The Transmission of Foreign Shocks in a Networked Economy

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April 29, 2025

We develop a multi-country New Keynesian model with rich sectoral heterogeneity and both national and international production networks to capture recent supply-side shocks. Calibrated to major Euro-Area economies and their trade partners, our model analyzes international energy price shocks. We show that input–output linkages amplify inflation by creating feedback between selling prices and production costs. High trade integration propagates inflationary pressures across borders. Heterogeneous production structures generate diverse inflation responses, with integrated economies experiencing persistent inflation and downstream sectors facing sharp, transient spikes. Our findings emphasize the critical role of monetary policy—and the trade-offs it faces—in a networked economy.

Keywords: Open Economy, Input-Output, New Keynesian, Energy Prices.
JEL Classifications: E31, E32, E52, E70.

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We thank Ignacio Hidalgo for excellent research assistance. We thank Anton Nakov, Dmitry Mukhin, Tommaso Monacelli, Mishel Ghassibe, Philip R. Lane, Diego Comín, numerous seminar and workshop participants, and colleagues at the Banco de España and the Eurosystem for helpful comments and discussions. The views expressed are those of the authors and do not necessarily represent the views of the Banco de España, the European Central Bank, and the Eurosystem.

1. Introduction

In recent years, the global economy has experienced a series of supply-side shocks that have significantly disrupted inflation dynamics and macroeconomic stability. Examples of these are energy price shocks, often triggered by geopolitical events, and supply chain disruptions. Despite their different underlying causes, these shocks share a key characteristic: they originate in specific sectors but quickly spread through complex production networks and international supply chains, ultimately affecting the whole economy. As a result, understanding how these shocks are transmitted through input-output (IO) linkages and spillover across countries and sectors has become a central focus of recent macroeconomic research.

In this paper, we investigate the transmission of supply-side shocks through production networks and their impact on inflation dynamics through the lens of a multi-country New Keynesian model of the global economy with rich sectoral heterogeneity and national and international production networks. Calibrating the model to the main Euro-Area countries and their trade partners, we use the model to analyze the macroeconomic effects of shocks to international energy prices and their transmission.

Our results show that production networks, by generating a feedback loop between increasing selling prices and rising production costs, are key in shaping inflation dynamics in response to the international energy price shock.

First, we find that IO linkages significantly amplify the response and persistence of headline and core inflation. Namely, we find that the cumulative response of headline inflation would be up to 60% smaller and substantially shorter-lived if production networks were absent. Additionally, we show that high trade integration across European economies propagates inflationary pressures across borders via IO linkages, with the interaction between national and international networks generating a larger inflationary impact than either alone.

Second, heterogeneity in production structures gives rise to differential inflation dynamics across countries: countries with more upstream industries and longer production chains (e.g., Germany) exhibit larger amplification and more persistence. In contrast, inflation is shorter-lived in countries with more downstream-oriented production structures and less complex production networks (e.g., Spain).

Third, we explore the implications of our findings for the conduct of monetary policy. We show that a weaker systematic monetary policy response increases inflation volatility more with production networks conditional on these supply-side shocks, despite IO linkages and intermediate goods dampening the response of inflation to monetary policy shocks (Nakamura and Steinsson 2010; Rubbo 2023). Additionally, we show that the presence of production networks worsens monetary policy trade-offs: stabilizing inflation in the presence of amplified supply shocks requires a larger contraction in output. The trade-off is particularly worsened by international production networks.

More in detail, we consider a model of the global economy with K countries and I sectors or industries within each country, and incomplete international financial markets. Depending on the monetary regime in place, countries may form part of currency unions or may have monetary autonomy. Firms in each sector use domestic labor alongside imported and domestically produced intermediate goods, leading to national and international production networks. At the sector level, we incorporate a high degree of heterogeneity to match observed data on labor shares, IO shares, and exposure to domestic and international markets through IO linkages. In addition, we also allow for nominal rigidities in nominal wages (Erceg *et al.* 2000) and staggered price setting à la Calvo (1983). We assume that nominal wage inflation is common across sectors, but we allow each sector's price-frequency probability to be heterogeneous across countries and industries.

We use our framework to provide analytical insights into how production networks alter inflation dynamics in response to foreign shocks. We derive a multi-sector Phillips Curve that illustrates how foreign price wedges are amplified via domestic and international IO linkages, with inflation persistence increasing due to nominal rigidities and network feedback. Notably, we show that, unlike in one-sector models (Gali and Monacelli 2005), the presence of IO structures breaks the Divine Coincidence: stabilizing inflation comes at the cost of a non-zero output gap, underscoring the complex trade-offs central banks face in networked economies. We additionally find that the international dimension worsens the trade-off, compared to the closed economy framework (Rubbo 2023).

We calibrate the model to 44 sectors per country and 6 regions: the four largest Euro-Area countries (Germany, France, Italy, and Spain), the rest of the Euro Area, and the rest of the world. We make sure that the model replicates observed trade flows between different sectors and countries using IO tables from the OECD and Eurostat. In addition, we use micro-level CPI data from Gautier *et al.* (2024) to calibrate the heterogeneous price-frequency adjustment probabilities across sectors and countries, allowing the model to capture the varying degrees of price rigidity observed in the data.

We next examine the effects and propagation of an increase in the international price of imported energy paid by European firms. We assume that this increase is driven by an exogenous wedge between the price charged by foreign exporting firms and the price paid by domestic importing firms (see, for example, Baqaee and Farhi 2024). This assumption aligns with the notion that, as seen in the recent energy crisis, fluctuations in international energy prices are often triggered by geopolitical rather than macroeconomic events.

In response to the increase in energy prices, we find that while Euro-Area headline inflation initially spikes due to the surge in energy prices, it declines very gradually over time, with core inflation becoming the primary driver of headline inflation, rather than energy prices. That is, we obtain a significant pass-through from headline to core inflation, which notably increases the persistence of inflationary pressures. Specifically, our results show that, on impact, core inflation increases by approximately 20% of the increase in headline inflation, consistent with previous empirical findings (Adolfsen *et al.* 2024). The increase in energy prices induces energy goods to become more expensive for households and energy production costs for firms to increase. As a result, firms respond by increasing the prices of their products. Therefore, through the production network, the costs of imported and domestically produced goods for firms increase further, leading to an additional increase in prices. This feedback between increasing selling prices and rising production costs results in a generalized increase in core and headline inflation.

The interaction between price rigidities and IO linkages adds more persistence to inflation dynamics. With staggered price-setting, the average selling price also incorporates the individual prices of those firms that have updated prices yet, in addition to those of updating firms. Next, in our model, a key component of firms' marginal costs is the price of the intermediate goods they purchase, and through the production network, also the costs faced by their suppliers. As a result, the stickiness and lagged adjustments in these prices are transmitted through firms' marginal costs and selling prices, ultimately amplifying and prolonging inflationary pressures.

We more formally quantify and isolate the role that production networks play through a series of counterfactuals. Namely, we consider three counterfactual economies where we sequentially turn off domestic, international, and national and international production networks.[1]

We find that without national and international production networks, cumulative headline inflation would be roughly up to 60% of our baseline, which includes a fully fledged production network structure. In particular, we find that despite headline inflation rising similarly on impact – driven by the rise of energy prices – it stabilizes and dies out much faster when production networks are absent, in line with the intuition provided above.

We further isolate the role played by national and international production networks separately. On the one hand, we find that the IO network contributes significantly to inflation persistence. Due to the high level of integration between industrial sectors across European economies, there are substantial spillovers from the shock, transmitted through cross-country links captured in the IO tables. On the other hand, it is crucial to account for both national and international production networks simultaneously. Specifically, the joint effect of these two dimensions is greater than the sum of their individual impacts in isolation. Intuitively, higher domestic inflation leads to increased export prices, which raises inflation abroad. In turn, increasing inflation abroad leads to higher import prices, further amplifying domestic

[1] In all these counterfactual economies we always keep energy as a production input for firms.

inflation.

We obtain that the increase in energy prices results in heterogeneous inflation developments across countries. For example, headline inflation increases sharply in Spain but it dissipates quickly. On the other hand, Germany shows the opposite dynamics. Namely, inflation in the German economy increases less than in Spain, but it displays more persistent dynamics. As a result, cumulative inflation in Germany surpasses that in Spain over time. The presence of heterogeneous production structures and consumption baskets can also rationalize this finding. That is, the energy share in the consumption baskets of Spanish households is greater than in Germany, which explains the initial heterogeneous inflation spike. However, the size of the IO network in Germany is significantly larger, meaning that the amplification effects previously described arise more strongly in the German economy, leading to a more persistent and over time larger response of inflation.

Finally, we analyze the implications of production networks and our findings for the conduct of monetary policy. Previous research has highlighted that the presence of intermediate goods and IO linkages leads to a higher degree of monetary non-neutrality in response to monetary policy shocks (Basu 1995; Nakamura and Steinsson 2010; Christiano 2016; Rubbo 2023). Namely, inflation tends to move less and output more in response to a monetary policy shock, as stickiness in the price of intermediate goods adds inertia to firms' marginal costs. Our model replicates these findings, with the presence of production networks dampening the response of inflation to a monetary policy shock.

We find that the systematic response of monetary policy becomes more relevant with IO linkages in the presence of energy price shocks, despite monetary policy shocks having a smaller effect on inflation. Namely, simulating the economy through a time series of shocks to the international price of energy, we find that a weaker systematic monetary policy response increases inflation volatility more when production networks are accounted for. Since international energy price shocks are amplified through the production network, a more passive monetary response allows the feedback between selling prices and production costs to build up further, resulting in a larger increase in inflation volatility. This finding shows that the implications of IO linkages for monetary policy are subtle: even though they may reduce the inflation effect of a given change in interest rates, such a monetary response becomes more relevant when it arrests the propagation of shocks that the production network severely amplifies.

Furthermore, and related to our previous result, we show that production networks exacerbate the trade-offs faced by monetary policy. Since IO linkages amplify and prolong the propagation of supply-side shocks—increasing inflation volatility when monetary policy is passive—stabilizing inflation in this environment requires larger output adjustments, worsening the policy trade-off relative to models without network amplification. We also find that the trade-off is further amplified by the international dimension of production networks. This underscores the heightened importance of a systematic monetary policy response in networked economies.

Related literature. Our paper contributes to several strands of the literature, at the intersection of the macroeconomic effects of production networks and the propagation of international macroeconomic shocks.

The seminal works of Acemoglu *et al.* (2012), Gabaix (2011), and Baqaee and Farhi (2019) study the propagation of granular shocks in production networks under flexible prices, abstracting from their inflationary effects. Building on these contributions, Pasten *et al.* (2020) and Rubbo (2023) incorporate IO linkages into frameworks with nominal rigidities. However, both papers focus on closed economies and thus cannot address international shocks, cross-country heterogeneity, and spillovers. In contrast, our paper analyzes these dynamics in an open economy, explicitly accounting for international trade and production network spillovers.

Earlier research examined the transmission of monetary policy in closed economies with simpler roundabout production structures. Nakamura and Steinsson (2010) develop a menu-cost model with heterogeneous sectoral nominal rigidities, showing that intermediate goods amplify monetary non-neutrality. While we replicate this finding, we also show that with supply-side shocks, the systematic component of monetary policy has a greater influence when IO linkages are present. Huang (2006) and Huang and Liu (2004) highlight that nominal rigidities and intermediate goods increase inflation persistence. Our model extends this by showing that a fully-fledged IO structure further amplifies persistence, beyond what would occur in a simpler roundabout economy.

Our paper also contributes to the growing literature that incorporates IO linkages in open economy models. Baqaee and Farhi (2024) study the propagation of shocks in an open economy model with production networks, but with a limited role for nominal price rigidities and monetary policy. Comin *et al.* (2023) develop a more tractable small open economy model with nominal rigidities, but focus on potentially binding capacity constraints. Ernst *et al.* (2023) consider a multi-country environmental model with flexible prices to study carbon taxes and climate clubs. Finally, Andrade *et al.* (2023) developed a 3-sector small open economy à la Gali and Monacelli (2005) to study the propagation of productivity shocks. Relative to this paper, and beyond focusing on energy price shocks, the quantitative nature of our model allows us, for example, to be able to uncover cross-country heterogeneity arising from diverse production structures.

Finally, we contribute to the literature exploring the transmission of energy shocks in macroeconomic models. An earlier contribution is Bodenstein *et al.* (2008), which studied optimal monetary policy in a closed-economy model with an energy sector. Gagliardone and Gertler (2023) explore the origins of the inflation surge in the US using a closed economy new Keynesian framework with oil, and find that oil price shocks were a key determinant. Auclert *et al.* (2023), Chan *et al.* (2024), and Bayer *et al.* (2023) explore the consequences of the recent energy crisis in open economy models with household heterogeneity, while the focus of the current paper is on the consequences of heterogeneity in the production sectors and across countries.

Roadmap. The paper proceeds as follows. Section 2 presents the international input-output New Keynesian framework. In Section 3 we inspect analytically the transmission mechanism of foreign price shocks. Later, in Section 4, we describe the model calibration, and we derive the main results. Section 5 concludes the paper.

2. General Model

We consider a world economy composed of K countries, indexed by k. The core of our model is a production structure characterized by national and international production networks through input-output (IO) linkages. Namely, each country is comprised of I production sectors, possibly heterogeneous within and between countries. Within each sector, there is a unit mass of monopolistically competitive firms indexed by $f \in (0, 1)$ that produce using labor and intermediate goods produced by other domestic and foreign sectors. In addition, we allow for heterogeneous nominal price rigidities at the sectoral and country levels.

In this section, we describe the general environment of the model. We then specialize it to an analytically tractable version in Section 3, which we use to derive intuition for the transmission mechanisms of foreign shocks via production networks and its implications for monetary policy. In Section 4, we consider a quantitative version of the model, where we calibrate it to the data and assess the macroeconomic consequences of shocks to the international price of energy in a networked economy.

2.1. Households

There is a representative household in each country k that derives utility from consumption and disutility from labor according to the following per-period utility function:

$$U_t = U(C_{k,t}) - N_{k,t}^{1+\varphi}/(1+\varphi), \tag{1}$$

where $N_{k,t}$ is aggregate labor supplied by the household, $U(C_{k,t})$ is a constant-relative-risk-aversion utility function over aggregate consumption, $C_{k,t}$, and φ denotes the inverse of the

Frisch elasticity.

Aggregate consumption is defined as a constant-returns-to-scale (CRS) composite of sectoral consumption, each of which is itself a CRS aggregation of country-specific consumption goods. This nested structure is summarized by

$$C_{k,t} = \mathcal{C}_k\left(\{C_{k,i,t}\}_{i=1}^{I}\right) \quad \text{and} \quad C_{k,i,t} = \mathcal{C}_{k,i}\left(\{C_{k,l,i,t}\}_{l=1}^{K}\right), \tag{2}$$

where $C_{k,i,t}$ is consumption of sector i at time t, and $C_{k,l,i,t}$ is country's k household's consumption of good produced by industry i in country l. Finally, we let $C_{k,l,i,t}$ be a Dixit and Stiglitz (1977) aggregator over differentiated goods produced by firms in sector i in country l:

$$C_{k,l,i,t} = \left(\int_0^1 C_{k,l,i,f,t}^{(\epsilon_{p,ki}-1)/\epsilon_{p,ki}} df\right)^{\epsilon_{p,ki}/(\epsilon_{p,ki}-1)} \tag{3}$$

where $\epsilon_{p,ki}$ denotes the sectoral constant elasticity of substitution between good varieties.

The household faces the following per-period budget constraint:

$$P_{C,k,t}C_{k,t} + \frac{B_{k,t}}{1+i_t} + \mathcal{E}_{k,K,t}\sum_{h\in\mathcal{H}} Q_t^h B_{k,t}^h \leq W_{k,t}N_{k,t} + \mathcal{E}_{k,K,t}\sum_{h\in\mathcal{H}}(Q_t^h + D_t^h)B_{k,t-1}^h + B_{k,t-1} + \Pi_{k,t} - T_{k,t} \tag{4}$$

where $W_{k,t}$ is the nominal wage received by the household, $\Pi_{k,t}$ are profits of domestic firms, and $T_{k,t}$ is a lump-sum tax or transfer. $P_{C,k,t}$ is the consumer price index in country k, implied by the consumption aggregators (2).

There are two sets of assets in the economy. The first of them, $B_{k,t}$, is a one-period nominal bond only traded domestically that pays a gross nominal interest rate $1+i_t$, set by the monetary authority. The second one is a set $\mathcal{H}$ of bonds traded internationally at price Q_t^h, with dividend payouts D_t^h.[2] These bonds are denominated in the currency of country K, and hence $\mathcal{E}_{k,K,t}$ is the bilateral nominal exchange rate between country k and country K.

Under the previous notation, we have the following first-order condition for labor supply:

$$N_{k,t}^{\varphi} = U'(C_{k,t})W_{k,t}/P_{C,k,t}, \tag{5}$$

and for the allocation of consumption across sectors, countries, and differentiated goods:

$$\frac{\partial\mathcal{C}_{k,i,t}}{\partial C_{k,i,t}} = \frac{P_{C,k,i,t}}{P_{C,k,t}}, \quad \frac{\partial\mathcal{C}_{k,i}}{\partial C_{k,l,i,t}} = \frac{(1+\tau_{k,l,i,t})P_{k,l,i,t}}{P_{C,k,i,t}}, \quad \text{and} \quad C_{k,l,i,f,t} = \left(\frac{P_{k,l,f,i,t}}{P_{k,l,i,t}}\right)^{-\epsilon_{p,ki}} C_{k,l,i,t}, \tag{6}$$

[2] This specification follows Egorov and Mukhin (2023). Note if $\mathcal{S}$ is the set of possible states of nature, we have that financial markets are complete if $\mathcal{H} = \mathcal{S}$. On the other hand, by setting $\mathcal{H} = 1$ we have the usual specification of incomplete financial markets with a single bond traded internationally.

where $P_{C,k,i,t}$ denotes the consumer price index of sector i faced by the household in country k, and $P_{k,l,i,t} \equiv \left(\int_0^1 P_{k,l,i,f,t}^{(\epsilon_{p,ki}-1)/\epsilon_{p,ki}} df \right)^{\epsilon_{p,ki}/(\epsilon_{p,ki}-1)}$ is the price in the currency of country k of the good produced by sector i in country l.[3] [4]

Our source of foreign shocks arises from $\tau_{k,l,i,t}$. In particular, the term $\tau_{k,l,i,t}$ is an exogenous *price wedge* between the price set by industry i in country l exporting to country k, $P_{k,l,i,t}$, and the actual price paid by domestic agents, $(1+\tau_{k,l,i,t})P_{k,l,i,t}$. Our motivation for these price wedges is to have a source of exogenous movements in import prices that are not necessarily triggered by changes in the economic activity of the exporting country.[5] These are reminiscent of, for example, energy price shocks arising as a consequence of geopolitical tensions, or even supply-chain disruptions as the ones witnessed after the COVID-19 pandemic. More in general, one can think of these wedges as part of the transmission mechanism of any foreign shock that leads to changes in international goods' prices.

Finally, the first-order condition for households' savings in domestic bonds results in the following Euler equation:

$$U'(C_{k,t}) = \beta \mathbb{E}_t \left[U'(C_{k,t+1})(1+i_t)/(1+\pi_{C,k,t+1}) \right], \tag{7}$$

where $1+\pi_{C,k,t+1} \equiv \frac{P_{C,k,t+1}}{P_{C,k,t}}$ is the gross consumer price inflation in country k.

2.2. Firms

There are I industries in each economy, indexed by $i \in \{1, 2, \ldots, I\}$, and within each industry there is a unit mass of firms.

Production. Each firm f, in sector i and country k, produces a differentiated good $Y_{k,i,f,t}$ using a CRS production function $F_{k,i}$ using labor $N_{k,i,f,t}$ and a basket of intermediate goods $X_{k,i,f,t}$ as inputs:

$$Y_{k,i,f,t} = F_{k,i}(N_{k,i,f,t}, X_{k,i,f,t}). \tag{8}$$

The bundle of intermediate goods $X_{k,i,f,t}$ is defined similar to the household's consump-

[3]Under the CRS assumptions, we have that $P_{C,k,t}C_{k,t} = \sum_{i=1}^{I} P_{C,k,i,t}C_{k,i,t}$ and $P_{C,k,i,t}C_{k,i,t} = \sum_{l=1}^{K} P_{k,l,i,t}C_{k,l,i,t}$.

[4]These theoretical price wedges are similar to import tariffs, since both elements distort international good prices faced by domestic agents. However, a key difference is that while the domestic government typically collects tariffs, and hence do not suppose a direct wealth transfer across countries, this is not the case in our specification. Namely, as it will be more apparent once we write the aggregate resource constraint, the price wedges here will generate a direct wealth loss for the importing country and a wealth gain for the exporting country.

[5]The price wedges are akin to the iceberg trade costs present in the trade literature (Baqaee and Farhi 2024).

tion basket, given by a CRS aggregator of sectoral intermediate goods, which are themselves defined as a CRS aggregator of country-specific intermediate goods:

$$X_{k,i,f,t} = \mathcal{X}_{k,i}\left(\{X_{k,i,j,f,t}\}_{j=1}^{I}\right) \quad \text{and} \quad X_{k,i,j,f,t} = \mathcal{X}_{k,i,j}\left(\{X_{k,l,i,j,f,t}\}_{l=1}^{K}\right), \tag{9}$$

where $X_{k,i,j,f,t}$ is firm's f demand in sector i of country k for goods produced in sector j, and $X_{k,l,i,j,f,t}$ is firm's f demand in sector i of country k for goods produced in sector j in country l. $X_{k,l,i,j,f,t}$ is itself a Dixit-Stiglitz aggregator over differentiated goods produced by firms in sector j in country l:

$$X_{k,l,i,j,f,t} = \left(\int_0^1 X_{k,l,i,j,f,f',t}^{(\epsilon_{p,kj}-1)/\epsilon_{p,kj}} df'\right)^{\epsilon_{p,ki}/(\epsilon_{p,ki}-1)} \tag{10}$$

Cost minimization by firms delivers the following first-order conditions for labor and intermediate goods demands:

$$W_{k,t} = \mathrm{MC}_{k,i,t}\frac{\partial F_{k,i}}{\partial N_{k,i,f,t}}, \quad P_{X,i,j,t} = \mathrm{MC}_{k,i,t}\frac{\partial F_{k,i}}{\partial X_{k,i,j,f,t}} \tag{11}$$

and the allocation of intermediate goods demand across sectors and countries:

(12)

$$\frac{\partial \mathcal{X}_{k,i}}{\partial X_{k,i,j,f,t}} = \frac{P_{X,i,j,t}}{P_{X,i,t}}, \quad \frac{\partial \mathcal{X}_{k,i,j}}{\partial X_{k,l,i,j,f,t}} = \frac{(1+\tau_{k,l,j,t})P_{k,l,j,t}}{P_{X,i,j,t}}, \quad X_{k,l,i,j,f,t} = \left(\frac{P_{l,j,t}}{P_{X,i,j,t}}\right)^{-\epsilon_{p,kj}} X_{k,i,j,t}$$

Above, $P_{X,i,t}$ denotes the price index of the intermediate input bundle $X_{k,i,f,t}$ faced by firms in sector i in country k, and $P_{X,i,j,t}$ is the price index of the sectoral intermediate input $X_{k,i,j,f,t}$ faced by firms in sector i in country k. As in the case of households, the prices faced by domestic industries are subject to price wedges $\tau_{k,l,j,t}$.[6]

Nominal marginal costs in sector i of country k are denoted by $\mathrm{MC}_{k,i,t}$. Note that, under CRS, all firms in a given sector choose the same combination of inputs. Hence, $\mathrm{MC}_{k,i,t}$ is common across firms and given by:

$$\mathrm{MC}_{k,i,t} = \min_{N_{k,i,t}, X_{k,l,i,j,t}} W_{k,i,t}N_{k,i,t} + \sum_{j=1}^{I}\sum_{l=1}^{K}(1+\tau_{k,l,j,t})P_{k,l,j,t}X_{k,l,i,j,t} \tag{13}$$

Price Setting. Firms set prices in a staggered manner (Calvo 1983). Specifically, firms in sector i of country k can reset their price with probability $1-\theta_{k,i}$ each period. Note that the

[6]As in the household case, our specification of the intermediate input aggregators implies that $P_{X,i,t}X_{k,i,f,t} = \sum_{j=1}^{I} P_{X,i,j,t}X_{k,i,j,f,t}$ and $P_{X,i,j,t}X_{k,i,j,f,t} = \sum_{l=1}^{K} P_{l,j,t}X_{k,l,i,j,f,t}$.

probability of price adjustment is country- and sector-specific.

We entertain two possibilities for the pricing decisions to foreign markets: producer currency pricing (PCP) or local currency pricing (LCP). Under PCP, the firm sets its export price in the domestic currency, and hence the selling price to the domestic and foreign market coincide. A firm that is resetting its price, chooses its optimal selling price to the domestic market $P^*_{k,k,i,t}$ by solving the following problem:

$$\max_{P^*_{k,k,i,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \mathrm{SDF}_{t,t+s} \theta^s_{k,i} \left(P^*_{k,k,i,t+s} - (1-\tau_{k,i}) \mathrm{MC}_{k,i,t+s} \right) \mathcal{D}_{k,k,i,t+s}, \tag{14}$$

where $\mathrm{SDF}_{t,t+s} = \beta^s U'(C_{k,t+s})/U'(C_{k,t})$ is the stochastic discount factor between periods t and $t+s$, and $\mathcal{D}_{k,k,i,t+s}$ denotes the demand for the firm's good from domestic agents given in equations (6) and (12). We further allow for having a country-sector specific production subsidy $\tau_{k,i}$.[7]

Under LCP, the firm sets its export price to country l in the currency of country l, denoted by $P^{l,*}_{k,l,i,t}$, solving the following problem:

$$\max_{P^{l,*}_{k,l,i,t}} \mathbb{E}_t \sum_{s=0}^{\infty} \mathrm{SDF}_{t,t+s} \theta^s_{k,i} \left(\mathcal{E}_{k,l,t+s} P^{l,*}_{k,l,i,t+s} - (1-\tau_{k,i}) \mathrm{MC}_{k,i,t+s} \right) \mathcal{D}_{l,k,i,t+s}, \tag{15}$$

where $\mathcal{D}_{l,k,i,t+s}$ is the demand for the firm's production from agents from country l.

2.3. Government

The government is composed of a fiscal authority and a central bank. The fiscal authority issues domestic debt $B_{k,t}$, provides production subsidies to firms, and collects lump-sum taxes $T_{k,t}$ from the household in order to balance its budget constraint:

$$\frac{B_{k,t}}{1+i_{k,t}} + T_{k,t} = B_{k,t-1} + \sum_{i=1}^{I} \tau_{k,i} \mathrm{MC}_{k,i,t} Y_{k,i,t} \tag{16}$$

where $Y_{k,i,t}$ denotes the production of sector i in country k.

The central bank sets the nominal interest rate, $i_{k,t}$. Since the specific monetary policy rule is inconsequential for our analytical results in Section 3, we defer its definition to Section 4.

[7]Production subsidies will be useful to derive our analytical results in section 3, as they allow us to eliminate steady-state distortions arising from monopolistic competition (Galí 2015).

2.4. Market Clearing and GDP

Market clearing in the goods market requires that the quantity produced of each good matches the quantity demanded at home and abroad, either for direct consumption or intermediate use. That is,

$$Y_{ki,t} = \sum_{l=1}^{K} C_{lki,t} + \sum_{l=1}^{K} \sum_{j=1}^{I} X_{lkji,t}. \tag{17}$$

Market clearing in the labor market requires that the aggregate labor supplied matches the sum of labor demand across sectors, for each country. That is,

$$N_{k,t} = \sum_{i=1}^{I} N_{ki,t}. \tag{18}$$

Finally, the aggregate resource constraint of the economy requires that the net foreign position of country k equals its trade balance:

$$\sum_{h\in\mathcal{H}} Q_t^h B_{k,t}^h - \sum_{h\in\mathcal{H}} (Q_t^h + D_t^h) B_{k,t-1}^h = P_{k\text{EXP},t}\text{EXP}_{k,t} - P_{k\text{IMP},t}\text{IMP}_{k,t} \tag{19}$$

where the first term in the right-hand side of (19) is the total nominal exports of country k,

$$P_{k\text{EXP},t}\text{EXP}_{k,t} = \sum_{l\neq k}^{K} \sum_{i=1}^{I} \left((1+\tau_{l,k,i,t}) P_{lki,t}^{k} C_{kli,t} + \sum_{j=1}^{I} (1+\tau_{l,k,i,t}) P_{lki,t}^{k} X_{lkji,t} \right), \tag{20}$$

where $P_{lki,t}^{k} = \mathcal{E}_{kl,t} P_{lki,t}$, and the second term is the total nominal imports,

$$P_{k\text{IMP},t}\text{IMP}_{k,t} = \sum_{l\neq k}^{K} \sum_{i\in I} \left((1+\tau_{k,l,i,t}) P_{kli,t} C_{kli,t} + \sum_{j=1}^{I} (1+\tau_{k,l,i,t}) P_{kli,t} X_{klji,t} \right). \tag{21}$$

Nominal GDP is defined as the sum of total household consumption and nominal net exports,

$$\mathcal{Y}_{k,t} = P_{kC,t} C_{k,t} + P_{k\text{EXP},t}\text{EXP}_{k,t} - P_{k\text{IMP},t}\text{IMP}_{k,t} \tag{22}$$

Defining the GDP deflator, $P_{kY,t}$, as the ratio between nominal GDP measured using time–t

prices and nominal GDP measured using steady-state prices

$$P_{kY,t} = \frac{P_{kC,t}C_{k,t} + P_{k\text{EXP},t}\text{EXP}_{k,t} - P_{k\text{IMP},t}\text{IMP}_{k,t}}{P_{kC}C_{k,t} + P_{k\text{EXP}}\text{EXP}_{k,t} - P_{k\text{IMP}}\text{IMP}_{k,t}},$$

we have that real GDP is given by:

$$Y_{k,t} = \mathcal{Y}_{k,t}/P_{kY,t}. \tag{23}$$

3. Analytical Insights

In order to derive the intuition behind the transmission of foreign shocks in an open economy with production networks, and the monetary policy implications therein, we consider a simplified version of the model outlined in Section 2. The following assumptions will not only render the model analytically tractable but they will also allow us to cleanly nest familiar cases in the literature, serving as useful benchmarks to highlight the novel mechanisms presented in this paper.

Assumptions. First, we assume that there are only two countries $K = 2$, with the home country being small relative to the foreign country. To ease notation, we will denote foreign variables with an asterisk, $*$. Under the small open economy assumption, we consider that aggregate variables in the foreign economy remain constant at their steady-state values. Furthermore, we focus on the case where the preferences and the technology are symmetric across countries.

Second, we make the following functional assumptions about preferences and technology. Regarding preferences, we assume that utility (1) is logarithmic in consumption:

$$U(C_t) = \log C_t. \tag{24}$$

In addition, we assume that the functional form of all consumption (2) and intermediate (9) goods aggregators are Cobb and Douglas (1928):

$$\mathcal{C}(\{C_{i,t}\}_{i=1}^{I}) = \prod_{i=1}^{I} C_{i,t}^{\beta_i}, \quad \text{and} \quad \mathcal{C}_i(\{C_{Hi,t}, C_{Fi,t}\}) = C_{H,i,t}^{1-\zeta_i} C_{F,i,t}^{\zeta_i}, \tag{25}$$

$$\mathcal{X}_i(\{X_{i,j,t}\}_{i=1}^{I}) = \prod_{i=1}^{I} X_{i,j,t}^{\gamma_i}, \quad \text{and} \quad \mathcal{X}_{i,j}(\{X_{H,i,j,t}, C_{F,i,j,t}\}) = X_{H,i,j,t}^{1-\zeta_{i,j}} X_{F,i,j,t}^{\zeta_{i,j}}, \tag{26}$$

as well as a unitary elasticity of substitution between labor and intermediate goods in the production function (8):

$$Y_{i,f,t} = N_{i,f,t}^{\alpha_i} X_{i,f,t}^{1-\alpha_i}. \tag{27}$$

Third, we assume that international financial markets are complete. This, together with log utility (24), implies the following risk-sharing condition holds:

$$C_t = Q_t C_t^*, \tag{28}$$

where Q_t is the real exchange rate given by:

$$Q_t = \mathcal{E}_t P_t^* / P_{C,t}. \tag{29}$$

Finally, regarding pricing we make the following assumptions. On the one hand, we assume PCP, so that the law of one price holds:

$$P_{F,i,t} = \mathcal{E}_t P_{i,t}^*, \tag{30}$$

where $P_{F,i,t}$ is the price of sectoral good i produced in the ROW, denominated in domestic currency, and $P_{i,t}^*$ is the same good's price denominated in the ROW currency. On the other hand, we also assume that the following set of production sectoral subsidies are in place: $\tau_i = 1/\epsilon_i$. The sectoral subsidy removes steady-state distortions arising from monopolistic competition, and ensures that the flexible price allocation is efficient.

3.1. Log-linearized equilibrium conditions

Under the previous set of assumptions, we derive a log-linear approximation of the model around a symmetric steady state with zero inflation. In what follows we focus on the set of equilibrium conditions of the domestic economy that guide our discussion on the transmission of foreign shocks in the domestic economy and its implications for monetary policy. Regarding notation, we will use small case letters to denote deviations of a variable from steady state. By way of example, $y_t = \frac{Y_t - Y}{Y}$ is the deviation of domestic real GDP from its steady-state level Y. Furthermore, we use bold letters and symbols (e.g., $\boldsymbol{x}$, $\boldsymbol{\Omega}$) to denote matrices and vectors throughout the text.

We first obtain the set of sectoral phillips curves in the domestic economy, by log-linearizing the first order conditions associated with (14):

$$\boldsymbol{\pi}_{H,t} = \kappa(\mathbf{mc}_t - \boldsymbol{p}_{H,t}) + \beta \mathbb{E}_t \boldsymbol{\pi}_{H,t+1}, \tag{31}$$

where $\boldsymbol{\pi}_{H,t} = [\pi_{H,1,t}, \ldots, \pi_{H,I,t}]^\top$ is a vector containing the domestic sectoral inflation rates, $\mathbf{mc}_t = [\mathrm{mc}_{1,t}, \ldots, \mathrm{mc}_{I,t}]^\top$ is the vector containing the sectoral nominal marginal costs and $\boldsymbol{p}_{H,t} = [p_{H,1,t}, \ldots, p_{H,I,t}]^\top$ contains domestic sectoral prices. The matrix $\boldsymbol{\kappa}$ is a $I \times I$ diagonal matrix containing the slopes of sectoral phillips curves.

The sectoral phillips curves (31) are reminiscent of the aggregate phillips curve present in one-sector small open economy frameworks (Gali and Monacelli 2005). However, note that heterogeneous price frequency adjustments deliver sector-specific phillips curve slopes, contained in $\boldsymbol{\kappa}$, which will generally deliver sectoral-specific prices, $\boldsymbol{p}_{H,t}$, and inflation rates, $\boldsymbol{\pi}_{H,t}$.

Moreover, note that differences in production structures across sectors, and in particular input-output linkages, do not affect directly (31). However, these features will be a key determinant shaping nominal marginal costs across sectors, $\mathbf{mc}_t$. To see this, we log-linearize the solution to the minimization problem (13) to obtain the following expression for the sectoral nominal marginal costs:

$$\mathbf{mc}_t = \boldsymbol{\alpha} w_t + \boldsymbol{\Omega}_H \, \boldsymbol{p}_{H,t} + \boldsymbol{\Omega}_F \left(\boldsymbol{\tau}_t + \boldsymbol{p}_{F,t} \right), \tag{32}$$

where $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_I]^\top$ denotes the vector of sectoral labor shares, $\boldsymbol{\Omega}_H$ is the IxI input-output matrix relative to domestic producers with elements $\boldsymbol{\Omega}_H(ij) = \frac{P_{H,j} X_{H,i,j}}{P_{H,i} Y_{H,i}}$, $\boldsymbol{\Omega}_F$ is the IxI input-output matrix relative to foreign producers with elements $\boldsymbol{\Omega}_F(ij) = \frac{P_{F,j} X_{F,i,j}}{P_{H,i} Y_{H,i}}$. $\boldsymbol{\tau}_t = [\tau_{1,t}, \ldots, \tau_{I,t}]^\top$ is the vector of price wedges faced by domestic agents, and $\boldsymbol{p}_{F,t} = [p_{F,1,t}, \ldots, p_{F,I,t}]^\top$ is the vector of foreign sectoral prices in domestic currency, that is $p_{F,i,t} = e_t + p^*_{i,t}$.[8]

The expression for marginal costs (32) helps to build intuition on how foreign price shocks propagate through the production network. When the price of imported input i increases through a price wedge $\tau_{i,t}$, its impact on domestic sectors varies according to their direct usage of that input, which is captured by the i-th column of the input-output matrix $\boldsymbol{\Omega}_F$.

