Fiscal Theory of the Price Level in Small and Open Economies

Juan Pablo Di Iorio[*] Javier García-Cicco[†]

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Abstract

We study the implications of the Fiscal Theory of the Price Level (FTPL) for the dynamics of an otherwise standard New Keynesian model of a small and open economy, paying special attention to the role played by the currency composition of government debt. We analyze the existence and uniqueness of a stationary equilibrium under interest rate rules, how the monetary and fiscal transmission mechanisms are altered when the FTPL is at play, and the extent to which alternative monetary and fiscal regimes influence the propagation of external shocks. We show that the share of debt denominated in foreign currency is crucial for understanding the role that the FTPL plays in these key aspects of open-economy dynamics.

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[*]Universidad de San Andrés. E-mail: jdiiorio@udesa.edu.ar
[†]Corresponding Author, Universidad de San Andrés. E-mail: jgcicco@udesa.edu.ar

1 Introduction

The worldwide spike in inflation following the COVID pandemic, preceded by an unprecedented fiscal expansion in response to that shock, reignited the debate about fiscal and monetary policy interactions —especially as the amount of government debt has significantly increased almost everywhere in recent years (for instance, IMF, 2025 reports that close to 70% of the 175 economies in their sample had, by 2024, heavier public debt burdens than before 2020). The Fiscal Theory of the Price Level (FTPL) has been one of the main theoretical frameworks for addressing these issues (foundational contributions include Leeper, 1991, Sims, 1994, and Woodford, 1994, with a detailed textbook treatment provided by Cochrane, 2023). Most studies in that literature have focused on closed-economy frameworks. Even the scattered papers analyzing open economies (discussed below) omit a critical aspect of public finances in emerging and developing economies: government debt denominated in foreign currency. For instance, according to Arslanalp and Tsuda (2014), in 2023, the average share of foreign currency debt in emerging countries was 30%, with a standard deviation of 21%, while for developing economies these values were 52% and 25%, respectively. Against this background, we study the implications of the FTPL for the dynamics of an otherwise standard New Keynesian model of a small open economy, with traded and non-traded goods, paying particular attention to the role played by the currency composition of government debt.

The FTPL focuses on the lifetime government budget constraint (a.k.a. the debt-valuation equation), which requires the discounted present value of current and expected primary surpluses to equal the real value of outstanding nominal debt. In a closed economy setup, where debt is fully denominated in local currency, a change in the net present value of primary surpluses (not compensated in the future) requires an opposite-sign movement in the price level to dilute (or revalue, depending on the required sign) the real burden of outstanding nominal debt.

An open economy setup allows us to broaden the analysis in several relevant dimensions. First, even if debt is fully denominated in domestic currency, FTPL considerations might induce non-trivial dynamics in the nominal exchange rate (as well as the real one, under nominal rigidities). Second, the larger the share of debt denominated in foreign currency, the smaller the amount of nominal debt that can be diluted via inflation, magnifying the effects on prices relative to a case where all liabilities are in domestic currency. Third, when part of the debt is in foreign currency, any movement in the real exchange rate also affects the debt-valuation equation; which might be particularly relevant for understanding the propagation of real shocks that would move the real exchange rate even under monetary neutrality. We aim to understand whether these potentially distinct channels are both qualitatively and quantitatively relevant, relative to the standard New-Keynesian open-economy analysis with passive fiscal policy frequently used in both academic and policy circles.

The model we use is an infinite horizon, small and open economy with incomplete asset markets and a representative household. There is an exogenous endowment of tradables, while non-tradables are produced using labor under price rigidities that generate a New-Keynesian Phillips curve. Fiscal policy levies lump-sum taxes to finance debt denominated in both domestic and foreign currencies, while monetary policy sets a rule for the nominal interest rate. Different combinations of active or passive fiscal and monetary rules modify the dynamics of the economy.

We begin by studying the (local) existence and uniqueness of a stationary equilibrium under interest rate rules. Results resemble those obtained in closed-economy setups, almost independently of the currency composition of debt. In particular, under a Ricardian/passive fiscal rule, the Taylor principle (i.e., the monetary policy rate reacting more than proportionally to inflation) is enough to guarantee a unique rational-expectations equilibrium; otherwise, multiplicity arises. Instead, if fiscal policy is non-Ricardian/active, the uniqueness of the equilibrium requires a passive monetary policy (i.e., the policy rate moving less than proportionally to inflation).

We next turn to analyzing the transmission of monetary and fiscal shocks. Dynamics not only vary depending on the fiscal and monetary regime (as studied elsewhere in closed-economy models), but our analysis also explores the consequences for the real and nominal exchange rates. For example, while under a passive fiscal setup an increase in the policy rate leads to both a nominal and real appreciation, if fiscal policy is active a nominal depreciation emerges.

Moreover, we show that these differences are exacerbated in the presence of debt denominated in foreign currency. When fiscal policy is passive, a smaller share of domestic-currency debt requires a larger change in the price level to compensate, which, in turn, magnifies the effects on the exchange rate. These results speak directly to the empirical literature documenting that an increase in the domestic monetary policy rate leads to an appreciation in developed economies, while the opposite seems to happen in emerging countries (see, for instance, Hnatkovska et al., 2016 and Bolhuis et al., 2024). Our analysis provides a novel interpretation of these differences based on FTPL considerations when foreign-currency debt is accounted for.

Finally, we study how alternative monetary and fiscal regimes alter the propagation of external shocks, such as surprises in trade-related income and the international cost of borrowing. We show that when debt is fully denominated in domestic currency, and as long as the policy rate responds to movements in inflation (even if the Taylor principle is not satisfied under a Ricardian fiscal regime), the particular fiscal/monetary configuration does not generate qualitatively different dynamics. In contrast, a larger fraction of debt in foreign currency significantly changes the dynamics, specially for variables related to the non-traded sector. In particular, while a jump in the nominal exchange rate helps dilute debt in domestic currency, it simultaneously increases the real burden of repaying dollar-denominated debt. Thus, a stronger reaction in non-traded inflation is generally required, leading to changes in non-traded activity; to the extent that it might even reverse the sign originally induced by the real shock under consideration.

This paper relates to the recent literature that theoretically studies how the New Keynesian propagation mechanism differs under alternative fiscal and monetary configurations, such as the textbook by Cochrane (2023), or the work by Caramp and Silva (2023). Our analysis complements these studies by considering the differences that may arise in an open economy setting. It also highlights that, under sticky prices, FTPL effectively becomes a fiscal theory of the real exchange rate, particularly if a fraction of government debt is denominated in foreign currency.

A few studies have previously analyzed how FTPL extends to open economies. Earlier examples include Loyo (1999), Dupor (2000), and Daniel (2001), while Bianchi (2021) is a more recent contribution. Many of these studies assume flexible prices in setups where purchasing power parity holds, leaving no role for the real exchange rate. In contrast, by considering non-traded goods and sticky prices, we are able to generate non-trivial dynamics for both the nominal and the real exchange rates. Moreover, once the currency composition of debt is taken into account (which these previous studies omit) the real exchange rate plays a crucial role in the debt valuation equation. Finally, while previous work either analyzes how alternative fiscal and monetary configurations determine the equilibrium in open economies or how the transmission of monetary and fiscal shocks is shaped by FTPL considerations, we also examine how the propagation of other real shocks is affected by different monetary and fiscal regimes, particularly when foreign-currency debt is present.

The rest of the paper is organized as follows. Section 2 describes the model, including the alternative fiscal and monetary configurations we consider. The analysis of the existence and uniqueness of equilibria is presented in Section 3. The propagation of monetary and fiscal shocks is discussed in

Section 4. Section 5 studies the impact of external shocks and the role played by the fiscal/monetary setup, while Section 6 presents several robustness exercises. Finally, Section 7 concludes.

2 The Model

We use a standard New Keynesian model, for instance, Schmitt-Grohé and Uribe (2017, Ch. 9.16). This is an infinite-horizon, small and open economy with discrete time and uncertainty. Domestic households derive utility from final-consumption goods and disutility from labor. Financial markets include one-period, non-contingent bonds denominated in both domestic and foreign currencies (pesos and dollars, for simplicity). There is an exogenous endowment of tradables, satisfying the law of one price, with its dollar price determined abroad. In contrast, non-tradable goods have a monopolistically competitive structure, with a continuum of varieties, each produced by a monopolist that demands labor and faces a Calvo pricing friction. Final-consumption goods are composed of both tradables and non-tradables.

The fiscal authority spends an exogenous amount of non-traded goods, financed by both lump-sum and proportional-income taxes, as well as by issuing debt in both currencies. We assume that local-currency bonds are held exclusively by domestic households, while dollar-denominated debt can also be purchased by foreigners (who also lend to domestic households). Finally, monetary policy follows an interest-rate rule.

2.1 Households

The domestic representative household seeks to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{(c_t)^{1-\sigma}}{1-\sigma} - \chi \frac{(h_t)^{1+\varphi}}{1+\varphi} \right] \right\},$$

where c_t denotes aggregate consumption, h_t hours worked, while $\beta \in (0,1)$ and $\sigma, \varphi, \chi > 0$ are parameters capturing, respectively, the discount factor, risk aversion, inverse-Frisch elasticity of labor supply, and relative dis-utility of hours worked. In every period t, they face the following budget constraint in domestic currency,

$$P_t c_t + S_t D_{t-1}^{H*} + \frac{B_t}{R_t} + T_t = (1-\nu_t)(W_t h_t + \Sigma_t) + B_{t-1} + S_t \frac{D_t^{H*}}{R_t^*}. \tag{1}$$

The left-hand side includes uses of income: final consumption (with price P_t), repayment of debt obligations in dollars decided at $t-1$ (D_{t-1}^{H*} is the amount in dollars to be repaid, and S_t is the nominal exchange rate), purchases of domestic-currency bonds B_t (with a gross nominal rate R_t), and lump-sum taxes T_t. The available resources on the right-hand side include labor income (W_t is the hourly nominal wage) and profits from the ownership of domestic companies (Σ_t), both taxed at a proportional rate ν_t; income from domestic bonds (B_{t-1}) purchased at $t-1$; and new debt in dollars (D_t^{H*}, with a gross rate R_t^*). Households also face No-Ponzi-game conditions (NPGC) for each financial asset.[1]

[1] These, as well as transversality conditions for fiscal policy, are discussed in depth in Appendix A. However, as we work with a linearized version of the model and focus only on the local existence and uniqueness of equilibrium (as in most of the literature working with New-Keynesian stochastic models), most of the issues discussed in the appendix are not consequential for the rest of the analysis.

The optimality conditions characterizing the solution to the household problem are the budget constraint (1), both NPGC holding with equality (transversality conditions), and

$$(1-\nu_t)w_t(c_t)^{-\sigma} = \chi(h_t)^{\varphi}, \quad (c_t)^{-\sigma} = \beta R_t E_t\left\{\frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}}\right\}, \quad (c_t)^{-\sigma} = \beta R_t^* E_t\left\{\frac{(c_{t+1})^{-\sigma}\pi_{t+1}^S}{\pi_{t+1}}\right\},$$

with $w_t \equiv W_t/P_t$, $\pi_t \equiv P_t/P_{t-1}$, and $\pi_t^S \equiv S_t/S_{t-1}$. These characterize the trade-offs between, respectively, consumption and hours worked, as well as current and future consumption through either type of financial asset.

2.2 Supply side

Final consumption goods are produced by competitive firms using the technology,

$$c_t = \left[\omega^{1/\eta}\left(c_t^N\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(c_t^T\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}},$$

where c_t^N and c_t^T denote, respectively, consumption of non-tradables and tradables, with prices P_t^N and P_t^T in local currency. The parameters $\eta > 0$ and $\omega \in [0,1]$ are, respectively, the elasticity of substitution between goods and the relative weight of non-tradables in the final-consumption basket. Profit maximization leads to the following demands,

$$c_t^N = \omega\left(p_t^N\right)^{-\eta} c_t, \quad c_t^T = (1-\omega)\left(p_t^T\right)^{-\eta} c_t,$$

with $p_t^T \equiv P_t^T/P_t$ and $p_t^N \equiv P_t^N/P_t$. The ratio of these two captures the intra-temporal trade-off between both goods as a function of their relative price.

Traded goods are produced by a stochastic endowment y_t^T, sold at an international dollar price P_t^*, with $\pi_t^* \equiv P_t^*/P_{t-1}^*$. Both y_t^T and π_t^* are assumed to be stationary. The local price satisfies the law of one price: $P_t^T = S_t P_t^*$. Given its previous definition, p_t^T is also the real exchange rate (rer_t) in this model.

Non-tradables are produced in two stages. First, a continuum of competitive firms produces non-traded goods by combining non-tradable varieties according to the Dixit-Stiglitz aggregator

$$y_t^N = \left[\int_0^1 \left(y_{jt}^N\right)^{\frac{\epsilon_N-1}{\epsilon_N}} \mathrm{d}j\right]^{\frac{\epsilon_N}{\epsilon_N-1}},$$

where y_{jt}^N is the demand for variety j, and $\epsilon_N > 1$ is the elasticity of substitution across varieties.

A monopolist for variety j produces using labor according to $y_{jt}^N = (h_{jt})^{\alpha}$, with $\alpha \in (0,1]$. It faces a Calvo problem in choosing prices: with probability θ_N it has to keep the previous-period price, while with probability $1-\theta_N$ it can freely choose. Log-linearization around the zero-inflation steady state, applied to the monopolist's optimality conditions (see Appendix B.1), leads to the Phillips curve

$$\widehat{\pi}_t^N = \beta E_t\{\widehat{\pi}_{t+1}^N\} + \kappa \cdot \widehat{mc}_t^N,$$

where $\widehat{\cdot}$ denotes log-deviations from the steady state, mc_t^N are real marginal costs in non-traded units, and $\kappa \geq 0$ is a function of parameters $\beta, \theta_N, \alpha, \epsilon_N$.[2]

[2] With this description of the supply side, profits in equilibrium are $\Sigma_t = P_t^N y_t^N - W_t h_t + P_t^T y_t^T$.

2.3 Fiscal policy

The government budget constraint in period t is,

$$\frac{D_t^G}{R_t} + S_t \frac{D_t^{G*}}{R_t^*} + T_t + \nu_t \left(W_t h_t + \Sigma_t \right) = D_{t-1}^G + S_t D_{t-1}^{G*} + P_t^N g_t. \tag{2}$$

Here, D_t^G and D_t^{G*} denote the repayment value at $t+1$ of non-contingent debt emission at t in, respectively, domestic and foreign currency. Notice that we assume the interest rate in dollars, R_t^*, is the same as that charged to households (we discuss the implication of this assumption below). Also, g_t denotes government consumption, assumed to be non-tradable. We focus on government choices satisfying transversality conditions for each type of debt, as discussed in Appendix A.

A fiscal policy is defined as *Ricardian* (a.k.a. passive) if instruments are set such that (2) holds for any value of predetermined debt $(D_{t-1}^G, D_{t-1}^{G*})$ as well as for any possible path of the endogenous variables that affect the government's resource constraint (such as $P_t^N, S_t, R_t, R_t^*, W_t, h_t, \Sigma_t$). In turn, fiscal policy is *Non-Ricardian* (a.k.a. active) if this condition is not satisfied.

The government's budget constraint (2) can be written in real terms as

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t \frac{d_t^{G*}}{R_t^*} + sp_t, \tag{3}$$

where $d_t^G \equiv D_t^G / P_t$, $d_t^{G*} \equiv D_t^{G*} / P_t^*$, and $\sigma_t \equiv \Sigma_t / P_t$, $\tau_t \equiv T_t / P_t$, and $sp_t \equiv \tau_t + \nu_t \left(w_t h_t + \sigma_t \right) - p_t^N g_t$, the latter being the primary surplus in real terms.

Most of the FTPL literature studies the case of lump-sum taxes/transfers and, as we mentioned, there is no discussion about the currency composition of debt. As such, we use a configuration for fiscal policy that can accommodate the assumptions in the literature to maintain comparability but can also be expanded to include other relevant cases.

First, we assume proportional taxes and government expenditures are determined by exogenous and stationary processes, with steady-state values given by $\bar{\nu}$ and $\bar{g}$ respectively. Second, lump-sum taxes are determined by the rule

$$\tau_t - \bar{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{sp}}{1-\beta} \right] + u_t^\tau, \tag{4}$$

where $\overline{sp}$ is the steady-state value of primary surpluses, and u_t^τ is an exogenous and stationary shock. In terms of the currency-composition of debt, let

$$\Omega_t \equiv \frac{rer_t d_t^{G*}}{d_t^G + rer_t d_t^{G*}}, \tag{5}$$

be the share of government debt denominated in dollars. We assume that the government maintains a constant currency composition: $\Omega_t = \overline{\Omega} \in [0, 1]$ for all t. From the perspective of a first-order approximation, like the one we will follow, this seems like a natural initial benchmark.

Overall, equations (3)-(5), plus the exogenous processes for proportional taxes and expenditures, characterize the evolution of the five fiscal variables: d_t^G, d_t^{G*}, T_t, ν_t and g_t. With the last two being exogenous and under a constant share of dollar debt, fiscal policy being Ricardian relies on the parameter ϕ_T. If $\phi_T = 0$, lump-sum taxes are also exogenous and we have a Non-Ricardian configuration. In the other extreme, $\phi_T = 1$ generates a Ricardian policy, while values between 0 and 1 are also possible.[3]

2.4 Monetary policy

We assume a rule for the short-term interest rate in domestic currency with a Taylor-type form:

$$\left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} u_t^R, \tag{6}$$

where $\overline{R}, \overline{\pi}$ are, respectively, steady-state values for R_t and π_t, u_t^R is an exogenous and stationary monetary shock, and $\phi_\pi \geq 0$. Under this configuration, a relevant question is which combination of parameters ϕ_π, ϕ_T, and $\overline{\Omega}$ delivers a unique stationary equilibrium.

2.5 Rest of the world

The interest rate for debt in dollars R_t^* is determined by

$$R_t^* = R_t^W \exp\left\{\psi\left(d_t^* - \bar{d}^*\right)\right\}. \tag{7}$$

where $d_t^* = d_t^{H*} + d_t^{G*}$ is the consolidated-net-foreign-debt position, R_t^W is an exogenous (stationary) process, and $\psi > 0$. Thus, locals are assumed to pay a premium for borrowing in dollars $(\exp\left\{\psi\left(d_t^* - \bar{d}^*\right)\right\})$ over the international rate R_t^W. This elastic premium serves as a "closing device" (Schmitt-Grohe and Uribe, 2003) ensuring that linear dynamics in the real version of the model are stationary. In turn, as discussed below, assuming a premium elastic to the *consolidated* net-foreign debt, in tandem with specific assumptions about preferences, simplifies the dynamics of this model.

2.6 Equilibrium, main channels and calibration

A stationary rational-expectations equilibrium is a set of stochastic processes for endogenous allocations and prices, given initial conditions for predetermined variables and stochastic processes for exogenous variables, satisfying (i) households and firms' optimality conditions, (ii) market clearing in all domestic markets, (iii) government's budget constraint and its transversality conditions (as discussed in Appendix A), and (iv) the determination of the interest rate in dollars (7). Appendix B.2 lists all the equilibrium conditions, while section 3 below analyzes conditions for local existence and uniqueness. Here, we discuss some relevant features to understand the results presented below.

By forward iteration of the government's budget constraint (3) and under the constant-currency composition assumption ($\Omega_t = \overline{\Omega}$), we get (see Appendix A for details)

$$\left[\frac{(1-\overline{\Omega}) + \overline{\Omega}\pi_t^S}{\pi_t}\right] d_{t-1} = E_t\left\{\sum_{j=0}^{\infty} \frac{sp_{t+j}}{rr_{t,t+j}}\right\} + h.o.t., \tag{8}$$

[3] Notice that we do not consider the possibility of outright default on debt, which is also the case in most of the related FTPL literature. An exception of default in the context of FTPL is Uribe (2006), who discusses a tension between price level determination and default in a closed economy model; without exploring explicitly the choice between the two. Of course, there is a large literature on sovereign default in open-economy models (e.g. see the survey by Aguiar and Amador, 2014). However, given the global analysis characterizing those models, the combination of monetary and fiscal policy are generally omitted, as well as the issue of currency composition of debt (for instance, Espino et al., 2025, present a model with default as well as distortionary fiscal and monetary policies, but exclude the possibility of peso-denominated debt). Moreover, discussing optimal policy choices in the context of FTPL should naturally consider time-inconsistency issues arising from incentives to dilute outstanding nominal debt with inflation, as shown by Calvo (1988) in a simple two-period model. These issues, while potentially relevant, are beyond the scope of the paper.

where $d_t \equiv d_t^G + rer_t d_t^{G*}$ is total government debt (in domestic consumption units), the real discount rate is $rr_{t,t+J} \equiv \prod_{j=0}^{J-1}(R_{t+j}/\pi_{t+1+j})$, while $h.o.t.$ includes terms (related to valuation effects) that are zero under certainty and thus vanish up to a first order.[4]

Without dollar-denominated debt ($\overline{\Omega} = 0$), (8) is the familiar valuation equation from closed-economy FTPL models. For a given path of the real rate, a change in primary surpluses (either contemporaneous or expected) that is not compensated by an offsetting change in the surplus at some other time (thus changing the net-present value on the right-hand side) needs to be met by an opposite-sign change in π_t on the left-hand side for (8) to hold (as d_{t-1} is predetermined). If prices are fully flexible, the real rate cannot be affected by monetary policy, so current prices are pinned down by fiscal policy alone (monetary policy still determines *expected* inflation). Under sticky prices, an additional channel emerges as monetary policy affects the real rate. Still, the degree of accommodation of primary surpluses is key (governed in the model by the parameter ϕ_T in (4), i.e. how Ricardian fiscal policy is). In section 4 we discuss in detail the monetary transmission mechanism in this context.

