Simple Implementable Financial Policy Rules*

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Abstract

How important, in terms of welfare, is the counter-cyclical capital buffer (CCyB) relative to other —higher and more permanent— bank capital requirements? While there is better understanding of the effect of a-cyclical higher capital requirements on banks' resilience and credit supply, much less is known about the marginal effects of introducing a macroprudential counter-cyclical capital requirement. In this paper, we study the welfare implications of introducing several simple and implementable financial policy (CCyB) rules that co-exist with monetary policy. We find that the institutional design of the financial-policy instruments matters for its welfare implications. In particular, a zero lower bound on the CCyB interacts with its counter-cyclical nature and provides a rationale for a positive *neutral* level. We build our analysis based on a quantitative macro-banking model with two main frictions, nominal rigidities and financial frictions, which we estimate for Chile; a representative small open economy.

JEL Codes: E12, E31, E44, E52

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1 Introduction

The 2008 financial crisis put forward the importance of financial intermediation, mainly through banking, in the potential origination and amplification of shocks to the macroeconomy. This observation catalyzed both, research on macro-financial linkages, and re-assessment of banking regulation. The latter materialized in the package of reforms we know as Basel III; with one of its main objectives being the incorporation of a system-wide approach to financial risk assessments, and financial policy; thereby explicitly introducing a macroprudential perspective to banks' capital regulation. Basel III introduces two buffers in this direction; the capital conservation buffer (CCoB) and the countercyclical capital buffer (CCyB) (Financial Stability Institute, 2019)[1]. While the CCoB has more automatic guidelines for its replenishment in case of loss-related draw downs, the CCyB can be activated and deactivated according to the decision of the authority. That is, the CCyB is a macroprudential tool. In this paper we examine the implications of different rules guiding this decision in terms of welfare and banks' resilience, how they interact with monetary policy, and emphasize the implications of the institutional design on the adequacy of a positive neutral level of CCyB.

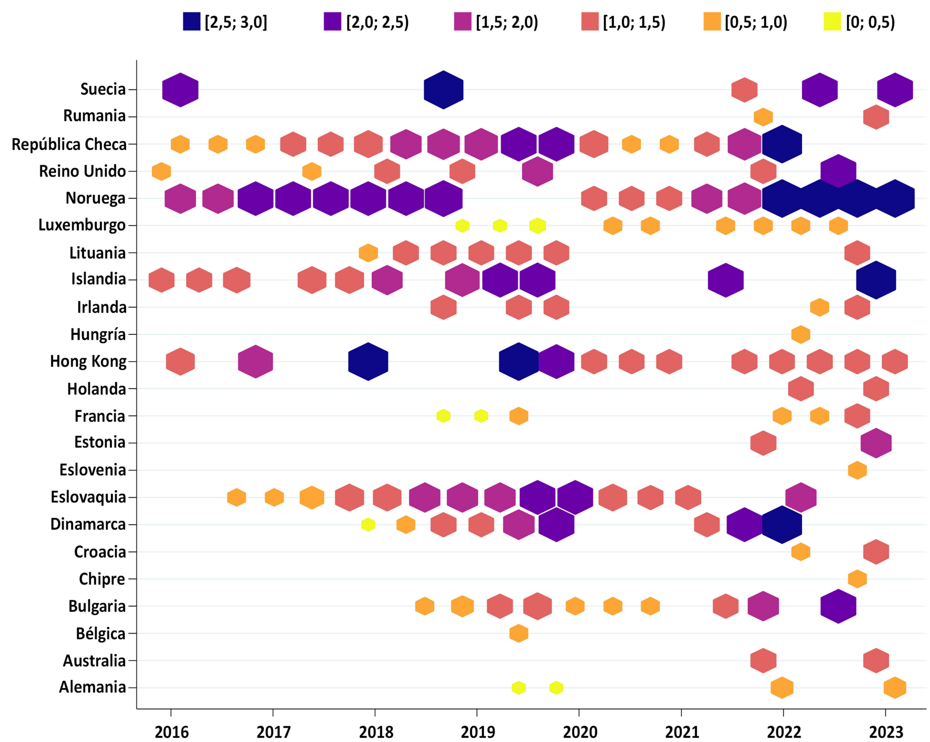
In order to comprehensively analyze the macroeconomic implications of different CCyB designs, we build a macro-banking model with two main inefficiencies as in Carrillo et al. (2021). Monetary policy addresses inefficiencies from staggered pricing by monopolistic input producers, and Financial policy addresses inefficiencies from financial frictions in the form of costly state verification. Drawing on the results of Carrillo et al. (2021) we abstract from a one-tool for two-objectives policy, and instead start from the Tinbergen rule. Our model includes both a monetary policy rule, and a countercyclical capital requirement rule, and features three levels of default by different agents in the economy, including the banking sector, as in Clerc et al. (2014). Hence our model is rich enough to analyze the interaction of monetary and financial policy, yet parsimonious enough to calculate welfare of different policy regimes. In particular, our model is based on a simplified version Calani et al. (2022), one of the main models used at the Central Bank of Chile. Notably, in the financial side, this model features financial frictions as in Bernanke et al. (1999) and Clerc et al. (2014); long term debt as in Woodford (2001); and a bank-related friction in which depositors do not price bank default risk at the margin, as in Mendicino et al. (2018) and Mendicino et al. (2020). Our model is more appropriate for small open economies with both monetary and financial policies, in which bank credit can be short- and long-term.

The literature on the effects of banks' capital requirements on financial and real variables, has grown significantly in the past years, in tandem with the number of countries adopting and implementing capital regulation, and the availability of micro-data. However, at least on its aggregate consequences, most of the focus of the literature has emphasized the effects of the higher levels of capital requirements. The main trade-off of higher, a-cyclical, capital requirements weights lower systemic risk —measured as banking sector default probability— and lower activity in credit and the ensuing lower economic activity (Van den Heuvel, 2008; Clerc et al., 2014; Mendicino et al., 2018, 2020). Our paper shares this main feature, but instead, its focus is on cyclical considerations of capital regulation,

[1]Both capital buffers must be met with Common Equity Tier 1 (CET1) capital only. The CCoB is meant to give banks and additional layer of usable capital when idiosyncratic losses are incurred. The CCyB is meant to be raised when system-wide risks, usually associated with high credit growth is perceived to become more important. Both buffers range from 0% to 2.5%.

i.e. the design of a CCyB rule and its macroeconomic effects. Thus, our paper is more related to Carrillo et al. (2021) and Malherbe (2020). We explore different implementable, simple, policy rules in terms of their welfare implications, exploring the relationship with monetary policy. Notably, we find that simply following a credit-gap rule may not be optimal.

Figure 1: Countercyclical capital buffer activation across countries



Note.– This figure reports activation of countercyclical buffer (CCyB) by date and size of requirement. Each hexagon shows the current level of CCyB. No hexagon means deactivated CCyB. Source: Financial Stability Report CBC 2023-S1

Further, the experience from the Covid-19 pandemic suggests that there might be important differences between CCoB and CCyB usability. In particular, banks might be reluctant to exhaust CCoB (Basel Committee on Banking Supervision, 2022), and instead might want to comply with capital requirements deleveraging. In contrast, a system-wide deactivation of the countercyclical capital buffer by instruction of the supervisor, would not attract adverse market reaction or stigma on any particular bank, and might better accomplish its countercyclical objective. Notably, before the Covid-19 pandemic many juristictions had activated the CCyB, and deactivated it in early 2020 (see Figure **??**). By the end of 2021, mostly the same economies started activating this buffer again, suggesting that its deactivation was useful during the worst moment of the sanitary crisis.

