, while they consider the IR- and Q-measures. The latter is also known as the pricing measure. Second, we estimate the nominal and real stochastic discount factors separately, as a way to avoid using the inflation series as an input. We note one could model the break-even inflation and estimate just one model. They estimate a nominal stochastic discount factor and an implicit real one jointly. Third, we aim to account for general risk premiums embedded in nominal and real interest rates. Accordingly, we do not use explicit proxies for any risks. They use an explicit proxy for liquidity risk.

Anticipating our main results, we have the following remarks. Market-based inflation expectations cause—in the Granger sense—survey-based ones, but not the other way around. This suggests that the former might have more informational content than the latter. We also assess the market-based approach forecasting performance by implementing Diebold-Mariano tests. It is comparable to survey-based measures.

2. Derivation of the Term Structure of Inflation Expectations
To analytically obtain the term structure of inflation expectations, consider the following decompositions for the nominal and real interest rates, respectively, with maturity $n$:

$$(1 + i_t^{(n)}) = (1 + R_t^{(n)})E_t[1 + \pi_{t,t+n}](1 + \rho_t^{(n)})$$

$$(1 + r_t^{(n)}) = (1 + R_t^{(n)})(1 + \gamma_t^{(n)})$$

where $i_t^{(n)}$ and $r_t^{(n)}$ are the nominal and real interest rates, respectively, in period $t$. We have that $R_t^{(n)}$ is the real interest rate that would prevail under the absence of risks. Consequently, $\rho_t^{(n)}$ is a risk premium for the holders of nominal bonds. Such a premium is a compensation for inflation and liquidity risks, among other possible ones. Similarly, $\gamma_t^{(n)}$ is a risk premium for the holders of real bonds, compensating for liquidity risks, among other ones.\(^5\) $E_t[\pi_{t,t+n}]$ is the expected inflation between periods $t$ and $t + n$.\(^6\)

---

\(^5\) If the other such risks were negligible, then one could directly identify the liquidity risk premium for real bonds.

\(^6\) We have that, for example, $E_t[\pi_{t,t+2}]$ refers to the expected inflation from $t$ to $t + 2$, and not to the expected inflation from $t + 1$ to $t + 2$, which is usually an object of more interest. From our model we can obtain the latter one. To see this, consider for example $E_t[\pi_{t,t+2}] = E_t[\pi_{t,t+1}] + E_t[\pi_{t+1,t+2}]$, where for ease of exposition, we have omitted the cross-terms. Note that one can obtain directly from the model $E_t[\pi_{t,t+1}]$ and $E_t[\pi_{t,t+2}]$. Then calculating $E_t[\pi_{t+1,t+2}]$ from them is direct. More generally, one can calculate $E_t[\pi_{t+k-1,t+k}]$ for $k = 1, 2, 3, \ldots K$ in an iterative fashion.
Note that in the expressions above we have omitted measuring errors, which are included explicitly in the interest rate affine models.

On the other hand, consider then the analogous interest rates decompositions in which we assume that agents are risk-neutral. If such were the case, then they would receive no risk compensations, making all premiums zero. This implies that:

\[
\left(1 + i_t^{(n,*)}\right) = \left(1 + R_t^{(n)}\right)E_t[1 + \pi_{t,t+n}],
\]

\[
\left(1 + r_t^{(n,*)}\right) = \left(1 + R_t^{(n)}\right),
\]

where we have used an asterisk to denote that such interest rates are associated with risk-neutral investors. To be clear, these interest rates and the inflation expectation are not, in general, associated with the \(\mathbb{Q}\)-measure.\(^7\)

Based on the above, we can obtain the expected inflation:

\[
E_t[\pi_{t,t+n}] = \left(1 + i_t^{(n,*)}\right)\left(1 + r_t^{(n,*)}\right)^{-1} - 1 
\approx i_t^{(n,*)} - r_t^{(n,*)}
\]

Several comments are in order. To estimate the interest rates assuming risk-neutral agents, one evidently needs a model. To that end, we use affine interest rate models, in which by setting the market prices of risk equal to zero, we obtain such interest rates.

It is worth reemphasizing that our main approach does not use inflation time series. Had we used it; our inflation expectation would not had been a \textit{bona fide} market-based one. In addition, inflation is only available at a lower frequency, while our approach obtains a daily expectation.\(^8\) As argued, it accounts for risk premiums, including inflation and liquidity ones, both of which can be sizeable (e.g., see Ang et al., [5], and Andreasen et al. [4]). Else, the estimation of inflation expectations could be biased (see Appendix A2.) For comparison purposes though, we estimate inflation expectations including inflation being an input to the model. The associated results are presented in Appendix A1.

We interpret the difference between the nominal and real risk-neutral interest rates as expected inflation. Being able to measure market-based inflation expectations has relative advantages, as mentioned. Moreover, it can be implemented in any economy in which both nominal and real interest rates are available for various maturities. One

\[^7\] In general, the \(\mathbb{P}\)-measure (aka, the objective measure) differs from the \(\mathbb{Q}\)-measure (aka, the equivalent martingale or risk-neutral measure). Moreover, if the market price of risk is zero, then the referred measures coincide (almost surely).

\[^8\] In principle, it could be used to obtain expectations on a higher frequency, but this depends on the availability of nominal and real interest rates at an intra-day frequency.
could think of a number of economies that have such bonds and do not have alternatives assets, such as inflation swaps, with which similar measure could be attained.

Our approach is close to Abrahams et al.'s [1], as we mentioned, and differs from others in the literature. For instance, Ang et al. [5] decompose the nominal interest rates into the real interest rate, inflation, and an inflation risk premium. They pay little attention whether their premium measures risks beyond inflationary ones. A related model is that of Garcia and Werner [18], who estimate the inflation risk premium using a joint macroeconomic and term structure model for the euro area. They account for the inflation risk premium and use inflation as an input variable. Chernov and Mueller [12] combine survey and interest rates data. They identify a hidden component in surveys that improves their model's forecasting capability. Aruoba [9] combines survey data to produce a term structure of inflation expectations and an associated term structure of real interest rates. In terms of forecast accuracy, he documents that they perform similarly or better than some known alternatives.

As for the literature on emerging market economies, Melo-Velandia and Granados-Castro [21] and Aguilar-Argaez et al. [8] estimate an inflation risk premium for Colombia and Mexico, respectively. To obtain inflation expectations, both papers use the same affine interest rate model. They account for the inflation risk premium implicitly assuming that other premiums are negligible. Moreover, as they use the inflation time series as a risk factor, their measures are not strictly market-based.

3. Data
We use nominal zero-coupon interest rates with one-month, one-, five-, ten-, and twenty-year maturities, and real zero-coupon interest rates for five-, ten-, and twenty-year maturities. Their source is Valor de Mercado and are reported as simple interest rates. While short-term nominal interest rates are readily available, short-term real interest rates are not, as there is less information about on-the-run short-term real bonds.

On their part, short-term real interest rates from price vendors, such as Valor de Mercado, could be extrapolations of longer-term ones. They are generally unreliable. Accordingly, we estimate the short-term real interest rates based on the short-term nominal interest rates and on survey-based inflation expectations. Specifically, we use the following construction. To begin with, we assume that the risk premiums for horizons equal or less than a year are negligible. In short, we assume that for such horizons the Fisher equation holds (the time period is a month).

\[ (1 + i_t^{(12)}) = (1 + r_t^{(12)})E_t[1 + \pi_{t,t+12}], \]

where \( i_t^{(12)} \) is the one-year nominal interest rate, \( r_t^{(12)} \) is the one-year real interest rate, and \( E_t[\pi_{t,t+12}] \) is the one-year expected inflation. This implies that the one-year real interest rate is:
\[ r_t^{(12)} = \left(1 + i_t^{(12)}\right) \mathbb{E}_t\left[1 + \pi_{t,t+12}\right]^{-1} - 1 \]

where the inflation expectation is survey-based, as mentioned.