This initial shock then triggers additional general equilibrium effects. First, domestic firms adjust their prices $\boldsymbol{p}_{H,t}$ in response to their higher costs, affecting other domestic producers through input-output linkages captured by $\boldsymbol{\Omega}_H$. Second, the exchange rate e_t responds to the shock, influencing the domestic-currency prices of all foreign goods $\boldsymbol{p}_{F,t}$ and creating an additional cost pressure on firms that use imported inputs. Third, changes in economic activity will also affect marginal costs through movements in wages, w_t.

For later reference, it will be useful to rewrite expression (32) in the following terms:

$$\mathbf{mc}_t = \boldsymbol{\alpha} w_t + \boldsymbol{\Omega} \, \boldsymbol{p}_{H,t} + \boldsymbol{\Omega}_F \boldsymbol{s}_t, \tag{33}$$

[8]The expression for $\boldsymbol{p}_{F,i,t}$ follows from log-linearizing the law of one price condition (30).

where $\boldsymbol{\Omega} = \boldsymbol{\Omega}_H + \boldsymbol{\Omega}_F$ is the total input-output matrix, and $\boldsymbol{s}_t = [s_{1,t}, \ldots, s_{I,t}]^\top$ is the vector containing the sectoral terms of trade, given by:

$$\boldsymbol{s}_t = \boldsymbol{\tau}_t + \boldsymbol{p}_{F,t} - \boldsymbol{p}_{H,t} \tag{34}$$

Let us now build intuition on how wages, and hence marginal costs, are affected. We start by considering the log-linearized version of the labor supply condition (5):

$$w_t - p_{C,t} = c_t + \varphi n_t = c_t + \varphi y_t, \tag{35}$$

where the second equality uses that aggregate hours worked, n_t, equal aggregate real value added, y_t, in the absence of physical capital or TFP. Next, in order to obtain an expression for consumption, c_t, we rely on a log-linear approximation of GDP (23), which in Appendix B we show to be given by:

$$(1 + \varphi\omega_X)\, y_t = c_t + (\boldsymbol{\nu}_F^\top + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F)\boldsymbol{s}_t - \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \boldsymbol{n}_t \tag{36}$$

The above equation shows that real GDP is affected by three components: aggregate consumption, sectoral terms of trade $\boldsymbol{s}_t$, and sectoral employment levels $\boldsymbol{n}_t = [n_{1,t}, \ldots, n_{I,t}]^\top$. The last two components capture movements in the trade balance of the domestic economy, resulting from aggregating sector-specific trade imbalances.

3.2. Sectoral Phillips Curves

In this section, we explore the implications and relevance of domestic and foreign production networks for the transmission mechanism of foreign price-wedge shocks to domestic inflation in the small open economy.

The Small Open Economy Phillips Curves. Under the previous set of assumptions, the domestic sectoral Phillips curves can be written as[9]

$$\boldsymbol{\pi}_{H,t} = \mathcal{B}\,(1 + \varphi + \varphi\omega_X)\,\widetilde{y}_t + \mathcal{B}\boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \widetilde{\boldsymbol{n}}_t + \mathcal{V}(\boldsymbol{I} - \boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F \widetilde{\boldsymbol{s}}_t - \mathcal{V}\boldsymbol{\chi}_t + \beta(\boldsymbol{I} - \mathcal{V})\mathbb{E}_t \boldsymbol{\pi}_{H,t+1} \tag{37}$$

where $\boldsymbol{\pi}_{H,t} = \boldsymbol{p}_{H,t} - \boldsymbol{p}_{H,t-1}$ denotes the vector of sectoral and domestic inflation rates and $\boldsymbol{p}_{H,t} = \begin{bmatrix} p_{H,1,t} & \cdots & p_{H,I,t} \end{bmatrix}^\top$ is a vector of sectoral domestic (log) prices, $\widetilde{y}_t$ denotes the output gap—computed as the log-deviation from the natural output prevailing under price flexibility—$\widetilde{\boldsymbol{n}}_t = \begin{bmatrix} \widetilde{n}_{1,t} & \ldots & \widetilde{n}_{I,t} \end{bmatrix}^\top$ denotes the vector of sectoral employment gaps, $\widetilde{\boldsymbol{s}}_t = \begin{bmatrix} \widetilde{s}_{1,t} & \ldots & \widetilde{s}_{I,t} \end{bmatrix}^\top$

[9] We relegate the derivation to Appendix B.2.

denotes the vector of sectoral terms of trade gaps, with $s_{i,t} = p_{Fi,t} - p_{Hi,t}$, and the vector $\chi_t \equiv \boldsymbol{p}_{H,t-1} - (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1}\boldsymbol{\Omega}_F\boldsymbol{\tau}_t$ denotes the wedge between lagged domestic prices $\boldsymbol{p}_{H,t-1}$ and the efficient foreign prices, with $\boldsymbol{\tau}_t = \begin{bmatrix}\tau_{1,t} & \dots & \tau_{I,t}\end{bmatrix}^\top$.

The matrices $\mathcal{B} = \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\alpha}/[1 - \boldsymbol{\beta}\boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}\boldsymbol{\alpha}]$ and $\mathcal{V} = \left\{\boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1} - \mathcal{B}[\boldsymbol{\lambda}^\top - \boldsymbol{\beta}^\top\boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega}\boldsymbol{\Delta})^{-1}]\right\}(\boldsymbol{I} - \boldsymbol{\Omega})$ exhibit the same structure to their equivalent version in Rubbo (2023), incorporating the open economy dimension. Denote by $\boldsymbol{\lambda}^\top = \boldsymbol{\beta}^\top(\boldsymbol{I} - \boldsymbol{\Omega})$ the vector of steady-state Domar weights, where the consumption share vector is given by $\boldsymbol{\beta} = \begin{bmatrix}\beta_1 & \beta_2 & \dots & \beta_I\end{bmatrix}^\top = \boldsymbol{\beta}_H + \boldsymbol{\beta}_F$ with elements $\beta_i = P_iC_i/(P_CC)$, and the input-output matrix is given by $\boldsymbol{\Omega} = \boldsymbol{\Omega}_H + \boldsymbol{\Omega}_F$,

$$\boldsymbol{\Omega}_H = \begin{bmatrix} \omega_{H11} & \omega_{H12} & \dots & \omega_{H1I} \\ \omega_{H21} & \omega_{H22} & \dots & \omega_{H2I} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{HI1} & \omega_{HI2} & \dots & \omega_{HII} \end{bmatrix}, \quad \boldsymbol{\Omega}_F = \begin{bmatrix} \omega_{F11} & \omega_{F12} & \dots & \omega_{F1I} \\ \omega_{F21} & \omega_{F22} & \dots & \omega_{F2I} \\ \vdots & \vdots & \ddots & \vdots \\ \omega_{FI1} & \omega_{FI2} & \dots & \omega_{FII} \end{bmatrix},$$

with elements $\omega_{Hij} = (P_jX_j^H)/(P_iY_i)$ and $\omega_{Fij} = (P_jX_j^F)/(P_iY_i)$ that denote the domestic and foreign input-output shares. Finally, $\boldsymbol{\alpha} = \begin{bmatrix}\alpha_1 & \alpha_2 & \dots & \alpha_I\end{bmatrix}^\top$ denotes the vector of sectoral labor shares $\alpha_i = WN_i/(P_iY_i)$, $\boldsymbol{K} = \text{diag}\begin{pmatrix}\kappa_1 & \kappa_2 & \dots & \kappa_I\end{pmatrix}$ with $\kappa_i = (1-\beta\theta_i)(1-\theta_i)/\theta_i$, and $\boldsymbol{\Delta} = (\boldsymbol{I} + \boldsymbol{K})^{-1}\boldsymbol{K}$, and $\boldsymbol{I}$ denotes the identity matrix.

Transmission Mechanism. Inspecting the Phillips curves (37), we observe that the direct effect of foreign price shocks $\boldsymbol{\tau}_t$ on domestic inflation, measured by $\boldsymbol{\Omega}_F$, is amplified through the domestic Leontief inverse $(\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1}$.

Notice that, in the small open economy framework, the direct impact of foreign price disturbances does not depend on nominal price rigidities. Intuitively, this result hinges on the small open economy assumption, where dynamics in the foreign Phillips curves do not affect the domestic economy.[10]

Nominal price rigidities play a role in the reaction of domestic sectoral prices. After an increase in foreign prices, the sectoral terms of trade gap will experience a heterogeneous reaction. On the one hand, sectors directly exposed to the foreign price increase will experience a negative terms of trade gap. To see this, we can write the sectoral terms of trade gap as $\tilde{s}_{it} = (p_{Fi,t} - p^n_{Fi,t}) - (p_{Hi,t} - p^n_{Hi,t})$. Given that prices are flexible in the natural equilibrium, after an increase in foreign prices the first term becomes negative, and the second term is negligible compared to the first term. On the other hand, sectors that are not directly affected by the foreign price increase will experience a positive terms of trade gap, since the second

[10]This finding vanishes in the more general open economy framework discussed in Section 4.

term is negative, and the first term becomes negligible.

The sectoral employment gaps will be negative after the increase in foreign prices, since production becomes relatively more expensive, which in turn leads firms to reduce production and labor demand. Unless the rise in foreign prices shifts the demand towards the sector in the domestic economy, and outweighs the previous channel, generating positive employment gaps.

Furthermore, the term $\omega_X = \sum_{i=1}^{I}\sum_{j=1}^{I} P_j X_{Fji}/(P_C C)$, which captures the expenditure on foreign inputs as a share of GDP, amplifies the pass-through of movements in the output gap to domestic prices, compared to the closed-economy framework.

Comparison to the One-Sector Small Open Economy. Compared to the one-sector small open economy benchmark (Gali and Monacelli 2005), production networks amplify the indirect effects of foreign price-wedge shocks. To see this, consider a simplification of our multi-sectoral framework, in which we set the number of sectors to $\boldsymbol{I} = 1$. In such case, the Phillips curve (37) collapses to

$$\pi_{H,t} = \kappa(1+\varphi)\widetilde{y}_t + \beta\mathbb{E}_t\pi_{H,t+1} + \omega_F\tau_t, \tag{38}$$

where $\kappa = (1-\beta\theta)(1-\theta)/\theta$ denotes the slope of the Phillips curve, and ω_F captures the exposition of the domestic economy to the foreign price-wedge shock.

The one-sector framework does not feature the amplification of direct effects through the domestic inverse Leontief matrix. Intuitively, the Leontief inverse captures how small and granular shocks can trigger a ripple effect throughout the entire production network. A minor change in one sector gets amplified as it passes along the supply chain, resulting in a much larger overall impact on production. As a result, the one-sector framework fails to capture the amplification of foreign price-wedge shocks.

An additional difference with the multi-sector economy is that, in the absence of supply disturbances, a monetary policy action that targets zero inflation simultaneously closes the output gap in the one-sector benchmark—the Divine Coincidence (Blanchard and Galí 2007). Setting $\pi_{H,t} = 0\ \forall t$ in (38) gives as a result $\widetilde{y}_t = 0$, and the central bank does not face any trade-off between stabilizing inflation and stabilizing the output gap. In contrast, targeting a given domestic inflation index $\boldsymbol{\Phi}^\top\boldsymbol{\pi}_{H,t} = 0$ in the multi-sector framework (37) results in

$$0 = \boldsymbol{\Phi}^\top\mathcal{B}\left(1+\varphi+\varphi\omega_X\right)\widetilde{y}_t + \boldsymbol{\Phi}^\top\mathcal{B}\boldsymbol{\lambda}^\top\boldsymbol{\Omega}_F\widetilde{\boldsymbol{n}}_t + \boldsymbol{\Phi}^\top\mathcal{V}(\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t - \boldsymbol{\Phi}^\top\mathcal{V}\boldsymbol{\chi}_t - \beta\boldsymbol{\Phi}^\top\mathcal{V}\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}.$$

Closing a domestic inflation index does not generally close the output gap. Intuitively, with one policy tool, the monetary authority cannot jointly close the I sectoral inflation rates and the output gap.

However, whether the resulting output gap is positive or negative will depend both on the location of the shock in the IO network and the price flexibility in the sectors experiencing the foreign inflation pressure. In Section 4.4 we make use of the quantitative model and show that, in order to stabilize headline inflation in the presence of foreign energy price shocks, the monetary authority must engineer a larger drop in the output gap, compared to the one-sector economy. That is, the presence of production networks worsens the trade-off faced by the monetary authority.

Comparison to the Multi-Sector Closed Economy. In the multi-sector and closed-economy framework (Rubbo 2023), the Phillips curve is collapsed to $\boldsymbol{\pi}_t = \mathcal{B}(1+\varphi)\widetilde{\boldsymbol{y}}_t - \mathcal{V}\chi_t + \beta(1-\mathcal{V})\mathbb{E}_t\boldsymbol{\pi}_{t+1}$, where $\boldsymbol{\beta} = \boldsymbol{\beta}_H$ and $\boldsymbol{\Omega} = \boldsymbol{\Omega}_H$, but we still allow for the presence of foreign price shocks.

Incorporating the international dimension introduces two new elements, compared to the closed economy. The domestic Phillips curves are extended to accommodate the employment gap $\widetilde{\boldsymbol{n}}_t$ and the terms of trade gap $\widetilde{\boldsymbol{s}}_t$. These two terms arise from the difference between the ROW consumption of domestic goods and the domestic consumption of foreign goods—explained through differentials in sectoral terms of trade—and the difference between the purchases from ROW of domestic inputs and the domestic purchases of foreign inputs—explained through differentials in the terms of trade and the domestic employment.

Notice that the closed-economy framework is also consistent with a failure of the Divine Coincidence. That is, such failure is directly attributable to the multi-sector extension. However, it remains an open question whether the open economy extension worsens the failure of the Divine Coincidence, amplifying the trade-off faced by the monetary authority in the closed economy. Compared to the closed economy, whether the inflation stabilization is more costly will ultimately depend on the relative changes of the sectoral employment gaps and terms of trade gaps, and on the larger pass-through of output gap changes to inflation ($\omega_X > 0$).

We study this quantitatively in Section 4.4. We find that, compared to the closed-economy framework, the trade-off faced by the monetary authority is worsened in the open economy. However, the difference between the two theoretical frameworks is not as stark as in the comparison with the one-sector economy.

4. Quantitative Analysis

A key determinant of the recent inflation surge in the Euro Area has been the increasing energy prices (Arce *et al.* 2024). Motivated by this, we next use our model to explore the aggregate effects of a shock to the price of imported energy paid by European firms.

For this purpose, we extend the analytical framework presented in Section 3 on several dimensions. First, we allow for an arbitrary $K \geq 2$ number of countries. Second, we allow for the presence of nominal wage rigidities, heterogeneous at the country level. Third, given our focus on energy price shocks, we model in detail the elasticity of substitution between energy and non-energy goods by introducing an additional layer for both households' and firms' baskets. Lastly, we specify the monetary stance in each economy, allowing for the presence of monetary unions.

4.1. Theoretical Model

After having derived intuition for the main transmission mechanisms and implications of foreign shocks in the previous simplified framework, we now turn to a quantitative analysis introducing features commonly present in workhorse New Keynesian open economy models. Motivated by the recent energy crisis, we will focus here on the effects of international energy price shocks. We highlight next the main differences relative to the simpler environment discussed in Section 3, and relegate to Appendix A the full characterization of the first-order and equilibrium conditions of the quantitative model.

4.1.1. Assumptions

First, as specified in the general environment in Section 2, we consider a $K \geq 2$ number of countries, which are potentially different in their preferences and technologies. More in particular, we no longer restrict utility (1) to be logarithmic: $U(C_{k,t}) = C_{k,t}^{1-\sigma}/(1-\sigma)$.

We consider more general CES production function (8), without imposing a unitary elasticity of substitution:

$$Y_{fki,t} = \left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{fki,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{fki,t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}. \tag{39}$$

Similarly, we assume a CES structure for the consumption (2) and intermediate (9) good aggregators. Given the quantitative focus on energy shocks, we introduce an additional layer to distinguish between energy and non-energy goods,[11]

$$C_{k,t} = \left[\widetilde{\beta}_{k}^{\frac{1}{\gamma}} C_{kE,t}^{\frac{\gamma-1}{\gamma}} + \left(1-\widetilde{\beta}_{k}\right)^{\frac{1}{\gamma}} C_{kM,t}^{\frac{\gamma-1}{\gamma}}\right]^{\frac{\gamma}{\gamma-1}}, \quad X_{ki,t} = \left[\widetilde{\beta}_{ki}^{\frac{1}{\phi}} X_{kiE,t}^{\frac{\phi-1}{\phi}} + (1-\widetilde{\beta}_{ki})^{\frac{1}{\phi}} X_{kiM,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{40}$$

[11]This specification allows us to introduce a specific elasticity of substitution of the energy consumption that does not necessarily need to be equal to the elasticity of substitution between the rest of goods and services.

where $C_{kE,t}$ and $C_{kM,t}$ denote the consumption of energy and non-energy goods, respectively, and $X_{kiE,t}$ and $X_{kiM,t}$ sector i intermediate goods' demand for energy and non-energy goods, respectively. These are given by:

$$(41) \qquad C_{kE,t} = \left[\sum_{i \in I_E} \tilde{\nu}_{ki}^{\frac{1}{\eta}} C_{ki,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \qquad C_{kM,t} = \left[\sum_{i \in I_M} \tilde{\upsilon}_{ki}^{\frac{1}{\iota}} C_{ki,t}^{\frac{\iota-1}{\iota}} \right]^{\frac{\iota}{\iota-1}},$$

$$(42) \qquad X_{kiE,t} = \left[\sum_{j \in I_E} \tilde{\nu}_{kij}^{\frac{1}{\chi}} X_{kij,t}^{\frac{\chi-1}{\chi}} \right]^{\frac{\chi}{\chi-1}}, \qquad X_{kiM,t} = \left[\sum_{j \in I_M} \tilde{\upsilon}_{kij}^{\frac{1}{\xi}} X_{kij,t}^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi}{\xi-1}},$$

where I_E and I_M denote the sets of sectors producing energy and non-energy goods, respectively. The aggregation over sectoral goods produced in different countries is given by CES aggregators:

$$(43) \qquad C_{ki,t} = \left[\sum_{l=1}^{K} \tilde{\zeta}_{kl\,i}^{\frac{1}{\delta}} C_{kl\,i,t}^{\frac{\delta-1}{\delta}} \right]^{\frac{\delta}{\delta-1}} \quad \text{and} \quad X_{kij,t} = \left[\sum_{j=1}^{I} \tilde{\zeta}_{kl\,ij}^{\frac{1}{\mu}} X_{kl\,ij,t}^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}.$$

Second, we consider the case of incomplete financial markets, where only one international bond is traded, denominated in country K's currency. This means that an analogous risk-sharing condition of the type (28) no longer holds.

Third, regarding the price-setting structure we implement the following features. We dispense with the assumption on production sectoral subsidies $\tau_{k,i}^{p}$, allowing for positive profits at steady state. In addition, we adopt the LCP paradigm (Devereux and Engel 2003), meaning that a law of one price similar to (30) no longer holds. We assume that the (log-)price wedge, introduced in (6), follows an AR(2) process:

$$(44) \qquad \tau_{l\,kj,t} = \rho_{1,l\,kj}^{\tau} \tau_{l\,kj,t-1} + \rho_{2,l\,kj}^{\tau} \tau_{l\,kj,t-2} + \varepsilon_{l\,kj,t}^{\tau}$$

where $\varepsilon_{l\,kj,t}^{\tau} \sim \mathcal{N}\left(0, (\sigma_{l\,kj}^{\tau})^2\right)$. Finally, we also allow for nominal wage rigidities (Erceg *et al.* 2000), heterogeneous across countries but homogeneous across sectors.

4.1.2. Monetary Authority

There is a monetary authority in each country $k \in K$. In terms of the monetary stance, we differentiate between those countries that belong to a monetary union and those countries that are member states of currency unions.

Non-members of Currency Unions. Each central bank follows a Taylor rule:

$$i_{k,t} = \rho_{kr} i_{k,t-1} + (1 - \rho_{kr}) \left(\phi_{k\pi} \pi_{k\phi,t} + \phi_{ky} \widehat{y}_{k,t} \right) + \varepsilon^r_{k,t} \tag{45}$$

where the coefficients are allowed to vary by country; ρ_{kr} denotes the degree of interest rate smoothing in the monetary instrument, coefficients $\{\phi_{k\pi}, \phi_{ky}\}$ modulate the elasticity of the policy rate with respect to changes in a given inflation index $\pi_{k\phi,t}$, and output $\widehat{y}_{k,t}$, measured as the log-deviation from its steady-state value. The Taylor rule features a monetary policy shock, which follows $\varepsilon^r_{k,t} \sim \mathcal{N}\left(0, (\sigma_{kr})^2\right)$.

Furthermore, we allow the monetary authority to choose the particular inflation measure (CPI, PCE, PPI, GDP deflator, etc.) that they aim to stabilize, $\pi_{k\phi,t} = \mathbf{\Phi}^\intercal \boldsymbol{\pi}_{k,t} = \sum_{l=1}^{K} \sum_{i=1}^{I} \phi_{kli} \pi_{kli,t}$, where $\sum_{l=1}^{K} \sum_{i=1}^{I} \phi_{kli} = 1$, and $\pi_{kki,t} = \pi_{ki,t}$. For example, when ϕ_{kli} is equal to the consumption share of sector i in the country k, then the central bank targets headline inflation.

Members of Currency Unions. Suppose that a subset $K^{MU} \subset K$ of countries belongs to a monetary union. Without loss of generality, we assume that the central bank of a country $k^{MU} \in K^{MU}$ sets the nominal interest rate to stabilize the *union-wide* price inflation index and output deviations, π^{MU}_t and $\widehat{y}^{MU}_t$,[12]

$$i_{k^{MU},t} = \rho^{\text{MU}r}_{k^{\text{MU}}} i_{k^{MU},t-1} + \left(1 - \rho^{\text{MU}r}_{k^{\text{MU}}}\right) \left(\phi_{\text{MU}\pi} \pi^{\text{MU}}_t + \phi_{\text{MU}y} \widehat{y}^{\text{MU}}_t \right) + \varepsilon^r_{\text{MU},t} \tag{46}$$

where π^{MU}_t and $\widehat{y}^{MU}_t$ are defined as the GDP-weighted sum of member states' price inflation and output deviations, $\pi^{\text{MU}}_t = \sum_{k=1}^{K} \phi^{\text{MU}}_k \pi_{k\phi,t}$ and $\widehat{y}^{\text{MU}}_t = \sum_{k=1}^{K} \phi^{\text{MU}}_k \widehat{y}_{k,t}$, where $\phi^{\text{MU}}_k = \mathcal{Y}_k / \sum_{l=1}^{K^{\text{MU}}} \mathcal{Y}_l$ is the measure of the (steady-state) relative size of country k in the monetary union in terms of nominal GDP.

The central banks in the rest of countries $l \neq k^{MU}$ that belong to the monetary union adopt a peg vis-a-vis the country k^{MU} that sets the monetary stance:

$$\mathcal{E}_{k,k^{MU},t} = \mathcal{E}_{k,k^{MU}} \quad \forall k \in K^{MU} \tag{47}$$

where $\mathcal{E}_{k,k^{MU}}$ is the bilateral nominal exchange rate in steady state.

4.2. Model Calibration

We calibrate the model economy presented in Section 4.1 at the quarterly frequency to $K = 6$ countries: Spain, France, Italy, Germany, the Rest of the EA (REA), and the Rest of the World

[12] The specific location of the union-wide central bank is innocuous as long as it targets union-wide variables.

(ROW). The production structure within each country contains $I = 44$ sectors.[13] We next discuss the calibration strategy and collect in Table 1 the main parameter values and the corresponding targets or sources.

Households. We set the household's discount factor β to 0.99, to target an annual real interest rate of 4.5%. The intertemporal elasticity of substitution σ is set to 1, a common value in the literature. The inverse of the Frisch elasticity φ is set to 1, in line with the estimates presented in Chetty *et al.* (2011). Households' borrowing premium γ_* is set to 0.001 so that the evolution of net foreign assets has only a small impact on the exchange rate and trade in the short run while guaranteeing that the net foreign asset position is stabilized at zero in the long run (Schmitt-Grohe and Uribe 2003).

The elasticity of substitution in consumption between energy and non-energy goods γ is set to 0.4 following Böhringer and Rivers (2021). The elasticity of substitution in consumption between energy sources η and between non-energy sectors ι is set to 0.9 following Atalay (2017). Household's trade elasticity δ is set to 1.[14]

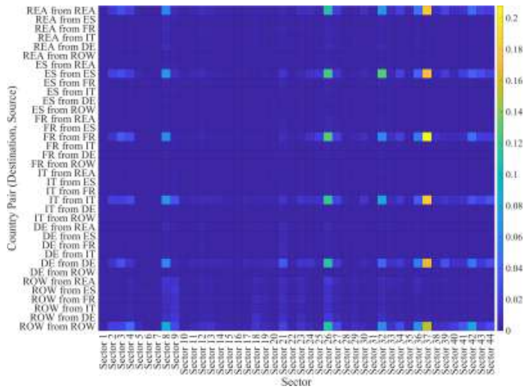
To calibrate the quasi-consumption shares $\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kl\,i}\}$ we rely on the linearized model to target the respective consumption sectoral consumption shares in each country. More precisely, in Appendix A we show that once the model has been linearized, it is possible to read directly consumption shares from the data as long as we have as many quasi-consumption shares parameters as data targets. Implementing this strategy, we obtain consumption shares by country from Inter-country Input Output (ICIO) tables produced by the OECD, using 2019 as our baseline period. Figure 1A reports a heatmap of the consumption share $\beta_{kl\,i} = P_{ki}C_{kl\,i}/(P_{kC}C_k)$, where each element denotes the consumption share of sector i of country l in households' basket of country k.

Regarding wage rigidities, ECB (2009) report limited cross-sectoral heterogeneity in wage frequency adjustments for Euro-Area countries. Therefore, we fix the Calvo frequency wage adjustment probability θ^w_k to 0.75 for all countries, in line with the evidence presented in Christoffel *et al.* (2008) for the EA.

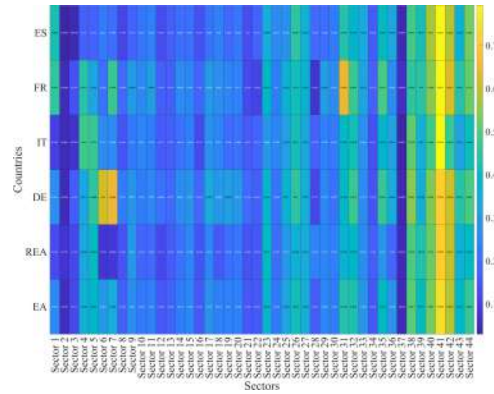
Production. The elasticity of substitution in production between labor and intermediate inputs ψ is set to 0.5 (Atalay 2017). The elasticity of substitution in production between energy and non-energy goods ϕ is set to 0.4 (Böhringer and Rivers 2021). The elasticity of substitution in production between energy sectors χ and between non-energy sectors ξ is set to 0.2,

[13] A detailed list of the sectors included in the analysis can be found on Table D.1, in Appendix D.

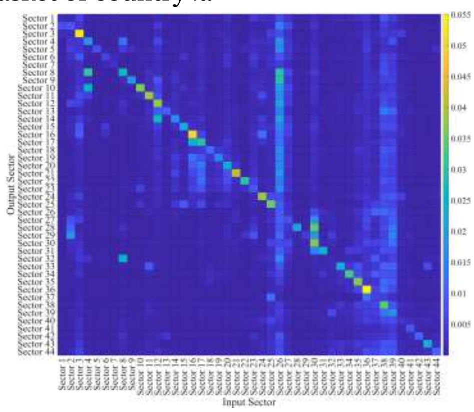
[14] A growing body of literature has estimated the value of these elasticities for different time horizons, finding that the values of trade elasticities are significantly greater than one in the long term but not in the short term, with values around 1 for horizons of up to two years (Boehm *et al.* (2023)). Given that the focus of our work is closer to a cyclical analysis rather than a long-term one, we choose the value of 1.



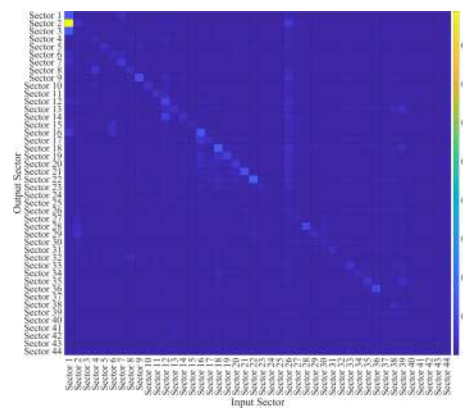
A. Heatmap of the Consumption matrix, where element β_{kli} denote the consumption share of sector i of country l in households' basket of country k.



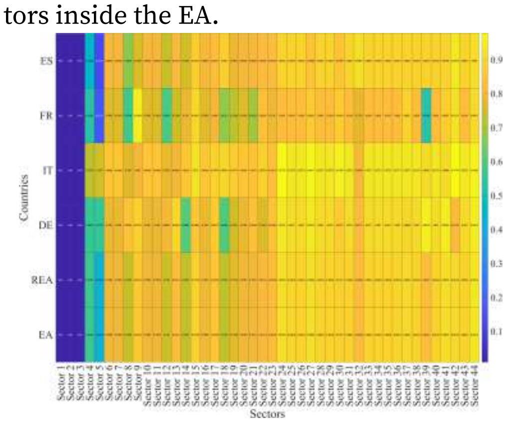
B. Heatmap of the Labor matrix, where element α_{ki} denotes the labor share of sector i in country k.



C. Heatmap of Home EA Input-Output matrix, where element ω_{ij} denote the input share of sector j for output sector i, both sectors inside the EA.



D. Heatmap of Foreign EA Input-Output matrix, where element ω_{ij} denotes the input share of sector j from ROW for output sector i inside EA.



E. Heatmap of the Calvo pricing rigidities matrix, where element θ^{p}_{ki} denotes the Calvo rigidity of sector i in country k.

FIGURE 1. Sectoral Heterogeneity on Consumption Shares, Labor Shares, Input-Output Network, and Nominal Price Rigidities

Notes: Panel 1A: heatmap of the consumption share. Panel 1B: heatmap of the labor share. Panel 1C: heatmap of the home input-output matrix of the EA. Panel 1D: heatmap of the foreign input-output matrix of the EA. Panel 1E: heatmap of Calvo rigidities.

Parameter	Description	Value	Target / Source
Households			
β	Discount factor	0.99	$R = 4.5\%$ p.a.
σ	Inv. Intertemp. Elast. Subs.	1	Standard Value
φ	Inv. Frisch Elasticity	1	Chetty *et al.* (2011)
γ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
η	Elast. Subst. E	0.9	Atalay (2017)
ι	Elast. Subst. M	0.9	Atalay (2017)
δ	Trade Elasticity	1	Standard value
$\{\widetilde{\beta}_k, \widetilde{\nu}_{ki}, \widetilde{\upsilon}_{ki}, \widetilde{\zeta}_{kl\,i}\}$	Quasi-shares consumption		ICIO tables (OECD)
θ^w_k	Calvo wage prob.	0.75	Christoffel *et al.* (2008)
Firms			
ψ	Elast. Subst. N and X	0.5	Atalay (2017)
ϕ	Elast. Subst. E and M	0.4	Böhringer and Rivers (2021)
χ	Elast. Subst. M	0.2	Atalay (2017)
ξ	Elast. Subst. E	0.2	Atalay (2017)
μ	Trade Elasticity	1	Standard value
$\{\widetilde{\alpha}_{ki}, \widetilde{\vartheta}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$	Quasi-shares production		ICIO tables (OECD)
$\mathcal{M}_{ki}$	Markups		Labor shares (Eurostat)
θ^p_{ki}	Calvo price prob.		Gautier *et al.* (2024)
Monetary Policy			
$\rho_{k,r}$	Interest Rate Smoothing	0.7	Standard Value
$\phi_{k,\pi}$	Reaction to Inflation	1.5	Galí (2015)
$\phi_{k,y}$	Reaction to real GDP	0.125	Galí (2015)
Exogenous Shock Process			
$\rho^\tau_{1,kl\,i}$	Persistence price wedge shock	1.17	Brent crude oil
$\rho^\tau_{2,kl\,i}$	Persistence price wedge shock	-0.2	Brent crude oil
$\sigma^\tau_{kl\,i}$	Std. Dev. price wedge shock	1	Standard Value
σ^r_k	Std. Dev. monetary shock	1	Standard Value

Notes: List of calibrated parameters. See the main text for a discussion on targets, values, and data used.

TABLE 1. Calibration

following the estimates of Atalay (2017). Finally, as with households, we set the trade elasticity for firms μ, equal to one.

We follow the same strategy as with households to calibrate the quasi-shares in production $\{\widetilde{\alpha}_{ki}, \widetilde{\beta}_{ki}, \widetilde{\nu}_{kij}, \widetilde{\upsilon}_{kij}, \widetilde{\zeta}_{kij}\}$. Namely, using the linearized model around the steady-state we directly read from the data shares in of each intermediate good in production as well as the shares of labor and production in total costs. Our data source here again is the 2019 ICIO tables from OECD. Figure 1C reports a heatmap of the home input-output matrix of the EA, $\omega_{kkij} = P_{kj}X_{kkij}/(P_{ki}Y_{ki})$, where each element denotes the input share of sector j for output sector i, both sectors inside the EA. Similarly, figure 1D reports a heatmap of the foreign input-output matrix of the EA, $\omega_{kl\,ij} = P_{l\,j}X_{kl\,ij}/(P_{ki}Y_{ki})$ for $l \neq k$, where each element denotes the input share of sector j from ROW for output sector i inside EA. We report in Appendix D the equivalent graphs for each country separately, in Figure D.1.

We complement the ICIO tables with the Figaro database by Eurostat to calibrate the labor share of each industry. Namely, once the quasi-shares in production have been used, we calibrate the sector-specific markups $\mathcal{M}_{ki}$ to target the wage-bill-over-sales observed in the data. Figure 1B reports a heatmap of the labor share $\alpha_{ki} = W_k N_{ki}/(P_{ki}Y_{ki})$, where each element denotes the labor share of sector i in country k.

Sectoral price rigidities are obtained from the PRISMA project conducted by the ECB (Gautier *et al.* 2024). Using CPI micro-data from several EA countries, the authors report the frequency of price adjustment by COICOP categories for each country separately, and from the aggregate EA. Using the COICOP–to–NACE correspondence tables (Kouvavas *et al.* 2021), we compute the frequency of price adjustment by each NACE category in each country, and obtain the heterogeneous price rigidities θ^p_{ki} for Spain, France, Italy, Germany and REA. Finally, we assume that the ROW price rigidities coincide with the aggregate REA price rigidities.

A drawback of the evidence presented in Gautier *et al.* (2024) is that it does not contain consistent price adjustment frequency data on energy goods. Therefore, we complement this with the evidence presented in Dhyne *et al.* (2006) on price adjustments for energy goods for Euro-Area countries. In line with the data presented there, and not surprisingly, energy sectors in the model have the steepest price Phillips Curves, with nearly fully flexible prices. Figure 1E reports a heatmap of the pricing rigidities θ^p_{ki}, where each element denotes nominal price-setting rigidity of sector i in country k.