By design, however, the CCyB ranges from 0 to 2.5 percent of risk weighted assets (RWA), which implies that if a shock which would be better addressed by deactivating the CCyB, hits the economy, and this instrument is currently not activated, then much of its benefits are not grasped. This mechanism provides a rationale for setting a positive neutral level in case deactivation is suddenly required. We explore this issue quantitatively.

The document is structured as follows. In Section 2 we present a detailed description of the theoretical structure of the model. Section 3 describes the estimation of the model, the calibration, the choice of priors and presents the results. Section 4 presents the results. Section 5 concludes.

2 A Small Open Economy Model with Financial Frictions

In the following section, we augment a standard New Keynesian small open economy model with financial frictions in the economy's entrepreneurial, banking, and housing sectors. To do this, we introduce new agents taking Clerc et al. (2014) as starting point: entrepreneurs and bankers. The former are the sole owners of capital, who finance their capital investment through banking loans, while the latter are the owners of the banks who lend resources for capital investment and housing investment.

Households are divided between patients, who save using the financial market, and impatients, who borrow using the financial market. We also introduce the segmented financial markets concept in the spirit of Vayanos and Vila (2009). Following Andres et al. (2004) and Chen et al. (2012), saving households can be unrestricted, who can save in short or long term financial assets, or restricted, who can save only in short term assets. All households derive utility from a consumption good, leisure, and housing stock.

From the production side, we use a simplified version of Garcia et al. (2019) in which a final good is produced using capital and labor and facing prices *à la* Calvo and a labor market facing quadratic adjustment cost in the style of Lechthaler and Snower (2011). In addition, we introduce three kinds of firms (capital producers, housing producers, and banks). Concerning debt, we include not only short-term deposits but also long-term government and bank bonds as perpetuities that pay exponentially decaying coupons as introduced by Woodford (2001)

2.1 Households

There are two continuums of households of measure one, risk-averse and infinitely lived. These agents differ in their discount factor: β_I for impatient households (I), and β_P for patient households (P), with $\beta_P > \beta_I$. In equilibrium, impatient households borrow from banks and are ex-ante identical in asset endowments and preferences to others of their same patience.

In terms of patient households, following Andres et al. (2004) and Chen et al. (2012), we allow for a distinction between two types of patient households: Restricted (R) and Unrestricted (U) depending on which assets they can access for saving purposes. While Unrestricted households can buy both long and short-term assets with a transaction cost, Restricted households can only buy long-term bonds but do not face any transaction cost. Their combined measure is of size one.

Restricted and Unrestricted households' preferences depend on consumption of a final good C_t relative to external habits $\tilde{C}_{t-1}$, their stock of housing from last period H_{t-1} relative to external habits $\tilde{H}_{t-2}$, and labor supplied (hours worked) n_t in each period. The consumption of the aggregate good $\hat{C}_t^i \equiv \hat{C}(C_t^i, \tilde{C}_{t-1}^i, H_{t-1}^i, \tilde{H}_{t-2}^i)$ for households of type $i = \{U, R, I\}$ is assumed to be a constant elasticity of substitution (CES) as shown in (1):

$$\hat{C}_t^i = \left[(1 - o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(C_t^i - \phi_c \tilde{C}_{t-1}^i \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi_t^h \left(H_{t-1}^i - \phi_{hh} \tilde{H}_{t-2}^i \right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{1}$$

where $o_{\hat{C}} \in (0,1)$ is the weight on housing in the aggregate consumption basket, $\eta_{\hat{C}}$ is the elasticity of substitution between the final good and the housing good, ξ_t^h is an exogenous preference shifter shock and $\phi_c, \phi_{hh} \geq 0$ are parameters guiding the strength of habits in consumption and housing respectively. Households of type $i = \{U, R, I\}$ maximize the following expected utility

$$\max_{\{\hat{C}_t^i, H_t^i\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=1}^{\infty} \beta_i^t \varrho_t \left[\frac{1}{1-\sigma} \left(\hat{C}_t^i \right)^{1-\sigma} - \Theta_t^i A_t^{1-\sigma} \xi_t^n \frac{\left(n_t^i\right)^{1+\varphi}}{1+\varphi} \right] \tag{2}$$

where $\beta_i \in (0,1)$ is the respective discount factor, ϱ_t is an exogenous shock to intertemporal preferences, ξ_t^n is a preference shock that affects the (dis)utility from labor, $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\varphi \geq 0$ is the inverse elasticity of labor supply.

As in Galí et al. (2012), we introduce an endogenous preference shifter Θ_t, that satisfies the following conditions

$$\Theta_t^i = \tilde{\chi}_t^i A_t^{\sigma} \left(\hat{C} \left(\tilde{C}_t^i, \tilde{C}_{t-1}^i, \tilde{H}_{t-1}^i, \tilde{H}_{t-2}^i \right) \right)^{-\sigma} \tag{3}$$

and

$$\tilde{\chi}_t^i = \left(\tilde{\chi}_{t-1}^i \right)^{1-v} A_t^{-\sigma v} \left(\hat{C} \left(\tilde{C}_t^i, \tilde{C}_{t-1}^i, \tilde{H}_{t-1}^i, \tilde{H}_{t-2}^i \right) \right)^{\sigma v} \tag{4}$$

where the parameter $v \in [0,1]$ regulates the strength of the wealth effect, and $\tilde{C}_t^i$ and $\tilde{H}_{t-1}^i$ are taken as given by the households. In equilibrium $C_t^i = \tilde{C}_t^i$ and $H_t^i = \tilde{H}_t^i$.

2.1.1 Patient Households

Unrestricted Households. This group is formed by fraction $\wp_U$ of the patient households. In equilibrium, they save in one-period government bond, BS_t^U, long-term government bonds, BL_t^U, short-term bank deposits D_t^U, long-term bank-issued bonds, BB_t^U, and one-period foreign bonds quoted in foreign currency $B_t^{\star U}$. All these assets being non-state-contingent.

