Measuring the Macroeconomic Impact of Capital Requirements*

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PRELIMINARY

Abstract

We quantitatively assess the implications of regulatory capital requirements in an open economy DSGE model with banks. The capital requirement consists of a fixed part and a countercyclical part (countercyclical buffers). Since we focus on the deterministic steady state, changes on the fixed part affect the long-term economy, while changes in the sensitivity of the buffers do not. In particular, we examine the aggregate dynamics resulting from a negative capital quality shock and assess the effectiveness of capital requirements as a tool to diminish real losses in the case of a financial crisis. We find that the higher the fixed capital requirements, the better able banks are, and hence economy, to handle a financial crisis. Furthermore, in general, countercyclical buffers that respond to deviations of the expected spread or the observed credit-to-GDP ratio from their long-term values, or to percentage deviations of the observed credit (or GDP) from its long-term values diminish the fluctuations in both financial and real variables. The conclusion is not clear when buffers respond to the observed deviations of asset prices, credit and output growth from their long-term levels.

Key Words: Capital requirements, countercyclical capital buffers, financial crisis.
JEL Codes: E32, G01, G21, G28.

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1 Introduction

The recent financial crisis proved the need for macroprudential policies targeting the probability of a financial crisis. In this context, Basel III appears with a proposal that is supposed to strengthen the capital of banks taken into account the deficiencies in financial regulation revealed for the 2008 US financial crisis.

The main objective of this paper is to introduce into an open economy DSGE model with banks a regulatory capital requirement rule in the spirit of Basel III. To do this, we incorporate banks as in Gertler and Karadi 2011 (henceforth GKa 2011) in a DSGE model. In particular, the regulatory capital requirement rule consists of two parts: The first part consists of a fixed capital requirement. This is, the regulator requires banks to hold as bank equity a fixed proportion of their total assets. It aims to promote an appropriate level of bank leverage so that the economy will be in good conditions to face a financial crisis.

The second part of the capital requirements are the countercyclical capital buffers. The role of this part is twofold: First, it prevents banks for taking excessive risk. And second, it reduces real losses once the crisis occurs. In other words, it relaxes the credit constraint (capital requirements) in downturns periods or equivalently it lessen the effects of the financial crisis whenever it occurs. In this paper, we mainly assess the second role. This second part of the rule, looks more like a “Taylor rule”.¹

The main conclusions that can be derived from this model are the following: we find that the higher the fixed capital requirements, the better able banks are, and hence economy, to handle a financial crisis. Furthermore, in general, countercyclical buffers that respond to deviations of the expected spread or the observed credit-to-GDP ratio from their long-term values, or to percentage deviations of the observed credit (or GDP) from its long-term values diminish the fluctuations in both financial and real variables.

In addition, if countercyclical buffers respond to the expected credit-to-GDP deviations or the credit or GDP percentage deviations (rather than to the observed variables) the results are not significantly different. Also, when studying the effects after a productivity (real) shock or a capital inflow shock, the results still hold. In other words, in general countercyclical buffers reduces the fluctuations of the economy independent of the source of the shock. The conclusion is not clear when buffers respond to the observed deviations of asset prices, credit and output growth from their long-term levels.

The paper is organized as follows: Section 2 presents the relevant literature on this topic. Section 3 describes the DSGE model and the regulatory capital requirement rule. Section 4 presents the calibration of the parameters. Section 5 presents the results of the

¹To do implement this rule we follow the conclusions of R 2013 and RS 2013
2 Literature Review

In what follows, we depict the picture of current thinking on theoretical models and empirical papers of financial frictions and capital requirements in order to communicate the interest that motivated this research.

To model financial frictions in a non-static perspective but in a dynamic stochastic general equilibrium model it is commonly required the presence of banks. Several authors have made efforts leading to the emergence of a new generation of models that attempt to incorporate banks in the analysis. These new models include GKa 2011, Gertler and Kiyotaki (2011) (henceforth GKi 2011), Gertler Kiyotaky and Queralo (2012) (henceforth GKQ 2012) and Akinci and Queralto (henceforth AQ 2014). In addition to introducing banks, they introduce financial frictions through a moral hazard problem between bankers and households, which raises an incentive constraint. In particular, bankers can divert a fraction of the resources under their management. The models provide, among its virtues, an intuitive rationale for (market-ruled) bank capital requirements.

In particular, in GKa 2011 assume nominal rigidities of prices. The contribution of GKi 2011 is to allow the existence of different investment opportunity shocks across islands. GKQ 2012 includes the possibility that banks can issue equity (outside equity). In contrast with the previous papers, in this paper the capital requirement rule is given by a regulatory authority. The main contributions of AQ 2014 are that the authors open the economy and solve the model assuming that the incentive constraint (originated by the moral hazard between bankers and households) is occasionally binding. GKQ 2012 and AQ 2014 solve the model focusing on the stochastic steady state. Our contribution is to assess the effect of an imposed capital requirement rule on an open economy DSGE model with banks. For simplicity we assume no rigidities. In addition, in this version of the paper we assume an always binding capital requirement constraint and we focus on the deterministic steady state.

As documented by some papers since the 1960s the leverage ratio of important segments of the financial intermediation sector has exhibited a markedly procyclical pattern. Kashyap and Stein (2004) (henceforth KS 2004) show by simulations and using different methodologies that Basel II capital requirements have the potential to create an amount of additional cyclicality in capital charges, which is economically significant. They explain that in a downturn bank’s capital are lower due to higher losses forcing banks to hold more capital. Since it is difficult or costly for banks to raise new capital in bad times, they reduce credits, and this contributes to a worsening of the initial downturn. This
describes the deleveraging process observed during the 2007-2009 financial crisis.

Nuno and Thomas (2013) using U.S. data of financial intermediaries find that leverage contributes at least as much as equity to cyclical movements in total assets. Leverage is negatively correlated with equity and positively correlated with assets and GDP. In theoretical models like GKa 2011, GKi 2011 and GKQ 2012 they also find a negative correlation of leverage and equity but they find a negative correlation of leverage and GDP when there is a market-based capital requirement.

Respect to regulatory issues, discussions on the potential business cycle amplification effects of Basel II started way before its approval in 2004 by the Basel Committee on Banking Supervision. The argument whereby these effects may occur is well-known. In recessions, losses erode banks’ capital, while risk-sensitive capital requirements such as those in Basel II become higher. Thus, if banks cannot quickly raise sufficient new capital, the standard financial accelerator mechanism will take place. In other words, Basel II might incentive large fluctuations of asset prices and real variables in the economy. This conjecture is assessed in DSGE models like Covas and Fujita (2010) and De Walque et al. (2010). Covas and Fujita (2010), comparing output fluctuations under Basel I and Basel II, find that the standard deviation of the second is higher. De Walque et al. (2010) develops a DSGE model and concludes that Basel I reduces long-term values for output and Basel II increases business cycle fluctuations. Recently, Rubio and Carrasco-Gallego (2016) study the interaction between Basel I, II and III with monetary policy. They conclude that the optimal implementation of capital buffers (Basel III) leads to higher financial stability with respect to Basel I and II.

