

Unexpected Expectations

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Monetary policy shocks are commonly identified using narrative and high-frequency instruments designed to isolate exogenous policy variation. Using the Michigan Survey of Consumers, Consensus Economics, and the Survey of Professional Forecasters, we find broad evidence that shocks identified with prominent instruments raise near-term beliefs about real activity even as agents anticipate higher interest rates. This result is robust across surveys, whereas the response of inflation beliefs is mixed. The belief responses indicate that existing shock measures may still carry information about economic fundamentals. We develop a New Keynesian model with dispersed information and estimate it by matching the macroeconomic and belief responses to an identified shock. The model reproduces the decline in activity and prices together with the short-lived optimism in beliefs. Isolating the pure monetary innovation generates nonpositive forecast revisions and, at an equal policy-rate impact, attenuates the responses of output and prices. We then identify a beliefs-adjusted monetary policy shock in a narrative sign-restriction VAR. The resulting macroeconomic responses are more precisely estimated and attenuated.

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JEL Classifications: E31, E32, E52, E70.

1. Introduction

Monetary policy shocks are commonly identified using high-frequency movements around policy announcements or narrative measures of policy intentions. These strategies are designed to isolate exogenous policy variation from the central bank’s systematic response to the economy. Yet a policy announcement can also reveal the central bank’s assessment of current and future conditions. An identified shock may therefore retain information that is relevant for private beliefs even when it removes other sources of endogeneity.

This possibility creates a basic tension in the empirical evidence. A prominent finding is that an identified contractionary monetary policy shock raises private forecasts of output growth, a response that is difficult to reconcile with a conventional monetary tightening (Nakamura and Steinsson 2018). The increase may reflect information held by the central bank, private agents’ response to public news released before the meeting, or a mixture of policy and information shocks (Romer and Romer 2000; Jarociński and Karadi 2020; Bauer and Swanson 2023). Each interpretation has different consequences for identification and for the transmission mechanism.

This paper studies the tension through the beliefs of the agents whose decisions transmit monetary policy. We ask three questions. First, do established monetary policy shocks produce similar belief responses across households and professional forecasters? Second, can a quantitative equilibrium model account simultaneously for optimistic belief revisions and contractionary macroeconomic outcomes? Third, can the response of beliefs help construct a monetary policy shock with less informational contamination?

Our first contribution is to provide systematic evidence on the response of beliefs across populations, survey designs, and monetary policy instruments. We combine three U.S. surveys. The Michigan Survey of Consumers (MSC) measures households’ views about business conditions, prices, and interest rates. Consensus Economics (CE) provides monthly forecasts from financial and research economists. The Survey of Professional Forecasters (SPF) supplies quarterly forecasts from professional macroeconomic forecasters. These sources differ in respondents’ expertise, incentives, question format, forecast horizon, and frequency. This variation allows us to assess which belief responses generalize across populations and measurement designs.

For every population, we estimate a Bayesian proxy VAR separately for each belief measure and each of six prominent monetary policy instruments. The instruments span narrative innovations, high-frequency surprises, decompositions of policy and information shocks, and measures that purge the Federal Reserve’s response to public news. Following the terminology of the source papers, we refer to the structural disturbance recovered by each proxy VAR as an identified monetary policy shock. The cross-instrument comparison asks which features of belief adjustment survive substantially different identifying assumptions.

One pattern is broad. Following an identified contractionary monetary policy shock, near-term beliefs about real activity generally improve even though expected short-term interest rates rise. Consumer sentiment increases on impact with posterior intervals above zero under five of the six instruments; the estimate based on *citetRomer2004* is positive but imprecise. CE economists revise real GDP growth upward in most specifications, and SPF forecasts of GDP growth increase on impact under every instrument. Expected rates rise across all three populations, showing that the optimistic activity revisions coexist with a perceived tightening. The same pattern among professional forecasters extends the evidence beyond household beliefs.

Inflation expectations provide a useful contrast. The monthly evidence is mixed: MSC responses are imprecisely positive in some specifications, while CE revisions are close to zero or negative. CPI revisions rise more clearly in the quarterly SPF system, where the timing of monetary policy is less precisely identified. The robust empirical puzzle therefore concerns expected activity; the inflation evidence is survey- and frequency-dependent. Across the three surveys, the positive response of activity expectations is concentrated at short horizons and fades rapidly.

The cross-instrument pattern shows that each identification strategy can remove its targeted source of contamination while retaining other information relevant to private beliefs. Controlling for the Federal Reserve's response to public data, for example, can leave information that agents infer from the policy decision itself. Similarly, decompositions based on high-frequency interest-rate and stock-price comovement may omit signals relevant to survey respondents. Forecast revisions therefore provide an additional observable for assessing the informational content of identified monetary policy shocks.

Our second contribution is to provide a quantitative equilibrium account of the joint response of beliefs and outcomes. We introduce dispersed information into an otherwise standard New Keynesian economy. Households and firms receive noisy private signals about monetary, demand, technology, and cost-push disturbances. The central bank observes the aggregate state and sets the interest rate according to a Taylor rule. Consequently, the policy rate is both an instrument and a signal. An unexpected increase can reflect an exogenous monetary disturbance, but it can also reveal that the central bank is responding systematically to stronger demand or other fundamentals.

Distinct household and firm information sets allow beliefs to affect aggregate demand and price setting through the margins relevant to each decision. Multiple latent disturbances make inference from the interest rate nondegenerate: when an unexpected rate movement can have several structural sources, agents must infer both the policy innovation and the fundamentals behind the systematic policy response. Higher-order uncertainty propagates this signal-extraction problem through two interacting strategic-complementarity blocks. The mechanism generates endogenous persistence without adding lagged output or inflation terms to the IS and Phillips curves and yields a well-defined pure-shock counterfactual.

We discipline the model with a minimum-distance exercise. All empirical targets correspond to the [Jarociński and Karadi \(2020\)](#) instrument and all model dynamics are quarterly. The macroeconomic block consists of the responses of an industrial-production gap, the price level, and the three-month Treasury bill rate from a macro-only proxy VAR. The belief block consists of SPF revisions for real GDP growth, CPI inflation, and the Treasury bill rate. We estimate the persistence and innovation volatility of four disturbances, the precision of households' and firms' signals, and five New Keynesian parameters.

We estimate the macroeconomic and belief blocks in separate VARs because the survey series differ in frequency, sample coverage, target definition, and measurement error. The macro-only VAR supplies a common transmission benchmark, while each survey VAR measures how a particular population updates. Both blocks use the same external instrument and therefore identify the same monetary policy shock from different sets of observables.

The estimated model reproduces both sides of the evidence. It tracks the hump-shaped contraction in the industrial-production gap, the decline in the price level, and the initial increase in the short-term rate. At the same time, it generates the sharp positive impact revisions in SPF output and inflation forecasts. Overall, the fitting errors are small relative to the uncertainty surrounding the empirical responses. The main discrepancy is the interest-rate forecast revision, whose sign differs from the data at intermediate horizons.

The estimates imply substantial heterogeneity in information precision across agents. In particular, the group mapped to SPF forecasts receives relatively noisy signals about monetary conditions, making it harder to distinguish a monetary innovation from the central bank's response to economic fundamentals.

We next use the model to separate the observed surprise from a pure monetary policy shock. We report two normalizations because a structural innovation and an observed surprise are different random variables. The first puts the pure shock on the variance scale of the surprise; the second equates their impact effects on the policy rate. A pure tightening contracts the output gap and generates nonpositive output and inflation forecast revisions under both normalizations. Under the equal-rate comparison, isolating the structural shock also attenuates the persistent responses of output and prices. The positive forecast revisions reflect favorable information about economic fundamentals contained in the observed policy surprise; they are not caused by the contractionary monetary innovation itself.

Our third contribution provides a complementary empirical identification based on belief restrictions. We augment a narrative sign-restriction VAR with short-run restrictions on output and inflation forecast revisions, while leaving the relative magnitude of the macroeconomic responses unrestricted. Conditional on the same impact increase in the policy rate, the beliefs-adjusted responses of output and prices are more precise and attenuated. The narrative VAR thus provides

complementary empirical evidence of attenuation.

Taken together, the results show how beliefs can diagnose residual information in monetary policy shock measures and sharpen structural identification. The survey evidence documents the informational content, the model traces its aggregate effects and constructs the pure-shock counterfactual, and the narrative VAR uses belief responses to distinguish the policy action from the news conveyed with it.

Related Literature. Our paper contributes first to the literature on central bank information and monetary policy identification. [Romer and Romer \(2000\)](#) show that Federal Reserve decisions reveal information not contained in commercial forecasts, while [Nakamura and Steinsson \(2018\)](#) use high-frequency movements in real rates and forecasts to document an information effect around FOMC announcements. [Jarociński and Karadi \(2020\)](#) exploit the joint response of interest rates and stock prices to separate policy and information shocks, and [Miranda-Agrippino and Ricco \(2021\)](#) purge policy innovations of predictable central bank responses. [Bauer and Swanson \(2023\)](#) emphasize private agents' response to public news before the announcement as an alternative explanation. We complement these approaches by observing revisions directly across three populations and by showing that the short-horizon real-activity response survives across instruments built to address different forms of contamination.

Second, we contribute to the literature that uses surveys to study expectation formation. Predictable forecast errors and sluggish revisions provide extensive evidence against full-information rational expectations ([Coibion and Gorodnichenko 2012, 2015](#)). Sticky-information and noisy-information models explain such behavior through infrequent updating or imperfect signals ([Mankiw and Reis 2002](#); [Sims 2003](#); [Woodford 2003](#); [Maćkowiak and Wiederholt 2009](#)). Our exercise differs in conditioning on monetary policy announcements and in requiring one estimated equilibrium model to account jointly for forecast revisions and aggregate impulse responses. The distinction between household and firm information sets also allows the same informational friction to operate through aggregate demand and price setting rather than treating survey expectations as an external wedge.

Third, the paper relates to work on higher-order beliefs and endogenous persistence. When agents act on private information, aggregate expectations need not satisfy the representative-agent law of iterated expectations, and strategic interaction can amplify sluggish learning ([Nimark 2008](#); [Angeletos and Huo 2018, 2021](#)). We embed this logic in a quantitative New Keynesian model with two decision-making groups and estimate the information structure from event-time responses. The resulting model is rich enough to distinguish the observed surprise from the underlying monetary innovation while remaining sufficiently tractable to derive both objects in closed form.

Finally, our beliefs-adjusted identification exercise is related to sign and narrative approaches that discipline monetary shocks with qualitative economic restrictions ([Uhlig 2005](#); [Antolín-Díaz](#)

and Rubio-Ramírez 2018a). We add restrictions on survey revisions consistent with the estimated model while leaving the relative responses of conventional macroeconomic variables unrestricted. This lets beliefs select among observationally admissible rotations and provides a direct bridge between expectation formation and structural identification.

Roadmap. Section 2 presents the survey evidence. Section 3 develops the dispersed-information New Keynesian model. Section 4 estimates the model, evaluates its fit, and reports the pure-shock counterfactual. Section 5 constructs the beliefs-adjusted monetary policy shock series. A final section concludes. Derivations, additional empirical results, and the exact macroeconomic targets used in estimation appear in the Appendix.

2. Empirical Puzzles on Expectations

We begin from the standard monetary benchmark. In the canonical New Keynesian model, a contractionary monetary policy shock raises the nominal interest rate above the systematic component of the policy rule. With sticky prices, the real interest rate rises and aggregate demand contracts. Output and inflation fall.

Full-information rational expectations (FIRE) extend these predictions to beliefs. If agents observe the shock and know the structure of the economy, they revise output and inflation forecasts downward and the nominal-rate forecast upward. The signs of realized outcomes and forecasts should therefore be aligned.

We confront this implication with evidence from three populations of U.S. forecasters. The MSC captures household beliefs at monthly frequency, CE surveys financial and research economists each month, and the SPF provides quarterly forecasts from professional forecasters. We estimate a Bayesian proxy-VAR separately for each belief measure and each of six monetary policy instruments. The common departure from FIRE is on the real side: identified contractionary monetary policy shocks improve beliefs about near-term activity while raising expected interest rates. The inflation response varies across surveys, a distinction we preserve throughout the analysis.

2.1. Data on Beliefs

We construct belief measures from three surveys that differ in population coverage, question format, and frequency. Our primary evidence uses MSC and CE at monthly frequency; SPF provides a quarterly cross-check. For CE and SPF, which collect quantitative point forecasts at specific target dates, we define the average forecast revision as

$$(1) \quad FR_t(x_{t+h}) = F_t x_{t+h} - F_{t-1} x_{t+h},$$

where $F_t x_{t+h}$ is the cross-sectional mean forecast formed at date t for variable x at horizon h . The revision measures the change in average beliefs about the same future outcome across two consecutive survey rounds. Two features of the MSC prevent applying the same construction. First, the MSC does not collect point forecasts: most questions yield qualitative responses or balance scores rather than numerical estimates. Second, the MSC asks about conditions “over the next twelve months,” a rolling horizon; consecutive surveys therefore target different end periods, violating the fixed-target condition embedded in FR_t .

The Michigan Survey of Consumers is conducted monthly by the University of Michigan using a rotating panel designed to represent U.S. households, with data available since the 1940s. We use three measures. The Index of Consumer Sentiment (ICS) aggregates responses to five questions about personal finances and aggregate business conditions, providing a composite indicator of perceived economic well-being. The mean expected inflation rate comes from the question “By about what percent do you expect prices in general to go up or come down during the next 12 months?” Because the question does not specify a price index, respondents may have in mind any combination of CPI, gasoline, food, or other prices; direct quantitative comparison with professional forecasters’ CPI or GDP deflator forecasts is therefore not appropriate, and we treat this measure as qualitative evidence on inflation expectations. The interest-rate balance is the net percentage expecting borrowing rates to rise rather than fall, so higher values indicate expectations of tighter rates.

Consensus Economics surveys professional forecasters from financial institutions, consulting firms, research centers, and universities each month. Respondents provide forecasts of approximately 20 macroeconomic variables anchored to specific calendar years rather than fixed horizons ahead. The CE data therefore require a rolling-horizon interpolation to form a consistent one-year-ahead revision, detailed in Section 2.2. We use forecast revisions for real GDP growth, CPI inflation, and the three-month Treasury bill rate.

The Survey of Professional Forecasters has been conducted quarterly since 1968Q4, first by the American Statistical Association and the National Bureau of Economic Research and, since 1990, by the Federal Reserve Bank of Philadelphia. Each quarter, the questionnaire is sent on the day the Bureau of Economic Analysis releases its advance GDP estimate for the prior quarter. We use forecast revisions for real GDP growth, CPI inflation, and the three-month Treasury bill rate, enabling comparison with the CE evidence at a lower frequency.

2.2. Estimation

To study the effects of monetary policy shocks on aggregate variables and beliefs, we estimate a Bayesian vector autoregression (VAR) following [Miranda-Agrippino and Ricco \(2021\)](#). The

reduced-form VAR is

$$(2) \quad \mathbf{x}_t = \mathbf{c} + \sum_{k=1}^p \beta_k \mathbf{x}_{t-k} + \mathbf{u}_t, \quad \mathbf{u}_t \sim \mathcal{N}(\mathbf{0}, \Sigma_u),$$

where $\mathbf{x}_t$ denotes the vector of endogenous variables at time t , $\mathbf{c}$ is a vector of intercepts, p is the lag length, and $\mathbf{u}_t$ is a Gaussian white-noise innovation with covariance matrix Σ_u . The vector $\mathbf{x}_t$ includes the one-year Treasury rate as the policy indicator,¹ the unemployment rate, the consumer price index, the excess bond premium (EBP) of [Gilchrist and Zakrajšek \(2012\)](#), and one measure of beliefs. We include each belief measure separately. For CE, we construct the rolling forecast revision

$$(3) \quad \text{FR}_t^{(12)} = \frac{h}{12} \text{FR}_t^{(h)} + \frac{12-h}{12} \text{FR}_t^{(12+h)}$$

following [Dovern et al. \(2012\)](#) and [Miranda-Agrippino and Ricco \(2021\)](#), because CE forecasts are anchored to calendar years rather than to a fixed horizon. We use fixed-horizon revisions for SPF and the corresponding survey indices for MSC. The sample spans 1978M1–2019M6 for MSC, 1989M11–2019M6 for CE, and 1990Q4–2019Q1 for SPF. The monthly VARs include nine lags, the quarterly VARs three lags, and all systems use a Jeffreys prior.

Identification of monetary policy shocks is central to our analysis. Reduced-form innovations to interest rates may reflect endogenous policy responses, anticipatory movements, central bank information effects, or a “Fed response to news” channel. To identify an exogenous monetary policy shock, we employ external instruments in the spirit of [Stock and Watson \(2012\)](#) and [Mertens and Ravn \(2013\)](#). Let $\varepsilon_t^{\text{mp}}$ denote the structural monetary policy shock and z_t an external instrument. The instrument must satisfy two moment conditions:

$$(4) \quad \text{Relevance:} \quad \mathbb{E}[z_t \varepsilon_t^{\text{mp}}] \neq 0,$$

$$(5) \quad \text{Exogeneity:} \quad \mathbb{E}[z_t \varepsilon_t^j] = 0 \quad \text{for all } j \neq \text{mp}.$$

The first condition requires the instrument to be correlated with the monetary policy shock. The second requires orthogonality with all other structural disturbances. Operationally, the covariance between the instrument and the reduced-form VAR innovations identifies the relative impact vector associated with the structural monetary policy shock. We use this impact vector to construct impulse responses. Thus, the responses below are responses to an identified monetary policy shock, not to the instrument itself.

We consider six instruments widely used in the literature. [Romer and Romer \(2004\)](#) construct narrative shocks from internal Federal Reserve documents and staff forecasts to isolate intended

¹We follow [Gertler and Karadi \(2015\)](#) and use the one-year nominal rate to account for the zero lower bound episode.

policy actions from endogenous and anticipatory responses. [Gertler and Karadi \(2015\)](#) employ high-frequency surprises around FOMC announcements, interpreting narrow-window interest rate changes as exogenous policy innovations. [Jarociński and Karadi \(2020\)](#) decompose high-frequency surprises by exploiting the joint movement of interest rates and stock prices to separate monetary policy shocks from central bank information shocks. [Miranda-Agrippino and Ricco \(2021\)](#) further refine high-frequency instruments by accounting explicitly for information rigidities. [Bauer and Swanson \(2023\)](#) introduce the “Fed response to news” channel, emphasizing that monetary surprises may reflect systematic responses to publicly available information. Finally, [Aruoba and Drechsel \(2025\)](#) develop a narrative-based approach using natural language processing to infer the Federal Reserve’s information set and extract policy shocks from textual communication.

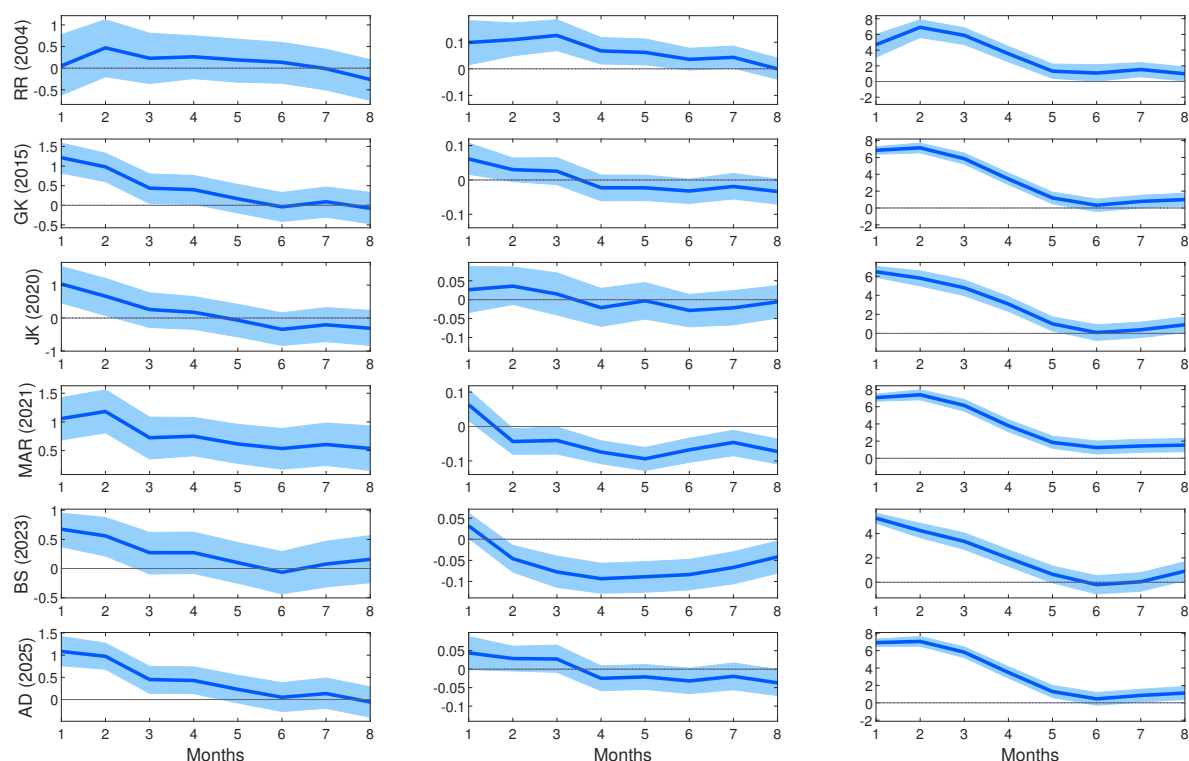
For CE, we retain only instruments associated with FOMC decisions that precede the survey date, leaving 144 instrument observations (40.33% of the original sample). For SPF, we cumulate within each quarter the monthly instruments that precede the survey date and discard subsequent observations. Individual MSC response dates are unavailable, so an analogous timing restriction cannot be applied to that survey.

2.3. Results

Each figure below displays impulse responses of belief measures to a contractionary one-standard-deviation monetary policy shock. The six rows correspond to the six instruments: [Romer and Romer \(2004\)](#), [Gertler and Karadi \(2015\)](#), [Jarociński and Karadi \(2020\)](#), [Miranda-Agrippino and Ricco \(2021\)](#), [Bauer and Swanson \(2023\)](#), and [Aruoba and Drechsel \(2025\)](#). Shaded bands are 68 percent posterior coverage intervals. The three columns in each figure report responses of expected real activity, expected inflation, and expected short-term interest rates. We impose the timing restriction for CE and SPF throughout; CE results without this restriction appear in Appendix Figure [A5](#) and are qualitatively unchanged.

2.3.1. Michigan Survey of Consumers

Figure [1](#) reports results for household beliefs. Three findings stand out. First, the Index of Consumer Sentiment rises on impact with 68 percent posterior intervals above zero for five of the six instruments. The response based on [Romer and Romer \(2004\)](#) is positive but imprecisely estimated, and its posterior interval includes zero. The precisely estimated responses span both high-frequency and narrative or text-based identification strategies. Second, mean expected inflation is positive in several specifications but is less precisely estimated. Moreover, the MSC question refers to “prices in general” rather than to a specific price index. Third, households anticipate higher interest rates, confirming that they process the rate signal while becoming more optimistic about economic conditions.