The structure of long term financial assets follows Woodford (2001), in this framework, long-term instruments are perpetuities, each paying a coupon of unitary value (in units of final goods) in the period after issuance, and a geometrically declining series of coupons (with a decaying factor $\kappa < 1$) thereafter. That is, a bond issued in period-t implies a series of coupon payments starting in $t+1$: $\{1, \kappa, \kappa^2, \ldots\}$. Also, let B_{t-1}, where $B_{t-1} = \left\{BL_{t-1}^U, BB_{t-1}^U\right\}$ represent the total liabilities due in period t from all past bond issues up to period $t-1$. That is

$$B_{t-1} = CI_{t-1} + \kappa CI_{t-2} + \kappa^2 CI_{t-3} + \ldots,$$

thus, $CI_{t-1} = B_{t-1} - \kappa B_{t-2}$. Let Q_t^B denote the period-t price of a new issue, then Q_t^B summarizes the prices at all maturities. For instance, $Q_{t|t-1}^B = \kappa Q_t^B$ is the price in t of a perpetuity issued in period $t-1$. Importantly, note that B_{t-1} denotes both, total liabilities in period-t from previous debt, and –because of the particular coupon structure– the total number of outstanding bonds. Then, the total value of financial asset debt in period t is given by $Q_t B_t$. Finally, the *real* yield to maturity of holding long term assets at period t, R_t^B, as,

$$R_t^B = \frac{P_t}{Q_t^B} + \kappa$$

Unrestricted households must pay a transaction cost ζ_t^L per unit of long-term bond purchased. These costs are paid to a financial intermediary as a fee. This financial intermediary distributes its nominal value profits Π^{FI}, as dividends to its shareholders. Then, unrestricted patient households' period budget constraint is

$$\begin{aligned} BS_t^U + \left(1+\zeta_t^L\right) Q_t^{BL} BL_t^U + D_t^U + \left(1+\zeta_t^L\right) Q_t^{BB} BB_t^U + S_t B_t^{\star U} + P_t C_t^U + Q_t^H H_t^U = \\ R_{t-1} BS_{t-1}^U + Q_t^{BL} R_t^{BL} BL_{t-1}^U + \tilde{R}_t^D D_{t-1}^U + \tilde{R}_t^{BB} Q_t^{BB} BB_{t-1}^U + S_t B_{t-1}^{\star U} R_{t-1}^{\star} + W_t n_t^U \\ + Q_t^H \left(1-\delta_H\right) H_{t-1}^U + \Psi_t \end{aligned} \tag{5}$$

where R_t^{BL} and R_t^{BB} are the real gross yield to maturity for long-term government and bank-issued bonds at time t, P_t denotes the price of the consumption good, Q_t^H denotes the nominal price of housing good, δ_H is depreciation rate of housing goods, S_t the nominal exchange rate (units of domestic currency per unit of foreign currency), $R_t^{\star}$ the foreign one-period bond return, and R_t denotes the short term nominal government bond rate.

Further, $\widetilde{R}_t^D = R_{t-1}^D(1-\gamma_D PD_t^B)$, $\widetilde{R}_t^{BB} = R_t^{BB}(1-\gamma_{BB} PD_t^B)$ denote the net return on resources loaned to banks in the form of deposits and bank-issued bonds, R_{t-1}^D is the gross interest rate received at t on the bank deposits at $t-1$, PD_t^B denotes the fraction of resources in banks that fail in period t and $\gamma_D(\gamma_{BB})$ is a linear transaction cost that households must pay in order to recover their funds. Finally, W_t denotes the nominal wage and, Ψ_t denotes lump-sum payments that include taxes T_t, dividend income from entrepreneurs C_t^e, bankers C_t^b, rents from ownership of foreign firms REN_t^*, profits from ownership of domestic firms, and profits from the financial intermediary in the long-term bond transactions, $\Pi^F = \zeta_t^L(Q_t^{BL} BL_t^U + Q_t^{BB} BB_t^U)$.

Chen et al. (2012) show that the discounted value of future transaction costs implies a term premium. We assume that the period transaction cost is a function of the ratio of the aggregate market value of long-term to short-term assets and a disturbance term. Further, households do not internalize the effect of their choices on this transaction cost, yet in equilibrium $\widetilde{BL}_t^U = BL_t^U$ and $\widetilde{BS}_t^U = BS_t^U$. This ratio captures the idea that holding long-term debt implies a loss of liquidity that households hedge by increasing the amount of short-term debt. Specifically, the functional form is given by

$$\zeta_t^L = \left(\frac{Q_t^{BL} \widetilde{BL}_t^U + Q_t^{BB} \widetilde{BB}_t^U}{\overline{Q}_t^{BL} \overline{BL}_t^U + \overline{Q}_t^{BB} \overline{BB}_t^U} \right)^{\eta_{\zeta_L}} \epsilon_t^L \tag{6}$$

Households supply differentiated labor services to a continuum of unions which act as wage setters on behalf of the households in monopolistically competitive markets. The unions pool the wage income of all households and then distribute the aggregate wage income in equal proportions among households, hence, they are insured against variations in household-specific wage income. Defining for convenience the multiplier on the budget constraint as $\lambda_t^U A_t^{-\sigma}/P_t$, then, Unrestricted Households solve (2) subject to (1), (3), (4), and (5). From this problem we obtain the following first-order conditions:

$$[C_t^U]: \qquad \lambda_t^U A_t^{-\sigma} = \left(\hat{C}_t^U\right)^{-\sigma} \left(\frac{\left(1-o_{\hat{C}}\right)\hat{C}_t^U}{\left(C_t^U - \phi_c \tilde{C}_{t-1}^U\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{7}$$

$$[H_t^P]: \qquad \varrho_t \frac{\lambda_t^U A_t^{-\sigma} Q_t^H}{P_t} = \beta_U \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{C}_{t+1}^U\right)^{-\sigma} \xi_{t+1}^h \left(\frac{o_{\hat{C}} \hat{C}_{t+1}^U}{\xi_{t+1}^h \left(H_t^U - \phi_{hh} \tilde{H}_{t-1}^U\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} + (1-\delta_H) \frac{\lambda_{t+1}^U A_{t+1}^{-\sigma} Q_{t+1}^H}{P_{t+1}} \right\} \tag{8}$$

$$[BS_t^U]: \qquad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U R_t \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda_{t+1}^U}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \tag{9}$$

$$[BL_t^U]: \qquad \varrho_t \lambda_t^U A_t^{-\sigma} (1+\zeta_t^L) \left(\frac{Q_t^{BL}}{P_t} \right) = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U A_{t+1}^{-\sigma} R_{t+1}^{BL} \left(\frac{Q_{t+1}^{BL}}{P_{t+1}} \right) \right\} \tag{10}$$

$$[B_t^{\star U}]: \qquad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U R_t^\star \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda_{t+1}^U \pi_{t+1}^s}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \tag{11}$$

$$[D_t^U]: \qquad \varrho_t \lambda_t^U A_t^{-\sigma} = \beta_U \mathbb{E}_t \left\{ \frac{\varrho_{t+1}\lambda_{t+1}^U}{\pi_{t+1}} \tilde{R}_{t+1}^D A_{t+1}^{-\sigma} \right\} \tag{12}$$

$$[BB_t^U]: \qquad \varrho_t \lambda_t^U A_t^{-\sigma} (1+\zeta_t^L) \left(\frac{Q_t^{BB}}{P_t} \right) = \beta_U \mathbb{E}_t \left\{ \varrho_{t+1} \lambda_{t+1}^U A_{t+1}^{-\sigma} \tilde{R}_{t+1}^{BB} \left(\frac{Q_{t+1}^{BB}}{P_{t+1}} \right) \right\} \tag{13}$$

In equilibrium, we have that $\tilde{C}_t^P = C_t^P$ and $\tilde{H}_t^P = H_t^P$, which applies for impatient households as well. The implied discount factor for nominal claims is, by iterating upon (9):

$$r_{t,t+s} = \frac{1}{\prod_{i=0}^{s-1} R_{t+i}} = \beta_U^s \frac{\varrho_{t+s}\lambda_{t+s}^U A_{t+s}^{-\sigma} P_t}{\varrho_t \lambda_t^U A_t^{-\sigma} P_{t+s}} \tag{14}$$

