

The Idea Multiplier: Government Spending in Innovation Networks*

Henrique S. Basso

Banco de España, CEMFI

Alessandro Pozzetti

IMT Lucca

Omar Rachedi

Esade, Universitat Ramon Llull

Iacopo Varotto

Banco de España

August 24, 2026

Abstract

When sectors invest in R&D, the resulting ideas spill over and feed innovation in other industries. We show empirically, analytically, and quantitatively that this innovation network shapes the propagation of government consumption spending. Using patent citations across U.S. industries, we find that defense expenditures raise a sector's private R&D both directly, when procurements target that sector, and indirectly, when they target upstream industries whose patents that sector cites. We rationalize this evidence with a multi-sector endogenous-growth model where sectoral R&D generates knowledge spillovers across industries. The fiscal multiplier is larger when government spending targets upstream industries in the innovation network.

Key Words: Fiscal Multipliers, Fiscal Policy, R&D, Endogenous Growth

JEL Classification Codes: E32, E62, H50, O30, O38.

*Addresses: henrique.basso@bde.es, alessandro.pozzetti@imtlucca.it, omar.rachedi@esade.edu, iacopo.varotto@bde.es. The opinions in this paper are those of the authors and do not necessarily reflect the views of the Banco de España or the Eurosystem.

1 Introduction

Ideas do not remain in the sector in which they are produced. When firms in one industry invest in R&D, the resulting knowledge can be used by innovators in other sectors, often far away in the production network. Patent citations reveal these knowledge links: a citation marks a flow of ideas from the cited industry to the citing one. The resulting innovation network is highly uneven: some industries act as knowledge hubs because their ideas are used directly and indirectly by many others (Bloom et al., 2013; Acemoglu et al., 2016b; Liu and Ma, 2026). This paper argues that this innovation network crucially shapes the propagation of government consumption spending.

The mechanism is simple. A fiscal expansion that raises government demand for a sector’s output increases its market size and profits, and with them the private return to R&D: a direct own-sector effect. If other industries build on that sector’s knowledge, the shock does not stop there: more R&D raises the sector’s stock of ideas, which makes R&D more productive in every sector that cites it, directly or indirectly. We call this additional propagation the *idea multiplier*: the part of the fiscal multiplier that arises because government spending expands the stock of ideas available to other sectors, beyond the direct demand effect.

We provide support for this mechanism empirically, theoretically, and quantitatively. Empirically, we document that government spending raises private R&D not only in the sectors that receive it, but also in the sectors that build on their knowledge. Theoretically, we develop a multi-sector New Keynesian model with endogenous innovation, in which sectors are connected by an innovation network, and show analytically that government spending propagates to R&D and output through citations, with larger aggregate effects when spending targets more upstream knowledge suppliers. Quantitatively, we find that the innovation network is a first-order determinant of the dispersion of *sectoral-spending multipliers*—the responses of aggregate GDP to government spending shocks in individual sectors.

We start by providing novel empirical evidence. We estimate dynamic responses using local projections on a panel of 66 U.S. industries from 1997 to 2024. The key variables are real sectoral private R&D expenditures and defense purchases. We construct the innovation network from patent citations across sectors: each entry of this matrix measures how much one sector builds on the knowledge produced by another. We regress each sector’s R&D on two variables: the government spending the sector receives directly—the *own* treatment—and the citation-

weighted spending received by its knowledge suppliers—the *upstream* treatment.

Identification comes from a shift-share design (Nekarda and Ramey, 2011; Nakamura and Steinsson, 2014). The instrument interacts an aggregate shifter—the national change in defense spending—with each sector’s predetermined exposure to defense purchases. The identifying variation thus reflects the differential incidence of national defense buildups rather than sector-level spending choices.

We find three main results. First, government spending raises R&D in the sector that receives it, and the response progressively goes up over time. Each dollar of government purchases raises the recipient sector’s annual private R&D by about 25 cents after two years.¹ Second, government spending propagates through the innovation network from upstream: a shock to a sector’s knowledge suppliers raises its private R&D with the same increasing profile, by about 7 cents, roughly 30% of the own-sector effect. Third, this channel operates above and beyond the propagation of government spending through the input-output network: the innovation and production networks are uncorrelated.

To rationalize these facts, we build a multi-sector New Keynesian model with knowledge accumulation. Sectors differ in their role in private and public demand, factor intensities, use of intermediate inputs, and degree of nominal rigidity. The government finances exogenous streams of sectoral purchases with lump-sum taxes. Absent knowledge spillovers, the model nests the workhorse multi-sector fiscal-policy economy (Bouakez et al., 2023, 2025; Cox et al., 2024).

The new ingredient is the innovation block. Each sector has a stock of knowledge that enters production as a Hicks-neutral productivity term and accumulates through R&D (i.e., research labor). Crucially, the productivity of a sector’s R&D depends not only on its own research effort, but also on the knowledge of the sectors it cites, with weights given by the innovation network: knowledge accumulated in a sector makes R&D more productive in its citers. Spillovers are not automatic, however. Absorbing external knowledge requires *adoption* capability, which producers derive from their own knowledge stock.

We then characterize analytically a tractable version of the model to clarify the economic mechanisms. A government spending shock raises demand for the targeted sector’s output, and with it the value of its knowledge. Producers respond by hiring more research labor, which expands sector’s knowledge. That stock enters the R&D technology of the citing sectors, lowering their effective

¹Our estimate is in line with the firm-level evidence on government demand and corporate R&D (Lichtenberg, 1988; Slavtchev and Wiederhold, 2016; Cioaca, 2025; Belenzon and Cioaca, 2026).

marginal cost of producing ideas: they too expand R&D, and—as their own knowledge grows—the shock keeps propagating along the innovation network. The responses of sectoral R&D and GDP inherit exactly this structure: a direct own effect plus an indirect network effect governed by the Leontief inverse of the citation matrix, which compounds knowledge spillovers along any citation chain.

We derive the idea multiplier as the difference in a sector’s spending multiplier with respect to a counterfactual economy in which each sector’s research builds only on its own knowledge. This comparison isolates the contribution of cross-sectoral spillovers via the citation matrix. The idea multiplier increases with the *upstreamness* of the targeted sector: the more upstream the sector, the further the knowledge created by the shock cascades along the innovation network.

The model is calibrated to U.S. data. Demand shares, factor intensities, and input-output linkages come from the Input-Output tables of the U.S. Bureau of Economic Analysis (BEA); and the innovation network is the empirical patent-citation matrix. To tie the model to the data, we discipline the parameter governing adoption by matching the estimated own response of private R&D at the two-year horizon. Although this is the only targeted moment, the model replicates the progressive build-up of the own response, and matches the magnitude and build-up of the upstream channel, all of which is untargeted. This exercise validates the model as an ideal laboratory to study the transmission of government spending in innovation networks.

The model delivers highly heterogeneous sectoral-spending multipliers: a dollar of government spending moves GDP by between -0.14 and 2.23 depending on the targeted sector. Multipliers are largest when spending targets the knowledge hubs: computers/electronics (2.23), publishing/software (1.37), and chemicals/pharmaceuticals (1.16). Relative to the economy without R&D, the innovation block compresses most of the distribution, stretches its top, and reorders the ranking: the largest multipliers no longer accrue to the sectors favored by demand composition and nominal rigidities. The heterogeneous citation matrix generates virtually all of this dispersion.

We then derive the idea multiplier: while small for most industries, it exceeds fifteen cents in five innovation-upstream industries, reaching 1.86 for computers and electronics. For this industry, the innovation networks explains more than 80% of the size of its multiplier. The idea multiplier accounts for 52% of the cross-sectional variance of sectoral-spending multipliers, as much as demand composition, factor intensities, input-output linkages, and nominal rigidities combined.

We confirm these results in several model extensions, including sticky wages, an investment network (vom Lehn and Winberry, 2022), and a two-agent economy with realistic intertemporal marginal propensity to consume (iMPC; Fagereng et al., 2021) and marginal propensity to earn (MPE; Auclert et al., 2023).

This paper builds on the work on the effects of government spending across sectors (Acemoglu et al., 2016a; Cox et al., 2024; Bouakez et al., 2023, 2025; Flynn et al., 2026; Peri et al., 2026; Basso et al., 2026), highlighting how sectoral heterogeneity and input-output linkages shape the transmission of fiscal policy. We add a novel margin: the innovation network. Embedding an endogenous-growth block in the tradition of Comin and Gertler (2006) and Anzoategui et al. (2019) into a multi-sector New Keynesian economy, we show that government spending propagates across sectors not only through the demand for intermediate inputs, but also through the knowledge that it induces the recipient sectors to create.

Our paper connects to the literature on knowledge spillovers and innovation networks (Bloom et al., 2013; Acemoglu et al., 2016b; Cai et al., 2022; Liu and Ma, 2026), which documents that ideas flow across industries, and studies their implications for long-run growth and the allocation of R&D. Our contribution is to bring the innovation network into the analysis of government spending.

2 Empirical Evidence

This section shows that public spending affects private R&D both directly and indirectly through the innovation network: a government spending shock to a sector raises not only its own R&D, but also that of the industries citing its patents.

Throughout, we focus only on the purchases of goods and services from private sectors that flow into defense *consumption* expenditures, and exclude defense *investment* expenditures, such as public R&D outlays.

2.1 Data

We build a sector-year panel from 1997 to 2024 on 66 private U.S. industries, a level of disaggregation that corresponds to the three-digit NAICS classification.

We use data on sectoral real value added, together with national data on defense expenditures and real GDP. We measure defense spending across sectors from the Use table, which gives sectors' contribution to defense expenditures, deflated with the sectoral gross output price index. All these data are from the BEA.

Sectoral private R&D is constructed from Compustat. We map each firm to a sector by bridging any historical industry code (SIC or NAICS) to NAICS 2017. Whenever a single code corresponds to multiple BEA sectors, we use *proportional*

allocation: each firm-year observation is distributed across all candidate sectors with weights summing to one. The procedure assigns a sector to all 527,642 firm-year observations covering 40,752 unique firms over 1950–2025. Nominal R&D expenditures are deflated using the sectoral gross output price index.

To capture knowledge flows across sectors, we build the innovation network from patent citation data, in the spirit of [Liu and Ma \(2026\)](#). First, we match all U.S. Patent and Trademark Office (USPTO) patents to Compustat firms by combining the patent–firm crosswalks of [Kogan et al. \(2017\)](#) and [Dyèvre and Seager \(2023\)](#): this yields 4,124,209 unique utility patents linked to 12,320 Compustat firms over 1949–2024. Second, we assign each (patent, firm) pair to a sector using the same concordance chain and proportional allocation applied to R&D. Third, we count patent-to-patent citations from the USPTO PatentsView file across sector pairs, dating each citation by the application year of the citing patent and weighting it by the proportional weights of the citing and cited patents, so that each citation contributes a total mass of one split across the relevant sector pairs. We exclude within-sector citations, so that the network captures only cross-sector knowledge flows, and row-normalize the citation matrix:

$$\omega_{A,s,x,t} = \frac{Cit_{s,x,t}}{\sum_{k \neq s} Cit_{s,k,t}}, \quad \sum_{x \neq s} \omega_{A,s,x,t} = 1, \quad (1)$$

where $Cit_{s,x,t}$ is the citations from patents of sector s to patents of sector x in year t , and $\omega_{A,s,x,t}$ is sector x 's share of sector s 's citations.

Our baseline innovation network, ω_{A,s,x,t_0} , averages the yearly matrices over the window t_0 , 1976–1996, a window that predates our sample and is therefore predetermined with respect to the fiscal shocks and R&D outcomes we study.

The innovation network is dominated by a handful of knowledge hubs: computers/electronics, chemicals/pharmaceuticals, machinery, publishing/software, and miscellaneous manufacturing.² These are the sectors whose patents are the most cited, directly and indirectly, by the rest of the economy.

The innovation network sharply differs from the production network, which we measure from the BEA Input-Output Use tables over 1997–2001, the first years of the sample: each entry gives the share of a supplying sector in a purchasing sector's intermediate-input basket. The two networks encode different economic linkages across the same industries: they are uncorrelated, and the same holds for their degree of upstreamness. Table 1 makes the point concrete: the five knowl-

²Machinery includes semiconductor manufacturing equipment, and miscellaneous manufacturing consists mostly of medical equipment.

Table 1: Innovation versus Production Network.

	Upstreamness rank	
	Innovation network	Production network
Computers/Electronics	1	29
Chemicals/Pharmaceuticals	2	55
Machinery	3	23
Publishing/Software	4	50
Miscellaneous Manufacturing	5	16

Note: The five most upstream industries of the innovation network, and their upstreamness rank in the production network, among the 66 BEA industries.

edge hubs spread widely across the production-network upstreamness ranking.

2.2 Econometric Specification

We estimate the dynamic response of R&D to government spending, and its cross-sectoral propagation through the innovation network, with a panel IV local-projection regression in the spirit of [Jordà \(2005\)](#). The baseline specification is

$$\begin{aligned}
 \frac{RD_{s,t+h} - RD_{s,t-1}}{Y_{s,t-1}} = & \alpha_s^{(h)} + \delta_t^{(h)} + \underbrace{\beta^{(h)} \frac{G_{s,t} - G_{s,t-1}}{Y_{s,t-1}}}_{\text{own effect}} + \dots \\
 & \dots + \underbrace{\gamma^{(h)} \sum_{x \neq s} \omega_{A,s,x,t_0} \frac{G_{x,t} - G_{x,t-1}}{Y_{s,t-1}}}_{\text{innovation-network indirect effect}} + \mathbf{Z}'_{s,t} \boldsymbol{\phi}^{(h)} + \varepsilon_{s,t}^{(h)},
 \end{aligned} \tag{2}$$

where $RD_{s,t}$ denotes real private R&D expenditures in sector s at year t , $Y_{s,t}$ is real sectoral value added, and $G_{s,t}$ is real defense purchases from sector s .

The dependent variable is the h -year-ahead change in real private R&D in sector s . The first treatment is the one-year change in real public spending in sector s , the (direct) own effect. The second, as in [Acemoglu et al. \(2016a\)](#) and [Liu and Ma \(2026\)](#), aggregates spending changes of the sectors cited by sector s with the citation shares ω_{A,s,x,t_0} of Equation (1), the (indirect) innovation-network effect. The treatments are scaled by lagged sectoral value added $Y_{s,t-1}$.

The controls $\mathbf{Z}_{s,t}$ collect three lags of sectoral R&D, the sectoral own and innovation-network treatments, and sectoral value-added growth, so as to capture the persistence of sectoral innovation and defense procurement, as well as sector-specific business-cycle conditions.³ The regression includes sector and year fixed effects, $\alpha_s^{(h)}$ and $\delta_t^{(h)}$, that absorb fixed unobserved heterogeneity across in-

³Appendix A.7 augments the control set with additional sectoral controls: employment growth, federal non-defense purchases, and investment growth.

dustries as well as common time-varying unobserved factors, such as the monetary policy stance.⁴ We estimate regression (2) separately for each horizon $h = 0, 1, 2$, and two-way cluster standard errors at the sector and year levels.

The own coefficient $\beta^{(h)}$ measures the h -year-ahead response of sector s 's R&D to a one-dollar increase in the defense spending it receives in the impact period; the indirect coefficient $\gamma^{(h)}$ measures the analogous response to a one-dollar increase in the spending received by sector s 's knowledge suppliers.⁵ Crucially, these coefficients capture the response of a given sector *relative to the others*, as is standard in cross-sectional designs (Nakamura and Steinsson, 2014; Chodorow-Reich, 2019): the year fixed effects $\delta_t^{(h)}$ absorb every force common to all sectors.⁶

The own and indirect effects in Equation (2) are, however, likely to be endogenous: the allocation of federal defense purchases across sectors may itself respond to sectoral R&D dynamics or industry business-cycle conditions. To address this concern, we follow an identification strategy that leverages instrumental variables, in the spirit of the shift-share designs of Ramey (2018) and Nakamura and Steinsson (2014). The idea is to isolate the variation in sectoral defense spending that is driven by aggregate, sector-neutral shifts in federal defense procurement, interacted with each sector's predetermined exposure to defense spending.

Specifically, we instrument the own treatment with the shift-share variable⁷

$$G_{s,t}^{\text{IV}} = \underbrace{\frac{G_{s,t_0}^{\text{nom}}}{\sum_x G_{x,t_0}^{\text{nom}}}}_{\text{share}_s} \cdot \underbrace{\frac{G_t - G_{t-1}}{Y_{t-1}}}_{\text{shift}_t}. \quad (3)$$

The share is the ratio of sector s 's nominal defense purchases, $G_{s,t}^{\text{nom}}$, to aggregate purchases, $\sum_x G_{x,t}^{\text{nom}}$. We evaluate this ratio at the base period t_0 , the first

⁴Appendix A.3 considers more granular fixed effects: industry-group $\times$ year effects, built by partitioning the 66 industries into five macro-sector groups, which absorb group-specific business cycles; and upstreamness-tercile $\times$ year effects, built by sorting industries into terciles of citation upstreamness, which absorb shocks common to the innovation-upstream sectors.

⁵These coefficients do not map into fiscal multipliers. For instance, the treatment is the one-period change in spending, but defense procurement is persistent: the same shock that raises a sector's spending by one dollar on impact also raises it in the following years, so the h -year-ahead R&D response reflects the entire spending path set in motion by the shock, not the impact dollar alone. When interpreting the estimates, we therefore convert them into per-dollar effects in three steps: (i) computing the cumulative response of annual R&D; (ii) computing the cumulative spending response; (iii) dividing the former by the latter.

⁶In the quantitative analysis we replicate the same regression on data simulated from the model. Once the model is consistent with this cross-sectional evidence, we use it to recover the general-equilibrium forces that the cross-sectional estimates cannot identify (Nakamura and Steinsson, 2014; Basso and Rachedi, 2021).

⁷Donaldson (2026) shows that cross-sectional designs of this type fail to identify partial-equilibrium elasticities when the exposure to the shock of interest is correlated with heterogeneous sensitivity to aggregate variables that move in general equilibrium—most notably monetary policy. Appendix A.8 implements her diagnostic test and shows that this concern does not threaten our estimates.

two years of the sample. The shift is the change in real aggregate defense expenditures, G_t , scaled by lagged real aggregate GDP, Y_{t-1} . For the measure of public expenditures, we use the intermediate-input component of defense gross output—the defense purchases of goods and services from the private sector.

We instrument the indirect treatment analogously, aggregating the sector-specific instrument across the citation weights:

$$G_{s,t}^{\text{IV,net}} = \sum_{x \neq s} \omega_{A,s,x,t_0} G_{x,t}^{\text{IV}}. \quad (4)$$

The key identifying assumption is that high-exposure sectors would not have experienced different R&D dynamics in years of large procurement changes, absent the induced change in their own purchases. The exclusion restriction is that the instrument affects private R&D only through government purchases, and not through other sector-specific channels correlated with sectors’ defense exposure, such as differential demand trends, technology shocks, or political influence.⁸

We also control for other forms of cross-sectoral propagation, such as the propagation via the production network emphasized by [Acemoglu et al. \(2016a\)](#) and [Bouakez et al. \(2023, 2025\)](#). Since the two networks are essentially uncorrelated, the two channels are separately identified. Appendix A estimates the own and upstream-innovation coefficients are estimated above and beyond alternative propagation channels, such as the downstream innovation network, and the downstream and upstream input-output networks.

2.3 Results

Table 2 reports the baseline IV local-projection estimates of the own treatment and of the upstream-innovation exposure at horizons $h = 0, 1, 2$, with p -values from standard errors two-way clustered at the sector and year levels in brackets.⁹

The own effect builds up progressively, reaching 0.992 at the two-year horizon, where it is significant at the 5% level. This result confirms previous evidence on the positive response of private R&D to government spending ([Lichtenberg, 1988](#); [Slavtchev and Wiederhold, 2016](#); [Cioaca, 2025](#); [Belenzon and Cioaca, 2026](#)).

The novel result is the upstream propagation of public spending through the innovation network: R&D rises not only in the sector that receives the spending,

⁸Appendix A.2 supports the instrumenting strategy with a series of placebo tests.

⁹Appendix A.4 considers alternative inference: clustering by year only—the shock-level convention that [Borusyak et al. \(2022\)](#) recommend for shift-share designs with a single shifter; two-way clustering by defense-exposure bin and year, implementing the exposure-correlation logic of [Adão et al. \(2019\)](#); and [Driscoll and Kraay \(1998\)](#) standard errors, which allow for arbitrary cross-sectional correlation.

Table 2: Own Effect and the Upstream Innovation Channel

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
Own effect (β)	0.213 [0.444]	0.663* [0.094]	0.992** [0.022]
Upstream innovation (γ)	0.167 [0.199]	0.266* [0.075]	0.280** [0.031]
Sector & Year FE	✓	✓	✓
Observations	1,149	1,087	1,031
Kleibergen–Paap F	34.7	43.7	37.0

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. The dependent variable is the h -year-ahead change in real private R&D. All regressions include sector and year fixed effects, and three lags of sectoral R&D, the sectoral own and innovation-network treatments, sectoral value-added growth. p -values from two-way clustered standard errors in brackets. * and **: significance at the 10% and 5% levels.

but also in the sectors that build on its ideas. The upstream innovation coefficient displays the same build-up as the own effect—it is significant at the 10% level after one year and at the 5% level at the two-year horizon, where it stabilizes at 0.280. Thus, it amounts to roughly 30% of the magnitude of the own effect. Converted into cumulative multipliers, each dollar of government purchases raises the recipient sector’s annual R&D by about 25 cents at the two-year horizon,¹⁰ and each dollar received by a sector’s knowledge suppliers raises its R&D by about 7 cents.

Appendix A reports the first-stage estimates and shows an extensive battery of robustness checks: replacing the upstream innovation with the downstream innovation or the downstream and upstream input-output treatments, none of which delivers any positive propagation of R&D; controlling for the production-network exposure, in either direction; controlling for heterogeneous exposure to monetary policy; alternative definitions of the control set; and measuring the innovation network over an earlier time window. The own and the upstream innovation effects always remain positive, of similar magnitude, and statistically significant.

All in all, these results provide causal evidence that government spending propagates across sectors through the upstream innovation network, from the sectors that receive the spending to the sectors that build on their ideas.

¹⁰The own effect is roughly half of the firm-level estimates in [Lichtenberg \(1988\)](#), who finds that each dollar of competitive federal procurement raises company-funded R&D by 54 cents, and [Belenzon and Cioaca \(2026\)](#), who estimate a 47-cent response to each dollar of government R&D contracts.

3 The Model

To quantify the propagation of fiscal policy through the innovation network, we build a multi-sector New Keynesian model that captures both the short-run transmission of fiscal policy and the medium-run propagation of knowledge spillovers.

The production side takes the form of a production network connecting S industries, denoted by $s \in 1, \dots, S$ and heterogeneous in demand composition, factor intensities, intermediate-input use, and price rigidity. Crucially, each sector also operates R&D activities that accumulate a sector-specific stock of knowledge, whose productivity is modulated by the knowledge of the sectors it cites, as specified by the innovation network. Finally, the fiscal authority uses lump-sum taxes to finance exogenous streams of sectoral government purchases.

We present the key parts of the model below; additional details and derivations are in [Appendix B](#).

3.1 Household

The economy is populated by a representative household. Labor markets are *segmented* at the sector and occupation level: the household supplies production labor $N_{P,s,t}$ and research labor $N_{R,s,t}$ to each sector s . The representative household chooses consumption C_t , the sectoral labor supplies, investment I_t , capital K_{t+1} , and one-period risk-free nominal bonds B_t to maximize lifetime utility:

$$\max_{C_t, \{N_{P,s,t}\}_{s=1}^S, \{N_{R,s,t}\}_{s=1}^S, I_t, K_{t+1}, B_t} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_t - \sum_{s=1}^S \theta_{P,s} \frac{N_{P,s,t}^{1+\eta_P}}{1+\eta_P} - \sum_{s=1}^S \theta_{R,s} \frac{N_{R,s,t}^{1+\eta_R}}{1+\eta_R} \right] \quad (5)$$

s.t. $P_t C_t + P_{I,t} I_t + B_t + T_t = \dots$

$$\dots = \sum_{s=1}^S W_{P,s,t} N_{P,s,t} + \sum_{s=1}^S W_{R,s,t} N_{R,s,t} + R_{K,t} K_t + R_{t-1} B_{t-1} + D_t, \quad (6)$$

where β is the discount factor, $\theta_{P,s}$ and $\theta_{R,s}$ are labor disutility shifts, and η_P and η_R denote the inverse of the Frisch elasticity for production and research labor.

The budget constraint in Equation (6) posits that households purchase consumption goods at price P_t and investment goods at price $P_{I,t}$, invest in nominal bonds, and pay a nominal lump-sum tax T_t . Households finance expenditures with the proceeds of nominal bonds, whose interest rate is R_t , nominal profits rebated from firms, D_t , sectoral production and research labor income at the nominal wages $W_{P,s,t}$ and $W_{R,s,t}$, and capital income at the nominal return $R_{K,t}$.

Physical capital depreciates at rate δ_K and evolves with investment as:

$$K_{t+1} = (1 - \delta_K) K_t + I_t. \quad (7)$$

Capital services reallocate across sectors, albeit imperfectly. Aggregate capital services are a CES basket of sectoral capital flows $K_{s,t}$:

$$K_t = \left[\sum_{s=1}^S \omega_{K,s}^{-\frac{1}{\nu_K}} K_{s,t}^{\frac{\nu_K+1}{\nu_K}} \right]^{\frac{\nu_K}{\nu_K+1}}, \quad (8)$$

where $\omega_{K,s}$ is the weight of sector s , and ν_K captures the elasticity of substitution of capital services across industries. When $\nu_K \rightarrow \infty$ capital is perfectly mobile and sectoral returns are equalized; for finite ν_K , return differentials emerge.

3.2 Firms

Each sector hosts a continuum of monopolistically competitive producers, which supply differentiated varieties subject to Rotemberg pricing frictions and undertake R&D; a representative wholesaler, which aggregates the varieties into a sectoral good; and retailers, which assemble the sectoral goods into the bundles demanded by households for consumption and investment, and by producers as intermediate inputs; the fiscal authority instead purchases sectoral goods directly.

3.2.1 Producers

Within each sector, a continuum of monopolistically competitive producers indexed by $i \in [0, 1]$ produces differentiated varieties of the sectoral good. Each producer combines production inputs and a producer-specific knowledge stock to generate gross output with the Cobb-Douglas technology:

$$Z_{s,t}^i = A_{s,t}^i (N_{P,s,t}^i)^{\alpha_{N,s}} (K_{s,t}^i)^{\alpha_{K,s}} (H_{s,t}^i)^{\alpha_{H,s}}, \quad (9)$$

where $Z_{s,t}^i$ is gross output of producer i in sector s at time t , $A_{s,t}^i$ is the producer's knowledge stock, and $N_{P,s,t}^i$, $K_{s,t}^i$, and $H_{s,t}^i$ denote production labor, capital, and intermediate inputs, respectively. The sector-specific parameters $\alpha_{N,s}$, $\alpha_{K,s}$, $\alpha_{H,s}$ denote the intensities of production labor, capital, and intermediates in gross output. We impose that $\alpha_{N,s} + \alpha_{K,s} + \alpha_{H,s} = 1$.

In addition to production, each producer undertakes R&D activity that accumulates its knowledge stock. Producer i hires research labor, $N_{R,s,t}^i$, at the sectoral wage $W_{R,s,t}$, and new knowledge is generated according to:

$$A_{s,t+1}^i = (1 - \delta_A) A_{s,t}^i + \chi N_{R,s,t}^i \lambda_s \prod_{x=1}^S A_{x,t}^{\omega_{A,s,x}} \left[\frac{A_{s,t}^i}{\prod_{x=1}^S A_{x,t}^{\omega_{A,s,x}}} \right]^\kappa, \quad (10)$$

where $A_{x,t} \equiv \int_0^1 A_{x,t}^i di$ denotes the sectoral aggregate knowledge stock in sector x .

Knowledge evolves as a function of four components. The first is the undepreciated part of the existing stock, $(1 - \delta_A) A_{s,t}^i$, where δ_A is the rate of knowledge

obsolescence. The second, $\chi N_{R,s,t}^{i \lambda_s}$, is the contribution of current R&D activity: χ pins down R&D efficiency, and λ_s is the sector-specific elasticity of new knowledge with respect to research labor. The third is the spillover bundle $\prod_{x=1}^S A_{x,t}^{\omega_{A,s,x}}$, which makes the productivity of sector s 's R&D depend on the knowledge stocks of all sectors. The weights $\omega_{A,s,x}$, with $\sum_{x=1}^S \omega_{A,s,x} = 1$ for every s , are the entries of the *innovation network*, which we collect in the matrix $\Omega_A \equiv [\omega_{A,s,x}]$. The fourth component, $\left[\frac{A_{s,t}^i}{\prod_{x=1}^S A_{x,t}^{\omega_{A,s,x}}} \right]^\kappa$, captures the *adoption* of external knowledge: producers benefit from the knowledge of other sectors only insofar as their own stock gives them the capability to absorb it, at a rate governed by $\kappa \in [0, 1)$.

Equation (10) builds on the specification of [Romer \(1990\)](#) and [Jones \(1995\)](#), in which research labor produces R&D against a stock-of-knowledge externality: producers are atomistic with respect to the spillover bundle and take it as given—the “standing-on-the-shoulders-of-giants” mechanism, whereby they benefit from past knowledge accumulation without internalizing their contribution to it.

Producers purchase production labor, research labor, and capital services from households at prices $W_{P,s,t}$, $W_{R,s,t}$, and $R_{K,s,t}$, and intermediate inputs from retailers at price $P_{H,s,t}$. Each producer sets its nominal variety price $P_{s,t}^i$ subject to a quadratic price-adjustment friction à la [Rotemberg \(1982\)](#): changes in $P_{s,t}^i$ are penalized by a real resource cost proportional to the sector's revenue, with the sector-specific parameter ϑ_s governing the intensity of nominal rigidities.

Specifically, given the household's nominal stochastic discount factor $\Lambda_{t,t+1}$, producer i in sector s chooses sequences of production inputs, research labor, and prices to maximize the expected present value of profits:

$$\begin{aligned} \max_{\{N_{P,s,t}^i, N_{R,s,t}^i, K_{s,t}^i, H_{s,t}^i, P_{s,t}^i\}} \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ P_{s,\tau}^i Z_{s,\tau}^i - W_{P,s,\tau} N_{P,s,\tau}^i - R_{K,s,\tau} K_{s,\tau}^i - P_{H,s,\tau} H_{s,\tau}^i - \dots \right. \\ \left. \dots - W_{R,s,\tau} N_{R,s,\tau}^i - \frac{\vartheta_s}{2} \left(\frac{P_{s,\tau}^i}{P_{s,\tau-1}^i} - 1 \right)^2 P_{s,\tau} Z_{s,\tau}^i \right\}, \quad (11) \end{aligned}$$

subject to the technology (9), the knowledge law of motion (10), and the variety-level demand schedule stemming from wholesalers' problem in the next section.

Henceforth we focus on the symmetric equilibrium, in which producers in a sector make identical choices, and drop the variety index i when not necessary.

The first-order condition for research labor equates its marginal cost to the discounted shadow value of the knowledge it will deliver next period:

$$W_{R,s,t} = \lambda_s \chi \mathbb{E}_t [\Lambda_{t,t+1} V_{s,t+1}] N_{R,s,t}^{i \lambda_s - 1} \prod_{x=1}^S A_{x,t}^{\omega_{A,s,x}} \left[\frac{A_{s,t}^i}{\prod_{x=1}^S A_{x,t}^{\omega_{A,s,x}}} \right]^\kappa, \quad (12)$$

where $V_{s,t+1}$ denotes the shadow value of an additional unit of producer's knowledge stock. The shadow value itself obeys the asset-value recursion:

$$V_{s,t} = \Theta_{s,t} + \left[(1 - \delta_A) + \kappa \frac{X_{s,t}}{A_{s,t}} \right] \mathbb{E}_t[\Lambda_{t,t+1} V_{s,t+1}], \quad (13)$$

where $X_{s,t} \equiv \chi N_{R,s,t}^{\lambda_s} \prod_x A_{x,t}^{\omega_{A,s,x}} [A_{s,t} / \prod_x A_{x,t}^{\omega_{A,s,x}}]^\kappa$ denotes the flow of new ideas, and $\Theta_{s,t}$ is the marginal flow of profits—in terms of cost savings—accruing to a unit of the producer's knowledge at time t :

$$\Theta_{s,t} = MC_{s,t} \frac{Z_{s,t}}{A_{s,t}}. \quad (14)$$

where $MC_{s,t}$ denotes the producer's nominal marginal cost of production. Since knowledge raises output one-for-one, an additional unit of knowledge saves production costs in proportion to the sector's gross output valued at marginal cost, spread over the existing stock. The term $\kappa X_{s,t}/A_{s,t}$ governs adoption: an extra unit of knowledge today raises the productivity of the producer's future R&D.

3.2.2 Wholesalers

In each sector, a perfectly competitive representative wholesaler purchases the varieties supplied by producers, $Z_{s,t}^i$, and bundles them into the sectoral good $Z_{s,t}$ with the CES technology:

$$Z_{s,t} = \left[\int_0^1 Z_{s,t}^i \frac{\epsilon-1}{\epsilon} di \right]^{\frac{\epsilon}{\epsilon-1}}, \quad (15)$$

where $\epsilon > 1$ is the elasticity of substitution across varieties within a sector.

The goods produced by wholesalers are destined for consumption, investment, government consumption, and intermediate-input use. The sectoral resource constraint, net of the real cost of price adjustment, reads:

$$Z_{s,t} \left[1 - \frac{\vartheta_s}{2} (\pi_{s,t} - 1)^2 \right] = C_{s,t} + I_{s,t} + G_{s,t} + \sum_{x=1}^S H_{s,x,t}, \quad (16)$$

where $P_{s,t}$ is the sectoral price, $\pi_{s,t} \equiv P_{s,t}/P_{s,t-1}$ is gross sectoral inflation, $C_{s,t}$ and $I_{s,t}$ denote the demand of sector s goods from consumption and investment retailers, $G_{s,t}$ is the demand from the fiscal authority, and $H_{s,x,t}$ corresponds to the demand of sector s goods from the retailer that assembles the intermediate inputs used by producers in sector x .

3.3 Retailers

A perfectly competitive representative consumption retailer bundles the sectoral consumption flows, $C_{s,t}$, into the consumption good, C_t , with the technology:

$$C_t = \prod_{s=1}^S C_{s,t}^{\omega_{C,s}}, \quad (17)$$

where $\omega_{C,s}$ is the share of private consumption expenditures on the consumption goods of sector s , such that $\sum_{s=1}^S \omega_{C,s} = 1$.

Analogously, the investment retailer assembles the sectoral investment flows, $I_{s,t}$, into the final investment good, I_t , with the technology:

$$I_t = \prod_{s=1}^S I_{s,t}^{\omega_{I,s}}, \quad (18)$$

with investment expenditure shares $\omega_{I,s}$ summing to one.

Intermediate inputs are assembled in the same fashion, the only difference being that there is one retailer for each sector. In each sector s , a perfectly competitive representative retailer assembles sectoral goods, $H_{x,s,t}$, into the bundle of intermediate inputs used by sector s producers, $H_{s,t}$, with the technology:

$$H_{s,t} = \prod_{x=1}^S H_{x,s,t}^{\omega_{H,s,x}}, \quad (19)$$

where $\omega_{H,s,x}$, with $\sum_{x=1}^S \omega_{H,s,x} = 1$ for every s , is the contribution of sector x to the intermediate-input bundle used by producers in sector s ; these weights form the entries of the Input-Output matrix.

3.4 Government

The government consists of a monetary authority and a fiscal authority. The former sets the nominal interest rate R_t with a standard Taylor rule that reacts to aggregate CPI inflation, $\pi_t = P_t/P_{t-1}$, and the output gap, Y_t/Y_t^f , defined as the ratio between aggregate real GDP, Y_t , and that in the flexible-price economy, Y_t^f :

$$\frac{R_t}{R^*} = \left(\frac{R_{t-1}}{R^*} \right)^{\phi_r} \left[\pi_t^{\phi_\pi} \left(\frac{Y_t}{Y_t^f} \right)^{\phi_x} \right]^{1-\phi_r}, \quad (20)$$

where R^* is the level of the nominal interest rate along the balanced growth path (BGP),¹¹ and ϕ_r , ϕ_π , and ϕ_x govern the degree of interest-rate inertia and the responsiveness to inflation and the output gap, respectively.

