Central bank digital currency, disintermediation and bank runs*

Preliminary Draft

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Abstract

This paper examines whether the introduction of a central bank digital currency (CBDC) makes the banking system more or less exposed to bank runs and disintermediation. To address this question, we build on the seminal paper of Diamond and Dybvig (1983). We show that CBDCs help prevent bank runs on real demand deposits, provided that the proportion of CBDC users is sufficiently high. In the event of excessive withdrawals, the central bank can lend to illiquid banks through an emergency liquidity assistance (ELA) policy. If enough withdrawals are made via CBDC, then sufficient funds are immediately available to the central bank to implement an ELA. If the central bank is expected to implement an ELA, bank runs are prevented. Full insurance equilibrium can be achieved as a unique Nash equilibrium, as long as the share of early consumers is non-stochastic. If the CBDC is strictly preferred to cash, bank runs are prevented without any assumption on CBDC use. Finally, we show that even when the share of early consumers is unknown, an ELA can still prevent bank runs, and also improves welfare compared to suspension of convertibility of deposits. Overall, the findings suggest that the introduction of CBDCs holds potential for mitigating bank runs and avoiding disintermediation.

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1 Introduction

Central bank digital currency (CBDC) is a form of digital money issued by the central bank, without any physical representation, intended to serve as legal tender. Unlike traditional currency, CBDC exists purely in digital form, typically in the form of tokens or accounts maintained by the central bank. Many central banks are actively engaged in developing or testing their own digital currencies. For instance, Uruguay launched an e-peso pilot in 2018, and The Bahamas started the Sand dollar pilot in 2019 (see Ponce (2020) and Bergara and Ponce (2019)). Furthermore, the European Central Bank has started its preparation phase for a digital euro European Central Bank (2023). Recent surveys reveal that a majority of Sub-Saharan central banks and many central banks in the Middle East and Central Asia are involved in CBDC initiatives (see Ricci et al. (2024) and Bouza et al. (2024)).

The growing interest in CBDCs stems from their potential to improve payment system efficiency, resilience, security, and promote financial inclusion, among other benefits, as suggested by Bank of England (2020), Federal Reserve (2022), and BIS (2020), among numerous other sources. CBDCs may also increase the availability and usability of central bank money, thereby reducing the risks associated with new forms of private money creation, like stablecoins, and counteracting the decline in cash usage. Moreover, the introduction of CBDC carries implications for consumer protection, financial integration, monetary policy, and more, as discussed in Griffoli and Adrian (2019). According to Mancini-Griffoli et al. (2019), the impact of introducing CBDCs remains uncertain and largely depends on their design and the specific characteristics of each country.

This paper addresses two pressing concerns regarding financial stability associated with the introduction of CBDCs, as highlighted by both the literature and policymakers: bank runs and disintermediation. Concerning bank runs, it is suggested that CBDC may exacerbate the propagation of bank runs and panics by offering a safe and liquid alternative to deposits. Unlike cash withdrawals, which face practical frictions, shifting to CBDC could be faster and easier, amplifying systemic risks. This phenomenon is referred to in the literature as "digital bank runs" Kumhof and Noone (2021). The recent case of the Silicon Valley Bank collapse, where clients withdrew over \$40 billion in a single day[1], highlights the concern for the rapidity of runs CBDCs could produce, underlining the impact of a highly digitalized depositor base. How-

[1] See the Review of the Federal Reserve's Supervision and Regulation of Silicon Valley Bank - April 2023 here https://www.federalreserve.gov/publications/2023-April-SVB-Evolution-of-Silicon-Valley-Bank.htm

ever, some argue, as seen in Griffoli and Adrian (2019), that this effect is often mitigated. First, CBDC does not facilitate idiosyncratic runs between banks, which can already occur electronically. Second, in a general economic crisis, funds will likely be withdrawn from all local assets, including CBDC. Finally, they suggest CBDCs could even help central banks to provide liquidity in case of a bank run, particularly in cases where cash transportation to bank branches and ATMs is costly and time-consuming.

Additionally, there is the concern of disintermediation, where the introduction of a CBDC could lead to a substantial shift of deposits away from traditional banks. This gives rise to two distinct worries: one related to "slow disintermediation" and the other to "fast disintermediation". Slow disintermediation refers to the possibility that an interest bearing CBDC could crowd out bank deposits, thereby shrinking banks' balance sheets, and disrupting credit availability Bidder et al. (2024). Nevertheless, it is also argued that banks are likely to respond to CBDC by raising deposit interest rates to retain depositors, as suggested in Griffoli and Adrian (2019). Fast disintermediation, on the other hand, refers to the phenomenon where, during a crisis, CBDC could serve as a safe option, prompting depositors to shift their funds from banks to CBDC due to safety concerns. In this work we only address the fast disintermediation issue, since we do not consider an interest bearing CBDC.

The question we try to answer directly arises from these two matters. We ask whether the introduction of a CBDC makes the system more or less exposed to systemic bank runs and disintermediation. To answer this question, we work within the Diamond-Dybvig bank run framework Diamond and Dybvig (1983). In their model, banks improve market allocation by reaching full insurance equilibrium through the transformation of illiquid assets into liquid liabilities. However, this mismatch in banks' balance sheets leaves them vulnerable to bank runs. Diamond and Dybvig show that certain policies, such as deposit insurance and stopping deposit conversion, prevent bank runs. We investigate whether CBDC could serve as a tool to achieve the same objective.