Restricted households. This group of households have a mass $\wp_R$ which complements the mass of unrestricted households $\wp_U$, then $\wp_R = 1 - \wp_U$. The main difference with Unrestricted Household is that they can only access long-term government bonds. In addition, Restricted Patient households do not face transaction costs. They are subject to the period-by-period budget constraint

$$P_t C_t^R + Q_t^H H_t^R + Q_t^{BL} BL_t^R = W_t n_t^R + Q_t^H (1-\delta_H) H_{t-1}^R + Q_t^{BL} R_t^{BL} BL_{t-1}^R \tag{15}$$

Let us define, for convenience, the multiplier on the budget constraint as $\lambda_t^R A_t^{-\sigma}/P_t$. Then, restricted households solve (2) subject to (1), (3), (4), and (15), from which we obtain the following first-order conditions:

$$[C_t^R]: \quad \lambda_t^R A_t^{-\sigma} = \left(\hat{C}_t^R\right)^{-\sigma} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{C}_t^R}{\left(C_t^R - \phi_c \tilde{C}_{t-1}^R\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{16}$$

$$[H_t^P]: \quad \varrho_t \frac{\lambda_t^R A_t^{-\sigma} Q_t^H}{P_t} = \beta_R \mathbb{E}_t \varrho_{t+1} \left\{ \left(\hat{C}_{t+1}^R\right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{C}_{t+1}^R}{\xi_{t+1}^h \left(H_t^R - \phi_{hh} \tilde{H}_{t-1}^R\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h \right. \tag{17}$$

$$\left. + \left(1 - \delta_H\right) \frac{\lambda_{t+1}^R A_{t+1}^{-\sigma} Q_{t+1}^H}{P_{t+1}} \right\}$$

$$[BL_t^R]: \quad \varrho_t \lambda_t^R A_t^{-\sigma} Q_t^{BL} = \beta_R \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^R}{\pi_{t+1}} R_{t+1}^{BL} Q_{t+1}^{BL} A_{t+1}^{-\sigma} \right\} \tag{18}$$

2.1.2 Impatient Households

Impatient households work, consume, and purchase housing goods. They take long-term loans in equilibrium from banks to finance their purchases of housing goods. Mortgage contracts are agreements on long-term debt and repayment plans which define an implicit yield to maturity R_t^I at date-t,

$$R_t^I = \left(\frac{P_t}{Q_t^L} + \kappa \right),$$

where Q_t^L is the price of one unit of long-term mortgage debt L_t^H issued in period-t, and κ is the geometric decline factor of long-term debt. At in any period $t' > t$ banks and households abide by the original contract agreement. In this way we are able to capture the fact that, for default decisions, households are concerned about face value of their debt and not necessarily the market value, which is a closer representation of the Chilean mortgage market with fixed-condition. Then, let the nominal-face-value of mortgage credit, $L_t^H \widehat{Q}_t^L$, be the sum of newly issued debt priced at current market conditions, and debt from previous periods priced at the moment when it was issued,

$$L_t^H \widehat{Q}_t^L = (L_t^H - \kappa L_{t-1}^H) Q_t^L + \kappa L_{t-1}^H \widehat{Q}_{t-1}^L \pi_t \tag{19}$$

Notably, observe that market value of mortgage debt, $L_t^H Q_t^L$, need not coincide with the value of debt priced at historic face-value, $L_t^H \widehat{Q}_t^L$. A second reason for using the latter is that this is the time series which we actually observe from the data to estimate the model

We follow the Clerc et al. (2014) by assuming that these mortgage loans are non-recourse and limited liability contracts, which enable the possibility of default for households. The only consequence of defaulting is losing the housing good on which the mortgage is secured on. Therefore default is optimal when total outstanding obligations is higher than the value of the assets posed as collateral,

$$(P_t + \kappa \widehat{Q}_{t-1}^L \pi_t) L_{t-1}^H > \omega_t^I Q_t^H \left(1 - \delta_H\right) H_{t-1}^I,$$

or,

$$\widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H > \omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I,$$

where we have used $\widehat{R}_t^I = \frac{P_t + \kappa \widehat{Q}_{t-1}^L \pi_t}{\widehat{Q}_t^L}$, and $R_t^H = \frac{Q_t^H(1-\delta_H)}{Q_{t-1}^H}$. Also, H_{t-1}^I denotes the housing units held by the impatient household at the beginning of period-t, and ω_t^I is an i.i.d idiosyncratic shock to the efficiency units of housing of impatient households, which follows a log-normal distribution with pdf $f_I\left(\omega_t^I\right)$ and cdf $F_I\left(\omega_t^I\right)$, and can be interpreted as a reduced-form representation of any shock to the value of houses.

Then, the default threshold $\bar{\omega}_t^I$ is given by

$$\bar{\omega}_t^I = \frac{\widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H}{R_t^H Q_{t-1}^H H_{t-1}^I}$$

If $\omega_t^I \geq \bar{\omega}_t^I$, the impatient household remains in good standing and repays the amount $\widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H$, which includes the coupon due in period-t and the remaining outstanding value of debt. Alternatively, if $\omega_t^I < \bar{\omega}_t^I$ the household defaults on its mortgage debt. This definition allows us to define $PD_t^I = F_I\left(\bar{\omega}_t^I\right)$ as the default rate of impatient households on housing loans. Notably, in case of repayment, the bank receives the fixed amount $\widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H$ from performing loans, and households walk away with $(\omega_t^I - \bar{\omega}_t^I) R_t^H Q_{t-1}^H H_{t-1}^I$. In case of default the bank recovers $(1-\mu_I)\omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I$ and the household walks away with nothing. This mechanism, a standard debt contract, is not only incentive compatible on the side of the bank but induces truth-telling on the side of the household.

Then the budget constraint for the impatient household is then given by:

$$P_t C_t^I + Q_t^H H_t^I - Q_t^L (L_t^H - \kappa L_{t-1}^H)\left[1 - \frac{\gamma_L}{2}\left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - \bar{a}\right)^2\right] - \pi_t \kappa L_{t-1}^H \widehat{Q}_{t-1}^L = W_t n_t^I + \int_0^\infty \max\left\{\omega_t^I R_t^H Q_{t-1}^H H_{t-1}^I - \widehat{R}_t^I \widehat{Q}_t^L L_{t-1}^H, 0\right\} dF_I(\omega_t^I) \tag{20}$$

where the expression $\left[1 - \frac{\gamma_L}{2}\left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - a_t\right)^2\right]$ represents the adjustment costs associated with the change in the level of debt L_t^H.

