Samba: Stochastic Analytical Model with a Bayesian Approach

DSGE Model Project for Brazil’s economy

Working in Progress - Preliminary results

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Outline

● Introduction

● Model

● Estimation results

● Challenges and next steps
Purposes of the project

- Provide the Banco Central do Brasil with a Dynamic Stochastic General Equilibrium (DSGE) model to be used as a tool for:
  - policy analysis
    - framework for policy discussions; qualitative and quantitative assessment of shock effects, monetary policy decisions and different scenarios, etc.
  - medium-term forecast

- “All models are wrong! Some are useful” George Box

- Models and judgement are complements, not substitutes
Model features

- Microfounded model developed for the inflation targeting period (started in mid-1999)

- Small open economy model

- Aggregate demand \((C + I + G + X - M)\):
  - Households \(\rightarrow\) private consumption and investment
  - Firms \(\rightarrow\) import demand
  - Government \(\rightarrow\) government consumption
  - Rest of the world \(\rightarrow\) export demand
Model features

- Supply side (Y)
  - Competitive firms -> assemble differentiated goods supplied by monopolistic competitive firms and sell them in
    * Local markets (domestic consumption and investment goods)
    * Abroad (export goods)
  - Monopolistic competitive firms -> production of differentiated goods
    * Inputs: labor, capital services, and imports
    * Price rigidity (à la Calvo) with forward- and backward-looking behavior (Galí and Gertler, 1999)
Model features

- Government:
  - Monetary policy: Taylor rule
  - Fiscal policy rule

- Rest-of-the-world variables: interest rate, inflation, world imports, and foreign investors’ "risk aversion".
Main loglinear equations

- **Aggregate Demand**: Consumption

  - Optimizing households
    \[
    c^o_t = \left( \frac{1}{1 + h} \right) E_t \left( c^o_{t+1} \right) + \left( \frac{h}{1 + h} \right) c^o_{t-1} - \frac{1}{\sigma} \left( \frac{1 - h}{1 + h} \right) E_t \left( r_t - \pi_{t+1} \right) + \\
    \ldots + \frac{1}{\sigma} \left( \frac{1 - h}{1 + h} \right) (1 - \rho_c) z^c_t
    \]

  - Rule-of-thumb households
    \[
    c^{rot}_t = w^r_t + n^{rot}_t
    \]

  - Aggregate consumption
    \[
    c_t = (1 - \omega_c) c^o_t + \omega_c c^{rot}_t
    \]

  \(r_t\) - interest rate; \(\pi_t\) - inflation; \(z^c_t\) - shock to consumption; 
  \(w^r_t\) - real wages; \(n^{rot}_t\) - employment
• **Aggregate Demand: Investment:**

\[
i_t = \frac{1}{\delta_s (1 + \beta)} q_t^I + \frac{\beta}{1 + \beta} E_t i_{t+1} + \frac{1}{1 + \beta} i_{t-1} + \left( \frac{1 - \rho I \beta}{1 + \beta} \right) z_t^I
\]

Shadow price of capital

\[
q_t^I = E_t \left\{ \beta (1 - \delta) q_{t+1}^I + (1 - \beta (1 - \delta)) \tilde{r}_t^k + (r_t - \pi_{t+1}) \right\}
\]

• **Aggregate Demand: Net Exports**

  – Exports

\[
x_t = m_t^* + \kappa q_t
\]

  – Imports

\[
m_t = y_t - \rho (q_t - m c_t)
\]

\(\tilde{r}_t^k\) - rental rate of capital; \(z_t^I\) - shock to investment; \(m_t^*\) - world imports; \(q_t\) - real exchange rate; \(y_t\) - (gross) output; \(m c_t\) - real marginal cost
• **Aggregate Supply**

  – Production function

    $$ y_t = f(k_t, u_t, n_t, m_t, a_t) $$

  – Labor market

    * Labor supply

      $$ n_t = (1 - \varpi_n) n_t^0 + \varpi nn_{rot} $$

    * Labor demand

      $$ n_t = y_t - [(1 - \varrho) + \varrho s_d]a_t - [\alpha + \varrho (1 + s_d) (1 - \alpha)] w_t^r + \\
          + \alpha [1 - \varrho (1 - s_d)] r_t^k + \varrho (1 - s_d) q_t $$

      $k_t$ - physical capital; $u_t$ - rate of capital utilization; $a_t$ - productivity shock
– Capital services