Monetary Policy. All Taylor rule parameters are set to standard values, and are homogeneous across countries. The interest-rate smoothing coefficient ρ_{rk} is set to 0.7. The coefficients for inflation and output, $\phi_{\pi k}$ and ϕ_{yk}, are set to their standard values of 1.5 and 0.125, respectively. Furthermore, we assume that central banks target headline inflation.

Exogenous Process. We fit the persistence coefficients of the energy price shock to the time-series data of the Brent crude oil. The variance of the innovation is set to 1. Lastly, the variance of the monetary policy shock is also set to 1.

4.3. Results

In this section, we first analyze the dynamics of EA variables, with a focus on how these dynamics are shaped by IO linkages. Second, we analyze the contribution of production networks to inflation dynamics through a series of counterfactuals. Third, we explore how heterogeneity in production structures gives rise to differential inflation dynamics across countries.

The energy price shock we analyze is structured as follows. In both the model and the data, the energy mining sector of the ROW extracts the main energy products.[15] These energy goods are then sold to EA firms that primarily belong to the energy sectors Coke and petrol refining and Electricity sector.[16] After being processed by these sectors, energy goods are then supplied to households as consumption goods, and to the remaining sectors of the economy as energy intermediate goods used in the production process.

In line with the previous reasoning, we consider a 10% increase in the price wedge τ_{klj} between the price charged by the energy mining sector located RoW and the price paid by EA firms. Formally, we set $k = \{ES, DE, FR, IT, REA\}$, l = ROW and j = energy mining.

4.3.1. The Macroeconomic Effects of Rising International Energy Prices

Figure 2 shows the IRFs of EA GDP (Panel 2B), real consumption (Panel 2C), headline inflation (Panel 2D), core inflation (Panel 2E), services inflation (Panel 2F), net exports (Panel 2G), MU central bank policy rate (Panel 2H), and the nominal exchange rate with respect to the ROW. The increase in the price of imported energy paid by EA firms is shown in Panel 2A.

The increase of production costs for EA firms induces them to decrease labor demand and hence production, with value-added (real GDP) falling. In addition, the increase in international energy prices means a negative wealth shock for households, reducing their demand for domestic goods. Overall, we obtain a fall in real consumption larger than the fall in real GDP. Both imports and exports fall, with net exports increasing, due to the relative larger increase in the price of imported goods compared to exported goods. The systematic monetary policy stance of the central bank of the monetary union reacts by increasing the policy rate to control inflation. As a result, the currency of the monetary union appreciates,

[15] In our data, this corresponds with the Mining and quarrying of energy products sectors, which accounts for sections B.5 and B.6 in the ISIC, Rev.4 classification.

[16] In our data, these correspond with sections C.19 and D.35 in the ISIC, Rev.4 , classification respectively.

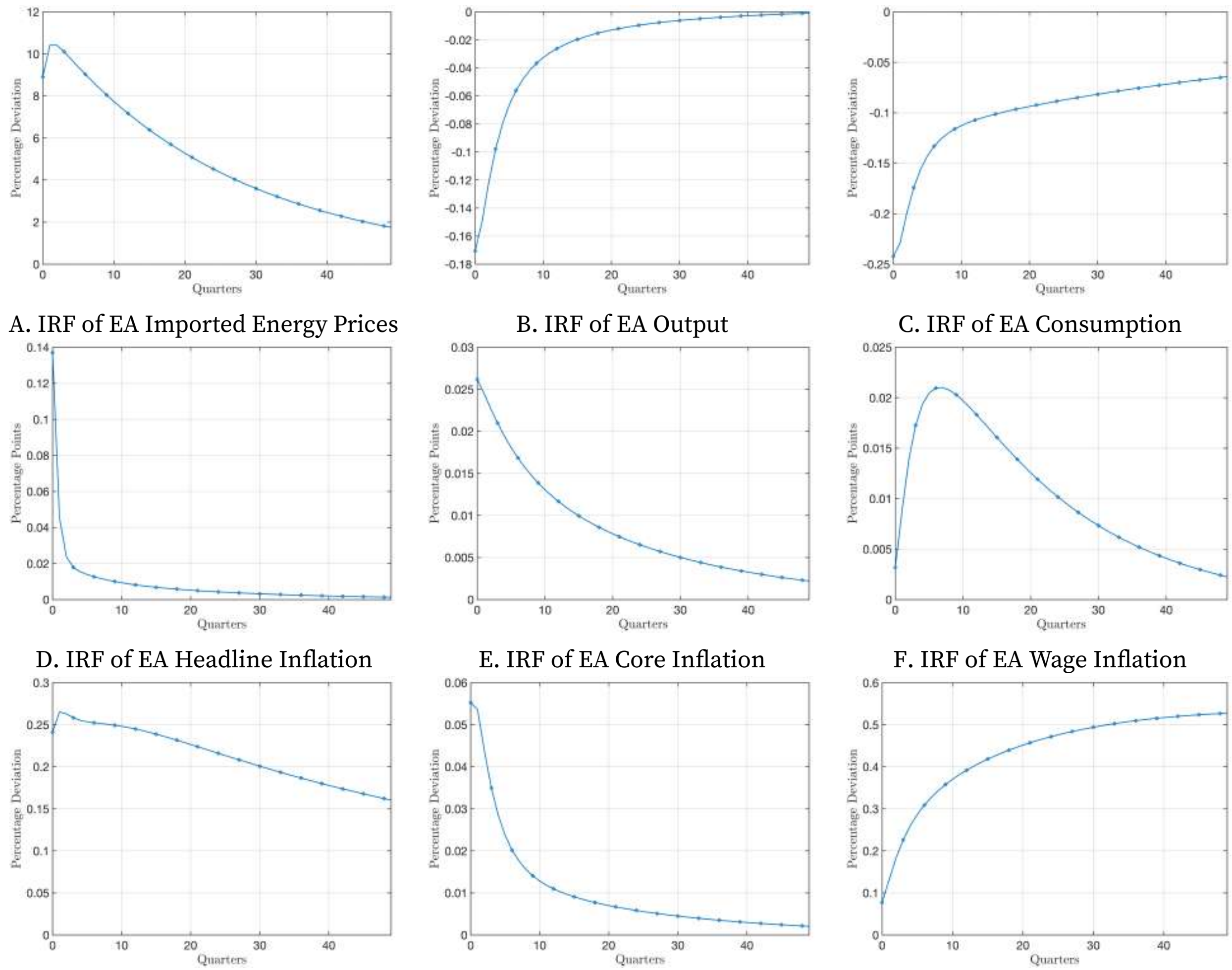
generating a rise in the nominal exchange rate with respect to the ROW.

Headline inflation responds immediately and sharply, reflecting the high price flexibility of energy sectors in the model and the non-negligible share of energy goods in households' consumption basket. The inflationary spike is followed by a more persistent rise in core inflation, which remains elevated long after the initial shock, contributing to the persistent increase of headline inflation. On impact, the pass-through of headline to core inflation is significant, amounting to roughly 20%, in line with the empirical evidence presented in Adolfsen *et al.* (2024). Intuitively, the increase in energy prices energy induces production costs for firms to increase. As a result, firms respond by increasing the prices of their products. Therefore, through the IO linkages, the costs of imported and domestically produced goods for firms further increase, leading to an additional rise in prices. This feedback between increasing selling prices and rising production costs results in a generalized increase in core and headline inflation. Finally, we obtain a remarkably persistent and hump-shaped response of wage inflation, which contributes to the persistent rise in core inflation.

Inspecting the Mechanism. Equation (37) in the analytical section reveals how the IO structure amplifies the persistence induced by the nominal rigidities. Any increase in the foreign price-wedge is passed-through to sectoral prices, with the intensity of the pass-through measured by the position of the shocked sector in the IO network and its price rigidities (direct effect). This direct effect is amplified through the ridigity-adjusted Leontief inverse, which additionally considers the transmission through the IO network of the direct intermediate purchases.

In addition to the impact effect of the price wedge, the persistence of nominal prices and wages, coming from the nominal rigidities, is amplified through the ridigity-adjusted Leontief inverse. Intuitively, the resulting sectoral price changes after the granular shock affect sectors in the economy differently, depending on their exposure through the IO network and the limited pass-through induced through price and wage rigidities, which builds up over time. This feedback between the persistence of selling prices and wages (marginal costs) builds up through the entire production network, resulting in the slow-decaying pattern of headline and core inflation displayed in Figure 2.

Sectoral Decomposition. In order to study the sectoral composition of EA headline inflation, we catalog each of the 44 sectors in the economy into three categories: energy, upstream or downstream. We consider *Mining* (NACE B), *Coke and refined petroleoum* (NACE C.19), and *Electricity* (NACE D.35) as the energy sectors. Out of the remanining non-energy sectors, we follow the methodology proposed in Antràs *et al.* (2012) to rank sectors according to their relative proximity to the final consumer. According to their measure, a sector is more downstream (i.e. closer to the final consumer) when a larger share of its output is used as



A. IRF of EA Imported Energy Prices
B. IRF of EA Output
C. IRF of EA Consumption
D. IRF of EA Headline Inflation
E. IRF of EA Core Inflation
F. IRF of EA Wage Inflation
G. IRF of EA Net Exports
H. IRF of EA Policy Rate
I. IRF of EA Nominal Exchange Rate vs. ROW

FIGURE 2. Effects of an International Energy Price Shock on EA Variables

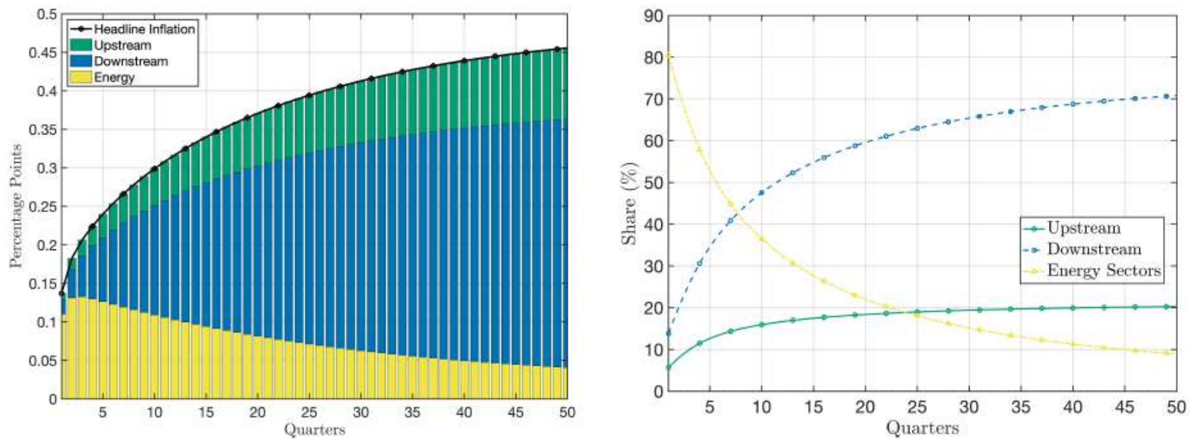
Notes: IRFs of EA macroeconomic variables: import energy prices (Panel 2A), real GDP (Panel 2B), real consumption (Panel 2C), headline inflation (Panel 2D), core inflation (Panel 2E), wage inflation (Panel 2F), net exports (Panel 2G), nominal interest rate (Panel 2H), and nominal exchange rate vs. Rest of the World (Panel 2I) to a 10% peak increase in imported energy prices.

final consumption. Conversely, more upstream sectors are the ones that sell a larger fraction its output as intermediate input for other sectors.[17] We label a sector as upstream if its upstreamness measure is above the median, and downstream otherwise.[18]

We plot in Figure 3 the decomposition of the cumulative impulse response function (CIRF)

[17] The upstreamness measure of an industry i, U_{ki}, is the solution to the system $U_{ki} = 1+\sum_{l\in K}\sum_{j\in I} X_{klji}U_{kj}/Y_{ki}$, where X_{klji}/Y_{ki} denotes the share of industry i output sold to industry j in country l and U_j is the upstreamness measure of industry j. This measure takes into account the upstreamness of the sectors to which industry i supplies intermediate inputs.

[18] The three most downstream sectors are *Health and Education Services* (NACE P-Q), *Public Administration* (NACE O), and *Accommodation and food services* (NACE I). The three most upstream sectors (apart from energy-related sectors) are *Basic metals* (NACE C.24), *Chemical products* (NACE C.20) and *Warehousing* (NACE H.52-H.53).



A. CIRF of Headline Inflation

B. Contributions of Upstream, Downstream and Energy sectors

FIGURE 3. Inflation Dynamics and its Sectoral Decomposition

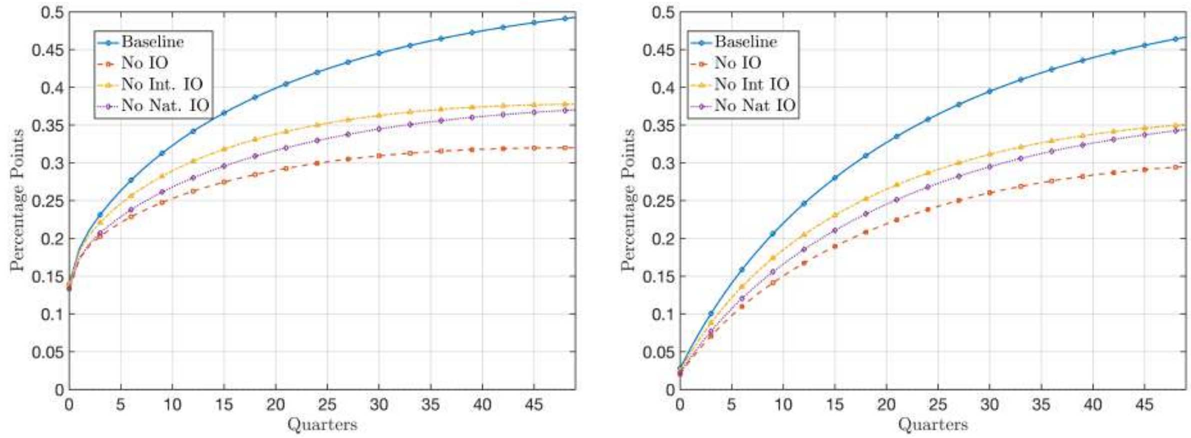
Notes: Panel 3A: CIRF of EA headline inflation and contributions of upstream and downstream sectors (Antràs *et al.* 2012). Panel 3B: contributions of upstream and downstream sectors as a percent of total headline inflation.

of the EA headline inflation after the energy price shock. In Panel 3A, we document that the initial increase in headline inflation is entirely driven by the increase in energy sectors, directly transmitted to consumption prices. Over time, the energy price falls, reverting to its initial level; and upstream and downstream sectors start contributing to headline inflation. Panel 3B reports each component's share in total headline inflation. We find that the share of inflation coming from upstream sectors stabilizes after 30 quarters, whilst the share coming from downstream sectors is more persistent, increasing steadily over time.

These findings can be interpreted through the intuition developed in the price dynamics in equation (37): upstream sectors are less affected by the energy price increase since their intermediate input share in production is small, while downstream sectors depend directly on the intermediate input purchases on other sectors, including energy, and indirectly through their customers' IO network. Given that the pass-through is limited through price rigidities along the supply chain, this further increases the persistence of headline inflation.

4.3.2. Dissecting the Role of Production Networks

We next assess the role played by both national and international production networks. Toward this end, we consider a series of counterfactual economies where we turn off the production network entirely ($\omega_{kl\,ij} = 0 \quad \forall k, l, i, j$), only the domestic production network ($\omega_{kl\,ij} = 0 \quad \forall l = k$), or only the international network ($\omega_{kl\,ij} = 0 \quad \forall l \neq k$). In all these counterfactual economies we always maintain energy as both a consumption good and production input for firms.



A. CIRF of Headline Inflation

B. CIRF of Core Inflation

FIGURE 4. Inflation Dynamics and Production Networks

Notes: Cumulative IRF of EA headline (Panel 4A) and core (Panel 4B) inflation for the baseline and turning off the full, international, or national IO structure. When turning off the IO structure, we always keep the use of energy as an intermediate input.

Figure 4 shows the CIRFs of headline inflation (Panel 4A) and core inflation (Panel 4B) for our baseline calibration and for each of the three counterfactual economies. The dashed red lines represent our counterfactual economy with national and international IO links removed altogether. On impact, headline inflation increases roughly by the same amount as in the baseline calibration (solid blue lines). This is a consequence of headline inflation being driven initially by the rise of international energy prices, which is common across counterfactuals.

However, the presence of IO linkages is key in explaining inflation dynamics beyond impact. When we conduct our first counterfactual by turning off the production networks, cumulative inflation increases only 60% of our baseline at the end of the simulation horizon (compare the blue and red lines). On the one hand, this is a consequence of the smaller increase in core inflation (Panel 4B). Without IO links, the feedback loop between increasing selling prices and rising production costs is absent, significantly dampening the increase in core inflation. On the other hand, inflation shows less persistence, dying out substantially quicker than in our baseline. More precisely, in our baseline simulation, inflation continues to rise steadily throughout the entire simulation horizon, whereas in the absence of production networks it stabilizes much earlier. This finding formalizes the intuition provided earlier, whereby the presence of intermediate goods in the marginal costs of firms in interaction with IO linkages leads to more persistent inflation dynamics.

The next two counterfactuals dissect the contribution of national (dotted purple line) and international (dashed yellow line) production networks. We find that the international IO network is a key factor that adds significant persistence on headline and core inflation.

Given the upstream position of the energy within production chains, the shock propagates particularly strongly to other productive sectors. Consequently, due to the high level of integration between industrial sectors across European economies, there are significant spillovers from the effects of the shock through the cross-country links captured in the input-output tables. This finding highlights the quantitative relevance of the multi-country dimension to account for international spillover effects, which explain around 20% of the overall response of headline inflation, and would be underestimated in the simpler small open economy framework in Section 3.

Interestingly, national and international production networks interact with each other. Without IO linkages, cumulative inflation increases by 0.32 percentage points (p.p.), while with only national or international production networks, it increases by roughly 0.37 p.p. In other words, the "marginal effect" of each of them is approximately 0.05 p.p., and adding those to the counterfactual without IO explains only 85% of the total cumulative response in the baseline.

In other words, it is crucial to account for both the national and international dimensions of production networks simultaneously. Intuitively, higher domestic inflation leads to increased export prices, which contributes to higher inflation abroad. In turn, rising inflation abroad translates into higher import prices, feeding back into domestic inflation. This interaction between national and international production networks amplifies the inflationary episode, resulting in a larger impact than if these dimensions were considered in isolation.

Alternative Foreign Shocks. In Appendix C, we analyze the effects of alternative shocks on aggregate EA variables, focusing on foreign perturbations (in ROW): internal demand, monetary policy, sectoral TFP, sectoral price and wage markups, and international price-wedge shocks to rigid-price sectors. For sectoral TFP and price cost-push disturbances, we examine shocks to the most upstream sector (*Basic Metals*) and the most downstream sector (*Health and Education Services*), as classified by Antràs *et al.* (2012). Production networks amplify the transmission of foreign demand and monetary policy shocks to real output and inflation, as well as the output effects of foreign TFP shocks regardless of sector position. Notably, upstream disturbances lose all impact on inflation once the IO network is shut down, and a similar pattern arises with sectoral cost-push shocks. We also compare the energy price shock to a price-wedge shock in the semiconductor industry—two upstream sectors with differing price rigidities. Despite both shocks being amplified by production networks, the semiconductor shock produces more persistent dynamics due to lower pass-through and higher rigidity.

In contrast, wage cost-push shocks are dampened by the network, reducing their impact on both output and inflation: under the no-IO counterfactual, the labor share is recalibrated to absorb the input share, amplifying the effect of wage shocks. We relegate the deeper analysis to Appendix C.

4.3.3. Heterogeneous Production Structures and Cross-country Heterogeneity

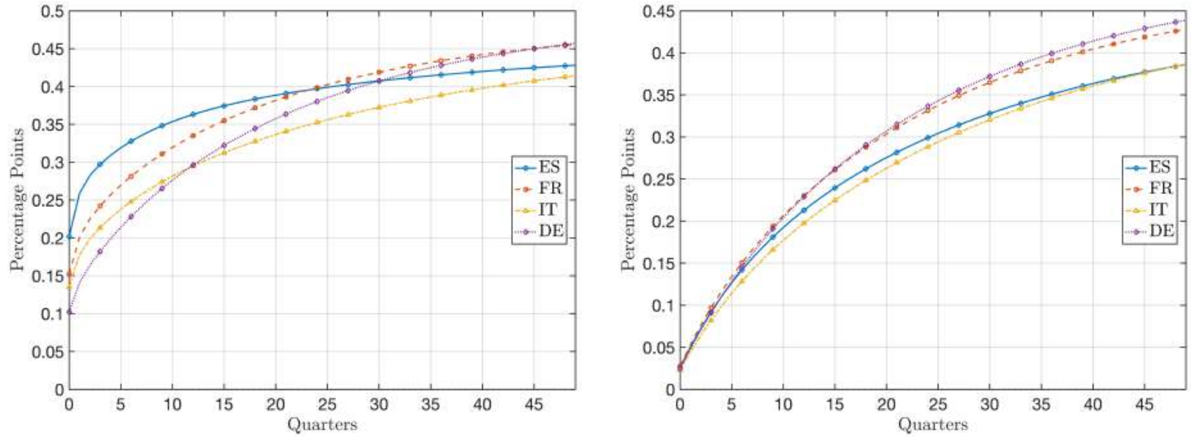
The previous sections have analyzed the effects of an international energy price shock on EA variables and the role played by production networks in its transmission. In this section, we instead show that such a common shock propagates differently across countries that differ in their production structures.

Figure 5 shows the CIRFs of headline (Panel 5A) and core inflation (Panel 5B) for the main EA countries: Spain (blue line), Germany (purple line), Italy (yellow line), and France (red line).

We find that the shock results in significantly heterogeneous inflation dynamics across countries, despite all European countries facing the same increase in imported energy prices. More precisely, we see that Spain suffers the largest spike in headline inflation in the first periods. However, note that this is also the country where inflation also stabilizes the fastest. In contrast, we observe the inflation dynamics in Germany. In response to the increase in energy prices, headline inflation increases the least in the German economy. In sharp contrast to Spain, headline inflation in Germany shows substantial persistence, increasing steadily over time. The dynamics of France and Italy sit somewhere between these two extremes.

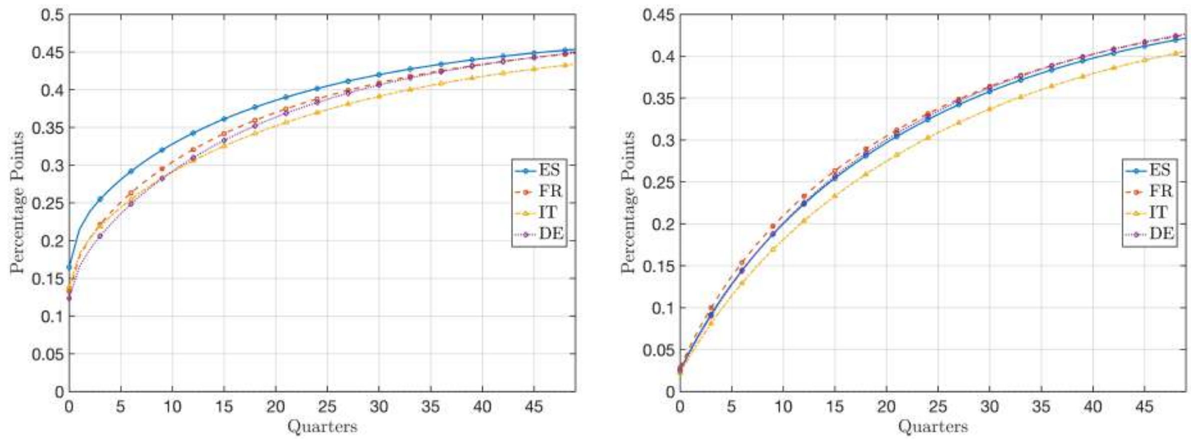
The dynamics of headline inflation can be better understood by looking at core inflation, shown in Panel 5B. Germany's core inflation rises gradually and remains elevated for a prolonged period. Spain, on the other hand, experiences a more transient rise in core inflation. This differential evolution in core inflation rates helps explain the varying persistence of headline inflation dynamics between the two countries, since most of the headline inflation at longer horizons is explained by core inflation dynamics.

The heterogeneity in production structures and households' consumption baskets across countries can rationalize these differentials in the inflation dynamics. In the model, as well as in the data, the consumption share of energy goods in Spain is the largest (4.49%). Therefore, headline inflation in Spain is the one that is most directly affected by the rise in energy prices. In contrast, Germany has a smaller share of energy consumption (4.09%), naturally leading to a smaller response of headline inflation on impact. However, Germany's production structure is characterized by a stronger industry exposure to the use of energy goods and long production chains. The longer production network structure of the German economy explains the persistent rise of inflation, as the feedback loops described in the previous sections apply more strongly here. On the other side, Spain has a more downstream-oriented production structure, with less complex IO linkages, resulting in lower amplification associated with



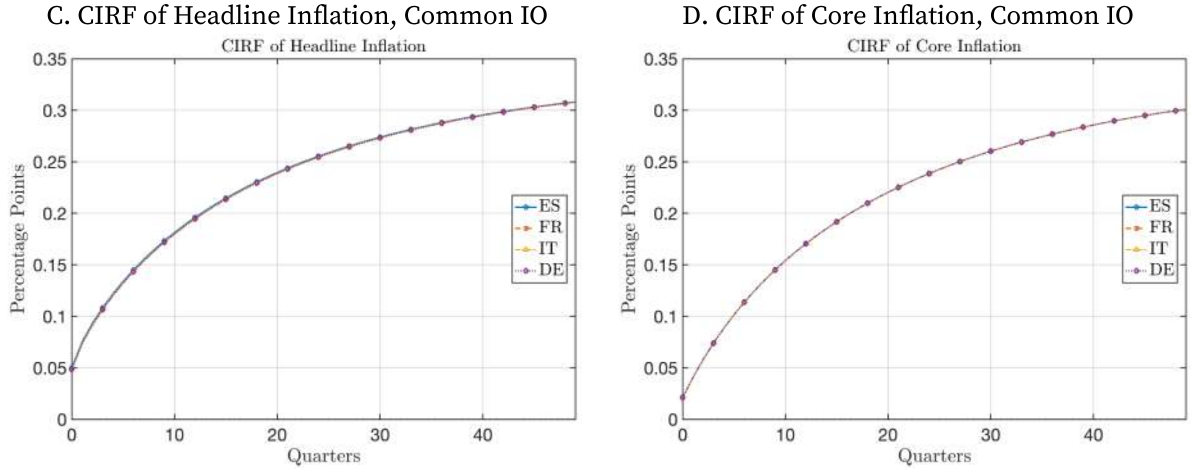
A. CIRF of Headline Inflation

B. CIRF of Core Inflation



C. CIRF of Headline Inflation, Common IO

D. CIRF of Core Inflation, Common IO



E. CIRF of Headline Inflation, Common IO and Consumption Shares

F. CIRF of Core Inflation, Common IO and Consumption Shares

FIGURE 5. Cross-country Heterogeneity

Notes: Cumulative IRF of headline (Panel 5A) and core (Panel 5B) inflation for Spain (ES), France (FR), Italy (IT), and Germany (DE). Panels 5C and 5D reproduce the analysis with an homogeneous IO network, and 5E and 5F reproduce the analysis under both homogeneous IO production network and consumption shares.

production networks in this case.[19]

To isolate the role of production networks, we consider the case in which the IO matrix is homogeneous across countries.[20] Panels 5C-5D present the resulting CIRFs of headline and core inflation. We find that equalization of the production network between countries reduces the gap in inflation dynamics between the different countries. The persistence induced by the network is the same across countries, and no CIRF crosses the others. Spain, which has a higher energy share in the CPI basket, reacts more initially and ends up with the highest cumulative inflation.

Finally, to eliminate the gap coming from the heterogeneous consumption shares, we consider the case in which the consumption share matrix is homogeneous across countries.[21] Panels 5E-5F present the resulting CIRFs of headline and core inflation. We find that equalizing the consumption shares across countries, on top of production network, further reduces the gap in inflation between the different countries. The remaining distance can be explained by the heterogeneous price rigidities: the average price duration in Spain is 4.18 quarters, having relatively flexible prices, whereas in Germany prices are more rigid, lasting for 4.50 quarters on average.[22]

4.4. Monetary Policy Implications

Our previous findings indicate that IO links play a central role in explaining the inflationary effects and cross-country propagation of international energy price shocks. Next, we investigate the implications of these findings for monetary policy.

4.4.1. Systematic Monetary Policy and Production Networks

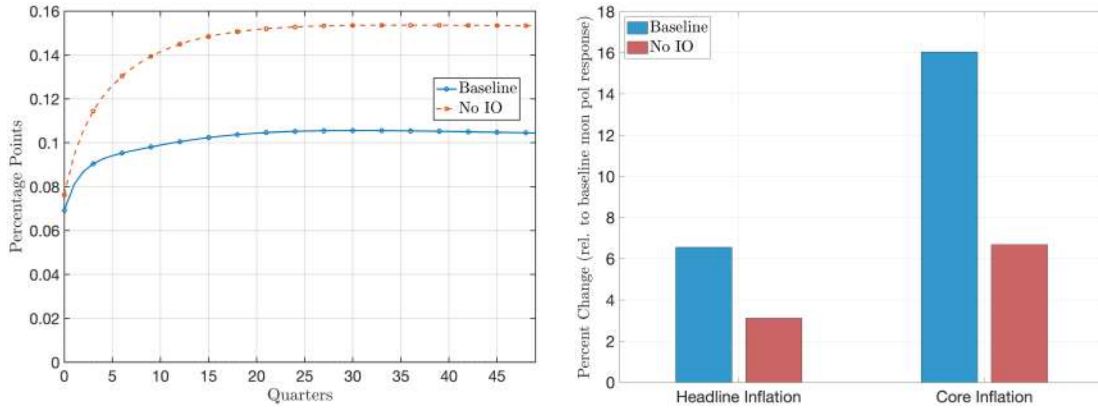
Previous research has shown that the presence of intermediate goods and IO links tends to increase the degree of monetary policy non-neutrality (see, for example, Nakamura and

[19]Although the values of the upstreamness measure do not have a straightforward quantitative interpretation, the sales-weighted average of the sectors in the Spanish and German economies shows that, on average, Spanish sectors are 6.4% closer to final demand than German sectors.

[20]This matrix assumes that, within EA economies, all sectors have the same productive structure. Therefore, within a given sector i, the weight of any sector j (υ_{kij} in our notation) as well as the different national varieties l ($\zeta_{kl\,ij}$ in our notation) is the same for all firms in the EA. Within each sector, these values are calibrated as the average of all EA countries. For consistency, this implies that the weight of a given sector in the GDP of its country is the same in all EA countries.

[21]In this case we set the different consumption shares by sector as well as by national varieties to be the equal to the mean for all households across the EA.

[22]The differences in sectoral price flexibility are particularly pronounced in energy and food sectors.The differences vanish when we consider only core CPI sectors (the average price duration is 6.45 quarters in Spain vs. 6.54 quarters in Germany), which explains the overlapping of core CPI inflation dynamics in the four countries in Panel 5F.



A. CIRF of EA Headline Inflation to a Monetary Policy Shock in the EA

B. Percent Change in EA Inflation Volatility under Weaker Monetary Policy Response

FIGURE 6. Monetary Policy and Production Networks

Notes: Panel 6A: CIRF of headline inflation to a monetary policy shock (easing) in the baseline and without IO. Panel 6B: percent change in inflation volatility (conditional on energy price shocks) with a lower coefficient on inflation in the Taylor rule.

Steinsson 2010; Rubbo 2023). That is, upon a monetary policy shock, inflation tends to respond less and output more than in a counterfactual where these features are absent.

This finding is also present in our framework. In Panel 6A of Figure 6, we show the CIRFs of headline inflation upon a monetary shock that increases EA interest rates. A monetary tightening leads to a larger drop in inflation when the IO structure is absent (red line), relative to our baseline calibration (blue line). Intuitively, the presence of intermediate goods with sticky prices reduces the volatility of marginal costs and reduces the pass-through of wages into prices (see expressio 37), leading to a muted inflation response.

The Panel 6B of Figure 6 considers the following exercise. We draw a series of shocks to the international price of imported energy faced by European firms. We then simulate the model with and without production networks, subject to those shocks, and compute the volatility of headline and core inflation. When doing so, we consider two different inflation coefficients in the Taylor rule (46): the first with our baseline calibration $\phi_{MU\pi} = 1.5$, and the second considering a weaker systematic response $\phi_{MU\pi} = 1.1$. Panel 6B shows the increase in EA inflation volatility when we move from $\phi_{MU\pi} = 1.5$ to $\phi_{MU\pi} = 1.1$ in the economy with production networks. The red bars show the same statistic in the economy without IO links.

First, we observe that monetary policy has a greater impact on core inflation than on headline inflation, both with and without IO links. Specifically, the increase in inflation volatility from a weaker monetary policy response is more than twice as large for core inflation compared to headline inflation. This difference arises from cross-sector heterogeneity in price flexibility and its interaction with import intensities. Sectors contributing to core inflation, such as services and manufacturing, have stickier prices compared to energy- and food-producing sectors in our dataset, which exhibit a higher pass-through from marginal costs to selling prices. In addition, the domestic energy and food sectors are heavily dependent on imported goods as key production inputs. Since domestic monetary policy has limited influence over the international prices of these imported goods, which strongly affect domestic prices, the monetary policy rule has a smaller impact on headline inflation than on core inflation.

Second, the results of this simulation show that the systematic response of monetary policy becomes more significant in the presence of IO linkages, despite the higher degree of monetary non-neutrality following monetary shocks. Specifically, by comparing the red and blue bars, we observe that inflation volatility increases by more than twice as much when production networks are included. This finding aligns with our earlier results: IO linkages amplify the inflationary response to international energy price shocks. Consequently, even though a given change in interest rates has a smaller direct effect (as shown in Panel 6A), a monetary policy response that fails to contain the propagation of such shocks—and allows the IO amplification to build up—will result in greater inflation volatility.

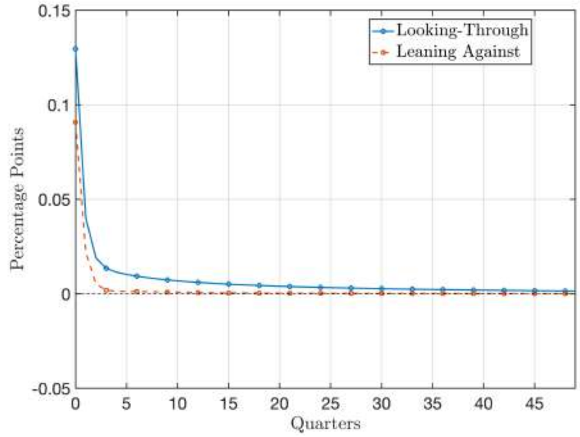
4.4.2. Production Networks and Monetary Policy Trade-offs

In the context of rising foreign energy prices that exert upward pressure on domestic inflation, the monetary authority faces a trade-off: pursuing strict inflation targeting may succeed in stabilizing inflation, but at the cost of inducing a decline in the output gap. In this section, we quantify this trade-off—analyzed analytically in Section 3—and examine how it is shaped by the multi-sectoral structure of the economy, which constitutes our main departure from the one-sector framework in Gali and Monacelli (2005), as well as by the open-economy dimension, which extends the closed-economy setting of Rubbo (2023).

