Interventions in the Foreign Exchange Market: Effectiveness of Derivatives and Other Instruments

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ABSTRACT

This paper discusses the effectiveness in Brazil of the traditional instruments of exchange rate interventions (spot interventions and interest rates) as well as instruments based on exchange rate derivatives (swaps and dollar indexed public bonds). We show that in periods of high volatility of the nominal exchange rate the instruments are not capable of significantly modifying the dynamics of the nominal exchange rate. In periods of low volatility of the nominal exchange rate, in contrast, both the traditional instruments and the derivative instruments are effective. These results are robust to the two techniques of estimation employed: GMM in continuous time and in discrete time.

Key Words: Central Bank, intervention in the foreign exchange market, foreign exchange derivatives

JEL E58, F31, E52
1. INTRODUCTION.

In January 1999, the Central Bank of Brazil adopted the floating exchange regime. Despite doing this, it continued to intervene in the foreign exchange market. The instruments of intervention were: interest rates, buying and selling dollars in the spot market and buying and selling public bonds indexed by the exchange rate and swap operations. A question naturally appears: were these instruments really effective?

This paper shows that the answer depends on the level of volatility of the nominal exchange rate. The answer is negative in periods where the volatility of the foreign exchange rate was high: first semester of 1999 and second semester of 2002. The answer, however, is positive in the other periods where this volatility is low.

The literature of interventions of the central bank in the foreign exchange market normally focus on only two types of instruments: interest rates and spot interventions. This paper contributes to the literature by studying the effectiveness of other types of instruments, exchange rate derivatives such as public bonds indexed by the foreign exchange rate and foreign exchange swaps.

A first step to analyze the effectiveness of the Central Bank instruments is to measure the effect of different instruments in the dynamics of the foreign exchange rate. To do this we specify a stochastic process of the foreign exchange rate, introduce the interventions of the Central Bank and finally estimate the process with and without the interventions.

We model these interventions following a vast literature of term structure of interest rates, that describes the interventions of the Central Bank in the open market when buying and selling public bonds. It chooses a continuous time process to model the dynamics of the interest rates and the interventions are considered as jump processes appended to the original diffusion process.

In an analogous way, we select a process in continuous time to model the dynamics of the foreign exchange rate and model the interventions of the Central Bank as corresponding jump processes. In this strategy, the instruments of intervention are effective if they are able to affect the conditional mean of the process of the foreign exchange rate. This criteria is frequently used as Sarno and Taylor (2001) and Edison (1993) point out.

To estimate the parameters of the dynamics of the foreign exchange rate we use the work of Hansen and Scheinkman (1995) that obtain moment conditions that allow the use of the Generalized Method of Moments (GMM). To test the robustness of the results we use a more common technique of estimation that transforms the continuous process in a discrete process (Chan et al (1993)).

Both techniques of estimation show that the interventions in the foreign exchange market are ineffective independent of the instrument used (spot, bonds indexed by the exchange rate and interest rates) in the periods of high volatility of the foreign exchange rate: after the change in the foreign exchange regime, the first semester of 1999 and in the period before the general election for president in the second semester of 2002. However, in the other periods the instruments are effective.

From these results we can extract two main lessons. As the interventions do not change the conditional expectations of the exchange rate in times of crisis in the foreign exchange market, the use of foreign exchange reserves and the of indebtedness in external currencies in these times is only justified if the interventions have other objectives other than

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1 See Sarno and Taylor (2001) for an excellent survey.
avoiding the depreciation of the foreign currency, for example to decrease the conditional volatility of the nominal exchange rate.

The second lesson derived from our results is that the demand for foreign exchange currency in Brazil is related to the demand for foreign exchange hedge that can be covered not only by foreign capital but also by public bonds indexed by the exchange rate or by foreign exchange derivatives.

The work that resembles ours most closely is Araújo and Goldfajn (2004) that analyses the impact of the interventions over the conditional volatility of the foreign exchange rate. Araújo and Goldfajn find that, considering the whole period from January 2000 to December 2003, the interventions help decrease the conditional volatility of the nominal exchange rate.

The rest of the paper is organized in the following manner. Section 2 discusses the models that we estimate. Section 3 describes the policy of interventions of the Central Bank in the second phase of the Real Plan and the data that we use. Section 4 discusses the two techniques used and and the empirical analyses, and Section 5 concludes.

2. The Dynamics of the Foreign Exchange Rate.

The main objective of this paper is to study the effectiveness of distinct instruments of intervention in the foreign exchange market. For this sake, we need a model that is sufficiently flexible to describe the dynamics of the nominal exchange rate with and without the intervention of the Central Bank. Although the literature about foreign exchange rates has not converged to a standard model for the dynamics of foreign exchange rate, a class of models based on continuous time has gained relevance in the last years. Among several advantages, with these models one can obtain closed expressions for the moments of the process that allow for statistical tests related to the impact of the interventions.

We model the dynamics of the depreciation of the exchange rate $X_t$, by the following process belonging to the class of diffusion processes with jumps:

$$dX_t = (\mu X_t + Ki_t) dt + \sigma X_t dW_t + \sum_f I_t^f X_t dJ_t^f,$$

where $i=$ interest rate (SELIC); $f =$ spot interventions $f =$ debt, if interventions are with bonds indexed by the foreign exchange rate, $f =$ swap if the interventions are foreign exchange swaps.

In an intuitive manner, the multiplicative term $dt$ is the predetermined trend of the process. This trend can be decomposed into two components: the first one reflects the impact of the depreciation of the nominal exchange rate in the instant immediately before. The second reflects the impact of the monetary policy of the Central Bank through the use of the nominal interest rate.

Two types of shocks disturb the predetermined trend. The first one, $\sigma X_t dW_t$, is not predictable. The second refers to the discontinuous interventions of the Central Bank in the foreign exchange market. These discontinuous interventions are modeled as Poisson processes. The Poisson processes are such that there is a probability $\lambda_f dt$, (f equal to the spot, debt, or swap) of occurring a discontinuous intervention, and once it occurs its intensity is know to be $I_t^f$, that is, the intervention is measured in units of the variation of the nominal exchange rate.

From equation (1), we have the following conditional expectation in $t$ of the depreciation of the nominal exchange rate:
\[ E_t[dX_t] = (\mu X_t + K_i + \lambda_{\text{spot}} X_t I_{\text{spot}} + \lambda_{\text{debt}} X_t I_{\text{debt}} + \lambda_{\text{swap}} X_t I_{\text{swap}}) dt, \quad (2) \]

Equation (2) shows that the depreciation of the nominal exchange rate is a function of the first difference of the lagged depreciation of the nominal exchange rate, the lagged nominal interest rate, the probability of interventions in the spot market, the probability of interventions in the public debt market indexed by the exchange rate, and the probability of interventions in the foreign exchange market.

In the case of the nominal interest rate, if the coefficient \( K \) is statistically significant and with a negative sign then we can state that the interest rate is effective. In an analogous way, if the probability associated with one of the non-continuous instruments is statistically significant than we can state that this instrument is capable of affecting the conditional expectation of the nominal exchange rate. For example, if in \( t \) the depreciation was 0.2% and the Central Bank supplies dollars in a value corresponding to R$10 Million and the probability of the foreign exchange intervention is \( 10^{-6} \), then the supply of foreign exchange by the Central Bank implies an increase in the appreciation of (or a decrease of expected depreciation) of 2.5%.

Note also that an effective policy, in the above sense, does not only affect the conditional expectation of the nominal exchange rate in the next period. Realizing that this expectation is affected by the interventions the market tries to infer the frequency and magnitudes. So as in Krugman (1991), Cadenillas and Zapatero (2000), Cadenillas and Zapatero (1999), and Jiang (1999), the policy of interventions affects all dynamics of the exchange rate.

From equation (2), we will test the capacity of the Central Bank to affect the conditional expectation of the depreciation of the nominal exchange rate. In these tests we will suppose that the market knows the decision process of the Central Bank that generates the Poisson process of interventions. That is, the process of interventions is a predetermined variable of the model. Using this hypothesis, an estimate significantly equal to zero – our null hypothesis- must be understood as evidence of the fact that the policy of interventions does not affect the conditional expectation of the depreciation of the nominal exchange rate instead of the more natural interpretation that the probability of intervention of the Central Bank is zero.

In addition to providing consistent interpretation (with the data) for the estimation of the probabilities of interventions of the Central Bank, this hypothesis of predetermining the policy of interventions allows us to ignore problems of endogeneity of the estimation of the parameters \( \lambda_i \) if as supposed in equation (1), the coefficients \( \mu \) e \( K \) do not change with the state variables that determine the interventions. Without these two hypotheses we would need to obtain instrumental variables for the policy interventions or estimate the reaction function of the Central Bank and the impact of the interventions as is done in Araújo and Goldfajn (2004).