As we analyze below, results from closed economy models are qualitatively similar in this open economy setup when all government debt is denominated in pesos ($\overline{\Omega} = 0$), although here we can also explore the exchange-rate consequences. When $\overline{\Omega} > 0$, several differences arise. For given values of d_{t-1} and π_t^S, a particular change on the right-hand side of (8), either through movements in primary surpluses or the real rate, requires (*ceteris paribus*) a larger change in π_t on the left-hand side. Thus, we would expect inflation to be more volatile the larger the share of dollar-denominated debt. This is true as long as $\overline{\Omega} < 1$, for dynamics are different if debt is fully denominated in foreign currency, as we analyze below.

Another relevant difference arises from the nominal depreciation π_t^S on the left-hand side of (8), which appears because rer_t affects the real value of outstanding debt. First, any shock inducing a nominal depreciation, for a given value of the right-hand side in (8), requires an increase in π_t. Of course, a nominal depreciation already induces a direct effect on prices (through tradable inflation) but, as long as the share of tradable in the consumption basket is less than one, the required rise in π_t needs to also affect non-trade inflation. Importantly, this channel is not present in the related literature, for most open-economy analyses of FTPL use one-good models. Second, any change in the net present value of primary surpluses can be accommodated not only by increasing inflation but also by a nominal appreciation if $\overline{\Omega} > 0$ (or a combination of both).

Therefore, the co-movement between π_t^S and π_t can be influenced by FTPL considerations when a fraction of government debt is denominated in foreign currency. In particular, the observed sign of this correlation is not obvious, and it is likely dependent on the type of shock affecting the economy. This creates a role for fiscal policy as a determinant of the real exchange rate, different from other channels that are present in the standard model.

In order to sharpen the intuition, our baseline specification imposes some additional simplifying assumptions, that we later modify in the robustness section 6. First, we assume $\sigma\eta = 1$ (i.e. equality between intra- and inter-temporal elasticities of substitution, η and $1/\sigma$ respectively). In tandem with a closing device specified as in (7), this leads to c_t^T, d_t^*, and R_t^* being determined only by the exogenous variables π_t^*, R_t^W, and y_t^T, with no influence from other domestic variables, monetary and fiscal policy in particular (see Appendix B.3).[5]

In addition, we assume a zero-inflation steady state ($\overline{\pi} = \overline{\pi}^* = 1$), and also simplify fiscal policy

[4]With no dollar debt, the term $h.o.t.$ is zero even without linearization, as shown in appendix A.

[5]Several authors argue that the constraint $\sigma\eta = 1$ is empirically plausible (e.g. Schmitt-Grohé and Uribe, 2016).

by making the primary surplus dependent only on lump-sum taxes in ($\nu_t = g_t = \overline{\nu} = \overline{g} = 0$). Thus, the log-linearized equilibrium conditions for the rest of the model simplify to (see Appendix B.4):

$$\widehat{\pi}_t^N = \beta E_t \left\{ \widehat{\pi}_{t+1}^N \right\} + \frac{\widetilde{\kappa}}{\omega} \left(\frac{\eta}{\omega} \widehat{rer}_t + \widehat{c}_t^T \right), \tag{9}$$

$$\widehat{\pi}_t = \omega(\widehat{\pi}_t^N) - (1-\omega) \left(\widehat{\pi}_t^S + \widehat{\pi}_t^* \right), \tag{10}$$

$$\widehat{rer}_t = \widehat{rer}_{t-1} - \omega \left(\widehat{\pi}_t^N - \widehat{\pi}_t^S - \widehat{\pi}_t^* \right), \tag{11}$$

$$\widehat{R}_t = \widehat{R}_t^* + E_t \left\{ \widehat{\pi}_{t+1}^S \right\}, \tag{12}$$

$$\widehat{R}_t = \phi_\pi \widehat{\pi}_t + \widehat{u}_t^R ., \tag{13}$$

$$(1-\phi_T) \left(\widehat{d}_{t-1} - \widehat{\pi}_t + \Omega \widehat{\pi}_t^S \right) = \beta \left(\widehat{d}_t - \widehat{R}_t + \Omega E_t \left\{ \widehat{\pi}_{t+1}^S \right\} \right) + \widehat{u}_t^\tau, \tag{14}$$

These are six equations for six endogenous variables $\widehat{\pi}_t^N, \widehat{\pi}_t, \widehat{rer}_t, \widehat{\pi}_t^S, \widehat{R}_t, \widehat{d}_t$. Equation (9) is the New-Keynesian Phillips curve for non-traded inflation, that in equilibrium is driven by movements in either $\widehat{rer}_t$ and $\widehat{c}_t^T$ (the relevant determinants of non-traded demand). Equation (10) expresses aggregate inflation as a weighted average of the evolution of traded and non-traded prices. In turn, (11) relates the evolution of the real exchange rate with the change of the relative prices between both goods. Equation (12) is the uncovered interest rate parity (UIP), while (13) is the Taylor rule for the domestic interest rate. Finally, equation (14) is the combination of the government's budget constraint and the rule for lump-sum taxes.

A relationship that will prove useful to analyze the dynamics of the nominal exchange rate arises from combining the UIP condition (12) with (10) and (11), and iterating forward,

$$\widehat{\pi}_t^S = \widehat{\pi}_t - \widehat{\pi}_t^* - \widehat{rer}_{t-1} + \sum_{j=0}^{\infty} \left[(\widehat{R}_{t+j}^* - \widehat{\pi}_{t+1+j}^*) - (\widehat{R}_{t+j} - \widehat{\pi}_{t+1+j}) \right], \tag{15}$$

As can be seen, everything else equal, an increase in the monetary policy rate (either today or expected) generates a nominal appreciation. The final effect, however, is determined by the endogenous response of the inflation path. If it falls, it reinforces the direct effect and the nominal exchange rate appreciates for sure. Instead, if inflation increases, the final effect on $\widehat{\pi}_t^S$ is ambiguous, possibly even increasing. As we will analyze, fiscal policy being Ricardian or not, as well as the share of government debt denominated in dollars, are key to determining the final effect.

Before moving to the main analysis, Table 1 presents the calibration of the parameters in the baseline specification. Those shared with the model in Schmitt-Grohé and Uribe (2017, Ch. 9.16) are taken from them. The parameters $\overline{R}^*$, $\overline{d}^*$, χ, and $\overline{\tau}$ are set endogenously in steady state to obtain shares of primary surplus and trade balance to GDP of 5%, as well as values for $\overline{h}$ and $\overline{rer}$ that generate a relative size of non-tradable GDP of 57% (as implicit in the calibration of Schmitt-Grohé and Uribe, 2017). The other parameters related to monetary and fiscal policy (ϕ_π, ϕ_T, and $\overline{\Omega}$) are set to specific values described below, depending on the particular exercise we investigate.

3 Existence and Uniqueness

In this section, we explore the determination of a locally-stationary equilibrium. As the system (9)-(14) contains only two endogenous predetermined/state variables ($\widehat{rer}_t$ and $\widehat{d}_t$), two stable eigenval-

Table 1: Baseline Calibration

β	σ	φ	η	ω	α	ϵ_N	θ_N	ψ
0.9694	2	0.5	0.5	0.6	0.75	6	0.7	0.000034
$\overline{\pi}$	$\overline{\pi}^*$	$\frac{\overline{sp}}{gdp}$	$\overline{\nu}$	$\overline{g}$	$\overline{h}$	$\frac{\overline{tb}}{gdp}$	$\overline{rer}$	
1	1	0.05	0	0	0.73	0.05	1	

ues are required for the existence of a unique equilibrium. While Appendix C provides a formal proof, here we provide an intuitive characterization. Notice first that equations (9), (10), and (11) can be combined to obtain a backward-looking equation for rer_t of the form

$$\widehat{rer}_t \left(1 + \frac{\widetilde{\kappa}\eta}{\omega}\right) = \widehat{rer}_{t-1} + \widehat{o}_t,$$

where $\widehat{o}_t$ collects all other terms (which, importantly, contain only non-predetermined/forward-looking variables). The eigenvalue for $\widehat{rer}_t$ in this equation is $\left(1 + \frac{\widetilde{\kappa}\eta}{\omega}\right)^{-1}$, which is always less than 1. Thus, to the extent that $\widehat{o}_t$ is stationary, $\widehat{rer}_t$ will not play a direct role in the discussion of how different combinations of fiscal and monetary policy determine the existence and uniqueness of equilibria.

In the case of government debt $\widehat{d}_t$, notice that equation (14) can be written as

$$(1 - \phi_T)\widehat{d}_{t-1} = \beta\widehat{d}_t + \widehat{e}_t,$$

where $\widehat{e}_t$ collects all other terms, and thus the eigenvalue for $\widehat{d}_t$ is $(1 - \phi_T)/\beta \geq 0$. Therefore, as long as $\widehat{e}_t$ is stationary, $\widehat{d}_t$ is non-explosive if its eigenvalue is less than one , or $\phi_T > 1 - \beta$. Alternatively, if $\phi_T \leq 1 - \beta$, $\widehat{d}_t$ could still be stationary if the endogenous behavior of variables in $\widehat{e}_t$ generates another stable eigenvalue that offsets the explosive dynamics that $\widehat{d}_t$ would otherwise display.

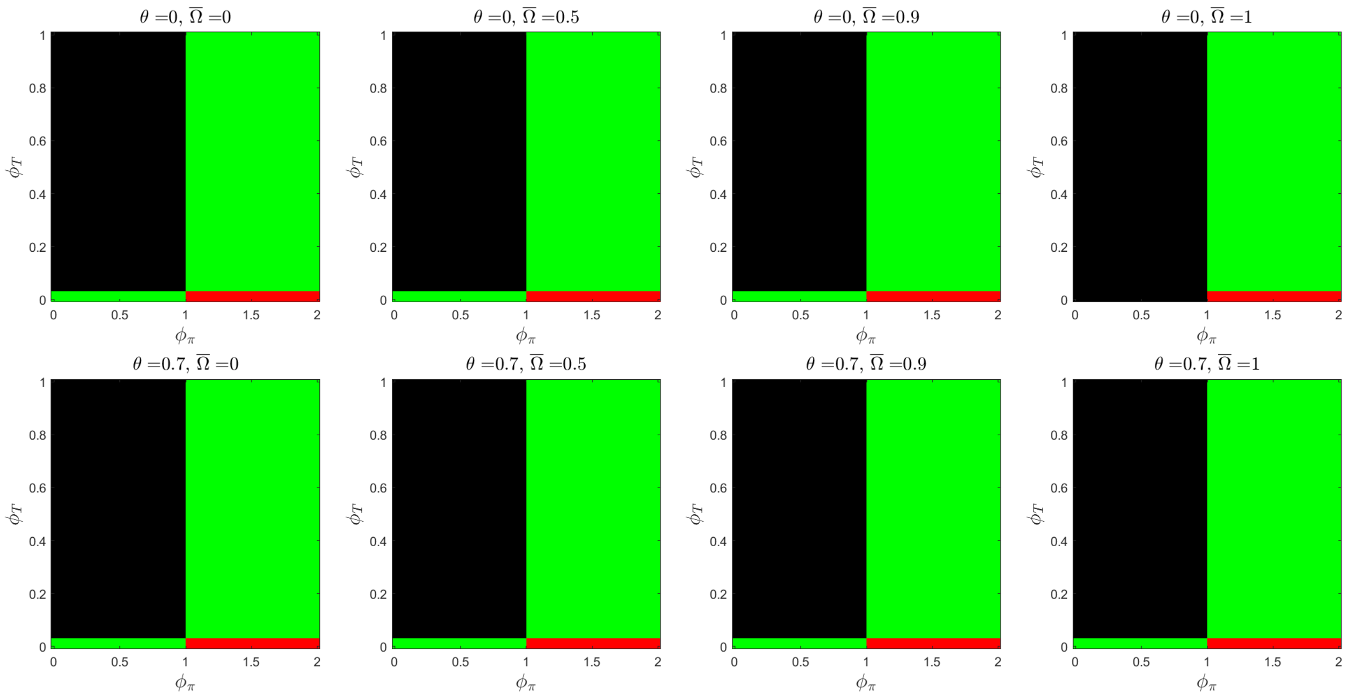
We mentioned that $\phi_T = 1$ corresponds to a Ricardian/passive fiscal policy, while $\phi_T = 0$ is Non-Ricardian/active. As it is well known from closed economy setups (e.g. Leeper, 1991), under Ricardian fiscal policy, the determination of inflation is up to monetary policy. Instead, in the Non-Ricardian case, monetary policy needs to be passive. This intuition can be generalized for intermediate values of ϕ_T. If ϕ_T is big enough ($\phi_T > 1 - \beta$), we will always have at least one stationary equilibrium. If the other parts of the model (monetary policy in particular) induce a unique stationary behavior for the other variables (e.g., if the Taylor principle is satisfied, $\phi_\pi > 1$) the equilibrium is unique. Otherwise, we have multiple stationary equilibria.

In contrast, if ϕ_T is relatively low ($\phi_T \leq 1 - \beta$), the existence of a stationary equilibrium is not guaranteed. If an additional stable eigenvalue is produced in other parts of the model (monetary policy in particular), the equilibrium would be unique (e.g., if $\phi_\pi < 1$), while there are no stationary equilibria otherwise.

Figure 1 provides a visual representation of the results. Each graph displays whether the particular combination of ϕ_π (horizontal axis) and ϕ_T (vertical axis) delivers a unique equilibrium (green), no equilibrium (red), or multiplicity/indeterminacy (black). Those in the upper panel correspond to cases under flexible prices ($\theta_N = 0$), while the others show cases with sticky prices ($\theta_N = 0.7$). Each column corresponds to a particular share of dollar-denominated government debt $\overline{\Omega}$.

In line with the previous intuition and the results from the proposition, determinacy arises either with $\phi_T < 1 - \beta$ and $\phi_\pi < 1$ (active fiscal, passive monetary), or $\phi_T > 1 - \beta$ and $\phi_\pi > 1$ (passive

Figure 1: Existence and Uniqueness under Taylor Rule



Notes: Green: Uniqueness. Red: No stable equilibrium. Black: Multiplicity

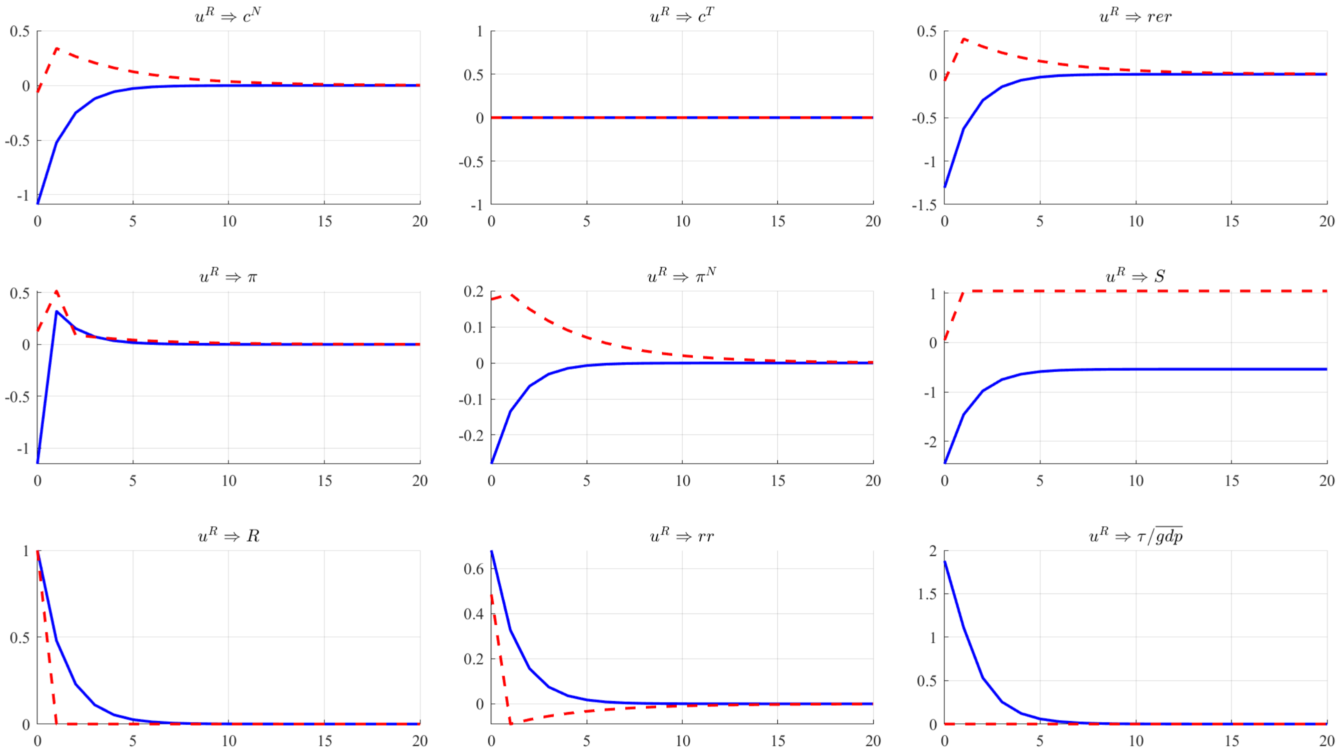
fiscal, active monetary). Instead, if $\phi_T < 1 - \beta$ and $\phi_\pi > 1$ (both active) there are no equilibria, while $\phi_T > 1 - \beta$ and $\phi_\pi > 1$ (both passive) generate multiplicity. The only exception to this pattern appears when prices are flexible ($\theta_N = 0$) and debt is fully dollarized ($\overline{\Omega} = 1$). In such a case, active fiscal policy is irrelevant for price determination under a passive monetary policy because, under flexible prices, the real exchange rate (the only relevant price affecting the current value of outstanding debt if $\overline{\Omega} = 1$) is independent of monetary policy. Thus, at least locally, in the case of $\phi_T < 1 - \beta$ and $\phi_\pi < 1$, there is also multiplicity if $\theta_N = 0$ and $\overline{\Omega} = 1$. However, as long as prices are sticky, even under 100% dollar-denominated debt, an active fiscal policy combined with passive monetary policy can affect the real exchange rate, thus the equilibrium is uniquely pinned down.

These results show that, in the baseline setup, the share of debt denominated in dollars is generally irrelevant to determine the conditions under which the equilibrium exists and is unique. However, in the following sections we show that, provided determinacy, dynamics are indeed significantly altered by the currency composition of debt in cases in which fiscal policy is Non-Ricardian/active.

4 Monetary and Fiscal Transmission Mechanism

We first analyze the dynamics generated by an i.i.d. shock to the monetary policy rate, u_t^R in equation (6); beginning with the $\overline{\Omega} = 0$ case (only domestic-currency debt) in Figure 2. The blue lines display the case with active monetary policy (with the usual parameter for a Taylor rule of $\phi_\pi = 1.5$) and a fully Ricardian/passive fiscal policy ($\phi_T = 0$); i.e., the frequently used configuration in NKSOE models that neglects fiscal policy. From the household's perspective, the shock induces (*ceteris paribus*, given the same for prices) an increase in the real interest rate. This leads to both an inter-temporal substitution as well as a negative wealth effect (as the net present value of after-tax real and financial income falls). Both of them tend to decrease current overall consumption.

Figure 2: Responses to a monetary-policy-rate shock, Ricardian vs. Non-Ricardian.



Notes: Each graph plots responses of the following variable: Real GDP ($gdp \equiv y^T + y^N$), non-traded consumption (c^N), traded consumption (c^T), real exchange rate (rer), total inflation (π), non-traded inflation (π^S), nominal exchange rate (S), monetary-policy rate (R), ex-ante real interest rate ($rr = \widehat{R}_t - E_t\{\widehat{\pi}_{t+1}\}$), and lump-sum taxes ($\tau/\overline{gdp}$). The shock is an increase in u_t^R, with zero persistence, normalized to increase R by 1% on impact. Solid-blue lines correspond to the case with $\phi_\pi = 1.5$ and $\phi_T = 1$, while dashed-red lines are the case with $\phi_\pi = 0$ and $\phi_T = 0$; all with $\overline{\Omega} = 0$. All responses are measured as percentage-point deviations relative to the steady state.