Out of all the loans, the share of the gross return that goes to the bank is denoted as $\Gamma_I(\bar{\omega}_t^I)$ whereas the share of gross return that goes to the impatient household is $(1 - \Gamma_I(\bar{\omega}_t^I))$ where:

$$\Gamma_I\left(\bar{\omega}_t^I\right) = \int_0^{\bar{\omega}_t^I} \omega_t^I f_I\left(\omega_t^I\right) d\omega_t^I + \bar{\omega}_t^I \int_{\bar{\omega}_t^I}^\infty f_I\left(\omega_t^I\right) d\omega_t^I$$

The first integral on the right denotes the share of the return that is defaulted while the second integral denotes the share of return that is paid in full. This allows us to rewrite the budget condition from (20) as

$$P_t C_t^I + Q_t^H H_t^I - Q_t^L (L_t^H - \kappa L_{t-1}^H) \left[1 - \frac{\gamma_L}{2} \left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - \bar{a} \right)^2 \right] - \pi_t \kappa L_{t-1}^H \widehat{Q}_{t-1}^L =$$

$$W_t n_t^I + \left[1 - \Gamma_I \left(\bar{\omega}_t^I\right)\right] R_t^H Q_{t-1}^H H_{t-1}^I \qquad (21)$$

Also, let

$$G_I\left(\bar{\omega}_t^I\right) = \int_0^{\bar{\omega}_t^I} \omega_t^I f_I\left(\omega_t^I\right) d\omega_t^I$$

denote the part of those returns that comes from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as $\mu_I G_I(\bar{\omega}_t^I)$, then the net share of return that goes to the bank is

$$\Gamma_I\left(\bar{\omega}_t^I\right) - \mu_I G_I\left(\bar{\omega}_t^I\right).$$

The terms of the loan must imply the net expected profits of the bank must equal its alternative use of funds, therefore it must satisfy a participation constraint:

$$\mathbb{E}_t \left\{ \left[1 - \Gamma^H\left(\bar{\omega}_{t+1}^H\right)\right] \left[\Gamma_I\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I\left(\bar{\omega}_{t+1}^I\right)\right] R_{t+1}^H Q_t^H H_t^I \right\} \geq \rho_{t+1}^H \phi_H Q_t^L L_t^H \qquad (22)$$

Where $\Gamma^H(\bar{\omega}_{t+1}^H)$ is the fraction of bank gross returns that is used to pay depositors or is lost due to bank defaults when their own idiosyncratic shock ω_{t+1}^H is too low. The rest of the left hand side expression is the total amount of returns on the housing project that goes to the lender bank. The right hand side indicates the opportunity cost, which is investing an amount of equity $\phi_H Q_t^L L_t^H$ at a market-determined rate of return of $\tilde{\rho}_{t+1}^H$, where ϕ_H is a regulatory capital constraint. We elaborate on the bank's problem on subsection 2.3, for now note that we can write (22) with equality without loss of generality.

Thus, following the timing described above, the impatient household's optimization problem can be written as maximizing (2) for $i = I$ subject to their budget constraint (21) and the bank participation constraint (22). For this, define for convenience ${}^{\lambda_t^I A_t^{-\sigma}}/_{P_t}$ and ${}^{\lambda_t^H A_t^{-\sigma}}/_{P_t}$ as the multipliers for each constraint respectively. Define also $x_t^I \equiv {}^{R_t^I L_t^H}/_{Q_t^H H_t^I}$, a measure of household leverage. This yields the following FOC's:

$$[C_t^I]: \quad \lambda_t^I A_t^{-\sigma} = \left\{ \left(\hat{C}_t^I \right)^{-\sigma} \right\} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{C}_t^I}{\left(C_t^I - \phi_c \tilde{C}_{t-1}^I \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \tag{23}$$

$$[H_t^I]: \quad \varrho_t \frac{\lambda_t^I A_t^{-\sigma} Q_t^H}{P_t} = \mathbb{E}_t \left\{ \begin{array}{l} \beta_I \varrho_{t+1} \left(\left(\hat{C}_{t+1}^I \right)^{-\sigma} \left(\frac{o_{\hat{C}} \hat{C}_{t+1}^I}{\xi_{t+1}^h \left(H_t^I - \phi_{hh} \tilde{H}_{t-1}^I \right)} \right)^{\frac{1}{\eta_{\hat{C}}}} \xi_{t+1}^h \\ + \frac{\lambda_{t+1}^I A_{t+1}^{-\sigma}}{P_{t+1}} \left[1 - \Gamma_I \left(\bar{\omega}_{t+1}^I \right) \right] R_{t+1}^H Q_t^H \right) \\ + \frac{\varrho_t \lambda_t^H A_t^{-\sigma}}{P_t} \left[1 - \Gamma^H \left(\bar{\omega}_{t+1}^H \right) \right] \left[\Gamma_I \left(\bar{\omega}_{t+1}^I \right) - \mu_I G_I \left(\bar{\omega}_{t+1}^I \right) \right] R_{t+1}^H Q_t^H \end{array} \right\} \tag{24}$$

$$[L_t^H]: \quad \varrho_t \frac{A_t^{-\sigma} Q_t^L}{P_t} \left\{ \lambda_t^I \left[1 - \frac{\gamma_L}{2} (\nabla \tilde{l}_t - \bar{a})^2 \right] - \lambda_t^I \nabla \tilde{l}_t \gamma_L (\nabla \tilde{l}_t - \bar{a}) - \lambda_t^H \rho_{t+1}^H \phi_H \right\} = ...$$

$$... \beta_I \mathbb{E}_t \left\{ \varrho_{t+1} \frac{\lambda_{t+1}^I A_{t+1}^{-\sigma}}{P_{t+1}} \left[Q_{t+1}^L \kappa \left[1 - \frac{\gamma_L}{2} (\nabla \tilde{l}_{t+1} - \bar{a})^2 \right] \right] \right\} +$$

$$... \beta_I \mathbb{E}_t \left\{ \varrho_{t+1} \frac{\lambda_{t+1}^I A_{t+1}^{-\sigma}}{P_{t+1}} \left[-Q_{t+1}^L \nabla \tilde{l}_{t+1} \gamma_L (\nabla \tilde{l}_{t+1} - \bar{a})(\nabla \tilde{l}_{t+1} + \kappa) - \kappa \pi_{t+1} \hat{Q}_t^L \right] \right\} \tag{25}$$

$$[x_t^I]: \quad \frac{\varrho_t \lambda_t^H A_t^{-\sigma}}{P_t} \mathbb{E}_t \left\{ \left[1 - \Gamma^H \left(\bar{\omega}_{t+1}^H \right) \right] \left[\Gamma_I' \left(\bar{\omega}_{t+1}^I \right) - \mu_I G_I' \left(\bar{\omega}_{t+1}^I \right) \right] \right\} = \beta_I \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^I A_{t+1}^{-\sigma}}{P_{t+1}} \Gamma_I' \left(\bar{\omega}_{t+1}^I \right) \right\} \tag{26}$$

Regarding the idiosyncratic shock, we assume that $\ln(\omega_t^I) \sim N(-\frac{1}{2}(\sigma_t^I)^2, (\sigma_t^I)^2)$, therefore its unconditional expectation is $\mathbb{E}\{\omega_t^I\} = 1$, and its average conditional on truncation is

$$\mathbb{E}_t \left\{ \omega_t^I | \omega_t^I \geq \bar{\omega}_t^I \right\} = \frac{1 - \Phi \left(z_t^I - \sigma_t^I \right)}{1 - \Phi \left(z_t^I \right)},$$

where Φ is the c.d.f. of the standard normal and z_t^I is an auxiliary variable defined as $z_t^I \equiv {}^{(\ln(\bar{\omega}_t^I) + 0.5(\sigma_t^I)^2)}/_{\sigma_t^I}$. Then, we can obtain the following functional forms:

$$\Gamma_I \left(\bar{\omega}_t^I \right) = \Phi \left(z_t^I - \sigma_t^I \right) + \bar{\omega}_t^I \left(1 - \Phi \left(z_t^I \right) \right)$$

and

$$\Gamma_I \left(\bar{\omega}_t^I \right) - \mu_I G_I \left(\bar{\omega}_t^I \right) = \left(1 - \mu_I \right) \Phi \left(z_t^I - \sigma_t^I \right) + \bar{\omega}_t^I \left(1 - \Phi \left(z_t^I \right) \right)$$

Finally, we allow for fluctuations in the variance of the idiosyncratic shock, as σ_t^I is modeled as an exogenous process.

