Structural Transformation and the Optimal Inflation Target

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This Draft: September 4, 2026

Abstract

This paper studies optimal monetary policy in a two-sector New Keynesian model with subsistence food consumption and imperfect labor mobility across sectors. These features - which are particularly relevant for emerging markets and developing economies - alter the welfare weights on sectoral inflation and create a wedge between the natural and the welfare-relevant output gap. Using a second-order approximation to aggregate welfare, the paper characterizes the resulting policy trade-offs. Limited labor mobility breaks the divine coincidence: even in the absence of cost push shocks, optimal monetary policy cannot fully stabilize both core inflation and the welfare-relevant output gap. Food-sector productivity shocks therefore generate a trade-off between inflation and output stabilization. As economies develop and the steady-state share of food consumption declines, this trade-off becomes quantitatively smaller, and full core inflation stabilization emerges as approximately optimal. Furthermore, by solving explicitly a problem of optimal discretionary policy delegation, we show that some form of strict *headline* inflation targeting yields higher social welfare than what obtained under flexible *core* inflation targeting, and can almost replicate what one would obtain under the truly optimal (yet unrealistic) policy with commitment.

Keywords: Inflation Targeting, Optimal Monetary Policy, Structural Transformation, New Keynesian Model, Food Inflation
JEL Classifications: C62, D83, E32, E52.

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1 Introduction

Economic development is typically accompanied by profound changes in both the structure of production and the conduct of macroeconomic policy. As countries grow richer and stabilize their macroeconomic environment, central banks often seek to modernize their monetary policy frameworks, moving away from regimes centered on monetary aggregates or exchange rate anchors toward more flexible, forward-looking strategies, such as inflation targeting (IT). This transition raises an important question: to what extent should the design and implementation of monetary policy depend on a country's stage of development and underlying structural characteristics?

These issues are particularly salient in low-income and emerging market economies, where recent decades have witnessed substantial progress in inflation stabilization. A prominent example is Latin America where inflation rates have been curbed from two (sometimes three or higher) to single-digit figures, with Chile - the most developed economy in the area - officially adopting IT in 1993, then followed by Brasil, Peru, Mexico, etc. as they climbed up the development ladder. Since the late 1990s and early 2000s, many countries in Sub-Saharan Africa have successfully reduced inflation to single-digit levels through fiscal-based stabilization efforts and monetary frameworks centered, at least de jure, on money targets. Having largely contained inflation and fiscal pressures, several of these economies are now seeking to modernize their monetary policy frameworks. Some have adopted explicit IT regimes (e.g., Ghana and Uganda), others have announced plans to do so (e.g., Nigeria and Angola), while a growing number are incorporating key elements of IT—such as the use of a policy rate to signal the monetary stance, enhanced central bank communication, and improved inflation forecasting and analytical capacity (e.g., Kenya and Rwanda). Similar developments have taken place in developing Asia, where countries such as Indonesia, the Philippines, Thailand, and India have adopted inflation targeting or flexible inflation-targeting frameworks, while others continue to debate further modernization of their monetary policy regimes.

The interest in more flexible and forward-looking monetary policy frameworks raises the question of whether development-related structural features are central to their effectiveness. In many developing economies, the food sector accounts for a large share of production, and households devote a substantial fraction of their expenditure to food consumption, in part because of subsistence requirements—a minimum level of food consumption below which utility cannot be maintained. At low levels of aggregate productivity, a large fraction of labor (and capital) must be allocated to food

production to satisfy subsistence needs. As economies develop and move away from subsistence, fewer resources are required in the food sector, allowing the relative expansion of manufacturing and services. The reallocation of labor away from agriculture was the defining feature of what Chenery and Syrquin (1971) termed structural transformation. Yet, the implications of these structural characteristics for monetary policy design remain relatively underexplored.

In this paper, we focus on one specific channel through which development matters for monetary policy. We study how the level of development—captured by labor-augmenting technological progress and its interaction with food subsistence—affects the stabilizing properties of monetary policy, whether implemented through Taylor-type interest rate rules or optimal targeting rules. To this end, we introduce subsistence food consumption into a two-sector New Keynesian model in which food prices are flexible, non-food prices are sticky, and labor is imperfectly mobile across sectors. Using a second-order approximation to aggregate welfare, we characterize the key policy trade-offs faced by the central bank, derive optimal discretionary policy, and examine how the optimal targeting rule can be implemented through simple instrumental interest rate rules.

Our analysis delivers several interesting results. First, when labor mobility across sectors is limited, the welfare-relevant (efficient) level of output differs from the natural level associated with flexible prices. Numerical evaluations indicate that the efficient level of output is systematically higher than the natural level. This reflects the fact that the natural allocation abstracts from labor immobility frictions, whereas the efficient allocation internalizes the welfare costs arising from sectoral labor misallocation.

Second, limited labor mobility implies a failure of the divine coincidence. Although the welfare-based loss function can still be expressed as a quadratic function of the squared deviations of core (non-food) inflation and the welfare-relevant output gap from their respective targets, optimal monetary policy cannot fully stabilize both objectives. In particular, a positive productivity shock in the food sector optimally calls for a negative output gap together with above-target core inflation, reflecting the trade-off between inflation stabilization and the mitigation of labor misallocation.

Third, the quantitative importance of this trade-off depends on the level of economic development. As the economy develops—captured by higher steady-state aggregate productivity—the share of food consumption, and hence the relative size of the food sector, declines. For any given degree of imperfect labor mobility, the departure from the divine coincidence becomes less pronounced, and full core price stability becomes increasingly predominant and less in conflict with

output gap stabilization. In the presence of food subsistence concerns, price stability is therefore harder to attain in poorer economies than in richer ones.

Fourth, we provide analytical results showing that by targeting an appropriate measure of *headline* inflation the central bank can exactly replicate the optimal policy under commitment. By targeting headline (instead of core) inflation, the central bank can induce endogenous inflation inertia in the model's equilibrium dynamics which, for the appropriate weight on (flexible) food inflation in the headline measure, generates the same degree of history-dependence obtained under commitment. Targeting CPI inflation does not replicate the commitment outcome but gets significantly close to that.

This paper connects to a long tradition of research on economic development and structural transformation, which examines how economies evolve as they grow richer and reallocate resources across sectors. Classic contributions emphasize shifts in sectoral labor and output shares as fundamental to growth: Simon Kuznets' early work documented the movement of labor out of agriculture and into industry and services as income rises, while Chenery and Syrquin (1975) and Syrquin (1988) formalized systematic patterns of structural change across countries. A theoretical literature embeds these patterns into general equilibrium frameworks, demonstrating how comparative advantage, demand shifts, and labor market frictions shape structural transformation. Seminal models by Matsuyama (1992) and by Ngai and Pissarides (2007) analyze how sectoral productivity and preferences drive resource reallocation across agriculture and non-agriculture, while Rogerson (2008) explores the interaction between labor market frictions and structural change. These contributions collectively highlight how the structural evolution of economies alters relative prices, labor allocation, and welfare outcomes.

The paper also builds on literature that highlights the role of subsistence consumption and food inflation in macroeconomic outcomes and policy design. A growing body of work recognizes that volatile food prices and subsistence requirements have disproportionately large welfare effects—especially in poorer economies where the food share of total expenditure is high. Empirical and theoretical studies by Catao and Chang (2010, 2012) document the macroeconomic consequences of food price shocks and subsistence constraints, while Anand, Prasad, and Zhang (2015) emphasize how inflation dynamics, particularly in food-intensive economies, shape monetary policy challenges. These contributions motivate incorporating food subsistence into dynamic macroeconomic models and underscore the importance of accounting for relative price effects in policy evaluation.

Finally, this paper contributes to the New Keynesian monetary policy literature, which provides the core analytical framework for modern policy analysis. Foundational work by Clarida, Galí, and Gertler (1999) and by Gali (2015) formalizes the dynamic general equilibrium approach to policy design under nominal rigidities and forward-looking expectations, characterizing optimal policy in terms of inflation and output stabilization. Subsequent research has explored the conditions under which the "divine coincidence" holds or fails, illustrating how various real frictions and market imperfections influence policy trade-offs. By introducing subsistence food consumption and imperfect labor mobility into a two-sector New Keynesian framework, this paper bridges these strands, highlighting how structural and developmental characteristics shape the monetary policy trade-offs faced by central banks.

The rest of the paper is organized as follows. Section 2 lays out the two-sector New Keynesian model. Section 3 defines the equilibrium and its steady state, presents the reduced-form (linearized) equations characterizing the equilibium dynamics around it, and describes the baseline calibration. Section 4 studies the design of optimal monetary policy (both under discretion and commitment), provides an in-depth derivation and discussion about the equivalence between (strict) headline and (flexible) core inflation targeting policies, and, more generally, studies the problem of optimal policy delegation. Section 5 extends the analysis to the case where labor mobility depends on the level of development. Section 6 concludes.

2 The Model

We build a two-sector New-Keynesian model, comprised of a flexible-price perfectly competitive food sector, and a sticky-price monopolistically competitive non-food sector. A continuum of infinitely-lived identical households purchases goods from both sectors, and supplies them sector-specific labor (labor is imperfectly mobile across sectors). A government/central bank conducts monetary policy through either an instrumental Taylor-type rule or an optimal targeting rule.

2.1 Households

The economy is populated by a continuum of infinitely-lived identical households. The representative household seeks to maximize the following lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\ln \tilde{c}_t - \frac{h_t^{1+\chi}}{1+\chi} \right], \tag{1}$$

where $\beta \in (0,1)$ is the subjective discount factor, and $\chi \geq 0$ is the (inverse) Frisch elasticity of labor supply. The term $\tilde{c}_t$ is a composite of food, $c_{F,t}$, and non-food, $c_{N,t}$, consumption. More specifically, we assume that

$$\tilde{c}_t \equiv \Upsilon \tilde{c}_{F,t}^{\alpha} c_{N,t}^{1-\alpha}, \qquad \text{for} \qquad \alpha \in (0,1), \tag{2}$$

$$\tilde{c}_{F,t} \equiv c_{F,t} - \bar{c}_F \tag{3}$$

where $\Upsilon \equiv \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}$ is a scaling factor, and $\bar{c}_F > 0$ indicates the subsistence level of food consumption, a threshold below which food consumption cannot decline. Following Horvath (2000), we capture the possibility of imperfect labor mobility across sectors by assuming the following CES aggregator for h_t:

$$h_t = \left[\psi^{-\frac{1}{\xi}} h_{F,t}^{\frac{1+\xi}{\xi}} + (1-\psi)^{-\frac{1}{\xi}} h_{N,t}^{\frac{1+\xi}{\xi}} \right]^{\frac{\xi}{1+\xi}}.$$

where $h_{j,t}$ denotes labor supplied to sector j, for $j = F, N$, and $\psi \in (0,1)$ corresponds to the steady state share of labor supplied to the food sector. The degree of imperfect mobility is governed by the intratemporal elasticity of substitution, $\xi > 0$: for $\xi \to \infty$, labor is perfectly mobile across sector (as h_t would just be the sum of $h_{F,t}$ and $h_{N,t}$), while for $\xi \to 0$, labor is completely immobile. For ξ positive but finite, mobiliy is limited, which will result in different equilibrium sectoral wages.[1]

The household's budget constraint is the following:

$$P_{F,t} c_{F,t} + P_{N,t} c_{N,t} + B_t = R_{t-1} B_{t-1} + W_{F,t} h_{F,t} + W_{N,t} h_{N,t} + \Delta_t^F + \Delta_t^N - T_t \tag{4}$$

where $P_{F,t}$ (respectively, $P_{N,t}$) is the per-unit price of food (respectively, non-food) consumption, B_t are units of a one-period nominal bond paying the riskless (gross) interest rate R_t, $W_{F,t}$ (respectively, $W_{N,t}$) is the nominal wage in the food (respectively, non-food) sector, Δ_t^F and Δ_t^N are profits from firms' ownership, and T_t are lump-sum taxes/transfers.

After taking first order conditions with respect to choice variables, we obtain the following

[1] Similar specifications - whereby ξ is treated as a standalone parameter unrelated to other structural features of the economy - appear in Iacoviello and Neri (2010), Bouakez et al. (2011), Petrella and Santoro (2011), and Katayama and Kim (2018). Later in the analysis (see Section 5) we will made ξ function of aggregate development to be more consistent with empirical evidence.

optimality conditions:

$$\tilde{c}_{F,t}^{-1} = \beta R_t E_t \frac{\tilde{c}_{F,t+1}^{-1}}{\pi_{F,t+1}}, \qquad c_{N,t}^{-1} = \beta R_t E_t \frac{c_{N,t+1}^{-1}}{\pi_{N,t+1}} \tag{5}$$

$$h_t^{\chi-\frac{1}{\xi}} \psi^{-\frac{1}{\xi}} h_{F,t}^{\frac{1}{\xi}} = \alpha \frac{W_{F,t}}{P_{F,t}} \tilde{c}_{F,t}^{-1}, \qquad h_t^{\chi-\frac{1}{\xi}} (1-\psi)^{-\frac{1}{\xi}} h_{N,t}^{\frac{1}{\xi}} = (1-\alpha) \frac{W_{N,t}}{P_{N,t}} c_{N,t}^{-1} \tag{6}$$

where $\pi_{j,t+1} \equiv \frac{P_{j,t+1}}{P_{j,t}}$, for $j = F, N$, denotes gross inflation in sector j. The two equations in (5) are good-specific Euler equations capturing the household's desire for intertemporal consumption smoothing. The two equations in (6) define instead the household's optimal supply of labor to both sectors. As the first order conditions with respect to consumption imply that $\alpha \frac{\tilde{c}_{F,t}^{-1}}{P_{F,t}} = (1-\alpha) \frac{c_{N,t}^{-1}}{P_{N,t}}$, these labor supply equations can be combined to yield:

$$\frac{h_{F,t}}{h_{N,t}} = \frac{\psi}{1-\psi} \left(\frac{W_{F,t}}{W_{N,t}} \right)^{\xi} \tag{7}$$

Clearly, under perfect mobility ($\xi \to \infty$), wages would be equalized - i.e. $W_{F,t} = W_{N,t}$ - and sectoral labor allocation would be indeterminate.

Since part of the analysis will focus on monetary policy rules reacting to/targeting CPI measures of inflation, we define the price index P_t as the expenditure weighted average price in the economy: namely,

$$P_t \equiv \gamma P_{F,t} + (1-\gamma) P_{N,t}, \qquad \text{for} \qquad \gamma \equiv \frac{P_F c_F}{P_F c_F + P_N c_N} \tag{8}$$

where γ corresponds to the steady state expediture share of food consumption in total consumption spending.[2] Based on this index P_t, real aggregate consumption c_t is defined as

$$c_t \equiv \frac{P_{F,t} c_{F,t} + P_{N,t} c_{N,t}}{P_t} = p_{F,t} c_{F,t} + p_{N,t} c_{N,t} \tag{9}$$

for $p_{F,t} \equiv \frac{P_{F,t}}{P_t}$ and $p_{N,t} \equiv \frac{P_{N,t}}{P_t}$. Finally, the relative price of food is defined as $q_t \equiv \frac{P_{F,t}}{P_{N,t}}$.

[2] As shown later, because of subsistence in food consumption, the weight γ will differ from α. The latter represents the steady state relative share of subsistence-adjusted food consumption, $\tilde{c}_F$, in the subsistence-adjusted consumption aggregator, $\tilde{c}$. Our construction of P_t is similar to what adopted by Ravn et al. (2012) in a two-sector model with deep-habits. As a matter of fact, one could think of "subsistence" as time-invariant habits.

2.2 Firms

Production comprises a food and a non-food sector. The food sector features perfect competition and flexible prices. A representative firm produces food using labor $h^d_{F,t}$ as the only input (with the superscript d standing for "demand"):

$$y_{F,t} = z_{F,t} A h^d_{F,t}, \tag{10}$$

where $z_{F,t}$ is (stochastic) TFP in the food sector with unconditional mean $E(z_{F,t})$ normalized to unity. Letting $\hat{z}_{F,t} \equiv \ln z_{F,t}$, we assume $\hat{z}_{F,t}$ follows a stationary AR(1) process $\hat{z}_{F,t} = \rho_z \hat{z}_{F,t-1} + \hat{\varepsilon}_{z,t}$ for $\rho_z \in [0,1)$ and $\hat{\varepsilon}_{z,t} \sim \text{iid} N\left(0, \sigma_z^2\right)$. The scaling factor A captures instead labor-augmenting technological progress, and can therefore be thought as an indicator of development in the economy. In the analysis, we will study how the stabilizing properties of optimal monetary policy and instrumental rules depend on A through its impact on the expediture share of food consumption γ. Letting $w_{F,t} \equiv \frac{W_{F,t}}{P_t}$ be the real wage, profit maximization in the food sector gives the following relationship:

$$\frac{w_{F,t}}{p_{F,t}} = z_{F,t} A \tag{11}$$

The non-food sector is divided in two subsectors: wholesale and retail. The retail sector produces the final non-food consumption good under perfect competition and flexible prices, out of a continuum of intermediate differentiated non-food products, according to the following CRS technology:

$$y_{N,t} = \left(\int_0^1 y_{N,t}(i)^{\frac{\epsilon_t}{\epsilon_t - 1}} di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}}, \qquad \epsilon_t > 1 \tag{12}$$

We allow for a (stochastic) time-varying elasticity of substitution ϵ_t to incorporate markup shocks, a documented major source of inflation volatility both in developed and developing economies. The optimal demand for the intermediate non-food good $y_{N,t}(i)$ is given by $y_{N,t}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t}} \right)^{-\epsilon_t} y_{N,t}$, where $P_{N,t} \equiv \left(\int_0^1 P_{N,t}(i)^{1-\epsilon_t} di \right)^{\frac{1}{1-\epsilon_t}}$ is the non-food price index.