As shown by Caramp and Silva (2023) in a closed economy setup, the size of the wealth effect heavily depends on the fiscal-policy response to the shock. From the perspective of the lifetime government budget constraint (8), the increase in the real rate induces a reduction in the net present value of primary surpluses. If fiscal policy is Ricardian ($\phi_T = 1$), lump-sum taxes need to increase to counteract this effect. In turn, this increase in taxes induces an even larger income effect for households, further decreasing desired consumption. According to Caramp and Silva (2023), this extra wealth effect can account for almost the totality of the contraction in consumption induced by the higher policy rate in closed-economy models.

In the open economy, the required change in taxes if $\phi_T = 1$ is also present. Thus, the contraction in aggregate consumption translates into a reduction in desired demand for both traded and non-traded goods (provided homothetic preferences). The contraction in c^N induces a fall in non-traded inflation (π^N). In turn, by (15), the increase in the policy rate induces a nominal appreciation if it is not compensated by an increase in inflation (indeed π^N falls). Overall, rer appreciates. Instead, c^T does not move in equilibrium under $\sigma\eta = 1$.[6]

After the initial period, the nominal appreciation is partially offset, as the UIP condition (12) requires a depreciation going forward if R increases today (i.e., the shock induces exchange-rate

[6] The real appreciation increases desired traded consumption (as it becomes cheaper), perfectly offsetting the desired fall previously discussed, when both elasticities (η and $1/\sigma$) are the same.

overshooting).[7] Overall, the shock contracts the economy (from the non-traded sector) and reduces both traded and non-traded inflation.

Dynamics are quite different if fiscal policy is Non-Ricardian ($\phi_T = 0$) and the interest rate is not responsive to inflation ($\phi_\pi = 0$), as shown in dashed-red lines in Figure 2. In this case, while the real rate increases (*ceteris paribus*) following the shock, lump-sum taxes remain constant. This has two important consequences. First, the negative wealth effect brought about by a Ricardian policy (through tax increases, as previously discussed) is absent, and therefore c^N only marginally drops at the moment the shock is realized. This extends the result in Caramp and Silva (2023) to this small and open economy setup.

Second, by the FTPL mechanics, the price level needs to increase to satisfy the government lifetime constraint equation (8), which leads to an increase in both non-traded prices and the nominal exchange rate. In terms of S, the initial-period response is only marginally positive. But we also know from UIP (12) that a further depreciation is expected if R increases today, so the new higher level in S is achieved with a delay, eliminating the overshooting dynamics. In turn, this could help explain the empirical result documented in the literature that, for emerging countries, an increase in the policy rate tends to be followed by a nominal depreciation, instead of an appreciation.

In terms of π^N, there are two channels at play. On the one hand, the fall in c^N pushes non-traded prices downward; though this channel should be small as the negative wealth effect induced by taxes is absent. On the other hand, the FTPL channel requires prices to increase. If prices were fully flexible, this would materialize instantaneously. Under sticky prices, it builds up over time, so it is natural to expect additional non-traded inflation in the future. Therefore, from the perspective of the New-Keynesian Phillips curve, the initial contraction in demand for non-tradables is compensated by the forward-looking channel that anticipates higher future inflation. As a result, π^N rises from the moment the shock hits, with an additional increase in the following period.

These dynamics for π^N and S also generate a different behavior for the real exchange rate, which initially increases, intensifying in the following periods as S jumps even further. This induces an additional substitution effect in favor of c^N, as traded goods become relatively more expensive if *rer* increases.[8] The dynamics of c^N can also be understood by the behavior of the ex-ante real rate *rr*. In the Ricardian case (solid-blue line), the real-rate path converges back to the steady state monotonically from above, explaining why consumption c^N displays a similar path back to steady state, with the opposite sign (due to intertemporal substitution). In turn, in the Non-Ricardian case (dashed-red line), the real rate falls below its steady state value after the initial increase due to the expected higher inflation, converging to the steady state from below. This is consistent with the dynamics displayed by c^N under that case.[9]

In the robustness appendix D.1, we explore the role of the monetary response parameter ϕ_π for a given fiscal regime. In the Ricardian case ($\phi_T = 1$), comparing the baseline $\phi_\pi = 1.5$ against a value just above the determinacy region ($\phi_\pi = 1.01$), only marginal differences arise, with responses being relatively muted for lower values of ϕ_π. Instead, in the Non-Ricardian regime ($\phi_T = 0$), setting the

[7]This also explains why, even though the shock is i.i.d., the policy rate slowly converges back to steady state and not instantaneously. Recall that the Taylor rule (6) is specified in terms of total inflation π, not just π^N. Thus, the overshooting behavior of the S generates total inflation to be above the steady state even after the shock materializes, leading to analogous dynamics for R in the period following the realization of the shock. This adds an extra push downwards to c^N (via the same channels previously discussed), and thus the effect of the shock does not simply vanish after the initial period (as it would be the case in the closed-economy version).

[8]Again, c^T is not affected in equilibrium as the several effects offset each other.

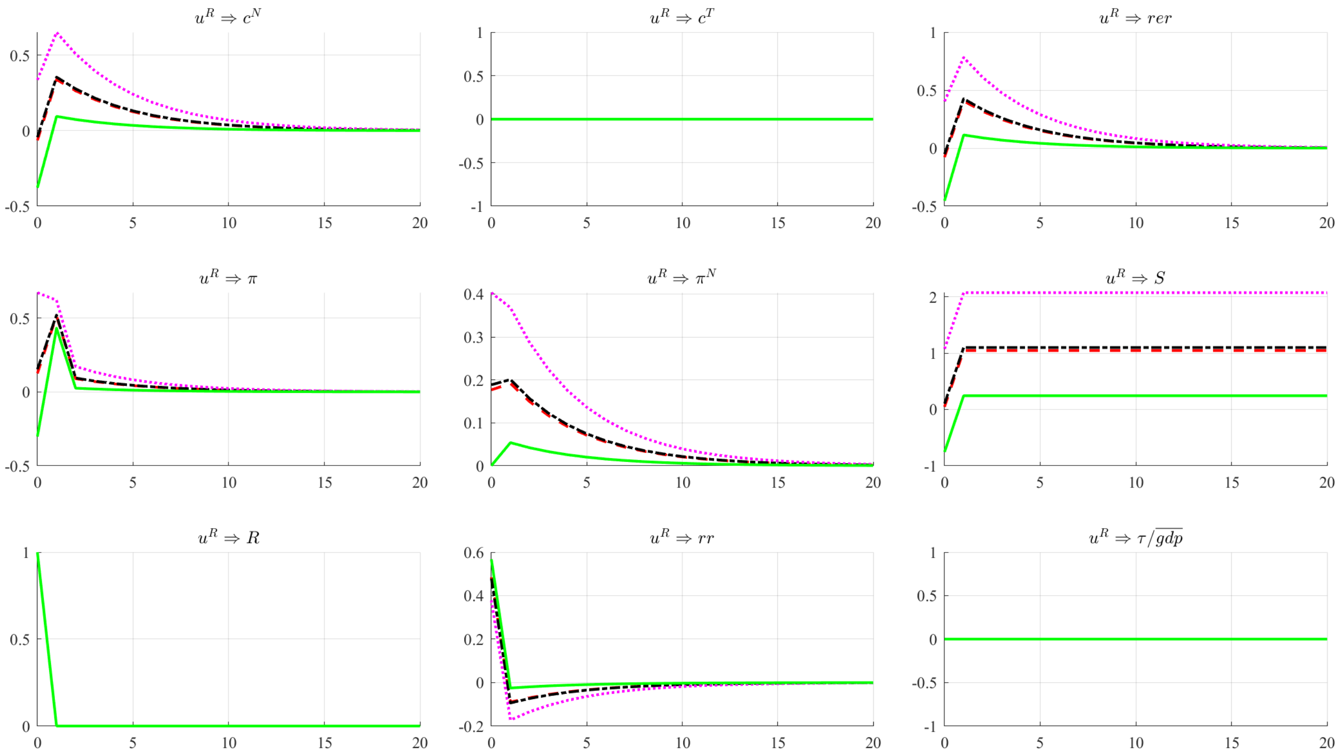
[9]Of course, explanations based on *rer* or *rr* are two sides of the same coin, as the behavior of these two variables is linked in equilibrium by the UIP condition expressed in terms of real rates.

monetary response parameter to a value marginally below that required for determinacy ($\phi_\pi = 0.99$) significantly alters the dynamics. In particular, the nominal policy stays above the steady state for much longer, despite the shock being i.i.d., as the Taylor rule dictates that the policy rate should be high if inflation is above the steady state. This, in turn, implies a more persistent higher path for the real rate which, from the FTPL channel, induces more non-traded inflation (which is also persistent) while nominal depreciation continues even after the initial periods. At the same time, the initial response of non-traded consumption and the real exchange rate is marginally smaller, but its convergence back to the steady state from above is much slower.

Overall, under this alternative policy configuration, an increase in the nominal rate leads to an increase in inflation and an expansion in output, similar to the effects shown, for instance, by Cochrane (2023) in a closed-economy setting. Our analysis extends the results to open economies, which also allows understanding the consequences for both the nominal and real exchange rates.

Figure 3 extends the analysis even further to cases with a positive fraction of dollar-denominated debt ($\overline{\Omega} > 0$). All cases displayed assume $\phi_\pi = 0$ and $\phi_T = 0$, considering alternative values for $\overline{\Omega}$ equal to 0, 0.5, 0.75, and 1. As long as $\overline{\Omega} < 1$, we can see that a higher share of dollar-denominated debt yields larger responses for both non-traded inflation and the nominal exchange rate. This is in line with the discussion of the FTPL equation (8) in section 2.6: as the share of debt in pesos is smaller, the rise in inflation required to dilute the real value of outstanding debt, in order to compensate for a given change in the net-present value of primary surpluses, is larger. As a result, the real exchange rate and non-traded consumption increase further under a larger share of dollar-denominated debt.

Figure 3: Responses to a monetary-policy-rate shock, Non-Ricardian, different values of $\overline{\Omega}$.



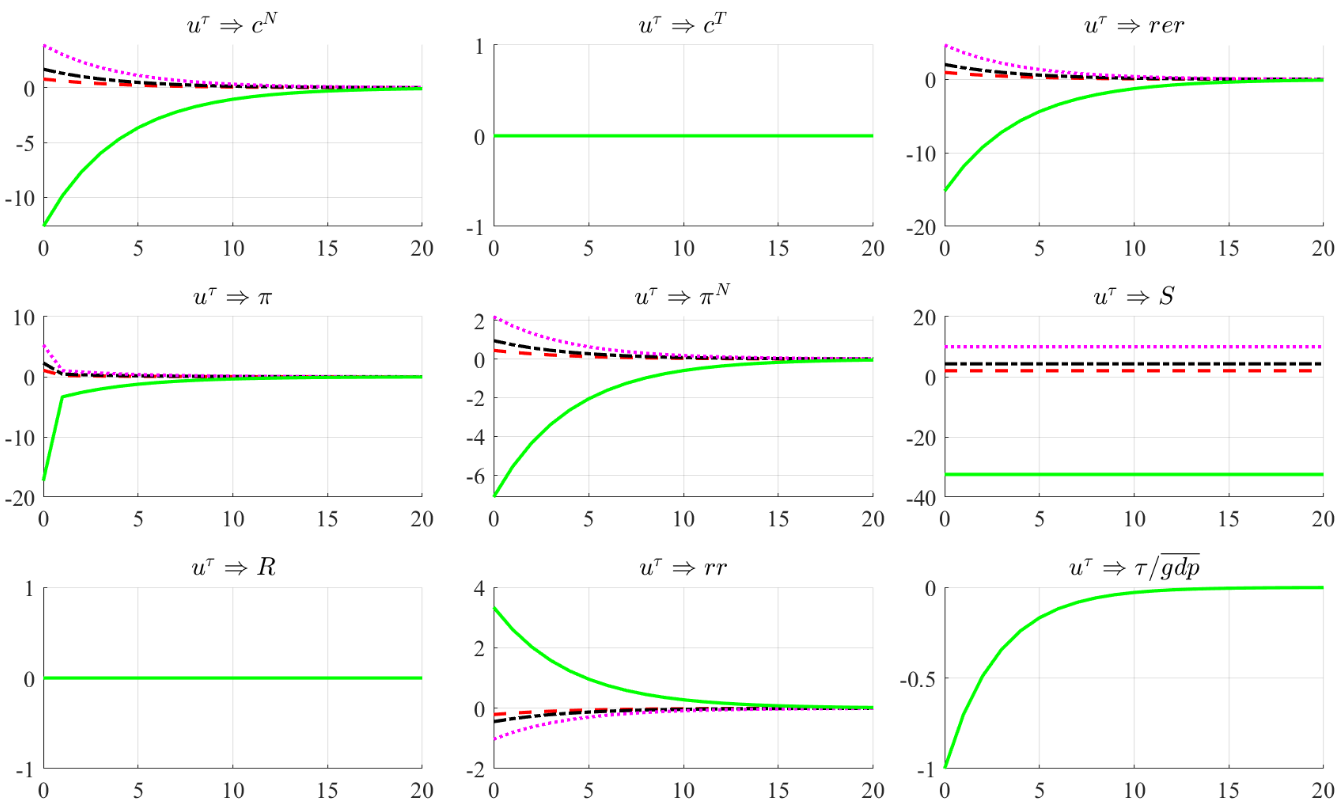
Notes: The figure is analogous to Figure 2, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for $\overline{\Omega}$: In dashed-red $\overline{\Omega} = 0$, in dashed-dotted black $\overline{\Omega} = 0.5$, in dotted magenta $\overline{\Omega} = 0.9$, and in solid-green $\overline{\Omega} = 1$.

Dynamics are different if debt is fully denominated in dollars ($\overline{\Omega} = 1$, solid-green lines in Figure

3). From the left-hand side of the FTPL equation (8), we see that a real appreciation is required to compensate for the drop in the net-present value of surpluses that the shock generates. As non-traded prices are sticky, this cannot materialize through an increase in π^N and therefore an initial nominal appreciation is induced. This, in turn, reduces the demand for non-tradables, generating a drop in c^N and gdp. After the initial period, given that UIP requires the nominal exchange rate to depreciate and also because the real exchange rate has to converge back to steady state, S jumps to a level above zero. Therefore, while a higher share of dollar denominated debt induces more inflation and a more depreciated currency, there is a discontinuity if $\overline{\Omega} = 1$.

We next turn to the dynamics generated by a fiscal shock, u^τ in the policy rule (4). Under a Ricardian/passive fiscal policy, a change in taxes today induces an opposite-sign modification in future taxes such that the net-present value in the right-hand side of (8) is unaltered, and therefore has zero effects. Instead, the shock induces non-trivial aggregate dynamics under non-Ricardian/active fiscal policy. Figure (4) shows the effects of a fall in lump-sum taxes, normalized to represent 1% of steady-state GDP (with an autocorrelation of 0.7), for different values of $\overline{\Omega}$. In all cases, the policy rate is kept fixed ($\phi_\pi = 0$) and $\phi_T = 0$.

Figure 4: Responses to a fiscal shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 3, except that is shows responses to a drop in lump-sum taxes, normalized to represent 1% of steady-state GDP, with an autocorrelation of 0.7. All cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for $\overline{\Omega}$: In dashed-red $\overline{\Omega} = 0$, in dashed-dotted black $\overline{\Omega} = 0.5$, in dotted magenta $\overline{\Omega} = 0.75$, and in solid-green $\overline{\Omega} = 1$.

Beginning with the case of full peso-denominated debt ($\overline{\Omega} = 0$, represented by dashed-red lines in the figure), if the policy rate is fixed, the FTPL equation (8) indicates that the shock requires either an increase in the price level today, an increase in expected inflation, or a combination of both. This is achieved by both a nominal depreciation and an increase in π^N, the latter being quantitatively smaller and spread over time due to sticky prices. As a result, a real depreciation materializes.

This real depreciation leads c^N to increase, through intra-temporal substitution, as non-traded goods become relatively cheaper. Moreover, an additional expansionary channel arises from the positive wealth effect that tax reduction generates. While the real interest rate is reduced (as expected inflation rises), this is not enough to compensate for the increase in the net present value of after-tax income. Overall, total output rises in equilibrium.[10]

If we turn to cases where a fraction of debt is denominated in dollars, as long as $\overline{\Omega} < 1$, the dynamics are magnified by a larger share $\overline{\Omega}$. Again, this can be explained by the additional inflation that is required to dilute the relatively smaller outstanding amount of peso-denominated debt. Finally, if all debt is denominated in dollars, we obtain dynamics with opposite effects; precisely for the same reasons discussed during the analysis of monetary-policy shocks under Non-Ricardian policies.

In summary, not only does the transmission of both monetary and fiscal shocks differ depending on the policy regime, but the share of dollar-denominated government debt is also relevant in determining the dynamics. In particular, a larger share of dollar-denominated debt magnifies the effects of both shocks (as long as the debt is not fully dollarized). These results are relevant, as they provide an interesting testable implication that future work can use to test the FTPL channel in the data.

5 The Effect of Real Shocks

In this section, we study the effects of shocks to traded output y^T (which could also be interpreted as terms-of-trade shocks in this simple setup) and to the world interest rate R^W (shocks to π^* induce similar effects to those generated by R^W). For each of them, we first describe the propagation under Ricardian/passive fiscal policy, then analyze the role played by a Non-Ricardian setup, and finally study the role played by the currency composition of government debt.

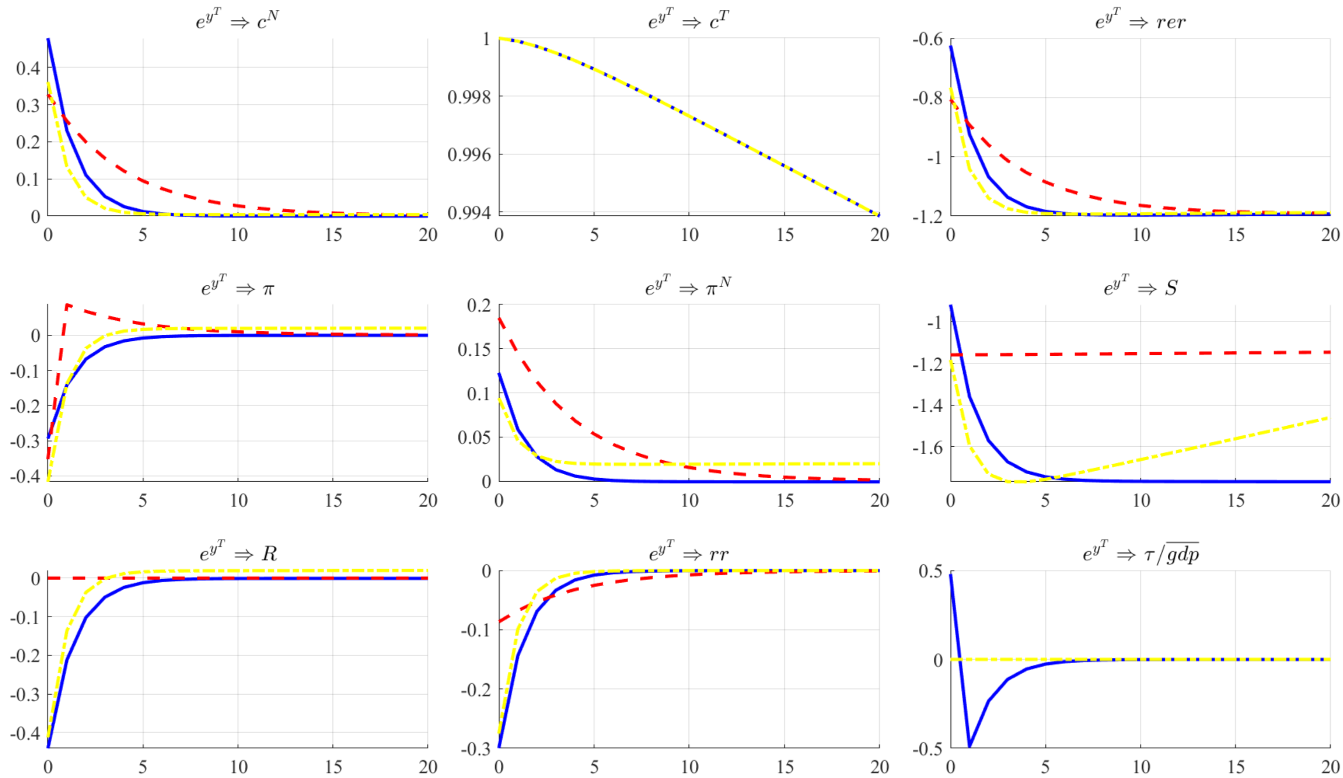
5.1 Traded output

Figure 5 displays the dynamics triggered by an increase in y^T, with an autocorrelation of 0.7, normalized to increase traded consumption by 1%. The shock induces a positive wealth effect, increasing desired consumption of both types of goods. The increase in c^T is quite persistent, as the interest rate in dollars R^* is calibrated to have a minor elasticity to the country's net-foreign debt position, and thus convergence back to steady state is quite slow. Additionally, the higher demand for non-tradables increases its relative price, leading to a real appreciation. If prices were fully flexible, this would materialize instantaneously, but as non-traded prices are sticky, it takes time to reach its maximum appreciated value. Instead, under sticky prices, the real appreciation comes about by a relatively small increase in non-trade inflation and, more significantly, by a nominal appreciation.

