PATACON
Policy Analysis Tool Applied to Colombian Needs

Department of Macroeconomic Modelling

Banco de la República

November 7th 2008
1. Introduction

2. Calibration

3. Data uncertainty and forecasting

4. Tools for calibrating the conditional forecast
References


Motivation

- Using a model as a main monetary policy forecasting tool is a very different exercise from simulating a model to answer particular questions.

- To forecast for policy, we need to match our model to as much of the useful information, even if that information comes in an awkward variety of shapes and forms.

- When forecasting with a DSGE model we have to take into account the following data problems:
  - Data uncertainty
  - Steady state uncertainty
  - Anticipated shocks that can be uncertain
Data availability

Diagram showing the availability of data:

- Employment
- Salaries
- CPI
- VAT Announcement
- Markets Rates
- Remittances
- IMF World GDP Forecast

Time periods: $t$, $T$
Brief description of the model

- PATACON is a DSGE model for policy analysis and forecasting designed for a Small Open Economy like Colombia.

- The model has the following set of nominal and real rigidities:
  1. Cascade of Calvo pricing
  3. Endogenous depreciation
  4. External habit in consumption
  5. Investment cost
Graphical description

Production firms

Differentiating firms

Domestic Investment

Investment Producers

Importers

Export

Foreign Sector

Households

Domestic Consumption

Production firms

Differentiating firms

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General methodology

- We think of calibration as an estimation process where we minimize an OLS-type objective function. That is, we find values of the model parameters that minimize an objective function depending on the difference between the model and the data. That is, the objective function will be

\[
    f_{\text{obj}}(\theta) = \sum_{i=1}^{n} \omega_i \left( \frac{x_{i}^{\text{ss}}(\theta) - x_{i}^{\text{d}-lr}}{x_{i}^{\text{d}-lr}} \right)^2
\]

where \(x^{ss}\) are the values of model moments and \(x_{i}^{d-lr}\) are the data values.

- We calibrate nominal ratios and relative prices including at least one level variable.
  - This allows us to get the real quantities correctly and to level the model.
Some results of our calibration

<table>
<thead>
<tr>
<th>CALIBRATION RESULTS</th>
<th>Model</th>
<th>OBJ</th>
<th>% Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment to GDP</td>
<td>0.22</td>
<td>0.22</td>
<td>-0.021</td>
</tr>
<tr>
<td>Domestic Investment to Total Investment</td>
<td>0.38</td>
<td>0.36</td>
<td>0.045</td>
</tr>
<tr>
<td>Dom. Inv. before Distribution to Gross Product</td>
<td>0.12</td>
<td>0.12</td>
<td>-0.017</td>
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<tr>
<td>Consumption to GDP</td>
<td>0.82</td>
<td>0.8</td>
<td>0.021</td>
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<tr>
<td>Dom. Cons. before Distribution to Gross Prod.</td>
<td>0.62</td>
<td>0.6</td>
<td>0.041</td>
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<td>Imported Consumption to Total Consumption</td>
<td>0.12</td>
<td>0.12</td>
<td>0.031</td>
</tr>
<tr>
<td>GDP</td>
<td>1.23</td>
<td>1.23</td>
<td>0.002</td>
</tr>
<tr>
<td>Real Wage</td>
<td>4.72</td>
<td>4.7</td>
<td>0.003</td>
</tr>
<tr>
<td>Labor Supply</td>
<td>0.3</td>
<td>0.3</td>
<td>-0.004</td>
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<tr>
<td>Raw Materials to Gross Product</td>
<td>0.1</td>
<td>0.1</td>
<td>0.034</td>
</tr>
<tr>
<td>Imports at Dock to Imports after Distribution</td>
<td>0.74</td>
<td>0.73</td>
<td>0.01</td>
</tr>
<tr>
<td>Total Imports to GDP</td>
<td>0.24</td>
<td>0.23</td>
<td>0.026</td>
</tr>
<tr>
<td>Distribution to Gross Product</td>
<td>0.1</td>
<td>0.1</td>
<td>-0.006</td>
</tr>
</tbody>
</table>
## More results ...

