Inflation Forecast with OutPut Gap Uncertainty*

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Abstract

One of the main indicators of inflationary pressures used in the monetary policy framework by the Banco Central do Brasil is the output gap. The output gap is also an important measure of the economic fluctuations. This paper uses quarterly data to estimate output gap using univariate and multivariate methods, and compares the value added in predicting inflation. A rolling forecasts experiment is used to assess the out of sample predictive accuracy of the alternative models in different horizons. There is a high degree of correlation between the methods as a whole. The results indicate that multivariate methods have best predictive power.

Resumo

Um dos principais indicadores de pressões inflacionária, usado na política monetária pelo Banco Central do Brasil, é o hiato do produto. O hiato é também uma medida importante de flutuações econômicas. Usamos neste artigo dados trimestrais para estimar diferentes medidas de hiato usando métodos univariados e multivariados. Um experimento de previsão fora da amostra é usado para medir a precisão das previsões de inflação de cada modelo em diferentes horizontes. Existe um alto grau de correlação entre as medidas. Os resultados indicam que métodos multivariados têm um poder preditivo maior do ponto de vista da medida esperada de inflação.

Key-words: output gap, state-space models, production function, Phillips curve.
Palavras chave: hiato, modelos de espaço estado, função de produção, curva de Phillips.
JEL classification: C32, E31, E52.

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1 Introduction

Along the last fifteen years we have seen a worldwide adoption of inflation targeting regime as reported at Mishkin and Schmidt-Hebbel (2001). Those countries explicitly adopt a Taylor Rule reaction function. Although Blinder (1997) claims that Federal Reserve Bank (FED)’s strategy is different from the “deliberate disinflation” followed by inflation targeters, Bernanke (2003) asserts there is little difference between inflation targeting and FED’s strategy, called by him “constrained discretion”. In fact, FED has a legal mandate to pursue “maximum employment” and “stable prices”. In line with this framework, European Central Bank (ECB) has a statutory objective of maintaining price stability, but without prejudicing this target, it shall support economic policies with a view to contributing to the achievement of high level of employment and sustainable and non-inflationary growth. These facts give evidence that both FED and ECB implicitly adopt a Taylor Rule type reaction function as described in Taylor (1993). Thereafter, it seems that all modern central bankers operate monetary policy with a view at the short-run trade-off between inflation and unemployment.

In order to identify the impact of economic activity level on inflation, all central banks need the output gap, which previously demands knowledge of the potential output. Saying it differently, measures of potential output and output gap are useful to identify the scope for sustainable noninflationary growth and to allow an assessment of the stance of macroeconomic policies. As a consequence, the two associated latent variables - along with core inflation and NAIRU - are all concepts whose have received increase attention over the past few years in central banks, international organizations and among academics researchers. St-Aman and van Norden (1997) discuss some methodologies for estimating potential output and the output gap for the Canadian economy. Claus (1999) uses a structural vector autoregression methodology to obtain a measure of potential output for the New Zealand economy. Kichian (1999) measures potential output using state-space models for the Canadian economy. Orphanides and van Nordem (1999) examine the reliability of alternative output detrending methods, with special attention to the accuracy of real-time estimates. Cerra and Saxena (2000) review different methodologies that can be used to estimate potential output for Sweden. Proietti, Musso and Westermann (2002) evaluate unobserved components models based production function approach for estimating the output gap and potential output for the Euro Area. Remnison (2003) uses a Monte Carlo experiment to evaluate the ability of a variety of output gap estimators to accurately measure the output gap in a model economy. Doménech and Gómez (2003) propose a new method to obtain estimates of the potential output, core inflation and the non-accelerating inflation rates of unemployment (NAIRU) as latent variables, using a standard Okum’s law, a forward-looking Phillips curve and an investment equation.

In July 1999, the Brazilian government officially adhered to an inflation targeting regime as described in Bogdanski, Tombini and Werlang (2000). This institutional change led the Central Bank of Brazil’s researches to further invest in developing tools
that appropriately delivery estimations of potential output and output gap. This effort is highlighted in Alves (2001), Bryan and Cecchetti (2001), Alves and Muinhos (2003) and Silva Filho (2002). This paper goes in the same line. We try to contribute to the debate, roughly speaking, following Arnold (2004), who examines both methods that rely on purely statistical techniques - filtering, simultaneous econometrics models, multivariate time-series models - and methods that rely on statistical procedures grounded in economic theory, highlighting the pros and cons of the various approaches. First of all we estimate potential output paths for the Brazilian economy through several different techniques, including time series filtering - such as Kalman filter (KF) and Beveridge-Nelson decomposition (BN) - and semi-structural approaches – a Hodrick-Prescott (HP) filter constrained by some structure. These time series filtering techniques have been largely used and basically extract a trend from Gross Domestic Product (GDP). Even though semi-structural approaches have been also popular, as far as we know, our approach is the only that allows simultaneously setting paths for two latent variables - non-accelerating inflation rates of unemployment (NAIRU) and non-accelerating capacity utilization (NAICU) - to obtain output gap as by-product.