The specific dynamics are shaped by the fiscal and monetary regime. The blue lines display the case with a Ricardian/passive tax response ($\phi_T = 1$) and an active Taylor rule for the interest rate ($\phi_\pi = 1.5$). Under such a configuration, the policy rate falls following the reduction in inflation. This, in turn, induces an even further appreciation: by the UIP condition (12), a fall in the domestic rate requires a further expected appreciation. In addition, as the policy rate falls by more than overall

[10] In the robustness appendix D.1, we see that if $\phi_T = 0$, but the monetary-response parameter is close to one, non-trivial differences arise; similar to those described before, in the case of a monetary shock. In particular, if $\phi_\pi = 0.99$, the policy rate remains high for a longer period, increasing non-traded inflation and the nominal exchange rate even further, resulting in the policy rate rising even more persistently. At the same time, the initial increases in both c^N and rer are smaller than with $\phi_\pi = 0$, but its convergence back to the steady state is much slower.

Figure 5: Responses to a traded-output shock, Ricardian vs. Non-Ricardian.



Notes: The figure show the responses to a positive shock to y^T, with an autocorrelation of 0.7, normalized to increase c^T by 1% at the moment the shock hits. All cases feature $\overline{\Omega} = 0$. In solid blue lines $\phi_T = 1$ and $\phi_\pi = 1.5$. in dashed-red lines $\phi_T = 0$ and $\phi_\pi = 0$, while in dashed-dotted yellow lines $\phi_T = 0$ and $\phi_\pi = 0.99$. See Figure 2 for variables' definitions.

inflation, the real rate also decreases. From the perspective of households, this increase non-traded consumption c^N further on impact, via inter-temporal substitution.

In turn, fiscal policy is also affected by the shock through two different channels. First, the reduction in inflation increases the real value of the outstanding nominal debt. Second, the reduction in the real rate increases the net present value of taxes. The former dominates in equation (8), as can be seen from the fact that lump-sum taxes increase in response to the shock.

The dashed-red lines in Figure 5 show the responses to the same shock under a Non-Ricardian fiscal policy ($\phi_T = 0$) and a constant policy rate ($\phi_\pi = 0$). While responses are qualitatively similar to those in the previous policy configuration, some quantitative differences emerge. The policy rate remaining fixed produces two differences. First, the further nominal appreciation in the period following the shock that we described in the blue lines disappears, which in turn eliminates the persistence of the initial fall in overall inflation. Second, as the nominal interest rate does not move but expected inflation is positive, the drop in the real rate is much smaller than in the previous policy configuration.

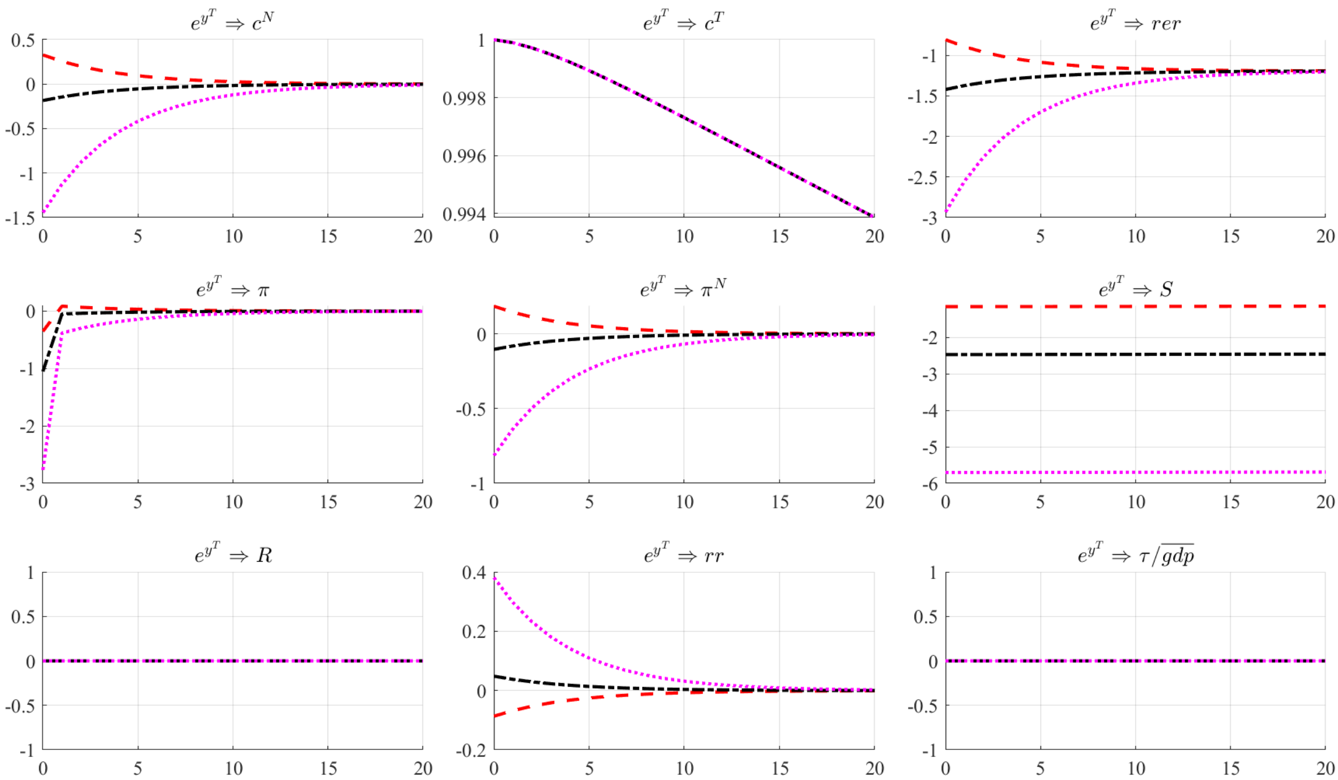
In turn, the smaller reduction in the real rate reduces the intertemporal substitution that influenced non-trade consumption, and therefore c^N increases by less on impact. This also implies that the path of c^N in the periods after the shock hits converges more slowly to the steady state. In turn, as π^N is forward-looking under Calvo prices, the initial increase in non-traded inflation is larger initially, anticipating the expected higher demand in the future.

In terms of fiscal policy, as lump-sum taxes are fixed, the right-hand side of the FTPL equation (8) increases as the real rate falls. This implies that, relative to the case with Ricardian policy, prices need to fall even more. We already described reasons for which this is not going to happen through π^N, which implies that the initial nominal appreciation is larger.

Notice there are two differences that distinguish the dashed-red lines from the solid-blue ones: both taxes and interest rates are kept constant if $\phi_T = \phi_\pi = 0$. In order to better understand the relative importance of both of them, Figure 5 includes an additional case (in dashed-dotted yellow lines) where taxes still do not move ($\phi_T = 0$) but the policy rate increases with inflation, using $\phi_\pi = 0.99$ (just below the value required for equilibrium uniqueness under an active fiscal policy). In this third case, the dynamics of both the nominal and the real interest rates, non-traded inflation and consumption, as well as the real exchange rate, are similar to those under a Ricardian/passive fiscal policy. Thus, to a large extent, the differences between the solid-blue and dashed-red lines are driven by the different behavior of the policy rate. Still, the FTPL channel continues to play a role. As the real rate drops even more in the dashed-dotted-yellow lines, further increasing the right-hand side of the FTPL equation (8). Thus, an initially larger nominal appreciation is still required to compensate if taxes do not change.

Dynamics are significantly altered once we allow for a positive share of debt denominated in foreign currency. Figure 6 shows three cases, all of them with $\phi_T = 0$ and $\phi_\pi = 0$, but with different values of $\overline{\Omega}$: 0 (dashed red), 0.5 (dashed-dotted black), and 0.75 (dotted magenta).[11] For larger values of $\overline{\Omega}$, further nominal and real appreciations materialize, and non-traded inflation and consumption now fall. What triggers these differences?

Figure 6: Responses to a traded-output shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 5, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for $\overline{\Omega}$: In dashed-red $\overline{\Omega} = 0$, in dashed-dotted black $\overline{\Omega} = 0.5$, in dotted magenta $\overline{\Omega} = 0.75$.

[11] In Appendix D.2 a similar figure is displayed with $\phi_\pi = 0.99$, where it can be seen that differences are magnified.

In the presence of foreign denominated debt, the real appreciation generated by the shock (which has a real origin, present even under flexible prices) tends to reduce the real value (in domestic consumption units) of outstanding dollar-denominated debt if $\overline{\Omega} > 0$. For a given path of lump-sum taxes (as $\phi_T = 0$), satisfying the FTPL equation (8) requires either an increase in the real value of outstanding peso-denominated debt (which can materialize if the price level drops as the shock hits), a reduction in the net-present value of primary surpluses (which requires the real rate to increase), or a combination of both. These two alternatives help each other in achieving the goal: a higher real rate pushes c^N downwards, which in turn reduces non-traded inflation today, helping to dilute the real value of outstanding debt in pesos. At the same time, the real-exchange rate appreciates even further, which also adds an intra-temporal substitution channel that further reduces c^N and thus π^N. Moreover, these effects are magnified for larger shares $\overline{\Omega} > 0$.[12]

Overall, while in the case of government debt fully denominated in pesos FTPL considerations do not qualitatively alter the propagation of shocks to y^T, this is not the case if a fraction of debt is denominated in foreign currency. In particular, a positive shock might lead to a contraction in the non-trade sector, even though the real exchange rate is appreciating. For here the appreciation is not mainly led by a larger demand in non-tradables, but it is produced by the FTPL channel that requires non-trade prices to fall, which materializes via a lower non-traded demand.[13]

5.2 World interest rate

We now turn to the analysis of the consequences generated by an increase in the world interest rate R^W. Figure 7 displays the dynamics of a shock with a persistence of 0.7, normalized to drop c^T by 1% on impact, under the assumption of $\overline{\Omega} = 0$. The real mechanisms behind the propagation of this shock work as follows. The increase in the international cost of borrowing induces an intertemporal substitution effect and a negative wealth effect (as the country is a net-foreign borrower in our calibration), both reducing consumption of both types of goods. In particular, the contraction in the demand for c^N reduces its relative price, leading to a real depreciation. In a world with sticky prices, the final outcome hinges on the fiscal and monetary configuration.

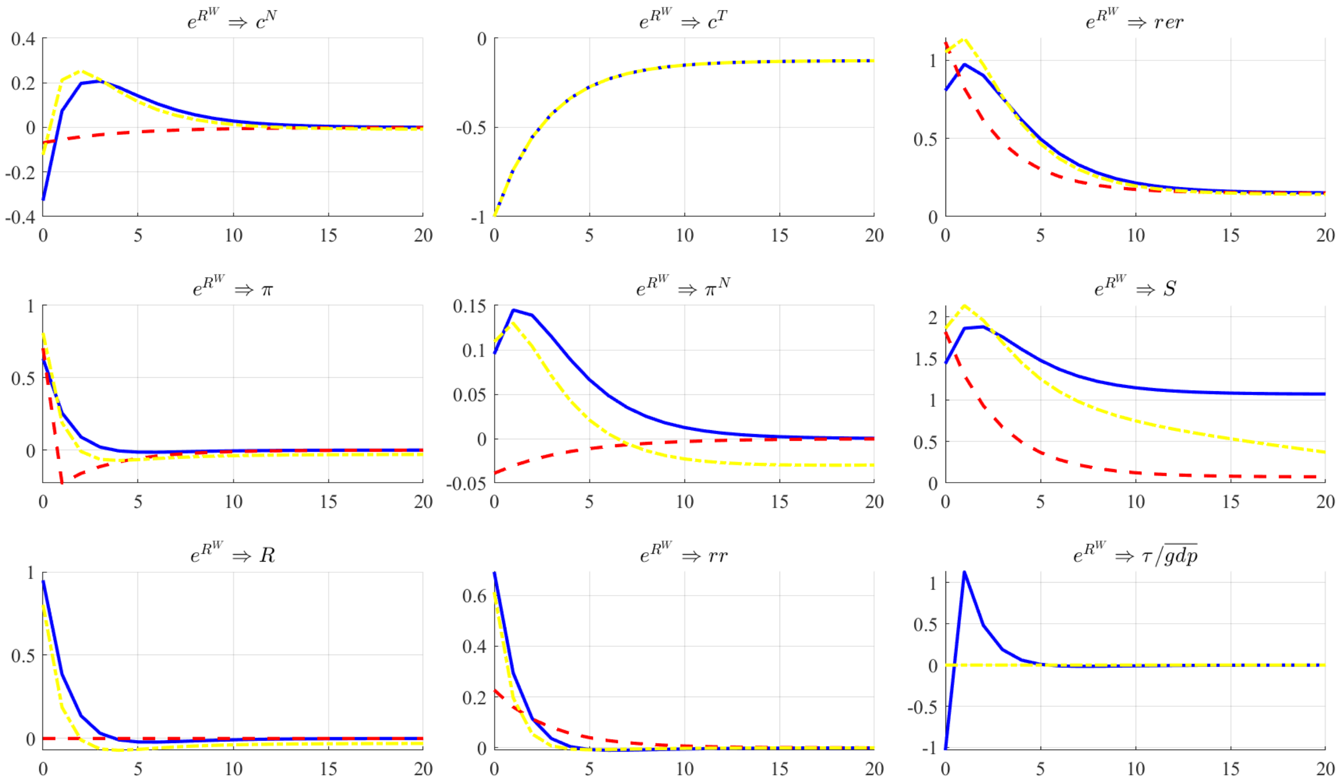
The solid-blue lines in Figure 7 correspond to the case passive-fiscal, active-monetary setup ($\phi_T = 1$ and $\phi_\pi = 1.5$). We can see that c^N falls on impact, and the real exchange rate depreciates via a nominal depreciation. We can also see that c^N rises above its steady state level in the periods following the realization of the shock, which also explains why non-trade inflation increases. This is generated by the fact that the domestic real rate rr, while increasing following the shock, returns back to the steady state at a faster rate than the world interest rate R^W.[14] This, in turn, implies that the real exchange rate (via the UIP condition) also takes longer to converge back to the steady state after the initial jump. As this relatively more depreciated path implies cheaper non-traded goods, c^N increases after the initial drop, increasing also π^N as firms anticipate this future higher demand in the presence of Calvo frictions. Finally, the Ricardian fiscal rule requires a reduction in taxes to compensate the effect on equation (8) of a higher inflation (reducing the outstanding value of debt) and the increase in the real rate.

[12]Notice that we have not included the case of $\overline{\Omega} = 1$, which is included in the Appendix D.2. As we mentioned before, if government debt is fully denominated in dollars, the dynamics have the opposite sign than those under positive but strictly smaller values for $\overline{\Omega}$, with a similar intuition as already discussed.

[13]As shown in Appendix D.2, having a policy rate that reacts to inflation with $\phi_\pi = 0.99$ only magnifies these differences, as the drop in the policy rate in such a case generates a path for the nominal exchange rate that appreciates even further, which requires an even larger and more persistent fall in π^N (and thus c^N).

[14]While that evolution of R^W is not shown in the figure, its autocorrelation of 0.7 implies that, for instance, five periods after the shock hits it still at a value of 0.08 above steady state, whereas rr is virtually zero at that point.

Figure 7: Responses to a world interest-rate shock, Ricardian vs. Non-Ricardian.



Notes: The figure show the responses to a positive shock to R^W, with an autocorrelation of 0.7, normalized to decrease c^T by 1% at the moment the shock hits. All cases feature $\overline{\Omega} = 0$. In solid blue lines $\phi_T = 1$ and $\phi_\pi = 1.5$. in dashed-red lines $\phi_T = 0$ and $\phi_\pi = 0$, while in dashed-dotted yellow lines $\phi_T = 0$ and $\phi_\pi = 0.99$. See Figure 2 for variables' definitions.

The dashed-red lines in Figure 7 correspond to the case of Non-Ricardian fiscal policy ($\phi_T = 0$) and a constant policy rate ($\phi_\pi = 0$). In that case, notice that c^N converges from below to the steady state, which shows how the previously analyzed dynamics in the periods following the shock in the blue lines were heavily influenced by the path of the policy rate. This, in turn, induces a fall in π^N, while the nominal exchange rate falls monotonically after the initial period.

From the FTPL perspective, this initial jump in the nominal exchange rate materializes to compensate for the fact that the increase in R^W puts upward pressure on the relevant discount rate. As lump-sum taxes are fixed, the net present value of primary surpluses drops, requiring a jump in the price level to compensate on the left-hand side of (8). Overall, dynamics under this alternative configuration are quite different.

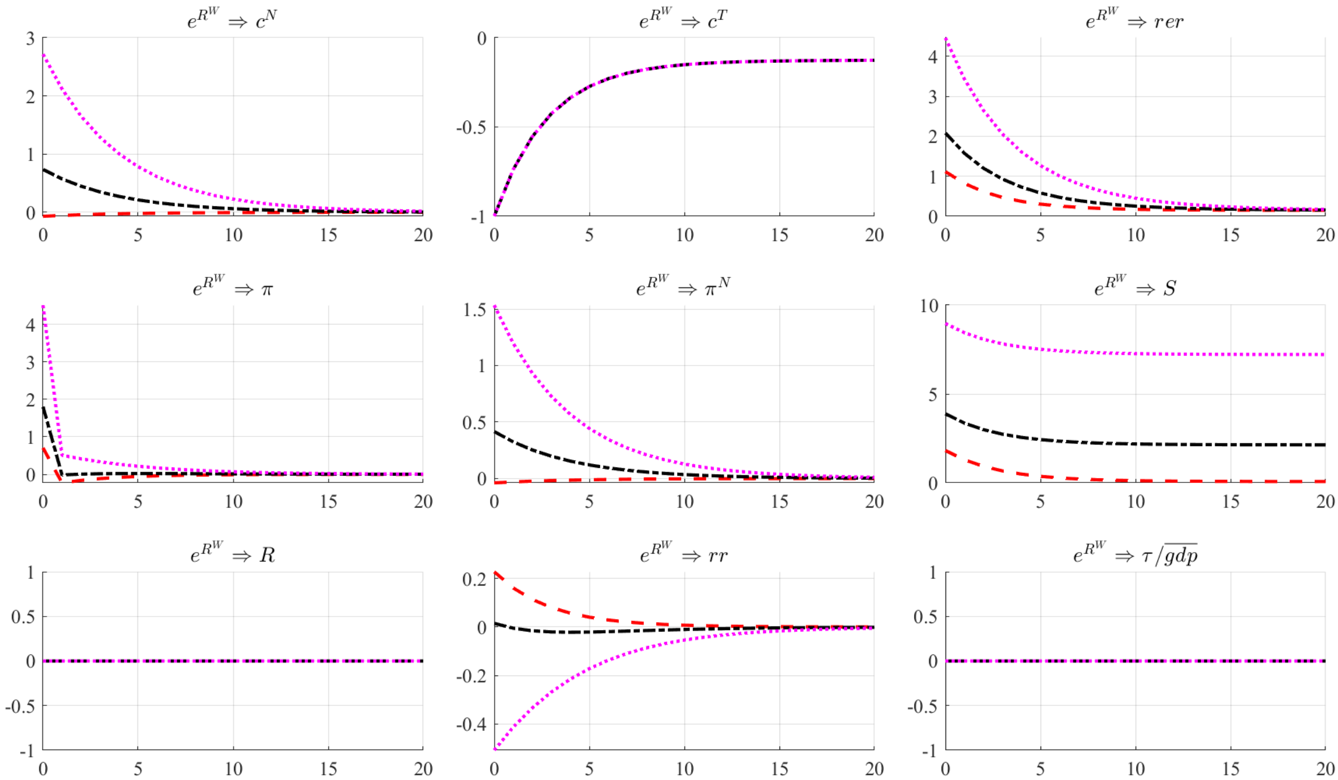
However, as we saw in the analysis of a y^T shock and also stressed here, these differences are, to a large extent, determined by the assumption of a constant policy rate. Indeed, the dashed-dotted yellow lines in Figure 7 report the case with an active fiscal policy ($\phi_T = 0$) but with a monetary-policy rate sensitive to inflation ($\phi_\pi = 0.99$). We can see that the dynamics are much closer to the case of a Ricardian fiscal policy in the solid-blue lines.