There is a related point when estimating the one-month real interest rate. The survey-based one-month inflation expectation is not available. Accordingly, we use the following approximation to obtain the one-month real interest rate.

\[ \mathbb{E}_t\left[1 + \pi_{t,t+1}\right] \approx \mathbb{E}_t\left[1 + \pi_{t,t+12}\right] \]

If its seasonable component is relatively small, then such an approximation is reasonable. Else, in a high (low) season, it will tend to underestimate (overestimate) the one-month expected inflation. In any case, our main interest is on longer-term inflation expectations. We use the following equation to obtain the one-month real interest rate.

\[ r_t^{(1)} = \left(1 + i_t^{(1)}\right) \mathbb{E}_t\left[1 + \pi_{t,t+12}\right]^{-1} - 1 \]

Regarding the use of survey-based measures, they are typically available only at a monthly frequency. Thus, we use a second approximation for our daily data as follows.

\[ \mathbb{E}_{t+d}\left[1 + \pi_{t,t+1}\right] \approx \mathbb{E}_t\left[1 + \pi_{t,t+12}\right] \text{ for each day } d \text{ in month } t. \]

The following comments are in order. Similar approaches to obtain short-term real interest rates have been used (e.g., in Magud and Tsounta [20]). In our case, they affect the short-term real interest rates. This is due to a characteristic of the data, and is unrelated to our approach to obtain the inflation expectations.

To obtain the interest rate for monthly maturities, we use cubic interpolation based on the one-month, and one, five, ten and twenty-year zero-coupon nominal interest rates. Similarly, we use the same interpolation approach based on the one-month, one, five, ten and twenty-year zero-coupon real interest rates. From this point forward, we refer to the interpolated interest rates as the interest rates data.

As mentioned, in Appendix A1, we estimate a term structure of inflation expectations using a model that includes inflation as a risk factor. We do so for comparative reasons. For this last model, we specifically use the year-to-year monthly inflation time series from the Mexican Statistics Institute (INEGI, Instituto Nacional de Estadística y Geografía).

---

9 There is a question on the survey regarding inflation for the following month. But, as the survey is published with lag, using it could have some drawbacks.
4. Model

We use an affine interest rate model for the nominal interest rates and, separately, another one for real (i.e., inflation-linked) interest rates. In terms of notation, we have hitherto used \( i_t \) for the nominal interest rates and \( r_t \) for the real ones. In what follows; however, we use \( y_t \) to denote a generic interest rate. We could be referring to either. We estimate such models following Adrian et al.'s [2], which we briefly describe next.

The model uses continuously compounding interest rates. We thus directly transform the data to such a compounding. As is common, we estimate the models with end-of-the-month data and obtain the model-implied daily interest rates using the daily risk factors. For the risk factors, we use the first \( k \) principal components of the interest rates, stacked in a vector \( F_t \). We then use a VAR(1) to model the referred \( k \) risk factors:

\[
F_{t+1} = \theta + \Phi F_t + v_{t+1},
\]

where \( \theta \) is a \( k \times 1 \) vector, \( \Phi \) is a \( k \times k \) matrix, \( v_{t+1} \) is a \( k \times 1 \) vector of shocks that follow a conditionally normal distribution with parameters \((0, \Sigma)\), where \( 0 \) is a \( k \times 1 \) vector and \( \Sigma \) a \( k \times k \) variance-covariance matrix, and \( t \) is the period.

If we assume the no arbitrage condition, then a stochastic discount factor (SDF) \( M_{t+1} \) exists that prices all financial assets (Duffie [16], Cochrane [13]), including government bonds. Thus,

\[
P_t^{(n)} = \mathbb{E}_t \left[ M_{t+1} P_{t+1}^{(n-1)} \right]
\]

where \( P_t^{(n)} \) is the price of a zero-coupon bond with maturity \( n \) in period \( t \). This equation links the price of the bond with maturity \( n \) in period \( t \) and \( t+1 \), i.e., the same bond one period ahead. Following Duffee [15], we assume that the SDF has the following functional form:

\[
M_{t+1} = \exp \left( -\gamma_t^{(1)} - \frac{\lambda_t^2}{2} - \lambda_t' \Sigma^{-1/2} v_{t+1} \right),
\]

in which \( \gamma_t^{(1)} \) is the risk-free interest rate. The market price of risk \( \lambda_t \) is linear with respect to the risk factors:

\[
\Sigma^{1/2} \lambda_t = (\lambda_0 + \lambda_1 F_t)
\]

Denoting the logarithm of the excess return for holding a bond with maturity \( n \) as \( r x_{t+1}^{(n-1)} \):

\[
r x_{t+1}^{(n-1)} \equiv \ln P_{t+1}^{(n-1)} - \ln P_t^{(n)} - y_t^{(1)}.
\]

One can then rewrite the pricing equation in terms of the excess return:
\[ 1 = \mathbb{E}_t \left[ \exp \left( rx_{t+1}^{(n-1)} - \frac{\lambda_t^2}{2} - \lambda_t^2 \Sigma^{-1/2} v_{t+1} \right) \right] \]

Assuming that \( rx_{t+1}^{(n-1)} \) and \( v_{t+1} \) are jointly normal, one can obtain:

\[ \mathbb{E}_t r x_{t+1}^{(n-1)} = \mathbb{E}_t \left( r x_{t+1}^{(n-1)}, v_{t+1} \Sigma^{-1/2} \lambda_t \right) = \frac{1}{2} v_t \mathbb{E}_t \left( r x_{t+1}^{(n-1)} \right). \]

Let \( (\beta_t^{(n-1)})' \equiv \mathbb{E}_t \left( r x_{t+1}^{(n-1)}, v_{t+1} \right) \Sigma^{-1/2}. \)

One then decomposes the unexpected excess returns into a constituent that correlates with \( v_{t+1} \) and another one that is conditionally orthogonal to it:

\[ r x_{t+1}^{(n-1)} - \mathbb{E}_t r x_{t+1}^{(n-1)} = \left( Y_t^{(n-1)} \right)' v_{t+1} + e_{t+1}^{(n-1)}, \]

where \( e_{t+1}^{(n-1)} \), the return pricing errors, are conditionally independent and identically distributed, with variance \( \sigma^2 \).

Furthermore, we have that \( (Y_t^{(n-1)})' = (\beta_t^{(n-1)})' \). This implies:

\[ r x_{t+1}^{(n-1)} = (\beta_t^{(n-1)})' (\lambda_0 + \lambda_1 F_t) - \frac{1}{2} (\beta_t^{(n-1)})' \Sigma (\beta_t^{(n-1)})' + (\beta_t^{(n-1)})' v_{t+1} + e_{t+1}^{(n-1)}. \]

One then stacks this expression across time and maturities to obtain the following expression:

\[ r x = \beta' (\lambda_0 t_T + \lambda_1 F_-) - \frac{1}{2} (B^* \text{vec}(\Sigma) + \sigma^2 t_N) t_T + \beta' V + E \quad (3) \]

where \( r x \) is \( N \times T \) matrix, \( \beta = [\beta^{(1)} \ldots \beta^{(N)}] \) is \( K \times N \), \( t_T \) and \( t_K \) are \( T \times 1 \) and \( N \times 1 \), respectively, vectors of ones, \( F_t = [F_0 F_1 \ldots F_{T-1}] \) is \( K \times T \), where \( B^* = [\text{vec} (\beta^{(1)} \beta^{(1)^*}) \ldots \text{vec} (\beta^{(N)} \beta^{(N)^*})] \) is \( N \times K^2 \), \( V = [v_1 \ldots v_T] \) is \( K \times T \), and \( E \) is a \( N \times T \) error matrix. Based on (3), one obtains \( \widehat{\Sigma}, \widehat{\sigma^2}, \) and \( \widehat{V} \), where a hat indicates an estimate of the variable beneath it. Therefore, one can rewrite (3) as:

\[ r x = \beta' \lambda_0 t_T - \frac{1}{2} (B^* \text{vec}(\Sigma) + \sigma^2 t_N) + \beta' \lambda_1 F_- + \beta' \widehat{V} + E. \]