As registered by Doménech and Gómez (2003), a useful decomposition of output into its low and high frequency components should account for three stylized facts: trade-off in the short run between inflation and unemployment rate or output gap - Phillips Curve; negative correlation between output and employment gap (deviation of the unemployment rate from NAIRU) - Okun’s Law; and comoviment of output and investment rate. Then in a second stage we care about the goodness of estimated paths of potential output for the Brazilian economy, investigating how they match those three stylized facts. We begin our checking task running Ordinary Least Square - OLS to specify the best Phillips Curve type equation for each output gap path generated before, which in turn used to make insample inflation forecast that finally serves to measure the mean squared errors - MSE. Following we calculate correlations between output and employment gap and between investment and output gap. The first criteria didn’t delivery us any material diferrence favoring whichever methodology, however, the second and third ones favored the semi-structural approaches.

The rest of the paper is organized as follows. Section 2 presents the econometric techniques considered to estimate potential output and output gap. Section 3 describes data used to estimate each model. Section 4 discusses the main results. Section 5 compares the inflation forecasts generated by a Phillips curve that is estimated using each output gap series. Section 6 concludes.

2 Econometric Modelling

This section reviews some methodologies for extracting potential output from observed output. The output can be decomposed in a permanent, the trend \((\tilde{y}_t)\), and cyclical

\(^{1}\)St-Amant and van Norden (1997) and Alves and Muı̈nos (2003) uses this approach.
components ($y^c_t$):

$$y_t = \bar{y}_t + y^c_t. \quad (1)$$

In general, the literature divides these methodologies into two categories, statistical methods and economic models. The first approach assumes that the trend and the cycle are unobservable components of the output and use itself different statistical methodologies for identify the permanent and transitory components. On the other hand, economic models use economic theory to identify the permanent and transitory components. The set of approaches utilized in this research are:

**Statistical Methods:**

- Deterministic Trends;
- Univariate unobserved component-methods;
- Hodrick-Prescott Filter;
- The Beveridge-Nelson Decomposition
- Band-pass filter.

**Economic Models:**

- Production-function approach;
- Multivariate Unobserved component.

Next we briefly discuss each of these group and variants of these methodologies which we consider in this research.

### 2.1 Deterministic Trends

The first set of statistical methodology to detrend output we use assume that the permanent component in output is satisfactorily approximated by a deterministic function of time (DT). We consider linear, quadratic and piece-wise linear functions.

The linear trend is the simplest of these models. It admits that the logarithm of output, $y_t$, may be decomposed into a linear function of time and a cyclical component:

$$y_t = \bar{y}_t + y^c_t = \alpha + \beta \cdot t + c_t, \quad (2)$$

where $c_t$ is the business cycle.
The quadratic trend (DQT) includes an additional term in the deterministic component to capture non-linearities of our measure of output in a simple way:

$$y_t = \alpha + \beta \cdot t + \gamma t^2 + c_t.$$  \hfill (3)

Because of the noticeable upturn in output growth after 2003, we include another simple technique allowing for a breaking linear trend. The Piece-wise linear trend function (DTPL) can be written as:

$$y_t = \alpha + \beta \cdot t + c_t \quad \text{for} \quad t \leq T_b$$  \hfill (4)

$$y_t = \alpha + \beta \cdot t + \gamma (t - T_b) + c_t \quad \text{for} \quad t \leq T_b$$

Perron (1989) allowed for multiple breaks in the permanent component. We assume that the location of the break is fixed and known.

### 2.2 Univariate Unobserved Component Models

Unobserved component models attempt to specify the time series properties of output and use the resulting model to identify cyclic and trend component. The simplest unobserved component is the Local Level Model (UC1):

$$y_t = \mu_t + \varepsilon_t$$

$$\mu_t = \mu_{t-1} + \eta_t$$

$$\varepsilon_t \sim iid \ N(0, \sigma^2_\varepsilon) \quad \text{and} \quad \eta_t \sim iid \ N(0, \sigma^2_\eta)$$

$$E[\varepsilon_t, \eta_s] = 0, \forall t, \forall s.$$  \hfill (5)

It assumes that the observed output series $y_t$ may be decomposed in a random walk component $\mu_t$ and a white noise process, $\varepsilon_t$. The increments of the random walk and the white noise error process, $\varepsilon_t$, are assumed to be mutually uncorrelated and follow independent Gaussian distributions.

If we assume that the increments to the trend follow a local level model (UC2), then we could write:

$$y_t = \mu_t + \varepsilon_t$$

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \quad \text{and} \quad \beta_t = \beta_{t-1} + \zeta_t$$

$$\varepsilon_t \sim iid \ N(0, \sigma^2_\varepsilon), \quad \eta_t \sim iid \ N(0, \sigma^2_\eta) \quad \text{and} \quad \zeta_t \sim iid \ N(0, \sigma^2_\zeta)$$

$$E[\varepsilon_t, \eta_s] = 0, \quad E[\eta_t, \zeta_s] = 0 \quad \text{and} \quad E[\varepsilon_t, \zeta_s] = 0 \quad \forall t, \forall s.$$  \hfill (6)
The third unobserved component model (UC3) we consider is

\[ y_t = \mu_t + c_t \]
\[ \mu_t = \delta + \mu_{t-1} + \eta_t \]
\[ c_t = \alpha_1 c_{t-1} + \alpha_2 c_{t-2} + \varepsilon_t \]
\[ \varepsilon_t \sim iid \ N(0, \sigma^2_\varepsilon) \]
\[ \eta_t \sim iid \ N(0, \sigma^2_\eta) \]
\[ E[\varepsilon_t, \eta_s] = 0, \forall t, \forall s \]

(7)
due to Watson (1986), where the output time series \( y_t \) is decomposed into a trend component, \( \mu_t \), and a cyclical component, \( c_t \). The trend component, \( \mu_t \), is assumed to follow a random walk with drift and the cyclical component, \( c_t \), is assumed to follow an AR(2) process, to allow for more persistence.