Figure 8 displays dynamics under the active-fiscal, passive-monetary setup ($\phi_\pi = 0$ and $\phi_T = 0$), for different values of the share of government debt denominated in dollars, $\overline{\Omega}$.[15] As the shock induces a real depreciation, it increases the real value (in domestic units) of outstanding dollar-

[15] Appendix D.2 also includes a case with $\phi_T = 0$ and $\phi_\pi = 0.99$, showing more exacerbated differences relative to Figure 8.

denominated debt. In order to compensate, either the price level should increase to dilute the burden of debt obligations in pesos, or the real rate needs to fall to increase the net-present value of primary surpluses (which is brought about by higher expected inflation), or a combination of both. As a result, the shock is more inflationary the higher the share $\overline{\Omega}$, and the expansion in non-traded consumption is even larger.[16]

Figure 8: Responses to a world interest-rate shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 7, except that here all cases feature $\phi_\pi = 0$ and $\phi_T = 0$, and they differ depending on the value for $\overline{\Omega}$: In dashed-red $\overline{\Omega} = 0$, in dashed-dotted black $\overline{\Omega} = 0.5$, in dotted magenta $\overline{\Omega} = 0.75$.

Overall, we see that the dynamics induced by a shock to the world interest rate are also significantly different if fiscal policy is active and part of the debt is denominated in foreign currency units.

[16] Again, as can be seen in Appendix D.2, the case of $\overline{\Omega}$ overturns these results.

6 Robustness

TBA

7 Conclusions

TBA

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Supplementary Appendix

A Transversality conditions

Define $rr_{t,t+J}$ such that $rr_{t,t} \equiv 1$ and $rr_{t,t+J} \equiv \prod_{j=0}^{J-1} \frac{R_{t+j}}{\pi_{t+1+j}}$ for $J \geq 1$. Under perfect foresight, this would be the cumulative (up to period J) ex-ante real rate, from the perspective of time t. In a stochastic world, this is the cumulative rate for a given history of realizations from t to $t+J$.

Let household's real assets be $b_t \equiv B_t/P_t$ and $d_t^{H*} \equiv D_t^{H*}/P_t^*$. We assume they face two No-Ponzi-game conditions (NPGC) for each financial asset,

$$\lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*} \right\} \leq 0, \quad \lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \frac{b_{t-1+J}}{\pi_{t+J}} \right\} \geq 0. \tag{A.1}$$

The discussion below clarifies why we impose one for each asset an instead of one for total financial wealth.

In real terms, the household's budget constraint (1) is

$$c_t + rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} + \frac{b_t}{R_t} + \tau_t = (1-\nu_t)(w_t h_t + \sigma_t) + \frac{b_{t-1}}{\pi_t} + rer_t \frac{d_t^{H*}}{R_t^*}, \tag{A.2}$$

Defining $sav_t \equiv (1-\nu_t)(w_t h_t + \sigma_t) - c_t - \tau_t$, the previous can be written as,

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = rer_t \frac{d_t^{H*}}{R_t^*} - \frac{b_t}{R_t} + sav_t.$$

From the real exchange rate's definition, $rer_t = rer_{t+1} \frac{\pi_{t+1}}{\pi_{t+1}^S \pi_{t+1}^*}$. Replacing this on the first-term on the right-hand side, and then multiplying and dividing by π_{t+1} the second-term, yields

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = \frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*} - \frac{\pi_{t+1}}{R_t} \frac{b_t}{\pi_{t+1}} + sav_t,$$

Adding and subtracting $\frac{\pi_{t+1}}{R_t} rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$ on the right hand side, we get

$$rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t} = \frac{\pi_{t+1}}{R_t} \left(rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*} - \frac{b_t}{\pi_{t+1}} \right) + sav_t + \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{\pi_{t+1}}{R_t} \right) rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$$

Let $so_{t+1}^H \equiv \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{\pi_{t+1}}{R_t} \right) rer_{t+1} \frac{d_t^{H*}}{\pi_{t+1}^*}$. The expectation $E_t\{so_{t+j}^H\}$ captures expected valuation changes due to deviations from perfect-foresight (or complete markets in a stochastic setting) uncovered interest rate parity which, in a non-linear stochastic model, generally does not hold because of covariance/premium terms. Notice also that this term arises only because we have more than one non-contingent asset and financial markets are incomplete; otherwise perfect-foresight non-arbitrage relationships would hold and $so_{t+1} = 0$. However, this term is zero up to a first order of approximation, for in the non-stochastic steady state valuation effects are nil.

Let $a_t^H \equiv rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t}$. Thus, we can write

$$a_t^H = \frac{a_{t+1}^H}{rr_{t,t+1}} + sav_t + so_{t+1}^H.$$

Replacing forward a_{t+1}^H yields

$$a_t^H = \frac{1}{rr_{t,t+1}} \left(\frac{a_{t+2}^H}{rr_{t+1,t+2}} + sav_{t+1} + so_{t+2}^H \right) + sav_t + so_{t+1}^H = \frac{a_{t+1}^H}{rr_{t,t+2}} + \sum_{j=0}^{1} \frac{\left(sav_{t+j} + so_{t+1+j}^H \right)}{rr_{t,t+j}},$$

where we have used the property $rr_{t,t+J} = rr_{t,t+N} \cdot rr_{t+N+1,t+J}$, for $0 \leq N < J$. Continuing iterating forward up to an arbitrary period J, we get

$$a_t^H = \frac{a_{t+J}^H}{rr_{t,t+J}} + \sum_{j=0}^{J-1} \frac{\left(sav_{t+j} + so_{t+1+j}^H \right)}{rr_{t,t+j}}$$

Finally, applying expectations conditional on time t information set on both sides, and then taking the limit for J up to infinity,

$$a_t^H = \lim_{J \to \infty} E_t \left\{ \frac{a_{t+J}^H}{rr_{t,t+J}} \right\} + \sum_{j=0}^{\infty} E_t \left\{ \frac{\left(sav_{t+j} + so_{t+1+j}^H \right)}{rr_{t,t+j}} \right\}$$

Recall that $a_t^H = rer_t \frac{d_{t-1}^{H*}}{\pi_t^*} - \frac{b_{t-1}}{\pi_t}$. Given the NPGC (A.1), the household optimal plan needs to satisfy the transversality condition,

$$\lim_{J \to \infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \left(rer_{t+J} \frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*} - \frac{b_{t-1+J}}{\pi_{t+J}} \right) \right\} = 0, \tag{A.3}$$

otherwise, if this limits was negative, consumption could be increased in every period and increase utility. Thus, it is not optimal to choose a plan in which this limit in (A.3) is not zero. For this condition to hold, and given the NPGC are imposed *individually* for each asset, both of the following transversality conditions need to hold,

$$\lim_{J \to \infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d_{t-1+J}^{H*}}{\pi_{t+J}^*} \right\} = 0, \quad \lim_{J \to \infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \frac{b_{t-1+J}}{\pi_{t+J}} \right\} = 0. \tag{A.4}$$

Regarding the government, its period t resource constraint in real terms is,

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t \frac{d_t^{G*}}{R_t^*} + sp_t, \tag{A.5}$$

where sp_t denotes the real primary surplus. Following similar steps as in the household case, this can be written as

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{\pi_{t+1}}{R_t} \frac{d_t^G}{\pi_{t+1}} + \frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*} + sp_t$$

Adding and subtracting $\frac{1}{rr_{t,t+1}} rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$ on the right hand side, we get

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{1}{rr_{t,t+1}} \left(\frac{d_t^G}{\pi_{t+1}} + rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*} \right) + sp_t + \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{1}{rr_{t,t+1}} \right) rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$$

Let $so_{t+1}^G \equiv \left(\frac{\pi_{t+1}}{\pi_{t+1}^S R_t^*} - \frac{1}{rr_{t,t+1}} \right) rer_{t+1} \frac{d_t^{G*}}{\pi_{t+1}^*}$. Similar to so_t^H, this represents valuation effects due to market incompleteness. Defining also, $a_t^G \equiv \frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*}$, we get

$$a_t^G = \frac{1}{rr_{t,t+1}} a_{t+1}^G + so_{t+1}^G + sp_t$$

Replacing forward a_{t+1}^G up until arbitrary period J we get

$$a_t^G = \frac{a_{t+J}^G}{rr_{t,t+J}} + \sum_{j=0}^{J-1} \frac{\left(sp_{t+j} + so_{t+1+j}^G \right)}{rr_{t,t+j}}$$

Finally, applying expectations conditional on information at time t on both sides, and then taking the limit for J up to infinity,

$$a_t^G = \lim_{J \to \infty} E_t \left\{ \frac{a_{t+J}^G}{rr_{t,t+J}} \right\} + \sum_{j=0}^{\infty} E_t \left\{ \frac{\left(sp_{t+j} + so_{t+1+j}^G \right)}{rr_{t,t+j}} \right\}$$

It follows that we need to impose the condition $\lim_{J \to \infty} E_t \left\{ \frac{a_{t+J}^G}{rr_{t,t+J}} \right\} = 0$ which, from the definition of a_t^G, is equivalent to

$$\lim_{J \to \infty} E_t \left\{ \frac{1}{rr_{t,t+J}} \left(\frac{d_{t+J-1}^G}{\pi_{t+J}} + rer_{t+J} \frac{d_{t-1+J}^{G*}}{\pi_{t+J}^*} \right) \right\} = 0. \tag{A.6}$$

While household's TVC (A.4) follow from optimization as we have discussed, the government's TVC does not. For that reason, transversality condition on government debt are part of the controversy surrounding FTPL in the literature.

In closed economy models with representative agents, the transversality condition for the government is not an extra requirement, for coincides with the household's TVC, which is an optimality condition. Here, as in equilibrium $b_t = d_t^H$ (as we are assuming that only domestic households hold peso bonds), the part of (A.6) corresponding to the debt in pesos is equivalent to household's TVC for its holding of domestic assets (i.e. the second equation in (A.4)) which, again, is an optimality condition given our assumptions.

However, if in an open economy we allow for *both* domestic and foreign agents to hold government bonds, the household's TVC related to foreign debt is not the same as that of the government's. This is discussed by earlier open-economy contributions such as Dupor (2000) and Daniel (2001). In particular, notice that a government not satisfying the TVC, but that still faces a NPGC condition (which is natural to assume), is wasting resources that could be used to either finance additional spending or to reduce taxes at some point. Therefore welfare could be improved if policy is constrained to satisfy the transversality condition with equality. A such, Daniel (2001) introduces the concept of "no-surplus fiscal policy" to describe schemes in which governments in a multi-country model do not waste resources in this way.

In our case, assuming that

$$\lim_{J\to\infty} E_t\left\{\frac{1}{rr_{t,t+J}} rer_{t+J}\frac{d^{G*}_{t-1+J}}{\pi^*_{t+J}}\right\} = 0, \tag{A.7}$$

in tandem with the household's TVC in pesos as previously discussed, implies that we focus on "no-surplus fiscal policies".

After imposing these transversality conditions, the lifetime government budget constraint is

$$\frac{d^G_{t-1}}{\pi_t} + rer_t\frac{d^{G*}_{t-1}}{\pi^*_t} = \sum_{j=0}^{\infty} E_t\left\{\frac{\left(sp_{t+j} + so^G_{t+1+j}\right)}{rr_{t,t+j}}\right\}$$

Using the definition of the share of government debt in dollars Ω_t in (5), plus the constant-currency composition rule ($\Omega_t = \overline{\Omega}$), we get

$$\left[\frac{1-\overline{\Omega}}{\pi_t} + \frac{rer_t}{rer_{t-1}}\frac{\overline{\Omega}}{\pi^*_t}\right] d_{t-1} = \left[\frac{(1-\overline{\Omega}) + \overline{\Omega}\pi^S_t}{\pi_t}\right] d_{t-1} = E_t\left\{\sum_{j=0}^{\infty}\frac{sp_{t+j}}{rr_{t,t+j}}\right\} + h.o.t.,$$

where $d_t \equiv d^G_t + rer_t d^{G*}_t$, in the first equality we have used $rer_t = rer_{t-1}\frac{\pi^S_t\pi^*_t}{\pi_t}$, and also $h.o.t. \equiv \sum_{j=0}^{\infty} E_t\left\{so^G_{t+1+j}/rr_{t,t+j}\right\}$. This is equation (8) in the text.

Finally, it is relevant to notice that the transversality condition for government debt in dollars (A.7) is not only relevant to describe the government problem, but also to guarantee that the balance of payments is sustainable. Consolidating the household and government's budget constraints (A.2) and (A.5), using also the market clearing condition for non-traded goods, the following represents the balance of payments in this model, expressed in domestic-consumption units,

$$rer_t\frac{d^*_{t-1}}{\pi^*_t} = rer_t\frac{d^*_t}{R^*_t} + tb_t$$

where $d^*_t \equiv d^{G*}_t + d^{H*}_t$ is the countries' net-foreign lending position, and $tb_t \equiv rer_t(y^T_t - c^T_t)$ is the trade balance. As before, this can be written as

$$rer_t\frac{d^*_{t-1}}{\pi^*_t} = \frac{\pi_{t+1}}{\pi^S_{t+1}R^*_t} rer_{t+1}\frac{d^*_t}{\pi^*_{t+1}} + tb_t$$

or,

$$rer_t\frac{d^*_{t-1}}{\pi^*_t} = \frac{1}{rr_{t,t+1}} rer_{t+1}\frac{d^*_t}{\pi^*_{t+1}} + \left(\frac{\pi_{t+1}}{\pi^S_{t+1}R^*_t} - \frac{1}{rr_{t,t+1}}\right) rer_{t+1}\frac{d^*_t}{\pi^*_{t+1}} + tb_t$$

Defining $so_{t+1} \equiv \left(\frac{\pi_{t+1}}{\pi^S_{t+1}R^*_t} - \frac{1}{rr_{t,t+1}}\right) rer_{t+1}\frac{d^*_t}{\pi^*_{t+1}}$, iterating forward up to period J, applying expectations conditional on information at time t on both sides, and taking the limit for J up to infinity, we get

$$rer_t\frac{d^*_{t-1}}{\pi^*_t} = \lim_{J\to\infty} E_t\left\{\frac{1}{rr_{t,t+J}} rer_{t+J}\frac{d^*_{t-1+J}}{\pi^*_{t+J}}\right\} + \sum_{j=0}^{\infty} E_t\left\{\frac{(tb_{t+j} + so_{t+1+j})}{rr_{t,t+J}}\right\}$$

It follows that for the balance of payments to be sustainable we need,

$$\lim_{J\to\infty} E_t \left\{ \frac{1}{rr_{t,t+J}} rer_{t+J} \frac{d^*_{t-1+J}}{\pi^*_{t+J}} \right\} = 0,$$

Therefore, as $d^*_t \equiv d^{G*}_t + d^{H*}_t$, the households optimality TVC for dollar assets, plus the assumption of "no-surplus" policy previously discussed, is equivalent to requiring the balance of payments' sustainability. In other words, in the open economy, arguing about whether the TVC condition for government's debt needs to hold is equivalent to discussing the balance-of-payment's sustainability.

Still, it should be highlighted that this discussion is more pertinent to a **global** equilibrium analysis, which previous open-economy papers had only tackle in deterministic models. Instead, in this paper, as we approximate the solution by linearization around a non-stochastic steady state, the **local** existence and uniqueness requirements implicitly assume these transversality conditions hold. Moreover, the valuation terms previously described vanish up to a first order of approximation. Future research could be devoted to study the requirements for global existence and uniqueness in stochastic small-and-open economy models.

B Equilibrium characterization

B.1 Production of non-tradable varieties and Calvo

A monopolist producing the variety j produces uses labor according to $y^N_{jt} = (h_{jt})^\alpha$, with $\alpha \in (0,1]$. It internalizes the demand for j (obtained from the maximization of the competitive firms aggregating non-trade goods), given by

$$y^N_{jt} = \left(\frac{P^N_{jt}}{P^N_t}\right)^{-\epsilon_N} y^N_t, \text{ for all } j.$$

In addition, it faces a Calvo problem in choosing its price P^N_{jt}: with probability θ_N it is forced to set $P^N_{jt} = P^N_{jt-1}$, while with probability $1-\theta_N$ it can freely choose a price $\widetilde{P}^N_{jt}$. Using well-known aggregation results (e.g. Schmitt-Grohé and Uribe, 2017, Ch. 9.16), in particular that all firms able to chooses set the same price $\widetilde{P}^N_t$, the following characterize dynamics of non-traded prices,

$$p^N_t mc^N_t = \frac{w_t}{\alpha}(y^N_t)^{\left(\frac{1}{\alpha}-1\right)}, \tag{A.8}$$

$$f_t = (\widetilde{p}^N_t)^{1-\epsilon_N} y^N_t \frac{(\epsilon_N - 1)}{\epsilon_N} + \theta_N E_t \left\{ \chi_{t,t+1} \left(\frac{\widetilde{p}^N_t}{\widetilde{p}^N_{t+1}}\right)^{1-\epsilon_N} (\pi^N_{t+1})^{\epsilon_N} f_{t+1} \right\}, \tag{A.9}$$

$$f_t = (\widetilde{p}^N_t)^{-\epsilon_N} y^N_t mc^N_t + \theta_N E_t \left\{ \chi_{t,t+1} \left(\frac{\widetilde{p}^N_t}{\widetilde{p}^N_{t+1}}\right)^{-\epsilon_N} (\pi^N_{t+1})^{1+\epsilon_N} f_{t+1} \right\}, \tag{A.10}$$

$$1 = \theta_N (\pi^N_t)^{\epsilon_N - 1} + (1-\theta_N)(\widetilde{p}^N_t)^{1-\epsilon_N}. \tag{A.11}$$

Equation (A.8) equates marginal costs to the ratio between wages and the marginal product of labor, where mc^N_t is the real marginal cost in non-traded units. Equations (A.9)-(A.10) provide a recursive representation to the optimal choice of price $\widetilde{P}^N_t$ for those allowed to chose (where $\widetilde{p}^N_t \equiv \widetilde{P}^N_t / P^N_t$), equating the net present value of marginal revenues in those states in which the price $\widetilde{P}^N_t$ still holds (called f_t in this notation) to that of marginal costs.[17] The last condition (A.11) relates the optimal price chosen by those allowed to do so and non-traded inflation $\pi_t^N \equiv P_t^N / P_{t-1}^N$.

These equations can be log-linearized around the zero-inflation steady state (see Schmitt-Grohé and Uribe, 2017, Ch. 9.16) to obtain,

$$\widehat{\pi}_t^N = \beta E_t\{\widehat{\pi}_{t+1}^N\} + \kappa \cdot \widehat{mc}_t^N,$$

where $\widehat{\cdot}$ denotes log-linear approximation, and $\kappa = \frac{(1-\theta_N)(1-\beta\theta_N)}{\theta_N\left(\frac{\epsilon_N}{\alpha}+1-\epsilon_N\right)}$.