2.2 Entrepreneurs

As in Clerc et al. (2014), we introduce risk-neutral entrepreneurs that follow an overlapping generations structure, where each generation lives across two consecutive periods. The entrepreneurs are the sole owners of productive capital, which is bought from capital producers to be, in turn, rented to the firms that produce different varieties of the home good.

Entrepreneurs born in period t draw utility in $t+1$ from transferring part of final wealth as dividends, C^e_{t+1}, to unrestricted patient households and from leaving the rest as bequests, N^e_{t+1}, to the next generation of entrepreneurs in the form:

$$\max_{C^e_{t+1},N^e_{t+1}} \left(C^e_{t+1}\right)^{\xi_{\chi_e}\chi_e} \left(N^e_{t+1}\right)^{1-\xi_{\chi_e}\chi_e} \text{ subject to}$$

$$C^e_{t+1} + N^e_{t+1} = \Psi^e_{t+1}$$

where Ψ^e_{t+1} is entrepreneurial wealth at $t+1$, explained below, and ξ_{χ_e} is a stochastic shock to their preferences. The first order conditions to this problem may be written as:

$$[C^e_{t+1}]: \ \xi_{\chi_e}\chi_e(C^e_{t+1})^{(\xi_{\chi_e}\chi_e-1)} \left(N^e_{t+1}\right)^{1-\xi_{\chi_e}\chi_e} - \lambda^{\chi_e}_t = 0$$

$$[N^e_{t+1}]: \ (1-\xi_{\chi_e}\chi_e)(C^e_{t+1})^{\xi_{\chi_e}\chi_e} \left(N^e_{t+1}\right)^{-\xi_{\chi_e}\chi_e} - \lambda^{\chi_e}_t = 0$$

$$[\lambda^{\chi_e}_t]: \ C^e_{t+1} + N^e_{t+1} - \Psi^e_{t+1} = 0$$

From first order conditions we get the following optimal rules

$$C^e_{t+1} = \chi_e \Psi^e_{t+1}$$
$$N^e_{t+1} = (1-\chi_e)\,\Psi^e_{t+1}$$

In their first period, entrepreneurs will try to maximize expected second period wealth, Ψ^e_{t+1}, by purchasing capital at nominal price Q^K_t, which will be productive (and rented) in the next period. These purchases are financed using the resources left as bequests by the previous generation of entrepreneurs and borrowing an amount L^F_t at nominal rate R^L_t from from F banks. In borrowing from banks, entrepreneurs also face an agency problem of the type faced by impatient households i.e. in $t+1$ entrepreneurs receive an idiosyncratic shock to the efficiency units of capital that will ultimately determine their ability to pay their liabilities to banks. Banks cannot observe these shock, but entrepreneurs can. Depreciated capital is sold in the next period to capital producers at Q^K_{t+1}. Entrepreneurial leverage, as measured by assets over equity, is $lev^e_t = {}^{Q^K_t K_t}/_{N^e_t}$.

In this setting, entrepreneurs solve, in their first period,

$$\max_{K_t,L^F_t} \mathbb{E}_t\left(\Psi^e_{t+1}\right) \text{ subject to}$$

$$Q^K_t K_t - L^F_t = N^e_t$$

$$\Psi^e_{t+1} = \max\left[\omega^e_{t+1}\left(R^k_{t+1} + (1-\delta_K)\,Q^K_{t+1}\right)K_t - R^L_t L^F_t,\ 0\right]$$

and a bank participation condition, which will be explained later. The factor ω^e_{t+1} represents the idiosyncratic shock to the entrepreneurs efficiency units of capital. This shock takes place after the loan with the bank has taken place but before renting capital to consumption goods producers. It is assumed that this shock is independently and identically distributed across entrepreneurs and follows a log-normal distribution with an expected value of one. Let

$$R_{t+1}^{e}=\left[\frac{R_{t+1}^{k}+\left(1-\delta_{K}\right)Q_{t+1}^{K}}{Q_{t}^{K}}\right] \tag{27}$$

be the gross nominal return per efficiency unit of capital obtained in period $t+1$ from capital obtained in period t. Then in order for the entrepreneur to pay for its loan the efficiency shock, ω_{t+1}^{e}, must exceed the threshold

$$\bar{\omega}_{t+1}^{e}=\frac{R_{t}^{L}L_{t}^{F}}{R_{t+1}^{e}Q_{t}^{K}K_{t}}$$

If $\omega_{t+1}^{e}\geq\bar{\omega}_{t+1}^{e}$ the entrepreneurs pays $R_{t}^{L}L_{t}^{F}$ to the bank and gets $(\omega_{t+1}^{e}-\bar{\omega}_{t+1}^{e})R_{t+1}^{e}Q_{t}^{K}K_{t}$. Otherwise, the entrepreneurs defaults and receives nothing. While F-banks only recover $(1-\mu_{e})\omega_{t+1}^{e}R_{t+1}^{e}Q_{t}^{K}K_{t}$ from non performing loans, and $R_{t}^{L}L_{t}^{F}$ from performing loans. With the threshold, we can define $PD_{t}^{e}=F_{e}(\bar{\omega}_{t}^{e})$ as the default rate of entrepreneurs on their loans.

The share of the gross return that goes to the bank is denoted as $\Gamma_{e}(\bar{\omega}_{t+1}^{e})$ whereas the share of gross return that goes to the entrepreneur is $(1-\Gamma_{e}(\bar{\omega}_{t+1}^{e}))$ where:

$$\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)=\int_{0}^{\bar{\omega}_{t+1}^{e}}\omega_{t+1}^{e}f_{e}\left(\omega_{t+1}^{e}\right)d\omega_{t+1}^{e}+\bar{\omega}_{t+1}^{e}\int_{\bar{\omega}_{t+1}^{e}}^{\infty}f_{e}\left(\omega_{t+1}^{e}\right)d\omega_{t+1}^{e}$$

also let

$$G_{e}\left(\bar{\omega}_{t+1}^{e}\right)=\int_{0}^{\bar{\omega}_{t+1}^{e}}\omega_{t+1}^{e}f_{e}\left(\omega_{t+1}^{e}\right)d\omega_{t+1}^{e}$$

denote the part of those returns that come from the defaulted loans. Taking into consideration the share of the return that is lost due to verification cost as $\mu_{e}G_{e}\left(\bar{\omega}_{t+1}^{e}\right)$, then the net share of return that goes to the bank is

$$\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)-\mu_{e}G_{e}\left(\bar{\omega}_{t+1}^{e}\right).$$