If one regresses \( r x \) on a constant \( t_T, \widehat{V}, \) and \( F_- \), one obtains

\[ r x = (a t_T + \beta' \widehat{V} + c F_-) + E = [a \beta' c] Z + E \]

As a result, \( \left[ a \beta' c \right] = r x Z'(ZZ')^{-1} \) and \( Z = [t_T \widehat{V}' F_-]' \). These finally lead to the following two equations to obtain the estimators of \( \lambda \).
\[ \lambda_0 = (\hat{\beta}\hat{\beta}')^{-1}\hat{\beta}\left(\hat{a} + \frac{1}{2}(B^*\text{vec}(\Sigma) + \sigma^2 \iota_N)\right), \]

\[ \lambda_1 = (\hat{\beta}\hat{\beta}')^{-1}\hat{\beta}\hat{\epsilon}. \]

In affine interest rate models, one can write the logarithm of a bond price as an affine function of the risk factors:

\[ \ln P_t(n) = A_n + B_n'F_t = -n y_t^{(n)}. \]  \( (4) \)

The coefficients \( A_n \) \((1 \times 1)\) and \( B_n \) \((k \times 1)\) have the following cross-sectional restrictions (Piazzesi [22]), a consequence of the pricing equation (2):

\[ A_n = A_{n-1} + B_{n-1}'(\Theta - \lambda_0) + \frac{1}{2}(B_{n-1}' \Sigma B_{n-1} + \sigma^2) - \delta_0; \]
\[ A_0 = 0; \]

\[ B_n' = B_{n-1}'(\Phi - \lambda_1) - \delta_1; \] and,

\[ B_0 = 0, \]

for maturities \( n = 1, \ldots, N \). We estimate \( y_t^{(1)} = \delta_0 + \delta_1' F_t \) with OLS. Relatedly, Adrian et al. [2] do not use the cross-sectional restrictions above, which are common to affine interest rate models’ estimations (Piazzesi [22], and Ang and Piazzesi [7]). However, the model’s estimates satisfy them. Note that, if \( \lambda_0 \) and \( \lambda_1 \) are set equal to zero, then there is no risk compensation. One can then obtain the associated interest rates and bond prices for risk-neutral agents, implied by the model.

\[ \ln P_t^{(n),*} = A_{n,*} + (B_{n,*}')' F_t. \]

The asterisks indicate that the coefficients and prices are associated with risk-neutral agents. Similarly, \( A_{n,*} \) is a scalar and \( B_{n,*} \) is \( k \times 1 \) vector, for each maturity \( n \). One can derive such coefficients from the following recursive relations:

\[ A_{n,*} = A_{n-1,*} + (B_{n-1,*}')'(\Theta' - \lambda_0) + \frac{1}{2}(B_{n-1,*}' \Sigma B_{n-1,*} + \sigma^2) - \delta_0; \]
\[ A_0,* = 0. \]

\[ (B_{n,*}')' = (B_{n-1,*}')'(\Phi - \delta_1); \]
\[ B_{0,*} = 0, \]

for \( n = 1, \ldots, N \).

Again, if agents are risk neutral, then all risk premiums are equal to zero. Specifically, if \( \lambda_0 \) and \( \lambda_1 \) are equal to zero, then \( y_t^{(n)} = 0 \) and \( \rho_t^{(n)} = 0 \) in
\[ i_t^{(n)} = R_t^{(n)} + \frac{1}{n} E_t[\pi_{t,t+n}] + \gamma_t^{(n)}; \text{ and,} \]

\[ r_t^{(n)} = R_t^{(n)} + \rho_t^{(n)}, \]

where, for simplicity, we have omitted the cross terms. As mentioned, we have estimated two separate models, having, two stochastic discount factors. We could have estimated one model, modelling the break-even inflation. We opt the former as we find it more intuitive. An alternative could have been using a latent inflation factor as in Chernov and Mueller [12]. Nonetheless, they use inflation surveys as inputs. By estimating two separate models, we possibly lose some statistical efficiency. We find our approach more intuitive.  

5. Estimation Results

Although we are mainly interested in the interest rates under risk-neutrality, we also explore the nominal and real interest rates from the model. In effect, one is more confident about a model when its broader implications are empirically verified. Thus, to set the stage, consider the data of nominal and the real interest rates and their model-based counterparts. These interest rates are associated with the five- and ten-year horizons (Figures 1 and 2). We underscore that both have a satisfactory fit, albeit the nominal interest rates do slightly better in this respect, as we further explore below.

---

10 A related issue is whether there can be two of such stochastic factors. Evidently, one is a nominal SDF and the other one is a real one. Thus, save for measuring errors, the inflation time series links them. In addition, the SDF is unique only under complete markets. If such were the case, they would keep the same relationship.

11 Adrian and Wu [3] argue that the covariance term between the stochastic discount factor and inflation is relevant to the estimation of expected inflation. Our approach accounts for such a covariance.
Figure 2. Real Interest Rates, 5- and 10-Year, Data and Model-Based.

*Note:* Simple Interest Rates.


*Source:* Valor de Mercado and own estimations.

Table 1. Interest Rates and Return Pricing Errors

<table>
<thead>
<tr>
<th>Horizon (months)</th>
<th>Nominal Rates</th>
<th>Real Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>Interest Rates Pricing Errors</td>
<td>Mean</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>Std. Dev.</td>
<td>0.68</td>
</tr>
<tr>
<td></td>
<td>Skewness</td>
<td>-0.45</td>
</tr>
<tr>
<td></td>
<td>Excess Kurtosis</td>
<td>0.82</td>
</tr>
<tr>
<td></td>
<td>Mean Absolute Error</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td>ρ(1)</td>
<td>0.34</td>
</tr>
<tr>
<td></td>
<td>ρ(6)</td>
<td>0.11</td>
</tr>
</tbody>
</table>

| Excess Return Pricing Errors | Mean | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
|                             | Std. Dev. | 0.00 | 0.00 | 0.01 | 0.00 | 0.02 | 0.11 | 0.00 | 0.02 | 0.02 | 0.03 | 0.03 | 0.12 |
|                             | Skewness | 0.65 | -0.62 | 0.55 | 0.82 | -0.52 | -0.62 | 0.37 | 0.34 | -0.05 | -0.04 | 0.09 | -1.00 |
|                             | Excess Kurtosis | 0.89 | 1.22 | 1.40 | 2.66 | 0.39 | 1.04 | 1.52 | 2.55 | 2.75 | 2.36 | 2.31 | 4.23 |
|                             | Mean Absolute Error | 0.00 | 0.00 | 0.01 | 0.00 | 0.02 | 0.09 | 0.00 | 0.02 | 0.02 | 0.02 | 0.03 | 0.08 |
|                             | ρ(1) | -0.15 | -0.28 | -0.29 | 0.38 | -0.28 | -0.23 | 0.09 | -0.16 | -0.19 | -0.13 | -0.15 | -0.15 |
|                             | ρ(6) | 0.02 | -0.01 | -0.01 | 0.10 | 0.00 | 0.02 | -0.05 | 0.00 | -0.04 | -0.01 | -0.06 | -0.09 |

Table 1. Interest Rates and Return Pricing Errors

*Notes:* An error is equal to the datum minus the model-implied one. Statistics based on simple rates. The mean and standard deviations are in basis points. ρ(m) denotes the autocorrelation with a lag of m months.