We build on the Diamond-Dybvig model and introduce CBDC as an alternative to the storage technology. Depositors are given the choice to have deposits, to self-keep, or to use CBDC. I find that bank runs on real demand deposits can be avoided through widespread CBDC adoption. The rationale is as follows: when banks experience excessive withdrawals, the central bank can provide liquidity to illiquid banks through an emergency liquidity assistance (ELA) policy, using bank assets as collateral. CBDC plays a crucial role; they provide the central bank with immediate availability of funds to lend to banks without introducing a tax. Banks, instead of transferring deposits, shift liabilities to depositors in the form of liabilities with the central bank, thereby keeping deposits within the banking system and preventing the liquidation of profitable investments. Consequently, the depletion of bank assets is avoided, eliminating incentives for depositors to engage in bank runs. Full insurance equilibrium can be achieved as a unique Nash equilibrium in dominant strategies, provided the share of CBDC users is large enough and as long as the number of early consumers is non-stochastic. A notable result is that, in equilibrium, banks experience no disintermediation.

Furthermore, we show that if CBDC is strictly preferred to cash, then all depositors would choose to withdraw into CBDC, eliminating the need to assume any share of CBDC users. Therefore, even if CBDC completely replaces cash outside the equilibrium path and increases the probability of a bank run, with an ELA, in equilibrium, there is no run, thus preventing disintermediation.

Lastly, we show that when the share of early consumers is unknown, an ELA can still prevent bank runs, and also improves welfare compared to suspending deposit convertibility. Overall, this work highlights how CBDCs can serve as a potential tool for enhancing financial stability, helping to prevent self-fulfilling systemic bank runs.

The remainder of the paper is organized as follows: the next section presents relevant literature. Section 3 introduces the model and defines the optimal allocation. In Section 4, it is shown that a sufficiently high use of CBDC helps implement an ELA, preventing bank runs and achieving optimal risk sharing. Additionally, it demonstrates that CBDC does not lead to disintermediation. Section 5 addresses stochastic early consumers. Section 6 concludes.

2 Related literature

This paper is closely grounded in the work of Diamond and Dybvig (1983). They show that deposit insurance prevents bank runs on real demand deposits. To later mention in the conclusion that:

> The Federal Reserve discount window can, as a lender of last resort, provide a service similar to deposit insurance. It would buy bank assets with (money creation) tax revenues at $T = 1$ for prices greater than their liquidating value. If the taxes and transfers were set to be identical to that of the optimal deposit insurance, it would have the same effect. (p.417)

Expanding on their observation, we contribute to the literature by demonstrating how a CBDC can provide funds to the central bank to implement an ELA, allowing the central bank to act as a lender of last resort. This effectively prevents bank runs without the need for money creation or taxes.

By integrating CBDCs into a framework with real demand deposits, we emphasize the potential of CBDCs to help mitigate the risk of bank runs, irrespective of nominal effects. Literature on bank runs with nominal deposits shows that bank runs caused solely by liquidity shocks do not occur Skeie (2004, 2006, 2008). Using a nominal version of Diamond-Dybvig's model, Skeie shows that for a liquidity-driven bank run to occur, there must be either some friction in the goods market or a coordination failure in the inter-bank market. Contrary to Diamond-Dybvig's findings regarding real demand deposits, with nominal deposits, policies like deposit insurance and suspension of convertibility are unnecessary to prevent bank runs.

There is also related work on nominal version of Diamond-Dybvig's model that introduces CBDC like this paper. Schilling et al. (2020) show that if a central bank with a price stability objective issues CBDC and acts as the intermediary, the bank run dilemma reappears in the nominal model, but it becomes a trilemma. They demonstrate that at most two of three objectives—a socially efficient allocation, absence of runs, and price stability—can be fulfilled by the central bank. Since my work operates within a real framework, we do not explicitly consider the objective of price stability by the central bank; however, we demonstrate that with an ELA, no taxation (money creation) is needed to prevent bank runs.

This paper is also related to recent expanding literature on CBDC[2], that analyzes potential effects its introduction could have on the propagation of bank runs and disintermediation.

2.1 Literature on CBDC and bank runs

Many papers suggest that bank deposits could be more prone to runs when a CBDC is introduced. Many refer to this vulnerability of the banking system as "digital bank runs". For instance, Kumhof and Noone (2021) state that the concern of a digital run is valid because CBDC enables runs from deposits into CBDC that could conceivably be near instantaneous and of an unprecedented scale. However, they suggest that if four design principles are respected, then this risk of a digital run is misguided. One of the principles they suggest is that there must be no obligation on banks to always convert deposits into CBDC. They mention that a credible obligation on banks to supply CBDC on demand for deposits requires the central bank, in turn, to pre-commit to supplying CBDC on demand to banks in financial stress times. This is one of

[2]See Infante et al. (2022) for a detail overview of the literature on CBDC.

the points we emphasize: if banks are obligated to supply CBDC, the central bank can credibly commit to providing an ELA since it will receive funds to do so. Overall, Kumhof and Noone (2021) find that if the four principles are respected, the digital run risk is mitigated. This aligns with Griffoli and Adrian (2019) views in that the design of the CBDC is crucial to predict its effect. In the same line, they mention that a design that involves caps on CBDC holdings could reduce the potential threat of fast digital runs.