Basel III ruled from 2010 requires banks to hold a higher proportion of common equity and “risk-weighted assets”. In addition, Basel III introduces “additional capital buffers”. In practice, Basel III introduces a series of measures to promote the build-up of capital buffer in good times that can be drawn upon in periods of stress. In this way, Basel III looks for “reducing procyclicality and promoting countercyclical buffers”. Some people disagree with this last measure since correcting the potential contractionary effect on credit supply by relaxing capital requirements in bad times may increase bank failure probabilities precisely when, because of high loan defaults, they are largest.

In Repullo (2013) (henceforth R 2013) is developed a static model to study this trade-off following the framework of KS 2004. In this framework a regulator maximizes the society’s welfare subject to the fixed capital supply where welfare captures the cost of bank failures. He states that capital requirements should be lowered in situations where capital is scarce such as in a recession. Repullo and Suarez (2013) show that Basel II is more procyclical than Basel I, but makes banks safer. They say Basel III seems to be even better since it increases capital requirement but it is less procyclical.
In addition, there are other papers that measure the effects of capital requirement on welfare. In general, all agree that capital requirements improve welfare, see for instance Begenau (2015), Cristiano and Ikeda (2013), Nguyen (2014) and Collard et al. (2015). In particular, capital requirements tackle a particular inefficiency modeled in each of these papers. Begenau (2015) presents a quantitative dynamic general equilibrium model where households have preferences for safe and liquid assets. It shows that an increased capital requirement can reduce bank funding costs and increase lending. Christiano and Ikeda (2013) develop a DSGE model with financial intermediaries and an agency problem. It shows that welfare increases when imposing binding capital requirements. In particular, a lower leverage reduces the risk of the creditors, then agency problems are mitigated and the efficiency of the banking system is improved. Nguyen (2014) develops a model with endogenous growth and a dynamic banking sector. It focuses in the distortions that bank bailouts create and the role of capital requirements in mitigating these distortions. In Collard et al. (2015) the interaction of limited liability and deposit insurance create excessive bank risk-taking. This motivates the use of capital requirements to mitigate the risk. This proposal departs from this literature since we do not explicitly model the inefficiency that capital requirements aim to tackle, and so the contribution to this literature of this paper is to assess the effects on real losses (or fluctuations) that several proposed capital requirement rules might have in the economy.

An empirical paper, Jiménez, Ongena, Peydro and Saurina (2013), using data for Spain shows that countercyclical capital buffers help to smooth credit supply cycles and in downturns those have positive effects on firm credit availability. In this line, Fillat and Montoriol-Garriga J. (2010) argue that if the USA had set aside general provisions in positive states of the economy, this would have been in a better position to absorb loan losses during the recent financial crisis. In the same line, Chan-Lau (2012) shows that countercyclical provisions might help to reduce procyclicality. However, this depends on the characteristics of the banking system of the country. In particular, a dynamic provision scheme as in Spanish would have improved bank’s solvency but not reduced the procyclicality.

3 The Baseline Model

3.1 Physical setup

Here we present the basic physical environment. There is a continuum of firms of mass unity. Each firm produces output using an identical constant returns to scale Cobb-Douglas production function with capital $K_t$ and labor $L_t$ as inputs. We can express...
aggregate output $Y_t$ as:

$$Y_t = A_t K_t^\alpha L_t^{1-\alpha}, 0 < \alpha < 1,$$

(1)

where $A_t$ is the aggregate productivity which follows an AR(1) process in logs: $\ln(A_t) = \rho_a \ln(A_{t-1}) + \epsilon_{a,t}$. There is a capital in process at $t$ for $t+1$, $S_t$, that is the sum of current investment $I_t$ and the stock of undepreciated capital, $(1 - \delta)K_t$:

$$S_t = (1 - \delta)K_t + I_t.$$  

(2)

Capital in process for period $t + 1$ is transformed into capital for production after the realization of a multiplicative shock to capital quality, $\psi_{t+1}$:

$$K_{t+1} = \psi_{t+1} S_t.$$  

(3)

We introduce the capital quality shock following the finance literature, for instance, Merton (1973), as a way to introduce an exogenous source of variation in the value of capital. As will become clear later, the market price of capital will be endogenous within this framework. The capital quality shock will serve as an exogenous trigger of asset price dynamics. The random variable $\psi_{t+1}$ is best thought of as capturing some form of economic obsolescence, as opposed to physical depreciation.\(^2\) We assume the log of the capital quality shock $\psi_t$ follows an AR(1) process:

$$\ln(\psi_t) = \rho_\psi \ln(\psi_{t-1}) + \epsilon_{\psi,t+1},$$

(4)

where $\rho_\psi \in (0,1)$ and $\epsilon_\psi \sim N(0, \sigma_\epsilon^2)$. Firms acquire new capital from capital goods producers. There are convex adjustment costs in the rate of change in investment goods output for capital goods producers.

Our preference structure follows GKQ (2012):

$$\mathbb{E}_t \sum_{i=t}^{\infty} \beta^{i-t} \frac{1}{1 - \gamma} \left( C_i - hC_{i-1} - \frac{X}{1 + \omega} L_i^{1+\omega} \right)^{1-\gamma},$$

(5)

where $\mathbb{E}_t$ is the expectation operator conditional on information at date $t$ and $\gamma > 0$. The preference specification allows for habit formation and abstracts from wealth effects on labor supply. We include the flow adjustment costs of investment and habit formation since they are standard features of many quantitative macro models. They improve the quantitative performance of the model considerably and can be added at relatively little cost in terms of model complexity. However, to keep the model tractable we abstract from other standard features that help account for employment volatility, such as price.

and wage rigidities.

### 3.2 Households

Following GKa 2011, we formulate the household sector in a way that permits maintaining the tractability of the representative agent approach. In particular, there is a representative household with a continuum of members of mass unity. Within the household, there are $1 - f$ "workers" and $f$ "bankers". Workers supply labor, $L_t$, and return their wages, $W_t$, to the household. Each banker manages a financial intermediary (bank) and transfers nonnegative dividends back to the household subject to its flow of fund constraint. There is perfect consumption insurance within the family. Households do not acquire capital and they do not provide funds directly to nonfinancial firms. Rather, they supply funds to banks. It may be best to think of them as providing funds to banks other than the ones they own. Banks offer non-contingent riskless short-term debt (deposits, $B_t$) to households. Let $Z_t$ be the flow returns at $t$ generated by one unit of the bank’s assets.