In the next section we will describe the data we used to estimate the parameters of the model, in particular those related to the forms of intervention that signal the capacity of the Central Bank to affect the dynamics of the nominal exchange rate in the period from 1999 to 2002.

3. The Data.

To investigate the effectiveness of the Central Bank’s interventions in the foreign exchange market in the recent period after the change in the exchange rate regime we use daily data. Our whole sample goes from January 1999 to October 2006. There are 1977 observations in the whole period.
There are two important variables for our analysis of the Central Bank’s interventions in the spot market: the date and volume of intervention. Starting July 1999 the Central Bank started to announce the dates when it intervened in the spot market. If we started our analysis beginning in July 1999, though we would lose the period immediately after the change in the foreign exchange regime (January 1999 to July 1990), for which there is evidence of frequent interventions in the spot market. For example, from January 1999 to June 1999, the daily average of the series of the variation of the monetary base due to exchange rate transactions was 355 million Brazilian reals, while from July 1999 on the daily average was 66 million.

Therefore, we consider as dates of intervention all dates before July 1999 and from then on the dates in which the Central Bank stated that it intervened. Following these criteria, we identify 526 interventions, being the first semester of 1999 the one with the greatest number of interventions.

Unfortunately, the Central Bank does not indicate the intensity of its intervention. To estimate them we consider the first difference of the monetary base due to exchange rate operations. This series does not consider a series of operations of the Central Bank in the foreign exchange market that are clearly not interventions, for instance those with multilateral organizations. Graph 1 shows the data and volumes of dollars.

Aside from intervening in the spot market, the Central Bank can intervene in the dynamics of the exchange rate by selling bonds indexed by the exchange rate. Not considering the default risk, these bonds are substitutes for foreign exchange capital from the perspective of those that demand foreign exchange hedge. Therefore, the issuance of these bonds is an alternative way of intervening in the foreign exchange market. The total stock in the market of these bonds during our sample period is illustrated in graph 2. It is the sum of NBC-E and NTN-D in the market. The first bond is a liability of the Central Bank while the second one is a liability of the Treasury. Both remunerate a fixed interest rate plus the depreciation of the nominal exchange rate in the period.

We consider as interventions of the Central Bank first difference of the stock in the market of public indexed bonds superior in modulus to the mean of the series plus two standard deviations. Using this criteria we identify 482 interventions from January 1999 to October 2006. The period of greatest use of this instrument was from April 2001 to December 2001.

The third type of intervention we consider are foreign exchange swaps. The law of Fiscal Responsibility made it illegal for the Central Bank to issue its own bonds. The law started to be operative in the second semester of 2001. From then on the Central Bank started to look for alternative instruments to implement its foreign exchange policy. From April 2002 on the Central Bank started to sell LFT, that is a fixed interest bond, together with foreign exchange swaps. In these swaps the Central Bank was short in the depreciation of the nominal exchange rate and long in a floating interest rate, DI. The combination of LFT and foreign exchange swaps is equivalent to issuing bonds indexed by the exchange rate.

The swaps contracts are recorded in Brazil’s clearing of derivatives, BMF. In Graph 3 we consider the stock in the market foreign exchange swaps in the period from April 2002 to October 2006. We consider as an intervention with this instrument first differences of this stock series greater in modulus to the mean of this series plus two standard deviations. There are 122 interventions with this instrument.

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2 This is confidential data of the Central Bank of Brazil. The Open Market Department of the Central Bank made it available.

3 This is also confidential data of the Central Bank made available once more by the Open Market Department.
The last form of intervention that we consider is the use of the interest rates that, through its impact on the flow of foreign exchange capital, can affect the dynamics of the nominal exchange rate. Different from other forms of intervention this can be done in a continuous fashion. In the literature of interventions this form of intervention is known as the monetary channel of intervention or as a sterilized form of intervention.

The short-term nominal interest rate, SELIC, is the natural candidate to capture these interventions. As we can see from Graph 4, during great part of our sample period, the Central Bank implemented a policy of gradually decreasing the interest rate. The exception was the period immediately after the change in the foreign exchange regime, in which there was an impressive increase in the nominal interest rate.

The tests of Perron and Dick-Fuler reject the hypotheses of stationarity of the nominal interest rate in the period from 1999 to 2002 (the p-value of the Perron test is 0.45 and that of the Dick-Fuler test is 0.37). Therefore, we will use the first difference of SELIC to estimate the impact of the interventions through monetary policy. In fact, the augmented Perron Dick-Fuler tests do not reject the stationarity of the variation of the SELIC.

Graphs 1 to 4 show that the Central Bank chooses its instruments of intervention depending on the period. For example, during the change of the foreign exchange regime in 1999, the Central Bank uses with more intensity interventions in the spot market and the interest rate. In 2001, the Central Bank uses more public foreign indexed bonds, and in the second semester of 2002 it used more foreign exchange swaps. The interventions are more pronounced during the foreign exchange crisis in the first semester of 1999 and in the second semester of 2002, and practically non-existent in the year 2000 when Brazil’s economy did not suffer any relevant shocks.


4.1 Hansen and Scheinkman (1995) Method

The usual technique to estimate a stochastic process in continuous time, as in equation (1), is to discretize it. The new process is called a discretized process or an Euler approximation. However, the discretization is merely an approximation of the specification in continuous time. Lo (1988) shows by means of a counter-example that the estimator for maximum likelihood of the differentiated process is in general not consistent.

Taking into account this problem, Hansen and Scheinkman (1995) propose a method to estimate continuous processes, which abstracts from the discretization of the time interval and which explores properties of the stationary Itô processes, moments that are constructed from the concept of infinitesimal generator. Intuitively, the infinitesimal generator of a continuous stochastic process shows the inclination of the stochastic process at a determined point. Or rather, the generator is the tendency (or dragging) of the process at a certain point. For example, in the process of diffusion, \( dX_t = \mu dt + \sigma dW_t \), the infinitesimal generator of the original process, \( \mathcal{A}(X_t) \), is \( \mu \).

In the same manner, the infinitesimal generator applied to a function of the process provides the tendency of the function of this process at said point. By means of an appropriate choice of these functions, known as test functions, Hansen and Scheinkman (1995) obtain the moment conditions that permit the application of the Generalized Method of Moments (GMM) to estimate the parameters of the original process.

Hansen and Scheinkman (1995) choose only stationary test functions and prove that the infinitesimal generator of these functions satisfies the first class of the moments below. The intuition for that class is that the average of the inclinations of a stationary process should be zero. More formally,
where $\phi(X_t)$ is a test function and $A\phi(X_t)$ is the infinitesimal generator of the test function.

A second class of moment conditions from Hansen and Scheinkman (1995) arises from a property of the infinitesimal generator. The property is that the infinitesimal generator commutes with the conditional expected operator one period ahead. More precisely, Hansen and Scheinkman prove that:

$$E_t[A\phi(X_{t+1})] = AE_t[\phi(X_{t+1})].$$

To implement conditions (3) and (4), Hansen and Scheinkman (1995) write the moments conditions in the form of their equivalent samples and find the estimators using GMM. For models similar to ours (Equation (1)), the authors suggest as test functions $X, X^2$ e $X^3$, which generate six moment conditions to estimate all the models. We use these six conditions, in addition to imposing that the standard deviation of the process of the variation in the nominal exchange rate be unitary to avoid problems with identification when using the GMM technique. Appendix 1 discusses in greater detail the technique used in Hansen and Scheinkman (1995) and Appendix 2 derives the infinitesimal generators and the moments conditions for the estimated models in this study.

4.2 Estimation of the Dynamics of the Depreciation of the Nominal Exchange Rate by Hansen and Scheinkman (1995)

In this subsection, we estimate the dynamics of the nominal exchange rate (1), using the Hansen and Scheinkman (1995) method. We divide the sample period of January 1999 to October 2006 according to the existence (or not) of currency crises. The periods of currency crises are the first semester of 1999 and the second semester of 2002. The periods without currency crisis are those from July 1999 to April 2002 and from December 2002 to October 2006. As Table 1 shows, the $J$-statistic does not reject the specification of the model in any of the periods that we consider.

The results of Table 1 show that the effectiveness of the intervention instruments varies depending on the existence of currency crises. Columns (A) and (B) show evidence that during periods of currency crises, none of the instruments of intervention are effective in affecting the conditional expectation of the variation of the exchange rate. On the contrary, columns (C) and (D) confirm the relevance of the interventions in the spot market of foreign currencies, public bonds indexed to the dollar and currency swaps to affect the conditional expectation of the variation of the nominal exchange rate in the periods without currency crises. At least one of the forms is significant (both times) in the periods considered, as can be noted by the p-values of the $\hat{\lambda}$ probabilities associated with the Poisson jumps of the intervention.

The coefficient of the first difference of the lagged nominal exchange rate, $\mu$, when statistically significant, has a negative sign. This sign indicates a mean reversion behavior of the first difference of the nominal exchange rate, which is to be expected on a priori basis. That is, depreciation in one moment is followed by an appreciation or a lower depreciation an instant later.