The wholesale sector is made of a continuum of firms indexed by i, for $i \in [0,1]$. Each firm acts

under monopolistic competition and is subject to nominal rigidities in price setting. The i-th firm hires labor to produce the i-th variety of a continuum of intermediate non-food products which are then sold to retailiers. Labor is subsidized at the constant rate $\tau \in (0,1)$ by the fiscal authority.[3] As for the food sector, the production technology is linear[4]:

$$y_{N,t}(i) = Ah^d_{N,t}(i), \tag{13}$$

so that labor-augmenting technological progress is the same across sectors. Prices cannot fully adjust due to Calvo contracts: in every period t, each firm will not be able to reset its price with probabiliy $\theta \in (0,1)$. The i-th firm objective is to choose the optimal price $P_{N,t}(i)$ in order to maximize

$$E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} \left[\frac{P_{N,t}(i)}{P_{N,t+k}} y_{N,t+k}(i) - (1-\tau) \frac{W_{N,t+k}}{P_{N.t+k}} h_{N,t+k}(i) \right] \tag{14}$$

suject to $y_{N,t+k}(i) = \left(\frac{P_{N,t}(i)}{P_{N,t+k}} \right)^{-\epsilon_t} y_{N,t+k}$, where $\Lambda_{t,t+k} \equiv \beta^k \left(\frac{\tilde{c}_{t+k}}{\tilde{c}_t} \right)^{-1}$ is the representative household's stochastic discount factor. After taking first order conditions, the optimal price $P^*_{N,t}(i)$ must satisfy the following condition:

$$E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} (1-\epsilon_{t+k}) \frac{y_{N,t,t+k}}{P_{N,t+k}} \left[P^*_{N,t} - \mu_{t+k} MC_{N,t+k} \right] \tag{15}$$

with relative demand $y_{N,t,t+k} \equiv \left(\frac{P^*_{N,t}}{P_{N,t+k}} \right)^{-\epsilon_{t+k}} y_{N,t+k}$, gross markup $\mu_{t+k} \equiv \frac{\epsilon_{t+k}}{\epsilon_{t+k}-1}$ and nominal marginal costs $MC_{N,t+k} \equiv (1-\tau) \frac{W_{N,t+k}}{A}$. Absent nominal rigidities, the optimal price in the non-food sector would just be a time-varying markup μ_t on nominal marginal costs: $P^*_{N,t} = \mu_t MC_{N,t}$. Following standard practice, we assume $\hat{\mu}_t \equiv \ln(\mu_t/\mu)$ - where $\mu \equiv \epsilon/(\epsilon-1)$ - also follows a stationary AR(1) process $\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \hat{\varepsilon}_{\mu,t}$ for $\rho_\mu \in [0,1)$ and $\hat{\varepsilon}_{\mu,t} \sim \text{iid}N\left(0, \sigma^2_\mu\right)$.

[3] As discussed below, the labor subsidy will be chosen to give an undistorted steady state and therefore allow the use of standard techniques to derive a second order approximation to the representative agent's welfare function for the optimal policy analysis. It is clearly unnecessary for the case of (sub-optimal) instrumental rules.

[4] Given the purpose of our analysis, without loss of generality, we are abstracting from stochatic variations in TFP in the non-food sector.

2.3 Government Policies

The fiscal government sets lump-sum taxes T_t to finance the the labor subsidy paid to firms: $T_t = \tau W_{N,t} h_{N,t}$ where $h_{N,t} \equiv \int_0^1 h_{N,t}(i)\, di$.

Monetary policy will take the form of either a) a Taylor-type interest rate rule, whereby the monetary authority moves the policy rate in response to deviations of inflation and other macro indicators from their respective targets, or b) a targeting rule, whereby the monetary authority implements the optimal inflation-output stabilization mix in every period.

3 Equilibrium, Steady State and Baseline Calibration

In equilibrium, all markets clear: in particular, $c_{N,t} = y_{N,t}$, $c_{F,t} = y_{F,t}$, $h_{N,t} \equiv \int_0^1 h^d_{N,t}(i)\, di$ for $i \in [0,1]$, $h_{F,t} = h^d_{F,t}$, and $B_t = 0$ (bonds are in zero net supply). From its definition in the previous section and Calvo-style pricing, the law of motion for $P_{N,t}$ is:

$$P_{N,t}^{1-\epsilon} = (1-\theta)\left(P^*_{N,t}\right)^{1-\epsilon} + \theta P_{N,t-1}^{1-\epsilon} \tag{16}$$

3.1 Steady State

Our economy features a unique zero-inflation non-stochastic steady state equilibrium[5]. To see that, let $z_{A,t}$ equal its unconditional mean, which, without loss of generality, we assume equal to unity, and let $\pi_N = 1$ (zero non-food inflation). Then, by equations (15) and (16), we have $P^*_N = P_N$, which in turns implies that $mc_N = \mu^{-1}$ for $\mu \equiv \frac{\epsilon}{\epsilon-1} > 1$. The latter, together with the definition of marginal costs, gives $P_N = (1-\tau)\,\mu \frac{W_N}{A}$. To eliminate the steady state distortion due to imperfect competition in the non-food sector, we assume that the fiscal authority chooses τ such that $(1-\tau)\,\mu = 1$. Using the pricing condition in the food sector (11), we have $P_F = \frac{W_F}{A}$, which then implies $\frac{W_F}{P_F} = \frac{W_N}{P_N} = A$.

To simplify the notation while allowing for analytical results, we consider a steady state equilibrium where the relative price of food is equal to unity - that is, $q = \frac{P_F}{P_N} = 1$ - such that from the definition of P_t in (8) we also have $p_F = p_N = 1$, which in turn implies $w_F = w_N = A$. With $q = 1$, the household's first order conditions with respect to consumption give $c_N = \frac{1-\alpha}{\alpha}\tilde{c}_F$,

[5] Lack of a time subscript for endogenous variables indicate a steady state value.

which, once combined with the definition of $\tilde{c}$ in (2), yields relative consumption levels $\tilde{c}_F = \alpha\tilde{c}$ and $c_N = (1-\alpha)\,\tilde{c}$. With $p_F = p_N = 1$, steady state aggregate consumption is $c = c_F + c_N$, from which it is straightforward to reach a relationship between c and $\tilde{c}$: $c = \tilde{c} + \bar{c}_F$. Moreover, the spending share γ defined in (8) is simply $\gamma = \frac{c_F}{c_F + c_N}$, such that $c_F = \gamma c$ and $c_N = (1-\gamma)\,c$.

From the labor supply conditions in (6), and relative demands $\tilde{c}_F = \alpha\tilde{c}$ and $c_N = (1-\alpha)\,\tilde{c}$, after simple manipulation of terms, we have that $h_F = \psi h$ and $h_N = (1-\psi)\,h$ for any finite ξ. The latter combined with market clearing conditions in the goods market, $c_F = y_F = Ah_F$ and $c_N = y_N = Ah_N$, and the spending shares $c_F = \gamma c$ and $c_N = (1-\gamma)\,c$, requires us to impose the following assumption.

Assumption 1: $\psi = \gamma$.[6]

Next consider the labor supply condition for the food sector in (6). Using $h = \gamma^{-1}h_F$, market clearing $h_F = c_F/A$, and $c_F = \gamma c$, we obtain an implicit equation for steady state consumption c:

$$c^{\chi} = \frac{A^{1+\chi}}{c - \bar{c}_F} \tag{17}$$

Proposition 1 *For any given $\bar{c}_F > 0$, there exists a unique solution $c > 0$ to equation (17), and hence a unique steady state equilibrium. Moreover, the steady state spending share on food consumption,*

$$\gamma = \frac{c_F}{c} = \alpha + \frac{(1-\alpha)\,\bar{c}_F}{c}, \tag{18}$$

satisfies the following properties: i) $\gamma = \alpha$ if $\bar{c}_F = 0$, ii) $\gamma \in (\alpha, 1)$ if $\bar{c}_F > 0$, with $\lim_{A\to\infty} \gamma = \alpha$, and iii) $\frac{\partial\gamma}{\partial A} < 0$.

Proof. See Appendix A.2.1. ■

The presence of subsistence implies that the spending share on food consumption γ is always larger than the utility share α, and converges to it only in the limit. Moreover, as the economy develops (higher A), the economy moves away from its subsistence level of food consumption, and, as a result, the (optimal) average share of food in the representative consumption basket declines. As shown later, this rather intuitive property of our model will have significant implications for the design of monetary policy.

[6] Without this assumption a unique steady state equilibrium would still exist, but it would not necessarily feature a relative price q equal to unity.

3.2 Log-Linearized Model

We log-linearize the equilibrium conditions of the decentralized economy around the unique steady state.[7] As shown in the Appendix, the Euler equation for aggregate output takes the following form:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}\left[\hat{R}_t - E_t\hat{\pi}_{t+1} + \delta\left(E_t\hat{p}_{F,t+1} - \hat{p}_{F,t}\right)\right], \tag{19}$$

$$\sigma \equiv \frac{1-\alpha}{1-\gamma}, \qquad \delta \equiv \frac{\gamma-\alpha}{1-\gamma} \tag{20}$$

As in the benchmark NK model, output depends negatively on the *ex-ante* real interest rate. However, due to the presence of subsistence, it also responds (negatively, since $\gamma > \alpha$) to expected changes in the relative price of food, $E_t\hat{p}_{F,t+1} - \hat{p}_{F,t}$. As shown in Appendix A.1.1, exploiting the relationship between $\hat{p}_{F,t}$ and $\hat{y}_t$, we can write (19) as follows:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\tilde{\sigma}}\left(\hat{R}_t - E_t\hat{\pi}_{N,t+1}\right) + \Psi_y\hat{z}_{F,t}, \tag{21}$$

$$\tilde{\sigma} \equiv \frac{\sigma\left[\vartheta + \frac{1}{\xi} + \gamma\left(\chi - \frac{1}{\xi}\right)\right]}{(1-\alpha)\left(\vartheta + \frac{1}{\xi}\right)}, \qquad \vartheta \equiv \frac{\gamma}{\alpha}\sigma \tag{22}$$

where Ψ_y is a composite parameter defined in the Appendix. Notice that without food consumption ($\alpha = 0$ and $\bar{c}_F = 0$, such that $\gamma = 0$ and $\sigma = 1$), we have $\tilde{\sigma} = 1$, in which case the model collapses to a benchmark one-sector sticky price New Keynesian model with log-utility. With food consumption and subsistence, but full labor mobility, $\xi \to \infty$, simple algebra gives $\tilde{\sigma} = \frac{\sigma+\alpha\chi}{1-\alpha}$. Hence, food consumption makes $\tilde{\sigma}$ larger than unity, and even more so with subsistence, since $\gamma > \alpha$. Therefore, the presence of subsistence levels in food consumption lowers the interest rate elasticity of output, $\tilde{\sigma}^{-1}$, with respect to the benchmark New Keynesian set-up. Simple calculus also shows that $\tilde{\sigma}^{-1}$ is strictly decreasaing in ξ. We summarize the results with the following Lemma.

Lemma 1 *The temporary elasticity of output to the real interest rate, i.e. $\tilde{\sigma}^{-1}$ in the IS curve (21), is strictly decreasing in the extent of subsistence and the degree of labor mobility ξ.*

[7] See Appendix A.1.1 for a detailed derivation.

Letting $\kappa \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ denote the temporary elasticity of non-food price inflation to marginal costs,the supply side is described by a linear Phillips curve for non-food inflation:

$$\hat{\pi}_{N,t} = \beta E_t \hat{\pi}_{N,t+1} + \kappa_y \hat{y}_t + \Psi_\pi \hat{z}_{F,t} + \kappa \hat{\mu}_t, \qquad \text{for} \qquad \kappa_y \equiv \kappa \frac{\gamma\chi + \vartheta\left[1 + (1-\gamma\chi)\right] + \frac{1}{\xi}\left[\gamma\vartheta + 1 - \gamma + \chi\right]}{(1-\gamma)\left(\vartheta + \frac{1}{\xi}\right)}. \tag{23}$$

where Ψ_π is also a composite parameter defined in the Appendix.

The model is closed by specifying monetary policy. In the analysis to follow, we will assess alternative policies with respect to aggregate welfare and equilibrium determinacy, for alternative parameterizations of development A.

3.3 Baseline Calibration

As most of the analysis will be numerical, before proceeding, we define our baseline calibration. Two key structural parameters are α (the share of subsistence-adjusted food consumption in utility) and $\bar{c}_F$ (the level of subsistence for food). For simplicity, we assume that these two, as well as all remaining structural parameters, are unchanged across different levels of development, such that countries are distinguished only because of A. We choose their values to match the following empirical observations: i) the share of food consumption in total spending (about 8% in the U.S. and 58% in India); ii) the real income per capita ratio of India with respect to the US (about 10%). Using the definition of γ in (18) and market clearing, we have $\gamma^i = \alpha + \frac{(1-\alpha)\bar{c}_F}{y^i}$ for $i = I, US$. After normalizing U.S. output to unity ($y^{US} = 1$), simple algebra gives $\alpha = \frac{\gamma^{US} - \gamma^I y^I}{1 - y^I}$. Given the latter, we retrive the subsistence level $\bar{c}_F = \frac{\gamma^{US} - \alpha}{1-\alpha}$. We use the latter, together with the steady state equation (17) to find the country-specific value for A: namely $A^i = \left[\left(y^i - \bar{c}_F\right)\left(y^i\right)^\chi\right]^{\frac{1}{1+\chi}}$, for $i = I, US$. This procedure gives $A^{US} \approx 0.99$ and $A^I \approx 0.09$.

The remaining structural parameters follow a textbook calibration.. We set $\beta = 0.99$, which corresponds to a 4% annual real interest rate. For the inverse elasticity χ we take unity as our baseline value. This is consistent with the micro-evidence on labor supply elasticity by Card (1991) in the U.S., and appears in line with the documented evidence of low labor elasticity in low income countries. The Calvo probability of no price change θ is set equal to 0.75. Finally, we fix the elasticity of substitution across non-food goods products ϵ equal to 6. For what concerns labor mobility, we have ξ equal to unity. This value is consistent with the original reduced form

estimation by Horvath (2000), as well as with the fully-fledged Bayesian estimation in Iacoviello and Neri (2010). It also corresponds to the mid-value used by Bouakez et al. (2011).[8]

4 Welfare-Based Optimal Policy

We study optimal monetary policy from the point of view of a benevolent central bank who seeks to maximize the representative household's lifetime utility. As shown in Appendix A.1.3, a second-order approximation to the representative household's welfare around the model's steady state allows us to express the central bank's objective as follows:

$$\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\lambda_\pi \hat{\pi}_{N,t}^2 + \lambda_x \hat{x}_t^2 \right) \tag{24}$$

The term $\hat{x}_t \equiv \hat{y}_t - \hat{y}_t^T$ represents the welfare-relevant output gap, with target value $\hat{y}_t^T = F^T \hat{z}_{F,t}$ for $F^T \equiv \gamma \frac{\Omega_{zy}}{\Omega_y}$, and welfare weights λ_π and λ_x defined as

$$\lambda_\pi \equiv \frac{(1-\gamma)\,\epsilon}{\kappa}, \qquad \lambda_x \equiv \Omega_y, \tag{25}$$

where Ω_y and Ω_{zy} are convoluted expressions of structural parameters. The Phillips curve for non-food inflation, the relevant policy constraint, can be easily expressed in terms of aggregate output deviations from its flexible price (natural) level, $\hat{y}_t^{gap} \equiv \hat{y}_t - \hat{y}_t^n$ - henceforth, the *natural gap* - where $\hat{y}_t^n = F^n \hat{z}_{F,t}$:[9]

$$\hat{\pi}_{N,t} = \beta E_t \hat{\pi}_{N,t+1} + \kappa_y \hat{y}_t^{gap} + \kappa \hat{\mu}_t \tag{26}$$

The analysis highlights a clear discrepancy between the welfare-relevant output gap $\hat{x}_t$ entering the objective (24), and the flexible-price output gap $\hat{y}_t^{gap}$ entering the constraint (26). More specifically:

$$\hat{x}_t = \hat{y}_t^{gap} - F \hat{z}_{F,t} \qquad \text{for} \qquad F \equiv F^T - F^n.$$

Absent markup shocks, non-food price stability ($\hat{\pi}_{N,t} = 0$ in every period t) would require the

[8] Katayama and Kim (2018) find evidence of much lower labor mobility across sectors, with ξ ranging between 1/4 and 1/3.

[9] See the Appendix A.1.2 for a detalied derivation of the flexible-price (natural) equilibrium - hence, for the explicit expression for F^n - and the reduced form system written in terms of natural gaps.

central bank to make output equal to its natural level at all times. However, such policy would not maximize aggregate welfare as it would generate inefficient deviations of output from its target $\hat{y}_t^T$. This tension is the direct consequence of imperfect mobility of labor across sectors. Proposition 2 shows that the target level of output converges to its flexible price level as labor get perfectly mobile.

Proposition 2 *Under perfect labor mobility, the target level of output,* $\hat{y}_t^T = F^T \hat{z}_{F,t}$*, coincides with its natural level,* $\hat{y}_t^n = F^n \hat{z}_{F,t}$*: i.e.,* $\lim_{\xi \to \infty} F^T = \lim_{\xi \to \infty} F^n$.

To solve for the optimal monetary policy allocation, we therefore express the NKPC in terms of the welfare relevant output gap $\hat{x}_t$:

$$\hat{\pi}_{N,t} = \beta E_t \hat{\pi}_{N,t+1} + \kappa_y \hat{x}_t + \kappa_z \hat{z}_{F,t} + \kappa \hat{\mu}_t, \qquad \text{for} \qquad \kappa_z \equiv \kappa_y F. \tag{27}$$

When labor is imperfectly mobile (namely, $F \neq 0$), food-specific productivity $\hat{z}_{F,t}$ acts like an additional cost-push shock, which, even in the absence of exogenous markup fluctuations (namely, $\hat{\mu}_t = 0$), makes the simultaneous stabilization of the output gap and (core) non-food inflation unattainable: divine coincidence fails. As discussed later, for our baseline calibration, the NKPC coefficient κ_z is positive, thus making a positive $\hat{z}_{F,t}$ acts similarly to a positive markup shock.

In what follows, we are going to compare the optimal policy outcomes under discretion and commitment, the latter corresponding to the first best in a decentralized environment[10]. We will then show that the optimal policy under commitment could be *fully replicated* through an appropriately designed *headline inflation targeting rule* and *closely approximated* by assigning a *flexible headling inflation targeting objective* to a discretionary central banker.

4.1 Discretion

Assuming the central bank acts under discretion, the optimal policy problem involves the minimization of the temporary loss $\frac{1}{2}\left(\lambda_\pi \hat{\pi}_{N,t}^2 + \lambda_x \hat{x}_t^2\right)$ subject to the static constraint $\hat{\pi}_{N,t} = \kappa_y \hat{x}_t + \kappa_z \hat{z}_{F,t} + \kappa \hat{\mu}_t$. As well known, all terms containining expectations of future variables, both in the objective (24) and the constraint (27), are taken as given by the policy-maker due its inability to

[10] Obviously, the true first best is the allocation implemented by a benevolent social planner in a centralized (frictionless) economy.

influence private expectations under lack of commitment[11]. The solution to this problem produces the following targeting rule[12]:

$$\hat{\pi}_{N,t} = -\frac{\lambda}{\kappa_y}\hat{x}_t, \qquad \text{for} \qquad \lambda \equiv \frac{\lambda_x}{\lambda_\pi} \tag{28}$$

which defines the optimal trade-off between non-food inflation and output gap stabilization.

