Monetary Policy and Sectoral Composition

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Abstract

We study what is the optimal measure of inflation that a central bank should target given the sectoral composition of the economy. The optimal measure of inflation is characterized by the weights assigned to inflation in each sector, such that welfare losses due to nominal frictions present in the economy are minimized. For this purpose, we build a two sector model including features from the structural change and the new keynesian literature. We calibrate the model to replicate the sectoral composition of both developing and developed countries. We consider flexible agricultural prices, sticky non-agricultural prices and sticky wages. We find that the optimal weights depend on the type of shocks hitting the economy. After agricultural productivity shocks, the optimal measure of inflation implies zero weight to agricultural inflation. This result holds independently of how sticky wages are. On the other hand, after non-agricultural productivity shocks, it is optimal to assign full weight to agricultural inflation. The reason is that by containing agricultural inflation the central bank indirectly contains wage inflation and non-agricultural price inflation, the main sources of welfare loss under sticky wages and non-agricultural prices.

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1 Introduction

One of the main objectives of monetary policy is stabilizing inflation. A key issue for central banks, seeking to meet this goal, is to define a measure of inflation to target. This measure should allow the monetary authority to minimize the welfare loss that arises due to nominal frictions present in the economy. In this chapter, we study what is the optimal measure of inflation that a central bank should target, given the sectoral composition of the economy in which it operates.

From the structural change literature, we know that sectoral composition changes as the economy grows. Developing countries have a larger share of agricultural consumption compared to developed nations. Therefore, a direct implication for monetary policy purposes is that agricultural prices have a larger weight in the aggregate price index in these economies. If we define the measure of inflation as a weighted average of inflation rates across sectors, where the weights are determined by the central bank, then, is it optimal for a central bank in a developing country to assign a large weight to agricultural inflation?

To answer this question, we build a multi sector model that includes features from the structural change and new Keynesian literature. We consider an economy with two sectors: agriculture and non-agriculture. In the model, agricultural goods have lower-than-one income elasticity and price elasticity is non-unitary. In a developing country, low aggregate productivity and low relative agricultural productivity imply low income level and high relative price of agricultural goods. Therefore, the shares of consumption and employment in agriculture are high. We refer to an economy with this feature as a country in an early stage of structural change. Clearly, as aggregate and relative agricultural productivity rise, sectoral composition shifts to non-agriculture.

Regarding the new Keynesian features of the model, we consider an economy with flexible prices in agriculture, sticky prices in non-agriculture and sticky wages in both sectors. We consider flexible prices in the agricultural sector, based on the findings of Bils and Klenow (2004) for the United States. These authors indicate that the frequency of price changes for unprocessed food is much higher than the average. We assume there are no labor mobility frictions across sectors, even at business cycle frequency. Wage stickiness is assumed to be equal across sectors based on evidence provided by Barattieri et al. (2004), who argued that there is little heterogeneity in the frequency of wage adjustment across industries and occupations in the United States. In the model,

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1Herrendorf et al. (2014) provide a review of the structural change literature.
countries in an early stage of structural change will have a high concentration of flexible prices in the aggregate price index, as the agricultural consumption share is high.

We set the parameters of the model to match the structural change features of a developing country, that is, large employment and consumption in agriculture as percentage of total employment and expenditure, respectively. We evaluate welfare losses when the economy is hit by sector specific productivity shocks, using a Taylor rule with different weights assigned to agricultural and non-agricultural inflation. We compute the optimal measure of inflation, that is, the weight of agricultural inflation that allows the central bank to minimize welfare loss. We then compare the baseline results of a developing country to those of a rich economy.

Results show that optimal weights depend on the type of shock hitting the economy. Productivity shocks to the flexible agricultural sector imply a zero weight on agricultural inflation. On the other hand, productivity shocks to the sticky non-agricultural sector imply a full weight on agricultural inflation. In addition, we find that optimal weights are equal for countries with different sectoral composition. However, the sectoral composition of the economy affects the welfare gain that a central bank can attain by targeting the optimal measure of inflation.

To understand these results we examine the impulse responses generated by the model and derive a welfare loss function to analyze the sources of welfare loss. The impulse responses, after non-agricultural productivity shocks, show that there is a link between agricultural price inflation and wage inflation. The reason is that, with perfect mobility across sectors, wages are proportional to agricultural prices. Targeting agricultural inflation allows the central bank to reduce wage inflation and, indirectly, non-agricultural inflation, since wages are part of marginal costs in this sector. In fact, using the welfare loss function we find that wage inflation is the main source of welfare loss, followed by non-agricultural price inflation, given the choice of parameters. On the other hand, the impulse responses, after agricultural productivity shocks, show that targeting agricultural prices actually increases wage and price inflation. Therefore, the central bank minimizes welfare loss by targeting non-agricultural inflation.

The most closely related findings in the literature are those of Aoki (2001), Mankiw and Reis (2003), Anand et al. (2015) and Portillo et al. (2016). Aoki (2001) uses a new Keynesian model with a flexible price sector and a sticky price sector. He shows that stabilizing sticky price inflation is sufficient to stabilize inflation around its efficient level. This analysis was expanded in Mankiw and Reis (2003), who ask what is the measure of inflation that central banks should target in order to stabilize the economy. They show that central banks should weight a sector in the price index given its characteristics, which include price stickiness, size, cyclical sensitivity and magnitude of
sectoral shocks.

More recently, Anand et al. (2015) consider segmented labor and incomplete credit markets. That is, workers cannot move across sectors in the economy, while households in the agricultural sector have no access to banking services. They find that, in these circumstances, it is optimal for the central bank to target headline inflation, defined as a broad measure including agricultural prices. The reason is that agricultural productivity shocks affect real wages of households in this sector, which in turn affect aggregate demand. To contain demand and price volatility, the central bank must include agricultural prices in the target. Portillo et al. (2016) consider a two sector model with subsistence consumption of food, and demand and agricultural productivity shocks. Their findings indicate that it is optimal for central banks to target only core inflation, a measure excluding volatile food prices, and losses from missing this target are larger for poorer countries.

In this chapter we find, as in Anand et al. (2015), that the central bank should target agricultural inflation to minimize welfare loss. As opposed to Anand et al. (2015), we do not need to consider financial frictions and immobile workers. We only have to consider sticky wages, a robust feature of the data. In addition, the model in this chapter can account for the results of Aoki (2001) and Portillo et al. (2016), when only agricultural productivity shocks hit the economy. In sum, the main contributions of this chapter are two-fold. First, we show how sticky wages affect the optimal measure of inflation that a central bank should target. Second, we show that welfare gains in economies with the sectoral composition of developed countries can be substantial, if central banks assign weight to agricultural inflation after shocks to non-agricultural productivity, and that this is equivalent to targeting wage inflation.

The rest of the chapter is organized as follows. The next section introduces the model. Section 3 describes the quantitative exercise, including parameter selection, results from the simulation and sensitivity analysis. Section 4 introduces the welfare loss function. Finally, Section 5 concludes.

2 Model

2.1 Firms

The economy consists of two sectors: agriculture and non-agriculture, denoted by $s \in \{a, n\}$. In each sector there is a continuum of firms, indexed by $i \in [0, 1]$, producing a single-differentiated
good and with monopoly power to set prices. Production technologies are given by

\[ Y_{s,t}(i) = A_{s,t} N_{s,t}(i)^{1-\alpha_s}, \]  

where \( Y_{s,t}(i) \) is output of firm \( i \) in sector \( s \). Productivity levels, denoted by \( A_{s,t} \), are common for all firms in the same sector. \( N_{s,t}(i) \) is an index of labor inputs demanded by firm \( i \) in sector \( s \), and is defined as

\[ N_{s,t}(i) \equiv \left( \int_0^1 N_{s,t}(i,j) \frac{\varepsilon_w - 1}{\varepsilon_w - 1} dj \right)^{\frac{\varepsilon_w - 1}{\varepsilon_w - 1}}, \]  

where \( N_{s,t}(i,j) \) denotes labor variety \( j \in [0,1] \). The firm regards different labor varieties as imperfect substitutes of each other. The parameter \( \varepsilon_w > 0 \) is the elasticity of substitution across labor varieties and is common in both sectors.

Firm \( i \) takes the wage of labor variety \( j \), \( W_t(j) \), as given in each period. Labor demand of firm \( i \) in sector \( s \) of labor variety \( j \) is determined by solving the firm’s cost minimization problem (derivations in the Appendix A.1). It is given by

\[ N_{s,t}(i,j) = \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_{s,t}(i), \]  

where the wage index \( (W_t) \) is defined as

\[ W_t \equiv \left( \int_0^1 W_t(j)^{1-\varepsilon_w} dj \right)^{\frac{1}{1-\varepsilon_w}}. \]  

2.1.1 Optimal price setting

In every period, firms in sector \( s \) reset prices with probability \((1 - \theta_s)\), as in Calvo (1983). The probability of resetting the price is sector specific. A firm in sector \( s \) that last reset prices in period \( t \), chooses the price that maximizes the following sum of discounted profits

\[ \max_{P_{s,t}} \sum_{k=0}^{\infty} \theta_s^k E_t \{ Q_{t,t+k} [ P_{s,t}^* Y_{s,t+k|t} - TC_{t+k}(Y_{s,t+k|t}) ] \} \]
subject to the demand constraint given by\(^3\)

\[
Y_{s,t+k|t} = \left( \frac{P_{s,t}^*}{P_{s,t+k}} \right)^{-\varepsilon_p} C_{s,t+k},
\]

where \(P_{s,t}^*\) is the optimal price of a firm that last reset its price at \(t\), \(Y_{s,t+k|t}\) is the output of that firm, \(P_{s,t+k}\) is the price of good \(s\) available at time \(t + k\) and \(C_{s,t+k}\) indicates total demand of that good. \(\varepsilon_p > 0\) is the elasticity of substitution across goods varieties. Firms discount profits by the state-contingent stochastic discount factor, \(Q_{t,t+k}\), as defined in Erceg et al. (2000), and by the probability that the firm will not reset prices next period, \(\theta_s\). The total cost of producing \(Y_{s,t+k|t}\) units of output is defined as

\[
TC_{t+k}(Y_{s,t+k|t}) \equiv W_{t+k}N_{s,t+k|t}.
\]

Maximization implies

\[
\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{s,t+k|t} \left[ P_{s,t}^* - \mu_p MC_{s,t+k|t} \right] \right\} = 0,
\]

where \(MC_{s}^n \equiv \partial TC(Y_s)/Y_s\) is the nominal marginal cost of producing one more unit of output in sector \(s\), and \(\mu_p \equiv \varepsilon_p / (\varepsilon_p - 1)\) is the desired markup, common to both sectors.

