Inflation and Economic Growth in the Short Run: Behavior under a Markov-Switching Approach*

Carolina Pagliacci†
Daniel Barráez§

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Abstract
In this paper, we analyze the dynamic of the business cycle in Venezuela, in the last twenty years, through a Markov-switching estimation of two models: a Phillips curve and an aggregate demand curve. The characterization of the different regimes recognized by these two models is based on the nature of the relationship established between the endogenous variables (inflation rate and output gap) and the explanatory variables. This task, although it is more complicated than classifying the economy in states of “recession” and “expansion”, provides a richer approach to understand the dynamic of these important variables. Estimation of the two proposed models is done through the implementation of an EM algorithm. Estimation results show that for inflation, the model basically distinguishes between a “normal” regime and a “prone to crisis” regime. The model for the output gap recognizes as well two “normal” regimes versus a “special” one associated to the oil strike.

Key words: regime switching. Business cycle, output-inflation tradeoff
JEL classification: C32, E31, and E32

* Opinions on this paper are full responsibility of the authors and do not compromise those of the Banco Central de Venezuela.
† Senior Researcher at the Banco Central de Venezuela.
§ Senior Researcher at the Banco Central de Venezuela and Professor at the Universidad Central de Venezuela.
I. Introduction

Since the seminal work of Hamilton (1989) many researchers have devoted to studying the economic growth from the perspective of regime changes, improving not only forecasts on growth, but also identifying two phases in the economic cycle: expansions and contractions. Naturally, through time, the estimation of Markov-switching models has experienced refinements in the applied econometric techniques, but it has mostly kept the original spirit of Hamilton’s work: distinguishing between regimes of recessions and expansions. In fact, later papers such as Kim and Murray (2002) and Kahn and Rich (2007) have incorporated the occurrence of regime switching in the non-observable components of growth (as in state-space models), but maintaining the same two underlying regimes: recessions and expansions. Another sophistication has been to suppose that the transition matrix that governs the process of switching is variable instead of being constant. An application to the Colombian case of this technique was carried out by Misas and Ramírez (2006), based on the previous work developed by Diebold and Rudebush (1996).

In a different strand of the literature, a basic comprehension of the dynamic and transmission mechanisms of an economy has been provided by the construction of small scale macroeconomic models. This is the case because these types of models consist of simplified reduced form equations that still describe the main theoretical relationships between variables, most commonly, inflation an output gap. In this paper, we have chosen to characterize inflation throughout the estimation of a basic Phillip, and to study the output growth with a simple aggregate demand equation, but incorporating the occurrence of regime switching. This strategy was chosen in order to combine two important facts: first, that fiscal and oil variables are tremendously relevant for explaining the fundamentals of the Venezuelan economy, and second, that the changes in the statistical properties of the inflation and output gap series are probably explained by the co-existence of at least two non-observable economic regimes.

The challenge that arises with the application of this approach is that the characterization of regimes can not be done ex-ante as in Hamilton’s model: is no longer clear that the regimes captured by these types of models consist of simplified reduced form equations that still describe the main theoretical relationships between variables, most commonly, inflation an output gap. In this paper, we have chosen to characterize inflation throughout the estimation of a basic Phillip, and to study the output growth with a simple aggregate demand equation, but incorporating the occurrence of regime switching. This strategy was chosen in order to combine two important facts: first, that fiscal and oil variables are tremendously relevant for explaining the fundamentals of the Venezuelan economy, and second, that the changes in the statistical properties of the inflation and output gap series are probably explained by the co-existence of at least two non-observable economic regimes.

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In order to estimate the type of Markov-switching models we are proposing, we need to implement the EM algorithm explained in Hamilton (1990), which typically has been applied to a two-regime setting where the endogenous variable has a constant mean, conditional on the regime. Although most of the economic literature applies numerical maximization techniques for the estimation of these switching models, we chose using the EM algorithm in order to
exploit its robustness and fast convergence property, as it is done in most of the non-economic literature, like for instance, the literature of speech pattern recognition.

In the analysis of the inflation dynamic, the estimated model shows that the Venezuelan economy switches from periods in which agents respond to past policy actions to periods in which agents adjust their pricing behavior only according to the expected future performance of the economy. These changes in the inflation’s behavior coincide with the development of macroeconomic crises, typically triggered by the occurrence of speculative attacks to the domestic currency. In this sense, one could synthetically claim that what this model does is to distinguish between a “normal” regime and a “prone to crisis” regime.

The model fitted for the output gap recognizes the co-existence of three regimes in the economy. The first one is mainly characterized by the response of the aggregate demand to past changes in the oil income and fiscal expenditure, two of the most important variables in the Venezuelan economy. The second regime differs from the first in two main features: a) the economy in the absence of policy actions tends to enter in a recession state, and b) the oil income does not longer affect the aggregate demand. Periods classified according to this second regime, although not entirely associated to the development of crises, mostly reflect a change in the way the government allocates oil income respect to domestic expenses, and therefore, the way aggregate demand is stimulated. The third regime is simply the acknowledgment that, the oil strike occurred in Venezuela between 2002 and 2003 conditioned the growth dynamic of the economy during the strike itself and few periods after.

One interesting element of the results is that, although the analyses of inflation and growth for the recognition of regimes are carried out separately, regimes can overlap and even exhibit common features. Nonetheless, it is important to study these two variables jointly in order to exploit their statistical correlation and to figure out if the new taxonomy of regimes adds more information to the process of regime recognition. In order to do so, we are developing a VAR estimation under a Markov switching approach, but results are still in progress.