*Source:* Own estimations with data from Valor de Mercado and Banco de México.

The pricing errors are defined as interest rate datum minus the model-implied one (Table 1). On these errors, their means are sensible to the horizon. Their standard deviations keep stable along the horizon. A positive skewness implies that the model falls short in capturing the data variability. When the skewness is negative, it is generally small. The kurtoses for the nominal rates appear close to zero. In the case of the real interest rates, their kurtoses are all positive, indicating that such errors’
densities have somewhat heavy tails. Their mean absolute errors fall under a reasonable range. Finally, their auto-correlations are notable.\footnote{12}

On the return pricing errors, their means maintain very low magnitudes. This is sensible given that they were used as the minimization criteria. Their standard deviations are in line with this observation. Their skewnesses do not seem to hold a pattern. Their kurtoses are all positive, indicating heavy densities’ tails. Their mean absolute errors are reasonable. The excess returns errors have small auto-correlations, in particular, compared to those of the interest rates’ errors.

As mentioned, the estimation method we use does not directly obtain the coefficient $B_n$. Under the estimation approach we have used, the $\beta^{(n)}$ coefficients (equation 3) stand for $B_n$ (equation 4), coefficient from the more common model estimation. Thus, it is illustrative to compare them. We do so for the first four risk factors of the nominal and real interest rates models. The related coefficients match well (Figures 3 and 4), except perhaps for the fourth components. While they are off by some tenths, the implications are minor given the quantitative relevance of the respective principal component.\footnote{13}

\footnote{12}This might be an issue when estimating such a model with maximum likelihood (Adrian et al. \cite{2}).\footnote{13}As one could have anticipated, the coefficients associated with the first principal components are negative and are greater (in absolute value) as their associated horizon $n$ grows. In general, investors demand higher returns for longer-term bonds.

Figure 3. $\beta(n)$ and $B(n)$ Associated with the Nominal Interest Rate Model.
Notes: Horizontal axes refer to maturities in months.
Source: Own estimations with data from Valor de Mercado and Banco de México.
A related issue is whether the model-implied interest rates maintain the statistical properties of the data interest rates. We explore this issue comparing the principal components from the model-implied interest rates and those from the data. We depict the first three principal components, for the model-implied nominal and real interest rate and those of the data, respectively (Figures 5 and 6). The match of the principal components is overall satisfactory.

In general, the model-based rates maintain relevant statistical properties. To provide support to this point, we have executed several exercises. We have their interest rate and excess return pricing errors, weighted on the estimated coefficients with those of traditional models, and compared the statistical properties of the model-implied interest rates with those obtained directly from the data. As we document favorable results, these lend us confidence in our general results.
Figure 5. PCA from Data Nominal Interest Rates and from Model-Implied Nominal Interest Rates.

Note: PC\(i\) refers to the \(i\)-th Principal Component.


Source: Own estimations with data from Valor de Mercado.
5.1. Statistical Analysis

Having estimated a market-based term structure of inflation expectations, we explore its statistical properties. We have two specificities. First, from this point onward, for the expected inflation with a horizon of $m$ months, we are referring specifically to $\mathbb{E}_t[\pi_{t+m-1:t+m}]$. Second, we present inflation expectations as simple (annualized) interest rates, as is commonly done.
We have the statistics of expected inflation for a selected set of horizons (Table 2). Their mean is a decreasing function of the horizon. A similar result holds for their volatility. At the 120-month horizon, it is fourteen basis points, less than the volatility at the 12-month horizon. Interestingly, its skewness is positive for most horizons. Hence, the inflation probability densities have fatter right-hand tails. In effect, abrupt increases in inflation are more probable than marked falls with respect to their mean. On its part, the kurtosis is relatively small for the twelve-month horizon, falls in a similar range for 24- to 60-month and becomes negative for the 120-month horizon.¹⁴

<table>
<thead>
<tr>
<th>Inflation Expectations Horizon</th>
<th>12</th>
<th>24</th>
<th>36</th>
<th>60</th>
<th>120</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>3.92</td>
<td>3.91</td>
<td>3.89</td>
<td>3.86</td>
<td>3.80</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.33</td>
<td>0.27</td>
<td>0.23</td>
<td>0.19</td>
<td>0.14</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.52</td>
<td>0.47</td>
<td>0.37</td>
<td>0.09</td>
<td>-0.20</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>0.08</td>
<td>0.61</td>
<td>1.01</td>
<td>0.63</td>
<td>-0.98</td>
</tr>
</tbody>
</table>

**Table 2. Term Structure of Expected Inflation Statistics.**

**Notes:** Annualized rates.

**Source:** Own Estimations with data from Valor de Mercado.

More generally, we depict the time series-average of expected inflation against the forecasting horizon (Figure 7). Initially, it is mildly concave and then turns convex. We also depict the time series volatility of expected inflation against the forecasting horizon (Figure 8). The bulk of the volatility takes place below the 48-month horizon mark, indicative of the inflationary shocks’ dissipating. Their relationship is concave.

![Figure 7. Mean Expected Inflation.](image1)

**Notes:** Annual interest rate (%).

**Source:** Own estimations with data from Valor de Mercado and Banco de México.

![Figure 8. Volatility Expected Inflation.](image2)

**Notes:** Annual interest rate (%).

**Source:** Own estimations with data from Valor de Mercado and Banco de México.

We next study the term structure of inflation expectations (Figure 9). It depicts end-of-period monthly estimates. Its level has considerably varied through time. In general, it has had a downward slope. Its curvature has notably varied, but tends to stay convex

¹⁴ If the kurtosis is positive (negative), the density's tails are heavier (lighter) than the tails of a normal density.
most of the time. Only in a few periods, it is concave. We think of this as something positive in terms of monetary policy, as we explain next.

We note that in general a convex term structure of inflation expectations is more favorable to a faster expected inflation convergence. Moreover, its slope and curvature are central to their dynamics, as they probably affect the speed of inflation reversion. Evidently, they reflect features of the inflation process, its expectation and, importantly, the nature of the shocks that could have affected them.

![Figure 9. Market-Based Term Structure of Expected Inflation.](image)

**Notes:** End-of-period monthly data. The time unit is in months. **Source:** Own estimations with data from Valor de Mercado and Banco de México.

6. **On the Dynamics of Inflation Expectations**

To gain intuition, consider the term structure of expected inflation contour plot (Figure 10). Its colors reflect the height of the inflation expectation. Its vertical axis indicates the forecasting horizon. The horizontal axis specifies the dates. In general, constant dark blue or fast vertical changes to dark blue are desirable. They reflect low levels of expected inflation or quick convergences to. Evenness in vertical colors different from dark blue is undesirable. It is indicative of persistent expected inflation levels above its target. The longer it takes for vertical lines to turn blue, the more persistence there is in expected inflation.

To explore the dynamics of the term structure of inflation expectations, we consider the *z*-scores of the following three time series based on our model. The *level* is the average of the expected inflations with horizon of 1, 2, ..., and 10 years; the *slope* is the difference between the expected inflations with horizon of 10 and 1 year, and the *curvature* is the
difference between the expected inflation with horizon of 10 and 7 years, minus the difference between the expected inflation with horizon of 5 and 1 year. To be sure, these are the level, slope and curvature of the inflation expectations term structure.

We have constructed the curvature with the opposite sign of the usual one. Intuitively, one can see the curvature as the difference between the slope at the short end of the term structure and the slope at the longer-end. Note that we have considered the z-scores of such time series (Figure 11).