The study by Williamson (2022) examines how replacing physical currency with CBDC would affect the incidence of banking panics. They find that the introduction of CBDC could increase the probability of a bank run by making it easier for depositors to flee at the first sign of trouble. However, they also find that such panics are less disruptive than in a world with physical currency.

Findings in Brunnermeier and Niepelt (2019) align with my own. They demonstrate that the introduction of CBDC need not destabilize the banking sector. Their findings suggest that, as long as the central bank is willing to acquire unsecured claims on the banking sector during bank runs, transferring funds from bank to central bank accounts would amount to an automatic substitution of one type of bank funding (deposits held by households and firms) by another one (central bank funding for banks). Depositors' run for CBDC, therefore, would not undermine financial stability.

As mentioned earlier, in their study, Bidder et al. (2024) propose two scenarios that could lead to different impacts of CBDC on bank runs. One scenario, described as "fast disintermediation", CBDC becomes a highly convenient asset during banking stress, potentially amplifying the likelihood of bank runs by offering safety at scale. An alternative scenario is defined as "slow disintermediation", wherein CBDC competes with bank deposits during normal times, diminishing the liquidity premium attainable by banks. This slower disintermediation suggests that banks may become smaller, thereby reducing the risk of bank runs. In this paper, we focus solely on addressing the issue of fast disintermediation and demonstrate how it can be prevented with an ELA. We do not address slow disintermediation, which would require the introduction of a deposit-like CBDC that bears interest.

The concern regarding slow disintermediation is also comprehensively explored in Fernández-Villaverde et al. (2021). Like this paper, they introduce CBDC in the Diamond-Dybvig model, but they focus on the risk of slow disintermediation rather than fast disintermediation. Their findings suggest that the contractual rigidity of CBDC with investment banks may deter runs, thereby potentially enhancing stability compared to commercial banks. Anticipating this feature, consumers lean towards CBDC, drawing all deposits away from the commercial banking sector. Subsequent literature further explores this phenomenon of disintermediation.

2.2 Literature on CBDC and disintermediation

Many recent studies suggest that the introduction of CBDC could crowd out deposits and thus disintermediate banks. The mechanism is that CBDC could act as a substitute for bank deposits. As mentioned earlier, Fernández-Villaverde et al. (2021) suggests that reliance of the central bank on investment banks renders central bank CBDC deposits safer and, thus, more attractive than deposit contracts at commercial banks, making the central become the monopolist of financial interemediation. Keister and Sanches (2022) also find that CBDC produces disintermediation of banks, they show that while a digital currency tends to improve efficiency, it also crowds out bank deposits, raises banks' funding costs, and decreases investment.

Other studies suggest that CBDC does not necessarily lead to disintermediation. For instance, Assenmacher et al. (2021) show that a positive interest spread on CBDC or stricter collateral or quantity constraints for CBDC, reduces welfare but can contain bank disintermediation. As mentioned previously, Kumhof and Noone (2021) find that if CBDC design follows the set of conservative core principles, bank funding is not necessarily reduced, and credit and liquidity provision to the private sector need not contract. Similarly, Burlon et al. (2022) show that welfare-maximizing CBDC policy rules are effective in mitigating the risk of bank disintermediation. Also, as mentioned before, Brunnermeier and Niepelt (2019) results imply that CBDC need not generate a credit crunch. A swap of CBDC for deposits would not reduce banks' funding; it would only change its composition.

Overall, while some studies suggest the introduction of CBDC is likely to lead to disintermediation, others highlight potential mitigating factors such as policy adjustments or changes in the composition of bank funding.

3 The Model

The model is one of the most commonly used frameworks for analyzing bank runs since Diamond and Dybvig (1983). In this study, we build on the seminal Diamond-Dybvig model by integrating two additional elements: Central Bank Digital Currency (CBDC) and Emergency Liquidity Assistance (ELA) policy. The setup is as follows:

There are three periods, $T = 0, 1, 2$. The economy is populated by consumers, who are ex-ante identical, and constitute a mass of 1. Consumers are initially endowed with a homogeneous divisible good, collectively amounting to a total mass of 1.

At $T = 1$, a proportion $t \in (0, 1)$ of consumers face an unverifiable liquidity shock and a need to consume within that period. These consumers, known as "early" consumers or type 1 consumers, derive utility solely from their consumption at $T = 1$. The remaining fraction $(1 - t)$ are "late" consumers or type 2 consumers, whose utility depends solely on their consumption at $T = 2$. Letting c_T represent goods "received" (to store or consume) by an agent at period T, then early consumers obtain utility from c_1 while late consumers from $c_1 + c_2$.

The period utility functions $u(.)$ are assumed to be twice continuously differentiable, increasing, strictly concave, and satisfying the Inada conditions $u'(0) = \infty$ and $u'(\infty) = 0$. Consumers' coefficient of relative risk aversion is assumed to be greater than one, implying that banks provide risk-decreasing insurance against liquidity shocks.

Consumers have access to a storage technology. Goods are imperishable, allowing consumers to store them at any time without any loss in value. Additionally, there is an investment technology, initiated at $T = 0$, which provides a return $R > 1$ at $T = 2$, but only a return of 1 if liquidated at $T = 1$. Moreover, there is a demand deposit technology, available only through banks. Consumers can deposit at $T = 0$. Bank deposits yield r_1 at $T = 1$. Banks operate at zero profit and are owned by consumers, meaning that what bank deposits offer at $T = 2$ equals the liquidation value of the banks' assets.