The paper is structured as follows: section II presents the general regression model for a single variable and explains the implementation of the EM algorithm. Section III and IV show the estimation results and their interpretation for the Phillips curve and the aggregate demand model respectively. Section V concludes.

II. The regression model and the EM estimation

Because of the paramount importance of fiscal and oil variables to explain the behavior of inflation and growth in Venezuela, we consider the following general regression model suggested in Hamilton (1994):

\[ y_t = z_t \beta_{it} + \varepsilon_t \quad \text{for } i=1,2,\ldots,N \text{ and } t=1,2,\ldots,T \]  

\( y_t \) refers to the model’s endogenous variable, \( \varepsilon_t \) is the error term i.i.d. according to \( \varepsilon_t \sim N\left(0,\sigma^2\right) \).
\( z_t \) is a \((1 \times k)\) vector that contains the explanatory variables (could include lagged values of \( y \)) , and \( \beta_{si} \) represents a \((k \times 1)\) vector of coefficients associated to regime \( si \), which by definition is unobservable. The total number of possible regimes or hidden states is given by \( N \), and the realizations of particular states are governed by the following first-order Markov process \( Q_t \), such that:

\[
\Pr(q_t = sj \mid q_{t-1} = si) = p_{ij} \\
\Pr(q_1 = si) = \pi_i \\
\sum_{j=1}^{N} p_{ij} = 1 , \quad \sum_{i=1}^{N} \pi_i = 1
\]  

These \( p_{ij} \)'s can be ordered in a so called transition probability matrix \( P \), while the unconditional probabilities of hidden states \( (\pi_i) \) are represented with a column vector \( \Pi \) of initial probabilities, as follows:

\[
P = \begin{bmatrix}
  p_{11} & p_{21} & \cdots & p_{N1} \\
  p_{12} & p_{22} & \cdots & p_{N2} \\
  \vdots & \vdots & \ddots & \vdots \\
  p_{1N} & p_{2N} & \cdots & p_{NN}
\end{bmatrix}, \quad \Pi = \begin{bmatrix}
  \pi_1 \\
  \pi_2 \\
  \vdots \\
  \pi_N
\end{bmatrix}
\]  

The above description implies that, once a realization of a regime occurs at a given point in time, the observable variable \( y_t \) exhibits a conditional mean equal to \( z_t \beta_{si} \). Then, the realization of the next hidden state is a random draw governed by the transition probabilities defined in \( P \). The complete model can be characterized by the set of parameters \( \Theta = \{P, \Pi, \beta\} \), where \( \beta = \{\beta_{s1}, \beta_{s2}, \ldots, \beta_{sN}, \sigma^2\} \) depicts the relationship between the endogenous and the explanatory variables of the model for all \( N \) different regimes.

The estimation of the above model is performed though the implementation of the EM algorithm, which finds the set of parameters that maximizes the likelihood function of the observed data through an iterative expectation process. We chose using the EM algorithm, as it is done in most of the non-economic literature, because “…this algorithm is quite robust with respect to poorly selected starting values and quickly moves to a reasonable region of the likelihood surface” (Hamilton 1990). This implies that for different starting values, the algorithm converges to the same solution with relatively few iterations and minimizes the problem of evaluating hundreds of initial values.

Given the structure of the model, the theoretical likelihood function for a sequence of observed data \( Y^T = \{y_1, y_2, \ldots, y_T\} \) has also to consider the possible sequence of hidden states that could have occurred, name it \( S^T = \{q_1, q_2, \ldots, q_T\} \). This is the case because hidden states condition the probability distributions of the endogenous variable, indicating that a joint probability of hidden states and observations must exist. Therefore, knowing the parameters of the model and a particular sequence \( S^T \), this joint probability can be defined as:
\[ \Pr(Y^T, S^T / \Theta) = \Pr(q_1) \prod_{t=2}^T \Pr(q_t / q_{t-1}) \prod_{t=1}^T \Pr(y_t / q_t, z_t) \] (4)

And the theoretical likelihood function for the entire sample \( L(Y / \Theta) \) can be simply written as:

\[ L(Y / \Theta) = \sum_{Y \in S} \Pr(Y^T, S^T / \Theta) = \sum_{Y \in S} \prod_{t=2}^T \Pr(q_t / q_{t-1}) \prod_{t=1}^T \Pr(y_t / q_t, z_t) \] (5)

where, for instance, \( \sum_{Y \in S} g(S) = \sum_{q_T=1}^{SN} \sum_{q_{T-1}=1}^{SN} \cdots \sum_{q_1=1}^{SN} g(q_1, q_2, \ldots, q_T) \). That is, the likelihood function must consider all possible sequences of hidden states, and not only a particular sequence.