When prices are flexible (\(\theta_s = 0\)), prices are given by the desired mark-up over the nominal marginal cost as

\[
P_{s,t}^* = P_{s,t} = \mu_p MC_{s,t}.
\]

### 2.2 Households

There is a continuum of households indexed by \(j \in [0, 1]\) with life-time utility given by

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\varphi}}{1+\varphi} \right) \right\},
\]

where \(C_t(j)\) is a consumption index and \(N_t(j)\) is labor supply. Each household \(j\) supplies a different variety of labor and has monopoly power to set wages. The consumption index is an aggregate of agricultural and non-agricultural goods consumption. It is defined as

\[
C_t(j) \equiv \left( \frac{1}{\omega_a} (C_{a,t}(j) - \tilde{C}_a)^{\frac{\gamma-1}{\gamma}} + \frac{1}{\omega_n} C_{n,t}(j)^{\frac{\gamma-1}{\gamma}} \right)^{\frac{1}{\gamma-1}},
\]

\(^3\)We derive the demand constraint, in Appendix A.2.
where \( C_{a,t}(j) \) and \( C_{n,t}(j) \) are in turn consumption indexes comprising the different varieties of goods available in each sector, and are defined as

\[
C_{s,t}(j) = \left( \int_0^1 C_{s,t}(i,j) \frac{\varepsilon_p - 1}{1 - \varepsilon_p} \frac{\varepsilon_p}{\varepsilon_p - 1} \, di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}},
\]

where \( C_{s,t}(i,j) \) denotes household \( j \)'s consumption of good variety \( i \) available in sector \( s \in \{ a, n \} \).

The parameter \( \beta \) indicates the discount factor, \( 1/\sigma \) is the intertemporal elasticity of substitution, \( 1/\varphi \) is the Frisch elasticity of labor supply. \( \omega_a \) and \( \omega_n \) are the utility weights of agriculture and non-agriculture and satisfy \( \omega_a + \omega_n = 1 \), \( \gamma \in (0, 1) \) is the elasticity of substitution between agricultural and non-agricultural goods (when preferences are homothetic), \( \tilde{C}_a \) is the agricultural minimum consumption requirement (when \( \tilde{C}_a \neq 0 \), preferences are non-homothetic), and \( \varepsilon_p > 1 \) is the elasticity of substitution across goods varieties, common in both sectors.

The budget constraint of household \( j \) is given by

\[
\int_0^1 P_{a,t}(i)C_{a,t}(i,j)\, di + \int_0^1 P_{n,t}(i)C_{n,t}(i,j)\, di + Q_tB_t(j) = W_t(j)N_t(j) + B_t - 1(j) + \Pi_t(j).
\]

Households receive labor income, \( W_t(j)N_t(j) \), and profits, \( \Pi_t(j) \), from equal ownership of firms. They spend income to consume and accumulate the state-contingent asset \( B_t \), valued at price \( Q_t \).

2.2.1 Intratemporal optimization

In each period, household \( j \) determines the optimal consumption allocation given total expenditure (derivations in the Appendix A.3) Optimization implies the following consumption demand function

\[
C_{s,t}(i,j) = C_{s,t}(j) \left( \frac{P_{s,t}(i)}{P_{s,t}} \right)^{-\varepsilon_p},
\]

where \( C_{s,t}(j) \) indicates total demand of good \( s \), and the price index in sector \( s \) is defined as

\[
P_{s,t} \equiv \left( \int_0^1 P_{s,t}(i)^{1-\varepsilon_p} \, di \right)^{\frac{1}{1-\varepsilon_p}}.
\]
The optimal consumption allocation satisfies the following condition

\[
\frac{C_{a,t}(j) - \tilde{C}_a}{C_{n,t}(j)} = \frac{\omega_a}{\omega_n} \left( \frac{P_{a,t}}{P_{n,t}} \right)^{-\gamma}.
\]

(8)

In turn, household j’s total demand of good \( s \in \{a,n\} \) is given by (derivations in the Appendix A.4)

\[
C_{n,t}(j) = \omega_n \left( \frac{P_{n,t}}{P_t} \right)^{-\gamma} C_t(j)
\]

(9)

and

\[
C_{a,t}(j) = \tilde{C}_a + \omega_a \left( \frac{P_{a,t}}{P_t} \right)^{-\gamma} C_t(j),
\]

(10)

where the aggregate price index is defined as

\[
P_t \equiv \left( \omega_a P_{a,t}^{1-\gamma} + \omega_n P_{n,t}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.
\]

(11)

Using equation (6) we can derive aggregate expenditure at sectoral level as \( \int_0^1 P_{s,t}(i)C_{s,t}(i,j)di = P_{s,t}C_{s,t}(j) \). Using this expression and equations (9)-(11), we can express the budget constraint of household j as

\[
P_tC_t(j) + Q_tB_t(j) = W_t(j)N_t(j) + B_{t-1}(j) + \Pi_t(j) - P_{a,t}\tilde{C}_a,
\]

(12)

where \( P_tC_t(j) \) is household j’s total expenditure excluding the value of the minimum consumption requirement, \( P_{a,t}\tilde{C}_a \).

### 2.2.2 Optimal wage setting

In every period, households reset wages with probability \( (1 - \theta_w) \), as in Erceg et al. (2000). Households set their optimal wage, \( W_t^* \), solving the following problem

\[
\max_{W_t^*} \mathbb{E}_t \left\{ \sum_{k=0}^\infty (\theta_w)^k \left( \frac{C_t^{1-\sigma} N_t^{1+\varphi}}{1 - \sigma - N_t^{1+\varphi}} \right) \right\}
\]
subject to the labor demand given by

\[ N_{t+k|t} = \left( \frac{W^*_t}{W^*_{t+k}} \right)^{-\varepsilon_w} N_{t+k} \]

and the budget constraint (12). \( C_{t+k|t} \) and \( N_{t+k|t} \) indicate consumption and labor supply of a household that last re-optimized wage in period \( t \).

Maximization implies

\[
\sum_{k=0}^{\infty} (\theta_w \beta)^k \mathbb{E}_t \left\{ C_{t+k|t} N_{t+k|t} \left( \frac{W^*_t}{P^*_t} - \mu_w MRS_{t+k|t} \right) \right\} = 0, \tag{13}
\]

where \( \mu_w \equiv \varepsilon_w / (\varepsilon_w - 1) \) is the desired wage markup and the marginal rate of substitution is defined as

\[ MRS_{t+k|t} \equiv -U_n(C_{t+k|t}, N_{t+k|t})/U_c(C_{t+k|t}, N_{t+k|t}) = C_{t+k|t} \frac{\sigma}{N_{t+k|t}}. \]

When wages are flexible (\( \theta_w = 0 \)), the real wage is given by the desired mark up over the marginal rate of substitution

\[ \frac{W^*_t}{P^*_t} = \frac{W_t}{P_t} = \mu_w MRS_t = \mu_w C_{t+k|t} N_{t+k|t}. \]

### 2.2.3 Intertemporal problem

Intertemporal optimization implies the following *Euler* equation

\[
Q_t = \beta \mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}, \tag{14}
\]

where \( Q_t \) is the stochastic discount factor. As in Erceg *et al.* (2000), we assume that households have access to complete assets markets for consumption, which implies identical consumption across households in very period \( (C_t(j) = C_t) \).

### 2.3 Price and wage dynamics

Since all firms and households that re-optimize choose the same price and wage, price dynamics in sector \( s \) is given by

\[
P_{s,t}^{1-\varepsilon_p} = \theta_s P_{s,t-1}^{1-\varepsilon_p} + (1 - \theta_s) P_{s,t}^{s1-\varepsilon_p} , \tag{15}
\]

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\(^4\)We derive the labor demand constraint in Appendix A.2.
and wage dynamics by
\[ W_t^{1-\varepsilon_w} = \theta_w W_{t-1}^{1-\varepsilon_w} + (1 - \theta_w) W_t^{1-\varepsilon_w}. \] (16)

Price inflation in sector \( s \) is defined as \( \Pi_{s,t} \equiv P_{s,t} / P_{s,t-1} \), while wage inflation is defined as \( \Pi_{w,t} \equiv W_t / W_{t-1} \).

### 2.4 Market clearing and aggregation

Goods market clearing implies
\[ Y_{s,t}(i) = \int_0^1 C_{s,t}(i,j) dj, \] (17)
that is, output of firm \( i \) in sector \( s \) satisfies demand of all households for that product variety.

Aggregate output in sector \( s \) is defined as \( Y_{s,t} \equiv (\int_0^1 Y_{s,t}(i) \frac{\varepsilon_p - 1}{\varepsilon_p} di)^\frac{\varepsilon_p}{\varepsilon_p - 1} \), which can be interpreted as a final good producer in sector \( s \) using as inputs the output of intermediate firms in the same sector. Combining this definition with equations (6), (17) and the definition of sectoral prices, \( P_{s,t} \), we obtain
\[ Y_{s,t} = \int_0^1 C_{s,t}(j) dj = C_{s,t}, \] (18)
where the last equality follows from the complete asset markets assumption.

Aggregate employment is given by
\[ N_t = N_{a,t} + N_{n,t}, \] (19)
while, aggregate employment at sectoral level is in turn given by
\[ N_{s,t} \equiv \int_0^1 \int_0^1 N_{s,t}(i,j) dj di. \] (20)

The sectoral production function is given by (derivations in Appendix A.5)
\[ Y_{s,t} = A_{s,t} N_{s,t}^{1-\alpha_s} (\Delta_{p,t}^{s} \Delta_{w,t}^{s})^{-(1-\alpha_s)}, \] (21)
where the price dispersion is defined as \( \Delta_{p,t}^{s} \equiv \int_0^1 (P_{s,t}(i)/P_{s,t})^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \) and wage dispersion as \( \Delta_{w,t}^{s} \equiv \int_0^1 (W_t(j)/W_t)^{-\varepsilon_w} dj \). As it is standard in the literature, the variables \( \Delta_{w,t}^{s} \) and \( \Delta_{p,t}^{s} \) are sources of inefficient output and employment variation, arising from inefficient price and wage dispersion due to nominal frictions present in the economy.
Finally, aggregate nominal output can be defined as \( P_tY_t \equiv P_{a,t}Y_{a,t} + P_{n,t}Y_{n,t} \).