Up to this point, the setup is identical to the Diamond-Dybvig model. We introduce two additional features:

CBDC: CBDCs are introduced as similar to storage technology, maintaining the value of goods. Instead of physically holding goods, consumers possess CBDCs, which can be redeemed for goods at any time. Additionally, we explore the possibility of CBDC being strictly preferred to storage.

ELA: The central bank can lend to an illiquid bank, accepting as collateral bank assets. We assume that the ELA has no cost for banks, though we conduct a robustness check for this assumption.

An ELA without CBDC would have to be financed through taxation or money creation. The presence of CBDC means that part of the withdrawals would go to the central bank to finance the ELA. Additionally, CBDC without ELA does not eliminate the bank run risk. We will demonstrate later that it actually exacerbates this risk when CBDC is strictly preferred to cash.

Together, CBDC and ELA close a circle, where bank assets do not need to be liquidated even if late consumers opt to withdraw their deposits early, thus eliminating incentives to do so. Interestingly, the equilibrium with no bank runs holds without imposing a tax, while still maintaining the promise of redeeming CBDC for consumers. The central bank's credibility plays a crucial role in this context.

Optimal risk sharing. For the sake of completeness, we reproduce the full information allocation in the Diamond-Dybvig model. The expected utility is

$$E[U(c_1, c_2)] = tu(c_1) + \rho(1-t)u(c_2),$$

s.t the budget constraint

$$tc_1 + (1-t)c_2/R = 1,$$

where ρ is the discount rate and is assumed that $\rho R > 1$. If we denote as c_k^i consumption in period k of an agent who is of type $i = 1, 2$, the optimal allocation is such that

$$c_1^{2*} = c_2^{1*} = 0,$$

marginal utilities equalize

$$u'(c_1^{1*}) = R\rho u'(c_2^{2*}),$$

and the budget constraint holds, which implies the following relationship between c_1^{1*} and c_2^{2*}

$$c_2^{2*} = \frac{(1-tc_1^{1*})R}{1-t} \tag{1}$$

As shown by Diamond and Dybvig, the fact that the relative risk aversion is greater than one and that $\rho R > 1$ implies that

$$1 < c_1^{1*} < \frac{(1-tc_1^{1*})R}{1-t} = c_2^{2*} < R \tag{2}$$

4 Bank run with CBDC

Let f be the share of depositors who withdraw at $T = 1$. The term "bank run" is used in this model to refer to situations where $f > t$, the number of withdrawers exceeds the number of early consumers, leading to the depletion of bank assets to cover withdrawals. A bank run equilibrium occurs when it is optimal for depositors to withdraw if they expect all depositors to be withdrawing.

Assume that at $T = 1$, a fraction f of depositors withdraws their deposits.

Withdrawals follow a sequential service constraint, with depositor j's place in line to withdraw, f_j, uniformly distributed in $[0, f]$. I start with the simplest case when a fixed fraction of withdrawals are in CBDC.

4.1 Fixed CBDC use

In this section, we assume that there is a constant fraction of depositors, θ, who choose to withdraw their deposits into CBDC. Since using CBDC does not provide any additional return (as assumed in this section), their preference could be interpreted as being due to exogenous factors such as user experience or access to information that encourages its use. As soon as one option becomes more preferred, demand can change. For now, a fixed share is a reasonable assumption given the indifference between the two. Later, we move to a scenario where CBDC is strictly preferred to cash.

A fraction θf of depositors withdraw in CBDC, and $(1-\theta)f$ in cash. Of the total fraction of depositors θf that withdrew via CBDC, those made by early consumers will be redeemed to be consumed at $T = 1$.

If CBDC is introduced and the central bank does not implement an ELA, the equilibrium with bank runs persists. As mentioned earlier, Diamond and Dybvig anticipate that a lender of last resort can, similar to deposit insurance, prevent a bank run. However, it requires a tax to satisfy the aggregate resource constraint. CBDC allows the central bank to satisfy the resource restriction by providing funds to implement an ELA. This ensures that banks do not stop investment, which is riskless in this model.

Banks offer deposit contracts that yields r_1 at $T = 1$. What is left after paying what is withdrawn at $T = 1$ is calculated as follows: Banks hold assets worth 1, and they have to pay r_1 to all early withdrawers, amounting to $r_1 f$. However, since $f > t$ the central bank activates the ELA, and lends the bank what it receives for deposits withdrawn in CBDC, which amounts to $r_1\theta(f-t)$. Therefore, the deposits that banks have left to continue investments are $1 - r_1[f - \theta(f - t)]$, which will yield a return of R at $T = 2$. At $T = 2$, banks first have to pay the central bank the loan provided through the ELA for $r_1\theta(f - t)$, since no penalty rate is assumed for now. Then, banks are liquidated and what is left of the banks' assets is distributed to each of the remaining $(1 - f)$ who did not withdraw early. If we denote V_1 and V_2 as the payoff depositors receive in period one and two respectively, they are:

$$V_1(f_j, r_1) = r_1 \quad \forall f_j \leq f \tag{3}$$

$$V_2(f, r_1) = \begin{cases} \frac{R(1-r_1 f)}{1-f} & \text{if } f \leq t \\ \frac{R[1-r_1(f-\theta(f-t))]-r_1\theta(f-t)}{1-f} & \text{if } f > t \end{cases} \tag{4}$$

If $V_2 > V_1$, late consumers will not have incentives to withdraw early. Then the equilibrium will be $f = t$. When $f = t$ and banks choose $r_1 = c_1^{1*}$, $V_2 = R(1 - c_1^{1*}t)/(1 - t) = c_2^{2*}$, then optimal risk sharing equilibrium is achieved.