The household chooses consumption, labor supply and riskless debt (or domestic bank deposits) $(C_t, L_t, B_t)$ to maximize expected discounted utility subject to the flow of funds constraint,

$$C_t + B_t = W_t L_t + \Pi_t - T_t + R_t B_{t-1}.$$  \hfill (6)

Here, $\Pi_t$ are the net funds from ownership of both banks and capital producing firms. Let $u_{Ct}$ denote the marginal utility of consumption and $\Lambda_{t,t+1}$ the household’s stochastic discount factor. Then, the household’s first-order conditions for labor supply and consumption/saving are given by,

$$E_t u_{Ct} W_t = \chi L_t^\omega (C_t - h C_{t-1} - \frac{\chi}{1 + \omega} L_t^{1+\omega})^{-\gamma}, \hfill (7)$$

$$E_t (\Lambda_{t,t+1}) R_{t+1} = 1, \hfill (8)$$

with,

$$u_{Ct} = (C_t - h C_{t-1} - \frac{\chi}{1 + \omega} L_t^{1+\omega})^{-\gamma} - \beta h (C_{t+1} - h C_t - \frac{\chi}{1 + \omega} L_{t+1}^{1+\omega})^{-\gamma},$$

$$\Lambda_{t,\tau} = \beta \frac{u_{C\tau}}{u_{Ct}}.$$  

Because banks may be financially constrained, bankers will retain earnings to accumulate assets. Absent some motive for paying dividends, they may find it optimal to accumulate to the point where the capital requirement constraint is no longer binding. To limit bankers’ ability to save to overcome financial constraints, a turnover between bankers and workers is introduced. In particular, there is a i.i.d. probability $1 - \sigma$, that a
banker exits next period, (i.e., an average survival time = \( \frac{1}{1-\sigma} \)). Upon exiting, a banker transfers retained earnings to the household and becomes a worker. Note that the expected survival time may be quite long (in our baseline calibration it is eight years.) It is critical, however, that the expected horizon is finite, in order to motivate payouts while the capital requirements are still binding.

Each period, \((1 - \sigma)f\) workers randomly become bankers, keeping the number in each occupation constant. Finally, because in equilibrium bankers will not be able to operate without any financial resources, each new banker receives a “startup” transfer from the family as a small constant fraction of the total assets of entrepreneurs. Thus, \(\Pi_t\) are net funds transferred to the household; that is, funds transferred from exiting bankers minus the funds transferred to new bankers (aside from small profits of capital producers).

### 3.3 Nonfinancial Firms

There are two types of nonfinancial firms: goods producers and capital producers.

#### 3.3.1 Goods Producers

Competitive goods producers operate a constant returns to scale technology, with capital and labor as inputs, given by equation (1). Firms choose labor to satisfy,

\[
W_t = (1 - \alpha) \frac{Y_t}{L_t}.
\]

(9)

Since goods producers face zero profits it follows that we may express gross profits per unit of capital \(Z_t\) as,

\[
Z_t = \frac{Y_t - W_t L_t}{K_t} = \alpha A_t \left( \frac{L_t}{K_t} \right)^{1-\alpha}.
\]

(10)

Goods producers can commit to pay all the future gross profits to the bank. In particular, we suppose that the bank is efficient at evaluating and monitoring nonfinancial firms and also at enforcing contractual obligations with these borrowers. That is why these borrowers rely exclusively on banks to obtain funds. Then, a goods producer who invests can obtain funds from a bank without any financial friction by issuing new state-contingent securities at the price \(Q_t\). The producer then uses the funds to buy new capital goods from capital goods producers. Each unit of the security is a state-contingent claim to the future returns from one unit of investment:

\[
\psi_{t+1} Z_{t+1}, (1 - \delta) \psi_{t+1} \psi_{t+2} Z_{t+2}, (1 - \delta)^2 \psi_{t+1} \psi_{t+2} \psi_{t+3} Z_{t+3}, \ldots
\]

(11)
Through perfect competition, the price of new capital goods is equal to $Q_t$, and goods producers earn zero residual profits in any state.

### 3.3.2 Capital Producers

They make new capital using input of final output and subject to adjustment costs. They sell new capital to firms at the price $Q_t$. Given that households own capital producers, the objective of a capital producer is to choose $I_t$ to solve:

$$\max \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \{Q_\tau I_\tau - [1 + f(I_{\tau-1}/I_\tau)]I_\tau\}. \quad (12)$$

From profit maximization, the price of the capital goods is equal to the marginal cost of investment goods as follows:

$$Q_t = 1 + f(I_t/I_{t-1}) + \frac{I_t}{I_{t-1}} f'(I_{t-1}) - E_t \Lambda_{t,t+1}(I_{t+1}/I_t)^2 f'(I_{t+1}/I_t). \quad (13)$$

Profits (which arise only outside of steady state), are redistributed lump sum to households.

### 3.4 Banks

Every period each bank raises funds by supplying deposits. In addition, the bank has its own net worth accumulated from retained earnings (which we refer to as equity). The bank then uses all its available funds to make loans to goods producers. As noted earlier, there is no friction in transferring funds between a bank and goods producers. As in case of investment banks, banks finance goods producers by purchasing perfectly state-contingent security. Their total value of loans is equal to the number $s_t$ times the price $Q_t$ of the state-contingent security (or asset). In other words, the bank’s claim on the future returns from one unit of a goods producer’s capital at the end of period $t$ (i.e., capital at $t$ in process for $t+1$).