### 2.5 Central Bank

The central bank sets the nominal interest rate following a simple and implementable Taylor rule, as in Schmitt-Grohe and Uribe (2006), given by

\[
R_t = \frac{1}{\beta} \left( \frac{\Pi^*_t}{\Pi} \right)^{\phi_\pi},
\tag{22}
\]

where \( R_t = Q_t^{-1} \) is the nominal interest rate and \( \phi_\pi > 1 \) is the weight assigned to the inflation target with respect to steady state, \( \Pi^*_t/\Pi \).

The measure of inflation that the central bank targets is defined as \( \Pi^*_t \equiv \Pi^\Omega_{a,t} \Pi^{1-\Omega}_{n,t} \), where \( \Omega \) is the weight assigned to agricultural inflation, \( \Pi_{a,t} \). When \( \Omega = 0 \) the target is interpreted as core inflation, a measure excluding flexible agricultural prices. When the weight is steady state agricultural consumption share, \( \Omega = P_aC_a/PY \), the target can be interpreted as headline inflation, the broadest measure available. In the quantitative exercise below, we compute the optimal level of \( \Omega \) that maximizes welfare. Note that there are three different measures of inflation: headline, core and the optimal.

### 2.6 Shocks

The model includes temporary shocks to agricultural and non-agricultural productivity. The exogenous process for sector \( s \in \{a,n\} \) is given by

\[
A_{s,t} = A_{s,0}e^{a_{s,t}},
\]

where the shock, \( a_{s,t} \), is given by

\[
a_{s,t} = \rho_s a_{s,t-1} + \nu_{s,t}.
\]

The variables \( \nu_{a,t} \) and \( \nu_{n,t} \) are IID shocks with zero mean and standard deviation \( \sigma_{\nu a} \) and \( \sigma_{\nu n} \). The parameter \( \rho_s \) indicates shock persistence. \( A_{a,0} \) and \( A_{n,0} \) are steady state productivity levels in agriculture and non-agriculture, respectively.
2.7 Welfare

To evaluate the optimal weight of agricultural inflation in the Taylor rule we introduce a welfare function. Welfare of household $j$ is defined as

$$W_t(j) \equiv \mathbb{E}_t \left\{ \sum_{k=0}^{\infty} \beta^k \left( \frac{C_{t+k}(j)^{1-\sigma}}{1-\sigma} - \frac{N_{t+k}(j)^{1+\varphi}}{1+\varphi} \right) \right\}.$$

Aggregating for all households and assuming complete markets ($C_t(j) = C_t$), it can be expressed recursively as

$$\int_0^1 W_t(j) dj = C_1 - \sigma - \int_0^1 \frac{N_t(j)^{1+\varphi}}{1+\varphi} dj + \beta \mathbb{E}_t \int_0^1 W_{t+1}(j) dj.$$

Aggregating equation (3) for all firms $i$ and using (19), we have that $N_t(j) = (W_t(j)/W_t)^{-\varepsilon_w} N_t$. Thus, aggregate welfare, $\mathbb{W}_t$, is given by

$$\mathbb{W}_t = C_t^{1-\sigma} - \frac{N_t^{1+\varphi} \Delta_{w,t}^*}{1+\varphi} + \beta \mathbb{E}_t \{\mathbb{W}_{t+1}\},$$

(23)

where $\mathbb{W}_t \equiv \int_0^1 W_t(j) dj$ and $\Delta_{w,t}^* \equiv \int_0^1 (W_t(j)/W_t)^{-\varepsilon_w(1+\varphi)} dj$. Note the similarity between the wage dispersion, $\Delta_{w,t}$, and the term $\Delta_{w,t}^*$. Both of these terms reflect inefficient employment variation arising from inefficient wage dispersion. Therefore, loss due to wage dispersion arises two times in the model. First, as a result of aggregating output at sector level. Second, after aggregating welfare across households.

3 Quantitative exercise

3.1 Parameter values

We set parameters for a baseline scenario and then perform sensitivity analysis. We set $\sigma = 1$ (log utility) and $\varphi = 1$, which are common in the literature. The elasticity of substitution is set to $\gamma = 0.3$, following Ngai and Pissarides (2004). We set $\beta = 0.99$ which implies an annual interest rate of 4%. The elasticity of substitution across goods and labor varieties are set to $\varepsilon_p = \varepsilon_w = 6$, as in Blanchard and Gali (2008), implying markups of 1.2 in steady state. We set $\theta_n = 2/3$ and $\theta_w = 3/4$, implying an average price duration of three quarters, as in Blanchard and Gali (2008),
and an average contract duration for wages of four quarters, as in Erceg et al. (2000). We set \( \theta_a \) to zero, so that prices in agriculture are flexible, as argued in Bils and Klenow (2004). For simplicity, we consider linear production technologies \( (\alpha_a = \alpha_n = 0) \).

Using the steady state equations of the model, we calibrate the structural change parameters to match data on employment, relative prices and income for rich and developing countries. For rich countries, we set the preference parameter \( \omega_a \) to 0.02 to match agricultural employment share in the U.S. \( (N_a/N = 2\%) \), and \( A_{a,0} = A_{n,0} = 1 \) to normalize income \( (Y) \) and relative agricultural prices \( (P_a/P) \) to 1. We then set the preference parameter \( \tilde{C}_a \) to 0.02808 and technology parameters \( A_{n,0} = 0.154 \) and \( A_{a,0} = 0.66 \times 0.154 \), to match agricultural the employment share \( (N_a/N = 30\%) \), income \( (15\% \text{ of the U.S.}) \) and relative agricultural prices \( (50\% \text{ higher than U.S.}) \) in a developing country.\(^5\)

We set the response to inflation in the Taylor rule, \( \phi_\pi \), to 1.5. The productivity shock parameters are set according to Anand et al. (2015), that is \( \rho_a = 0.25, \rho_n = 0.9, \sigma_{va} = 0.03 \) and \( \sigma_{vn} = 0.02 \). Shocks in agriculture are assumed less persistent than in non-agriculture, which results from the dependence of the sector on weather, as argued in Anand et al. (2015). In the simulation exercise, \( \Omega \) is computed to maximize welfare. We summarize the parameter values in Table 1.

**3.2 Results**

We solve the model numerically using a second-order approximation to the system of non-linear equations around its steady state. We summarize the model equations, including recursive formulations of equations (5) and (13), price dispersion \( (\Delta_s^{p,t}) \) and wage dispersion \( (\Delta_{w,t}) \) in Appendix A.6. Using the baseline parameter set up for a developing country, we compute the weight of agricultural inflation, \( \Omega \), in the Taylor rule (22) that maximizes welfare (23).

We find that the type of productivity shock hitting the economy is key to determine the optimal weight that the central bank should assign to agricultural inflation. After a shock to agricultural productivity, it is optimal for the central bank to assign full weight to non-agricultural (core) inflation \( (\Omega = 0) \). After a temporary shock to non-agricultural productivity, we find that the opposite is true. That is, the full weight should be allocated to agricultural inflation \( (\Omega = 1) \). These results are summarized in Figures 1 and 2. Furthermore, if both agricultural and non-agricultural productivity shocks occur simultaneously it is still optimal for the central bank to assign a non-zero weight to

\(^5\)According to Alvarez-Cuadrado and Poschke (2011), currently rich countries had higher-than-one relative prices of agricultural goods (in units of manufacturing goods) in an early stage of development.
agricultural inflation. This exercise is summarized in Figure 3. To understand why this is the case we take a look at the impulse responses.

After an agricultural productivity shock, it is optimal to respond only to inflation in the sticky price sector. We can infer the reasoning by looking at the impulse responses in Figure 4. After a positive agricultural productivity shock, there is an immediate and sharp decline in agricultural inflation. This is followed by a period of positive agricultural inflation, resulting from the temporary productivity shock fading away. Headline inflation increases as well, as 35% of this aggregate index corresponds to agricultural inflation. If the central bank responds to headline inflation (the continuous line) by rising the interest rate, it destabilizes the economy. That is, we observe higher volatility in wage inflation, price inflation in the non-agricultural sector and output gap. The central bank stabilizes the economy by targeting core inflation (dashed line).

After a non-agricultural productivity shock, it is desirable to assign non-zero weight in the Taylor rule to agricultural inflation. In fact, the optimal weight is the maximum allowed ($\Omega = 1$). The reason is that the evolution of wage inflation follows closely that of agricultural inflation. This can be observed in the impulse responses in Figure 5. After a productivity shock in non-agriculture, both wages and non-agricultural prices are affected. It is not possible for the central bank to stabilize both variables if it reacts to non-agricultural inflation only (dashed line). However, the central bank can contain wage inflation by containing agricultural inflation (continuous line), since there is a link between these variables.

To clarify this result, we use the optimal price equation (5). We consider the case where only non-agricultural productivity shocks are present and technologies are linear. Then, flexible agricultural prices are given by

$$P_{a,t} = \mu_p \frac{W_t}{A_{a,0}},$$

while non-agricultural sticky prices are given by

$$P^*_n, t = \mu_p \frac{\sum_{k=0}^{\infty} (\theta_n)^k E_t \{ Q_{t,t+k} Y_n,t+k | t \} W_{t+k}}{\sum_{k=0}^{\infty} (\theta_n)^k E_t \{ Q_{t,t+k} Y_n,t+k | t \}}.$$

Note that wages in both sectors are equal and proportional to agricultural prices (since $A_{a,0}$ and $\mu_p$ are constant parameters). Therefore, if the central bank targets agricultural price inflation it can contain wage inflation. Moreover, it indirectly contains non-agricultural price inflation, as wages are part of marginal costs in this sector. Conversely, if the central bank targets core inflation (that is, sticky non-agricultural prices), it can not simultaneously contain wage inflation. Since wage
inflation and non-agricultural price inflation are the main sources of welfare loss, the central bank can minimize losses by targeting agricultural inflation.

We now examine an economy with the sectoral composition of a developed country. Results show that the optimal weight of agricultural inflation remains unchanged in a rich economy with lower agricultural employment and consumption share. Figures 6 to 8 show that the optimal weights for an advanced economy are almost the same as in a developing economy. To understand these results, we compare the impulse responses between rich and developing economies after a non-agricultural productivity shock. Figure 9 shows impulse responses when the central bank targets core inflation, while Figure 10 when it assigns full weight to agricultural inflation. We consider two developing countries, one with \( N_a/N = 30\% \) the other with \( N_a/N = 50\% \), in addition to a rich country with \( N_a/N = 2\% \). We find that the response of wage inflation and price inflation (in both sectors) is amplified in a rich country, after a productivity shock in non-agriculture. Since income and price elasticity of agricultural goods consumption is lower in developing countries, employment in this sector can not fluctuate as much as in rich countries. This explains the higher wage fluctuation in developed countries. However, this does not change the fact that it is optimal to set \( \Omega = 1 \) in order to reduce wage and price inflation, in both developing and rich countries.