Proposition 1. *Demand deposit contracts achieve the unconstrained optimum as a unique Nash equilibrium (in fact, a dominant strategies equilibrium) if the share of CBDC users is large enough, i.e.* $\theta > \underline{\theta}$.

Proof. To prove proposition 1, it is enough to show the condition under which V_2 expressed in equation (4) is greater than V_1 when $f > t$. If that relation holds, then all late consumers would prefer waiting no matter what other depositors do, so bank runs do not occur.

V_2 exceeds V_1 provided that the proportion of CBDC users surpasses the specified lower bound.[3]

$$\theta > \frac{(1-f) - Rr_1^{-1}(1-r_1 f)}{(f-t)(R-1)} = \underline{\theta} \tag{5}$$

For this restriction to potentially hold, $\underline{\theta}$ should be less than 1. This occurs if the following condition on the return of investment holds.[4]

$$R \geq \frac{1-t}{1-r_1 t} r_1 \tag{6}$$

This restriction can be interpreted as follows: if the return of investment is sufficiently large such that what is left inside banks after the withdrawal of early consumers, $(1 - r_1 t)$, can produce a return $(1 - r_1 t)R$ large enough for all late consumers to receive more than withdrawing their deposits early r_1, then there is no incentive to withdraw, provided $\theta \in (\underline{\theta}, 1]$. Since the relative risk aversion is assume to be greater than 1, if $r_1 = c_1^{1*}$ equation (6) holds strictly, as stated in equation (2).

Therefore, if $r_1 = c_1^{1*}$ and $\theta > \underline{\theta}$, then $V_2 > V_1$ for all f and $f_j \leq f$. Under this contract, $f = t$ emerges as the sole Nash equilibrium in dominant strategies, where $V_1 = c_1^{1*}$ and $V_2 = c_2^{2*}$, unconstrained optimum is attained. ■

Two additional noteworthy outcomes arise. Firstly, in equilibrium, all de-

[3] See appendix A.1 for proof.

[4] See appendix A.2 for proof.

posits remain within the banking system, thus negating any potential disintermediation stemming from the introduction of CBDC. Secondly, it is assumed that the central bank does not charge banks for the loan provided. However, even if such charges were applied, the same conclusion holds, although with an increased restriction on CBDC usage.[5]

4.2 Convenience of CBDC

In this subsection, we depart from the assumption that storage technology and CBDC are indifferent to consumers. Instead, we assume that CBDC may be strictly preferred to cash. This reflects the fact that CBDC may be more convenient for consumers (e.g., safer, better user experience, easier to access) than storing goods themselves. We model this by letting the less attractive withdrawal method bear a cost $\delta < 1$, which reflects the opportunity cost for withdrawers of not using a more convenient method.

Under this assumption, bank runs are still an equilibrium without an ELA. In fact, we show that the introduction of CBDC makes bank runs more likely compared to a system without CBDC, when the central bank does not implement an ELA.[6] However, if the central bank is expected to implement an ELA, since CBDC is strictly preferred than cash, all withdrawals will be made via CBDC, placing us in the specific case of section 4.1 where $\theta = 1$. For ELA to be effective in preventing bank runs, meaning $V_2 > V_1$ for all f and $f_j \leq f$, only equation (6) needs to hold. As shown before, this inequality is true when $r_1 = c_1^{1*}$. Then, the unique Nash equilibrium is $f = t$ and full insurance equilibrium is achieved.

A notable result is that even when CBDC is strictly preferred to cash and replaces it completely outside the equilibrium path, in equilibrium, with an ELA there is no run and therefore no fast disintermediation.

5 Stochastic early consumers

In this section, we move away from assuming that the number of early consumers t is common knowledge. Now, the share of early consumers is an unobserved random variable $\tilde{t}$, that ranges from 0 to $\bar{t}$. Consequently, contracts established in period one cannot be a function of the specific realization of $\tilde{t}$. The optimal consumption levels under full information are given by the realization $\tilde{t} = t$, namely $c_1^{1*}(t)$ and $c_2^{2*}(t)$.

[5] See appendix A.3 to see robustness check for ELA with cost for banks.
[6] See appendix B.1 for proof.

Just as Diamond and Dybvig (1983) demonstrate for suspension of convertibility, optimal allocation cannot be achieved in this context either. However, similar to suspension of convertibility, an ELA can generally improve demand deposit contract by preventing bank runs. Unlike suspension of convertibility, which may leave some early consumers without access to their deposits, an ELA ensures that all early consumers will be able to withdraw their funds, thereby improving welfare ex-post. Deposit insurance can achieve optimal insurance even when the share of early consumers is unknown, making it a superior policy in this context. Nevertheless, as mentioned by Diamond and Dybvig, if a non-optimal tax is needed and the share of early consumers is stochastic, this will cause tax distortions and resource costs for government deposit insurance. In such cases, where an inefficient tax funds the insurance, social welfare might be higher without it.