![Figure 10. A Market-Based Term Structure of Expected Inflation. Notes: End-of-the month data. The time unit is in months. Source: Own estimations with data from Valor de Mercado and Banco de México.](image)

A rise in its level is bad news, evidently. It implies a general deterioration of inflation expectations. Given an increase in its level, a decrease in the slope is good news. It would be indicative of a swifter inflation convergence. A decrease in the curvature is good news, as it would imply that the bulk of such a convergence would take place in the shorter-term. An increase in the curvature implies that more of the convergence process will take place in the longer-term. Of course, the slower the convergence, another adverse shock would tend to deteriorate such a process. Conversely, an increase in the slope is bad news, as it be indicative of a sluggish convergence process.

\[15\] For convenience, we have constructed the curvature with the opposite sign of the second derivative. Thus, if its curvature is negative (positive), then it is a convex (concave) function with respect to the forecasting horizon. Thus, generally, negative signs for the slope and curvature are beneficial for the convergence or reversion speed, countering a positive expected inflation level.
and, in addition, it would possibly increase the probability of second round effects. In contrast, a decrease in the level is in general good news. If such a decrease were below the inflation target, a mild positive slope would be good news for similar explanations.

The slope and curvature tend to react to changes in the level in desirable ways. To be precise, in general, as its level increases, the slope and curvature decrease in tandem. Conversely, as its level decrease, the slope and curvature tend to increase. We interpret such dynamics as if the slope and curvature responded to stave off changes in the level. As an example, in December 2009, as its level increased markedly, both the slope and curvature promptly responded by decreasing. In stark contrast, consider November 2008, it seems that the level, slope, and curvature increased together. This did not bode well for the convergence of expected inflation. We explore more economic episodes captured by the inflation expectation dynamics in Appendix A4.

![Z-scores for the Level, Slope, and Curvature of Expected Inflation.](image)

**Figure 11.** Z-scores for the Level, Slope, and Curvature of Expected Inflation.  
**Notes:** We consider the z-scores of the following statistics based on the term structure of inflation expectations. The *level* is the average of the expected inflations with horizon of 1, 2, ..., and 10 years; the *slope* is the difference between the expected inflations with horizon of 10 and 1 year, and the *curvature* is the difference between the expected inflation with horizon of 10 and 7 years, minus the difference between the expected inflation with horizon of 5 and 1 year.  
**Source:** Own estimations with data from Valor de Mercado and Banco de México.

7. Further Analysis

We next instrument three exercises. First, we compare our market-based expected inflation dynamics with that of the survey. This is relevant as both approaches, in some sense, *aggregate* expectations of a group of agents. In the market-based approach, market participants implicitly convey their inflation expectation. If an agent were wrong about its expectations, she could incur in financial costs. In the survey-based
approach, surveyees directly provide their individual expectation. If a surveyee were wrong about her appraisal, she would incur in a reputational cost.

Second, we explore whether market-based inflation expectations cause, in the Granger sense, survey-based expectations, and the other way around. These exercises shed light upon the informational content of market-based inflation expectations. Finally, we implement Diebold-Mariano [14] tests to assess and compare their forecasting performance. Altogether, there is no a priori reason why market-based measures have to coincide with survey-based ones. On the contrary, they could differ for several reasons one of which is simply the diverse approaches and information sets used to obtain them. In particular, as the horizon increases, more factors affect expectations and, thus, there are more reasons for such expectations to differ.

Importantly, as we have estimated the model using the full-sample in a given period, we are using lead information to formulate inflation expectations. To be sure, we use the models’ coefficients estimated with the full sample to calculate the expected inflation. Of course, to formulate such an expectation we use the contemporaneous risk factors. In contrast, the survey-based data only use information available up to the time of their publication. Thus, when comparing both expectations, we are implicitly assuming that the interest rate models are stable through time. We briefly explore such an assumption in Appendix A3. On a related matter, surveys’ information is publicly available on the first days of the following month, not contemporaneously to its content. So, the inflation surveys are lagged by one month. This respects the unfolding of the information.

Having said that, the twelve-month ahead inflation expectations co-move with the survey-based inflation expectations (Figure 12). One can see such a match as a support to the consistency of the model. From time to time, however, the market-based measure seems to move ahead of the survey-based measure. This suggests that the former might have more information.

With respect to the medium-term inflation expectations (Figure 13), its proximity to the survey-based level appears reasonable. Their average difference is 33 basis points. Its level tends to hover just above the survey-based measure. Except for a handful of months, the market-based measures fall within the survey-based confidence intervals.\textsuperscript{16} We underscore two results that are not evident from the figure. The correlation for the first half of the sample is higher than that for the whole sample. The gap between them has closed in in the last periods, relative to their difference in January 2017.

\textsuperscript{16} We have built such confidence intervals based on the surveys’ standard deviations directly.
Figure 12. Short-Term Expected Inflation vs. Survey’s Measure (12-month).
Notes: Their mean absolute difference is 13 basis points. Confidence intervals (CI) are 2.6 standard deviations in each direction. The max and min indicate the maximum and minimum forecasts in each survey.
Source: Own estimations with data from Valor de Mercado and Banco de México.

Figure 13. Medium-Term Expected Inflation vs. Survey’s Measure (1-4 years).
Notes: Their average difference is 33 basis points. Confidence intervals (CI) are 2.6 standard deviations in each direction. The max and min indicate the maximum and minimum forecasts in each survey.
Source: Own estimations with data from Valor de Mercado and Banco de México.
As for the long-term expected inflation (Figure 14), their level differs with that of survey-based by 46 basis points on average. Their dynamics are to some extent similar. We underscore three points. Akin to the medium-term expectations, they have a high initial correlation, which lessens once considering the whole sample. Moreover, the market-based inflation expectations are within 2.6 standard deviations of the survey-based measure. They fall within the max-min survey range most of the time. Market-based inflation expectations have gotten closer to the survey-based measures in the last periods. Since about November 2010, market-based inflation expectations have been below the 4% level.

All in all, the market-based inflation expectations share several characteristics with the survey-based measures. However, we conjecture that they might provide additional information beyond that contained in the survey-based measure. Thus, we explore the extent to which market-based inflation expectations have information that the surveys might not have, and vice versa. To that end, we implement Granger Causality tests between market-based inflation expectations and survey-based measures. We consider short-, medium- and long-term expectations from both sources. These correspond to 1-year, 1- to 4-year, and 5- to 8-year, respectively, based on Banco de México’s surveyed horizons.

Our key results are as follows. For the short-term, we find evidence that the market-based expectation causes, in the Granger sense, the survey-based measure. On the other hand, the mean of the survey-based inflation expectations does not appear to cause, in the Granger sense, the market-based inflation expectation.
Table 3. Granger Wald Causality Tests. Short-term Expectations (12-months).

Notes: The null hypothesis is that the ‘excluded’ variable does not cause in the Granger sense the equation variable. Thus, rejecting the null is evidence of the ‘excluded’ variable Granger-causing the equation variable. Based on a set of model selection criteria, we have considered one and two lags. The data have a monthly frequency. The market-based expectation seems to Granger-cause the survey-based measure.

Source: Own estimations with data from Valor de Mercado and Banco de México.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Excluded</th>
<th>$\chi^2$</th>
<th>DF</th>
<th>$Pr &gt; \chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>Survey</td>
<td>0.57</td>
<td>1</td>
<td>0.45</td>
</tr>
<tr>
<td>Survey</td>
<td>Market</td>
<td>23.03</td>
<td>1</td>
<td>0.00</td>
</tr>
<tr>
<td>Market</td>
<td>Survey</td>
<td>3.63</td>
<td>2</td>
<td>0.16</td>
</tr>
<tr>
<td>Survey</td>
<td>Market</td>
<td>18.25</td>
<td>2</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 4. Granger Wald Causality Tests. Medium-term Expectations (1-4 year).