The sectoral composition affects the welfare gain that central banks can attain by targeting the optimal measure of inflation. In panels (a) and (b) of Figure 11, we observe the optimal weight for agricultural inflation after non-agricultural productivity shocks. As discussed above, it is optimal to set \( \Omega = 1 \) in both rich and poor countries. In a developing country, targeting headline inflation (vertical dashed line) improves welfare substantially with respect to core inflation. This is not the case in rich countries where the weight of agriculture in headline inflation is small. However, notice that rich countries experience the largest gain in welfare by setting \( \Omega = 1 \), with respect to headline inflation targeting. In panels (c) and (d) of the same figure, we observe the opposite result after an agricultural productivity shock. In this case, it is the developing country that observes larger welfare gains after setting core inflation as the target (\( \Omega = 0 \)), with respect to headline inflation targeting (vertical dashed line). The later result was already discussed in Portillo et al. (2016), the former however is new, to the best of our knowledge.

3.3 Role of sticky wages

In the previous subsection, we saw that setting a full weight to agricultural inflation is optimal for the central bank, after non-agricultural productivity shocks. This is a consequence of the link
between agricultural prices and wages. The result is robust to changes in sectoral composition. It is however sensitive to how sticky wages are. In fact, if we reduce the parameter controlling the degree of wage stickiness ($\theta_w$), we find that the optimal weight of agricultural inflation is reduced to a value close to zero and generates only small welfare gains compared to core inflation targeting.

To explore the link between agricultural and wage inflation further, we introduce the following modified Taylor rule

$$R_t = \frac{1}{\beta} \left( \frac{\Pi^*_t}{\Pi} \right)^{\phi_{\pi}} \left( \frac{\Pi_{w,t}}{\Pi_{w}} \right)^{\phi_{w}},$$

where $\Pi_{w,t}/\Pi_{w}$ indicates wage inflation in deviations from steady state and $\phi_{w}$ the weight assigned by the central bank to wage inflation. As before, the measure of inflation is given by $\Pi^*_t \equiv \Pi^*_{a,t} \Pi^*_{n,t}$.

We find that the optimal weight for agricultural prices declines, in both rich and poor countries, using the modified Taylor rule. In fact, for $\phi_{w} \geq 3$, the optimal $\Omega$ is lower than 0.1 and welfare gains from setting $\Omega > 0$ are largely reduced. This supports our previous claim that targeting agricultural prices in a simple Taylor rule is a proxy for targeting wage inflation.

### 3.4 Sensitivity analysis

We perform a series of tests to check how sensitive is the optimal weight of agricultural inflation to changes in the baseline parameters. First, we test changes to parameters that determine the steady state of the model. We find that changes in the elasticity of substitution across agricultural and non-agricultural goods, $\gamma$, in the elasticity of substitution across goods and labor varieties, $\varepsilon_p$ and $\varepsilon_w$, in the Frisch elasticity of labor supply, $1/\varphi$, and considering decreasing return to scale technologies $\alpha_a, \alpha_n \in \{0, 1\}$ have no effect on the optimal $\Omega$.

Second, we test changes to the parameters underlying productivity shocks. As expected, these changes affect the optimal value of $\Omega$. We set values for relative standard deviation $\sigma_a/\sigma_n$ from 0.01/0.05 to 0.05/0.01, and values for relative autocorrelation $\rho_a/\rho_n$ from 0.1/0.9 to 0.7/0.3 and compute the optimal $\Omega$. Results are summarized in Figure 12. They indicate that, as the relative standard deviation and the relative autocorrelation of agricultural productivity shocks increase, the optimal weight that the central bank should assign to agricultural inflation approaches zero. That is, the central bank should assign higher weight to agricultural inflation when shocks to this sector are less persistent and of lower magnitude than in non-agriculture.

Finally, we simulate the model with flexible wages ($\theta_w = 0$), flexible agricultural prices, sticky non-agricultural prices and productivity shocks in both sectors with the same persistence and mag-
nitude \(\rho_a/\rho_n = \sigma_a/\sigma_n = 1\). We find that the optimal \(\Omega\) decreases to 0.1, a value much lower than the baseline scenario including both shocks (Figure 3). In this case, welfare loss due to wage dispersion is reduced. Therefore, the central bank does not need to contain wage inflation through agricultural inflation.

4 Welfare loss function

We derive a welfare loss function to analyze the sources of welfare loss. Since our objective is to understand why should the central bank assign non-zero weight to agricultural inflation, we simplify the model by setting \(\gamma\) to 1 (Cobb-Douglas case), and drop the productivity shock in the agricultural sector. In Section 3.4, we showed that these simplifications have no effect on the optimal \(\Omega\). To derive the welfare loss function, we take a second order approximation to the utility function and use the optimality conditions from households and firms and the market clearing conditions. We obtain the following function (derivations in Appendix A.7)

\[
W_0 = -\frac{1}{2} \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left( \psi_{ya} \tilde{y}_{a,t}^2 + \psi_{yn} \tilde{y}_{n,t}^2 + \psi_{\pi a} \pi_{a,t}^2 + \psi_{\pi n} \pi_{n,t}^2 + \psi_w \pi_w^2 + \psi_p \hat{y}_{a,t} \hat{y}_{n,t} \right) \right\}.
\]

(24)

Welfare losses can be decomposed in variance of output gap in agriculture \(\tilde{y}_{a,t}^2\), output gap in non-agriculture \(\tilde{y}_{n,t}^2\), agricultural inflation \(\pi_{a,t}^2\), non-agricultural inflation \(\pi_{n,t}^2\), wage inflation \(\pi_w^2\) and the cross product of output in both sectors. We interpret (24) as welfare losses in units of total real expenditure, \(Y\), that arise after a non-agricultural productivity shock. The weights of each component are given by \(\psi_{ya}, \psi_{yn}, \psi_{\pi a}, \psi_{\pi n}, \psi_w\) and \(\psi_p\). The weights are determined by price and wage frictions parameters, sectoral composition in steady state, in addition to preference and technology parameters. Welfare losses are increasing in price and wage stickiness.

To compute welfare losses (24), we use a log-linear approximation to the non-linear system of equations of the model. Results are summarized in Table 2. When the country targets core inflation, we find that wage inflation and non-agricultural price inflation generate the bulk of welfare loss. If the central bank sets the full weight to agricultural inflation, it reduces welfare losses generated by these two components, and thus total welfare loss. This is the case in both rich and developing countries.
5 Concluding remarks

In this chapter, we study what is the optimal measure of inflation that a central bank should target, given the sectoral composition of the economy. The optimal measure of inflation is defined as the weights assigned to inflation in each sector, such that the central bank minimizes welfare losses that arise due to nominal frictions in the economy.

We consider a two sector model including features from the structural change and new Keynesian literature. The steady state of the model is calibrated to replicate the sectoral composition of a developing and a rich economy. We assume flexible agricultural prices and sticky non-agricultural prices and wages. In developing countries, where the agricultural consumption share is large, the aggregate price index includes a large fraction of flexible agricultural prices.

The model shows that the type of shock hitting the economy is key to determine the optimal weight of agricultural inflation. When only agricultural productivity shocks are present, it is optimal to target core inflation, that is non-agricultural sticky prices. This result holds independently of how sticky wages are. On the other hand, when only non-agricultural productivity shocks are present and wages are sticky, it is optimal to assign full weight to flexible agricultural prices. If the central bank contains agricultural inflation it can contain wage inflation, since agricultural prices are proportional to marginal costs and, therefore, wages. Since wages are equal in both sectors, by containing wage inflation, the central bank indirectly contains non-agricultural prices through marginal costs. Therefore, the central bank can reduce wage inflation and non-agricultural price inflation, the two main sources of welfare loss, using agricultural inflation as a proxy of wage inflation.

When the sectoral composition changes, the optimal weight remains unchanged. The reason is that, as long as wages are sticky and equal across sectors, the central bank can always reduce welfare loss arising from inefficient wage dispersion by containing agricultural prices. In addition, we find that changes in sectoral composition due to structural change have important consequences for welfare, when the central bank targets the optimal measure of inflation. Developing countries experience larger welfare gain by targeting core inflation after agricultural productivity shocks. Rich countries, on the other hand, experience larger welfare gain by fully targeting agricultural inflation after non-agricultural productivity shocks, which is equivalent to targeting wage inflation.

Finally, we acknowledge that the results in this chapter depend on our assumptions of equal degree of wage stickiness across sectors, free labor mobility across sectors, and the shock parameters. Future work will include investigating the effects of different degree of wage stickiness across
sectors, including imperfect labor mobility, and providing further empirical support to the shock structure in developing countries.
References


Appendix A. Derivations

A.1. Firm’s cost minimization

We derive labor demand equation (3). A firm $i$ in sector $s$ solves the following minimization problem

$$
\min_{N_{s,t}(i,j)} \int_0^1 W_t(j) N_{s,t}(i,j) dj - \lambda \left[ A_{s,t} N_{s,t}(i) \right]^{1-\alpha_s} - Y_{s,t}(i)
$$

subject to (2). First order conditions for labor variety $j$ imply

$$
W_t(j) = \lambda (1 - \alpha_s) A_{s,t} \left( \int_0^1 N_{s,t}(i,j) \frac{\varepsilon_w - 1}{\varepsilon_w} dj \right)^{-\frac{\varepsilon_w (1 - \alpha_s)}{\varepsilon_w - 1}} N_{s,t}(i,j)^{-\frac{1}{\varepsilon_w}},
$$

combining optimality conditions for labor variety $j$ and $j'$ in sector $s$, we obtain

$$
N_{s,t}(i,j) = N_{s,t}(i,j') \left( \frac{W_t(j)}{W_t(j')} \right)^{-\varepsilon_w},
$$

plugging this expression in (2), we obtain equation (3) as

$$
N_{s,t}(i,j) = \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_{s,t}(i)
$$

where the wage index $W_t$ is defined by equation (4).
A.2 Demand constraint in the optimal price and wage setting problem

The demand constraint for sector $s$ in the optimal price setting problem results from aggregating demand equation (6) for all households $j$ as

$$\int_{0}^{1} C_{s,t}(i,j) dj = \left( \frac{P_{s,t}(i)}{P_{s,t}} \right)^{-\varepsilon_p} \int_{0}^{1} C_{s,t}(j) dj.$$

Using marketing clearing conditions (17) and (18) we obtain the demand constraint as

$$Y_{s,t}(i) = \left( \frac{P_{s,t}(i)}{P_{s,t}} \right)^{-\varepsilon_p} C_{s,t}.$$