However, the expressions to implement the EM algorithm are derived, not by directly maximizing the likelihood function in (5), but by maximizing an alternative expression \( \mathcal{Q}(\Theta^{(i)}, \Theta^{(i-1)}) \) that makes explicit the fact that maximization is achieved iteratively by considering diverse parameter values for the model. The proof of this equivalence can be read either in Hamilton (1990) or Welch (2003). The particular form for this alternative expression is given by:

\[ \mathcal{Q}(\Theta^{(i)}, \Theta^{(i-1)}) = \sum_{Y \in S} \ln \Pr(Y^T, S^T / \Theta^{(i)}) \Pr(Y^T, S^T / \Theta^{(i-1)}) \] (6)

where the arguments of the function denote the existence of a sequence parameters \( \{ \Theta^{(i)}, \Theta^{(2)}, \ldots, \Theta^{(i)} \} \) that are used in the different iterations of the maximization process. This function, according to Hamilton (1990) can be interpreted as the expected log-likelihood (for all sequences of hidden states) of the observable variable parameterized by \( \Theta^{(i)} \), where the weights of the expectation operator are given by the joint probability of data and hidden states parameterized by \( \Theta^{(i-1)} \).

Therefore, the application of the EM algorithm entails finding a sequence of estimated parameters \( \{ \hat{\Theta}^{(1)}, \hat{\Theta}^{(2)}, \ldots, \hat{\Theta}^{(i)} \} \) such that \( L(\hat{\Theta}^{(i)}) \geq L(\hat{\Theta}^{(i-1)}) \) is always satisfied for any \( i^{th} \) iteration of the algorithm. The recursive application of this procedure leads eventually to find a fixed point where \( \hat{\Theta}^{(i)} = \hat{\Theta}^{(i-1)} \) is satisfactorily approximated, and \( \hat{\Theta}^{(i)} = \arg \max L(\hat{\Theta}) \), that is, \( \hat{\Theta}^{(i)} \) is the maximum likelihood estimator.

In the EM algorithm, the analytical functional forms for the parameter estimates are obtained by solving the first order conditions that maximize expression (6) respect to \( \Theta^{(i)} \). Among these FOCs, Hamilton (1990) shows that the estimation of the regression parameters in (1) satisfies:
Since the sequences of hidden states are not directly observed by the econometrician, then they are inferred from the sequence of realizations of the observed variable $Y^T$, which entails to re-writing \[ \Pr(Y^T, S^T / \hat{\Theta}^{(l-1)}) = \Pr(S^T / Y^T, \hat{\Theta}^{(l-1)}) \Pr(Y^T / \hat{\Theta}^{(l-1)}) \]. After several algebraic manipulations, and a change of representation of the sequences of hidden states, Hamilton (1990) shows that the maximum likelihood estimator $\hat{B}^{(l)}$ must satisfy:

\[
\sum_{t=1}^{T} \sum_{q_i=1}^{SN} \frac{\partial \ln f(y_t / q_i = s_i, z_t, B^{(l)})}{\partial B^{(l)}} \bigg|_{\hat{\Theta}^{(l)}} \Pr(q_t = s_i / Y^T, Z^T, \hat{\Theta}^{(l-1)}) = 0 \tag{8}
\]

where $f(y_t / q_i = s_i, z_t, B)$ is the density function of $y_t$ conditional on the parameters of the regression model, on the assumed hidden state $q_i$, and on $z_t$, which is a row vector of dimension $k$ containing information on the lagged endogenous variable and on the exogenous variables of the model $(x_t)$, such that $z_t = [y_{t-1}, y_{t-2}, \ldots, y_{t-p}, x_t, x_{t+1}, \ldots, x_{(k-p)+1}]$ and $p$ is the number of lags for the endogenous variable. On the other hand, $\Pr(q_t = s_i / Y^T, Z^T, \hat{\Theta}^{(l-1)})$ is the probability that the hidden state $s_i$ has occurred at time $t$, conditional on the entire data sample: $Y^T = \{y_1, y_2, \ldots, y_T\}$ and $Z^T = \{z_1, z_2, \ldots, z_T\}$, evaluated in the parameter estimates from the preceding iteration. In our model, as already stated, we assume that the conditional density of $y_t$ is normal, such that $f(y_t / q_i = s_i, z_t, B) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{(y_t - z_t\beta_t)^2}{2\sigma^2} \right\}$.

In order to estimate the model (1)-(3), the implementation of the EM algorithm in any $l^{th}$ iteration implies following the next four steps:

1. Given the estimated parameters in the preceding iteration $(\hat{\Theta}^{(l-1)})$ and the sequence of the observable variable until time $t$ ($Y^t$), estimate the probability that each possible state $s_i$ has occurred at time $t$, computing recursively, from $t=1$ through $t=T$, the following expressions:

   (a) $\xi_{t/0} = \hat{\Gamma}^{(l-1)}$

   (b) $\xi_{t/t} = \frac{\eta_t \circ \xi_{t/t-1}}{j_N (\eta_t \circ \xi_{t/t-1})}$

   (c) $\xi_{t+1/t} = \hat{\Pi}^{(l-1)} \xi_{t/t}$

   (d) $f(y_t / z_t, \hat{B}^{(l-1)}) = j_N (\xi_{t/t-1} \circ \eta_t)$

where $\eta_t$, $\xi_{t/t}$, $\xi_{t+1/t}$ and $j_N$ are column vectors of dimension $N$, defined as:
\[ f(y_t / q_t, z_t, \hat{B}^{(t-1)}) \]

\[ \eta_t = \begin{bmatrix} f(y_t / q_t, s1, z_t, \hat{B}^{(t-1)}) \\ \vdots \\ f(y_t / q_t, sN, z_t, \hat{B}^{(t-1)}) \end{bmatrix}, \quad \xi_{t|t} = \begin{bmatrix} \Pr(q_t = s1 / \Omega^t, \hat{\Theta}^{(t-1)}) \\ \vdots \\ \Pr(q_t = sN / \Omega^t, \hat{\Theta}^{(t-1)}) \end{bmatrix}, \quad j_N = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \]