The fiscal authority purchases the sectoral goods, $G_{s,t}$, from wholesalers. This flow is determined by a sequence of autoregressive processes:

¹¹Throughout, we refer to X_t^* as the value of variable X_t along the BGP.

$$\log \left(\frac{G_{s,t}}{G_{s,t}^*} \right) = \rho \log \left(\frac{G_{s,t-1}}{G_{s,t-1}^*} \right) + \varepsilon_{s,t}, \quad (21)$$

where ρ is the persistence of the AR(1), and $\varepsilon_{s,t}$ is a sector-specific shock.¹²

The fiscal authority finances public purchases with lump-sum taxes, T_t , so that the public budget constraint reads:

$$\sum_{s=1}^S P_{s,t} G_{s,t} = T_t. \quad (22)$$

3.5 Closing the model

Sectoral nominal value added, $\mathcal{Y}_{s,t}$, is the value of gross output net of intermediate-input purchases,

$$\mathcal{Y}_{s,t} = P_{s,t} Z_{s,t} - P_{H,s,t} H_{s,t}. \quad (23)$$

Summing across sectors and using market clearing delivers nominal GDP, $\mathcal{Y}_t$:

$$\mathcal{Y}_t = P_t C_t + P_{I,t} I_t + \sum_s P_{s,t} G_{s,t}. \quad (24)$$

GDP is defined as the sum of consumption, investment, and public spending valued at their BGP prices (relative to the BGP consumption price):

$$Y_t = C_t + \frac{P_{I,t}^*}{P_t^*} I_t + \sum_{s=1}^S \frac{P_{s,t}^*}{P_t^*} G_{s,t}. \quad (25)$$

We report in Appendix B.5 the market clearing conditions and the aggregation of sectoral quantities into aggregate GDP. Importantly, the model features an exact balanced growth path, sustained by the endogenous accumulation of knowledge. Appendix B.6 derives the balanced-growth conditions in detail.

4 Analytical Results

We characterize analytically the sectoral-spending multiplier—the response of aggregate GDP to a sectoral government spending shock—and the idea multiplier, its innovation-network component. We do so in a stripped-down economy in which the innovation network is the *only* source of heterogeneity: sectors differ solely in where they sit in the network of knowledge flows. Section 4.1 presents the framework and its mapping into the full model; Sections 4.2–4.5 build from

¹²Sectoral government spending evolves around its BGP value $G_{s,t}^*$. We define the latter implicitly as $P_{s,t}^* G_{s,t}^* = \omega_{G,s} \varphi_G \mathcal{Y}_t^*$, where $\mathcal{Y}_t^*$ is nominal aggregate GDP at BGP, φ_G is the share of aggregate nominal government spending as a fraction of nominal GDP at BGP, and $\omega_{G,s}$ captures the fraction of nominal government spending in sector s over total government spending, such that $\sum_{s=1}^S \omega_{G,s} = 1$.

the log-linearized equilibrium up to the sectoral-spending multiplier; Section 4.6 derives the idea multiplier.

Throughout, we state the propositions in matrix form for an arbitrary number of sectors S . To make the economic mechanism transparent, each result is accompanied by an explicit two-sector example. All proofs are relegated to Appendix C.

To fix notation, throughout the analytical block, hats denote log-deviations from the deterministic steady state, and bold letters collect the corresponding sectoral vectors. Thus, for any sectoral variable $X_{s,t}$, we write $\hat{X}_{s,t} \equiv \log(X_{s,t}/\bar{X}_s)$ and $\hat{\mathbf{X}}_t \equiv (\hat{X}_{1,t}, \dots, \hat{X}_{S,t})'$.

4.1 Tractable Framework

The analytical economy is obtained from the full quantitative model by imposing the following restrictions:

- (i) *No physical capital and no intermediate inputs*: $\alpha_{N,s} = 1$, $\alpha_{K,s} = 0$, $\alpha_{H,s} = 0$ for every s , so that output is assembled only with labor and knowledge: $Z_{s,t}^i = A_{s,t}^i N_{P,s,t}^i$. Thus, the production-network matrix plays no role.
- (ii) *Indivisible labor*: $U(C_t, N_{P,t}, N_{R,t}) = \log C_t - \theta_P N_{P,t} - \theta_R N_{R,t}$. As in Hansen (1985) and Rogerson (1988), this delivers a perfectly elastic labor supply, so that—normalizing $\theta_P = \theta_R$ —wages are equalized across sectors and occupations in equilibrium, $W_{P,s,t} = W_{R,s,t} = W_t$.
- (iii) *Flexible prices*: $\vartheta_s = 0$ for every s , so producers price at a constant flexible markup $\mu = \epsilon/(\epsilon - 1)$ over marginal cost.
- (iv) *Homogeneous final-demand shares*: $\omega_{C,s} = \omega_{G,s} = 1/S$ for every s , so that all sectors contribute equally to private consumption and government spending.
- (v) *Homogeneous R&D elasticity*: $\lambda_s = \lambda$ for every s , so that the elasticity of new knowledge with respect to research labor is symmetric across sectors.
- (vi) *i.i.d. government spending shocks*: $\rho = 0$, so that the date-0 shock vector $\hat{\mathbf{G}}_0 = \hat{G}_{s,0} \mathbf{E}_s$ targets a single sector s and returns to steady state thereafter, with $\mathbf{E}_s$ denoting the s -th column of the identity matrix.
- (vii) *Contemporaneous R&D implementation*:

$$A_{s,t}^i = (1 - \delta_A) A_{s,t-1}^i + \chi N_{R,s,t}^\lambda \left[\prod_{x=1}^S A_{x,t-1}^{\omega_{A,s,x}} \right]^\zeta (A_{s,t-1}^i)^\kappa,$$

so R&D raises the knowledge stock available within the same period.¹³

¹³This timing departs from the law of motion of Equation (10) because the i.i.d. shock would leave the forward-looking shadow value $V_{s,t+1}$ at its steady-state value, generating no R&D response.

- (viii) *Elasticity to knowledge spillovers*: the spillover elasticity is $\zeta \in (0, 1 - \lambda - \kappa)$, so that $\lambda + \zeta + \kappa < 1$: the semi-endogenous restriction of [Jones \(1995\)](#), which delivers a stationary steady state.

In deriving the equilibrium, we take each period's consumption basket as the numeraire, deflating all date- t nominal variables by the date- t consumption price index: $q_{s,t} \equiv P_{s,t}/P_{C,t}$ denotes the relative price of the sector- s good, $w_t \equiv W_t/P_{C,t}$ the real wage, and $v_{s,t} \equiv V_{s,t}/P_{C,t}$ the real value of knowledge.

Despite the heterogeneity of the innovation network, the deterministic steady state of the analytical economy is sector-symmetric:¹⁴

$$\bar{A}_s = \bar{A}, \quad \bar{N}_{R,s} = \bar{N}_R, \quad \bar{Z}_s = \bar{Z}, \quad \bar{q}_s = \bar{q}, \quad \bar{v}_s = \bar{v} \quad \text{for every } s. \quad (26)$$

The homogeneity of the steady state bears two implications: (i) the [Hulten \(1978\)](#)-[Domar \(1961\)](#) sectoral sales weights equal $\mu_s = 1/S$, and (ii) the share of government spending in each sector's revenue (i.e., the sector's contribution to total government spending divided by its contribution to GDP) collapses to a sector-invariant constant ξ^G , with private-consumption complement $\xi^C \equiv 1 - \xi^G$.

4.2 Log-Linearized Equilibrium

Two static identities follow directly from the simplifications. First, indivisible labor makes aggregate real consumption move one-for-one with the real wage:

$$\hat{C}_t = \hat{w}_t. \quad (27)$$

Second, constant markup over labor-only marginal cost makes each sectoral price the ratio of wage to sectoral productivity:

$$\hat{q}_{s,t} = \hat{w}_t - \hat{A}_{s,t}. \quad (28)$$

A higher wage raises the sectoral price; a higher productivity lowers it.

Sectoral consumption. Without investment and intermediates, the sectoral resource constraint reduces to $Y_{s,t} = Z_{s,t} = C_{s,t} + G_{s,t}$. Throughout, we treat $\hat{G}_{s,t}$ as exogenous. Sectoral consumption moves one-for-one with sectoral knowledge:

$$\hat{C}_{s,t} = \hat{A}_{s,t}. \quad (29)$$

Cobb-Douglas demand with homogeneous shares allocates spending equally across sectors, i.e. $\hat{C}_{s,t} = \hat{C}_t - \hat{q}_{s,t}$. Substituting using Equations (27)-(28) yields (29).

Real wage. The Cobb-Douglas consumption price index implies $\prod_s q_{s,t}^{1/S} = 1/S$

¹⁴While heterogeneity in the innovation network would in principle break sector symmetry of the steady state, Appendix C.1 shows that the contribution of the knowledge spillover at steady state cancels exactly between the R&D first-order condition and the knowledge law of motion.

at all times, so that in log deviations $\frac{1}{S} \sum_s \hat{q}_{s,t} = 0$. Substituting (28) and rearranging makes the real wage equal to the average sectoral knowledge:

$$\hat{w}_t = \frac{1}{S} \sum_{s=1}^S \hat{A}_{s,t} = \frac{1}{S} \mathbf{1}^\top \hat{\mathbf{A}}_t. \quad (30)$$

Sectoral real value added. Real sectoral value added is the share-weighted average of real components, $\hat{Y}_{s,t} = \xi^C \hat{C}_{s,t} + \xi^G \hat{G}_{s,t}$. Substituting Equation (29) gives

$$\hat{Y}_{s,t} = \xi^C \hat{A}_{s,t} + \xi^G \hat{G}_{s,t}. \quad (31)$$

Sectoral GDP increases with sectoral knowledge and sectoral public consumption.

Shadow value of knowledge. The wage-deflated shadow value of sectoral knowledge, $\tilde{v}_{s,t} \equiv \hat{v}_{s,t} - \hat{w}_t$, drives sectoral R&D incentives. Log-linearizing the asset-value recursion of Equation (13) around the symmetric steady state gives

$$\begin{aligned} \tilde{v}_{s,t} = & (1 - \tilde{\beta}_\kappa) \xi^G \hat{G}_{s,t} - (1 - \tilde{\beta}_\kappa)(1 + \xi^G) \hat{A}_{s,t} + \dots \\ & \dots + \beta \kappa (\hat{A}_{s,t} - \hat{A}_{s,t-1}) + \tilde{\beta}_\kappa \mathbb{E}_t \tilde{v}_{s,t+1}, \end{aligned} \quad (32)$$

where $\tilde{\beta}_\kappa \equiv \beta(1 - \delta_A(1 - \kappa))$ is the obsolescence- and adoption-adjusted discount factor, which reduces to $\tilde{\beta} \equiv \beta(1 - \delta_A)$ in the pure-network benchmark $\kappa = 0$.

The shadow value of knowledge depends on four components. The first, $(1 - \tilde{\beta}_\kappa) \xi^G \hat{G}_{s,t}$, is positive and captures *market size*: a public-spending shock raises the volume of output the firm sells, so the cost saving each unit of knowledge generates applies to a larger base. The second, $-(1 - \tilde{\beta}_\kappa)(1 + \xi^G) \hat{A}_{s,t}$, is negative and captures *saturation*: a larger existing stock spreads the same sectoral revenue across more units of knowledge, eroding the marginal value of new ones. The third, the *momentum* term $\beta \kappa (\hat{A}_{s,t} - \hat{A}_{s,t-1})$, is positive and arises from internalized adoption: the value of knowledge rises when the sector's stock is growing, because fresh knowledge feeds the producer's own future idea production. The fourth is the continuation value, discounted at $\tilde{\beta}_\kappa$.

Adoption offsets saturation on two margins. It makes knowledge a longer-lived asset ($\tilde{\beta}_\kappa > \tilde{\beta}$), shrinking the weight of current flows in the asset value; and the momentum term enters with the opposite sign of saturation. Equating the momentum loading $\beta \kappa$ to the saturation loading $(1 - \tilde{\beta}_\kappa)(1 + \xi^G)$ delivers an adoption level at which the two forces offset exactly, which we now impose.

Assumption 1 (Saturation-neutral adoption). *The adoption elasticity equals*

$$\kappa^* \equiv \frac{(1 - \beta + \beta \delta_A)(1 + \xi^G)}{\beta[1 + \delta_A(1 + \xi^G)]},$$

at which adoption offsets the saturation channel in the value of knowledge (32).

Under Assumption 1, the net loading of the shadow value on the *current* own stock is zero. Although a residual term in the *lagged* own stock survives, we abstract from it from now on: Appendix C.3 works out the full derivations retaining the term, and shows that our analytical result also go through in that case.

Sectoral research labor. R&D firms hire research labor up to the point where its marginal cost—the real wage—equals its marginal benefit, the marginal product of research labor times the asset value of new knowledge. Log-linearizing and substituting $\hat{v}_{s,t} = \tilde{v}_{s,t} + \hat{w}_t$, the wage on the marginal-cost side cancels against the wage embedded in the asset value, leaving the wage-deflated shadow value and the cross-sectoral knowledge bundle as the drivers of research labor:

$$\hat{N}_{R,s,t} = \frac{1}{1-\lambda} \tilde{v}_{s,t} + M \sum_{x=1}^S \omega_{A,s,x}^{\kappa} \hat{A}_{x,t-1}, \quad \omega_{A,s,x}^{\kappa} \equiv (1-\hat{\kappa}) \omega_{A,s,x} + \hat{\kappa} [I]_{s,x}, \quad (33)$$

where $[I]_{s,x}$ denotes the (s, x) entry of the identity matrix, equal to one if $s = x$ and zero otherwise, $M \equiv (\zeta + \kappa)/(1 - \lambda)$ is the total knowledge-feedback discount, and the mixing weight $\hat{\kappa} \equiv \kappa/(\zeta + \kappa)$ is the share of the internalized own stock in total knowledge feedback. Thus, research labor responds to the shadow value of sectoral knowledge and to the adoption-adjusted knowledge bundle: citation-network spillovers from the sectors that s builds on, with weight $1 - \hat{\kappa}$, plus the internalized own stock, with weight $\hat{\kappa}$.

Knowledge law of motion. Substituting the R&D first-order condition of Equation (33) into the knowledge-accumulation equation of simplification (viii) delivers a first-order vector recursion for the sectoral knowledge vector:

$$\hat{\mathbf{A}}_t = \mathbf{T} \hat{\mathbf{A}}_{t-1} + \varpi \tilde{\mathbf{V}}_t. \quad (34)$$

The matrix $\mathbf{T} \equiv (1 - \delta_A)I + \delta_A M \Omega_A^{\kappa}$ is the discounted knowledge-propagation matrix: sectoral knowledge depreciates at rate δ_A and is replenished by R&D, with one-step feedback weighted by $M \Omega_A^{\kappa}$ —citation spillovers carrying weight $1 - \hat{\kappa}$ and internalized own-stock adoption adding $\hat{\kappa}$ to the diagonal. The forcing vector $\tilde{\mathbf{V}}_t \equiv (\tilde{v}_{1,t}, \dots, \tilde{v}_{S,t})^\top$ collects the sectoral wage-deflated shadow values, and the coefficient $\varpi \equiv \frac{\delta_A \lambda}{1 - \lambda}$ is the *value-to-knowledge pass-through*: a 1% rise in $\tilde{v}_{s,t}$ raises sectoral knowledge $\hat{A}_{s,t}$ by ϖ percentage points before network propagation.

Closing the model. Under Assumption 1, and abstracting from the residual lagged term, Equation (32) reduces to

$$\tilde{v}_{s,t} = (1 - \tilde{\beta}_{\kappa}) \xi^G \hat{G}_{s,t} + \tilde{\beta}_{\kappa} \mathbb{E}_t \tilde{v}_{s,t+1} : \quad (35)$$

the value of knowledge responds only to the sector's fiscal shock and to its own

expected continuation. Appendix C.2 shows that the propagation of knowledge across sectors is governed by the Leontief inverse of the innovation network:

$$\mathcal{L}_A \equiv (I - M\Omega_A^\kappa)^{-1} = \sum_{k \geq 0} M^k (\Omega_A^\kappa)^k. \quad (36)$$

The entry $[\mathcal{L}_A]_{s,x}$ measures the cumulative reliance of sector s on the knowledge produced in sector x : a one-percent rise in A_x ultimately lifts the productivity of R&D in s by $[\mathcal{L}_A]_{s,x}$ percent, summing direct citations and indirect chains of any length. Adoption enters only through the adjusted network Ω_A^κ , which combines the citation links, with weight $1 - \hat{\kappa}$, and the internalized own-stock loop, with weight $\hat{\kappa}$; in the product $M\Omega_A^\kappa$ the citation links retain their total loading $\zeta/(1 - \lambda)$, so cross-sector propagation operates at full strength.

We summarize sector x 's position in the innovation network with its *upstreamness*,

$$\mathcal{U}_x \equiv \mathbf{1}^\top \mathcal{L}_A \mathbf{E}_x = \sum_{s=1}^S [\mathcal{L}_A]_{s,x}, \quad (37)$$

the column sum of $\mathcal{L}_A$: the total weight with which the knowledge produced in sector x reaches the rest of the economy, adding up direct and indirect citations. A sector with a high $\mathcal{U}_x$ is upstream in the innovation network: many other sectors build on its knowledge, either directly or through chains of indirect citations.

The cumulative knowledge response to a sectoral public-spending shock is

$$\sum_{t=0}^{\infty} \hat{\mathbf{A}}_t = \lambda \Gamma \mathcal{L}_A \hat{\mathbf{G}}_0, \quad (38)$$

where $\Gamma \equiv \frac{(1-\tilde{\beta}_\kappa)\xi^G}{1-\lambda}$ is the *impact pass-through*: a one-percent real public-spending shock at sector s raises its research labor by Γ percent on impact.¹⁵

4.3 The Response of Sectoral R&D

This section characterizes the cumulative response of R&D expenditure—defined as the research wage bill in consumption units, $\mathcal{N}_{R,s,t} \equiv w_t N_{R,s,t}$ —to a government spending shock. We decompose the response into three transparent components: a direct *own effect*, present only in the shocked sector, generated by the spending it receives; an indirect *innovation-network effect*, which travels along the citation matrix so that a sector's R&D reacts to public spending in any other sector it cites; and a *common wage-bill effect* that affects all sectors

¹⁵The impact pass-through coefficient combines the market-size channel loading $(1 - \tilde{\beta}_\kappa)\xi^G$ with the R&D-leverage scale $1/(1 - \lambda)$ from the first-order condition in Equation (33).

uniformly. Intuitively, knowledge expansions raise the real wage that all R&D firms in any industry must pay, inflating the research wage bill.

A two-sector example. To build intuition before stating the proposition, consider a stylized two-sector economy, which we use as a running example throughout, in which sector 1 is fully upstream and sector 2 fully downstream:

$$\omega_{A,1,1} = 1, \quad \omega_{A,1,2} = 0, \quad \omega_{A,2,1} = 1, \quad \omega_{A,2,2} = 0. \quad (39)$$

Both sectors cite sector 1, while no sector cites sector 2.¹⁶ The Leontief inverse of the innovation network and its column sums are then

$$\mathcal{L}_A = \begin{pmatrix} \frac{1}{1-M} & 0 \\ \frac{M}{1-M} & 1 \end{pmatrix}, \quad \mathcal{U}_1 = \frac{1+M}{1-M}, \quad \mathcal{U}_2 = 1. \quad (40)$$

Consider a shock $\hat{G}_{1,0}$ in the upstream sector 1. Using Equation (38), we can pin down the sectoral knowledge responses as

$$\sum_t \hat{A}_{1,t} = \frac{\lambda\Gamma}{1-M} \hat{G}_{1,0}, \quad \sum_t \hat{A}_{2,t} = \frac{\lambda\Gamma M}{1-M} \hat{G}_{1,0}. \quad (41)$$

Although the shock hits sector 1, knowledge expands in both industries: sector 2 benefits indirectly because its R&D builds on sector 1's ideas.

Combining the wage response implied by Equation (30), $\sum_t \hat{w}_t = \frac{\lambda\Gamma}{2} \frac{1+M}{1-M} \hat{G}_{1,0}$, with the cumulated first-order condition (33), in which $\sum_t \tilde{v}_{1,t} = (1 - \tilde{\beta})\xi^G \hat{G}_{1,0}$ and $\sum_t \tilde{v}_{2,t} = 0$, delivers the sectoral R&D response:

$$\begin{aligned} \sum_{t=0}^{\infty} \hat{\mathcal{N}}_{R,1,t} &= \underbrace{\Gamma \hat{G}_{1,0}}_{\text{own effect}} + \underbrace{\frac{\lambda\Gamma M}{1-M} \hat{G}_{1,0}}_{\text{innovation-network effect}} + \underbrace{\frac{\lambda\Gamma}{2} \frac{1+M}{1-M} \hat{G}_{1,0}}_{\text{wage-bill effect}}, \\ \sum_{t=0}^{\infty} \hat{\mathcal{N}}_{R,2,t} &= \underbrace{0}_{\text{own effect}} + \underbrace{\frac{\lambda\Gamma M}{1-M} \hat{G}_{1,0}}_{\text{innovation-network effect}} + \underbrace{\frac{\lambda\Gamma}{2} \frac{1+M}{1-M} \hat{G}_{1,0}}_{\text{wage-bill effect}}. \end{aligned}$$

Thus, a shock to the upstream sector increases R&D expenditures in the downstream sector via the innovation-network and the wage-bill effects. While the wage-bill effect would always apply symmetrically across industries insofar as research wages are equalized across sectors, the fact that the innovation-network channel is also symmetric is an artifact of the extreme shape of the citation matrix, in which a single sector is the sole knowledge supplier for both industries. With a more general (and empirically relevant) citation matrix, the innovation-network effect would also be heterogeneous.

¹⁶To keep the formulas transparent, we display the example for $\kappa = 0$ and disregard the saturation term.

By contrast, a downstream shock, $\hat{G}_{2,0}$, in sector 2 generates an own-sector R&D incentive at sector 2 (the demand shock raises the value of sector 2's knowledge), but no knowledge accumulation in sector 1 since sector 1 does not cite sector 2. There is therefore no innovation-network effect in either direction, and the R&D responses collapse to

$$\begin{aligned}\sum_{t=0}^{\infty} \hat{\mathcal{N}}_{R,1,t} &= \underbrace{0}_{\text{own effect}} + \underbrace{0}_{\text{innovation-network effect}} + \underbrace{\frac{\lambda\Gamma}{2} \hat{G}_{2,0}}_{\text{wage-bill effect}}, \\ \sum_{t=0}^{\infty} \hat{\mathcal{N}}_{R,2,t} &= \underbrace{\Gamma \hat{G}_{2,0}}_{\text{own effect}} + \underbrace{0}_{\text{innovation-network effect}} + \underbrace{\frac{\lambda\Gamma}{2} \hat{G}_{2,0}}_{\text{wage-bill effect}},\end{aligned}$$

so that sector 2 reacts via its own direct effect and a common wage-bill effect, while sector 1 reacts only via the wage-bill effect.

The proposition below generalizes this decomposition to any citation matrix, with $\mathcal{L}_A$ collecting the network channel.

Proposition 1. *The cumulative response of sector s 's R&D expenditures to a sectoral government spending shock to sector x , $\hat{G}_{x,0}$, is*

$$\sum_{t=0}^{\infty} \hat{\mathcal{N}}_{R,s,t} = \underbrace{\Gamma [I]_{s,x} \hat{G}_{x,0}}_{\text{own effect}} + \underbrace{\lambda\Gamma [\mathcal{L}_A - I]_{s,x} \hat{G}_{x,0}}_{\text{innovation-network effect}} + \underbrace{\frac{\lambda\Gamma}{S} \mathcal{U}_x \hat{G}_{x,0}}_{\text{wage-bill effect}}. \quad (42)$$

For a positive shock $\hat{G}_{x,0} > 0$, all three components are non-negative: the own effect is strictly positive at the shocked sector and zero elsewhere; the innovation-network effect is strictly positive at every sector connected to x through a directed citation path in Ω_A and zero otherwise; and the wage-bill effect is always positive.

Proof. See Appendix C.5.1. □

The proposition states that a spending shock that targets industry x propagates to sector s 's R&D through the innovation-network channel if and only if a citation path runs from s to x . Sectors disconnected from x in the citation network still see their R&D expenditures rise, but only through the wage-bill channel.

4.4 The Response of Sectoral GDP

This section characterizes the response of sectoral real GDP to a government spending shock, and decomposes it into own and innovation-network effects.

A two-sector example. Continuing with the 2-sector upstream-downstream network, we map the cumulative knowledge responses of Section 4.3 into sectoral

real value added through Equation (31).

The sectoral GDP responses to a shock in upstream sector 1, $\hat{G}_{1,0}$:

$$\begin{aligned}\sum_{t=0}^{\infty} \hat{Y}_{1,t} &= \underbrace{(\xi^G + \xi^C \lambda \Gamma) \hat{G}_{1,0}}_{\text{own effect}} + \underbrace{\frac{\xi^C \lambda \Gamma M}{1-M} \hat{G}_{1,0}}_{\text{innovation-network effect}}, \\ \sum_{t=0}^{\infty} \hat{Y}_{2,t} &= \underbrace{0}_{\text{own effect}} + \underbrace{\frac{\xi^C \lambda \Gamma M}{1-M} \hat{G}_{1,0}}_{\text{innovation-network effect}}.\end{aligned}$$

Relative to the responses of sectoral R&D, two differences arise. First, the own effect bundles two distinct mechanisms, mapping onto the two components of the real-GDP identity $\hat{Y}_{s,t} = \xi^C \hat{A}_{s,t} + \xi^G \hat{G}_{s,t}$. The first is a mechanical *public-demand channel*, $\xi^G \hat{G}_{1,0}$: government spending in a sector raises its GDP. The second is a *private-demand channel*, $\xi^C \lambda \Gamma \hat{G}_{1,0}$: the shock raises sector 1's knowledge stock, which lifts sectoral productivity and lowers the sectoral price; households respond by consuming more of sector 1's good.

Second, there is no common wage-bill effect.¹⁷ As a result, sectors disconnected from the shocked sector in the citation matrix see no response at all. In other words, the only indirect effect operates via the innovation network.

The responses of sectoral GDP to a shock in downstream sector 2, $\hat{G}_{2,0}$, are:

$$\begin{aligned}\sum_{t=0}^{\infty} \hat{Y}_{1,t} &= \underbrace{0}_{\text{own effect}} + \underbrace{0}_{\text{innovation-network effect}}, \\ \sum_{t=0}^{\infty} \hat{Y}_{2,t} &= \underbrace{(\xi^G + \xi^C \lambda \Gamma) \hat{G}_{2,0}}_{\text{own effect}} + \underbrace{0}_{\text{innovation-network effect}}.\end{aligned}$$

The lack of the wage-bill effect implies that downstream shocks do not alter the GDP of the upstream sector at all.

The proposition below generalizes this decomposition to any network.

Proposition 2. *The cumulative response of sector s 's real value added to a sectoral government spending shock to sector x , $\hat{G}_{x,0}$, is*

$$\sum_{t=0}^{\infty} \hat{Y}_{s,t} = \underbrace{(\xi^G + \xi^C \lambda \Gamma) [I]_{s,x} \hat{G}_{x,0}}_{\text{own effect}} + \underbrace{\xi^C \lambda \Gamma [\mathcal{L}_A - I]_{s,x} \hat{G}_{x,0}}_{\text{innovation-network effect}}. \quad (43)$$

For a positive shock $\hat{G}_{x,0} > 0$, both components are non-negative: the own effect is

¹⁷R&D expenditures are a cost in consumption units, $w_t N_{R,s,t}$: the real wage paid to research labor times the labor input. When aggregate knowledge raises the real wage uniformly across sectors, R&D expenditures rise with it through the wage component. Real sectoral GDP, $Y_s = C_s + G_s$, is by contrast a quantity-side measure that does not depend on the wage.

strictly positive at the shocked sector and zero elsewhere; the innovation-network effect is strictly positive at every sector connected to x through a directed citation path in Ω_A and zero otherwise.

Proof. See Appendix C.5.2. □

4.5 The Response of Aggregate GDP

This section aggregates the sectoral GDP responses of Proposition 2 into the *sectoral-spending multiplier*: the cumulative response of aggregate GDP per dollar of government spending in sector x .

A two-sector example. Aggregating the two-sector real GDP responses derived above via the log-linearized identity $\hat{Y}_t = \sum_s (Y_s/Y) \hat{Y}_{s,t}$, with Domar weights $Y_s/Y = 1/2$ in the homogeneous two-sector economy, the cumulative response of aggregate GDP to a shock in the upstream and downstream sectors equals:

$$\begin{aligned} \sum_{t=0}^{\infty} \hat{Y}_t &= \left[\frac{\xi^G}{2} + \frac{\xi^C \lambda \Gamma}{2} \cdot \frac{1+M}{1-M} \right] \hat{G}_{1,0}, \\ \sum_{t=0}^{\infty} \hat{Y}_t &= \left[\frac{\xi^G}{2} + \frac{\xi^C \lambda \Gamma}{2} \right] \hat{G}_{2,0}. \end{aligned}$$

The two responses share a common fiscal component, $\xi^G/2$: the mechanical contribution of any sectoral shock to aggregate GDP is the same, because each sector features the same Domar weight. The two responses also share a private-demand channel, but its size depends on the position of the shocked sector in the innovation network: a shock at sector 1 raises knowledge in both industries because sector 2 cites sector 1, whereas a shock at sector 2 raises knowledge only at sector 2, since sector 1 does not cite sector 2. As a result, the upstream shock generates a strictly larger private-demand response than the downstream shock.

The difference between the two responses equals $\xi^C \lambda \Gamma M/(1-M)$, which can be rewritten as $(\xi^C \lambda \Gamma/2) \cdot (\mathcal{U}_1 - \mathcal{U}_2)$, exactly the upstreamness gap between the two sectors. The proposition below shows that this connection between upstreamness and the aggregate response generalizes to any innovation network.

Proposition 3. *The cumulative response of aggregate real GDP to a government spending shock to sector x , $\hat{G}_{x,0}$, is*

$$\sum_{t=0}^{\infty} \hat{Y}_t = \left[\frac{\xi^G}{S} + \frac{\xi^C \lambda \Gamma}{S} \mathcal{U}_x \right] \hat{G}_{x,0}. \quad (44)$$

The associated sectoral-spending multiplier—the cumulative GDP response per

dollar of government spending, obtained by converting the log deviations of Equation (44) into dollars with the steady-state spending share $\bar{G}_x/\bar{Y} = \xi^G/S$ —equals

$$\mathcal{M}_x \equiv \frac{\sum_{t=0}^{\infty} \Delta Y_t}{\Delta G_{x,0}} = 1 + \frac{\xi^C \lambda \Gamma}{\xi^G} \mathcal{U}_x : \quad (45)$$

the direct fiscal component of the multiplier is one—each dollar of purchases adds one dollar of GDP—and the innovation component is increasing in the upstreamness of the targeted sector. For any pair of sectors x, x' , the multiplier is larger when government spending occurs in the sector with the higher upstreamness:

$$\mathcal{M}_x > \mathcal{M}_{x'} \iff \mathcal{U}_x > \mathcal{U}_{x'}. \quad (46)$$

Proof. See Appendix C.5.3. □

Thus, the cross-sectional ranking of sectoral-spending multipliers is governed entirely by the position of the targeted sectors in the innovation network.

4.6 The Idea Multiplier

We now derive the *idea multiplier*: the component of the sectoral-spending multiplier that stems from *cross-sectoral* knowledge spillovers—as distinct from the own-sector R&D incentive, which would arise even in the absence of an innovation network. We do so by comparing the sectoral-spending multiplier in our network economy with its counterpart in a counterfactual economy in which the innovation network is diagonal: knowledge is still accumulated through R&D, but each sector’s research builds only on its own stock, so ideas no longer spill over across industries.

Definition 1 (Idea multiplier). Let $\mathcal{M}_x$ denote the sectoral-spending multiplier of Equation (45) in the network economy, and let $\mathcal{M}_x^{\text{diag}}$ denote the same multiplier in the counterfactual economy obtained from the network economy by replacing the innovation-network matrix Ω_A with its diagonal counterpart $\Omega_A^{\text{diag}} \equiv I$, holding all other parameters fixed. The idea multiplier of sector x is

$$\mathcal{M}_x^{\text{idea}} \equiv \mathcal{M}_x - \mathcal{M}_x^{\text{diag}}. \quad (47)$$

Proposition 4. The idea multiplier of sector x equals

$$\mathcal{M}_x^{\text{idea}} = \frac{\xi^C \lambda \Gamma}{\xi^G} [\mathcal{U}_x - \bar{\mathcal{U}}], \quad (48)$$

where

$$\bar{\mathcal{U}} \equiv \frac{1}{S} \sum_{s=1}^S \mathcal{U}_s = \frac{1}{1-M} \quad (49)$$

denotes the average upstreamness of the innovation network across sectors. The idea multiplier is positive if and only if the upstreamness of sector x is larger than the average value in the economy:

$$\mathcal{M}_x^{\text{idea}} > 0 \iff \mathcal{U}_x > \bar{\mathcal{U}}. \quad (50)$$

Proof. See Appendix C.5.4. □

The idea multiplier is governed entirely by the upstreamness of the sector that receives spending: the direct fiscal component ξ^G/S in Proposition 3 cancels in the difference, leaving only the cross-sectoral innovation channel. The innovation network thus amplifies the aggregate effects of government spending when the targeted sector’s ideas feed the rest of the economy more than average.

5 Quantitative Results

We now take the full model of Section 3 to the data. We first present the calibration, showing how we leverage adoption to tie the model to the data, and validate the calibration. We then use the model to quantify the sectoral-spending multipliers, trace their heterogeneity to the innovation network, and measure the idea multiplier.

5.1 Calibration

The model is calibrated to the U.S. economy at monthly frequency. The economy features the same $S = 66$ industries as the empirical analysis of Section 2.

The household discount factor $\beta = 0.9984$ targets a 4% annual real interest rate along the balanced growth path. We set $\eta_P = \eta_R = 1/2$, so that the Frisch elasticities of production and research labor are 2, in line with the aggregate labor-supply elasticity of Erosa et al. (2016). The labor disutility levels are set so that total steady-state production and research hours each equal 0.33. Capital depreciates at a 10% annual rate, so that $\delta_K = 0.0087$, and the capital mobility elasticity is $\nu_K = 1$, as in Bouakez et al. (2025).

The parametrization of the Taylor rule follows Clarida et al. (2000): the interest-rate smoothing is $\phi_r = 0.93$, and the responses to inflation and the output gap are $\phi_\pi = 1.5$ and $\phi_x = 0.5/12$. Government spending absorbs 20% of GDP in steady state, and its AR(1) process has a persistence $\rho_G = 0.9916$, corresponding to 0.975 at quarterly frequency, in line with Kormilitsina and Zubairy (2018).

On the production side, the within-sector elasticity of substitution across varieties is $\epsilon = 4$, implying a gross markup of 1.33, as in De Loecker et al. (2020).

Factor intensities $(\alpha_{N,s}, \alpha_{K,s}, \alpha_{H,s})$, the shares of private consumption, private investment, and government spending $(\omega_{C,s}, \omega_{I,s}, \omega_{G,s})$, and the production-network linkages $\omega_{H,s,x}$ come from the BEA Input-Output tables. The sectoral degrees of price rigidity ϑ_s match the frequencies of price adjustment of [Pasten et al. \(2020\)](#).

Turning to the innovation block, the innovation network, $\omega_{A,s,x}$, is the patent-citation matrix described in Section 2.1. The R&D elasticities λ_s are pinned down by sectoral R&D intensities, the ratio of sectoral R&D expenditures to value added. Knowledge becomes obsolete at a 15% annual rate, $\delta_A = 0.01345$ at the monthly frequency, in line with the BEA depreciation rates of R&D capital. Finally, we posit that the economy grows along a balanced growth path at 2% annually, and back out the R&D efficiency $\chi = 0.0176$ from this restriction.

To tie the model to the data, we discipline the adoption parameter κ . We simulate the calibrated economy under sectoral government spending shocks, aggregate the simulated series into a sector-year panel with the same structure as the data, and estimate on the simulated panel the *same* local-projection specification of Section 2.2. The estimation uses a single moment: we choose κ so that the own-sector response of private R&D at the two-year horizon equals the estimated coefficient of 0.992 in Table 2, which yields $\kappa = 0.62$. We report the estimates in Table 3, together with their empirical counterparts.