For an individual bank, the balance sheet equation implies the value of loans funded within a given period, $Q_t s_t$, must equal the sum of bank net worth $n_t$, and funds raised from deposits, $d_t$,

$$Q_t s_t = n_t + d_t, \quad (14)$$

where,

$$d_t = b_t + b_t^*, \quad (15)$$

where $b_t$ are the external funds that a bank can obtain from domestic households (or
domestic deposits) and $b_t^r$ are the funds obtained from foreign investors (or foreign deposits). Notice that banks raise equity only through retained earnings. Since equity involves management and control of the firm’s assets, we suppose it is prohibitively costly for the existing insiders to bring in new ones with sufficient wealth. In particular, the bank’s net worth $n_t$ at $t$ is the gross payoff from assets funded at $t-1$, net of returns to depositors. Let $R_{kt}$ denote the gross rate of return on a unit of the bank’s assets from $t-1$ to $t$. Then:

$$n_t = R_{kt}Q_{t-1}s_{t-1} - R_t d_{t-1},$$

with,

$$R_{kt} = \left[ \frac{Z_t + (1-\delta)Q_t}{Q_{t-1}} \right] \psi_t.$$ 

Given that the bank faces a financing constraint, it is in its interest to retain all earnings until the time it exits, at which point it pays out its accumulated retained earnings as dividends. Accordingly, the objective of the bank at the end of period $t$ is the expected present value of the future terminal dividend,

$$\mathbb{E}_t \left[ \sum_{i=t+1}^{\infty} (1-\sigma)\sigma^{i-t-1} \Lambda_{t,i}n_i \right].$$

Combining (14) and (16) yields the evolution of $n_t$ as a function of $s_{t-1}$ and $n_{t-1}$ as,

$$n_t = (R_{kt} - R_t)Q_{t-1}s_{t-1} + R_t n_{t-1}.$$ 

Next, we introduce the regulatory capital requirements.

### 3.5 Regulatory Capital Requirement Rule

There is a regulatory authority that imposes a capital requirement rule. This rule dictates that banks are required to maintain as net worth (equity) at least a fraction of the value of their total assets.

We assume the regulator is efficient at monitoring and ensuring that all banks will satisfy the capital requirement rule and the costs of doing so are negligible. At the same time, if a bank does not satisfy capital requirements, it is charged a huge penalty which will not allow the bank to continue with its activity. Thus, in equilibrium all banks in the market will satisfy the capital requirement rule for all $t$.

The capital requirement rule for each bank is expressed in general as,

$$\kappa_t Q_t s_t \leq n_t.$$ 

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where $\kappa_t$ is the regulatory capital requirement ratio, which is the sum of a fixed capital requirement ratio, $\kappa^{FCR} > 0$, and a countercyclical capital buffer ratio, $\kappa^{CCB}_t$, i.e.,

$$\kappa_t = \kappa^{FCR} + \kappa^{CCB}_t,$$

where $FCR$ stands for fixed capital requirements and $CCB$ stands for countercyclical capital buffer. And the countercyclical capital buffers are defined as follows,

$$\kappa^{spread,CCB}_t = \kappa^{spread,CCB}[(E_t R_{kt+1} - E_t R_{t+1}) - (ER_k - R)].$$

This countercyclical capital requirement rule, $\kappa^{CCB}_t$, is similar to a “Taylor rule”. Since there is an infinite number of banks, bankers know they cannot affect $\kappa^{CCB}_t$ and thus this value will be taken as given. This rule states that capital requirements will react to any deviation of the expected spread from its steady state. We define the spread as the difference between the return of the assets and the risk-free interest rate: $R_{kt} - R_t$. Here, we propose a countercyclical capital requirement rule that responds to the deviation of the expected (and no the observed) spread from its long-term values in order to ensure the model converges, i.e., we assume $\kappa^{CCB} < 0$. The intuition is the following: high spreads, which capture any solvency problem in the banking sector, induce regulator to be more flexible and thus to reduce the minimum amount of capital held for one unit of total assets. Conversely, lower spreads will induce a more strong (precautionary) response from the regulator.

In section 5.1, we propose some capital buffers that we are also interested in since these are also in the interest of policy makers.

### 3.6 Banks’ Optimization Problem

Now, we can continue solving the Banks’ optimization problem. From (18) it follows that, in general, the franchise value of the bank at the end of period $t-1$ satisfies the Bellman equation,

$$V_{t-1}(s_{t-1}, n_{t-1}) = E_{t-1} \Lambda_{t-1,t} \{(1 - \sigma)n_t + \sigma \max_{s_t}[V_t(s_t, n_t)]\},$$

where the right side takes into account that the bank exits with probability $1 - \sigma$ and continues with probability $\sigma$. Thus, at each time $t$, the bank chooses $s_t$ to maximize $V_t(s_t, n_t)$ subject to the capital requirement constraint (20) and the law of motion for net worth (19). We conjecture the value function,

$$V_t(s_t, n_t) = \mu_t Q_t s_t + \nu_t n_t,$$
where $\mu_{st}$ and $\nu_t$ are time-varying parameters, and verify this guess later. Note that $\mu_{st}$ is the marginal value of assets at the end of the period $t$, and $\nu_t$ is the marginal value of bank net worth at the end of the period $t$. Appendix A provides a detailed derivation of the first order conditions. We assume the capital requirements are always binding. In the equilibrium, we construct below, under reasonable parameter values the constraint always binds within a local region of the deterministic steady state. Then,

$$\kappa_t Q_t s_t = n_t. \quad (24)$$

Equation (20) is a key relation of the banking sector: it indicates that when the borrowing constraint binds, the total quantity of private assets that a bank can intermediate is limited by its net worth, $n_t$. Let define $\phi_t = \frac{Q_t}{n_t}$ be the maximum ratio of bank assets to net worth (leverage ratio) that satisfies the capital requirement constraint. Then, by construction:

$$\phi_t = \frac{1}{\kappa_t} \quad (25)$$

Then, after combining the conjectured value function with the Bellman equation, we can verify that the value function is linear in $(s_t, n_t)$ if $\nu_t$ and $\nu_{st}$ satisfy:

$$\nu_t = E_t(\Lambda_{t,t+1} \Omega_{t+1}) R_{t+1}, \quad (26)$$

$$\mu_{st} = E_t[\Lambda_{t,t+1} \Omega_{t+1}(R_{kt+1} - R_{t+1})], \quad (27)$$

where $\Omega_1$ is the shadow value (or marginal value) of a unit of net worth to the bank at $t + 1$ and is given by,

$$\Omega_{t+1} = 1 - \sigma + \sigma[\nu_{t+1} + \phi_{t+1} \mu_{st+1}]. \quad (28)$$