A. Consumer Sentiment

B. Inflation Expectations

C. Interest Rate Expectations

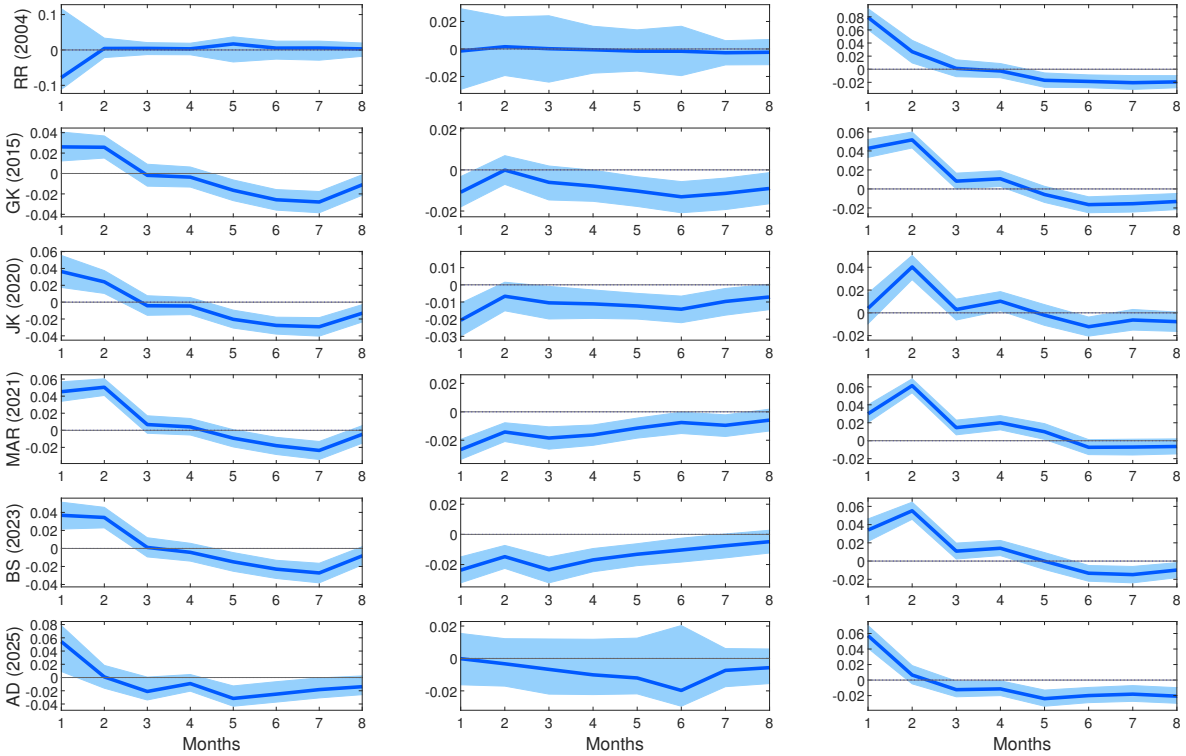
Note: Each row corresponds to a different monetary policy instrument. RR (2004): [Romer and Romer \(2004\)](#); GK (2015): [Gertler and Karadi \(2015\)](#); JK (2020): [Jarociński and Karadi \(2020\)](#); MAR (2021): [Miranda-Agrippino and Ricco \(2021\)](#); BS (2023): [Bauer and Swanson \(2023\)](#); AD (2025): [Aruoba and Drechsel \(2025\)](#). Horizontal axis: months after the shock. Shaded areas are 68% posterior coverage bands. Consumer Sentiment is the Index of Consumer Sentiment. Inflation expectations are the mean expected annual price change. Interest rate expectations are the balance score for expected rate changes.

FIGURE 1. Impulse Responses to a Monetary Policy Shock: Michigan Survey of Consumers

The increase in consumer sentiment is concentrated at short horizons and fades over the following three to five months. Household optimism therefore appears as a short-lived revision around the announcement rather than a persistent shift in the outlook. Appendix Figure A2 provides related evidence from one-year business expectations, which rise on impact with 68 percent posterior intervals above zero for four of the six instruments.

2.3.2. Consensus Economics

Figure 2 reports the CE evidence from financial and research economists. Five of the six estimates show a positive impact response of real GDP growth forecast revisions, ranging from approximately 0.02 to 0.06 percentage points, with 68 percent posterior intervals above zero. The estimate obtained



A. Real GDP Growth

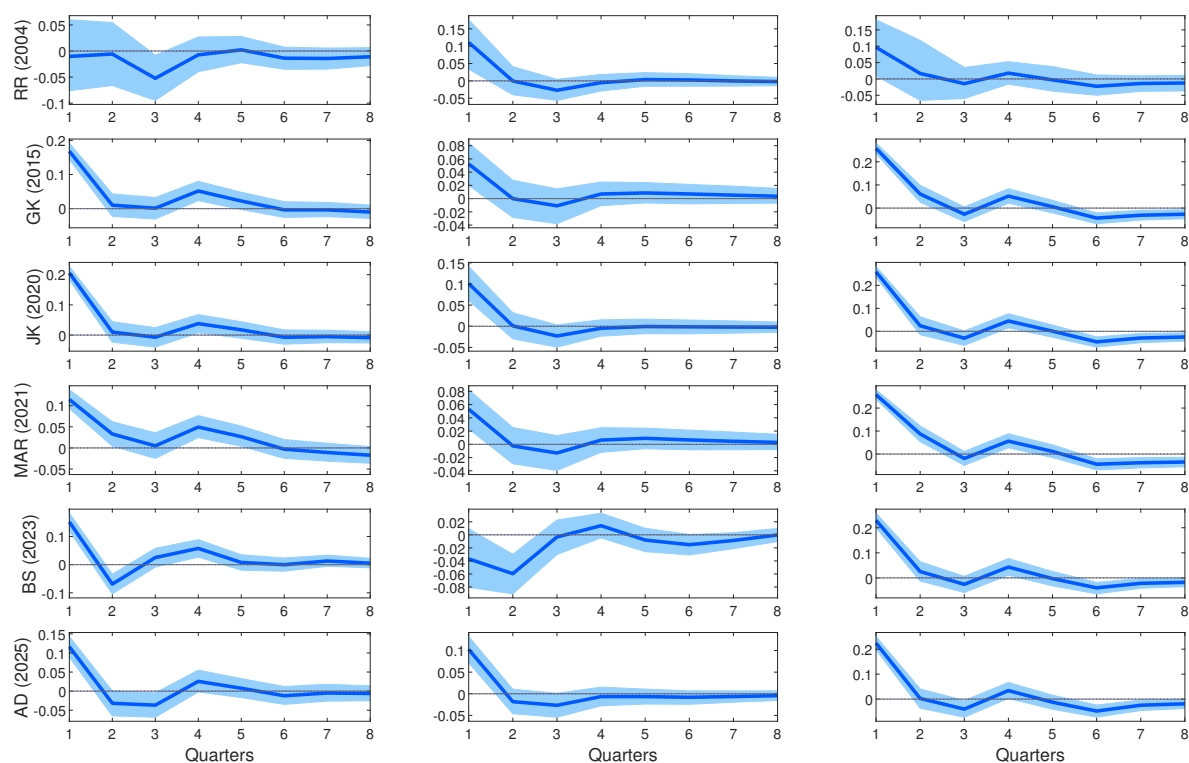
B. CPI Inflation

C. 3-Month T-Bill Rate

Note: Sample restricted to months in which the FOMC decision precedes the survey date. Each row corresponds to a different instrument. RR (2004): [Romer and Romer \(2004\)](#); GK (2015): [Gertler and Karadi \(2015\)](#); JK (2020): [Jarociński and Karadi \(2020\)](#); MAR (2021): [Miranda-Agrippino and Ricco \(2021\)](#); BS (2023): [Bauer and Swanson \(2023\)](#); AD (2025): [Aruoba and Drechsel \(2025\)](#). Horizontal axis: months. Shaded areas are 68% posterior coverage bands.

FIGURE 2. Impulse Responses to a Monetary Policy Shock: Consensus Economics

with the [Romer and Romer \(2004\)](#) instrument is centered near zero and imprecise. Across the informative specifications, CE forecasters revise GDP upward following identified monetary policy shocks. CPI inflation revisions are centered near zero or slightly negative on impact. Thus, the positive GDP revisions coexist with little or no inflationary news. Three-month Treasury bill forecasts rise by approximately 0.05 percentage points on impact, so the activity revisions also coexist with an anticipated tightening. Appendix Figure A5 shows qualitatively similar output and rate responses without the timing restriction. CPI revisions become somewhat more negative, indicating that within-month timing matters more for inflation revisions than for the GDP and interest-rate responses.



A. Real GDP Growth

B. CPI Inflation

C. 3-Month T-Bill Rate

Note: Each row corresponds to a different instrument. RR (2004): [Romer and Romer \(2004\)](#); GK (2015): [Gertler and Karadi \(2015\)](#); JK (2020): [Jarociński and Karadi \(2020\)](#); MAR (2021): [Miranda-Agrippino and Ricco \(2021\)](#); BS (2023): [Bauer and Swanson \(2023\)](#); AD (2025): [Aruoba and Drechsel \(2025\)](#). Horizontal axis: quarters. Shaded areas are 68% posterior coverage bands. Sample restricted to include only shocks falling before the survey date within each quarter.

FIGURE 3. Impulse Responses to a Monetary Policy Shock: Survey of Professional Forecasters

2.3.3. Survey of Professional Forecasters

Figure 3 reports quarterly evidence from the SPF. Real GDP growth forecast revisions are positive on impact across all six instruments, ranging from 0.10 to 0.20 percentage points. CPI inflation revisions are also positive in several specifications, while the three-month Treasury bill forecast rises by roughly 0.20 percentage points on impact. The GDP result reinforces the monthly evidence. The CPI result is more tentative because it is not shared by CE and is estimated in a quarterly system.

First, the sign and impact profile of the GDP revision are consistent with the monthly CE evidence: both surveys show upward revisions concentrated at short horizons. Their magnitudes are not directly comparable because survey frequency, forecast targets, and the aggregation of policy announcements differ.

Second, SPF respondents are professional macroeconomists who release their projections publicly. The positive activity revision therefore also appears among forecasters with strong incentives and substantial expertise in processing monetary and macroeconomic news.

Third, as with MSC and CE, the responses are concentrated at the impact horizon. GDP and CPI revisions lose statistical significance after one to two quarters. This profile is consistent with a short-lived informational component that is strongest at the announcement and dissipates as subsequent data arrive.

2.4. Summary of Empirical Findings

The impulse responses reveal a systematic departure from the FIRE benchmark, but not a uniform one across variables. The most consistent pattern is the response of expected activity: identified contractionary monetary policy shocks generally improve the near-term outlook across households, financial economists, and professional forecasters, even as expected short-term rates rise. Inflation beliefs are imprecisely positive in parts of the MSC evidence, close to zero or negative in CE, and more clearly positive only in the quarterly SPF system. The real-activity result appears across identification strategies ranging from narrative measures ([Romer and Romer 2004](#)) to high-frequency instruments designed to separate policy and information components ([Jarociński and Karadi 2020](#); [Miranda-Agrippino and Ricco 2021](#)).

This pattern is consistent with announcements revealing information about the state of the economy that is absent from private priors. If the central bank’s information set differs from that of private agents, an unexpected rate increase can reveal favorable fundamentals even when the policy innovation itself is contractionary. [Jarociński and Karadi \(2020\)](#) show that mixing these components distorts inference about monetary transmission. In our data, the increase in output beliefs is consistent with good news inferred from the central bank’s systematic response. The heterogeneous inflation evidence suggests that the informational component is not a single one-dimensional signal.

A second disciplining fact is that the unusual belief response is short-lived. Revisions are largest on impact and generally return toward zero within a few months or quarters. This profile is consistent with agents extracting a signal from the announcement and subsequently incorporating macroeconomic releases that reduce uncertainty. The next section formalizes this mechanism in an equilibrium model in which private signals are noisy and the policy rate is informative about several latent disturbances.

3. Theoretical Framework

Section 2 established that contractionary monetary policy shocks identified with standard external instruments can improve the near-term outlook even as agents anticipate higher rates. We now

develop an equilibrium model in which the policy rate is both an instrument and a signal. An empirically identified disturbance can then combine a contractionary monetary innovation with news about the fundamentals to which policy responds, allowing favorable belief revisions and contractionary macroeconomic effects to coexist.

3.1. Motivation Behind the Theoretical Model

The empirical evidence places four requirements on the theoretical framework. First, it must explain why a disturbance identified as monetary can generate optimistic belief revisions. We allow the policy-rate innovation observed by private agents to contain both a monetary disturbance and information about the systematic policy response. The proxy-VAR shock remains the empirical object of interest; the model characterizes the information about fundamentals that can remain associated with it. Under full information, agents observe the source of the rate movement and this signal-extraction channel disappears.

Second, beliefs must affect the decisions that determine aggregate outcomes. We distinguish households from firms because expectations enter the two central New Keynesian margins differently. Households' beliefs about real rates and income determine spending, whereas firms' beliefs about marginal costs, demand, and other prices determine price setting. Group-specific signal precision lets the model confront belief heterogeneity without disconnecting forecasts from behavior.

Third, agents must be uncertain about more than the monetary disturbance. We include monetary, demand, technology, and markup shocks so that an observed rate movement has competing structural interpretations. A higher rate attributed to an exogenous monetary innovation is bad news for activity, whereas the same movement can convey favorable news when it is associated with stronger demand or productivity. The estimated shock processes and signal precisions determine the relative importance of these interpretations.

Fourth, the framework must propagate beliefs through general equilibrium and deliver a pure-shock counterfactual. Noisy private signals generate higher-order uncertainty: agents care not only about fundamentals, but also about how others perceive them. The resulting spending–income and pricing interactions create endogenous persistence, as in [Angeletos and Huo \(2018\)](#) and [Angeletos and Huo \(2021\)](#). At the same time, the linear solution lets us remove the nonmonetary news from the observed surprise and trace the structural monetary innovation. These features allow the model to confront the six empirical impulse responses within a common equilibrium framework.

We implement these requirements by adding dispersed information to an otherwise standard New Keynesian economy. Households and firms observe noisy private signals about the four aggregate disturbances. The central bank observes the aggregate state and sets the policy rate according to a Taylor rule. Private agents therefore cannot perfectly distinguish an exogenous policy innovation from the endogenous response of policy to fundamentals. An unexpected increase in the rate changes

their posterior beliefs about both, affecting aggregate demand through household spending and inflation through firms' price setting.

3.2. Households

There is a unit continuum of ex ante identical households, indexed by $i \in \mathcal{I}_c = [0, 1]$. Household i chooses consumption, labor, and bond holdings to maximize $\mathbb{E}_{i0} \sum_{t=0}^{\infty} \beta^t U(C_{it}, N_{it}; Z_t)$, where Z_t is a common preference shifter and $U(C, N; Z) = \left[C^{1-\sigma}/(1-\sigma) - N^{1+\varphi}/(1+\varphi) \right] Z$. The parameter $1/\sigma$ is the intertemporal elasticity of substitution and φ is the inverse Frisch elasticity. Households are identical in preferences and endowments but can choose differently because their private signals, and hence their posterior beliefs, differ.

Households' Problem. The household consumption index is $C_{it} = \left(\int_{\mathcal{J}_f} C_{ijt}^{(\epsilon_t-1)/\epsilon_t} dj \right)^{\epsilon_t/(\epsilon_t-1)}$, where C_{ijt} is household i 's consumption of variety $j \in \mathcal{J}_f = [0, 1]$ and ϵ_t is the elasticity of substitution across varieties. Cost minimization yields

$$(6) \quad C_{ijt} = (P_{jt}/P_t)^{-\epsilon_t} C_{it}$$

where the aggregate price index is defined as $P_t \equiv \left(\int_{\mathcal{J}_f} P_{jt}^{1-\epsilon_t} dj \right)^{\frac{1}{1-\epsilon_t}}$. Using the above conditions, one can show that $\int_{\mathcal{J}_f} P_{jt} C_{ijt} dj = P_t C_{it}$.

In real terms, households choose consumption, labor, and savings subject to

$$(7) \quad C_{it} + B_{it} = R_{t-1} B_{i,t-1} + W_{it}^r N_{it} + D_{it},$$

where B_{it} denotes savings (or bond purchases) by household i , R_{t-1} denotes the gross real return on savings, W_{it}^r denotes the real wage at time t , and D_{it} denotes dividends received from the profits produced by firms.

The optimality conditions from the household problem satisfy $C_{it}^{-\sigma} = \beta \mathbb{E}_{it} \left(R_t Z_{t+1} / Z_t C_{i,t+1}^{-\sigma} \right)$ and $C_{it}^{\sigma} N_{it}^{\varphi} = \mathbb{E}_{it} W_t^r$.

Auxiliary Shocks. If individual wages and dividends revealed the aggregate state without noise, households could reconstruct other agents' information from equilibrium outcomes, undoing the intended information friction. Following [Angeletos and Huo \(2018\)](#), we therefore overlay each household's observed labor and dividend income with high-variance idiosyncratic auxiliary shocks, and impose the analogous structure on firms. Private realizations then remain noisy signals of aggregate conditions.

Let W_t and D_t denote, respectively, the average real wage and the average real dividend from firms' profits. The real wage and the dividend received by consumer i at time t are $W_{it} = \zeta_{it}^W W_t$ and $D_{it} = \zeta_{it}^D D_t$, where $\{\zeta_{it}^W, \zeta_{it}^D\}$ are i.i.d. across i and t . The auxiliary shocks enter only at the individual level, average out in the aggregate, and do not affect equilibrium dynamics.

3.3. Firms

Each variety is produced by a monopolistically competitive firm $j \in \mathcal{J}_f = [0, 1]$ using labor, $Y_{jt} = A_t N_{jt}^{1-\alpha}$. Common technology follows the log-AR(1) process

$$(8) \quad a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

where $0 \leq \rho_a < 1$ and the innovation satisfies $\varepsilon_t^a \sim \mathcal{N}(0, \sigma_a^2)$.

Price-Setting. Nominal price rigidities take the form of a Calvo price-setting friction. In every period, each firm can reset its price with probability $1 - \theta$, independent of the time of the last price change. A measure $1 - \theta$ of firms can reset their prices in a given period, and the average duration of a price is given by $1/(1 - \theta)$.

A firm that reoptimizes in period t chooses P_{jt}^* to maximize the expected discounted profits generated while that price remains effective. Formally,

$$P_{jt}^* = \arg \max_{P_{jt}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt} \left\{ \Lambda_{t,t+k} / P_{t+k} \left[P_{jt} Y_{j,t+k|t} - \mathcal{C}_{t+k}(Y_{j,t+k|t}) \right] \right\},$$

subject to the demand schedule in (6). Here $\Lambda_{t,t+k} \equiv \beta^k (C_{t+k}/C_t)^{-\sigma}$ is the stochastic discount factor, $\mathcal{C}_t(\cdot)$ is nominal cost, and $Y_{j,t+k|t}$ is output at $t+k$ for a firm that last reset its price at t . The first-order condition is

$$(9) \quad \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt} \left[\Lambda_{t,t+k} Y_{j,t+k|t} / P_{t+k} (P_{jt} - \mathcal{M}_t \Psi_{j,t+k|t}) \right] = 0,$$

where $\mathcal{M}_t = \epsilon_t/(\epsilon_t - 1)$ denotes the time-varying markup and $\Psi_{j,t+k|t} \equiv \mathcal{C}'_{t+k}(Y_{j,t+k|t})$ denotes nominal marginal cost for firm j .

Auxiliary Shocks. On the firm side, nominal wages include the auxiliary component $W_{jt}^n = \zeta_{jt}^F W_t^n$, where ζ_{jt}^F is i.i.d. across j and t .

3.4. Monetary Policy

Monetary policy is conducted following a Taylor rule of the form

$$(10) \quad i_t = \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t$$

where π_t denotes the inflation rate, $\tilde{y}_t$ denotes the output gap, and the monetary policy shock v_t follows the AR(1) process

$$(11) \quad v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

where $0 \leq \rho_v < 1$ and $\varepsilon_t^v \sim \mathcal{N}(0, \sigma_v^2)$.

3.5. Information Structure

Households and firms do not observe the four aggregate disturbances directly. Their information sets consist of the histories of their private signals.²

Each period, agent l in group g receives one Gaussian signal about each disturbance:

$$(12) \quad x_{lgt}^v = v_t + u_{lgt}^v, \quad u_{lgt}^v \sim \mathcal{N}(0, \sigma_{gv}^2),$$

$$(13) \quad x_{lgt}^z = z_t + u_{lgt}^z, \quad u_{lgt}^z \sim \mathcal{N}(0, \sigma_{gz}^2),$$

$$(14) \quad x_{lgt}^a = a_t + u_{lgt}^a, \quad u_{lgt}^a \sim \mathcal{N}(0, \sigma_{ga}^2),$$

$$(15) \quad x_{lgt}^\mu = \mu_t + u_{lgt}^\mu, \quad u_{lgt}^\mu \sim \mathcal{N}(0, \sigma_{g\mu}^2),$$

where $g \in \{c, f\}$ indexes households and firms and $n \in \{v, z, a, \mu\}$ indexes the monetary, demand, technology, and markup disturbances. The standard deviation σ_{gn} governs the precision of group g 's signal about disturbance n . Allowing it to vary across groups and shocks is the model's source of information heterogeneity.

3.6. Equilibrium Dynamics

We solve the log-linearized model around a zero-inflation steady state. Appendix C provides the full derivation; this section summarizes the main steps.

DEFINITION 1. *Given an initial price level P_{-1} , a general equilibrium is a path for prices $\{P_t, W_t, \pi_t, R_t, i_t\}_{t=0}^\infty$, aggregates $\{Y_t, C_t, B_t, D_t\}_{t=0}^\infty$, and individual allocation rules $\{C_{it}, P_{jt}\}_{t=0, \forall i, j}^\infty$*

²The idiosyncratic components of the auxiliary shocks introduced in Sections 3.2–3.3 are assumed to have large variance relative to their aggregate components.

such that consumers and firms optimize, taking their information sets as given; monetary policy follows (10); and the goods, labor, and bond markets clear.

DIS curve. The optimal intratemporal labor supply condition can be log-linearized to

$$(16) \quad \mathbb{E}_{it} w_t^r = \sigma c_{it} + \varphi n_{it},$$

where a lowercase variable denotes the log deviation of its uppercase counterpart from steady state, $x_t = \log X_t - \log X$, and the intertemporal Euler condition can be log-linearized to

$$(17) \quad c_{it} = -1/\sigma \mathbb{E}_{it} r_t + (1 - \rho_z)/\sigma \mathbb{E}_{it} z_t + \mathbb{E}_{it} c_{it+1},$$

where we define the ex-post real interest rate as the difference between the nominal interest rate and one-period-ahead inflation, $r_t = i_t - \pi_{t+1}$, and $z_t \equiv \log Z_t$ follows an exogenous AR(1) process

$$(18) \quad z_t = \rho_z z_{t-1} + \varepsilon_t^z,$$

where $0 \leq \rho_z < 1$ and $\varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$. The individual household DIS curve is then

$$(19) \quad c_{it} = -\beta/\sigma \mathbb{E}_{it} r_t + \beta(1 - \rho_z)/\sigma \mathbb{E}_{it} z_t - \beta\psi(1 - \rho_a) \mathbb{E}_{it} a_t + (1 - \beta) \mathbb{E}_{it} c_t + \beta \mathbb{E}_{it} c_{i,t+1},$$

with $c_t = \int c_{it} di$ and $\psi \equiv (1 + \varphi)/[\sigma(1 - \alpha) + \varphi + \alpha]$.³ Iterating forward and aggregating across households, the aggregate DIS curve can be written as

$$(20) \quad \tilde{y}_t = -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \bar{\mathbb{E}}_t^c r_{t+k} + \frac{\beta(1 - \rho_z)}{\sigma} \sum_{k=0}^{\infty} \beta^k \bar{\mathbb{E}}_t^c z_{t+k} - \beta\psi(1 - \rho_a) \sum_{k=0}^{\infty} \beta^k \bar{\mathbb{E}}_t^c a_{t+k} + (1 - \beta) \sum_{k=0}^{\infty} \beta^k \bar{\mathbb{E}}_t^c \tilde{y}_{t+k}$$

where $\tilde{y}_t = \tilde{c}_t$ denotes the output gap, and $\bar{\mathbb{E}}_t^c(\cdot) = \int_0^1 \mathbb{E}_{it}(\cdot) di$ denotes the cross-sectional average forecast across households.⁴ As in the textbook NK model, the demand curve (20) can be summarized as a single equation; however, under incomplete information, it cannot be collapsed into a first-order stochastic difference equation because the hierarchy of beliefs prevents the law of iterated expectations (LIE) from holding at the aggregate level.