\( f(y_t / q_t, z_t, \hat{B}^{(t-1)}) \) is the conditional density for a given time period evaluated in parameters estimates from the preceding iteration, and the operator \((\odot)\) indicates an element by element multiplication. Notice that because of the recursive nature of \( \hat{\xi}_{t|t} \), the set of information used is \( \Omega^t \equiv Y^t \cup Z^t \), which also includes the sequence of realizations of the lagged endogenous and exogenous variables of the model until time \( t \) (\( Z^t \)). This must be the case, because at each time \( t \), the algorithm needs to evaluate the likelihood that a particular hidden state has occurred, but taking into consideration that its transition could have taken place from any possible sequence of \( t-1 \) hidden states.

2. Use the complete sequence of the observable variable (\( Y^T \) instead of \( Y^t \)) to re-estimate the probabilities that each possible state \( si \) has occurred at time \( t \). These new probabilities are computed with the Kim’s algorithm and are referred by Hamilton (1990, 1994) as smooth probabilities. This algorithm is applied recursively, from \( t+1=T \) backward to \( t=1 \), calculating the following expressions:

(a) \( \xi_{t+1|T} = \xi_{t+1|t} \quad \text{for} \quad t+1=T \)

(b) \( \xi_{t|T} = \xi_{t|t} \odot \left( \hat{\pi}^{(t-1)} \odot \left( \xi_{t+1|T} \div \xi_{t+1|t} \right) \right) \)

(c) \( \xi_{t|T} = \Pr(q_t = si / \Omega^t, \hat{\Theta}^{(t-1)}) \left( \hat{\pi}^{(t-1)} \odot \left( \xi_{t+1|T} \div \xi_{t+1|t} \right) \right) \quad i=1,\ldots,N \)

where \((\div)\) indicates an element by element division, \( \xi_{t|T} \) is a column vector of dimension \( N \), and \( \hat{\pi}_i \) is the estimated \( i^{th} \) column of matrix \( \hat{P} \):

\[ \xi_{t|T} = \begin{bmatrix} \Pr(q_t = si, q_{t+1} = s1 / \Omega^T, \hat{\Theta}^{(t-1)}) \\ \vdots \\ \Pr(q_t = si, q_{t+1} = sN / \Omega^T, \hat{\Theta}^{(t-1)}) \end{bmatrix}, \quad \hat{\pi} = \begin{bmatrix} \hat{\pi}_1 \\ \vdots \\ \hat{\pi}_N \end{bmatrix} \]

3. Re-estimate the model parameters \( \hat{\Theta} \) for this \( l^{th} \) iteration, by solving the different FOCs that maximize (6). According to Hamilton (1990, 1994) this procedure is equivalent to computing:

(a) Transition probabilities using the equations:
\[
\hat{p}_{i}^{(t)} = \left( \sum_{t=2}^{T} \xi_{i,t} \right) \div \left( \sum_{t=2}^{T} \xi_{i,t} \right) \quad \text{for } i = 1, \ldots, N
\]

(b) Unconditional probabilities of being at each state following:
\[
\hat{\Pi}^{(t)} = \frac{\sum_{t=1}^{T} \xi_{i,t}}{T}
\]

(c) Parameters of the regression model in (1), by solving the FOCs stated in (8), such that:
\[
\hat{\beta}_{si}^{(t)} = \left( z' \hat{\Gamma}_{si} z \right)^{-1} z' \hat{\Gamma}_{si} y \quad \text{for } i = 1, \ldots, N
\]
\[
\hat{\sigma}^{2} = \sum_{si=s1}^{SN} \frac{\left( y - z \hat{\beta}_{si}^{(t)} \right)' \hat{\Gamma}_{si} \left( y - z \hat{\beta}_{si}^{(t)} \right)}{T - k}
\]
\[
\text{Var} \left( \hat{\beta}_{si}^{(t)} \right) = \hat{\sigma}^{2} \left( z' \hat{\Gamma}_{si} z \right)^{-1}
\]

where:
\[
y = \begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_T
\end{bmatrix}, \quad z = \begin{bmatrix}
z_{11} & z_{12} & \cdots & z_{1k_2} \\
z_{21} & z_{22} & \cdots & z_{2k_2} \\
\vdots & \vdots & \ddots & \vdots \\
z_{1T} & z_{2T} & \cdots & z_{kT}
\end{bmatrix}, \quad \beta_{si} = \begin{bmatrix}
\beta_{1,si} \\
\beta_{2,si} \\
\vdots \\
\beta_{k,si}
\end{bmatrix}, \quad \epsilon_{si} = \begin{bmatrix}
\epsilon_{1,si} \\
\epsilon_{2,si} \\
\vdots \\
\epsilon_{k,si}
\end{bmatrix}
\]
\[
\hat{\Gamma}_{si} = \begin{bmatrix}
\Pr \left( q_1 = si / \Omega^T, \hat{\Theta}^{(t-1)} \right) & 0 & \cdots & 0 \\
0 & \Pr \left( q_2 = si / \Omega^T, \hat{\Theta}^{(t-1)} \right) & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \Pr \left( q_T = si / \Omega^T, \hat{\Theta}^{(t-1)} \right)
\end{bmatrix}
\]

4. Evaluate if the parameter estimates have attained a fixed point, that is \( \left| \hat{\Theta}^{(t)} - \hat{\Theta}^{(t-1)} \right| \leq 10^{-8} \).