Taking this into account then the maximization problem of the entrepreneur can be written as

$$\max_{\bar{\omega}_{t+1}^{e},K_{t}}\mathbb{E}_{t}\left\{\Psi_{t+1}^{e}\right\}=\mathbb{E}_{t}\left\{\left[1-\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)\right]R_{t+1}^{e}Q_{t}^{K}K_{t}\right\},\ \text{subject to}$$

$$\mathbb{E}_{t}\left\{\left[1-\Gamma_{F}\left(\bar{\omega}_{t+1}^{F}\right)\right]\left[\Gamma_{e}\left(\bar{\omega}_{t+1}^{e}\right)-\mu_{e}G_{e}\left(\bar{\omega}_{t+1}^{e}\right)\right]R_{t+1}^{e}Q_{t}^{K}K_{t}\right\}\geq\rho_{t+1}^{F}\phi_{F}L_{t}^{F}, \tag{28}$$

that says that banks will participate in the contract only if its net expected profits are at least equal to their alternative use of funds. This yields the following optimality conditions

$$\left(1-\Gamma_{t+1}^{e}\right)=\lambda_{t}^{e}\left(\frac{\rho_{t+1}^{F}\phi_{t}^{F}}{R_{t+1}^{e}}-\left(1-\Gamma_{t+1}^{F}\right)\left[\Gamma_{t+1}^{e}-\mu^{e}G_{t+1}^{e}\right]\right) \tag{29}$$

$$\Gamma_{t+1}^{e'}=\lambda_{t}^{e}\left(1-\Gamma_{t+1}^{F}\right)\left[\Gamma_{t+1}^{e'}-\mu^{e}G_{t+1}^{e'}\right] \tag{30}$$

Further, it is assumed that $\ln(\omega_t^e) \sim N(-0.5(\sigma_t^e)^2, (\sigma_t^e)^2)$, leading to analogous properties as with impatient households for $\bar{\omega}_t^e$, Γ_e and G_e.

2.3 Bankers and Banks

2.3.1 Bankers

Bankers are modeled as in Clerc et al. (2014) and in a similar way to entrepreneurs: They belong to a sequence of overlapping generations of risk-neutral agents who live 2 periods and have exclusive access to the opportunity of investing their wealth as banks' inside equity capital.

In the first period, the banker receives a bequest N_t^b from the previous generation of bankers and must distribute it across the two types of existing banks: banks specializing in corporate loans (F banks) and banks specializing in housing loans (H banks). That is, a banker who chooses to invest an amount E_t^F of inside equity in F banks will invest the rest of her bequest in H banks, $E_t^H = N_t^b - E_t^F$. Then, in the second period bankers receive their returns from both investments, and must choose how to distribute their net worth Ψ_{t+1}^b between transferring dividends C_{t+1}^b to households and leaving bequests N_{t+1}^b to the next generation. Additionally, disturbances to the exogenous variable $\xi_t^{\chi_b}$ capture transitory fluctuations in the banker's dividend policy

Given Ψ_{t+1}^b, the banker will distribute it by solving the following maximization problem:

$$\max_{C_{t+1}^b, N_{t+1}^b} \left(C_{t+1}^b\right)^{\xi_{t+1}^{\chi_b}\chi^b} \left(N_{t+1}^b\right)^{1-\xi_{t+1}^{\chi_b}\chi^b} \text{ , subject to}$$

$$C_{t+1}^b + N_{t+1}^b = \Psi_{t+1}^b$$

which leads to the following optimal rules

$$C_{t+1}^b = \xi_{t+1}^{\chi_b}\chi^b \Psi_{t+1}^b \tag{31}$$

$$N_{t+1}^b = \left(1 - \xi_{t+1}^{\chi_b}\chi^b\right) \Psi_{t+1}^b \tag{32}$$

In turn, net worth in the second period is determined by the returns on bankers' investments in period-t:

$$\Psi_{t+1}^b = \rho_{t+1}^F E_t^F + \xi_t^{b,roe} \rho_{t+1}^H \left(N_t^b - E_t^F\right)$$

where $\xi_t^{b,roe}$ is a shock to the bankers' required return to equity invested in the housing branches, ρ_{t+1}^j is the period $t+1$ ex-post gross return on inside equity E_t^j invested in period t in bank of class j. In order to capture the fact that most of mortgage debt takes the form of non endorsable debt —meaning the issuer bank retains it in its balance sheet to maturity— we assume that the banker $j = H$ invests in the banking project H through a mutual fund which pays the expected average return to housing equity ρ_{t+1}^H every period. Thus, letting $\tilde{\rho}_t^H$ represent the period return on housing portfolio, then $\rho_t^H = \kappa\tilde{\rho}_t^H + (1-\kappa)\rho_{t+1}^H$. The banker then chooses

$$\max_{E_t^F} \mathbb{E}_t \left\{\Psi_{t+1}^b\right\} = \mathbb{E}_t \left\{\rho_{t+1}^F E_t^F + \xi_t^{b,roe} \rho_{t+1}^H \left(N_t^b - E_t^F\right)\right\}$$

An interior equilibrium in which both classes of banks receive strictly positive inside equity from bankers will require the following equality to hold:

$$\mathbb{E}_t \left\{\rho_{t+1}^F\right\} = \mathbb{E}_t \left\{\xi_t^{b,roe} \rho_{t+1}^H\right\} = \bar{\rho}_t$$

where $\bar{\rho}_t$ denotes banks' required expected gross rate of return on equity investment undertaken at time t.

2.3.2 Banks

Banks are institutions specialized in extending either corporate or housing loans drawing funds through deposits, and bonds from unconstrained household, and equity from bankers. We assume a continuum of identical banking institutions of j class banks $j = \{F, H\}$. In particular, banks of class j are investment projects created in period-t that in $t+1$ generate profits Π_{t+1}^j before being liquidated with:

$$\Pi_{t+1}^F = \max\left[\omega_{t+1}^F \tilde{R}_{t+1}^F L_t^F - R_t^D D_t^F, 0\right], \quad \Pi_{t+1}^H = \max\left[\omega_{t+1}^H \tilde{R}_{t+1}^H Q_t^L L_t^H - R_{t+1}^{BB} Q_{t+1}^{BB} BB_t, 0\right]$$

where $\tilde{R}_{t+1}^j$ is the realized return on a well-diversified portfolio of loans to entrepreneurs or households and ω_{t+1}^j is an idiosyncratic portfolio return shock, which is i.i.d across banks of class j with a cdf of $F_j(\omega_{t+1}^j)$ and pdf $f_j(\omega_{t+1}^j)$. Due to limited liability, the equity payoff may not be negative, which defines thresholds $\bar{\omega}_{t+1}^j$:

$$\bar{\omega}_{t+1}^F \equiv \frac{R_t^D D_t^F}{\tilde{R}_{t+1}^F L_t^F}, \qquad \bar{\omega}_{t+1}^H \equiv \frac{R_{t+1}^{BB} Q_{t+1}^{BB} BB_t}{\tilde{R}_{t+1}^H Q_t^L L_t^H}$$

Similar to households and entrepreneurs, $\Gamma_j(\bar{\omega}_{t+1}^j)$ denotes the share of gross returns to bank j investments which are either paid back to depositors or bond holders, implying that $[1-\Gamma_j(\bar{\omega}_{t+1}^j)]$ is the share that the banks will keep as profits. We also define $G_j(\bar{\omega}_{t+1}^j)$ as the share of bank j assets which belong to defaulting j banks, and thus $\mu_j G_j(\bar{\omega}_{t+1}^j)$ is the total cost of bank j defaults expressed as a fraction of total bank j assets.