Observe that the household discounts the returns by the stochastic factor $\Lambda_{t,t+1}$ while the banker uses a discount factor $\Lambda_{t,t+1} \Omega_{t+1}$. This latter is defined in the literature as “augmented stochastic discount factor”. The marginal value of net worth is a weighted average of marginal values for exiting and for continuing banks. If a continuing bank has an additional net worth, it can save the cost of deposits and can increase assets by the leverage ratio $\phi_t$, where assets have an excess value equal to $\mu_{st+1}$ per unit. Equation (26) claims that the marginal cost of net worth at the end of $t$ is the expected product of the augmented stochastic discount factor and the deposit rate. Similarly equation (27) state that the excess value per unit of assets is the expected product of the augmented stochastic discount factor and the excess return, $R_{kt+1} - R_{t+1}$. Since $\phi_t$ does not depend on bank-specific factors, we can aggregate equation (24) and (25) to obtain a relation between the aggregate demand or securities by banks $S_t$ and aggregate net worth in the banking sector $N_t$,

$$Q_t S_t = \phi_t N_t. \quad (29)$$
3.7 Evolution of bank net worth

Let total net worth for banks, $N_t$, equals the sum of the net worth of existing bankers $N_{ot}$ (o for old) and of entering bankers $N_{yt}$ (y for young),

$$N_t = N_{ot} + N_{yt}.$$

Net worth of existing bankers equals earnings on assets held in the previous period net the cost of deposit finance, multiplied by the fraction that survive until the current period, $\sigma$,

$$N_{ot} = \sigma \{ [Z_t + (1 - \delta)Q_t] \psi_t S_{t-1} - R_t D_{t-1} \}.$$

We assume that the family transfers to each new banker the fraction $\xi/(1 - \sigma)$ of the total value assets of exiting bankers, implying,

$$N_{yt} = \xi [Z_t + (1 - \delta)Q_t] \psi_t S_{t-1}.$$

Total net worth of bank is now,

$$N_t = (\xi + \sigma) [Z_t + (1 - \delta)Q_t] \psi_t S_{t-1} - \sigma R_t D_{t-1}. \quad (30)$$

Finally, by the balance sheet of the entire banking sector, deposits equal the difference between total assets and bank net worth as follows,

$$D_t = Q_t S_t - N_t. \quad (31)$$

Observe that the evolution of net worth depends on fluctuations in the return to assets. Further, the higher the leverage of the bank, the larger the percentage impact of return fluctuations on net worth will be. Note also that a deterioration of capital quality (a decline in $\psi_t$) directly reduces net worth.

3.8 International Capital Markets

As it is done in AQ 2014, who follow Schmitt-Grohé and Uribe (2003), we assume that the small open economy is subject to debt-elastic interest rate premium in the international markets,

$$R_t = \frac{1}{\beta} + \varphi \left( e^{\frac{b^*}{\beta}} - 1 \right) + e^{R_t^*-1} - 1, \quad (32)$$

where $b$ governs the steady state foreign debt to GDP ratio $R_t^*$ is the risk-free world interest rate, which is assumed to follow an AR(1) process in natural logs, $\ln(R_t^*) = \rho_R \ln(R_{t-1}^*) + \epsilon_{R,t}$, where $\epsilon_{R,t} \sim N(0, \sigma_R)$. 

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Equation (32) suggests, turning off the world interest rate shock, that if the foreign debt to GDP ratio is above its long-term value (i.e., above its “sustainable level”), the deposit market assigns a positive country risk premium due to a high risk or a positive probability that the domestic economy fails to honor its foreign debt. As in AQ 2014, we do not model this friction; however, equation (32) aims to capture this issue in a reduced form. In addition, from equation (32) for a given $R_t$ after a negative world interest rate shock (or a transitory capital inflow shock), the foreign debt to GDP ratio should move above its long-term value.

3.9 Resource Constraint and Market Clearing

To close the model (in the case without government policy), we require market clearing in the market for securities and the labor market. In the market for securities we say that the supply the securities of firms equals the demand of securities of banks. Finally, the condition that labor demand equals labor supply requires that,

$$
(1 - \alpha) \frac{Y_t}{L_t} \mathbb{E}_t \left[ \frac{u_{Ct}}{(C_t - hC_{t-1} - \frac{X_{Nt} L^{1+\omega}_{t}}{1+\omega})^{-\gamma}} \right] = \chi L^\omega_t. \quad (33)
$$

Aggregate output is divided between household consumption $C_t$, investment expenditures, and net exports $N X_t$.

$$
Y_t = C_t + \left[ 1 + f(I_t / I_{t-1}) \right] I_t + G_t + N X_t, \quad (34)
$$

where net exports are given by,

$$
R_t B^*_{t-1} - B^*_t = N X_t, \quad (35)
$$

where $f(I_t / I_{t-1})I_t$ reflects physical adjustment costs, with $f(1) = f'(1) = 0$ and $f''(I_t / I_{t-1}) > 0$, and $X N_t$ stands for net exports.

Equations (1, 2, 3, 6, 8, 10, 13, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35) determine the seventeen endogenous variables ($Y_t, L_t, C_t, I_t, S_t, K_t, D_t, B^*_t, Q_t, N_t, R_t, R_{kt}, Z_t, \mu_{st}, \nu_t, \Omega_t, X N_t$) as a function of the state exogenous variables ($A_t, \psi_t, R^*_t$).

The equilibrium is different from the RBC equilibrium since capital requirements (or the credit constraint), faced by banks, limit investing spending, affecting aggregate real activity. As usual in models with banks that accumulate net worth and face a leverage constraint bank leverage creates an amplification effect, see BGG (1999).
4 Calibration

Without including the standard deviation of the exogenous disturbances, there are seventeen parameters for which we need to assign values. Eight parameters are standard preference and technology parameters. These include the discount factor $\beta$, the coefficient of relative risk aversion $\gamma$, the habit parameter $h$, the utility weight on labor $\xi$, the inverse elasticity of the Frisch elasticity of labor supply $\omega$, the capital share parameter $\alpha$, the deterministic depreciation $\delta$ and the elasticity of the price of capital with respect to investment $\eta = If''/f$. For these parameters we assign the same values used in GKQ 2012, see table 1.3

Two additional parameters are specific to financial intermediaries: $\sigma$, the quarterly survival probability of bankers, and $\xi$, the transfer parameter for new bankers. We set $\sigma = 0.975$, implying that bankers survive for ten years on average as in GKi 2011. We set $\xi$ to 0.0540 to have an average credit spread of 100 basis points per year (i.e., $ER_k - R = 1\%$).

There are two parameters associated with the capital requirement rule, $\kappa^{FCR}$ and $\kappa^{spread,CCB}$. We set $\kappa^{FCR}$ to 0.25 to hit the following target: an aggregate leverage ratio of four in the steady state as in AQ 2014. Recall that $\kappa^{spread,CCB}$ measures the degree of countercyclicality of the capital requirement rule. We are going to start with $\kappa^{spread,CCB} = -12$. Implying that for 100 basis points (bps) of a positive deviation of expected spread from its steady state, the regulatory authority reacts reducing the requirement of capital by 12 percent bps. In other words, the regulator requires 0.12 units less of bank net worth per unit of the total value of bank assets. Finally, we set the debt elasticity of interest rate, $\varphi$, to 0.05 and we set the reference foreign debt to output ratio, $b$, to 60% as in AQ 2014.