It is important to highlight some key implications of food subsistence for optimal monetary policy. For this purpose, let $ORV \equiv \frac{SD\left(\hat{\pi}_{N,t}\right)}{SD(\hat{x}_t)}$ denote the optimal relative volatility of core inflation with respect to the output gap. Under the assumption of perfect labor mobility across sectors, we can obtain analytical results on the relationshipt between the level of development A and ORV.

Proposition 3 ***May work also for*** $\xi = 1$. ***Check!****Assume labor is perfectly mobile across sectors, i.e.* $\xi \to +\infty$. *As the economy develops - i.e., larger* A *- stabilizing non-food core inflation becomes more important (larger* λ*), but also more costly (smaller* κ_y*) in terms of output. Overall, an increase in* A *lowers the optimal relative volatility* ORV*.*

The economic intuition for Proposition 3 is the following. To start with, recall that the cost of inflation stabilization is inversely related to the slope of the Phillips curve. Namely, a one percent reduction in core inflation requires, other things equal, a $1/\kappa_y$ drop in the output gap. Then, as the economy develops the share of consumption expenditure going to food (hence, the size of the food sector, as well as the share of labor allocated to it) shrinks. On the one hand, this diminishes the equilibrium response of marginal costs paid by non-food firms to aggregate activity, as the latter becomes less sensitive to fluctuations in relative prices. As a result, the Phillips curve for non-food inflation flattens (lower κ_y), making inflation stabilization more costly. On the other hand, the corresponding increase in the size of the non-food sector hampers the negative welfare consequences of its own Calvo-driven price dispersion, thus making non-food (core) price stability more desirable. As stated in the proposition, with perfect labor mobility, the benefits outweigh the

[11] The solution under discretion corresponds to a Markov Perfect Equilibrium whereby endogenous variables are (linear) functions of pay-off relevant state variables only. In the current context, those state variables are $\hat{z}_{F,t}$ and $\hat{\mu}_t$.

[12] Under discretion, the equilibrium is locally determinate, for any parameterization of the model. Substituting the targeting rule (28) into the Phillips curve (23), while leaving aside stochastic terms, we find

$$\hat{\pi}_{N,t} = \frac{\beta\lambda}{\lambda + \kappa_y^2} E_t \hat{\pi}_{N,t+1}.$$

Since i) the coefficient multiplying $E_t\hat{\pi}_{N,t+1}$ is always between 0 and 1, and ii) $\hat{\pi}_{N,t}$ is a jump variable, the equilibrium under discretion is always locally unique.

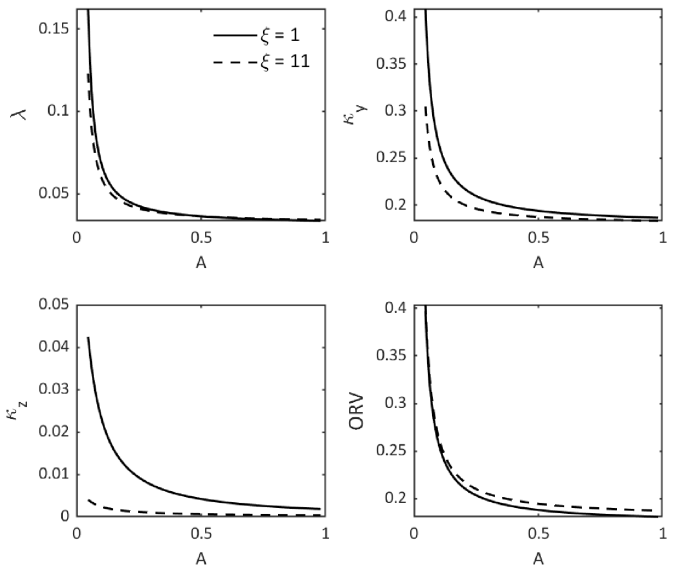

Figure 1: Key Coefficients and Development. Solid line: low labor mobility. Dashed line: high labor mobility.

costs. Consequently, under the optimal policy, the equilibrium volatility of core inflation relative to that of the output gap is a decreasing function of A.

Figure 1 displays key composite coefficients related to the conduct of optimal policy as function of development A: namely, the relative weight on output gap stabilization λ, the slope of the Phillips curve κ_y, its slope with respect to food-sector TFP κ_z, and the optimal relative volatility ORV. We consider both the case of low ($\xi = 1$) and high ($\xi = 11$) labor mobility[13]. For what concerns the role of development A, the numerical results are consistent with Proposition 3 focusing on full labor mobility. In particular, more developed economies should be more concerned with stabilizing non-food (core) inflation - i.e. they feature a lower λ - although that appears to be a more costly task - i.e. lower κ_y, hence a flatter Phillips curve. As the ORV is strictly decreasing in A, the higher benefits outweigh the higher costs. A lower degree of mobility makes the Phillips curve slightly steeper, as well as magnifies the negative relationship between κ_y and A. On the contrary, neither the relative welfare weight λ nor the ORV appear significantly sensitive to ξ. This allows us to safely conclude that, irrespective of the degree of labor mobility, as the economy develops the

[13] As discussed later in the paper, the values $\xi = 1$ and $\xi = 11$ reflect the estimated degree of labor mobility in, respectively, India and the U.S. Higher values for ξ- even extreme ones - do not yield quantitatively different results. Hence, the case of $\xi = 11$ is already a good approximation of what happens under perfect mobility.

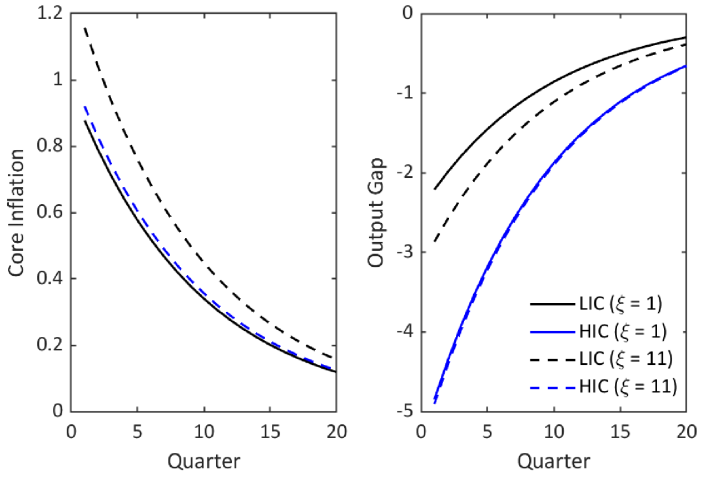

Figure 2: Optimal monetary policy under discretion: responses to a 1% mark-up shock. Bold lines display the responses of a LIC (black) and a HIC (blue) under limited labor mobility ($\xi = 1$). Dashed lines are their counterparts under high labor mobility ($\xi = 11$).

optimal relative volatility of core inflation with respect to the output gap declines.

The impulse responses of non-food (core) inflation $\hat{\pi}_{N,t}$ and the welfare-relevant output gap $\hat{x}_t$ to a 1% markup shock are displayed in Figure 2, for both the low income country (henceforth, LIC) and the high income country (henceforth, HIC), corresponding, respectively, to India and the U.S. First, under low mobility, the response of inflation is unaffected by the level of development (the solid black and blue lines in the left panel coincide, hence only the black one is displayed). This is not the case for the output gap where instead the shock-induced recession is harsher in the HIC. Second, the adverse consequences of a markup shock are amplified by labor mobility: increasing ξ from 1 (limited mobility) to 11 (close to full mobility) generates more positive inflation and a more negative output gap. To gain a wider perspective on the role of development and labor mobility, Figure 3 shows the on-impact responses as functions of A and ξ.

As previously argued, under imperfect mobility (finite ξ), the food-sector specific TFP behaves similarly to a cost-pusch shock. This is shown in Figure 4. A positive $\hat{z}_{F,t}$ generates inflation and a real contraction, but its effect is significantly weaker in more developed (higher A) and/or more mobile (higher ξ) economies. The result is obviously related to the value of κ_z, the temporary

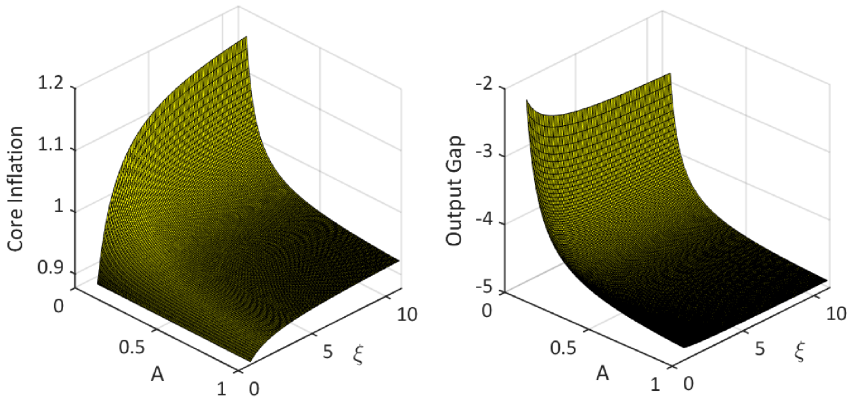

Figure 3: Optimal monetary policy under discretion: on-impact responses to a 1% mark-up shock.

elasticity of core inflation to $\hat{z}_{F,t}$: as shown in Figure 1, the latter is always positive and significantly larger in an economy which is less developed (lower A) and features lower mobility (lower ξ). Figure 4 shows the on-impact responses as functions of A and ξ

4.2 Commitment

Under commitment the policy-maker announces and implements the optimal state-contingent Ramsey plan that maximizes aggregate welfare, taking into account its direct impact on individual expectations. This allows the policy-maker to attain a better trade-off between stabilizing inflation and the output-gap in the face of any shock affecting price setting by individual firms. From the Ramsey's plan first order conditions we obtain the following targeting rule under commitment:

$$\hat{\pi}_{N,t} = -\frac{\lambda}{\kappa_y}\left(\hat{x}_t - \hat{x}_{t-1}\right) \tag{29}$$

Following a positive shock to the markup or to the food-specific TFP, the responses of core inflation and the output gap i) are *qualitatively* similar to the case of discretion - i.e. they are both

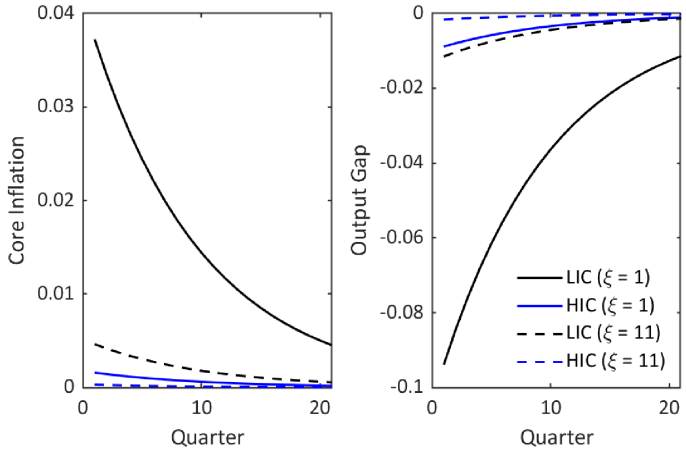

Figure 4: Optimal monetary policy under discretion: responses to a 1% food-specific TFP shock. Bold lines display the responses of a LIC (black) and a HIC (blue) under limited labor mobility ($\xi = 1$). Dashed lines are their counterparts under high labor mobility ($\xi = 11$).

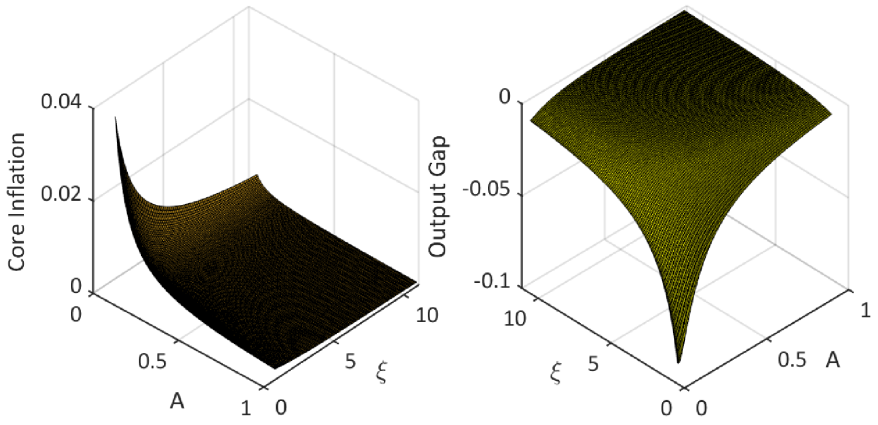

Figure 5: Optimal monetary policy under discretion: on-impact responses to a 1% food-sector specific TFP shock.

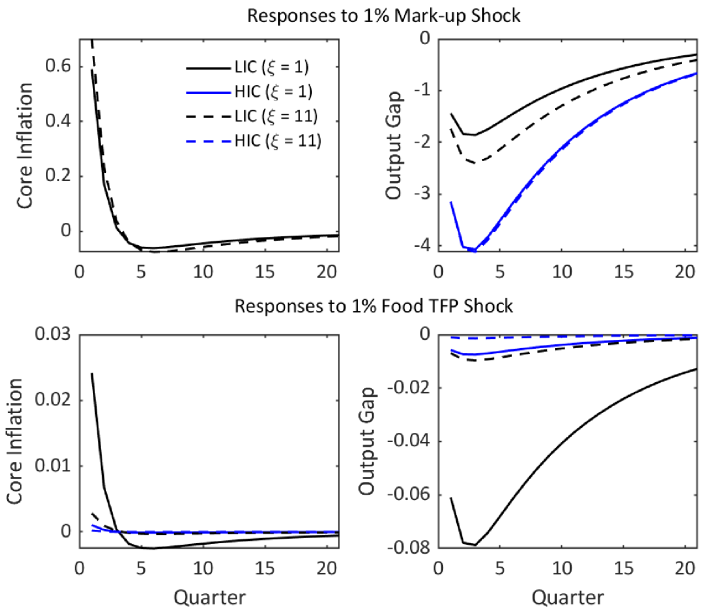

Figure 6: Optimal monetary policy under commitment: responses to a 1% markup shock (top panels) and food-specific TFP shock (bottom panels). Bold lines display the responses of a LIC (black) and a HIC (blue) under limited labor mobility ($\xi = 1$). Dashed lines are their counterparts under high labor mobility ($\xi = 11$).

inflationary and contractionary - but *quantitatively* significantly more contained, and ii) display endogenous persistence due to the history dependence induce by the targeting rule (29) As displayed in the top panels of Figure 6, in regard to the markup shock, the inflation responses are equivalent in both countries, while the output gap responses are larger and more persistent in the HIC. From the bottom panels, the inflationary and contractionary consequences of the food-specific shock are more sizable in the LIC.

4.3 (Strict) Headline vs (Flexible) Core Targeting: An Equivalence Result

As just shown, through its better control of inflation expectations, commitment allows the central bank to contain the adverse consequences of both shocks and therefore attain higher welfare. In this section, we show that such outcome can be fully replicated by targeting an appropriate measure of headline inflation. Recalling the relationship between the *welfare relevant* and the *natural* gaps, $\hat{x}_t = \hat{y}_t^{gap} - F\hat{z}_{F,t}$, the optimal targeting rule under commitment (29) can be equivalently written

as follows:

$$\hat{\pi}_{N,t} = -\frac{\lambda}{\kappa_y}\Delta\hat{y}_t^{gap} + \frac{\lambda}{\kappa_y}F\Delta\hat{z}_{F,t}, \tag{30}$$

namely, core inflation should react negatively to the growth rate of the natural gap and positively (since $F > 0$) to food-specific TFP growth. Next, consider a general headline inflation measure as the weighted average of food and non-food inflation:

$$\hat{\pi}_t^o \equiv \omega\hat{\pi}_{F,t} + (1-\omega)\,\hat{\pi}_{N,t} \tag{31}$$

for arbitrary weight $\omega \in [0,1]$, which nests the cases of *core inflation* for $\omega = 0$ and *CPI inflation* for $\omega = \gamma$. As shown in Appendix A.1.4, $\hat{\pi}_t^o$ can be written as:

$$\begin{aligned} \hat{\pi}_t^o &= \hat{\pi}_{N,t} + m_{\pi y}\Delta\hat{y}_t^{gap} + m_{\pi z}\Delta\hat{z}_{F,t} \\ m_{\pi y} &\equiv \frac{\omega}{1-\gamma}\Upsilon_{py}, \qquad m_{\pi z} \equiv \frac{\omega}{1-\gamma}\left(\Upsilon_{py}F^n - \Upsilon_{pz}\right) \end{aligned} \tag{32}$$

where Υ_{py} and Υ_{pz} are composite coefficients appearing in the linear relationships $\hat{p}_{F,t} = \Upsilon_{py}\hat{y}_t - \Upsilon_{pz}\hat{z}_{F,t}$[14].

Proposition 4 *A headline inflation targeting rule $\hat{\pi}_t^o = \phi_z\Delta\hat{z}_{F,t}$ exactly replicates the optimal policy under commitment if and only if*

$$\omega = \omega^* \equiv \frac{\lambda}{\kappa_y}\frac{1-\gamma}{\Upsilon_{py}} \qquad \textit{and} \qquad \phi_z = \phi_z^* \equiv \frac{\lambda}{\kappa_y}F + m_{\pi z} \tag{33}$$

Clearly, if markup shocks were the sole source of economic fluctuations - i.e. $\hat{z}_{F,t} = 0$ at all times - the optimal policy under commitment would be replicable through strict headline targeting, $\hat{\pi}_t^o = 0$. Although, for general parameterizations of the model, it is not possible to establish the sign of ϕ_z^* as well as that indeed $\omega^* \in [0,1]$ - making it a proper weight - as shown in Corollary 1, a sharper analytical characterization is attainable for the case of a unitary Frisch elasticity of composite labor supply[15].

Corollary 1 *Assume a unitary Frisch elasticity of composite labor supply, $\chi = 1$.*

[14] See Appendix A.1.1 for their explicit expressions.

[15] A unitary Frisch elasticity is rather common in the New Keynesian literature as it is intermediate between the macro-based and the micro-based evidence of, respectively high ($\chi < 1$) and low ($\chi > 1$) labor supply elasticity.