Proposition 2. *Bank contracts cannot achieve optimal risk sharing when t is stochastic and has a non-degenerate distribution. However, an ELA can prevent bank runs if the share of CBDC use is large enough. Moreover, it ensures that all early consumers can withdraw their deposits.*

Proof. The first part is established in Diamond and Dybvig (1983), where they prove that any type of contract that has V_1 as a function of f_j and V_2 as a function of f, cannot achieve optimal allocation when t is stochastic. The final points can be clarified as follows: If banks use $\bar{t}$ instead of t in their contracts, this establishes a new condition, which requires a higher value for θ, i.e. $\theta > \underline{\theta}(\bar{t})$. If banks choose $r_1 = c_1^{1*}(\bar{t})$, this condition can be met, since the higher lower bound for θ will be less than 1. Therefore, an ELA can prevent bank runs and ensure that early consumers can withdraw their deposits, provided there are enough CBDC users. ■

Fixed CBDC use. For ELA to prevent bank runs, the previous conditions on θ and R, Equations (5) and (6), must hold for all realizations of $\tilde{t}$. Therefore, since $\theta(t)$ is an increasing function of t, it must be that

$$\theta > \frac{(1-f) - Rr_1^{-1}(1-r_1 f)}{(f-\bar{t})(R-1)} = \underline{\theta}(\bar{t}) \tag{7}$$

If banks choose $r_1 = c_1^{1*}(\bar{t})$ then $\underline{\theta}(\bar{t})$ is between 0 and 1. Under these conditions, the same logic applies, and then $f = t$ comes as the unique Nash equilibrium.[7] However, since optimal consumption now depends on t, denoted as $c_1^{1*}(t)$, which is unknown, banks cannot choose r_1 equal to $c_1^{1*}(t)$ to

[7] See appendix C.1 for the rest of the proof.

achieve the optimal allocation. Nevertheless, even if the optimal allocation is not achieved, ELA improves welfare over the equilibrium with the policy of suspension of convertibility shown in Diamond and Dybvig (1983). This is because, even if banks choose $r_1 = c_1^{1*}(\bar{t})$, some early consumers may not receive their deposits if convertibility is suspended too early relative to the realization of t, while with ELA, early consumers are able to withdraw for all realizations of $\tilde{t}$.

Convenience of CBDC. If withdrawing using cash is costly, then all early withdrawals are made via CBDC regardless of the realization of $\tilde{t}$. If $r_1 = c_1^{1*}(\bar{t})$ equation (7) holds since $\theta = 1$, then an ELA prevents bank runs. Once again, banks cannot choose $r_1 = c_1^{1*}(t)$, so the optimal allocation is not achieved. However, it remains true in this case that all early consumers can withdraw at $T = 1$ and receive r_1, since $f = t$ is the equilibrium.

6 Final Remarks

The introduction of CBDC has been extensively studied in the literature to evaluate whether it increases or decreases the banking system's exposure to bank runs and disintermediation. In this paper, we show that introducing a cash-like CBDC in the Diamond-Dybvig model can serve as a viable tool to prevent liquidity-driven systemic bank runs.

The findings in this paper suggest that when CBDC is available as an alternative means of storing goods, bank runs on real demand deposits can be effectively prevented with emergency liquidity assistance by the central bank. If enough depositors choose to withdraw their deposits via CBDC, the central bank receives sufficient funds to support illiquid banks, acting as a lender of last resort and maintaining liquidity in the banking system, thus avoiding the termination of profitable investments. This policy prevents bank runs without the need for taxation associated with deposit insurance, which can create distortions and resource costs that may even reduce welfare.

Moreover, although CBDC makes the system more susceptible to bank runs when strictly preferred to cash, it also facilitates the implementation of an ELA by providing the central bank with additional funds to do so. We have shown that even when CBDC is strictly preferred over storage technology and completely replaces it outside the equilibrium path, in equilibrium, with an ELA, there is no run and therefore no fast disintermediation.

I established that under certain conditions, i.e., the share of early consumers is known, CBDC can achieve a full insurance equilibrium as a unique Nash equilibrium. This implies that optimal risk-sharing can be achieved without the need for deposit insurance.

Furthermore, this paper extends its analysis to scenarios where the share of early consumers is stochastic. While optimal risk-sharing may not be attainable in such cases, an ELA still proves effective in preventing bank runs and ensuring that all early consumers can withdraw their deposits, unlike the suspension of convertibility.

In conclusion, this paper demonstrates that CBDC has the potential to facilitate an effective implementation of ELA, thereby preventing liquidity-driven bank runs. It reaffirms Diamond and Dybvig's prediction that a lender of last resort could function similarly to deposit insurance in their model, with the distinction that taxation is not necessary. Finally, this paper supports existing literature by placing the risk of digital bank runs in perspective and tempering concerns about fast disintermediation caused by CBDC. However, it does not address concerns about slow disintermediation that could potentially arise from an interest-bearing CBDC.