Then, verify that the empirical expected log-likelihood function of the dependent variable (for all hidden states), has also achieved a maximum.

This verification implies observing, for:
\[
E \left[ g \left( y / \hat{\Theta}^{(t)} \right) \right] = \sum_{i=1}^{T} \sum_{q_{i,j}} \ln f \left( y_{i,j} / q_{i,j}, \hat{\Theta}^{(t)} \right) \Pr \left( q_{i,j} = si / \Omega^T, \hat{\Theta}^{(t-1)} \right),
\]
that \( E \left[ g \left( y / \hat{\Theta}^{(t)} \right) \right] - E \left[ g \left( y / \hat{\Theta}^{(t-1)} \right) \right] \leq \text{tolerance value} \). The maximization of this expected log-likelihood function should be easily confirmable since it is a by-product of the estimation process, and in particular, of the imposition of the FOCs stated in (8).
III. A Phillips curve estimation with Markov-switching

In this section, inflation is characterized throughout the estimation of a two-regime New Keynesian Phillips curve. We model inflation strictly as a function of lagged inflation, indicating that only inflationary inertia (and not inflation expectations) determines the level of the structural or underlying inflation. This simplifying assumption, not only will allow fitting the model within the class of models presented in (1), but also will enable interpreting for how long diverse shocks hitting the economy will have an impact in the different regimes. Theoretically, it is argued that inertia is related to the existence of staggered price setting. In particular, if firms change prices at different times, adjustment of the aggregate price level to shocks takes longer than the time of adjustment between firms, even if individuals change prices frequently (Ball et al 1988). This is equivalent to stating that during periods of high inflationary (or price level) inertia, shocks have long lasting and larger effects. For this type of explanation, an increase in inflationary inertia would imply a higher dispersion in the timing of price adjustments by individual firms, or equivalently, a larger coordination failure between firms in acknowledging the occurrence of aggregate demand shocks.

The pressures of aggregate demand on inflation are summarized by the inclusion of the output gap (the IS component) and a variable that measures the quantity of money created by the public sector (the LM component) as explanatory variables. This money variable represents the main source of money supply in the economy and it is the result of combining the state monopoly of the oil activity with the fact that an important size of domestic public expenditures is financed with oil resources. Its inclusion as an additional aggregate demand factor tries to find out if an excess of money supply respect to the size of the nominal output will impinge a positive pressure on the inflation rate. Inflation also depends on the nominal depreciation of the domestic currency, as a way to acknowledge the potential impact of cost-push elements (supply shifters) on the inflation dynamic.

The particular regression model for the Phillips curve is given by:

\[ Inf_t = a_d + \rho_d \ Inf_{t-1} + \alpha_d \ (Gdp_{t-1} - Gdp_{t-1}^*) + \delta_d \ M_{t-2} + \gamma_d \ \hat{E}_{t-1} + \epsilon, \quad \text{for } i=1,2 \quad (9) \]

where \( Inf \) represents the annual average inflation rate, \( Gdp_{t-1} - Gdp_{t-1}^* \) is the output gap computed as the difference between the log of the annual real GDP and its Hodrick-Prescott tendency, \( M \) is the ratio between the money created by the public sector\(^1\) in a year span and the nominal GDP, and \( \hat{E} \) is the rate of depreciation of the domestic currency, measured as the log difference of the yearly average of the nominal exchange rate (Bs per U.S. dollar)\(^2\).

The lag structure of the regression model in (9) was chosen by running several linear regressions for the complete estimation period. The recursive procedure implied starting with a

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\(^1\) For this case, the public sector is defined as the sum of the Central Government, the state oil industry (PDVSA) and the Central Bank.

\(^2\) During periods of exchange rate controls (1994-1996 and 2003 to the present), this exchange rate refers to the value of the dollar in the non-official market.
general model of four lags (for all explanatory variables) and reducing all non significant variables until obtaining a parsimonious model that contained only significant lags. The estimation period is defined from 1990:4 to 2007:04, for a total of 69 quarterly observations.

The number of regimes or hidden states was selected using a mixed criterion: a statistical one and an economic one. First, we evaluated the value of the likelihood function for two and three regimes respectively. Second, given that the differences in the likelihood functions seemed statistically insignificant, we observed the classification of regimes provided by each model. A two-state model was preferred over a three-state model because of the few time periods classified in the third regime (barely three) and the lower power of the three-state model to distinguish among diverse regimes.

Initial values for the $\beta^{(0)}$ parameters to implement the EM algorithm were chosen by imposing, in each regime, variations to the estimated linear regression parameters. Such variations were constructed taking into account that each regime might contain extreme values of the parameters, but within their expected theoretical range. In this way, OLS estimates are simply interpreted as average estimates of the true two underlying regimes prevailing in the economy. Initial values for the transition matrix ($P^{(0)}$) assumed that all elements of the matrix were equal. Initial unconditional probabilities ($\Pi^{(0)}$) were set as the ergodic probabilities of the Markov process, as suggested by Hamilton (1994).

After applying the EM algorithm, estimation results are summarized in table1.