1. *If labor is perfectly mobile across sectors, i.e.* $\xi \to +\infty$*, then,*

$$\omega^* \equiv \frac{\sigma + \alpha}{\epsilon\,(1+\sigma)} \in (0,1)\,, \tag{34}$$

which is strictly decreasing in development A. Letting $\epsilon^L \equiv \frac{1+\alpha}{2\alpha}$*, by the properties of the consumption share of food* γ*, it follows that*

i) *if* $\epsilon \geq \epsilon^L$*, then* $\omega^* < \gamma$ *for any* $A > 0$*;*

ii) *if* $\epsilon < \epsilon^L$*, then there exists a development threshold* $A^* > 0$ *such that* $\omega^* \gtreqless \gamma$ *for* $A \gtreqless A^*$*.*

2. *If labor is very (but not completely) immobile, i.e.* $\xi = 1$*, then*

$$\omega^* \equiv \epsilon^{-1} \in (0,1)\,. \tag{35}$$

Letting $\epsilon^H \equiv \alpha^{-1}$*, by the properties of the consumption share of food* γ*, it follows that*

i) *if* $\epsilon \geq \epsilon^H$*, then* $\omega^* < \gamma$ *for any* $A > 0$*;*

ii) *if* $\epsilon < \epsilon^H$*, then there exists a development threshold* $A^* > 0$ *such that* $\omega^* \gtreqless \gamma$ *for* $A \gtreqless A^*$*.*

Let's look at these results more closely. Under perfect mobility, the weight ω^*on food price inflation required to replicated the commitment solution is a strictly decreasing function of development. Such weight is larger or smaller than the expenditure-based CPI weight γ depending on the intratemporal elasticity of substitution across goods varieties ϵ. A similar result obtains under low mobility, although, in this case, the weight ω^* does not depend on development. Under the baseline calibration, the utility weight α on subsistence adjusted food consumption is rather small (about 0.04), which, in turn, makes the thresholds ϵ^L and ϵ^H rather large (respectively, about 12 and 23). With $\epsilon = 6$, it follows that statements 1.ii) and 2.ii) of Corollary 1 apply. The relationship between ω^* and γ is displayed in Figure 7, with the development threshold A^* strictly increasing in labor mobility. When the latter is perfect, the headline weight ω^* replicating the optimal policy under commitment is smaller than the CPI weight γ unless the economy belongs to the upper 20th percentile of development.

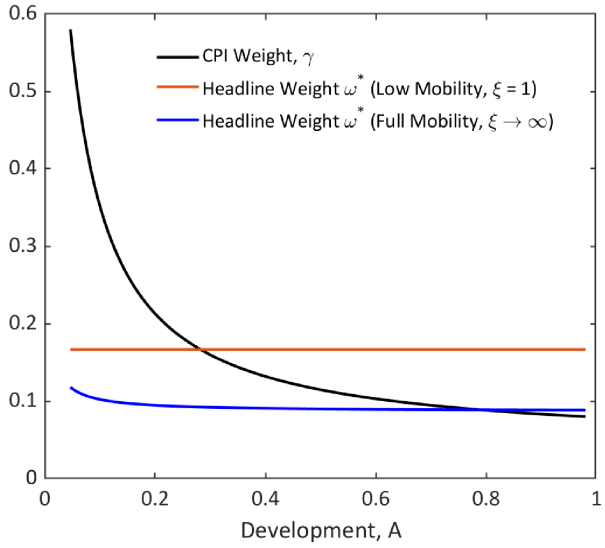

Figure 7: Replicating commitment with headline inflation targeting.

4.4 Optimal Delegation

Motivated by the analytical results of Proposition 4 and Corollary 1, we study optimal monetary policy delegation, in the spirit of Rogoff (1983). Namely, we consider a *discretionary* central banker who is assigned a policy objective - i.e., a loss function - which may differ from the welfare-based one. Namely, we define the delegated loss $\mathcal{L}_t^D$ as

$$\mathcal{L}_t^D \equiv \frac{1}{2}\left[(\hat{\pi}_t^o)^2 + \lambda^D \left(\hat{y}_t^{gap}\right)^2\right] \tag{36}$$

with the arbitrary headline inflation objective $\hat{\pi}_t^o$ specified in (31), which, as already discussed, nests the extreme cases of core non-food inflation targeting ($\omega = 0$), food inflation targeting ($\omega = 1$), as well as all intermediate cases, including CPI inflation ($\omega = \gamma$). For what concerns the output gap, we assume the central banker targets the natural gap. This seems a reasonable choice as i) the flexible price level of output appears more in line with the notion of potential output available in the data, and ii) the correct welfare-based gap is measured with respect to the efficient output level which is instead a much more complex notion.[16] It is worth noting that the policy objective (36) is in line with the flexible inflation targeting mandate of several central banks, in both developed

[16] Replacing $\hat{y}_t^{gap}$ with $\hat{x}_t$ has minor quantitative consequences for the results since we restrict the analysis to the range $\lambda^D \in [0, \lambda]$, and the welfare-based weight λ is rather small.

and developing economies.

Our analysis proceed as follows. First, for given ω and λ^D, we solve for the optimal policy under discretion associated with the delegated loss $\mathcal{L}_t^D$. In doing so, we restrict to a Markov Perfect Equilibrium (MPE) solution where all endogenous variables are functions of pay-off relevant state variables only, which, in our specific context include all current exogenous shocks, as well as the past natural gap $\hat{y}_{t-1}^{gap}$ and lagged food sector TFP $\hat{z}_{F,t-1}$.[17] Then, we compute the correct welfare-based loss associated with the MPE. We repeat the process for all possible definitions of headline inflation - i.e. for all $\omega \in [0,1]$ - and for any $\lambda^D \in [0,\lambda]$. To conclude, we identify the optimal weight ω^* on food inflation and the optimal relative weight λ^{D*} on the natural gap solving

$$\min_{\omega\in[0,1],\ \lambda^D\in[0,\lambda]} (1-\beta)\,\mathcal{L} \equiv \frac{1}{2}\left[Var\left(\hat{\pi}_{N,t}\right) + \lambda Var\left(\hat{x}_t\right)\right]$$

To start with, it is informative to compare, for all possible combinations of $\omega \in [0,1]$ and $\lambda^D \in [0,\lambda]$, the unconditional social welfare losses obtained from solving the correct optimal policy under discretion problem with the one obtained from giving the central banker the loss objective specified in (36). As shown in Figure 8, the parametric space for which the delegated optimal policy problem yields a lower loss compared to the correct optimal policy problem is rather wide. In particular, given a sufficiently small relative weight on output gap stabilization λ^D, for both countries (left panel for the LIC, right panel for the HIC), any specification of headline inflation with positive weight on food specific inflation - including the CPI weight $\omega = \gamma$ - can yield a smaller loss compared to the one attained by targeting non-food core inflation.

Among all possible parameterizations, which one does deliver the smallest loss? We find that for the LIC, the smallest loss is obtained for $\omega = 0.13$ and $\lambda^D = 0.01$. That is, the optimal weight on food inflation is smaller that what dictated by CPI ($\gamma = 0.58$) while the delegated central banker should be more conservative (the welfare-based weight on output is $\lambda = 0.16$). Similar results apply to the HIC where $\omega = 0.16$ and $\lambda^D = 0$, although in this case the CPI weight is $\gamma = 0.08$ and $\lambda = 0.03$. The impulse responses to a 1% markup shock under the optimally delegated policy - in comparrison with those obtained earlier for the truely optimal policies under discretion and commitment - are displayed in Figure 9. For both the LIC and the HIC, under optimal delegation, a

[17] To see why, recall that for any arbitrary weight ω, the headline inflation objective $\hat{\pi}_t^o$ defined in (32) is a linear function of $\Delta\hat{y}_t^{gap}$ and $\Delta\hat{z}_{F,t}$. When computing the optimal policy solution, expectations are no longer taken parametrically as current choices affect future outcomes. We find the unique MPE by adapting Soderlind (1999) computational algorithm to our setting.

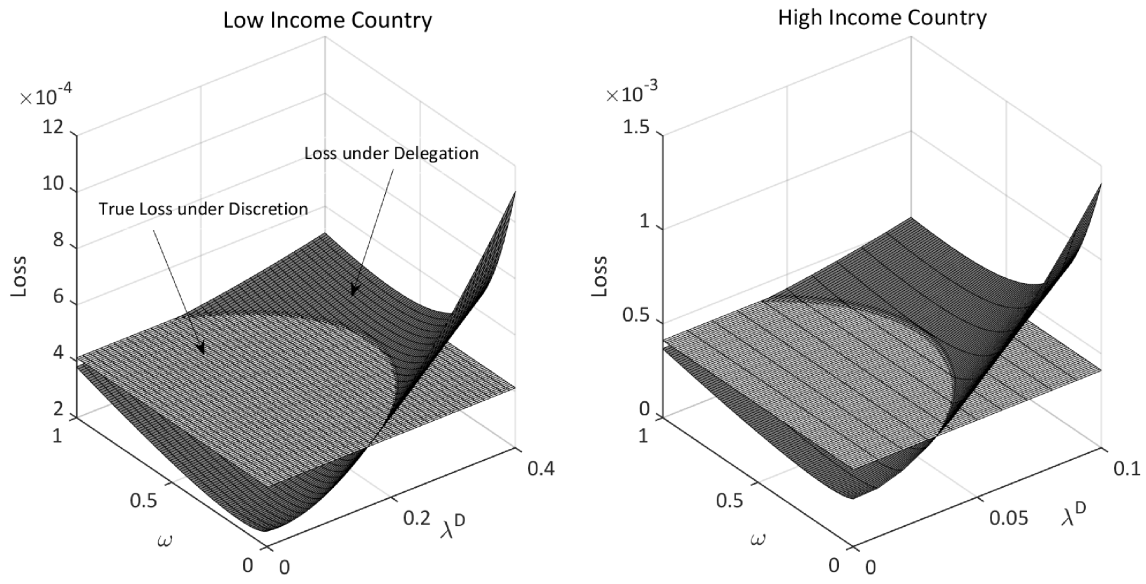

Figure 8: Welfare comparison: correct (core) vs delegated (headline) inflation targeting. Flat lighter grey surface: unconditional social loss under correct discretionary policy. Darker grey surface: unconditional social loss under delegated discretionary policy, as function of headline weight ω and output gap weight λ^D.

discretionary central banker can significantly tame the equilibrium responses to the adverse markup shock obtained if minimizing the welfare-based loss, and mimic very closely what it would have obtained under commitment.

Since $\lambda^D \approx 0$, for both countries, the optimal delegation scheme is almost equivalent to strict headline inflation targeting with a weight on (flexible) food inflation lower (respectively, larger) than the expenditure-based CPI weight in the low income (respectively, high income) economy. We conclude this section by comparing the optimal policies under discretion and commitment with what would obtain under the sub-optimal strict CPI inflation targeting. As shown in Figure 10, the simpler strict CPI inflation targeting policy almost delivers the impulse responses obtained with the optimal policy under commitment, in particular for what concerns the high income economy.

5 Development-Dependent Labor Mobility

So far, we have treated development (indexed by A) and labor mobility (indexed by ξ) as two independent structural features of the economy. In reality, as documented by Artuc et al. (2015),

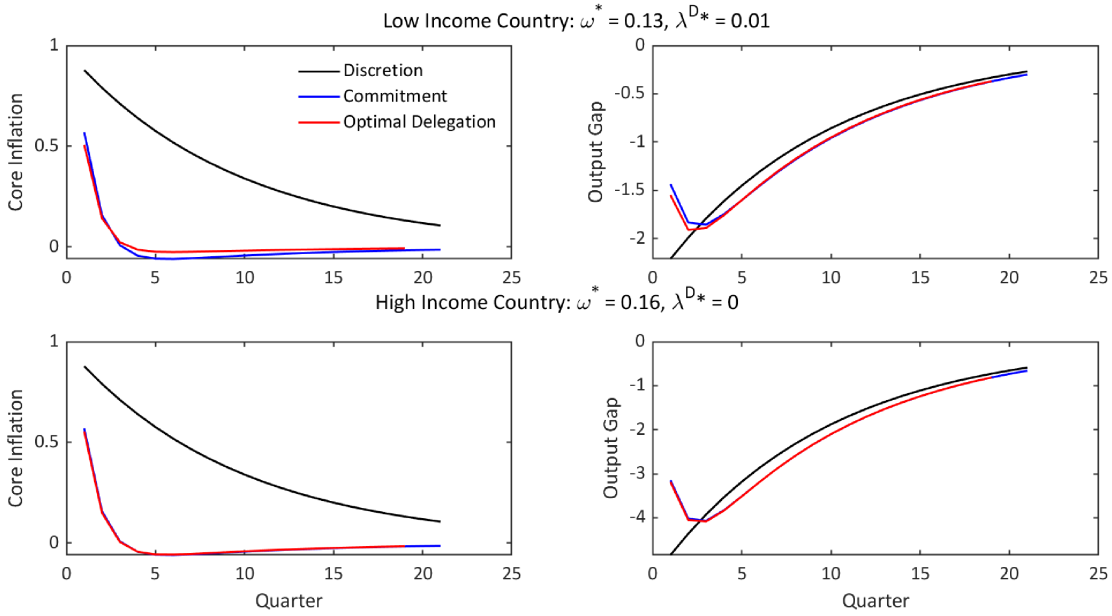

Figure 9: Impulse responses to 1% markup shock under alternative policies: optimal policy under discretion (black), optimal policy under commitment (blue) and optimal delegation under discretion (red). Top panels for LIC, bottom panels for HIC.

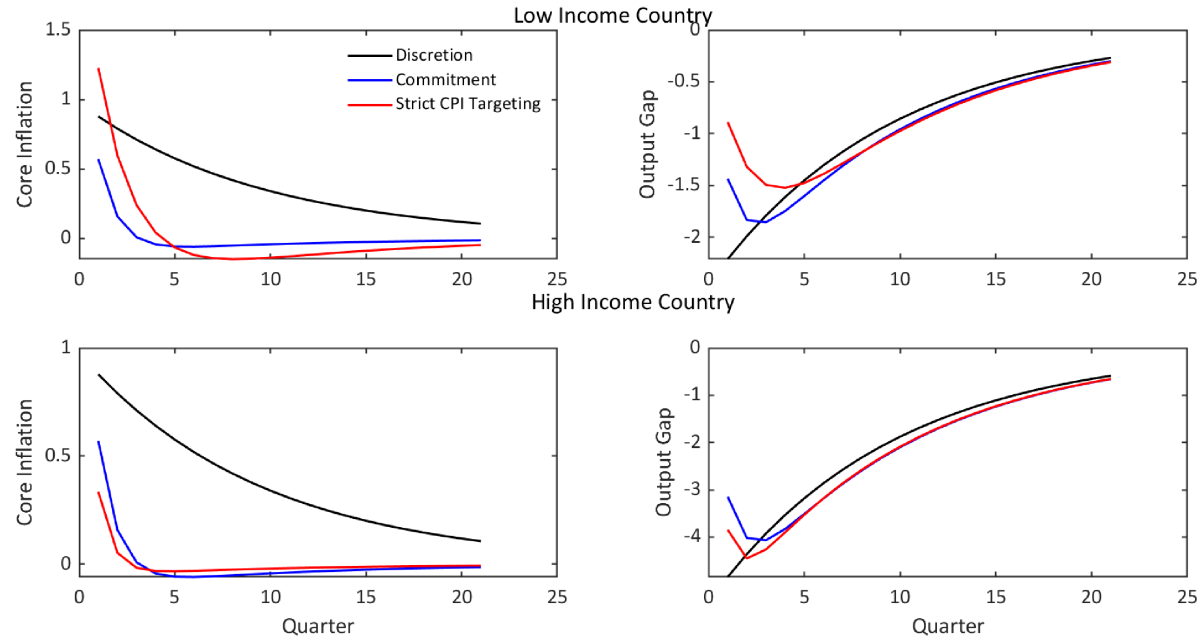

Figure 10: Impulse responses to 1% markup shock under alternative policies: optimal policy under discretion (black), optimal policy under commitment (blue) and strict CPI inflation targeting (red). Top panels for LIC, bottom panels for HIC.

mobility is positively correlated with income per capita, i.e. the economic costs of moving across sectors are lower in more developed economies, making sector-specific labor supply more elastic to relative wages.[18] We would like to incorporate this relationship - which, in what follows, we are going to simply refer to as "varying mobility", to distinguish it from the fixed mobility case studied so far - into our general equilibrium model in a parsimomious, yet empirically grounded, manner.

To do so, we first provide direct estimates for the across-sector labor elasticity ξ for a sample of developing and developed economies, and quantitatively assess its relationship with income per capita. Our analysis follows the Roy (1951) model's adaptation to a two-sector general equilibrium framework of labor allocation by Lagakos and Waugh (2013). That set-up gives a log-linear relationship between the ratio of the agriculture to non-agriculture labor shares and the relative wage: namely

$$\ln\left(\frac{\text{Share of Labor in Agriculture}}{\text{Share of Labor in Non-Agriculture}}\right) = \theta^{\text{Roy}} \ln\left(\frac{\text{Real Wage in Agriculture}}{\text{Real Wage in Non-Agriculture}}\right), \qquad (37)$$

where θ^{Roy} is a Frechet distibution parameter (equal across sectors) indexing the dispersion of individual workers' productivity in each sector. Notice that equation (37) is isomorphic to (the log of) (7) obtained earlier, where logged relative labor supply is proportional to the logged relative wage, $\ln\left(h_{F,t}/h_{N,t}\right) \propto \xi \ln\left(W_{F,t}/W_{N,t}\right)$. Hence, in reduced form, the Roy model's parameter θ_{Roy} is equivalent to the labor mobility index ξ in our framework.

Using sectoral data on output and employment from the Economic Tranformation Database (ETD) of the GGDC/UNU-WIDER for developing/low-income countries (e.g. Latin America, Asia and Africa) and from the Structural Analysis Database (STAN) for OECD countries, we compute long-run average labor shares in agriculture and non-agriculture, as well as related wages, for all countries included in the sample over the period 1990-2018.[19] We then use (37) to retrieve the ith country specific Roy's parameter θ_i^{Roy}, hence, the country specific labor mobility index ξ_i:

$$\theta_i^{\text{Roy}} = \frac{\ln \bar{w}_{F,i} - \ln \bar{w}_{N,i}}{\ln \bar{h}_{F,i} - \ln \bar{h}_{N,i}} \approx \xi_i$$

where a "bar" on top of a variable denotes its long-run average, with the food subscript F referring

[18] See Panel A) in Figure 4 in thier work.

[19] GGDC and UNU-WIDER stand for, respectively, the Groningen Growth and Development Center and the United Nations University World Institute for Development Economics Research.