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A Appendix

A Fixed CBDC use

A.1 Condition for $V_2 > V_1$

Here, we derive the condition under which $V_2 > V_1$, given the case $f > t$. From equation (4), we have that:

$$
\begin{aligned}
V_2 = \frac{R[1 - r_1(f - \theta(f - t))] - r_1\theta(f - t)}{1 - f} &> r_1 = V_1 \\
R[1 - r_1 t - r_1(1 - \theta)(f - t)] - r_1(f - t)\theta &> r_1(1 - f) \\
R[1 - r_1 t - r_1(1 - \theta)(f - t)] - r_1(f - t)\theta - r_1(1 - f) &> 0 \\
R(1 - r_1 f) + Rr_1(f - t)\theta - r_1(f - t)\theta - r_1(1 - f) &> 0 \\
Rr_1(f - t)\theta - r_1(f - t)\theta &> r_1(1 - f) - R(1 - r_1 f) \\
(R - 1)(f - t)r_1\theta &> r_1(1 - f) - R(1 - r_1 f) \\
\theta &> \frac{r_1(1 - f) - R(1 - r_1 f)}{r_1(R - 1)(f - t)} \\
\theta &> \frac{(1 - f) - Rr_1^{-1}(1 - r_1 f)}{(R - 1)(f - t)} = \underline{\theta}
\end{aligned}
$$

this represents the lower bound as stated in equation (5).

A.2 Proof that $\underline{\theta} \leq 1$

We show the condition that investment returns must satisfy for $V_2 > V_1$ if the share of depositors using CBDC is greater than $\underline{\theta}$. To ensure this possibility, it is necessary that $\underline{\theta} \leq 1$.

$$
\begin{aligned}
\underline{\theta} = \frac{(1 - f) - Rr_1^{-1}(1 - r_1 f)}{(R - 1)(f - t)} &\leq 1 \\
(1 - f) - Rr_1^{-1}(1 - r_1 f) &\leq (R - 1)(f - t) \\
(1 - f) + (f - t) &\leq Rr_1^{-1}(1 - r_1 f) + R(f - t) \\
r_1(1 - t) &\leq R(1 - r_1 f) + R(r_1 f - r_1 t) \\
r_1(1 - t) &\leq R(1 - r_1 t) \\
r_1 \frac{(1 - t)}{(1 - r_1 t)} &\leq R
\end{aligned}
$$

this condition on R is expressed in equation (6). Importantly, it is not an additional restriction, as it holds true under the assumptions of the Diamond-Dybvig model if $r_1 = c_1^{1*}$, as stated in equation (2).

A.3 Robustness check for costly ELA

We conduct a robustness check and demonstrate that the central bank can charge banks for the ELA, while still preventing bank runs and achieving an optimal risk-sharing equilibrium. Let e denote the cost per unit of deposit loaned through ELA to banks, then

$$
\begin{aligned}
V_2 = \frac{R[1 - r_1 t - r_1(1-\theta)(f-t)] - e r_1 (f-t)\theta}{1-f} &> r_1 = V_1 \\
R[1 - r_1 t - r_1(1-\theta)(f-t)] - e r_1(f-t)\theta - r_1(1-f) &> 0 \\
R(1 - r_1 f) + R r_1 (f-t)\theta - e r_1 (f-t)\theta - r_1(1-f) &> 0 \\
R r_1 (f-t)\theta - e r_1 (f-t)\theta &> r_1(1-f) - R(1-r_1 f) \\
(R-e)(f-t) r_1 \theta &> r_1(1-f) - R(1-r_1 f) \\
\theta &> \frac{r_1(1-f) - R(1-r_1 f)}{r_1(R-e)(f-t)} \\
\theta &> \frac{(1-f) - R r_1^{-1}(1-r_1 f)}{(R-e)(f-t)} = \underline{\theta}(e) > \underline{\theta}
\end{aligned}
$$

The cost for each unit of deposit loaned through ELA has to be less than R. Also, for $\underline{\theta}(e)$ to be lower than 1 it has to be that:

$$
\begin{aligned}
\underline{\theta} = \frac{(1-f) - R r_1^{-1}(1-r_1 f)}{(R-e)(f-t)} &\leq 1 \\
(1-f) - R r_1^{-1}(1-r_1 f) &\leq (R-e)(f-t) \\
(1-f) + e(f-t) &\leq R r_1^{-1}(1-r_1 f) + R(f-t) \\
r_1[(1-f) + e(f-t)] &\leq R(1-r_1 f) + R(r_1 f - r_1 t) \\
r_1[(1-f) + e(f-t)] &\leq R(1-r_1 t) \\
r_1 \frac{[(1-f) + e(f-t)]}{(1-r_1 t)} &\leq R \\
r_1 \frac{(1-t)}{(1-r_1 t)} < r_1 \frac{[(1-f) + e(f-t)]}{(1-r_1 t)} &\leq R
\end{aligned}
$$

since equation (6) holds strictly when banks choose $r_1 = c_1^{1*}$, it exist $e > 1$ such that:

$$
c_1^{1*} \frac{[(1-f) + e(f-t)]}{(1-c_1^{1*} t)} \leq R
$$

Then, we demonstrated that it is possible for the central bank to impose a penalty rate on banks for ELA and still prevent bank runs. However, a greater share of CBDC users is needed, since $\underline{\theta}(e) > \underline{\theta}(1) = \underline{\theta}$.