Estimated coefficients in regime 1 show that inflation responds significantly to all the explanatory variables of the model in the expected magnitude and direction. Among of aggregate demand factors, the output gap has the greatest explanatory power, while the pass-through coefficient indicates that a 100% depreciation of the domestic currency will cause 29 p.p. of increase in the rate of inflation. The autoregressive component of the inflation is positive and less than one. According to the relationship established among the variables, one could characterize this regime as the “normal” state of the economy, or at least, as a regime in which inflation is appropriately described by the theory.

On the contrary, estimated coefficients in regime 2 indicate that only lagged inflation and the output gap are statistically significant to explain current inflation. However, the most striking characteristic of this regime is that the autoregressive coefficient of inflation is positive, but strictly greater than one. Trying to understand the underlying causes of why this coefficient takes over in the explanation of the current inflation rate becomes a difficult task, especially when the literature defines inertia as a positive, but smaller than one, autoregressive coefficient.

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3 Robustness of the EM-estimation results to the selection of the initial parameter values was tested by imposing diverse structures of initial parameters. Convergence to the same estimates was always achieved.

4 A standard contrast of hypothesis rejected the null that the $\rho_{12} \leq 1$. 
**Table 1.- Two-Regime Coefficient Estimates for the Phillips Curve**

Dependent Variable: Inf
Estimation Method: EM
Sample (adjusted): 1991Q2 2007Q4
Included observations: 67 after adjustments
Iterations: 74

<table>
<thead>
<tr>
<th>Regime</th>
<th>Parameters</th>
<th>Estimates</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\alpha_{s1}$</td>
<td>0.006459</td>
<td>0.000284</td>
<td>0.383236</td>
<td>0.702856</td>
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<td></td>
<td>$\rho_{s1}$</td>
<td>0.676060</td>
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<td></td>
<td>$\alpha_{s1}$</td>
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<td>0.016194</td>
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<td>0.008381</td>
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<td>-4.125895</td>
<td>0.000112</td>
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<td>$\rho_{s2}$</td>
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<td>0.003162</td>
<td>0.177132</td>
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<td>S.E. of regression</td>
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<td>F-statistic</td>
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<tr>
<td>Prob (F-statistic)</td>
<td>0.000000</td>
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</table>

Several works have analyzed inflation in Venezuela, but only two of them have explicitly referred to the problem of inflationary inertia. Dorta et al. (1998) in their analysis of the inflation for the period 1970 to 1997, state that inflationary inertia has increased since 1984 mainly due to the reduced credibility of agents in the performed economic policy. On the other hand, Guerra and Pineda (2004), when studying the implementation of a bound system for the exchange rate (1997:01 to 2002:1) claim that, although the inflation rate had shown a descending path during the whole period, a further decreased was precluded, exactly because of the existence of a greater inflationary inertia. This empirical evidence, although it could relate intuitively to our findings, it does not really support the fact that the estimated coefficient on lagged inflation could be greater than one.

Another way to proceed to characterize regimes, it is by writing inflation as the particular
solution of the implicit first-order difference equation proposed, so that its current level can be explained by the dynamic of the other explanatory variables of the model. That is, consider the regression model in (9), take its expected value and solve it according to the estimated two sets of parameters.

In regime 1, inflation can be characterized as:

\[
\text{Inf}_t = 0.33 \left[ \sum_{m=0}^{\infty} 0.68^m (\text{Gdp}_{t-m-1} - \text{Gdp}^*_t) \right] + 0.16 \left[ \sum_{m=0}^{\infty} 0.68^m M_{t-m-2} \right] + 0.29 \left[ \sum_{m=0}^{\infty} 0.68^m \hat{E}_{t-m-1} \right]
\]  

(10)

Instead, in regime 2, after solving the difference equation forward, inflation can be described by:

\[
\text{Inf}_t = 0.25 - 0.34 \left[ \sum_{m=0}^{\infty} 0.88^m E(\text{Gdp}_{t+m} - \text{Gdp}^*_{t+m}) \right]
\]  

(11)

where \( E(\text{Gdp}_{t+m} - \text{Gdp}^*_{t+m}) \) denotes agent’s expectations on the future values of the output gap.

In regime 1, the mean of inflation responds to the past values of the output gap, money creation and currency depreciation, and changes in these variables might significantly affect the inflation rate for approximately 11 quarters\(^5\). In regime 2, the mean of the inflation rate depends on agent’s expectations about the output gap, which allows labeling this regime as a “rational expectation” regime. In this case, if the economy is expected to grow above its potential level, then inflation will tend to drop below 25%, while if the economy is expected to be in a recession, inflation will tend to rise above 25%. Also, in this regime, the effect of expected changes in the output gap will last for approximately 36 quarters, implying that bad expectations regarding the future of the economy will have at least a three times more persistent effect than past shocks in regime 1.

Theoretically, the fact that in regime 2 inflation depends on agents’ expectations, can be easily supported by the literature, and in particular by Woodford (1991) when he explains that, without requiring any objective change in economic circumstances, the degree of optimism of economic actors can have an important role in understanding recurrent cyclical fluctuations of the business activity. He additionally states that, by all means, this is a well taken stand within the economic tradition, even before the presentation of the Keynesian theories of investment. However, a more challenging task is to justify why the output gap is the variable that agents take into consideration for establishing their pricing strategy. One could argue that, in regime 2, the output gap becomes the best proxy for the size of the demand that sellers of goods will face in the future. Therefore, as demand is expected to rise, revenues will be obtained by increasing the amount of goods sold or produced and prices could be allowed to increase less. At the micro-

\(^5\)This intuitive form of characterizing the duration of a change in any explanatory variable results from assuming that the effect over inflation disappears when the factor \( \rho^m = 0.01 \).
level, as in Stiglitz (1991) and Rotemberg and Saloner (1991), this could imply that the downward-slopping demand faced by sellers and producers would shift outward and become more elastic as competition in the market is expected to kick in.