The labor demand constraint in the optimal wage setting problem results from integrating equation (3) with respect to $i$ as

$$\int_{0}^{1} N_{s,t}(i,j) di = \left( \frac{W_{t}(j)}{W_{t}} \right)^{-\varepsilon_w} \int_{0}^{1} N_{s,t}(i) di,$$

where $\int_{0}^{1} N_{s,t}(i,j) di = N_{s,t}(j)$ is employment of labor variety $j$ by all firms in sector $s$ and $\int_{0}^{1} N_{s,t}(i) di = N_{s,t}$ is aggregate employment at sectoral level. Therefore $N_{s,t}(j) = (W_{t}(j)/W_{t})^{-\varepsilon_w} N_{s,t}$. Adding up this expression for both sectors, we obtain demand for labor variety $j$ as

$$N_{t}(j) = N_{a,t}(j) + N_{n,t}(j)$$

$$= \left( \frac{W_{t}(j)}{W_{t}} \right)^{-\varepsilon_w} (N_{a,t} + N_{n,t})$$

$$= \left( \frac{W_{t}(j)}{W_{t}} \right)^{-\varepsilon_w} N_{t}.$$
A.3 Optimal consumption allocation of household $j$

We derive household demand equations (6) for sector $s$. Household $j$ maximizes $C_t(j)$ conditional on expenditure level $E_t$ in every period $t$ as

$$
\max_{\{C_{a,t}(i,j), C_{n,t}(i,j)\}_{i \in (0,1)}} \left\{ \frac{1}{\omega_a} \left[ \left( \int_0^1 C_{a,t}(i,j)^{\gamma p-1} \frac{\varepsilon p-1}{\varepsilon p} \, di \right)^{\gamma-1} - \tilde{C}_a \right]^{\gamma-\frac{1}{\gamma}} + \frac{1}{\omega_n} \left[ \left( \int_0^1 C_{n,t}(i,j)^{\gamma p-1} \frac{\varepsilon p-1}{\varepsilon p} \, di \right)^{\gamma-1} - \tilde{C}_n \right]^{\gamma-\frac{1}{\gamma}} - \lambda \left( \int_0^1 P_{a,t}(i)C_{a,t}(i,j)di + \int_0^1 P_{n,t}(i)C_{n,t}(i,j)di - E_t(j) \right) \right\}.
$$

First order conditions for good $i$ in sectors $a$ and $n$ imply

$$
C_t(j)^{\gamma} \omega_a \left[ C_{a,t}(j) - \tilde{C}_a \right]^{1-\gamma} C_{a,t}(j)^{1-\gamma} C_{a,t}(i,j)^{\gamma-1} = \lambda P_{a,t}(i) \tag{25}
$$

and

$$
C_t(j)^{\gamma} \omega_n \left[ C_{n,t}(j) - \tilde{C}_n \right]^{1-\gamma} C_{n,t}(j)^{1-\gamma} C_{n,t}(i,j)^{\gamma-1} = \lambda P_{n,t}(i). \tag{26}
$$

Combining the first order conditions for good $i$ and $i'$ in sector $s$, we obtain

$$
\frac{C_{s,t}(i,j)}{C_{s,t}(i',j)} = \left( \frac{P_{s,t}(i)}{P_{s,t}(i')} \right)^{-\varepsilon p},
$$

combining this last equation with the definition of $C_{s,t}(j)$, we obtain equation (6) as

$$
C_{s,t}(i,j) = C_{s,t}(j)\left( \frac{P_{s,t}(i)}{P_{s,t}} \right)^{-\varepsilon p}
$$

where the relative price $P_{s,t}$ is given by equation (7). Finally, combining (6), (25) and (26) we obtain equation (8).
A.4 Sectoral demand

We derive household demand equations (9) and (10). First, we define total expenditure, $E_t(j) \equiv P_{a,t}C_{a,t}(j) + P_{n,t}C_{n,t}(j)$. Plugging equation (8) into this definition, we obtain

$$C_{n,t}(j) = \omega_n\left(\frac{P_{n,t}}{P_t}\right)^{1-\gamma} \frac{E_t(j) - P_{a,t}\tilde{C}_a}{P_{n,t}}$$ (27)

and

$$C_{a,t}(j) - \tilde{C}_a = \omega_a\left(\frac{P_{a,t}}{P_t}\right)^{1-\gamma} \frac{E_t(j) - P_{a,t}\tilde{C}_a}{P_{a,t}}$$ (28)

where the price index $P_t$ is defined in equation (11). Plugging (27) and (28) into the definition of $C_t(j)$ we obtain

$$P_tC_t(j) = E_t(j) - P_{a,t}\tilde{C}_a.$$ (29)

Combining equations (27) and (28) with (29), we obtain household demand equations (9) and (10).
A.5 Sectoral production function

The sectoral production function can be obtained combining equations (3) and (20) as

\[
N_{s,t} \equiv \int_0^1 \int_0^1 N_{s,t}(i,j) dj di = \int_0^1 \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_{s,t}(i) dj di = \Delta_{w,t} \int_0^1 N_{s,t}(i) di
\]

where \( \Delta_{w,t} \equiv \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj \). Using equations (1), (6), (17) and (18), we obtain equation (21) as

\[
N_{s,t} = \Delta_{w,t} \frac{1}{A_{s,t}^{1-\alpha_s}} \int_0^1 Y_{s,t}(i) i^{1-\alpha_s} di
\]

\[
= \Delta_{w,t} \frac{1}{A_{s,t}^{1-\alpha_s}} \int_0^1 \left( \int_0^1 C_{s,t}(i,j) dj \right) i^{1-\alpha_s} di
\]

\[
= \Delta_{p,t} \Delta_{w,t} \left( \frac{Y_{s,t}}{A_{s,t}} \right)^{1-\alpha_s},
\]

where \( \Delta_{p,t} \equiv \int_0^1 \left( \frac{P_{s,t}(i)}{P_{s,t}} \right)^{-\epsilon_p} i^{1-\alpha_s} di \). Rearranging the last equality we obtain expression (21).
A.6 System of non-linear equations

The system of nonlinear equations of the model is given by

Demand equations:

\[ Q_t = \beta \mathbb{E}_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} \]

\[ C_{n,t} = \omega_n (P_{n,t}/P_t)^{-\gamma} C_t \]

\[ C_{a,t} = \tilde{C}_a + \omega_a (P_{a,t}/P_t)^{-\gamma} C_t \]

Price equations:

\[ 1 = \omega_a (P_{a,t}/P_t)^{1-\gamma} + \omega_n (P_{n,t}/P_t)^{1-\gamma} \]

\[ 1 = \theta_a \Pi_{a,t}^{\varepsilon p - 1} + (1 - \theta_a) \left( \frac{P_{a,t}^*/P_t}{P_{a,t}/P_t} \right)^{1-\varepsilon p} \]

\[ 1 = \theta_n \Pi_{n,t}^{\varepsilon p - 1} + (1 - \theta_n) \left( \frac{P_{n,t}^*/P_t}{P_{n,t}/P_t} \right)^{1-\varepsilon p} \]

\[ 1 = \theta_w \Pi_{w,t}^{\varepsilon w - 1} + (1 - \theta_w) \left( \frac{W_{t}^*/P_t}{W_{t}/P_t} \right)^{1-\varepsilon w} \]

Market clearing:

\[ Y_{a,t} = C_{a,t} \]

\[ Y_{n,t} = C_{n,t} \]

\[ N_t = N_{a,t} + N_{n,t} \]

\[ N_{a,t} = \Delta_{w,t} \Delta_{p,t}^n \left( \frac{Y_{a,t}}{A_{a,t}} \right)^{1-\alpha_a} \]

\[ N_{n,t} = \Delta_{w,t} \Delta_{p,t}^n \left( \frac{Y_{n,t}}{A_{n,t}} \right)^{1-\alpha_n} \]

Price setting:

\[ \kappa_{1,t}^{a} = Y_{a,t} \left( \frac{P_{a,t}^*/P_t}{P_{a,t}/P_t} \right)^{-\varepsilon_p} + \theta_a \mathbb{E}_t Q_t \left( \frac{P_{a,t}^*/P_t}{P_{a,t+1}/P_{t+1}} \frac{1}{\Pi_{t+1}} \right)^{-\varepsilon_p} \kappa_{1,t+1}^{a} \]
\[ \kappa_{2,t}^a = Y_{a,t}MC_{a,t} \left( \frac{P_{a,t}^*/P_t}{P_{a,t}/P_t} \right)^{\frac{1-\alpha_a+\varepsilon_p}{1-\alpha_a}} + \theta_a \mathbb{E}_t Q_t \left( \frac{P_{a,t}^*/P_t}{P_{a,t}/P_t} \right)^{\frac{1}{1-\alpha_a}} \kappa_{2,t+1}^a \]

\[ \kappa_{1,t}^a = \mu_p \kappa_{2,t}^a \]

\[ \kappa_{1,t}^n = Y_{n,t} \left( \frac{P_{n,t}^*/P_t}{P_{n,t}/P_t} \right)^{-\varepsilon_p} + \theta_n \mathbb{E}_t Q_t \left( \frac{P_{n,t}^*/P_t}{P_{n,t}/P_t} \right)^{-\varepsilon_p} \kappa_{1,t+1}^n \]

\[ \kappa_{2,t}^n = Y_{n,t}MC_{n,t} \left( \frac{P_{n,t}^*/P_t}{P_{n,t}/P_t} \right)^{\frac{1-\alpha_n+\varepsilon_p}{1-\alpha_n}} + \theta_n \mathbb{E}_t Q_t \left( \frac{P_{n,t}^*/P_t}{P_{n,t}/P_t} \right)^{\frac{1}{1-\alpha_n}} \kappa_{2,t+1}^n \]

\[ \kappa_{1,t}^n = \mu_p \kappa_{2,t}^n \]

Wage setting:

\[ \kappa_{1,t}^w = C_t^{-\sigma} N_t \left( \frac{W_t^*/P_t}{W_t/P_t} \right)^{-\varepsilon_w} W_t^*/P_t + (\theta_w \beta) \mathbb{E}_t \left( \frac{W_t^*/P_t}{W_{t+1}^*/P_{t+1}} \right)^{\frac{1-\varepsilon_w}{1-\varepsilon_w}} \kappa_{1,t+1}^w \]

\[ \kappa_{2,t}^w = C_t^{-\sigma} N_t MRS_t \left( \frac{W_t^*/P_t}{W_t/P_t} \right)^{-\varepsilon_w (1+\varphi)} \]

\[ (\theta_w \beta) \mathbb{E}_t \left( \frac{W_t^*/P_t}{W_{t+1}^*/P_{t+1}} \right)^{-\varepsilon_w (1+\varphi)} \kappa_{2,t+1}^w \]

\[ \kappa_{1,t}^w = \mu_w \kappa_{2,t}^w \]