The adoption exponent controls the effective duration of knowledge as an asset: a higher κ makes the value of knowledge load more on the sector's own accumulated stock and less on the transitory demand shock, damping the own R&D response and, at the same time, tilting R&D incentives toward the slow-moving knowledge inherited through citations. A single parameter thus rotates the model's R&D response between the own and the upstream channel. To confirm this effect, we consider three alternative parametrizations—no adoption ($\kappa = 0$), half the estimated value ($\kappa = 0.31$), and one and a half times it ($\kappa = 0.93$)—reported in the last three columns of Table 3. As κ rises, the own effect falls monotonically while the upstream effect strengthens.

Although the estimation targets a single moment—the own effect at the two-year horizon—the model replicates the progressive build-up of the own response, and matches the magnitude and build-up of the upstream channel, all of which is untargeted. In particular, the model upstream coefficient at the two-year horizon is 95% of that estimated in the data. The build-up arises because the return to R&D loads on knowledge stocks—own and cited—which take time to accumulate. This validates the model as a laboratory to study the transmission of

Table 3: Model versus Data.

	Data	Model	Model: counterfactual adoption		
		$\kappa = 0.62$	$\kappa = 0$	$\kappa = 0.31$	$\kappa = 0.93$
<i>Panel A: Own effect</i>					
$h = 0$	0.213	0.072	0.137	0.110	0.007
$h = 1$	0.663	0.584	1.416	1.074	−0.257
$h = 2$	0.992	0.992 [†]	2.558	1.915	−0.586
<i>Panel B: Upstream effect</i>					
$h = 0$	0.167	0.008	−0.005	0.000	0.024
$h = 1$	0.266	0.139	−0.033	0.033	0.332
$h = 2$	0.280	0.266	−0.057	0.066	0.631

Note: Responses of private R&D to government spending at horizons of h years, estimated with the local-projection specification of Section 2.2 on the data (point estimates of Table 2) and on the model-simulated panel. [†]Targeted moment: the adoption parameter κ is chosen so that the model matches the own effect at the two-year horizon. All remaining entries are untargeted. The last three columns report the same statistics, computed with the identical simulation and estimation steps, for economies that differ from the baseline only in the adoption exponent.

government spending in innovation networks.

5.2 Sectoral-Spending Multipliers

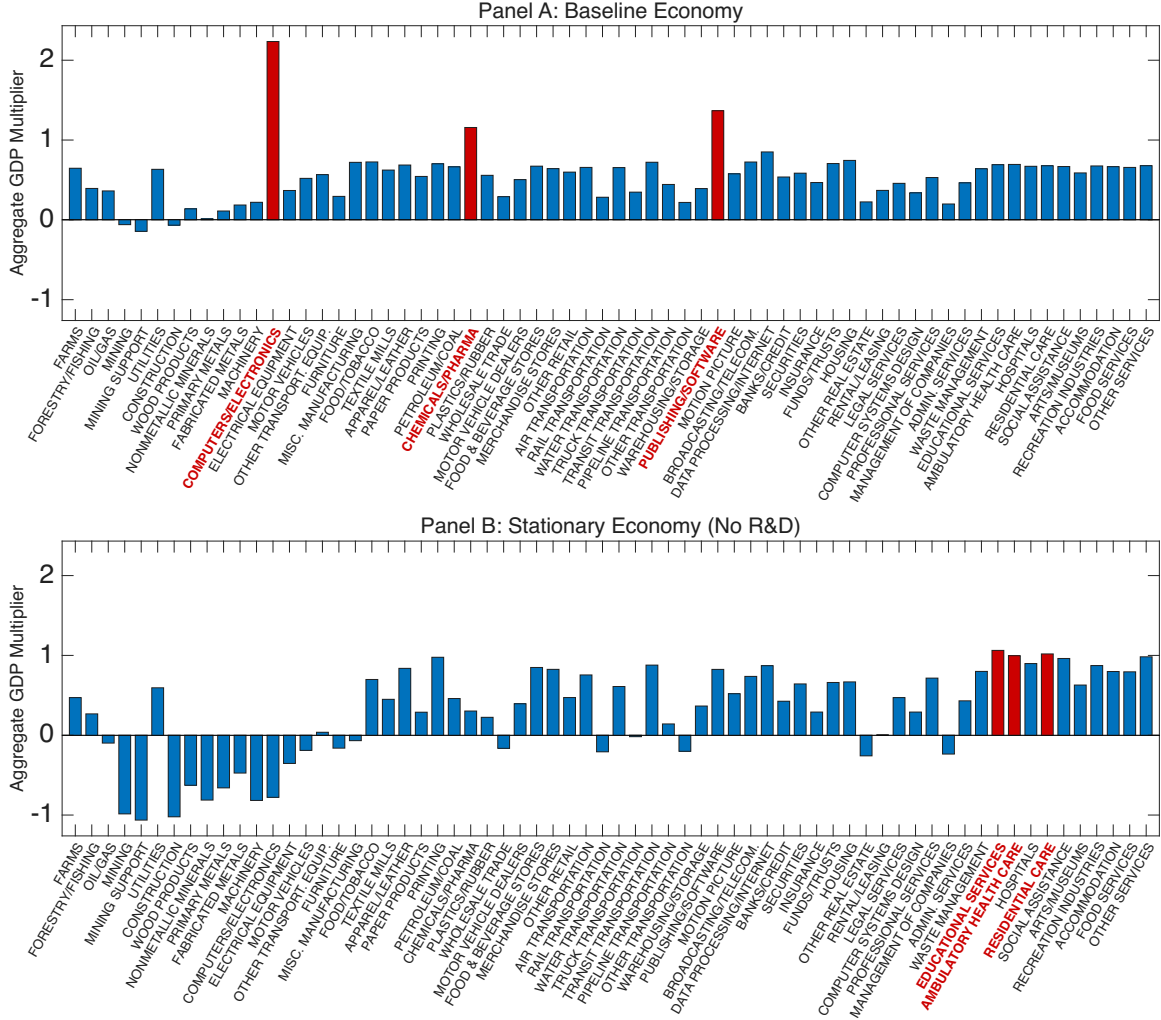
We study sectoral government spending shocks: each experiment raises government purchases of one industry’s output and traces the response of aggregate real value added, computed as deviations around the balanced growth path of the model. For each sector s , we compute the present-value cumulative multiplier

$$\mathcal{M}_s = \frac{\sum_{h=0}^{\infty} (r^*)^{-h} \Delta Y_{t+h}}{\sum_{h=0}^{\infty} (r^*)^{-h} \frac{P_s^*}{P_t^*} \Delta G_{s,t+h}}, \quad (51)$$

where ΔY_{t+h} and $\Delta G_{s,t+h}$ denote, at horizon h after the spending shock in sector s , the differences of aggregate real value added—as defined in Section 3.5—and of real government purchases from their balanced-growth paths absent the shock, discounted by the gross real interest rate along the BGP r^* .

Panel A of Figure 1 reports the aggregate value-added multiplier associated with government spending in each of the 66 industries. Three features stand out. First, sectoral-spending multipliers are highly heterogeneous, ranging from −0.14 to 2.23, with a mean of 0.54. Second, the upper tail is dominated by the knowledge hubs of the innovation network: spending in computers/electronics

Figure 1: Sectoral-Spending Multipliers.



Note: Sectoral-spending multipliers: the long-run present-value cumulative response of aggregate real value added to a government spending shock in each of the 66 BEA industries, discounted at the BGP real interest rate. Panel A refers to the baseline economy. Panel B refers to the stationary economy, in which the R&D block is shut down and every other source of sectoral heterogeneity is kept at its baseline value. The two panels share the same scale. In each panel, the three industries with the largest multipliers are highlighted.

yields a multiplier of 2.23, followed by publishing/software at 1.37 and chemicals/pharmaceuticals at 1.16. These are precisely the industries whose patents are most cited by the rest of the economy, that is, the most upstream sectors in the innovation network. Third, the correlation between multipliers and innovation-network upstreamness is 0.65, exactly the pattern predicted by the analytical results of Section 4: the fiscal component of the multiplier is common across sectors, and the cross-sectional variation is generated by the innovation component, which is increasing in the upstreamness of the targeted sector.

Our model adds to the workhorse multi-sector fiscal framework of [Bouakez et al. \(2023\)](#) an R&D block with heterogeneous R&D intensities and the innovation network. Shutting down innovation altogether—holding every other source of sectoral heterogeneity fixed—makes the model stationary and takes us back to that economy, whose sectoral-spending multipliers we report in Panel B of Figure 1; the comparison isolates the overall contribution of the innovation block.

Without R&D, sectoral-spending multipliers range between -1.06 and 1.06 . The three largest multipliers no longer belong to the knowledge hubs of Panel A, but to educational services (1.06), residential care (1.02), and ambulatory health care (1.00)—downstream service sectors combining rigid prices with a high labor share, the strongest transmitters of government spending in the multi-sector fiscal literature ([Bouakez et al., 2025, 2023](#)). The innovation block therefore changes not only the level of the multipliers, but also the identity of the sectors that transmit fiscal policy the most: it compresses most industries into a narrow positive band, while stretching the top of the distribution towards the sectors that supply knowledge to the rest of the economy.

5.3 Inspecting the Mechanism

To isolate what drives the results of Panel A of Figure 1, we study a sequence of increasingly rich economies, whose sectoral-spending multipliers are in Table 4.

We start from a fully homogeneous model, in which all sectors share identical technologies, demand shares, price rigidities, and homogeneous input-output linkages, and feature no R&D: sectoral-spending multipliers are identical by construction, at 0.45 . Adding a homogeneous R&D block—identical R&D intensities and a homogeneous innovation network—raises the common multiplier to 0.51 but generates no dispersion: endogenous innovation per se only marginally raises the level of the multiplier. Allowing for the observed heterogeneity in sectoral R&D intensities, while keeping the innovation network homogeneous, barely moves the cross-section: multipliers range between 0.49 and 0.70 . Replacing the homogeneous network with a roundabout network—each sector builds only on its own knowledge—delivers a nearly identical cross-section, with a range of $[0.49, 0.69]$.¹⁸

The heterogeneous citation network is the key driver: replacing the homo-

¹⁸The near-equivalence of the homogeneous and roundabout configurations is in line with our analytical results. By Proposition 3, the sectoral-spending multiplier depends on the innovation network only through the upstreamness of the targeted sector. Because both configurations treat all sectors symmetrically and the citation matrix is row-stochastic, every sector’s upstreamness equals $1/(1 - M)$ in both economies, delivering the same multiplier sector by sector.

Table 4: Inspecting the Mechanism.

	Sectoral-spending multipliers
Homogeneous economy, no R&D	[0.45, 0.45]
+ homogeneous R&D block	[0.51, 0.51]
+ heterogeneous R&D intensities (homogeneous network)	[0.49, 0.70]
+ roundabout innovation network	[0.49, 0.69]
+ empirical innovation network	[0.54, 2.55]

Note: Sectoral-spending multipliers across the sequence of economies of Section 5.3. All economies feature homogeneous technologies, demand shares, and price rigidities, and homogeneous input-output linkages.

geneous network with the empirical citation matrix amplifies the range of multipliers from [0.49, 0.70] to [0.54, 2.55]. The cross-section concentrates in the knowledge hubs, which detach from the rest of the economy, and the ranking lines up with citation upstreamness. Neither endogenous R&D per se, nor the observed differences in R&D intensity, generate meaningful dispersion until the empirical structure of knowledge flows is put in place.¹⁹

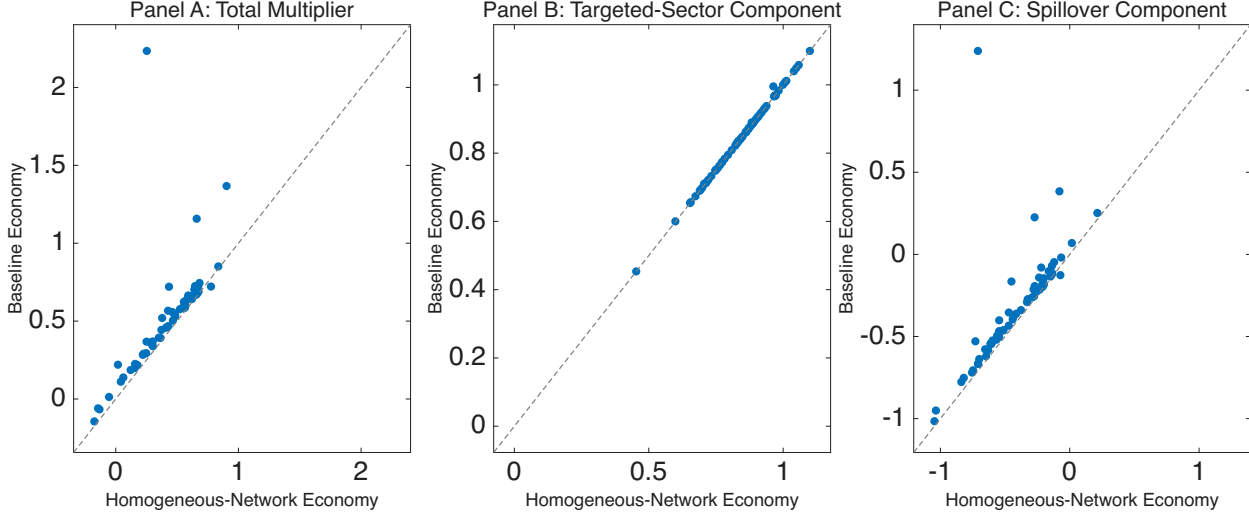
To quantify the role of the innovation network in the full model, we split each sectoral-spending multiplier into a *targeted-sector* component—the response of the shocked sector’s own value added per dollar of spending—and a *spillover* component—the contribution of all other sectors. We then compare both components with those implied by an otherwise identical economy in which the empirical citation matrix is replaced by a homogeneous network, $\omega_{A,s,x} = 1/S$.²⁰

This split maps directly into the decomposition of Proposition 2. The targeted-sector component bundles the own effect—which involves no network objects—with the innovation-network effect on the targeted sector itself. The spillover component, by contrast, is pure innovation-network effect: the value added of any non-targeted sector responds only through the citation paths that connect it to the shocked sector. The analytics thus predict an asymmetric role for the innovation network across the two components: in the targeted-sector component, the network merely adds to the own effect, and thus plays a minor role; the spillover component is instead entirely shaped by it. Replacing the empirical

¹⁹The production network plays a separate role. Appendix D isolates its contribution by re-solving the baseline and the stationary economies with a homogeneous IO matrix: while the production network is an important channel for the standard demand transmission of fiscal policy, it barely modulates the R&D responses to fiscal shocks—confirming that it plays no confounding role in the empirical analysis of Section 2.

²⁰The results of the exercise are virtually identical if the comparison is carried out against the roundabout innovation network (in which the citation matrix is replaced by the identity matrix) rather than the homogeneous network.

Figure 2: Targeted-Sector versus Spillover Components.



Note: Each panel scatters the 66 sectoral values of the sectoral-spending multipliers (Panel A) and of their targeted-sector (Panel B) and spillover (Panel C) components in the baseline economy (vertical axis) against the economy in which the empirical citation matrix is replaced by a homogeneous network, $\omega_{A,s,x} = 1/S$, holding every other parameter fixed (horizontal axis). The targeted-sector component is the response of the shocked sector's own value added per dollar of spending; the spillover component is the contribution of all other sectors. Dashed lines are 45-degree lines.

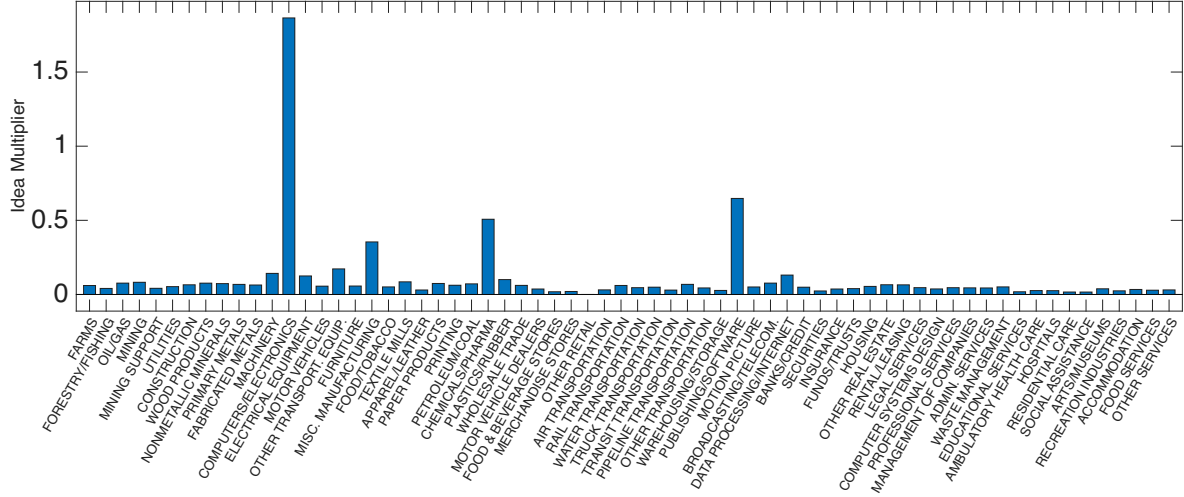
citation matrix with a homogeneous network should therefore barely move the former, and fully reorganize the latter.

Figure 2 confirms the prediction. Each panel scatters the 66 sectoral values in the baseline economy against their homogeneous-network counterparts: the total multipliers in Panel A, the targeted-sector components in Panel B, and the spillover components in Panel C. The targeted-sector components sit on the 45-degree line: their correlation across the two economies is 0.999. The spillover components, by contrast, are completely reorganized (correlation 0.68), and their reorganization accounts for the entire reshaping of the total multipliers (correlation 0.67): the spillover component of computers/electronics jumps from -0.71 to 1.24 , that of publishing/software from -0.08 to 0.38 , and that of chemicals/pharmaceuticals from -0.27 to 0.23 , lifting their total multipliers from 0.25 , 0.90 , and 0.66 to 2.23 , 1.37 , and 1.16 .

5.4 The Idea Multiplier

The comparison between the baseline and its roundabout-network counterfactual is precisely the quantitative counterpart of the idea multiplier of Definition 1: the difference, sector by sector, between the sectoral-spending multiplier in the network economy and in the economy in which the citation matrix is replaced by

Figure 3: The Idea Multiplier.



Note: Sectoral idea multipliers: for each of the 66 BEA industries, the difference between the aggregate value-added multiplier in the baseline economy and the corresponding multiplier in the counterfactual economy in which the innovation network is replaced by the identity matrix, holding all other parameters fixed.

the identity matrix, holding all other parameters fixed. Figure 3 reports it.

The idea multiplier is positive in 65 of the 66 industries, yet it is negligible for most of them: the average across all industries is 0.10, and it exceeds ten cents in only 9 industries. Five industries, however, stand above 15 cents: computers/electronics (1.86), publishing/software (0.65), chemicals/pharmaceuticals (0.51), miscellaneous manufacturing (0.35), and other transportation equipment (0.17). Four of the five are knowledge hubs of the citation network; machinery, the remaining hub, sits just below the threshold, at 0.14. The concentration is starkest at the top: the multiplier of computers/electronics is 0.37 in the roundabout economy and 2.23 in the baseline, so more than four-fifths of it flows through the innovation network.

The idea multiplier is also quantitatively central to the *dispersion* of sectoral-spending multipliers. Decomposing each baseline multiplier into its roundabout component and its idea component, the idea multiplier accounts for 52% of the cross-sectional variance of sectoral-spending multipliers; equivalently, shutting down the innovation network halves the variance and shrinks the range of multipliers from 2.4 to 0.9, cutting more than three-fifths of the overall spread. Cross-sectoral knowledge spillovers are thus not a refinement at the margin: they are the single largest source of heterogeneity in the effects of government spending across industries, on par with all conventional sources—demand composition, factor intensities, input-output linkages, and nominal rigidities—combined.

5.5 Robustness

Appendix D shows that these results are robust to an extensive battery of alternative specifications: an economy with perfectly mobile labor across industries; alternative Frisch elasticities of labor supply; sticky wages—for both labor types, for production wages only, and for research wages only; an economy in which government consumption also employs public labor; an economy with sector-specific capital stocks and an investment network; and a two-agent economy disciplined by the micro evidence on intertemporal marginal propensities to consume and the marginal propensity to earn. Across all these variants, the cross-sectoral heterogeneity (and in particular the dominance of innovation-upstream sectors) is preserved throughout.

6 Concluding Remarks

This paper shows that the innovation network shapes the transmission of fiscal policy. Government spending raises private R&D not only in the sectors that receive it, but also in the sectors that cite their patents. A multi-sector New Keynesian model with endogenous innovation rationalizes this evidence, and reveals that cross-sectoral knowledge spillovers—the idea multiplier—account for about half of the dispersion of sectoral-spending multipliers, concentrating the largest fiscal effects in the knowledge hubs of the economy. The gains from innovation-centered spending packages thus accrue not only to their direct recipients, but also to the network of sectors that build on their ideas.

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A Empirical Evidence: Robustness

This appendix provides additional details on, and an extensive battery of robustness checks of, the results of Section 2.3. Section A.1 reports the first-stage regressions behind the baseline specification. Section A.2 validates the instrumenting strategy through placebo tests. Section A.3 tightens the fixed effects to industry-group $\times$ year and upstreamness-tercile $\times$ year. Section A.4 reports inference under alternative clustering conventions and Driscoll–Kraay standard errors. Section A.5 pairs the own effect with alternative propagation channels: the downstream innovation network, and the downstream and upstream input-output networks. Section A.6 re-estimates the baseline specification controlling for the propagation of government spending through the production network, in both directions. Section A.7 considers alternative definitions of the control set and augments it with additional sectoral controls. Section A.8 addresses the concern that the cross-sectional variation used for identification may be correlated with heterogeneous exposure to monetary policy. Section A.9 varies the measurement window of the innovation network and the sample period.

All tables follow the format of the body of the paper: each column corresponds to a projection horizon $h = 0, 1, 2$, the dependent variable is the h -year-ahead response of real private R&D in sector s between $t - 1$ and $t + h$ scaled by lagged sectoral value added, all treatments are instrumented with the shift-share design of Equations (3), and p -values from two-way clustered (sector and year) standard errors are reported in brackets.

A.1 The First Stage

Table A.1 reports the first-stage estimates behind the baseline specification of Table 2: at each horizon, each endogenous treatment is regressed on the two shift-share instruments, with the same fixed effects and control set as the second stage. The two first-stage regressions read

$$\frac{G_{s,t} - G_{s,t-1}}{Y_{s,t-1}} = \alpha_s + \delta_t + \pi G_{s,t}^{IV} + \pi^{net} G_{s,t}^{IV,net} + \mathbf{Z}_{s,t}' \boldsymbol{\psi} + u_{s,t}, \quad (\text{A.1})$$

$$\sum_{x \neq s} \omega_{A,s,x,t} \frac{G_{x,t} - G_{x,t-1}}{Y_{s,t-1}} = \alpha_s + \delta_t + \rho G_{s,t}^{IV} + \rho^{net} G_{s,t}^{IV,net} + \mathbf{Z}_{s,t}' \boldsymbol{\varrho} + v_{s,t}. \quad (\text{A.2})$$

The identification displays a sharp diagonal structure. Each treatment loads strongly, and almost exclusively, on its own instrument: the own treatment on the own instrument, with a coefficient of about 0.42–0.45, and the upstream innovation treatment on the upstream instrument, with a coefficient of about 0.69,

Table A.1: First-Stage Estimates

Dependent Variable:	Own and upstream innovation treatments		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
<i>First stage: own treatment</i>			
Own instrument	0.450*** [0.000]	0.433*** [0.000]	0.422*** [0.000]
Upstream instrument	-0.193*** [0.000]	-0.192*** [0.000]	-0.194*** [0.000]
<i>First stage: upstream innovation treatment</i>			
Own instrument	0.150 [0.160]	0.157 [0.157]	0.163 [0.156]
Upstream instrument	0.694*** [0.000]	0.690*** [0.000]	0.691*** [0.000]
Sector & Year FE	✓	✓	✓
Observations	1,149	1,087	1,031
Kleibergen–Paap F	34.7	43.7	37.0

Note: First-stage regressions behind the baseline specification of Table 2, across 66 U.S. industries, 1997–2024. At each horizon, each treatment is regressed on the own instrument of Equation (3) and the upstream instrument of Equation (A.2), with the same fixed effects and controls as the second stage. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

both significant at the 1% level at every horizon. The cross-loadings are economically small, and the two instruments are nearly orthogonal cross-sectionally, with a correlation of 0.06: the own instrument does not enter the upstream first stage significantly, and the upstream instrument enters the own first stage with a small negative coefficient.

The negative sign may indicate that, during defense buildups, government purchases grow relatively less in the sectors whose knowledge suppliers are defense-intensive. Whatever its source, this force is conservative for our estimates: high upstream exposure comes bundled with slower own spending, which depresses R&D through the own channel.

A complementary way to characterize the identification is to ask which industries provide the variation behind the shift-share design. Following the logic of Goldsmith-Pinkham et al. (2020), we compute the contribution of each industry to the own-treatment first stage—the share-weighted covariance between the instrument and the residualized treatment, normalized to sum to one. The five largest weights belong to miscellaneous professional, scientific, and technical services (0.31); other transportation equipment (0.30), the aircraft and military-hardware industry; petroleum and coal products (0.21); data processing and internet services (0.12); and computers/electronics (0.10). The identification is thus driven by precisely the canonical suppliers of defense procurement, mapping onto the institutional composition of military purchases.

Two additional diagnostics confirm the strength and the separation of the two first stages. First, the Sanderson and Windmeijer (2016) conditional F statistics—which measure the identifying power of the instruments for each treatment *conditional* on the other endogenous treatment—are far above conventional thresholds at every horizon: 126, 234, and 185 for the own treatment, and 155, 428, and 270 for the upstream innovation treatment at $h = 0, 1, 2$. Each treatment is thus separately identified, rather than jointly supported by a single source of variation. Second, after partialling out the fixed effects and the controls, the covariance matrix of the residualized instruments is close to diagonal: their correlation ranges between -0.09 and -0.10 across horizons, so the own and upstream first stages draw on essentially independent variation.

A.2 Placebo Tests

This section reports three placebo exercises that validate the instrumenting strategy: future instruments do not predict current R&D, defense exposure carries no

differential R&D trends, and the upstream effect is not reproduced by randomly rewired innovation networks.

A first concern is anticipation: if defense exposure proxied for expected demand, R&D would move ahead of the procurement shifts. We add the one-year-ahead own and upstream instruments to the reduced form of the baseline specification. Table A.2 reports the coefficients on the lead instruments: they are individually and jointly insignificant at every horizon (joint p -values of 0.53 and 0.51 at $h = 1, 2$). R&D responds to procurement shifts that have occurred, not to those about to occur.

Table A.2: Placebo: Lead Instruments

Dependent Variable:	Private R&D	
	(1)	(2)
	$h = 1$	$h = 2$
Own instrument at $t + 1$	-0.061 [0.764]	0.092 [0.713]
Upstream instrument at $t + 1$	0.058 [0.281]	0.035 [0.286]
Joint p -value	0.526	0.514
Sector & Year FE	✓	✓
Observations	1,087	1,029

Note: Reduced form of the baseline specification augmented with the one-year-ahead own and upstream instruments; the table reports the coefficients on the lead instruments. The impact projection is not reported because the lead own instrument is collinear with the current instrument set on that sample. All regressions include the baseline controls and fixed effects. p -values from two-way clustered standard errors in brackets.

A second concern is that high-exposure sectors may sit on different secular innovation trajectories, which the shift-share coefficients would pick up mechanically. We regress the one-year R&D response on the interaction of the defense-exposure share with a linear and a quadratic time trend, with the baseline controls and fixed effects. Table A.3 shows that the interactions are individually and jointly insignificant (p -values of 0.30 and 0.41): there is no differential trend in R&D by defense exposure, in line with the identifying assumption of Section 2.2.

The third exercise asks whether the upstream effect reflects the measured structure of knowledge flows, or whether any network-shaped aggregation of defense spending would deliver it. We reassign at random which sectors act as each industry’s knowledge suppliers, rebuild the upstream treatment and its instru-

Table A.3: Placebo: Defense Exposure and Trends in R&D

Dependent Variable:	Private R&D	
	(1)	(2)
Exposure $\times$ trend	−0.005 [0.303]	0.011 [0.211]
Exposure $\times$ trend ²		−0.005 [0.185]
Joint p -value		0.408
Sector & Year FE	✓	✓
Observations	1,149	1,149

Note: The one-year R&D response regressed on the interaction of the defense-exposure share of the shift-share design with a linear time trend (column 1) and with linear and quadratic trends (column 2), with the baseline controls and fixed effects. p -values from two-way clustered standard errors in brackets.

ment on the permuted network, and re-estimate the baseline specification at the two-year horizon, repeating the exercise across 100 permutations. Because many permuted networks generate weak first stages, we compare t -statistics. Table A.4 summarizes the placebo distribution: only 9 of the 100 permutations deliver a t -statistic on the upstream effect at least as large as the one estimated on the actual citation network, a permutation p -value of 0.09. The upstream propagation of government spending is thus tied to the empirical citation network, and not to the mechanics of network aggregation.

Table A.4: Placebo: Randomly Rewired Innovation Networks

Permutations	100
t -statistic, actual citation network	2.32
Placebo t -statistics: mean	0.98
median	1.27
90th percentile	2.31
Share of placebo $t \geq$ actual (permutation p -value)	0.09

Note: Distribution of the t -statistic on the upstream innovation effect at the two-year horizon across 100 random permutations of the supplier identities in the citation network. For each permutation, the upstream treatment and its instrument are rebuilt on the permuted network and the baseline specification is re-estimated.

A.3 Fixed Effects

The baseline specification absorbs sector and year fixed effects. This section tightens the comparison group along two dimensions. First, we replace the year fixed effects with industry-group $\times$ year fixed effects, where the 66 industries are partitioned into five broad groups: the primary sector, utilities, and construction; manufacturing; trade and transportation; information, finance, and real estate; and services. The coefficients are then identified only by differential defense exposure *within* a group, so that group-specific business cycles (recessions or technology cycles that hit manufacturing differently from services) cannot confound the estimates. Second, we replace the year fixed effects with upstreamness-tercile $\times$ year fixed effects, where industries are grouped by their citation upstreamness: this absorbs any shock common to the innovation-upstream sectors in a given year, the sharpest confounder for the upstream channel.

Tables A.5 and A.6 report the estimates. In both cases the own and upstream innovation effects preserve the baseline build-up and remain statistically significant at the two-year horizon, if anything, they are slightly larger under the more granular fixed effects. The upstream propagation of government spending is thus identified neither by cross-group comparisons nor by shocks common to upstream sectors.

Table A.5: Industry-Group $\times$ Year Fixed Effects

Dependent Variable:	Private R&D		
	(1) $h = 0$	(2) $h = 1$	(3) $h = 2$
Own effect (β)	0.165 [0.623]	0.729* [0.068]	1.225* [0.068]
Upstream innovation (γ)	0.138 [0.220]	0.270** [0.035]	0.364* [0.070]
Sector & Group $\times$ Year FE	✓	✓	✓
Observations	1,149	1,087	1,031

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024, replacing the year fixed effects with industry-group $\times$ year fixed effects, for five broad industry groups. All regressions include the baseline controls. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

Table A.6: Upstreamness-Tercile $\times$ Year Fixed Effects

Dependent Variable:	Private R&D		
	(1) $h = 0$	(2) $h = 1$	(3) $h = 2$
Own effect (β)	0.290 [0.441]	0.989* [0.072]	1.497* [0.057]
Upstream innovation (γ)	0.177 [0.174]	0.351* [0.053]	0.442* [0.058]
Sector & Upstreamness $\times$ Year FE	✓	✓	✓
Observations	1,149	1,087	1,031

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024, replacing the year fixed effects with upstreamness-tercile $\times$ year fixed effects, where industries are grouped into terciles of citation upstreamness. All regressions include the baseline controls. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

A.4 Inference

Shift-share designs raise the concern that residuals may be correlated across observations with similar exposures, beyond what conventional clustering captures (Adão et al., 2019; Borusyak et al., 2022). Our design features a single aggregate shifter (the change in national defense procurement) interacted with predetermined sectoral exposure shares. In such a setting, the shock-level inference recommended by Borusyak et al. (2022) corresponds to clustering at the year level, since each period contributes one aggregate shock: under this convention, the own and upstream innovation effects are significant at the 1% and 5% levels at the two-year horizon (p -values of 0.004 and 0.012). The concern of Adão et al. (2019) is instead cross-sectional: industries with similar exposures may be hit by common shocks other than the fiscal one, in which case clustering by sector understates the standard errors. We implement their logic by grouping the 66 industries into sixteen bins of defense exposure and clustering two-way by exposure bin and year, which allows the residuals of industries in the same bin to be arbitrarily correlated, both within and across periods. The baseline inference is unaffected: the own and upstream innovation effects remain significant at the 5% level at the two-year horizon (p -values of 0.018 and 0.027). Finally, we compute Driscoll and Kraay (1998) standard errors with a three-lag bandwidth, which allow for arbitrary cross-sectional correlation of the residuals together with serial correlation, without taking a stand on which industries co-move: the own effect is

significant at the 1% level at the two-year horizon, and the upstream innovation effect at every horizon (p -values of 0.004 and 0.008 at $h = 2$). Table A.7 collects the p -values of the baseline coefficients under the alternative conventions.

Table A.7: Inference under Alternative Clustering Conventions

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
<i>Own effect (β)</i>			
Two-way: sector and year	[0.444]	[0.099]	[0.022]
Year	[0.478]	[0.055]	[0.004]
Two-way: exposure bin and year	[0.416]	[0.060]	[0.018]
Driscoll–Kraay	[0.480]	[0.068]	[0.004]
<i>Upstream innovation (γ)</i>			
Two-way: sector and year	[0.199]	[0.075]	[0.031]
Year	[0.090]	[0.002]	[0.012]
Two-way: exposure bin and year	[0.000]	[0.001]	[0.027]
Driscoll–Kraay	[0.033]	[0.000]	[0.008]
Sector & Year FE	✓	✓	✓
Observations	1,149	1,087	1,031

Note: p -values of the baseline coefficients of Table 2—which are identical across rows—under alternative clustering conventions: two-way by sector and year (the baseline), by year only (the shock-level convention of [Borusyak et al., 2022](#)), two-way by defense-exposure bin (sixteen groups) and year (the exposure-correlation logic of [Adão et al., 2019](#)), and [Driscoll and Kraay \(1998\)](#) standard errors with a three-lag bandwidth. Observations refer to the baseline specification.

A.5 Alternative Propagation Channels

The baseline specification transmits government spending across sectors through a single indirect channel: the upstream innovation network. This section tests the alternative channels through which spending in one sector could reach the R&D of another: the *downstream* innovation network (the exposure to spending received by the sectors that cite one’s patents), and the *downstream* and *upstream* input-output networks (the exposure to spending received by one’s customers and suppliers, respectively). The downstream input-output channel is the leading alternative: it is the channel through which the literature on sectoral fiscal multipliers documents the propagation of government spending to sectoral *output* ([Acemoglu et al., 2016a](#); [Bouakez et al., 2023, 2025](#)). We test each channel for private R&D, pairing it with the own effect, and find that upstream innovation is the only channel for which the data provide evidence of positive propagation.

We measure each alternative channel exactly like the upstream-innovation indirect effect, replacing the citation weight $\omega_{A,s,x,t}$ in Equation (2) with an alternative network weight $W_{s,x,t}$,

$$\sum_{x \neq s} W_{s,x,t} \frac{G_{x,t} - G_{x,t-1}}{Y_{s,t-1}}. \quad (\text{A.3})$$

The downstream innovation network uses the transpose of the citation matrix, $W_{s,x,t}^{\text{down-innov}} = \text{Cit}_{x,s,t} / \sum_k \text{Cit}_{k,s,t}$, the share of the citations received by s that originate in sector x . The downstream input-output network uses the transpose of the BEA Use entries, $W_{s,x}^{\text{down-io}} = \omega_{H,x,s}$ —the weight of sector s in the input bundle of its customer x —while the upstream input-output network uses the Use entries directly, $W_{s,x}^{\text{up-io}} = \omega_{H,s,x}$, the weight of supplier x in the input bundle of sector s . The three alternative treatments—the downstream innovation and the upstream and downstream input-output exposures—are instrumented analogously to the upstream-innovation treatment in Equation (4), by aggregating the shift-share instrument across the corresponding network weights.