2.3 The Hodrick-Prescott Filter

The Hodrick-Prescott Filter (1997), commonly called HP filter, is a simple smoothing procedure that has become increasingly popular because of its flexibility in tracking the characteristics of the fluctuations in trend output. Output trend, \( \bar{y}_t \), derived using the HP filter is obtained by minimizing a combination of the gap between actual output, \( y_t \), trend output and the rate of change in trend output for the whole sample of observations, \( N \). Formally, the HP-filtered trend is given by

\[ \{\bar{y}_t\}_{t=1}^N = \arg \min \sum_{t=1}^N (y_t - \bar{y}_t)^2 + \lambda \sum_{t=2}^{T-1} [(\bar{y}_{t+1} - \bar{y}_t) - (\bar{y}_t - \bar{y}_{t-1})]^2 \]

(8)

and \( y_t^c \) is the resulting measure of the output gap,

\[ \{y_t^c\}_{t=1}^N = \{y_t - \bar{y}_t\}_{t=1}^N \]

(9)

\( \lambda \) is called the “smoothing parameter” and penalizes the variability in the growth component. The larger the value of \( \lambda \), the smoother the growth component and the greater the variability of the output gap. As \( \lambda \) approaches infinity, the growth component correspond to a linear time trend. For quarterly data, Hodrick and Prescott propose setting \( \lambda \) equal to 1600.

St-Aman and van Nordem (1997) and Cerra and Saxena (2002) stress the high filter’s end-sample biases. Mise, Kim and Newbold (2002) demonstrate that the HP filter is the optimal decomposition in mid-sample, under certain conditions, but it is suboptimal at time-series endpoints. This flaw is particularly severe when the focus of attention is directed at the recent observations in the sample.
2.4 The Beveridge-Nelson Decomposition.

Beveridge-Nelson (1981) decomposition provides a convenient way to estimate the permanent and transitory components of an integrated time series. Given a forecasting model for the first-difference of the series, the Beveridge-Nelson (BN) estimate of the trend is defined to be the limiting forecast as horizon goes to infinity, adjusted for the mean rate of growth,

\[ \tau_{t}^{BN} = \lim_{J \to \infty} E_t[y_{t+j} - J \mu], \]

the BN cycle is the difference between the present level of the series and its long-run forecast:

\[ c_{t}^{BN} = y_t - \tau_{t}^{BN} \]

The calculation of the exact BN trend and cycle is complicated by the presence of infinite sums in the long-run forecast. Morley (2002) showed that the exact calculation of the BN trend and cycle is straightforward if the forecasting model can be written in a state-space form. Suppose that \( y_t \) follows an ARIMA\((p, 1, q)\) process and \( \Delta y_t \) is a linear combination of the elements of the state vector \( \xi_t \):

\[ (\Delta y_t - \mu) = G \cdot \xi_t, \]

the state vector evolves according to the following first-order stochastic difference equation:

\[ \xi_t = F \cdot \xi_{t-1} + \nu_t \]

where \( \nu_t \sim N(0, \Omega) \), \( \mu \) is the unconditional mean of \( \Delta y_t \) and the eigenvalues of \( F \) are supposed to lie inside the unit root circle, \( (I - F) \) is always an invertible matrix. As suggested in Morley (2002), the state-space representation expressed in equations 12 and 13 can be used to calculate the exact trend and cycle:

\[ \tau_t = y_t + \lim_{J \to \infty} \sum_{j=1}^{J} E_t[\Delta y_{t+j} - \mu] = y_t + G \lim_{J \to \infty} \sum_{j=1}^{J} F^j E_t[\xi_t] = y_t + GF(I-F)^{-1} E_t[\xi_t] \] (14)

and

\[ c_t = y_t - \tau_t = -GF(I-F)^{-1} E_t[\xi_t] \] (15)

Additionally, using equations 12 and 13, it is easy to see that:

\[ \tau_t - \tau_{t-1} = \mu + G(I-F)^{-1} \nu_t \] (16)

proving that the trend is a random walk with a drift.
2.5 Band-pass filter

The band-pass filter (BPF) is another common tool used to extract information from a time series. A band-pass filter is a device that passes frequencies within a certain range and rejects (attenuates) frequencies outside that range. The observed time series, in this framework, is the result of weighted sum of different time series with varying cyclical frequencies. The variance of the observed series is the result of the sum over all frequencies, since the cycles of underlying series are uncorrelated in the long run. The band-pass filter proposed by Burns and Mitchell (1946), which admits frequency components between 6 and 32 quarters for quarterly data, removes low-frequency trend variations, smooths high-frequency irregular variation and retains the major features of business cycles.