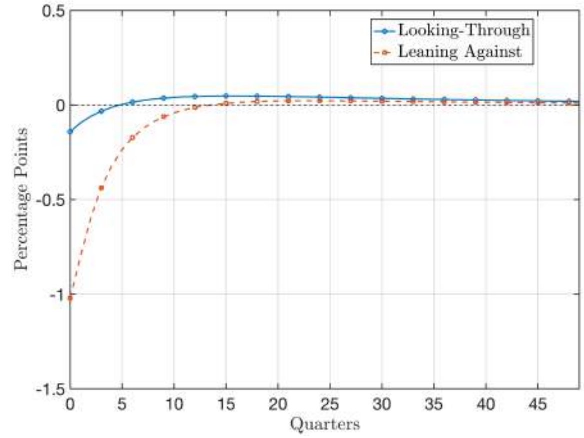
We consider two counterfactual monetary regimes that differ in the systematic component of the Taylor rule. In the *Looking-Through* (LT) regime, central banks set the policy rate according to equations (45) and (46) with coefficients $\{\phi_{k\pi}, \phi_{ky}\} = \{1.5, 0\}$ for all k. We interpret this as standard monetary policy in the face of supply-side disturbances, and we treat it as our baseline scenario. In contrast, the *Leaning Against the Wind* (LATW) regime corresponds to strict inflation targeting, modeled as $\{\phi_{k\pi}, \phi_{ky}\} = \{10, 0\}$ for all k, representing a more aggressive stance against inflation deviations.

Under both regimes, we examine the dynamics of the headline inflation and the output gap in the EA, in response to the energy price shock discussed in Section 4.3. Figure 7 presents the IRFs of headline inflation (panel 7A) and the output gap (panel 7B) for both monetary policy rules.

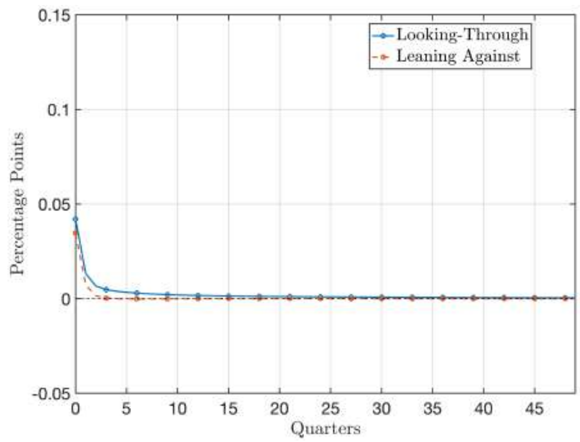
Under the LT regime (blue solid line), cumulative headline inflation increases by approx-



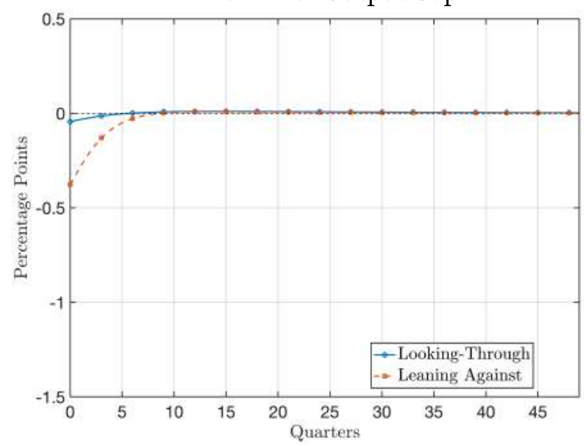
A. IRF of EA Headline Inflation



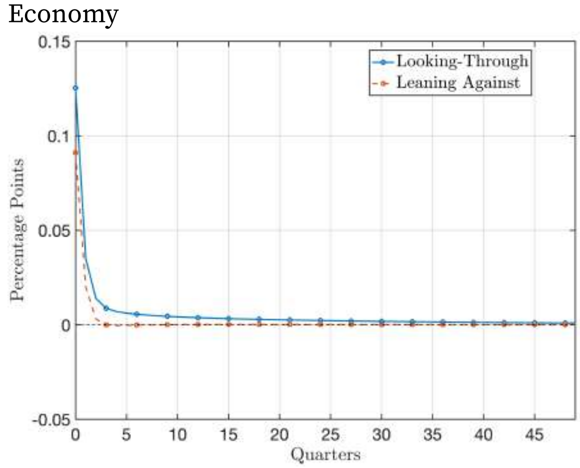
B. IRF of EA of Output Gap



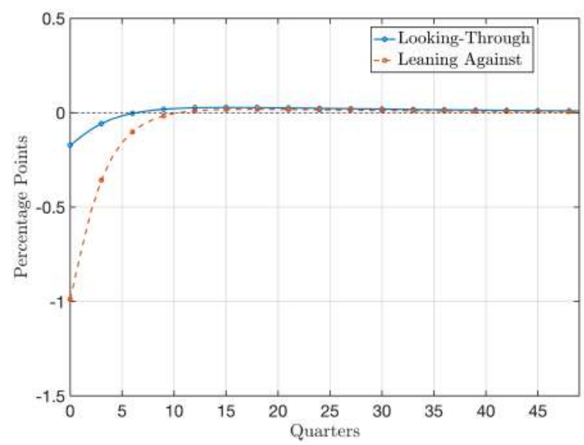
C. IRF of EA of Headline Inflation, One-sector Economy



D. IRF of EA of Output Gap, One-sector Economy



E. IRF of EA of Headline Inflation, Closed Economy



F. IRF of EA of Output Gap, Closed Economy

FIGURE 7. Monetary Policy Trade-Offs

Notes: IRFs of EA headline inflation (panel 7A) and output gap (panel 7B) after an increase in imported energy prices (10%) under Looking-through monetary policy $\{\phi_\pi, \phi_y\} = \{1.5, 0\}$ (blue and solid line) and Leaning Against the Wind $\{\phi_\pi, \phi_y\} = \{10, 0\}$ (red and dashed line) monetary policy. Panels 7C and 7D reproduce the analysis with common parameters across sectors and countries (the one-sector-economy limit). Panels 7E and 7F reproduce the analysis without the foreign IO network and consumption shares (the closed-economy limit).

imately 0.4 p.p. after the shock. The output gap initially turns negative, but then becomes positive as the energy price starts declining, and eventually returns to baseline. (Recall that the energy shock includes a deflationary phase after the fourth period, see Figure 2A.) In contrast, the LATW regime (red dashed line) achieves significantly lower and less persistent inflation, at the cost of a much sharper and prolonged decline in the output gap. This illustrates a clear policy trade-off: stricter inflation stabilization entails a substantially larger output gap contraction.

To evaluate how this trade-off is shaped by production networks and international linkages, we conduct two counterfactual exercises.

First, we eliminate the multi-sectoral dimension by imposing full symmetry across sectors and countries.[23] These assumptions collapse the multi-sector model into a one-sector economy, akin to the small open economy setup in Gali and Monacelli (2005) discussed analytically in Section 3.2.

Panels 7C and 7D show the IRFs of headline inflation and the output gap, in this one-sector economy, to foreign prices. The trade-off between inflation and output stabilization is noticeably milder. With the amplification effect of production networks removed, stabilizing inflation becomes less costly in terms of the output gap. Quantitatively, the magnitude of the trade-off is reduced by roughly three-fourths relative to the baseline.

Second, we eliminate the international dimension.[24] This environment resembles the closed-economy setup of Rubbo (2023) discussed analytically in Section 3.2, extended to include foreign price shocks. Crucially, this framework lacks the international spillovers that amplify the domestic response to foreign shocks.

Panels 7E and 7F display the corresponding IRFs. As in the one-sector case, the trade-off between inflation and output stabilization becomes less severe. With international amplification shut down, the contraction of the output gap under LATW policy is considerably smaller, and the difference with the LT regime narrows. Overall, the trade-off is reduced by about one-third compared to the baseline.

Taken together, these findings highlight the importance of modeling production structures and cross-border linkages in detail. The presence of sectoral heterogeneity and international IO spillovers significantly amplifies the macroeconomic effects of external shocks and exacerbates the policy trade-off. Ignoring these dimensions would underestimate the costs of strict inflation stabilization in an open and interconnected economy.

[23] Specifically, we assume homogeneous production networks, consumption shares, labor shares, and nominal rigidities: $\omega_{kl\,ij} = \overline{\omega}$, $\beta_{kl\,i} = \overline{\beta}$, $\alpha_{ki} = \overline{\alpha}$, and $\theta^{p}_{ki} = \overline{\theta}^{p}$ for all k, l, i, j, where the bar denotes the average value in the baseline calibration.

[24] We shut down cross-border input-output linkages ($\omega_{kl\,ij} = 0$ for all $l \neq k$) and foreign consumption shares ($\beta_{kl\,i} = 0$ for all $l \neq k$), while preserving energy as a consumption good and production input.

5. Conclusions

This paper highlights the critical role of production networks in shaping the transmission and persistence of inflation in response to international energy price shocks.

Using a multi-country New Keynesian model with rich sectoral heterogeneity and input-output linkages, we show that production networks significantly amplify inflation through a feedback loop between rising selling prices and production costs. The interaction between national and international networks intensifies inflationary pressures, with cross-border spillovers further magnifying the persistence of inflation. This effect is particularly pronounced in countries with more integrated production structures, such as Germany, where inflation lasts longer, while countries with less complex networks, like Spain, experience shorter-lived inflation spikes.

Our findings highlight the importance of an active monetary policy response to supply-side shocks, as weaker stabilization policies lead to greater inflation volatility when production networks are present. Furthermore, we find that production networks exacerbate the trade-off between inflation and output gap stabilization after an energy price shock.

Our framework is well-suited for analyzing the inflationary impact of macroeconomic shocks and policies characterized by a high degree of sectoral granularity. Examples include trade policies like tariffs, environmental measures such as carbon pricing, supply-side bottlenecks, and disruptions in global value chains. On the modeling side, further progress can be made by incorporating the complexity of investment input-output networks (vom Lehn and Winberry 2021; Quintana 2024). These represent promising avenues for future research, offering potential insights into how capital investments and production structures interact to shape the transmission of shocks and policies across sectors and countries.

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Appendix

Appendix A. Model Derivation and Log-linearization

In this section, we derive model equilibrium conditions, outlining the final set of log-linearized equations. We further enlarge the theoretical model presented in the main text—which only contains exogenous perturbations to the foreign price wedges and the monetary policy shock—to accommodate the standard shocks in the literature: an internal demand shock, sectoral TFP, sectoral price cost-push shocks, and wage cost-push shocks.

A.1. Households

Each household is made up of a continuum of members, each specialized in a different labor service, indexed by $g \in [0,1]$. Income is pooled within each household, acting as a risk-sharing mechanism. The per-period utility function (1) is modified to accommodate nominal wage rigidities and a discount factor disturbance. Namely,

$$U_t = \left(\frac{C_{k,t}^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{\mathcal{N}_{gk,t}^{1+\varphi}}{1+\varphi} \, dg \right) Z_{k,t}, \tag{A.1}$$

where $\mathcal{N}_{gk,t}$ denotes the labor supply of labor service g, $Z_{k,t}$ is an exogenous preference shifter, and σ denotes the inverse of the intertemporal elasticity of substitution.[25]

Consumption Demand Curves. The allocation optimal allocation between energy and non-energy goods is the result of a cost minimization programme $\min P_{kE,t}C_{kE,t} + P_{kM,t}C_{kM,t}$ subject to (40). Similarly, the optimal allocation between energy (non-energy) consumption is the result of a cost minimization programme $\min \sum_{i \in I_E} P_{kiC,t}C_{ki,t}$ ($\min \sum_{i \in I_M} P_{kiC,t}C_{ki,t}$) subject to (41). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme $\min \sum_{l=1}^{K} (1+\tau_{kl\,i,t})P_{kl\,i,t}C_{kl\,i,t}$ subject to (43). The implied demand curves are given by

$$P_{kE,t} = P_{kC,t} \left(\frac{\widetilde{\beta}_k C_{k,t}}{C_{kE,t}} \right)^{\frac{1}{\gamma}} \quad \text{and} \quad P_{kM,t} = P_{kC,t} \left(\frac{(1-\widetilde{\beta}_k) C_{k,t}}{C_{kM,t}} \right)^{\frac{1}{\gamma}} \tag{A.2}$$

(A.3)

$$P_{kiC,t} = P_{kE,t} \left(\frac{\widetilde{\nu}_{ki} C_{kE,t}}{C_{ki,t}} \right)^{\frac{1}{\eta}} \quad \forall \quad i \in I_M \quad \text{and} \quad P_{kiC,t} = P_{kM,t} \left(\frac{\widetilde{\upsilon}_{ki} C_{kM,t}}{C_{ki,t}} \right)^{\frac{1}{\iota}} \quad \forall \quad i \in I_E$$

$$(1+\tau_{kl\,i,t})P_{kl\,i,t} = P_{kiC,t} \left(\frac{\widetilde{\zeta}_{kl\,i} C_{ki,t}}{C_{kl\,i,t}} \right)^{\frac{1}{\delta}} \quad \forall \quad l \in K. \tag{A.4}$$

[25] Each household takes as given labor income since wages are set by labor unions and employment is decided by firms. Thus, the only decisions made by the household are the optimal allocation of consumption expenditures among different good varieties across different countries, and the optimal intertemporal allocation of consumption.

The log-linearized versions of the consumption demand curves (A.2)-(A.4) are given by

$$\text{(A.5)} \qquad \widehat{p}_{kE,t} = \frac{1}{\gamma}(\widehat{c}_{k,t} - \widehat{c}_{kE,t}) \quad \text{and} \quad \widehat{p}_{kM,t} = \frac{1}{\gamma}(\widehat{c}_{k,t} - \widehat{c}_{kM,t})$$

$$\text{(A.6)} \qquad \widehat{p}_{kiC,t} - \widehat{p}_{kE,t} = \frac{1}{\eta}(\widehat{c}_{kE,t} - \widehat{c}_{ki,t}) \quad \text{and} \quad \widehat{p}_{kiC,t} - \widehat{p}_{kM,t} = \frac{1}{\iota}(\widehat{c}_{kM,t} - \widehat{c}_{ki,t})$$

$$\text{(A.7)} \qquad \tau_{kli,t} + \widehat{p}_{kli,t} - \widehat{p}_{kiC,t} = \frac{1}{\delta}\left(\widehat{c}_{ki,t} - \widehat{c}_{kli,t}\right)$$

where $\widehat{p}_{kE,t} = p_{kE,t} - p_{kC,t}$, $\widehat{p}_{kM,t} = p_{kM,t} - p_{kC,t}$, $\widehat{p}_{kiC,t} = p_{kiC,t} - p_{kC,t}$, and $\widehat{p}_{kli,t} = p_{kli,t} - p_{kC,t}$ are well-defined as a ratio of prices.[26]

Consumption Baskets. The log-linearized consumption aggregator (40) is given by

$$\text{(A.8)} \qquad \widehat{c}_{k,t} = \beta_k \widehat{c}_{kE,t} + (1 - \beta_k)\widehat{c}_{kM,t}$$

where $\beta_k = \frac{P_{kE}C_{kE}}{P_{kC}C_k} = \widetilde{\beta}_k^{\frac{1}{\gamma}}\left(\frac{C_{kE}}{C_k}\right)^{\frac{\gamma-1}{\gamma}}$ and $(1-\beta_k) = \frac{P_{kM}C_{kM}}{P_{kC}C_k} = (1-\widetilde{\beta}_k)^{\frac{1}{\gamma}}\left(\frac{C_{kM}}{C_k}\right)^{\frac{\gamma-1}{\gamma}}$ can be verified using the steady-state consumption aggregator (40) and the demand curves (A.2).

The log-linearized versions of the energy and non-energy consumption aggregators (41) are given by

$$\text{(A.9)} \qquad \widehat{c}_{kE,t} = \sum_{i \in I_E} \nu_{ki}\widehat{c}_{ki,t} \qquad \text{and} \qquad \widehat{c}_{kM,t} = \sum_{i \in I_M} \upsilon_{ki}\widehat{c}_{ki,t}$$

where $\nu_{ki} = \frac{P_{kiC}C_{ki}}{P_{kE}C_{kE}} = \widetilde{\nu}_{ki}^{\frac{1}{\eta}}\left(\frac{C_{ki}}{C_{kE}}\right)^{\frac{\eta-1}{\eta}}$ and $\upsilon_{ki} = \frac{P_{kiC}C_{ki}}{P_{kM}C_{kM}} = \widetilde{\upsilon}_{ki}^{\frac{1}{\iota}}\left(\frac{C_{ki}}{C_{kM}}\right)^{\frac{\iota-1}{\iota}}$ can be verified using the steady-state energy and non-energy consumption aggregators (41) and the demand curves (A.3).

The log-linearized version of the final layer of the consumption aggregator, (43), is given by

$$\text{(A.10)} \qquad \widehat{c}_{ki,t} = \sum_{l=1}^{K} \zeta_{kli}\widehat{c}_{kli,t}$$

where $\zeta_{kli} = \frac{P_{kli}C_{kli}}{P_{kiC}C_{ki}} = \widetilde{\zeta}_{kli}^{\frac{1}{\delta}}\left(\frac{C_{kli}}{C_{ki}}\right)^{\frac{\delta-1}{\delta}}$ can be verified using the steady-state international consumption aggregator (43) and the consumption demand curves (A.4).

[26] The individual price levels are not well-defined in steady state, but their ratio is.

Price Indices. The different price indices can be derived by combining the consumption demand curves previously derived with the different consumption aggregators. The consumption price index, the energy and non-energy price index, and the consumption import price index are given by $P_{kC,t} = \left[\widetilde{\beta}_k P_{kE,t}^{1-\gamma} + (1-\widetilde{\beta}_k) P_{kM,t}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}$, $P_{kE,t} = \left[\sum_{i \in I_E} \widetilde{\nu}_{ki} P_{kiC,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$, $P_{kM,t} = \left[\sum_{i \in I_M} \widetilde{\upsilon}_{ki} P_{kiC,t}^{1-\iota}\right]^{\frac{1}{1-\iota}}$, and $P_{kiC,t} = \left\{\sum_{l=1}^{K} \widetilde{\zeta}_{kl\,i}[(1+\tau_{kl\,i,t})P_{kl\,i,t}]^{1-\delta}\right\}^{\frac{1}{1-\delta}}$. Their log-linearized counterparts are given by

$$0 = \beta_k \widehat{p}_{kE,t} + (1-\beta_k)\widehat{p}_{kM,t} \tag{A.11}$$

$$\widehat{p}_{kE,t} = \sum_{i \in I_E} \nu_{kj} \widehat{p}_{kiC,t} \quad \text{and} \quad \widehat{p}_{kM,t} = \sum_{i \in I_M} \upsilon_{ki} \widehat{p}_{kiC,t} \tag{A.12}$$

$$\widehat{p}_{kiC,t} = \sum_{l=1}^{K} \zeta_{kl\,i}(\widehat{p}_{kl\,i,t} + \tau_{kl\,i,t}) \tag{A.13}$$

Intertemporal Household Problem. International financial markets are incomplete, $\mathcal{H} = 1$ in (4), with households in each country only having access to two risk-free bonds. More precisely, the household in country k has access to a domestic bond, and an internationally traded bond, $B_{k,t}^K$, issued, without loss of generality, by country K and denominated in country K's currency. The agent maximizes the present discounted value of per-period utility flows (A.1), with discount factor β, subject to her budget constraint,

$$\begin{aligned} &P_{kC,t}C_{k,t} + B_{k,t} + B_{k,t}^K \left[1 - \Gamma(\text{NFA}_{k,t}^K)\right]^{-1} \mathcal{E}_{kK,t} + \Xi_{k,t} \leq \\ &B_{k,t-1}(1+i_{k,t-1}) + B_{k,t-1}^K \mathcal{E}_{kK,t}(1+i_{K,t-1}) + \int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg + \Pi_{k,t} - T_{k,t} \end{aligned} \tag{A.14}$$

where $\int_0^1 W_{gk,t}\mathcal{N}_{gk,t}dg$ is the nominal labor income received by the representative household, $\text{NFA}_{k,t}^K = B_{k,t}^K \mathcal{E}_{kK,t}$ is the net foreign asset position of households in the country k, and where $\Gamma(x) = \gamma_* \left(\exp\left\{x/\mathcal{Y}_{k,t}\right\} - 1\right)$ is an external financial intermediary premium that depends on the economy-wide net holdings of internationally traded foreign bonds as a ratio to the national nominal GDP $\mathcal{Y}_{k,t}$, with $\gamma_* > 0$.[27] The incurred intermediation premium is rebated to households in a lump-sum manner through the fiscal instrument $\Xi_{k,t}$. Finally, $T_{k,t}$ denotes government transfers, also rebated to households in lump sum.

[27]The role of this intermediation premium is to stabilize the net foreign asset position in response to transitory shocks, a common practice in open economies with incomplete financial markets (Schmitt-Grohe and Uribe 2003). Furthermore, this specification guarantees that, in the non-stochastic steady state, households have no incentive to hold foreign bonds and the economy's net foreign asset position is zero.

The above program delivers two sets of different Euler conditions,

$$C_{k,t}^{-\sigma} = \mathbb{E}_t \beta C_{k,t+1}^{-\sigma} \frac{1+i_{k,t}}{1+\pi_{kC,t+1}} \frac{Z_{k,t+1}}{Z_{k,t}} \tag{A.15}$$

$$C_{k,t}^{-\sigma} = \mathbb{E}_t \beta C_{k,t+1}^{-\sigma} \frac{1+i_{K,t}}{1+\pi_{kC,t+1}} \left[1-\Gamma(\text{NFA}_{k,t}^{K})\right] \frac{\mathcal{E}_{kK,t+1}}{\mathcal{E}_{kK,t}} \frac{Z_{k,t+1}}{Z_{k,t}} \quad \forall k \neq K \tag{A.16}$$

where we assume that the (log-)demand shock follows an AR(1) process:

$$z_{k,t} = \rho_k^z z_{k,t-1} + \varepsilon_{k,t}^z \tag{A.17}$$

where $z_{k,t} := \log Z_{k,t}$, and $\varepsilon_{k,t}^z \sim \mathcal{N}\left(0, \sigma_{kz}^2\right)$.

The log-linearized version of the household's first-order conditions (A.15)-(A.16) are given by

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{k,t} - \mathbb{E}_t\pi_{kc,t+1}) + \mathbb{E}_t\widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho_k^z)z_{k,t} \tag{A.18}$$

$$\widehat{c}_{k,t} = -\frac{1}{\sigma}(i_{K,t} - \mathbb{E}_t\pi_{kC,t+1}) + \mathbb{E}_t\widehat{c}_{k,t+1} + \frac{1}{\sigma}(1-\rho_k^z)z_{k,t} - \frac{1}{\sigma}\mathbb{E}_t\Delta e_{kK,t+1} - \frac{1}{\sigma}\gamma_*\text{nfa}_{k,t}^K \quad \forall k \neq K \tag{A.19}$$

where we define the different log-linear NFA positions as $\text{nfa}_{k,t}^K = B_{k,t}^K \mathcal{E}_{kK,t}/\mathcal{Y}_k$ and $\text{nfa}_{k,t}^{\text{MU}} = B_{k,t}^{\text{MU}} \mathcal{E}_{k\text{MU},t}/\mathcal{Y}_k$ since $B_{k,t}^K = 0$ and $B_{k,t}^{\text{MU}} = 0$ in the steady state.

Combining the log-linearized first-order conditions for the holdings of domestic and internationally traded bonds (A.18)-(A.19), yields a risk-adjusted Uncovered Interest Parity (UIP) condition $i_{k,t} - i_{K,t}^* = \mathbb{E}_t\Delta e_{kK,t+1} + \gamma_*\text{nfa}_{k,t}^K$.

A.2. Firms

We augment the production function (39) to include a sectoral TFP shock $A_{ki,t}$,

$$Y_{fki,t} = A_{ki,t}\left[\widetilde{\alpha}_{ki}^{\frac{1}{\psi}} N_{fki,t}^{\frac{\psi-1}{\psi}} + \widetilde{\vartheta}_{ki}^{\frac{1}{\psi}} X_{fki,t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}, \tag{A.20}$$

where the (log-)TFP shock follows an AR(1) process:

$$a_{ki,t} \equiv \log A_{ki,t} = \rho_{ki}^a a_{ki,t-1} + \varepsilon_{ki,t}^a, \tag{A.21}$$

where $a_{ki,t} := \log A_{ki,t}$, and $\varepsilon_{ki,t}^a \sim \mathcal{N}\left(0, \sigma_{kia}^2\right)$.

Intermediate Input Demand Curves. The optimal allocation between labor and intermediate inputs is the result of a cost minimization programme min $W_{k,t}N_{fki,t} + P_{kiX,t}X_{fki,t}$ subject to (A.20). The optimal allocation between energy and non-energy intermediate goods is the result of a cost minimization programme min $P_{kiXE,t}X_{kiE,t} + P_{kiXM,t}X_{kiM,t}$ subject to (40). Similarly, the optimal allocation between energy (non-energy) intermediate inputs is the result of a cost minization programme min $\sum_{j\in I_E} P_{kijX,t}X_{kij,t}$ (min $\sum_{j\in I_M} P_{kijX,t}X_{kij,t}$) subject to (42). Finally, the optimal allocation between the different consumption goods is the result of a cost minimization programme min $\sum_{l=1}^{K}(1+\tau_{klj,t})P_{klj,t}X_{kl\,ij,t}$ subject to (43). The implied intermediate input demand curves are given by

(A.22)
$$W_{k,t} = \mathrm{MC}_{fki,t}A_{ki,t}^{\frac{\psi-1}{\psi}}\left(\frac{\widetilde{\alpha}_{ki}Y_{fki,t}}{N_{fki,t}}\right)^{\frac{1}{\psi}}$$

(A.23)
$$P_{kiX,t} = \mathrm{MC}_{ki,t}A_{ki,t}^{\frac{\psi-1}{\psi}}\left(\frac{\widetilde{\vartheta}_{ki}Y_{fki,t}}{X_{fki,t}}\right)^{\frac{1}{\psi}}$$

(A.24)
$$P_{kiXE,t} = P_{kiX,t}\left(\frac{\widetilde{\beta}_{ki}X_{ki,t}}{X_{kiE,t}}\right)^{\frac{1}{\phi}} \quad \text{and} \quad P_{kiXM,t} = P_{kiX,t}\left(\frac{(1-\widetilde{\beta}_{ki})X_{ki,t}}{X_{kiM,t}}\right)^{\frac{1}{\phi}}$$

(A.25)
$$P_{kijX,t} = P_{kiXE,t}\left(\frac{\widetilde{\nu}_{kij}X_{kiE,t}}{X_{kij,t}}\right)^{\frac{1}{\chi}} \quad \forall \quad j\in I_E \quad \text{and} \quad P_{kijX,t} = P_{kiXM,t}\left(\frac{\widetilde{\upsilon}_{kij}X_{kiM,t}}{X_{kij,t}}\right)^{\frac{1}{\xi}} \quad \forall \quad j\in I_M$$

(A.26)
$$(1+\tau_{klj,t})P_{klj,t} = P_{kijX,t}\left(\frac{\zeta_{kl\,ij}X_{kij,t}}{X_{kl\,ij,t}}\right)^{\frac{1}{\mu}} \quad \forall \quad l\in K$$

The log-linearized versions of the labor and intermediate inputs demand curves (A.22)-(A.26) are given by

(A.27)
$$\widehat{w}_{k,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi}a_{ki,t} + \frac{1}{\psi}\left(\widehat{y}_{fki,t} - \widehat{n}_{fki,t}\right)$$

(A.28)
$$\widehat{p}_{kiX,t} - \widehat{\mathrm{mc}}_{ki,t} = \frac{\psi-1}{\psi}a_{ki,t} + \frac{1}{\psi}\left(\widehat{y}_{fki,t} - \widehat{x}_{fki,t}\right)$$

(A.29)
$$\widehat{p}_{kiXE,t} - \widehat{p}_{kiX,t} = \frac{1}{\phi}\left(\widehat{x}_{ki,t} - \widehat{x}_{kiE,t}\right) \quad \text{and} \quad \widehat{p}_{kiXM,t} - \widehat{p}_{kiX,t} = \frac{1}{\phi}\left(\widehat{x}_{ki,t} - \widehat{x}_{kiM,t}\right)$$

(A.30)
$$\widehat{p}_{kijX,t} - \widehat{p}_{kiXE,t} = \frac{1}{\chi}\left(\widehat{x}_{kiE,t} - \widehat{x}_{kij,t}\right) \quad \forall \quad j\in I_E \quad \text{and} \quad \widehat{p}_{kijX,t} - \widehat{p}_{kiXM,t} = \frac{1}{\xi}\left(\widehat{x}_{kiM,t} - \widehat{x}_{kij,t}\right) \quad \forall \quad j\in I_M$$

(A.31)
$$\tau_{klj,t} + \widehat{p}_{klj,t} - \widehat{p}_{kijX,t} = \frac{1}{\mu}\left(\widehat{x}_{kij,t} - \widehat{x}_{kl\,ij,t}\right) \quad \forall \quad l\in K$$

where $\widehat{w}_{k,t} = w_{k,t} - p_{kC,t}$, $\widehat{\mathrm{mc}}_{ki,t} = \widehat{\mathrm{mc}}^{n}_{ki,t} - p_{kC,t}$, $\widehat{p}_{kiX,t} = p_{kiX,t} - p_{kC,t}$, $\widehat{p}_{kiXE,t} = p_{kiXE,t} - p_{kC,t}$, $\widehat{p}_{kiXM,t} = p_{kiXM,t} - p_{kC,t}$, $\widehat{p}_{kijX,t} = p_{kijX,t} - p_{kC,t}$, and $\widehat{p}_{klj,t} = p_{klj,t} - p_{kC,t}$ are well-defined as a ratio of prices.

Intermediate Inputs Baskets. The log-linearized intermediary input aggregator (40) is given by

$$\widehat{x}_{ki,t} = \beta_{ki}\widehat{x}_{kiE,t} + (1-\beta_{ki})\widehat{x}_{kiM,t} \tag{A.32}$$

where $\beta_{ki} = \frac{P_{kiXE}X_{kiE}}{P_{kiX}X_{ki}} = \widetilde{\beta}_{ki}^{\frac{1}{\phi}}\left(\frac{X_{kiE}}{X_{ki}}\right)^{\frac{\phi-1}{\phi}}$ and $(1-\beta_{ki}) = \frac{P_{kiXM}X_{kiM}}{P_{kiM}X_{ki}} = (1-\widetilde{\beta}_{ki})^{\frac{1}{\phi}}\left(\frac{X_{kiM}}{X_{ki}}\right)^{\frac{\phi-1}{\phi}}$ can be verified using the steady-state intermediate input aggregator (40) and the input demand curves (A.24).

The log-linearized versions of the energy and non-energy intermediate input aggregators (42) are given by

$$\widehat{x}_{kiE,t} = \sum_{j\in I_E}\nu_{kij}\widehat{x}_{kij,t} \qquad \text{and} \qquad \widehat{x}_{kiM,t} = \sum_{j\in I_M}\upsilon_{kij}\widehat{x}_{kij,t} \tag{A.33}$$

where $\nu_{kij} = \frac{P_{kijX}X_{kij}}{P_{kiXE}X_{kiE}} = \widetilde{\nu}_{kij}^{\frac{1}{\chi}}\left(\frac{X_{kij}}{X_{kiE}}\right)^{\frac{\chi-1}{\chi}}$ and $\upsilon_{kij} = \frac{P_{kijX}X_{kij}}{P_{kiXM}X_{kiM}} = \widetilde{\upsilon}_{kij}^{\frac{1}{\xi}}\left(\frac{X_{kij}}{X_{kiM}}\right)^{\frac{\xi-1}{\xi}}$ can be verified using the steady-state energy and non-energy intermediate input aggregators (42) and the demand curves (A.25).

The log-linearized version of the final layer of the intermediate input aggregator, (43), is given by

$$\widehat{x}_{kij,t} = \sum_{l=1}^{K}\zeta_{klij}\widehat{x}_{klij,t} \tag{A.34}$$

where $\zeta_{klij} = \frac{P_{klj}X_{klij}}{P_{kijX}X_{kij}} = \widetilde{\zeta}_{klij}^{\frac{1}{\mu}}\left(\frac{X_{klij}}{X_{kij}}\right)^{\frac{\mu-1}{\mu}}$ can be verified using the steady-state international intermediate input aggregators (43) and the demand curve (A.26).

Price Indices. The different price indices can be derived by combining the intermediate input demand curves previously derived with the other intermediate input aggregators. The marginal cost of production, the intermediate input price index, the energy and non-energy input price index, and the input import price index are given by $MC_{ki,t} = A^{-1}_{ki,t}\left[\widetilde{\alpha}_{ki}W^{1-\psi}_{k,t} + \widetilde{\vartheta}_{ki}P^{1-\psi}_{kiX,t}\right]^{\frac{1}{1-\psi}}$, $P_{kiX,t} = \left[\widetilde{\beta}_{ki}P^{1-\phi}_{kiXE,t} + (1-\widetilde{\beta}_{ki})P^{1-\phi}_{kiXM,t}\right]^{\frac{1}{1-\phi}}$,

$P_{kiXE,t} = \left[\sum_{j\in I_E}\widetilde{\nu}_{kij}P_{kijX,t}^{1-\chi}\right]^{\frac{1}{1-\chi}}$, $P_{kiXM,t} = \left[\sum_{j\in I_M}\widetilde{\upsilon}_{kij}P_{kijX,t}^{1-\xi}\right]^{\frac{1}{1-\chi}}$, and $P_{kijX,t} = \left\{\sum_{l=1}^{K}\widetilde{\zeta}_{kl\,ij}[(1+\tau_{kl\,j,t})P_{kl\,j,t}]^{1-\mu}\right\}^{\frac{1}{1-\mu}}$. Their log-linearized counterparts are given by

$$\widehat{\mathrm{mc}}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}}\widehat{w}_{k,t} + \mathcal{M}_{ki}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}}\widehat{p}_{kiX,t} \tag{A.35}$$

$$\widehat{p}_{kiX,t} = \widetilde{\beta}_{ki}\widehat{p}_{kiXE,t} + (1-\widetilde{\beta}_{ki})\widehat{p}_{kiXM,t} \tag{A.36}$$

$$\widehat{p}_{kiXE,t} = \sum_{j\in I_E}\nu_{kij}\widehat{p}_{kijX,t} \quad \text{and} \quad \widehat{p}_{kiXM,t} = \sum_{j\in I_M}\upsilon_{kij}\widehat{p}_{kijX,t} \tag{A.37}$$

$$\widehat{p}_{kijX,t} = \sum_{l=1}^{K}\zeta_{kl\,ij}(\widehat{p}_{kl\,j,t} + \tau_{kl\,j,t}) \tag{A.38}$$

where $\mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} = \left(\frac{W_k}{\mathrm{MC}_{ki}}\right)^{1-\psi}\widetilde{\alpha}_{ki}$ and $\mathcal{M}_{ki}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}} = \left(\frac{P_{kiX}}{\mathrm{MC}_{ki}}\right)^{1-\psi}\widetilde{\vartheta}_{ki}$ can be derived using (A.22)-(A.23) in steady-state.