What is curious is the behavior of the coefficient of the first difference of the interest rate. The coefficient is not significant in any period. Thus, the interest rate as an instrument is not effective in any of the periods considered. One possible explanation for this result is the fact that the Central Bank of Brazil uses this instrument in its inflation targeting system, which started to be implemented in Brazil in January 1999. Or rather, gradual changes in the
The basic interest rate were intended to alter the dynamics of the inflation rate and not the
dynamics of the nominal exchange rate.

In summary, the results presented in Table 1 clearly show that the Central Bank’s
instruments of intervention are not effective in periods of great volatility in the nominal
exchange rate. On the contrary, in periods of low volatility an instrument or a combination of
instruments is shown to be effective.

### 4.3 Test of Robustness Using the Method of Differentiation

In this section we investigate the robustness of the presented results with the Hansen
and Scheinkman (1995) method, estimating the processes using the most common GMM
technique based on the discretization of the process.\(^4\)

Let the stationary process in continuous time be described as:

\[
dX_t = (\mu X_t + K_i) dt + X_t dW_t + \sum I_t^f X_t dI_t^f
\]

We discretize this process according to the Euler approximation, dividing the time
interval in 252 parts (\( h = 1/252 \)), as follows: \(^5\)

\[
X_{t+1} - X_t = (\mu X_t + K_i) h + \sum I_t^f X_t (N_t^f - N_{t-1}^f) + \epsilon_{t+1},
\]

\[
\sum_f (N_t^f - N_{t-1}^f) = \begin{cases} 1 \text{ with probability } \lambda_f \\ 0 \text{ with probability } 1 - \lambda_f \end{cases}
\]

\( f = \text{ spot, indexed bonds or foreign exchange swaps} \)

In order to discretize the process (6), we must impose that:

\[
E[\epsilon_{t+1}] = 0 \quad \text{and} \quad \text{Var}(\epsilon_{t+1}) = X_t^2.
\]

As instruments for the estimation, we use the constant, the variation in the lagged
exchange rate, \( X_{t-1} \), and the variation in the lagged SELIC index, \( \Delta i_{t-1} \). The moment
conditions are that these instruments are orthogonal to the mean and variance of \( \epsilon_{t+1} \), as
follows:

\[
E \begin{bmatrix} \epsilon_{t+1} \\ \epsilon_{t+1} X_{t-1} \\ \epsilon_{t+1} \Delta i_{t-1} \\ \epsilon_{t+1}^2 - \sigma^2 X_t^2 \\ (\epsilon_{t+1}^2 - \sigma^2 X_t^2) X_{t-1} \\ (\epsilon_{t+1}^2 - \sigma^2 X_t^2) \Delta i_{t-1} \end{bmatrix} = 0.
\]

In Table 2 we present the results of the estimation with this technique. The results are
clearly in line with those we obtain using Hansen and Scheinkman (1995). They confirm, in

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\(^4\) See Chang et al (1993) for some examples of the use of this technique.

\(^5\) As shown in Lo (1988), this differentiation is not exact.
general terms, the relevance of the interventions in the spot market, in the market of public debt indexed by the exchange rate, and operations with currency swaps in periods without foreign exchange crises, and the ineffectiveness of the instruments of intervention in periods of currency crises. When significant, the coefficient of the lagged depreciation of the nominal exchange rate is negative, suggesting a mean reversion behavior, similar to that of the Hansen and Scheinkman (1995) estimation described earlier. Finally, the coefficient of the first difference of the interest rate is once again not significant in any period, which reveals that the use of the interest rate as an instrument of intervention was not effective in considered periods.

If we compare the results of Tables 1 and 2, we can observe that the Hansen and Scheinkman (1995) and the discretization technique diverge with respect to the effectiveness of public debt indexed by the exchange rate and the foreign exchange swaps in the period December 2002 to October 2006, a period without a currency crisis. For Hansen and Scheinkman (1995), the interventions through currency swaps were effective, whereas for the discretization technique the only effective instrument is public debt indexed by the exchange rate. This observed difference might be related to multicollinearity between the instruments and to the fact that they are close substitutes. In fact, we verify that the correlation between the series of the variations of stock in the market for public indexed bonds to dollars and the net supply of currency swaps is -0.81 between December 2002 and October 2006.

Taking into account the above observations, we re-estimate the model for the periods in which both currency swaps and public debt indexed to the dollar were used as instruments of intervention by the Central Bank. To do so, we sum the stock in the market for currency swaps in dollars with the stock in the market for public indexed bonds in dollars. We then consider as interventions of the Central Bank the variations in modulus greater than two standard deviations plus the mean of this new series. Using this criterion, we identify 198 interventions in the period of April 2002 to October 2006.

The results of the estimations for the two techniques are presented in Table 3. For periods of foreign exchange crises, columns (A) and (D), we see that the instruments of intervention continue to be ineffective. However, in periods without currency crisis, columns (B) and (E), the instruments are effective, and the instrument public debt indexed to the dollar and currency swaps is effective for both techniques (p-values of 0.0 for both techniques).

4.4 Estimation of Other Models

In the previous section we showed that the effectiveness of the instruments of foreign exchange policy is not an artifact of the Hansen and Scheinkman (1995) estimation method. The results are robust to a more common technique of discretization of the stochastic process in continuous time. However, there is still a risk that a bad specification of the stochastic process may have contaminated the two techniques of estimation analyzed. To investigate this possibility we consider alternative models for the dynamics of the variation of the nominal exchange rate.

Table 4 estimates again the basic model of the dynamics of the foreign exchange rate, eq (1), including the variation of the foreign exchange coupon in the trend of the process. Everything else constant, the greater the foreign exchange coupon the less the opportunity cost of holding bonds indexed by the exchange rate. However, as Table 5 shows both for Hansen and Scheinkman (1995) and for the discretization, the foreign exchange coupon is not significant for practically all periods.

We estimate two additional models that frequently appear in the literature about interventions: the standard Brownian geometric model and the diffusion model with the inclusion of the interest rate in the trend. Our interest when estimating these models is to
verify if the variables of the deterministic trend of the general model, without the foreign exchange swaps, continue to yield the same behavior we observed earlier.

To verify the relative importance of the behavior of these two models in relation to the models that includes the non-continuous interventions we followed Chan et al and realized an R test (generating a new R statistic), known as the Newey and West test.\(^6\)

Table 5 presents the results of the estimations of Hansen and Scheinkman (1995) and the discretization for the Brownian geometric model. What is most relevant in these results is the fact that the coefficient of the lagged variation of the foreign exchange rate, \(\mu\), is statistically significant in almost all periods and has a negative sign. This is in line with the results we obtained earlier and confirm a mean reversion behavior.

In Table 6 we show the results of the estimation of the diffusion model with the inclusion of a trend. The coefficient for the variation of the interest rate is never significant and the mean reversion behavior is still present, in accordance with both techniques.

Finally, we verify the predictive capacity of the five models we estimate using the two techniques. To do so we discretize the processes estimated using the Hansen and Scheinkman (1995) technique. We observe both the forecast within the sample by means of the R2 and also the out-of-sample forecast, by means of the Theil inequation coefficient and the decomposition of the mean quadratic error of the forecast. To save space we do not report the results but they confirm in general terms that the models were not good in these forecasts. The models estimated by Hansen and Scheinkman (1995) were slightly better than the models estimated by differentiation.

### 4.5 Structural Breaks

In our previous analyses we assumed that in the period between foreign exchange crises there are no structural breaks that could justify the significant changes in the dynamics of the foreign exchange rate. However, during the period between January 1999 and April 2002, several shocks occurred in the Brazilian economy, such as the energy crisis and the September 11 crisis, which could have altered these dynamics. To verify if the dynamics are affected by these shocks we repeat the estimation for the period between foreign exchange crises, dividing the period in several sub-periods, for example the second semester of 1999, the years 2000 and 2001, the period from January 2002 to October 2006.

The results of the estimations for the two techniques are presented in table 7. The results show that the instruments of intervention, with exception of the interest rate, continue to be effective in periods without currency crises, and in fact the shocks that occurred in the economy did provoke structural breaks in the dynamics of the exchange rate, as indicated by the estimated coefficients of the parameter \(\mu\), which measures the mean reversion of the variation of the nominal exchange rate and the variation of the interest rate, \(K\).

### 4.6 Stationarity of the Interest Rate

Despite the fact that the tests of Perron and Dick-Fuler reject the hypothesis of stationarity of the SELIC rate in the period from January 1999 to December 2006, there are arguments that suggest that it is unlikely that SELIC is not stationary. For example, non-stationarity means that with probability one that the nominal interest rate will be infinite in a finite interval time. This is quite unlikely in economic terms.