Notes: The null hypothesis is that the ‘excluded’ variable does not cause in the Granger sense the equation variable. Thus, rejecting the null is evidence of the ‘excluded’ variable Granger-causing the equation variable. Based on a set of model selection criteria, we have considered two lags. The data have a monthly frequency. The market-based expectation seems to Granger-cause the survey-based measure.

Source: Own estimations with data from Valor de Mercado and Banco de México.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Excluded</th>
<th>$\chi^2$</th>
<th>DF</th>
<th>$Pr &gt; \chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market</td>
<td>Survey</td>
<td>&lt;0.00</td>
<td>1</td>
<td>0.98</td>
</tr>
<tr>
<td>Survey</td>
<td>Market</td>
<td>22.59</td>
<td>1</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 5. Granger Wald Causality Tests. Long-term Expectations (5-8 year).

Notes: The null hypothesis is that the ‘excluded’ variable does not cause in the Granger sense the equation variable. Thus, rejecting the null is evidence of the ‘excluded’ variable Granger-causing the equation variable. Based on a set of model selection criteria, we have considered one and two lags. The data have a monthly frequency. There is no evidence that one of the time series is causing in the Granger sense the other one.

Source: Own estimations with data from Valor de Mercado and Banco de México.

This result is, in general, robust to different lags (up to two lags as indicated by the Bayesian Information Criteria (BIC)), and hold at the 1% statistical significance level (Table 3). This suggests that the market-based inflation expectations have relevant informational content to forecast the survey-based inflation expectations. For other medium- and long-term inflation expectations, we have comparable results. Overall, we provide evidence that the market-based inflation expectation causes, in the Granger sense, the survey-based measure (Tables 4 and 5).
We also implement the Diebold and Mariano test [14]. We do so to assess the expected inflation forecasting performance. This test compares the error series of two possible forecasts. The errors are included in a loss function $g$ and the null hypothesis is that the difference between the corresponding loss functions is zero.\textsuperscript{17} Under the null hypothesis, the Diebold-Mariano (DM) statistic has a standard normal distribution. A significant positive (negative) statistic indicates that the expected inflation from surveys (model) forecast performs better. We test for the three horizons considered in the previous exercises (12 months, 1 to 4 years, and 5 to 8 years).

To compare surveys’ expected inflation from 1 to 4 years and from 5 to 8 years, we take the average of our model’s expected inflation that correspond to the 12, 24, 36, and 48-month horizons for the former set of survey-based expected inflation, and those that correspond to the 60, 72, 84, and 96-month horizons for the latter one. Similarly, we assume that the realized inflation corresponds to the mean of 12, 24, 36, and 48-month, for the first set, and the mean of 60, 72, 84, and 96-month, for the second one.

The results are presented in Table 6. For 12-month, 1 to 4-year and 5 to 8-year horizons. We like to underscore two results from these statistics. First, survey-based measures seem to be no better than our market-based measure. Second, the performance of long-term inflation forecast appears tilting toward the market-based expectations. Sure enough, it is not statistically significant at the conventional statistical levels, it hints that one could improve its forecasting performance by including additional risk factors to the model. This is similar approach to the one in Gimeno and Márques [19].\textsuperscript{18}

<table>
<thead>
<tr>
<th>Horizon</th>
<th>DM statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-month</td>
<td>1.09</td>
<td>0.28</td>
</tr>
<tr>
<td>1 to 4-year</td>
<td>−0.80</td>
<td>0.42</td>
</tr>
<tr>
<td>5 to 8-year</td>
<td>−1.38</td>
<td>0.17</td>
</tr>
</tbody>
</table>

\textbf{Table 6. Diebold-Mariano Statistics, Market-based Model}

\textbf{Notes:} Diebold-Mariano (DM) test on the mean squared errors. A positive DM statistic means that surveys’ expected inflation is a better forecaster for inflation, and vice versa.

\textbf{Source:} Own estimations with data from \textit{Valor de Mercado} and Banco de México.

In effect, when we include inflation time series as a risk factor in the model, its long-term inflation forecast is statistically better than that of the survey (Table 7). Of course, we lose the high frequency of the estimation, but such an exercise points to the potential forecasting capabilities of our approach. In effect, for 5 to 8-year horizon, the market-

\textsuperscript{17} The function $g$ has to be such that $g(0) = 0$, $g$ is strictly increasing and $g > 0$. The squared error, which we use, satisfies these criteria.

\textsuperscript{18} They use an affine interest rate model to decompose the Spanish nominal interest rate into their risk-neutral interest rates, expected inflation, and term premium components. Although their approach is similar to ours, the authors explicitly use inflation as a risk factor. Thus, theirs is not a \textit{bona fide} market-based measure.
based forecast is significantly better relative to the survey-based measure at the 99% confidence level.\textsuperscript{19, 20} As mentioned, we assume that the model estimation is stable. Naturally, we do not use lead risk factors to formulate our forecasts, but we do use the whole sample to estimate the model’s coefficients. Nonetheless, the stability results in Appendix A3 and the low p-value for the long-term expectations (Table 7) lends us confidence on our results. Our point is that market-based expectations are comparable to survey-based measures.

<table>
<thead>
<tr>
<th>Horizon</th>
<th>DM statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12-months</td>
<td>0.05</td>
<td>0.96</td>
</tr>
<tr>
<td>1 to 4-year</td>
<td>-1.26</td>
<td>0.21</td>
</tr>
<tr>
<td>5 to 8-year</td>
<td>-2.59</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 7. Diebold-Mariano Statistics, Market-based Model + Inflation as a Risk Factor

Notes: Diebold-Mariano (DM) test on the mean squared errors. A positive DM statistic means that surveys’ expected inflation is a better forecaster for inflation, and vice versa.

Source: Own estimations with data from Valor de Mercado and Banco de México.

8. Final Remarks

Market-based measures can complement and, possibly, improve other more common approaches.\textsuperscript{21} To obtain one, we use the difference of nominal and real (i.e., inflation-linked) interest rates, assuming risk-neutral investors, as a measure of expected inflation. We are able obtain a daily term structure of inflation expectations. This accounts for liquidity and inflation risk premiums.

As a case study, we implement this approach with Mexican government bonds. Its daily availability makes it a timely indicator and a useful gauge for policy announcements. We document that market-based expectations Granger-cause survey-based means, but not conversely. This suggests that the former have more informational content. We further evaluate its forecasting performance by implementing Diebold-Mariano tests, concluding that market-based- and survey-measures are on comparable performance footing.

We have left several matters unexplored, among which, we underscore the following ones. We have just started to explore how its information could be relevant to improve inflation forecasts. Moreover, it is worth exploring what statistics of the term structure of inflation expectations could be informative in this regard. We have underscored that our aim has not been to improve upon a certain approach to forecast inflation but

---

\textsuperscript{19} Inflation time series for month $t$ is only available on the first days of month $t+1$. Therefore, one cannot implement the model contemporaneously with the end-of-period interest rate information.

\textsuperscript{20} In contrast, Ang et al. [6] argue that, in the case of the U.S., surveys tend to perform better than other approaches. We attribute such a result on the richness of surveys in the U.S.

\textsuperscript{21} It is worth emphasizing that we have used the means of the survey-based expectations, and other statistics from survey-based measures might possess additional information.
mainly to offer an alternative measure of inflation expectations, which is applicable in several economies.