We first consider the reverse direction of the innovation network: the downstream innovation treatment, which captures the exposure of sector s to the government spending received by the sectors that cite its patents. The model predicts no propagation through this channel: knowledge flows from cited to citing sectors, so spending received by a sector’s citers changes neither its market size nor the productivity of its R&D. Table A.8 reports the estimates of the specification that replaces the upstream innovation treatment with the downstream one, instrumented with its own shift-share instrument. No coefficient is statistically significant at any horizon.

We then replace the upstream innovation treatment with the downstream input-output treatment, which captures the exposure of sector s to the government spending received by its customers, instrumented with its own shift-share instrument. Table A.9 reports the estimates. The downstream input-output channel never enters positively: the point estimates are negative at every horizon, and significantly so at the two-year horizon—if anything, the demand received by a sector’s customers tilts the sector’s activity toward production and away from R&D. The own effect remains positive and statistically significant from the one-year horizon onward. Hence, whereas the production network is the relevant transmission channel for sectoral output, it is not the channel through which government spending stimulates private R&D.

We finally turn to the upstream direction of the input-output network: the

Table A.8: The Downstream Innovation Channel

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
Own effect (β)	0.772 [0.194]	1.093 [0.131]	1.048 [0.188]
Downstream innovation (γ)	1.143 [0.196]	1.030 [0.209]	0.430 [0.724]
Sector & Year FE	✓	✓	✓
Observations	1,149	1,087	1,031

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. The downstream innovation treatment, built as in Equation (A.3) from the transpose of the citation matrix, is instrumented with its own shift-share instrument. All regressions include sector and year fixed effects, and three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added growth, all dated $t - 1$ or earlier. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

Table A.9: The Downstream Input-Output Channel

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
Own effect (β)	0.099 [0.614]	0.435** [0.037]	0.752** [0.029]
Downstream input-output (γ)	−0.164 [0.849]	−1.495 [0.100]	−1.826** [0.049]
Sector & Year FE	✓	✓	✓
Observations	1,073	1,016	965

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. The downstream input-output treatment, built as in Equation (A.3) from the transpose of the BEA Use entries, is instrumented with its own shift-share instrument. All regressions include sector and year fixed effects, and three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added growth, all dated $t - 1$ or earlier. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

exposure of sector s to the government spending received by its input suppliers. Demand propagates in the opposite direction in the production network—spending on a sector’s suppliers does not raise the demand for its output—so, if anything, the channel should operate through costs, with higher supplier prices

Table A.10: The Upstream Input-Output Channel

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
Own effect (β)	0.562 [0.327]	1.131 [0.237]	1.271 [0.219]
Upstream input-output (γ)	0.384 [0.377]	0.527 [0.467]	0.409 [0.628]
Sector & Year FE	✓	✓	✓
Observations	1,149	1,087	1,031

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. The upstream input-output treatment, built as in Equation (A.3) from the BEA Use entries, is instrumented with its own shift-share instrument. All regressions include sector and year fixed effects, and three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added growth, all dated $t - 1$ or earlier. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

weighing on the sector’s activity. Table A.10 reports the estimates of the specification that replaces the upstream innovation treatment with the upstream input-output treatment, instrumented with its own shift-share instrument. No coefficient is statistically significant at any horizon, and the shift-share instrument for this exposure carries essentially no independent identifying variation.

All in all, these results confirm the relevance of the upstream propagation of government spending through the innovation network: it is the only channel through which spending in one sector generates a positive R&D response in another.

A.6 Controlling for the Production Network

We now verify that the baseline results hold above and beyond the production network. Recall from Section 2.1 that the citation matrix and the input-output matrix are essentially unrelated—their off-diagonal correlation is 0.03—so the two channels are separately identified, and this exercise is a meaningful test rather than a horse race between two versions of the same network. Table A.11 reports the estimates of the specification that adds the downstream input-output exposure to the own and upstream innovation treatments; to save space, we do not report the coefficients on the input-output exposure, which are never statistically significant, in line with Table A.9. Both effects of interest are unaffected—if any-

thing, they are slightly larger and estimated more precisely: the own effect reaches 1.054 at the two-year horizon and is significant at the 10% level already after one year, and the upstream innovation effect is significant at the 10% level at one year and at the 5% level at two years, where it reaches 0.298. The stability of the coefficients confirms that the upstream innovation channel captures knowledge linkages, and is neither absorbed by, nor confounded with, production linkages.

Table A.11: Controlling for the Downstream Input-Output Channel

Dependent Variable:	Private R&D		
	(1) $h = 0$	(2) $h = 1$	(3) $h = 2$
Own effect (β)	0.198 [0.473]	0.682* [0.069]	1.054** [0.020]
Upstream innovation (γ)	0.162 [0.199]	0.272* [0.067]	0.298** [0.027]
Sector & Year FE	✓	✓	✓
Observations	1,073	1,016	965

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. All three treatments are instrumented with their own shift-share instruments. All regressions include sector and year fixed effects, and three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added growth, all dated $t - 1$ or earlier. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

Table A.12 adds the upstream input-output treatment to the baseline specification, so that the own and upstream innovation effects are estimated above and beyond it. The upstream innovation effect is unaffected: it retains the baseline build-up and reaches 0.289 at the two-year horizon, where it is significant at the 5% level. Also in this case the upstream input-output coefficient is never statistically significant. The own effect keeps its dynamic profile and remains statistically significant at the 2-year horizon. Together with Tables A.9 and A.11, this exercise completes the picture: neither direction of the production network transmits government spending to private R&D, and the upstream innovation channel is robust to controlling for both.

A.7 The Control Set

This section assesses the sensitivity of the baseline results to the definition of the control set. The baseline specification controls for three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added

Table A.12: Controlling for the Upstream Input-Output Channel

Dependent Variable:	Private R&D		
	(1) $h = 0$	(2) $h = 1$	(3) $h = 2$
Own effect (β)	0.244 [0.331]	0.553 [0.121]	0.643* [0.077]
Upstream innovation (γ)	0.165 [0.188]	0.269* [0.091]	0.289** [0.043]
Upstream input-output	0.027 [0.824]	-0.092 [0.402]	-0.305 [0.251]
Sector & Year FE	✓	✓	✓
Observations	1,149	1,087	1,031

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. All three treatments are instrumented with their own shift-share instruments. All regressions include sector and year fixed effects, and three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added growth, all dated $t - 1$ or earlier. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

growth. We first consider two variations: using two lags of every control instead of three (Table A.13), and dropping the value-added-growth controls altogether, so that the control set only comprises the lags of the dependent variable and of the treatments (Table A.14). In both cases, the own and upstream innovation effects preserve the same dynamic profile and remain statistically significant at the two-year horizon.

We then augment the baseline control set with additional sectoral variables that capture demand and factor-market conditions: (i) the growth rate of sectoral employment, from the BEA full-time-equivalent employees by industry; (ii) the change in federal *non-defense* purchases of the sector’s goods, from the BEA Use tables, scaled by lagged sectoral value added like the defense treatment; and (iii) the growth rate of real sectoral investment, from the BEA fixed-assets tables. Each control enters with three lags, dated $t - 1$ or earlier, consistent with the baseline control set. Table A.15 reports the estimates adding the three controls one at a time, and then all of them jointly. The own and upstream innovation effects preserve the baseline build-up in every case; in the most demanding specification, with all nine additional controls, the point estimates are if anything larger than in the baseline, and the upstream innovation effect remains significant at the 10% level at the two-year horizon.

Table A.13: Two Lags of All Controls

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
Own effect (β)	0.180 [0.567]	0.559 [0.192]	0.900** [0.027]
Upstream innovation (γ)	0.141 [0.212]	0.239* [0.086]	0.239** [0.042]
Sector & Year FE	✓	✓	✓
Observations	1,214	1,149	1,089

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. The control set comprises two lags, instead of three, of sectoral R&D, the sectoral own and innovation-network treatments, and sectoral value-added growth. All regressions include sector and year fixed effects. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

Table A.14: Excluding the Value-Added-Growth Controls

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
Own effect (β)	0.235 [0.373]	0.757 [0.109]	1.060* [0.063]
Upstream innovation (γ)	0.149 [0.152]	0.249* [0.055]	0.274** [0.038]
Sector & Year FE	✓	✓	✓
Observations	1,149	1,087	1,031

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. The control set comprises three lags of sectoral R&D and of the sectoral own and innovation-network treatments, and excludes the value-added-growth controls. All regressions include sector and year fixed effects. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

A.8 Heterogeneous Exposure to Monetary Policy

Donaldson (2026) shows that cross-sectional designs exploiting heterogeneous exposure to aggregate shocks fail to identify partial-equilibrium elasticities when the exposure to the shock of interest is correlated with the sensitivity to other aggregate variables that move in general equilibrium. In our setting, the natural concern is monetary policy: defense-intensive industries may be differentially sen-

Table A.15: Additional Sectoral Controls

Dependent Variable:	Private R&D			
	(1)	(2)	(3)	(4)
	Employment	Non-Defense	Investment	All
<i>Own effect (β)</i>				
$h = 0$	0.223 [0.516]	0.190 [0.568]	0.183 [0.519]	0.171 [0.655]
$h = 1$	0.626* [0.082]	0.685 [0.104]	0.624 [0.102]	0.647* [0.083]
$h = 2$	0.996*** [0.001]	1.078** [0.027]	0.918** [0.025]	1.073*** [0.003]
<i>Upstream innovation (γ)</i>				
$h = 0$	0.235 [0.126]	0.165 [0.151]	0.178 [0.191]	0.249 [0.116]
$h = 1$	0.310* [0.062]	0.251* [0.060]	0.288* [0.077]	0.335* [0.071]
$h = 2$	0.304** [0.046]	0.274* [0.060]	0.295** [0.043]	0.331* [0.092]
Sector & Year FE	✓	✓	✓	✓
Observations ($h = 2$)	975	1,025	1,025	975

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024, augmenting the baseline control set with three lags of, respectively: sectoral employment growth (column 1), the change in federal non-defense purchases scaled by lagged sectoral value added (column 2), sectoral real investment growth (column 3), and all three sets jointly (column 4). Employment is from the BEA full-time-equivalent employees by industry, available from 1998, which shortens the sample in columns (1) and (4). p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

sitive to interest rates, and monetary policy responds to defense buildups, so the differential monetary response would load onto the shift-share coefficients despite the year fixed effects. We address the concern in two steps, on the 1997–2023 sample dictated by the availability of the monetary policy shocks of [Jarociński and Karadi \(2020\)](#).

First, we implement the diagnostic test proposed by [Donaldson \(2026\)](#): we regress the outcome on the interaction between the defense-exposure share of the shift-share design and the change in the federal funds rate, instrumented with the interaction between the same share and the annual monetary policy shock, with sector and year fixed effects. A significant coefficient would indicate that sectors more exposed to defense spending respond differently to interest-rate movements. The interaction coefficient is never significant at the 5% level (0.004, -0.028 , and -0.037 at $h = 0, 1, 2$, with p -values of 0.58, 0.19, and 0.07), and its sign is un-

Table A.16: Controlling for Monetary-Policy Exposure

Dependent Variable:	Private R&D		
	(1) $h = 0$	(2) $h = 1$	(3) $h = 2$
Own effect (β)	0.417 [0.201]	0.649* [0.074]	0.948*** [0.002]
Upstream innovation (γ)	0.192* [0.092]	0.262* [0.054]	0.267** [0.010]
Exposure $\times$ monetary shock	0.071** [0.035]	−0.013 [0.860]	−0.035 [0.731]
Sector & Year FE	✓	✓	✓
Observations	1,099	1,087	1,031

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2023 (the sample dictated by the availability of the monetary policy shocks). The specification adds to the baseline the interaction between the defense-exposure share of the shift-share design and the annual monetary policy shock of Jarociński and Karadi (2020), included directly as a control. All regressions include sector and year fixed effects, and three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added growth, all dated $t - 1$ or earlier. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

stable across horizons and specifications: there is no evidence of a systematic differential exposure.

Second, Table A.16 re-estimates the baseline specification adding the interaction between the exposure share and the monetary policy shock as a control, so that the own and upstream innovation effects are estimated net of any differential response to monetary shocks. Both effects are essentially unchanged relative to the baseline—the own effect reaches 0.948 and the upstream innovation effect 0.267 at the two-year horizon, both estimated more precisely—while the interaction itself matters only on impact. The cross-sectional variation behind our estimates is thus not proxying for heterogeneous exposure to monetary policy.

A.9 Alternative Specifications

This section assesses the sensitivity of the baseline results to the window over which the innovation network is measured and to the sample period. We start with the window. The baseline network averages the citation matrices over 1976–1996. Table A.17 reports the estimates when the network is measured over the earlier 1976–1985 window, which precedes the estimation sample by more than a decade. The results are virtually unchanged: the own effect reaches 0.905 at the

Table A.17: Innovation Network Measured over 1976–1985

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
Own effect (β)	0.148 [0.581]	0.574 [0.115]	0.905** [0.024]
Upstream innovation (γ)	0.057 [0.216]	0.105* [0.093]	0.119** [0.025]
Sector & Year FE	✓	✓	✓
Observations	1,145	1,085	1,031

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries, 1997–2024. The innovation network is the average of the yearly citation matrices over 1976–1985, instead of the baseline 1976–1996 window. All regressions include sector and year fixed effects, and three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added growth, all dated $t - 1$ or earlier. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

Table A.18: Ending the Sample in 2019

Dependent Variable:	Private R&D		
	(1)	(2)	(3)
	$h = 0$	$h = 1$	$h = 2$
Own effect (β)	0.438 [0.193]	0.921* [0.099]	1.151* [0.093]
Upstream innovation (γ)	0.158 [0.140]	0.280 [0.116]	0.331* [0.096]
Sector & Year FE	✓	✓	✓
Observations	909	899	892

Note: IV local-projection estimates of Equation (2) across 66 U.S. industries on the 1997–2019 sample, which excludes the COVID-19 years. All regressions include sector and year fixed effects, and three lags of sectoral R&D, of the sectoral own and innovation-network treatments, and of sectoral value-added growth, all dated $t - 1$ or earlier. p -values from two-way clustered standard errors in brackets. *, **, and ***: significance at the 10%, 5%, and 1% levels.

two-year horizon and the upstream innovation effect 0.119, both significant at the 5% level. The smaller magnitude of the upstream coefficient reflects the sparser citation counts of the earlier period, which spread the same exposure over fewer

sector pairs; its statistical significance and dynamic profile are unaffected. The stability of the estimates across windows separated by a decade confirms that the innovation network captures slow-moving, structural knowledge linkages across industries.

Then, we end the sample in 2019, excluding the COVID-19 years. Table [A.18](#) reports the estimates. The coefficients are unchanged relative to the baseline—if anything slightly larger, with the own effect reaching 1.151 and the upstream innovation effect 0.331 at the two-year horizon—and both remain significant at the 10% level in the shorter sample.

B The Model: Details and Derivations

This appendix collects the model details and derivations omitted from the body of the paper: the household's first-order conditions, the producers' optimality conditions for production inputs and prices, the wholesalers' demand schedule and price index, the retailers' price indices and demand schedules, and the market-clearing and aggregation conditions. The first-order conditions of the R&D block are reported in the body, Equations (12)–(13).

B.1 Household

Let $\Lambda_{t,t+1} \equiv \beta \frac{P_t C_t}{P_{t+1} C_{t+1}}$ denote the household's nominal stochastic discount factor implied by log utility. The first-order condition for nominal bonds delivers the Euler equation:

$$1 = R_t \mathbb{E}_t[\Lambda_{t,t+1}]. \quad (\text{B.1})$$

The first-order conditions for sectoral production and research labor equate the marginal disutility of labor to the corresponding wage in marginal-utility units:

$$\theta_{P,s} N_{P,s,t}^{\eta_P} = \frac{W_{P,s,t}}{P_t C_t}, \quad \theta_{R,s} N_{R,s,t}^{\eta_R} = \frac{W_{R,s,t}}{P_t C_t}. \quad (\text{B.2})$$

The first-order conditions for investment and capital deliver:

$$P_{I,t} = \mathbb{E}_t\{\Lambda_{t,t+1} [R_{K,t+1} + (1 - \delta_K) P_{I,t+1}]\}. \quad (\text{B.3})$$

Finally, the household allocates capital services across sectors to maximize rental income given the CES basket of Equation (8), delivering the sectoral supply schedule

$$K_{s,t} = \omega_{K,s} \left(\frac{R_{K,s,t}}{R_{K,t}} \right)^{\nu_K} K_t, \quad R_{K,t} = \left[\sum_{s=1}^S \omega_{K,s} R_{K,s,t}^{1+\nu_K} \right]^{\frac{1}{1+\nu_K}}, \quad (\text{B.4})$$

where the aggregate rental rate $R_{K,t}$ is the price index dual to the capital basket.

B.2 Producers

The first-order conditions for production labor, capital, and intermediate inputs follow from cost-minimization at each instant and equate factor prices to their marginal revenue products. Letting $MC_{s,t}$ denote the producers' nominal marginal cost, the producer's demand for production labor, capital, and intermediate inputs equals

$$W_{P,s,t} N_{P,s,t} = \alpha_{N,s} MC_{s,t} Z_{s,t}, \quad (\text{B.5})$$

$$R_{K,s,t} K_{s,t} = \alpha_{K,s} MC_{s,t} Z_{s,t}, \quad (\text{B.6})$$

and

$$P_{H,s,t}H_{s,t} = \alpha_{H,s} MC_{s,t} Z_{s,t}. \quad (\text{B.7})$$

The first-order condition for the price $P_{s,t}$ trades off the static markup gain against current and future price-adjustment costs, delivering the sector-specific New Keynesian Phillips curve:

$$\vartheta_s(\pi_{s,t} - 1)\pi_{s,t} = (1 - \epsilon) + \epsilon \frac{MC_{s,t}}{P_{s,t}} + \vartheta_s \mathbb{E}_t \left[\Lambda_{t,t+1} (\pi_{s,t+1} - 1) \pi_{s,t+1} \frac{P_{s,t+1} Z_{s,t+1}}{P_{s,t} Z_{s,t}} \right], \quad (\text{B.8})$$

where $\pi_{s,t} = \frac{P_{s,t}}{P_{s,t-1}}$ denotes the sectoral gross inflation rate, and the forward term is scaled by the growth of nominal sectoral revenue, the base of the price-adjustment cost.

B.3 Wholesalers

In each sector, the wholesaler's cost minimization implies the variety-level demand schedule

$$Z_{s,t}^i = (P_{s,t}^i / P_{s,t})^{-\epsilon} Z_{s,t}, \quad (\text{B.9})$$

and the price index of the sectoral good is:

$$P_{s,t} = \left[\int_0^1 P_{s,t}^i{}^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}. \quad (\text{B.10})$$

B.4 Retailers

The technology of the private consumption retailer, Equation (17), implies that the private consumption price index, P_t , is a function of sectoral prices:

$$P_t = \prod_{s=1}^S \frac{P_{s,t}^{\omega_{C,s}}}{\omega_{C,s}}. \quad (\text{B.11})$$

As a result, the retailer's optimal demand of goods from sector s is:

$$C_{s,t} = \omega_{C,s} \frac{P_t}{P_{s,t}} C_t. \quad (\text{B.12})$$

Analogously, the technology of the private investment retailer, Equation (18), implies that the private investment price index, $P_{I,t}$, is a function of sectoral prices:

$$P_{I,t} = \prod_{s=1}^S \frac{P_{s,t}^{\omega_{I,s}}}{\omega_{I,s}}, \quad (\text{B.13})$$

and the retailer's optimal demand of goods from sector s is:

$$I_{s,t} = \omega_{I,s} \frac{P_{I,t}}{P_{s,t}} I_t. \quad (\text{B.14})$$

The same conditions hold for the intermediate-input retailers, one for each sector. The price of the intermediate-input bundle demanded by sector s producers is:

$$P_{H,s,t} = \prod_{x=1}^S \frac{P_{x,t}^{\omega_{H,s,x}}}{\omega_{H,s,x}}, \quad (\text{B.15})$$

and the optimal demand of sector x goods from the retailer assembling intermediate inputs for sector s producers reads:

$$H_{x,s,t} = \omega_{H,s,x} \frac{P_{H,s,t}}{P_{x,t}} H_{s,t}. \quad (\text{B.16})$$

B.5 Market Clearing and Aggregation

Labor markets. Production and research labor markets are segmented at the sector and occupation level: for every sector s , the labor supplied by the household equals the corresponding demand across producers,

$$N_{P,s,t} = \int_0^1 N_{P,s,t}^i di, \quad N_{R,s,t} = \int_0^1 N_{R,s,t}^i di,$$

and total hours of each type are $N_{P,t} = \sum_{s=1}^S N_{P,s,t}$ and $N_{R,t} = \sum_{s=1}^S N_{R,s,t}$.

Capital market. The capital services allocated by the household to each sector equal the demand across producers, $K_{s,t} = \int_0^1 K_{s,t}^i di$, and the sectoral flows exhaust the aggregate stock through the CES basket of Equation (8).

Bond market. Nominal bonds are in zero net supply: $B_t = 0$ in every period.

Goods markets. For each sector, the wholesaler's output net of the price-adjustment resource cost equals total demand from the consumption and investment retailers, the fiscal authority, and the S intermediate-input retailers, as stated by the sectoral resource constraint (16). The retailers' zero-profit conditions imply that the value of each bundle equals the value of its inputs: $P_t C_t = \sum_s P_{s,t} C_{s,t}$, $P_{I,t} I_t = \sum_s P_{s,t} I_{s,t}$, and $P_{H,x,t} H_{x,t} = \sum_s P_{s,t} H_{s,x,t}$ for every x .

Dividends. Producer i in sector s earns nominal profits equal to the value of gross output net of production costs, research labor costs, and price-adjustment costs:

$$D_{s,t}^i = \left[1 - \frac{\vartheta_s}{2} \left(\frac{P_{s,t}^i}{P_{s,t-1}^i} - 1 \right)^2 \right] P_{s,t}^i Z_{s,t}^i - W_{P,s,t} N_{P,s,t}^i - R_{K,s,t} K_{s,t}^i - P_{H,s,t} H_{s,t}^i - W_{R,s,t} N_{R,s,t}^i.$$

Wholesalers and retailers are perfectly competitive and earn zero profits. Aggregate dividends, rebated lump sum to the household in the budget constraint (6),

are therefore

$$D_t = \sum_{s=1}^S \int_0^1 D_{s,t}^i di.$$

Aggregation. Nominal value added at the sectoral level is the value of gross output, net of the Rotemberg costs, minus the value of intermediate inputs:

$$\mathcal{Y}_{s,t} = \left[1 - \frac{\vartheta_s}{2} \left(\frac{P_{s,t}}{P_{s,t-1}} - 1 \right)^2 \right] P_{s,t} Z_{s,t} - P_{H,s,t} H_{s,t}. \quad (\text{B.17})$$

Summing across industries, using the sectoral resource constraints and the retailers' zero-profit conditions, the intermediate-input flows cancel and aggregate nominal GDP equals total final expenditure:

$$\mathcal{Y}_t = \sum_{s=1}^S \mathcal{Y}_{s,t} = P_t C_t + P_{I,t} I_t + \sum_{s=1}^S P_{s,t} G_{s,t}. \quad (\text{B.18})$$

Combining the household budget constraint (6) with aggregate dividends, the government budget constraint (22), and the bond-market clearing condition delivers the same identity from the income side, so Walras' law holds. Finally, real GDP is defined as the sum of consumption, investment, and public spending valued at their BGP relative prices:

$$Y_t = C_t + \frac{P_{I,t}^*}{P_t^*} I_t + \sum_{s=1}^S \frac{P_{s,t}^*}{P_t^*} G_{s,t}. \quad (\text{B.19})$$

B.6 The Balanced Growth Path

This subsection derives the conditions under which the model admits an exact balanced growth path, and characterizes the growth rates of all variables along it. Throughout, stars denote BGP values, $\gamma_X \equiv X_{t+1}^*/X_t^*$ denotes the trend factor of variable X_t , and $q_{s,t} \equiv P_{s,t}/P_t$ denotes sectoral prices relative to the consumption price index. Consistently with the monetary rule, aggregate trend inflation is zero, $\pi^* = 1$, so trends in relative prices coincide with trends in nominal sectoral prices.

Common knowledge growth. The knowledge technology (10) is homogeneous of degree one in the vector of knowledge stocks: the spillover bundle carries exponent $1 - \kappa$, the internalized own stock carries exponent κ , and the innovation network is row-stochastic. Dividing the law of motion by $A_{s,t}$ delivers the growth

rate of sectoral knowledge:

$$\frac{A_{s,t+1}}{A_{s,t}} = (1 - \delta_A) + \chi N_{R,s,t}^{\lambda_s} \prod_{x=1}^S \left(\frac{A_{x,t}}{A_{s,t}} \right)^{(1-\kappa)\omega_{A,s,x}}. \quad (\text{B.20})$$

A BGP with stationary research labor therefore requires the relative knowledge stocks $A_{x,t}/A_{s,t}$ to be constant, which forces a *common* knowledge growth rate γ_A across all sectors. The innovation network is the reason: if a sector's stock grew more slowly than the stocks of the sectors it cites, its spillover bundle would outgrow its own knowledge, raising the productivity of its R&D without bound and pulling its growth back up—and symmetrically for a sector growing too fast. Imposing $A_{s,t+1}/A_{s,t} = \gamma_A$ and taking logs of Equation (B.20),

$$(1 - \kappa) \left[\log A_{s,t}^* - \sum_{x=1}^S \omega_{A,s,x} \log A_{x,t}^* \right] = \log \chi + \lambda_s \log N_{R,s}^* - \log (\gamma_A - 1 + \delta_A), \quad (\text{B.21})$$

a system of S conditions that pins down the $S - 1$ relative knowledge levels together with the common growth rate. Since the left Perron eigenvector $\boldsymbol{\mu}$ of the innovation network ($\boldsymbol{\mu}^\top \Omega_A = \boldsymbol{\mu}^\top$, $\boldsymbol{\mu}^\top \mathbf{1} = 1$) satisfies $\boldsymbol{\mu}^\top (I - \Omega_A) = \mathbf{0}^\top$, aggregating the system with $\boldsymbol{\mu}$ eliminates the knowledge levels and isolates the growth condition:

$$\log (\gamma_A - 1 + \delta_A) = \log \chi + \sum_{s=1}^S \mu_s \lambda_s \log N_{R,s}^*. \quad (\text{B.22})$$

Knowledge growth is thus pinned down by *centrality-weighted research labor*, with weights given by the eigenvector centralities of the innovation network. Growth is endogenous—it depends on the equilibrium level of research employment—and the BGP is well defined because the R&D elasticities satisfy $\lambda_s < 1$, so research labor is stationary. In the calibration, we use these conditions in reverse: the aggregate trend γ_Y is targeted at 2% annually, γ_A follows from the aggregation condition (B.25) below, and the R&D efficiency is backed out from Equation (B.22) evaluated at the targeted allocation, with the relative knowledge levels solving Equation (B.21).

Sectoral price trends. Along the BGP, markups are constant and each sectoral relative price is proportional to real marginal cost. From the Cobb-Douglas technology (9), real marginal cost is proportional to $A_{s,t}^{-1} w_{P,s,t}^{\alpha_{N,s}} r_{K,t}^{\alpha_{K,s}} q_{H,s,t}^{\alpha_{H,s}}$, where $w_{P,s,t}$, $r_{K,t}$, and $q_{H,s,t}$ denote the production wage, the rental rate, and the intermediate-bundle price, all relative to the consumption price index. The

trends of the cost components follow from the rest of the model: real wages grow at γ_Y in every sector—stationary hours in the labor supply condition require the wage to trend with consumption; the rental rate trends with the investment price, $\gamma_{r_K} = \gamma_{q_I}$ with $\log \gamma_{q_I} = \sum_x \omega_{I,x} \log \gamma_{q,x}$, since the return to capital and the depreciation rate are stationary; the intermediate-bundle prices obey $\log \gamma_{qH,s} = \sum_x \omega_{H,s,x} \log \gamma_{q,x}$ from Equation (B.15); and knowledge grows at γ_A . Collecting the sectoral relative-price trends in the vector γ_q and equating price and cost trends sector by sector:

$$\log \gamma_q = \mathcal{B} \log \gamma_q + \alpha_N \log \gamma_Y - \mathbf{1} \log \gamma_A, \quad \mathcal{B} \equiv \alpha_K \omega_I^\top + \text{diag}(\alpha_H) \Omega_H, \quad (\text{B.23})$$

where $\alpha_N, \alpha_K, \alpha_H$ collect the sectoral factor intensities and row s of the matrix $\mathcal{B}$ measures the pass-through of capital and intermediate-input cost trends into the marginal cost of sector s . The rows of $\mathcal{B}$ sum to $\alpha_{K,s} + \alpha_{H,s} = 1 - \alpha_{N,s} < 1$, so its spectral radius is below one, the Leontief-type inverse $(I - \mathcal{B})^{-1}$ exists, and

$$\log \gamma_q = (I - \mathcal{B})^{-1} [\alpha_N \log \gamma_Y - \mathbf{1} \log \gamma_A]. \quad (\text{B.24})$$

Sectoral relative prices trend at heterogeneous rates, shaped by two opposing forces propagated through the production network: a labor-cost push—labor-intensive sectors see their prices rise with real wages—and a uniform knowledge deflation, amplified for sectors that buy intermediate inputs and capital from industries whose own prices fall. With zero aggregate trend inflation, the sectoral trend inflation rates that enter the sectoral Phillips curves coincide with the entries of γ_q .

Aggregate growth. The relative-price trends must be consistent with the definition of the consumption price index: from Equation (B.11), $\sum_s \omega_{C,s} \log \gamma_{q,s} = 0$. Applying the consumption weights to Equation (B.24) and solving for γ_Y :

$$\log \gamma_Y = \Xi \log \gamma_A, \quad \Xi \equiv \frac{\omega_C^\top (I - \mathcal{B})^{-1} \mathbf{1}}{\omega_C^\top (I - \mathcal{B})^{-1} \alpha_N} > 1. \quad (\text{B.25})$$

The coefficient Ξ is the balanced-growth knowledge multiplier. Its numerator is the consumption-weighted total requirement of a uniform Hicks-neutral improvement—the direct effect plus its amplification through the intermediate-input and investment linkages—while its denominator aggregates the labor intensities along the same network paths. Since knowledge augments gross output while labor commands only a share of it, Ξ exceeds one: aggregate GDP grows faster than knowledge, the network counterpart of the usual gross-output-versus-

value-added amplification of technical progress à la Hulten.

Remaining trends and the real interest rate. The trends of all remaining variables follow. Aggregate consumption, real wages, and real GDP grow at γ_Y ; hours are stationary, as log consumption utility makes the income and substitution effects of trend wage growth cancel. Sector- s gross output grows at $\gamma_Y/\gamma_{q,s}$, so that nominal expenditure shares are constant, and real sectoral government spending trends in the same way, given that sectoral purchases are indexed to BGP nominal GDP. The capital stock and investment grow at $\gamma_K = \gamma_Y/\gamma_{q_I}$, deflating output growth by the investment-price trend. The real value of knowledge $V_{s,t}/P_t$ grows at γ_Y/γ_A —its dividend, the cost-saving flow $\Theta_{s,t}$ of Equation (14), scales with real revenues over the knowledge stock—and the asset-value recursion (13) converges provided $\beta[(1 - \delta_A) + \kappa(\gamma_A - 1 + \delta_A)] < \gamma_A$, a condition satisfied at the calibration. Finally, the household’s Euler equation delivers the BGP real interest rate,

$$r^* = \frac{\gamma_Y}{\beta} > \gamma_Y, \quad (\text{B.26})$$

the rate at which the present-value multipliers of Section 5 discount future GDP and spending flows; all present values are finite because $\beta < 1$.

Requirements and stationarization. The exact BGP relies on five features of the model: log consumption utility, of the King-Plosser-Rebelo class; Cobb-Douglas technologies and bundles throughout, which keep expenditure shares constant while relative prices trend; the degree-one homogeneity of the knowledge technology in the vector of knowledge stocks, combined with the row-stochasticity of the innovation network; adjustment costs expressed on stationary objects, as the price-adjustment cost is proportional to sectoral revenue; and policy scaling, with sectoral government purchases indexed to BGP nominal GDP and a monetary rule that responds to inflation and a stationary output gap. Dividing each variable by its trend delivers a stationary system, whose deterministic steady state is the object we solve in the calibration of Section 5.1; the first-order perturbation of Section 5 is taken around this steady state.

C Proofs of the Analytical Results

Notation and the adoption margin. As in Section 4, all prices and values are expressed in units of the current consumption basket: w_t denotes the real wage, $v_{s,t} \equiv V_{s,t}/P_{C,t}$ the real value of knowledge, and $\Theta_{s,t}$ its real marginal profit flow. Throughout this appendix, the knowledge technology carries a free spillover elasticity ζ and a free internalized own-stock elasticity κ , with $\lambda + \zeta + \kappa < 1$. The total knowledge-feedback discount is $M \equiv (\zeta + \kappa)/(1 - \lambda) < 1$; the adoption-adjusted network is $\Omega_A^\kappa \equiv (1 - \dot{\kappa})\Omega_A + \dot{\kappa}I$ with mixing weight $\dot{\kappa} \equiv \kappa/(\zeta + \kappa)$; the adoption-adjusted discount is $\tilde{\beta}_\kappa \equiv \beta(1 - \delta_A(1 - \kappa))$; the momentum coefficient is $b \equiv \beta\kappa$; and the knowledge-propagation matrix is $\mathbf{T} \equiv (1 - \delta_A)I + \delta_A M \Omega_A^\kappa$. At $\kappa = 0$ these reduce to the pure-network objects. Because Ω_A^κ is an affine function of Ω_A —and hence every matrix appearing in the analysis is a polynomial in Ω_A , so that all operators commute—the lemmas below apply under these substitutions: Ω_A^κ is row-stochastic, $\mathbf{1}$ remains a common eigenvector with $\mathbf{T}\mathbf{1} = \varrho\mathbf{1}$, $\varrho \equiv 1 - \delta_A(1 - M)$. Throughout, the adoption elasticity is fixed at the saturation-neutral value $\kappa = \kappa^*$ of Assumption 1, and all objects above are evaluated there. The generic- κ notation is used only where it must be: in deriving the value equation of Appendix C.2, whose saturation and momentum terms have to appear explicitly before Assumption 1 disposes of them. Every dynamic result is then established under the saturation-neutral adoption $\kappa = \kappa^*$ of Assumption 1: the body’s propositions under the benchmark that additionally drops the residual lagged-momentum term of coefficient ϖb , and Appendix C.7 for the exact system that retains it. Displayed formulas are written in the benchmark $\dot{\kappa} = 0$ notation wherever adoption enters only through these substitutions. This appendix proves Propositions 1, 2, 3, and 4 under the simplifications of Section 4.1 and Assumption 1. The appendix is organized as follows:

- Appendix C.1 derives the homogeneous steady state.
- Appendix C.2 log-linearizes the tractable framework around the steady state and derives the cumulative knowledge response.
- Appendix C.3 characterizes the saturation-neutral benchmark, and bounds the correction induced by the residual lagged-momentum term.
- Appendix C.4 collects matrix lemmas on the Leontief inverse, the knowledge-propagation matrix $\mathbf{T}$, and the matrix quadratic underlying the exact system of Appendix C.7.
- Appendix C.5 proves the body propositions under the simplified value equa-

tion (35) (no saturation).

- Appendix C.6 works out the two-sector example under the no-saturation benchmark.
- Appendix C.7 states the exact second-order system under Assumption 1—retaining the lagged momentum leg—and solves it, delivering the exact propagation operator that Appendix C.3 compares with the benchmark.

C.1 Homogeneous Steady State

We show that the deterministic steady state of the analytical economy—with the adoption technology of simplification (vii)—features a sector-symmetric profile $\bar{A}_s = \bar{A}$, $\bar{N}_{R,s} = \bar{N}_R$, $\bar{v}_s = \bar{v}$, $\bar{Z}_s = \bar{Z}$, $\bar{q}_s = \bar{q}$ for all s , and that this profile is the unique solution of the joint system formed by the steady-state R&D first-order condition and the steady-state knowledge law of motion, given the aggregate block. We do not assume symmetry of any quantity a priori. The remaining aggregate equilibrium (the levels of consumption, the wage, and aggregate labor) reduces to standard scalar conditions, which we do not characterize further here.