Price and wage dispersion:

\[ \Delta_{p,t}^a = (1-\theta_a) \left( \frac{P_{a,t}^*/P_t}{P_{a,t}/P_t} \right)^{\frac{-\varepsilon_p}{1-\alpha_a}} \theta_a \Pi_{a,t}^{\frac{\varepsilon_p}{1-\alpha_a}} \Delta_{p,t-1}^a \]
\[
\Delta_{p,t}^n = (1 - \theta_n) \left( \frac{P_{a,t}^*/P_t}{P_{a,t-1}^*/P_{t-1}} \right)^{\varepsilon_p} + \theta_n \Pi_{n,t}^{\varepsilon_{\Pi}} \Delta_{p,t-1}^n
\]
\[
\Delta_{w,t} = (1 - \theta_w) \left( \frac{W_{t}^*/P_t}{W_{t-1}^*/P_{t-1}} \right)^{-\varepsilon_w} + \theta_w \Pi_{w,t}^{\varepsilon_{\Pi}} \Delta_{w,t-1}
\]

Sectoral inflation, wage inflation, marginal costs, and marginal rate of substitution:

\[
\Pi_{a,t} = \frac{P_{a,t}/P_t}{P_{a,t-1}/P_{t-1}} \Pi_t
\]
\[
\Pi_{n,t} = \frac{P_{n,t}/P_t}{P_{n,t-1}/P_{t-1}} \Pi_t
\]
\[
\Pi_{w,t} = \frac{W_t/P_t}{W_{t-1}/P_{t-1}} \Pi_t
\]
\[
MC_{a,t} = \frac{1}{1 - \alpha_a} \frac{W_t/P_t}{P_{a,t}/P_t} \frac{N_{a,t}}{Y_{a,t}}
\]
\[
MC_{n,t} = \frac{1}{1 - \alpha_n} \frac{W_t/P_{n,t}}{P_{n,t}/P_t} \frac{N_{n,t}}{Y_{n,t}}
\]
\[
MRS_t = C^\sigma_t N^\varphi_t
\]

Taylor rule:
\[
\frac{1}{Q_t} = \frac{1}{\beta} (\Pi_t^* / \Pi)^{\phi_{\pi}}
\]
\[
\Pi_t^* = \Pi_{a,t}^{\Omega} \Pi_{n,t}^{1-\Omega}
\]

Aggregate output:
\[
Y_t = (P_{a,t}/P_t) Y_{a,t} + (P_{n,t}/P_t) Y_{n,t}
\]

Shocks:
\[
\ln A_{a,t} = \ln A_{a,0} + a_{a,t}
\]
\[
\ln A_{n,t} = \ln A_{n,0} + a_{n,t}
\]
\[
a_{a,t} = \rho_a a_{a,t-1} + \nu_{a,t}
\]
\[
a_{n,t} = \rho_n a_{n,t-1} + \nu_{n,t}
\]
A.7 Welfare loss function

We derive a second order approximation to households’ welfare, following essentially the same procedure described in Gali (2008). We simplify the model and consider \( \sigma = 1 \) (log utility), \( \alpha_a = \alpha_n = 0 \) and \( \gamma = 1 \). Therefore, household \( j \) utility at time \( t \) is given by

\[
U(C_t(j), N_t(j)) = \ln C_t(j) - \frac{N_t(j)^{1-\varphi}}{1-\varphi},
\]

where \( C_t(j) = \omega_a^{-\omega_a}(1-\omega_a)^{-1-\omega_a}(C_{a,t}(j) - \hat{C}_a)^{\omega_a} C_{n,t}(j)^{(1-\omega_a)} \). The term \( \omega_a^{-\omega_a}(1-\omega_a)^{-1-\omega_a} \) is introduced to reduce notation. We aggregate utility across households and take a second order Taylor expansion of the utility, \( U_t \), around its steady state, \( \bar{U}_t \), to obtain the following expression

\[
\int_0^1 (U_t - \bar{U}) \, dj \approx \frac{P_a C_a}{PC} \left( \frac{C_{a,t} - C_a}{C_a} \right) - \frac{1}{2 \omega_a} \left( \frac{P_a C_a}{PC} \right)^2 \left( \frac{C_{a,t} - C_a}{C_a} \right)^2 \\
+ \frac{P_n C_n}{PC} \left( \frac{C_{n,t} - C_n}{C_n} \right) - \frac{1}{2} \frac{P_n C_n}{PC} \left( \frac{C_{n,t} - C_n}{C_n} \right)^2 \\
+ \frac{P_a C_a}{PC} \frac{P_n C_n}{PC} \left( \frac{C_{a,t} - C_a}{C_a} \right) \left( \frac{C_{n,t} - C_n}{C_n} \right) \\
+ U_n N \frac{N_a}{N} \int_0^1 \frac{N_{a,t}(j) - N_a}{N_a} \, dj + U_n N \frac{N_n}{N} \int_0^1 \frac{N_{n,t}(j) - N_n}{N_n} \, dj \\
+ \frac{1}{2} U_{nn} N^2 \int_0^1 \left( \frac{N_t(j) - N}{N} \right)^2 \, dj. \tag{30}
\]

We consider the second order approximation given by

\[
\frac{Z - \bar{Z}}{\bar{Z}} \approx \hat{z}_t + \frac{1}{2} \hat{z}_t^2,
\]

where \( \hat{z}_t \) indicates the log deviation of variable \( Z_t \) from its steady state \( Z \). In addition, we use the market clearing condition \( \hat{s}_{s,t} = \hat{y}_{s,t} \), for \( s \in \{a,n\} \), to express equation (30) in log deviations as

\[
\int_0^1 (U_t - \bar{U}) \, dj \approx \frac{P_a C_a}{PC} \hat{y}_{a,t} + \frac{1}{2} \left( \frac{P_a C_a}{PC} - \frac{1}{\omega_a} \left( \frac{P_a C_a}{PC} \right)^2 \right) \hat{y}_{a,t}^2 \\
+ \frac{P_n C_n}{PC} \hat{y}_{n,t} + \frac{P_a C_a}{PC} \frac{P_n C_n}{PC} (\hat{y}_{a,t} \hat{y}_{n,t}) \\
+ U_n N \int_0^1 \left( \frac{N_a}{N} (\hat{n}_{a,t}(j) + \frac{1}{2} \hat{n}_{a,t}(j)^2) \\
+ \frac{N_n}{N} (\hat{n}_{n,t}(j) + \frac{1}{2} \hat{n}_{n,t}(j)^2) + \frac{1}{2} \hat{n}_t(j)^2 \right) \, dj, \tag{31}
\]

where \( \varphi = \frac{U_{nn} N}{U_n} \). Note that we discard terms of order higher than 2.
First, we focus on the last part of the previous equation that includes employment. Notice that the definition of aggregate employment at the sectoral level, \( N_{s,t} \equiv \int_0^1 N_{s,t}(j) dj \) for \( s \in \{a,n\} \), can be expressed in terms of log deviation from steady state as 
\[
\hat{n}_{s,t} + \frac{1}{2} \hat{n}_{s,t}^2 = \int_0^1 \hat{n}_{s,t}(j) dj + \frac{1}{2} \int_0^1 \hat{n}_{s,t}(j)^2 dj.
\]
Additionally, aggregate total employment, \( N_t \equiv \int_0^1 N_t(j) dj \), can be expressed in terms of log deviation from steady state as
\[
\int_0^1 \hat{n}_t(j)^2 = \int_0^1 (\hat{n}_t(j) - \hat{n}_t + \hat{n}_t)^2 dj
\]
\[
= \int_0^1 (-\epsilon_w (w_t(j) - w_t) + \hat{n}_t)^2 dj
\]
\[
= \hat{n}_t^2 - 2\hat{n}_t \epsilon_w \int_0^1 (w_t(j) - w_t) dj + \epsilon_w^2 \int_0^1 (w_t(j) - w_t)^2 dj
\]
\[
= \hat{n}_t^2 + \epsilon_w^2 \int_0^1 (w_t(j) - w_t)^2 dj
\]
\[
= \hat{n}_t^2 + \epsilon_w^2 var_j \{w_t(j)\},
\]
where we use the labor demand equation expressed in deviation from steady state, \( \hat{n}_t(j) = \hat{n}_t - \epsilon_w (w_t(j) - w_t) \). Following Gali (2008), we consider that \( \int_0^1 (w_t(j) - w_t)^2 dj \approx \int_0^1 (w_t(j) - E_j \{w_t(j)\})^2 dj \equiv var_j \{w_t(j)\} \) holds up to a second order approximation. We discard the term \( 2\hat{n}_t \epsilon_w \int_0^1 (w_t(j) - w_t) dj = 2\hat{n}_t \epsilon_w \frac{\epsilon_w - 1}{2} var_j \{w_t(j)\} \) since it of order higher than two. Replacing in equation (31), we have
\[
\int_0^1 (U_t - U) dj = \frac{P_a C_a}{P C} \hat{y}_{a,t} + \frac{1}{2} \left( \frac{P_a C_a}{P C} - \frac{1}{\omega_a} \left( \frac{P_a C_a}{P C} \right)^2 \right) \hat{y}_{a,t}^2
\]
\[
+ \frac{P_n C_n}{PC} \hat{y}_{n,t} + \frac{P_a C_a P_n C_n}{PC} (\hat{y}_{a,t} \hat{y}_{n,t})
\]
\[
+ U_n N \left( \frac{N_p}{N} \left( \hat{n}_{a,t} + \frac{1}{2} \hat{n}_{a,t}^2 \right) + \frac{N_p}{N} \left( \hat{n}_{n,t} + \frac{1}{2} \hat{n}_{n,t}^2 \right) \right)
\]
\[
+ \frac{\epsilon_w}{2} \left( \hat{n}_t^2 + \epsilon_w^2 var_j \{w_t(j)\} \right).
\]