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\(^6\) The R statistic is defined as: \(T [\text{simplified J model} – \text{complete J model}]\), where \(T\) is a number of observations. The statistic is \(\chi^2\) with the number of degrees of freedom equal to the difference between the number of parameters of the complete model and the number of parameters in the simplified model. High values for R indicate that the simplified model is poorly specified in relation to the complete model.
Based on this argument, we re-estimate the previous models in all periods using the Hansen and Scheinkman (1995) and discretization techniques, substituting the variance of the interest rate for its actual value.

In general, the previous results continue to be valid. In particular, we observe that the interest rate instrument is not effective in almost all periods including the periods of high volatility in the nominal exchange rate, and that the other instruments of intervention in the foreign exchange market are effective in periods of low volatility of the of the nominal exchange rates and not in periods of high volatility.

4.7 The Discretization Technique and the Choice of Other Instruments

In the estimation based on discretization technique, we used the first difference of the nominal exchange rate as one of the instruments to build orthogonal conditions. Due to the fact that this instrument may be correlated to the estimation error if is shows any sign of serial correlation, said instrument may not satisfy orthogonal conditions with the error, making GMM inconsistent.

Taking into consideration this potential problem, we repeat the GMM estimations using the discretization technique for all models and in all periods in which we observed autocorrelations in the residuals. When repeating the estimations, we use the first difference of the contemporary interest rate as an instrument in place of the depreciation of the lagged nominal exchange rate. As this new variable is not correlated to the error of the depreciation of the nominal exchange rate, given that it is used as a control variable in the inflation targeting system, it guarantees that the GMM estimator is consistent and permits that the Newey West standard errors take into consideration eventual autocorrelations of residuals in the statistical tests.

In general, the results of section 4.3 continue to be valid. In particular, the instruments of intervention in the foreign exchange market are effective only in periods without foreign exchange crises. The only differences observed are that, in some periods between currency crises, the estimated coefficient for the variation of the interest rate is negative and significant. Or rather, the interest rate becomes an effective instrument to reduce the conditional expectation of the currency depreciation outside periods of crisis.

4. Conclusion

This paper analyses the effectiveness of the interventions of the Central Bank of Brazil in the foreign exchange market during the period following the currency devaluation in January 1999. We estimate several models in continuous time of the depreciation of the nominal exchange rate, using two distinct econometric techniques, GMM according to Hansen and Scheinkman (1995), and GMM based on the differentiation process.

Both techniques show that, in periods of foreign exchange crises, the first semester of 1999 and the second semester of 2002, none of the instruments of intervention are capable of affecting the conditional expectation of the depreciation of the nominal exchange rate. However, spot market interventions, interventions by bonds indexed by the exchange rate and by swaps proved to be effective in other periods.

The main lesson that we take away from this study, therefore, is that in periods of high volatility (e.g. currency crises), interventions in the spot market are not justifiable as attempts to alter the conditional expectation of the equilibrium nominal exchange rate, despite the possibility that the interventions may diminish the conditional volatility of the nominal exchange rate, as shown by Araujo and Goldfajn (2004).

One relevant question that is not discussed in this paper is the analysis of the relative effectiveness of the instruments, that is, of the cost-benefit relationship of these interventions. Discussions surrounding this question involve a definition of a loss function of the Central
Bank in the foreign exchange market and an optimal choice of instruments of intervention in order to minimize the loss function. The definition of the loss function involves, among other questions, a discussion of whether the objective of the Central Bank is to reduce the volatility or the conditional average of the nominal exchange rate, or analyze that imperfections in the Brazilian foreign currency market justify the supply of foreign exchange derivatives by the Central Bank.

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Appendix 1


A.1.1 Introduction.

The Hansen and Scheinkman (1995) technique is different from the usual practice of GMM because of the following reasons: the asymptotic identification is not obvious; the identification in small sample, uniqueness and the existence of an estimator is not an autonomous problem with respect to the asymptotic identification; the lack of asymptotic identification is such that implies lack of small sample identification.

The Hansen and Scheinkman (1995) paper can be divided in two parts. In the first part, the author’s intent is to characterize the infinitesimal generator and to find the moment conditions related to this generator. In the second part, the authors analyze asymptotic proprieties of the infinitesimal generator and find moment conditions that assure that the law of large numbers and the central limit theorem apply to processes whose samples are discrete and obtained in regular intervals. In this appendix, we will discuss questions related to the first part of the paper.

Hansen and Scheinkman (1995) is used for the estimation of markov processes of strictly stationary continuous time processes, observed in regular discrete intervals whose frequencies one normalizes to one.

A1.2 Concept of an Infinitesimal Generator.

(A1.2.1) Definition of $L^2(Q)$: Let $Q$ be the true distribution of probability of the stationary stochastic process $X_t$ defined over $\mathbb{R}^n$: $L^2(Q) = \{\text{functions } \mathbb{R}^n \rightarrow \mathbb{R}, \text{Q square integrable}\}$

(A1.2.2) Definition of a Semi group $T_t$: $T_t$, $t \geq 0$ is a semi group over $L^2(Q)$ if and only if:

i) $T_t$ is a linear operator $L^2(Q)$, $t \geq 0$

ii) $T_{t+s} = T_tT_s$, $s \leq t \geq 0$

Given a semi group $T_t$ over $L^2(Q)$, we can define:

(A1.2.3) Definition of a set of test functions $\phi$: $D = \{\phi \in L^2(Q): \lim_{t \to 0} \frac{T_t\phi - \phi}{t}, \exists \phi \}$

(A1.2.4) Definition of a infinitesimal generator $A\phi$: $A\phi = \lim_{t \to 0} \frac{T_t\phi - \phi}{t}, \forall \phi \in D$

The infinitesimal generator defined in (A1.2.4) describes locally the stationary stochastic process in continuous time. The motivation for the moment conditions based on the local behavior of the continuous process $X_t$ characterized by the infinitesimal generator is derived from the fact that this behavior is sometimes specified in the first place. Normally, we start with the stochastic differential that must be satisfied for $X_t$. There is a known correspondence between the coefficients of the equation and the infinitesimal generator.

The main advantage to characterize the moment conditions in terms of infinitesimal generators is the fact that it is not necessary for computing the sample analog that transition functions of the process be found. This would require that partial differential equations be solved or conditional density functions of the process be built.

A1.3 Infinitesimal Generator Characteristics and Other Definitions.

(A1.3.1) Continuity: $\forall \phi \in L^2(Q), T_t\phi \to \phi, \forall \phi \in D, t \to 0$
We will consider strictly stationary markov processes with marginal distribution \( q \) and semi groups or correspondent transition functions\(^7\) \{\( X_t, t \geq 0 \)\}:

\( (A1.3.2) \) Definition of \( T_t \) e \( T_t^* \)

\[
T_t \phi = E[\phi(X_{t+t})|X_s = y] \\
T_t^* \phi = E[\phi(X_s)|X_{t+t} = y]
\]

\( \Phi \) is the true infinitesimal generator of the process \( X_t \), using the semi group \( T_t \) above.

\( A1.4 \) Moment Conditions.

The moment conditions suggested by Hansen and Scheinkman (1995) for the true candidates of the true infinitesimal generator of the continuous stochastic process are:

\( (A1.4.1) \) (C1) \( E^Q [\hat{\Phi}(X_0)] = 0, \nabla \Phi \in \hat{D} \)

\( (A1.4.2) \) (C2) \( E^Q [\hat{\Phi}(X_{t+1}) \Phi^*(X_t) - \Phi(X_{t+1})\hat{\Phi}^*(X_t)] = 0, \nabla \Phi \in \hat{D} \)

\( \hat{\Phi} \) is the candidate generator, \( \hat{\Phi}^* \) is the joint operator of \( \hat{\Phi} \) and \( \hat{D} \) is a set candidate of test functions and \( Q \) is the distribution of the true process.\(^8\)

Condition (C1) above explores the informational content of the generator candidate. The second condition eliminates the candidate whose marginal distributions are incorrect. The formal demonstrations for the two conditions can be found on the paper.

The inspiration or intuition for the first condition comes from the fact that \( X_t \) is a stationary process. In particular, it must be true that:

\( (A1.4.3) \) \( E[\hat{\Phi}(X_t)] = E[E[\hat{\Phi}(X_t)|X_0] = E[T_t \hat{\Phi}(X_0)] \)

\( (A1.4.4) \) \( \frac{d}{dt} E[\hat{\Phi}(X_t)] = 0 \Rightarrow \frac{d}{dt} E[T_t \hat{\Phi}(X_0)] = E[A \hat{\Phi}(X_0)] = E[A \hat{\Phi}(X_t)] \)

\( A1.5 \) Identification.

The moment conditions are in the ideal format to be replicated by their sample counterparts. However, to understand the identification problem it is convenient to obtain a set of equivalent conditions. We show that:

\( (A1.5.1) \) Proposition (P1): \( X_t \) satisfies (C1) \( \iff \hat{Q} = Q \), where \( \hat{Q} \) is a candidate stationary distribution

\(^7\) In our paper this process is the depreciation of the nominal exchange rate.