Inflation expectations have been important macroeconomic indicators. We speculate that more information might be embedded in the term structure of inflation expectations relative to the surveys. These could be revealing on to the macroeconomic state of the economy. As a conjecture, we think that the slope of the term structure of inflation expectations could be capturing expected price level pressures and, thus, might be indicative of the expected monetary policy stance. We leave such issues for future research.

References


Appendices
We briefly explore four issues. First, we estimate the model using the inflation as a risk-factor. Although this limits the available frequency of expected inflation, the model seems to predict better than survey-based measures. Second, we briefly discuss some of the analytics when accounting for different types of risk premium and estimating the expected inflation. Third, we briefly assess the interest rate model's stability. Specifically, we reduce our estimation sample and consider the extent to which our expected inflations differ, compared to those derived from a full sample. Fourth, we match selected economic episodes to the dynamics of the term structure of expected inflation.

A1. Estimation using Inflation as a Risk Factor
Although we did not use inflation as an input in our original model, we still think it is a natural complementary exercise to include inflation as a risk factor. In doing so, we are able to compare our expected inflation estimations with those from the model with inflation. We use a similar procedure to that in the main text. The key difference is that we include inflation in the affine model for nominal interest rates.

One of the principal features of our inflation estimates is that they are pure market-based measure. Evidently, to formulate all of their expectations, agents consider the publicly available information about the state of economy. Therefore, we estimate an affine model using nominal interest rates and inflation time series as a risk factor. We then compare its estimates of the expected inflation with those of our pure market-based approach.

To construct the risk factors for this model, we first orthogonalize the nominal interest rates with respect to the inflation data. To that end, we run the following regressions:

\[ i_t^{(n)} = \beta_{0,n} + \beta_{1,n} \pi_t + \epsilon_{t,n} \text{ for } n = 1, 2, 3, \ldots, N, \]

where \( i_t^{(n)} \) is the zero-coupon nominal interest rate with maturity \( n \), \( \beta_{i,n} \) with \( i = 0 \) or 1, are the regression coefficients, \( \pi_t \) is the inflation, and \( \epsilon_{t,n} \) are the error terms, all at time \( t \). Of course, \( \text{cov} [\epsilon_{n, \pi}] = 0 \) for \( n = 1, 2, 3, \ldots, N \).

The intuition behind is as follows. We extract from the nominal interest rates the information they could have on the inflation, leaving the rest in the residuals. In contrast, as real interest rates are inflation indexed, there is no reason to use inflation as a risk factor in this case.

As a next step, all residuals \( \epsilon_{n} \) are subject to a principal analysis decomposition. The risk factors of the interest rate model are then the first \( k - 1 \) principal components and the inflation, which we similarly stack in vector \( F_t \). Then, we follow the approach described in the main text. Of course, as inflation time series are used, we are limited by their frequency; for instance, we use monthly inflation data.
We present a comparison of the mean and standard deviations between our main estimates, and those that use inflation as a risk factor in Figures A3 and A4. It is worth mentioning that the mean levels of the ones that use inflation as a risk factor are lower than those from the pure market-based model. Nonetheless, the standard deviations of the former are, in general, higher than those of the latter.

![Figure A1. Mean Expected Inflation (%), as Function of the Horizon (in months).](image)

**Notes:** Annual interest rate. RF stands for Risk-Factor.

**Source:** Own estimations with data from Valor de Mercado and Banco de México.

With respect to the estimates that use inflation as a risk factor, we find similar results in the Granger causality test than those presented in the main text.

In addition, we estimate the Diebold-Mariano test over to compare the expected inflation from the model that uses inflation as a risk factor with those of the surveys. We find that, for the 5 to 8 years case, inflation expectations from the model are better forecaster of inflation with a confidence level of 99%. In the other two cases, we do not find a statistically significant difference between the inflation expectations from the model, and those from the surveys.

**A2. Accounting for Different Risk Premiums**

We have used the following decompositions:

\[
\begin{align*}
(1 + n l_t^{(n)}) &= (1 + n R_t^{(n)})(1 + E_t[\pi_{t,t+n}])(1 + n \rho_t^{(n)}), \\
(1 + n r_t^{(n)}) &= (1 + n R_t^{(n)})(1 + n \gamma_t^{(n)}).
\end{align*}
\]

To simplify the discussion, in what follows, we will use their linear approximations.
\[ i_t^{(n)} = R_t^{(n)} + \mathbb{E}_t[\pi_{t,t+n}] + \rho_t^{(n)}, \]
\[ r_t^{(n)} = R_t^{(n)} + \gamma_t^{(n)}. \]

To recap, \( i_t^{(n)} \) and \( r_t^{(n)} \) are the nominal and real interest rates, respectively, associated with a \( n \)-period maturity. \( \mathbb{E}_t[\pi_{t,t+n}] \) is the expected inflation for the following \( n \) periods.

We consider the following decompositions for the \( \rho_t^{(n)} \) and \( \gamma_t^{(n)} \) premiums:

\[
\rho_t^{(n)} = i rp_t^{(n)} + lr p_t^{(n)} + or p_t^{(n)} \quad \text{(Inflation risk + Liquidity risks + Other risks)},
\]
\[
\gamma_t^{(n)} = rlr p_t^{(n)} + ror p_t^{(n)} \quad \text{(Real liquidity risk + Real Other Risks)}.
\]

We are assuming that linear decompositions are feasible and that:

\[ rir p_t^{(n)} = 0 \] (There is no inflation risk premium for real bonds).

In addition, we assume that:

\[
lr p_t^{(n)} < rlr p_t^{(n)} \quad \text{(Liquidity risk are higher for real bonds)}.
\]
\[
or p_t^{(n)} < ror p_t^{(n)} \quad \text{(Other risks are higher for real bonds)}.
\]

We then have the following. It is ambiguous if \( \rho_t^{(n)} \) is greater or less than \( \gamma_t^{(n)} \).

Break-even:
\[
i_t^{(n)} - r_t^{(n)} - \rho_t^{(n)} = \frac{1}{n} \mathbb{E}_t[\pi_{t,t+n}] + \rho_t^{(n)} - \gamma_t^{(n)} \neq \frac{1}{n} \mathbb{E}_t[\pi_{t,t+n}].
\]

Correcting for inflation risk premium measured as \( \rho_t^{(n)} \) we have that:
\[
i_t^{(n)} - r_t^{(n)} - \rho_t^{(n)} = \frac{1}{n} \mathbb{E}_t[\pi_{t,t+n}] - \gamma_t^{(n)} \neq \frac{1}{n} \mathbb{E}_t[\pi_{t,t+n}].
\]

Correcting for inflation risk premium measured as \( ir p_t^{(n)} \):
\[
i_t^{(n)} - r_t^{(n)} - ir p_t^{(n)} = \frac{1}{n} \mathbb{E}_t[\pi_{t,t+n}] + lr p_t^{(n)} + or p_t^{(n)} - \gamma_t^{(n)} \neq \frac{1}{n} \mathbb{E}_t[\pi_{t,t+n}].
\]

Finally, as we argued in a footnote, the covariance between the real stochastic discount factor and inflation can be seen as the difference of \( \rho_t^{(n)} \) and \( \gamma_t^{(n)} \).

**A3. Model’s Stability**

We briefly explore the affine interest rate model’s stability. To be sure, we explore two aspects of it. First, we estimate our models but omitting the last two years of the estimation sample and compare some results with the model using the full sample. As a first result, we have that the model, in terms of the nominal interest rates, fit is satisfactory (Figure A3). Next, in terms of expected inflation, we have that both the
market-based and survey-based expected inflations in the short-term co-move closely. Altogether, the model largely maintains a close co-movement, and seems to capture comparable inflation expectations (Figure A4).

Figure A3. 5- and 10-Year Nominal Interest Rates.
Note: Simple Interest Rates.
Source: Valor de Mercado and own estimations.