Empirically, the frequent occurrence of regime 2 would imply that agents’ expectations on inflation are inversely related to the expected economic growth. In fact, looking at the polls on economic outlook collected by the Central Bank, we verified that, in average, there is a significant negative correlation (-0.81) between inflation and growth expectations. This can be verified by eyeballing figure 1.

![Figure 1.- Inflation and Growth Expectations](image)

Estimated (smooth) probabilities that each hidden state had occurred allowed classifying each quarter of the estimation period according to one of the regimes, as shown in figure 2.

Looking at the classification of periods that the estimated model provides, it can be stated that it is more likely to be in the “rational expectation” regime during periods of macroeconomic instability or at the end of non-floating exchange rate regimes, when the economy usually faces speculative attacks against the domestic currency. In fact, the first long period of regime 2 detected by the model (1991:02 to 1995:02) corresponds to a period of general (political and economic) instability coupled with a financial crisis and an exchange rate control in 1994. The second period of regime 2 (1996:01 to 1997:01) corresponds to the end

---

6 In 1992, the government in charge confronted a military coup, and during the outset of the financial
of the exchange rate control started in 1994 and the beginning of the implementation of a system of exchange rate bounds. This system consisted on establishing an upper and a lower bound to the trajectory of the exchange rate, such that deviations of the exchange rate outside such bounds triggered additional interventions of the Central Bank in the market. The “rational expectation” state is again detected by the model at the end of the system of bounds (2000:2 to 2002:2), just before the implementation of a floating exchange rate in March 2002.

Figure 2.- Two-Regime Inflation Classification

Although the characterization of regime 2 appears to be consistent with the history of collapses of non-floating exchange rate systems, it is still a question, why agents under such circumstances revert from looking at the past behavior of economic variables to forming expectations on the aggregate demand. Since regime 2 seems to conform to the verification of propitious conditions for the occurrence of currency crises, the explanation for switching from regime 1 to regime 2 could be found in the problems of information faced by agents when the economic environment becomes more vulnerable.

Currently, the economy is under an exchange rate control since February 2003. According to our model, since 2004:2, inflation behaves according to regime 2, implying that inflation will show a descending path only until agents’ expectations continue being positive regarding the future performance of the economy. As a matter of fact, most indicators of forecasted and

crisis in 1994, the president of the Central Bank resigned as the result of existing contradictory policy intentions between the Central Bank and the Executive Power. Also, in the second quarter of 1994, an exchange rate control was implemented as consequence of the persistent speculative attacks faced at the time.

\footnote{In practice, this period was a type of fixed exchange rate since the chosen distance between the bounds was relatively small.}
actual inflation started to show higher levels since the end of year 2006 and in particular, since
the last quarter of 2007. These events have coincided with a slower economic growth,
especially in the first quarter of 2008.

IV. An Aggregate demand estimation with Markov-switching

In this section, we estimate a three-regime aggregate demand equation. The structure of the
equation corresponds to the one typically assumed in small scale macroeconomic models,
where the output gap responds to the variables that ultimately drive consumption and
investment in the economy. In our basic specification, the output gap is function of its lagged
value, the growth of government expenditure and the wealth associated to the oil business. This
last variable emphasizes the importance that oil income and transfers can have on consumption
and investment decisions of both, private agents and the public sector.

The particular specification of the regression model for the aggregate demand is:

\[
(Gdp_t - Gdp_t^*) = b_{si} + \lambda_{si} (Gdp_{t-1} - Gdp_{t-1}^*) + \phi_{si} \hat{G}_{t-1} + \varphi_{si} \hat{W}_{t-2} + \epsilon_t \quad \text{for } i=1,2 \tag{12}
\]

where \(\hat{G}\) is the annual growth of the public sector real expenditure\(^8\), and \(\hat{W}\) is the annual
growth of oil export proceeds in U.S. dollars.

As in the estimation of the price equation, the lag structure of (12) was chosen through the
recursive elimination of non significant variables, starting with a general linear regression
model of four lags. As in the previous exercise, the estimation period runs from 1990:4 to
2007:04, for a total of 69 quarterly observations.

Due to the occurrence of a general oil strike from 2002:04 through 2003:1, selecting the
number of hidden states or regimes for this case was a difficult task. This event, not only
dramatically reduced domestic output during the strike, but also affected the complete growth
dynamic for several more periods. For this reason, when fitting a two-regime regression, the
model was only capable to recognize the special period associated to the oil strike and to
classify the rest of the periods as a single homogeneous regime. This result, although intuitive,
lacks of any interest for the characterization of the aggregate demand dynamic for most part of
the Venezuelan economic history. Given this result, the natural way to proceed was to let the
model identify, not only the special period around the oil strike, but also other types of regimes
that might have taken place. This reasoning led to the selection of a three-regime model for the
aggregate demand instead of a two-regime model.

The model was estimated using the EM algorithm and the initial parameter values were

\(^8\) This variable is defined as the non-investment domestic expenses of the Central government and
the state oil company (PDVSA), deflated by the domestic consumer price index.
selected in the same manner as in the case of the Phillips curve estimation. Estimation results are summarized in table 2.