\(^8\) \( L^2(Q) \) is a Hilbert space. \( \hat{\Phi} : L^2(Q) \rightarrow L^2(Q) \) is a linear bounded operator. Let \( \Phi \) be a fixed point of \( L^2(Q) \). In can be shown because of the riez representation theorem (see Naylor e Sell (1982) ) that there is only one \( \Phi^* \) such that : \( <\hat{\Phi}x, \Phi^*> =<x, \Phi^*> \), where \( <,> \) is the internal product defined as \( L^2(Q) \) and \( \Phi^* = \hat{\Phi}^* \Phi \). \( \hat{\Phi} \) is the joint operator and is unique.
Proof

\( \Leftarrow \)

let us suppose that \( Q \) is a stationary distribution and \( \hat{Q} = Q \). Then, \( \forall \phi \in L^2(Q) : \)

\[
\text{(A1.5.1.1)} \quad E[T_t(\phi(X_0))] = E[E[\phi(X_t)|X_0]] = E[\phi(X_0)] = E[\phi(X_0)]
\]

\[
\text{(A1.5.1.2)} \quad E[\hat{A}(\phi(X_t))] = E[\hat{A}(\phi(X_t) | X_0)] = E[\hat{A}(\phi(X_0))] = \lim_{t \to \infty} \frac{E[T_t(\phi(X_0))] - E[\phi(X_0)]}{t} = 0
\]

\( \Rightarrow \)

To show the converse, consider the markov process, \( \{X_t, t \geq 0\} \), initialized with a distribution \( \hat{Q} \) and having transition functions \( T e T^* \). We need to show that if \( \hat{A} \) satisfies (C1) then it has a stationary distribution \( Q \).

\( \text{(A1.5.1.3)} \quad \int \hat{T}_t \phi ds \in D \)

\( \text{(A1.5.1.4)} \quad \hat{A} \int \hat{T}_t \phi ds = T \phi - \phi \)

Integrating the right side of the above equation with respect to \( Q \) we have:

\( \text{(A1.5.1.5)} \quad \int \hat{T}_t \phi ds Q = \int \hat{A}[\int \hat{T}_t \phi ds] dQ = E[\hat{A} \int \hat{T}_t \phi ds] = 0 \)

In particular the above equation is valid for \( \phi = 1_B \) Borean in \( R^n \). Opening the first left term we have: \( \hat{Q}(B) - Q(B) = 0, \hat{Q} \) is the unconditional distribution of \( X_t \). C.Q.D

\( \text{(A1.5.2) Proposition (P2):} \ X_t \text{ satisfies (C2) } \Leftrightarrow \hat{A}T_t(\phi(X_0)) = T_t \hat{A}(\phi(X_0)), \text{ or } \hat{A} e T_t \) commute

Proof

\( \Rightarrow \)

Let us suppose that \( \hat{A} \) satisfies (C1), therefore it exists as a stationary distribution \( Q \) and \( T_t, t \geq 0 \) is a semi group in \( L^2(Q) \). \( \forall \phi \in \hat{D}, \phi^* \in \hat{D}^* : \)

\( \text{(A1.5.2.1)} \quad E[\hat{A}(\phi(X_{t+1}))(\phi^*(X_{t+1}))] = E[E[\hat{A}(\phi(X_{t+1})| X_{t+1}))(\phi^*)] = E[T_t \hat{A}(\phi(X_t))(\phi^*)] = <T_t \hat{A}, \phi, \phi^*>^9 \)

\(^9 \leftrightarrow \text{Scalar product} \)
Therefore (C2) can be written as:

\[ (A1.5.2.3) \quad <T_i \hat{A} \phi, \phi^* > = <T_i \hat{A}^* \phi, \phi^* >, \quad \forall \phi \in \hat{D}, \phi^* \in \hat{D}*. \]

Therefore,

\[ (A1.5.2.4) \quad <\hat{A} T_i \phi, \phi^* > = <T_i \phi, \phi^* >, \quad \forall \phi \in \hat{D}, \phi^* \in \hat{D}*. \quad \text{This equality is only valid if} \]

\[ \forall \phi \iff A e T_i \text{ commute.} \]

\[ \iff \]

\[ A e T_i \text{ commute in } \hat{D} \text{ then:} \]

\[ (A1.5.2.5) \quad <T_i \hat{A} \hat{A} \phi, \phi^* > = <\hat{A} T_i \hat{A} \phi, \phi^* >, \quad \forall \phi \in \hat{D}, \phi^* \in \hat{D}*. \quad \text{C.Q.D.} \]

Among the candidates with the right distribution condition P2 eliminates the one that do not commute with the true generator T1. The true infinitesimal generator commutes with T1. Therefore, the candidate of infinitesimal generator must also commute. The two conditions must be used together for estimation and identification.

The identification may not be complete. In particular, KA commutes with T1. If there are multiple candidates with the same true marginal distribution and with infinitesimal generators that are multiples of one another then they must be equivalent.

(A1.5.3) Definition of markov reversible processes: markov continuous reversible processes are those in which A=A*, that is the infinitesimal generator is equal to its adjoint.

Hansen and Scheinkman (1995) show that for reversible processes there does not exist the phenomenon of aliasing. This phenomenon corresponds to the fact that different continuous stochastic processes seem identical when estimated by discrete samples. Diffusion scalar processes are reversible. However, processes that are a combination of diffusion and jump processes are not reversible. (Duffie e Glynn (2001))

A.1.6 Infinitesimal Generator of Jump-Diffusion Processes.

We use as candidates for the infinitesimal generators of a jump diffusion process the sum of the candidates of infinitesimal generators of a diffusion process and the candidates of infinitesimal generators of the jump processes. (Duffie e Glynn (2001))

(A1.6.1) Proposition (P3): The candidate infinitesimal generator above satisfies the moment conditions specified by Hansen as Scheinkman (1995)

Proof
Let

\[ A\phi = A'\phi + A''\phi \]

\( A'\phi \) is the infinitesimal generator of the diffusion process

\( A''\phi \) is the infinitesimal generator of the jump process

We will show that the infinitesimal generator satisfies conditions (P1) and (P2)

(P1)

\[ E[ A'] = 0; E[ A''] = 0, \text{ thus } E = [ A' + A''] = 0 \]

We must show that \( AT_1 = T_1A \)

\[ A = A' + A'' \]

Therefore, \( AT_1 = A'T_1 + A''T_1 = T_1A' + T_1A'' = T_1[A + A'' \]

The first equality comes from the linearity of the infinitesimal generator and the second equality derives from the fact that \( A' \) and \( A'' \) are the infinitesimal generators of the diffusion and jump processes respectively. The third equality derives from the linearity of \( T_1 \). The extension to processes with more than one jump process is trivial.

We modeled the jump process as Merton (see Jinag (1998)): there is a probability that a jump occurs, and if it occurs its intensity is a random variable whose probability density function is lognormal.

(A1.6.2) Infinitesimal generators of continuous, ergodic diffusion processes: \( dX_t = \mu(X_t)dt + \sigma(X_t)dW_t, \mu(X_t), \sigma(X_t), dW_t \in \mathbb{R} \)

\[ A(X_t) = \mu \frac{\partial}{\partial X} (X_t) + \frac{1}{2} \sigma^2 \frac{\partial^2}{\partial X^2} (X_t) \]

(A1.6.3) Infinitesimal generators of jump processes with probability density functions of a Merton type:

\[ A\Phi(X_t) = \lambda \frac{1}{\sqrt{2\pi} s} \int (\phi(X + Y) - \phi(X)) e^{-\frac{(y-\mu)^2}{2\sigma^2}} dy \]


We start with a parametric family of candidates, supposedly including the true candidate. We choose test functions and estimate the parameters of interest completely identifies by its sample counterparts. We find the six moment conditions to estimate the parameters of interest in all models presented in section 3.
A1.8 The Choice of Test Functions.

Hansen and Scheinkman (1995) suggest a simple polynomial for the scalar diffusion with polynomials drifts. Duffie e Glynn (2001) also suggest simple polynomials for jump-diffusion processes in which the Laplace transform of the jump probability is known. This is the case of the processes we are modeling. Considering these facts we chose as test functions the simple polynomials $X$, $X^2$ e $X^3$.\(^{10}\)

The intuition for the choice of such functions is related to the fact that the drift of the process is a simple polynomial of the first degree. As the infinitesimal generator describes the process locally, infinitesimal generators built from these polynomials seem to be more adequate.