We calibrate the persistence of the productivity and capital quality shock to 0.66 and the persistence of the foreign interest rate shock to 0.9 as in AQ 2014. The model solved under this calibration is going to be called the baseline model. We solve the DSGE model using a first order approximation around the deterministic steady state. In section 5.2 we discuss the advantages of studying the stochastic steady state and also the advantages of dropping the assumption of an always binding capital requirement rule.

3Indeed, these values are similar to those used in the literature, see, for instance, GKa 2011 and GKi 2011
Table 1: Parameter values

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk aversion</td>
<td>$\gamma$ 2</td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$ 0.99</td>
</tr>
<tr>
<td>Capital share</td>
<td>$\alpha$ 0.33</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>$\delta$ 0.025</td>
</tr>
<tr>
<td>Utility weight of labor</td>
<td>$\chi$ 0.25</td>
</tr>
<tr>
<td>Inverse Frish elast. of labor supply</td>
<td>$\omega$ $1/3$</td>
</tr>
<tr>
<td>Inverse elasticity of investment to Q</td>
<td>$\eta$ 1</td>
</tr>
<tr>
<td>Habit parameter</td>
<td>$h$ 0.75</td>
</tr>
<tr>
<td>Survival rate of bankers</td>
<td>$\sigma$ 0.975</td>
</tr>
<tr>
<td>Transfer to entering Banks</td>
<td>$\xi$ 0.054</td>
</tr>
<tr>
<td>Debt elast. of interest rate</td>
<td>$\varphi$ 0.05</td>
</tr>
<tr>
<td>Reference debt/output ratio</td>
<td>$h$ 0.60</td>
</tr>
<tr>
<td>Capital requirement rule</td>
<td>$\kappa^{\text{FCR}}$ 0.25</td>
</tr>
<tr>
<td></td>
<td>$\kappa^{\text{spread,CCB}}$ -12.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Shock processes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Persistence of capital quality</td>
<td>$\rho_\psi$ 0.66</td>
</tr>
<tr>
<td>Persistence of productivity</td>
<td>$\rho_a$ 0.66</td>
</tr>
<tr>
<td>Persistence of interest rate</td>
<td>$\rho_R$ 0.90</td>
</tr>
</tbody>
</table>

Next, we discuss the impulses responses after several shocks. A negative capital quality shock characterizes a financial crisis. GK1i 2011 motivates this shock as a decline in real estate observed in the 2008-2009 US financial crisis. A negative productivity shock aims to characterize a real shock. Finally, a negative world interest rate shock characterizes a capital inflow shock. Since the base of this model is an RBC model many of the dynamics have been already studied. Hence, we mainly focus on the effects of the capital requirements.

5 Simulations and sensitivity analysis

Here, figures 1 and 2 plots the impulse response functions for main variables after a one-time negative shock of capital quality of 2% with a persistence of $\rho_\psi = 0.66$. Figure 1 and 2 aim to compare the effects of changing the parameters associated with the fixed capital requirements, $\kappa^{\text{spread,FCR}}$, and with the countercyclical capital buffers, $\kappa^{\text{CCB}}$, respectively.

Figure 1 plots the impulse response functions in the baseline calibration (solid black
It also plots the impulse responses for a higher (than the baseline) fixed capital requirement ratio (blue dashed lines) and for a lower (than the baseline) capital requirement ratio (red dotted lines) that respectively yield to a long-term leverage ratio of 1.5 and 12, being 4 the leverage in the baseline. In these two latter cases, only $\kappa^{spread, FCR}$ is modified while keeping the other parameters unchanged.

As expected the larger the fixed capital requirement ratio, the smaller the fluctuations in the economy. This feature is more evident in financial variables as spread, bank net worth and leverage. The intuition is the following: the negative quality capital shock will not only reduce the value of loans but it will also affect bank net worth. Since we assume the capital requirements bind, a lower net worth deteriorates bank capacity to lend, which in turn reduces even more the value of the loans. Hence, the capital requirements (or lending constraint) create an amplification effect of the negative capital quality shock. As a result, the higher the capital requirements (or the tighter the lending constraint) the stronger the amplification effect, which in turn leads to larger fluctuations in the economy.

In the baseline calibration in the short-term asset prices shrink by 5%, and loans (capital) decreases by 2%. These drive the almost 30% reduction of bank equity. The higher the fixed capital requirements, the lower the leverage and hence the higher bank’s capacity to absorb losses, which in turn leads to a smaller fluctuation in bank net worth. Note that bankers (bank equity) absorb the whole losses (and gains if there are) since depositors receive a risk-free payment. Thus, an economy governed with a more cautious regulatory authority will be in a better position to handle a financial crisis produced by a capital quality shock. However, the long-term values for this economy will be lower. This creates a trade off that challenges regulator’s actions. Hence, one pending question to be assessed in the future is how this fixed capital requirement affect domestic welfare.
Figure 1: Negative capital quality shock and $\kappa^{FCR}$ sensitivity

Figure 2 plots the impulse responses for the baseline calibration, where the counter-cyclical capital buffer ratio is $\kappa^{spread,CCB} = -12$. In order to model more countercyclical capital buffers (blue dashed lines) we set $\kappa^{spread,CCB} = -24$. And in order to model less countercyclical capital buffers (red dotted lines) we set $\kappa^{spread,CCB} = -4$. Notice that moving the countercyclical capital buffers ratio does not affect the deterministic steady state of the model since at the steady state any capital buffers, which depends on the cycle, are turned off.\textsuperscript{4}

In general, economic fluctuations associated with highly countercyclical capital buffers (blue solid lines) are lower, excepting for bank net worth and leverage. The intuition is the following: recall the countercyclical rule consists of reducing the capital requirements each time the expected spread is above its long-term value. This is because the regulator aims to alleviate the bank solvency issues reflected in a higher expected spread. Clearly, after a negative capital quality shock the spread increases, which drives the reduction of capital requirements per unit of assets. This allows banks to lend more for unit of net worth, which counteracts the negative effects on credit produced by the negative capital quality shock. As a result, a more countercyclical capital buffer reduces in a higher degree the proportion of equity required in banks and thus counteracts in a higher degree the

\textsuperscript{4}In contrast, the stochastic steady state, which is not studied here yet, is going to be affected when moving $\kappa^{spread,CCB}$.  

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negative effects of the capital quality shock.