Production Structure. The log-linearized version of the production function (A.20) is given by

$$\widehat{y}_{fki,t} = a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\widehat{n}_{fki,t} + \mathcal{M}_{ki}\vartheta_{ki}\widehat{x}_{fki,t} \tag{A.39}$$

where the identities $\mathcal{M}_{ki}\alpha_{ki} = \mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} = \frac{N_{ki}}{Y_{ki}}\left(\frac{\widetilde{\alpha}_{ki}Y_{ki}}{N_{ki}}\right)^{\frac{1}{\psi}}$ and $\mathcal{M}_{ki}\vartheta_{ki} = \mathcal{M}_{ki}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}} = \frac{X_{ki}}{Y_{ki}}\left(\frac{\widetilde{\vartheta}_{ki}Y_{ki}}{X_{ki}}\right)^{\frac{1}{\psi}}$ can be verified using the first-order conditions from the firms' problem (A.22) and (A.23) in steady-state and the standard monopolistic competition pricing condition in steady-state, $P_{ki} = \mathcal{M}_{ki}\mathrm{MC}_{ki}^{n}$. Using the previous identities, together with the production function (A.20) in steady-state, one can verify that

$$\mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}} = \frac{W_k N_{ki}}{\mathrm{MC}_{ki}^{n}Y_{ki}} + \frac{P_{kiX}X_{ki}}{\mathrm{MC}_{ki}^{n}Y_{ki}} = 1. \tag{A.40}$$

A.3. Price–Setting

We extend our framework to allow for a time-varying elasticity of substitution between different good varieties, in order to micro-found price cost-push shocks. We extend (3) to

$$Y_{ki,t} = \left(\int_0^1 Y_{fki,t}^{\frac{\epsilon_{pk,t}-1}{\epsilon_{pk,t}}}\,df\right)^{\frac{\epsilon_{pk,t}}{\epsilon_{pk,t}-1}}, \tag{A.41}$$

The implied sectoral price index is

$$P_{ki,t} = \left(\int_0^1 P_{fki,t}^{1-\epsilon_{pki,t}} df \right)^{\frac{1}{1-\epsilon_{pki,t}}} . \tag{A.42}$$

Producers of each differentiated variety face the demand function

$$Y_{ik,t+l|t} = \left(\frac{P_{fki,t}}{P_{ki,t+l}} \right)^{-\epsilon_{pki,t}} Y_{ki,t+l} \tag{A.43}$$

Firms set prices à la Calvo, which implies that the aggregate price dynamics are described by the equation

$$\Pi_{ki,t}^{1-\epsilon_{pki,t}} = \theta_{ki}^{p} + (1-\theta_{ki}^{p}) \left(\frac{P_{ki,t}^{*}}{P_{ki,t-1}} \right)^{1-\epsilon_{pki,t}} \tag{A.44}$$

Log-linearized: $\pi_{ki,t} = (1-\theta_{ki}^{p})(p_{ki,t}^{*} - p_{ki,t-1})$. A firm that resets its price at time t faces the following problem $\max_{P_{ki,t}^{*}} \sum_{l=0}^{\infty} \theta_{ki}^{pl} \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+l}}{P_{ki,t+l}} [P_{ki,t}^{*} Y_{ki,t+l|t} - \mathcal{C}_{ki,t+l}(Y_{ki,t+l|t})] \right\}$ subject to the sequence of demand constraints (A.43). The optimality condition associated with the problem takes the form

$$\sum_{l=0}^{\infty} \theta_{ki}^{pl} \mathbb{E}_t \left\{ \frac{\Lambda_{t,t+l} Y_{ki,t+l|t}}{P_{ki,t+l}} [P_{ki,t}^{*} - \mathcal{M}_{ki,t} \mathrm{MC}_{ki,t+l|t}^{n}] \right\} = 0 \tag{A.45}$$

where $\mathrm{MC}_{ki,t+l|t}^{n}$ denotes the nominal marginal cost in period $t+l$ for a firm which last reset its price in period t, and $\mathcal{M}_{ki,t} = \frac{\epsilon_{pki,t}}{\epsilon_{pki,t}-1}$. A first-order Taylor expansion of (A.45) around the zero inflation steady state yields

$$p_{ki,t}^{*} = (1-\beta\theta_{ki}^{p}) \sum_{l=0}^{\infty} (\beta\theta_{ki}^{p}) \mathbb{E}_t \left(\mathrm{mc}_{ki,t+l|t}^{n} + \mu_{ki,t}^{n} \right) \tag{A.46}$$

where $\mathrm{mc}_{ki,t+l|t}^{n} \equiv \log \mathrm{MC}_{ki,t+l|t}^{n}$ is the log marginal cost, and $\mu_{ki,t}^{n} := \log \mathcal{M}_{ki,t}$ is the log of the desired gross markup.

The log marginal cost for an individual firm that last set its price in period t is given by

$$\mathrm{mc}_{ki,t+l|t}^{n} = w_{k,t+l} - \frac{\psi-1}{\psi} a_{ki,t+l} - \frac{1}{\psi} \left[\log \tilde{\alpha}_{ki} + y_{ki,t+l|t} - n_{ki,t+l|t} \right] ,$$

where $n_{ki,t+l|t}$ denotes the log employment in period $t+l$ for a firm that last reset its price in period t, and where we have made use of (A.27).

Letting $\mathrm{mc}^n_{ki,t} = \int_0^1 \mathrm{mc}^n_{fki,t}\, df = (1-\theta^p_{ki}) \sum_{l=0}^{\infty} \theta^{pl}_{ki}\, \mathrm{mc}^n_{ki,t|t-l}$ represent the log average marginal cost, it follows that

$$\mathrm{mc}^n_{ki,t} = w_{k,t} - \frac{\psi-1}{\psi} a_{ki,t} - \frac{1}{\psi}\left[\log\tilde{\alpha}_{ki} + y_{ki,t} - n_{ki,t}\right].$$

Thus, the following relation holds between firm-specific and economy-wide marginal costs

$$\mathrm{mc}^n_{ki,t+l|t} = \mathrm{mc}^n_{ki,t+l} - \frac{1}{\psi}\left[\left(y_{ki,t+l|t} - y_{ki,t+l}\right) - \left(n_{ki,t+l|t} - n_{ki,t+l}\right)\right].$$

Notice that, making use of both marginal cost expressions (A.22)-(A.23), the identity $x_{ki,t+l|t} - x_{ki,t+l} = n_{ki,t+l|t} - n_{ki,t+l}$ is satisfied. Hence, we can write

$$y_{ki,t+l|t} - y_{ki,t+l} = \left[\mathcal{M}_{ki}\frac{W_k N_{ki}}{P_{ki}Y_{ki}} + \mathcal{M}_{ki}\frac{P_{kiX}X_{ki}}{P_{ki}Y_{ki}}\right](n_{ki,t+l|t} - n_{ki,t+l}) = (n_{ki,t+l|t} - n_{ki,t+l}),$$

where we have used the identity (A.40), and where we have used the linearized production function (A.39). Hence, we can finally write the relation between marginal costs as $\mathrm{mc}^n_{ki,t+l|t} = \mathrm{mc}^n_{ki,t+l}$.

Introducing this last expression into the log-linearized firms' FOC (A.46), we can write

$$p^*_{ki,t} = (1-\beta\theta^p_{ki})\sum_{l=0}^{\infty}(\beta\theta^p_{ki})\mathbb{E}_t\left[p_{ki,t+l} - \left(\hat{\mu}_{ki,t+l} - \hat{\mu}^n_{ki,t+l}\right)\right],$$

where $\hat{\mu}_{ki,t} \equiv \mu_{ki,t} - \mu_{ki}$ is the deviation between the average and desired markups, with $\mu_{ki,t} = p_{ki,t} - \mathrm{mc}^n_{ki,t}$, and $\hat{\mu}^n_{ki,t} \equiv \mu^n_{ki,t} - \mu_{ki}$. Combining the (linearized) inflation dynamics (A.44) with the above expression, we can write

$$\pi_{ki,t} = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \hat{p}_{ki,t}\right) + \beta\mathbb{E}_t\pi_{ki,t+1} + u^p_{ki,t}, \tag{A.47}$$

where $\pi_{ki,t} = p_{ki,t} - p_{ki,t-1}$ denotes price inflation in sector i, and the price variables (real marginal costs $\widehat{\mathrm{mc}}_{ki,t} = \widehat{\mathrm{mc}}^n_{ki,t} - p_{kC,t}$ and the real price level $\hat{p}_{ki,t} = p_{ki,t} - p_{kC,t}$) appear in real terms so that they are stationary. Furthermore, $\kappa_{ki} = (1-\theta^p_{ki})(1-\beta\theta^p_{ki})/\theta^p_{ki}$.

An equivalent log-linearization of the LCP condition (15) yields:

$$\pi^l_{ki,t} = \kappa_{ki}\left(\widehat{\mathrm{mc}}_{ki,t} - \hat{p}^l_{ki,t} - \hat{q}_{kl,t}\right) + \beta\mathbb{E}_t\pi^l_{ki,t+1} + u^p_{ki,t}, \tag{A.48}$$

where $\pi^l_{ki,t}$ denotes the export price inflation produced in country k and sold to country l, and $\hat{q}_{kl,t}$ denotes the log-deviation of the real exchange rate between country k and country

l:

$$\mathcal{Q}_{kl,t} = \frac{P_{lC,t}\mathcal{E}_{kl,t}}{P_{kC,t}}. \tag{A.49}$$

We assume that the sectoral price cost-push shocks $u^p_{ki,t} = \kappa_{ki}\widehat{\mu}^n_{ki,t}$, micro-founded through a time-varying elasticity of substitution $\epsilon_{pki,t}$ in (3), follow independent AR(1) processes:

$$u^p_{ki,t} = \rho^p_{ki} u^p_{ki,t-1} + \varepsilon^p_{ki,t}, \tag{A.50}$$

where $u^p_{ki,t} \sim \mathcal{N}\left(0, \sigma^2_{kip}\right)$.

In this open input-output (IO) economy, the price Phillips curve (A.47) depends on the international supply network through the real marginal costs faced by firm i in country k, $\widehat{\text{mc}}_{ki,t}$. Combining the log-linearized intermediate input prices indices (A.35)-(A.38), we obtain the marginal cost equation,

$$\widehat{\text{mc}}_{ki,t} = -a_{ki,t} + \mathcal{M}_{ki}\alpha_{ki}\widehat{w}_{k,t} + \sum_{l=1}^{K}\sum_{j=1}^{I} \mathcal{M}_{ki}\omega_{klij}\widehat{p}_{klij,t} \tag{A.51}$$

where, in the absence of a production subsidy, $\alpha_{ki} = \frac{W_k N_{ki}}{P_{ki}Y_{ki}} = \frac{W_k N_{ki}}{\mathcal{M}_{ki}\text{MC}_{ki}Y_{ki}}$ denotes the (steady-state) labor income share of total sales of firm i, $\omega_{klij} = \frac{P_{klj}X_{klij}}{P_{ki}Y_{ki}} = \frac{P_{klj}X_{klij}}{\mathcal{M}_{ki}\text{MC}_{ki}Y_{ki}}$ denotes the (steady-state) IO expenditure share of total sales of firm i, and $\mathcal{M}_{ki} = \epsilon_{pki}/(\epsilon_{pki} - 1)$ denotes the steady-state markup charged by firm i.

Note that sectoral-inflation rates and sectoral-level real prices ($\widehat{p}_{ki,t}$) are related through the identity

$$\pi_{ki,t} = \widehat{p}_{ki,t} - \widehat{p}_{ki,t-1} + \pi_{kC,t}. \tag{A.52}$$

Writing (A.11)-(A.13) in first-differences, we can obtain consumer price inflation,

$$\pi_{kC,t} = \sum_{i=1}^{I}\sum_{l=1}^{K} \beta_{kli}\pi_{kli,t} \tag{A.53}$$

where $\beta_{kli} = \frac{P_{kli}C_{kli}}{P_{kC}C_k} = \zeta_{kli}\left[\nu_{ki}\beta_k \mathbb{1}_{\{i\in I_E\}} + \upsilon_{ki}(1-\beta_k)\left(1-\mathbb{1}_{\{i\in I_E\}}\right)\right]$.

A.4. Wage–Setting

Following Erceg *et al.* (2000), wage stickiness is introduced in a way analogous to price stickiness. Labor unions specialized in any given labor type can reset their nominal wage only with probability $1-\theta_k^w$ each period, independently of the time elapsed since they last adjusted their wage. We assume that firms employ a continuum of differentiated labor services. In particular, $N_{fki,t}$ is an index of labor input used by firm f, and defined by

$$N_{fki,t} = \left(\int_0^1 N_{fgki,t}^{\frac{\epsilon_{wk}-1}{\epsilon_{wk}}} dg\right)^{\frac{\epsilon_{wk}}{\epsilon_{wk}-1}}, \tag{A.54}$$

where $N_{fgki,t}$ denotes the quantity of type-g labor employed by firm f in period t. Note that $\epsilon_{wk,t}$ represents the elasticity of substitution among labor varieties. Note also the assumption of a continuum of labor types, indexed by $g \in [0,1]$.

Let $W_{gk,t}$ denote the nominal wage for type-g labor prevailing in period t. Nominal wages are set by workers of each type (or a union representing them) and taken as given by firms. Given the wages effective at any point in time for the different types of labor services, cost minimization yields a corresponding set of demand schedules for each firm f and labor type g, given the firm's total employment $N_{fk,t}$,

$$N_{fgki,t} = \left(\frac{W_{gk,t}}{W_{k,t}}\right)^{-\epsilon_{wk,t}} N_{fki,t}, \tag{A.55}$$

where

$$W_{k,t} \equiv \left(\int_0^1 W_{gk,t}^{1-\epsilon_{wk,t}} dg\right)^{\frac{1}{1-\epsilon_{wk,t}}} \tag{A.56}$$

is an aggregate wage index. Combining the previous conditions, one can obtain a convenient aggregation result, $\int_0^1 W_{gk,t} N_{fgki,t} dg = W_{k,t} N_{fki,t}$. That is, the wage bill of any given firm can be expressed as the product of the wage index and the firm's employment index.

Consider a union resetting its members' wage in period t, and let $W_{k,t}^*$ denote the newly set wage. The union chooses $W_{k,t}^*$ in a way consistent with utility maximization of its members' households, taking as given the decisions of other unions as well as the paths of aggregate consumption and prices. Specifically, the union seeks to maximize

$$\max_{W_{k,t}^*} \mathbb{E}_t \sum_{l=0}^{\infty} (\beta\theta_k^w)^l \left(\frac{C_{k,t+l} W_{k,t}^* N_{k,t+l|t}}{P_{t+l}^c} - \frac{N_{k,t+l|t}^{1+\varphi}}{1+\varphi}\right)$$

subject to the sequence of labor demand schedules

$$N_{k,t+l|t} = \left(\frac{W^*_{k,t}}{W_{k,t+l}}\right)^{-\epsilon_{wk,t}} \int_0^1 N_{gk,t} dg,$$

where $N_{k,t+l|t}$ denotes the level of employment in period $t+l$ among workers that last reset their wage in period t. The first-order condition is given by

$$\sum_{l=0}^{\infty} (\beta\theta^w_k)^l \mathbb{E}_t \left[N_{k,t+l|t} C^{-\sigma}_{t+l} \left(\frac{W^*_{k,t}}{P^c_{t+l}} - \mathcal{M}_{wk,t} \text{MRS}^{\varphi}_{k,t+l|t} \right) \right] = 0,$$

where $\mathcal{M}_{wk,t} = \frac{\epsilon_{wk,t}}{\epsilon_{wk,t}-1}$, and $\text{MRS}_{k,t+l|t} = C^{-\sigma}_{t+l} N^{\varphi}_{k,t+l|t}$ denotes the marginal rate of substitution between household consumption and employment in period $t+l$ relevant to the workers resetting their wage in period t. Log-linearizing the above expression around a zero inflation steady-state yields the wage setting rule

$$w^*_{k,t} = (1-\beta\theta^w_k) \sum_{l=0}^{\infty} (\beta\theta^w_k)^l \mathbb{E}_t \left(\text{mrs}_{t+l|t} + \mu^n_{wk,t} + p_{kc,t+l} \right) \tag{A.57}$$

where $\mu^n_{wk,t} = \log \mathcal{M}_{wk,t}$ and $\text{mrs}_{t+l|t} = \sigma c_{k,t+l} + \varphi n_{k,t+l|t}$.

Letting $\text{mrs}_{t+l} = \sigma c_{k,t+l} + \varphi n_{k,t+l}$ define the economy's average marginal rate of substitution, where $n_{k,t+l} = \log \int_0^1 \int_0^1 N_{fgk} df dg$ denotes the log aggregate employment. Up to a first-order approximation,

$$\text{mrs}_{t+l|t} = \text{mrs}_{t+l} + \varphi(n_{k,t+l} - n_{k,t+l|t}) = \text{mrs}_{t+l} - \epsilon_{wk}\varphi(w^*_{k,t} - w_{k,t+l}).$$

Hence, we can write (A.57) as

$$w^*_{k,t} = \frac{1-\beta\theta^w_k}{1+\epsilon_{wk}\varphi} \sum_{l=0}^{\infty} (\beta\theta^w_k)^l \mathbb{E}_t \left[(1+\epsilon_{wk}\varphi) w_{k,t+l} - \left(\widehat{\mu}_{wk,t+l} - \widehat{\mu}^n_{wk,t+l} \right) \right] \tag{A.58}$$

where $\widehat{\mu}_{wk,t+l} = \mu_{wk,t+l} - \mu_{wk}$ denotes the deviations of the economy's log average wage markup $\mu_{wk,t+l} = w_{k,t+k} - p_{kc,t+l} - \text{mrs}_{k,t+l}$ from its steady-state level, and $\widehat{\mu}^n_{wk,t+l} = \mu^n_{wk,t+l} - \mu_{wk}$.

Given the assumed wage setting structure, the evolution of the aggregate wage index is given by

$$W_{k,t} = \left(\theta^w_k W^{1-\epsilon_{wk}}_{k,t-1} + (1-\theta^w_k)(W^*_{k,t})^{1-\epsilon_{wk}} \right)^{\frac{1}{1-\epsilon_{wk}}}.$$

Log-linearized,

$$w_{k,t} = \theta^w_k w_{k,t-1} + (1-\theta^w_k) w^*_{k,t}.$$

Combining the last expression with (A.58), and letting $\pi_{wk,t} = w_{k,t} - w_{k,t-1}$, we obtain the wage inflation equation:

$$\pi_{wk,t} = \kappa_{wk}\left(\sigma\widehat{c}_{k,t} + \varphi\widehat{n}_{k,t} - \widehat{w}_{k,t}\right) + \beta\mathbb{E}_t\pi_{wk,t+1} + u^w_{ki,t}, \tag{A.59}$$

where $\pi_{wk,t} = w_{k,t} - w_{k,t-1} = \widehat{w}_{k,t} - \widehat{w}_{k,t-1} + \pi_{Ck,t}$ denotes wage inflation, with , $\widehat{w}_{k,t} = w_{k,t} - p_{kC,t}$ denoting the real wage; $\widehat{\mu}_{wk,t} = \widehat{w}_{k,t} - \sigma\widehat{c}_{k,t} - \varphi\widehat{n}_{k,t}$, where both aggregate consumption $\widehat{c}_{k,t}$ and employment $\widehat{n}_{k,t}$ appear in log-deviations from their steady-state values, and $u^w_{k,t} = \kappa_{wk}\widehat{\mu}^n_{wk,t}$. We assume that the wage cost-push shock, micro-founded through a time-varying elasticity of substitution in the labor demand aggregator, follows an AR(1) processes:

$$u^w_{k,t} = \rho^w_k u^w_{k,t-1} + \varepsilon^w_{k,t} \tag{A.60}$$

where $u^w_{k,t} \sim \mathcal{N}\left(0, \sigma^2_{kw}\right)$.

A.5. Monetary Authority

The log-linearized bilateral nominal exchange rate (47) is given by $e_{k,k^{MU},t} = e_{k,k^{MU}}, \forall k \in K^{MU}$. In stationary terms, taking first differences, this can be written as

$$\Delta e_{k,k^{MU},t} = 0 \quad \forall k \in K^{MU} \tag{A.61}$$

Log-linearizing the expression for the real exchange rate (A.49) and first-differencing, we obtain

$$\Delta q_{kl,t} = \Delta e_{kl,t} + \pi_{l,t} - \pi_{k,t}. \tag{A.62}$$

Similarly, log-linearizing and first-differencing the symmetry of nominal exchange rates condition $\mathcal{E}_{kl,t} = \mathcal{E}^{-1}_{lk,t}$ yields

$$\Delta e_{kl,t} = -\Delta e_{lk,t} \tag{A.63}$$

A.6. Market Clearing, GDP, and Trade Balance

Market Clearing. We first consider the goods market clearing condition (17). Pre-multiplying by $\frac{P_{ki,t}}{P_{k,t}C_{k,t}} = \frac{P_{ki,t}}{E_{k,t}}$, and making use of (A.9),

$$
\begin{aligned}
\frac{P_{ki,t}Y_{ki,t}}{E_{k,t}} &= \sum_{l=1}^{K} \frac{P_{ki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}X_{lkji,t}}{E_{k,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P_{lki,t}} \frac{P_{lki,t}C_{lki,t}}{E_{k,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P_{lki,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{k,t}} \frac{P_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \\
&= \sum_{l=1}^{K} \frac{P_{ki,t}}{P_{lki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lki,t}C_{lki,t}}{E_{l,t}} + \sum_{l=1}^{K}\sum_{j=1}^{I} \frac{P_{ki,t}}{P_{lki,t}} \frac{E_{l,t}}{E_{k,t}} \frac{P_{lj,t}Y_{lj,t}}{E_{l,t}} \frac{P_{lki,t}X_{lkji,t}}{P_{lj,t}Y_{lj,t}} \quad \forall i \in I
\end{aligned} \tag{A.64}
$$

which we can write in steady-state as

$$
\lambda_{ki} = \sum_{l=1}^{K} y_{lk}\beta_{lki} + \sum_{l=1}^{K}\sum_{j=1}^{I} y_{lk}\lambda_{lj}\omega_{lkji} \quad \forall i \in I
$$

where the Domar weight for sector i in country k is $\lambda_{ki} = \frac{P_{ki}Y_{ki}}{y_k}$, the nominal GDP ratio between countries l and k is defined as $y_{lk} = \frac{P_{lC}C_l}{P_{kC}C_k}$, and the IO share is given by $\omega_{lkji} = \frac{P_{lki}X_{lkji}}{P_{lj}Y_{lj}} = \frac{P_{ki}X_{lkji}}{P_{lj}Y_{lj}} = \zeta_{lkji}\vartheta_{lj}\left[\nu_{lji}\beta_{lj}\mathbb{1}_{\{i\in I_E\}} + \upsilon_{lji}(1-\beta_{lj})\left(1-\mathbb{1}_{\{i\in I_E\}}\right)\right]$, where we have made use of the law of one price in steady-state, $P_{klj} = P_{lj}$. Notice that β_{lki} and ω_{lkji} can be extracted directly from the data.

Hence, we can write the log-linearized version of the goods market clearing condition (17), $Y_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^{K}\left(C_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I} X_{lkji}\widehat{x}_{lkji}\right)$. Pre-multiplying the expression by P_{ki}/E_k, we can write

$$
\lambda_{ki}\widehat{y}_{ki,t} = \sum_{l=1}^{K} y_{lk}\left(\beta_{lki}\widehat{c}_{lki,t} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\widehat{x}_{lkji}\right)
$$

where we have pre-multiplied the first expression by $\frac{P_{ki}}{E_k}$.

The log-linearized version of the labor market clearing condition (18) is given by

$$
\widehat{n}_{k,t} = \sum_{i=1}^{I} \delta_{ki}\widehat{n}_{ki,t} \tag{A.65}
$$

where $\delta_{ki} = \frac{N_{ki}}{N_k} = \frac{W_k N_{ki}}{P_{ki}Y_{ki}} \frac{P_{ki}Y_{ki}}{P_{kC}C_k} \frac{P_{kC}C_k}{W_k N_k} = \frac{\alpha_{ki}\lambda_{ki}}{1-\sum_{j=1}^{I}\left(1-\frac{\psi_{kj}}{\mathcal{M}_{kj}}\right)\lambda_{kj}}$ can be derived using (A.70).

The NFA from the "global" country K (19) can be log-linearized to

$$\text{(A.66)} \qquad \frac{1}{\beta}\sum_{k=1}^{K-1} \text{nfa}_{k,t-1}^{K} - \sum_{k=1}^{K-1} \text{nfa}_{k,t}^{K} = \Upsilon_K \left(\widehat{\text{exp}}_{K,t} - \widehat{\text{imp}}_{K,t} + \widehat{p}_{K\text{EXP},t} - \widehat{p}_{K\text{IMP},t}\right),$$

the NFA from country $k \neq K$ $k \notin$ MU, (19) can be log-linearized to

$$\text{(A.67)} \qquad \text{nfa}_{k,t}^{K} - \frac{1}{\beta}\text{nfa}_{k,t-1}^{K} = \Upsilon_k \left(\widehat{\text{exp}}_{k,t} - \widehat{\text{imp}}_{k,t} + \widehat{p}_{k\text{EXP},t} - \widehat{p}_{k\text{IMP},t}\right)$$

where the linearized export and import price deflators are given by:

$$\widehat{p}_{k\text{IMP},t} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\frac{P_{kli}C_{kli} + \sum_{j=1}^{I} P_{kli}X_{klji}}{P_{k,IMP}IMP_k}(\widehat{p}_{kli,t} + \tau_{kli,t})\right]$$

$$\text{(A.68)} \qquad = \sum_{l\neq k}\sum_{i=1}^{I}\Upsilon_k^{-1}\left(\beta_{kli} + \sum_{j=1}^{I}\lambda_{kj}\omega_{klji}\right)(\widehat{p}_{kli,t} + \tau_{kli,t})$$

$$\widehat{p}_{k\text{EXP},t} = \sum_{l\neq k}\sum_{i=1}^{I}\left[\frac{P_{ki}C_{lki} + \sum_{j=1}^{I} P_{ki}X_{lkji}}{P_{k,EXP}EXP_k}(\widehat{p}_{lki,t}^{k} + \tau_{lki,t})\right]$$

$$\text{(A.69)} \qquad = \sum_{l\neq k}\sum_{i=1}^{I}\frac{\mathcal{Y}_{lk}}{\Upsilon_k}\left(\beta_{lki} + \sum_{j=1}^{I}\lambda_{lj}\omega_{lkji}\right)(e_{kl,t} + \widehat{p}_{lki,t} + \tau_{lki,t})$$

Gross Domestic Product and Net Exports. Let us now move to nominal GDP (22). In steady state, assuming zero net exports, $P_{k\text{EXP}}\text{EXP}_k - P_{k\text{IMP}}\text{IMP}_k = 0$, we can write $\mathcal{Y}_k = P_{kC}C_k$. Using the household's budget constraint (A.14) in steady state, we can write

$$\text{(A.70)} \qquad \mathcal{Y}_k = P_{kC}C_k = W_k N_k + \Pi_k = W_k N_k + \sum_{i=1}^{I}\left(1 - \frac{1}{\mathcal{M}_{ki}}\right)P_{ki}Y_{ki}$$

where the last equality makes use of (A.40).

Log-linearizing the real GDP (23) definition,

$$\widehat{y}_{k,t} = \frac{P_{kC}C_k}{\mathcal{Y}_k}\widehat{c}_{k,t} + \frac{P_{k\text{EXP}}\text{EXP}_k}{\mathcal{Y}_k}\widehat{\text{exp}}_{k,t} - \frac{P_{k\text{IMP}}\text{IMP}_k}{\mathcal{Y}_k}\widehat{\text{imp}}_{k,t} = \widehat{c}_{k,t} + \Upsilon_k\left(\widehat{\text{exp}}_{k,t} - \widehat{\text{imp}}_{k,t}\right)$$

where second equality uses that nominal consumption expenditures will be equal nominal GDP in steady state, and $\Upsilon_k = P_{k\text{EXP}}\text{EXP}_k/\mathcal{Y}_k = P_{k\text{IMP}}\text{IMP}_k/\mathcal{Y}_k$ is the export (or import) share of nominal GDP.

The nominal exports expression (20) can be log-linearized to:

$$\widehat{\exp}_{k,t} = \sum_{l \neq k} \sum_{i \in I} \left(\frac{P_{ki} C_{lki}}{P_{k\text{EXP}} \text{EXP}_k} \hat{c}_{lki,t} + \sum_{j=1}^{I} \frac{P_{ki} X_{lki}}{P_{k\text{EXP}} \text{EXP}_k} \hat{x}_{lkji,t} \right)$$

$$= \sum_{l \neq k} \sum_{i \in I} \frac{y_{lk}}{\Upsilon_k} \left(\beta_{lki} \hat{c}_{lki,t} + \sum_{j=1}^{I} \lambda_{lj} \omega_{lkji} \hat{x}_{lkji,t} \right) \tag{A.71}$$

where the export share of nominal GDP is given by

(A.72)

$$\Upsilon_k = \frac{P_{k\text{EXP}} \text{EXP}_k}{y_k} = \sum_{l \neq k} \sum_{i=1}^{I} y_{lk} \left[\beta_{lki} + \sum_{j=1}^{I} \lambda_{lj} \omega_{lkji} \right] = \left(\sum_{i=1}^{I} \lambda_{ki} \right) - \left(\beta_{kki} + \sum_{j=1}^{I} \lambda_{kj} \omega_{kkji} \right)$$

(A.73)

$$= \frac{P_{k\text{IMP}} \text{IMP}_k}{y_k} = \sum_{l \neq k} \sum_{i=1}^{I} \left[\beta_{kli} + \sum_{j=1}^{I} \lambda_{kj} \omega_{klji} \right]$$

Similarly, the nominal imports expression (21) can be log-linearized to

$$\widehat{\text{imp}}_{k,t} = \sum_{l \neq k} \sum_{i \in I} \left(\frac{P_{kli} C_{kli}}{P_{k\text{IMP}} \text{IMP}_k} \hat{c}_{kli,t} + \sum_{j=1}^{I} \frac{P_{kli} X_{klji}}{P_{k\text{IMP}} \text{IMP}_k} \hat{x}_{klji,t} \right)$$

$$= \sum_{l \neq k} \sum_{i \in I} \Upsilon_k^{-1} \left(\beta_{kli} \hat{c}_{kli,t} + \sum_{j=1}^{I} \lambda_{kj} \omega_{klji} \hat{x}_{klji,t} \right) \tag{A.74}$$

Now we can combine the linearized expression for real GDP, que the expressions for real imports and exports:

$$\hat{y}_{k,t} = \hat{c}_{k,t} + \sum_{l \neq k} \sum_{i \in I} \left(\frac{P_{ki} C_{lki}}{y_k} \hat{c}_{lki,t} + \sum_{j=1}^{I} \frac{P_{ki} X_{lkji}}{y_k} \hat{x}_{lkji,t} - \frac{P_{kli} C_{kli}}{y_k} \hat{c}_{kli,t} - \sum_{j=1}^{I} \frac{P_{kli} X_{klji}}{y_k} \hat{x}_{klji,t} \right)$$

$$= \hat{c}_{k,t} + \sum_{l \neq k} \sum_{i \in I} \left(y_{lk} \beta_{lki} \hat{c}_{lki,t} + \sum_{j=1}^{I} y_{lk} \lambda_{lj} \omega_{lkji} \hat{x}_{lkji,t} - \beta_{kli} \hat{c}_{kli,t} - \sum_{j=1}^{I} \lambda_{kj} \omega_{klji} \hat{x}_{klji,t} \right)$$

Appendix B. Inspecting the Mechanisms

B.1. Production Networks and Inflation

Starting from a general Phillips curve, where $\pi_{Hi,t} = p_{Hi,t} - p_{Hi,t-1}$ denotes the inflation rate of a good in sector i in country H, $\pi_{Hi,t} = \kappa_i \left(\mathrm{mc}^n_{i,t} - p_{Hi,t}\right) + \beta\mathbb{E}_t\pi_{Hi,t+1}$. The nominal marginal cost faced by sector i are given by (A.35) in the full model, and under the assumptions are simplified to

$$\mathrm{mc}^n_{i,t} = \alpha_i w_t + (1-\alpha_i)\, p_{Xi,t}, \tag{A.75}$$

where $\alpha_i = WN_i/(P_iY_i)$ is the steady-state ratio of labor costs to sales in sector i, and $1-\alpha_i = P_{Xi}X_i/(P_iY_i)$ denotes the steady-state ratio of input costs to sales in sector i. The price index of intermediate goods faced by sector i is given by:

$$p_{Xi,t} = \sum_{j=1}^{I} \nu_{ij}\left[(1-\zeta_{ij})\, p_{Hj,t} + \zeta_{ij}\, p_{Fj,t}\right] = \sum_{j=1}^{I} \nu_{ij}\left(p_{Hj,t} + \zeta_{ij}s_{j,t}\right), \tag{A.76}$$

where $\nu_{ij} = P_{ij}X_{ij}/(P_{Xi}X_i)$ denotes the purchases of sector i from sector j as a share of total intermediate goods' purchases of sector i, and $1-\zeta_{ij} = (P_jX_{Hij})/(P_{ij}X_{ij})$ denotes the purchases of sector i from Home's sector j over total intermediate goods' purchases of sector i from sector j.

Introducing condition (A.76) into the marginal cost equation (A.75), and stacking over sectors, we obtain:

$$\mathbf{mc}^n_t = \alpha w_t + \boldsymbol{\Omega}_H\, \boldsymbol{p}_{H,t} + \boldsymbol{\Omega}_F\, \boldsymbol{p}_{F,t} = \alpha w_t + \boldsymbol{\Omega}\, \boldsymbol{p}_{H,t} + \boldsymbol{\Omega}_F \boldsymbol{s}_t \tag{A.77}$$

where $\mathbf{mc}^n_t = \begin{bmatrix} mc^n_{1,t} & \dots & mc^n_{I,t}\end{bmatrix}^\top$ denotes the vector of sectoral nominal marginal costs, $\boldsymbol{p}_{F,t} = \begin{bmatrix} p_{F,1,t} & \cdots & p_{F,I,t}\end{bmatrix}^\top$ is a vector of sectoral prices of imports from ROW, $\boldsymbol{s}_t = [s_{1,t}, \dots, s_{I,t}]^\top$, $\boldsymbol{\Omega} = \boldsymbol{\Omega}_H + \boldsymbol{\Omega}_F$ is the input-output matrix with elements $\omega_{ij} = (1-\alpha_i)\nu_{ij}$, and $\boldsymbol{\Omega}_F$ is the imported input-output matrix with elements $\omega_{Fij} = \omega_{ij}\zeta_{ij}$.

We now seek to obtain the Phillips curve in price terms. We can write the previous Phillips curve in matrix form,

$$\begin{aligned}
\boldsymbol{\pi}_{H,t} &= \boldsymbol{K}\left(\mathbf{mc}^n_t - \boldsymbol{p}_{H,t}\right) + \beta\mathbb{E}_t\boldsymbol{\pi}_{H,t+1} = \boldsymbol{K}\left[\mathbf{mc}^n_t - \left(\boldsymbol{\pi}_{H,t} + \boldsymbol{p}_{H,t-1}\right)\right] + \beta\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}\\
&= (\boldsymbol{I}+\boldsymbol{K})^{-1}\boldsymbol{K}\left(\mathbf{mc}^n_t - \boldsymbol{p}_{H,t-1}\right) + (\boldsymbol{I}+\boldsymbol{K})^{-1}\beta\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}\\
&= \boldsymbol{\Delta}\left[\alpha\boldsymbol{w}_t + \boldsymbol{\Omega}_H\, \boldsymbol{p}_{H,t} + \boldsymbol{\Omega}_F\, \boldsymbol{p}_{F,t} - \boldsymbol{p}_{H,t-1}\right] + (\boldsymbol{I}-\boldsymbol{\Delta})\beta\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}
\end{aligned}$$

$$= \boldsymbol{\Delta}\left(\alpha \boldsymbol{w}_t + \boldsymbol{\Omega}_F\, \boldsymbol{p}_{F,t} + \boldsymbol{\Omega}_H \boldsymbol{\pi}_{H,t} + \boldsymbol{\Omega}_H\, \boldsymbol{p}_{H,t-1} - \boldsymbol{p}_{H,t-1}\right) + (\boldsymbol{I} - \boldsymbol{\Delta})\beta \mathbb{E}_t \boldsymbol{\pi}_{H,t+1}$$

$$= (\boldsymbol{I} - \boldsymbol{\Delta}\boldsymbol{\Omega}_H)^{-1}\boldsymbol{\Delta}\left[\alpha \boldsymbol{w}_t + \boldsymbol{\Omega}_F\, \boldsymbol{p}_{F,t} - (\boldsymbol{I} - \boldsymbol{\Omega}_H)\, \boldsymbol{p}_{H,t-1}\right] + (\boldsymbol{I} - \boldsymbol{\Delta}\boldsymbol{\Omega}_H)^{-1}(\boldsymbol{I} - \boldsymbol{\Delta})\beta \mathbb{E}_t \boldsymbol{\pi}_{H,t+1} \tag{A.78}$$

where we have introduced nominal marginal costs (A.77), and the different objects are defined in section 3.2. We can also rewrite (A.78) in terms of the price level as

$$\boldsymbol{p}_{H,t} = (\boldsymbol{I} - \Delta\boldsymbol{\Omega}_H)^{-1}(\boldsymbol{I} - \Delta)\, \boldsymbol{p}_{H,t-1} + (\boldsymbol{I} - \Delta\boldsymbol{\Omega}_H)^{-1}\Delta\left(\alpha \boldsymbol{w}_t + \boldsymbol{\Omega}_F\, \boldsymbol{p}_{F,t}\right) + (\boldsymbol{I} - \Delta\boldsymbol{\Omega}_H)^{-1}(\boldsymbol{I} - \Delta)\beta \mathbb{E}_t \boldsymbol{\pi}_{t+1}$$

B.2. Production Networks and Monetary Policy

We now seek to write the sectoral Phillips curves in gaps from the flexible-price equilibrium. Under no nominal wage rigidities, the labor supply condition is given by:

$$w_t - p_{C,t} = c_t + \varphi n_t \tag{A.79}$$

where $p_{C,t}$ is the consumption price index given by:

$$p_{C,t} = \sum_{i=1}^{I} \beta_i \left[(1 - \zeta_i)\, p_{Hi,t} + \zeta_i\, p_{Fi,t}\right] = \sum_{i=1}^{I} \beta_i \left(p_{Hi,t} + \zeta_i s_{i,t}\right) = \boldsymbol{\beta}^\top \boldsymbol{p}_{H,t} + \boldsymbol{\beta}_F^\top \boldsymbol{s}_t \tag{A.80}$$

where $\beta_i = P_i C_i/(P_C C)$ denotes the consumption share of households in H, and $1 - \zeta_i = (P_i C_{Hi})/(P_i C_i)$ denotes the consumption from sector i in H over total consumption of sector i, $\boldsymbol{\beta}^\top = [\nu_1\zeta_1, \ldots, \beta_I\zeta_I]^\top$ is the vector of steady-state sectoral consumption shares, and $\boldsymbol{\beta}_F^\top = [\nu_1\zeta_1, \ldots, \beta_I\zeta_I]^\top$ denotes the vector of steady-state sectoral foreign consumption shares.