In appendix 2 we show the infinitesimal generators for each test function as well as the respective moment conditions. To find the moment conditions we use the sample counterparts of propositions (P1) e (P2) above.
Appendix 1 – Infinitesimal Generators of Diffusion Processes

1.1) Infinitesimal generator of diffusion processes

\[ \mu(X) = (\mu X + K_i + K_i c) \]
\[ \sigma(X_i) = \sigma X_i \]

Test Functions: \( \phi \in [X, X^2, X^3] \)

Test Function \( X \)

\( (A.1) A \phi = (\mu X + K_i + K_i c) \)

Test Function \( X^2 \)

\( \phi' = 2X \)
\( \phi'' = 2 \)

\( (A.2) A \phi = 2\mu X^2 + 2KXi + 2XKic + \sigma^2 X^2 \)

Test Function \( X^3 \)

\( \phi' = 3X^2 \)
\( \phi'' = 6X \)

\( (A.3) A \phi = 3\mu X^3 + 3X^2 Ki + KicX^2 + 3\sigma^2 X^3 \)

1.2) Infinitesimal generator of jump processes

\( f = \text{spot(spot market of exchange rate), debt or swap} \)

Test Function \( X \)

\( (A.4) A \phi = \lambda_f \frac{1}{\sqrt{2\pi v_f}} \int \left( (X + Y) - X \right) e^{-\frac{(y-y_f)^2}{2v_f}} dy = \lambda_f \mu_f \)

Test Function \( X^2 \)

\( (A.5) A \phi = \lambda_f \frac{1}{\sqrt{2\pi v_f}} \int \left( (X + Y)^2 - X^2 \right) e^{-\frac{(y-y_f)^2}{2v_f}} dy = 2X\lambda_f \mu_f + \lambda_f \mu_f^2 + \lambda_f v_f^2 \)

Test Function \( X^3 \)

\( (A.6) A \phi = \lambda_f \frac{1}{\sqrt{2\pi v_f}} \int \left( (X + Y)^3 - X^3 \right) e^{-\frac{(y-y_f)^2}{2v_f}} dy = 3X^2 \lambda_f \mu_f + 3\lambda_f X (\mu_f^2 + v_f^2) + 3v_f^2 \mu_f + \mu_f^3 \)

1.3) Class of Moment Conditions (C1): \( E[\phi] = 0 \)

Test Function \( X \)

\( (A.7) E[ (\mu X + K_i + K_i c) + \lambda_{\text{spot}} \mu_{\text{spot}} + \lambda_{\text{debt}} \mu_{\text{debt}} + \lambda_{\text{swap}} \mu_{\text{swap}} ] = 0 \)
Test Function $X^2$

\[(A.8)\] \(E[2\mu X^2 + 2KK_i + 2KKic + \sigma^2 X^2 + \sum_{f\in\{\text{spot,debt,swap}\}} 2X\lambda_j \mu_j + \lambda_j \mu_j^2 + \lambda_j v_j^2] = 0\)

Test Function $X^3$

1.4) Moment conditions of class (C2) : $T_1A = AT_1$

Tets function $X$

\[(A.9)\] \(E_t[\mu(X_{t+1} - X_t)] + K(i_{t+1} - i_t) + K'(i_{t+1} - ic_t) + \sum_{f\in\{\text{spot,debt,swap}\}} \lambda_f (Y_{f_{t+1}} - Y_{f_t}) = 0\)

Test Function $X^2$

\[(A.10)\] \(E_t[2\mu X^2_{t+1} + 2KK_{t+1}i_{t+1} + X_{t+1}K'ic_{t+1} + \sigma^2 X^2_{t+1} + \sum_{f\in\{\text{spot,div,swap}\}} 2X_{t+1}\lambda_f \mu_f + \lambda_f v_f^2 + \lambda_f \mu_f^2] = (\mu\mu_t + Ki_t + K'ic_t E_t[2X_{t+1}]) + 2\sigma^2 X^2_t + \sum_{f\in\{\text{spot,debt,swap}\}} \lambda_f (2E_t[X_{t+1}Y_{f_{t+1}}] + E_t[Y_{f_{t+1}}^2])\)

Test Function $X^3$

\[(A.11)\] \(E_t[3\mu X^3_{t+1} + 3X^2_{t+1}Ki_{t+1} + 3K'X^2_{t+1}ic_{t+1}] + \sum_{f\in\{\text{spot,div,swap}\}} E_t[3X^2_{t+1}\lambda_f \mu_f + 3\lambda_f X_{t+1}(v_f^2 + \mu_f^2)] = (\mu X_{t+1}K'ic_t E_t[3X^2_{t+1}] + 6E_t[\sigma^2 X^3_{t+1}] + \sum_{f\in\{\text{spot,debt,swap}\}} \lambda_f (3E_t[X^2_{t+1}Y_{f_{t+1}}] + 3E_t[X^2_{t+1}Y_{f_{t+1}}^2] + E_t[Y_{f_{t+1}}^3])\)

\(Y_{f\in\{\text{spot,debt,swap}\}}\) are the correspondent sample counterparts of the interventions
Graph 1 - Spot Interventions (Millions $)

Jan-99  
Mar-99  
Jun-99  
Aug-99  
Oct-99  
Dec-99  
Mar-00  
Apr-00  
Jun-00  
Jul-00  
Aug-00  
Sep-00  
Oct-00  
Nov-00  
Dec-00  
Jan-01  
Feb-01  
Mar-01  
Apr-01  
May-01  
Jun-01  
Jul-01  
Aug-01  
Sep-01  
Oct-01  
Nov-01  
Dec-01  
Jan-02  
Feb-02  
Mar-02  
Apr-02  
May-02  
Jun-02  
Jul-02  
Aug-02  
Sep-02  
Oct-02  
Nov-02  
Dec-02  
Jan-03  
Feb-03  
Mar-03  
Apr-03  
May-03  
Jun-03  
Jul-03  
Aug-03  
Sep-03  
Oct-03  
Nov-03  
Dec-03  
Jan-04  
Feb-04  
Mar-04  
Apr-04  
May-04  
Jun-04  
Jul-04  
Aug-04  
Sep-04  
Oct-04  
Nov-04  
Dec-04  
Jan-05  
Feb-05  
Mar-05  
Apr-05  
May-05  
Jun-05  
Jul-05  
Aug-05  
Sep-05  
Oct-05  
Nov-05  
Dec-05  
Jan-06  
Feb-06  
Mar-06  
Apr-06  
May-06  
Jun-06  
Jul-06  
Aug-06  
Sep-06  
Oct-06  
Nov-06  
Dec-06

Y-axis:
- 0.000
- 0.500
- 1.000
- 1.500
- 2.000
- 2.500
Graph 2 - Public Debt indexed by the Exchange Rate Interventions
Graph 3 - Swap Interventions
Table 1

Estimation using Hansen and Scheinkman (1995) of the model:

\[ dX_t = (\mu X_t + K_i)dt + X_t dW_t + I_t^\text{spot} X_t dJ_{t}^\text{spot} + I_t^\text{debt} X_t dJ_{t}^\text{debt} + I_t^\text{swap} X_t dJ_{t}^\text{swap} \]

The horizon of estimation extends from January 1999 to October 2006. The data are daily. The periods of estimation are: the foreign exchange crisis, first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 through April 2002, December 2002 to October 2006, and the entire sample period. We fix \( \sigma = 1 \) as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: \( \mu, K, \lambda_{\text{div}}, \lambda_{\text{vista}}, \lambda_{\text{swap}} \). In parenthesis are the p-values. The moment conditions are obtained with two classes of moment conditions (C1) e (C2), described in the text, using \( X, X^2 \) e \( X^3 \) as test functions. The J-statistic is \( \chi^2(2) \) in the estimations related to columns (A) e (C) and \( \chi^2(1) \) in the estimations related to columns (B), (D) e (E). Presented below the statistics, in parentheses, are the probabilities of not rejecting the model specification.