Figure 2: Negative capital quality shock and $\kappa^{\text{spread,CCB}}$ sensitivity

Figures 6, 7, 8 and 9 in Appendix B show the results for a negative productivity shock and a negative world interest rate shock. From figures 6 and 7, as in the case of a capital quality shock, after a productivity or a world interest rate shock, stronger fixed capital requirements reduce fluctuations more markedly in financial variables. Similarly, from figures 8 and 9, as in the case of a capital quality shock, after a productivity or a world interest rate shock, stronger countercyclical buffers reduce fluctuations more markedly except for bank net worth and leverage.

5.1 Countercyclical capital buffers

Here, we assess the effects of two different forms of countercyclical capital buffers. The first form consists in buffers as in Basel III (BCBS, 2010). Basel III uses the gap between the credit-to-GDP ratio and its long-term trend as a guide for setting countercyclical capital buffers, i.e., the additional capital buffers ratio under Basel III recommendations takes the form of,

$$
\kappa_{t}^{S/Y,\text{CCB}} = \kappa^{S/Y,\text{CCB}} \left[ \frac{S_{t-1}}{Y_{t}} - \frac{S}{Y} \right],
$$

(36)
where \( \kappa_{S/Y,CCB} > 0 \) and hence the buffers are expected to be countercyclical. This is, if the credit-to-GDP ratio is above its long-term value, regulator asks banks to hold more bank net worth per unit of assets or equivalently to reduce their assets given their net worth levels.

The second form of capital buffers we propose follows Rubio and Carrasco-Gallego (2016). They propose a capital buffer ratio that responds to percentage deviations of bank credit from its steady state. Similarly, we propose capital buffers that respond to percentage deviations of GDP or asset prices from its steady state. In any of these cases, the additional capital buffer ratio takes the following form,

\[
\kappa_t^{X,CCB} = \kappa^{X,CCB} \left[ \ln(X_t) - \ln(X) \right],
\]

(37)

where \( X_t \in \{ \text{credit } S_t - 1, \text{ gross domestic product } Y_t, \text{ asset prices } Q_t - 1 \} \) and \( \kappa^{X,CCB} > 0 \) and hence buffers are expected to be countercyclical. For instance, for \( X_t = Y_t \), each time output is higher than its long-term value, the regulator mandates banks to increases their buffers.

Importantly, the strategy to assess the consequences of each of these countercyclical capital buffer ratios is to add each of these ratios to equation (21). This is to ensure the model always converges to the steady state. In other words, to assess the effects of \( \kappa_{S/Y,CCB} \), the capital requirement ratio becomes,

\[
\kappa_t = \kappa^{FCR} + \kappa^{spread,CCB} + \kappa_{S/Y,CCB}.
\]

Similarly, in order to assess the effects of \( \kappa_t^{X,CCB} \), the capital requirement ratio becomes,

\[
\kappa_t = \kappa^{FCR} + \kappa^{spread,CCB} + \kappa^{X,CCB}.
\]

Note that the proposed additional buffers depends on information of variables already observed in the economy, which are easy to obtain, and thus buffers do not depend on expectations that in real life are hard to fairly measure.

Figures 3, 4 and 5 plot the impulse response functions for main variables after a one-time negative shock of capital quality of 2% with a persistence of \( \rho_\psi = 0.66 \). As usual, it plots the impulse responses in the baseline calibration (black solid lines), where there are not additional capital buffers, i.e., \( \kappa_{S/Y,CCB} = 0 \) and \( \kappa_{X,CCB} = 0 \).

In figure 3 we observe the effects of additional capital buffers that respond to the observed credit-to-GDP deviations from its stead state. It compares the effects of a more countercyclical additional capital buffer (\( \kappa_{S/Y,CCB} = 0.20 \), blue dashed lines) with a less

\footnote{Technically, we keep \( \kappa_t^{spread,CCB} \) in any rule such that the Blanchard Kahn conditions are satisfied in order to have a stable equilibrium.}
countercyclical additional capital buffer \( (\kappa_{S/Y,CCB} = 0.08, \text{ red dotted lines}) \). In general, this additional capital buffers reduce the cyclicality. Thus, the more countercyclical the additional buffer is, the higher the reduction of the fluctuations, except for the bank net worth and the bank leverage ratio.

Figure 4 compares the effects of additional capital buffers that responds to the observed percentage deviations of bank credit from its steady state. We set \( \kappa_{S,CCB} = 0.20 \) in order to have a relatively more countercyclical additional capital buffer (blue dashed lines). And then we set \( \kappa_{S,CCB} = 0.08 \) in order to have a relatively less countercyclical additional capital buffer (red dotted lines). The results are, qualitatively speaking, similar when studying the buffers that responds to credit-to-GDP deviations. The same occurs when studying the buffers that responds to the percentage deviations of GDP from its steady state, figure 5. This is, the more countercyclical the buffers (i.e., the larger \( \kappa_{S,CCB} > 0 \) or the larger \( \kappa_{Y,CCB} > 0 \)), the smaller the fluctuations. However, when assessing the buffers that respond to the percentage deviations of asset price from its steady state, figure 10, the results are inconclusive. In other words, it is not clear if the more countercyclical buffers (i.e., the larger \( \kappa_{Q,CCB} > 0 \)) leads to smaller fluctuations.

Finally, a further discussion of other possible countercyclical capital buffer ratios is developed in section 5.2.

Figure 3: Negative capital quality shock and \( \kappa_{S/Y,CCB} \) sensitivity

High \( \kappa_{S/Y,CCB} = 0.20 \). Low \( \kappa_{S/Y,CCB} = 0.08 \). Baseline: \( \kappa_{S/Y,CCB} = 0 \) and \( \kappa_{X,CCB} = 0 \).
Figure 4: Negative capital quality shock and $\kappa^{S,CCB}$ sensitivity

High $\kappa^{S,CCB} = 0.20$. Low $\kappa^{S,CCB} = 0.08$. Baseline: $\kappa^{S/Y,CCB} = 0$ and $\kappa^{X,CCB} = 0$.

Figure 5: Negative capital quality shock and $\kappa^{Y,CCB}$ sensitivity

High $\kappa^{Y,CCB} = 0.20$. Low $\kappa^{Y,CCB} = 0.08$. Baseline: $\kappa^{S/Y,CCB} = 0$ and $\kappa^{X,CCB} = 0$. 

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5.2 Discussion of key issues

5.2.1 A frictionless economy:

Ignore for a minute the investment adjustment costs. Then, if we remove the regulatory capital requirements we are in a frictionless economy. The competitive equilibrium would correspond to a solution of the planner’s problem that involves choosing aggregate quantities \((Y_t, L_t, C_t, I_t, S_t, B_t^*)\) as a function of the exogenous variables \((\psi_t, A_t, R^*_t)\) in order to maximize the expected discounted utility of the representative household subjects to the resource constraints. This frictionless economy corresponds to an open economy RBC model.