Using a second order approximation of aggregate output at sectoral level (21) we obtain
\[
\hat{n}_{s,t} = \hat{y}_{s,t} - a_{s,t} + d_{w,t} + d_{s,t},
\]
where, as shown in Gali (2008), \( d_{s,t} \equiv log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{- \epsilon_s} di \approx \epsilon_p \frac{1}{2} var \{p_{s,t}(i)\}, d_{w,t} \equiv log \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{- \epsilon_w} dj \approx \frac{\epsilon_w}{2} var \{w_t(j)\} \). We can combine (34) with the expression for aggregate employment, \( \hat{n}_t = \).
\[
\frac{N_n}{N} \dot{n}_{a,t} + \frac{N_n}{N} \dot{n}_{n,t}, \text{ and use } \gamma_a = \frac{N_n}{N} \text{ and } \gamma_n = \frac{N_n}{N}, \text{ to obtain }
\]

\[
\dot{n}_t^2 = (\gamma_a \dot{n}_{a,t} + \gamma_n \dot{n}_{n,t})^2 \\
= \gamma_a^2 \dot{n}_{a,t}^2 + \gamma_n^2 \dot{n}_{n,t}^2 + 2 \gamma_a \gamma_n \dot{n}_{a,t} \dot{n}_{n,t} \\
= \gamma_a^2 (\dot{y}_{a,t} - a_{a,t})^2 + \gamma_n^2 (\dot{y}_{n,t} - a_{n,t})^2 + 2 \gamma_a \gamma_n \dot{y}_{a,t} \dot{y}_{n,t},
\]

where the terms of order higher than 2 are discarded. Replacing in equation (33), we obtain

\[
\int_0^1 (U_t - U) dj = \frac{P_a C_a}{PC} \dot{y}_{a,t} + \frac{1}{2} \left( \frac{P_a C_a}{PC} - \frac{1}{\omega_a} \left( \frac{P_a C_a}{PC} \right)^2 \right) \dot{n}_a^2 \\
+ \frac{P_n C_n}{PC} \dot{y}_{n,t} + \frac{P_a C_a}{PC} \frac{P_n C_n}{PC} (\dot{y}_{a,t} \dot{y}_{n,t}) \\
+ U_n N \left[ \gamma_a (\dot{y}_{a,t} + \frac{\epsilon_w}{2} \text{var}_j \{ w_t(j) \}) + \frac{\epsilon_p}{2} \text{var}_i \{ p_{a,t}(i) \} \\ + \frac{1}{2} (\dot{y}_{a,t} - a_{a,t})^2 + \frac{1}{2} (\dot{y}_{n,t} - a_{n,t})^2 \\ + \frac{\epsilon_p}{2} \text{var}_i \{ p_{n,t}(i) \} + \frac{1}{2} (\dot{y}_{n,t} - a_{n,t})^2 \\ + \frac{\epsilon_w}{2} \text{var}_j \{ w_t(j) \} \right].
\] (35)

Using the following steady state relations \( U_n N \frac{N_n}{N} = -\frac{P_a C_a}{PC}, U_n N \frac{N_n}{N} = -\frac{P_n C_n}{PC}, \frac{1}{\omega_a} \frac{P_a}{P} = \left( \frac{C_a - \tilde{C}_a}{C} \right)^{-1}, \)

and assuming productivity shocks to non-agriculture only, equation (35) can be expressed as
\[
\int_0^1 (U_t - U) dj = \frac{1}{2} \left( \frac{P_a}{P} \chi_a - \beta_a^{-1} \frac{P_a}{P} \right) \chi_a y^2_{a,t} \frac{1}{2} \left( \frac{P_a}{P} \chi_a + \frac{P_a}{P} \chi_a \gamma_a \varphi \right) y^2_{a,t} \\
- \frac{1}{2} \left( \frac{P_n}{P} \beta_n + \frac{P_n}{P} \beta_n \varphi \gamma_n \right) (\dot{y}_{n,t} - a_{n,t})^2 + \\
\left( \frac{P_a}{P} \chi_a \frac{P_n}{P} \beta_n - \frac{P_a}{P} \chi_a \gamma_a \varphi \gamma_n \right) \dot{y}_{a,t} \dot{y}_{n,t} \\
- \frac{P_n C_a}{P \epsilon_p} \text{var}_i \{p_{a,t}(i)\} - \frac{P_n C_n}{P \epsilon_p} \text{var}_i \{p_{n,t}(i)\} \\
- \frac{\epsilon_w}{2} \left( \frac{P_a C_a + P_n C_n}{P C} + N^{1+\varphi} \varphi \epsilon_w \right) \text{var}_j \{w_t(j)\} \\
= -\frac{1}{2} \left( \beta_a^{-1} \frac{P_a}{P} \chi_a^2 + \frac{P_a}{P} \chi_a \gamma_a \varphi \right) \dot{y}_{a,t}^2 \\
- \frac{1}{2} \left\{ \left( \frac{P_n}{P} \beta_n + \frac{P_n}{P} \beta_n \varphi \gamma_n \right) \dot{y}_{n,t}^2 - 2 \left( \frac{P_n}{P} \beta_n + \frac{P_n}{P} \beta_n \varphi \gamma_n \right) \dot{y}_{n,t} \dot{a}_{n,t} \right\} \\
+ \left( \frac{P_a}{P} \chi_a \frac{P_n}{P} \beta_n - \frac{P_a}{P} \chi_a \gamma_a \varphi \gamma_n \right) \dot{y}_{a,t} \dot{y}_{n,t} - \frac{P_a C_a}{P \epsilon_p} \text{var}_i \{p_{a,t}(i)\} \\
- \frac{P_n C_n}{P \epsilon_p} \text{var}_i \{p_{n,t}(i)\} \\
- \frac{\epsilon_w}{2} \left( \frac{P_a C_a + P_n C_n}{P C} + N^{1+\varphi} \varphi \epsilon_w \right) \text{var}_j \{w_t(j)\} \\
= \frac{\epsilon_w}{2} \left( \frac{P_a C_a + P_n C_n}{P C} + N^{1+\varphi} \varphi \epsilon_w \right) \text{var}_j \{w_t(j)\} \\
= -\frac{1}{2} \left( \beta_a^{-1} \frac{P_a}{P} \chi_a^2 + \frac{P_a}{P} \chi_a \gamma_a \varphi \right) \dot{y}_{a,t}^2 \\
- \frac{1}{2} \left( \frac{P_n}{P} \beta_n + \frac{P_n}{P} \beta_n \varphi \gamma_n \right) \left( \dot{y}_{n,t}^2 - 2 \dot{y}_{n,t} \dot{y}_{n,t} \right) \\
+ \left( \frac{P_a}{P} \chi_a \frac{P_n}{P} \beta_n - \frac{P_a}{P} \chi_a \gamma_a \varphi \gamma_n \right) \dot{y}_{a,t} \dot{y}_{n,t} - \frac{P_a C_a}{P \epsilon_p} \text{var}_i \{p_{a,t}(i)\} \\
- \frac{P_n C_n}{P \epsilon_p} \text{var}_i \{p_{n,t}(i)\} \\
- \frac{\epsilon_w}{2} \left( \frac{P_a C_a + P_n C_n}{P C} + N^{1+\varphi} \varphi \epsilon_w \right) \text{var}_j \{w_t(j)\} \\
\]

where \(\beta_n = \frac{C_n}{C^*}, \chi_a = \frac{C_a}{C^*} \) and \(\beta_a = \frac{C_a - C_a}{C^*} \). \(\dot{y}^n_{a,t}\) indicates natural output in sector \(s\) (that is, output in absence of nominal frictions) in deviation from steady state. Denoting sectoral output in deviation
from natural level as $\bar{y}_{s,t} = \bar{y}_{n,t} - \hat{y}_{s,t}$, we obtain

$$
\int_0^1 (U_t - U) dj = -\frac{1}{2} \left( \beta_a^{-1} P_a \frac{\partial^2}{\partial a^2} + P_a \frac{\partial^2}{\partial a} \gamma_a \varphi \right) \bar{y}_{a,t}^2 - \frac{1}{2} \left( \frac{P_a}{P} \beta_n + \frac{P_n}{P} \beta_n \gamma_n \right) \bar{y}_{n,t}^2
$$

$$
+ \left( \frac{P_a}{P} \frac{\partial}{\partial a} \gamma_a \varphi \right) \bar{y}_{a,t} \bar{y}_{n,t} - \frac{P_a C_n}{PC} \frac{\epsilon_p}{2} \text{var}_i \{p_{a,t}(i)\}
$$

$$
- \frac{P_a C_n}{PC} \frac{\epsilon_p \text{var}_i \{p_{n,t}(i)\}}{2}
$$

$$
- \frac{\epsilon_w}{2} \left( \frac{P_a C_n}{PC} + \frac{P_n C_n}{PC} + N^{1+\varphi} \varphi \epsilon_w \right) \text{var}_j \{w_t(j)\},
$$

where the last equations results from $\hat{y}_{a,t}^2 = 0$ (we consider shocks to non-agriculture only) which implies $\bar{y}_{a,t}^2 = y_{a,t}^2$. The households’ welfare loss can therefore be expressed as

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 (U_t - U) dj =
$$

$$
- \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \beta_a^{-1} P_a \frac{\partial^2}{\partial a^2} + P_a \frac{\partial^2}{\partial a} \gamma_a \varphi \right) \bar{y}_{a,t}^2 + \left( \frac{P_a}{P} \beta_n + \frac{P_n}{P} \beta_n \gamma_n \right) \bar{y}_{n,t}^2 \right. 
$$

$$
- \frac{1}{2} \left( \frac{P_a}{P} \frac{\partial}{\partial a} \gamma_a \varphi \right) \bar{y}_{a,t} \bar{y}_{n,t} + \frac{P_a C_n}{PC} \frac{\epsilon_p \text{var}_i \{p_{a,t}(i)\}}{2} + \frac{P_n C_n}{PC} \frac{\epsilon_p \text{var}_i \{p_{n,t}(i)\}}{2}
$$

$$
+ \frac{\epsilon_w}{2} \left( \frac{P_a C_n}{PC} + \frac{P_n C_n}{PC} + N^{1+\varphi} \varphi \epsilon_w \right) \text{var}_j \{w_t(j)\} \right\},
$$