<table>
<thead>
<tr>
<th>Dependent Variable: First Difference of the Nominal Exchange Rate</th>
<th>Foreign exchange crisis</th>
<th>No Foreign exchange crisis</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend coefficient for the first difference of the lagged nominal exchange rate ( (\mu) )</td>
<td>-0.53 (0.0)</td>
<td>-0.0041 (0.99)</td>
<td>-0.35 (0.02)</td>
</tr>
<tr>
<td>Trend coefficient of the first difference of the interest rate ( (K) )</td>
<td>-2.77 (0.70)</td>
<td>15.9 (0.99)</td>
<td>-11.30 (0.47)</td>
</tr>
<tr>
<td>Probability of intervention by indexed bond ( (\lambda_{\text{div}}) )</td>
<td>0.009 (0.40)</td>
<td>0.13 (0.57)</td>
<td>0.6 (0.75)</td>
</tr>
<tr>
<td>Probability of intervention in the spot market ( (\lambda_{\text{vista}}) )</td>
<td>0.023 (0.63)</td>
<td>0.19 (0.98)</td>
<td>0.021 (0.01)</td>
</tr>
<tr>
<td>Probability of intervention in the swap market ( (\lambda_{\text{swap}}) )</td>
<td>N.A</td>
<td>0.0 (0.96)</td>
<td>N.A</td>
</tr>
<tr>
<td>Observations</td>
<td>120</td>
<td>126</td>
<td>730</td>
</tr>
<tr>
<td>J-Statistic</td>
<td>0.05 (0.98)</td>
<td>0.05 (0.99)</td>
<td>0.005 (0.99)</td>
</tr>
</tbody>
</table>
Estimation by Discretization of the Model:
\[ dX_t = (\mu X_t + Ki_t)dt + X_t W_t + I_t^{\text{spot}} X_t dJ_t^{\text{spot}} + I_t^{\text{debt}} X_t dJ_t^{\text{div}} + I_t^{\text{swap}} X_t dJ_t^{\text{swap}} \]

The horizon of estimation goes from January 1999 to October 2006. The data are daily. The periods of estimation are: the foreign exchange crisis, the first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 through April 2002, December 2002 to October 2006, and the entire sample period. We fix \( \sigma = 1 \) as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: \( \mu, K, \lambda_{\text{div}}, \lambda_{\text{à vista}}, \lambda_{\text{swap}} \). In parenthesis are the p-values. The moment conditions are obtained using as instruments the constant, the first lag of the nominal exchange rate and the first lag of the variation of the interest rate. The J-statistic is presented at the end, and presented in parentheses are the probabilities of not rejecting the model specification.

### Table 2

**Dependent variable: First Difference of the Foreign Exchange Rate**

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Foreign exchange crisis</th>
<th>No foreign exchange crisis</th>
<th>Whole sample</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A)</td>
<td>(B)</td>
<td>(C)</td>
</tr>
<tr>
<td>Trend coefficient for the first difference of the lagged nominal exchange rate (( \mu ))</td>
<td>0.68 (0.42)</td>
<td>-0.18 (0.76)</td>
<td>-0.79 (0.0)</td>
</tr>
<tr>
<td>Trend coefficient of the first difference of the interest rate (K)</td>
<td>2.92 (0.53)</td>
<td>-0.17 (0.95)</td>
<td>-1.72 (0.50)</td>
</tr>
<tr>
<td>Probability of intervention by indexed bond (( \lambda_{\text{div}} ))</td>
<td>0.03 (0.69)</td>
<td>0.04 (0.22)</td>
<td>0.012 (0.15)</td>
</tr>
<tr>
<td>Probability of intervention in the spot market (( \lambda_{\text{à vista}} ))</td>
<td>0.0099 (0.66)</td>
<td>0.16 (0.84)</td>
<td>0.05 (0.0)</td>
</tr>
<tr>
<td>Probability of intervention in the swap market (( \lambda_{\text{swap}} ))</td>
<td>N.A</td>
<td>0.0 (0.79)</td>
<td>N.A</td>
</tr>
<tr>
<td>Observations</td>
<td>120</td>
<td>126</td>
<td>730</td>
</tr>
<tr>
<td>J-Statistic</td>
<td>0.05 (0.96)</td>
<td>0.03 (0.98)</td>
<td>0.01 (0.99)</td>
</tr>
</tbody>
</table>
Table 3

Estimation by Hansen and Scheinkman (1995) and Discretization of the Model:

\[ dX_t = (\mu X_t + K_t) dt + X_t \, W_t + I_t^{\text{spot}} \, X_t \, dJ_t^{\text{spot}} + I_t^{\text{debt + swap}} \, X_t \, dJ_t^{\text{debt + swap}} \]

The horizon of estimation goes from January 1999 to October 2006. The data are daily. The periods of estimation are: the foreign exchange crisis, the first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 to April 2002, December 2002 to October 2006, and the entire sample period. We fix \( \sigma = 1 \) as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: \( \mu, K, \lambda_{\text{div}}, \lambda_{\text{spot}}, \lambda_{\text{swap}} \). In parenthesis are the p-values. The moment conditions for Hansen and Scheinkman (1995) are obtained with two classes of moment conditions (C1) e (C2), described in the text, using \( X, X^2, X^3 \) as test functions. The moment conditions for the discretization are obtained using as instruments the constant, the first lag of the nominal exchange rate and the first lag of the variation of the interest rate. The \( J \)-statistic is presented at the end and is described as \( \chi^2(2) \) in the estimations related to columns (A) e (C) and \( \chi^2(1) \) in the estimations related to columns (B), (D) and (E). Below the statistics, presented in parentheses, are the probabilities of not rejecting the model specification.

<table>
<thead>
<tr>
<th>Independent variables</th>
<th>Hansen and Scheinkman</th>
<th>Discretization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Foreign exchange crisis</td>
<td>No foreign exchange crisis</td>
</tr>
<tr>
<td>Trend coefficient for the first difference in the lagged nominal exchange rate (( \mu ))</td>
<td>-0.31 (0.47)</td>
<td>-0.76 (0.0)</td>
</tr>
<tr>
<td>Trend coefficient for the first difference in the interest rate (( K ))</td>
<td>15.77 (0.52)</td>
<td>-7.00 (0.36)</td>
</tr>
<tr>
<td>Probability of intervention by indexed bonds and foreign exchange swaps (( \lambda_{\text{debt + swaps}} ))</td>
<td>0.0 (0.19)</td>
<td>0.01 (0.0)</td>
</tr>
<tr>
<td>Probability of intervention in the spot market (( \lambda_{\text{spot}} ))</td>
<td>0.000004 (0.18)</td>
<td>0.000003 (0.0)</td>
</tr>
<tr>
<td>Observations</td>
<td>126</td>
<td>1001</td>
</tr>
<tr>
<td>J-Statistic</td>
<td>0.02 (0.97)</td>
<td>0.10 (0.95)</td>
</tr>
</tbody>
</table>
Table 4

Estimation by Hansen and Scheinkman (1995) and Discretization of the Model:

\[ dX_t = (\mu X_t + K_t + K' t dW_t + I_t^{\text{spot}} X_t dJ_t^{\text{spot}} + I_t^{\text{debt}} X_t dJ_t^{\text{debt}} + I_t^{\text{swap}} X_t dJ_t^{\text{swap}} ) dt + X_t dW_t + I_t^{\text{spot}} X_t dJ_t^{\text{spot}} + I_t^{\text{debt}} X_t dJ_t^{\text{debt}} + I_t^{\text{swap}} X_t dJ_t^{\text{swap}} \]

The horizon of estimation goes from January 1999 to October 2006. The data are daily. The estimated periods are: the foreign exchange crisis, the first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 to April 2002, December 2002 to October 2006, and the entire sample period. We fix \( \sigma = 1 \) as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: \( \mu, K, \lambda_{\text{div}}, \lambda_{\text{vista}}, \lambda_{\text{swap}} \). In parenthesis are the p-values. The moment conditions for Hansen and Scheinkman are obtained with two classes of moment conditions \((C1)\) e \((C2)\), described in the text, using \( X, X^2 \) e \( X^3 \) as test functions. The moment conditions for the discretization are obtained using as instruments the constant, the first lag of the nominal exchange rate and the first lag of the variation of the interest rate. The J-statistic is presented at the end, and is described as \( \chi^2(2) \) in the estimations related to columns (A) e (C) and \( \chi^2(1) \) in the estimations related to columns (B), (D) e (E). Below the statistics, presented in parentheses, are the probabilities of not rejecting the model specification.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Hansen and Scheinkman</th>
<th>Discretization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Foreign Exchange Crisis</td>
<td>No foreign exchange crisis</td>
</tr>
<tr>
<td>Trend Coefficient for the variation of the lagged nominal exchange rate ( (\mu) )</td>
<td>-0.18 (0.39)</td>
<td>-0.22 (0.09)</td>
</tr>
<tr>
<td>Trend coefficient of the variation of the interest rate ( (K) )</td>
<td>13.33 (0.72)</td>
<td>-19.22 (0.66)</td>
</tr>
<tr>
<td>Foreign exchange coupon ( (K') )</td>
<td>0.19 (0.71)</td>
<td>-0.02 (0.95)</td>
</tr>
<tr>
<td>Probability of intervention by indexed debt ( (\lambda_{\text{debt}}) )</td>
<td>0.00 (0.54)</td>
<td>0.004 (0.62)</td>
</tr>
<tr>
<td>Probability of intervention in the spot market ( (\lambda_{\text{spot}}) )</td>
<td>0.00 (0.51)</td>
<td>0.001 (0.84)</td>
</tr>
<tr>
<td>Probability of intervention in the swap market ( (\lambda_{\text{swap}}) )</td>
<td>N.A (0.99)</td>
<td>0.002 (0.67)</td>
</tr>
<tr>
<td>Observations</td>
<td>120</td>
<td>126</td>
</tr>
<tr>
<td>J-Statistic</td>
<td>0.0 (1.0)</td>
<td>0.26 (0.65)</td>
</tr>
</tbody>
</table>
The horizon of estimation goes from January 1999 to April 2003. The data are daily. The periods of estimation are: the foreign exchange crisis, the first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 to April 2002, December 2002 to April 2003, and the entire sample period. We fix $\sigma=1$ as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: $\mu$, $K$, $\lambda_{d_{\text{err}}}$, $\lambda_{\text{rate}}$, $\lambda_{\text{swap}}$. In parenthesis are the p-values. The moment conditions for Hansen and Scheinkman are obtained with two classes of moment conditions (C1) and (C2), described in the text, using $X$, $X^2$, and $X^3$ as test functions. The moment conditions for the discretization are obtained using as instruments the constant, the first lag of the nominal exchange rate and the first lag of the variation of the interest rate. The $J$-statistic is presented at the end, and presented in parentheses are the probabilities of not rejecting the model specification. The $J \equiv \chi^2(1)$. The R-statistic is $\chi^2(4)$. Below this statistic in parentheses is the probability of not rejecting the specification of the more simplified model in relation to the more complete model.