B.2 Equilibrium conditions

Besides the transversality conditions in Appendix A, the following characterize the equilibrium,

$$(1-\nu_t)w_t(c_t)^{-\sigma} = \chi(h_t)^{\varphi}, \tag{Eq.1}$$

$$(c_t)^{-\sigma} = \beta R_t E_t\left\{\frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}}\right\}, \tag{Eq.2}$$

$$(c_t)^{-\sigma} = \beta R_t^* E_t\left\{\frac{(c_{t+1})^{-\sigma}\pi_{t+1}^S}{\pi_{t+1}}\right\}, \tag{Eq.3}$$

$$c_t = \left[\omega^{1/\eta}\left(c_t^N\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(c_t^T\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}}, \tag{Eq.4}$$

$$c_t^N = \omega\left(p_t^N\right)^{-\eta} c_t. \tag{Eq.5}$$

$$c_t^T = (1-\omega)\left(p_t^T\right)^{-\eta} c_t, \tag{Eq.6}$$

$$p_t^N mc_t^N = \frac{w_t}{\alpha}(y_t^N)^{\left(\frac{1}{\alpha}-1\right)}, \tag{Eq.7}$$

$$f_t = (\widetilde{p}_t^N)^{1-\epsilon_N} y_t^N \frac{(\epsilon_N-1)}{\epsilon_N} + \theta_N E_t\left\{\beta\frac{(c_{t+1})^{-\sigma}}{(c_t)^{-\sigma}\pi_{t+1}}\left(\frac{\widetilde{p}_t^N}{\widetilde{p}_{t+1}^N}\right)^{1-\epsilon_N}(\pi_{t+1}^N)^{\epsilon_N} f_{t+1}\right\}, \tag{Eq.8}$$

$$f_t = (\widetilde{p}_t^N)^{-\epsilon_N} y_t^N mc_t^N + \theta_N E_t\left\{\beta\frac{(c_{t+1})^{-\sigma}}{(c_t)^{-\sigma}\pi_{t+1}}\left(\frac{\widetilde{p}_t^N}{\widetilde{p}_{t+1}^N}\right)^{-\epsilon_N}(\pi_{t+1}^N)^{1+\epsilon_N} f_{t+1}\right\}, \tag{Eq.9}$$

$$1 = \theta_N(\pi_t^N)^{\epsilon_N-1} + (1-\theta_N)(\widetilde{p}_t^N)^{1-\epsilon_N}. \tag{Eq.10}$$

$$\frac{d_{t-1}^G}{\pi_t} + rer_t\frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t\frac{d_t^{G*}}{R_t^*} + sp_t, \tag{Eq.11}$$

$$sp_t = \tau_t + \nu_t\left(p_t^N y_t^N + p_t^T y_t^T\right) - p_t^N g_t, \tag{Eq.12}$$

$$\tau_t - \overline{\tau} = \phi_T\left[\frac{d_{t-1}^G}{\pi_t} + rer_t\frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{sp}}{1-\beta}\right] + u_t^{\tau}, \tag{Eq.13}$$

$$\overline{\Omega} = \frac{\overline{rer}d_t^{G*}}{d_t^G + \overline{rer}d_t^{G*}}, \tag{Eq.14}$$

[17] In these, $\chi_{t,t+h} \equiv \beta^h \frac{(c_{t+h})^{-\sigma}P_t}{(c_t)^{-\sigma}P_{t+h}}$ is the households' stochastic discount factor for nominal claims.

$$\left(\frac{R_t}{\overline{R}}\right) = \left(\frac{\pi_t}{\overline{\pi}}\right)^{\phi_\pi} u_t^R, \tag{Eq.15}$$

$$R_t^* = R_t^W \exp\left\{\psi\left(d_t^* - \bar{d}^*\right)\right\}, \tag{Eq.16}$$

$$d_t^* = d_t^{*G} + d_t^{*H}, \tag{Eq.17}$$

$$y_t^N = (h_t)^\alpha, \tag{Eq.18}$$

$$y_t^N = (\Delta_t)^\alpha (c_t^N + g_t), \tag{Eq.19}$$

$$\Delta_t = \theta_N \Delta_t (\pi_t^N)^{\frac{\epsilon_N}{\alpha}} + (1-\theta_N)(\widetilde{p}_t^N)^{-\frac{\epsilon_N}{\alpha}}, \tag{Eq.20}$$

$$rer_t = p_t^T, \tag{Eq.21}$$

$$rer_t = rer_{t-1}\frac{\pi_t^S \pi_t^*}{\pi_t}, \tag{Eq.22}$$

$$p_t^N = p_{t-1}^N \frac{\pi_t^N}{\pi_t}, \tag{Eq.23}$$

$$\frac{d_{t-1}^*}{\pi_t^*} = \frac{d_t^*}{R_t^*} + (y_t^T - c_t^T). \tag{Eq.24}$$

All variables were described before, except for Δ_t that captures the potential inefficiency induced by price dispersion in the presence of sticky prices. However, as we will assume zero steady-state inflation, it is not relevant up to first order.

Overall, we have 24 endogenous variables:

$$\begin{matrix} w_t & c_t & h_t & R_t & \pi_t & R_t^* & \pi_t^S & c_t^N & c_t^T & p_t^N & p_t^T & rer_t \\ mc_t^N & y_t^N & f_t & \widetilde{p}_t^N & \pi_t^N & d_t^G & d_t^{G*} & sp_t & \tau_t & d_t^* & d_t^{*H} & \Delta_t \end{matrix}$$

and 7 exogenous variables:

$$y_t^T \quad \pi_t^* \quad R_t^W \quad g_t \quad \nu_t \quad u_t^\tau \quad u_t^R$$

For $x_t = \{y_t^T, \pi_t^*, R_t^W, u_t^\tau, u_t^R\}$ we assume the following AR(1) process in logs

$$\log\left(\frac{x_t}{\overline{x}}\right) = \rho_x \log\left(\frac{x_{t-1}}{\overline{x}}\right) + \sigma_x \varepsilon_t^x,$$

where ε_t^x is an i.i.d. shock, $\rho_x \in [0,1)$ and $\sigma_x > 0$. Instead, for the other exogenous variables that can take the value zero, we assume

$$\frac{(g_t - \overline{g})}{\overline{gdp}} = \rho_g \frac{(g_{t-1} - \overline{g})}{\overline{gdp}} + \sigma_g \varepsilon_t^g, \quad \frac{(u_t^\tau - \overline{u}^\tau)}{\overline{gdp}} = \rho_\tau \frac{(u_{t-1}^\tau - \overline{u}^\tau)}{\overline{gdp}} + \sigma_\tau \varepsilon_t^\tau, \quad (\nu_t - \overline{\nu}) = \rho_\nu (\nu_{t-1} - \overline{\nu}) + \sigma_\nu \varepsilon_t^\nu$$

where $\overline{gdp} = \overline{y}^N + \overline{y}^T$ is real GDP in steady state, $\rho_g, \rho_\tau, \rho_\nu \in [0,1)$ and $\sigma_g, \sigma_\tau, \sigma_\nu > 0$.

B.3 Tradable block

Combining equations (Eq.3), (Eq.6), (Eq.21) and (Eq.22) we get

$$(c_t^T)^{-\sigma}(p_t^T)^{-\sigma\eta} = \beta R_t^* E_t \left\{ \frac{(c_{t+1}^T)^{-\sigma}(p_{t+1}^T)^{-\sigma\eta} p_{t+1}^T}{\pi_{t+1}^* p_t^T} \right\}, \tag{B.1}$$

If in addition we assume $\eta\sigma = 1$, this yields

$$(c_t^T)^{-\sigma} = \beta R_t^* E_t \left\{ \frac{(c_{t+1}^T)^{-\sigma}}{\pi_{t+1}^*} \right\}, \tag{B.2}$$

Equations (B.2), (Eq.16) and (Eq.24) form a system for the endogenous variables c_t^T, d_t^* and R_t^*, which can be solved as a function of exogenous variables π_t^*, R_t^W, y_t^T alone. Thus, in this setup, the tradable block is isolated from non-tradables, as well as from monetary and fiscal policy. The intuitive reason for this result is that any effect on the real exchange rate (p_t^T), originated from exogenous variables different from π_t^*, R_t^W, y_t^T, induces both intra- and inter-temporal substitution effects. By the former, an increase in p_t^T produces (*ceteris paribus*) a desire for substituting away from tradables, as these become more expensive; an effect determined by the intra-temporal elasticity of substitution η. By the latter, a temporary increase in p_t^T produces (*ceteris paribus*) a fall in the real return of saving in tradables, increasing desired tradable consumption today (an effect determined by the intra-temporal substitution elasticity $1/\sigma$). Thus, when $\eta\sigma = 1$, the two effects offset each other.

B.4 Non-tradable and policy block

Taking out 3 equations for the 3 traded variables that are determined independently, eliminating c_t, d_t^{*H} and p_t^T to simplify, using also the assumption $\eta\sigma = 1$, assuming $\Delta_t = 1$ (which holds up to first order), and re-ordering some equations, the remainder equilibrium conditions are:

Non-Policy

$$(1 - \nu_t) \frac{w_t}{p_t^N} \left(\frac{c_t^N}{\omega} \right)^{-\sigma} = \chi (h_t)^{\varphi}, \tag{B.3}$$

$$c_t^N = \frac{\omega}{1 - \omega} \left(\frac{p_t^N}{rer_t} \right)^{-\eta} c_t^T, \tag{B.4}$$

$$1 = \omega \left(p_t^N \right)^{1-\eta} + (1 - \omega) \left(rer_t \right), ^{1-\eta} \tag{B.5}$$

$$p_t^N mc_t^N = \frac{w_t}{\alpha} (y_t^N)^{(\frac{1}{\alpha} - 1)}, \tag{B.6}$$

$$y_t^N = (h_t)^{\alpha}, \tag{B.7}$$

$$y_t^N = (c_t^N + g_t), \tag{B.8}$$

$$rer_t = rer_{t-1} \frac{\pi_t^S \pi_t^*}{\pi_t}, \tag{B.9}$$

$$p_t^N = p_{t-1}^N \frac{\pi_t^N}{\pi_t}, \tag{B.10}$$

$$R_t E_t \left\{ \frac{(c_{t+1})^{-\sigma}}{\pi_{t+1}} \right\} = R_t^* E_t \left\{ \frac{(c_{t+1})^{-\sigma} \pi_{t+1}^S}{\pi_{t+1}} \right\}, ^{18} \tag{B.11}$$

$$\widehat{\pi}_t^N = \beta E_t \{ \widehat{\pi}_{t+1}^N \} + \kappa \cdot \widehat{mc}_t^N, \tag{B.12}$$

Policy

$$\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t \frac{d_t^{G*}}{R_t^*} + sp_t \tag{B.13}$$

[18] This equation also features c_{t+1}, but it will disappear once we log-linearize.

$$sp_t = \tau_t + \nu_t \left(p_t^N y_t^N + p_t^T y_t^T \right) - p_t^N g_t, \tag{B.14}$$

$$\tau_t - \overline{\tau} = \phi_T \left[\frac{d_{t-1}^G}{\pi_t} + rer_t \frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{sp}}{1-\beta} \right] + u_t^\tau, \tag{B.15}$$

$$\overline{\Omega} = \frac{rer_t d_t^{G*}}{d_t^G + rer_t d_t^{G*}}, \tag{B.16}$$

$$\left(\frac{R_t}{\overline{R}} \right) = \left(\frac{\pi_t}{\overline{\pi}} \right)^{\phi_\pi} u_t^R, \tag{B.17}$$

Overall, we have 15 endogenous variables:

$$\begin{array}{cccccccc} w_t & h_t & R_t & \pi_t & \pi_t^S & c_t^N & p_t^N & rer_t \\ mc_t^N & y_t^N & \pi_t^N & d_t^G & d_t^{G*} & sp_t & \tau_t & \end{array}$$

and 10 exogenous variables:

$$y_t^T \quad \pi_t^* \quad R_t^W \quad g_t \quad \nu_t \quad u_t^\tau \quad u_t^R \quad d_t^* \quad c_t^T \quad R_t^*$$

(recall d_t^*, c_t^T, R_t^* are determined independently of other endogenous variables)

We begin by log-linearizing the Non-Policy block (except for ν_t and g_t for which we just linearize)

$$-\frac{\widehat{\nu}_t}{1-\overline{\nu}} + \widehat{w}_t - \widehat{p}_t^N - \sigma \widehat{c}_t^N = \varphi \widehat{h}_t,$$

$$\widehat{c}_t^N = -\eta \left(\widehat{p}_t^N - \widehat{rer}_t \right) + \widehat{c}_t^T,$$

$$0 = \omega \widehat{p}_t^N + (1-\omega) \widehat{rer}_t,$$

$$\widehat{p}_t^N + \widehat{mc}_t^N = \widehat{w}_t + \left(\frac{1}{\alpha} - 1 \right) \widehat{y}_t^N,$$

$$\widehat{y}_t^N = \alpha \widehat{h}_t,$$

$$\widehat{y}_t^N = \left(1 - \frac{\overline{g}}{\overline{y}^N} \right) \widehat{c}_t^N + \left(\frac{1}{\overline{y}^N} \right) \widehat{g}_t,$$

$$\widehat{p}_t^N = \widehat{p}_{t-1}^N + \widehat{\pi}_t^N - \widehat{\pi}_t,$$

$$\widehat{rer}_t = \widehat{rer}_{t-1} + \widehat{\pi}_t^S + \widehat{\pi}_t^* - \widehat{\pi}_t,$$

$$\widehat{R}_t = \widehat{R}_t^* + E_t \left\{ \widehat{\pi}_{t+1}^S \right\},$$

$$\widehat{\pi}_t^N = \beta E_t \{ \widehat{\pi}_{t+1}^N \} + \kappa \cdot \widehat{mc}_t^N.$$

The first six equations (the ones that are just static) can be used to write marginal costs as,

$$\widehat{mc}_t^N = \left[\left(\frac{1+\varphi}{\alpha} - 1 \right) \left(1 - \frac{\overline{g}}{\overline{y}^N} \right) + \sigma \right] \widehat{c}_t^N + \frac{\widehat{\nu}_t}{1-\overline{\nu}} + \left(\frac{1}{\alpha} - 1 \right) \left(\frac{1}{\overline{y}^N} \right) \widehat{g}_t$$

Notice also that, from $0 = \omega \widehat{p}_t^N + (1-\omega) \widehat{rer}_t$, we have

$$\widehat{p}_t^N - \widehat{rer}_t = -\frac{1}{\omega} \widehat{rer}_t,$$

Thus, from $\widehat{c}_t^N = -\eta\left(\widehat{p}_t^N - \widehat{rer}_t\right) + \widehat{c}_t^T$ we get,

$$\widehat{c}_t^N = \frac{\eta}{\omega}\widehat{rer}_t + \widehat{c}_t^T$$

In addition, again using $0 = \omega\widehat{p}_t^N + (1-\omega)\widehat{rer}_t$ and the equations describing the evolution of $\widehat{p}_t^N$ and $\widehat{rer}_t$, we can write inflation as

$$\widehat{\pi}_t = \omega(\widehat{p}_{t-1}^N + \widehat{\pi}_t^N) + (1-\omega)\left(\widehat{rer}_{t-1} + \widehat{\pi}_t^S + \widehat{\pi}_t^*\right).$$

But, as it also holds that $0 = \omega\widehat{p}_{t-1}^N + (1-\omega)\widehat{rer}_{t-1}$, we get

$$\widehat{\pi}_t = \omega(\widehat{\pi}_t^N) + (1-\omega)\left(\widehat{\pi}_t^S + \widehat{\pi}_t^*\right) \Rightarrow \widehat{\pi}_t^N = \frac{1}{\omega}\widehat{\pi}_t - \frac{(1-\omega)}{\omega}\left(\widehat{\pi}_t^S + \widehat{\pi}_t^*\right)$$

If we further simplify by assuming $\nu_t = g_t = \overline{\nu} = \overline{g} = 0$, defining $\widetilde{\kappa} \equiv \omega\kappa\left[\frac{(1+\varphi)}{\alpha} - 1 + \sigma\right]$, we can reduce the non-policy block to:

$$\widehat{\pi}_t^N = \beta E_t\left\{\widehat{\pi}_{t+1}^N\right\} + \frac{\widetilde{\kappa}}{\omega}\left(\frac{\eta}{\omega}\widehat{rer}_t + \widehat{c}_t^T\right), \tag{B.18}$$

$$\widehat{\pi}_t = \omega(\widehat{\pi}_t^N) + (1-\omega)\left(\widehat{\pi}_t^S + \widehat{\pi}_t^*\right), \tag{B.19}$$

$$\widehat{rer}_t = \widehat{rer}_{t-1} + \widehat{\pi}_t^S + \widehat{\pi}_t^* - \widehat{\pi}_t, \tag{B.20}$$

$$\widehat{R}_t = \widehat{R}_t^* + E_t\left\{\widehat{\pi}_{t+1}^S\right\}, \tag{B.21}$$

These are 4 equations for 4 non-policy endogenous variables $\widehat{\pi}_t^N, \widehat{\pi}_t, \widehat{rer}_t, \widehat{\pi}_t^S$.

The fiscal policy block, with the simplifying assumptions $\nu_t = g_t = \overline{\nu} = \overline{g} = 0$, is

$$\frac{d_{t-1}^G}{\pi_t} + rer_t\frac{d_{t-1}^{G*}}{\pi_t^*} = \frac{d_t^G}{R_t} + rer_t\frac{d_t^{G*}}{R_t^*} + \tau_t$$

$$\tau_t - \overline{\tau} = \phi_T\left[\frac{d_{t-1}^G}{\pi_t} + rer_t\frac{d_{t-1}^{G*}}{\pi_t^*} - \frac{\overline{\tau}}{1-\beta}\right] + u_t^\tau,$$

$$\overline{\Omega} = \frac{rer_t d_t^{G*}}{d_t^G + rer_t d_t^{G*}},$$

These are 3 equations for 3 fiscal-policy endogenous variables, d_t^G, d_t^{G*}, τ_t. Notice that the government budget constraint can be written as

$$\left[\frac{(1-\overline{\Omega})}{\pi_t} + \frac{rer_t}{rer_{t-1}}\frac{\overline{\Omega}}{\pi_t^*}\right]d_{t-1} = \left[\frac{(1-\overline{\Omega})}{R_t} + \frac{\overline{\Omega}}{R_t^*}\right]d_t + \tau_t$$

where $d_t = d_t^G + rer_t d_t^{G*}$ is the total amount of government debt, expressed in domestic consumption units, evaluated at the steady-state real exchange rate.

Define $A_t \equiv \left[\frac{(1-\overline{\Omega})}{\pi_t} + \frac{rer_t}{rer_{t-1}}\frac{\overline{\Omega}}{\pi_t^*}\right]$, $B_t \equiv \left[\frac{(1-\overline{\Omega})}{R_t} + \frac{\overline{\Omega}}{R_t^*}\right]$, $\widehat{A}_t \equiv \log(A_t/\overline{A})$ and $\widehat{B}_t \equiv \log(B_t/\overline{B})$. Also, we focus on cases with $\overline{d}, \overline{\tau} > 0$, so we also define $\widehat{d}_t \equiv \log(d_t/\overline{d})$ and $\widehat{\tau}_t \equiv \log(\tau_t/\overline{\tau})$. Thus, applying the change of variables, we get

$$\overline{A}e^{\widehat{A}_t}\overline{d}e^{\widehat{d}_{t-1}} = \overline{B}e^{\widehat{B}_t}\overline{d}e^{\widehat{d}_t} + \overline{\tau}e^{\widehat{\tau}_t}.$$

Taking a first-order approximation

$$\overline{A}\overline{d}(\widehat{A}_t + \widehat{d}_{t-1}) = \overline{B}\overline{d}(\widehat{B}_t + \widehat{d}_t) + \overline{\tau}\widehat{\tau}_t.$$

In a steady state with $\overline{\pi} = \overline{\pi}^* = \overline{\pi}^S = 1$, then $R = R^* = 1/\beta$, so we have

$$\overline{A} = 1, \quad \overline{B} = \beta, \quad \overline{d}(1-\beta) = \overline{\tau}.$$

Therefore, the first-order approximation simplifies to

$$\widehat{A}_t + \widehat{d}_{t-1} = \beta(\widehat{B}_t + \widehat{d}_t) + (1-\beta)\widehat{\tau}_t. \tag{B.22}$$

Similarly, the rule for lump-sump taxes can be written as

$$\tau_t - \overline{\tau} = \phi_T \left(A_t d_{t-1} - \frac{\overline{\tau}}{1-\beta} \right) + u_t^\tau \quad \Rightarrow \quad \overline{\tau}e^{\widehat{\tau}_t} - \overline{\tau} = \phi_T \left(\overline{A}e^{\widehat{A}_t}\overline{d}e^{\widehat{d}_{t-1}} - \frac{\overline{\tau}}{1-\beta} \right) + u_t^\tau,$$

or

$$e^{\widehat{\tau}_t} - 1 = \phi_T \left(\frac{1}{(1-\beta)} e^{\widehat{A}_t + \widehat{d}_{t-1}} - \frac{1}{1-\beta} \right) + \frac{1}{(1-\beta)} \widehat{u}_t^\tau,$$

where $\widehat{u}_t^\tau \equiv u_t^\tau(1-\beta)/\overline{\tau}$. Taking a first order approximation,

$$\widehat{\tau}_t = \phi_T \frac{1}{(1-\beta)} (\widehat{A}_t + \widehat{d}_{t-1}) + \frac{1}{(1-\beta)} \widehat{u}_t^\tau. \tag{B.23}$$

Thus, combining (B.22) and (B.23), we can write

$$(1-\phi_T)(\widehat{d}_{t-1} + \widehat{A}_t) = \beta(\widehat{d}_t + \widehat{B}_t) + \widehat{u}_t^\tau. \tag{B.24}$$

In addition, from the definitions $A_t \equiv \left[\frac{(1-\overline{\Omega})}{\pi_t} + \frac{rer_t}{rer_{t-1}} \frac{\overline{\Omega}}{\pi_t^*} \right]$, $B_t \equiv \left[\frac{(1-\overline{\Omega})}{R_t} + \frac{\overline{\Omega}}{R_t^*} \right]$, first notice that using (Eq.22),

$$A_t \equiv \left[\frac{(1-\overline{\Omega})}{\pi_t} + \frac{rer_t}{rer_{t-1}} \frac{\overline{\Omega}}{\pi_t^*} \right] = \left[\frac{(1-\overline{\Omega})}{\pi_t} + \frac{\pi_t^S}{\pi_t} \overline{\Omega} \right] = \left[\frac{(1-\overline{\Omega}) + \overline{\Omega}\pi_t^S}{\pi_t} \right]$$

Thus, we can log-linearize to obtain,

$$\widehat{A}_t = -\widehat{\pi}_t + \Omega\widehat{\pi}_t^S \quad \widehat{B}_t = -(1-\overline{\Omega})\widehat{R}_t - \Omega\widehat{R}_t^*.$$

Thus, (B.24) can be written as

$$(1-\phi_T)\left[\widehat{d}_{t-1} - \widehat{\pi}_t + \Omega\widehat{\pi}_t^S \right] = \beta \left[\widehat{d}_t - \widehat{R}_t - \Omega(\widehat{R}_t^* - \widehat{R}_t) \right] + \widehat{u}_t^\tau,$$

or, using (B.20) and (B.21),

$$(1-\phi_T)\left(\widehat{d}_{t-1} - \widehat{\pi}_t + \Omega\widehat{\pi}_t^S \right) = \beta \left[\widehat{d}_t - \widehat{R}_t + \Omega E_t \left\{ \widehat{\pi}_{t+1}^S \right\} \right] + \widehat{u}_t^\tau, \tag{B.25}$$

Notice also that, using (B.19), this can be written as

$$(1-\phi_T)\left[\widehat{d}_{t-1}-\omega(\widehat{\pi}_t^N)-(1-\omega)\left(\widehat{\pi}_t^S+\widehat{\pi}_t^*\right)+\Omega\widehat{\pi}_t^S\right]=\beta\left[\widehat{d}_t-\widehat{R}_t+\Omega E_t\left\{\widehat{\pi}_{t+1}^S\right\}\right]+\widehat{u}_t^\tau,$$

or

$$(1-\phi_T)\left[\widehat{d}_{t-1}-\omega(\widehat{\pi}_t^N)+(\Omega+\omega-1)\widehat{\pi}_t^S-(1-\omega)\widehat{\pi}_t^*\right]=\beta\left[\widehat{d}_t-\widehat{R}_t+\Omega E_t\left\{\widehat{\pi}_{t+1}^S\right\}\right]+\widehat{u}_t^\tau,$$

Finally, the log-linearization of the Taylor rule is

$$\widehat{R}_t=\phi_\pi\widehat{\pi}_t+\widehat{u}_t^R. \tag{B.26}$$

Summarizing, the non-traded plus policy blocks can be reduced, up to first order, to equations (B.18), (B.19), (B.20), (B.21), (B.25) and (B.26), corresponding to the endogenous variables $\widehat{\pi}_t^N$, $\widehat{\pi}_t$, $\widehat{rer}_t$, $\widehat{\pi}_t^S$, $\widehat{R}_t$, and $\widehat{d}_t$.