Phillips curve. Following Angeletos and Huo (2018) and Angeletos and Huo (2021), we simplify the model by assuming that firms observe aggregate prices through period $t - 1$ but do not extract

³Appendix C provides the model derivation.

⁴Appendix C shows that the expected natural real interest rate is $\mathbb{E}_{it} r_t^n = -\sigma\psi(1 - \rho_a) \mathbb{E}_{it} a_t$.

information from them.⁵ This allows us to write the Phillips curve in inflation terms.

Combining the firms' pricing condition and the goods market clearing, the (log-linearized) firm-level Phillips curve is given by

$$(21) \quad \pi_{jt} = \theta \lambda \mathbb{E}_{jt} \mu_t + \kappa \theta \mathbb{E}_{jt} \tilde{y}_t + (1 - \theta) \mathbb{E}_{jt} \pi_t + \beta \theta \mathbb{E}_{jt} \pi_{j,t+1}$$

where $\pi_t = \int \pi_{jt} dj$, $\lambda = (1 - \theta)(1 - \beta\theta)(1 - \alpha)/[\theta(1 - \alpha + \alpha\epsilon)]$, $\kappa = \lambda [\sigma + (\varphi + \alpha)/(1 - \alpha)]$, and μ_t is a markup shock, which follows an exogenous AR(1) process

$$(22) \quad \mu_t = \rho_\mu \mu_{t-1} + \varepsilon_t^\mu$$

where $0 \leq \rho_\mu < 1$ and $\varepsilon_t^\mu \sim \mathcal{N}(0, \sigma_\mu^2)$.⁶ Iterating forward and aggregating across firms, the aggregate Phillips curve can be written as

$$(23) \quad \pi_t = \lambda \theta \sum_{k=0}^{\infty} (\beta\theta)^k \bar{\mathbb{E}}_t^f \mu_{t+k} + \kappa \theta \sum_{k=0}^{\infty} (\beta\theta)^k \bar{\mathbb{E}}_t^f \tilde{y}_{t+k} + (1 - \theta) \sum_{k=0}^{\infty} (\beta\theta)^k \bar{\mathbb{E}}_t^f \pi_{t+k}$$

where $\bar{\mathbb{E}}_t^f(\cdot) = \int_0^1 \mathbb{E}_{jt}(\cdot) dj$ is the cross-sectional average forecast across firms.

Equilibrium Dynamics. The economy is described by two interacting dynamic beauty contests: the spending–income feedback in households' decisions, summarized by (19), and strategic complementarity in firms' price setting, summarized by (21).

The equilibrium dynamics must satisfy the individual-level optimal policy functions (19) and (21). Expectations must be consistent with the Taylor rule (10), the exogenous shock processes, and the signals in (12)–(15). Proposition 1 characterizes the resulting fixed point.

PROPOSITION 1. *In equilibrium, the aggregate outcome obeys the following law of motion*

$$(24) \quad c_t = \sum_{n \in \{v, z, a, \mu\}} \sum_{j=1}^2 \frac{\psi_{1jn}}{\rho_n} \sum_{k=0}^{\infty} \left(\rho_n^{k+1} - \vartheta_{jn}^{k+1} \right) \varepsilon_{t-k}^n$$

$$(25) \quad \pi_t = \sum_{n \in \{v, z, a, \mu\}} \sum_{j=1}^2 \frac{\psi_{2jn}}{\rho_n} \sum_{k=0}^{\infty} \left(\rho_n^{k+1} - \vartheta_{jn}^{k+1} \right) \varepsilon_{t-k}^n$$

where the coefficients ψ_{kjn} and the stable roots ϑ_{jn} are functions of the calibrated parameters, the persistence and variance of disturbance n , and the two groups' signal precision. Appendix B gives the complete mapping while allowing $\sigma_{cn} \neq \sigma_{fn}$.

⁵An alternative formulation lets firms remain uncertain about the history of the aggregate price level, p^{t-1} .

⁶Appendix C provides the model derivation.

PROOF. See Appendix B. □

The representation separates fundamental persistence, ρ_n , from the endogenous roots ϑ_{jn} generated by dispersed information and strategic interaction. Each impulse response is a weighted difference between these decay rates. As signals become uninformative, the endogenous roots approach the persistence of the underlying state: aggregate responses become more delayed and persistent. As signal noise vanishes, the endogenous roots converge toward zero and this additional source of persistence disappears. The model can therefore match sluggish aggregate dynamics and sharp forecast revisions without imposing ad hoc lagged output or inflation terms.

4. Estimation and Results

Section 2 estimates responses to monetary policy shocks identified by proxy VARs. To interpret their informational content, we match these responses to the model's policy-rate surprise. Unlike the pure monetary innovation, this surprise can also load on news about nonmonetary fundamentals. Let the preannouncement rate forecast based on the household information set be

$$\widehat{i}_{t|t-}^c = \phi_\pi \bar{\mathbb{E}}_{t-}^c \pi_t + \phi_y \bar{\mathbb{E}}_{t-}^c \tilde{y}_t + \bar{\mathbb{E}}_{t-1}^c v_t.$$

The timing convention treats output and inflation as current nowcast objects, whereas the contemporaneous monetary innovation is realized with the policy decision. The surprise is therefore

$$(26) \quad \text{mps}_t^c = i_t - \widehat{i}_{t|t-}^c = \phi_\pi \left(\pi_t - \bar{\mathbb{E}}_{t-}^c \pi_t \right) + \phi_y \left(\tilde{y}_t - \bar{\mathbb{E}}_{t-}^c \tilde{y}_t \right) + v_t - \bar{\mathbb{E}}_{t-1}^c v_t.$$

The superscript c indexes the household information set. We refer to mps_t^c as the composite policy surprise. The first two terms are nowcast errors about the endogenous state and capture news embedded in the systematic policy response. The last term is the monetary component of the surprise; under incomplete information it can include the current innovation and previously unlearned monetary innovations. The pure structural shock considered below is the current innovation ε_t^v , not the entire last term. Under full information, previously unlearned components vanish and the surprise collapses to the current monetary innovation. With noisy information, by contrast, a positive surprise can be associated with several orthogonal fundamentals. Proposition A1 in Appendix B derives the corresponding linear-projection responses of aggregate outcomes and forecast revisions.

4.1. Minimum-Distance Estimation

We estimate the model at quarterly frequency and require every target to correspond to the same identified shock. The macroeconomic targets are the responses to the monetary policy shock

identified with the [Jarociński and Karadi \(2020\)](#) instrument in a monthly proxy VAR containing an industrial-production gap, the CPI, and the three-month Treasury bill rate. We aggregate the monthly posterior impulse responses to quarterly frequency; the output gap and interest rate remain in levels, while CPI inflation responses are cumulated to obtain the price-level response. The belief targets are the output-growth, CPI-inflation, and Treasury-bill forecast revisions from the quarterly SPF VAR reported in Figure 3, again using the [Jarociński and Karadi \(2020\)](#) instrument and the three-lag quarterly specification.

The two target blocks come from complementary systems. The macro-only VAR provides a common benchmark for aggregate transmission across survey specifications, which differ in frequency, sample period, target definition, and measurement error. The SPF VAR measures the revision of professional forecasts. Both systems use the [Jarociński and Karadi \(2020\)](#) instrument and therefore recover responses to the same identified monetary policy shock under different estimating information sets. Appendix Figure A1 reports the exact macroeconomic responses entering the criterion.

We map the SPF responses to firms’ average expectations, the belief block that enters price setting in the model. The discount factor, labor share, and elasticity across varieties remain calibrated. We estimate the four shock persistences, the four innovation standard deviations, the eight signal-noise standard deviations, and five New Keynesian parameters. Persistences are parameterized with a logistic transformation and standard deviations in logs.

The criterion is quadratic in the distance between empirical and model impulse responses. Its weighting matrix uses regularized covariance blocks for the horizons of each response. We use multiple starting values because the criterion is nonconvex and report parameter uncertainty from a 400-draw bootstrap.

4.2. Model Fit and Estimated Information Frictions

Figure 4 compares the empirical targets with the model-implied responses. The top row contains the macroeconomic block and the bottom row the SPF forecast revisions. The model reproduces the persistent contraction in the industrial-production gap, the negative price-level response, and the initial increase in the short-term rate. In the belief block, it captures the sharp positive impact revisions for output, inflation, and the interest rate together with their rapid decay.

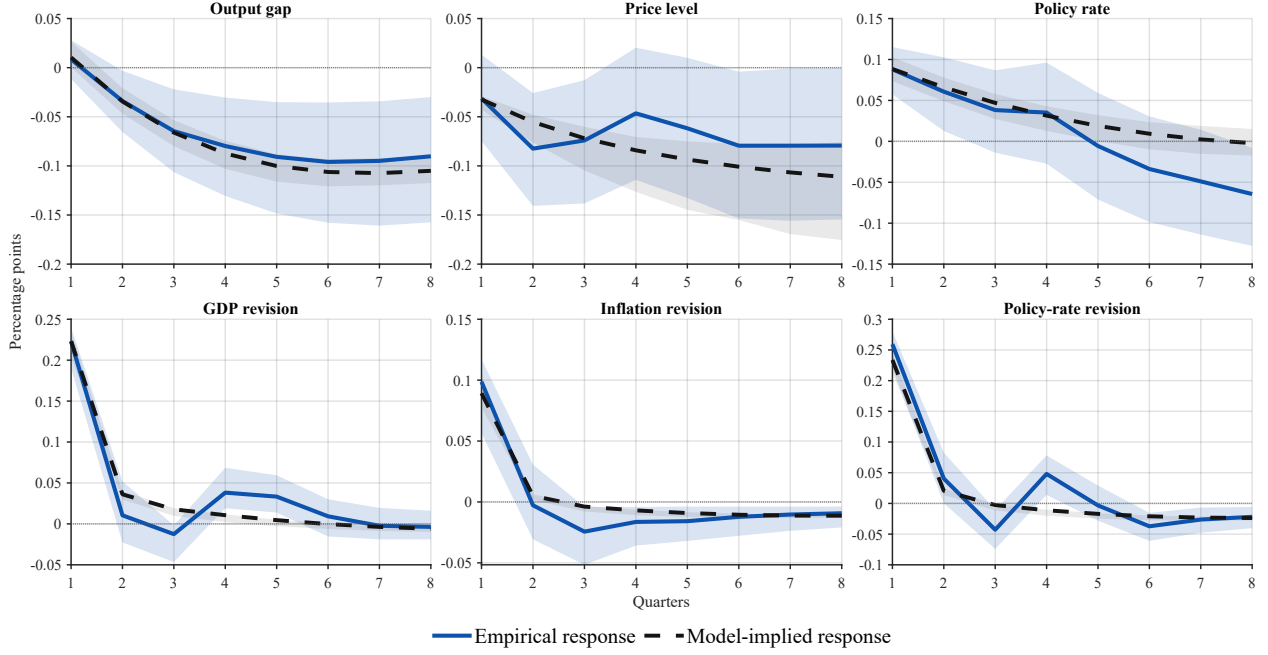
The average fitting errors are small relative to empirical uncertainty. The ratio of the root mean squared fitting error to the average 68 percent empirical half-width equals 0.15 for output, 0.37 for the price level, and 0.54 for the interest rate. The corresponding ratios are 0.82 for the output forecast revision and 0.44 for the inflation forecast revision. The largest ratio, 0.92, corresponds to the interest-rate forecast revision: the model captures its positive impact response while smoothing the intermediate-horizon oscillations. These ratios summarize model fit relative to the uncertainty surrounding each empirical response.

Parameter	Description	Value	Source
<i>Calibrated parameters</i>			
β	Discount factor	0.990	Gali (2015)
$1 - \alpha$	Labor share	0.750	Gali (2015)
ϵ	Elasticity across varieties	9.000	Gali (2015)
<i>Estimated New Keynesian parameters</i>			
σ	Inverse intertemporal elasticity	1.511 (0.058)	Estimated
φ	Inverse Frisch elasticity	6.256 (0.217)	Estimated
θ	Calvo non-reset probability	0.710 (0.013)	Estimated
ϕ_π	Inflation coefficient, Taylor rule	1.991 (0.114)	Estimated
ϕ_y	Output-gap coefficient, Taylor rule	0.209 (0.025)	Estimated
<i>Shock processes</i>			
ρ_v	Monetary-shock persistence	0.789 (0.036)	Estimated
ρ_z	Demand-shock persistence	0.861 (0.025)	Estimated
ρ_a	Technology-shock persistence	0.424 (0.037)	Estimated
ρ_μ	Cost-push-shock persistence	0.870 (0.019)	Estimated
σ_v	Monetary innovation s.d.	0.0029 (0.0003)	Estimated
σ_z	Demand innovation s.d.	0.048 (0.009)	Estimated
σ_a	Technology innovation s.d.	0.013 (0.002)	Estimated
σ_μ	Cost-push innovation s.d.	0.048 (0.005)	Estimated
<i>Signal-noise standard deviations</i>			
σ_{1v}	Monetary signal, households	0.0054 (0.0008)	Estimated
σ_{1z}	Demand signal, households	0.095 (0.012)	Estimated
σ_{1a}	Technology signal, households	0.0026 (0.0005)	Estimated
$\sigma_{1\mu}$	Cost-push signal, households	1.102 (0.178)	Estimated
σ_{2v}	Monetary signal, firms	0.018 (0.006)	Estimated
σ_{2z}	Demand signal, firms	0.156 (0.042)	Estimated
σ_{2a}	Technology signal, firms	0.0056 (0.0010)	Estimated
$\sigma_{2\mu}$	Cost-push signal, firms	0.214 (0.031)	Estimated
<i>Note:</i> Parentheses report bootstrap standard deviations based on 400 replications. All parameters are quarterly; shock and signal entries are standard deviations.			

TABLE 1. Calibrated and Estimated Parameters

Table 1 summarizes the parameter estimates. The monetary, demand, and cost-push disturbances are persistent, with quarterly autoregressive coefficients of 0.789, 0.861, and 0.870. Technology is less persistent, at 0.424.

The relative signal precisions are central to the mechanism. For the group whose expectations are matched to SPF forecasts, represented by firms in the model, monetary-signal noise is large relative to the monetary innovation: $\sigma_{2v} = 0.0179$ compared with $\sigma_v = 0.00288$. Firm demand-signal noise is 0.156, compared with 0.095 for households, while the largest asymmetry concerns the cost-push signal. The estimated information structure is therefore heterogeneous across groups and fundamentals. In particular, professional forecasters face uncertainty about whether a rate increase



Note: Blue lines and bands report empirical posterior medians and 68 percent intervals; black dashed lines and gray bands report model responses and 68 percent bootstrap intervals. The price level is cumulative inflation. The horizontal axis reports quarters 1 through 8, with quarter 1 denoting impact.

FIGURE 4. Empirical Responses to an Identified Monetary Policy Shock and Their Model-Implied Counterpart

reflects a monetary innovation or the central bank's response to economic conditions. This ambiguity allows the announcement to raise their forecasts while aggregate demand subsequently contracts.

4.3. Treating Beliefs as Beliefs

The information structure is introduced to account jointly for aggregate dynamics and survey forecast revisions. As an external diagnostic, we summarize its unconditional implications using the population regression of Coibion and Gorodnichenko (2015). For information set g and variable x , consider

$$(27) \quad x_{t+h} - \bar{\mathbb{E}}_{g,t} x_{t+h} = \alpha_{gx} + \beta_{gx}^Q (\bar{\mathbb{E}}_{g,t} x_{t+h} - \bar{\mathbb{E}}_{g,t-1} x_{t+h}) + u_{gx,t+h}.$$

A positive coefficient means that an upward revision is followed, on average, by a positive forecast error: agents initially adjust too little to the information they receive. Proposition A2 derives β_{gx}^Q from the stationary distribution of the estimated model, accounting for all four fundamentals and their group-specific signal precisions.

The model is quarterly, whereas the benchmark coefficients in Coibion and Gorodnichenko (2015)

are reported for annual forecast objects. For comparability, we apply the reporting transformation

$$(28) \quad \tilde{\beta}_{gx} = \left(\beta_{gx}^Q \right)^{1/4}.$$

This monotone transformation places the quarterly coefficient on the reporting scale used for the annual comparison; it is not an exact temporal aggregation of equation (27). Appendix B reports the state-space expression from which the quarterly coefficient is computed.

Information set	Activity	Inflation
Households	0.470 (0.065)	0.871 (0.065)
Firms	0.667 (0.057)	1.062 (0.080)
SPF benchmark	0.73–0.76	1.193

Note: Model entries report $(\beta^Q)^{1/4}$ and parentheses contain bootstrap standard deviations. The SPF activity range applies the same transformation to the output-growth and industrial-production estimates in Coibion and Gorodnichenko (2015); the inflation benchmark is their annual year-ahead estimate.

TABLE 2. Forecast Underreaction in the Estimated Model

Table 2 delivers two messages. First, all four model coefficients are positive, making forecast underreaction an unconditional implication of the estimated information structure. The household activity coefficient is 0.470 and the firm coefficient is 0.667, with the latter reasonably close to the SPF range of 0.73–0.76. Second, the firm inflation coefficient is 1.062, close to the annual SPF estimate of 1.193, while the household coefficient is 0.871. The external evidence is therefore reproduced more closely for the firm information set used to map SPF revisions than for households.

This comparison complements the shock-conditional evidence in Figure 4. While the figure describes belief revisions around a monetary announcement, equation (27) summarizes forecast errors and revisions across all structural disturbances in the stationary distribution. We next use the estimated model to isolate the response to a pure monetary policy innovation.

4.4. Counterfactual: The Pure Monetary Policy Shock

The fitted response in Figure 4 concerns the composite policy surprise in equation (26). This object is a linear projection on the unexpected policy-rate movement, not an impulse response to a single structural innovation. To see its composition, let $\text{mps}_t^{c,(n)}$ denote the part of the surprise generated by fundamental $n \in \{v, z, a, \mu\}$. Orthogonality of the structural innovations implies

$$(29) \quad \text{mps}_t^c = \sum_n \text{mps}_t^{c,(n)}, \quad \mathbb{V}(\text{mps}_t^c) = \sum_n \mathbb{V}(\text{mps}_t^{c,(n)}).$$

Define

$$\omega_n \equiv \frac{\mathbb{V}(\text{mps}_t^{c,(n)})}{\mathbb{V}(\text{mps}_t^c)}, \quad \beta_{h,n}^x \equiv \frac{\mathbb{C}(x_{t+h}, \text{mps}_t^{c,(n)})}{\mathbb{V}(\text{mps}_t^{c,(n)})}.$$

By covariance linearity and cross-component orthogonality, the fitted response then satisfies

$$(30) \quad \beta_h^x \equiv \frac{\mathbb{C}(x_{t+h}, \text{mps}_t^c)}{\mathbb{V}(\text{mps}_t^c)} = \sum_{n \in \{v,z,a,\mu\}} \omega_n \beta_{h,n}^x.$$

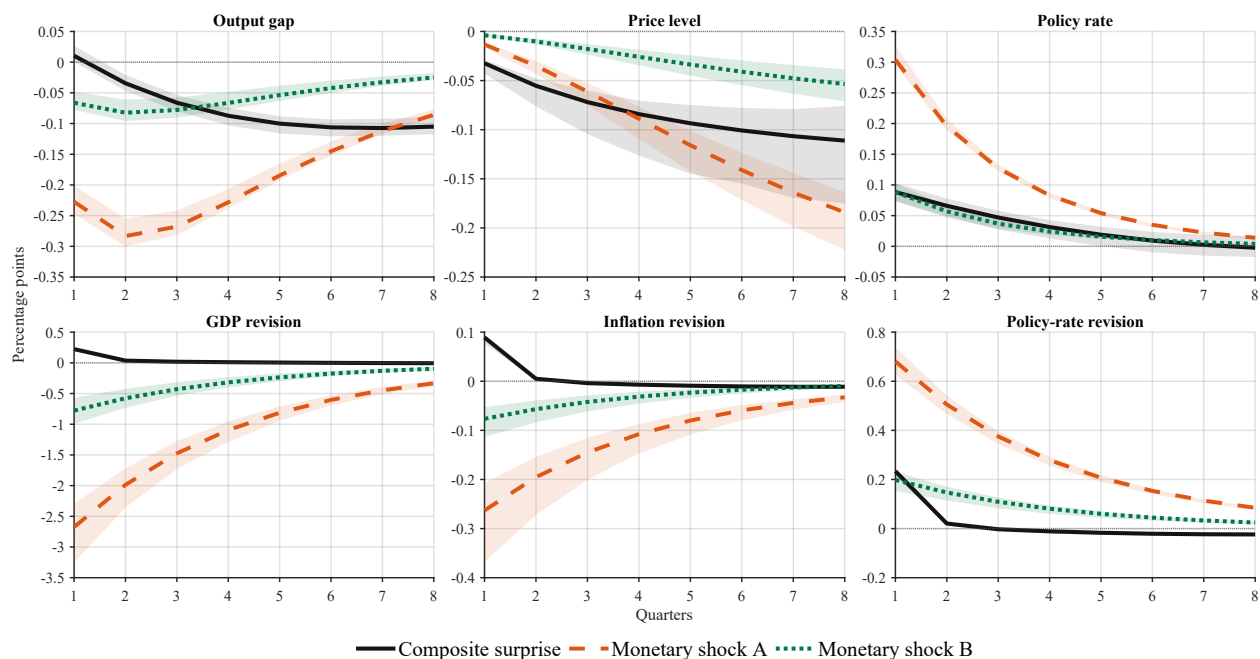
Thus, every response to the composite surprise is a variance-share-weighted average of the responses associated with monetary, demand, technology, and cost-push news. The pure-shock counterfactual retains only the current monetary innovation ε_t^v and sets the three nonmonetary innovations to zero. Corollary A1 in Appendix B provides the closed-form responses. Because the composite surprise and the structural innovation are distinct random variables, their quantitative comparison requires a normalization.

Figure 5 reports two normalizations. Normalization A places the pure shock on the variance scale of the composite surprise, using the factor $\phi_v^A = \sqrt{\mathbb{V}(\text{mps}_t^c)}/\sigma_v$. It compares equally sized random innovations in unconditional standard-deviation units. Normalization B chooses ϕ_v^B so that the impact response of the policy rate to $\phi_v^B \varepsilon_t^v$ equals its impact response to the composite surprise. By holding fixed the observed policy-rate movement, normalization B isolates differences in transmission from differences in shock scale.

Two results are common to both normalizations. First, a pure tightening is contractionary. The output gap falls on impact, and the forecast revisions for output and inflation turn negative rather than positive. By contrast, the composite surprise raises output and inflation forecast revisions by 0.223 and 0.089 percentage points on impact. The optimism following the announcement is therefore generated by the nonmonetary information contained in the composite surprise.

Second, the normalization matters for magnitudes. Under normalization A, the pure shock raises the interest rate by 0.304 percentage points on impact, compared with 0.088 percentage points for the composite surprise, and lowers the output gap by 0.228 percentage points. Under normalization B, both interest rates rise by 0.088 percentage points and the pure shock lowers the output gap by 0.066 percentage points on impact. Favorable nonmonetary news makes the impact output response to the composite surprise slightly positive, so the equal-rate pure response is initially more contractionary. The ranking reverses as the different components propagate: by quarter 8, the output gap falls by 0.105 percentage points after the composite surprise and by 0.025 after the pure shock. The equal-rate price-level response is attenuated throughout, moving from -0.004 on impact to -0.054 in quarter 8, compared with -0.032 and -0.111 for the composite surprise.