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<td>0.1</td>
<td>0.1</td>
<td>-0.006</td>
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<tr>
<td>Exports before Dist. to GP</td>
<td>0.16</td>
<td>0.17</td>
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<tr>
<td>Exports to Exports Share in World’s Consumption</td>
<td>1</td>
<td>1.03</td>
<td>-0.028</td>
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<td>Relative price: Exports to World’s Consumption</td>
<td>1</td>
<td>0.98</td>
<td>0.021</td>
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<tr>
<td>Exchange rate</td>
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<td>1.19</td>
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<tr>
<td>Price: Domestic Consumption</td>
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<td>0.99</td>
<td>0.006</td>
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<tr>
<td>Price: Imports After Dist.</td>
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<td>1.03</td>
<td>0.002</td>
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<tr>
<td>Price: Investment</td>
<td>1.19</td>
<td>1.19</td>
<td>0.002</td>
</tr>
<tr>
<td>Price: Exports</td>
<td>1.19</td>
<td>1.19</td>
<td>0.001</td>
</tr>
<tr>
<td>Price: Gross Product</td>
<td>1.06</td>
<td>1.06</td>
<td>0.003</td>
</tr>
<tr>
<td>Price: Imports at Dock</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
<td>Price: Raw Materials at Dock</td>
<td>0.9</td>
<td>0.9</td>
<td>-0.002</td>
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## CALIBRATION RESULTS

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<tr>
<td>Price: Raw Materials at Dock</td>
<td>0.9</td>
<td>0.9</td>
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<tr>
<td>Price: Domestic Investment</td>
<td>1.25</td>
<td>1.26</td>
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<tr>
<td>Relative price: Domestic Inv. to Total Inv.</td>
<td>1.05</td>
<td>1.05</td>
<td>0.001</td>
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<td>Relative price: Imports to Total Inv.</td>
<td>0.87</td>
<td>0.87</td>
<td>-0.005</td>
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<td>Relative price: Distribution to Imports</td>
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<td>0.91</td>
<td>-0.001</td>
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<tr>
<td>Relative price: Imp. at Dock to Imports</td>
<td>0.97</td>
<td>0.97</td>
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<tr>
<td>Relative price: Raw materials at Dock to Imports</td>
<td>0.87</td>
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<tr>
<td>Relative price: Dom. Cons. to Product</td>
<td>0.94</td>
<td>0.93</td>
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<td>Relative price: Dom. Inv. to Product</td>
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<td>Relative price: Distribution to Product</td>
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<td>Relative price: Exports to Product</td>
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<td>Transfers to GDP</td>
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<td>Model</td>
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<td>---------------------------------------------------------</td>
<td>-------</td>
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<td>Capital to GDP</td>
<td>6.79</td>
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<td>Rel. Price: Dom. Cons. before Dist. to Dom. Cons.</td>
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<td>Rel. Price: Dom. Inv. before Dist. to Dom. Inv.</td>
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<td>1.08</td>
<td>0.002</td>
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<td>Rel. Price: Exp. before Dist. to Exp.</td>
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<td>1.18</td>
<td>-0.002</td>
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<td>0.06</td>
<td>0.025</td>
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<td>Distribution of Exp. to Exp</td>
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<td>Dom. Inv. before Dist. to Dom. Inv.</td>
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<td>0.95</td>
<td>-0.041</td>
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<tr>
<td>Exp. before Dist. to Exp</td>
<td>0.83</td>
<td>0.88</td>
<td>-0.055</td>
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<td>Domestic Consumption to Gross Product</td>
<td>0.69</td>
<td>0.64</td>
<td>0.078</td>
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<tr>
<td>Domestic Investment to Gross Product</td>
<td>0.13</td>
<td>0.12</td>
<td>0.08</td>
</tr>
<tr>
<td>Exports to Gross Product</td>
<td>0.2</td>
<td>0.19</td>
<td>0.033</td>
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</tbody>
</table>
Outline

1. Introduction
2. Calibration
3. Data uncertainty and forecasting
4. Tools for calibrating the conditional forecast
Our forecasting method allows us to bring unbalanced and uncertainty data into a policy forecast, and especially for a DSGE model.

1. Solve the model for rational expectations as if all data at $t$ was perfectly known.

2. Use that solution as the state equation in a Kalman filter to solve the data uncertainty problem.

3. Use the whole data set, not only the known at $t$, and apply a smoothing filter.
We use a state space representation of the model where the measurement equation relates the available data to the model variables and the transition equation contains the structure of the model.

- The measurement equation allows us to include the uncertainty of the data through a time varying variance of the measurement error.
- Also, it allows us to handle an unbalanced data set through the use of a selection matrix.