Finally, notice that combining (B.19), (B.20) and (B.21) we can derive,

$$\widehat{R}_t=\widehat{R}_t^*+E_t\left\{\widehat{\pi}_{t+1}^S\right\}\Rightarrow\widehat{rer}_t=\widehat{R}_t^*-\widehat{R}_t+E_t\left\{\widehat{rer}_{t+1}+\widehat{\pi}_{t+1}-\widehat{\pi}_{t+1}^*\right\}$$
$$\Rightarrow\widehat{rer}_t=\sum_{j=0}^{\infty}\left[(\widehat{R}_{t+j}^*-\widehat{\pi}_{t+1+j}^*)-(\widehat{R}_{t+j}-\widehat{\pi}_{t+1+j})\right]\Rightarrow$$
$$\widehat{\pi}_t^S=\widehat{\pi}_t-\widehat{\pi}_t^*-\widehat{rer}_{t-1}+\sum_{j=0}^{\infty}\left[(\widehat{R}_{t+j}^*-\widehat{\pi}_{t+1+j}^*)-(\widehat{R}_{t+j}-\widehat{\pi}_{t+1+j})\right],$$

which is (15) in the text.

C Existence and Uniqueness

C.1 Case A: $\theta_N\in(0,1)$

Combining the equations (B.18), (B.19), (B.20), (B.21), (B.25) and (B.26) to eliminate $\widehat{\pi}_t$ and $\widehat{R}_t$, we get the following equations characterizing the dynamics of $\widehat{\pi}_t^N,\widehat{rer}_t,\widehat{\pi}_t^S,\widehat{d}_t$:[19]

$$\widehat{\pi}_t^N=\beta\widehat{\pi}_{t+1}^N+K\widehat{rer}_t, \tag{B.27}$$

$$\widehat{rer}_t=\widehat{rer}_{t-1}+\omega(\widehat{\pi}_t^S-\widehat{\pi}_t^N), \tag{B.28}$$

$$\phi_\pi\left[\omega\widehat{\pi}_t^N+(1-\omega)\widehat{\pi}_t^S\right]=\widehat{\pi}_{t+1}^S, \tag{B.29}$$

$$(1-\phi_T)\left(\widehat{d}_{t-1}-\omega\widehat{\pi}_t^N-(1-\omega-\Omega)\widehat{\pi}_t^S\right)=\beta\left[\widehat{d}_t-(1-\Omega)\widehat{\pi}_{t+1}^S\right], \tag{B.30}$$

with $K\equiv\frac{\tilde{\kappa}}{\omega}\frac{\eta}{\omega}$. Here, $\widehat{rer}_t$ and $\widehat{d}_t$ are predetermined/state endogenous variables, while $\widehat{\pi}_t^N$ and $\widehat{\pi}_t^S$ are non-predetermined/jumping. Local equilibrium uniqueness thus requires 2 stable and 2 non-stable eigenvalues. If there are more than 2 stable eigenvalues, there are multiple local equilibria. Otherwise, there is no locally stationary equilibrium.

[19] We have eliminated uncertainty and exogenous variables, as these are not relevant for analysis of local existence and uniqueness

These equations can be re-arranged as

$$\widehat{\pi}^S_{t+1} = \phi_\pi \omega \widehat{\pi}^N_t + \phi_\pi (1-\omega) \widehat{\pi}^S_t \tag{B.31}$$

$$\widehat{\pi}^N_{t+1} = \frac{1}{\beta}(1+\omega K)\widehat{\pi}^N_t - \frac{1}{\beta} K \widehat{rer}_{t-1} - \frac{1}{\beta}\omega K \widehat{\pi}^S_t, \tag{B.32}$$

$$\widehat{rer}_t = \widehat{rer}_{t-1} + \omega \widehat{\pi}^S_t - \omega \widehat{\pi}^N_t, \tag{B.33}$$

$$\widehat{d}_t = \frac{(1-\phi_T)}{\beta}\widehat{d}_{t-1} + \left[(1-\Omega)(1-\omega)\phi_\pi - \frac{(1-\phi_T)}{\beta}(1-\omega-\Omega)\right]\widehat{\pi}^S_t + ... \\ + \left[(1-\Omega)\phi_\pi - \frac{(1-\phi_T)}{\beta}\right]\omega \widehat{\pi}^N_t \tag{B.34}$$

Defining $w_t \equiv [\widehat{\pi}^S_t, \widehat{\pi}^N_t, \widehat{rer}_{t-1}, \widehat{d}_{t-1}]'$, the system can be written as

$$w_{t+1} = F w_t$$

where

$$F = \begin{pmatrix} a_{\pi^S\pi^S} & a_{\pi^S\pi^N} & 0 & 0 \\ a_{\pi^N\pi^S} & a_{\pi^N\pi^N} & a_{\pi^N rer} & 0 \\ a_{rer\pi^S} & a_{rer\pi^N} & a_{rerrer} & 0 \\ a_{d\pi^S} & a_{d\pi^N} & 0 & a_{dd} \end{pmatrix}$$

Given that the matrix F is a block lower triangular matrix of the form

$$F = \begin{pmatrix} A & 0 \\ C & D \end{pmatrix}$$

we get

$$det(F - \lambda I) = det(A - \lambda I) \times det(D - \lambda I)$$

This means that we can analyze the eigenvalues of A and D separately. First, notice that the eigenvalue of D is equal to $\frac{1-\phi_T}{\beta}$, which is less than one if $1-\beta < \phi_T$.

In terms of A, notice that

$$det(A-\lambda I) = \begin{vmatrix} a_{\pi^S\pi^S} - \lambda & a_{\pi^S\pi^N} & 0 \\ a_{\pi^N\pi^S} & a_{\pi^N\pi^N} - \lambda & a_{\pi^N rer} \\ a_{rer\pi^S} & a_{rer\pi^N} & a_{rerrer} - \lambda \end{vmatrix} =$$

$$(a_{\pi^S\pi^S} - \lambda)\left[(a_{\pi^N\pi^N} - \lambda)(a_{rerrer} - \lambda) - a_{rer\pi^N} a_{\pi^N rer}\right] - (a_{\pi^S\pi^N})\left[a_{\pi^N\pi^S}(a_{rerrer} - \lambda) - a_{rer\pi^S} a_{\pi^N rer}\right] =$$

$$\lambda^3 - (a_{\pi\pi} + a_{rerrer})\lambda^2 + (a_{\pi\pi}a_{rerrer} - a_{\pi rer}a_{rer\pi} - a_{\pi^S\pi}a_{\pi\pi^S})\lambda + (a_{\pi^S\pi}a_{\pi\pi^S}a_{rerrer} - a_{\pi^S\pi}a_{\pi rer}a_{rer\pi^S})$$

Thus, we need to compute the solutions to,

$$\lambda^3 - \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)\lambda^2 + \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \phi_\pi \frac{1}{\beta}(1+\omega K) - \omega\phi_\pi\frac{1}{\beta}\right)\lambda - \phi_\pi \frac{(1-\omega)}{\beta} = 0$$

In other words, the characteristic equation of matrix A takes the following form:

$$\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 = 0.$$

We are interested in determining *sufficient conditions* under which this polynomial has:

- Exactly **two roots outside** the unit circle (i.e., $|\lambda| > 1$) and **one inside** ($|\lambda| < 1$),
- Or, exactly **two roots inside** the unit circle and **one outside**.

Sufficient Conditions for 2 Roots Outside and 1 Root Inside

Any one of the following sets of conditions is sufficient to ensure this root configuration (see, for instance, Woodford, 2003, Appendix C):

Condition Set 1:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 > 0. \end{cases}$$

Condition Set 2:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0A_2 + A_1 - 1 > 0. \end{cases}$$

Condition Set 3:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0A_2 + A_1 - 1 < 0, \\ |A_2| > 3. \end{cases}$$

Sufficient Conditions for 2 Roots Inside and 1 Root Outside

Any one of the following sets of conditions is sufficient to ensure this alternative configuration:

Condition Set A:

$$\begin{cases} 1 + A_2 + A_1 + A_0 > 0, \\ -1 + A_2 - A_1 + A_0 > 0. \end{cases}$$

Condition Set B:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 < 0, \\ A_2^2 - A_0A_2 + A_1 - 1 > 0. \end{cases}$$

Condition Set C:

$$\begin{cases} 1 + A_2 + A_1 + A_0 < 0, \\ -1 + A_2 - A_1 + A_0 > 0, \\ A_2^2 - A_0A_2 + A_1 - 1 < 0, \\ |A_2| > 3. \end{cases}$$

Case I: $\phi_\pi > 1$

We need

$$1 + A_2 + A_1 + A_0 = (\phi_\pi - 1)\frac{\omega K}{\beta} > 0$$

$$-1 + A_2 - A_1 + A_0 = -2\left(1 + \frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\phi_\pi(1-\omega)}{\beta}\right) - (1+\phi_\pi)\frac{\omega K}{\beta}$$

Given that $\phi_\pi > 0$:

$$-1 + A_2 - A_1 + A_0 < 0$$

$$|A_2| = |-\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)|$$

Since $\beta \in (0, 1)$:

$$1 + \frac{1}{\beta} > 2$$

Then, to prove that $|A_2| > 3$, it is sufficient to show that:

$$(1-\omega)\phi_\pi + \frac{\omega K}{\beta} \geq 1$$

Which implies that:

$$\phi_\pi \geq \frac{1 - \omega\frac{K}{\beta}}{1-\omega}$$

$$A_2^2 - A_0A_2 + A_1 - 1 = \left[-\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)\right]^2 - \phi_\pi\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)$$
$$+ \left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \phi_\pi\frac{1}{\beta}(1+\omega K) - \omega\phi_\pi\frac{1}{\beta}\right)$$

Therefore, $A_2^2 - A_0A_2 + A_1 - 1$ takes the form of a quadratic equation in ϕ_π:

$$A_2^2 - A_0A_2 + A_1 - 1 = a\phi_\pi^2 + b\phi_\pi + c$$

where:

$$a = (1-\omega)^2(1 - \frac{1}{\beta}) < 0$$

In order to prove that $A_2^2 - A_0A_2 + A_1 - 1 > 0$ it is sufficient to show that:

$$\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right)^2 - \phi_\pi\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega)\phi_\pi + \frac{\omega K}{\beta} + 1\right) + (\phi_\pi - 1)\frac{\omega K}{\beta} + \frac{\phi_\pi}{\beta}(1-\omega) > 0$$

For the case where $\phi_\pi > 1$ and $\frac{1-\frac{\omega K}{\beta}}{1-\omega} \geq 1$, we know that for $\phi_\pi \in \left(1, \frac{1-\frac{\omega K}{\beta}}{1-\omega}\right)$, it is not guaranteed that $|A_2| > 3$. Therefore, we must show that within this interval, the condition $A_2^2 - A_0A_2 + A_1 - 1 > 0$ holds:

If $\phi_\pi = 1$:

$$\left(\frac{1}{\beta} + (1-\omega) + \frac{\omega K}{\beta} + 1\right)^2 - \frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega) + \frac{\omega K}{\beta} + 1\right) + \frac{1}{\beta}(1-\omega) =$$

$$= (1-\omega) + \frac{\omega K}{\beta} + 1 + \frac{\omega}{\beta} + \frac{1}{\beta}\frac{(1-\omega)}{\frac{1}{\beta} + (1-\omega) + \frac{\omega K}{\beta} + 1} > 0$$

If $\phi_\pi = \frac{1-\frac{\omega K}{\beta}}{1-\omega}$:

$$\left(\frac{1}{\beta} + (1-\omega)\frac{1-\frac{\omega K}{\beta}}{1-\omega} + \frac{\omega K}{\beta} + 1\right)^2 - \frac{1-\frac{\omega K}{\beta}}{1-\omega}\frac{(1-\omega)}{\beta}\left(\frac{1}{\beta} + (1-\omega)\frac{1-\frac{\omega K}{\beta}}{1-\omega} + \frac{\omega K}{\beta} + 1\right)$$

$$+ \left(\frac{1-\frac{\omega K}{\beta}}{1-\omega} - 1\right)\frac{\omega K}{\beta} + \frac{1-\frac{\omega K}{\beta}}{1-\omega}\frac{(1-\omega)}{\beta} =$$

$$= 2 + \frac{\omega K}{\beta^2} + \left[\left(\frac{1-\frac{\omega K}{\beta}}{1-\omega} - 1\right)\frac{\omega K}{\beta} + \frac{1-\frac{\omega K}{\beta}}{1-\omega}\frac{(1-\omega)}{\beta}\right]\frac{1}{\frac{1}{\beta} + (1-\omega)\frac{1-\frac{\omega K}{\beta}}{1-\omega} + \frac{\omega K}{\beta} + 1} > 0$$

On the other hand, if $\frac{1-\frac{\omega K}{\beta}}{1-\omega} < 1$ then:

$$|A_2| > 3 \quad \forall\, \phi_\pi \in (1, +\infty)$$

We can now establish the stability conditions for all values of $\phi_\pi > 1$. The expression $A_2^2 - A_0A_2 + A_1 - 1$ has been shown to be a strictly concave function of ϕ_π (i.e., a downward-opening parabola).

On the interval where $\phi_\pi \in \left(1, \frac{1-\frac{\omega K}{\beta}}{1-\omega}\right)$, which is the region where the condition $|A_2| > 3$ is not guaranteed, the function evaluates to a strictly positive value at the boundary points of this interval (i.e., at $\phi_\pi = 1$ and $\phi_\pi = \frac{1-\frac{\omega K}{\beta}}{1-\omega}$).

A fundamental property of strictly concave functions dictates that:

Lemma. Let f be a strictly concave function on the closed interval $[a, b]$. If $f(a) > 0$ and $f(b) > 0$, then $f(x) > 0$ for all x in the open interval (a, b).

Then, it follows that $A_2^2 - A_0A_2 + A_1 - 1 > 0$ holds for all ϕ_π in the specified range.

For the case where $\phi_\pi > 1$, we show that the sufficient conditions are satisfied for the case in which there are two roots outside the unit circle and one inside[20]. Given that $\widehat{d}_t$ and $\widehat{rer}_t$ are predetermined variables, and $\widehat{\pi}_t^S$ and $\widehat{\pi}_t$ are jumping variables, we require two eigenvalues with modulus less than one and two with modulus greater than one in order to ensure the *local uniqueness of equilibrium*. This implies that the remaining eigenvalue must lie inside the unit circle:

$$det(D - \lambda I) = |a_{dd} - \lambda|$$

[20]This holds whether $A_2^2 - A_0A_2 + A_1 - 1 > 0$ or $A_2^2 - A_0A_2 + A_1 - 1 < 0$; we avoid discussion of certain non-generic boundary cases.

$$\therefore \lambda_4 = a_{dd} = \frac{(1-\phi_T)}{\beta}$$

$$\lambda_4 < |1| \iff \phi_T > 1-\beta$$

Case II: $\phi_\pi < 1$

$$1 + A_2 + A_1 + A_0 = (\phi_\pi - 1)\,\frac{\widetilde{\kappa}}{\beta}\frac{\eta}{\omega} < 0$$

As previously stated, for $\phi_\pi > 0$, we know that:

$$-1 + A_2 - A_1 + A_0 < 0$$

Now, for the case where $\phi_\pi \in (0,1)$ we need to prove that:
$A_2^2 - A_0 A_2 + A_1 - 1 > 0$
If $\phi_\pi = 0$:

$$\left(\frac{1}{\beta} + \frac{\omega K}{\beta} + 1\right)^2 - \frac{\omega K}{\beta} = \frac{1}{\beta} + \left(1 - \frac{1}{\frac{1}{\beta} + \frac{\omega K}{\beta} + 1}\right)\frac{\omega K}{\beta} + 1 > 0$$

We previously prove that the quadratic function is positive when ϕ_π is valued at 1. Therefore, given that the quadratic function is a strictly concave function, we can state that:

$$A_2^2 - A_0 A_2 + A_1 - 1 > 0 \quad \forall\, \phi_\pi \in (0,1)$$

Thus, we have shown that when $\phi_\pi < 1$, the sufficient conditions are met to establish the existence of two roots inside the unit circle and one outside. Accordingly, the remaining eigenvalue must lie outside the unit circle:

$$\lambda_4 < |1| \iff \phi_T < 1-\beta$$

Case B: $\theta_N = 0 \ \wedge \ \Omega \in [0,1)$

From the system (B.27)-(B.30) and the fact that $\theta_N = 0$ and, thus, $K \to \infty$:

$$\widehat{\pi}^N_{t+1} = \phi_\pi \widehat{\pi}^N_t, \tag{B.35}$$

$$\widehat{d}_t = (1-\Omega)\left[\phi_\pi - \frac{(1-\phi_T)}{\beta}\right]\pi^N_t + \frac{(1-\phi_T)}{\beta}\widehat{d}_{t-1}, \tag{B.36}$$

Defining $z_t \equiv [\widehat{\pi}^N_t, \widehat{d}_{t-1}]'$, the system can be written as

$$z_{t+1} = J z_t$$

where J:

$$J = \begin{pmatrix} a_{\pi^N\pi^N} & 0 \\ a_{d\pi^N} & a_{dd} \end{pmatrix}$$

Given that the matrix J is a block lower triangular matrix:

$$J = \begin{pmatrix} E & 0 \\ G & H \end{pmatrix}$$

where:

$$det(J - \lambda I) = det(E - \lambda I) \times det(H - \lambda I)$$

Once again, we can split the analysis of eigenvalues of E and H. In this sense, it can be seen that the eigenvalue of E is equal to ϕ_π and the eigenvalue of H to $\frac{(1-\phi_T)}{\beta}$. Consequently, given that π_t^N is a jumping variable and d_t is a predetermined variable, one eigenvalue must lie outside and one inside the unit circle, which leads to the existence of two regions of *local uniqueness of equilibrium*:

– **Passive Fiscal - Active Monetary:** $\phi_T \in [0, 1-\beta) \ \wedge \ \phi_\pi > 1$

– **Active Fiscal - Passive Monetary:** $\phi_T > 1-\beta \ \wedge \ \phi_\pi \in (0, 1)$