The decomposition in Appendix Table A1 explains this reversal. At the point estimate, monetary, demand, technology, and cost-push news account for 14.6, 12.9, 0.1, and 72.4 percent of the variance



Note: Black lines report responses to the composite policy surprise mps_t^c . Orange dashed and green dotted lines report responses to the structural monetary innovation ε_t^y under the variance normalization (A) and the equal-impact policy-rate normalization (B), respectively. Shaded regions are 68 percent bootstrap intervals. The price level is cumulative inflation. The horizontal axis reports quarters 1 through 8, with quarter 1 denoting impact.

FIGURE 5. Model Responses to the Composite Policy Surprise and the Structural Monetary Policy Shock

of the composite surprise. Technology news supplies most of the positive impact revision in GDP forecasts, with an additional positive contribution from demand. At the same time, monetary and cost-push news offset the expansionary impact of demand on output, leaving the aggregate impact response close to zero.

The medium-run attenuation reflects the different propagation of these components. In quarter 8, the cost-push contribution to output is -0.155 percentage points, the demand contribution is 0.083 , and the monetary contribution is -0.034 . Their sum yields the -0.105 response to the composite surprise. The equal-rate counterfactual removes both the positive demand contribution and the larger negative cost-push contribution, while rescaling the remaining monetary innovation to match the impact rate increase. The resulting pure-shock response is -0.025 . The price-level response follows the same logic: the persistent negative cost-push contribution slightly dominates the positive demand contribution in the composite surprise, whereas both disappear in the pure-shock counterfactual.

5. Beliefs-Adjusted Monetary Policy Shocks

We now return to the data and use survey revisions to refine the narrative identification of monetary policy shocks. Starting from sign and narrative restrictions, we add restrictions on the short-run response of beliefs. Output and prices remain unrestricted, so their responses can be compared with the model's equal-rate counterfactual.

We estimate a six-variable monthly Bayesian VAR containing real GDP, the GDP deflator, a commodity price index, the federal funds rate, and CE forecast revisions for GDP growth and CPI inflation. The final two variables are the GDP-belief and inflation-belief measures introduced in Section 2.1. The baseline identification combines the sign and narrative restrictions in Uhlig (2005) and Antolín-Díaz and Rubio-Ramírez (2018b).

ASSUMPTION 1 (sign). *A contractionary monetary policy shock raises the federal funds rate for six months and lowers the commodity price index for one month.*

ASSUMPTION 2 (narrative). *The shock is positive in February 1994 and is the main driver of the unexpected increase in the federal funds rate in that month.*⁷

The baseline leaves output, the GDP deflator, and both belief measures unrestricted. Figure 6 reports the resulting responses, normalized to raise the federal funds rate by 25 basis points on impact.

The conventional macroeconomic responses have the expected signs: real GDP and the GDP deflator decline after the tightening. GDP and inflation forecast revisions instead rise on impact, although only the GDP response is robust across the surveys in Section 2. Both belief responses are unrestricted outcomes of the baseline identification.

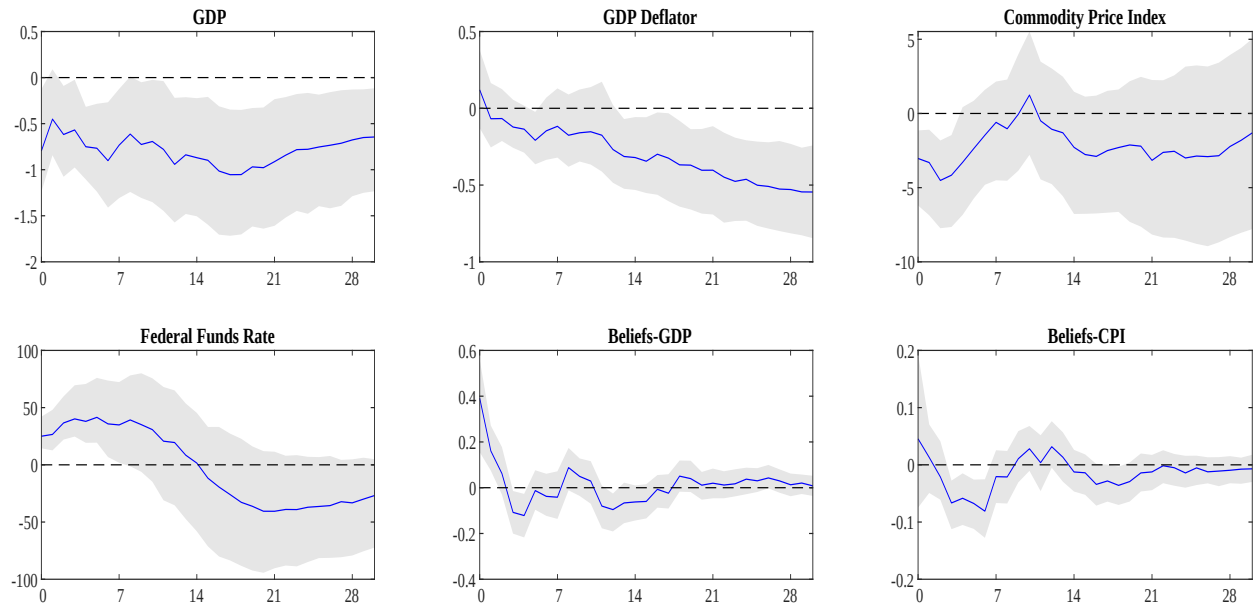
We next add the identifying implication of the pure-shock counterfactual:

ASSUMPTION 3 (beliefs). *A contractionary monetary policy shock lowers GDP and inflation forecast revisions during the first two months.*

This restriction targets the short-run optimism associated with the informational component. The inflation restriction follows the model's pure-shock implication, while the two-month window reflects the impact profile of the belief responses.

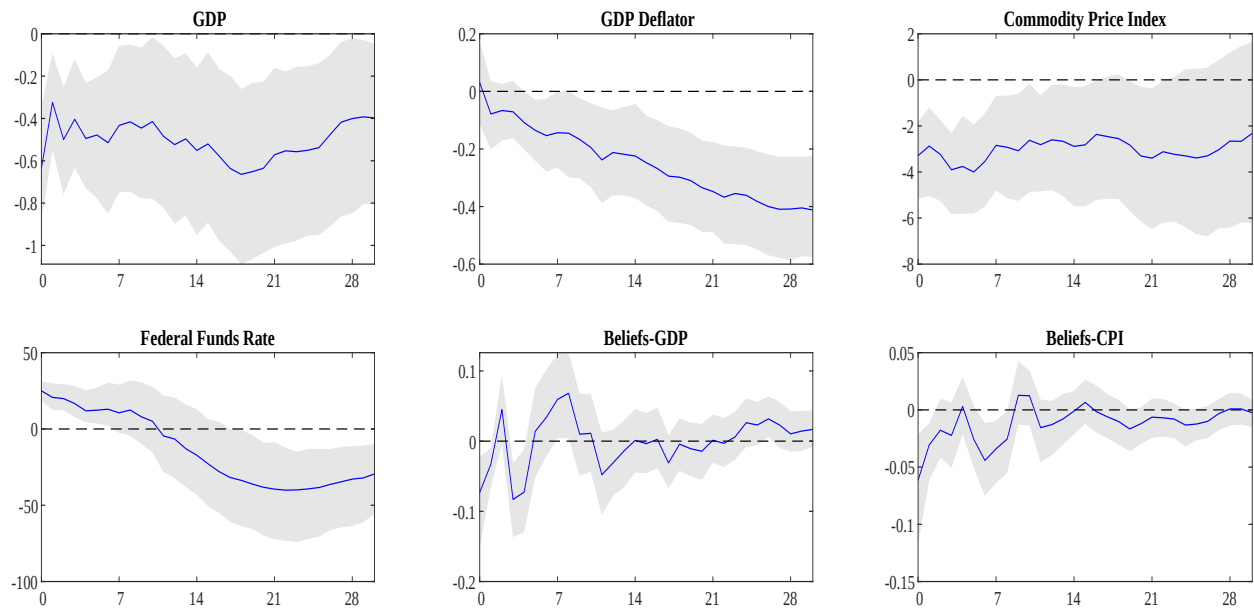
Figure 7 reports the beliefs-adjusted responses, again normalized to the same 25-basis-point impact increase in the federal funds rate. The restriction selects rotations with declining forecast revisions at short horizons; output, prices, and their posterior uncertainty remain unrestricted. Relative to the baseline, the macroeconomic responses are more precisely estimated and attenuated, particularly beyond impact.

⁷The CE data begin in 1989, so we do not impose narrative restrictions on earlier episodes.



Note: Lines report posterior medians and shaded areas are 68 percent posterior intervals. The federal funds rate is measured in basis points; all other responses are in percentage points. Horizons are measured in months.

FIGURE 6. Dynamic Responses to a Monetary Policy Shock



Note: Lines report posterior medians and shaded areas are 68 percent posterior intervals. The federal funds rate is measured in basis points; all other responses are in percentage points. Horizons are measured in months.

FIGURE 7. Dynamic Responses to a Monetary Policy Shock: Beliefs-Adjusted

Table 3 provides a simple historical check. It compares the standardized beliefs-adjusted series with five prominent monetary policy shock measures in four episodes emphasized by [Antolín-Díaz](#)

and Rubio-Ramírez (2018b). The beliefs-adjusted series is positive in February 1994 and negative in October 1998, April 2001, and November 2002, matching the narrative signs and the Jarociński and Karadi (2020) series in all four cases.

The historical comparison aligns the beliefs-adjusted series with the narrative signs of the selected episodes. Under the common impact-rate normalization, the attenuation of the unrestricted macroeconomic responses also parallels normalization B in Figure 5.

Date	GMS (2025)	MAR (2021)	JK (2020)	AD (2025)	GK (2015)	RR (2004)
<i>February 1994</i>	1.80	-0.33	1.93	0.74	0.26	1.80
<i>October 1998</i>	-2.63	-4.55	-3.77	-0.02	-1.21	-0.04
<i>April 2001</i>	-1.39	-6.58	-6.64	-0.02	-3.98	-0.04
<i>November 2002</i>	-0.69	-0.88	-0.22	-3.10	-1.20	-0.04

Note: Entries are standardized to have mean zero and standard deviation one. *GMS* denotes the beliefs-adjusted series; the remaining columns report the measures described in Section 2.2.

TABLE 3. Monetary Policy Shocks in Selected Episodes

6. Conclusion

Monetary policy announcements affect the economy through both policy and information. Survey forecasts make the distinction visible. Across households, financial economists, and professional forecasters, identified contractionary monetary policy shocks generally improve the near-term outlook for real activity even as expected interest rates rise. This is the central empirical puzzle. Inflation beliefs are less systematic: the monthly evidence is mixed, while clearer increases arise mainly in the quarterly SPF system.

A New Keynesian model with dispersed information reproduces this disconnect. The estimated model matches the contraction in activity and prices together with the impact optimism in SPF forecasts because the identified disturbance remains correlated with several nonmonetary fundamentals. Favorable technology and demand news drive the optimistic revision, while the monetary and cost-push components shape the contractionary macroeconomic response. Removing the nonmonetary components leaves a pure tightening that contracts output and generates nonpositive forecast revisions. Under the equal-rate comparison, the persistent responses of output and prices are also attenuated.

Finally, a narrative VAR uses short-run belief restrictions to identify a beliefs-adjusted monetary policy shock. The resulting responses of activity and prices are more precise and attenuated under a common impact-rate normalization. Beliefs therefore provide information about both monetary transmission and the content of identified policy shocks.

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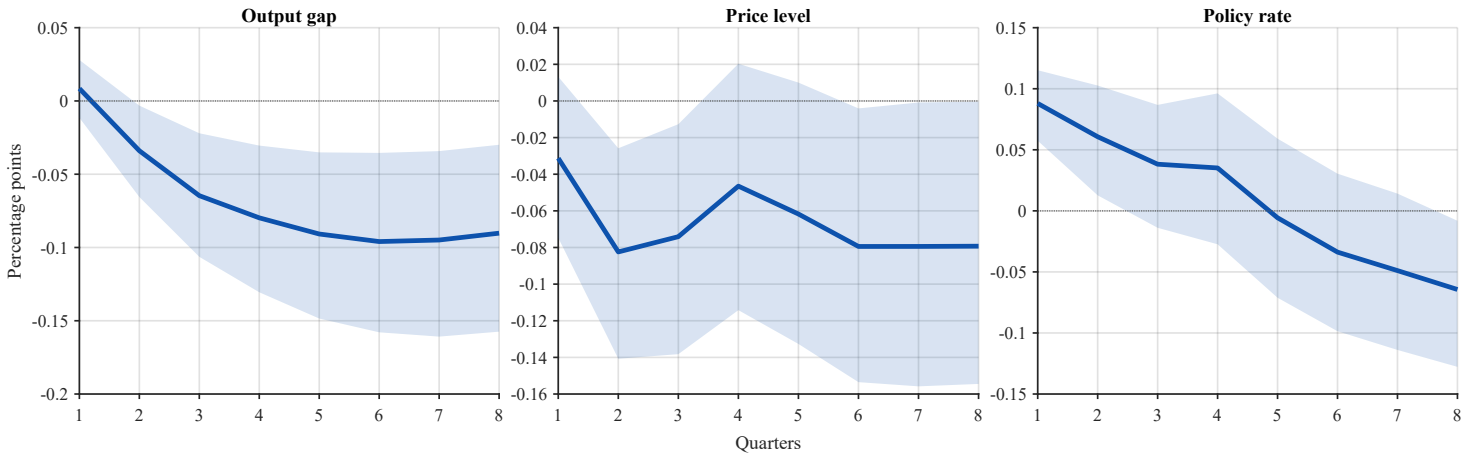
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Appendix A. Empirical Appendix

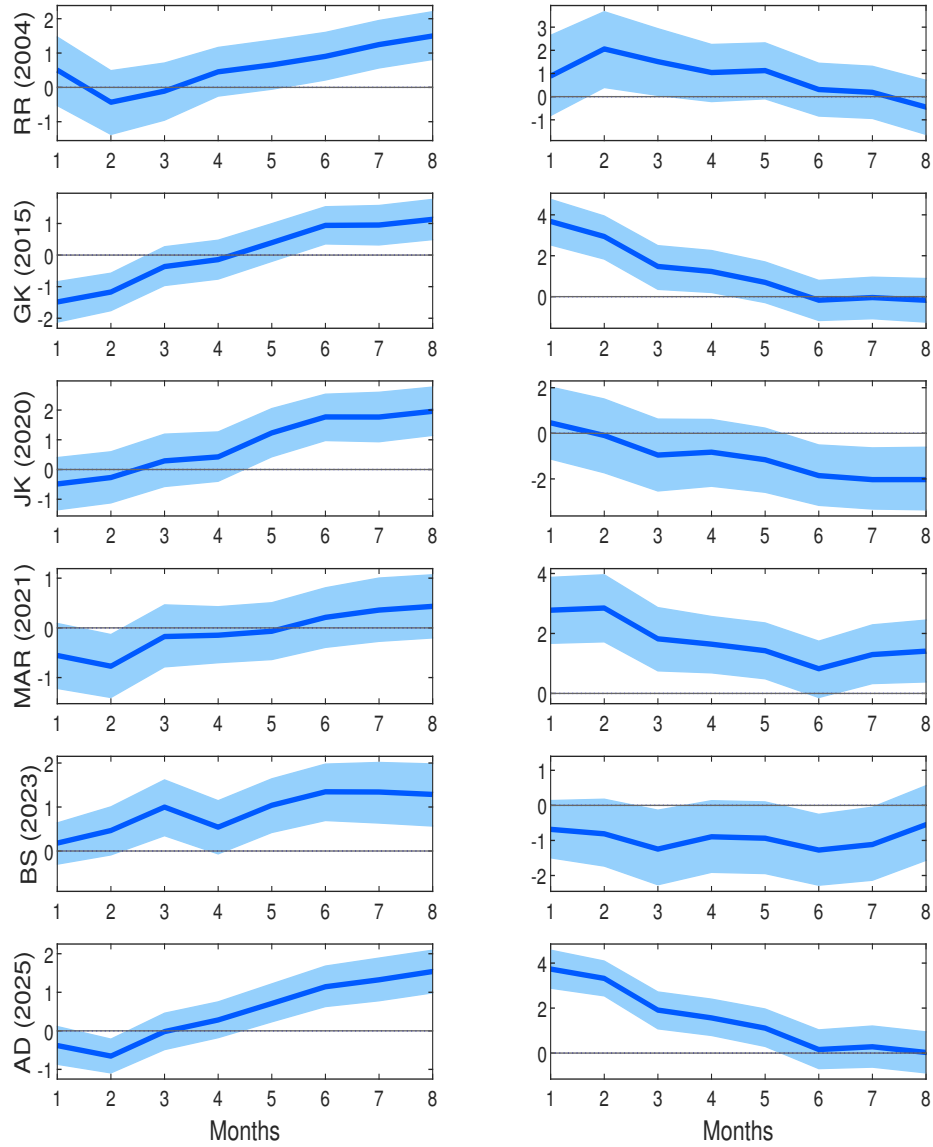
A.1. Macroeconomic Targets Used in Model Estimation

Figure A1 reports the macroeconomic impulse responses matched in Section 4. They come from the macro-only monthly proxy VAR and use the [Jarociński and Karadi \(2020\)](#) instrument. Posterior draws are aggregated to quarterly means, and monthly CPI responses are cumulated before aggregation so that the target is the price level. No survey or forecast-revision variable enters this VAR. The belief targets are estimated separately in the SPF systems reported in Figure 3; the common external instrument identifies the same monetary policy shock in both blocks.



Note: Blue lines and bands report posterior medians and 68 percent coverage intervals. The responses use the [Jarociński and Karadi \(2020\)](#) instrument in the macro-only monthly proxy VAR and are aggregated to quarterly means. CPI responses are cumulated to obtain the price level. The VAR contains no belief measure. Quarter 1 denotes impact.

FIGURE A1. Macroeconomic Responses Used in the Model Estimation

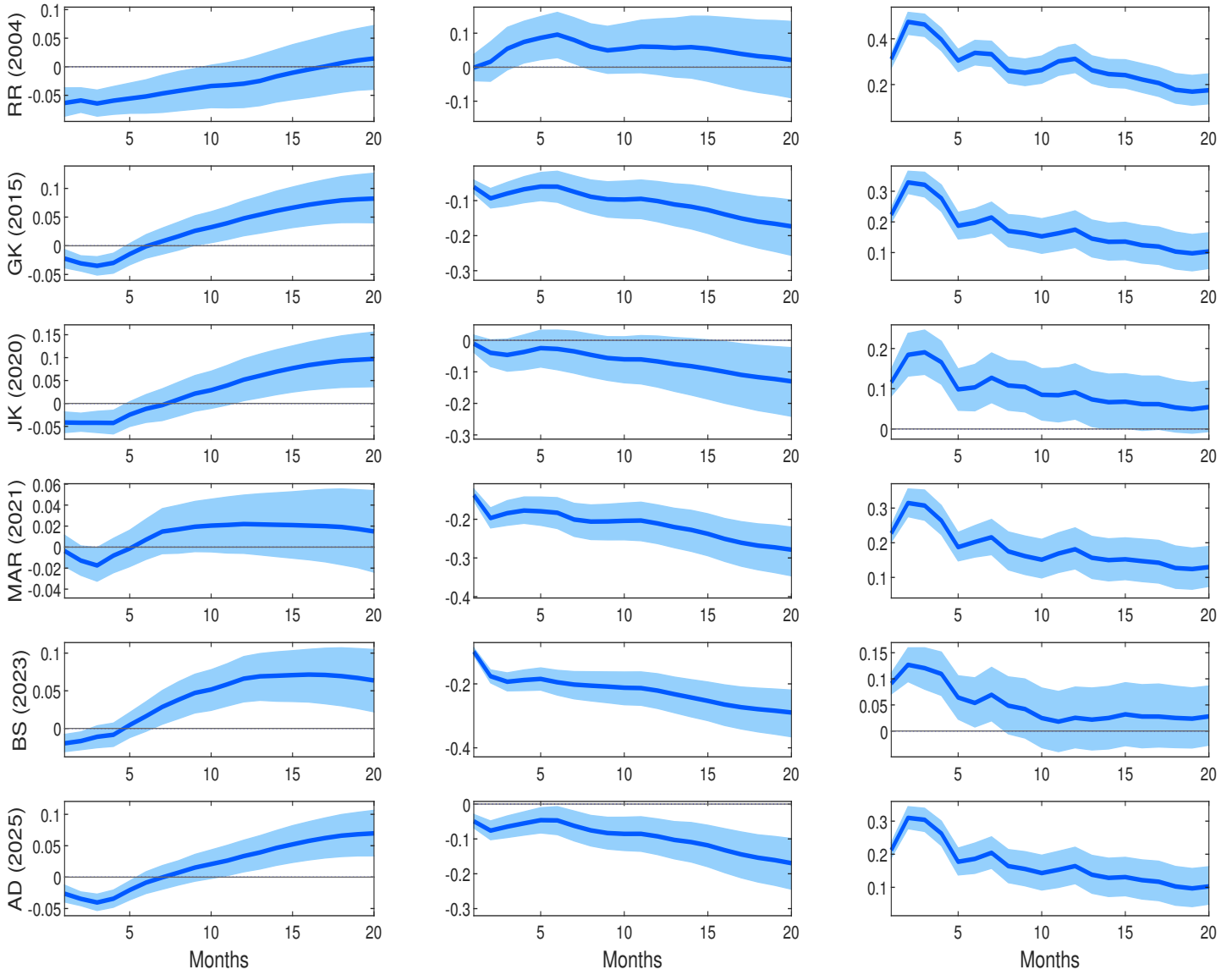


A. Unemployment

B. One-Year Business Expectations

Note: RR (2004): [Romer and Romer \(2004\)](#); GK (2015): [Gertler and Karadi \(2015\)](#); JK (2020): [Jarociński and Karadi \(2020\)](#); MAR (2021): [Miranda-Agrippino and Ricco \(2021\)](#); BS (2023): [Bauer and Swanson \(2023\)](#); AD (2025): [Aruoba and Drechsel \(2025\)](#). Horizontal axes measure months after the shock; vertical axes measure percentage points. Shaded areas are 68 percent posterior coverage bands.