Figure 11: Labor mobility and development. Panel a): labor mobility is measured by the (log of the) "moving costs" displayed in Table 1 of Artuc et al. (2015). Panel b): labor mobility is measured by our own estimate of the (log of the) across-sector labor elasticity ξ. The dashed red lines in both panels are the respective fitted OLS regressions. NOTE 1: the sample of countries is not the same in the same across panels. NOTE 2: the sample in Artuc et al. (2015) has been truncated such that the U.S. is the country with the largest GDP per capita and the highest labor mobility, i.e. lowest $\ln C$ and highest $\ln \xi$.

to agriculture.[20]

Country specific estimates for ξ (in logs) are displayed in panel b) of Figure 11, in relationship with the underlying real GDP per capita relative to the U.S. y (also in logs). The scatter diagram highlights a clear positive relationship between labor mobiliy and economic development. Panel a) displays instead the across-sector "moving cost" C (also in logs) reported in Table 1 of Artuc et al. (2015). Similarly to panel b), workers' explicit costs of moving across sectors are lower in more developed economies. In both panels, the dashed red lines are the fitted (log-on-log) OLS regressions corresponding to the results in Table 2 below. The estimated value for b - the coefficient on $\ln y$ - is statistically significant in both regression, taking a negative 0.09 for the case of moving costs and a positive 0.73 for the case of labor elasticity.[21]

[20] Sector-specific real wages are computed using sectoral value added and related employment. Table 3 in Appendix A.3 reports all estimates.

[21] For a subsample of countries (about 30) for which both measures are available, the correlation coefficient between C and ξ is -0.32. The related log-on-log regression yields $\ln \xi = 1.92 - 1.05 \ln C$. Hence, the non-linear relationship between ξ and C is approximately $\xi \approx 7/C$, i.e. a 1% increase in C implies a 1% decline in ξ.

Table 1: Labor Mobility and Development			
Moving Cost Regression (Panel a. of Figure 11): $\ln C_i = a + b \ln y_i + e_i$			
Coefficient	**Estimate**	**Std. Dev.**	**t-stat (p-value)**
a	1.13	0.05	22.62 (0.00)
b	-0.09	0.032	-2.85 (0.01)
N. of Obs. $= 46$, $R^2 = 0.16$			
Elasticity Regression (Panel b. of Figure 11): $\ln \xi_i = a + b \ln y_i + e_i$			
Coefficient	**Estimate**	**Std. Dev.**	**t-stat (p-value)**
a	1.55	0.12	12.78 (0.00)
b	0.73	0.06	12.02 (0.00)
N. of Obs. $= 69$, $R^2 = 0.68$			

The empirical analysis suggests the following power function relationship between ξ_i and y_i:

$$\xi_i = e^a y_i^b, \qquad a = 1.55, \qquad b = 0.73. \tag{38}$$

The latter corresponds to the blue line in Figure 12. Despite the rather good fit, this specification significantly underestimates the labor mobility for the U.S., i.e. it yields $\xi = 4.5$ instead of $\xi = 11.05$. To overcome this significant drawback, we substitute (38) with a regularized incomplete beta cumulative distribution function whereby, given income per capita y, the implied elasticity ξ is

$$\xi = \xi_{US} I_y(a, b) \equiv \xi(y) \tag{39}$$

where $I_y(a, b)$ is the regularized incomplete beta function and ξ_{US} denotes the labor elasticity for the U.S. We parametrize (39) such that i) it passes exactly through India and the U.S. (the two anchor points of our income distribution), ii) it closely tracks middle and high income countries, and iii) it is globally convex in income. While, by construction, $\xi(0) = 0$ and $\xi(1) = \xi_{US}$, the parameters a and b are estimated by non-linear least squares (see Appendix A.4 for a detailed description), which yields $a = 0.5$ and $b = 0.18$. As shown by the red line in Figure 12, these values imply pronounced curvature at low income levels with a steep but smooth increase near the unity upper-bound. The resulting function is indeed globally convex, thus capturing the empirical pattern that labor mobility is rather flat at low-to-middle income levels but then rises rapidly as

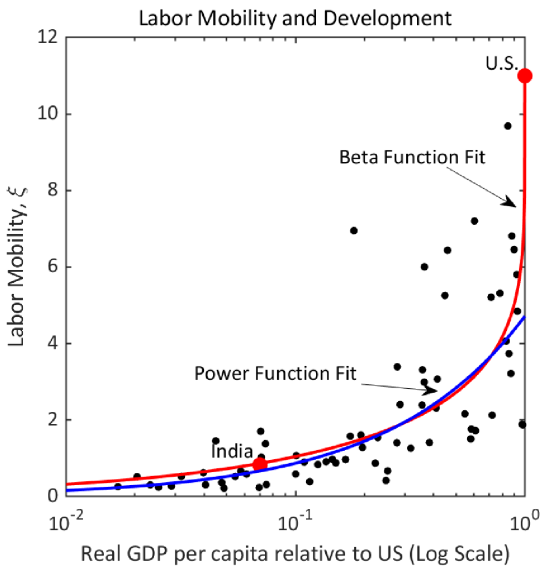

Figure 12: Labor mobility and development: reduced-form functional fit. Blue line: power function $\xi = e^a y^b$ for $a = 1.55$ and $b = 0.73$. Red line: incomplete beta function

countries approach the U.S. level of development.

With varying mobility, development amplifies the positive and the negative on-impact responses of, respectively, core inflation and output to the mark-up shock. This is shown in the top panels of Figure 13. With respect to inflation, this differs from the case of fixed mobility previously analyzed where development dampened the positive response. Looking back at the results displayed in Figure 3, the positive output response was in fact increasing in mobility but decreasing in development. Accounting for the positive relationship between the two in the data, labor mobility seems to dominate, in particular for less developed economies (for A lower than 0.2, the on-impact core inflation reponse function is rather steep). The negative response of output drops instead at a faster pace since, as displayed in Figure 3 as well, development and labor mobility mutually reinforce each other. With varying mobility, as the economy develops, the respectively positive and negative responses of core inflation and output to the food-sector specific TFP shock fade away at a much faster rate compared to the fixed mobility set-up.

When labor mobility varies with development, the weight ω^* on flexible food inflation making strict headline targeting equivalent to the optimal policy under commitment is significantly more decreasing in development. This can be seen by comparing the red line in Figure 14 with the red (the fixed low mobility case) and the blue (the perfect mobility case) lines in Figure 7.

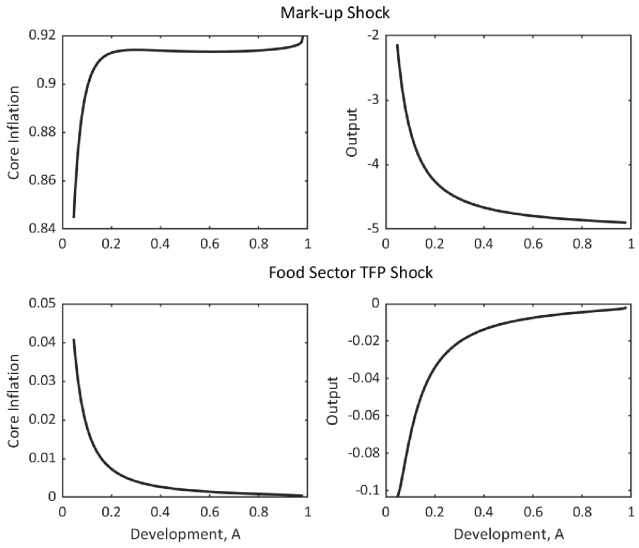

Figure 13: Optimal monetary policy under discretion with varying labor mobility: on impact responses to a 1% mark-up shock (top panels) and a 1% food-sector specific TFP shock (bottom panels).

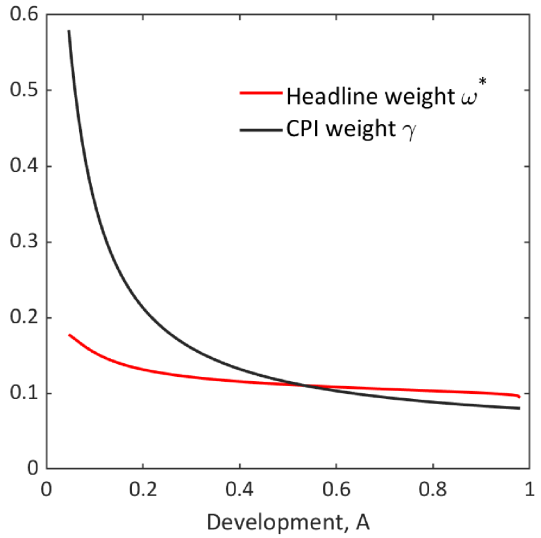

Figure 14: Replicating commitment with headline inflation targeting: the case of varying labor mobility.

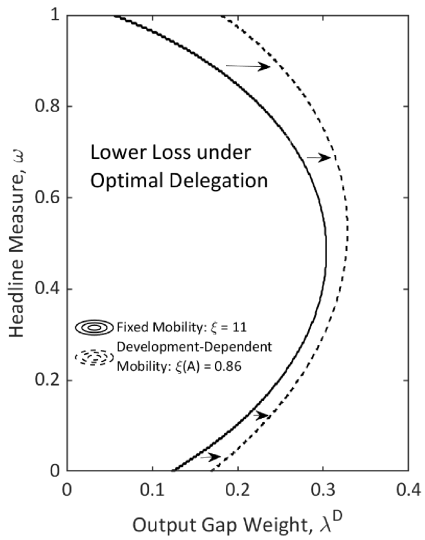

Figure 15: Delegation and mobility in LIC. Solid line boundary: combination of headline weight ω and output gap weight λ^D used for optimal delegation yielding a lower social loss than correct optimal policy for case of fixed mobility (e.g. $\xi = 11$). Dashed line: expanded region when mobility for LIC is function of development.

To conclude, we also find a slight improvement for what concerns optimal delegation. Focusing on the LIC, as shown in Figure 15, the set of values for the headline weight ω and the output gap weight λ^D for which the optimal delegated policy delivers a smaller social loss than the correct optimal policy (under discretion) expands rightward as labor mobility is parametrized according to the empirical evidence.

6 Conclusions

This paper studies the design of optimal monetary policy in a two-sector New Keynesian economy featuring a subsistence level of food consumption and imperfect labor mobility across sectors. These two ingredients, which are particularly relevant for low- and middle-income economies, fundamentally alter the welfare properties of inflation and output stabilization. By deriving a second-order approximation to aggregate welfare, the analysis characterizes the welfare-relevant inflation index and output gap and examines how optimal policy and its implementable interest rate rules depend on the degree of economic development and labor market frictions.

The presence of subsistence food consumption implies that food price fluctuations receive a

welfare weight exceeding their expenditure share, especially at early stages of development. At the same time, limited labor mobility introduces sectoral misallocation distortions that drive a wedge between the natural (flexible-price) level of output and the welfare-relevant efficient level. As a result, the divine coincidence breaks down: even though the welfare loss function can be expressed in terms of core inflation and the welfare-relevant output gap, optimal policy cannot fully stabilize both. In particular, food-sector productivity shocks optimally generate a trade-off between core inflation stabilization and output gap stabilization.

A key contribution of the paper is to show that this policy trade-off is not invariant to structural transformation. As economies develop and the steady-state share of food consumption declines, the welfare importance of food price fluctuations diminishes and the departure from the divine coincidence becomes quantitatively smaller. Core inflation stabilization becomes increasingly compatible with output gap stabilization, providing a welfare-based rationale for why monetary policy in advanced economies can place greater emphasis on core inflation without significant welfare losses. In this sense, subsistence concerns make price stability inherently more difficult to achieve in poorer economies than in richer ones.

Interestingly, we find that by targeting an appropriate measure of *headline* inflation the central bank can exactly replicate the optimal policy under commitment. By targeting headline (instead of core) inflation, the central bank can induce endogenous inflation inertia in the model's equilibrium dynamics which, for the appropriate weight on (flexible) food inflation in the headline measure, generates the same degree of history-dependence obtained under commitment. Targeting CPI inflation does not replicate the commitment outcome but gets significantly close to that. We see these results as a model-based justification for why many developing and emerging market economies explicitly target CPI inflation.

Current work in progress studies whether the equilibrium resulting from the optimal policy under commitment could be proxied by simpler Taylor-type instrumental policy rules. Our preliminary results show that a rule responding to (some measure of) headline inflation can get rather close to the first best and be welfare superior to a rule responding exclusively to core.

A Appendix

A.1 Additional Derivations

A.1.1 Log-Linearized Equilibrium

We provide a detailed derivation of the log-linearized equilibrium, letting $\widehat{v_t}$ denote the log-deviation of a generic variable v_t from its steady state value. Before moving to the optimality conditions for households' and firms' decisions, it is useful to derive some key relationships between the different components of aggregate consumption, as well as relative prices. First of all, from the definitions of $\tilde{c}_t$ in (2) and c_t in (9), we have:

$$\begin{aligned}
\widehat{\tilde{c}}_t &= \alpha \widehat{\tilde{c}}_{F,t} + (1-\alpha)\,\hat{c}_{N,t} && \text{(A.1)}\\
\hat{c}_t &= \frac{p_F c_F}{c}\left(\hat{p}_{F,t} + \hat{c}_{F,t}\right) + \frac{p_N c_N}{c}\left(\hat{p}_{N,t} + \hat{c}_{N,t}\right)\\
&= \gamma \hat{c}_{F,t} + (1-\gamma)\,\hat{c}_{N,t} && \text{(A.2)}
\end{aligned}$$

where the second equality in (A.2) follows from the definition of $\gamma \equiv \frac{c_F}{c}$, the fact that $p_F = p_N = 1$ in steady state, as well as $\gamma \hat{p}_{F,t} + (1-\gamma)\,\hat{p}_{N,t} = 0$ (as, from the definition of P_t, we have that $\gamma p_{F,t} + (1-\gamma)\,p_{N,t} = 1$, for $p_{F,t} \equiv \frac{P_{F,t}}{P_t}$ and $p_{N,t} \equiv \frac{P_{N,t}}{P_t}$). Then, from the definition of $\tilde{c}_{F,t}$ in (3), the steady state ratio $\frac{c_N}{\tilde{c}_F} = \frac{1-\alpha}{\alpha}$, and the steady state consumption shares $\tilde{c}_F = \alpha\tilde{c}$, $c_F = \gamma c$ and $c_N = (1-\alpha)\,\tilde{c} = (1-\gamma)\,c$, it is straightforward to derive

$$\widehat{\tilde{c}}_{F,t} = \vartheta \hat{c}_{F,t}, \qquad \text{for} \qquad \vartheta \equiv \frac{\gamma}{\alpha}\frac{1-\alpha}{1-\gamma} \tag{A.3}$$

From the household's first order conditions with respect to $\tilde{c}_{F,t}$ and $\hat{c}_{N,t}$, and the definition of $q_t \equiv \frac{P_{F,t}}{P_{N,t}}$, we have that $\widehat{\tilde{c}}_{F,t} = \hat{c}_{N,t} - \hat{q}_t$. Using the latter and equation (A.3) in (A.2), simple algebra gives

$$\begin{aligned}
\hat{c}_{N,t} &= \sigma \hat{c}_t + \alpha \hat{q}_t && \text{(A.4)}\\
\hat{c}_{F,t} &= \frac{\alpha}{\gamma}\hat{c}_t - \frac{\alpha}{\gamma}(1-\alpha)\,\hat{q}_t \qquad \text{for} \qquad \sigma \equiv \frac{1-\alpha}{1-\gamma} && \text{(A.5)}
\end{aligned}$$

Once plugged into (A.1), these equations give

$$\widehat{\tilde{c}}_t = \sigma \hat{c}_t \tag{A.6}$$

From (5), we obtain the linearized Euler equations:

$$\hat{c}_{N,t} = E_t\hat{c}_{N,t} - \left(\hat{R}_t - E_t\hat{\pi}_{N,t+1}\right) \tag{A.7}$$

$$\widehat{\tilde{c}}_{F,t} = E_t\widehat{\tilde{c}}_{F,t+1} - \left(\hat{R}_t - E_t\hat{\pi}_{F,t+1}\right) \tag{A.8}$$

Multiplying them by, respectively, $1-\alpha$ and α, and summing up, we find

$$\widehat{\tilde{c}}_t = E_t\widehat{\tilde{c}}_t - \left(\hat{R}_t - E_t\widehat{\tilde{\pi}}_{t+1}\right), \qquad \text{for} \qquad \widehat{\tilde{\pi}}_t \equiv \alpha\hat{\pi}_{F,t} + (1-\alpha)\,\hat{\pi}_{N,t} \tag{A.9}$$

Next, we look for a relationship between $\widehat{\tilde{\pi}}_t$, as defined in (A.9), and CPI inflation, $\hat{\pi}_t \equiv \gamma\hat{\pi}_{F,t} + (1-\gamma)\,\hat{\pi}_{N,t}$:

$$\begin{aligned}\widehat{\tilde{\pi}}_t &= \hat{\pi}_t + (\alpha-\gamma)\,(\hat{\pi}_{F,t} - \hat{\pi}_{N,t}) \\ &= \hat{\pi}_t - \frac{\gamma-\alpha}{1-\gamma}\,(\hat{p}_{F,t} - \hat{p}_{F,t-1})\end{aligned} \tag{A.10}$$

where the second equality follows from the definition of the relative price $\hat{q}_t = \hat{p}_{F,t} - \hat{p}_{N,t}$ (hence, $\hat{\pi}_{F,t} - \hat{\pi}_{N,t} = \hat{q}_t - \hat{q}_{t-1}$), together with $\gamma\hat{p}_{F,t} + (1-\gamma)\,\hat{p}_{N,t} = 0$ such that $\hat{q}_t = \frac{\hat{p}_{F,t}}{1-\gamma}$. Substituting (A.6) and (A.10) in (A.9), together with the market clearing condition $\hat{c}_t = \hat{y}_t$, we obtain the following Euler equation for aggregate ouput:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}\left[\hat{R}_t - E_t\hat{\pi}_{t+1} + \frac{\gamma-\alpha}{1-\gamma}\,(E_t\hat{p}_{F,t+1} - \hat{p}_{F,t})\right]. \tag{A.11}$$

Using the definition of $\hat{\pi}_t$, the latter can also be written as:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\sigma}\left[\hat{R}_t - E_t\hat{\pi}_{N,t+1} - \frac{\alpha}{1-\gamma}\,(E_t\hat{p}_{F,t+1} - \hat{p}_{F,t})\right]. \tag{A.12}$$