Step 1: Sectoral revenue is sector-invariant. Under simplification (iv), households' Cobb-Douglas demand with shares $\omega_{C,s} = 1/S$ allocates a fraction $1/S$ of total nominal consumption to each sector, so $\bar{q}_s \bar{C}_s = (1/S) \bar{C}$. The government allocates a fraction $\omega_{G,s} = 1/S$ of nominal spending to each sector, so $\bar{G}_s = (1/S) \bar{G}$. The sectoral resource constraint $Z_{s,t} = C_{s,t} + G_{s,t}$ implies $\bar{q}_s \bar{Z}_s = \bar{q}_s \bar{C}_s + \bar{q}_s \bar{G}_s = \bar{q}_s \bar{C}_s + \bar{G}_s$. Hence

$$\bar{R}_s \equiv \bar{q}_s \bar{Z}_s = (1/S) \bar{C} + (1/S) \bar{G} = (1/S) (\bar{C} + \bar{G}) \equiv \bar{R},$$

independent of s . No assumption on $\bar{A}_s$ has been used at this stage.

Step 2: Envelope value of knowledge is inversely proportional to $\bar{A}_s$. With internalized adoption, the asset-value recursion is $v_{s,t} = \Theta_{s,t} + [(1 - \delta_A) + \kappa X_{s,t}/A_{s,t-1}] \mathbb{E}_t[\Lambda_{t,t+1} v_{s,t+1}]$, where $\Theta_{s,t} = (\epsilon - 1) \Pi_{s,t}/A_{s,t}$ is the envelope-derived marginal cost-saving flow, with $\Pi_{s,t}$ the producer's flow profits, of one extra unit of knowledge under constant markup $\mu = \epsilon/(\epsilon - 1)$, and $X_{s,t}$ is the idea flow, with steady-state ratio $\bar{X}_s/\bar{A}_s = \delta_A$ from the law of motion. At the deterministic steady state, with stationary Lagrange ratio $\bar{\Lambda} = \beta$, the continuation factor is $(1 - \delta_A) + \kappa \delta_A = 1 - \delta_A(1 - \kappa)$, so the effective discount is the adoption-adjusted

$\tilde{\beta}_\kappa \equiv \beta[1 - \delta_A(1 - \kappa)]$, and

$$\bar{v}_s = \frac{\bar{\Theta}_s}{1 - \tilde{\beta}_\kappa} = \frac{(\epsilon - 1)\bar{\Pi}_s}{(1 - \tilde{\beta}_\kappa)\bar{A}_s} = \frac{(\epsilon - 1)\bar{R}}{\epsilon(1 - \tilde{\beta}_\kappa)\bar{A}_s},$$

using $\bar{\Pi}_s = \bar{R}_s/\epsilon = \bar{R}/\epsilon$ from constant-markup pricing and the sector-invariance of $\bar{R}_s = \bar{R}$ established in Step 1. Hence

$$\bar{v}_s \bar{A}_s = \frac{(\epsilon - 1)\bar{R}}{\epsilon(1 - \tilde{\beta}_\kappa)}$$

is sector-invariant, and $\bar{v}_s \propto 1/\bar{A}_s$.

Step 3: Steady-state first-order condition and law of motion. Under indivisible labor—simplification (ii)—every sectoral wage is pinned by aggregate consumption, so the steady-state real wage $\bar{w}$ is uniform across sectors and occupations. Define the composite knowledge input $\bar{B}_s \equiv (\prod_x \bar{A}_x^{\zeta\omega_{A,s,x}}) \bar{A}_s^\kappa$, which includes the internalized own-stock adoption factor and may depend on s at this stage because no symmetry of $\bar{A}_s$ has been assumed. The steady-state R&D first-order condition and the steady-state knowledge law of motion read

$$\bar{w} = \lambda\chi \bar{v}_s \bar{N}_{R,s}^{\lambda-1} \bar{B}_s, \quad \delta_A \bar{A}_s = \chi \bar{N}_{R,s}^\lambda \bar{B}_s. \quad (\text{C.1})$$

Step 4: Eliminating the spillover bundle. Dividing the second equation in (C.1) by the first eliminates $\bar{B}_s$ exactly, regardless of its sector dependence:

$$\frac{\delta_A \bar{A}_s}{\bar{w}} = \frac{\chi \bar{N}_{R,s}^\lambda \bar{B}_s}{\lambda\chi \bar{v}_s \bar{N}_{R,s}^{\lambda-1} \bar{B}_s} = \frac{\bar{N}_{R,s}}{\lambda \bar{v}_s} \iff \bar{N}_{R,s} = \frac{\lambda \delta_A}{\bar{w}} \bar{v}_s \bar{A}_s.$$

Note that the division eliminates the *entire* composite input $\bar{B}_s$, own-stock adoption factor included. Using Step 2, $\bar{v}_s \bar{A}_s$ is sector-invariant, so

$$\bar{N}_{R,s} = \frac{\lambda \delta_A (\epsilon - 1) \bar{R}}{\epsilon (1 - \tilde{\beta}_\kappa) \bar{w}} \equiv \bar{N}_R, \quad (\text{C.2})$$

independent of s . We have established sector-invariance of research labor without yet assuming $\bar{A}_s$ is uniform.

Step 5: $\bar{A}_s$ is sector-invariant. Substituting $\bar{N}_{R,s} = \bar{N}_R$ from (C.2) back into the steady-state law of motion in (C.1) gives

$$\delta_A \bar{A}_s = \chi \bar{N}_R^\lambda \left(\prod_x \bar{A}_x^{\zeta\omega_{A,s,x}} \right) \bar{A}_s^\kappa.$$

Taking logs and collecting the own-stock term,

$$(1 - \kappa) \log \bar{A}_s = \log \left(\frac{\chi \bar{N}_R^\lambda}{\delta_A} \right) + \zeta \sum_x \omega_{A,s,x} \log \bar{A}_x,$$

or in vector form,

$$[(1 - \kappa)I - \zeta \Omega_A] \log \bar{\mathbf{A}} = c \mathbf{1}, \quad c \equiv \log \left(\frac{\chi \bar{N}_R^\lambda}{\delta_A} \right). \quad (\text{C.3})$$

Under simplification (viii), $\zeta + \kappa < 1 - \lambda < 1$, so $\zeta/(1 - \kappa) < 1$ and $(1 - \kappa)I - \zeta \Omega_A = (1 - \kappa)[I - \frac{\zeta}{1 - \kappa} \Omega_A]$ is invertible by Lemma 1 (applied with M replaced by $\zeta/(1 - \kappa)$). Row-stochasticity gives $\Omega_A \mathbf{1} = \mathbf{1}$, hence $[(1 - \kappa)I - \zeta \Omega_A]^{-1} \mathbf{1} = \mathbf{1}/(1 - \kappa - \zeta)$. The unique solution of (C.3) is therefore

$$\log \bar{\mathbf{A}} = \frac{c}{1 - \kappa - \zeta} \mathbf{1}, \quad \bar{A}_s = \bar{A} \equiv \exp \left(\frac{c}{1 - \kappa - \zeta} \right) \quad \text{for every } s,$$

which reduces to the pure-network solution $c/(1 - \zeta)$ at $\kappa = 0$.

Step 6: Implied sector-symmetric quantities. With $\bar{A}_s = \bar{A}$ uniform, sectoral relative prices $\bar{q}_s = \mu \bar{w}/\bar{A} = \bar{q}$, sectoral output $\bar{Z}_s = \bar{R}/\bar{q} = \bar{Z}$, production labor $\bar{N}_{P,s} = \bar{Z}/\bar{A} = \bar{N}_P$, and the asset value $\bar{v}_s = (\epsilon - 1)\bar{R}/[\epsilon(1 - \tilde{\beta}_\kappa)\bar{A}] = \bar{v}$ are all sector-invariant. The Hulten/Domar weight is $\mu_s = \bar{q}_s \bar{Z}_s / (\sum_x \bar{q}_x \bar{Z}_x) = 1/S$. The steady-state share of government spending in sectoral revenue is $\xi^G \equiv \bar{\mathcal{G}}/(\bar{C} + \bar{\mathcal{G}})$, also sector-invariant, with private-consumption complement $\xi^C \equiv 1 - \xi^G$. $\square$

C.2 Log-Linearization of the Tractable Framework

This appendix derives the dynamic equations of Section 4.2—the value equation (32), the R&D first-order condition (33), and the knowledge recursion (34)—by log-linearizing the tractable framework around the homogeneous steady state of Appendix C.1.

Adoption: the internalized premium and the momentum term. With free-elasticity adoption, the composite knowledge input carries exponent ζ on the network bundle and exponent κ on the producer's own lagged stock, and the symmetric steady state of Appendix C.1 is unaffected: the steady-state cancellation between the R&D first-order condition and the law of motion applies with the total elasticity $\zeta + \kappa$ in place of ζ . Since the own-stock margin is internalized, the asset-value recursion becomes $v_{s,t} = \Theta_{s,t} + [(1 - \delta_A) + \kappa X_{s,t}/A_{s,t-1}] \mathbb{E}_t[\Lambda_{t,t+1} v_{s,t+1}]$, where $X_{s,t}$ is the idea flow with steady-state ratio $\bar{X}/\bar{A} = \delta_A$; the steady-state continuation factor is therefore $1 - \delta_A(1 - \kappa)$ and

the effective discount is $\tilde{\beta}_\kappa = \beta(1 - \delta_A(1 - \kappa))$. Log-linearizing, the premium contributes $\beta\kappa\delta_A(\hat{X}_{s,t} - \hat{A}_{s,t-1})$ to the shadow value.^{A.1} The law of motion delivers the identity $\hat{X}_{s,t} = \delta_A^{-1}(\hat{A}_{s,t} - (1 - \delta_A)\hat{A}_{s,t-1})$, and substituting,

$$\beta\kappa\delta_A(\hat{X}_{s,t} - \hat{A}_{s,t-1}) = \beta\kappa(\hat{A}_{s,t} - (1 - \delta_A)\hat{A}_{s,t-1} - \delta_A\hat{A}_{s,t-1}) = \beta\kappa(\hat{A}_{s,t} - \hat{A}_{s,t-1}),$$

the momentum term of Equation (32): the internalized complementarity premium loads exactly on the *growth* of the sectoral knowledge stock, with coefficient $b \equiv \beta\kappa$. Equating the contemporaneous momentum loading b to the saturation loading $(1 - \tilde{\beta}_\kappa)(1 + \xi^G)$ and solving for κ delivers the saturation-neutral value κ^* of Assumption 1. All remaining derivations below go through with $\Omega_A \mapsto \Omega_A^\kappa$ and $\tilde{\beta} \mapsto \tilde{\beta}_\kappa$; we display them in the benchmark notation.

The value equation. The static block delivers the inputs. Indivisible labor and log utility give $\hat{C}_t = \hat{w}_t$ (Equation (27)) and a stochastic discount factor $\hat{\Lambda}_{t,t+1} = \hat{C}_t - \mathbb{E}_t \hat{C}_{t+1} = \hat{w}_t - \mathbb{E}_t \hat{w}_{t+1}$; constant-markup pricing gives $\hat{q}_{s,t} = \hat{w}_t - \hat{A}_{s,t}$ (Equation (28)); and sectoral demand and the resource constraint give $\hat{C}_{s,t} = \hat{A}_{s,t}$ and $\hat{Y}_{s,t} = \xi^C \hat{A}_{s,t} + \xi^G \hat{G}_{s,t}$ (Equations (29)-(31)). Sectoral revenue in consumption units, $R_{s,t} \equiv q_{s,t} Z_{s,t}$, therefore obeys $\hat{R}_{s,t} = \hat{q}_{s,t} + \hat{Y}_{s,t} = \hat{w}_t + \xi^G(\hat{G}_{s,t} - \hat{A}_{s,t})$, and the marginal cost-saving flow $\Theta_{s,t} = (\epsilon - 1)\Pi_{s,t}/A_{s,t}$, with $\Pi_{s,t} = R_{s,t}/\epsilon$, obeys

$$\hat{\Theta}_{s,t} = \hat{R}_{s,t} - \hat{A}_{s,t} = \hat{w}_t + \xi^G \hat{G}_{s,t} - (1 + \xi^G) \hat{A}_{s,t}.$$

Log-linearizing the asset-value recursion around the steady state of Appendix C.1, where $\bar{\Theta}/\bar{v} = 1 - \tilde{\beta}_\kappa$, and adding the internalized-adoption premium $\beta\kappa(\hat{A}_{s,t} - \hat{A}_{s,t-1})$ derived in the previous paragraph,

$$\hat{v}_{s,t} = (1 - \tilde{\beta}_\kappa) \hat{\Theta}_{s,t} + \beta\kappa(\hat{A}_{s,t} - \hat{A}_{s,t-1}) + \tilde{\beta}_\kappa \mathbb{E}_t [\hat{\Lambda}_{t,t+1} + \hat{v}_{s,t+1}].$$

Subtracting $\hat{w}_t$ from both sides to obtain the wage-deflated value $\tilde{v}_{s,t} \equiv \hat{v}_{s,t} - \hat{w}_t$, the wage terms cancel exactly: the current wage in $\hat{\Theta}_{s,t}$ carries weight $1 - \tilde{\beta}_\kappa$, the discount-factor term contributes $\tilde{\beta}_\kappa(\hat{w}_t - \mathbb{E}_t \hat{w}_{t+1})$, and the continuation value contributes $\tilde{\beta}_\kappa \mathbb{E}_t \hat{w}_{t+1}$, which sum to $\hat{w}_t$. The result is the value equation (32).

^{A.1}The adoption bracket is dated as in the full-model recursion (13), with the current idea flow evaluated against the inherited stock. Under the contemporaneous timing of simplification (vii), the marginal adoption payoff of date- t knowledge accrues through $X_{s,t+1}$, which would date the bracket at $(X_{s,t+1}, A_{s,t})$ and shift the momentum term one period forward, to $\beta\kappa \mathbb{E}_t(\hat{A}_{s,t+1} - \hat{A}_{s,t})$. The two datings share the same steady state ($\bar{X}/\bar{A} = \delta_A$) and the same cumulative sum, so the propositions—which are proved under the no-saturation benchmark—are unaffected; Assumption 1 is stated for the contemporaneous dating.

The R&D first-order condition. Under the contemporaneous R&D implementation, the optimality condition for research labor equates the real wage to the marginal product of research labor valued at the shadow value of knowledge, $w_t = \lambda (X_{s,t}/N_{R,s,t}) v_{s,t}$, where $X_{s,t} = \chi N_{R,s,t}^\lambda [\prod_x A_{x,t-1}^{\zeta \omega_{A,s,x}}] A_{s,t-1}^\kappa$ is the idea flow. In log-deviations, $\hat{w}_t = \hat{X}_{s,t} - \hat{N}_{R,s,t} + \hat{v}_{s,t}$ with

$$\hat{X}_{s,t} = \lambda \hat{N}_{R,s,t} + \zeta \sum_{x=1}^S \omega_{A,s,x} \hat{A}_{x,t-1} + \kappa \hat{A}_{s,t-1}.$$

Solving for $\hat{N}_{R,s,t}$ and substituting $\hat{v}_{s,t} = \tilde{v}_{s,t} + \hat{w}_t$, the wage on the marginal-cost side cancels against the wage embedded in the asset value, and, collecting the knowledge terms with $\zeta + \kappa = (1 - \lambda)M$ and the adoption-adjusted weights $\omega_{A,s,x}^\kappa$, the condition collapses to the first-order condition (33).

The knowledge recursion. Log-linearizing the law of motion $A_{s,t} = (1 - \delta_A)A_{s,t-1} + X_{s,t}$ around the steady-state ratio $\bar{X}/\bar{A} = \delta_A$ gives $\hat{A}_{s,t} = (1 - \delta_A)\hat{A}_{s,t-1} + \delta_A \hat{X}_{s,t}$. Substituting the idea flow, with research labor given by Equation (33),

$$\hat{X}_{s,t} = \frac{\lambda}{1 - \lambda} \tilde{v}_{s,t} + M \sum_{x=1}^S \omega_{A,s,x}^\kappa \hat{A}_{x,t-1},$$

where the knowledge bundle collects the direct loading $(1 - \lambda)M$ and the loading λM passing through research labor. Stacking sectors delivers the vector recursion (34), with $\mathbf{T} = (1 - \delta_A)I + \delta_A M \Omega_A^\kappa$ and $\varpi = \delta_A \lambda / (1 - \lambda)$.

The cumulative knowledge response. Under Assumption 1, and dropping the residual lagged-momentum term, the value equation reduces to Equation (35), which solves forward as $\tilde{v}_{s,t} = (1 - \tilde{\beta}_\kappa) \xi^G \sum_{j \geq 0} \tilde{\beta}_\kappa^j \mathbb{E}_t \hat{G}_{s,t+j}$. With the i.i.d. spending process, the date-0 shock $\hat{\mathbf{G}}_0$ implies $\tilde{\mathbf{V}}_0 = (1 - \tilde{\beta}_\kappa) \xi^G \hat{\mathbf{G}}_0$ and $\tilde{\mathbf{V}}_t = \mathbf{0}$ for $t \geq 1$. The recursion (34) then gives $\hat{\mathbf{A}}_t = \mathbf{T}^t \varpi \tilde{\mathbf{V}}_0$, and cumulating with Lemma 2,

$$\sum_{t=0}^{\infty} \hat{\mathbf{A}}_t = (I - \mathbf{T})^{-1} \varpi (1 - \tilde{\beta}_\kappa) \xi^G \hat{\mathbf{G}}_0 = \frac{\varpi}{\delta_A} (1 - \tilde{\beta}_\kappa) \xi^G \mathcal{L}_A \hat{\mathbf{G}}_0 = \lambda \Gamma \mathcal{L}_A \hat{\mathbf{G}}_0,$$

using $\varpi / \delta_A = \lambda / (1 - \lambda)$ and the impact pass-through $\Gamma = (1 - \tilde{\beta}_\kappa) \xi^G / (1 - \lambda)$: the cumulative-knowledge identity used throughout Appendix C.5.

C.3 The Saturation-Neutral Benchmark: Exact versus Approximate Dynamics

At $\kappa = \kappa^*$ the value equation (32) reads exactly

$$\tilde{v}_{s,t} = (1 - \tilde{\beta}_{\kappa^*})\xi^G \hat{G}_{s,t} - \beta\kappa^* \hat{A}_{s,t-1} + \tilde{\beta}_{\kappa^*} \mathbb{E}_t \tilde{v}_{s,t+1},$$

whereas the body works with the no-saturation form (35), which omits the lagged momentum leg $-\beta\kappa^* \hat{A}_{s,t-1}$. This subsection characterizes and quantifies the approximation. Substituting the exact value equation into the knowledge recursion delivers the second-order system of Appendix C.7, whose lag matrix is $\bar{\mathbf{T}} \equiv \mathbf{T} - \varpi\beta\kappa^* I$: the omitted term is a *uniform diagonal reduction in the persistence of the lagged own stock*—writing $\bar{\mathbf{T}} = (1 - \delta_A - \varpi\beta\kappa^*)I + \delta_A M\Omega_A^\kappa$, the wedge lowers the diagonal survival coefficient while holding the network replenishment term $\delta_A M\Omega_A^\kappa$ fixed. It is therefore a reduced-form obsolescence wedge, not literally equivalent to raising the primitive rate δ_A , which would also rescale the network term. Two implications follow. First, because the wedge is diagonal and commutes with the network, it introduces no new propagation paths and preserves the network’s eigenvectors; it does, however, shift the eigenvalues, and can thereby reweight the relative contribution of short and long citation paths—sectors whose upstreamness derives mainly from long chains can in principle be affected differently from sectors cited directly. The expansion below bounds this reweighting analytically and shows that it enters at the same order as the common rescaling. Second, because the cumulative operator compounds the wedge through the knowledge resolvent—whose largest eigenvalue is $1/[\delta_A(1-M)]$ —the effect on the *level* of cumulative responses is first order and grows without bound as the economy approaches the semi-endogenous boundary $\lambda + \zeta + \kappa \rightarrow 1$; the expansion below makes both statements exact.

We characterize the approximation analytically, by expanding the exact dynamics to first order in the wedge $\varpi b^* \equiv \varpi\beta\kappa^*$. Solving the exact value equation forward and evaluating the lagged-stock term along the benchmark path $\hat{\mathbf{A}}_t = \mathbf{T}^t \hat{\mathbf{A}}_0$, the cumulative shadow-value correction is

$$\sum_t \Delta \tilde{\mathbf{v}}_t = -\frac{b^*}{1 - \tilde{\beta}^*} \left[(I - \mathbf{T})^{-1} - \tilde{\beta}^{*2} (I - \tilde{\beta}^* \mathbf{T})^{-1} \right] \hat{\mathbf{A}}_0 + O((\varpi b^*)^2),$$

and propagating it through the knowledge recursion delivers the exact cumulative

response as a correction to the benchmark:

$$\sum_{t=0}^{\infty} \hat{\mathbf{A}}_t^{\text{exact}} = [I - \varpi b^* \mathbf{\Upsilon}] \sum_{t=0}^{\infty} \hat{\mathbf{A}}_t^{\text{bench}} + O((\varpi b^*)^2), \quad \mathbf{\Upsilon} \equiv \frac{1}{1 - \tilde{\beta}^*} \left[(I - \mathbf{T})^{-1} - \tilde{\beta}^{*2} (I - \tilde{\beta}^* \mathbf{T})^{-1} \right]. \quad (\text{C.4})$$

Three analytical properties of the correction operator $\mathbf{\Upsilon}$ pin down everything one needs to know about the approximation.

(i) *Sign: the approximation overstates responses, entrywise.* Expanding, $\mathbf{\Upsilon} = \frac{1}{1 - \tilde{\beta}^*} \sum_{s \geq 0} (1 - \tilde{\beta}^{*s+2}) \mathbf{T}^s$: every coefficient is strictly positive and $\mathbf{T}^s$ is entrywise non-negative, so $\mathbf{\Upsilon} \geq 0$ entrywise, with the same citation-path support as the Leontief operator. The benchmark therefore overstates every cumulative response, and does not distort responses of sector pairs unconnected by citation paths, which remain zero in both the exact and the approximate dynamics.

(ii) *Size: an infinity-norm error bound.* Since $\mathbf{T}\mathbf{1} = \varrho\mathbf{1}$, the row sums of $\mathbf{\Upsilon}$ are identical across sectors, $\mathbf{\Upsilon}\mathbf{1} = \ell(\varrho)\mathbf{1}$, where

$$\ell(\tau) \equiv \frac{1}{1 - \tilde{\beta}^*} \left[\frac{1}{1 - \tau} - \frac{\tilde{\beta}^{*2}}{1 - \tilde{\beta}^* \tau} \right].$$

Because $\mathbf{\Upsilon}$ is entrywise non-negative, this is precisely its infinity norm, and the first-order error obeys the norm bound

$$\left\| \sum_t \hat{\mathbf{A}}_t^{\text{bench}} - \sum_t \hat{\mathbf{A}}_t^{\text{exact}} \right\|_{\infty} \leq \mathcal{E} \left\| \sum_t \hat{\mathbf{A}}_t^{\text{bench}} \right\|_{\infty} + O((\varpi b^*)^2), \quad \mathcal{E} \equiv \varpi b^* \|\mathbf{\Upsilon}\|_{\infty} = \frac{\varpi \beta \kappa^*}{1 - \tilde{\beta}^*} \left[\frac{1}{\delta_A(1 - \tilde{\beta}^*)} - \frac{1}{\delta_A(1 - \tilde{\beta}^* \varrho)} \right] \quad (\text{C.5})$$

Two clarifications are important. First, this bounds the correction relative to the *largest* component of the benchmark response; it is not an entrywise proportional bound, and small components can be attenuated by proportionally more than $\mathcal{E}$. Second, uniform row sums do not imply uniform *column* sums— $\mathbf{1}^{\top} \mathbf{\Upsilon} \neq \ell(\varrho) \mathbf{1}^{\top}$ in general—so the correction is *not* a common rescaling of aggregate responses across target sectors: the proportional aggregate correction for a shock to sector x is $\varpi b^* \mathbf{1}^{\top} \mathbf{\Upsilon} \mathbf{B} \mathbf{E}_x / \mathbf{1}^{\top} \mathbf{B} \mathbf{E}_x$ with $\mathbf{B} \equiv (I - \mathbf{T})^{-1}$, and it varies with x . The leading term of $\mathcal{E}$ is $\lambda \beta \kappa^* / [(1 - \lambda)(1 - \tilde{\beta}^*)(1 - M)]$, so

$$\lambda \beta \kappa^* \ll (1 - \lambda)(1 - \tilde{\beta}^*)(1 - M) \quad (\text{C.6})$$

is a transparent *sufficient* leading-order condition for $\mathcal{E} \ll 1$ (the exact condition is $\mathcal{E} \ll 1$ itself, which also carries the subtractive second term). The condition is demanding only in two limits: high R&D elasticity λ , and proximity to the semi-endogenous boundary $M \rightarrow 1$, where the knowledge resolvent diverges and *any* perturbation of the dynamics is amplified without bound; accordingly, the

$O((\varpi b^*)^2)$ remainder of (C.4) is uniform only on parameter sets bounded away from $M = 1$.

(iii) *Cross-section: longer and more indirect responses are attenuated more.* The walk decomposition makes the heterogeneity of the correction exact for a directed, possibly non-diagonalizable network. Writing $\Upsilon = \sum_{s \geq 0} a_s \mathbf{T}^s$ with $a_s = (1 - \tilde{\beta}^{*s+2})/(1 - \tilde{\beta}^*)$ and $\mathbf{B} = \sum_{j \geq 0} \mathbf{T}^j$,

$$\Upsilon \mathbf{B} = \sum_{k \geq 0} \eta_k \mathbf{T}^k, \quad \eta_k \equiv \sum_{s=0}^k a_s,$$

and η_k is strictly increasing in k : relative to the benchmark—which weights every walk of length k equally in $\mathbf{B}$ —the first-order correction loads more heavily on longer walks. Responses that arrive through longer and more indirect citation chains are therefore attenuated proportionally *more* than direct responses. The paper’s two-sector example makes this concrete: with upstream sector 1 and downstream sector 2, the propagation matrix is triangular with diagonal (ϱ, ϱ_2) , $\varrho_2 < \varrho$, and for a shock to sector 1 the proportional attenuation of sector 1’s own knowledge response is $\varpi b^* \ell(\varrho)$, while that of sector 2’s propagated response is $\varpi b^* [\ell(\varrho_2) + (\ell(\varrho) - \ell(\varrho_2))/m] > \varpi b^* \ell(\varrho)$, where $m \in (0, 1)$ is the benchmark ratio of the propagated to the own response. The residual momentum term thus attenuates the propagated response more than the targeted sector’s own response, and target-specific aggregate corrections likewise differ across shocked sectors. Two consequences follow. First, exact own-to-upstream response ratios and target rankings are not preserved identically at first order: when they are the object of interest, they should be computed from the exact system of Appendix C.7. Second, path support is preserved exactly, and entrywise positivity of the exact knowledge operator is preserved wherever the benchmark entry is bounded away from zero relative to $\mathcal{E}$; entries of order $\mathcal{E}$ —for instance, active-R&D kernel entries that vanish in the benchmark—can change sign, so strict entrywise positivity of *active-R&D* responses is a property of the benchmark that survives only up to $O(\mathcal{E})$ terms.

Summarizing: at the saturation-neutral adoption, the residual lagged momentum term lowers cumulative knowledge responses to first order and preserves the network’s zero-path structure exactly. Its overall size is bounded in the infinity norm by the closed-form factor $\mathcal{E}$ of (C.5), with (C.6) a transparent sufficient condition on primitives; its target- and sector-specific proportional effects depend on network position and path length—longer and more indirect responses are attenu-

ated more—so exact own-to-upstream ratios and target rankings must be checked on the exact system of Appendix C.7 when they are the object of interest.

C.4 Matrix Facts

Maintained parameter restrictions. The lemmas of this subsection and all subsequent appendix results are stated under the following primitive parameter restrictions, which are maintained throughout the analysis and correspond to the economically interpretable region: discount factor $\beta \in (0, 1)$, knowledge depreciation rate $\delta_A \in (0, 1)$ with $\delta_A(1 + \lambda) \leq 1$, R&D share parameter $\lambda \in (0, 1)$, spillover elasticity $\zeta > 0$, and adoption elasticity $\kappa \geq 0$ with $\lambda + \zeta + \kappa < 1$. The depreciation bound—implied by $\delta_A \leq 1/2$ and satisfied at any monthly or annual calibration—is used only in the spectral bound for the momentum-shifted lag matrix of the full-envelope system. These imply $M \in (0, 1)$ (the spectral radius of $M\Omega_A^\kappa$ is then $M < 1$), $\tilde{\beta} \equiv \beta(1 - \delta_A) \in (0, 1)$, $1 - \delta_A \in (0, 1)$ (used in Lemma 2 below), and $\varrho \equiv 1 - \delta_A(1 - M) \in (0, 1)$.

Lemma 1 (Innovation Leontief Inverse). *Under simplification (viii), the matrix $I - M\Omega_A$ is invertible and*

$$\mathcal{L}_A = (I - M\Omega_A)^{-1} = \sum_{k=0}^{\infty} M^k \Omega_A^k = I + M\Omega_A + M^2\Omega_A^2 + \cdots, \quad (\text{C.7})$$

with the Neumann series converging absolutely. The network component admits the path decomposition $\mathcal{L}_A - I = \sum_{k \geq 1} M^k \Omega_A^k$, and for $M > 0$ the entry $[\mathcal{L}_A - I]_{s,x}$ is non-negative, strictly positive iff a positive-length directed citation path from s to x exists in Ω_A .

Proof. Row-stochasticity gives $\rho(\Omega_A) = 1$, so $\rho(M\Omega_A) = M < 1$. The Neumann series converges absolutely. For $k \geq 1$, $[\Omega_A^k]_{s,x}$ is a sum of nonnegative products over length- k directed walks; summing over $k \geq 1$ preserves the equivalence with path existence. $\square$

Lemma 2 (Stable Knowledge-Propagation Matrix). *The matrix $\mathbf{T} = (1 - \delta_A)I + \delta_A M\Omega_A$ has its spectrum contained in the closed disk*

$$\sigma(\mathbf{T}) \subset \{\tau \in \mathbb{C} : |\tau| \leq \varrho\}, \quad \varrho \equiv 1 - \delta_A(1 - M) < 1. \quad (\text{C.8})$$

In particular $\rho(\mathbf{T}) \leq \varrho < 1$, and

$$(I - \mathbf{T})^{-1} = \frac{1}{\delta_A} \mathcal{L}_A. \quad (\text{C.9})$$

Proof. Since Ω_A is row-stochastic and non-negative, $\sigma(\Omega_A) \subset \{z \in \mathbb{C} : |z| \leq 1\}$ (Perron-Frobenius), so any eigenvalue τ of $\mathbf{T}$ has the form $\tau = (1 - \delta_A) + \delta_A M z$ with $|z| \leq 1$, hence $|\tau| \leq (1 - \delta_A) + \delta_A M = \varrho < 1$. The identity $I - \mathbf{T} = \delta_A(I - M\Omega_A)$ together with Lemma 1 delivers $(I - \mathbf{T})^{-1} = \delta_A^{-1} \mathcal{L}_A$. $\square$

Lemma 3 (Scalar Riccati root separation). *Let $a \equiv 1 + \varpi(1 - \tilde{\beta})(1 + \xi^G) > 1$. For every complex τ with $|\tau| < 1$, the scalar quadratic*

$$\tilde{\beta} r^2 - (a + \tilde{\beta}\tau) r + \tau = 0 \quad (\text{C.10})$$

has exactly one root inside the unit circle and exactly one root outside. The stable root, denoted $r(\tau)$, is

$$r(\tau) = \frac{(a + \tilde{\beta}\tau) - \sqrt{(a + \tilde{\beta}\tau)^2 - 4\tilde{\beta}\tau}}{2\tilde{\beta}}, \quad (\text{C.11})$$

holomorphic on the open unit disk with $r(0) = 0$; the unstable root is $\tau/(\tilde{\beta} r(\tau))$, satisfying $\tau/(\tilde{\beta} r(\tau)) \rightarrow a/\tilde{\beta} > 1$ as $\tau \rightarrow 0$.

Proof. The product of the two roots of (C.10) equals $\tau/\tilde{\beta}$, so if one root is $r(\tau)$ the other is $\tau/(\tilde{\beta} r(\tau))$.

No unit-circle roots when $|\tau| < 1$. Suppose a root r of (C.10) satisfies $|r| = 1$. Solving (C.10) for τ gives $\tau = r(a - \tilde{\beta}r)/(1 - \tilde{\beta}r)$. Writing $r = e^{iv}$,

$$|\tau|^2 = \frac{a^2 + \tilde{\beta}^2 - 2a\tilde{\beta}\cos v}{1 + \tilde{\beta}^2 - 2\tilde{\beta}\cos v} = 1 + \frac{(a-1)(a+1-2\tilde{\beta}\cos v)}{1 + \tilde{\beta}^2 - 2\tilde{\beta}\cos v} > 1, \quad (\text{C.12})$$

because $a > 1$, $\tilde{\beta} \in (0, 1)$, the denominator $1 + \tilde{\beta}^2 - 2\tilde{\beta}\cos v \geq (1 - \tilde{\beta})^2 > 0$, and the numerator $(a-1)(a+1-2\tilde{\beta}\cos v) > 0$ (since $a+1-2\tilde{\beta} > 0$). Hence every root of (C.10) with $|r| = 1$ corresponds to some $|\tau| > 1$.

Continuity argument. At $\tau = 0$ the two roots of (C.10) are $\{0, a/\tilde{\beta}\}$ with $|0| < 1 < |a/\tilde{\beta}|$. The roots are continuous in τ on the connected open unit disk and never cross the unit circle (no unit-modulus roots when $|\tau| < 1$), so exactly one root remains inside and one outside for every $|\tau| < 1$. The stable root (C.11) is the inside-circle solution.

Holomorphic branch. The discriminant $\Delta(\tau) = (a + \tilde{\beta}\tau)^2 - 4\tilde{\beta}\tau = \tilde{\beta}^2\tau^2 + 2\tilde{\beta}(a-2)\tau + a^2$ is a quadratic in τ whose two zeros have modulus $a/\tilde{\beta} > 1$ (the product of the zeros of $\Delta(\tau)$ is $a^2/\tilde{\beta}^2$ and their sum is $-2(a-2)/\tilde{\beta}$, giving a conjugate pair with squared modulus $a^2/\tilde{\beta}^2$). Hence $\Delta(\tau) \neq 0$ on the open unit disk, a single-valued holomorphic branch of $\sqrt{\Delta(\tau)}$ with $\sqrt{\Delta(0)} = a$ is well-defined there, and $r(\tau)$ in (C.11) is holomorphic on $\{|\tau| < 1\}$ with $r(0) = 0$. $\square$

Lemma 4 (Blanchard-Kahn root separation for the matrix Riccati).

The matrix pencil $\mathcal{H}(z) \equiv \begin{pmatrix} \mathbf{0} & I \\ -\mathbf{T}/\tilde{\beta} & \mathbf{D}^{\text{env}}/\tilde{\beta} \end{pmatrix} - zI$ associated with the matrix quadratic $\tilde{\beta}\mathbf{R}^2 - \mathbf{D}^{\text{env}}\mathbf{R} + \mathbf{T} = \mathbf{0}$ has exactly S generalized eigenvalues of magnitude strictly less than one, and exactly S of magnitude strictly greater than one (no unit-modulus eigenvalues).

Proof. The statement is the standard Blanchard-Kahn root-count condition for the second-order linear rational-expectations system associated with the pencil $\mathcal{H}(z)$ (Blanchard and Kahn, 1980; Klein, 2000). We verify it directly from Lemmas 2 and 3, without reference to Lemma 5 below.