The risk-sharing condition (under complete markets and constant rest of the world variables, i.e. $c_t^* = 0$) is given by:

$$c_t = q_t - c_t^* = e_t - p_{C,t}. \tag{A.81}$$

Furthermore, the definition of nominal GDP is given by:

$$P_{Y,t}Y_t = P_{C,t}C_t + \sum_{i=1}^{I}\left(P_{Hi,t}C_{Hi,t}^* - P_{Fi,t}C_{Fi,t}\right) + \sum_{i=1}^{I}\sum_{j=1}^{I}\left(P_{Hi,t}X_{Hji,t}^* - P_{Fi,t}X_{Fji,t}\right) \tag{A.82}$$

where $P_{Y,t}$ is the GDP deflator, $C_{Hi,t}^*$ and $X_{Hij,t}^*$ denote Home's sectoral exports to households and firms in ROW, and $C_{Fi,t}$ and $X_{Fij,t}$ are Home's sectoral imports from households and firms.

We start by finding an expression for *real* GDP. In order to do so, we first linearize the definition of nominal GDP (A.82):

$$p_{Y,t} + y_t = p_{C,t} + c_t + \sum_{i=1}^{I} \beta_i \zeta_i \left(p_{Hi,t} + c^*_{Hi,t} - p_{Fi,t} - c_{Fi,t} \right) + \sum_{i=1}^{I} \sum_{j=1}^{I} \lambda_j (1-\alpha_j) \nu_{ji} \zeta_{ji} \left(p_{Hi,t} + x^*_{Hji,t} - p_{Fi,t} - x_{Fji,t} \right) \tag{A.83}$$

Next, note that we have the following expression for the GDP deflator:

$$\begin{aligned} p_{Y,t} &= p_{C,t} + \sum_{i=1}^{I} \beta_i \zeta_i \left(p_{Hi,t} - p_{Fi,t} \right) + \sum_{i=1}^{I} \sum_{j=1}^{I} \lambda_j (1-\alpha_j) \nu_{ji} \zeta_{ji} \left(p_{Hi,t} - p_{Fi,t} \right) \\ &= p_{C,t} + \sum_{i=1}^{I} \left(\beta_i \zeta_i + \sum_{j=1}^{I} \lambda_j (1-\alpha_j) \nu_{ji} \zeta_{ji} \right) \left(p_{Hi,t} - p_{Fi,t} \right) \\ &= p_{C,t} - (\boldsymbol{\beta}_F^\top + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F) \boldsymbol{s}_t \end{aligned} \tag{A.84}$$

and hence we have that real GDP is given by:

$$y_t = c_t + \sum_{i=1}^{I} \beta_i \zeta_i \left(c^*_{Hi,t} - c_{Fi,t} \right) + \sum_{i=1}^{I} \sum_{j=1}^{I} \lambda_j \nu_{ji} (1-\alpha_j) \zeta_{ji} \left(x^*_{Hji,t} - x_{Fji,t} \right). \tag{A.85}$$

Consider first the term $\sum_{i=1}^{I} \beta_i \zeta_i \left(c^*_{Hi,t} - c_{Fi,t} \right)$. Note that we have

$$\begin{aligned} c^*_{Hi,t} &= e_t + p^*_{Fi} - p_{Hi,t} + c^*_i = e_t + p^*_{Fi} - p_{Hi,t} + (e_t + p^*_{C,t}) - (e_t + p^*_{Fi,t}) + c^*_t \\ &= -p_{Hi,t} + (e_t + p^*_{C,t}) + c_t - q_t = -p_{Hi,t} + p_{C,t} + c_t, \end{aligned}$$

where we have made use of the risk-sharing condition (A.81). In addition, note that we have

$$c_{Fi,t} = p_{Ci,t} - p_{Fi,t} + c_{i,t} = p_{Ci,t} - p_{Fi,t} + (p_{C,t} - p_{Ci,t}) + c_t = p_{C,t} + c_t - p_{Fi,t}.$$

Therefore, we have that:

$$\sum_{i=1}^{I} \beta_i \zeta_i \left(c^*_{Hi,t} - c_{Fi,t} \right) = \sum_{i=1}^{I} \beta_i \zeta_i \left(p_{Fi,t} - p_{Hi,t} \right) = \boldsymbol{\beta}_F^\top \boldsymbol{s}_t. \tag{A.86}$$

Next, we work with the term $x^*_{Hji,t} - x_{Fji,t}$. First, note that we have

$$
\begin{aligned}
x^*_{Hji,t} &= e_t + p^*_{Fi} - p_{Hi,t} + x^*_{ji,t} = e_t + p^*_{Fi} - p_{Hi,t} + (e_t + p^*_{Xj}) - (e_t + p^*_{Fi}) + x^*_j \\
&= -p_{Hi,t} + e_t + p^*_{C,t} + (w^*_t - p^*_{C,t}) + n^*_{j,t} = -p_{Hi,t} + p_{Ct} + q_t + c^*_t + \varphi n^*_t + n^*_{j,t} \\
&= -p_{Hi,t} + p_{Ct} + q_t + c_t - q_t + \varphi n^*_t + n^*_{j,t} = -p_{Hi,t} + p_{Ct} + c_t,
\end{aligned}
$$

where we have made use of the first-order conditions of the firms for labor demand and intermediate goods' demand, $x^*_{j,t} + p^*_{Xj,t} = n^*_{j,t} + w^*_t$, and of the fact that foreign variables are constant, in addition to the risk-sharing condition (A.81). Next, note that we have

$$
\begin{aligned}
x_{Fji,t} &= p_{ji,t} - p_{Fi,t} + x_{ji,t} = p_{ji,t} - p_{Fi,t} + (p_{Xj,t} - p_{ji,t}) + x_{j,t} = -p_{Fi,t} + p_{C,t} + (w_t - p_{C,t}) + n_{j,t} \\
&= -p_{Fi,t} + p_{C,t} + c_t + \varphi n_t + n_{j,t}.
\end{aligned}
$$

Therefore, we have that:

(A.87)

$$
\begin{aligned}
\sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_j\nu_{ji}(1-\alpha_j)\zeta_{ji}\left(x^*_{Hji,t} - x_{Fji,t}\right) &= \sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_j\nu_{ji}(1-\alpha_j)\zeta_{ji}\left(p_{Fi,t} - p_{Hi,t} - \varphi n_t - n_{j,t}\right) \\
&= \sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_j\nu_{ji}(1-\alpha_j)\zeta_{ji}\left(s_{i,t} - \varphi n_t - n_{j,t}\right) \\
&= \boldsymbol{\lambda}^\intercal\boldsymbol{\Omega}_F\boldsymbol{s}_t - \boldsymbol{\lambda}^\intercal\boldsymbol{\Omega}_F\boldsymbol{n}_t - \omega_X\varphi y_t,
\end{aligned}
$$

where $\boldsymbol{n}_t = [n_{1,t}, \ldots, n_{I,t}]^T$ is the vector of sectoral employment. In addition, we have made use of the fact that aggregate GDP equals the sales-weighted sum of sectoral value-added, and hence aggregate employment (using the labor market clearing condition $n_t = \sum_{i=1}^{I}\lambda_i\alpha_i n_{i,t}$), $y_t = \sum_{i=1}^{I}\lambda_i\alpha_i n_{i,t} = n_t$. Furthermore, $\omega_X = \sum_{i=1}^{I}\sum_{j=1}^{I}\lambda_j\nu_{ji}(1-\alpha_j)\zeta_{ji}$.

Next, plugging in (A.86) and (A.87) into (A.85) we obtain:

$$(1+\varphi\omega_X)y_t = c_t + (\boldsymbol{\beta}_F^\intercal + \boldsymbol{\lambda}^\intercal\boldsymbol{\Omega}_F)\boldsymbol{s}_t - \boldsymbol{\lambda}^\intercal\boldsymbol{\Omega}_F\boldsymbol{n}_t. \tag{A.88}$$

Natural Equilibrium. In the natural allocation with flexible prices (denoted by superscript n) where prices equal marginal costs and nominal wages remain constant (Rubbo 2023), we have that $\boldsymbol{p}^n_{H,t} = \boldsymbol{\alpha} w^n_t + \boldsymbol{\Omega}\,\boldsymbol{p}^n_{H,t} + \boldsymbol{\Omega}_F\boldsymbol{s}^n_t$. Therefore, we can write:

$$\boldsymbol{p}^n_{H,t} = (\boldsymbol{I}-\boldsymbol{\Omega})^{-1}\boldsymbol{\Omega}_F\boldsymbol{s}^n_t = (\boldsymbol{I}-\boldsymbol{\Omega}_H)^{-1}\boldsymbol{\Omega}_F\,\boldsymbol{p}^n_{F,t}. \tag{A.89}$$

That is, in response to an increase in import prices, domestic prices adjust through a direct effect via its exposure to foreign markets (measured by $\boldsymbol{\Omega}_F$) and an indirect effect via input-output linkages mediated by the domestic Leontief inverse $(\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1}$.

Note also that making use of this results and of the relationship between consumption shares and Domar weights in steady state, $\boldsymbol{\lambda}^\top = \boldsymbol{\beta}^\top (\boldsymbol{I} - \boldsymbol{\Omega})^{-1}$, we have that the consumer price level in the flexible price equilibrium is given by:

(A.90)
$$p^n_{C,t} = \boldsymbol{\beta}^\top \boldsymbol{p}^n_{H,t} + \boldsymbol{\beta}^\top_F \boldsymbol{s}^n_t = \left[\boldsymbol{\beta}^\top_F + \boldsymbol{\beta}^\top (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \boldsymbol{\Omega}_F\right] \boldsymbol{s}^n_t = \left[\boldsymbol{\beta}^\top_F + \boldsymbol{\lambda}^\top (\boldsymbol{I} - \boldsymbol{\Omega}) (\boldsymbol{I} - \boldsymbol{\Omega})^{-1} \boldsymbol{\Omega}_F\right] \boldsymbol{s}^n_t = (\boldsymbol{\beta}^\top_F + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F) \boldsymbol{s}^n_t.$$

Next, writing the expression for real GDP (A.88) in natural terms and using (A.90) we obtain:

$$(1 + \varphi\omega_X)\, y^n_t = c^n_t + p^n_{C,t} - \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \boldsymbol{n}^n_t. \tag{A.91}$$

In addition, make use of the labor supply condition with flexible prices and constant nominal wages, together with $y^n_t = n^n_t$ and $-\varphi\, y^n_t = c^n_t + p^n_{C,t}$ to obtain:

$$\left[1 + \varphi(\omega_X + 1)\right] y^n_t = -\boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \boldsymbol{n}^n_t \tag{A.92}$$

Phillips Curve in Gaps from Natural Equilibrium. First, we start by writing the labor supply condition (A.79), using $y_t = n_t$ together with (A.88) to substitute out c_t:

$$w_t - p_{C,t} = \left[1 + \varphi(\omega_X + 1)\right] y_t - \left(\boldsymbol{\beta}^\top_F + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F\right) \boldsymbol{s}_t + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \boldsymbol{n}_t \tag{A.93}$$

Next, we substract $p^n_{C,t}$ from both sides and use (A.90) to obtain:

$$w_t - \widetilde{p}_{C,t} = \left[1 + \varphi(\omega_X + 1)\right] y_t - \left(\boldsymbol{\beta}^\top_F + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F\right) \widetilde{\boldsymbol{s}}_t + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \boldsymbol{n}_t \tag{A.94}$$

where tildes denote deviations from the flexible price equilibrium: $\widetilde{x}_t = x_t - x^n_t$ for a variable x. Finally, make use of and add and subtract $\left[1 + \varphi(\omega_X + 1)\right] y^n_t$ on the right-hand side to obtain:

$$w_t - \widetilde{p}_{C,t} = \left[1 + \varphi(\omega_X + 1)\right] \widetilde{y}_t - \left(\boldsymbol{\beta}^\top_F + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F\right) \widetilde{\boldsymbol{s}}_t + \boldsymbol{\lambda}^\top \boldsymbol{\Omega}_F \widetilde{\boldsymbol{n}}_t \tag{A.95}$$

We next work with the stacked sectoral Phillips curves (A.78). First, note that we have that:

$$(\boldsymbol{I} - \boldsymbol{\Delta}\boldsymbol{\Omega})\boldsymbol{\pi}_{H,t} = \boldsymbol{\Delta}(\boldsymbol{\alpha} w_t - (\boldsymbol{I} - \boldsymbol{\Omega})\, \boldsymbol{p}_{H,t-1} + \boldsymbol{\Omega}_F \boldsymbol{s}_t) + \beta(\boldsymbol{I} - \boldsymbol{\Delta})\mathbb{E}_t \boldsymbol{\pi}_{H,t+1} \tag{A.96}$$

Next, we subtract $\boldsymbol{\Delta\alpha} p_{C,t}$ on both sides, and use (A.80) to write $p_{C,t} = \boldsymbol{\beta}^\top \boldsymbol{p}_{H,t} + \boldsymbol{\beta}_F^\top \boldsymbol{s}_t = \boldsymbol{\beta}^\top \boldsymbol{\pi}_{H,t} + \boldsymbol{\beta}^\top \boldsymbol{p}_{H,t-1} + \boldsymbol{\beta}_F^\top \boldsymbol{s}_t$, obtaining:

(A.97)
$$(\boldsymbol{I}-\boldsymbol{\Delta\Omega}-\boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_{H,t} = \boldsymbol{\Delta}\left[\boldsymbol{\alpha}(w_t - p_{C,t}) - (\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)\,\boldsymbol{p}_{H,t-1} + (\boldsymbol{\Omega}_F + \boldsymbol{\alpha\beta}_F^\top)\boldsymbol{s}_t\right] + \beta(\boldsymbol{I}-\boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}$$

Adding and subtracting $p^n_{C,t}$ on the right-hand side, we obtain:

(A.98)
$$(\boldsymbol{I} - \boldsymbol{\Delta\Omega} - \boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_{H,t} = \boldsymbol{\Delta}\left[\boldsymbol{\alpha}(w_t - \widetilde{p}_{C,t}) - (\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)\,\boldsymbol{p}_{H,t-1} + (\boldsymbol{\Omega}_F + \boldsymbol{\alpha\beta}_F^\top)\boldsymbol{s}_t - \boldsymbol{\alpha\beta}^\top\,\boldsymbol{p}^n_{H,t} - \boldsymbol{\alpha\beta}_F^\top \boldsymbol{s}^n_t\right]$$
$$+ \beta(\boldsymbol{I} - \boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}$$

Next, adding and subtracting $(\boldsymbol{I}-\boldsymbol{\Omega})\,\boldsymbol{p}^n_{H,t}$ on the right-hand side and use that $(\boldsymbol{I}-\boldsymbol{\Omega})\,\boldsymbol{p}^n_{H,t} = \boldsymbol{\Omega}_F\boldsymbol{s}^n_t$ to obtain:

(A.99)
$$(\boldsymbol{I} - \boldsymbol{\Delta\Omega} - \boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_{H,t} = \boldsymbol{\Delta}\left[\boldsymbol{\alpha}(w_t - \widetilde{p}_{C,t}) - (\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)(\boldsymbol{p}_{H,t-1} - \boldsymbol{p}^n_{H,t}) + (\boldsymbol{\Omega}_F + \boldsymbol{\alpha\beta}_F^\top)\widetilde{\boldsymbol{s}}_t\right]$$
$$+ \beta(\boldsymbol{I} - \boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}$$

Finally, using $\boldsymbol{p}^n_{H,t} = (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1}\,\boldsymbol{\Omega}_F\,\boldsymbol{p}^n_{F,t}$ we obtain:

(A.100)
$$(\boldsymbol{I} - \boldsymbol{\Delta\Omega} - \boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_{H,t} = \boldsymbol{\Delta}\left[\boldsymbol{\alpha}(w_t - \widetilde{p}_{C,t}) - (\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)(\boldsymbol{p}_{H,t-1} - (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1}\,\boldsymbol{\Omega}_F\,\boldsymbol{p}^n_{F,t}) + (\boldsymbol{\Omega}_F + \boldsymbol{\alpha\beta}_F^\top)\widetilde{\boldsymbol{s}}_t\right]$$
$$+ \beta(\boldsymbol{I} - \boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}$$

Next, plugging in the expression for the real wage gap (A.95) into (A.97) to obtain:

$$(\boldsymbol{I} - \boldsymbol{\Delta\Omega} - \boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_{H,t} = \boldsymbol{\Delta\alpha}\left[1 + \varphi(\omega_X + 1)\right]\widetilde{y}_t - \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)(\boldsymbol{p}_{H,t-1} - (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1}\,\boldsymbol{\Omega}_F\,\boldsymbol{p}^n_{F,t})$$
(A.101)
$$+ \boldsymbol{\Delta\alpha\lambda}^\top\boldsymbol{\Omega}_F\widetilde{n}_t + \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\alpha\lambda}^\top)\boldsymbol{\Omega}_F\widetilde{\boldsymbol{s}}_t + \beta(\boldsymbol{I} - \boldsymbol{\Delta})\mathbb{E}_t\boldsymbol{\pi}_{H,t+1}$$

Social Planner's Problem. We now seek to obtain the natural equilibrium variables as a function of exogenous variables. To do so, we first solve the social planner problem. The planner faces the following problem:

(A.102)
$$\max_{\{C^n_{Hi,t}, C^n_{Fi,t}, N^n_{i,t}, X^n_{Hji,t}, X^n_{Fji,t}, \mathcal{E}^n_t, D^n_{t+1}\}^I_{i=1,j=1}} \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left\{ \log C^n_t - \frac{\left(\sum_{i=1}^I N^n_{i,t}\right)^{1+\varphi}}{1+\varphi} \right\}$$

subject to the set of market clearing conditions and the aggregate resource constraint:

(A.103)

$$(N_{i,t}^n)^{\alpha_i}(X_{i,t}^n)^{1-\alpha_i} \geq C_{Hi,t}^n + \zeta_i \frac{\mathcal{E}_t^n P_{Fi,t}^{n*}}{\mathcal{E}_t^n P_{Hi,t}^{n*}} C_{i,t}^{n*} + \sum_{j=1}^{I} \left(\zeta_{ji} \frac{\mathcal{E}_t^n P_{Fi,t}^{n*}}{\mathcal{E}_t^n P_{Hi,t}^{n*}} X_{ji,t}^{n*} + X_{Hji,t}^n \right) \quad \forall i \in I, t$$

(A.104)

$$\sum_{i=1}^{I} \mathcal{E}_t^n P_{Fi,t}^{n*} \left[\zeta_i C_{i,t}^{n*} - (1+\tau_{i,t}) C_{Fi,t}^n + \sum_{j=1}^{I} \left(\zeta_{ji} X_{ji,t}^{n*} - (1+\tau_{i,t}) X_{Fji,t}^n \right) \right] \geq \mathbb{E}_t Q_{t,t+1}^n D_{t+1}^n - D_t^n \quad \forall t$$

together with definitions of the consumption and intermediate goods' aggregators.

Let $\xi_{i,t}$ and ξ_t denote the lagrange multipliers on the constraints (A.103) and (A.104), respectively. The first-order conditions of the planner's problem for $\{C_{Hi,t}^n, C_{Fi,t}^n, N_{i,t}^n, X_{Hji,t}^n, X_{Fji,t}^n\}_{i=1,j=1}^I$ are, respectively, given by:

(A.105)
$$(N_{i,t}^n)^{\varphi} = \xi_{i,t} \alpha_i \frac{Y_{i,t}^n}{N_{i,t}^n}$$

(A.106)
$$\frac{1}{C_t^n} \frac{\partial C_t^n}{\partial C_{i,t}^n} \frac{\partial C_{i,t}^n}{\partial C_{Hi,t}^n} = \xi_{i,t}$$

(A.107)
$$\frac{1}{C_t^n} \frac{\partial C_t^n}{\partial C_{i,t}^n} \frac{\partial C_{i,t}^n}{\partial C_{Fi,t}^n} = \xi_t (1+\tau_{i,t}) \mathcal{E}_t^n P_{Fi,t}^{n*}$$

(A.108)
$$\xi_{j,t}(1-\alpha_j) \frac{Y_{j,t}^n}{X_{j,t}^n} \frac{\partial X_{j,t}^n}{\partial X_{ji,t}^n} \frac{\partial X_{ji,t}^n}{\partial X_{Hji,t}^n} = \xi_{i,t}$$

(A.109)
$$\xi_{j,t}(1-\alpha_j) \frac{Y_{j,t}^n}{X_{j,t}^n} \frac{\partial X_{j,t}^n}{\partial X_{ji,t}^n} \frac{\partial X_{ji,t}^n}{\partial X_{Fji,t}^n} = \xi_t (1+\tau_{i,t}) \mathcal{E}_t^n P_{Fi,t}^{n*}$$

(A.110)
$$\xi_t \mathbb{E}_t Q_{t,t+1}^n = \mathbb{E}_t \xi_{t+1}$$

Employing the definitions of the intermediate goods' and consumption aggregators, we can rewrite the first-order (A.106) - (A.109) conditions as:

(A.111)
$$\beta_i (1-\zeta_i) \frac{1}{C_{Hi,t}^n} = \xi_{i,t}$$

(A.112)
$$\beta_i \zeta_i \frac{1}{C_{Fi,t}^n} = \xi_t (1+\tau_{i,t}) \mathcal{E}_t^n P_{F,t}^{n*}$$

(A.113) $$\xi_{j,t}(1-\alpha_j)\frac{Y^n_{j,t}}{X^n_{Hji,t}}\nu_{ji}(1-\zeta_{ji}) = \xi_{i,t}$$

(A.114) $$\xi_{j,t}(1-\alpha_j)\frac{Y^n_{j,t}}{X^n_{Fji,t}}\nu_{ji}\zeta_{ji} = \xi_t(1+\tau_{i,t})\mathcal{E}^n_t P^{n*}_{F,t}$$

Combining (A.105) and (A.106) we have that:

(A.115) $$(N^n_{i,t})^{\varphi} = \beta_i(1-\zeta_i)\frac{1}{C^n_{Hi,t}}\alpha_i\frac{Y^n_{i,t}}{N^n_{i,t}}.$$

Noting that in the flexible price equilibrium we have that $\beta_i(1-\zeta_i)\frac{1}{C^n_{Hi,t}} = \frac{P^n_{Hi,t}}{P^n_{C,t}C^n_t}$ and using the labor supply condition, we obtain that (A.115) is equivalent to the labor demand condition of firms.

Next, combining (A.105) and (A.108) we have that:

(A.116) $$\beta_j(1-\zeta_j)\frac{1}{C^n_{Hj,t}}(1-\alpha_j)\frac{Y^n_{j,t}}{X^n_{Hji,t}}\nu_{ji}(1-\zeta_{ji}) = \beta_i(1-\zeta_i)\frac{1}{C^n_{Hi,t}}.$$

Again, noting that in the flexible price equilibrium we have that $\beta_j(1-\zeta_j)\frac{1}{C^n_{Hj,t}} = \frac{P^n_{Hj,t}}{P^n_{C,t}C^n_t}$ and $\beta_i(1-\zeta_i)\frac{1}{C^n_{Hi,t}} = \frac{P^n_{Hi,t}}{P^n_{C,t}C^n_t}$ we observe that (A.116) is equivalent to the intermediate goods demand of domestic goods condition of firms.

Next, inserting (A.106) and (A.107) in (A.109), we obtain:

(A.117) $$\beta_j(1-\zeta_j)\frac{1}{C^n_{Hj,t}}(1-\alpha_j)\frac{Y^n_{j,t}}{X^n_{Fji,t}}\nu_{ji}\zeta_{ji} = \beta_i\zeta_i\frac{1}{C^n_{Fi,t}}$$

and again noting that in the flexible price equilibrium we have that $\beta_j(1-\zeta_j)\frac{1}{C^n_{Hj,t}} = \frac{P^n_{Hj,t}}{P^n_{C,t}C^n_t}$ and $\beta_i\zeta_i\frac{1}{C^n_{Fi,t}} = \frac{P^n_{Fi,t}}{P^n_{C,t}C^n_t}$ we observe that (A.117) is equivalent to the intermediate goods demand of foreign goods condition of firms.

Finally, using (A.106) into (A.110) to obtain:

(A.118) $$\frac{1}{C^n_{Fi,t}}\frac{1}{(1+\tau_{i,t})\mathcal{E}^n_t P^{n*}_{Fi,t}}\mathbb{E}_t Q^n_{t,t+1} = \mathbb{E}_t\frac{1}{C^n_{Fi,t+1}}\frac{1}{(1+\tau_{i,t+1})\mathcal{E}^n_t P^{n*}_{Fi,t+1}}$$

and again noting that in the flexible price equilibrium we have that $\frac{1}{C^n_{Fi,t}}\frac{1}{(1+\tau_{i,t})\mathcal{E}^n_t P^{n*}_{Fi,t}} =$

$\frac{1}{\beta_i \zeta_i P^n_{C,t} C^n_t}$ we observe that (A.118) is equivalent to Euler Equation in the flexible price equilibrium.

Next, note that under the following definitions of the Lagrange multipliers:

$$\xi_{i,t} = \frac{P^n_{Hi,t}}{P^n_{C,t} C^n_t} \tag{A.119}$$

$$\xi_t = \frac{1}{P^n_{C,t} C^n_t} \tag{A.120}$$

and using the labor supply condition $\frac{W_t}{P_{C,t}} = C^n_t N^{\varphi}_t$ we have that the first-order conditions coincide with the first-order conditions of households and firms in the flexible price equilibrium:

$$W^n_t = P^n_{Hi,t} \alpha_i \frac{Y^n_{i,t}}{N^n_{i,t}} \tag{A.121}$$

$$\beta_i (1 - \zeta_i) \frac{1}{C^n_{Hi,t}} = \frac{P^n_{Hi,t}}{P^n_{C,t} C^n_t} \tag{A.122}$$

$$\beta_i \zeta_i \frac{1}{C^n_{Fi,t}} = \frac{P_{Fi,t}}{P^n_{C,t} C^n_t} \tag{A.123}$$

$$P^n_{Hj,t} (1 - \alpha_j) \frac{Y^n_{j,t}}{X^n_{Hji,t}} \nu_{ji} (1 - \zeta_{ji}) = P^n_{Hi,t} \tag{A.124}$$

$$P^n_{Hj,t} (1 - \alpha_j) \frac{Y^n_{j,t}}{X^n_{Fji,t}} \nu_{ji} \zeta_{ji} = P^n_{Fi,t} \tag{A.125}$$

$$\mathbb{E}_t Q^n_{t,t+1} = \mathbb{E}_t \left(\frac{P^n_{C,t+1} C^n_{t+1}}{P^n_{C,t} C^n_t} \right) \tag{A.126}$$

Finally, the first-order condition of the planner with respect the exchange rate is given by:

$$\xi_t \sum_i P^{n*}_{Fi,t} \left[\zeta_i C^{n*}_{i,t} - (1 + \tau_{i,t}) C^n_{Fi,t} + \sum_j \left(\zeta_{ji} X^{n*}_{ji,t} - (1 + \tau_{i,t}) X^n_{Fji,t} \right) \right] = 0$$

Note that since $\xi_t = \frac{1}{P^n_{C,t} C^n_t} > 0$ we have that nominal exports are equal to zero at all times under the planner's allocation:

$$\sum_i \mathcal{E}^n_t P^{n*}_{Fi,t} \left[\zeta_i C^{n*}_{i,t} - (1 + \tau_{i,t}) C^n_{Fi,t} + \sum_j \left(\zeta_{ji} X^{n*}_{ji,t} - (1 + \tau_{i,t}) X^n_{Fji,t} \right) \right] = 0. \tag{A.127}$$

To see the implications of this condition for the flexible price allocation, we linearize equation (A.127) to obtain:

$$-(\beta_F^\top + \lambda^\top \boldsymbol{\Omega}_F)\boldsymbol{s}_t + \sum_i \beta_i \zeta_i \left(c^{n*}_{Hi,t} - c^n_{Fi,t}\right) + \sum_i \sum_j (1-\alpha_j)\nu_{ji}\zeta_{ji}\left(x^{n*}_{ji,t} - x^n_{Fji,t}\right) = 0. \tag{A.128}$$

Using our derivations in section B.2 we can write (A.128) as:

$$-(\beta_F^\top + \lambda^\top \boldsymbol{\Omega}_F)\boldsymbol{s}^n_t + (\beta_F^\top + \lambda^\top \boldsymbol{\Omega}_F)\boldsymbol{s}^n_t - \omega_X \varphi n^n_t - \lambda^T \boldsymbol{\Omega}_F \boldsymbol{n}^n_t = 0 \tag{A.129}$$

which implies that:

$$-\lambda^T \boldsymbol{\Omega}_F \boldsymbol{n}^n_t = \omega_X \varphi n^n_t \tag{A.130}$$

Next, using (A.130) into the expression for real GDP (A.88) we have that:

$$y^n_t = c^n_t + (\beta_F^\top + \lambda^\top \boldsymbol{\Omega}_F)\boldsymbol{s}^n_t$$

and using that in the flexible price equilibrium $p^n_{C,t} = (\beta_F^\top + \lambda^\top \boldsymbol{\Omega}_F)\boldsymbol{s}^n_t$ (equation A.90), together with the labor supply condition, we obtain:

$$y^n_t = -\varphi\, y^n_t \tag{A.131}$$

which only holds if $y^n_t = 0$.

In addition, using the risk-sharing condition $c^n_t = e^n_t - p^n_{C,t}$ we have that $e^n_t = 0$. Under these conditions, we obtain that the response of domestic prices in the flexible price equilibrium is given by:

$$\boldsymbol{p}^n_{H,t} = (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1} \boldsymbol{\Omega}_F \boldsymbol{\tau}_t, \tag{A.132}$$

and that the response of the terms of trade is given by:

$$\boldsymbol{s}^n_t = \left[\boldsymbol{I} - (\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1} \boldsymbol{\Omega}_F\right] \boldsymbol{\tau}_t. \tag{A.133}$$

We next show that $y^n_t = 0$—and hence, from the labor supply condition, $p^n_{C,t} + c^n_t = 0$—implies that sectoral-level employment remains constant (that is, $\boldsymbol{n}^n_t = \boldsymbol{0}$). To see this, we start from the market clearing condition for good i:

$$Y^n_{i,t} = C^n_{Hi,t} + C^{n*}_{Hi,t} + \sum_j \left(X^n_{Hji,t} + X^{n*}_{Hji,t}\right). \tag{A.134}$$

Substituing out the demands for good i of domestic and foreign households and firms, we have that:

(A.135)

$$Y^n_{i,t} = \beta_i \frac{P^n_{C,t} C^n_t}{P^n_{Hi,t}} + \sum_j \left(\nu_{ji}(1-\zeta_{ji}) \frac{1-\alpha_j}{\alpha_j} \frac{P^n_{C,t} C^n_t}{P^n_{Hi,t} C^n_t} N^{n\varphi}_t N_{j,t} + \nu_{ji}\zeta_{ji} \frac{1-\alpha_j}{\alpha_j} \frac{P^n_{C,t} C^n_t}{P^n_{Hi,t} C^n_t} (N^{n*}_t)^{\varphi} N^{n*}_{j,t} \right).$$

Linearizing this expression, and dividing by steady state GDP, we have that:

(A.136)

$$\lambda_i y^n_{i,t} = \beta_i \left(p^n_{C,t} + c^n_t - p^n_{Hi,t} \right) + \sum_j \left[\lambda_j \nu_{ji}(1-\zeta_{ji})(1-\alpha_j)(p^n_{C,t} + c^n_t - p^n_{Hi,t} + \varphi n^n_t + n^n_{j,t}) + \lambda_j \nu_{ji}\zeta_{ji}(1-\alpha_j)(p^n_{C,t} + c^n_t - p^n_{Hi,t}) \right]$$

Next, using that in the flexible price equilibrium $p^n_{C,t} + c^n_t = -\varphi n^n_t = 0$, and collecting terms, the above expression simplifies to:

$$\lambda_i y^n_{i,t} = -\left[\beta_i + \sum_j \lambda_j \nu_{ji}(1-\zeta_{ji})(1-\alpha_j) \right] p^n_{Hi,t} + \sum_j \lambda_j \nu_{ji}(1-\zeta_{ji})(1-\alpha_j) n^n_{j,t} \tag{A.137}$$

In addition, note that at the steady state we have $\lambda_i = [\beta_i + \sum_j \lambda_j \nu_{ji}(1-\zeta_{ji})(1-\alpha_j)]$, leading to:

$$\lambda_i (p^n_{Hi,t} + y^n_{i,t}) = \sum_j \lambda_j \nu_{ji}(1-\zeta_{ji})(1-\alpha_j) n^n_{j,t}. \tag{A.138}$$

Using the result that with flexible prices we have that $p^n_{Hi,t} + y^n_{i,t} = n^n_{i,t}$ from the labor demand condition of firms and stacking over i we have that:

$$\boldsymbol{\Lambda} \boldsymbol{n}^n_t = \boldsymbol{\Omega}^\top_H \boldsymbol{\Lambda} \boldsymbol{n}^n_t \tag{A.139}$$

where $\boldsymbol{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$. Since both $\boldsymbol{\Lambda}$ and $(\boldsymbol{I} - \boldsymbol{\Omega_H})$ are invertible, the only solution for (A.139) is $\boldsymbol{n}^n_t = \boldsymbol{0}$.

Finally, using $\boldsymbol{e}^n_t = \boldsymbol{0}$ in the flexible price equilibrium we can write $\boldsymbol{p}^n_{F,t} = \boldsymbol{\tau}_t$. Introducing this expression into the Phillips curve (A.101),

$$\begin{aligned}(\boldsymbol{I} - \boldsymbol{\Delta\Omega} - \boldsymbol{\Delta\alpha\beta}^\top)\boldsymbol{\pi}_{H,t} =& \boldsymbol{\Delta\alpha} \left[1 + \varphi(\omega_X + 1)\right] \widetilde{y}_t + \boldsymbol{\Delta\alpha\lambda}^\top \boldsymbol{\Omega}_F \widetilde{n}_t + \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\alpha\lambda}^\top)\boldsymbol{\Omega}_F \widetilde{\boldsymbol{s}}_t + \beta(\boldsymbol{I} - \boldsymbol{\Delta})\mathbb{E}_t \boldsymbol{\pi}_{H,t+1} \\ & - \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top) \boldsymbol{p}_{H,t-1} + \boldsymbol{\Delta}(\boldsymbol{I} - \boldsymbol{\Omega} - \boldsymbol{\alpha\beta}^\top)(\boldsymbol{I} - \boldsymbol{\Omega}_H)^{-1} \boldsymbol{\Omega}_F \boldsymbol{\tau}_t\end{aligned} \tag{A.140}$$

Following the same transformations as in Rubbo (2023) to invert the term $(\boldsymbol{I} - \boldsymbol{\Delta}\Omega - \boldsymbol{\Delta\alpha\beta}^\top)$, we obtain the expression in the main text (37).