Consider now the equation for labor supply to the food sector, the leftward expression in (6). Its log-linearized form gives:

$$\begin{aligned}\hat{w}_{F,t} &= \widehat{\tilde{c}}_{F,t} + \hat{p}_{F,t} + \left(\chi - \frac{1}{\xi}\right)\hat{h}_t + \frac{1}{\xi}\hat{h}_{F,t} \\ &= \vartheta\hat{c}_{F,t} + \hat{p}_{F,t} + \left[\chi\gamma + \frac{1-\gamma}{\xi}\right]\hat{h}_{F,t} + (1-\gamma)\left(\chi - \frac{1}{\xi}\right)\hat{h}_{N,t} \\ &= \sigma\hat{c}_t + \frac{\alpha-\gamma}{1-\gamma}\hat{p}_{F,t} + \varkappa_F\,(\hat{y}_{F,t} - \hat{z}_{F,t}) + \varkappa_N\hat{y}_{N,t}\end{aligned} \tag{A.13}$$

for

$$\varkappa_F \equiv \chi\gamma + \frac{1-\gamma}{\xi} \qquad \text{and} \qquad \varkappa_N \equiv (1-\gamma)\left(\chi - \frac{1}{\xi}\right), \tag{A.14}$$

where the second equality in (A.13) follows from (A.3) and $\hat{h}_t = \gamma\hat{h}_{F,t} + (1-\gamma)\,\hat{h}_{N,t}$, while the third one makes use of (A.5) and $\hat{q}_t = \frac{\hat{p}_{F,t}}{1-\gamma}$. Combining this with the profit maximization condition in the food sector, $\hat{w}_{F,t} = \hat{p}_{F,t} + \hat{z}_{F,t}$ (coming from equation 11) and the definition of aggregate output $\hat{y}_t = \gamma\hat{y}_{F,t} + (1-\gamma)\,\hat{y}_{N,t}$, while recalling that $\sigma \equiv \frac{1-\alpha}{1-\gamma}$, after simple rearrangement of terms, we obtain

$$\hat{p}_{F,t} = \hat{y}_t + \frac{\varkappa_F}{\sigma}\hat{y}_{F,t} + \frac{\varkappa_N}{\sigma}\hat{y}_{N,t} - \frac{1+\varkappa_F}{\sigma}\hat{z}_{F,t} \tag{A.15}$$

Using the household's first order condition $\widehat{\widehat{c}}_{F,t} = \hat{c}_{N,t} - \hat{q}_t$, equation (A.3) and market clearing, we have $\vartheta\hat{y}_{F,t} = \hat{y}_{N,t} - \frac{\hat{p}_{F,t}}{1-\gamma}$. Substituting (A.15) into the latter, we obtain a relationship between sectoral outputs:

$$\hat{y}_{F,t} = -\frac{\varkappa_N}{\vartheta+\varkappa_F}\hat{y}_{N,t} + \frac{1+\varkappa_F}{\vartheta+\varkappa_F}\hat{z}_{F,t} \tag{A.16}$$

This together with the definition of aggregate output allows us to obtain

$$\hat{y}_t = \frac{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)}{\vartheta+\varkappa_F}\hat{y}_{N,t} + \gamma\frac{1+\varkappa_F}{\vartheta+\varkappa_F}\hat{z}_{F,t}, \tag{A.17}$$

or, alternatively

$$\hat{y}_{N,t} = \frac{\vartheta+\varkappa_F}{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)}\hat{y}_t - \gamma\frac{1+\varkappa_F}{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)}\hat{z}_{F,t}, \tag{A.18}$$

A straightforward combination of (A.16)-(A.17) also gives

$$\hat{y}_{F,t} = -\frac{\chi-\frac{1}{\xi}}{\vartheta+\frac{1}{\xi}}\hat{y}_t + \frac{1+\varkappa_F}{\vartheta+\frac{1}{\xi}}\hat{z}_{F,t} \tag{A.19}$$

Using (A.18) and (A.19) in (A.15), we find an expression for $\hat{p}_{F,t}$:

$$\hat{p}_{F,t} = \Upsilon_{py}\hat{y}_t - \Upsilon_{pz}\hat{z}_{F,t} \tag{A.20}$$

$$\Upsilon_{py} \equiv \frac{\sigma\left(\vartheta+\xi^{-1}\right)+\vartheta\left(\chi-\xi^{-1}\right)}{\sigma\left(\vartheta+\xi^{-1}\right)}, \qquad \Upsilon_{pz} \equiv \frac{(1+\varkappa_F)\,\vartheta}{\sigma\left(\vartheta+\frac{1}{\xi}\right)} \tag{A.21}$$

Plugging the latter into (A.12), after some simplification of terms, we obtain:

$$\hat{y}_t = E_t\hat{y}_{t+1} - \frac{1}{\tilde{\sigma}}\left(\hat{R}_t - E_t\hat{\pi}_{N,t+1}\right) + \Psi_y\hat{z}_{F,t}, \tag{A.22}$$

where

$$\begin{aligned}\tilde{\sigma} &\equiv \frac{\sigma\left[\vartheta + \frac{1}{\xi} + \gamma\left(\chi - \frac{1}{\xi}\right)\right]}{(1-\alpha)\left(\vartheta + \frac{1}{\xi}\right)} \\ \Psi_y &\equiv \frac{(1-\rho)\,\gamma\,(1+\varkappa_F)}{\vartheta + \varkappa_F}\end{aligned}$$

Letting $mc_{N,t} \equiv MC_{N,t}/P_t = w_{N,t}/p_{N,t}$ denote real marginal costs with real wage $w_{N,t} \equiv W_{N,t}/P_{N,t}$, the log-linearization of the optimal pricing rule for the non-food sector, (**??**), gives rise to a dynamic Phillips curve:

$$\begin{aligned}\hat{\pi}_{N,t} &= \beta E_t\hat{\pi}_{N,t+1} + \kappa\left(\widehat{mc}_{N,t} + \hat{\mu}_t\right) &\text{(A.23a)}\\ &= \beta E_t\hat{\pi}_{N,t+1} + \kappa\left(\hat{w}_{Nt} - \hat{p}_{N,t}\right) + \kappa\hat{\mu}_t \\ \kappa &\equiv \frac{(1-\beta\theta)\,(1-\theta)}{\theta} &\text{(A.23b)}\end{aligned}$$

where $\hat{\mu}_t = (1-\mu)\hat{\epsilon}_t$. From the log-linearization of the labor supply condition for the non-food sector (rightward expression in (6)), and market clearing conditions, marginal costs $\hat{w}_{Nt} - \hat{p}_{N,t}$ are

$$\hat{w}_{Nt} - \hat{p}_{N,t} = \varphi_F\hat{y}_{F,t} + (1+\varphi_N)\,\hat{y}_{N,t} - \varphi_F\hat{z}_{F,t} \tag{A.24}$$

for

$$\varphi_F \equiv \gamma\left(\chi - \frac{1}{\xi}\right) \qquad \text{and} \qquad \varphi_N \equiv \chi\,(1-\gamma) + \frac{\gamma}{\xi} \tag{A.25}$$

Hence, after combining (**??**) with (A.24), and using market clearing conditions, we obtain

$$\hat{\pi}_{N,t} = \beta E_t\hat{\pi}_{N,t+1} + \kappa\varphi_F\hat{y}_{F,t} + \kappa\,(1+\varphi_N)\,\hat{y}_{N,t} - \kappa\varphi_F\hat{z}_{F,t} + \kappa\hat{\mu}_t \tag{A.26}$$

Using the relationships (A.18) and (A.19), simple algebra yields the following expression:

$$\hat{\pi}_{N,t} = \beta E_t\hat{\pi}_{N,t+1} + \kappa_y\hat{y}_t + \Psi_\pi\hat{z}_{F,t} + \kappa\hat{\mu}_t \tag{A.27}$$

where

$$\kappa_y \equiv \kappa \frac{\gamma\chi + \vartheta\left[1+(1-\gamma)\chi\right] + \frac{1}{\xi}\left[\gamma\vartheta + 1 - \gamma + \chi\right]}{(1-\gamma)\left(\vartheta + \frac{1}{\xi}\right)}$$

$$\Psi_\pi \equiv -\gamma \frac{1+\chi+\frac{(1-\gamma)(1-\vartheta)(1-\chi\xi)}{1+\xi}}{\chi\gamma + \vartheta + (1-\gamma)\chi\vartheta + \frac{(1-\gamma)(1-\vartheta)(1+\chi)}{1+\xi}} \kappa_y$$

A.1.2 Flexible Price Equilibrium and Gap Representation

We characterize the flexible price (or, *natural*) equilibrium of the model, by setting the non-food sector's Calvo probability θ equal to zero. All household's first order condition stand. On the firms' side, the optimal price in the non-food sector becomes $P_{N,t} = (1-s)\,\mu \frac{W_{N,t}}{A} = \frac{W_{N,t}}{A}$ given the mark-up neutralizing subsidy s. It is straightforward to show that the steady state in the natural equilibrium is the same as in the sticky price economy, and that, under Assumption 1, it features a relative price of food, q, equal to unity.

We proceed by log-linearizing the natural equilibrium around the steady state. From the pricing decisions of firms we obtain real wages:

$$\hat{w}^n_{F,t} - \hat{p}^n_{F,t} = \hat{z}_{F,t} \qquad \text{and} \qquad \hat{w}^n_{N,t} - \hat{p}^n_{N,t} = 0 \tag{A.28}$$

From (A.16) and (A.24), we have, respectively:

$$\hat{y}^n_{F,t} = -\frac{\varkappa_N}{\vartheta + \varkappa_F}\hat{y}^n_{N,t} + \frac{1+\varkappa_F}{\vartheta+\varkappa_F}\hat{z}_{F,t} \tag{A.29}$$

$$\hat{y}^n_{F,t} = -\frac{1+\varphi_N}{\varphi_F}\hat{y}^n_{N,t} + \hat{z}_{F,t} \tag{A.30}$$

where (A.30) follows from $\hat{w}^n_{N,t} - \hat{p}^n_{N,t} = 0$, and the composite parameters $\varkappa_i$ and φ_i for $i = F, N$ are defined in equations (A.14) and (A.25). Combining (A.29)-(A.30), simple but tedious algebra gives:

$$\hat{y}^n_{F,t} = \frac{1+\chi}{\chi\gamma + \vartheta + (1-\gamma)\chi\vartheta + \frac{(1-\gamma)(1-\vartheta)(1+\chi)}{1+\xi}}\hat{z}_{F,t} \tag{A.31}$$

$$\hat{y}^n_{N,t} = \frac{\gamma\frac{(1-\vartheta)(1-\chi\xi)}{1+\xi}}{\chi\gamma + \vartheta + (1-\gamma)\chi\vartheta + \frac{(1-\gamma)(1-\vartheta)(1+\chi)}{1+\xi}}\hat{z}_{F,t} \tag{A.32}$$

We can then get the natural level of aggregate output $\hat{y}_t^n = \gamma \hat{y}_{F,t}^n + (1-\gamma)\,\hat{y}_{N,t}^n$:

$$\hat{y}_t^n = F^n \hat{z}_{F,t} \tag{A.33}$$

$$F^n \equiv \gamma \frac{1+\chi+\frac{(1-\gamma)(1-\vartheta)(1-\chi\xi)}{1+\xi}}{\chi\gamma+\vartheta+(1-\gamma)\chi\vartheta+\frac{(1-\gamma)(1-\vartheta)(1+\chi)}{1+\xi}} \tag{A.34}$$

From (A.20), we have then

$$\begin{aligned} \hat{p}_{F,t}^n &= \left(\gamma+\frac{\varkappa_F}{\sigma}\right)\hat{y}_{F,t}^n + \left(1-\gamma+\frac{\varkappa_N}{\sigma}\right)\hat{y}_{N,t}^n - \frac{1+\varkappa_F}{\sigma}\hat{z}_{F,t} \\ &= \frac{\sigma\left(\vartheta+\xi^{-1}\right)+\vartheta\left(\chi-\xi^{-1}\right)}{\sigma\left(\vartheta+\xi^{-1}\right)}\hat{y}_t^n - \frac{1+\varkappa_F}{\sigma}\vartheta\frac{\vartheta+\xi^{-1}+\gamma\left(\chi-\xi^{-1}\right)}{\left(\vartheta+\varkappa_F\right)\left(\vartheta+\xi^{-1}\right)}\hat{A}_{F,t} \end{aligned} \tag{A.35}$$

where the second equality follows from (A.31), (A.32) and the definition of $\hat{y}_t^n$.

Next, we define variables in deviation from their natural level, letting the superscrit *gap* denote such deviation. From (A.16) and (A.29), we have that

$$\hat{y}_{F,t}^{gap} = -\frac{\varkappa_N}{\vartheta+\varkappa_F}\hat{y}_{N,t}^{gap} \tag{A.36}$$

The latter together with $\hat{y}_t^{gap} = \gamma\hat{y}_{F,t}^{gap} + (1-\gamma)\,\hat{y}_{N,t}^{gap}$ yields

$$\hat{y}_t^{gap} = \frac{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)}{\vartheta+\varkappa_F}\hat{y}_{N,t}^{gap} \tag{A.37}$$

Substracting (A.35) from (A.20), we find

$$\hat{p}_{F,t}^{gap} = \frac{\sigma\left(\vartheta+\xi^{-1}\right)+\vartheta\left(\chi-\xi^{-1}\right)}{\sigma\left(\vartheta+\xi^{-1}\right)}\hat{y}_t^{gap} \tag{A.38}$$

Using all these definitions, it is possible to write the aggregate Euler equation (A.22) and the Phillips curve (**??**) in *gap* terms:

$$\hat{y}_t^{gap} = E_t\hat{y}_{t+1}^{gap} - \frac{1}{\tilde{\sigma}}\left(\hat{R}_t - \hat{R}_t^n - E_t\hat{\pi}_{N,t+1}\right) \tag{A.39}$$

$$\hat{\pi}_{N,t} = \beta E_t\hat{\pi}_{N,t+1} + \kappa_y\hat{y}_t^{gap} + \kappa\hat{\mu}_t \tag{A.40}$$

where $\hat{R}_t^n$, the natural interest rate, is an appropriately defined linear function of $\hat{z}_{F,t}$.

A.1.3 Second-Order Welfare Approximation

The benevolent policy-maker seeks to maximize the representative household's lifetime utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[U\left(\tilde{c}_t\right) - V\left(h_t\right) \right] \tag{A.41}$$

Throughout the analysis, we will make use of the following approximations: for a generic variables $x_{1,t}$ (with unconditional mean x_1) and $x_{2,t}$ (with unconditional mean x_2), up to second order, we have that $\frac{x_{1,t}-x_1}{x_1} \approx \hat{x}_{1,t} + \frac{\hat{x}_{1,t}^2}{2}$, $\left(\frac{x_{1,t}-x_1}{x_1}\right)^2 \approx \hat{x}_{1,t}^2$, and $\left(\frac{x_{1,t}-x_1}{x_1}\right)\left(\frac{x_{2,t}-x_2}{x_2}\right) \approx \hat{x}_{1,t}\hat{x}_{2,t}$. Consider the consumption term $U_t \equiv U\left(\tilde{c}_t\right)$. Its second order Taylor expansion gives:

$$\begin{aligned} U_t &\approx U + \frac{\alpha}{\tilde{c}_F} \underbrace{\left(\tilde{c}_{F,t} - \tilde{c}_F\right)}_{=c_{F,t}-c_F} + \frac{1-\alpha}{c_N}\left(c_{N,t} - c_N\right) - \frac{\alpha}{2\tilde{c}_F^2} \underbrace{\left(\tilde{c}_{F,t} - \tilde{c}_F\right)^2}_{=\left(c_{F,t}-c_F\right)^2} - \frac{1-\alpha}{2c_N^2}\left(c_{N,t} - c_N\right)^2 \\ &= U + \alpha\vartheta\left(\hat{c}_{F,t} + \frac{\hat{c}_{F,t}^2}{2}\right) + (1-\alpha)\left(\hat{c}_{N,t} + \frac{\hat{c}_{N,t}^2}{2}\right) - \frac{\alpha}{2}\vartheta^2\hat{c}_{F,t}^2 - \frac{1-\alpha}{2}\hat{c}_{N,t}^2 \end{aligned}$$

where the second equality makes use of the steady state relationship $\frac{c_F}{\tilde{c}_F} = \vartheta$. Recalling that $\vartheta = \frac{\gamma}{\alpha}\sigma$ and $\sigma \equiv \frac{1-\alpha}{1-\gamma}$, we can write the above expression as:

$$\begin{aligned} U_t &\approx U + \sigma\left[\gamma\hat{c}_{F,t} + (1-\gamma)\,\hat{c}_{N,t} - \frac{\gamma\left(\vartheta-1\right)}{2}\hat{c}_{F,t}^2\right] \\ &= U + \sigma\left[\gamma\hat{y}_{F,t} + (1-\gamma)\,\hat{y}_{N,t} - \frac{\gamma\left(\vartheta-1\right)}{2}\hat{y}_{F,t}^2\right] \end{aligned} \tag{A.42}$$

where the second equality follows from market clearing conditions.