* Demand

\[ k_t + u_t = y_t - [(1 - \varrho (1 - s_d)] a_t - [(1 - \alpha) + \alpha \varrho (1 - s_d)] \hat{r}^k_t + \ldots + (1 - \alpha) [(1 - \varrho (1 - s_d)] w_t + \varrho (1 - s_d) q_t \]

* Supply

\[ u_t = \frac{1}{\delta a} \hat{r}^k_t \]

* Law of motion for capital

\[ k_{t+1} = (1 - \delta) k_t + \left( \frac{I}{K} \right) i_t \]
- Phillips curve

\[
\pi_t = \lambda mc_t + \lambda_b \pi_{t-1} + \lambda_f E_t \pi_{t+1}
\]

where:

\[
mc_t = s_d \left[ \alpha \hat{r}_t^k + (1 - \alpha) w_t^r - a_t \right] + (1 - s_d) q_t
\]

\[
(\lambda, \lambda_b, \lambda_f) = f (\theta, \omega_b, \beta)
\]
• Financial variables

  – Real exchange rate (UIP)

  \[ q_t = E_t q_{t+1} - \left[ \left( r_t - E_t \pi_{t+1} \right) - \left( r^*_t + \phi_t - E_t \pi^*_{t+1} \right) \right] \]

  – Country-risk premium

  \[ \phi_t = -\psi b^*_t + \nu z^*_t + z^*_t \]

  \( r^*_t \) - world interest rate; \( \pi^*_t \) - world inflation;

  \( z^*_t \) - international investors’ risk aversion; \( z^*_t \) - shock to country-risk premium
- **Government**

  - Monetary policy (Taylor rule)
    \[
    r_t = γr r_{t-1} + (1 - γr) \left[ γπ E_t (π_{t+1} - \bar{π}_{t+1}) + \bar{π}_t + γ_y y_t V A \right] + z^r_t
    \]

  - Fiscal policy rule
    \[
    g^y_t = γ_g g^y_{t-1} + (1 - γ_g) \left( γ_s s^y_{t-1} - γ_b b^y_{t} \right) + z^g_t
    \]

  \( \bar{π}_t \) - inflation target; \( z^r_t \) - shock to monetary policy;

  \( g^y_t \) - government consumption-to-GDP ratio; \( s^y_t \) - primary fiscal surplus target;

  \( z^g_t \) - shock to fiscal policy; \( s^y_{t-1} \) - primary fiscal surplus deviation from the target.
- **Shocks and rest-of-the-world variables:**

\[ z_t = \rho z_{t-1} + \varepsilon_t \]

- **Value added (GDP) - Equilibrium:**

\[ y_t^V = s_{cc} c_t + s_{it} i_t + s_{gg} g_t + s_{xx} x_t - s_{mm} m_t \]
Estimation technique

- Bayesian estimation:

Estimated parameter distribution = prior distribution + likelihood information from the data

It is a bridge between calibration and maximum likelihood

Results: Model + Data + Priors
Estimation

- Sample period: 1999Q2 to 2008Q1 (36 obs)

- Data: 25 series:

- Data treatment: HP filter

- Number of model parameters: 58
  - 41 estimated: 17 structural parameters and 24 shock parameters
  - 17 calibrated: 3 structural parameters and 14 steady-state relationships
Posterior distributions for selected parameters

- **h**
  - Distribution range from 0 to 2
  - Values at 0.2, 0.4, 0.6, 0.8

- **θ**
  - Distribution range from 0.4 to 1
  - Values at 0.4, 0.6, 0.8, 1
  -高峰值约为10

- **γ_π**
  - Distribution range from 1 to 2
  - Values at 1.5
  -高峰值约为2
Impulse responses to a consumption shock \((z^C=1\%)\)

- Interest Rate
- Inflation
- Real Exchange Rate
- GDP
- Consumption
- Investment
- Net Exports
- Exports
- Imports
Impulse responses to a monetary policy shock ($z^r=1\% \text{ p.q.}$)
Impulse responses to a (negative) world GDP shock
Challenges

• Common to DSGE models and their estimation:
  
  – Generation of slower and more persistent dynamics (enough propagation mechanisms, lags in the transmission mechanisms, etc.)
  
  – Identification of the main model channels in place
  
  – Large number of parameters to be estimated – calibration versus estimation
Challenges

- Brazilian economic features:
  - Small sample size
  - Specific features: administered prices
  - Large changes in some ratios over the sample (ex.: net external debt-to-GDP ratio)
Next steps

- Refining model setup:
  - Add nominal and real rigidities: wage rigidity, price rigidity in the import and export sectors, firm-specific capital
  - Disaggregate CPI inflation into administered and non-administered prices

- New estimation and model implementation
Thank you for your attention!