An optimal filter passes all frequencies in the specified frequency range and leaves out all other frequencies. The optimal filter can be derived, it requires an infinite number of data points, making it impractical in empirical work. We use the band-pass filter proposed by Baxter and King (1999), whose filter is a moving average of 25 quarters,

\[ y^c_t = \sum_{i=-12}^{12} \alpha_i y_{t-i}, \]  

(17)

The weights of the filter, \( \alpha_i \), are derived from the inverse Fourier transform of the frequency response function.

2.6 Production-function approach

In line with the approach presented in Alves and Muinhos (2003), the methodology proposed in Areosa (2004) uses a Cobb-Douglas production function with constant return to scale is used to assess output and potential output. That is:

\[ Y_t = A_t(K_tC_t)^\alpha(L_t(1-U_t))^{(1-\alpha)} \]  

(18)

\[ \bar{Y}_t = A_t(K_tNAICU_t)^\alpha(L_t(1-NAIRU_t))^{(1-\alpha)} \]  

(19)

where \( Y_t \) is the output, \( \bar{Y}_t \) is the potential output, \( A_t \) is the productivity factor, \( K_t \) is the capital stock, \( L_t \) is the labor force, \( \alpha \) is the income capital share, \( C_t \) is the capacity utilization, \( U_t \) is the rate of unemployment, \( NAICU_t \) is the non-accelerating inflation capacity utilization and \( NAIRU_t \) is the non-accelerating inflation rate of unemployment.

Defining \( y_t = \ln Y_t, \bar{y}_t = \ln \bar{Y}_t, c_t = \ln C_t, e_t = \ln(1-U_t), naire_t = \ln(1-NAIRU_t), naicu_t = \ln NAICU_t \), it is possible to derive the following equations:

\[ \bar{y}_t = y_t + \alpha (naicu_t - c_t) + (1-\alpha) (naire_t - e_t) \]  

(20)

\[ h_t \equiv \ln \left( \frac{Y_t}{\bar{Y}_t} \right) = \alpha (c_t - naicu_t) + (1-\alpha)(e_t - naire_t) \]  

(21)
These relations highlight the fact that the difficulty of estimating $y_t$ and $h_t$ comes from the difficulty of estimating their unobserved components - $nairu_t$ and $naicu_t$. Areosa (2004) estimate these components by solving following problem:

$$
\min_{\{nairu_t\}_{t=1}^N, \{naicu_t\}_{t=1}^N} \left\{ \begin{array}{l}
\beta_1 \left[ \sum_{t=1}^N (nairu_t - u_t)^2 + \lambda \sum_{t=2}^{N-1} (\Delta^2 nairu_t)^2 \right] + \\
\beta_2 \left[ \sum_{t=1}^N (naicu_t - c_t)^2 + \lambda \sum_{t=2}^{N-1} (\Delta^2 naicu_t)^2 \right] + \\
\beta_3 \left[ \sum_{t=1}^N (\bar{y}_t - y_t)^2 + \lambda \sum_{t=2}^{N-1} (\Delta^2 \bar{y}_t)^2 \right]
\end{array} \right\}
$$

s.t.

$$\bar{y}_t = y_t + \alpha (naicu_t - c_t) + (1 - \alpha) (naire_t - e_t)$$

It is importat to notice that equation (20) appears as a restriction for the problem. Without this restriction, the filter would give the same solution as if the Hodrick-Prescott filter were used on series $\{nairu_t\}_{t=1}^N$, $\{naicu_t\}_{t=1}^N$, $\{\bar{y}_t\}_{t=1}^N$.

### 2.7 Multivariate unobserved component-methods

An advantage of multivariate unobserved component-method (MVUC) over univariate methods is the use of more information.

#### 2.7.1 Multivariate unobserved component-method 1

The univariate unobserved component method can be enhanced by including variables that are presumed to contain information about the output. Scotts (2000) extend Clark’s (1989) approach explicitly linking inflation and product through an equation and including variable capacity utilisation as observed. First, decompose the output in three unobserved components:

$$y_t = \bar{y}_t + z_t + e_{1t} \sim (0, \sigma_{e_1}^2)$$

where $y_t$ is output, $\bar{y}_t$ is the permanent trend for output, $z_t$ is a temporary trend reverting component and $e_{1t}$ is an irregular component. Observed unemployment and capacity utilisation are also decomposed in the same way:

$$u_t = \bar{u}_t + D(L) \cdot z_t + e_{2t} \sim (0, \sigma_{e_2}^2)$$

and

$$cap_t = \bar{cap}_t + G(L) \cdot z_t + e_{3t} \sim (0, \sigma_{e_3}^2)$$

On the following problem $\Delta^2$ represents the second centred difference. For instance, $\Delta^2 y_t = y_{t+1} - 2y_t + y_{t-1}$.
where \( u_t \) is output, \( \bar{u}_t \) is its permanent trend, \( \text{cap}_t \) is capacity utilisation, \( \bar{\text{cap}}_t \) is its permanent trend and \( D(L) \) and \( G(L) \) are polynomials in the lag operator.