Consider the labor term $V\left(h_t\right)$, letting $n_t \equiv \psi^{-\frac{1}{\xi}} h_{F,t}^{\frac{1+\xi}{\xi}} + (1-\psi)^{-\frac{1}{\xi}} h_{N,t}^{\frac{1+\xi}{\xi}}$. Its second order Taylor expansion gives:

$$\begin{aligned} V_t \approx\ & V + \gamma^{-\frac{1}{\xi}} n^{\xi\frac{1+\chi}{1+\xi}-1} h_F^{\frac{1+\xi}{\xi}} \left(\frac{h_{F,t}-h_F}{h_F}\right) + (1-\gamma)^{-\frac{1}{\xi}} n^{\frac{\xi(1+\chi)}{1+\xi}-1} h_N^{\frac{1+\xi}{\xi}} \left(\frac{h_{N,t}-h_N}{h_N}\right) \\ &+ \frac{\gamma^{-\frac{1}{\xi}}}{2}\left[\frac{1}{\xi} h_F^{\frac{1+\xi}{\xi}} n^{\xi\frac{1+\chi}{1+\xi}-1} + \left(\xi\frac{1+\chi}{1+\xi} - 1\right)\gamma^{-\frac{1}{\xi}}\frac{1+\xi}{\xi} n^{\xi\frac{1+\chi}{1+\xi}-2} h_F^{2\frac{1+\xi}{\xi}}\right]\left(\frac{h_{F,t}-h_F}{h_F}\right)^2 \\ &+ \frac{(1-\gamma)^{-\frac{1}{\xi}}}{2}\left[\frac{1}{\xi} h_N^{\frac{1+\xi}{\xi}} n^{\xi\frac{1+\chi}{1+\xi}-1} + \left(\xi\frac{1+\chi}{1+\xi} - 1\right)(1-\gamma)^{-\frac{1}{\xi}}\frac{1+\xi}{\xi} n^{\xi\frac{1+\chi}{1+\xi}-2} h_N^{2\frac{1+\xi}{\xi}}\right]\left(\frac{h_{N,t}-h_N}{h_N}\right)^2 \\ &+ \gamma^{-\frac{1}{\xi}}(1-\gamma)^{-\frac{1}{\xi}}\frac{1+\xi}{\xi}\left(\xi\frac{1+\chi}{1+\xi} - 1\right) n^{\xi\frac{1+\chi}{1+\xi}-2} h_F^{\frac{1+\xi}{\xi}} h_N^{\frac{1+\xi}{\xi}} \left(\frac{h_{F,t}-h_F}{h_F}\right)\left(\frac{h_{N,t}-h_N}{h_N}\right) \end{aligned}$$

Making use of the fact that $n^{\frac{\xi(1+\chi)}{1+\xi}-1} = h^{\frac{\chi\xi-1}{\xi}}$, $h_F = \gamma h$ and $h_N = (1-\gamma)\, h$, we can simplify some of the terms in the previous expression as follows:

$$\begin{aligned} n^{\frac{\xi(1+\chi)}{1+\xi}-1} h_F^{\frac{1+\xi}{\xi}} &= \left(\frac{1}{\gamma}\right)^{\frac{\chi\xi-1}{\xi}} h_F^{1+\chi}, \qquad n^{\frac{\xi(1+\chi)}{1+\xi}-1} h_N^{\frac{1+\xi}{\xi}} = \left(\frac{1}{1-\gamma}\right)^{\frac{\chi\xi-1}{\xi}} h_N^{1+\chi} \\ n^{\xi\frac{1+\chi}{1+\xi}-2} h_F^{2\frac{1+\xi}{\xi}} &= \left(\frac{h_F}{\gamma}\right)^{1+\chi}, \qquad n^{\xi\frac{1+\chi}{1+\xi}-2} h_N^{2\frac{1+\xi}{\xi}} = \left(\frac{h_N}{1-\gamma}\right)^{1+\chi} \end{aligned}$$

Hence, we have:

$$\begin{aligned} V_t \approx\ & V + \left(\frac{1}{\gamma}\right)^{\chi} h_F^{1+\chi} \left(\hat{h}_{F,t} + \frac{\hat{h}_{F,t}^2}{2}\right) + \left(\frac{1}{1-\gamma}\right)^{\chi} h_N^{1+\chi} \left(\hat{h}_{N,t} + \frac{\hat{h}_{N,t}^2}{2}\right) \\ & + \frac{1}{2}\left[\frac{1}{\xi}\left(\frac{1}{\gamma}\right)^{\chi} + \frac{\chi\xi-1}{\xi}\left(\frac{1}{\gamma}\right)^{\chi-1}\right] h_F^{1+\chi} \hat{h}_{F,t}^2 \\ & + \frac{1}{2}\left[\frac{1}{\xi}\left(\frac{1}{1-\gamma}\right)^{\chi} + \frac{\chi\xi-1}{\xi}\left(\frac{1}{1-\gamma}\right)^{\chi-1}\right] h_N^{1+\chi} \hat{h}_{N,t}^2 \\ & + \gamma\left(\frac{1}{1-\gamma}\right)^{\chi} \frac{\chi\xi-1}{\xi} h_N^{1+\chi} \left(\frac{h_{F,t}-h_F}{h_F}\right) \hat{h}_{F,t} \end{aligned}$$

From the labor supply conditions in (6) with $\psi = \gamma$, and given the steady state labor shares $h = \gamma^{-1} h_F = (1-\gamma)^{-1} h_N$, we have $\left(\frac{h_F}{\gamma}\right)^{\chi} \tilde{c}_F = \alpha A$ and $\left(\frac{h_N}{1-\gamma}\right)^{\chi} c_N = (1-\alpha) A$. These last two equations, together with market clearing conditions $c_i = A h_i$ for $i = F, N$, and the relationship $c_F = \vartheta \tilde{c}_F$, allow the following simplifications:

$$\begin{aligned} \left(\frac{1}{\gamma}\right)^{\chi} h_F^{1+\chi} &= \gamma\sigma, \qquad \left(\frac{1}{1-\gamma}\right)^{\chi} h_N^{1+\chi} = (1-\gamma)\,\sigma \\ \left(\frac{1}{\gamma}\right)^{\chi-1} h_F^{1+\chi} &= \gamma^2\sigma, \qquad \left(\frac{1}{1-\gamma}\right)^{\chi-1} h_N^{1+\chi} = (1-\gamma)^2\,\sigma \end{aligned}$$

All of this and some additional algebra allow us to write V_t as follows

$$\begin{aligned} V_t &\approx V+\gamma\sigma\left(\hat{h}_{F,t}+\frac{\hat{h}_{F,t}^2}{2}\right)+(1-\gamma)\,\sigma\left(\hat{h}_{N,t}+\frac{\hat{h}_{N,t}^2}{2}\right) \\ &\quad +\frac{\sigma\chi}{2}\left[\gamma\hat{h}_{F,t}+(1-\gamma)\,\hat{h}_{N,t}\right]^2+\frac{\sigma\,(1-\gamma)\,\gamma}{2\xi}\left(\hat{h}_{F,t}-\hat{h}_{N,t}\right)^2 \\ &= V+\gamma\sigma\left[\hat{y}_{F,t}-\hat{z}_{F,t}+\frac{(\hat{y}_{F,t}-\hat{z}_{F,t})^2}{2}\right]+(1-\gamma)\,\sigma\left[\hat{y}_{N,t}+\hat{d}_{N,t}+\frac{\left(\hat{y}_{N,t}+\hat{d}_{N,t}\right)^2}{2}\right] \quad \text{(A.43)} \\ &\quad +\frac{\sigma\chi}{2}\left[\gamma\left(\hat{y}_{F,t}-\hat{z}_{F,t}\right)+(1-\gamma)\left(\hat{y}_{N,t}+\hat{d}_{N,t}\right)\right]^2+\frac{\sigma\,(1-\gamma)\,\gamma}{2\xi}\left(\hat{y}_{F,t}-\hat{y}_{N,t}-\hat{z}_{F,t}-\hat{d}_{N,t}\right) \end{aligned}$$

where the second equality makes use of aggregate technology, with $\hat{d}_{N,t}$ denoting (log) price dispersion across non-food firms.

After combining (A.42) with (A.43), dropping all terms that are either independent from policy, *t.i.p* (e.g. $\hat{z}_{F,t}$), or of order higher than two, *h.o.t.*, we obtain:

$$\begin{aligned} & U\left(\tilde{c}_t\right)-V\left(h_t\right)-(U-V) \\ \approx\ & -\frac{\sigma}{2}\left[2\,(1-\gamma)\,\hat{d}_{N,t}+\gamma\vartheta\hat{y}_{F,t}^2+\gamma\left(\chi\gamma+\frac{1-\gamma}{\xi}\right)\hat{y}_{F,t}^2+(1-\gamma)\,\hat{y}_{N,t}^2\right. \\ & +(1-\gamma)\left(\chi\,(1-\gamma)+\frac{\gamma}{\xi}\right)\hat{y}_{N,t}^2-2\gamma\left(1+\chi\gamma+\frac{1-\gamma}{\xi}\right)\hat{y}_{F,t}\hat{z}_{F,t} \\ & \left.-2\gamma\,(1-\gamma)\,\frac{\chi\xi-1}{\xi}\hat{y}_{N,t}\hat{z}_{F,t}+2\gamma\,(1-\gamma)\,\frac{\chi\xi-1}{\xi}\hat{y}_{F,t}\hat{y}_{N,t}\right]+t.i.p.+h.o.t. \\ =\ & -\frac{\sigma}{2}\left[2\,(1-\gamma)\,\hat{d}_{N,t}+\gamma\vartheta\hat{y}_{F,t}^2+(1-\gamma)\,\hat{y}_{N,t}^2+\chi\hat{y}_t^2-2\gamma\hat{y}_{F,t}\hat{z}_{F,t}-2\gamma\chi\hat{y}_t\hat{z}_{F,t}\right. \\ & \left.+\frac{\gamma\,(1-\gamma)}{\xi}\left(\hat{y}_{F,t}-\hat{y}_{N,t}\right)^2-2\frac{\gamma\,(1-\gamma)}{\xi}\left(\hat{y}_{F,t}-\hat{y}_{N,t}\right)\hat{z}_{F,t}\right]+t.i.p.+h.o.t. \qquad \text{(A.44)} \end{aligned}$$

Next, we exploit the relationships between $\hat{y}_{F,t}$, $\hat{y}_{N,t}$ and $\hat{y}_t$ stated in (A.18)-(A.19), as well as the fact that

$$\hat{y}_{F,t}-\hat{y}_{N,t}=-\frac{\vartheta+\chi}{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)}\hat{y}_t+\frac{\gamma\,(1+\varkappa_F)}{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)}\hat{z}_{F,t}.$$

Using them in (A.44) and collecting terms, the household's lifetime utility can be written as follows:

$$E_0\sum_{t=0}^{\infty}\beta^t\left[U\left(\tilde{c}_t\right)-V\left(h_t\right)\right]\approx\frac{U-V}{1-\beta}-\frac{\sigma}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[2\,(1-\gamma)\,\hat{d}_{N,t}+\Omega_y\hat{y}_t^2-2\gamma\Omega_{zy}\hat{y}_t\hat{z}_{F,t}\right]+t.i.p.+h.o.t.$$

where

$$\Omega_y \equiv \frac{\chi(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)^2+(\vartheta+\varkappa_F)^2+\gamma\vartheta(1-\gamma)\left(\chi-\frac{1}{\xi}\right)^2+\frac{\gamma}{\xi}(\vartheta+\chi)^2}{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)^2} \tag{A.45}$$

$$\Omega_{zy} \equiv \chi-\frac{\chi-\frac{1}{\xi}}{\vartheta+\frac{1}{\xi}}+\frac{(\vartheta+\varkappa_F)(1+\varkappa_F)}{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)^2}+\frac{\vartheta\left(\chi-\frac{1}{\xi}\right)(1+\varkappa_F)}{\left(\vartheta+\frac{1}{\xi}\right)^2} \tag{A.46}$$

As standard in the literature, the Calvo-style nominal rigidity gives the following law of motion for price dispersion $\hat{d}_{N,t}$:

$$\begin{aligned}\hat{d}_{N,t} &= \theta\hat{d}_{N,t-1}+\frac{\epsilon\theta}{2(1-\theta)}\hat{\pi}_{N,t}^2 \\ &= \theta^{t+1}\hat{d}_{N,-1}+\frac{\epsilon\theta}{2(1-\theta)}\sum_{s=0}^{t}\theta^{t-s}\hat{\pi}_{N,s}^2\end{aligned}$$

Assuming $\hat{d}_{N,-1}=0$ (no initial price dispersion), we have that

$$\begin{aligned}E_0\sum_{t=0}^{\infty}\beta^t\hat{d}_{N,t} &= \frac{\epsilon\theta}{2(1-\theta)(1-\beta\theta)}E_0\sum_{t=0}^{\infty}\beta^t\hat{\pi}_{N,t}^2 \\ &= \frac{\epsilon}{2\kappa}E_0\sum_{t=0}^{\infty}\beta^t\hat{\pi}_{N,t}^2\end{aligned}$$

where the second equality makes use of the definition of κ in (**??**). Hence:

$$\begin{aligned}&E_0\sum_{t=0}^{\infty}\beta^t\left[U(\tilde{c}_t)-V(h_t)\right] \\ \approx\ &\frac{U-V}{1-\beta}-\frac{\sigma}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{(1-\gamma)\epsilon}{\kappa}\hat{\pi}_{N,t}^2+\Omega_y\left[\hat{y}_t^2-2\gamma\frac{\Omega_{zy}}{\Omega_y}\hat{z}_{F,t}\hat{y}_t\right]\right\}+t.i.p.+h.o.t. \\ =\ &\frac{U-V}{1-\beta}-\frac{\sigma}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{(1-\gamma)\epsilon}{\kappa}\hat{\pi}_{N,t}^2+\Omega_y\left[\hat{y}_t^2-2\gamma\frac{\Omega_{zy}}{\Omega_y}\hat{z}_{F,t}\hat{y}_t+\left(\gamma\frac{\Omega_{zy}}{\Omega_y}\hat{z}_{F,t}\right)^2-\left(\gamma\frac{\Omega_{zy}}{\Omega_y}\hat{z}_{F,t}\right)^2\right]\right\}+t.i.p.+h. \\ =\ &\frac{U-V}{1-\beta}-\frac{\sigma}{2}E_0\sum_{t=0}^{\infty}\beta^t\left[\frac{(1-\gamma)\epsilon}{\kappa}\hat{\pi}_{N,t}^2+\Omega_y\left(\hat{y}_t-\hat{y}_t^T\right)^2\right]+t.i.p.+h.o.t.\end{aligned}$$

where we have defined the welfare-relevant target level of output

$$\hat{y}_t^T \equiv F^T\hat{z}_{F,t} \qquad \text{for} \qquad F^T \equiv \gamma\frac{\Omega_{zy}}{\Omega_y} \tag{A.47}$$

It therefore follows that maximizing the representative household's lifetime utility is isomorphic to

minimizing the following intertemporal loss function

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_\pi \hat{\pi}_{N,t}^2 + \lambda_x \hat{x}_t^2 \right]$$

for

$$\lambda_\pi \equiv \frac{(1-\gamma)\,\epsilon}{\kappa}, \qquad \lambda_x \equiv \Omega_y, \qquad \hat{x}_t \equiv \hat{y}_t - \hat{y}_t^T,$$

where the latter defines the welfare-relevant output gap.

A.1.4 Headline Inflation Representation

This section provides a detailed derivation of the representation of headline inflation given by equation (32). Starting with its definition in (31), we have that

$$\begin{aligned} \hat{\pi}_t^o &= \hat{\pi}_{N,t} + \omega \left(\hat{q}_t - \hat{q}_{t-1} \right) \\ &= \hat{\pi}_{N,t} + \frac{\omega}{1-\gamma} \left(\hat{p}_{F,t} - \hat{p}_{F,t-1} \right) \end{aligned} \tag{A.48}$$

where the first equality follows from the relative price $\hat{q}_t = \hat{P}_{F,t} - \hat{P}_{N,t}$ (hence, $\hat{\pi}_{F,t} - \hat{\pi}_{N,t} = \hat{q}_t - \hat{q}_{t-1}$), while the second one makes use of $\hat{q}_t = (1-\gamma)^{-1} \hat{p}_{F,t}$. Next, recall from Appendix A.1.1 that

$$\begin{aligned} \hat{p}_{F,t} &= \Upsilon_{py} \hat{y}_t - \Upsilon_{pz} \hat{z}_{F,t} \\ \Upsilon_{py} &\equiv \frac{\sigma \left(\vartheta + \xi^{-1} \right) + \vartheta \left(\chi - \xi^{-1} \right)}{\sigma \left(\vartheta + \xi^{-1} \right)}, \qquad \Upsilon_{pz} \equiv \frac{(1+\varkappa_F)\,\vartheta}{\sigma \left(\vartheta + \frac{1}{\xi} \right)} \end{aligned} \tag{A.49}$$

Plugging the latter togethet with $\hat{y}_t = \hat{y}_t^{gap} + F^n \hat{z}_{F,t}$ into (A.48), after simple rearrangement of terms we obtain equation (32) in the text:

$$\hat{\pi}_t^o = \hat{\pi}_{N,t} + \frac{\omega \Upsilon_{py}}{1-\gamma} \left(\hat{y}_t^{gap} - \hat{y}_{t-1}^{gap} \right) + \frac{\omega}{1-\gamma} \left(\Upsilon_{py} F^n - \Upsilon_{pz} \right) \left(\hat{z}_{F,t} - \hat{z}_{F,t-1} \right)$$

A.2 Lemmata and Proofs of the Propositions

Lemma 2 *Define the following threshold values for the response coefficient to inflation ϕ_π:*

$$\breve{\phi}_\pi \equiv \frac{\sigma\left[\vartheta+\frac{1}{\xi}+\gamma\left(\chi-\frac{1}{\xi}\right)\right]}{\gamma\left[\sigma\left(\vartheta+\frac{1}{\xi}\right)+\vartheta\left(\chi-\frac{1}{\xi}\right)\right]}, \qquad \bar{\phi}_\pi \equiv \frac{2(1+\beta)\sigma\left[\vartheta+\frac{1}{\xi}+\gamma\left(\chi-\frac{1}{\xi}\right)\right]+(1-\alpha)\kappa_y\left(\vartheta+\frac{1}{\xi}\right)}{2(1+\beta)\gamma\left[\sigma\left(\vartheta+\frac{1}{\xi}\right)+\vartheta\left(\chi-\frac{1}{\xi}\right)\right]+(1-\alpha)\kappa_y\left(\vartheta+\frac{1}{\xi}\right)}$$

1. *Assume that $\chi > \frac{1+\xi}{\xi}$. Then, there exist a threshold $A^* > 0$ such that $0 \leq \breve{\phi}_\pi \leq \bar{\phi}_\pi \leq 1$ if $A \leq A^*$, and $1 < \bar{\phi}_\pi < \breve{\phi}_\pi$ if $A > A^*$.*

2. *Assume that $\chi \in \left[\frac{1}{\xi}, \frac{1+\xi}{\xi}\right]$. Then, $1 < \bar{\phi}_\pi < \breve{\phi}_\pi$ for any $A \geq 0$.*

Proof. Assume that $\chi > \frac{1+\xi}{\xi}$, such that the thresholds $\bar{\phi}_\pi$ and $\breve{\phi}_\pi$ are both strictly positive. Simple algebra shows that i) $\bar{\phi}_\pi \gtreqless \breve{\phi}_\pi$ if and only if $\breve{\phi}_\pi \lesseqgtr 1$, and ii) $\bar{\phi}_\pi \lesseqgtr 1$ if and only if $\breve{\phi}_\pi \lesseqgtr 1$ (*). Focusing on $\breve{\phi}_\pi$, making use of the definitions of σ and ϑ, simple algebra gives

$$\breve{\phi}_\pi \gtreqless 1 \qquad \text{iff} \qquad \underbrace{1-\alpha+(\alpha-\gamma)\left(\chi-\frac{1}{\xi}\right)}_{LHS(\gamma)} \gtreqless \underbrace{-\frac{\alpha}{\xi}\frac{1-\gamma}{\gamma}}_{RHS(\gamma)} \tag{A.50}$$

Recalling that $\gamma \in (\alpha, 1)$, simple calculus shows that $RHS(\gamma)$ and $LHS(\gamma)$ satisfy the following properties:

- $\lim_{\gamma\to\alpha} RHS(\gamma) = -\frac{1-\alpha}{\xi} < 0$, $\lim_{\gamma\to 1} RHS(\gamma) = 0$, $RHS'(\gamma) > 0$, and $RHS''(\gamma) < 0$;
- $\lim_{\gamma\to\alpha} LHS(\gamma) = 1-\alpha > 0$, $\lim_{\gamma\to 1} LHS(\gamma) = (1-\alpha)\left(\frac{1+\xi}{\xi}-\chi\right) < 0$, and $LHS'(\gamma) = -\left(\chi-\frac{1}{\xi}\right) < 0$.