Case C: $\theta_N = 0 \ \wedge \ \Omega = 1$

If the debt is entirely denominated in foreign currency and prices are flexible, the system (B.35)-(B.36) takes the form of:

$$\widehat{\pi}_{t+1}^N = \phi_\pi \widehat{\pi}_t^N, \tag{B.37}$$

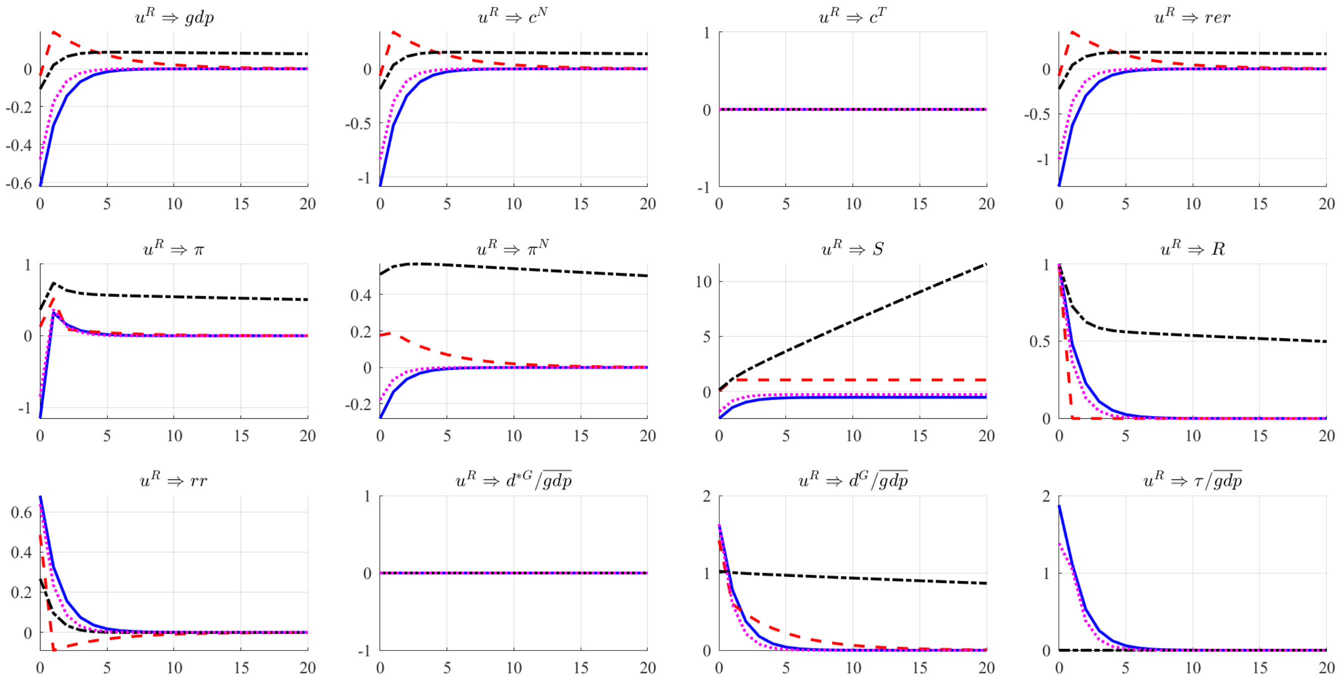
$$\widehat{d}_t = \frac{(1-\phi_T)}{\beta} \widehat{d}_{t-1}, \tag{B.38}$$

As it may be seen, the system becomes dichotomous. Consequently, the **Active Fiscal - Passive Monetary** regime does not ensure *local uniqueness of equilibrium*, with the **Passive Fiscal - Active Monetary** regime being the only one that guarantees *local uniqueness of equilibrium*.

D Robustness Exercises

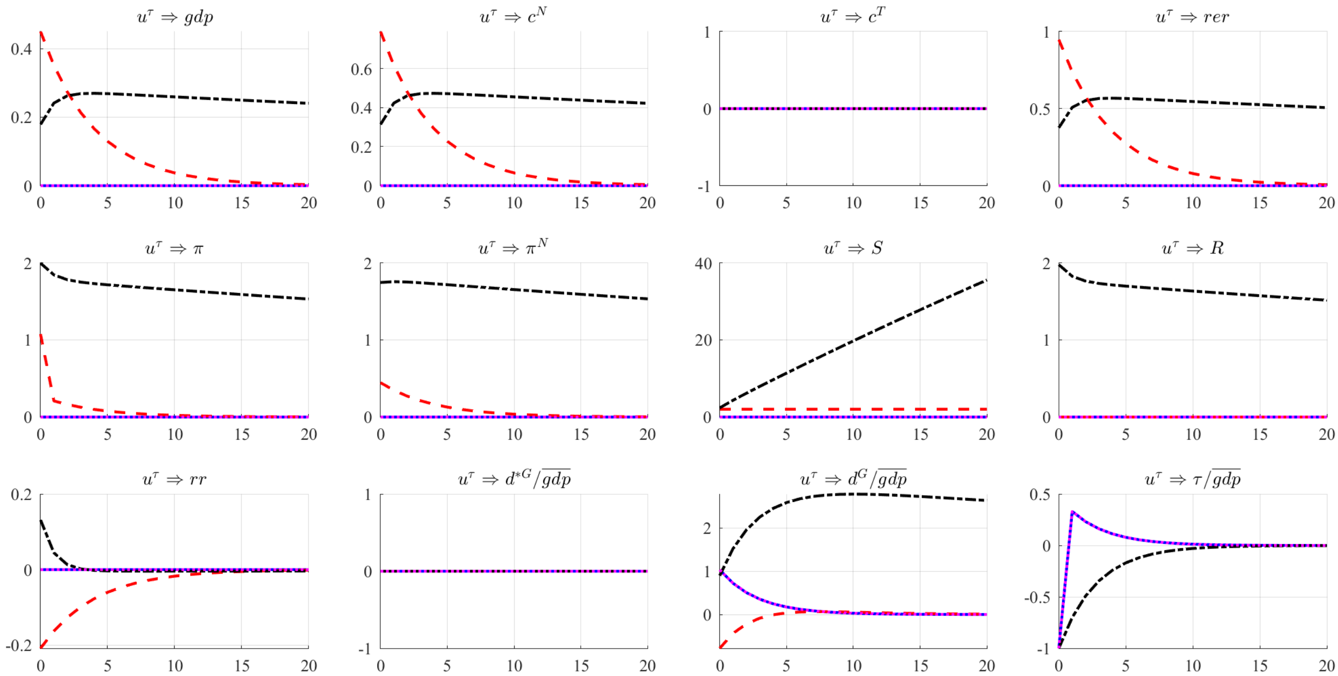
D.1 Monetary and fiscal shocks, the role of ϕ_π

Figure D.1: Responses to a monetary-policy-rate shock, Ricardian vs. Non-Ricardian, the role of ϕ_π.



Notes: The shock is an increase in u_t^R, with zero persistence, normalized to increase R by 1% on impact. Solid-blue and dashed red lines are the same as in Figure 2 for comparison (i.e. respectively, $\phi_\pi = 1.5, \phi_T = 1$, and $\phi_\pi = 0, \phi_T = 0$). In addition, the dashed-dotted black lines correspond to the case of $\phi_\pi = 0.99, \phi_T = 0$, while dotted-magenta lines use $\phi_\pi = 1.01, \phi_T = 1$. In all cases, $\overline{\Omega} = 0$. See Figure 2 for variables' definitions.

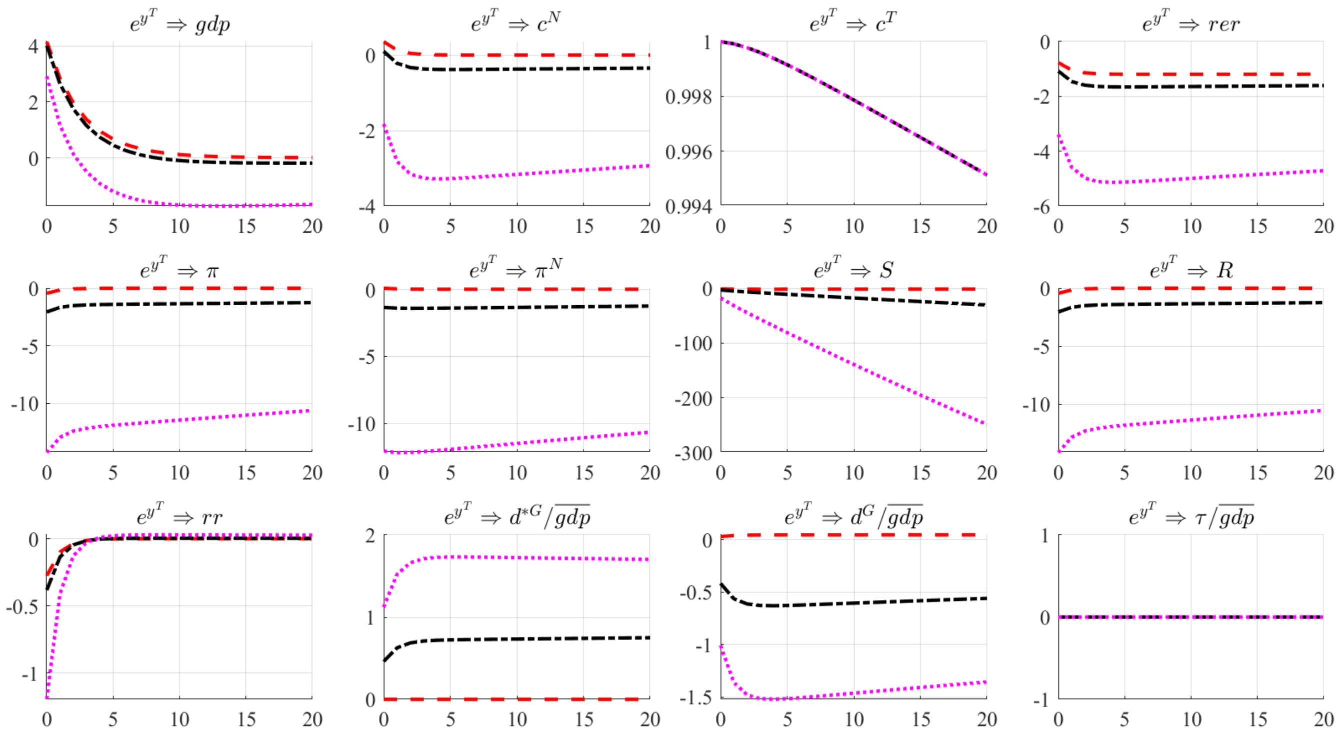
Figure D.2: Responses to a monetary-policy-rate shock, Ricardian vs. Non-Ricardian, the role of ϕ_π.



Notes: The shock is a drop in lump-sum taxes, normalized to represent 1% of steady-state GDP, with an autocorrelation of 0.7. All cases feature $\overline{\Omega} = 0$, differing in the fiscal and monetary configuration: solid-blue lines use $\phi_\pi = 1.5$ and $\phi_T = 1$, dashed-red lines $\phi_\pi = 0, \phi_T = 0$, dashed-dotted black lines correspond to the case of $\phi_\pi = 0.99$ and $\phi_T = 0$, and dotted-magenta lines use $\phi_\pi = 1.01$ and $\phi_T = 1$. See Figure 2 for for variables' definitions.

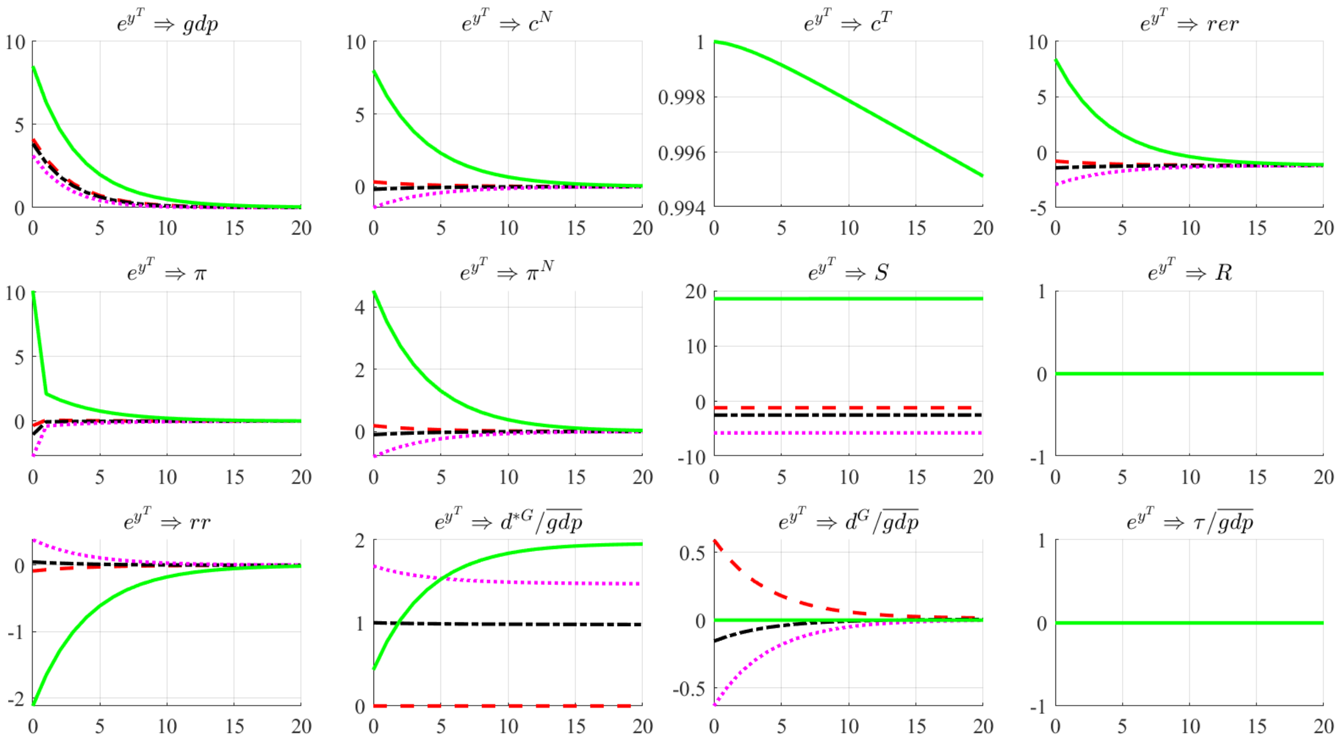
D.2 The effect of real shocks, alternative policy configurations

Figure D.3: Responses to a traded-output shock, Non-Ricardian, different values of $\overline{\Omega}$, with $\phi_\pi = 0.99$.



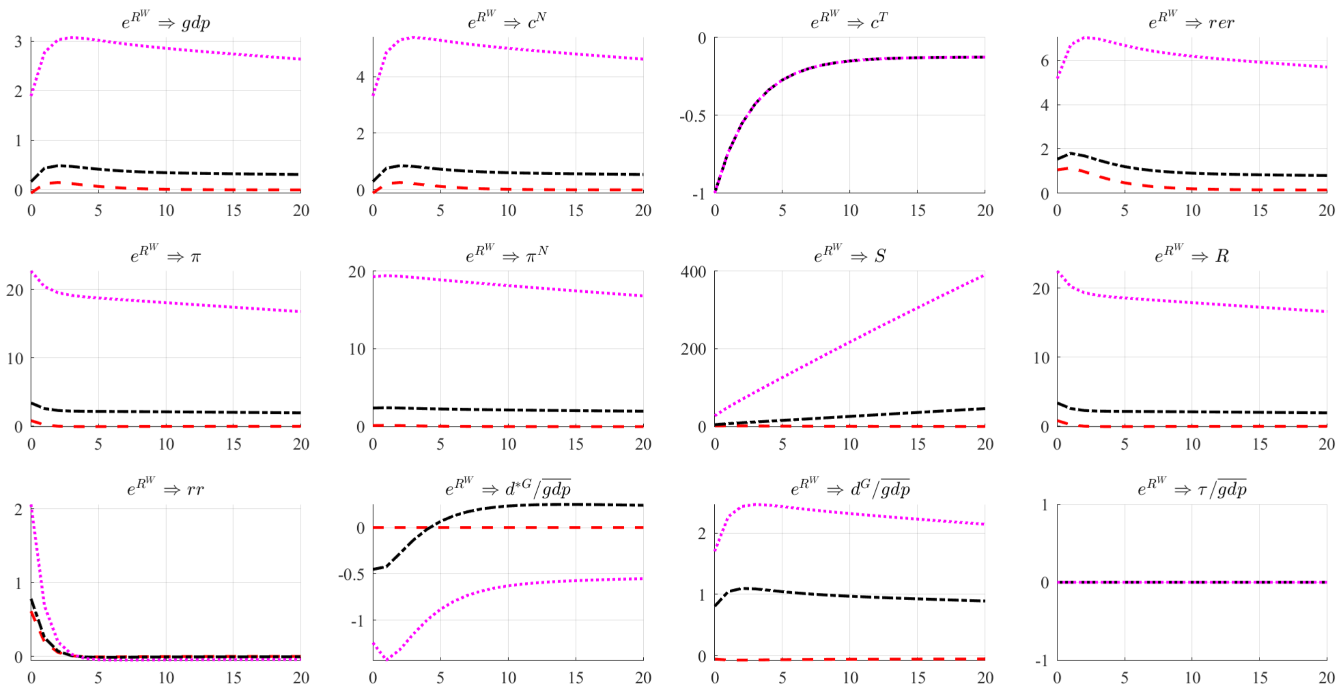
Notes: The figure is analogous to Figure 6, but using a value for $\phi_\pi = 0.99$.

Figure D.4: Responses to a traded-output shock, Non-Ricardian, different values of $\overline{\Omega}$.



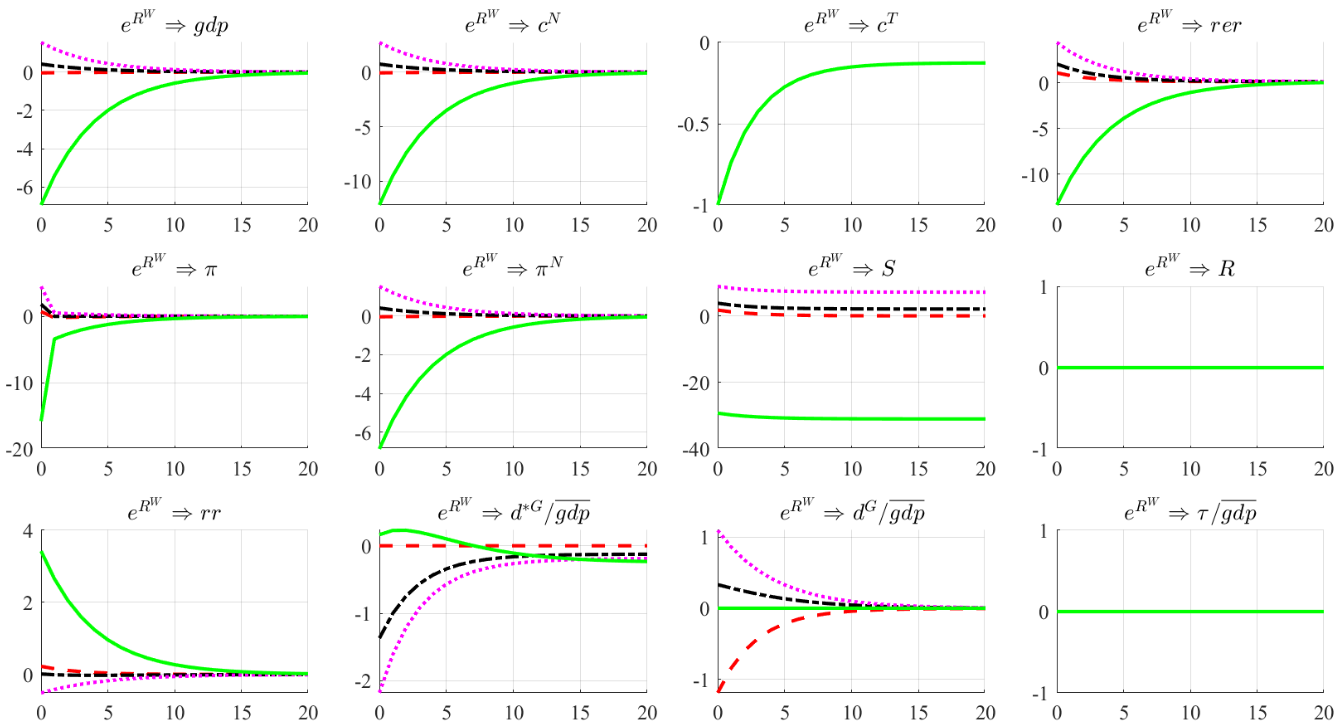
Notes: The figure is analogous to Figure 6, adding also the case of $\overline{\Omega} = 1$ in solid-green lines.

Figure D.5: Responses to a world interest-rate shock, Non-Ricardian, different values of $\overline{\Omega}$, with $\phi_\pi = 0.99$.



Notes: The figure is analogous to Figure 8, but using a value for $\phi_\pi = 0.99$.

Figure D.6: Responses to a world interest-rate shock, Non-Ricardian, different values of $\overline{\Omega}$.



Notes: The figure is analogous to Figure 8, adding also the case of $\overline{\Omega} = 1$ in solid-green lines.