Figure A4. Short-Term Expected Inflation Term Structure using a subsample (12-month horizon).
Source: Own estimations with data from Valor de Mercado and Banco de México.
A4. Selected Economic Episodes and the Term Structure of Expected Inflation

An illustrative exercise is to match the expected inflation dynamics to selected economic episodes. Specifically, we describe several economic episodes related to the inflation process, which possibly had an effect on the expected inflation dynamics. The numbers in parentheses refer to the episodes marked in Figure A5, which depicts the z-scores of the level, slope, and curvature of inflation expectations. To be sure, these statistics do not refer to the term structure of interest rates, for which they are commonly used. They refer to the term structure of expected inflation.

In 2004, a notable disinflationary process took place, largely steered by the local monetary policy. However, a marked rise in commodity prices brought inflationary pressure, a rise which was probably the repercussion of solid economic growth of commodity importers. Accordingly, the level of inflation expectations reached a local maximum by the end of the year (1).

In 2006, there was a renewed rise in commodity prices. At the time, the monetary authority stance had been accommodating as inflation reached a local minimum near 3.00% in May. However, in October, inflation had reached 4.29%. The monetary authority set its reference rate on hold, at 7.00%. Correspondingly, while the level of the inflation expectations spiked, the slope stayed constant and the curvature increased (2).

Figure A5. Z-scores for the Level, Slope, and Curvature of Expected Inflation.

Notes: We consider the z-scores of the following statistics based on the term structure of inflation expectations. The level is the average of the expected inflations with horizon of 1, 2, ..., and 10 years; the slope is the difference between the expected inflations with horizon of 10 and 1 year, and the curvature is the difference between the expected inflation with horizon of 10 and 7 years, minus the difference between the expected inflation with horizon of 5 and 1 year.

Source: Own estimations with data from Valor de Mercado and Banco de México.

---

22 We have partly based the economic episodes’ descriptions on Banco de Mexico’s Quarterly Inflation Reports [10].
In 2007, the merchandise price index markedly increased. There were significant distortions in the food chain, adding pressure to the associated prices. In this context, one can observe a spike in the level of expected inflation. Accordingly, its slope and curvature decreased in response (3). The oil price set at 70 USD by mid-year. In September, the government approved duties such as the flat rate business tax (*IETU, its acronym in Spanish*). Accordingly, the level of inflation expectations mildly increased, albeit its slope lessened (4).

In 2008, in mid-September the Lehman Brothers’ demise shook the financial world. The level of the inflation expectations promptly reached a local maximum (5). In November, the Federal Reserve started its quantitative easing program for the first time (i.e., QE1). In the process, the level of inflation expectations increased, and in stark contrast with other episodes, the slope and curvature stood near flat. While they presented some variability during this episode, their trends co-moving positively and adversely affected the expectations’ persistence.

In 2009, their level initially decreased and maintained a positive sign. Yet, the slope was close to zero and the curvature was only somewhat positive, indicative of expected inflation persistence. The European Debt Crisis started toward the end of 2009 (6), and deteriorated in early 2010, leading to a noticeable increase in the level of inflation expectations. Even so, its slope and curvature responded in the opposite direction, enabling a swift reversion of inflation expectations.

During 2010, the bond spreads of the economies in the European periphery increased. The subsequent repricing of sovereign risk brought about a drop in the pace of capital inflows to EMEs. Their effects soon reverted and, in tandem, Mexico’s fiscal consolidation’s efforts paid off. Possibly as a result, the level of inflation expectations dropped (7), and eventually reached a value close to zero.

In 2012, financial markets deteriorated in the context of the Greek election outcome in May. Specifically, it increased the uncertainty regarding its government’s willingness to maintain its commitments with the IMF and European Union back in March. Locally, unfavorable weather conditions affected vegetables and the bird flu adversely affected local livestock productions. Notwithstanding such factors, the level of inflation expectations maintained close to zero (8).

In 2013, toward the end of the first quarter, there was a supply shock, which led to an inflation spike. In this context, frosts considerably damaged some plantations and the bird flu put pressure on eggs and poultry prices. In line with this, there was an increase in the inflation expectations’ level. Their slope and curvatures dropped in tandem (9). A few months after, the “Taper Tantrum” went off in May, intensifying in June (10). EMEs’ capital outflows markedly increased. The level of inflation expectations became positive. Although its slope initially decreased, it assumed a near-flat value, and its curvature stood positive (10).

---

23 The Federal Reserve carried on with the implementation of the rest of U.S. UMPs.
curvature increased. These two elements did not contribute toward a swifter inflation convergence. These contrast with the common dynamics.

In 2016, the Banco de México and the Ministry of Finance called a press conference on February 17. The former announced a fifty basis points rise in its reference rate, the latter a cut in government expenditures. In line with such announcements, the level of inflation expectations stayed in place, and its slope and curvature dropped, both favorable outcomes (11). On June 23, the Brexit referendum outcome took the financial markets by surprise. The day after, Banco de México and the Ministry of Finance held a press conference on the solid prospects of the Mexican economy. In this context, the level of inflation expectations increased, albeit from a relatively low level (12). On November 9, one day after the U.S. Presidential Election, Banco de México and the Ministry of Finance called a press conference to underscore the favorable economic conditions for Mexico. While the level of inflation expectation marginally increased, the slope and curvature responded rapidly, leading to a swifter convergence of inflation expectations (13).

In 2017, in January, gasoline prices markedly increased. Accordingly, the level of inflation expectations conspicuously increased. Opportunely, its slope and curvature decreased (14). On February 21, the Foreign Exchange Commission announced that the Banco de Mexico would sell non-deliverable forwards on U.S. dollars. The level of inflation expectations kept dropping, supported by the government policy responses (15). In December, gasoline prices increased once more. Its effects were milder than those at the beginning of the year. While the level of inflation expectations increased, its slope decreased (16).

Overall, with a handful of exceptions, we have that inflation expectations’ dynamics tend to react in such a way that they have facilitated the convergence process. We think of their dynamics as supporting well-anchored inflation expectations. More generally, although these statistical regularities are not the product of a monetary model in which some stable equilibrium is explicitly derived, they are obtained based on a no-arbitrage condition and, hence, have an associated equilibrium (see Duffie [16]).

---

24 Consider the following decomposition of the nominal interest rates:

\[
\left( 1 + n \ i_t^{(n)} \right) = \left( 1 + n \ R_t^{(n)} \right) \left( 1 + E_t^{[\pi_{t,t+n}]} \right) \left( 1 + n \ \rho_t^{(n)} \right)
\]

Recall that \( \rho_t^{(n)} \) is a compensation for overall risks. One could then further decompose such a premium in the following elements.

\[
\rho_t^{(n)} = \Pi_t^{(n)} + \Lambda_t^{(n)} + E_t^{(n)}
\]

We have assumed that such a decomposition is plausible as a sum. In other words, different risk compensations are not necessarily linear with respect to the general risk premium nor necessarily equally weighted. We think of \( \Pi_t \) as inflation risk premium, \( \Lambda_t \) as nominal bond liquidity premium and \( E_t \) as the residual risk unrelated to inflation and liquidity. Even under the assumption that \( E_t \) is equal to zero, we have that identifying \( \Pi_t \) from \( \rho_t \) is not direct. It is relatively common to see that researchers and practitioners think of \( \rho_t \) as the inflation risk premium. This implicitly assumes that \( \Lambda_t + E_t \) are equal to zero at all times. Thus, the estimation of the inflation risk premium commonly has a positive bias. It would be a relatively
small problem if they were constant. Nonetheless, quite possible, such a bias probably tends to increase in times of financial stress, precisely when there is a greater need to assess the macroeconomic conditions more closely, an elusive problem.