### Table 5

**Estimation by Hansen and Scheinkman (1995) and Discretization of the Model:**

\[
\frac{dX_t}{(\mu X_t)} = dt + X_t dW_t
\]

The horizon of estimation goes from January 1999 to April 2003. The data are daily. The periods of estimation are: the foreign exchange crisis, the first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 to April 2002, December 2002 to April 2003, and the entire sample period. We fix $\sigma=1$ as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: $\mu$, $K$, $\lambda_{d_{\text{err}}}$, $\lambda_{\text{rate}}$, $\lambda_{\text{swap}}$. In parenthesis are the p-values. The moment conditions for Hansen and Scheinkman are obtained with two classes of moment conditions (C1) and (C2), described in the text, using $X$, $X^2$, and $X^3$ as test functions. The moment conditions for the discretization are obtained using as instruments the constant, the first lag of the nominal exchange rate and the first lag of the variation of the interest rate. The $J$-statistic is presented at the end, and presented in parentheses are the probabilities of not rejecting the model specification. The $J \equiv \chi^2(1)$. The R-statistic is $\chi^2(4)$. Below this statistic in parentheses is the probability of not rejecting the specification of the more simplified model in relation to the more complete model.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>First Difference of the Nominal Exchange Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Hansen and Scheinkman</td>
</tr>
<tr>
<td></td>
<td>Foreign exchange crises</td>
</tr>
<tr>
<td>Trend coefficient for variation of the lagged nominal exchange rate ($\mu$)</td>
<td>-0.01 (0.04)</td>
</tr>
<tr>
<td>Observations</td>
<td>300</td>
</tr>
<tr>
<td>J-Statistic</td>
<td>0.06 (0.95)</td>
</tr>
<tr>
<td>R-statistic</td>
<td>1.20 (0.60)</td>
</tr>
</tbody>
</table>
The horizon of estimation goes from January 1999 to October 2006. The data are daily. The estimated periods are: the foreign exchange crisis, the first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 to April 2002, December 2002 to October 2006, and the entire sample period. We fix $\sigma=1$ as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: $\mu$, $K$, $\lambda_{diss}$, $\lambda_{casa}$, $\lambda_{swap}$. In parenthesis are the $p$-values. The moment conditions for Hansen and Scheinkman are obtained with two classes of moment conditions (C1) e (C2), described in the text, using $X$, $X^2$ e $X^3$ as test functions. The moment conditions for the discretization are obtained using as instruments the constant, the first lag of the nominal exchange rate and the first lag of the variation of the interest rate. The J-statistic is presented at the end, and in parentheses are the probabilities of not rejecting the model specification $\chi^2(1)$. The R-statistic is $\chi^2(3)$. Below this statistic in parentheses is the probability of not rejecting the specification of the more simplified model in relation to the more complete model.

### Table 6

**Estimation by Hansen and Scheinkman (1995) and Discretization of the Model:**

$$dX_t = (\mu X_t + K_t)dt + X_t dW_t$$

The horizon of estimation goes from January 1999 to October 2006. The data are daily. The estimated periods are: the foreign exchange crisis, the first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 to April 2002, December 2002 to October 2006, and the entire sample period. We fix $\sigma=1$ as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: $\mu$, $K$, $\lambda_{diss}$, $\lambda_{casa}$, $\lambda_{swap}$. In parenthesis are the $p$-values. The moment conditions for Hansen and Scheinkman are obtained with two classes of moment conditions (C1) e (C2), described in the text, using $X$, $X^2$ e $X^3$ as test functions. The moment conditions for the discretization are obtained using as instruments the constant, the first lag of the nominal exchange rate and the first lag of the variation of the interest rate. The J-statistic is presented at the end, and in parentheses are the probabilities of not rejecting the model specification $\chi^2(1)$. The R-statistic is $\chi^2(3)$. Below this statistic in parentheses is the probability of not rejecting the specification of the more simplified model in relation to the more complete model.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Hansen and Scheinkman (1995)</th>
<th>Differentiation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Foreign Exchange Crisis</td>
<td>No foreign exchange crisis</td>
</tr>
<tr>
<td>Trend coefficient for variation of the lagged nominal exchange rate ($\mu$)</td>
<td>-0.11 (0.0)</td>
<td>-0.065 (0.0)</td>
</tr>
<tr>
<td>Interest Rate</td>
<td>-0.14 (0.63)</td>
<td>0.34 (0.80)</td>
</tr>
<tr>
<td>Observations</td>
<td>120</td>
<td>126</td>
</tr>
<tr>
<td>J-Statistic</td>
<td>0.06 (0.95)</td>
<td>0.09 (0.93)</td>
</tr>
<tr>
<td>R-Statistic</td>
<td>1.20 (0.75)</td>
<td>28 (0.0)</td>
</tr>
</tbody>
</table>
Table 7
Estimation by Hansen and Scheinkman (1995) and Discretization
\[ dX_t = (\mu X_t + K_i + K'ic_i)dt + X_t^\text{spot} dW_t + I_t^\text{spot} X_t dJ_{t}^{\text{spot}} + I_t^\text{debt} X_t dJ_{t}^{\text{debt}} \]

The horizon of estimation goes from January 1999 to October 2006. The data are daily. The estimated periods are: the foreign exchange crisis, the first semester of 1999 and second semester of 2002, all the periods without foreign exchange crisis, July 1999 to April 2002, December 2002 to October 2006. We define \( \sigma = 1 \) as an identifying assumption when estimating by GMM. We are estimating the following set of parameters: \( \mu, K, \lambda_{\text{div}}, \lambda_{\text{vista}}, \lambda_{\text{swap}} \). In parenthesis are the p-values. The moment conditions for Hansen and Scheinkman are obtained with two classes of moment conditions (C1) e (C2), described in the text, using \( X, X^2, X^3 \) as test functions. The moment conditions for the discretization are obtained using as instruments the constant, the first lag of the nominal exchange rate and the first lag of the variation of the interest rate. The J-statistic is presented at the end, and in parentheses are the probabilities of not rejecting the model specification.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Hansen and Scheinkman</th>
<th>Discretization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trend coefficient for variation of the lagged nominal exchange rate (( \mu ))</td>
<td>-0.96 (0.0) -0.011 (0.93) -0.02 (0.0) -0.80 (0.10)</td>
<td>0.012 (0.96) 0.09 (0.54) -0.74 (0.0) -0.63 (0.26)</td>
</tr>
<tr>
<td>Trend coefficient of the variation of the interest rate (( K ))</td>
<td>3.37 (0.78) 0.13 (0.93) 33.20 (0.20) -15.39 (0.25)</td>
<td>4.84 (0.86) 3.3 (0.06) 0.22 (0.97) -11.68 (0.61)</td>
</tr>
<tr>
<td>Probability of intervention by indexed bond (( \lambda_{\text{div}} ))</td>
<td>0.002 (0.0) 0.000003 (0.63) 0.0 0.004 (0.11)</td>
<td>0.026 (0.0) 0.002 (0.87) 0.00019 (0.27) 0.002 (0.02)</td>
</tr>
<tr>
<td>Probability of intervention in the spot market (( \lambda_{\text{vista}} ))</td>
<td>0.01 (0.0) 0.000009 (0.03) 0.0 0.008 (0.0)</td>
<td>0.25 (0.0) 0.09 (0.0) 0.11 (0.04) 0.005 (0.56)</td>
</tr>
<tr>
<td>Observations</td>
<td>300 300 300 1077</td>
<td>300 300 300 1077</td>
</tr>
<tr>
<td>J-Statistic</td>
<td>0.03 (0.98) 0.04 (0.97) 0.0 0.30 (1.0)</td>
<td>0.03 (0.97) 0.04 (0.97) 0.02 (0.98) 0.17 (0.92)</td>
</tr>
</tbody>
</table>