FIGURE A2. Dynamic Responses to a Monetary Policy Shock: Other Beliefs from the MSC



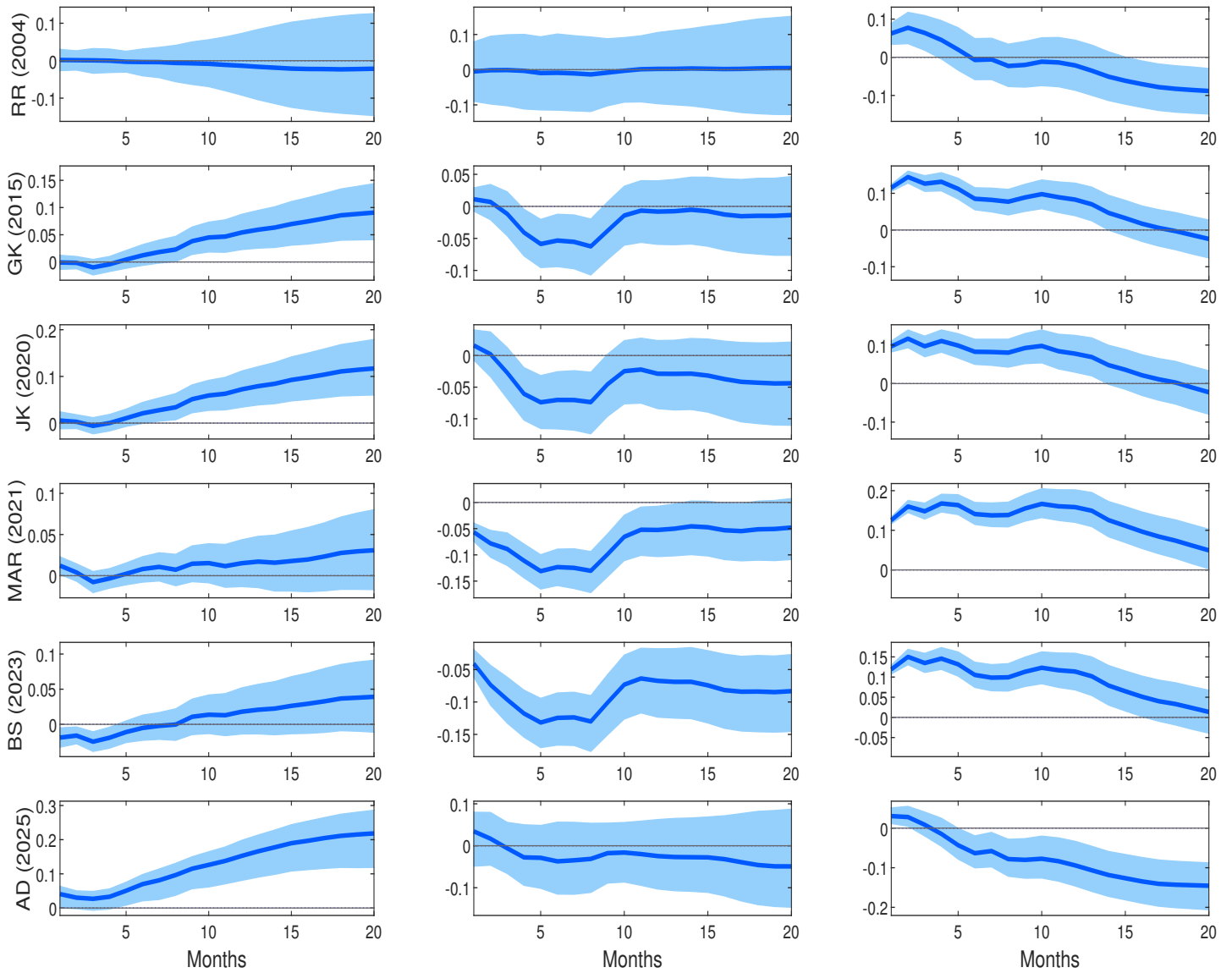
A. Unemployment

B. CPI Level

C. One-Year Treasury Rate

Note: RR (2004): [Romer and Romer \(2004\)](#); GK (2015): [Gertler and Karadi \(2015\)](#); JK (2020): [Jarociński and Karadi \(2020\)](#); MAR (2021): [Miranda-Agrippino and Ricco \(2021\)](#); BS (2023): [Bauer and Swanson \(2023\)](#); AD (2025): [Aruoba and Drechsel \(2025\)](#). Horizontal axes measure months after the shock; vertical axes measure percentage points. Shaded areas are 68 percent posterior coverage bands.

FIGURE A3. Dynamic Responses to a Monetary Policy Shock: Macroeconomic Variables with MSC Beliefs



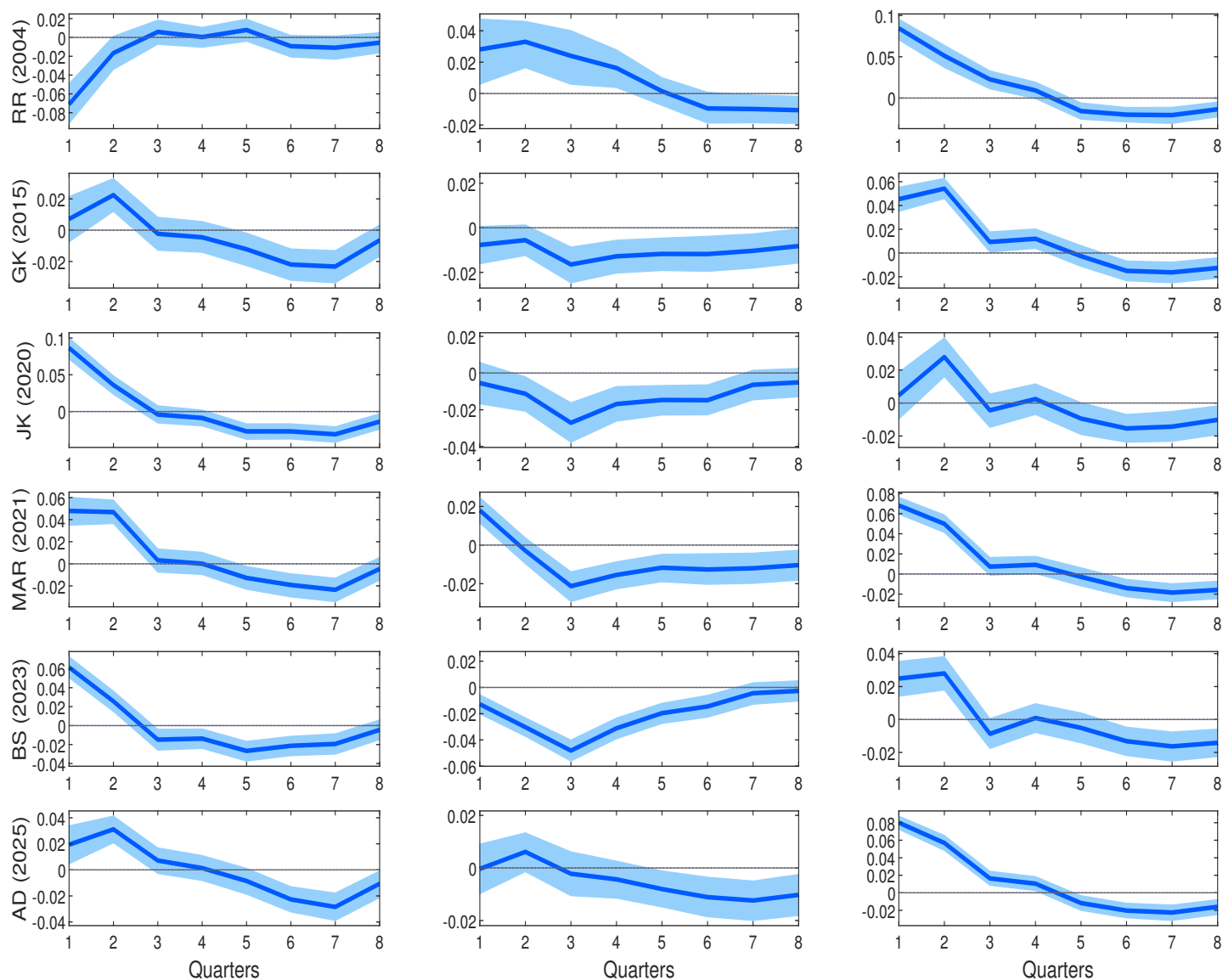
A. Unemployment

B. CPI Level

C. One-Year Treasury Rate

Note: RR (2004): [Romer and Romer \(2004\)](#); GK (2015): [Gertler and Karadi \(2015\)](#); JK (2020): [Jarociński and Karadi \(2020\)](#); MAR (2021): [Miranda-Agrippino and Ricco \(2021\)](#); BS (2023): [Bauer and Swanson \(2023\)](#); AD (2025): [Aruoba and Drechsel \(2025\)](#). Horizontal axes measure months after the shock; vertical axes measure percentage points. Shaded areas are 68 percent posterior coverage bands.

FIGURE A4. Dynamic Responses to a Monetary Policy Shock: Macroeconomic Variables with CE Beliefs



A. Real GDP Growth

B. Inflation

C. Three-Month T-Bill Rate

Note: RR (2004): [Romer and Romer \(2004\)](#); GK (2015): [Gertler and Karadi \(2015\)](#); JK (2020): [Jarociński and Karadi \(2020\)](#); MAR (2021): [Miranda-Agrippino and Ricco \(2021\)](#); BS (2023): [Bauer and Swanson \(2023\)](#); AD (2025): [Aruoba and Drechsel \(2025\)](#). Horizontal axes measure months after the shock; vertical axes measure percentage points. Shaded areas are 68 percent posterior coverage bands.

FIGURE A5. Dynamic Responses to a Monetary Policy Shock: Monthly CE Beliefs without the Timing Restriction

Appendix B. Model Appendix

B.1. Decomposing the Composite Policy Surprise

Equations (29)–(30) express the composite surprise as the sum of four orthogonal components and each fitted response as their variance-share-weighted average. Table A1 reports the weighted contribution $\omega_n \beta_{h,n}^x$ of each component at the estimated parameters. These contributions sum to the composite-surprise response in the penultimate row. The final row reports the pure monetary response under normalization B, which holds fixed the impact response of the policy rate.

Component	Surprise variance	GDP revision	Output gap		Price level	
	share (percent)	impact	impact	quarter 8	impact	quarter 8
Monetary	14.6 [13.8, 17.1]	−0.009 [−0.011, −0.008]	−0.113 [−0.125, −0.103]	−0.034 [−0.036, −0.030]	−0.008 [−0.010, −0.007]	−0.084 [−0.107, −0.075]
Demand	12.9 [11.1, 13.4]	0.043 [0.037, 0.049]	0.320 [0.308, 0.348]	0.083 [0.080, 0.088]	0.157 [0.126, 0.163]	1.180 [0.984, 1.225]
Technology	0.10 [0.09, 0.14]	0.208 [0.190, 0.223]	0.019 [0.018, 0.022]	0.000 [0.000, 0.000]	0.008 [0.007, 0.009]	0.015 [0.013, 0.016]
Cost-push	72.4 [70.9, 73.5]	−0.018 [−0.023, −0.017]	−0.216 [−0.239, −0.199]	−0.155 [−0.169, −0.133]	−0.190 [−0.199, −0.165]	−1.222 [−1.276, −1.031]
Composite policy surprise	100.0	0.223 [0.204, 0.235]	0.010 [0.000, 0.027]	−0.105 [−0.117, −0.090]	−0.032 [−0.043, −0.029]	−0.111 [−0.175, −0.076]
Pure monetary shock (B)	−	−0.778 [−0.994, −0.571]	−0.066 [−0.078, −0.048]	−0.025 [−0.029, −0.018]	−0.004 [−0.005, −0.003]	−0.054 [−0.071, −0.039]

Note: Entries are percentage points except for the variance shares. Brackets contain 16th and 84th percentiles from the 400 bootstrap replications. Component responses are covariance contributions and can be larger than the total when contributions of opposite sign offset. Normalization B scales the pure monetary innovation to match the impact policy-rate response to the composite policy surprise.

TABLE A1. Decomposition of the Model-Implied Policy Surprise

B.2. Proofs of Propositions in the Main Text

Proof of Proposition 1. The best response of agent l in group g can be written as

$$(A.1) \quad a_{lgt} = \varphi_{gv} \mathbb{E}_{lgt} v_t + \varphi_{gz} \mathbb{E}_{lgt} z_t + \varphi_{ga} \mathbb{E}_{lgt} a_t + \varphi_{g\mu} \mathbb{E}_{lgt} \mu_t + \beta_g \mathbb{E}_{lgt} a_{lgt,t+1} + \sum_{j=1}^2 \gamma_{gj} \mathbb{E}_{lgt} a_{jt} + \sum_{j=1}^2 \alpha_{gj} \mathbb{E}_{lgt} a_{j,t+1}$$

Let $\mathbf{a}_t = (a_{gt})$ collect the two aggregate actions, consumption and inflation. The 2×4 matrix $\boldsymbol{\varphi} = (\varphi_{gn})$ collects the direct loading of each group on the four fundamentals. The diagonal matrix $\boldsymbol{\beta} = \text{diag}(\beta_g)$ contains the discount factors, while the 2×2 matrices $\boldsymbol{\gamma} = (\gamma_{gj})$ and $\boldsymbol{\alpha} = (\alpha_{gj})$ collect contemporaneous and future interactions, respectively. Finally, define $\boldsymbol{\delta} \equiv \boldsymbol{\beta} + \boldsymbol{\alpha}$:

$$\mathbf{a}_t = \begin{bmatrix} a_{1t} \\ a_{2t} \end{bmatrix}, \quad \boldsymbol{\varphi} = \begin{bmatrix} \varphi_{1v} & \varphi_{1z} & \varphi_{1a} & \varphi_{1\mu} \\ \varphi_{2v} & \varphi_{2z} & \varphi_{2a} & \varphi_{2\mu} \end{bmatrix}, \quad \boldsymbol{\beta} = \begin{bmatrix} \beta_1 & 0 \\ 0 & \beta_2 \end{bmatrix}, \quad \boldsymbol{\gamma} = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix}, \quad \boldsymbol{\alpha} = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}$$

We now solve the expectation terms in (A.1). The shock processes and the private signals in (12)–(15) admit the following state-space representation:

$$(A.2) \quad \begin{aligned} \mathbf{Z}_t &= \mathbf{F} \mathbf{Z}_{t-1} + \boldsymbol{\Phi} \mathbf{S}_{lgt} \\ \mathbf{X}_{lgt} &= \mathbf{H} \mathbf{Z}_t + \boldsymbol{\Psi}_g \mathbf{S}_{lgt} \end{aligned}$$

with $\mathbf{Z}_t = \begin{bmatrix} v_t & z_t & a_t & \mu_t \end{bmatrix}^\top$, $\mathbf{S}_{lgt} = \begin{bmatrix} \varepsilon_t^v & \varepsilon_t^z & \varepsilon_t^a & \varepsilon_t^\mu & u_{lgt}^v & u_{lgt}^z & u_{lgt}^a & u_{lgt}^\mu \end{bmatrix}^\top$, $\mathbf{H} = \mathbf{I}_4$, $\mathbf{F} = \begin{bmatrix} \rho_v & 0 & 0 & 0 \\ 0 & \rho_z & 0 & 0 \\ 0 & 0 & \rho_a & 0 \\ 0 & 0 & 0 & \rho_\mu \end{bmatrix}$, and

$$\Phi = \begin{bmatrix} \tau_v^{-1/2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tau_z^{-1/2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau_a^{-1/2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tau_\mu^{-1/2} & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Psi_g = \begin{bmatrix} 0 & 0 & 0 & 0 & \tau_{gv}^{-1/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \tau_{gz}^{-1/2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \tau_{ga}^{-1/2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tau_{g\mu}^{-1/2} \end{bmatrix}.$$

The signal system can be written as

(A.3)

$$\mathbf{X}_{lgt} = \begin{bmatrix} \frac{\tau_v^{-1/2}}{1-\rho_v L} & 0 & 0 & 0 & \tau_{gv}^{-1/2} & 0 & 0 & 0 \\ 0 & \frac{\tau_z^{-1/2}}{1-\rho_z L} & 0 & 0 & 0 & \tau_{gz}^{-1/2} & 0 & 0 \\ 0 & 0 & \frac{\tau_a^{-1/2}}{1-\rho_a L} & 0 & 0 & 0 & \tau_{ga}^{-1/2} & 0 \\ 0 & 0 & 0 & \frac{\tau_\mu^{-1/2}}{1-\rho_\mu L} & 0 & 0 & 0 & \tau_{g\mu}^{-1/2} \end{bmatrix} \mathbf{S}_{lgt} = \mathbf{M}_g(L) \mathbf{S}_{lgt}, \quad \mathbf{S}_{lgt} \sim \mathcal{N}(0, I)$$

The Wold decomposition theorem provides a representation of the signal process (A.3), $\mathbf{X}_{lgt} = \mathbf{B}_g(L) \mathbf{w}_{lgt}$, such that $\mathbf{B}_g(L)$ is invertible and $\mathbf{w}_{lgt} \sim (0, \mathbf{V}_g)$ is white noise. Hence,

(A.4)

$$\mathbf{X}_{lgt} = \mathbf{M}_g(L) \mathbf{S}_{lgt} = \mathbf{B}_g(L) \mathbf{w}_{lgt}$$

In the Wold representation of $\mathbf{X}_{lgt}$, observing $\{\mathbf{X}_{lgt}\}$ is equivalent to observing $\{\mathbf{w}_{lgt}\}$, and $\{\mathbf{X}_{lgt}^t\}$ and $\{\mathbf{w}_{lgt}^t\}$ contain the same information. Furthermore, note that the Wold representation has the property that, using the equivalence (A.4), both processes share the autocovariance generating function, $\rho_{xx,g}(L) = \mathbf{M}_g(L) \mathbf{M}_g^\top(L^{-1}) = \mathbf{B}_g(L) \mathbf{V}_g \mathbf{B}_g^\top(L^{-1})$. Computing this last term,

$$\begin{aligned} \mathbf{M}_g(L) \mathbf{M}_g^\top(L^{-1}) &= \begin{bmatrix} \tau_{gv}^{-1} + \frac{\tau_v^{-1} L}{(L-\rho_v)(1-\rho_v L)} & 0 & 0 & 0 \\ 0 & \tau_{gz}^{-1} + \frac{\tau_z^{-1} L}{(L-\rho_z)(1-\rho_z L)} & 0 & 0 \\ 0 & 0 & \tau_{ga}^{-1} + \frac{\tau_a^{-1} L}{(L-\rho_a)(1-\rho_a L)} & 0 \\ 0 & 0 & 0 & \tau_{g\mu}^{-1} + \frac{\tau_\mu^{-1} L}{(L-\rho_\mu)(1-\rho_\mu L)} \end{bmatrix} \\ &= \begin{bmatrix} \frac{1-\lambda_{gv} L}{1-\rho_v L} \frac{\rho_v}{\lambda_{gv} \tau_{gv}} \frac{L-\lambda_{gv}}{L-\rho_v} & 0 & 0 & 0 \\ 0 & \frac{1-\lambda_{gz} L}{1-\rho_z L} \frac{\rho_z}{\lambda_{gz} \tau_{gz}} \frac{L-\lambda_{gz}}{L-\rho_z} & 0 & 0 \\ 0 & 0 & \frac{1-\lambda_{ga} L}{1-\rho_a L} \frac{\rho_a}{\lambda_{ga} \tau_{ga}} \frac{L-\lambda_{ga}}{L-\rho_a} & 0 \\ 0 & 0 & 0 & \frac{1-\lambda_{g\mu} L}{1-\rho_\mu L} \frac{\rho_\mu}{\lambda_{g\mu} \tau_{g\mu}} \frac{L-\lambda_{g\mu}}{L-\rho_\mu} \end{bmatrix} \end{aligned}$$

where, in the second equality, we have used the result that λ_{gn} , for $n \in \{v, z, a, \mu\}$, are the four inside roots of the determinant of $\mathbf{M}_g(L) \mathbf{M}_g^\top(L^{-1})$, $L^2 - \left(\frac{1}{\rho_n} + \rho_n + \frac{\tau_{gn}}{\rho_n \tau_n} \right) L + 1$, satisfying $(\rho_n - \lambda_{gn})(1 - \rho_n \lambda_{gn}) = \lambda_{gn} \tau_{gn} / \tau_n$. We can

then identify the unknown matrices,

$$\mathbf{B}_g(L) = \begin{bmatrix} \frac{1-\lambda_{gv}L}{1-\rho_vL} & 0 & 0 & 0 \\ 0 & \frac{1-\lambda_{gz}L}{1-\rho_zL} & 0 & 0 \\ 0 & 0 & \frac{1-\lambda_{ga}L}{1-\rho_aL} & 0 \\ 0 & 0 & 0 & \frac{1-\lambda_{g\mu}L}{1-\rho_\mu L} \end{bmatrix}, \quad \mathbf{V}_g = \begin{bmatrix} \frac{\rho_v}{\lambda_{gv}\tau_{gv}} & 0 & 0 & 0 \\ 0 & \frac{\rho_z}{\lambda_{gz}\tau_{gz}} & 0 & 0 \\ 0 & 0 & \frac{\rho_a}{\lambda_{ga}\tau_{ga}} & 0 \\ 0 & 0 & 0 & \frac{\rho_\mu}{\lambda_{g\mu}\tau_{g\mu}} \end{bmatrix},$$

$$\mathbf{B}_g^\top(L^{-1}) = \begin{bmatrix} \frac{L-\lambda_{gv}}{L-\rho_v} & 0 & 0 & 0 \\ 0 & \frac{L-\lambda_{gz}}{L-\rho_z} & 0 & 0 \\ 0 & 0 & \frac{L-\lambda_{ga}}{L-\rho_a} & 0 \\ 0 & 0 & 0 & \frac{L-\lambda_{g\mu}}{L-\rho_\mu} \end{bmatrix}.$$

Consider a variable $f_t = A(L)\mathbf{S}_{lgt}$. Applying the Wiener–Hopf prediction filter gives

$$\mathbb{E}_{lgt}f_t = \left[A(L)\mathbf{M}_g^\top(L^{-1})\mathbf{B}_g(L^{-1})^{-1} \right]_+ \mathbf{V}_g^{-1}\mathbf{B}_g(L)^{-1}\mathbf{X}_{lgt},$$

where $[\cdot]_+$ denotes the annihilator operator. Recall from condition (19) that we are interested in obtaining $\mathbb{E}_{lgt}v_t$, $\mathbb{E}_{lgt}z_t$, $\mathbb{E}_{lgt}a_t$, $\mathbb{E}_{lgt}\mu_t$, $\mathbb{E}_{lgt}a_{jt}$, $\mathbb{E}_{lgt}a_{j,t+1}$, and $\mathbb{E}_{lgt}a_{lg,t+1}$. We therefore need the $A(L)$ polynomial for each forecasted variable. Starting with the exogenous monetary policy shock,

$$v_t = \begin{bmatrix} \frac{\tau_v^{-1/2}}{1-\rho_vL} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{S}_{lgt} = A_v(L)\mathbf{S}_{lgt},$$

$$A_z(L) = \begin{bmatrix} 0 & \frac{\tau_z^{-1/2}}{1-\rho_zL} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_a(L) = \begin{bmatrix} 0 & 0 & \frac{\tau_a^{-1/2}}{1-\rho_aL} & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_\mu(L) = \begin{bmatrix} 0 & 0 & 0 & \frac{\tau_\mu^{-1/2}}{1-\rho_\mu L} & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For the endogenous variables, we guess and verify that agent l 's policy function takes the form $a_{lgt} = h_{gv}(L)x_{lgt}^v + h_{gz}(L)x_{lgt}^z + h_{ga}(L)x_{lgt}^a + h_{g\mu}(L)x_{lgt}^\mu$.⁸ Aggregate action can then be expressed as

$$\begin{aligned} a_{gt} &= \int_{l \in g} a_{lgt} dl = \int_{l \in g} [h_{gv}(L)x_{lgt}^v + h_{gz}(L)x_{lgt}^z + h_{ga}(L)x_{lgt}^a + h_{g\mu}(L)x_{lgt}^\mu] dl \\ &= h_{gv}(L) \frac{\tau_v^{-1/2}}{1-\rho_vL} \varepsilon_t^v + h_{gz}(L) \frac{\tau_z^{-1/2}}{1-\rho_zL} \varepsilon_t^z + h_{ga}(L) \frac{\tau_a^{-1/2}}{1-\rho_aL} \varepsilon_t^a + h_{g\mu}(L) \frac{\tau_\mu^{-1/2}}{1-\rho_\mu L} \varepsilon_t^\mu \\ &= \begin{bmatrix} h_{gv}(L) \frac{\tau_v^{-1/2}}{1-\rho_vL} & h_{gz}(L) \frac{\tau_z^{-1/2}}{1-\rho_zL} & h_{ga}(L) \frac{\tau_a^{-1/2}}{1-\rho_aL} & h_{g\mu}(L) \frac{\tau_\mu^{-1/2}}{1-\rho_\mu L} & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{S}_{lgt} = A_{ag0}(L)\mathbf{S}_{lgt} \\ a_{g,t+1} &= \begin{bmatrix} h_{gv}(L) \frac{\tau_v^{-1/2}}{L(1-\rho_vL)} & h_{gz}(L) \frac{\tau_z^{-1/2}}{L(1-\rho_zL)} & h_{ga}(L) \frac{\tau_a^{-1/2}}{L(1-\rho_aL)} & h_{g\mu}(L) \frac{\tau_\mu^{-1/2}}{L(1-\rho_\mu L)} & 0 & 0 & 0 & 0 \end{bmatrix} \mathbf{S}_{lgt} = A_{ag1}(L)\mathbf{S}_{lgt}. \end{aligned}$$