Table 2:- Three-Regime Coefficient Estimates for the Aggregate Demand Curve

<table>
<thead>
<tr>
<th>Regime</th>
<th>Parameters</th>
<th>Estimates</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
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R-squared 0.960360  Log likelihood 285.3687
Adjusted R-squared 0.958472  F-statistic 508.7655
S.E. of regression 0.000781  Prob. (F-statistic) 0.000000
Sum squared resid 0.002420

Estimation (smooth) probabilities that each hidden state had occurred allowed classifying each quarter of the estimation period according to one of three regimes, as shown in figure 3.

Results show that estimated coefficients for regime 1 correspond to the ones expected according to the theory. That is, output in the economy will tend to increase above its potential
level as long as the fiscal expenditure and the oil income perceived will rise. In regime 2, oil proceedings are no longer significant to affect the aggregate demand of the economy and the impact of the lagged output gap becomes greater. Since regime 3 corresponds to the special period around the oil strike, the impact of the lagged output gap becomes of outmost importance, while the rest of the coefficients tend to behave differently than in the other two regimes.

**Figure 3.- Three-Regime Output Gap Classification**

To improve our intuition regarding the estimated results, let us solve the particular solution of the three difference equations implicit in the model. For regime 1, the output gap can be characterized as follows:

$$Gdp_t - Gdp_t^* = 0.04 \left[ \sum_{m=0}^{\infty} 0.56^m \hat{G}_{t-m-1} \right] + 0.01 \left[ \sum_{m=0}^{\infty} 0.56^m \hat{W}_{t-m-2} \right]$$

(13)

In regime 2, we have instead:

$$Gdp_t - Gdp_t^* = -0.02 + 0.08 \left[ \sum_{m=0}^{\infty} 0.85^m \hat{G}_{t-m-1} \right]$$

(14)

While in the special regime, we observe:

$$Gdp_t - Gdp_t^* = -0.04 + 0.33 \left[ \sum_{m=0}^{\infty} 0.61^m \hat{G}_{t-m} \right] - 0.19 \left[ \sum_{m=0}^{\infty} 0.61^m \hat{W}_{t+m-1} \right]$$

(15)
In regime 1, the mean of the output gap responds to the past growth of real expenditure and oil proceeds, and changes in these variables might significantly affect the aggregate demand for approximately 8 quarters.

In regime 2, the mean of the output will tend to fall below its potential level, unless there is a boost in the expenses undertaken by the public sector. In this case, changes in the past values of the government expenditure might have an effect on the aggregate demand of around 38 quarters. The fact that in this regime oil proceeds no longer affect the aggregate demand might suggest that, during these periods, the government substantially changed the way in which such income is managed: that is, government expenditure becomes less pro-cyclical respect to oil income. This phenomenon could be particularly relevant in situations of economic instability or speculative attacks, when oil proceeds might be mainly financing capital outflows instead of turning into in domestic expenses\(^9\). Indeed, all but one of the episodes classified within this regime also belong to the inertial state detected in the inflation analysis. The exceptional episode we are referring to corresponds to 1997:04-1999:02, when the value of oil income fell importantly due to the plunge of oil prices, but the levels of domestic fiscal expenditure were kept high. Therefore, although this period cannot be labeled as one of economic instability or speculative attacks, it still represents a case in which the pro-cyclical relationship between the government expenditure and the oil income is broken.

Around the periods of the special regime (regime 3), not only the mean of output will significantly fall below its potential level, but also expected changes in the government expenditure will have tremendous effects on the aggregate demand. In fact, after the oil strike concluded, the government advocated to increasing the level of its spending, boosting the recovery of the economy. The sign associated to the expected flows of oil income might indicate that future increases in export proceeds were perceived as a mean to augment the potential output of the economy, and therefore reduce the aggregate demand pressures. As already mentioned, according to the classification of regimes provided by this model, this special period not only included the strike itself, but also the year following the strike, revealing the importance of this event in the economic dynamic.

\(^9\) Typically the way this occurs is by keeping the nominal exchange rate at which the government converts foreign exchange fiscal income into domestic currency fixed at a low level. In this way, although the oil income in dollars of the economy might be increasing, the government’s capacity to spend in domestic currency is weakened, especially in real terms.
V. Conclusions

In this paper we have analyzed the dynamic of the business cycle in Venezuela in the last twenty years through a Markov-switching estimation of a Phillips curve and an aggregate demand curve.

The characterization of the different regimes recognized by these two models is based on the nature of the relationship established between the endogenous variables (inflation rate and output gap) and the explanatory variables. This task, although it is more complicated than classifying the economy in states of “recession” and “expansion”, as it is typically done in the literature, reveals a richer approach to understand the dynamic of these important variables.

In the analysis of the inflation dynamic, the model basically distinguishes between a “normal” regime and a “prone to crisis” regime. The model fitted for the output gap recognizes as well two “normal” regimes versus a “special” one associated to the oil strike that took place in Venezuela. The main difference between these two “normal” hidden states is the change in the way the government allocates the oil income, and therefore in the factors that ultimately boost aggregate demand and economic growth in the short run.

The separate analysis of inflation and growth has shown common features in the recognition of regimes. Nonetheless, it is important to study these two variables jointly, as it is done in a work currently in progress.

References