The cyclical component, \( z_t \), is common to all three equations and it has a dynamic that is described by a \( p \)th-order autoregressive process,

\[
z_t = C(L) \cdot z_t + \varepsilon_t \sim (0, \sigma_e^2).
\]

The underlying trends for all observed variables are independent and they are modelled as local linear trends. The model can be written in the state space representation as,

\[
\begin{bmatrix}
  y_t \\
  u_t \\
  \text{cap}_t 
\end{bmatrix}
= \begin{bmatrix}
  1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  d_0 & d_1 & d_2 & 0 & 0 & 1 & 0 & 0 & 0 \\
  g_0 & g_1 & g_2 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
  \beta^y_t \\
  y_{t-1} \\
  y_{t-2} \\
  \bar{y}_t \\
  \beta^y_t \\
  \text{cap}_t \\
  \beta^\text{cap}_t
\end{bmatrix}
+ \begin{bmatrix}
  \varepsilon_t \\
  \varepsilon_{1t} \\
  \varepsilon_{2t} \\
  \varepsilon_{3t}
\end{bmatrix},
\]

(27)

equation (27) is the observation equation and the state equation can be written as,

\[
\begin{bmatrix}
  z_t \\
  z_{t-1} \\
  z_{t-2} \\
  \bar{y}_t \\
  \beta^y_t \\
  \text{cap}_t \\
  \beta^\text{cap}_t
\end{bmatrix}
= \begin{bmatrix}
  c_1 & c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
  \beta^y_t \\
  \bar{y}_{t-1} \\
  \beta^y_{t-1} \\
  \text{cap}_{t-1} \\
  \beta^\text{cap}_{t-1}
\end{bmatrix}
\begin{bmatrix}
  \varepsilon_t \\
  z_{t-1} \\
  z_{t-2} \\
  \bar{y}_{t-1} \\
  \beta^y_{t-1} \\
  \text{cap}_{t-1} \\
  \beta^\text{cap}_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
  \eta_{1t} \\
  \eta_{2t} \\
  \eta_{3t} \\
  \xi_{1t} \\
  \xi_{2t} \\
  \xi_{3t}
\end{bmatrix},
\]

(28)

2.7.2 Multivariate unobserved component-method 2

Following Bjornland, Brubakk e Jore (2006), we decompose the output in three unobserved components,

\[
y_t = \bar{y}_t + y^c_t + s_t
\]

(29)

where \( y_t \) is output, \( \bar{y}_t \) is the permanent trend for output, \( y^c_t \) is the cyclical component (output gap) and \( s_t \) is a seasonal component of output. If we apply the lag operator to equation (29),

\[
\Delta y_t = \Delta \bar{y}_t + y^c_t - \bar{y}_{t-1} + s_t - s_{t-1},
\]

(30)

the growth rate of output is equal to the growth in potential output plus the change in the output gap plus the change in seasonality. Inflation and output gap are linked
by a backward-looking Phillips curve,

\[ \pi_t = \delta_1 \pi_{t-1} + \delta_2 \pi_{t-2} + \theta_1 y^c_{t-1} + \varepsilon_{1,t} \]  

(31)

\( \pi_t \) is the actual inflation and \( \varepsilon_{1,t} \) is an irregular component. A version of Okun’s law connects unemployment rate and output gap,

\[ (u_t - u^*_t) = \alpha_1 (u_{t-1} - u^*_{t-1}) + \theta_2 y^c_{t-1} + \varepsilon_{2,t} \]  

(32)

where \( u_t \) is the unemployment rate, \( u^*_t \) is the NAIRU and \( \varepsilon_{1,t} \) is an irregular component. The dynamics of the output gap is described by a second-order autoregressive process,

\[ y^c_t = \phi_1 y^c_{t-1} + \phi_2 y^c_{t-2} + \xi_{1,t} \]  

(33)

The growth in potential output are modelled as local linear trend model,

\[ \Delta y_t = \beta \Delta y_{t-1} + \omega_{t-1} + \xi_{2,t} \]  

\[ \omega_t = \omega_{t-1} + \xi_{3,t} \]  

(34)

The process for the NAIRU is also a local linear trend model,

\[ u^*_t = u^*_{t-1} + v_{t-1} + \xi_{4,t} \]  

\[ v_t = v_{t-1} + \xi_{5,t} \]  

(35)

Finally, the seasonal component is,

\[ s_t = \cos(\rho)s_{t-1} + \sin(\rho)s^*_t + \xi_{6,t} \]  

\[ s^*_t = -\sin(\rho)s_{t-1} + \cos(\rho)s^*_t + \xi_{7,t} \]  

(36)

The model can be written in the state space representation as,

\[
\begin{bmatrix}
\Delta y_t \\
\pi_t \\
u_t
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
\delta_1 & \delta_2 & 0 \\
0 & 0 & \alpha_1
\end{bmatrix}
\begin{bmatrix}
\pi_{t-1} \\
\pi_{t-2} \\
u_{t-1}
\end{bmatrix}
\]