Finally,

$$a_{lg,t+1} = \frac{1}{L} \begin{bmatrix} h_{gv}(L) & h_{gz}(L) & h_{ga}(L) & h_{g\mu}(L) \end{bmatrix} \mathbf{X}_{lgt} = \frac{1}{L} \begin{bmatrix} h_{gv}(L) & h_{gz}(L) & h_{ga}(L) & h_{g\mu}(L) \end{bmatrix} \mathbf{M}_g(L)\mathbf{S}_{lgt}$$

⁸Agents observe only signals, so their policy functions can depend only on current and past signals.

$$\begin{aligned}
&= \begin{bmatrix} \frac{h_{gv}(L)}{L} \frac{\tau_v^{-1/2}}{1-\rho_v L} & \frac{h_{gz}(L)}{L} \frac{\tau_z^{-1/2}}{1-\rho_z L} & \frac{h_{ga}(L)}{L} \frac{\tau_a^{-1/2}}{1-\rho_a L} & \frac{h_{g\mu}(L)}{L} \frac{\tau_\mu^{-1/2}}{1-\rho_\mu L} & \frac{h_{gv}(L)}{L} \tau_{gv}^{-1/2} & \frac{h_{gz}(L)}{L} \tau_{gz}^{-1/2} & \frac{h_{ga}(L)}{L} \tau_{ga}^{-1/2} & \frac{h_{g\mu}(L)}{L} \tau_{g\mu}^{-1/2} \end{bmatrix} \mathbf{S}_{lgt} \\
&= \mathbf{A}_{gi}(L) \mathbf{S}_{lgt}.
\end{aligned}$$

We now have the objects needed to obtain the relevant forecasts. Starting with the exogenous variables,

$$\begin{aligned}
\mathbb{E}_{lgt} v_t &= \left[\mathbf{A}_v(L) \mathbf{M}_g^\top(L^{-1}) \mathbf{B}_g(L^{-1})^{-1} \right]_+ \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt} = \begin{bmatrix} \frac{L}{\tau_v(1-\rho_v L)(L-\lambda_{gv})} & 0 & 0 & 0 \end{bmatrix}_+ \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt} \\
&= \left[\left[\frac{\phi_v(L)}{L-\lambda_{gv}} \right]_+ \quad 0 \quad 0 \quad 0 \right] \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt}, \quad \phi_v(L) = \frac{L}{\tau_v(1-\rho_v L)} \\
&= \left[\frac{\phi_v(L)-\phi_v(\lambda_{gv})}{L-\lambda_{gv}} \quad 0 \quad 0 \quad 0 \right] \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt} = \begin{bmatrix} \frac{1}{\tau_v(1-\rho_v L)(1-\rho_v \lambda_{gv})} & 0 & 0 & 0 \end{bmatrix} \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt} \\
&= \begin{bmatrix} \frac{\lambda_{gv} \tau_{gv}}{\rho_v \tau_v(1-\rho_v \lambda_{gv})} \frac{1}{1-\lambda_{gv} L} & 0 & 0 & 0 \end{bmatrix} \mathbf{X}_{lgt} = \frac{\lambda_{gv} \tau_{gv}}{\rho_v \tau_v(1-\rho_v \lambda_{gv})} \frac{1}{1-\lambda_{gv} L} x_{lgt}^v = \left(1 - \frac{\lambda_{gv}}{\rho_v} \right) \frac{1}{1-\lambda_{gv} L} x_{lgt}^v \\
(A.5) \quad &= G_{gv}(L) x_{lgt}^v.
\end{aligned}$$

In the same manner, we can obtain

$$(A.6) \quad \mathbb{E}_{lgt} z_t = \left(1 - \frac{\lambda_{gz}}{\rho_z} \right) \frac{1}{1-\lambda_{gz} L} x_{lgt}^z = G_{gz}(L) x_{lgt}^z$$

$$(A.7) \quad \mathbb{E}_{lgt} a_t = \left(1 - \frac{\lambda_{ga}}{\rho_a} \right) \frac{1}{1-\lambda_{ga} L} x_{lgt}^a = G_{ga}(L) x_{lgt}^a$$

$$(A.8) \quad \mathbb{E}_{lgt} \mu_t = \left(1 - \frac{\lambda_{g\mu}}{\rho_\mu} \right) \frac{1}{1-\lambda_{g\mu} L} x_{lgt}^\mu = G_{g\mu}(L) x_{lgt}^\mu.$$

For the endogenous variables,

$$\begin{aligned}
\mathbb{E}_{lgt} a_{kt} &= \left[\mathbf{A}_{ak0}(L) \mathbf{M}_g^\top(L^{-1}) \mathbf{B}_g(L^{-1})^{-1} \right]_+ \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt} \\
&= \begin{bmatrix} \frac{L h_{kv}(L)}{\tau_v(1-\rho_v L)(L-\lambda_{gv})} & \frac{L h_{kz}(L)}{\tau_z(1-\rho_z L)(L-\lambda_{gz})} & \frac{L h_{ka}(L)}{\tau_a(1-\rho_a L)(L-\lambda_{ga})} & \frac{L h_{k\mu}(L)}{\tau_\mu(1-\rho_\mu L)(L-\lambda_{g\mu})} \end{bmatrix}_+ \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt} \\
&= \left[\left[\frac{\phi_{ak0v}(L)}{L-\lambda_{gv}} \right]_+ \quad \left[\frac{\phi_{ak0z}(L)}{L-\lambda_{gz}} \right]_+ \quad \left[\frac{\phi_{ak0a}(L)}{L-\lambda_{ga}} \right]_+ \quad \left[\frac{\phi_{ak0\mu}(L)}{L-\lambda_{g\mu}} \right]_+ \right] \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt}, \quad \phi_{ak0n}(L) = \frac{L h_{kn}(L)}{\tau_n(1-\rho_n L)} \\
&= \begin{bmatrix} \frac{\phi_{ak0v}(L)-\phi_{ak0v}(\lambda_{gv})}{L-\lambda_{gv}} & \frac{\phi_{ak0z}(L)-\phi_{ak0z}(\lambda_{gz})}{L-\lambda_{gz}} & \frac{\phi_{ak0a}(L)-\phi_{ak0a}(\lambda_{ga})}{L-\lambda_{ga}} & \frac{\phi_{ak0\mu}(L)-\phi_{ak0\mu}(\lambda_{g\mu})}{L-\lambda_{g\mu}} \end{bmatrix} \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt} \\
&= \begin{bmatrix} \frac{L h_{kv}(L)-\lambda_{gv} h_{kv}(\lambda_{gv}) \frac{1-\rho_v L}{1-\rho_v \lambda_{gv}}}{\tau_v(1-\rho_v L)(L-\lambda_{gv})} & \frac{L h_{kz}(L)-\lambda_{gz} h_{kz}(\lambda_{gz}) \frac{1-\rho_z L}{1-\rho_z \lambda_{gz}}}{\tau_z(1-\rho_z L)(L-\lambda_{gz})} & \frac{L h_{ka}(L)-\lambda_{ga} h_{ka}(\lambda_{ga}) \frac{1-\rho_a L}{1-\rho_a \lambda_{ga}}}{\tau_a(1-\rho_a L)(L-\lambda_{ga})} & \frac{L h_{k\mu}(L)-\lambda_{g\mu} h_{k\mu}(\lambda_{g\mu}) \frac{1-\rho_\mu L}{1-\rho_\mu \lambda_{g\mu}}}{\tau_\mu(1-\rho_\mu L)(L-\lambda_{g\mu})} \end{bmatrix} \mathbf{V}_g^{-1} \mathbf{B}_g(L)^{-1} \mathbf{X}_{lgt} \\
&= \begin{bmatrix} \frac{\lambda_{gv} \tau_{gv} \left[L h_{kv}(L)-\lambda_{gv} h_{kv}(\lambda_{gv}) \frac{1-\rho_v L}{1-\rho_v \lambda_{gv}} \right]}{\rho_v \tau_v(1-\rho_v \lambda_{gv})(L-\lambda_{gv})} & \frac{\lambda_{gz} \tau_{gz} \left[L h_{kz}(L)-\lambda_{gz} h_{kz}(\lambda_{gz}) \frac{1-\rho_z L}{1-\rho_z \lambda_{gz}} \right]}{\rho_z \tau_z(1-\rho_z \lambda_{gz})(L-\lambda_{gz})} & \frac{\lambda_{ga} \tau_{ga} \left[L h_{ka}(L)-\lambda_{ga} h_{ka}(\lambda_{ga}) \frac{1-\rho_a L}{1-\rho_a \lambda_{ga}} \right]}{\rho_a \tau_a(1-\rho_a \lambda_{ga})(L-\lambda_{ga})} & \frac{\lambda_{g\mu} \tau_{g\mu} \left[L h_{k\mu}(L)-\lambda_{g\mu} h_{k\mu}(\lambda_{g\mu}) \frac{1-\rho_\mu L}{1-\rho_\mu \lambda_{g\mu}} \right]}{\rho_\mu \tau_\mu(1-\rho_\mu \lambda_{g\mu})(L-\lambda_{g\mu})} \end{bmatrix} \mathbf{X}_{lgt} \\
&= \sum_{n \in \{v,z,a,\mu\}} \frac{\lambda_{gn} \tau_{gn} \left[L h_{kn}(L) - \lambda_{gn} h_{kn}(\lambda_{gn}) \frac{1-\rho_n L}{1-\rho_n \lambda_{gn}} \right]}{\rho_n \tau_n} \frac{1}{(1-\lambda_{gn} L)(L-\lambda_{gn})} x_{lgt}^n \\
&= \sum_{n \in \{v,z,a,\mu\}} \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \left[L h_{kn}(L)(1-\rho_n \lambda_{gn}) - \lambda_{gn} h_{kn}(\lambda_{gn})(1-\rho_n L) \right] \frac{1}{(1-\lambda_{gn} L)(L-\lambda_{gn})} x_{lgt}^n \\
(A.9) \quad &= \sum_{n \in \{v,z,a,\mu\}} G_{0gkn}(L) x_{lgt}^n \\
&= \sum_{n \in \{v,z,a,\mu\}} \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \left[h_{kn}(L)(1-\rho_n \lambda_{gn}) - h_{kn}(\lambda_{gn})(1-\rho_n L) \right] \frac{1}{(1-\lambda_{gn} L)(L-\lambda_{gn})} x_{lgt}^n
\end{aligned}$$

$$\begin{aligned}
(A.10) \quad &= \sum_{n \in \{v, z, a, \mu\}} G_{lgn}(L) x_{lgt}^n \\
\mathbb{E}_{lgt} a_{lgt+1} &= \left[A_{gi}(L) \mathbf{M}_g^\top (L^{-1}) \mathbf{B}_g (L^{-1})^{-1} \right]_+ \mathbf{V}_g^{-1} \mathbf{B}_g (L)^{-1} \mathbf{X}_{lgt} \\
&= \left[\frac{h_{gv}(L)}{L - \lambda_{gv}} \left(\frac{1}{\tau_v(1 - \rho_v L)} + \frac{L - \rho_v}{L \tau_{gv}} \right) \quad \frac{h_{gz}(L)}{L - \lambda_{gz}} \left(\frac{1}{\tau_z(1 - \rho_z L)} + \frac{L - \rho_z}{L \tau_{gz}} \right) \quad \frac{h_{ga}(L)}{L - \lambda_{ga}} \left(\frac{1}{\tau_a(1 - \rho_a L)} + \frac{L - \rho_a}{L \tau_{ga}} \right) \quad \frac{h_{g\mu}(L)}{L - \lambda_{g\mu}} \left(\frac{1}{\tau_\mu(1 - \rho_\mu L)} + \frac{L - \rho_\mu}{L \tau_{g\mu}} \right) \right]_+ \mathbf{V}_g^{-1} \mathbf{B}_g (L)^{-1} \mathbf{X}_{lgt} \\
&= \left[\left[\frac{\phi_{igv1}(L)}{L - \lambda_{gv}} \right]_+ + \left[\frac{\phi_{igv2}(L)}{L(L - \lambda_{gv})} \right]_+ \quad \left[\frac{\phi_{igz1}(L)}{L - \lambda_{gz}} \right]_+ + \left[\frac{\phi_{igz2}(L)}{L(L - \lambda_{gz})} \right]_+ \quad \left[\frac{\phi_{iga1}(L)}{L - \lambda_{ga}} \right]_+ + \left[\frac{\phi_{iga2}(L)}{L(L - \lambda_{ga})} \right]_+ \quad \left[\frac{\phi_{ig\mu1}(L)}{L - \lambda_{g\mu}} \right]_+ + \left[\frac{\phi_{ig\mu2}(L)}{L(L - \lambda_{g\mu})} \right]_+ \right] \mathbf{V}_g^{-1} \mathbf{B}_g (L)^{-1} \mathbf{X}_{lgt} \\
&= \sum_{n \in \{v, z, a, \mu\}} \left(\frac{h_{gn}(L)}{L - \lambda_{gn}} \left(\left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \frac{1 - \rho_n \lambda_{gn}}{1 - \rho_n L} + \frac{\lambda_{gn}(L - \rho_n)}{\rho_n L} \right) - \frac{h_{gn}(0)}{L} \right) \frac{1 - \rho_n L}{1 - \lambda_{gn} L} x_{lgt}^n
\end{aligned}$$

$$\begin{aligned}
(A.11) \quad &= \sum_{n \in \{v, z, a, \mu\}} G_{ign}(L) x_{lgt}^n \\
\text{where } \phi_{ign1}(L) &= \frac{h_{gn}(L)}{\tau_n(1 - \rho_n L)} \text{ and } \phi_{ign2}(L) = \frac{h_{gn}(L)(L - \rho_n)}{\tau_{gn}}. \text{ Substituting these expressions into (A.1), collecting the} \\
&\text{coefficients on } x_{lgt}^n, \text{ and rearranging gives}
\end{aligned}$$

$$\begin{aligned}
&h_{gn}(L) \left\{ 1 - \beta_g \left[\left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \frac{1 - \rho_n \lambda_{gn}}{1 - \rho_n L} + \frac{\lambda_{gn}(L - \rho_n)}{\rho_n L} \right] \frac{1 - \rho_n L}{(1 - \lambda_{gn} L)(L - \lambda_{gn})} \right\} \\
&- \sum_{k=1}^2 h_{kn}(L) \frac{(\rho_n - \lambda_{gn})(1 - \rho_n \lambda_{gn})}{\rho_n} \frac{\gamma_{gk} L + \alpha_{gk}}{(1 - \lambda_{gn} L)(L - \lambda_{gn})} \\
&= \varphi_{gn} \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \frac{1}{1 - \lambda_{gn} L} - \beta_g \frac{1 - \rho_n L}{L(1 - \lambda_{gn} L)} h_{gn}(0) - \sum_{k=1}^2 h_{kn}(\lambda_{gn}) \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \frac{\gamma_{gk} \lambda_{gn} + \alpha_{gk}}{(1 - \lambda_{gn} L)(L - \lambda_{gn})} (1 - \rho_n L)
\end{aligned}$$

Multiplying both sides by $L(L - \lambda_{gn})(1 - \lambda_{gn} L)$,

$$\begin{aligned}
&h_{gn}(L) [L(L - \lambda_{gn})(1 - \lambda_{gn} L) - \beta_g(L - \lambda_{gn})(1 - \lambda_{gn} L)] - \sum_{k=1}^2 h_{kn}(L) \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) (1 - \rho_n \lambda_{gn}) L (\gamma_{gk} L + \alpha_{gk}) \\
&= \varphi_{gn} \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) L(L - \lambda_{gn}) - \beta_g(1 - \rho_n L)(L - \lambda_{gn}) h_{gn}(0) - \sum_{k=1}^2 h_{kn}(\lambda_{gn}) \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) (\gamma_{gk} \lambda_{gn} + \alpha_{gk}) L (1 - \rho_n L)
\end{aligned}$$

We can write the above system of equations in terms of $\mathbf{h}(L)$ in matrix form

$$(A.12) \quad \mathbf{C}_n(L) \mathbf{h}_n(L) = \mathbf{d}_n(L)$$

where

$$\mathbf{C}_n(L) \equiv \text{diag} \{ \lambda_{gn} \} \left[(\beta - IL) \text{diag} \left\{ \left(L - \frac{1}{\rho_n} \right) (L - \rho_n) \right\} - (\beta - IL) L \text{diag} \left\{ \frac{\tau_{gn}}{\rho_n \tau_n} \right\} - L \text{diag} \left\{ \frac{\tau_{gn}}{\rho_n \tau_n} \right\} (L\gamma + \alpha) \right]$$

Equivalently, $\mathbf{C}_n(L) = \begin{bmatrix} C_{n11}(L) & C_{n12}(L) \\ C_{n21}(L) & C_{n22}(L) \end{bmatrix}$, where

$$\begin{aligned} C_{n11}(L) &= \lambda_{1n} \left[(\beta_1 - L) \left(L - \frac{1}{\rho_n} \right) (L - \rho_n) + \frac{\tau_{1n}}{\rho_n \tau_n} L [L(1 - \gamma_{11}) - \delta_{11}] \right], & C_{n12}(L) &= -\lambda_{1n} L \frac{\tau_{1n}}{\rho_n \tau_n} (L\gamma_{12} + \delta_{12}) \\ C_{n22}(L) &= \lambda_{2n} \left[(\beta_2 - L) \left(L - \frac{1}{\rho_n} \right) (L - \rho_n) + \frac{\tau_{2n}}{\rho_n \tau_n} L [L(1 - \gamma_{22}) - \delta_{22}] \right], & C_{n21}(L) &= -\lambda_{2n} L \frac{\tau_{2n}}{\rho_n \tau_n} (L\gamma_{21} + \delta_{21}) \end{aligned}$$

We can also write $\mathbf{d}_n(L) = \begin{bmatrix} d_{1n}[L; h_{1n}(\cdot)] \\ d_{2n}[L; h_{2n}(\cdot)] \end{bmatrix}$, where

$$\begin{aligned} d_{gn}(L) &= \varphi_{gn} \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) L(L - \lambda_{gn}) \\ &\quad - \beta_g (1 - \rho_n L)(L - \lambda_{gn}) h_{gn}(0) \\ &\quad - \sum_{k=1}^2 h_{kn}(\lambda_{gn}) \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) (\gamma_{gk} \lambda_{gn} + \alpha_{gk}) L(1 - \rho_n L) \end{aligned}$$

From (A.12), the policy function satisfies $\mathbf{h}_n(L) = \mathbf{C}_n(L)^{-1} \mathbf{d}_n(L) = \frac{\text{adj } \mathbf{C}_n(L)}{\det \mathbf{C}_n(L)} \mathbf{d}_n(L)$. The determinant $\det \mathbf{C}_n(L)$ is a sixth-degree polynomial in L . Denote its four inside roots by $\{\zeta_{1n}, \zeta_{2n}, \zeta_{3n}, \zeta_{4n}\}$ and its two outside roots by $\{\vartheta_{1n}^{-1}, \vartheta_{2n}^{-1}\}$. Because agents cannot condition on future signals, the inside roots must be removed. The four free constants in $\mathbf{d}_n(L)$ are $\{h_{gn}(0)\}$ and $\{\tilde{h}_n(\lambda_{gn}) = \sum_{k=1}^2 h_{kn}(\lambda_{gn}) \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) (\gamma_{gk} \lambda_{gn} + \alpha_{gk})\}$ for each $g \in \{1, 2\}$. Uniqueness therefore requires two outside roots. By Cramer's rule, $h_{gn}(L)$ is given by

$$h_{1n}(L) = \frac{\det \begin{bmatrix} d_{1n}(L) & C_{n12}(L) \\ d_{2n}(L) & C_{n22}(L) \end{bmatrix}}{\det \mathbf{C}_n(L)}, \quad h_{2n}(L) = \frac{\det \begin{bmatrix} C_{n11}(L) & d_{1n}(L) \\ C_{n21}(L) & d_{2n}(L) \end{bmatrix}}{\det \mathbf{C}_n(L)}$$

These are the policy functions for groups 1 (consumers) and 2 (firms). Each numerator has degree five, one degree below $\det \mathbf{C}_n(L)$. Choosing $\{h_{1n}(0), \tilde{h}_n(\lambda_{1n}), h_{2n}(0), \tilde{h}_n(\lambda_{2n})\}$ to remove the four inside roots yields the following system of four linear equations:

$$\det \begin{bmatrix} d_{1n}(\zeta_{sn}) & C_{n12}(\zeta_{sn}) \\ d_{2n}(\zeta_{sn}) & C_{n22}(\zeta_{sn}) \end{bmatrix} = 0, \quad \text{for } \{\zeta_{sn}\}_{s=1}^4$$

After removing the inside roots, the numerator has degree one and the denominator degree two. Solving the four equations for the four free constants gives $h_{gn}(L) = \frac{\tilde{\psi}_{gn,1} + \tilde{\psi}_{gn,2}L}{(1 - \vartheta_{1n}L)(1 - \vartheta_{2n}L)}$. Hence,

$$a_{gnt} = h_{gn}(L)n_t = \frac{\tilde{\psi}_{gn,1} + \tilde{\psi}_{gn,2}L}{(1 - \vartheta_{1n}L)(1 - \vartheta_{2n}L)} n_t = \sum_{j=1}^2 \psi_{gjn} \left(1 - \frac{\vartheta_{jn}}{\rho_n} \right) \frac{1}{1 - \vartheta_{jn}L} n_t = \sum_{j=1}^2 \psi_{gjn} \tilde{\vartheta}_{jnt}$$

We can write

$$\mathbf{a}_{nt} = \begin{bmatrix} a_{1nt} \\ a_{2nt} \end{bmatrix} = \Psi_n \tilde{\vartheta}_t = \begin{bmatrix} \psi_{11,n} & \psi_{12,n} \\ \psi_{21,n} & \psi_{22,n} \end{bmatrix} \begin{bmatrix} \tilde{\vartheta}_{1nt} \\ \tilde{\vartheta}_{2nt} \end{bmatrix} = \begin{bmatrix} \psi_{11,n} \tilde{\vartheta}_{1nt} + \psi_{12,n} \tilde{\vartheta}_{2nt} \\ \psi_{21,n} \tilde{\vartheta}_{1nt} + \psi_{22,n} \tilde{\vartheta}_{2nt} \end{bmatrix}$$

The relation $\tilde{\vartheta}_{gnt}(1 - \vartheta_{gn}L) = \left(1 - \frac{\vartheta_{gn}}{\rho_n}\right) n_t$ implies $\tilde{\vartheta}_{gnt} = \vartheta_{gn} \tilde{\vartheta}_{gn,t-1} + \left(1 - \frac{\vartheta_{gn}}{\rho_n}\right) n_t$. In vector form, $\tilde{\vartheta}_{nt} = \Lambda_n \tilde{\vartheta}_{n,t-1} + \Gamma_n n_t$, where $\Lambda_n = \begin{bmatrix} \vartheta_{1n} & 0 \\ 0 & \vartheta_{2n} \end{bmatrix}$, $\Gamma_n = \begin{bmatrix} 1 - \frac{\vartheta_{1n}}{\rho_n} \\ 1 - \frac{\vartheta_{2n}}{\rho_n} \end{bmatrix}$. Hence, $\mathbf{a}_{nt} = \Psi_n \tilde{\vartheta}_{nt} = \Psi_n (\Lambda_n \tilde{\vartheta}_{n,t-1} + \Gamma_n n_t) = \Psi_n \Lambda_n \Psi_n^{-1} \mathbf{a}_{n,t-1} + \Psi_n \Gamma_n n_t = A_n \mathbf{a}_{n,t-1} + B_n n_t$. Finally, $\mathbf{a}_t = \sum_n \mathbf{a}_{nt}$. $\square$