5.2.2 Capital requirements in a frictionless economy:

In real life, capital requirements are imposed to provide banks with enough resources to respond to bad unexpected events in the economy. Hence, it is expected that the capital requirements prevent banks to take excessive risk. Note, however, that we are not explicitly modeling this friction. In that sense, we motivate the presence of capital requirements in this economy by assuming banks take excess bank risk-taking or chose an inefficiently high leverage without explicitly modeling this latter. Consequently, since we are not explicitly modeling such inefficiency, it is not the interest of this paper to provide a socially optimal capital requirement rule, but rather to study the effects of different capital requirements rules.

5.2.3 Other countercyclical capital buffers:

We have tried to cover and report the most discussed forms of capital buffers. However, it is worth to mention other exercises performed but no reported. For example, when considering that the capital buffers of the form, as in equations (36) and (37), respond to the expected deviations rather than to the observed deviations, we find very similar impulse responses. This suggests that apparently there are not significant gains of being forward looking in this economy.

Also, countercyclical capital buffers that respond to (observed or expected) credit growth or output growth require a large sensitivity parameter to affect fluctuations and it is not clear if the higher the sensitivity, the smaller the fluctuations in the economy. This result is aligned with the literature that suggests that it is no ideal to build buffers that respond to the dynamics of the variables but to their long-term deviations in order to reduce procyclicality in the economy.
5.2.4 Future agenda:

Even though, this paper highlights important characteristics and effects of fixed capital requirements and countercyclical capital buffers in a small open economy, we are not still quantitatively measuring the effects of capital requirements and capital buffers on domestic welfare, which is also one of the main goals as stated in the proposal.

In other words, the model is missing some key elements that are going to be added in a future version of the paper. A first element is to study the stochastic steady state instead of the deterministic steady state. By doing so, we will be able to capture the effects of the countercyclical capital buffers (presented in section 3.5) on the long-term economy. Recall that the stochastic steady state is an equilibrium where there are not shocks in the economy but agents expect that shocks might occur in the future, while in the deterministic steady state agents does not expect shocks might occur in the future.

The second element is to drop the assumption of an always binding capital requirement constraint and assume an occasionally binding capital requirement. From a technical point of view, this feature allows to qualitatively measure the effect of countercyclical buffers on domestic welfare independently, i.e., without the presence of the countercyclical buffers that depend on the expected spread that ensures convergence in an always binding constraint environment.

6 Conclusions

In this document we quantitatively assess the implications of regulatory capital requirements in an open economy DSGE model with financial intermediaries. The capital requirements consist of a fixed capital requirement ratio and a countercyclical capital buffer ratio.

Since we focus on the deterministic steady state, changes on the fixed part affect the long-term economy, while changes in the sensitivity of the countercyclical buffers do not. In particular, we examine the aggregate dynamics resulting from a negative capital quality shock (financial crisis), a negative productivity shock and a capital inflow shock and assess the effectiveness of capital requirements as a tool to diminish the size of the fluctuations.

We find that the higher the fixed capital requirement ratio, the better able banks are, and hence economy, to handle a financial crisis, a real crisis and capital inflows. Furthermore, in general, countercyclical buffers that respond to deviations of the expected spread or the observed credit-to-GDP ratio from their long-term values, or to percentage deviations of the observed credit (or GDP) from its long-term values diminish the fluctuations in both financial and real variables. The conclusion is not clear when buffers respond to
the observed deviations of asset prices, credit and output growth from their long-term levels.

References


Appendices

A First order conditions

Recall the expressions of the capital requirement constraint, bank net worth and the guess of the value function:

\[ \kappa_t Q_t s_t \leq n_t, \]  
(38)

\[ n_t = Q_t s_t - d_t, \]  
(39)

\[ V_t = (\mu_{st} + x_t \mu_{et}) Q_t s_t + \nu_t n_t. \]  
(40)

We insert the conjectured solution, equation (40), into the Bellman equation (22). Then, we maximize it, with respect to the to the capital requirement constraint (38). Using the Lagrangian,

\[ L = (\mu_{st} + x_t \mu_{et}) Q_t s_t + \nu_t n_t + \lambda_t [n_t - \kappa_t Q_t s_t], \]  
(41)

where \( \lambda_t \) is the Lagrangian multiplier with respect to the incentive constraint, the first order necessary conditions for \( s_t \) and \( \lambda_t \) yield, respectively: The first order condition for \( s_t \),

\[ \mu_{st} = \lambda_t \kappa_t, \]  
(42)

and the first order condition for \( \lambda_t \) (or complementary slackness condition),

\[ (n_t - \kappa_t Q_t s_t) \lambda_t = 0, \]  
(43)

such that \( 0 \leq \lambda_t \) and \( \kappa_t Q_t s_t \leq n_t \). Note that banks take \( \kappa_t \) as given. The first order condition for \( s_t \), equation (42), state that the marginal benefit from increasing a unit of asset, \( \mu_{st} \), is equal to the marginal cost of tightening the capital requirement constraint \( \lambda_t \kappa_t \). In other words, the incentive constraint binds (i.e., \( \lambda_t \) is positive) if and only if the excess value of bank assets \( \mu_{st} \) is positive. Finally, the first order condition for \( \lambda_t \) yields the incentive constraint.
B Additional figures

Figure 6: Negative productivity shock and $\kappa^{FCR}$ sensitivity

IRFs after a one-time negative productivity shock of 5% with a persistence of $\rho_a = 0.66$.

Figure 7: Negative interest rate shock and $\kappa^{FCR}$ sensitivity

IRFs after a one-time negative world interest rate shock of 0.8% with a persistence of $\rho_R = 0.90$. 
Figure 8: Negative productivity shock and $\kappa^{spread,CCB}$ sensitivity

IRFs after a one-time negative productivity shock of 5% with a persistence of $\rho_a = 0.66$.

Figure 9: Negative interest shock shock and $\kappa^{spread,CCB}$ sensitivity

IRFs after a one-time negative world interest rate shock of 0.8% with a persistence of $\rho_R = 0.90$. 

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Figure 10: Negative capital quality shock and $\kappa^{Q,CCB}$ sensitivity

High $\kappa^{Q,CCB} = 0.20$. Low $\kappa^{Q,CCB} = 0.08$. Baseline: $\kappa^{S/Y,CCB} = 0$ and $\kappa^{X,CCB} = 0$. 