B Convenience of CBDC

B.1 Proof of bank runs without ELA

Here, we show that when CBDC is strictly preferred to cash as a method of withdrawal in $T = 1$, and the central bank does not implement an ELA, then the bank run risk still exist. Additionally, we show that without an ELA, bank runs are more likely to occur with CBDC than without it. The payoffs for withdrawing using cash and CBDC are as follows:

$$V_1^{CASH}(f_j, r_1) = \begin{cases} r_1 - \delta & \text{if } f_j \leq r_1^{-1} \\ 0 & \text{if } f_j > r_1^{-1} \end{cases} \tag{8}$$

$$V_1^{CBDC}(f_j, r_1) = \begin{cases} r_1 & \text{if } f_j \leq r_1^{-1} \\ 0 & \text{if } f_j > r_1^{-1} \end{cases} \tag{9}$$

At $T = 2$, banks are liquidated and give payoff:

$$V_2(f, r_1) = \max\{R(1 - r_1 f)/(1 - f), 0\} \tag{10}$$

Without an ELA, banks can only fulfil their obligations until $f_j = r_1^{-1}$. If f is expected to be grater than $\underline{f} = (R - r_1)/[r_1(R - 1)]$, then for all $f > \underline{f}$ and $f_j \leq f$, $V_2 < V_1^{CBDC}$, meaning late consumers have incentives to withdraw early via CBDC. However, since withdrawing in $T = 1$ using cash gives $V_1^{CASH} = r_1 - \delta$, then for all $f \in (\underline{f}, \underline{f} + \delta)$, it holds that $V_1^{CASH} < V_2$ but $V_1^{CBDC} > V_2$. Therefore, the threshold for withdrawals that trigger the bank run equilibrium is lower in the presence of a CBDC compared to when a CBDC is absent. If $f > \underline{f} + \delta$ then both methods of withdrawals induce the bank run equilibrium if there is no ELA.

C Stochastic early consumers

C.1 Equilibrium decisions and proof that $\underline{\theta}(\bar{t}) \leq 1$

I conclude the proof that no bank run occurs, provided the restriction mentioned in equation (7) holds and if banks choose $r_1 = c_1^{1*}(\bar{t})$. V_2 is defined as in equation (4). However, banks do not know t, but they do know $\bar{t}$. Therefore, the central bank implements the ELA only when $f > \bar{t}$. Thus, the payoffs are as follows:

$$V_1(f_j, r_1) = r_1 \; \forall f_j \leq f \tag{11}$$

$$V_2(f, r_1) = \begin{cases} \frac{R(1-r_1 f)}{1-f} & \text{if } f \leq \bar{t} \\ r_2(\bar{t}) = \frac{R[1-r_1(f-\theta(f-\bar{t}))]-r_1(f-\bar{t})\theta}{1-f} & \text{if } f > \bar{t} \end{cases} \tag{12}$$

I show that for V_2 to be always greater than V_1, equation (7) must holds. First, if $f \leq \bar{t}$ then:

$$\frac{R(1-r_1 f)}{1-f} \geq r_1 \iff R \geq \frac{1-\bar{t}}{1-r_1\bar{t}} r_1$$

since $(1-r_1 f)/(1-f)$ is a decreasing function of f and has to be true for all $f \leq \bar{t}$. If banks choose $r_1 = c_1^{1*}(\bar{t})$ then that inequality holds strictly as shown in equation (2). If $f > \bar{t}$, then $r_2(\bar{t})$, which is equal to (4) but with t substituted by $\bar{t}$, must be greater then r_1. Since $r_2(t)$ is a decreasing function of t, there are always enough assets to cover liquidation, i.e. $r_2(t) > r_2(\bar{t})$. Then, for $r_2(\bar{t})$ to be greater than r_1, the following inequality must hold:

$$\begin{aligned}
r_2(\bar{t}) = \frac{R[1-r_1\bar{t}-r_1(1-\theta)(f-\bar{t})]-r_1(f-\bar{t})\theta}{1-f} &> r_1 \\
R[1-r_1\bar{t}-r_1(1-\theta)(f-\bar{t})]-r_1(f-\bar{t})\theta - r_1(1-f) &> 0 \\
R(1-r_1 f) + Rr_1(f-\bar{t})\theta - r_1(f-\bar{t})\theta - r_1(1-f) &> 0 \\
Rr_1(f-\bar{t})\theta - r_1(f-\bar{t})\theta &> r_1(1-f) - R(1-r_1 f) \\
(R-1)(f-\bar{t})r_1\theta &> r_1(1-f) - R(1-r_1 f) \\
\theta &> \frac{r_1(1-f)-R(1-r_1 f)}{r_1(R-1)(f-\bar{t})} \\
\theta &> \frac{(1-f)-Rr_1^{-1}(1-r_1 f)}{(R-1)(f-\bar{t})} = \underline{\theta}(\bar{t})
\end{aligned}$$

As shown in A.1, but substituting t for $\bar{t}$, we need $\underline{\theta}(\bar{t})$ to be a fraction between 0 and 1, which requires

$$R \geq \frac{1-\bar{t}}{1-r_1\bar{t}} r_1$$

this inequality holds strictly, as shown above, when $r_1 = c_1^{1*}(\bar{t})$. Therefore, if enough withdrawals are made via CBDC, i.e. $\theta > \underline{\theta}(\bar{t})$, then $V_2 > V_1$ for all f. The dominant strategy for late consumers is to withdraw in $T = 2$. Early consumers always withdraw at $T = 1$, since their utility only depends on c_1. Thus, $f = t$ emerges as the unique Nash equilibrium in dominant strategies, and all early consumers can withdraw at $T = 1$, concluding the proof of Proposition 2. ■