PROPOSITION A1. (i) The reaction of beliefs to the composite policy surprise, mps_t , is given by the OLS coefficient

$$\begin{aligned} \beta_{\text{FR},c}^h &= \frac{\mathbb{C}[\text{optimism}_{t+h}^y, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} = \frac{\sum_n \left(\frac{\gamma_n \lambda_n^h}{1 - \lambda_n^2} + \sum_{j=1}^2 \frac{\phi_{jn} \vartheta_{jn}^h}{1 - \lambda_n \vartheta_{jn}} \right) \zeta_{cn} \sigma_n^2}{\sum_n \left(\frac{\gamma_n^2}{1 - \lambda_n^2} + 2 \sum_{j=1}^2 \frac{\gamma_n \phi_{jn}}{1 - \lambda_n \vartheta_{jn}} + \sum_{j=1}^2 \frac{\phi_{jn}^2}{1 - \vartheta_{jn}^2} + \frac{2\phi_{1n}\phi_{2n}}{1 - \vartheta_{1n}\vartheta_{2n}} \right) \sigma_n^2} \\ \beta_{\text{FR},\pi}^h &= \frac{\mathbb{C}[\text{optimism}_{t+h}^\pi, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} = \frac{\sum_n \left(\frac{\gamma_n \lambda_n^h}{1 - \lambda_n^2} + \sum_{j=1}^2 \frac{\phi_{jn} \vartheta_{jn}^h}{1 - \lambda_n \vartheta_{jn}} \right) \zeta_{\pi n} \sigma_n^2}{\sum_n \left(\frac{\gamma_n^2}{1 - \lambda_n^2} + 2 \sum_{j=1}^2 \frac{\gamma_n \phi_{jn}}{1 - \lambda_n \vartheta_{jn}} + \sum_{j=1}^2 \frac{\phi_{jn}^2}{1 - \vartheta_{jn}^2} + \frac{2\phi_{1n}\phi_{2n}}{1 - \vartheta_{1n}\vartheta_{2n}} \right) \sigma_n^2} \\ \beta_{\text{FR},i}^h &= \frac{\mathbb{C}[\text{optimism}_{t+h}^i, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} \\ &= \phi_\pi \beta_{\text{FR},\pi}^h + \phi_y \beta_{\text{FR},c}^h + \frac{(\rho_v - \lambda_v) \sigma_v^2 \left[\frac{\gamma_v \lambda_v^h}{1 - \lambda_v^2} + \sum_{j=1}^2 \frac{\phi_{jv} \vartheta_{jv}^h}{1 - \lambda_v \vartheta_{jv}} \right]}{\sum_n \left(\frac{\gamma_n^2}{1 - \lambda_n^2} + 2 \sum_{j=1}^2 \frac{\gamma_n \phi_{jn}}{1 - \lambda_n \vartheta_{jn}} + \sum_{j=1}^2 \frac{\phi_{jn}^2}{1 - \vartheta_{jn}^2} + \frac{2\phi_{1n}\phi_{2n}}{1 - \vartheta_{1n}\vartheta_{2n}} \right) \sigma_n^2} \end{aligned}$$

where $\zeta_{cn} = \frac{(\rho_n - \lambda_n)}{\rho_n^2 \lambda_n} \sum_{j=1}^2 \frac{\psi_{cjn}(\rho_n - \vartheta_{jn})}{1 - \vartheta_{jn} \lambda_n}$, $\zeta_{\pi n} = \frac{(\rho_n - \lambda_n)}{\rho_n^2 \lambda_n} \sum_{j=1}^2 \frac{\psi_{\pi jn}(\rho_n - \vartheta_{jn})}{1 - \vartheta_{jn} \lambda_n}$, $\gamma_{gn} = \sum_{j=1}^2 \frac{\lambda_n^2 \psi_{gjn}(\rho_n - \vartheta_{jn})(1 - \rho_n \vartheta_{jn})}{\rho_n^2 (1 - \lambda_n \vartheta_{jn})(\lambda_n - \vartheta_{jn})}$, and $\gamma_{gjn} = -\frac{\lambda_n \vartheta_{jn} \psi_{gjn}(\rho_n - \vartheta_{jn})(1 - \rho_n \vartheta_{jn})}{\rho_n^2 (1 - \lambda_n \vartheta_{jn})(\lambda_n - \vartheta_{jn})}$, $\gamma_v = \phi_\pi \gamma_{\pi v} + \phi_y \gamma_{yv} + 1$, $\gamma_n = \phi_\pi \gamma_{\pi n} + \phi_y \gamma_{yn}$ for $n \neq v$, and $\phi_{jn} = \phi_\pi \gamma_{\pi jn} + \phi_y \gamma_{yjn}$.

(ii) The reaction of aggregate macro variables to the composite policy surprise is given by the OLS coefficient

$$\begin{aligned} \beta_{\text{IRF},c}^h &= \frac{\mathbb{C}[c_{t+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} \\ &= \frac{\sum_n \left\{ \left[\frac{\gamma_n}{1 - \rho_n \lambda_n} + \sum_{j=1}^2 \frac{\phi_{jn}}{1 - \rho_n \vartheta_{jn}} \right] \rho_n^h \beta_{yn} + \sum_{j=1}^2 \left[\frac{\gamma_n}{1 - \lambda_n \vartheta_{jn}} + \frac{\phi_{1n}}{1 - \vartheta_{1n} \vartheta_{jn}} + \frac{\phi_{2n}}{1 - \vartheta_{2n} \vartheta_{jn}} \right] \vartheta_{jn}^h \beta_{yjn} \right\} \sigma_n^2}{\sum_n \left(\frac{\gamma_n^2}{1 - \lambda_n^2} + 2 \sum_{j=1}^2 \frac{\gamma_n \phi_{jn}}{1 - \lambda_n \vartheta_{jn}} + \sum_{j=1}^2 \frac{\phi_{jn}^2}{1 - \vartheta_{jn}^2} + \frac{2\phi_{1n}\phi_{2n}}{1 - \vartheta_{1n}\vartheta_{2n}} \right) \sigma_n^2} \\ \beta_{\text{IRF},\pi}^h &= \frac{\mathbb{C}[\pi_{t+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} \\ &= \frac{\sum_n \left\{ \left[\frac{\gamma_n}{1 - \rho_n \lambda_n} + \sum_{j=1}^2 \frac{\phi_{jn}}{1 - \rho_n \vartheta_{jn}} \right] \rho_n^h \beta_{\pi n} + \sum_{j=1}^2 \left[\frac{\gamma_n}{1 - \lambda_n \vartheta_{jn}} + \frac{\phi_{1n}}{1 - \vartheta_{1n} \vartheta_{jn}} + \frac{\phi_{2n}}{1 - \vartheta_{2n} \vartheta_{jn}} \right] \vartheta_{jn}^h \beta_{\pi jn} \right\} \sigma_n^2}{\sum_n \left(\frac{\gamma_n^2}{1 - \lambda_n^2} + 2 \sum_{j=1}^2 \frac{\gamma_n \phi_{jn}}{1 - \lambda_n \vartheta_{jn}} + \sum_{j=1}^2 \frac{\phi_{jn}^2}{1 - \vartheta_{jn}^2} + \frac{2\phi_{1n}\phi_{2n}}{1 - \vartheta_{1n}\vartheta_{2n}} \right) \sigma_n^2} \end{aligned}$$

$$\beta_{\text{IRF},i}^h = \frac{\mathbb{C}[i_{t+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} = \phi_\pi \beta_{\text{IRF},\pi}^h + \phi_y \beta_{\text{IRF},c}^h + \frac{\left(\frac{\gamma_v}{1-\rho_v \lambda_v} + \sum_{j=1}^2 \frac{\phi_{jv}}{1-\rho_v \vartheta_{jv}} \right) \rho_v^h \sigma_v^2}{\sum_n \left(\frac{\gamma_n^2}{1-\lambda_n^2} + 2 \sum_{j=1}^2 \frac{\gamma_n \phi_{jn}}{1-\lambda_n \vartheta_{jn}} + \sum_{j=1}^2 \frac{\phi_{jn}^2}{1-\vartheta_{jn}^2} + \frac{2\phi_{1n}\phi_{2n}}{1-\vartheta_{1n}\vartheta_{2n}} \right) \sigma_n^2}$$

where $\beta_{gn} = \sum_{j=1}^2 \psi_{gjn}$ and $\beta_{gjn} = -\frac{\psi_{gjn}\vartheta_{jn}}{\rho_n}$ for $g \in \{y, \pi\}$ and $j \in \{1, 2\}$.

PROOF. Following the results in the proof of Proposition 1, we have that

$$\mathbb{E}_t^g a_{j,t+k} = \sum_n \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \frac{L^{1-k} h_{jn}(L)(1 - \rho_n \lambda_{gn}) - \lambda_{gn}^{1-k} h_{jn}(\lambda_{gn})(1 - \rho_n L)}{(1 - \lambda_{gn} L)(L - \lambda_{gn})} n_t$$

for $k \geq 0$ and $n \in \{v, z, a, \mu\}$. Hence, we can write the forecast revision as

$$\begin{aligned} \mathbb{E}_{t+h}^g a_{j,t+1+h} - \mathbb{E}_{t-1+h}^g a_{j,t+1+h} &= \sum_n \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \frac{h_{jn}(\lambda_{gn})}{\lambda_{gn}} \frac{1 - \rho_n L}{1 - \lambda_{gn} L} n_{t+h} = \sum_n \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \frac{h_{jn}(\lambda_{gn})}{\lambda_{gn}} \sum_{k=0}^{\infty} \lambda_{gn}^k \varepsilon_{t-k+h}^n \\ &= \sum_n \zeta_{gjn} \sum_{k=0}^{\infty} \lambda_{gn}^k \varepsilon_{t-k+h}^n \end{aligned}$$

where $\zeta_{gjn} = \frac{(\rho_n - \lambda_{gn})}{\rho_n^2 \lambda_{gn}} \sum_{s=1}^2 \frac{\psi_{jns}(\rho_n - \vartheta_{sn})}{1 - \vartheta_{sn} \lambda_{gn}}$. Our baseline monetary policy surprise is

$$\text{mps}_t^g = \phi_\pi(\pi_t - \mathbb{E}_t^g \pi_t) + \phi_y(\tilde{y}_t - \mathbb{E}_t^g \tilde{y}_t) + v_t - \mathbb{E}_{t-1}^g v_t.$$

For completeness, an alternative contemporaneous-information convention replaces the final term by $v_t - \mathbb{E}_t^g v_t$:

$$\widetilde{\text{mps}}_t^g = \phi_\pi(\pi_t - \mathbb{E}_t^g \pi_t) + \phi_y(\tilde{y}_t - \mathbb{E}_t^g \tilde{y}_t) + v_t - \mathbb{E}_t^g v_t.$$

Aggregate endogenous variables evolve according to

$$\begin{aligned} a_{kt} &= \sum_n \sum_{j=1}^2 \left(1 - \frac{\vartheta_{jn}}{\rho_n} \right) \psi_{kjn} \frac{1}{(1 - \vartheta_{jn} L)(1 - \rho_n L)} \varepsilon_t^n = \sum_n \sum_{j=1}^2 \frac{\psi_{kjn}}{\rho_n} \sum_{s=0}^{\infty} (\rho_n^{s+1} - \vartheta_{jn}^{s+1}) \varepsilon_{t-s}^n \\ &= \sum_n \left[\beta_{kn} \sum_{l=0}^{\infty} \rho_n^l \varepsilon_{t-l}^n + \sum_{j=1}^2 \beta_{kjn} \sum_{l=0}^{\infty} \vartheta_{jn}^l \varepsilon_{t-l}^n \right] \end{aligned}$$

where $\beta_{kn} = \sum_{j=1}^2 \psi_{kjn}$ and $\beta_{kjn} = -\frac{\psi_{kjn}\vartheta_{jn}}{\rho_n}$. Their nowcast is

$$\mathbb{E}_t^g a_{jt} = \sum_n \left(1 - \frac{\lambda_{gn}}{\rho_n} \right) \frac{L h_{jn}(L)(1 - \rho_n \lambda_{gn}) - \lambda_{gn} h_{jn}(\lambda_{gn})(1 - \rho_n L)}{(1 - \lambda_{gn} L)(L - \lambda_{gn})} n_t$$

Hence,

$$\begin{aligned} a_{g,t} - \mathbb{E}_t^k a_{g,t} &= \sum_n \left\{ h_{gn}(L) \left[1 - \frac{(\rho_n - \lambda_{kn})L(1 - \rho_n \lambda_{kn})}{\rho_n(1 - \lambda_{kn} L)(L - \lambda_{kn})} \right] + h_{gn}(\lambda_{kn}) \frac{(\rho_n - \lambda_{kn})\lambda_{kn}(1 - \rho_n L)}{\rho_n(1 - \lambda_{kn} L)(L - \lambda_{kn})} \right\} n_t \\ &= \sum_n \frac{\lambda_{kn}(1 - \rho_n L)}{\rho_n(1 - \lambda_{kn} L)(L - \lambda_{kn})} [h_{gn}(L)(L - \rho_n) + h_{gn}(\lambda_{kn})(\rho_n - \lambda_{kn})] n_t \end{aligned}$$

$$\begin{aligned}
&= \sum_n \frac{\lambda_{kn}}{\rho_n(1 - \lambda_{kn}L)(L - \lambda_{kn})} [h_{gn}(L)(L - \rho_n) + h_{gn}(\lambda_{kn})(\rho_n - \lambda_{kn})] \varepsilon_t^n \\
&= \sum_n \frac{\lambda_{kn}}{\rho_n(1 - \lambda_{kn}L)(L - \lambda_{kn})} \sum_{j=1}^2 \left[\frac{\psi_{gin}(\rho_n - \vartheta_{jn})(L - \rho_n)}{\rho_n(1 - \vartheta_{jn}L)} + \frac{\psi_{gin}(\rho_n - \vartheta_{jn})(\rho_n - \lambda_{kn})}{\rho_n(1 - \vartheta_{jn}\lambda_{kn})} \right] \varepsilon_t^n \\
&= \sum_n \frac{\lambda_{kn}}{\rho_n(1 - \lambda_{kn}L)(L - \lambda_{kn})} \sum_{j=1}^2 \frac{\psi_{gin}(\rho_n - \vartheta_{jn})}{\rho_n} \left[\frac{(L - \rho_n)}{1 - \vartheta_{jn}L} + \frac{\rho_n - \lambda_{kn}}{1 - \vartheta_{jn}\lambda_{kn}} \right] \varepsilon_t^n \\
&= \sum_n \frac{\lambda_{kn}}{\rho_n(1 - \lambda_{kn}L)(L - \lambda_{kn})} \sum_{j=1}^2 \frac{\psi_{gin}(\rho_n - \vartheta_{jn})}{\rho_n} \frac{(L - \lambda_{kn})(1 - \rho_n\vartheta_{jn})}{(1 - \vartheta_{jn}L)(1 - \lambda_{kn}\vartheta_{jn})} \varepsilon_t^n \\
&= \sum_n \sum_{j=1}^2 \frac{\lambda_{kn}\psi_{gin}(\rho_n - \vartheta_{jn})(1 - \rho_n\vartheta_{jn})}{\rho_n^2(1 - \lambda_{kn}\vartheta_{jn})} \frac{1}{(1 - \vartheta_{jn}L)(1 - \lambda_{kn}L)} \varepsilon_t^n \\
&= \sum_n \sum_{j=1}^2 \frac{\lambda_{kn}\psi_{gin}(\rho_n - \vartheta_{jn})(1 - \rho_n\vartheta_{jn})}{\rho_n^2(1 - \lambda_{kn}\vartheta_{jn})} \left(\frac{\lambda_{kn}}{\lambda_{kn} - \vartheta_{jn}} \frac{1}{1 - \lambda_{kn}L} - \frac{\vartheta_{jn}}{\lambda_{kn} - \vartheta_{jn}} \frac{1}{1 - \vartheta_{jn}L} \right) \varepsilon_t^n \\
&= \sum_n \left[\sum_{j=1}^2 \frac{\lambda_{kn}^2 \psi_{gin}(\rho_n - \vartheta_{jn})(1 - \rho_n\vartheta_{jn})}{\rho_n^2(1 - \lambda_{kn}\vartheta_{jn})(\lambda_{kn} - \vartheta_{jn})} \sum_{l=0}^{\infty} \lambda_{kn}^l \varepsilon_{t-l}^n - \sum_{j=1}^2 \frac{\lambda_{kn}\vartheta_{jn} \psi_{gin}(\rho_n - \vartheta_{jn})(1 - \rho_n\vartheta_{jn})}{\rho_n^2(1 - \lambda_{kn}\vartheta_{jn})(\lambda_{kn} - \vartheta_{jn})} \sum_{l=0}^{\infty} \vartheta_{jn}^l \varepsilon_{t-l}^n \right] \\
&= \sum_n \gamma_{gkn} \sum_{l=0}^{\infty} \lambda_{kn}^l \varepsilon_{t-l}^n + \sum_n \sum_{j=1}^2 \gamma_{gkjn} \sum_{l=0}^{\infty} \vartheta_{jn}^l \varepsilon_{t-l}^n
\end{aligned}$$

where $\gamma_{gkn} = \sum_{j=1}^2 \frac{\lambda_{kn}^2 \psi_{gin}(\rho_n - \vartheta_{jn})(1 - \rho_n\vartheta_{jn})}{\rho_n^2(1 - \lambda_{kn}\vartheta_{jn})(\lambda_{kn} - \vartheta_{jn})}$ and $\gamma_{gkjn} = -\frac{\lambda_{kn}\vartheta_{jn} \psi_{gin}(\rho_n - \vartheta_{jn})(1 - \rho_n\vartheta_{jn})}{\rho_n^2(1 - \lambda_{kn}\vartheta_{jn})(\lambda_{kn} - \vartheta_{jn})}$. We also have

$$v_t - \mathbb{E}_{t-1}^k v_t = \left[\frac{1}{1 - \rho_v L} - \left(1 - \frac{\lambda_{kv}}{\rho_v} \right) \frac{\rho_v L}{(1 - \lambda_{kv}L)(1 - \rho_v L)} \right] \varepsilon_t^v = \sum_{l=0}^{\infty} \lambda_{kv}^l \varepsilon_{t-l}^v$$

and

$$\begin{aligned}
v_t - \mathbb{E}_t^k v_t &= \left[\frac{1}{1 - \rho_v L} - \left(1 - \frac{\lambda_{kv}}{\rho_v} \right) \frac{1}{(1 - \lambda_{kv}L)(1 - \rho_v L)} \right] \varepsilon_t^v \\
&= \left\{ \frac{1}{1 - \rho_v L} - \left(1 - \frac{\lambda_{kv}}{\rho_v} \right) \left[\frac{\lambda_{kv}}{\lambda_{kv} - \rho_v} \frac{1}{1 - \lambda_{kv}L} - \frac{\rho_v}{\lambda_{kv} - \rho_v} \frac{1}{1 - \rho_v L} \right] \right\} \varepsilon_t^v \\
&= \frac{\lambda_{kv}}{\rho_v} \sum_{l=0}^{\infty} \lambda_{kv}^l \varepsilon_{t-l}^v
\end{aligned}$$

We can therefore write the monetary policy surprise as

$$\begin{aligned}
\text{mps}_t &= \phi_\pi \left[\sum_n \gamma_{\pi kn} \sum_{l=0}^{\infty} \lambda_{kn}^l \varepsilon_{t-l}^n + \sum_n \sum_{j=1}^2 \gamma_{\pi kjn} \sum_{l=0}^{\infty} \vartheta_{jn}^l \varepsilon_{t-l}^n \right] \\
&\quad + \phi_y \left[\sum_n \gamma_{ykn} \sum_{l=0}^{\infty} \lambda_{kn}^l \varepsilon_{t-l}^n + \sum_n \sum_{j=1}^2 \gamma_{ykjn} \sum_{l=0}^{\infty} \vartheta_{jn}^l \varepsilon_{t-l}^n \right] + \sum_{l=0}^{\infty} \lambda_{kv}^l \varepsilon_{t-l}^v
\end{aligned}$$

$$= \sum_n \left[\gamma_{kn} \sum_{l=0}^{\infty} \lambda_{kn}^l \varepsilon_{t-l}^n + \sum_{j=1}^2 \gamma_{kjn} \sum_{l=0}^{\infty} \vartheta_{jn}^l \varepsilon_{t-l}^n \right]$$

where $\gamma_{kv} = \phi\pi\gamma_{\pi kv} + \phi_y\gamma_{ykv} + 1$ ($\gamma_{kv} = \phi\pi\gamma_{\pi kv} + \phi_y\gamma_{ykv} + \frac{\lambda_{gv}}{\rho_v}$ in the case we consider the second definition of the monetary surprise), $\gamma_{kn} = \phi\pi\gamma_{\pi kn} + \phi_y\gamma_{ykn}$ for $n \neq v$, and $\gamma_{kjn} = \phi\pi\gamma_{\pi kn} + \phi_y\gamma_{ykn}$. We can thus write

$$\begin{aligned} \mathbb{C} [\bar{\mathbb{E}}_{t+h}^k a_{g,t+1+h} - \bar{\mathbb{E}}_{t-1+h}^k a_{g,t+1+h}, \text{mps}_t] &= \sum_n \left(\frac{\gamma_{kn} \lambda_{kn}^h}{1 - \lambda_{kn}^2} + \sum_{j=1}^2 \frac{\gamma_{kjn} \vartheta_{jn}^h}{1 - \lambda_{kn} \vartheta_{jn}} \right) \zeta_{kgn} \sigma_n^2 \\ \mathbb{V} [\text{mps}_t] &= \sum_n \left(\frac{\gamma_{kn}^2}{1 - \lambda_{kn}^2} + 2 \sum_{j=1}^2 \frac{\gamma_{kn} \gamma_{kjn}}{1 - \lambda_{kn} \vartheta_{jn}} + \sum_{j=1}^2 \frac{\gamma_{kjn}^2}{1 - \vartheta_{jn}^2} + \frac{2\gamma_{k1n} \gamma_{k2n}}{1 - \vartheta_{1n} \vartheta_{2n}} \right) \sigma_n^2 \end{aligned}$$

The OLS coefficient is given by

$$\beta_{\text{FR},kg}^h = \frac{\mathbb{C} [\bar{\mathbb{E}}_{t+h}^k a_{g,t+1+h} - \bar{\mathbb{E}}_{t-1+h}^k a_{g,t+1+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} = \frac{\sum_n \left(\frac{\gamma_{kn} \lambda_{kn}^h}{1 - \lambda_{kn}^2} + \sum_{j=1}^2 \frac{\gamma_{kjn} \vartheta_{jn}^h}{1 - \lambda_{kn} \vartheta_{jn}} \right) \zeta_{kgn} \sigma_n^2}{\sum_n \left(\frac{\gamma_{kn}^2}{1 - \lambda_{kn}^2} + 2 \sum_{j=1}^2 \frac{\gamma_{kn} \gamma_{kjn}}{1 - \lambda_{kn} \vartheta_{jn}} + \sum_{j=1}^2 \frac{\gamma_{kjn}^2}{1 - \vartheta_{jn}^2} + \frac{2\gamma_{k1n} \gamma_{k2n}}{1 - \vartheta_{1n} \vartheta_{2n}} \right) \sigma_n^2}$$