(37)

\[
+ \begin{bmatrix}
1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \theta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\begin{bmatrix}
y^c_t \\
y^c_{t-1} \\
\Delta y_t \\
\omega_t \\
u^*_t \\
u^*_{t-1} \\
v_t \\
s_t \\
s^*_t \\
s^*_{t-1} \\
\end{bmatrix}
+ \begin{bmatrix}
0 \\
\varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{bmatrix}
\]
equation (37) is the observation equation and the state equation can be written as,

\[
\begin{bmatrix}
    y_t^c \\
    y_{t-1}^c \\
    \Delta y_t \\
    \omega_t \\
    u_t^* \\
    u_{t-1}^* \\
    v_t \\
    s_t \\
    s_{t-1} \\
    s_t^c
\end{bmatrix} =
\begin{bmatrix}
    \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & \beta & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & \cos(\rho) & 0 & \sin(\rho) & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos(\rho) & 0 \\
\end{bmatrix}
\begin{bmatrix}
    y_{t-1}^c \\
    y_{t-2}^c \\
    \Delta y_{t-1} \\
    \omega_{t-1} \\
    u_{t-1}^* \\
    u_{t-2}^* \\
    v_{t-1} \\
    s_{t-1} \\
    s_{t-2} \\
    s_{t-1}^c
\end{bmatrix} +
\begin{bmatrix}
    \xi_{1,t} \\
    0 \\
    \xi_{2,t} \\
    \xi_{3,t} \\
    \xi_{4,t} \\
    0 \\
    \xi_{5,t} \\
    \xi_{6,t} \\
    0 \\
    \xi_{7,t}
\end{bmatrix}
\]

(38)

3 Data

The following set of time-series (1995 to 2007) was used in our exercises, which from now on will be represented by the letter in brackets.

- Gross Domestic Product – GDP ($y_t$) - quarterly; Fundação Instituto Brasileiro de Geografia e Estatística (IBGE)\(^3\).
- Unemployment Rate ($u_t$) - monthly; IBGE. IBGE changed its measuring methodology, discontinuing its original unemployment series at December 2002\(^4\) and starting another one at September 2001\(^5\). We extended the former old series supposing that all variation occurred in the new series.
- Installed Capacity Utilization ($c_t$) - monthly; Conselho Nacional da Indústria (CNI)\(^6\).
- Income Capital Share ($\alpha_t$) - yearly IBGE. Estimated from an yearly series based on the empirical labor share obtained from national accounts\(^7\).
- Broad Consumer Price Index Inflation Rate ($\pi_t$) - monthly; IBGE measures the IPCA (Índice de preços ao consumidor amplo), the official consumer price index used in the inflation targeting regime\(^8\).

\(^3\)series 1232 available at http://www4.bcb.gov.br/?TIMESERIESSEARCH  
\(^4\)series 1629 available at http://www4.bcb.gov.br/?TIMESERIESSEARCH  
\(^5\)series 10777 available at http://www4.bcb.gov.br/?TIMESERIESSEARCH  
\(^6\)series 1341 available at http://www4.bcb.gov.br/?TIMESERIESSEARCH  
\(^7\)available at tab04.xls, extracted from sinoticas.zip at www.ibge.gov.br  
\(^8\)series 433 available at www4.bcb.gov.br/pec/series/ingl/
• Inflation of Market Prices ($\pi_t^f$) - monthly; Banco Central do Brasil - DEPEC\(^9\).
  This series excludes the managed price items from the IPCA headline index.

• Expected Headline Inflation($E_t[\pi_{t+1}]$) - monthly; Focus, Banco Central do Brasil.

We used the average of observations for frequency conversion (monthly to quarterly).

4 Results

We use full sample (1995.1 - 2007.4) to obtain potential output and output gap estimating the models presented in Econometric Modelling section. The calculated output gaps are shown in Figure 1 in appendix 1. The different methodologies show common features for Brazilian Economy that should be highlighted. All measures of output gap indicate steep declines (early 1998, 2000 and 2002) and a mild decline at the end of 2002. On the other hand, we have four periods of rising output gaps (end of 1998, 2001, 2003 and 2006). In general the linear trend and the Beveridge-Nelson decomposition estimate more severe changes. Since 2003 the estimated distance of the measures is generally small, indicating a concordance between them.

Table 1 shows the descriptive statistics of different measures of the output gap calculated by alternative methods. In general it is reasonable to expect that the average over the time of the output gaps should be close to zero. The unobserved components models have mean less than zero The BN filter has the highest mean and median between the methodologies. Excluding the BN filter, the HP filter has the highest median, while the medians of DT and BPF are negative. The BN filter and DT model present the greatest and lowest values. The DT model, the BN filter, the BPF and the MVUC2 exhibit the highest standard deviation. DT and TDQ have positive skewness. All distribution of the measures of output gap are flat (platykurtic) relative to the normal.