Under the real-spending specification, $\mathbf{D}^{\text{env}} = aI + \tilde{\beta}\mathbf{T}$ commutes with $\mathbf{T}$. Let $\mathbf{T} = \mathbf{Q}\mathbf{U}\mathbf{Q}^{-1}$ be a Schur triangularization with $\mathbf{U}$ upper-triangular and $\mathbf{Q}$ unitary; then $\mathbf{D}^{\text{env}} = \mathbf{Q}(aI + \tilde{\beta}\mathbf{U})\mathbf{Q}^{-1}$ is simultaneously triangularized. Substituting in $\mathcal{H}(z)$ and conjugating by $\text{diag}(\mathbf{Q}, \mathbf{Q})$ shows that the eigenvalues of $\mathcal{H}$ are exactly the union over $\tau \in \sigma(\mathbf{T})$ (the diagonal entries of $\mathbf{U}$, counted with algebraic multiplicity) of the eigenvalues of the 2×2 scalar pencil whose roots solve $\tilde{\beta}r^2 - (a + \tilde{\beta}\tau)r + \tau = 0$. The two scalar roots are $\{r(\tau), \tau/[\tilde{\beta}r(\tau)]\}$ for $\tau \neq 0$, and $\{0, a/\tilde{\beta}\}$ for $\tau = 0$ (since at $\tau = 0$ the scalar quadratic factors as $r(\tilde{\beta}r - a) = 0$).

By Lemma 2, $\sigma(\mathbf{T}) \subset \{|\tau| \leq \varrho < 1\}$. For $\tau \neq 0$, Lemma 3 gives $|r(\tau)| < 1 < |\tau/[\tilde{\beta}r(\tau)]|$. For $\tau = 0$, the two roots $\{0, a/\tilde{\beta}\}$ also separate strictly: $|0| < 1 < a/\tilde{\beta}$ since $a > 1 > \tilde{\beta}$. Counting eigenvalues with algebraic multiplicity over $\sigma(\mathbf{T})$ yields exactly S stable and S unstable generalized eigenvalues of $\mathcal{H}$, with no unit-modulus eigenvalues. $\square$

Lemma 5 (Stabilizing matrix root and exact propagation operator). Let $\mathbf{D}^{\text{env}} = aI + \tilde{\beta}\mathbf{T}$ with $a \equiv 1 + \varpi(1 - \tilde{\beta})(1 + \xi^G) > 1$. The matrix quadratic

$$\tilde{\beta}\mathbf{R}^2 - \mathbf{D}^{\text{env}}\mathbf{R} + \mathbf{T} = \mathbf{0} \tag{C.13}$$

admits a unique stabilizing matrix solution $\mathbf{R}_A$ with $\rho(\mathbf{R}_A) < 1$, defined via the holomorphic functional calculus as $\mathbf{R}_A = r(\mathbf{T})$, where $r(\cdot)$ is the stable scalar root of Lemma 3,

$$r(\tau) = \frac{(a + \tilde{\beta}\tau) - \sqrt{(a + \tilde{\beta}\tau)^2 - 4\tilde{\beta}\tau}}{2\tilde{\beta}}. \tag{C.14}$$

The spectral mapping theorem gives $\sigma(\mathbf{R}_A) = r(\sigma(\mathbf{T}))$; combining with $\sigma(\mathbf{T}) \subset \{|\tau| \leq \varrho < 1\}$ (Lemma 2) and $|r(\tau)| < 1$ on the open unit disk (Lemma 3) yields

$\rho(\mathbf{R}_A) < 1$. The exact propagation operator is

$$\mathcal{P}_A^{\text{env}} = (I - \mathbf{R}_A)^{-1} (\mathbf{D}^{\text{env}} - \tilde{\beta} \mathbf{R}_A)^{-1} = p(\mathbf{T}), \quad (\text{C.15})$$

where the scalar propagation function is

$$p(\tau) \equiv \frac{1}{[1 - r(\tau)] [a + \tilde{\beta}\tau - \tilde{\beta}r(\tau)]}, \quad (\text{C.16})$$

holomorphic on the open unit disk and defined at $\mathbf{T}$ by the holomorphic functional calculus. No diagonalizability of $\mathbf{T}$ or restriction to real spectrum is required.

Proof. *Existence.* By Lemma 3, the stable scalar root $r(\tau)$ in (C.14) is holomorphic on the open unit disk and satisfies the scalar Riccati $\tilde{\beta}r(\tau)^2 - (a + \tilde{\beta}\tau)r(\tau) + \tau = 0$ for every τ therein. By Lemma 2, $\sigma(\mathbf{T})$ is contained in the disk $\{|\tau| \leq \varrho\}$ with $\varrho < 1$, a compact subset of the open unit disk, so $\mathbf{R}_A \equiv r(\mathbf{T})$ is well-defined via the holomorphic functional calculus on $\sigma(\mathbf{T})$ (no diagonalizability of $\mathbf{T}$ required). Substituting $\tau \mapsto \mathbf{T}$ in the scalar Riccati and using the algebra-homomorphism property of the functional calculus,

$$\tilde{\beta} \mathbf{R}_A^2 - (aI + \tilde{\beta}\mathbf{T})\mathbf{R}_A + \mathbf{T} = \tilde{\beta} r(\mathbf{T})^2 - \mathbf{D}^{\text{env}} r(\mathbf{T}) + \mathbf{T} = \mathbf{0}, \quad (\text{C.17})$$

so $\mathbf{R}_A$ solves (C.13). The spectral mapping theorem combined with $|r(\tau)| < 1$ for $|\tau| < 1$ (Lemma 3) and $\sigma(\mathbf{T}) \subset \{|\tau| \leq \varrho < 1\}$ (Lemma 2) gives $\rho(\mathbf{R}_A) = \max_{\tau \in \sigma(\mathbf{T})} |r(\tau)| < 1$, so $\mathbf{R}_A$ is stabilizing.

Uniqueness. Let $\tilde{\mathbf{R}}$ be any other matrix satisfying (C.13) with $\rho(\tilde{\mathbf{R}}) < 1$. The graph $\mathcal{G} \equiv \{(\mathbf{v}, \tilde{\mathbf{R}}\mathbf{v}) : \mathbf{v} \in \mathbb{C}^S\} \subset \mathbb{C}^{2S}$ is invariant under the companion linearization $\mathcal{H} + zI$ of (C.13), because $(\mathcal{H} + zI)|_{\mathcal{G}}$ acts as multiplication by $\tilde{\mathbf{R}}$ on the first component. Since $\rho(\tilde{\mathbf{R}}) < 1$, all eigenvalues of $\mathcal{H} + zI$ restricted to $\mathcal{G}$ are inside the open unit disk, so $\mathcal{G}$ is contained in the stable invariant subspace of $\mathcal{H}$. By Lemma 4, the stable invariant subspace has dimension exactly S ; $\mathcal{G}$ has dimension S by construction, so the two coincide. The same argument applied to $\mathbf{R}_A = r(\mathbf{T})$ shows that its graph also coincides with the stable invariant subspace. Both graphs therefore coincide with the same S -dimensional stable invariant subspace; two graphs over the first S coordinates that coincide as subspaces agree as functions, so $\tilde{\mathbf{R}} = \mathbf{R}_A = r(\mathbf{T})$.

Propagation operator. Substituting $\mathbf{R}_A = r(\mathbf{T})$ in (C.15) and using $\mathbf{D}^{\text{env}} - \tilde{\beta} \mathbf{R}_A = (aI + \tilde{\beta}\mathbf{T}) - \tilde{\beta}r(\mathbf{T}) = [a + \tilde{\beta}\tau - \tilde{\beta}r(\tau)](\mathbf{T})$ via the functional calculus delivers $\mathcal{P}_A^{\text{env}} = p(\mathbf{T})$. Inversions are well-defined on $\sigma(\mathbf{T}) \subset \{|\tau| < 1\}$ since $|r(\tau)| < 1$ implies $1 - r(\tau) \neq 0$, and $a + \tilde{\beta}\tau - \tilde{\beta}r(\tau) = \tau/r(\tau)$ for $\tau \neq 0$ (and equals $a > 0$ at $\tau = 0$) is also nonzero throughout $\sigma(\mathbf{T})$.

No-saturation benchmark. Under the simplified value equation (35) (saturation channel absent), the constant a collapses to 1, the scalar Riccati reduces to $r(\tau) = \tau$, $\mathbf{R}_A = \mathbf{T}$, and $p(\tau) = 1/(1 - \tau)$. Hence

$$\mathcal{P}_A^{\text{env}}|_{\text{no-sat}} = (I - \mathbf{T})^{-1} = \frac{1}{\delta_A} \mathcal{L}_A, \quad (\text{C.18})$$

recovering the no-saturation propagation operator of Appendix C.2. $\square$

C.5 Proofs of Propositions Without Saturation

This subsection proves Propositions 1, 2, and 3 under the simplified value equation (35). All three proofs use the cumulative knowledge response of Appendix C.2,

$$\sum_{t=0}^{\infty} \hat{\mathbf{A}}_t = \lambda \Gamma \mathcal{L}_A \hat{\mathbf{G}}_0, \quad (\text{C.19})$$

and the wage-aggregation identity $\hat{w}_t = (1/S) \mathbf{1}^\top \hat{\mathbf{A}}_t$, which yields $\sum_t \hat{w}_t = (\lambda \Gamma / S) \mathcal{U}_x \hat{G}_{x,0}$ for a one-period shock at sector x .

C.5.1 Proof of Proposition 1

Cumulating the R&D first-order condition (33) and using $\sum_t \hat{\mathbf{A}}_{t-1} = \sum_t \hat{\mathbf{A}}_t$ (since $\hat{\mathbf{A}}_{-1} = \mathbf{0}$),

$$\sum_{t=0}^{\infty} \hat{N}_{R,s,t} = \frac{1}{1 - \lambda} \sum_t \tilde{v}_{s,t} + M [\Omega_A]_{s,\cdot} \sum_t \hat{\mathbf{A}}_t.$$

Under (35), $\sum_t \tilde{v}_{s,t} = (1 - \tilde{\beta}) \xi^G [I]_{s,x} \hat{G}_{x,0}$ for a one-period shock at sector x , so the first term equals $\Gamma [I]_{s,x} \hat{G}_{x,0}$. Substituting (C.19) into the second term and using $M \Omega_A \mathcal{L}_A = \mathcal{L}_A - I$,

$$\sum_{t=0}^{\infty} \hat{N}_{R,s,t} = \Gamma [I]_{s,x} \hat{G}_{x,0} + \lambda \Gamma [\mathcal{L}_A - I]_{s,x} \hat{G}_{x,0}.$$

The R&D wage bill is $\hat{\mathcal{N}}_{R,s,t} = \hat{w}_t + \hat{N}_{R,s,t}$; cumulating and substituting the wage-aggregation identity gives (42). $\square$

C.5.2 Proof of Proposition 2

From the sectoral real-GDP identity (31), $\hat{Y}_{s,t} = \xi^C \hat{A}_{s,t} + \xi^G \hat{G}_{s,t}$. For a one-period shock $\hat{G}_{x,0}$ at sector x , $\sum_t \hat{G}_{s,t} = [I]_{s,x} \hat{G}_{x,0}$. Cumulating and substituting (C.19),

$$\sum_{t=0}^{\infty} \hat{Y}_{s,t} = \xi^C \lambda \Gamma [\mathcal{L}_A]_{s,x} \hat{G}_{x,0} + \xi^G [I]_{s,x} \hat{G}_{x,0}.$$

Splitting $[\mathcal{L}_A]_{s,x} = [I]_{s,x} + [\mathcal{L}_A - I]_{s,x}$ yields (43). $\square$

C.5.3 Proof of Proposition 3

Aggregating the sectoral identity of Proposition 2 via $\hat{Y}_t = (1/S)\mathbf{1}^\top \hat{\mathbf{Y}}_t$ and using $\sum_s [I]_{s,x} = 1$ and $\mathbf{1}^\top \mathcal{L}_A \mathbf{E}_x = \mathcal{U}_x$:

$$\sum_{t=0}^{\infty} \hat{Y}_t = \frac{\xi^G + \xi^C \lambda \Gamma}{S} \hat{G}_{x,0} + \frac{\xi^C \lambda \Gamma}{S} (\mathcal{U}_x - 1) \hat{G}_{x,0} = \left[\frac{\xi^G}{S} + \frac{\xi^C \lambda \Gamma}{S} \mathcal{U}_x \right] \hat{G}_{x,0}.$$

Converting into dollars, with $\sum_t \Delta Y_t = \bar{Y} \sum_t \hat{Y}_t$ and $\Delta G_{x,0} = \bar{G}_x \hat{G}_{x,0}$, and using the sector-invariant nominal spending share $\bar{G}_x / \bar{Y} = \xi^G / S$ of the homogeneous steady state, delivers the multiplier (45). The upstreamness ranking follows because the multiplier is strictly increasing in $\mathcal{U}_x$, with strictly positive coefficient $\xi^C \lambda \Gamma / \xi^G$. $\square$

C.5.4 Proof of Proposition 4

The proof applies Proposition 3 in the network economy and in the diagonal counterfactual, takes the difference, and uses the row-stochastic structure of Ω_A to identify the constant in (48) with the cross-sectional average upstreamness.

Step 1: Aggregate response in the network economy. By Proposition 3 (equation (44)), the cumulative aggregate GDP response to a sector- x government spending shock $\hat{G}_{x,0}$ in the network economy reads

$$\sum_{t=0}^{\infty} \hat{Y}_t = \left[\frac{\xi^G}{S} + \frac{\xi^C \lambda \Gamma}{S} \mathcal{U}_x \right] \hat{G}_{x,0}, \quad (\text{C.20})$$

where $\mathcal{U}_x = \mathbf{1}^\top \mathcal{L}_A \mathbf{E}_x$ and $\mathcal{L}_A = (I - M \Omega_A)^{-1}$ as in (36)–(37).

Step 2: Aggregate response in the diagonal counterfactual. Replace Ω_A by $\Omega_A^{\text{diag}} \equiv I$ in the simplified economy of Section 4.1, holding all other primitives $(\beta, \delta_A, \lambda, \zeta, \kappa, \chi, \xi^G, S)$ fixed. Three facts make Proposition 3 carry over verbatim with Ω_A replaced by I . (i) The steady state remains sector-symmetric: equation (C.3) becomes $(1 - \kappa - \zeta) \log \bar{\mathbf{A}} = c \mathbf{1}$, still solved by $\log \bar{A}_s = c / (1 - \kappa - \zeta)$ for every s , so the Hulten weights $\mu_s = 1/S$ and the constant ξ^G are unchanged. (ii) The impact pass-through $\Gamma \equiv (1 - \tilde{\beta}) \xi^G / (1 - \lambda)$ is independent of Ω_A . (iii) None of the derivations leading to Proposition 3—the wage-aggregation identity $\hat{w}_t = (1/S) \mathbf{1}^\top \hat{\mathbf{A}}_t$, the cumulative-knowledge identity $\sum_t \hat{\mathbf{A}}_t = \lambda \Gamma \mathcal{L}_A \hat{\mathbf{G}}_0$, and the cross-sectoral GDP aggregation $\hat{Y}_t = (1/S) \mathbf{1}^\top \hat{\mathbf{Y}}_t$ —use any property of Ω_A beyond its appearance in $\mathcal{L}_A$ via the Leontief inverse, and all go through verbatim with Ω_A replaced by Ω_A^{diag} . Proposition 3 therefore applies in the diagonal

counterfactual with $\mathcal{L}_A$ replaced by

$$\mathcal{L}_A^{\text{diag}} = (I - M \Omega_A^{\text{diag}})^{-1} = (I - M I)^{-1} = \frac{1}{1 - M} I, \quad (\text{C.21})$$

and the upstreamness of sector x replaced by

$$\mathcal{U}_x^{\text{diag}} = \mathbf{1}^\top \mathcal{L}_A^{\text{diag}} \mathbf{E}_x = \frac{1}{1 - M} \mathbf{1}^\top \mathbf{E}_x = \frac{1}{1 - M}. \quad (\text{C.22})$$

The cumulative aggregate GDP response in the diagonal counterfactual is therefore

$$\sum_{t=0}^{\infty} \hat{Y}_t^{\text{diag}} = \left[\frac{\xi^G}{S} + \frac{\xi^C \lambda \Gamma}{S} \cdot \frac{1}{1 - M} \right] \hat{G}_{x,0}, \quad (\text{C.23})$$

where the bracketed coefficient is the same regardless of which sector x receives the shock: in the absence of cross-sectoral knowledge spillovers, every sectoral target generates the same per-dollar aggregate response.

Step 3: Closed-form expression for $\mathcal{M}_x^{\text{idea}}$. Converting (C.20) and (C.23) into dollar multipliers—dividing each response by the same dollar spending change, $\Delta G_{x,0} = (\xi^G/S) \bar{Y} \hat{G}_{x,0}$, legitimate because the two economies share the same homogeneous steady state—and subtracting, the common direct component, equal to one, cancels, leaving

$$\mathcal{M}_x^{\text{idea}} = \frac{\xi^C \lambda \Gamma}{\xi^G} \mathcal{U}_x - \frac{\xi^C \lambda \Gamma}{\xi^G} \cdot \frac{1}{1 - M} = \frac{\xi^C \lambda \Gamma}{\xi^G} \left[\mathcal{U}_x - \frac{1}{1 - M} \right]. \quad (\text{C.24})$$

The cancellation is exact: the diagonal counterfactual leaves the own-sector demand mechanism untouched, so the idea multiplier isolates the cross-sectoral spillover channel.

Step 4: The constant equals the cross-sectional average upstreamness.

The innovation-network matrix Ω_A is row-stochastic, $\Omega_A \mathbf{1} = \mathbf{1}$, hence $\Omega_A^k \mathbf{1} = \mathbf{1}$ for every $k \geq 0$. The Neumann expansion (36) then gives

$$\mathcal{L}_A \mathbf{1} = \sum_{k \geq 0} M^k \Omega_A^k \mathbf{1} = \sum_{k \geq 0} M^k \mathbf{1} = \frac{1}{1 - M} \mathbf{1}, \quad (\text{C.25})$$

where the geometric series converges because $0 < M < 1$ (Section 4.1). Pre-multiplying by $(1/S) \mathbf{1}^\top$,

$$\bar{\mathcal{U}} \equiv \frac{1}{S} \sum_{s=1}^S \mathcal{U}_s = \frac{1}{S} \mathbf{1}^\top \mathcal{L}_A \mathbf{1} = \frac{1}{S} \mathbf{1}^\top \frac{1}{1 - M} \mathbf{1} = \frac{1}{1 - M}, \quad (\text{C.26})$$

which is the equality stated in (49). Substituting back into the closed form of Step 3 yields (48).

Step 5: Sign condition. Since $\xi^C > 0$, $\lambda > 0$, $\Gamma > 0$, and $\xi^G > 0$, the prefactor $\xi^C \lambda \Gamma / \xi^G$ in (48) is strictly positive, so the sign of $\mathcal{M}_x^{\text{idea}}$ coincides with the sign of $\mathcal{U}_x - \bar{\mathcal{U}}$, yielding the sign condition (50). $\square$

C.6 Two-Sector Calculations Without Saturation

This appendix presents the two-sector example formulas of Sections 4.3–4.6, displayed in the pure-network benchmark $\kappa = 0$ (Appendix C records the substitutions delivering the general- κ expressions). The upstream-downstream network of (39) has $\omega_{A,1,1} = 1$, $\omega_{A,2,1} = 1$, $\omega_{A,1,2} = \omega_{A,2,2} = 0$. The transition matrix $\mathbf{T} = (1 - \delta_A)I + \delta_A M \Omega_A$ is lower-triangular with eigenvalues $\varrho \equiv 1 - \delta_A + \delta_A M$ (right eigenvector $\mathbf{1}$) and $d \equiv 1 - \delta_A$ (right eigenvector $\mathbf{E}_2$).

Cumulative responses under (35). Under (35), $\mathbf{D}^{\text{env}} = I + \tilde{\beta} \mathbf{T}$, $\mathbf{R}_A = \mathbf{T}$, and $\mathcal{P}_A^{\text{env}} = \delta_A^{-1} \mathcal{L}_A$ with

$$\mathcal{L}_A = \begin{pmatrix} 1/(1-M) & 0 \\ M/(1-M) & 1 \end{pmatrix}, \quad \mathcal{U}_1 = \frac{1+M}{1-M}, \quad \mathcal{U}_2 = 1. \quad (\text{C.27})$$

The matrix $(I - \zeta \Omega_A) \mathcal{L}_A = (1 - \lambda)I + \lambda \mathcal{L}_A$ has diagonal $((1 - M + \lambda M)/(1 - M), 1)$ and off-diagonal $(2, 1)$ entry $\lambda M/(1 - M)$, both strictly positive. The benchmark formulas for sectoral R&D, sectoral GDP, and aggregate GDP follow.

C.6.1 Knowledge dynamics, no-saturation benchmark

The cumulative knowledge response is $\sum_t \hat{\mathbf{A}}_t = \varpi(1 - \tilde{\beta}) \xi^G \delta_A^{-1} \mathcal{L}_A \hat{\mathbf{G}}_0 = \lambda \Gamma \mathcal{L}_A \hat{\mathbf{G}}_0$. For an upstream shock $\hat{G}_{1,0} > 0$, $\sum_t \hat{A}_{1,t} = \lambda \Gamma \hat{G}_{1,0}/(1 - M)$ and $\sum_t \hat{A}_{2,t} = \lambda \Gamma M \hat{G}_{1,0}/(1 - M)$. For a downstream shock, $\sum_t \hat{A}_{1,t} = 0$ and $\sum_t \hat{A}_{2,t} = \lambda \Gamma \hat{G}_{2,0}$.

C.6.2 R&D wage bill, no-saturation benchmark

Substituting into (42):

Upstream shock.

$$\begin{aligned} \sum_t \hat{\mathcal{N}}_{R,1,t} \Big|_{\text{no-sat}} &= \Gamma \left(1 + \frac{\lambda M}{1 - M} \right) \hat{G}_{1,0} + \frac{\lambda \Gamma}{2} \frac{1 + M}{1 - M} \hat{G}_{1,0}, \\ \sum_t \hat{\mathcal{N}}_{R,2,t} \Big|_{\text{no-sat}} &= \frac{\Gamma \lambda M}{1 - M} \hat{G}_{1,0} + \frac{\lambda \Gamma}{2} \frac{1 + M}{1 - M} \hat{G}_{1,0}. \end{aligned}$$

Downstream shock.

$$\sum_t \hat{\mathcal{N}}_{R,1,t} \Big|_{\text{no-sat}} = \frac{\lambda \Gamma}{2} \hat{G}_{2,0}, \quad \sum_t \hat{\mathcal{N}}_{R,2,t} \Big|_{\text{no-sat}} = \Gamma \hat{G}_{2,0} + \frac{\lambda \Gamma}{2} \hat{G}_{2,0}. \quad (\text{C.28})$$

C.6.3 Sectoral and aggregate real GDP, no-saturation benchmark

From (43): $\sum_t \hat{Y}_{s,t} = (\xi^G + \xi^C \Gamma \lambda) [I]_{s,x} \hat{G}_{x,0} + \xi^C \Gamma \lambda [\mathcal{L}_A - I]_{s,x} \hat{G}_{x,0}$.

Upstream shock.

$$\begin{aligned}\sum_t \hat{Y}_{1,t} \Big|_{\text{no-sat}} &= \left[\xi^G + \frac{\xi^C \Gamma \lambda}{1-M} \right] \hat{G}_{1,0}, \\ \sum_t \hat{Y}_{2,t} \Big|_{\text{no-sat}} &= \frac{\xi^C \Gamma \lambda M}{1-M} \hat{G}_{1,0}.\end{aligned}$$

Downstream shock.

$$\sum_t \hat{Y}_{1,t} \Big|_{\text{no-sat}} = 0, \quad \sum_t \hat{Y}_{2,t} \Big|_{\text{no-sat}} = [\xi^G + \xi^C \Gamma \lambda] \hat{G}_{2,0}. \quad (\text{C.29})$$

Sector 1's response to a downstream shock is exactly zero: no citation path runs from sector 1 to sector 2, and under real-spending shocks there is no wage-offset channel that would otherwise leak the aggregate wage rise into a price decline at sector 1.

Aggregate, no-saturation benchmark. With $\mu_1 = \mu_2 = 1/2$:

$$\frac{1}{\hat{G}_{1,0}} \sum_t \hat{Y}_t \Big|_{\text{no-sat}} = \frac{\xi^G}{2} + \frac{\xi^C \lambda \Gamma}{2} \cdot \frac{1+M}{1-M}, \quad \frac{1}{\hat{G}_{2,0}} \sum_t \hat{Y}_t \Big|_{\text{no-sat}} = \frac{\xi^G}{2} + \frac{\xi^C \lambda \Gamma}{2}, \quad (\text{C.30})$$

with upstream-downstream gap $\xi^C \lambda \Gamma M / (1-M) > 0$, growing without bound as $M \uparrow 1$.

C.7 The Exact System Under Saturation-Neutral Adoption

This appendix states and solves the *exact* dynamic system under Assumption 1, retaining the lagged momentum leg that the body drops. Write $\tilde{\beta}^* \equiv \tilde{\beta}_{\kappa^*}$ and $b^* \equiv \beta \kappa^* = (1 - \tilde{\beta}^*)(1 + \xi^G)$, the momentum coefficient at the saturation-neutral adoption. Combining the exact value equation—Equation (32) at $\kappa = \kappa^*$, where the contemporaneous own-stock loading vanishes—with the R&D first-order condition (33) and the knowledge law of motion (34), and eliminating $\tilde{\mathbf{V}}_t$, delivers the second-order matrix system

$$\tilde{\beta}^* \mathbb{E}_t \hat{\mathbf{A}}_{t+1} - \mathbf{D}^* \hat{\mathbf{A}}_t + \bar{\mathbf{T}} \hat{\mathbf{A}}_{t-1} + \varpi (1 - \tilde{\beta}^*) \xi^G \hat{\mathbf{G}}_t = \mathbf{0}, \quad (\text{C.31})$$

where $\bar{\mathbf{T}} \equiv \mathbf{T} - \varpi b^* \mathbf{I}$ is the momentum-shifted lag matrix and, regrouping the diagonal,

$$\mathbf{D}^* = \bar{a}^* \mathbf{I} + \tilde{\beta}^* \bar{\mathbf{T}}, \quad \bar{a}^* \equiv 1 + \varpi (1 - \tilde{\beta}^*) \tilde{\beta}^* (1 + \xi^G) > 1,$$

using $1 + \xi^G - b^* = (1 + \xi^G) \tilde{\beta}^*$. Two spectral facts make the lemmas of Appendix C.4 applicable to the triple $(\tilde{\beta}^*, \bar{a}^*, \bar{\mathbf{T}})$ in place of $(\tilde{\beta}, a, \mathbf{T})$. First, $\bar{a}^* > 1$ holds unconditionally. Second, $\rho(\bar{\mathbf{T}}) < 1$: in the admissible region $\kappa^* < 1 - \lambda$, so

$\varpi b^* = \frac{\delta_A \lambda}{1-\lambda} \beta \kappa^* < \delta_A \lambda \leq 1 - \delta_A$ by the maintained depreciation bound $\delta_A(1+\lambda) \leq 1$, whence every eigenvalue of $\bar{\mathbf{T}}$ satisfies $|\bar{\tau}| \leq (1 - \delta_A - \varpi b^*) + \delta_A M = \varrho - \varpi b^* < 1$. Since $\bar{\mathbf{T}}$ is an affine function of Ω_A , all operators commute, the scalar root-separation argument of Lemma 3 applies, and Lemma 5 delivers the unique stabilizing root $\mathbf{R} = r(\bar{\mathbf{T}})$ and the exact propagation operator

$$\mathcal{P}^* \equiv (I - \mathbf{R})^{-1}(\mathbf{D}^* - \tilde{\beta}^* \mathbf{R})^{-1}, \quad \sum_{t=0}^{\infty} \hat{\mathbf{A}}_t = \varpi (1 - \tilde{\beta}^*) \xi^G \mathcal{P}^* \hat{\mathbf{G}}_0.$$

Setting the wedge ϖb^* to zero—the approximation adopted in the body—gives $\bar{\mathbf{T}} = \mathbf{T}$, $\bar{a}^* \rightarrow 1$ in the relevant grouping, the quadratic factorizes with stable root $\mathbf{R} = \mathbf{T}$, and $\mathcal{P}^* = (I - \mathbf{T})^{-1} = \delta_A^{-1} \mathcal{L}_A$: the Leontief benchmark underlying Propositions 1–4. Appendix C.3 characterizes the distance between the exact operator $\mathcal{P}^*$ and the benchmark analytically, to first order in the wedge, and states the accuracy condition under which the benchmark analysis is reliable.

D Quantitative Results: Robustness

This appendix provides additional results on the quantitative analysis of Section 5. We first assess the robustness of the results to a battery of alternative model specifications; we then isolate the role of the production network in the transmission of sectoral government spending.

In the robustness of the quantitative results, we consider ten alternative specifications: (i) perfect labor mobility across industries; (ii)–(iii) Frisch elasticities of labor supply of 1 and of 4; (iv)–(vi) sticky wages, for both labor types, for production wages only, and for research wages only; (vii) government consumption employing public labor; (viii) sector-specific capital stocks accumulated through the investment network as in vom Lehn and Winberry (2022); and (ix)–(x) a two-agent economy à la Cantore and Freund (2021), in which workers save subject to portfolio adjustment costs disciplined by the micro evidence on marginal propensities to consume, in a flexible-wage and a sticky-wage configuration.

Each variant modifies one block of the model, holding the rest of the calibration fixed, and is solved with the same first-order perturbation as the baseline; all multipliers follow the convention of Section 5.2. For each variant we additionally solve the stationary counterpart without the R&D block and the roundabout-network counterfactual—the same economy with the innovation network being the identity matrix—which delivers the variant-specific idea multiplier of Section 5.4.

Before describing each case in detail, we summarize the overall picture that emerges from these exercises in Tables D.19 and D.20. Table D.19 reports the range of the sectoral-spending multipliers in the variant’s baseline economy and in its counterpart without the R&D block, together with the correlation between the variant’s and the baseline economy’s multipliers; Table D.20 reports the range of the variant’s idea multipliers, together with their correlation with the baseline idea multipliers.

Across cases (i)–(viii), the ranges move moderately around the baseline, and the correlation between each variant’s multipliers and the baseline’s is never below 0.89; the idea multipliers are essentially unchanged, with a correlation with the baseline idea multipliers of 0.99 or above, a mean between 0.06 and 0.12, and between 37% and 72% of the cross-sectional variance of the variant’s multipliers accounted for. In the two-agent economy of cases (ix) and (x), the demand of the high-MPC workers shifts multipliers up and reshuffles the middle of the

Table D.19: Robustness: Sectoral-Spending Multipliers.

	Without R&D	Baseline economy	Corr. with baseline
Baseline	[−1.06, 1.06]	[−0.14, 2.23]	—
Mobile labor	[−0.42, 0.95]	[0.09, 3.09]	0.89
Frisch elasticity 1	[−1.28, 0.78]	[−0.49, 1.48]	0.96
Frisch elasticity 4	[−0.90, 1.27]	[0.13, 2.79]	0.98
Sticky wages, both types	[−1.09, 1.07]	[−0.15, 2.27]	1.00
Sticky wages, production only	[−1.09, 1.07]	[−0.14, 2.23]	1.00
Sticky wages, research only	[−1.06, 1.06]	[−0.16, 2.27]	1.00
Public labor	[−0.12, 1.03]	[0.36, 2.03]	0.98
Sector-specific capital & inv. network	[−0.86, 1.02]	[−0.06, 1.84]	0.92
Two-agent economy, flexible wages	[−1.05, 2.00]	[−0.26, 2.68]	0.75
Two-agent economy, sticky wages	[−0.73, 1.98]	[−0.19, 2.42]	0.65

Note: Ranges of the long-run present-value sectoral-spending multipliers across the 66 industries, for the economy without the R&D block and the baseline economy, under each alternative specification. The last column reports the correlation between the sectoral-spending multipliers of the variant’s baseline economy and those of the baseline model.

Table D.20: Robustness: Idea Multipliers.

	Range	Corr. with baseline
Baseline	[−0.00, 1.86]	—
Mobile labor	[−0.10, 2.44]	0.99
Frisch elasticity 1	[0.01, 1.57]	1.00
Frisch elasticity 4	[−0.01, 2.06]	1.00
Sticky wages, both types	[−0.00, 1.89]	1.00
Sticky wages, production only	[−0.00, 1.87]	1.00
Sticky wages, research only	[−0.01, 1.90]	1.00
Public labor	[0.00, 1.36]	1.00
Sector-specific capital & inv. network	[0.00, 1.42]	0.99
Two-agent economy, flexible wages	[−0.07, 2.16]	0.97
Two-agent economy, sticky wages	[−0.07, 2.16]	0.97

Note: Range of the sectoral idea multipliers across the 66 industries under each alternative specification: the difference between the variant’s sectoral-spending multipliers and those of the same economy with the innovation network Ω_A replaced by the identity matrix. The last column reports the correlation between the variant’s idea multipliers and those of the baseline model.

distribution, yet the correlation with the baseline multipliers remains very high, at 0.75 with flexible wages and 0.65 with sticky wages, and the idea multipliers stay closely aligned with the baseline ones, with a correlation of 0.97 in both configurations. In every variant, the largest idea multiplier still belongs to computers/electronics. We now describe each modification, and discuss its results, in turn.

Mobile labor. We replace the segmented labor markets of Section 3.1 with perfect mobility of both labor types across industries. Sectoral wages are equalized, $W_{P,s,t} = W_{P,t}$ and $W_{R,s,t} = W_{R,t}$ for every s , and the household supplies the two aggregate labor bundles $N_{P,t} = \sum_s N_{P,s,t}$ and $N_{R,t} = \sum_s N_{R,s,t}$ according to

$$\theta_P N_{P,t}^{\eta_P} = \frac{W_{P,t}}{P_t} C_t^{-1}, \quad \theta_R N_{R,t}^{\eta_R} = \frac{W_{R,t}}{P_t} C_t^{-1}, \quad (\text{D.1})$$

with the sectoral allocation determined by labor demand at the common wage. The steady state is unchanged, since sectoral wages are equalized there in any case: mobility alters only the propagation of shocks.

Under segmentation, a sectoral spending shock bids up the sector's own wages, which dampens the expansion of production and research employment in the shocked sector; under mobility, labor flows in from the other 65 industries at an essentially flat wage, so both the own response and the reallocation toward the knowledge hubs are amplified. This is visible in Table D.19 and Figure D.1: the upper tail of the sectoral distribution stretches, with the largest multiplier rising from 2.23 to 3.09, as mobility redistributes the response across sectors toward the knowledge hubs. The correlation with the baseline multipliers, at 0.89, is the lowest across cases (i)–(viii), precisely because mobility amplifies the innovation-network channel rather than muting it: the idea multipliers stretch to a maximum of 2.44, still attained by computers/electronics, followed by publishing/software and chemicals/pharmaceuticals, and the heterogeneous innovation network accounts for 72% of the cross-sectional variance of the multipliers, the largest contribution across all variants.

Frisch elasticity. We consider two alternative values of the Frisch elasticity of both labor types. The first is a Frisch elasticity of 1 ($\eta_P = \eta_R = 1$), which is a standard value in the business-cycle literature, consistent with the macro elasticity based on the aggregation of micro-level labor supply responses of Kneip et al. (2020). The second is a Frisch elasticity of 4 ($\eta_P = \eta_R = 1/4$), in line with the

higher values typically used by the fiscal-multiplier literature (Ramey, 2021). In both cases, the labor disutility levels are re-derived so that steady-state hours remain at 0.33 for each labor type, leaving the steady state otherwise identical to the baseline.

The elasticity of labor supply scales the output response to any demand impulse (Figures D.2 and D.3): all multipliers shift down under the Frisch elasticity of 1—the range compresses to $[-0.49, 1.48]$ —and up under the Frisch elasticity of 4—the range moves to $[0.13, 2.79]$ —while the correlations with the baseline multipliers remain high, at 0.96 and 0.98, respectively. The idea multipliers scale in the same direction: computers/electronics remains the sector with the largest idea multiplier in both cases (1.57 and 2.06), followed by publishing/software and chemicals/pharmaceuticals, and the heterogeneous innovation network accounts for 37% of the cross-sectional variance of the multipliers under the elasticity of 1 and 59% under the elasticity of 4.

Sticky wages. We relax the assumption of flexible wages of the baseline model by introducing nominal wage rigidity, analogously to the way the baseline model treats prices: the wages of both labor types are set by monopolistically competitive unions, subject to a quadratic adjustment cost à la Rotemberg (1982), the wage-setting counterpart of the producers' price-adjustment friction.