To proceed in a similar way with the OLS coefficient of policy rate forecast revisions, we first compute

$$\bar{\mathbb{E}}_{t+h}^k v_{t+1+h} - \bar{\mathbb{E}}_{t-1+h}^k v_{t+1+h} = \left[\frac{\rho_v - \lambda_{kv}}{1 - \lambda_{kv} L} v_{t+h} - \frac{\rho_v(\rho_v - \lambda_{kv})}{1 - \lambda_{kv} L} v_{t-1+h} \right] = \frac{\rho_v - \lambda_{kv}}{1 - \lambda_{kv} L} \varepsilon_{t+h}^v = (\rho_v - \lambda_{kv}) \sum_{j=0}^{\infty} \lambda_{kv}^j \varepsilon_{t-j+h}^v$$

and hence,

$$\begin{aligned} \beta_{\text{FR},ki}^h &= \frac{\mathbb{C} [\bar{\mathbb{E}}_{t+h}^k i_{t+1+h} - \bar{\mathbb{E}}_{t-1+h}^k i_{t+1+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} = \phi\pi\beta_{\text{FR},k,\pi}^h + \phi_y\beta_{\text{FR},k,y}^h + \frac{\mathbb{C} [\bar{\mathbb{E}}_{t+h}^k v_{t+1+h} - \bar{\mathbb{E}}_{t-1+h}^k v_{t+1+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} \\ &= \phi\pi\beta_{\text{FR},k,\pi}^h + \phi_y\beta_{\text{FR},k,y}^h + \frac{(\rho_v - \lambda_{kv}) \sigma_v^2 \left[\frac{\gamma_{kv} \lambda_{kv}^h}{1 - \lambda_{kv}^2} + \sum_{j=1}^2 \frac{\gamma_{kjh} \vartheta_{jh}^h}{1 - \lambda_{kv} \vartheta_{jh}} \right]}{\sum_n \left(\frac{\gamma_{kn}^2}{1 - \lambda_{kn}^2} + 2 \sum_{j=1}^2 \frac{\gamma_{kn} \gamma_{kjn}}{1 - \lambda_{kn} \vartheta_{jn}} + \sum_{j=1}^2 \frac{\gamma_{kjn}^2}{1 - \vartheta_{jn}^2} + \frac{2\gamma_{k1n} \gamma_{k2n}}{1 - \vartheta_{1n} \vartheta_{2n}} \right) \sigma_n^2} \end{aligned}$$

For the aggregate response to the composite policy surprise,

$$\begin{aligned} \mathbb{C} [a_{g,t+h}, \text{mps}_t] &= \sum_n \left\{ \left[\frac{\gamma_{kn} \rho_n^h}{1 - \rho_n \lambda_{kn}} + \sum_{j=1}^2 \frac{\gamma_{kjn} \rho_n^h}{1 - \rho_n \vartheta_{jn}} \right] \beta_{gn} + \left[\frac{\gamma_{kn} \vartheta_{1n}^h}{1 - \lambda_{kn} \vartheta_{1n}} + \frac{\gamma_{k1n} \vartheta_{1n}^h}{1 - \vartheta_{1n}^2} + \frac{\gamma_{k2n} \vartheta_{1n}^h}{1 - \vartheta_{1n} \vartheta_{2n}} \right] \beta_{g1n} \right. \\ &\quad \left. + \left[\frac{\gamma_{kn} \vartheta_{2n}^h}{1 - \lambda_{kn} \vartheta_{2n}} + \frac{\gamma_{k1n} \vartheta_{2n}^h}{1 - \vartheta_{1n} \vartheta_{2n}} + \frac{\gamma_{k2n} \vartheta_{2n}^h}{1 - \vartheta_{2n}^2} \right] \beta_{g2n} \right\} \sigma_n^2 \end{aligned}$$

where $\beta_{gn} = \sum_{j=1}^2 \psi_{gjn} \vartheta_{jn}$ and $\beta_{gin} = -\frac{\psi_{gin} \vartheta_{jn}}{\rho_n}$. The OLS coefficient is given by

$$\beta_{\text{IRF},g}^h = \frac{\mathbb{C} [a_{g,t+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)}$$

To proceed in a similar way with the OLS coefficient of policy rate IRFs, we first compute

$$\mathbb{C}[v_{t+h}, \text{mps}_t] = \left(\frac{\gamma_{kv}}{1 - \rho_v \lambda_{kv}} + \sum_{j=1}^2 \frac{\gamma_{k j v}}{1 - \rho_v \vartheta_{j v}} \right) \rho_v^h \sigma_v^2$$

hence,

$$\begin{aligned} \beta_{IRF,k,i}^h &= \frac{\mathbb{C}[i_{t+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} = \phi \pi \beta_{IRF,k,\pi}^h + \phi_y \beta_{IRF,k,y}^h + \frac{\mathbb{C}[v_{t+h}, \text{mps}_t]}{\mathbb{V}(\text{mps}_t)} \\ &= \phi \pi \beta_{IRF,k,\pi}^h + \phi_y \beta_{IRF,k,y}^h + \frac{\left(\frac{\gamma_{kv}}{1 - \rho_v \lambda_{kv}} + \sum_{j=1}^2 \frac{\gamma_{k j v}}{1 - \rho_v \vartheta_{j v}} \right) \rho_v^h \sigma_v^2}{\sum_n \left(\frac{\gamma_{kn}^2}{1 - \lambda_{kn}^2} + 2 \sum_{j=1}^2 \frac{\gamma_{kn} \gamma_{k j n}}{1 - \lambda_{kn} \vartheta_{j n}} + \sum_{j=1}^2 \frac{\gamma_{k j n}^2}{1 - \vartheta_{j n}^2} + \frac{2 \gamma_{k 1 n} \gamma_{k 2 n}}{1 - \vartheta_{1 n} \vartheta_{2 n}} \right) \sigma_n^2} \end{aligned}$$

□

COROLLARY A1. (i) The reaction of beliefs to the structural monetary policy shock, $\phi_v \varepsilon_t^v$, is given by the OLS coefficient

$$\begin{aligned} \beta_{FR,c,\varepsilon}^h &= \frac{\mathbb{C}[\text{optimism}_{t+h}^y, \phi_v \varepsilon_t^v]}{\mathbb{V}(\phi_v \varepsilon_t^v)} = \frac{\zeta_{cv} \lambda_v^h}{\phi_v} \\ \beta_{FR,\pi,\varepsilon}^h &= \frac{\mathbb{C}[\text{optimism}_{t+h}^\pi, \phi_v \varepsilon_t^v]}{\mathbb{V}(\phi_v \varepsilon_t^v)} = \frac{\zeta_{\pi v} \lambda_v^h}{\phi_v} \\ \beta_{FR,i,\varepsilon}^h &= \frac{\mathbb{C}[\text{optimism}_{t+h}^i, \phi_v \varepsilon_t^v]}{\mathbb{V}(\phi_v \varepsilon_t^v)} = \phi \pi \beta_{FR,\pi,\varepsilon}^h + \phi_y \beta_{FR,c,\varepsilon}^h + \frac{(\rho_v - \lambda_v) \lambda_v^h}{\phi_v} \end{aligned}$$

(ii) The reaction of aggregate macro variables to the structural monetary policy shock is given by the OLS coefficient

$$\begin{aligned} \beta_{IRF,c,\varepsilon}^h &= \frac{\mathbb{C}[c_{t+h}, \phi_v \varepsilon_t^v]}{\mathbb{V}(\phi_v \varepsilon_t^v)} = \frac{\beta_{yv} \rho_v^h + \sum_{j=1}^2 \beta_{y j v} \vartheta_{j v}^h}{\phi_v} \\ \beta_{IRF,\pi,\varepsilon}^h &= \frac{\mathbb{C}[\pi_{t+h}, \phi_v \varepsilon_t^v]}{\mathbb{V}(\phi_v \varepsilon_t^v)} = \frac{\beta_{\pi v} \rho_v^h + \sum_{j=1}^2 \beta_{\pi j v} \vartheta_{j v}^h}{\phi_v} \\ \beta_{IRF,i,\varepsilon}^h &= \frac{\mathbb{C}[i_{t+h}, \phi_v \varepsilon_t^v]}{\mathbb{V}(\phi_v \varepsilon_t^v)} = \phi \pi \beta_{IRF,\pi,\varepsilon}^h + \phi_y \beta_{IRF,c,\varepsilon}^h + \frac{\rho_v^h}{\phi_v} \end{aligned}$$

PROPOSITION A2. For group g and target k , let D_{kn} map the state associated with fundamental n into the object being forecast. The model-implied [Coibion and Gorodnichenko \(2015\)](#) coefficient is

$$(A.13) \quad \beta_{gk}^D = \frac{\sum_n \left(E_{gkn} V_{gn} R'_{gkn} + \sigma_n^2 e_{gkn} r_{gkn} \right)}{\sum_n \left(R_{gkn} V_{gn} R'_{gkn} + \sigma_n^2 r_{gkn}^2 \right)},$$

where the matrices on the right-hand side are defined in the proof below. For the quarterly SPF objects used in the text, the activity and inflation targets are $D_{yn} = C_{yn}(A_n - I)$ and $D_{\pi n} = C_{\pi n} A_n$, respectively. The annual-comparison metric is $\tilde{\beta}_{gk}^D = (\beta_{gk}^D)^{1/4}$. This is the reporting convention in equation (28); it is distinct from exact four-quarter aggregation.

PROOF. For each fundamental, collect the equilibrium state in $s_{nt} = (s_{nt}^\rho, s_{nt}^{\vartheta_1}, s_{nt}^{\vartheta_2})'$ so that

$$s_{nt} = A_n s_{n,t-1} + B \varepsilon_t^n, \quad a_{knt} = C_{kn} s_{nt}, \quad A_n = \text{diag}(\rho_n, \vartheta_{1n}, \vartheta_{2n}),$$

with $B = (1, 1, 1)'$. Let K_{gn} denote the steady-state Kalman gain, $H = (1, 0, 0)$, and $F_{gn} = (I - K_{gn}H)A_n$. Because idiosyncratic signal noise averages to zero within each group, the average posterior evolves as

$$\hat{s}_{gnt} = K_{gn}HA_n s_{n,t-1} + F_{gn}\hat{s}_{gn,t-1} + K_{gn}\varepsilon_t^n.$$

Thus, for $q_{gnt} = (s'_{nt}, \hat{s}'_{gnt})'$, define

$$\mathcal{A}_{gn} = \begin{bmatrix} A_n & 0 \\ K_{gn}HA_n & F_{gn} \end{bmatrix}, \quad \mathcal{B}_{gn} = \begin{bmatrix} B \\ K_{gn} \end{bmatrix},$$

and let V_{gn} be the unique stationary covariance satisfying

$$V_{gn} = \mathcal{A}_{gn}V_{gn}\mathcal{A}_{gn}' + \sigma_n^2\mathcal{B}_{gn}\mathcal{B}_{gn}'.$$

The average forecast of a fixed target is $D_{kn}\hat{s}_{gnt}$. Its revision between $t-1$ and t can therefore be written as

$$\Delta\bar{\mathbb{E}}_{gt}a_k^D = R_{gkn}q_{gn,t-1} + r_{gkn}\varepsilon_t^n, \quad R_{gkn} = D_{kn} \begin{bmatrix} K_{gn}HA_n & F_{gn} - A_n \end{bmatrix}, \quad r_{gkn} = D_{kn}K_{gn}.$$

The component of the forecast error correlated with this revision is

$$a_k^D - \bar{\mathbb{E}}_{gt}a_k^D = E_{gkn}q_{gn,t-1} + e_{gkn}\varepsilon_t^n + u_{k,t}, \quad E_{gkn} = D_{kn} \begin{bmatrix} F_{gn} & -F_{gn} \end{bmatrix}, \quad e_{gkn} = D_{kn}(B - K_{gn}),$$

where $u_{k,t}$ contains future innovations and is orthogonal to the time- t revision. Taking covariances, summing across independent fundamentals, and dividing by the revision variance gives equation (A.13). $\square$

Appendix C. Model Derivation

C.1. Households

Consider the budget constraint. Define $A_{it} = R_{t-1}B_{i,t-1}$ as consumer i 's initial asset position in period t . At $t+1$, equation (7) becomes

$$(A.14) \quad C_{it+1} + B_{it+1} = R_t B_{i,t} + W_{it+1}^r N_{it+1} + D_{it+1}$$

Combining (7) and (A.14) gives $C_{it} + (C_{it+1} + B_{it+1})R_t^{-1} = A_{it} + W_{it}^r N_{it} + D_{it} + (W_{it+1}^r N_{it+1} + D_{it+1})R_t^{-1}$. Iterating forward and imposing the transversality condition yields $\sum_{k=0}^{\infty} \prod_{j=1}^k \frac{1}{R_{t+j-1}} C_{it+k} = A_{it} + \sum_{k=0}^{\infty} \prod_{j=1}^k \frac{1}{R_{t+j-1}} (W_{it+k}^r N_{it+k} + D_{it+k})$. Log-linearizing this condition around a zero-inflation steady state gives

$$(A.15) \quad \sum_{k=0}^{\infty} \beta^k c_{it+k} = a_{it} + \Omega_i \sum_{k=0}^{\infty} \beta^k (w_{it+k}^r + n_{it+k}) + (1 - \Omega_i) \sum_{k=0}^{\infty} \beta^k d_{it+k}$$

where a lowercase letter denotes the log deviation from steady state, $x_t = \log X_t - \log X$, except for the initial asset position, defined as $a_{it} = A_{it}/C_i$. The term Ω_i denotes household i 's labor-income share.

We obtain household i 's optimal expenditure in period t as a function of current and expected future wages, dividends,

and real interest rates. Using (16) and taking expectations, we can rearrange (A.15) as

$$\begin{aligned}
\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} c_{it+k} &= a_{it} + \Omega_i \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} \left(\frac{1+\varphi}{\varphi} w_{it+k}^r - \frac{\sigma}{\varphi} c_{it+k} \right) + (1 - \Omega_i) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} d_{it+k} \\
(A.16) \quad &= \frac{\varphi}{\varphi + \sigma \Omega_i} a_{it} + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} \left[\frac{\Omega_i(1+\varphi)}{\varphi + \sigma \Omega_i} w_{it+k}^r + \frac{(1 - \Omega_i)\varphi}{\varphi + \sigma \Omega_i} d_{it+k} \right]
\end{aligned}$$

Taking individual expectations on the left-hand side gives $\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} c_{it+k}$. Rearranging (17) yields $\mathbb{E}_{it} c_{it+1} = c_{it} + \frac{1}{\sigma} \mathbb{E}_{it} r_t$. Iterating forward gives $\mathbb{E}_{it} c_{it+2} = c_{it} + \frac{1}{\sigma} \mathbb{E}_{it}(r_t + r_{t+1})$ and, for a general $k \geq 1$, $\mathbb{E}_{it} c_{it+k} = c_{it} + \frac{1}{\sigma} \sum_{j=0}^{k-1} \mathbb{E}_{it} r_{t+j}$. Therefore,

$$\sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} c_{it+k} = \sum_{k=0}^{\infty} \beta^k c_{it} + \frac{1}{\sigma} \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \beta^k \mathbb{E}_{it} r_{t+j} = \frac{1}{1-\beta} c_{it} + \frac{\beta}{\sigma(1-\beta)} \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} r_{t+k}$$

Inserting this last condition into (A.16), we can write

$$c_{it} = -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} r_{t+k} + \frac{\varphi(1-\beta)}{\varphi + \sigma \Omega} a_{it} + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_{it} \left[\frac{\Omega_i(1+\varphi)(1-\beta)}{\varphi + \sigma \Omega} w_{it+k}^r + \frac{(1 - \Omega_i)\varphi(1-\beta)}{\varphi + \sigma \Omega} d_{it+k} \right]$$

Aggregating and using the fact that assets are in zero net supply, $\int_{\mathcal{C}} a_{it} di = a_t = 0$,

$$(A.17) \quad c_t = -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \bar{\mathbb{E}}_t^h r_{t+k} + \sum_{k=0}^{\infty} \beta^k \left[\frac{\Omega(1+\varphi)(1-\beta)}{\varphi + \sigma \Omega} \bar{\mathbb{E}}_t^h w_{t+k}^r + \frac{(1-\Omega)\varphi(1-\beta)}{\varphi + \sigma \Omega} \bar{\mathbb{E}}_t^h d_{t+k} \right]$$

where $\bar{\mathbb{E}}_t^h(\cdot) = \int_{\mathcal{C}} \mathbb{E}_{it}(\cdot) di$ is the average household expectation operator in period t .

C.2. Firms

Aggregate Price Level Dynamics. As in the benchmark NK model, price rigidities take the form of a Calvo lottery. In each period, a firm can reset its price with probability $(1 - \theta)$, independently of the time since its previous price change. Thus, a measure $(1 - \theta)$ of firms resets prices in each period, and the average price duration is $1/(1 - \theta)$. In log-linear terms, aggregate price-level dynamics are

$$(A.18) \quad \pi_t = \int_{\mathcal{J}_f} \pi_{jt} dj = (1 - \theta) \left[\int_{\mathcal{J}_f} p_{jt}^* dj - p_{t-1} \right] = (1 - \theta) (p_t^* - p_{t-1})$$

Optimal Price Setting. A firm reoptimizing in period t chooses the price P_{jt}^* that maximizes the current market value of the profits generated while the price remains effective. Formally, $P_{jt}^* = \arg \max_{P_{jt}} \sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt} \left\{ \Lambda_{t,t+k} \frac{1}{P_{t+k}} [P_{jt} Y_{j,t+k|t} - \mathcal{C}_{t+k}(Y_{j,t+k|t})] \right\}$ subject to the sequence of demand schedules $Y_{j,t+k|t} = \left(\frac{P_{jt}}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$, where $\Lambda_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma}$ is the stochastic discount factor, $\mathcal{C}_t(\cdot)$ is the nominal cost function, and $Y_{j,t+k|t}$ denotes output in period $t+k$ for firm j , which last reset its price in period t . The first-order condition is $\sum_{k=0}^{\infty} \theta^k \mathbb{E}_{jt} \left[\Lambda_{t,t+k} Y_{j,t+k|t} \frac{1}{P_{t+k}} \left(P_{jt}^* - \mathcal{M}\Psi_{j,t+k|t} \right) \right] = 0$, where $\Psi_{j,t+k|t} \equiv \mathcal{C}'_{t+k}(Y_{j,t+k|t})$ denotes nominal marginal cost for

firm j , and $\mathcal{M} = \frac{\epsilon}{\epsilon-1}$. Log-linearizing around the zero-inflation steady state yields the familiar price-setting rule

$$(A.19) \quad p_{jt}^* = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_{jt} (\psi_{j,t+k|t} + \mu)$$

where $\psi_{j,t+k|t} = \log \Psi_{j,t+k|t}$ and $\mu = \log \mathcal{M}$.

C.3. Equilibrium

Goods-market clearing implies $Y_{jt} = C_{jt} = \int_{\mathcal{J}_c} C_{ijt} di$ for each variety j . Because assets are in zero net supply and there is no capital, investment, government consumption, or net exports, aggregation across varieties gives $\int_{\mathcal{J}_f} Y_{jt} dj = \int_{\mathcal{J}_c} \int_{\mathcal{J}_f} C_{ijt} dj di$, so $Y_t = C_t$. Aggregate labor demand equals aggregate labor supply, $N_t = \int_{\mathcal{J}_c} N_{it} di = \int_{\mathcal{J}_f} N_{jt} dj$. Using $Y_{jt} = A_t N_{jt}^{1-\alpha}$ and (6), $N_t = \int_{\mathcal{J}_f} \left(\frac{Y_{jt}}{A_t}\right)^{\frac{1}{1-\alpha}} dj = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_{\mathcal{J}_f} \left(\frac{P_{jt}}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} dj$. Log-linearizing yields $n_t = \frac{1}{1-\alpha}(y_t - a_t)$.

The (log) marginal cost for firm j at time $t+k|t$ is $\psi_{j,t+k|t} = w_{t+k} - m p n_{j,t+k|t} = w_{t+k} - [a_{t+k} - \alpha n_{j,t+k|t} + \log(1 - \alpha)]$, where $m p n_{j,t+k|t}$ and $n_{j,t+k|t}$ denote log marginal product of labor and log employment in period $t+k$ for a firm that last reset its price at time t , respectively. Let $\psi_t \equiv \int_{\mathcal{J}_f} \psi_{jt} dj$ denote log average marginal cost. We can then write $\psi_t = w_t - [a_t - \alpha n_t + \log(1 - \alpha)]$. Thus, the following relation holds:

$$(A.20) \quad \psi_{j,t+k|t} = \psi_{t+k} + \alpha(n_{j,t+k|t} - n_{t+k}) = \psi_{t+k} + \frac{\alpha}{1-\alpha}(y_{j,t+k|t} - y_{t+k}) = \psi_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_{jt}^* - p_{t+k})$$

Introducing (A.20) into (A.19), we can rewrite the firm price-setting condition as

$$(A.21) \quad p_{jt}^* = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \mathbb{E}_{jt} (p_{t+k} - \Theta \hat{\mu}_{t+k}),$$

where $\hat{\mu}_t = \mu_t - \mu$ is the deviation between the average and desired markups, $\mu_t = -(\psi_t - p_t)$, and $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

Individual and Aggregate Phillips curve. Note that we can write the deviation between average and desired markups as

$$\begin{aligned} \mu_t &= p_t - \psi_t = p_t - w_t + w_t - \psi_t = -(w_t - p_t) + w_t - [w_t - a_t + \alpha n_t - \log(1 - \alpha)] \\ &= -(\sigma y_t + \varphi n_t) + [a_t - \alpha n_t + \log(1 - \alpha)] = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t + \frac{1 + \varphi}{1 - \alpha} a_t + \log(1 - \alpha) \end{aligned}$$

As in the benchmark model, under flexible prices ($\theta = 0$) the average markup is constant and equal to the desired markup μ . Let y_t^n denote the natural level of output, defined as equilibrium output under flexible prices and FIRE. The markup condition in the natural equilibrium is $\mu = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) y_t^n + \frac{1 + \varphi}{1 - \alpha} a_t + \log(1 - \alpha)$, which can be written as $y_t^n = \psi a_t + \psi_y y$, where $\psi = \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha}$ and $\psi_y = -\frac{(1 - \alpha)[\mu - \log(1 - \alpha)]}{\sigma(1 - \alpha) + \varphi + \alpha}$. Therefore, $\hat{\mu}_t = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \tilde{y}_t$, where $\tilde{y}_t = y_t - y_t^n$ is the output gap. Substituting this expression into (A.21) yields the Phillips curve in (23).

Individual and Aggregate DIS curve. To derive the DIS curve, first log-linearize firm profits. Firm j earns $D_{jt} = \frac{1}{P_t} (P_{jt} Y_{jt} - W_t N_{jt}) = \frac{P_{jt}}{P_t} Y_{jt} - W_t^r N_{jt}$. Log-linearizing around a zero-inflation steady state gives $D_j d_{jt} = \frac{P_j}{P} Y_j (p_{jt} + y_{jt} -$

$p_t) - W^r N_j(w_t^r + n_{jt})$. Aggregating across firms yields

$$(A.22) \quad y_t = \frac{W^r N}{Y}(w_t^r + n_t) + \frac{D}{Y}d_t = \Omega(w_t^r + n_t) + (1 - \Omega)d_t$$

Aggregating the labor-supply condition (16) across households and imposing goods-market clearing gives $w_t^r = \sigma y_t + \varphi n_t$. Substituting this condition into (A.22) yields $y_t = \frac{\Omega(1+\varphi)}{\varphi+\Omega\sigma} w_t^r + \frac{(1-\Omega)\varphi}{\varphi+\Omega\sigma} d_t$. Substituting this expression into the aggregate consumption function (A.17) and again imposing goods-market clearing gives

$$(A.23) \quad y_t = -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t^h r_{t+k} + (1 - \beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t^h y_{t+k}$$

Subtracting the natural level of output from (A.23) gives the DIS curve:

$$(A.24) \quad \tilde{y}_t = -\frac{\beta}{\sigma} \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t^h (r_{t+k} - r_{t+k}^n) + (1 - \beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t^h \tilde{y}_{t+k}$$

In the natural equilibrium without price or information frictions, the natural real interest rate is

$$(A.25) \quad r_t^n = \sigma \mathbb{E}_t \Delta y_{t+1}^n = \sigma \psi \mathbb{E}_t \Delta a_{t+1} = \sigma \psi (\rho_a - 1) a_t$$