Appendix C. The Transimission of (Other) Foreign Shocks in a Networked Economy

In this section, we study the macroeconomic dynamics after alternative foreign shocks, both in the EA and in the ROW. Namely, we consider an aggregate demand shock to ROW, a monetary policy shock in ROW, a TFP shock to the most upstream and downstream sectors in ROW, a price cost-push shock to the most upstream and downstream sectors in ROW, a wage cost-push shock in ROW, and a price-wedge shock to the sector C.26 (Manufacture of computer, electronic and optical products) in ROW.

In the different exercises, we assume three alternative input-output structures. First, we consider the baseline production network used in the main text of the paper. Second, we consider a rupture in national input-output linkages, but allow for the transmission through international input-output links. Third, we consider a complete rupture in production networks, both national and international.[28]

We report the parametric assumptions on the exogenous shock processes in Table C.1. We set all persistence coefficients to their benchmark counterpart in Galí (2015), considering supply shocks more persistent than demand shocks. Finally, we set all standard deviations equal to one, and consider one standard deviation shocks.

Aggregate Demand. In figure C.1 we report the CIRFs of the different aggregate macroeconomic variables to a positive aggregate demand shock in ROW, under the three different input-output scenarios.

In terms of the local effects on ROW, we find the standard results: a positive aggregate demand shock generates an increase in real output (panel C.1G) and consumption (panel C.1J), headline inflation (panel C.1H), and the policy rate through the systematic component of the monetary stance (panel C.1C). Furthermore, net exports fall on impact since households demand excess consumption, and decrease over time (panel C.1K).

We next focus on the effects on the EA. A positive foreign demand shock increases EA real output (panel C.1A) and consumption (panel C.1D), increasing the policy rate through the systematic component of monetary policy (panel C.1C). Net exports in the EA increase on impact, and decrease over time (panel C.1E). We find that the foreign aggregate demand shock does not generate economically relevant effects on EA headline inflation (panel C.1B).

[28] For the case of aggregate shocks—such as the demand, the monetary or the wage cost-push shocks—we set the input-output links to zero. For the case of a sectoral shock—such as the TFP, price cost-push or price-wedge shocks—we set the input-output links to zero except for the shocked sector.

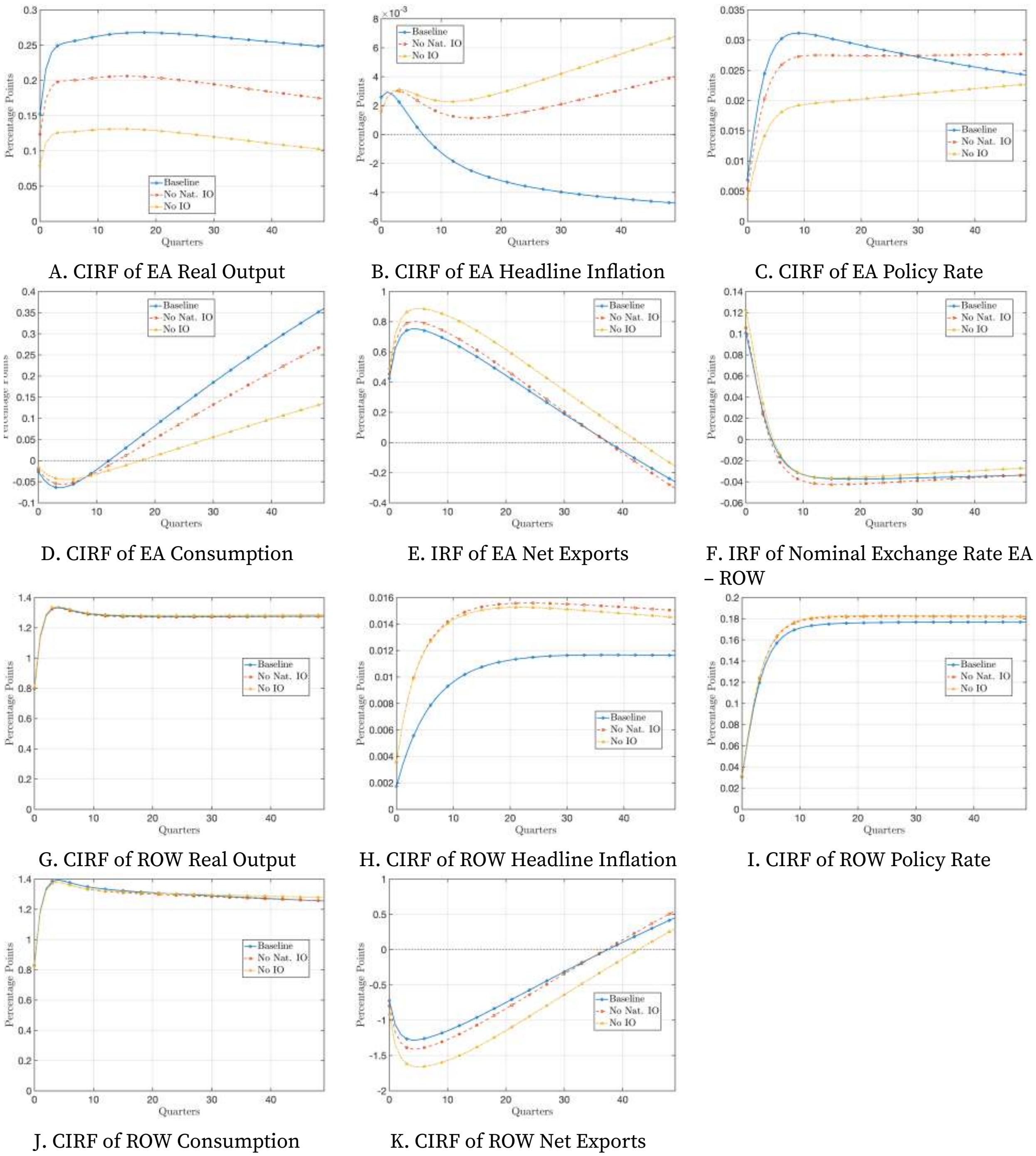


FIGURE C.1. Effects of a Foreign Demand Shock on Euro-Area Variables

Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive demand shock in ROW.

Parameter	Description	Value	Target / Source
Exogenous Shock Processes			
Persistence:			
ρ_{kz}	Persistence demand shock	0.5	Galí (2015)
ρ_{kia}	Persistence TFP shock	0.9	Galí (2015)
ρ_{pki}	Persistence price cost-push shock	0.7	Standard Value
ρ_{kw}	Persistence wage cost-push shock	0.7	Standard Value
$\rho^{\tau}_{1,kli}$	Persistence price wedge shock	1.17	Identical to Energy
$\rho^{\tau}_{2,kli}$	Persistence price wedge shock	-0.2	Identical to Energy
Standard Deviation			
σ^{z}_{k}	Std. Dev. demand shock	1	Standard Value
σ^{r}_{k}	Std. Dev. monetary policy shock	1	Standard Value
σ^{a}_{ki}	Std. Dev. TFP shock	1	Standard Value
σ^{p}_{ki}	Std. Dev. price cost-push shock	1	Standard Value
σ^{w}_{k}	Std. Dev. wage cost-push shock	1	Standard Value
σ^{τ}_{kli}	Std. Dev. price wedge shock	1	Standard Value

Notes: List of calibrated parameters. See the main text for a discussion on targets, values, and data used.

TABLE C.1. Calibration

Furthermore, the increase in the policy rate is larger in the ROW (where both real output and headline inflation increase by a larger magnitude), which generates an initial increase in the nominal exchange rate, which undershoots and reverts slowly to its stationary value (panel C.1F).

We find that production networks do not significantly alter the local (in ROW) effects on real output and real consumption. However, production networks dampen the effect of demand shocks on inflation. This finding is akin to the result in the monetary economics literature: production networks effectively reduce the pass-through of changes in demand to prices. We also find that production networks dampen the local effects of demand shocks on net exports.

Turning to the international effects, we find that production networks amplify the effect of foreign aggregate demand shocks on real output and consumption, and dampen the effect on headline inflation—although this last result is not economically significant. Finally, we find that production networks dampen the international effects of demand shocks on net exports.

Monetary Policy Shock. Figure C.2 reports the CIRFs of several macroeconomic variables following a contractionary monetary policy shock in ROW, under three different IO scenarios (no IO linkages, only domestic IO, and full domestic and international IO). The figure is organized into panels, each depicting the response of a particular variable over time. Below, we summarize the main findings sector by sector and discuss how production networks shape both the local (ROW) and international (EA) transmission of the shock.

As in most standard open-economy New Keynesian settings, the positive monetary policy shock in ROW leads to an initial decrease in ROW real output (panel C.2G) and consumption (panel C.2J).

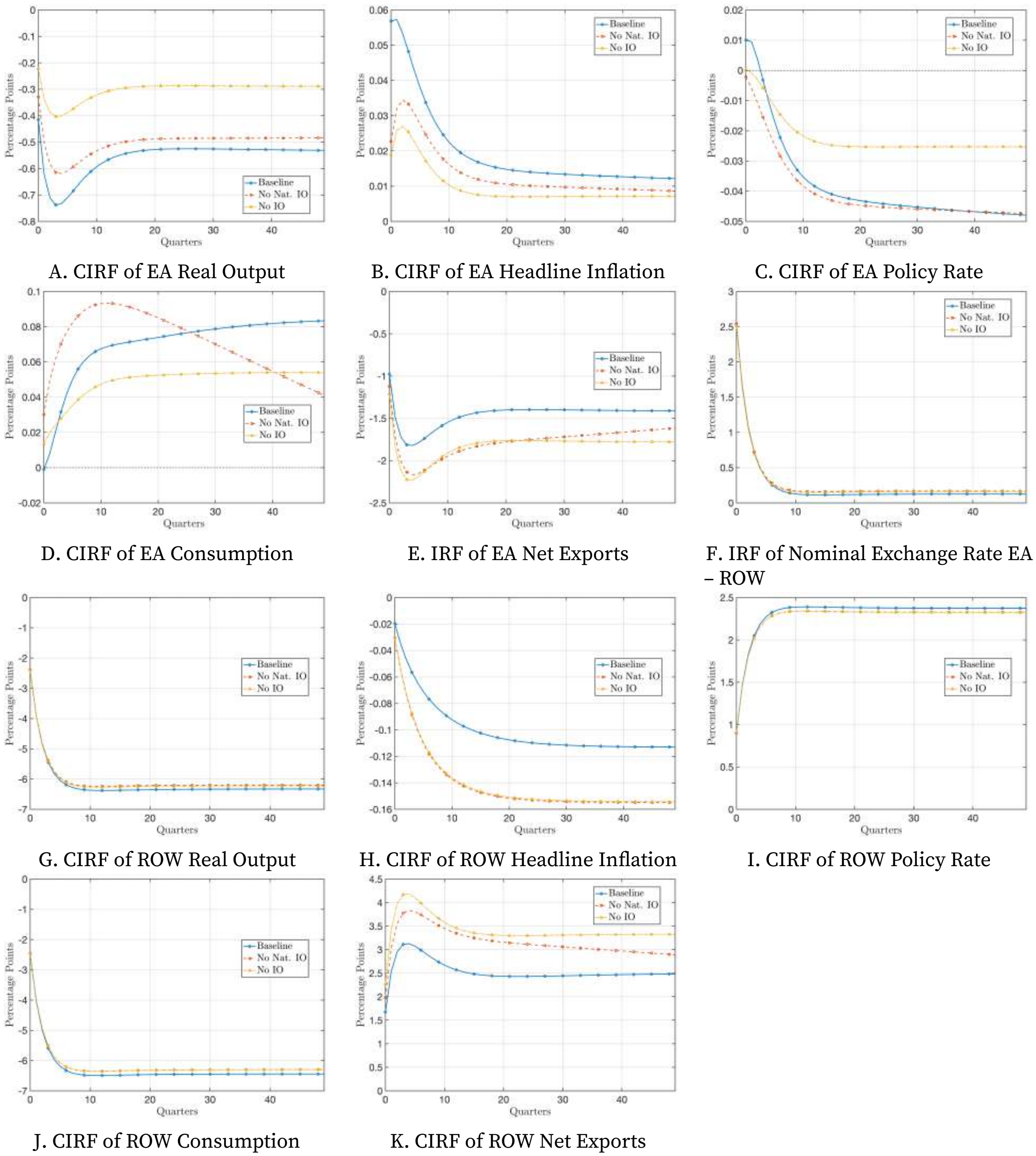
Across the three IO scenarios, these local responses remain broadly similar. In line with the intuition from Figure C.1, production networks do not substantially alter the size of the impact on ROW's own real activity: the dominant driver there is the direct effect of the shock itself.

The shock triggers a response in ROW headline inflation (panel C.2H). Under the systematic monetary rule, the ROW policy rate (panel C.2I) moves to stabilize inflation. When we allow for richer production networks, the pass-through from demand pressures to prices is somewhat dampened (relative to a scenario without input-output linkages). This aligns with the usual finding that having multiple sectors can reduce the pass-through of demand changes on prices. Panel C.2K shows the standard pattern following a contractionary monetary policy shock shock: net exports increase on impact, reflecting lower import demand from ROW's own households and firms. Over time, depending on the shock's persistence and the policy reaction, net exports recover. The presence of production networks moderates the amplitude of the responses.

Turning to the EA panels, we see that ROW's positive monetary policy shock spills over and reduces EA real GDP (panel C.2A) and net exports (panel C.2E). Consistent with Figure C.1's lesson, production networks amplify the effect of foreign demand shocks on EA output: the non-neutrality introduced through a smaller pass-through of demand changes to prices exacerbates the response of real activity.

The panels for EA headline inflation and the EA policy rate, C.2B and C.2C, show that the cross-border transmission of foreign monetary policy shocks can be meaningful, but that the net impact on EA inflation often remains smaller than the local impact in ROW. This again reflects the fact that ROW's shock directly affects its own production costs and demand, with only a portion of that feeding into EA's pricing decisions. In line with our findings for Figure C.1, production networks dampen the effect on EA inflation because the pass-through from upstream price changes to final consumer prices involves multiple stages of price adjustments.

The monetary authority in the EA reacts by decreasing the policy rate, which in turn



A. CIRF of EA Real Output
B. CIRF of EA Headline Inflation
C. CIRF of EA Policy Rate
D. CIRF of EA Consumption
E. IRF of EA Net Exports
F. IRF of Nominal Exchange Rate EA – ROW
G. CIRF of ROW Real Output
H. CIRF of ROW Headline Inflation
I. CIRF of ROW Policy Rate
J. CIRF of ROW Consumption
K. CIRF of ROW Net Exports

FIGURE C.2. Effects of a Foreign Monetary Policy Shock on Euro-Area Variables

Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a contractionary monetary policy shock in ROW.

generate the positive response of real consumption (panel C.2D).

The nominal exchange rate (panel C.2F) highlights the systematic policy reaction on both economies. While the ROW policy rate increases as a result of the contractionary monetary shock, the EA policy rate decreases to stabilize the EA economy. As a result, the EA currency depreciates, boosting EA net exports. Over time, these dynamics revert as interest rates and inflation rates gradually normalize.

As in Figure C.1, the panels compare the responses across three IO scenarios (e.g., dotted vs. dashed vs. solid lines). Generally: Local (ROW) variables (output, consumption, local inflation) do not change drastically across scenarios, as local effects remain dominated by the direct shock.

Inflation variables (especially abroad in EA) are dampened by the presence of input-output production chains. Real output and consumption are amplified by production networks, since cross-border linkages spread the additional demand changes to other countries' industries.

In sum, Figure C.2 confirms the qualitative patterns already seen in Figure C.1 (both after aggregate demand shocks): production networks modestly dampen the local impact of the shock on inflation, but they amplify the cross-border propagation to real activity. These results underscore the importance of accounting for multi-country input-output structures in open-economy models when assessing how foreign shocks reverberate through domestic variables.

Sectoral TFP Shock. Figures C.3 and C.4 examine the macroeconomic responses to positive TFP shocks—but in two different sectors. Figure C.3 reports the CIRFs following a TFP shock in the most upstream sector (Basic metals), while Figure C.4 presents the responses after a TFP shock in the most downstream sector (Health and Education Services). The comparison between these figures highlights how the position of a sector within the production network fundamentally alters both the propagation and persistence of shocks.

A positive TFP shock in the most upstream sector, Basic metals, leads to a substantial improvement in productivity in a sector that serves as a key supplier of intermediate inputs across many industries. Consequently, the reduction in production costs in Basic metals propagates through the input–output network, yielding a robust and persistent boost in real output (panel C.3G) in ROW.

As the lower production costs spread through the network, headline inflation experiences a deflationary pressure (panel C.3H). However, due to the extensive interlinkages of upstream sectors, the shock's deflationary impact on inflation is gradual and persistent. This reflects the slower adjustment process in sectors that are heavily embedded in longer production chains.

The deflationary spiral generated by the productivity increase enhances the international competitiveness of ROW, stimulating net exports in ROW (panel C.3K). As a result, although the EA real consumption increases after the shock (panel C.3D), real output falls (panel C.3A) due to the loss in the relative international competitiveness, with falling net exports.

In contrast, the positive TFP shock in the most downstream sector, Health and Education Services, directly affects final consumption. The immediate effect is more pronounced in terms of a rapid boost to output (panel C.4G) and an almost instantaneous deflationary effect on consumer prices (panel C.3H). However, because downstream sectors have a smaller role as suppliers of intermediate inputs, the propagation through the production network is less amplified and less persistent.

The CIRFs show that while the shock in a downstream sector generates a sharp initial response in both output and inflation, these effects tend to dissipate more quickly. In other words, the improvement in productivity in a downstream sector translates into a short-lived boost in economic activity and a temporary moderation in inflation, with variables reverting to their steady states relatively rapidly.

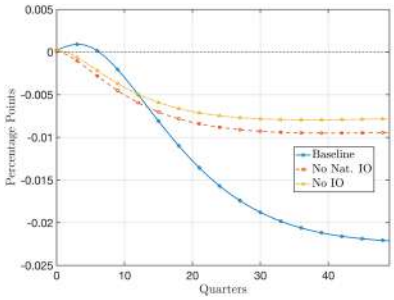
The localized nature of the downstream shock means that the spillovers to other sectors are more muted. As a result, the impact on macroeconomic aggregates—such as net exports (panel C.4K) or the systematic monetary response (panel C.4I)—is less persistent compared to an upstream shock.

The key difference in the macroeconomic dynamics after an upstream or downstream shock lies in the network role: upstream sectors like Basic metals have a high degree of intersectoral connectivity. Thus, productivity improvements here generate a cascade of cost reductions that travel across a wide array of industries, leading to a more sustained expansion in output and a more gradual deflation of prices. In summary, production networks unequivocally amplify the effects of TFP shocks on upstream sectors.

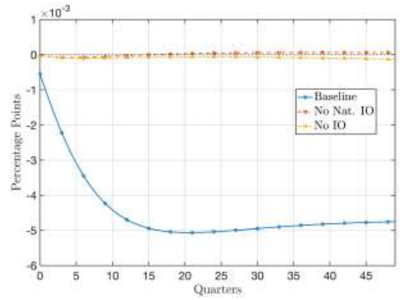
Downstream sectors, by contrast, are closer to the final consumption basket; shocks here affect consumer prices and output directly but with limited cross-sectoral amplification. For instance, local (ROW) macroeconomic variables are dampened by production networks; whereas EA macroeconomic dynamics after a foreign TFP shock are amplified by production networks.

The differential dynamics suggest that policy responses may need to be tailored to the origin of the shock. Upstream shocks, due to their persistent and widespread effects, might require a more cautious and prolonged monetary response. On the other hand, shocks originating in downstream sectors, with their immediate but short-lived impacts, may call for more temporary or targeted measures.

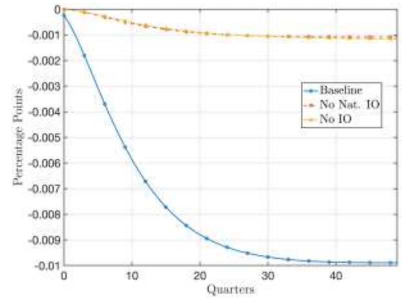
In summary, Figures C.3 and C.4 together underscore the importance of sectoral heterogeneity in understanding shock transmission through production networks. While both upstream and downstream TFP shocks are expansionary and deflationary in nature, their



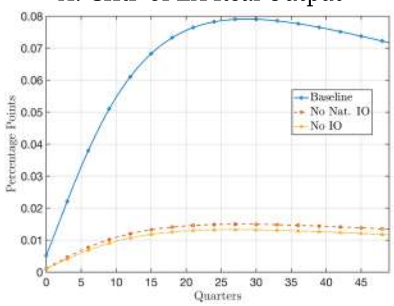
A. CIRF of EA Real Output



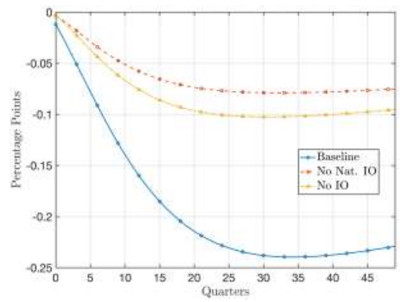
B. CIRF of EA Headline Inflation



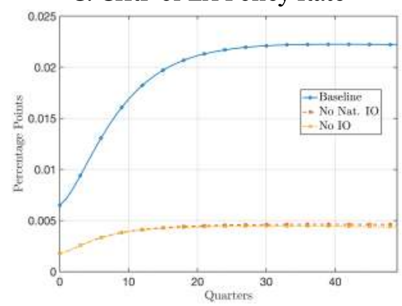
C. CIRF of EA Policy Rate



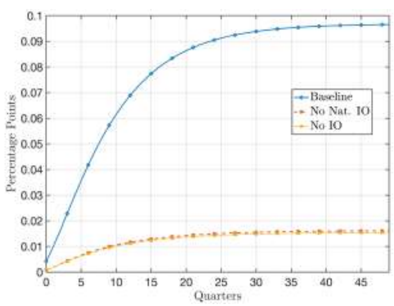
D. CIRF of EA Consumption



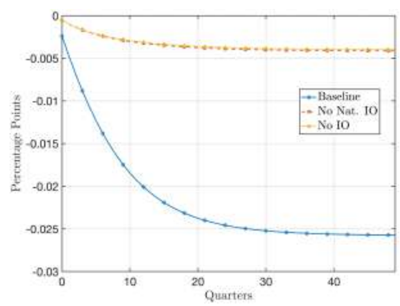
E. IRF of EA Net Exports



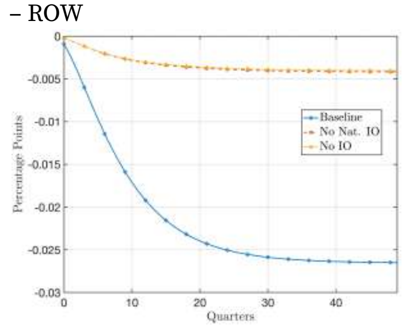
F. IRF of Nominal Exchange Rate EA – ROW



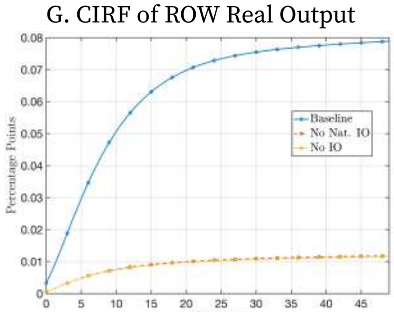
G. CIRF of ROW Real Output



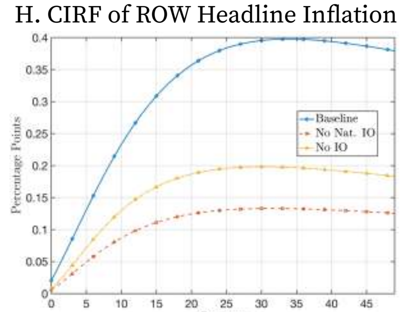
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



J. CIRF of ROW Consumption



K. CIRF of ROW Net Exports

FIGURE C.3. Effects of a Foreign TFP Shock (Upstream) on Euro-Area Variables

Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive TFP shock (most upstream sector) in ROW.

effects differ markedly in terms of magnitude, persistence, and propagation pathways. The upstream shock in Basic metals exerts a more prolonged influence across the economy, amplified by its central role in the input–output network, whereas the downstream shock in Health and Education Services delivers a more immediate yet transient impact. These findings highlight that the sectoral origin of shocks must be carefully considered when designing macroeconomic stabilization policies in an interconnected economy.

Sectoral Price Cost-Push Shock. Figures C.5 and C.6 illustrate the economy-wide effects of positive price cost–push shocks, but they do so for two very distinct sectors. In Figure C.5, the shock originates in the most upstream sector—Basic metals—whereas in Figure C.6 the shock is applied to the most downstream sector—Health and Education Services. While both shocks are inflationary in nature, the differences in sectoral position within the production network give rise to markedly different transmission mechanisms and macroeconomic responses.

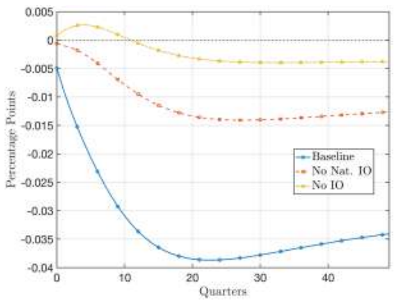
The Basic metals sector plays a crucial role as an intermediate input supplier to many industries. A cost–push shock here increases the production costs for a wide range of downstream firms. However, because the shock originates in a sector with relatively lower direct exposure to final consumption, its initial impact on headline inflation is more muted (panel C.5H).

As the increased costs propagate via the input–output linkages, the inflationary pressure builds gradually. The cost increases are transmitted over multiple sectors, resulting in a persistent upward drift in headline inflation. In addition, the extensive cross–border linkages in the upstream network amplify these effects internationally, leading to a more sustained response in variables such as the EA central bank policy rate (panel C.5C), and even exchange rates (panel C.5F).

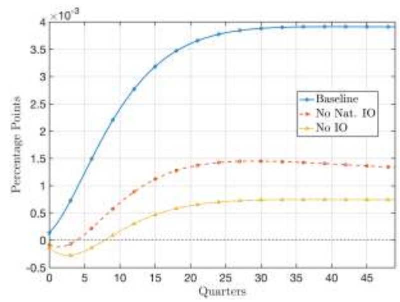
The persistence of the inflationary impulse, coupled with the gradual spread of higher production costs, also translates into delayed adjustments in output (panel C.5G) and consumption (panel C.5H). Firms, facing rising input costs, eventually reduce labor demand and production, thereby impacting real GDP. The central bank's response (panel C.5I) tends to be more prolonged as it attempts to counteract the persistent cost–driven inflation dynamics.

In contrast, a cost–push shock in Health and Education Services—a sector directly linked to final consumption—produces an immediate and sharp rise in consumer prices (panel C.6H). Although the immediate inflationary spike is pronounced, the downstream shock tends to be more transient. With limited propagation through the production network, the initial high inflationary impulse dissipates relatively quickly. Output (panel C.6G) and consumption (panel C.6J), while initially affected by the surge in costs, recover more rapidly as the shock is not reinforced through extensive intermediate linkages.

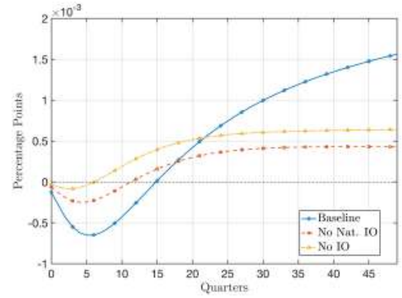
The direct exposure of the downstream sector to final consumption means that the cost in-



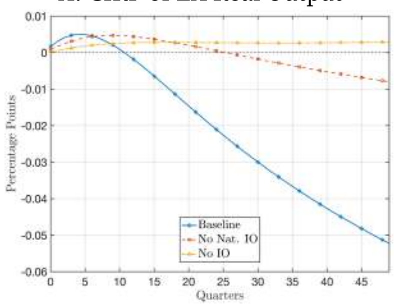
A. CIRF of EA Real Output



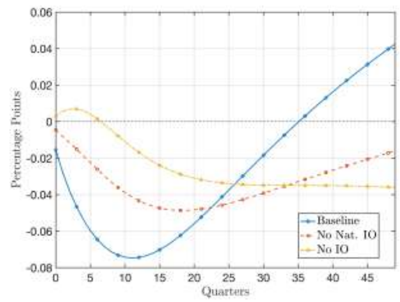
B. CIRF of EA Headline Inflation



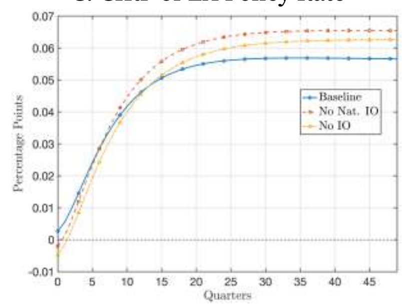
C. CIRF of EA Policy Rate



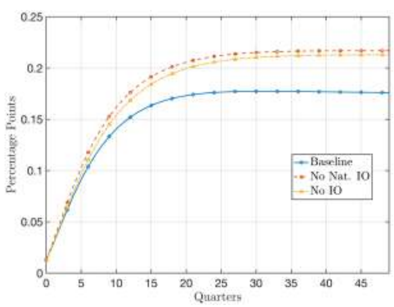
D. CIRF of EA Consumption



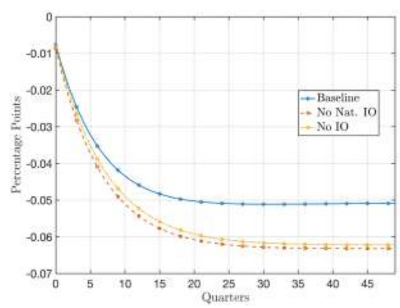
E. IRF of EA Net Exports



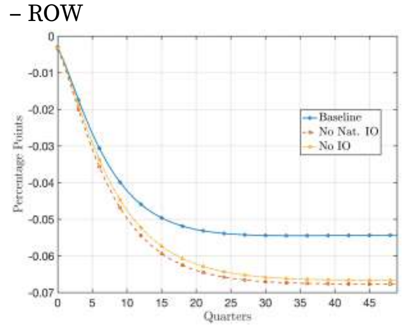
F. IRF of Nominal Exchange Rate EA – ROW



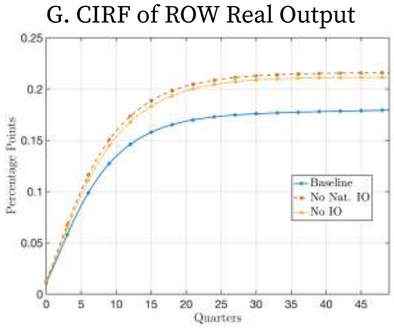
G. CIRF of ROW Real Output



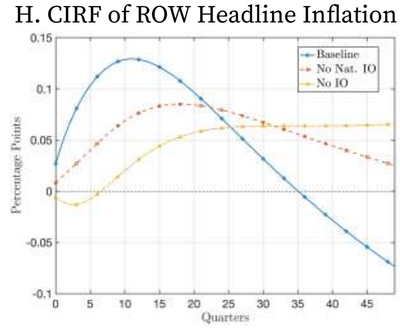
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



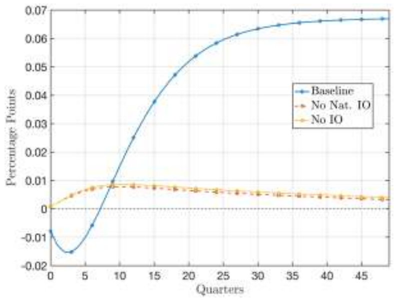
J. CIRF of ROW Consumption



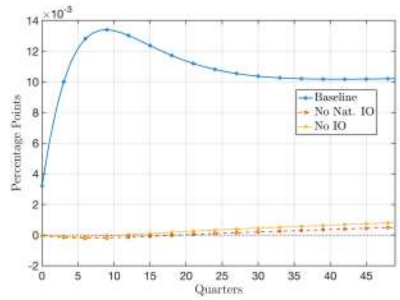
K. CIRF of ROW Net Exports

FIGURE C.4. Effects of a Foreign TFP Shock (Downstream) on Euro-Area Variables

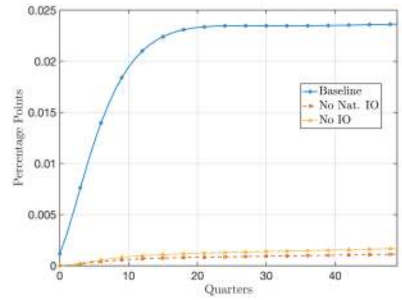
Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive TFP shock (most downstream sector) in ROW.



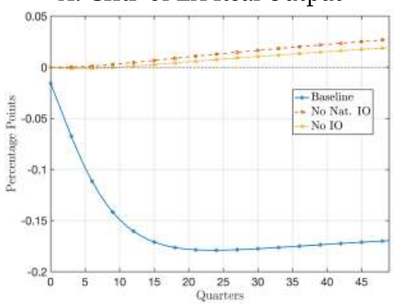
A. CIRF of EA Real Output



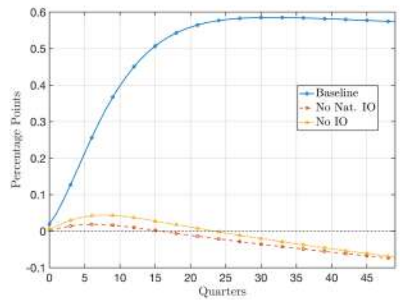
B. CIRF of EA Headline Inflation



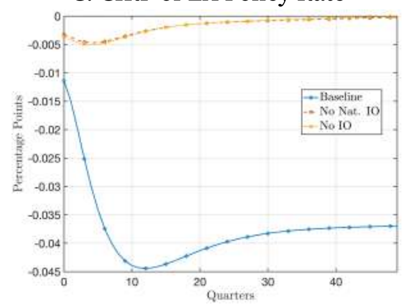
C. CIRF of EA Policy Rate



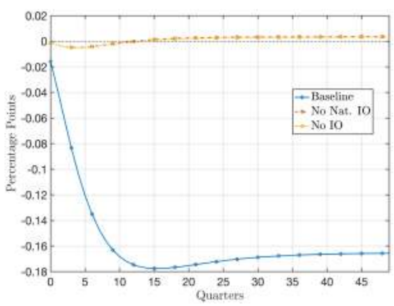
D. CIRF of EA Consumption



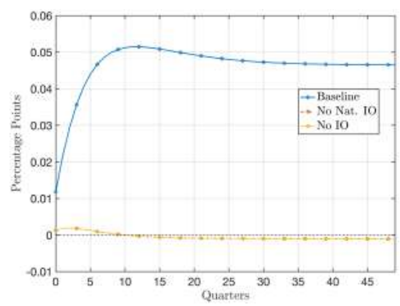
E. IRF of EA Net Exports



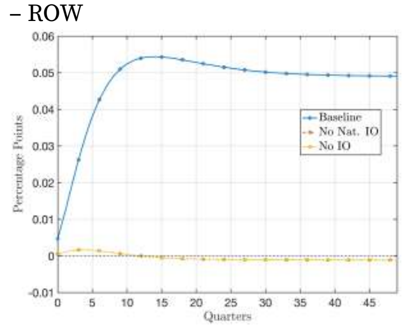
F. IRF of Nominal Exchange Rate EA – ROW



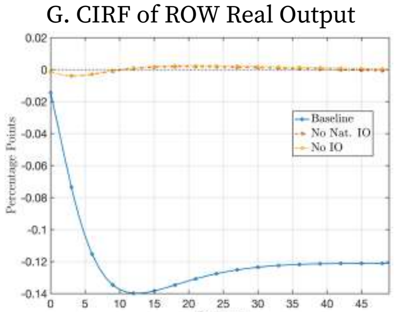
G. CIRF of ROW Real Output



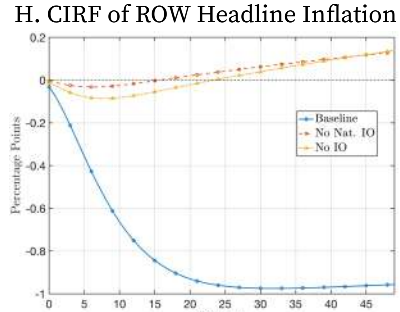
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



J. CIRF of ROW Consumption



K. CIRF of ROW Net Exports

FIGURE C.5. Effects of a Foreign Price Cost-Push Shock (Upstream) on Euro-Area Variables

Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive price cost-push shock (most upstream sector) in ROW.

crease does not circulate widely as an intermediate input. Consequently, while the short–term impact on consumer prices and demand is significant, the shock's spillover into other sectors and across countries is relatively modest, leading to a faster reversion towards the steady state.