The block adds one layer between the household and the producers, separately for each sector and each labor type. In each sector s , a continuum of unions indexed by $j \in [0, 1]$ differentiates the household's production labor into varieties $N_{P,s,t}^j$, which a competitive labor packer assembles into the bundle demanded by producers with the CES technology

$$N_{P,s,t} = \left[\int_0^1 (N_{P,s,t}^j)^{\frac{\epsilon_W - 1}{\epsilon_W}} dj \right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \quad (\text{D.2})$$

where ϵ_W is the elasticity of substitution across labor varieties.

Cost minimization by the packer delivers the demand schedule for each variety

$$N_{P,s,t}^j = \left(\frac{W_{P,s,t}^j}{W_{P,s,t}} \right)^{-\epsilon_W} N_{P,s,t}, \quad (\text{D.3})$$

with the sectoral wage index

$$W_{P,s,t} = \left[\int_0^1 (W_{P,s,t}^j)^{1-\epsilon_W} dj \right]^{\frac{1}{1-\epsilon_W}}. \quad (\text{D.4})$$

Union j sets the nominal wage of its variety on behalf of the household: it

maximizes the real wage income of the hours it commits, net of their disutility—converted into consumption units by the household’s marginal utility, which under log preferences amounts to multiplying by C_t —and of a quadratic cost of adjusting the nominal wage, proportional to the sectoral wage bill. Since all pay-offs are expressed in consumption units, the union discounts them with the real stochastic discount factor:

$$\begin{aligned} \max_{\{W_{P,s,t}^j\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^{\mathcal{R}} \left[\frac{W_{P,s,t}^j N_{P,s,t}^j}{P_t} - \theta_{P,s} \frac{(N_{P,s,t}^j)^{1+\eta_P}}{1+\eta_P} C_t - \dots \right. \\ \left. \dots - \frac{\vartheta_W}{2} \left(\frac{W_{P,s,t}^j / W_{P,s,t-1}^j}{\bar{\pi}^W} - 1 \right)^2 \frac{W_{P,s,t} N_{P,s,t}}{P_t} \right], \quad (\text{D.5}) \end{aligned}$$

subject to the demand schedule for its variety, where $\bar{\pi}^W$ is nominal wage inflation along the balanced growth path, so that no adjustment cost is paid on the balanced path.

The first-order condition, evaluated at the symmetric equilibrium in which all unions of a given sector and labor type set the same wage, delivers the sectoral wage Phillips curve

$$\begin{aligned} \vartheta_W \left(\frac{\pi_{s,t}^W}{\bar{\pi}^W} - 1 \right) \frac{\pi_{s,t}^W}{\bar{\pi}^W} = (1 - \epsilon_W) + \epsilon_W \frac{\theta_{P,s} N_{P,s,t}^{\eta_P} C_t}{W_{P,s,t} / P_t} + \dots \\ \dots + \vartheta_W \mathbb{E}_t \left[\Lambda_{t,t+1}^{\mathcal{R}} \left(\frac{\pi_{s,t+1}^W}{\bar{\pi}^W} - 1 \right) \frac{\pi_{s,t+1}^W}{\bar{\pi}^W} \frac{W_{P,s,t+1} N_{P,s,t+1} / P_{t+1}}{W_{P,s,t} N_{P,s,t} / P_t} \right], \quad (\text{D.6}) \end{aligned}$$

where $\pi_{s,t}^W \equiv W_{P,s,t} / W_{P,s,t-1}$ is gross nominal wage inflation, $\bar{\pi}^W$ its balanced-growth value—the trend growth of nominal wages, which at zero trend inflation equals the real-wage growth factor γ_Y —and the fraction in the second term is the ratio of the marginal rate of substitution to the real wage, the wage-setting analogue of the marginal-cost-to-price ratio in the price Phillips curve (B.8). An analogous condition holds for research labor, with the corresponding disutility level and Frisch elasticity.

The parameters are homogeneous across industries and labor types: the wage elasticity of substitution across labor varieties is $\epsilon_W = 4$, implying a steady-state wage markup of 1.33, and $\vartheta_W = (\epsilon_W - 1)\psi_W / [(1 - \psi_W)(1 - \beta\psi_W)]$ maps a Calvo non-reset probability of $\psi_W = 11/12$ —a twelve-month average wage duration, the value typically estimated for U.S. wages—into the equivalent Rotemberg adjustment cost. Two implementation details preserve comparability with the baseline. First, the labor disutility levels are rescaled by $(\epsilon_W - 1)/\epsilon_W$, so that the steady

state carries the union wage markup while leaving all steady-state quantities and prices at their baseline values. Second, the flexible-wage benchmark used to construct the output gap in the monetary rule retains the same markup, so that the sticky- and flexible-wage blocks share the same steady state and the gap is well defined.

We consider three configurations: wage rigidity on both labor types, on production wages only, and on research wages only. In the one-sided configurations, the union block above applies verbatim to the rigid labor type, the other type retains the competitive labor supply condition of Section 3.1, and no other part of the model changes. In the economy without the R&D block, which employs production labor only, the production-only configuration coincides with the both-types one and the research-only configuration with the flexible-wage benchmark.

Table D.19 and Figures D.4–D.6 show that the three configurations deliver nearly identical results: the multipliers span at most $[-0.16, 2.27]$ and correlate with the baseline ones at 1.00; the largest idea multiplier belongs to computers/electronics in every configuration (1.87–1.90), followed by publishing/software and chemicals/pharmaceuticals; and the heterogeneous innovation network accounts for 52% of the cross-sectional variance of the multipliers in all three cases, exactly as in the baseline. Nominal wage rigidity is thus close to neutral for the cross-section of multipliers, whichever labor market it operates in.

Public labor. In our model, as is standard in the literature on fiscal multipliers, government spending consists of purchases of goods and services from the private sector. However, Moro and Rachedi (2022) show that such purchases account for only around 25% of total government consumption spending, with virtually the entirety of the remainder consisting of the compensation of public employees.

To account for the role of public employment, in this case we follow Moro and Rachedi (2022) and let government consumption combine purchases of goods with public labor. Government consumption output is produced with the technology

$$Y_{G,t} = N_{G,t}^{\alpha_G} G_t^{1-\alpha_G}, \quad (\text{D.7})$$

where G_t is the bundle of goods purchased from the private sectors, which retains the Cobb-Douglas sectoral composition and the spending process of Section 3.4, and $N_{G,t}$ is public employment, hired in a separate segmented labor market at wage $W_{G,t}$ with household supply

$$\theta_G N_{G,t}^{\eta_P} = (W_{G,t}/P_t) C_t^{-1}, \quad (\text{D.8})$$

symmetric to the private sectoral labor supplies. Cost minimization by the government sets the compensation bill proportional to goods purchases,

$$W_{G,t}N_{G,t} = \frac{\alpha_G}{1 - \alpha_G} P_{G,t}G_t, \quad (\text{D.9})$$

so public employment co-moves endogenously with the fiscal shock, and lump-sum taxes cover both components.

The closure of the economy is otherwise unchanged. The goods bundle is the baseline Cobb-Douglas aggregator,

$$G_t = \prod_{s=1}^S G_{s,t}^{\omega_{G,s}}, \quad (\text{D.10})$$

and the sectoral resource constraints of Equation (16) are unaffected: wholesalers keep selling $G_{s,t}$ to the government, which now uses these goods as inputs into $Y_{G,t}$ rather than consuming them directly. National accounting follows from valuing government output at cost: under constant returns, government consumption expenditure equals

$$P_{G,t}^Y Y_{G,t} = W_{G,t}N_{G,t} + \sum_{s=1}^S P_{s,t}G_{s,t}, \quad (\text{D.11})$$

where $P_{G,t}^Y$ is the cost-based deflator of government output, and nominal GDP adds it to private absorption:

$$\mathcal{Y}_t = P_t C_t + P_{I,t} I_t + P_{G,t}^Y Y_{G,t}. \quad (\text{D.12})$$

The goods purchased by the government therefore enter GDP only once, through government consumption expenditure—they appear in neither C_t nor I_t —so private goods purchases and public output are not double counted; equivalently, GDP equals private value added plus the public compensation bill. The multiplier denominator is total government consumption expenditure, consistent with this identity.

We set $\alpha_G = 0.6$, matching the share of the compensation of general government employees in the sum of compensation and intermediate purchases of the general government, the same target used by [Moro and Rachedi \(2022\)](#). The steady state is re-derived under this technology: total government consumption expenditure remains 20% of GDP, now split 60%–40% between compensation and goods purchases; the government wage equals the production wage, which pins down steady-state public employment; and both GDP and the multiplier denominator include government output, valued at cost.

Relative to the baseline, a spending shock now stimulates the economy through

two channels—the purchases of goods from the private industries, as before, and the public wage bill, which accrues directly to households—which is why the multipliers rise in both economies in Table D.19 and Figure D.7: the range shifts up to $[0.36, 2.03]$, with a correlation of 0.98 with the baseline multipliers. The idea multipliers instead compress—their mean falls to 0.06 and their maximum to 1.36, still attained by computers/electronics, followed by publishing/software and chemicals/pharmaceuticals—because only the goods component of government consumption reaches the private industries, and hence the innovation network, while the compensation component bypasses it. Even so, the heterogeneous innovation network accounts for 62% of the cross-sectional variance of the multipliers.

Sector-specific capital and the investment network. The baseline model features a single aggregate capital stock—the household accumulates

$$K_{t+1} = (1 - \delta_K)K_t + I_t, \quad (\text{D.13})$$

allocates capital services across industries through the CES scheme of Equation (8) with mobility elasticity ν_K , and assembles investment as a single Cobb-Douglas bundle over sectoral goods,

$$I_t = \prod_s I_{s,t}^{\omega_{I,s}}. \quad (\text{D.14})$$

This variant replaces both objects with their fully sectoral counterparts, changing the model in two ways. First, capital becomes sector-specific: each industry operates its own capital stock, which only that industry can rent. The household accumulates the 66 stocks according to

$$K_{s,t+1} = (1 - \delta_K) K_{s,t} + I_{s,t} - \frac{\phi_I}{2} \left(\frac{I_{s,t}}{K_{s,t}} - \iota_s \right)^2 K_{s,t}, \quad (\text{D.15})$$

where $I_{s,t}$ is investment installed in industry s and $\iota_s \equiv \gamma_{K,s} - 1 + \delta_K$ is the investment-capital ratio along the balanced growth path, so that the quadratic installation cost and its derivative vanish along the balanced path: the friction affects dynamics without altering the steady state. Each stock earns its own rental rate $R_{K,s,t}$, and the household's optimality conditions deliver one Tobin's- q pair per industry—an installation margin equating the sectoral investment price to the shadow value of installed capital net of the marginal installation cost, and an Euler equation pricing that shadow value off the discounted rental rate and continuation value. The CES reallocation scheme of the baseline, and with it the mobility elasticity ν_K , drops out of the model.

We set $\phi_I = 2$; a strictly positive installation cost is required for determinacy—with freely adjustable sector-specific capital and an interest-rate rule, the equilibrium becomes indeterminate—and the results are insensitive to its exact value.

Second, the investment installed in each industry is assembled from a sector-specific bundle of goods purchased from the producing industries,

$$I_{s,t} = \prod_{m=1}^S I_{s,m,t}^{\omega_{I,s,m}}, \quad (\text{D.16})$$

with the implied sector-specific investment price

$$P_{I,s,t} = \prod_{m=1}^S \left(\frac{P_{m,t}}{\omega_{I,s,m}} \right)^{\omega_{I,s,m}}, \quad (\text{D.17})$$

where $I_{s,m,t}$ denotes the purchases of industry- m goods by industry s for investment purposes, and the shares $\omega_{I,s,m}$ —the weight of producing industry m in the investment expenditure of purchasing industry s —are taken from the investment network Ω_I of vom Lehn and Winberry (2022), built from the BEA capital-flow data. The rows of Ω_I thus replace the common vector $\omega_{I,s}$ of the baseline: industries no longer buy investment goods in the same proportions, but each according to its own supplier mix.

We map the 41-sector network of vom Lehn and Winberry (2022) to our 66 industries (splitting supplier shares within groups in proportion to the aggregate investment column) and average it over 1997–2001, the same window as the production network. Accordingly, each sectoral resource constraint replaces the single retailer demand of Equation (B.14) with the investment demand of all purchasing industries:

$$Z_{s,t} \left[1 - \frac{\vartheta_s}{2} (\pi_{s,t} - 1)^2 \right] = C_{s,t} + \sum_{x=1}^S \omega_{I,x,s} \frac{P_{I,x,t}}{P_{s,t}} I_{x,t} + G_{s,t} + \sum_{x=1}^S H_{s,x,t}. \quad (\text{D.18})$$

The balanced growth path changes accordingly. Since sectoral relative prices trend at heterogeneous rates, each investment price inherits its own trend from the supplier mix, and each capital stock grows at the sector-specific rate $\gamma_{K,s}$ given by the common output trend deflated by the trend of the sector’s investment price. The steady state is re-derived with sector-specific investment prices, rental rates, and capital trends; all calibration targets, and every other block of the model, are unchanged, and GDP now aggregates the 66 sectoral investment expenditures.

Table D.19 and Figure D.8 show that the transmission is essentially unaffected: the multipliers span $[-0.06, 1.84]$ and correlate with the baseline ones

at 0.92, the largest sectoral multiplier still belongs to computers/electronics, and the correlation between sectoral multipliers and citation upstreamness stays high, at 0.61. The sectoral idea multipliers retain the baseline shape—a mean of 0.09 and a maximum of 1.42, attained by computers/electronics, followed by publishing/software and chemicals/pharmaceuticals—and the heterogeneous innovation network accounts for 57% of the cross-sectional variance of the multipliers. The innovation-network channel thus operates independently of how investment is sourced across industries.

Two-agent economy. The last case relaxes the assumption of a representative household: following [Cantore and Freund \(2021\)](#), we split households into workers and capitalists, and discipline the household block with the micro evidence on marginal propensities to consume.

A share ς_W of households are *workers*, denoted by $\mathcal{W}$. They supply all production and research labor in every sector, own no firms or capital, and can save only in government bonds, subject to a quadratic portfolio adjustment cost that penalizes deviations of their real detrended per-capita holdings $b_{\mathcal{W},t} \equiv B_{\mathcal{W},t}/(P_t \gamma_Y^t)$ from the steady-state level $\bar{b}_{\mathcal{W}} = 0$. Their problem is

$$\begin{aligned} \max_{\{C_{\mathcal{W},t}, \{N_{\mathcal{W},P,s,t}, N_{\mathcal{W},R,s,t}\}_{s=1}^S, B_{\mathcal{W},t}\}} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log C_{\mathcal{W},t} - \sum_{s=1}^S \theta_{P,s} \frac{(N_{\mathcal{W},P,s,t})^{1+\eta_P}}{1+\eta_P} - \sum_{s=1}^S \theta_{R,s} \frac{(N_{\mathcal{W},R,s,t})^{1+\eta_R}}{1+\eta_R} \right] \\ \text{s.t.} \quad & P_t C_{\mathcal{W},t} + B_{\mathcal{W},t} + \frac{\phi_{\mathcal{W}}}{2} (b_{\mathcal{W},t} - \bar{b}_{\mathcal{W}})^2 P_t + T_t = \dots \\ & \dots = R_{t-1} B_{\mathcal{W},t-1} + \sum_{s=1}^S (W_{P,s,t} N_{\mathcal{W},P,s,t} + W_{R,s,t} N_{\mathcal{W},R,s,t}), \end{aligned} \quad (\text{D.19})$$

where T_t is the uniform per-capita lump-sum tax.

The first-order conditions for labor are the sectoral supply schedules of the baseline model evaluated at the workers' consumption, and the first-order condition for bonds delivers the Euler equation

$$\frac{1}{C_{\mathcal{W},t}} [1 + \phi_{\mathcal{W}} (b_{\mathcal{W},t} - \bar{b}_{\mathcal{W}})] = \frac{\beta}{\gamma_Y} \mathbb{E}_t \left[\frac{1}{C_{\mathcal{W},t+1}} \frac{R_t}{\pi_{t+1}} \right], \quad (\text{D.20})$$

in which the factor β/γ_Y reflects the deterministic detrending—as throughout the stationarized system, all real quantities are expressed net of the output trend γ_Y^t —and the return to saving is taxed at the margin by the portfolio friction $\phi_{\mathcal{W}}$: workers behave as target savers who smooth windfalls only partially—the feature that generates realistic intertemporal marginal propensities to consume.

The remaining share $1 - \varsigma_W$ of households are *capitalists*, denoted by $\mathcal{C}$. They supply no labor, own the economy's capital stock and firms, and trade bonds freely:

$$\max_{\{C_{\mathcal{C},t}, I_{\mathcal{C},t}, K_{\mathcal{C},t+1}, B_{\mathcal{C},t}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log C_{\mathcal{C},t} \quad (\text{D.21})$$

$$\text{s.t. } P_t C_{\mathcal{C},t} + B_{\mathcal{C},t} + P_{I,t} I_{\mathcal{C},t} + T_t = R_{t-1} B_{\mathcal{C},t-1} + R_{K,t} K_{\mathcal{C},t} + D_{\mathcal{C},t}, \quad (\text{D.22})$$

where $I_{\mathcal{C},t}$, $K_{\mathcal{C},t}$, and $D_{\mathcal{C},t}$ denote the capitalists' per-capita investment, capital, and rebated firm profits, and per-capita capital accumulates as in the baseline model. Their first-order conditions are the standard Euler and investment conditions of Section 3.1, and their stochastic discount factor is the one that prices producers' decisions.

Aggregation and market clearing change accordingly. Aggregate consumption averages the two types in per-capita terms, $C_t = \varsigma_W C_{\mathcal{W},t} + (1 - \varsigma_W) C_{\mathcal{C},t}$; sectoral labor is supplied by workers only, $N_{P,s,t} = \varsigma_W N_{\mathcal{W},P,s,t}$ and $N_{R,s,t} = \varsigma_W N_{\mathcal{W},R,s,t}$; the capital market clears as in the baseline, with the entire stock held by capitalists, $K_t = (1 - \varsigma_W) K_{\mathcal{C},t}$, $I_t = (1 - \varsigma_W) I_{\mathcal{C},t}$, and $D_t = (1 - \varsigma_W) D_{\mathcal{C},t}$; and the bond market clears against the public supply, $\varsigma_W B_{\mathcal{W},t} + (1 - \varsigma_W) B_{\mathcal{C},t} = B_t$.

On the fiscal side, the government no longer balances its budget period by period: it finances purchases with the uniform per-capita lump-sum taxes and one-period nominal bonds, with steady-state debt at 30% of annual GDP: at the monthly frequency of the model, $\bar{b} = 0.30 \times 12 \bar{Y}$, where $\bar{Y}$ is steady-state monthly GDP. Real government debt (in units of the consumption good and detrended by the output trend, like all quantities) evolves as

$$b_t = \frac{R_{t-1}}{\pi_t \gamma_Y} b_{t-1} + g_t - \tau_t,$$

where $g_t \equiv \sum_s (P_{s,t}/P_t) G_{s,t}$ is real government spending. Taxes follow the rule

$$\tau_t = \bar{\tau} + \phi_b (b_{t-1} - \bar{b}) + \phi_g (g_t - \bar{g}),$$

where $\bar{\tau}$ balances the steady-state budget inclusive of debt service, and the coefficients follow Galí et al. (2007), adapted to the monthly frequency: $\phi_b = 0.33/3$ and $\phi_g = 0.1/3$, so that a spending shock is deficit-financed on impact and taxes rise only gradually as debt accumulates. Every other block of the model is unchanged, and we consider two wage configurations: flexible wages, and sticky wages for both labor types, with the union block of the sticky-wage case evaluated at the workers' marginal rate of substitution and discount kernel.

The calibration involves two new parameters: the share of workers and the

portfolio friction. We set $\varsigma_W = 0.5$, as in [Cantore and Freund \(2021\)](#), and calibrate ϕ_W in closed form to the iMPC evidence of [Fagereng et al. \(2021\)](#), exploiting the exact solution of the worker’s partial-equilibrium consumption-saving problem: a worker who receives a windfall consumes the share $x_0 = (R - \lambda_\phi + \phi_W C_W)/(1 + R - \lambda_\phi + \phi_W C_W)$ on impact and saves the rest, decumulating the bonds at the stable root

$$\lambda_\phi = \frac{1}{2} \left[(1 + R + \phi_W C_W) - \sqrt{(1 + R + \phi_W C_W)^2 - 4R} \right],$$

where R is the growth-adjusted monthly gross real rate and C_W the worker’s steady-state consumption, while capitalists, who are permanent-income consumers, contribute the annuity value of the windfall. Setting $\phi_W = 0.028$ makes the aggregate quarterly impact marginal propensity to consume out of an unanticipated windfall match the empirical value of 0.1942 exactly. The implied first-year cumulative propensity is 0.44: it matches the annual marginal propensities to consume out of transitory income estimated by [Jappelli and Pistaferri \(2014\)](#) and [Auclert et al. \(2024\)](#), at 0.48 and 0.45, respectively, and is only slightly lower than the 0.55 of [Fagereng et al. \(2021\)](#). In the steady state, workers hold zero bonds, so worker consumption equals per-capita labor income net of the per-capita tax, capitalists absorb the remainder of aggregate consumption, and the labor disutility levels are re-derived so that the sectoral steady state is identical to the baseline.

The two wage configurations differ starkly in their implied marginal propensity to earn—the partial-equilibrium earnings response to the same windfall, which the micro evidence places near zero ([Auclert et al., 2023](#)). With flexible wages, the windfall raises worker consumption and lowers hours through the labor-supply condition, delivering an aggregate quarterly MPE of $-\varsigma_W (1/\eta_P) (E_W/C_W)$ times the workers’ quarterly MPC, where E_W denotes the workers’ steady-state per-capita labor earnings; at our calibration, the MPE equals -0.54 : the tension between matching MPCs and MPEs that [Auclert et al. \(2023\)](#) identify in New Keynesian models. With sticky wages, employment is demand-determined and the partial-equilibrium MPE is exactly zero.

Table [D.19](#) and Figures [D.9–D.10](#) show that the workers’ demand recycling makes the economy visibly more expansionary—the range shifts up to $[-0.26, 2.68]$ with flexible wages and $[-0.19, 2.42]$ with sticky wages—and reshuffles the middle of the cross-section toward rigid-price consumption sectors, which is why the correlation with the baseline multipliers, while remaining high, declines to 0.75 and 0.65: with flexible wages, housing overtakes computers/electronics at the top

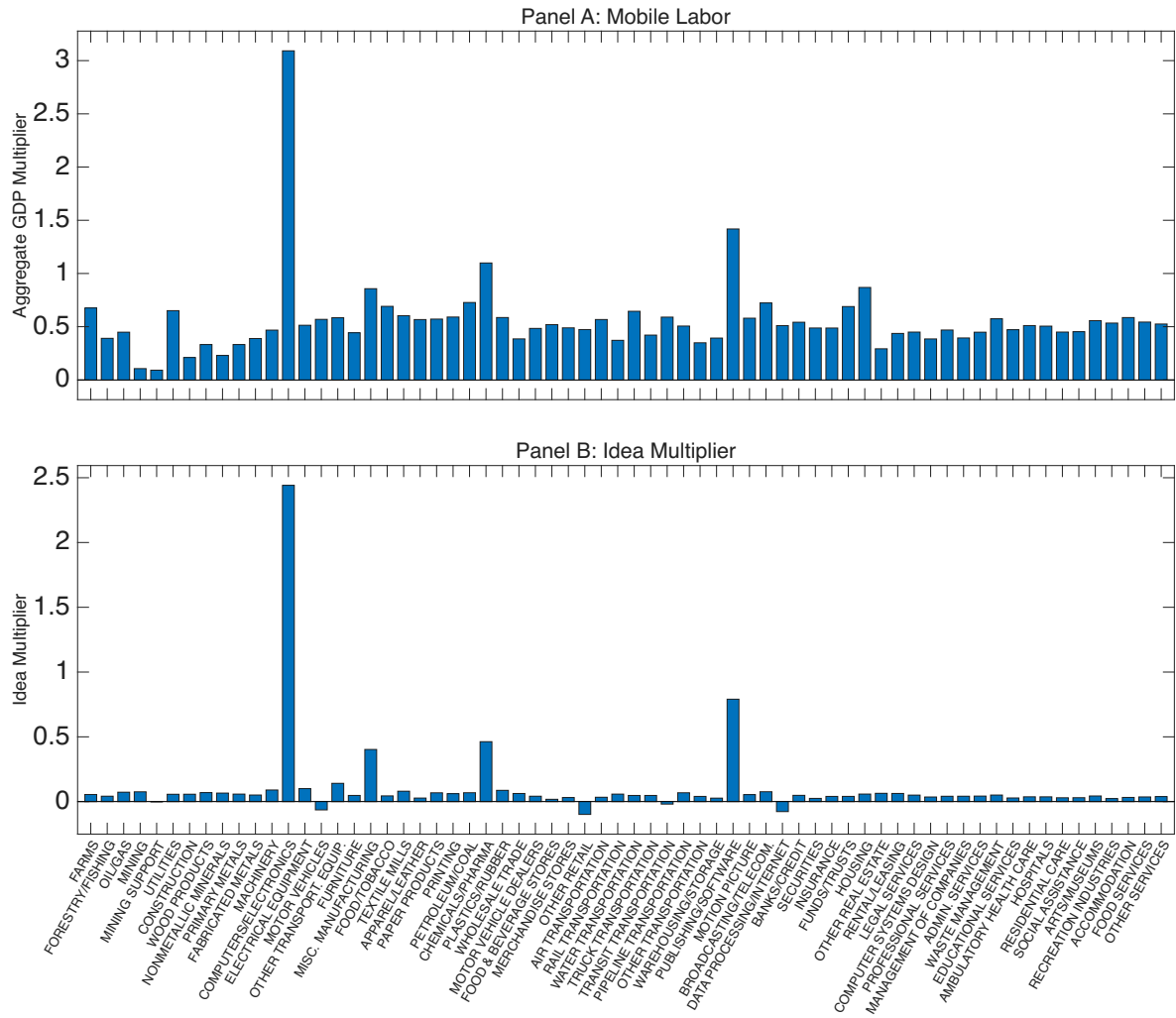
of the total-multiplier ranking, while with sticky wages computers/electronics remains first. The innovation mechanism, however, is intact: the idea multipliers are essentially identical across the two wage configurations, ranging up to 2.16 with a mean of 0.19, computers/electronics has the largest idea multiplier in both cases, followed by publishing/software and chemicals/pharmaceuticals, their correlation with the baseline idea multipliers is 0.97, and the heterogeneous innovation network accounts for 20% and 37% of the cross-sectional variance of the multipliers, respectively.

The role of the production network. We conclude by isolating the contribution of the production network: we re-solve both the baseline economy and the stationary economy without the R&D block, replacing the empirical input-output matrix with a homogeneous one, $\omega_{H,s,x} = 1/S$, holding every other parameter fixed.

Figure D.11 replicates the comparison of Figure 1 under the homogeneous IO matrix, and Figure D.12 reports, for each economy, the sector-by-sector difference between the heterogeneous-IO and the homogeneous-IO multipliers. For the baseline economy, the multipliers with the homogeneous IO matrix are quite similar to those with the empirical one: the differences in Panel A of Figure D.12 are small—a mean absolute difference of 0.14—and sizable in only one case, publishing/software, at 0.55. The exception is instructive: publishing/software combines a dense within-sector supply chain—it purchases 71% of its intermediate inputs from its own industry—with the role of second-largest knowledge supplier of the citation network, so the input-output loop amplifies precisely the R&D engine that the innovation network broadcasts.

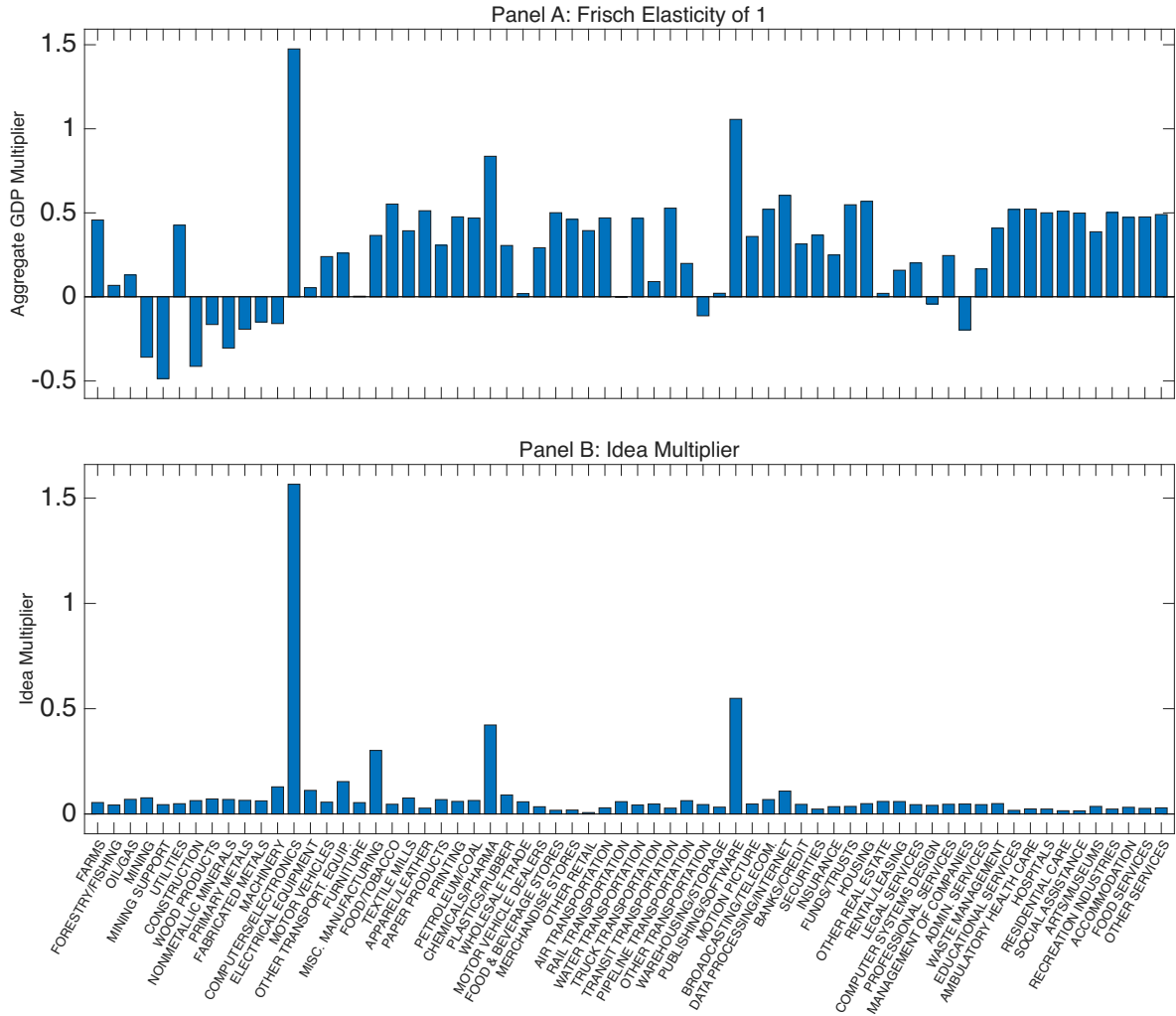
The differences are instead much larger in the stationary economy: their dispersion is more than twice that of the baseline economy, and over half of the industries move by more than 0.25 in absolute value. The comparison delivers a sharp message: while the production network is an important propagation channel for the standard demand transmission of fiscal policy, it barely modulates the R&D responses to fiscal shocks. This is the model counterpart of the empirical results of Section 2: the input-output network does not play a confounding role in the estimation of the innovation-network channel.

Figure D.1: Robustness: Mobile Labor.



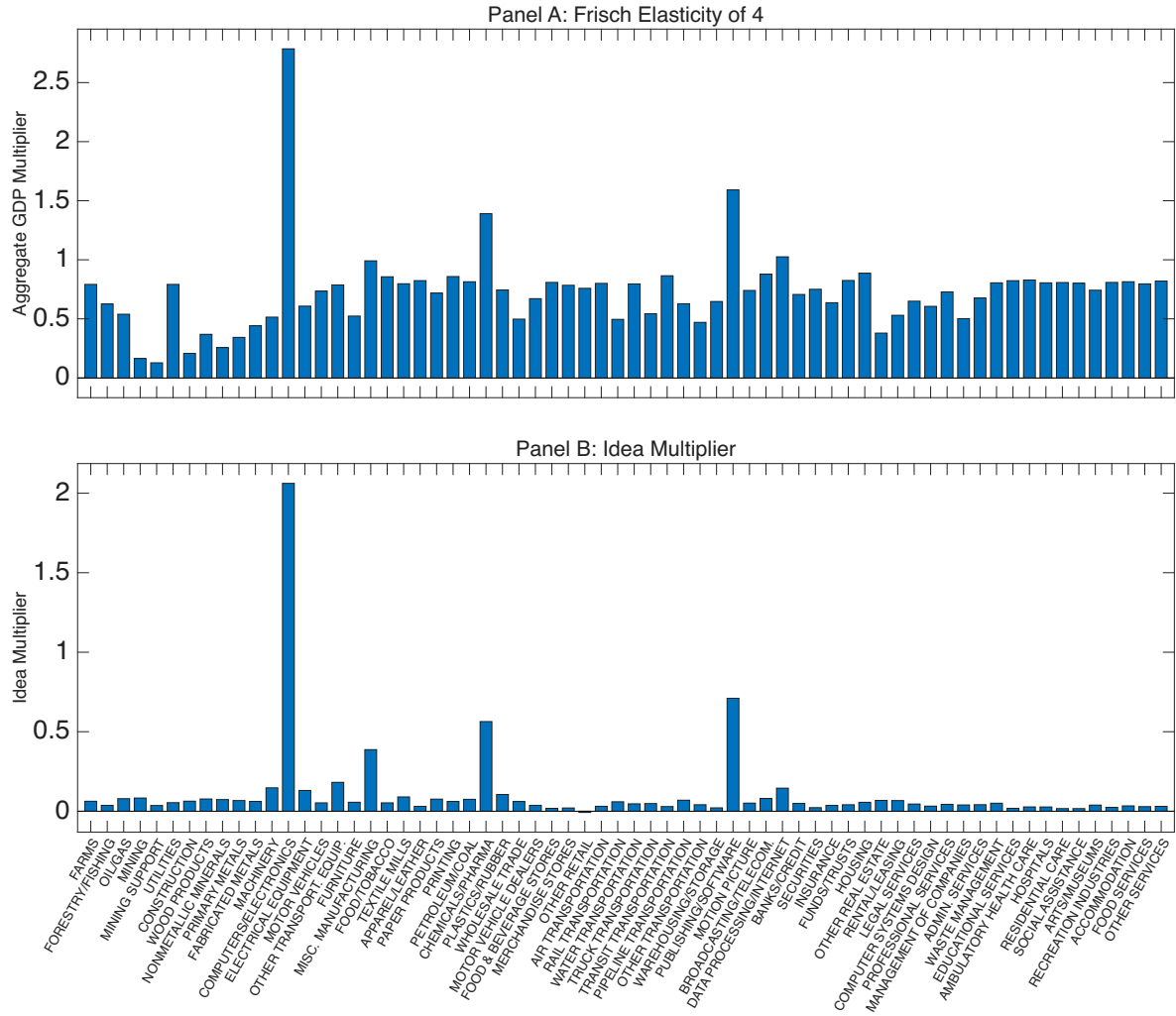
Note: Panel A reports the sectoral-spending multipliers of the economy in which both labor types are perfectly mobile across industries and sectoral wages are equalized. Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.2: Robustness: Frisch Elasticity of 1.