Hence, there exist a threshold value $\gamma^* \in (\alpha, 1)$ such that $\breve{\phi}_\pi \gtreqless 1$ for $\gamma \lesseqgtr \gamma^*$. Given the properties of γ stated in Proposition 1, there exists then a threshold level of development $A^* > 0$ such that $\gamma \lesseqgtr \gamma^*$ if and only if $A \gtreqless A^*$. It follows that when $A \leq A^*$, we have that $\gamma \geq \gamma^*$ and therefore $\breve{\phi}_\pi \leq 1$. This, together with property (*) above, allows us to conclude that $0 < \breve{\phi}_\pi \leq \bar{\phi}_\pi \leq 1$. On the contrary, when $A > A^*$, we have that $\gamma < \gamma^*$ and therefore $\breve{\phi}_\pi > 1$. This, together with property (*) above, allows us to conclude that $1 < \bar{\phi}_\pi < \breve{\phi}_\pi$. This concludes the proof of statement 1. in the proposition.

Assume now that $\chi \in \left[\frac{1}{\xi}, \frac{1+\xi}{\xi}\right]$. As for the previous case, both thresholds $\bar{\phi}_\pi$ and $\breve{\phi}_\pi$ are strictly positive, $\bar{\phi}_\pi \gtreqless \breve{\phi}_\pi$ if and only if $\breve{\phi}_\pi \lesseqgtr 1$, and the statement in (A.50) still holds. However, we now have that $\lim_{\gamma\to 1} LHS(\gamma) \geq 0$ (while all remaining properties of $RHS(\gamma)$ and $LHS(\gamma)$ still hold).

Hence, $\breve{\phi}_\pi > 1$ for any $\gamma \in (\alpha, 1)$. Appealing to (*) again, we have that $1 < \bar{\phi}_\pi < \breve{\phi}_\pi$.This proves statement 2 in the proposition. ■

A.2.1 Proof of Proposition 1

Consider the equation $c^\chi = \frac{A^{1+\chi}}{c-\bar{c}_F}$ (*), where both sides are functions of c. Let its left and the right hand sides be denoted, respectively, by $f(c)$ and $g(c)$, and consider the domain $c > \bar{c}_F$. It is straightforward to show that f is strictly increasing in c, with $\lim_{c\to\bar{c}_F} f(c) > 0$ and $\lim_{c\to\infty} f(c) = +\infty$, while $g(c)$ is strictly decreasing in c, with $\lim_{c\to\bar{c}_F} g(c) = +\infty$ and $\lim_{c\to\infty} g(c) = 0$. It follows that there exists a unique $c > \bar{c}_F$ that solves (*). Given c, the steady state value of all remaining endogenous variables are uniquely defined. A straightforward application of the Implicit Function Theorem shows that c is a strictly increasing function of A, with $\lim_{A\to\infty} c = \infty$.

The steady state spending share on food consumption, $\gamma = \frac{c_F}{c}$, can be written as follows:

$$\gamma = \frac{\bar{c}_F + \tilde{c}_F}{c} = \frac{\bar{c}_F + \alpha(c - c_F)}{c} = \alpha + \frac{(1-\alpha)\bar{c}_F}{c} \tag{A.51}$$

Clearly, without food subsistence, we have that $\gamma = \alpha$. With subsistence, we have instead $\gamma > \alpha$ but less than unity. From the relationship between c and A stated above, equation (A.51) allows us to infer that γ is a strictly decreasing function of A, as well as that $\lim_{A\to\infty} \gamma = \alpha$.

A.2.2 Proof of Proposition 2

The natural and the target levels of output are given, respectively, by the following expressions

$$\hat{y}_t^n = \gamma \frac{1 + \chi + \frac{(1-\gamma)(1-\vartheta)(1-\chi\xi)}{1+\xi}}{\chi\gamma + \vartheta + (1-\gamma)\chi\vartheta + \frac{(1-\gamma)(1-\vartheta)(1+\chi)}{1+\xi}} \hat{z}_{F,t} \tag{A.52}$$

$$\hat{y}_t^T \equiv \gamma \frac{\Omega_{zy}}{\Omega_y} \hat{z}_{F,t} \tag{A.53}$$

with Ω_y and Ω_{zy} defined in (A.45)-(A.46). Let $\hat{y}_t^{FM,n} \equiv \lim_{\xi\to\infty} \hat{y}_t^n$ and $\hat{y}_t^{FM,T} \equiv \lim_{\xi\to\infty} \hat{y}_t^n$ denote, respectively, the natural and the target levels of output under full labor mobility. It is straightforward to show that

$$\hat{y}_t^{FM,n} = \gamma \frac{1 + \chi - (1-\gamma)(1-\vartheta)\chi}{\chi\gamma + \vartheta + (1-\gamma)\chi\vartheta} \hat{z}_{F,t}$$

Next, consider the terms Ω_y and Ω_{zy} separately. Given that $\lim_{\xi\to\infty} \varkappa_F = \chi\gamma$, after simplification of terms, we find:

$$\begin{aligned}\lim_{\xi\to\infty} \Omega_{zy} &= \chi - \frac{\chi}{\vartheta} + \frac{(\vartheta + \varkappa_F)(1+\varkappa_F)}{(1-\gamma)\vartheta^2} + \frac{\vartheta\chi(1+\varkappa_F)}{\vartheta^2} \\ &= \frac{(\vartheta+\gamma\chi)[1+\chi-\chi(1-\gamma)(1-\vartheta)]}{(1-\gamma)\vartheta^2}\end{aligned}$$

For Ω_y we have instead:

$$\begin{aligned}\lim_{\xi\to\infty} \Omega_{y} &= \frac{\chi(1-\gamma)\vartheta^2 + (\vartheta+\chi\gamma)^2 + \gamma\vartheta(1-\gamma)\chi}{(1-\gamma)\vartheta^2} \\ &= \frac{(\vartheta+\gamma\chi)[\chi\gamma+\vartheta+(1-\gamma)\chi\vartheta]}{(1-\gamma)\vartheta^2}\end{aligned}$$

Merging these results, we have indeed that $\hat{y}_t^{FM,T} = \hat{y}_t^{FM,n}$.

A.2.3 Proof of Proposition 3

To start with, recall the definition of κ_y in (23). Taking the limit for $\xi \to \infty$, while using the definitions of σ and ϑ, we obtain $\kappa_y \equiv \kappa\frac{\sigma+\chi}{1-\alpha}$. Given $\sigma \equiv \frac{1-\alpha}{1-\gamma}$ and the fact that γ is a strictly decreasing function of A, simple calculus implies that κ_y is strictly decreasing in A. With the Phillips curve getting flatter, a higher A make inflation stabilization costlier.

Next, recall the definitions of the welfare weights λ_π and λ_x from (25) and (A.45), where the former does not depend on ξ. Taking the limit for $\xi \to \infty$, while using the definitions of σ and ϑ again, after simple algebra, we obtain

$$\lambda_x = \frac{(1-\gamma)(\sigma+\chi)(\sigma+\alpha\chi)}{(1-\alpha)^2}$$

It therefore follows that, under full labor mobility, the relative weight on (core) inflation stabilization in the policy objective is:

$$\frac{\lambda_\pi}{\lambda_x} = \frac{(1-\alpha)^2\epsilon}{\kappa(\sigma+\chi)(\sigma+\alpha\chi)}$$

Given $\sigma \equiv \frac{1-\alpha}{1-\gamma}$ and the fact that γ is a strictly decreasing function of A - hence, σ is strictly decreasing in A - simple calculus implies that the relative weight on output gap stabilization $\lambda \equiv \lambda_x/\lambda_\pi$ is strictly decreasing in A: in a more developed economy (higher A), the central bank should be more (respectively, less) concerned about stabilizing core inflation (respectively, the output gap).

The targeting rule $\hat{\pi}_{N,t} = -\frac{\lambda}{\kappa_y}\hat{x}_t$ in (28) and simple algebra give the following expression for ORV:

$$ORV = \frac{\lambda}{\kappa_y} = \frac{(\sigma + \alpha\chi)}{(1-\alpha)\,\epsilon}$$

By the previous result - i.e., σ is strictly decreasing in A - it immediately follows that ORV is a strictly decreasing function of A.

A.2.4 Proof of Proposition 4

The proof is straightforward. The targeting rule characterizing the optimal policy under commitment is

$$\hat{\pi}_{N,t} + \frac{\lambda}{\kappa_y}\Delta\hat{y}_t^{GAP} = \frac{\lambda}{\kappa_y}F\Delta\hat{z}_{F,t}. \tag{A.54}$$

Making use of the headline representation given in (32), the strict headline targeting rule $\hat{\pi}_t^o = \phi_z\Delta\hat{z}_{F,t}$ becomes

$$\hat{\pi}_{N,t} + m_{\pi y}\Delta\hat{y}_t^{GAP} = (\phi_z - m_{\pi z})\,\Delta\hat{z}_{F,t} \tag{A.55}$$

The two are equivalent if and only if

$$m_{\pi y} = \frac{\lambda}{\kappa_y} \qquad \text{and} \qquad (\phi_z - m_{\pi z}) = \frac{\lambda}{\kappa_y}F.$$

Using the definitions of $m_{\pi y}$ and $m_{\pi z}$, the first equality holds if and only if a) the weight on food inflation ω is set equal to $\omega^* \equiv \frac{\lambda}{\kappa_y}\frac{1-\gamma}{\Upsilon_{py}}$, and b) given $\omega = \omega^*$, $\phi_z = \frac{\lambda}{\kappa_y}F + m_{\pi z}$.

A.2.5 Proof of Corollary 1

Consider the expressions for ω^* from Proposition 4:

$$\omega^* \equiv \frac{\lambda}{\kappa_y}\frac{1-\gamma}{\Upsilon_{py}} \tag{A.56}$$

and recall the definitions of $\lambda \equiv \frac{\lambda_x}{\lambda_\pi} = \frac{\Omega_y \kappa}{(1-\gamma)\epsilon}$,

$$\begin{aligned}
\Omega_y &\equiv \frac{\chi(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)^2 + (\vartheta+\varkappa_F)^2 + \gamma\vartheta(1-\gamma)\left(\chi-\frac{1}{\xi}\right)^2 + \frac{\gamma}{\xi}(\vartheta+\chi)^2}{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)^2} \\
\kappa_y &\equiv \kappa\frac{\gamma\chi + \vartheta[1+(1-\gamma\chi)] + \frac{1}{\xi}[\gamma\vartheta+1-\gamma+\chi]}{(1-\gamma)\left(\vartheta+\frac{1}{\xi}\right)} \\
\Upsilon_{py} &\equiv \frac{\sigma\left(\vartheta+\xi^{-1}\right)+\vartheta\left(\chi-\xi^{-1}\right)}{\sigma\left(\vartheta+\xi^{-1}\right)}
\end{aligned}$$

Suppose labor is fully mobile ($\xi \to \infty$). Simple calculus gives:

$$\begin{aligned}
\lim_{\xi\to\infty}\Omega_y &= \frac{\vartheta+\gamma\chi}{(1-\gamma)\vartheta^2}\left[\frac{\gamma}{\alpha}(\sigma+\chi)\right] \\
\lim_{\xi\to\infty}\kappa_y &\equiv \kappa\frac{\sigma+\chi}{1-\alpha} \\
\lim_{\xi\to\infty}\Upsilon_{py} &\equiv \frac{\sigma+\chi}{\sigma}
\end{aligned}$$

Plugging them back into (A.56) and simplifying, we obtain:

$$\omega^* \equiv \frac{\sigma+\alpha\chi}{\epsilon(\sigma+\chi)}.$$

Since $\epsilon > 1$ and $\alpha \in [0,1)$, indeed we have that $\omega^* \in (0,1)$. By chain rule, ω^* is strictly decreasing in A: namely,

$$\frac{\partial\omega^*}{\partial A} = \frac{\partial\omega^*}{\partial\sigma}\frac{\partial\sigma}{\partial\gamma}\frac{\partial\gamma}{\partial A} < 0$$

given that $\partial\omega^*/\partial\sigma > 0$, $\partial\sigma/\partial\gamma > 0$ and $\partial\gamma/\partial A < 0$. Next we compare the headline weight ω^* with the CPI weight γ. To do so, it is convenient to define $F(A) \equiv \omega^* - \gamma$. Simple calculus shows that

$$\begin{aligned}
&\lim_{A\longrightarrow\infty} F = \frac{1+\alpha}{2\epsilon} - \alpha, \qquad \lim_{A\longrightarrow 0} F = \frac{1}{\epsilon} - 1 < 0 \\
&\frac{\partial F}{\partial A} = \frac{\partial\gamma}{\partial A}\left(\frac{\partial\omega^*}{\partial\sigma}\frac{\partial\sigma}{\partial\gamma} - 1\right) = \frac{\partial\gamma}{\partial A}\left[\frac{1}{\epsilon}\left(\frac{\sigma}{1+\sigma}\right)^2 - 1\right] > 0.
\end{aligned}$$

Let $\epsilon^L \equiv \frac{1+\alpha}{2\alpha}$, such that $F^{\lim} \equiv \lim_{A\longrightarrow\infty} F \gtreqless 0$ for $\epsilon \lesseqgtr \epsilon^L$. Based on all these properties of F, we can conclude that

i) if $\epsilon \geq \epsilon^L$, then $\omega^* < \gamma$ for any $A > 0$;

ii) if $\epsilon < \epsilon^L$, then there exists a development threshold $A^* > 0$ such that $\omega^* \gtreqless \gamma$ for $A \gtreqless A^*$.

Consider now the case of low labor mobility, $\xi = 1$, while retaining the assumption $\chi = 1$. Because $\varkappa_F = \chi\gamma + \frac{1-\gamma}{\xi} = 1$, we now have that

$$\Omega_y = \frac{2}{1-\gamma}, \qquad \kappa_y = \kappa\frac{2}{1-\gamma}, \qquad \Upsilon_{py} = 1.$$

Plugging these back into ω^*, we obtain:

$$\omega^* = \epsilon^{-1} \in (0,1)$$

If labor is very (but not completely) immobile, i.e. $\xi = 1$, then

$$\omega^* \equiv \frac{1}{\epsilon} \in (0,1)\,.$$

Letting $\epsilon^H \equiv \alpha^{-1}$, by the properties of the consumption share of food γ, it follows that:

i) if $\epsilon \geq \epsilon^H$, then $\omega^* < \gamma$ for any $A > 0$;

ii) if $\epsilon < \epsilon^H$, then there exists a development threshold $A^* > 0$ such that $\omega^* \gtreqless \gamma$ for $A \gtreqless A^*$.

A.3 Labor Mobility Estimates

Table 3: Labor Mobility and (Relative) Income per Capita

Country	ξ	Real GDP p.c.	Country	ξ	Real GDP p.c.	Country	ξ	Real GDP p.c.
Zambia	0.219	0.049	Morocco	0.902	0.109	Lithuania	2.315	0.41
Laos	0.237	0.069	Indonesia	0.913	0.136	Mexico	2.392	0.356
Rwanda	0.247	0.025	Namibia	0.97	0.165	Costa Rica	2.406	0.285
Mozambique	0.261	0.017	Sri Lanka	0.972	0.145	Latvia	2.993	0.364
Burkina Faso	0.275	0.029	Ghana	1.029	0.071	Slovakia	3.071	0.415
Malawi	0.307	0.023	Philippines	1.07	0.1	Italy	3.218	0.869
Tanzania	0.31	0.04	Turkey	1.265	0.317	Chile	3.314	0.357
Cameroon	0.316	0.074	Ecuador	1.284	0.196	Mauritius	3.392	0.277
Nepal	0.368	0.048	Kenya	1.382	0.074	Finland	3.739	0.852
Nigeria	0.39	0.115	Brazil	1.407	0.277	France	4.067	0.828
Botswana	0.421	0.248	Poland	1.413	0.384	Germany	4.844	0.927
Ethiopia	0.519	0.02	Myanmar	1.456	0.045	Spain	5.212	0.712
Cambogia	0.527	0.054	Slovenia	1.507	0.581	Hungary	5.257	0.448
Uganda	0.527	0.031	Colombia	1.544	0.228	U.K.	5.316	0.778
Bangladesh	0.588	0.061	Peru	1.58	0.173	Belgium	5.803	0.921
Vietnam	0.59	0.1	Egypt	1.61	0.192	Malysia	6.006	0.364
Lesotho	0.625	0.039	Pakistan	1.707	0.071	Estonia	6.439	0.46
Senegal	0.667	0.058	Portugal	1.725	0.61	Sweden	6.457	0.897
Thailand	0.671	0.25	Greece	1.764	0.584	Canada	6.812	0.877
India	0.836	0.071	Austria	1.878	0.978	Tunisia	6.951	0.179
China	0.839	0.125	Ireland	1.887	0.973	Czech Rep.	7.205	0.602
South Africa	0.873	0.223	Japan	2.129	0.719	Australia	9.687	0.843
Bolivia	0.881	0.15	Korea	2.16	0.547	U.S.	11.05	1

A.4 Calibration of Incomplete Beta Function

Letting y_i denote income per capita for a generic country i in the sample I, the implied labor elasticity ξ_i^p (with the superscript p standing for "predicted") is

$$\xi_i^p = \xi_{US} \mathcal{I}_{y_i}(a,b)$$

where ξ_{US} denotes the elasticity estimate for the U.S. while $\mathcal{I}_{y_i}(a,b)$ is the regularized incomple beta cumulative density function defined by

$$\mathcal{I}_{y_i}(a,b) \equiv \frac{\mathcal{B}(y_i;a,b)}{\mathcal{B}(1;a,b)}, \qquad \mathcal{B}(y_i;a,b) \equiv \int_0^{y_i} x^{a-1}(1-x)^{b-1}\,dx, \tag{A.57}$$

for $a > 0$, $b > 0$ and $y_i \in [0,1]$. For $i =$US for which $y_{\text{US}} = 1$ (namely, the U.S.), we have $\mathcal{B}(y_{\text{US}};a,b) = \mathcal{B}(1;a,b)$, such that $\mathcal{I}_{y_{\text{US}}}(a,b) = 1$ and hence $\xi_{US}^p = \xi_{US}$, while $\mathcal{I}_{y_i}(a,b) \in [0,1)$ for any $y_i \in [0,1)$.

The shape parameters a and b are chosen by solving the nonlinear least squares problem:

$$\min_{a>0,\ b>0} \sum_{i \in \mathcal{S}} \left[\left(\xi_i^p - \xi_i^o\right)^2 + \iota \left(\xi_I^p - \xi_I^o\right)^2 \right] \tag{A.58}$$

where $\mathcal{S}$ is the set of countries, ι is a penalty parameter, while ξ_I^p and ξ_I^o are, respectively, the predicted and observed elasticities for India. A sufficiently large value of ι ensures that $\xi_I^p \approx \xi_I^o$ such that, to numerical precision, the functional specification $\xi = \xi_{US}\mathcal{I}_y(a,b)$ exactly passes through the scatter points representing India and the U.S. The solution to (A.58) yields $a = 0.5$ and $b = 0.18$.