<table>
<thead>
<tr>
<th>TD</th>
<th>TDQ</th>
<th>TDPL</th>
<th>UC2</th>
<th>UC3</th>
<th>HP</th>
<th>BN</th>
<th>BPF</th>
<th>PF</th>
<th>MVUC1</th>
<th>MVUC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.0014</td>
<td>0.0007</td>
<td>-0.0001</td>
<td>-0.0007</td>
<td>-0.0004</td>
<td>0.0000</td>
<td>0.0064</td>
<td>-0.0015</td>
<td>-0.0002</td>
<td>-0.0001</td>
</tr>
<tr>
<td>Median</td>
<td>-0.0012</td>
<td>0.0005</td>
<td>0.0000</td>
<td>0.0005</td>
<td>0.0008</td>
<td>0.0017</td>
<td>0.0119</td>
<td>-0.0020</td>
<td>0.0003</td>
<td>0.0000</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.0528</td>
<td>0.0211</td>
<td>0.0196</td>
<td>0.0181</td>
<td>0.0190</td>
<td>0.0192</td>
<td>0.0630</td>
<td>0.0164</td>
<td>0.0141</td>
<td>0.0075</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.0358</td>
<td>-0.0203</td>
<td>-0.0233</td>
<td>-0.0217</td>
<td>-0.0201</td>
<td>-0.0195</td>
<td>-0.0525</td>
<td>-0.0204</td>
<td>-0.0164</td>
<td>-0.0084</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.0218</td>
<td>0.0119</td>
<td>0.0108</td>
<td>0.0097</td>
<td>0.0094</td>
<td>0.0110</td>
<td>0.0279</td>
<td>0.0104</td>
<td>0.0071</td>
<td>0.0030</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.3740</td>
<td>0.0899</td>
<td>-0.1748</td>
<td>-0.0711</td>
<td>-0.1330</td>
<td>-0.0099</td>
<td>-0.1533</td>
<td>-0.0135</td>
<td>-0.3712</td>
<td>-0.3064</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>2.4669</td>
<td>1.9851</td>
<td>2.2823</td>
<td>2.4225</td>
<td>2.4010</td>
<td>1.9361</td>
<td>2.3279</td>
<td>2.0409</td>
<td>2.7982</td>
<td>3.5816</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>1.6797</td>
<td>2.1691</td>
<td>1.3013</td>
<td>0.7222</td>
<td>0.8771</td>
<td>2.3115</td>
<td>1.1143</td>
<td>1.8979</td>
<td>1.2081</td>
<td>1.4572</td>
</tr>
<tr>
<td>Probability</td>
<td>0.4318</td>
<td>0.3381</td>
<td>0.5217</td>
<td>0.6369</td>
<td>0.6450</td>
<td>0.3148</td>
<td>0.5728</td>
<td>0.3907</td>
<td>0.5466</td>
<td>0.4826</td>
</tr>
<tr>
<td>Sum</td>
<td>-0.0707</td>
<td>0.0327</td>
<td>-0.0030</td>
<td>-0.0333</td>
<td>-0.0200</td>
<td>0.0020</td>
<td>0.3132</td>
<td>-0.0725</td>
<td>-0.0104</td>
<td>-0.0036</td>
</tr>
<tr>
<td>Sum Sq. Dev.</td>
<td>0.0229</td>
<td>0.0067</td>
<td>0.0056</td>
<td>0.0045</td>
<td>0.0042</td>
<td>0.0058</td>
<td>0.0374</td>
<td>0.0052</td>
<td>0.0024</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

\(^9\)series 11428 available at www4.bcb.gov.br/pec/series/ingl/
Table 2 shows the correlation between different measures of output gap. We found high correlations between univariate models as was expected. The filter HP has high correlation with the linear trend and the univariate unobserved component models, while this measure falls when compared with economic models. The MVUC1 model has negative correlation with the other models indicating misspecification.

<table>
<thead>
<tr>
<th></th>
<th>TD</th>
<th>TDQ</th>
<th>TDPL</th>
<th>UC2</th>
<th>UC3</th>
<th>HP</th>
<th>BN</th>
<th>BPF</th>
<th>PF</th>
<th>MVUC1</th>
<th>MVUC2</th>
</tr>
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<tbody>
<tr>
<td>TD</td>
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<td>TDQ</td>
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<td></td>
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<tr>
<td>TDPL</td>
<td>0.48</td>
<td>0.83</td>
<td>1</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UC2</td>
<td>0.55</td>
<td>0.89</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UC3</td>
<td>0.53</td>
<td>0.90</td>
<td>0.94</td>
<td>0.99</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HP</td>
<td>0.68</td>
<td>0.97</td>
<td>0.91</td>
<td>0.96</td>
<td>0.96</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BN</td>
<td>0.65</td>
<td>0.56</td>
<td>0.67</td>
<td>0.68</td>
<td>0.70</td>
<td>0.69</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BPF</td>
<td>0.68</td>
<td>0.84</td>
<td>0.78</td>
<td>0.83</td>
<td>0.85</td>
<td>0.87</td>
<td>0.57</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PF</td>
<td>0.42</td>
<td>0.63</td>
<td>0.65</td>
<td>0.74</td>
<td>0.77</td>
<td>0.70</td>
<td>0.57</td>
<td>0.72</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MVUC1</td>
<td>-0.04</td>
<td>-0.42</td>
<td>-0.48</td>
<td>-0.51</td>
<td>-0.52</td>
<td>-0.45</td>
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<td>-0.30</td>
<td>-0.32</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>MVUC2</td>
<td>0.78</td>
<td>0.70</td>
<td>0.65</td>
<td>0.78</td>
<td>0.76</td>
<td>0.78</td>
<td>0.64</td>
<td>0.86</td>
<td>0.79</td>
<td>-0.17</td>
<td>1</td>
</tr>
</tbody>
</table>

5 Comparative Analysis

After estimating the output gap measures through different methodologies, we compare the predictive power in terms of inflation. Following Proietti, Musso and Westermann (2002), we use as comparative criterium their accuracy in forecasting. Since output gap is to represents a measure of inflation pressures, we expect it to increase accuracy in our inflation forecasts.