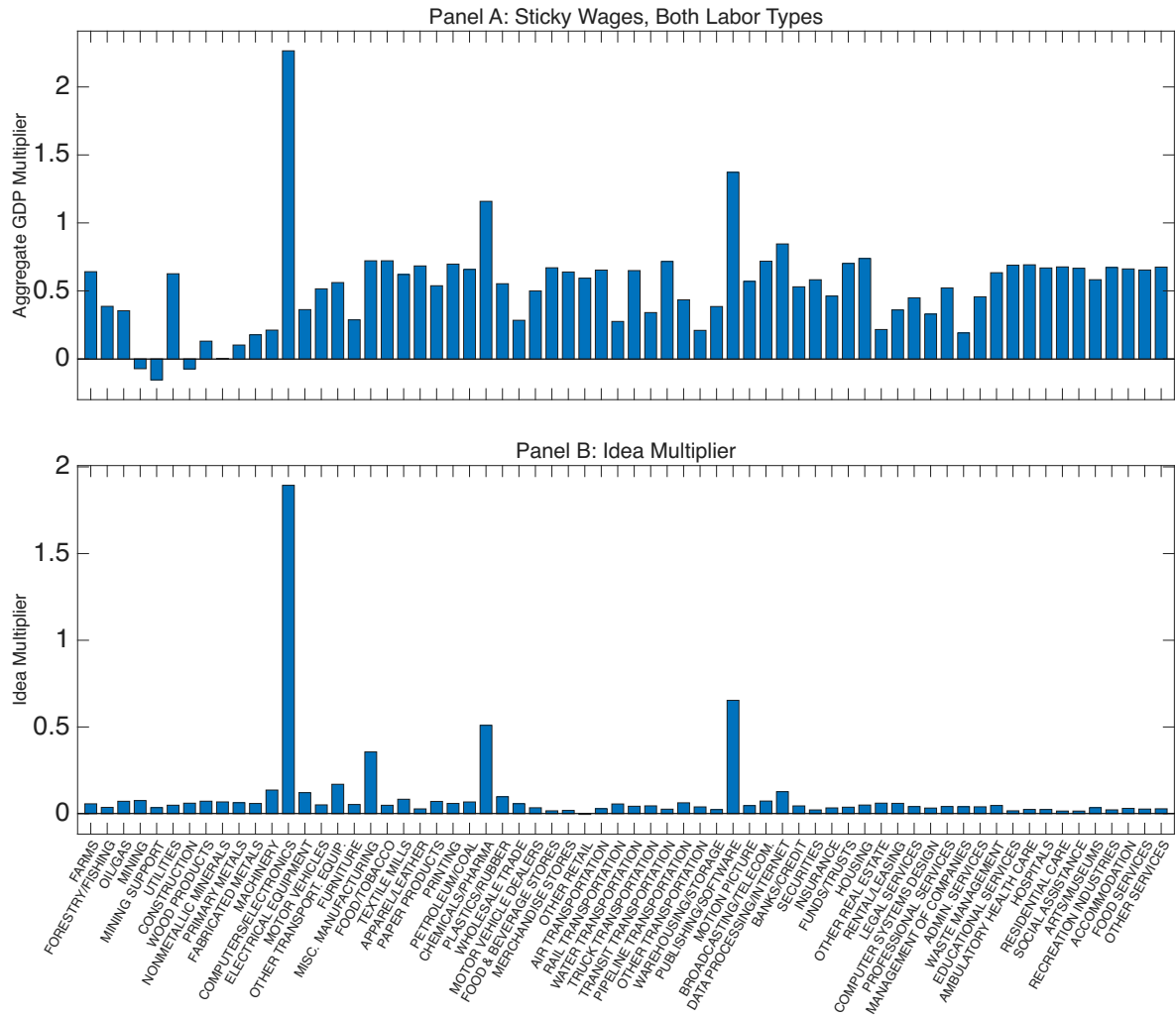
Note: Panel A reports the sectoral-spending multipliers of the economy with a Frisch elasticity of production and research labor equal to 1, $\eta_P = \eta_R = 1$. Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.3: Robustness: Frisch Elasticity of 4.



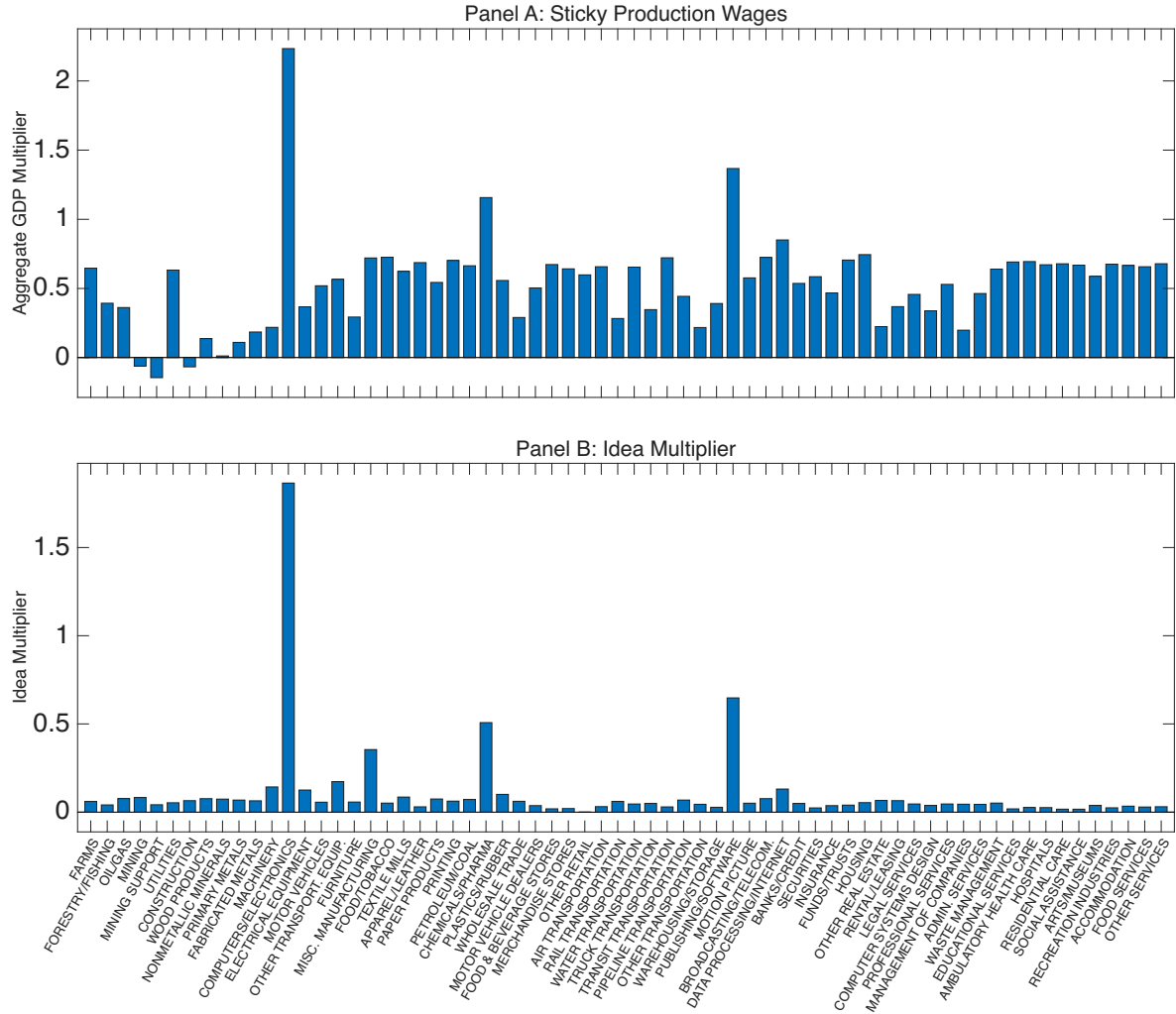
Note: Panel A reports the sectoral-spending multipliers of the economy with a Frisch elasticity of production and research labor of 4, $\eta_P = \eta_R = 1/4$. Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.4: Robustness: Sticky Wages, Both Labor Types.



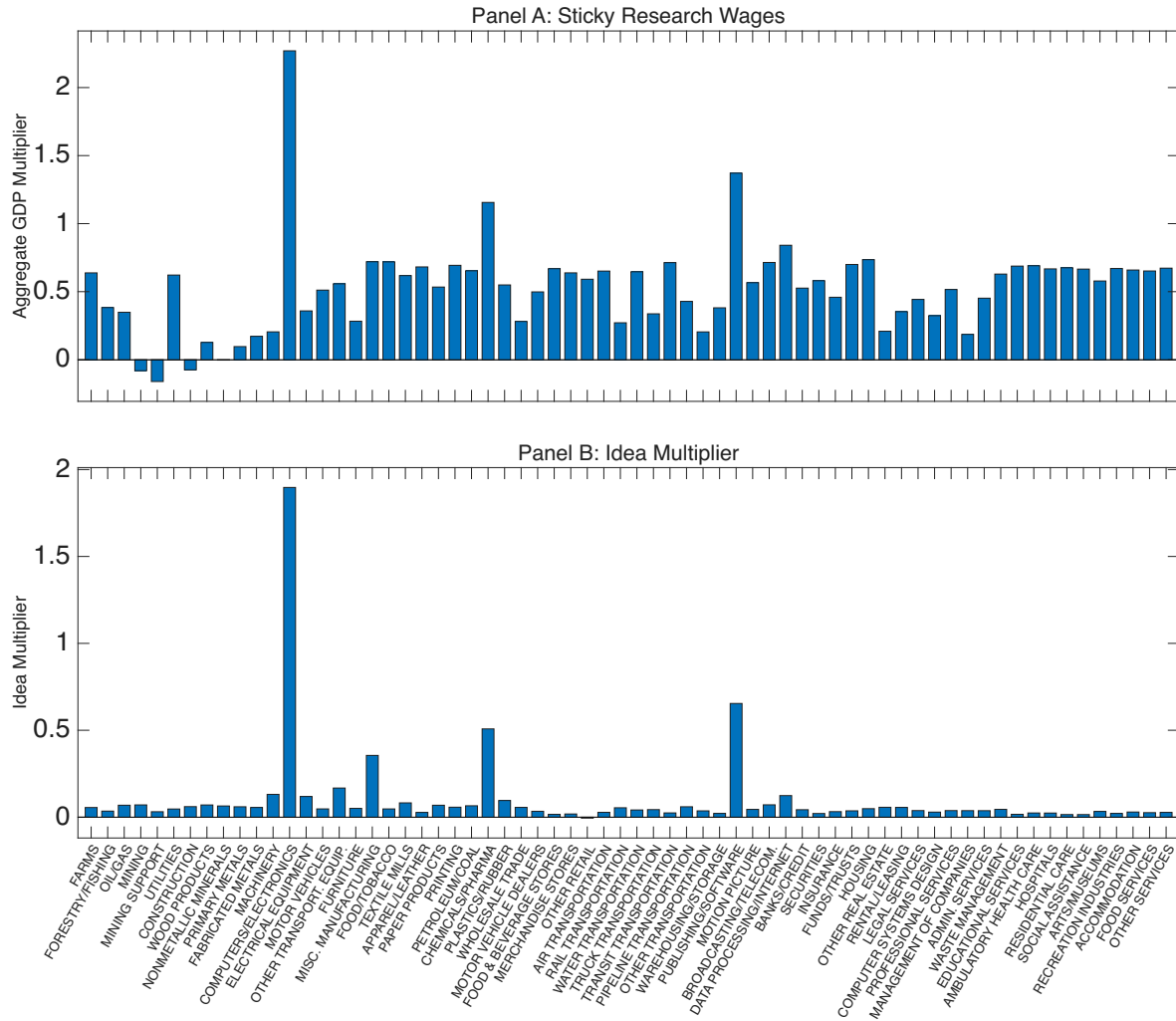
Note: Panel A reports the sectoral-spending multipliers of the economy in which nominal wages of both labor types adjust à la [Rotemberg \(1982\)](#), with a twelve-month average wage duration homogeneous across industries. Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.5: Robustness: Sticky Production Wages.



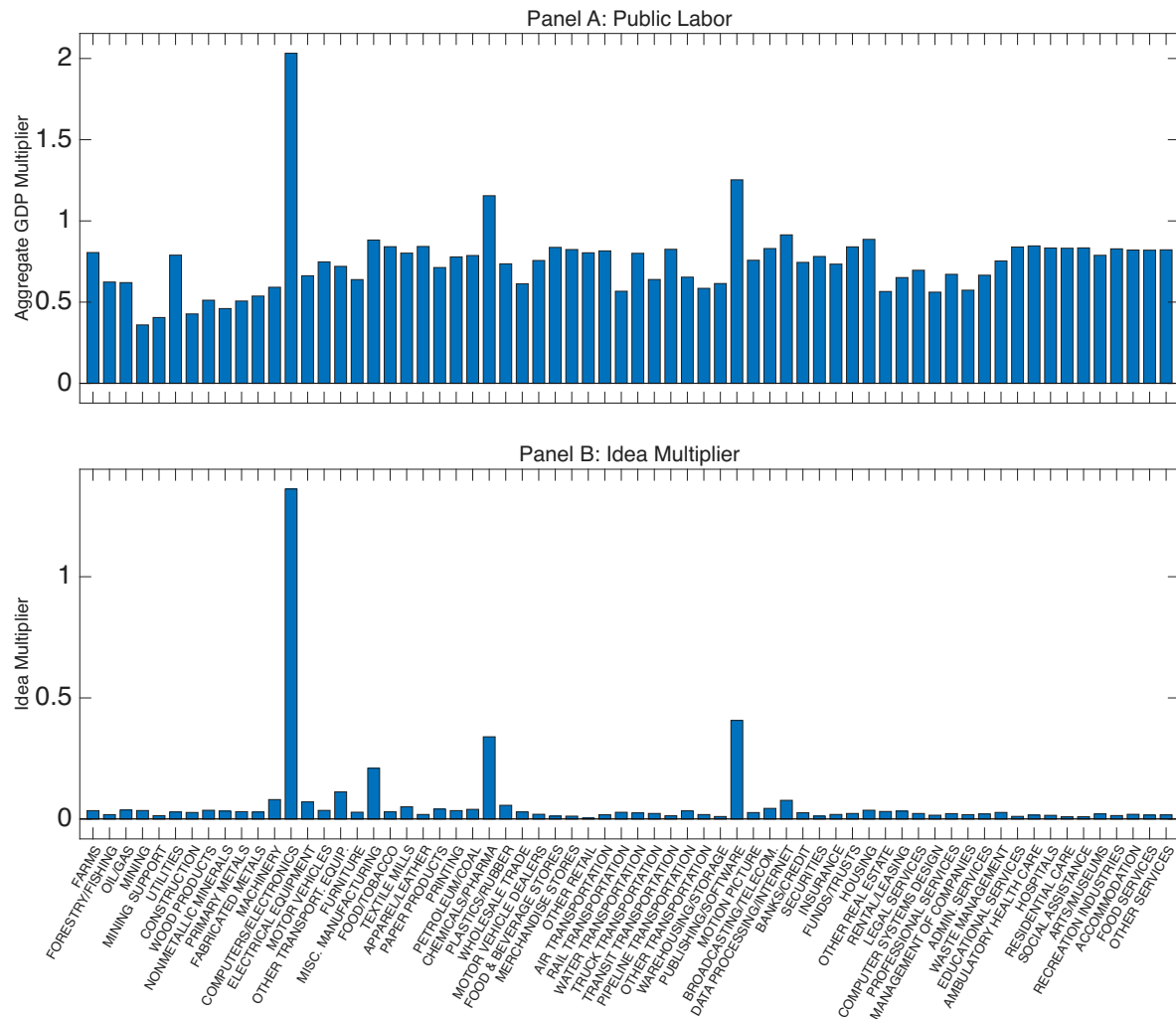
Note: Panel A reports the sectoral-spending multipliers of the economy in which only the nominal wages of production labor adjust à la [Rotemberg \(1982\)](#), with a twelve-month average wage duration homogeneous across industries, while research wages are flexible. Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.6: Robustness: Sticky Research Wages.



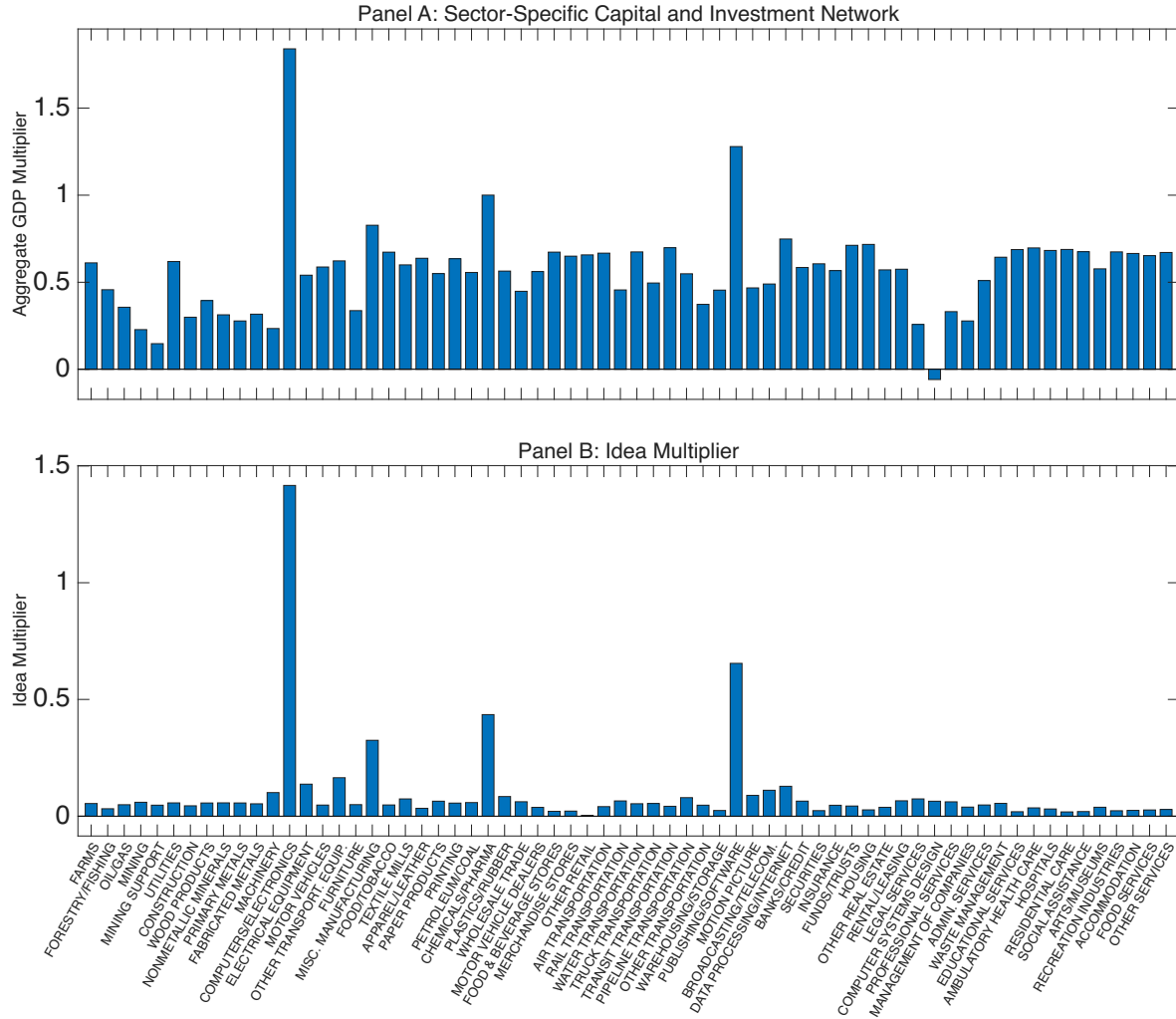
Note: Panel A reports the sectoral-spending multipliers of the economy in which only the nominal wages of research labor adjust à la [Rotemberg \(1982\)](#), with a twelve-month average wage duration homogeneous across industries, while production wages are flexible. Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.7: Robustness: Public Labor.



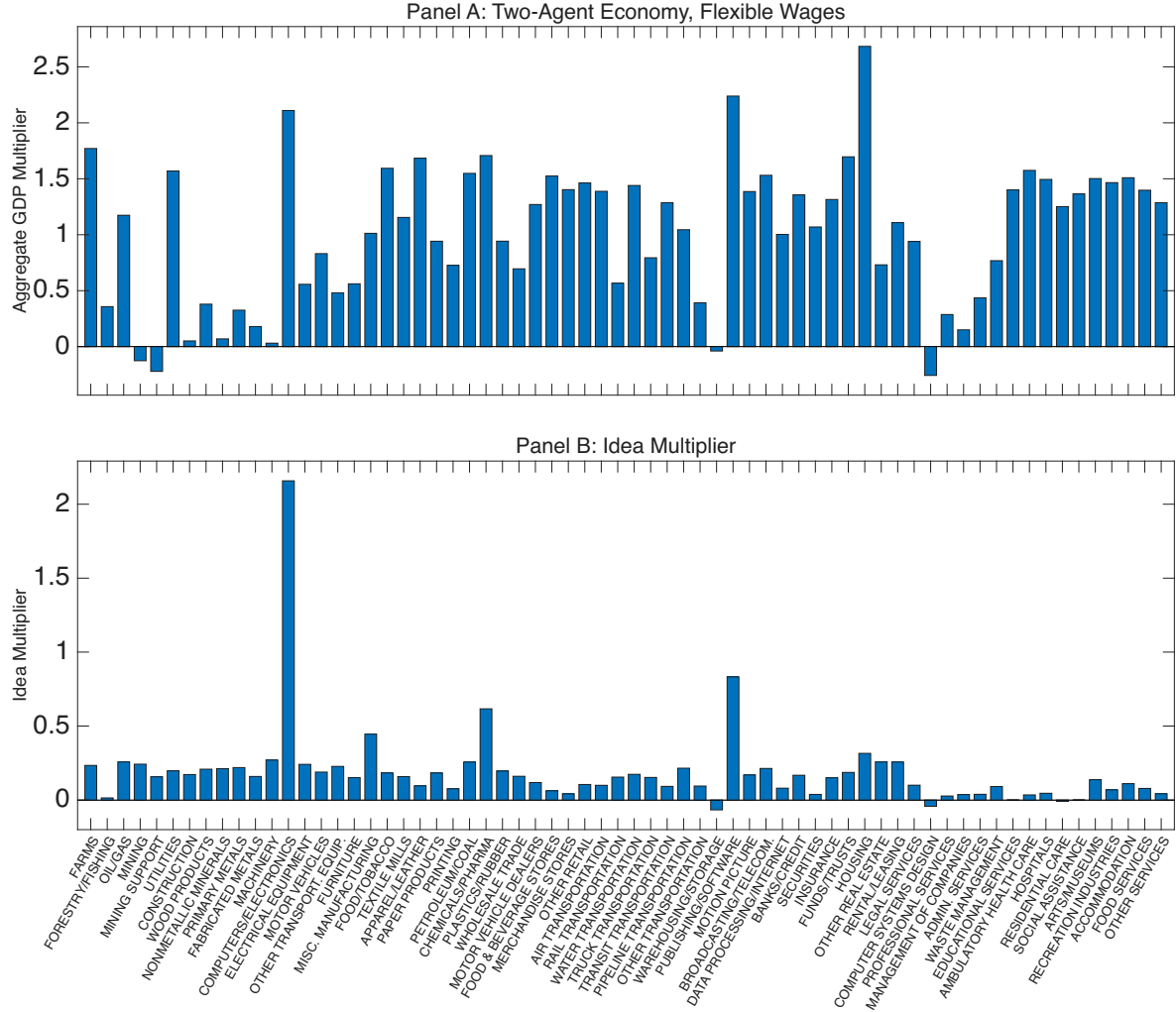
Note: Panel A reports the sectoral-spending multipliers of the economy in which government consumption combines purchases of goods with public employment, with a compensation share of 60%, following [Moro and Rachedi \(2022\)](#). Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.8: Robustness: Sector-Specific Capital and the Investment Network.



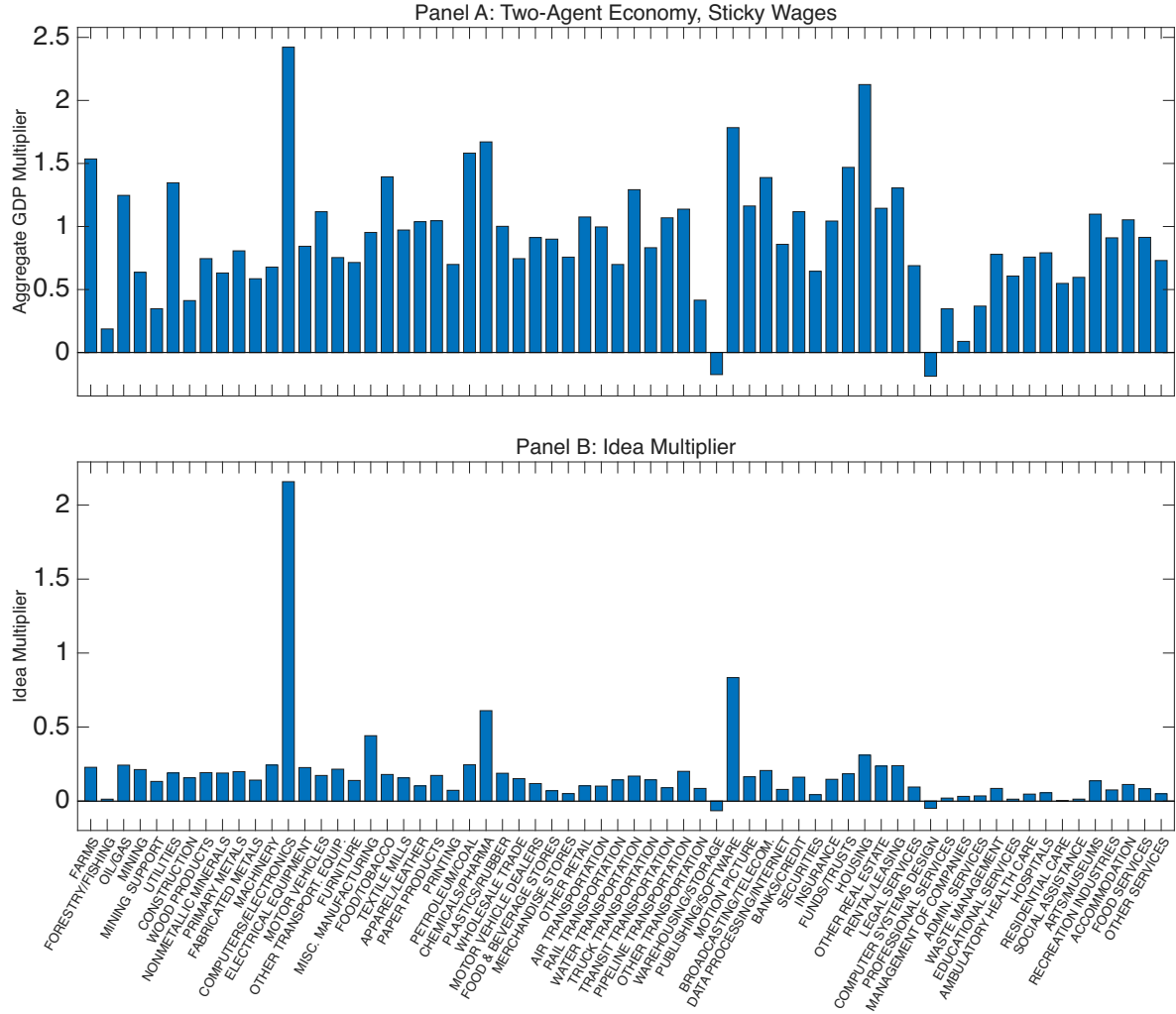
Note: Panel A reports the sectoral-spending multipliers of the economy with sector-specific capital stocks, each accumulated through a sector-specific investment bundle whose composition is given by the investment network of [vom Lehn and Winberry \(2022\)](#). Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.9: Robustness: Two-Agent Economy, Flexible Wages.



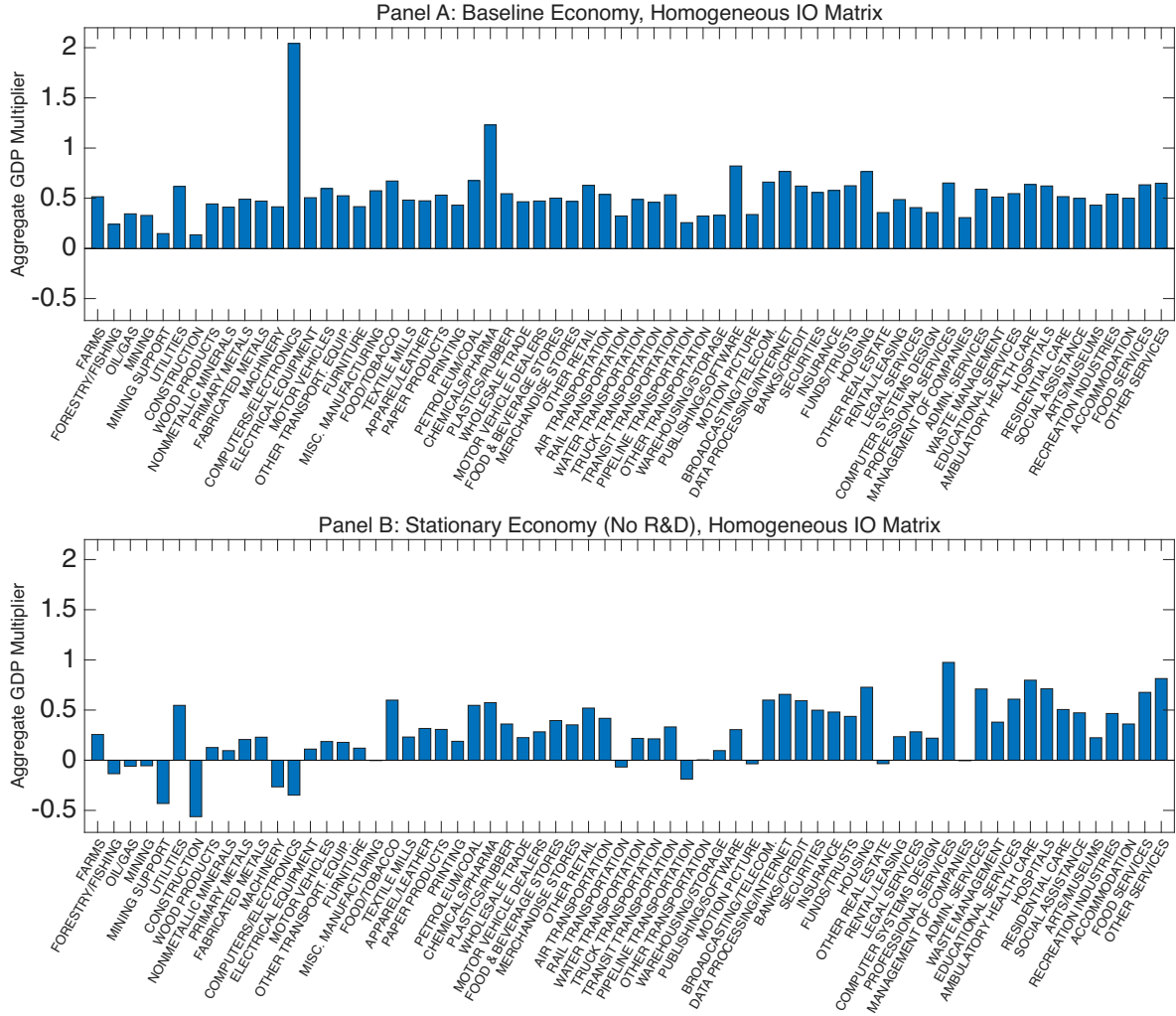
Note: Panel A reports the sectoral-spending multipliers of the two-agent economy à la [Cantore and Freund \(2021\)](#) with flexible wages: workers—half of households—supply all labor and save subject to portfolio adjustment costs calibrated to match the quarterly impact MPC of [Fagereng et al. \(2021\)](#), and the government follows the tax rule of [Galí et al. \(2007\)](#) with public debt at 30% of annual GDP. Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.10: Robustness: Two-Agent Economy, Sticky Wages.



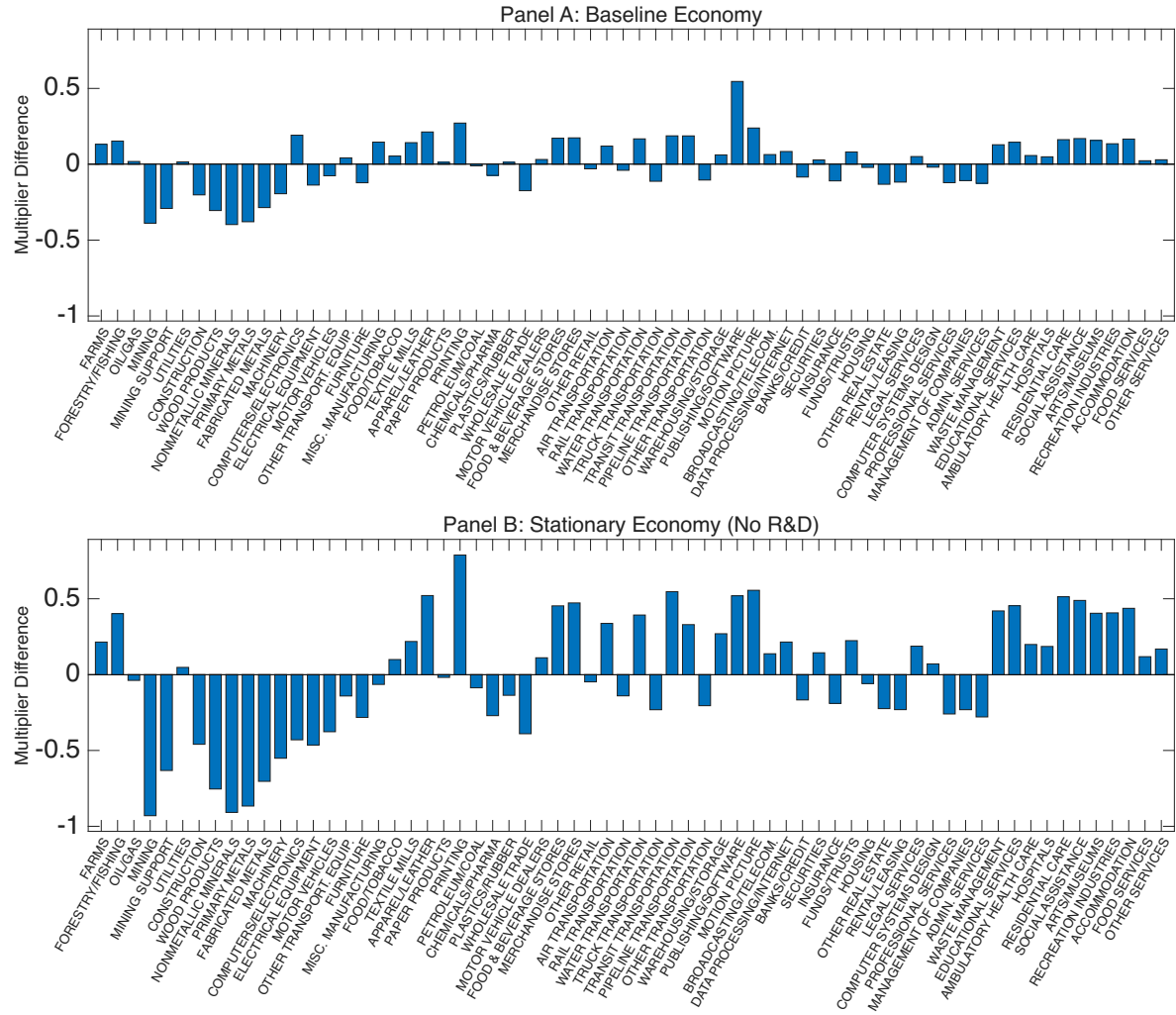
Note: Panel A reports the sectoral-spending multipliers of the two-agent economy à la Cantore and Freund (2021) with sticky wages for both labor types, the configuration with a zero partial-equilibrium marginal propensity to earn. Panel B reports the corresponding sectoral idea multipliers: the difference between the multipliers of Panel A and those of the same economy with the innovation network replaced by the identity matrix.

Figure D.11: Sectoral-Spending Multipliers with a Homogeneous IO Matrix.



Note: Sectoral-spending multipliers when the empirical input-output matrix is replaced by the homogeneous matrix $\omega_{H,s,x} = 1/S$, keeping every other parameter at its baseline value. Panel A refers to the baseline economy and Panel B to the stationary economy without the R&D block. The two panels share the same scale.

Figure D.12: The Contribution of the Production Network.



Note: Sector-by-sector difference between the sectoral-spending multipliers computed with the empirical (heterogeneous) input-output matrix and those computed with the homogeneous matrix $\omega_{H,s,x} = 1/S$. Panel A refers to the baseline economy and Panel B to the stationary economy without the R&D block. The two panels share the same scale.