The key distinction between the two shocks lies in the role of the sector within the production network. Upstream shocks in Basic metals generate a slower but more persistent increase in inflation as cost increases are gradually amplified and diffused through multiple intermediate stages. Conversely, downstream shocks in Health and Education Services lead to an immediate but short–lived impact on inflation, as the shock is less embedded in the network and thus dissipates faster.

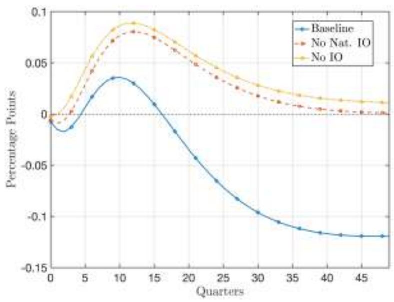
In the upstream case, persistent inflation feeds back into wage setting and the monetary policy stance, resulting in more gradual adjustments in output, consumption, and even exchange rates. Production networks unequivocally amplify the effects of the price cost-push shock, both in ROW and internationally. In the downstream scenario, the rapid spike in inflation is met with a swift adjustment in consumer behavior, leading to a sharper but more transient drop in real activity. Furthermore, production networks dampen amplify the effects of the price cost-push shock on local real output, consumption, and headline inflation in ROW, while they amplify the effects of the price cost-push shock internationally.

These findings suggest that the origin of a cost–push shock should be carefully considered when designing policy responses. Persistent, network–amplified shocks from upstream sectors may require a more sustained and cautious monetary response to anchor inflation expectations. By contrast, transient shocks originating in downstream sectors might be addressed with more flexible, short–term measures to avoid over–tightening.

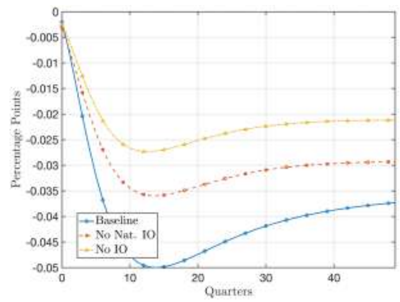
In sum, Figures C.5 and C.6 underscore that the position of a sector within the production network is crucial in determining the dynamics of cost–push shocks. While both types of shocks are inflationary, those emanating from upstream sectors (Basic metals) lead to a gradual, persistent, and internationally amplified response, whereas shocks in downstream sectors (Health and Education Services) deliver a sharp, immediate, yet fleeting impact on the economy. This nuanced understanding reinforces the need for targeted policy interventions that account for the underlying structure of production networks.

Wage Cost-Push Shock. Figure C.7 examines the macroeconomic responses to a positive aggregate wage cost–push shock in the ROW. In line with our previous analyses, this figure illustrates how wage-driven cost increases propagate through both domestic and international production networks, but with nuances that distinguish wage shocks from demand, TFP, or price cost–push shocks.

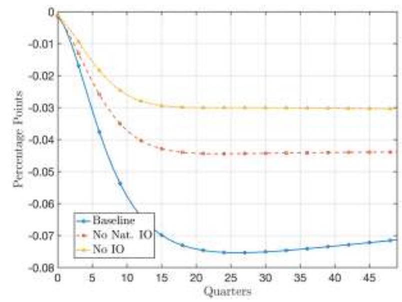
A positive wage cost–push shock in the ROW immediately raises the unit labor costs,



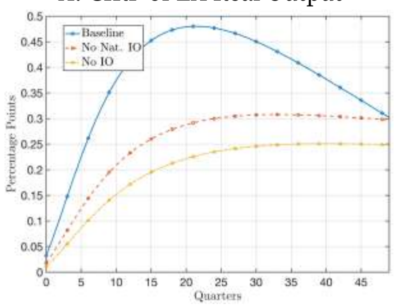
A. CIRF of EA Real Output



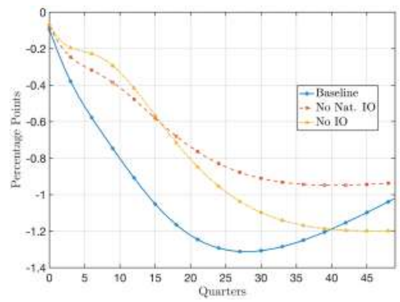
B. CIRF of EA Headline Inflation



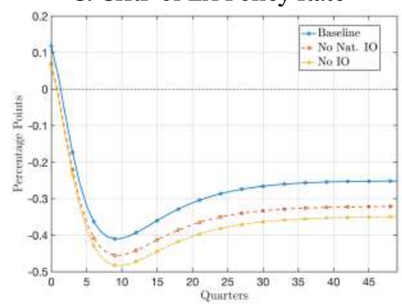
C. CIRF of EA Policy Rate



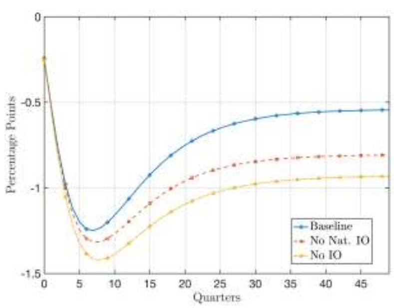
D. CIRF of EA Consumption



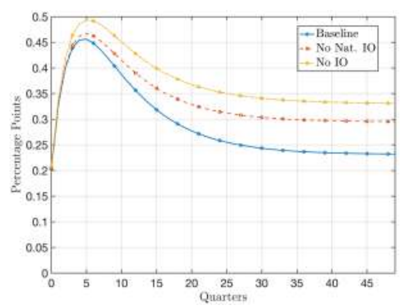
E. IRF of EA Net Exports



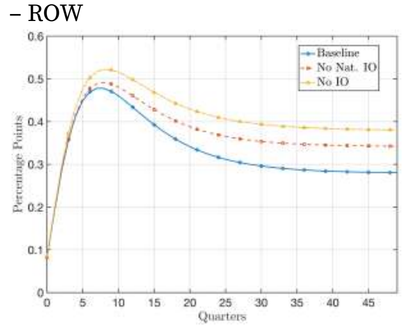
F. IRF of Nominal Exchange Rate EA – ROW



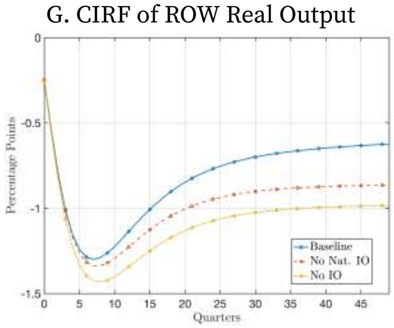
G. CIRF of ROW Real Output



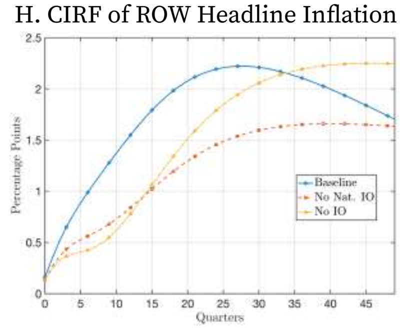
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



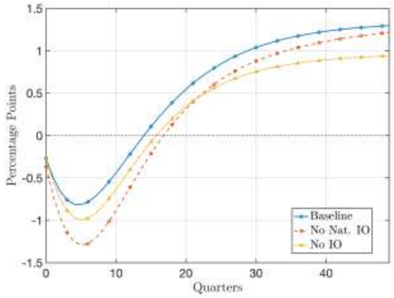
J. CIRF of ROW Consumption



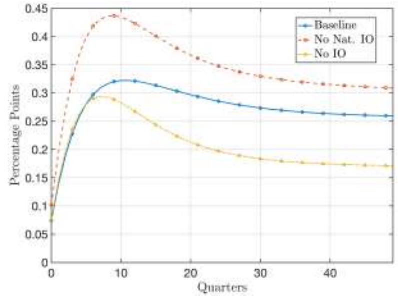
K. CIRF of ROW Net Exports

FIGURE C.6. Effects of a Foreign Price Cost-Push Shock (Downstream) on Euro-Area Variables

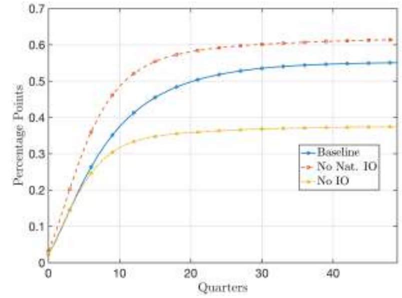
Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive price cost-push shock (most downstream sector) in ROW.



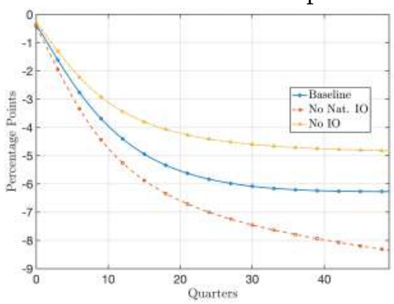
A. CIRF of EA Real Output



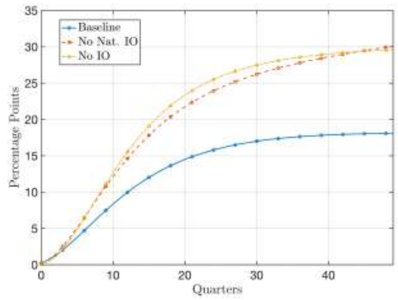
B. CIRF of EA Headline Inflation



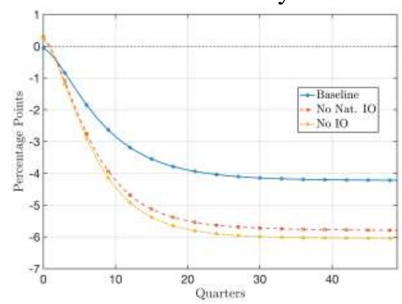
C. CIRF of EA Policy Rate



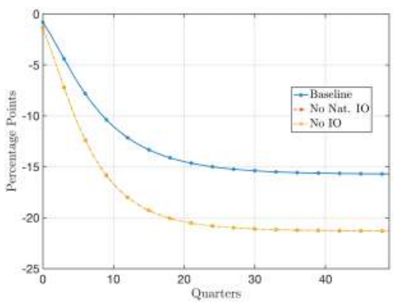
D. CIRF of EA Consumption



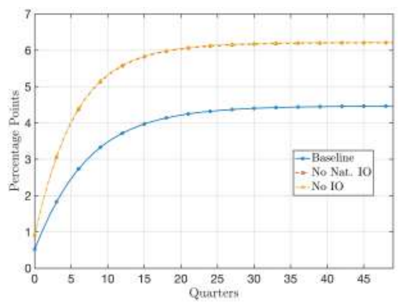
E. IRF of EA Net Exports



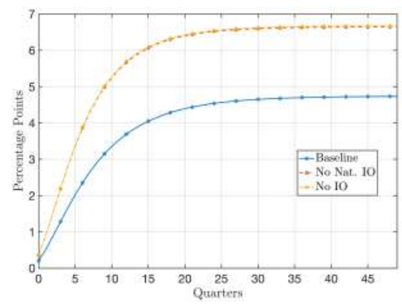
F. IRF of Nominal Exchange Rate EA – ROW



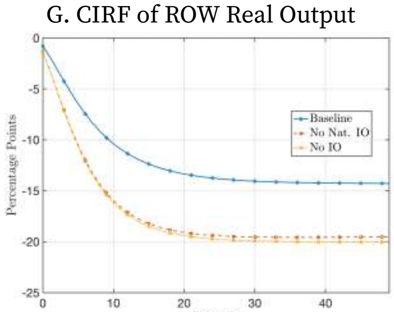
G. CIRF of ROW Real Output



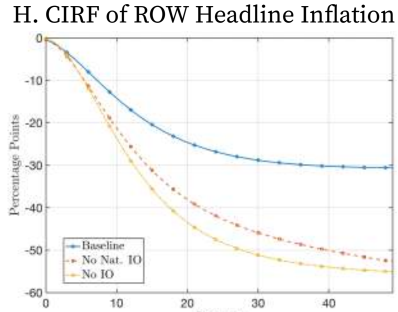
H. CIRF of ROW Headline Inflation



I. CIRF of ROW Policy Rate



J. CIRF of ROW Consumption



K. CIRF of ROW Net Exports

FIGURE C.7. Effects of a Foreign Wage Cost-Push Shock on Euro-Area Variables

Notes: Cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive wage cost-push shock in ROW.

leading to a direct upward pressure on firms' marginal costs. In our model, this initial cost increase is transmitted to final prices through the firms' pricing decisions—captured by the wage and price Phillips curves—resulting in an immediate spike in headline inflation (panel C.7B). However, unlike a pure price cost–push shock, the propagation of a wage shock is mediated by the structure of the wage bill within the production network.

On the one hand, if the production network is highly interconnected—where wage costs constitute a significant share of intermediate input costs—the shock is amplified. As higher wages raise costs across the production network, the feedback loop between rising selling prices and further cost adjustments becomes more pronounced. This mechanism not only heightens the initial inflationary response but also prolongs its persistence, much like the amplification observed with upstream cost–push shocks in Figures C.5 and C.6.

On the other hand, production networks can also dampen the immediate impact of a wage shock. In sectors where wage costs represent a relatively lower share of total production costs or where firms can partially substitute away from labor inputs, the direct transmission of higher wages to final prices is less effective. In these cases, while there may still be a noticeable inflationary effect, the response is more muted and tends to die out sooner as the shock dissipates through the network.

The insights from Figures C.1–C.6 guide our interpretation of Figure C.7. For instance, in our earlier figures, we observed that production networks tend to amplify shocks that affect intermediate input costs—such as upstream price cost–push shocks—by reinforcing the feedback between cost increases and price adjustments. However, in the case of the wage shock in the ROW, when wage adjustments are pervasive across multiple sectors with high interlinkages, the cumulative inflationary response in ROW is significantly smaller. This pattern is prevalent across macroeconomic variables in the ROW.

Instead, production networks predict heterogeneous responses in the EA. For instance, the effect of a foreign wage markup shock on EA real output is amplified (panel C.7A). Interestingly, removing only the national IO linkages amplifies the response of headline inflation (panel C.7B), real consumption (panel C.7D), and the policy rate (panel C.7C) in the EA.

From a policy perspective, Figure C.7 underscores the importance of understanding the structure of production networks when designing responses to wage cost–push shocks. In economies where production networks amplify wage shocks, a passive monetary response may allow inflation to persist, thereby necessitating a more proactive policy stance. In contrast, if the networks dampen the immediate impact, policymakers might opt for a less aggressive intervention, focusing instead on measures that prevent the shock from generating adverse second-round effects through global linkages.

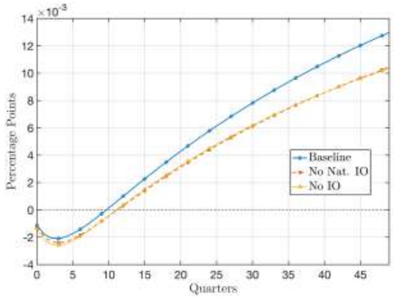
In summary, Figure C.7 shows that a positive aggregate wage cost–push shock in the ROW leads to an immediate increase in inflation, with the overall response being critically determined by the configuration of production networks. When these networks are characterized by high intersectoral interdependencies and a significant share of wage costs in production, the shock is amplified and persists longer. Conversely, in environments with a greater degree of substitution or a more downstream production focus, the networks serve to dampen the shock's effects. This dual role of production networks—amplifying or dampening depending on the context—remains a key insight in understanding the transmission of wage shocks in an interconnected global economy.

International Price-Wedge Shock. Figure C.8 presents CIRFs for key macroeconomic variables in the EA and the ROW following an international price–wedge shock in two upstream sectors of ROW. In both cases, EA faces a higher import price than ROW due to the imposed wedge; however, the transmission mechanisms differ markedly because of the underlying price rigidities. Given the structure of the shock, the local effects are minimal, and we focus on the international pass-through to the EA.

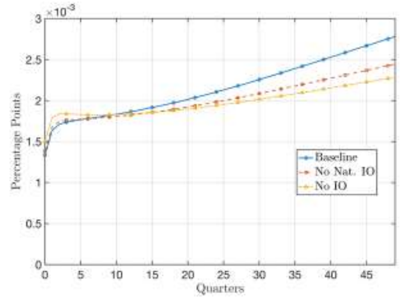
The first two rows of figure C.8 present the macroeconomic dynamics after a price-wedge shock to the NACE sector C.26 *Manufacture of computer, electronic and optical products,* which contains the semiconductors industry. In the semiconductor sector, which is characterized by rigid prices ($\theta^{p}_{\text{ROW, C.26}} = 0.72$), the shock is absorbed more gradually. The rigidity delays the immediate pass-through of the increased price to intermediate and final goods, resulting in a more muted initial response. For instance, headline inflation in the EA increases by 0.0013 percentage points on impact (panel C.8B), a magnitude four times smaller than the increase in EA inflation after the energy shock studied in Section 4.3 (reproduced for convenience in panel C.8H).

However, once adjustments begin, the higher prices persist over time. This persistence leads to a gradual but sustained rise in headline inflation in EA. For instance, after the shock in the semiconductor sector, the cumulative price increase in the EA increases exponentially even after 50 quarters, whereas it begins its convergence to steady-state inflation after the shock in the energy sector. The slow adjustment process amplifies the shock's propagation through the production network, as intermediate input costs remain elevated for longer periods. Consequently, real consumption tends to decline more gradually. Real output follows markedly different dynamics after both exogenous disturbances. The large increase in net exports generates an increase in real output, whereas real output decreases after the energy shock. This generates a smaller response of systematic monetary policy after a shock in the semiconductor industry.

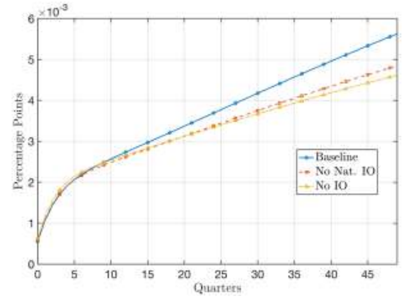
The mining and quarrying sector features flexible prices, enabling firms to adjust prices almost immediately in response to the shock. This flexibility leads to a sharper and more pronounced initial impact on inflation as the cost shock is quickly transmitted to downstream



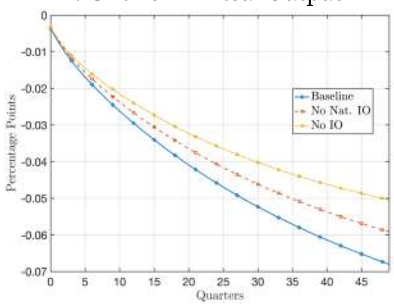
A. CIRF of EA Real Output



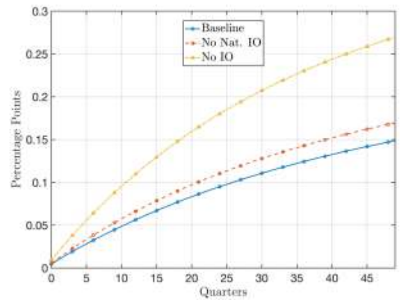
B. CIRF of EA Headline Inflation



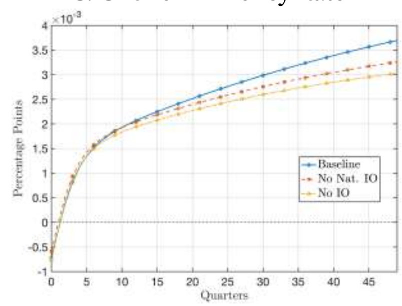
C. CIRF of EA Policy Rate



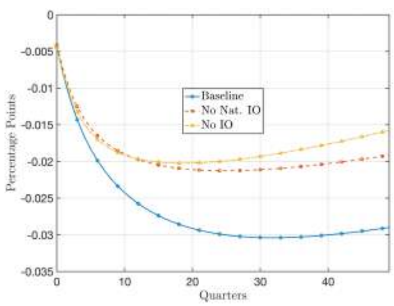
D. CIRF of EA Consumption



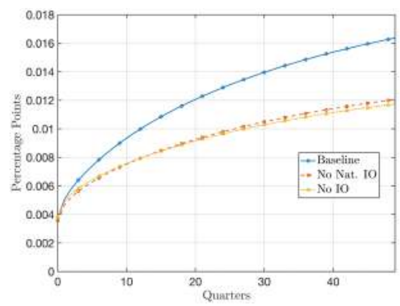
E. IRF of EA Net Exports



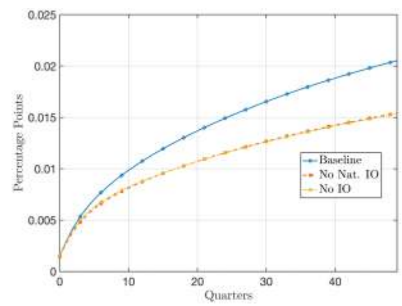
F. IRF of Nominal Exchange Rate EA – ROW



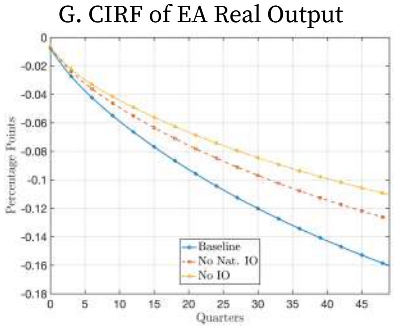
G. CIRF of EA Real Output



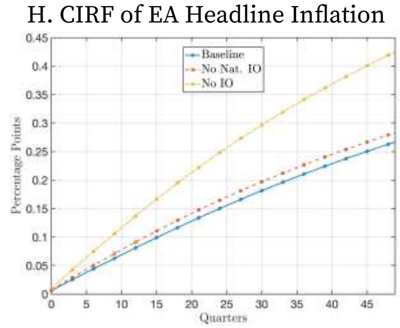
H. CIRF of EA Headline Inflation



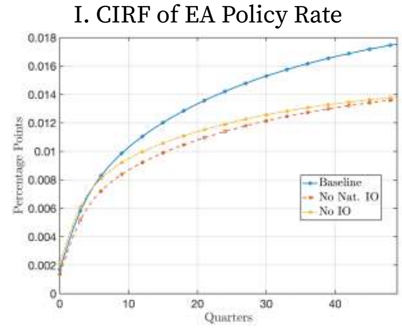
I. CIRF of EA Policy Rate



J. CIRF of EA Consumption



K. IRF of EA Net Exports



L. IRF of Nominal Exchange Rate EA – ROW

FIGURE C.8. Effects of a Foreign Price-Wedge Shock (Semiconductors and Energy) on Euro-Area Variables

Notes: First two rows: cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive price-wedge shock in sector C.26 Manufacture of computer, electronic and optical products. Last two rows: cumulative Impulse response functions (CIRF) of macroeconomic variables to a positive price-wedge shock in sector B.6 Mining and quarrying.

sectors. However, because prices adjust rapidly, the inflationary impulse is more transitory, and the cumulative response—while initially stronger—is less persistent over time. The swift pass-through limits the duration of adverse effects on output and consumption, and the faster adjustment allows monetary authorities to stabilize the economy more quickly compared to the scenario with rigid prices.

The contrasting dynamics captured in Figure C.8 underscore that the role of production networks in shock transmission is highly dependent on sectoral pricing behavior. In the case of rigid prices, as in semiconductors, the production network amplifies and prolongs the inflationary response, with persistent cost pressures filtering through various stages of production. Conversely, when prices are flexible, as seen in mining and quarrying, the shock is passed through rapidly and then dissipates, resulting in a more immediate yet short-lived impact on macroeconomic variables. These findings imply that when designing stabilization policies, policymakers must consider the specific pricing characteristics of the affected sectors: more aggressive and sustained interventions may be warranted in cases of rigid price adjustments to counteract the prolonged propagation of shocks, while more measured responses could suffice for shocks in sectors with flexible prices.

Appendix D. Additional Figures and Tables

Table D.1 lists the 44 NACE sectors used in the analysis.

Figure D.1 presents a series of heatmaps that illustrate the structure of the production networks at a detailed, country-specific level. In these visualizations, each cell represents the intensity of the input–output linkage between sectors—whether for domestic (home) or international (foreign) transactions. Lighter shades indicate higher input shares, revealing which sectors are more interconnected within a country and highlighting the key channels through which shocks can propagate. For instance, clusters of lighter cells in certain regions of the heatmaps point to sectors that rely heavily on inputs from specific domestic industries. The heatmaps provide a comparative view across different countries, capturing heterogeneities in both the domestic production structures and the degree of integration with foreign supply chains.

TABLE D.1. NACE Rev. 2 sectors used in the Analysis

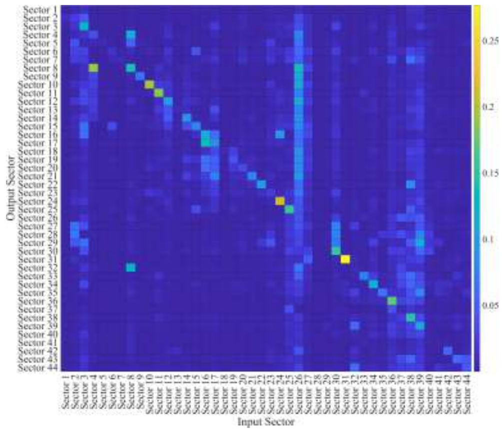
NACE Code	**Sector Name**
A.01–02	Agriculture, forestry and fishing

Continued on next page

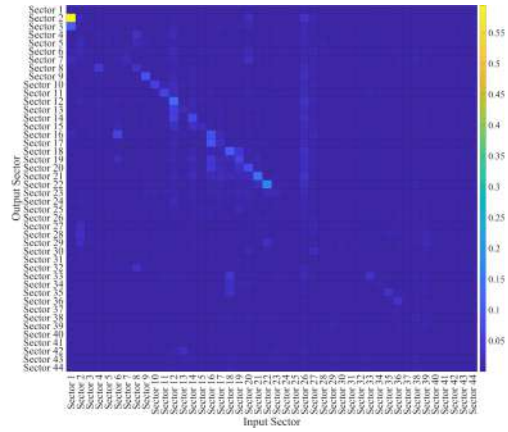
NACE Code	Sector Name
A.03	Fishing and aquaculture
B.05–06	Mining of coal, lignite, crude petroleum and natural gas
B.07–08	Mining of metal ores and other mining and quarrying
B.09	Mining support service activities
C.10–12	Manufacture of food, beverages and tobacco
C.13–15	Manufacture of textiles, wearing apparel and leather products
C.16	Manufacture of wood and products of wood and cork
C.17–18	Manufacture of paper, and printing and media reproduction
C.19	Manufacture of coke and refined petroleum products
C.20	Manufacture of chemicals and chemical products
C.21	Manufacture of basic pharmaceutical products
C.22	Manufacture of rubber and plastic products
C.23	Manufacture of other non-metallic mineral products
C.24	Manufacture of basic metals
C.25	Manufacture of fabricated metal products
C.26	Manufacture of computer, electronic and optical products
C.27	Manufacture of electrical equipment
C.28	Manufacture of machinery and equipment n.e.c.
C.29	Manufacture of motor vehicles, trailers and semi-trailers
C.30	Manufacture of other transport equipment
C.31–33	Other manufacturing and repair and installation of machinery
D.35	Electricity, gas, steam and air conditioning supply
E.36–39	Water supply; sewerage, waste management and remediation
F.41–43	Construction
G.45–47	Wholesale and retail trade; repair of motor vehicles
H.49	Land transport and transport via pipelines
H.50	Water transport
H.51	Air transport
H.52	Warehousing and support activities for transportation
H.53	Postal and courier activities

Continued on next page

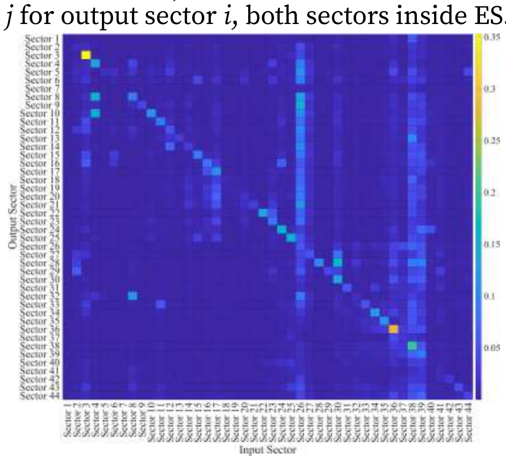
NACE Code	Sector Name
I.55–56	Accommodation and food service activities
J.58–60	Publishing, audiovisual and broadcasting activities
J.61	Telecommunications
J.62–63	IT and other information services
K.64–66	Financial and insurance activities
L.68	Real estate activities
M.69–75	Professional, scientific and technical activities
N.77–82	Administrative and support service activities
O.84	Public administration and defence; compulsory social security
P.85	Education
Q.86–88	Human health and social work activities
R.90–93	Arts, entertainment and recreation
S.94–96	Other personal service activities



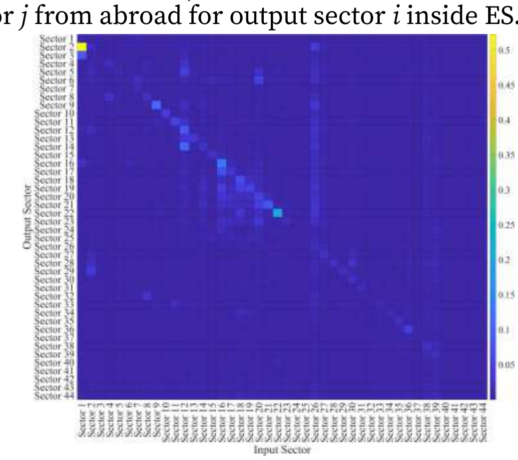
A. Heatmap of Home ES Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside ES.



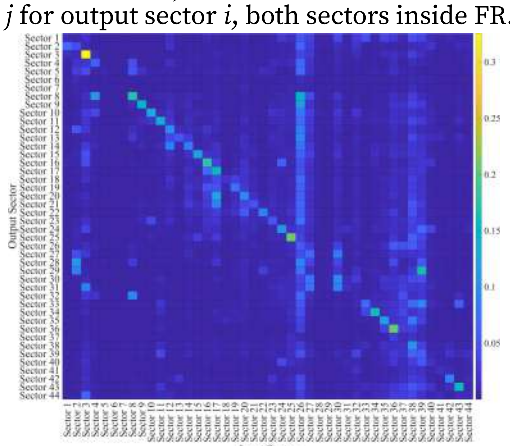
B. Heatmap of Foreign ES Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside ES.



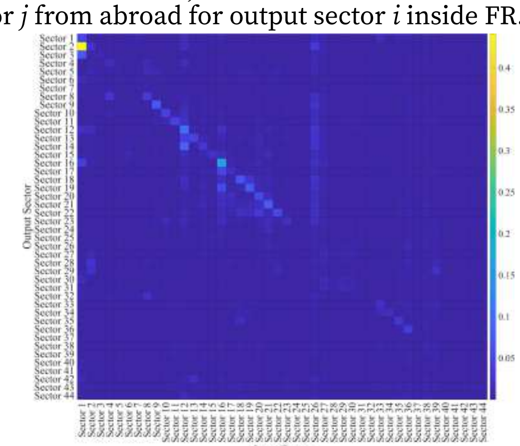
C. Heatmap of Home FR Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside FR.



D. Heatmap of Foreign FR Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside FR.



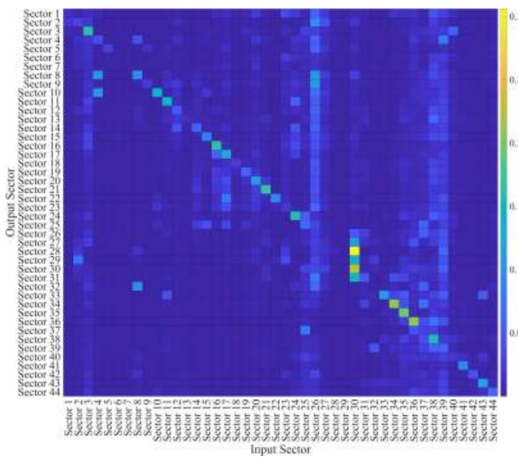
E. Heatmap of Home IT Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside IT.



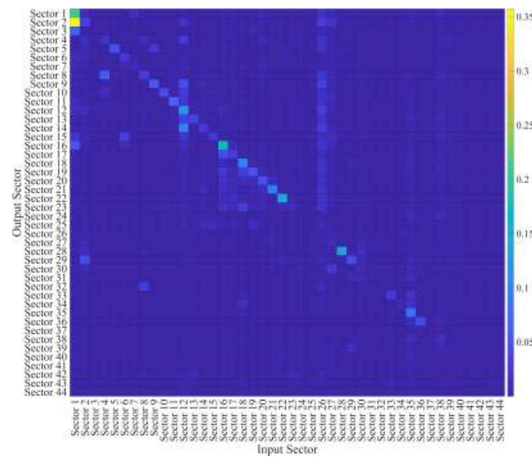
F. Heatmap of Foreign IT Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside IT.

FIGURE D.1. Heatmaps of the Input-Output Structure

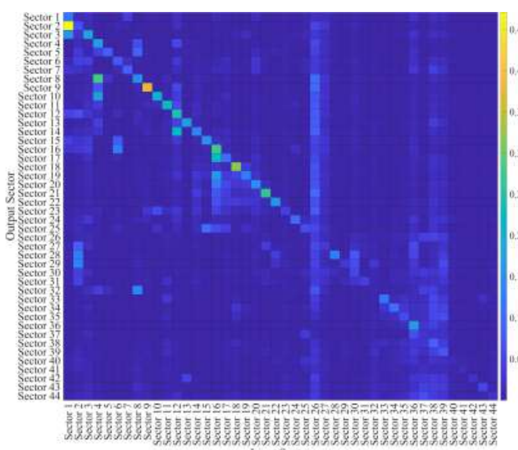
Notes: Heatmaps of the home (left column) and foreign (right column) input-output matrices of the Rest of the Euro Area, Spain, France, Italy, Germany, and the Rest of the world.



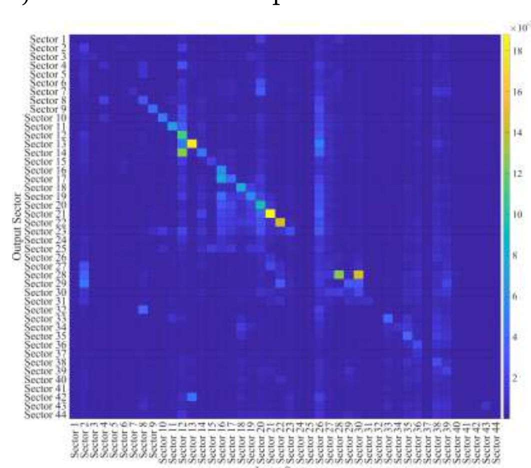
G. Heatmap of Home DE Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside DE.



H. Heatmap of Foreign DE Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside DE.



I. Heatmap of Home ROW Input-Output matrix, where element ω_{ij} denotes the input share of sector j for output sector i, both sectors inside ROW.



J. Heatmap of Foreign ROW Input-Output matrix, where element ω_{ij} denotes the input share of sector j from abroad for output sector i inside ROW.

FIGURE D.1. (Continued)