Analyzing the predictive power of each output gap series is a complex task. First of all, we should estimate a plain Phillips curve that captures the trade-off between inflation and unemployment. Based on Bodanski, Tombini and Werlang (2000), we have chosen the following specification:

$$\pi_t^f = \alpha_1 \pi_{t-1} + \alpha_2 E_t[\pi_{t+1}] + \alpha_3 h_{t-2} + \varepsilon_t,$$

where $\pi_t^f$ is the (log of ) inflation of market prices, $\pi_t$ is the (log of) headline inflation, $E_t[\pi_{t+1}]$ is the conditional expected value of inflation and $h_t$ is the output gap.

It is important to highlight that the dependent variable in this equation is the free price inflation. This specification is in line with Banco Central do Brasil Inflation Report (2002) and it can be understood considering that a relevant part of inflation is due to managed prices and consequently does not respond to monetary policy. It is also important to note that expected inflation takes an important part on this specification.

We used a rolling forecast experiment as a test of forecast accuracy as proposed in Proietti, Musso and Westermann (2002). For each output gap series, we estimated the Phillips curve changing the sample from 1995:1 - 2002:4 to 1995:1 - 2007:3 and, each
time, forecasting up to 4 steps ahead. The mean square error of these forecasts was chosen as a measure of accuracy for each model.

Table 3 shows the mean square error of each forecast from 1 to 4 steps ahead. The results of this exercise indicate that the multivariate unobserved component model, with exogenous treatment of seasonality, offers the best performance for all horizons. The band-pass filter performs well in all horizons of forecasting.

<table>
<thead>
<tr>
<th>Model</th>
<th>1 step</th>
<th>2 steps</th>
<th>3 steps</th>
<th>4 steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>DT</td>
<td>0.0080</td>
<td>0.0073</td>
<td>0.0068</td>
<td>0.0068</td>
</tr>
<tr>
<td>UC2</td>
<td>0.0080</td>
<td>0.0070</td>
<td>0.0067</td>
<td>0.0067</td>
</tr>
<tr>
<td>UC3</td>
<td>0.0080</td>
<td>0.0071</td>
<td>0.0068</td>
<td>0.0068</td>
</tr>
<tr>
<td>HP</td>
<td>0.0080</td>
<td>0.0073</td>
<td>0.0069</td>
<td>0.0069</td>
</tr>
<tr>
<td>BPF</td>
<td>0.0069</td>
<td>0.0062</td>
<td>0.0061</td>
<td>0.0061</td>
</tr>
<tr>
<td>PF</td>
<td>0.0088</td>
<td>0.0075</td>
<td>0.0075</td>
<td>0.0072</td>
</tr>
<tr>
<td>CNM1</td>
<td>0.0074</td>
<td>0.0067</td>
<td>0.0066</td>
<td>0.0068</td>
</tr>
<tr>
<td>CNM2</td>
<td>0.0067</td>
<td>0.0061</td>
<td>0.0060</td>
<td>0.0062</td>
</tr>
<tr>
<td>CNM3</td>
<td>0.0082</td>
<td>0.0076</td>
<td>0.0070</td>
<td>0.0068</td>
</tr>
</tbody>
</table>

The production function presents made the biggest error. Univariate methods produce poor output gaps from the standpoint of forecasting inflation.

6 Conclusions

In this paper we use different measures of output gap. They were calculated through different methodologies widely used in the literature. To compare the predictive power of each measure from the point of view of forecast of inflation, we propose an experiment involving out of sample forecasts with mobile sample obtained by a forward-looking Phillips curve.

The output gap measures indicate jointly historical reversals of output gap of the Brazilian economy and in recent years the distance between these measures do not vary substantially. The descriptive statistics are within the range expected. The correlation analysis shows high correlations between univariate models as expected. The filter HP has high correlation with the linear trend and the univariate unobserved component models, while this measure falls when compared with economic models.

We used a rolling forecast experiment as a test of forecast accuracy as proposed in Proietti, Musso and Westermann (2002). The mean square error of these forecasts was chosen as a measure of accuracy for each model. The results of this exercise indicate that the multivariate unobserved component model, with exogenous treatment of seasonality, offers the best performance for all horizons. The band-pass filter performs well in all horizons of forecasting. The production function presents made
the biggest error. Univariated methods produce poor output gaps from the standpoint of forecasting inflation.
References


Appendix 1 - Figures

Figure 1 - Output gaps
Figura 2 - Produtos potenciais e hiatos (TD, CNO1 e CNO2)
Figura 3 - Produtos potenciais e hiatos (HP, FPB e FP)
Figura 4 - Produtos potenciais e hiatos (CNOM1, CNOM2 e CNOM3)