As shown in Woodford (2003)

$$
\sum_{t=0}^{\infty} \beta^t \text{var}_i \{p_{a,t}(i)\} = \frac{\theta_{pa}}{(1 - \beta \theta_{pa})(1 - \beta_{pa})} \sum_{t=0}^{\infty} \beta^t \pi_{a,t},
$$

$$
\sum_{t=0}^{\infty} \beta^t \text{var}_i \{p_{n,t}(i)\} = \frac{\theta_{pn}}{(1 - \beta \theta_{pn})(1 - \beta_{pn})} \sum_{t=0}^{\infty} \beta^t \pi_{n,t},
$$

$$
\sum_{t=0}^{\infty} \beta^t \text{var}_j \{w_t(j)\} = \frac{\theta_w}{(1 - \beta \theta_w)(1 - \beta_w)} \sum_{t=0}^{\infty} \beta^t \pi_{w,t},
$$

then we have

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 (U_t - U) dj =
$$

$$
- \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( \beta_a^{-1} P_a \frac{\partial^2}{\partial a^2} + P_a \frac{\partial^2}{\partial a} \gamma_a \varphi \right) \bar{y}_{a,t}^2 + \left( \frac{P_a}{P} \beta_n + \frac{P_n}{P} \beta_n \gamma_n \right) \bar{y}_{n,t}^2 \right. 
$$

$$
- \frac{1}{2} \left( \frac{P_a}{P} \frac{\partial}{\partial a} \gamma_a \varphi \right) \bar{y}_{a,t} \bar{y}_{n,t} + \frac{P_a C_n}{PC} \frac{\epsilon_p \theta_{pa}}{(1 - \beta \theta_{pa})(1 - \beta_{pa})} \pi_{a,t} + \frac{P_n C_n}{PC} \frac{\epsilon_p \theta_{pn}}{(1 - \beta \theta_{pn})(1 - \beta_{pn})} \pi_{n,t}
$$

$$
+ \frac{\epsilon_w}{2} \left( \frac{P_a C_n}{PC} + \frac{P_n C_n}{PC} + N^{1+\varphi} \varphi \epsilon_w \right) \pi_{w,t} \right\},
$$

33
Noting that $N^{1+\phi} = \frac{WN}{PC} = \frac{WN_a + WN_n}{PC} = \frac{P_a Y_a + P_n Y_n}{PC} = \frac{P_a C_a + P_n C_n}{PC}$ we obtain

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 (U_t - U) dj$$

$$= -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l}
(\beta_a^{-1} P_a \chi_a + \frac{P_a}{P_n} \chi_a \gamma_a \phi) \tilde{y}_{a,t} + \left( P_a \beta_n + \frac{P_a}{P_n} \beta_n \phi \gamma_n \right) \tilde{y}_{n,t} \\
-2 \left( P_a \beta_n - \frac{P_a}{P_n} \chi_a \gamma_n \phi \right) \tilde{y}_{a,t} \hat{y}_{n,t} \\
+ \frac{P_a C_a}{PC} \frac{\epsilon_p \theta_{pa}}{(1 - \beta \theta_{pa})(1 - \theta_{pa})} \hat{\Pi}_{n,t}^2 \\
+ \frac{P_n C_n}{PC} \frac{\epsilon_p \theta_{pn}}{(1 - \beta \theta_{pn})(1 - \theta_{pn})} \hat{\Pi}_{n,t}^2 \\
+ \frac{P_a C_a + P_n C_n}{PC} \epsilon_w (1 + \phi \epsilon_w) \theta_w \hat{\Pi}_{w,t}^2
\end{array} \right\}$$

Finally, we divide both sides by $U Y Y = U c Y = C^{-1}(P a C_a + P_n C_n)$, and obtain welfare loss as percentage of steady state expenditure

$$\mathbb{W}_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \int_0^1 (U_t - U) dj \overline{U Y Y}$$

$$= -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l}
\Xi \left( \beta_a^{-1} \chi_a + \gamma_a \phi \right) \tilde{y}_{a,t} + \left( \frac{P_a}{P_n} \beta_n (1 + \phi \gamma_n) \right) \tilde{y}_{n,t} \\
-2 \Xi \left( \frac{P_a}{P_n} \chi_a \beta_n - \frac{P_a}{P_n} \chi_a \gamma_n \phi \right) \tilde{y}_{a,t} \hat{y}_{n,t} \\
+ \frac{P_a C_a + P_n C_n}{PC} \frac{\epsilon_p \theta_{pa}}{(1 - \beta \theta_{pa})(1 - \theta_{pa})} \hat{\Pi}_{n,t}^2 \\
+ \frac{P_n C_n}{PC} \frac{\epsilon_p \theta_{pn}}{(1 - \beta \theta_{pn})(1 - \theta_{pn})} \hat{\Pi}_{n,t}^2 \\
+ \left( \frac{P_a C_a + P_n C_n}{PC} \epsilon_w (1 + \phi \epsilon_w) \theta_w \right) \hat{\Pi}_{w,t}^2
\end{array} \right\}$$

where $\Xi \equiv \frac{PC}{P_a C_a + P_n C_n}$.
Appendix B. Figures and tables

Figure 1: Shock to agriculture in developing country
Figure 2: Shock to non-agriculture in developing country
Figure 3: Both shocks in developing country
The continuous line shows impulse responses when the central bank targets headline inflation, $\Omega = 0.35$, while the dashed line shows the impulse response when the central bank targets core inflation, $\Omega = 0$. $Y_{\text{gap}}$ is output gap, $R$ is the nominal interest rate, $C_n$ is consumption in non-agriculture, $C_a$ is consumption in agriculture, $P_{\text{ia}}$ is agricultural inflation, $P_{\text{in}}$ is non-agricultural inflation, $P_{\text{iw}}$ is wage inflation, $P_i$ is headline inflation when $\Omega = 0.35$ and core inflation when $\Omega = 0$, $N$ is total employment, $N_a$ is agricultural employment, $N_n$ is non-agricultural employment and $\text{shock}$ is the agricultural productivity shock.
The continuous line shows impulse responses when the central bank targets headline inflation, $\Omega = 0.35$, while the dashed line shows the impulse response when the central bank targets core inflation, $\Omega = 0$. $Y_{gap}$ is output gap, $R$ is the nominal interest rate, $Cn$ is consumption in non-agriculture, $Ca$ is consumption in agriculture, $P_{ia}$ is agricultural inflation, $Pin$ is non-agricultural inflation, $Piw$ is wage inflation, $Pi$ is headline inflation when $\Omega = 0.35$ and core inflation when $\Omega = 0$, $N$ is total employment, $Na$ is agricultural employment, $Nn$ is non-agricultural employment and $\text{shock}$ is the non-agricultural productivity shock.
Figure 6: Shock to agriculture in rich country
Figure 7: Shock to non-agriculture in rich country
Figure 8: Both shocks in rich country
Figure 9: Shock in non-agriculture: rich vs developing ($\Omega = 0$)

The figure shows impulse responses when the central bank targets core inflation, $\Omega = 0$. The continuous line shows impulse responses of a rich country with agricultural employment share of 2%, while the dashed line and the dashed and dotted line show impulse responses in developing countries with agricultural employment share of 30% and 50%, respectively. $Y_{gap}$ is output gap, $R$ is the nominal interest rate, $C_n$ and $C_a$ are consumption in non-agriculture and agriculture, $P_{ia}$, $P_{in}$, $P_{iw}$ and $P_i$ are agricultural, non-agricultural, wage and core inflation, $N$, $N_a$ and $N_n$ are total, agricultural and non-agricultural employment, and shock is the non-agricultural productivity shock.
The figure shows impulse responses when the central bank assigns full weight to agricultural inflation, $\Omega = 1$. The continuous line shows impulse responses of a rich country with agricultural employment share of 2%, while the dashed line and the dashed and dotted line show impulse responses in developing countries with agricultural employment share of 30% and 50%, respectively. $Y_{gap}$ is output gap, $R$ is the nominal interest rate, $C_n$ and $C_a$ are consumption in non-agriculture and agriculture, $P_{ia}$, $P_{in}$, $P_{iw}$ and $P_i$ are agricultural, non-agricultural, wage and core inflation, $N$, $N_a$ and $N_n$ are total, agricultural and non-agricultural employment, and $shock$ is the non-agricultural productivity shock.
In this figure we compare welfare loss in rich and developing countries, given the measure of inflation, $\Omega$. The vertical dashed line indicates headline inflation targeting, $\Omega = \frac{P_aC_a}{PY}$. 
Figure 12

(a) Optimal $\Omega$ for different $\sigma_a/\sigma_n$

(b) Optimal $\Omega$ for different $\rho_a/\rho_n$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_a$</td>
<td>Weight of agricultural goods in utility in rich countries</td>
<td>0.02</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Elasticity of substitution between agric. and non-agric. goods</td>
<td>0.3</td>
</tr>
<tr>
<td>$C_{\text{ag}}$</td>
<td>Employment in agriculture in developing country (30%)</td>
<td>0.02808</td>
</tr>
<tr>
<td>$A_{\text{ag}}$</td>
<td>Relative agricultural price in developing country (1.5 of U.S.)</td>
<td>0.66 * $A_n$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Intertemporal elasticity of substitution</td>
<td>1</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse of the Frisch elasticity of labor supply</td>
<td>0.99</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Elasticity of substitution of different goods varieties</td>
<td>6</td>
</tr>
<tr>
<td>$\epsilon_n$</td>
<td>Elasticity of substitution of different labor varieties</td>
<td>6</td>
</tr>
<tr>
<td>$\theta_{\text{ag}}$</td>
<td>Probability of not reseting prices next period in sector a</td>
<td>0</td>
</tr>
<tr>
<td>$\theta_{\text{n}}$</td>
<td>Probability of not reseting prices next period in sector n</td>
<td>2/3</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Probability of not reseting wages next period</td>
<td>3/4</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Autocorrelation of productivity shock in a</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_{\text{ag}}$</td>
<td>Weight assigned to price inflation in Taylor rule</td>
<td>0.93</td>
</tr>
<tr>
<td>$\rho_{\text{n}}$</td>
<td>Autocorrelation of productivity shock in n</td>
<td>0.03</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>Standard deviation of shock in a</td>
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</tr>
<tr>
<td>Welfare Loss</td>
<td>Developing country</td>
<td>Rich country</td>
</tr>
<tr>
<td>-------------</td>
<td>--------------------</td>
<td>--------------</td>
</tr>
<tr>
<td></td>
<td>$\Omega = 0$</td>
<td>$\Omega = 1$</td>
</tr>
<tr>
<td>$\hat{y}_{\Delta t}^2$</td>
<td>0.0215</td>
<td>0.000</td>
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<tr>
<td>$\hat{y}_{\Delta t}^n$</td>
<td>1.6828</td>
<td>1.0881</td>
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<tr>
<td>$\pi^2_{\Delta t}$</td>
<td>0.0001</td>
<td>0.0000</td>
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<td>$\pi^n_{\Delta t}$</td>
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<td>3.4677</td>
</tr>
<tr>
<td>$\pi^n_w$</td>
<td>36.8258</td>
<td>0.2030</td>
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<tr>
<td>Total</td>
<td>44.9181</td>
<td>4.7589</td>
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