The forecast of the model is a mix of the Kalman filter and the filter smoother.
Solution of the DSGE model

- The log-linearized version of the model can be written as a system of linear expectational difference equations of the following form:

\[ AE[x_{t+1}] = Bx_t, \quad t = 1, 2, \ldots \]

where \( x_t \) is a \( n \times 1 \) vector containing the variables of the model. \( x_t = (p_t', c_t')' \) where \( c_t \) is a vector with forward-looking variables and \( p_t = (k_t', z_t')' \) a vector with backward-looking variables (\( k_t \)) and exogenous variables (\( z_t \)).

- We include anticipated shocks by writing out the exogenous states as

\[ z_{t+1}^j = \rho_i z_t^j + \sum_{i=0}^{q} \eta_{t-i}^j \]

where shocks to \( \eta_{t-i}^j \) will be anticipated \( i \)-periods ahead.
Kalman Filter representation

The solution to the log-linearized first order conditions can be written as

\[
\begin{align*}
    c_t &= Gp_t \\
    p_{t+1} &= Hp_t + \epsilon_{t+1}
\end{align*}
\]

Consequently, our transition equation is

\[
\begin{align*}
    x_t &= \begin{pmatrix} c_t \\ p_t \end{pmatrix} = \begin{pmatrix} 0 & GH \\ 0 & H \end{pmatrix} \begin{pmatrix} c_{t-1} \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} G \\ I \end{pmatrix} \epsilon_t \\
    W_t y_t &= W_t I_s x_t + W_t \Gamma + W_t v_t
\end{align*}
\]

and our measurement equation is

where \( W_t \) is a selection matrix and \( I_s \) is matrix where every row contains only one entry different from zero (=1) and every column has at most one entry different from zero. \( \Gamma \) is vector with the steady-state values.
Data uncertainty and off model information
Forecasting method

Our forecasts for the variables contained in the $y_t$ vector is

$$y_{t+h}^f = l_s x_{t+h}^f + \Gamma$$

with

$$x_{t+h}^f = \begin{cases} x_{t+h}^T & \text{if } t + h \leq T \quad \text{Smoother} \\ \Phi^h x_T^T & \text{if } t > T \quad \text{Standard Kalman filter forecast} \end{cases}$$

where $x_t^s = E [x_t|Y_s]$ and $Y_s = (y_1 \ldots, y_t, \ldots, y_s)$.

- $E [x_t|Y_s]$ is the Kalman smoother for $s > t$.
- $T$ is the last period for which there’s at least data available for one variable in vector $y_t$. 
Real GDP Growth Forecast

Unconditional forecast

Conditional on inflation

Conditional on inflation and interest rate

Conditional on inflation, interest rate and real exchange rate
Consumption Growth Forecast

Unconditional forecast

Conditional on inflation

Conditional on inflation and interest rate

Conditional on inflation, interest rate and real exchange rate

DMM (Banco de la República)

 PATACON

November 7th 2008
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We are interested in forecasting a variable $X$ in period 45 with sample observations up to period 40.

We got some extra data up to period 53 (out sample information).

How to explain the forecast based on the available data?
Filter weights

- It is possible to write the state estimated by Kalman Filter as

\[ a_t = \sum_{j=1}^{t} w_j Y_j \]

where \( Y_j \) is a vector of observed variables at \( t \) and \( w_j \) are the weights given to the data at time \( j \) when estimating \( a_t \) with information up to time \( t \).

- Similarly, the states estimated by the filter smoother can also be written as

\[ a_t = \sum_{j=1}^{T} w_j Y_j \]

where now we have weights for data after time \( t \).
Weights for a consumption series

In this example we have data on consumption for 53 periods. The figures below show the weights that the available data has on explaining consumption in period 45.
Inflation weights for consumption forecast

In this example we have data on inflation up to 53 and consumption only up to period 40. The inflation equation has measurement error after period 40. The figure below shows the weights of the inflation series in the consumption forecast at time 45.
Exogenous shocks can be both anticipated and unanticipated. The conditional forecast of the model depends on the assumption made about the nature of the shocks.

Next figure shows the weights on the forecast of consumption in period 45 of the remittances series when shocks are

- surprises,
- anticipated four periods in advance
- or both.
Remittances graphs

All shocks are surprises

All shocks are anticipated shocks without measurement error

Shocks can be both surprises and anticipated shocks
Conclusions

- To improve the forecast of our model we need to model the different trends in the data. This can be done, following Canova (2008), by adding stochastic trends to the transition equation.
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- To improve the forecast of our model we need to model the different trends in the data. This can be done, following Canova (2008), by adding stochastic trends to the transition equation.

- We need to estimate our model to take account of parameter uncertainty in our forecasting exercises.