The balance sheet of banks of class F is given by $L_t^F = E_t^F + D_t^F$, and they face a regulatory capital constraint given by $E_t^F \geq \phi_F L_t^F$, where ϕ_F is the capital-to-asset ratio, and is binding at all times in equilibrium so that the loans can be written as $L_t^F = E_t^F/\phi_F$ and the deposits as $D_t^F = (1-\phi_F/\phi_F)E_t^F$. Likewise, balance sheet of banks of class H is given by $Q_t^L L_t^H = E_t^H + Q_t^{BB} BB_t$, with binding capital regulation determining $E_t^H = \phi_H Q_t^L L_t^H$, and

$Q_t^{BB} BB_t = {}^{(1-\phi_H)}/_{\phi_H} E_t^H$. Further, using the threshold definitions and the binding capital constraints, we obtain[2]

$$\bar{\omega}_{t+1}^F = (1-\phi_F)\,\frac{R_t^D}{\tilde{R}_{t+1}^F}$$
$$\bar{\omega}_{t+1}^H = (1-\phi_H)\,\frac{R_{t+1}^{BB}}{\tilde{R}_{t+1}^H}\left(\frac{Q_{t+1}^{\widehat{BB}}}{Q_t^{\widehat{BB}}}\right)$$

Finally, we define the realized rate of return of equity invested in a bank of class j:

$$\rho_{t+1}^j = \left[1-\Gamma_j\left(\bar{\omega}_{t+1}^j\right)\right]\frac{\tilde{R}_{t+1}^j}{\phi_j} \tag{33}$$

For completeness, notice that derivations in prior sections imply that following expressions for $\tilde{R}_{t+1}^j,\ j=\{F,H\}:$

$$\tilde{R}_{t+1}^F = \left(\Gamma_e\left(\bar{\omega}_{t+1}^e\right) - \mu_e G_e\left(\bar{\omega}_{t+1}^e\right)\right)\frac{R_{t+1}^e Q_t^K K_t}{L_t^F}$$

$$\tilde{R}_{t+1}^H = \left(\Gamma_I\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I\left(\bar{\omega}_{t+1}^I\right)\right)\frac{R_{t+1}^H Q_t^H H_t^I}{Q_t^L L_t^H}$$

As with households and entrepreneurs, it is assumed that the bank idiosyncratic shock follows a log-normal distribution: $\log(\omega_t^j) \sim N(-\frac{1}{2}(\sigma_t^j)^2, (\sigma_t^j)^2)$, leading to analogous properties for $\bar{\omega}_t^j$, Γ_j and G_j.

2.4 Production

The supply side of the economy is composed by different types of firms that are all owned by the households. Monopolistically competitive unions act as wage setters by selling household's differentiated varieties of labor supply n_{it} to a perfectly competitive firm, which packs these varieties into a composite labor service $\widetilde{n}_t$. There is a set of monopolistically competitive firms producing different varieties of a home good, Y_{jt}^H, using wholesale good X_t^Z as input; a set of monopolistically competitive importing firms that import a homogeneous foreign good to transform it into varieties, X_{jt}^F; and three groups of perfectly competitive firms that aggregate products: one packing different varieties of the home good into a composite home good, X_t^H, one packing the imported varieties into a composite foreign good, X_t^F, and, finally, another one that bundles the composite home and foreign goods to create a final good, Y_t^C. This final good is purchased by households (C_t^P,C_t^I), capital and housing producers (I_t^K,I_t^H), and the government (G_t).

Similarly to Clerc et al. (2014), we model perfectly competitive capital-producing and housing-producing firms. Both types of firms are owned by patient households and their technology is subject to an adjustment cost. They produce new units of capital and housing from the final good and sell them to entrepreneurs and households respectively. However, we depart from Clerc et al. (2014) by assuming time-to-build frictions in housing investment. Finally, there is a set of competitive firms producing a homogeneous commodity good that is exported abroad (and

[2] As with impatient households, to avoid excessive volatility of the default threshold due to the influence of the revaluation of long term debt, we model the default decision based on a smoothed valuation of the outstanding debt, $Q_t^{\widehat{BB}}$, where $\log Q_t^{\widehat{BB}} \equiv \alpha_{Q^{BB}}^1(\alpha_{Q^{BB}}^2 \log Q_{t-1}^{\widehat{BB}} + (1-\alpha_{Q^{BB}}^2)\log Q^{BB}) + (1-\alpha_{Q^{BB}}^1)\log Q_t^{BB}$.

which follows an exogenous process). The total mass of firms in each sector is normalized to one.

2.4.1 Capital goods

There is a continuum of competitive capital firm producers who buy an amount I_t of final goods at price P_t and use their technology to satisfy the demand for new capital goods not covered by depreciated capital, i.e. $K_t - (1-\delta_K)K_{t-1}$, where new units of capital are sold at price Q_t^K. As is usual in the literature, we assume that the aggregate stock of new capital considers investment adjustment costs and evolves according to following law of motion:

$$K_t = (1-\delta_K)\,K_{t-1} + \left[1 - \frac{\gamma_K}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2\right]\xi_t^i I_t$$

Where ξ_t^i is a shock to investment efficiency. Therefore a representative capital producer chooses how much to invest in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i}\left\{Q_{t+i}^K\left[1 - \frac{\gamma_K}{2}\left(\frac{I_{t+i}}{I_{t+i-1}} - a\right)^2\right]\xi_{t+i}^i I_{t+i} - P_{t+i}I_{t+i}\right\}$$

Discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of capital to the level of investment

$$\begin{aligned} P_t &= Q_t^K\left\{\left(1 - \frac{\gamma_K}{2}\left(\frac{I_t}{I_{t-1}} - a\right)^2\right) - \gamma_K\left(\frac{I_t}{I_{t-1}} - a\right)\frac{I_t}{I_{t-1}}\right\}\xi_t^i \\ &\quad + E_t\left\{r_{t,t+1}Q_{t+1}^K\gamma_K\left(\frac{I_{t+1}}{I_t} - a\right)\left(\frac{I_{t+1}}{I_t}\right)^2\xi_{t+1}^i\right\} \end{aligned} \tag{34}$$

2.4.2 Housing goods

The structure of housing producers is similar to that of capital good producers with the difference that housing goods also face investment adjustment costs in the form of time to build Kydland and Prescott (1982) and Uribe and Yue (2006). As such, there is a continuum of competitive housing firm producers who authorize housing investment projects I_t^{AH} in period t, which will increase housing stock N_H periods later, the time it takes to build.[3] Thus, the law of motion for the aggregate stock of housing in H_t will consider projects authorized N_H periods before, and includes investment adjustment costs,

$$H_t = (1-\delta_H)\,H_{t-1} + \left[1 - \frac{\gamma_H}{2}\left(\frac{I_{t-N_H}^{AH}}{I_{t-N_H-1}^{AH}} - a\right)^2\right]\xi_{t-N_H}^{ih} I_{t-N_H}^{AH}$$

where ξ_t^{ih} is a shock to housing investment efficiency, and the sector covers all demand for new housing, $H_t - (1-\delta_H)H_{t-1}$, by selling units at price Q_t^H.

The firm's effective expenditure is spread out during the periods that new housing is being built. In particular,

[3]Notice that if $N_H = 0$, the structure is symmetric to the capital producers.

the amount of final goods purchased (at price P_t) by the firm in t to produce housing is given by

$$I_t^H = \sum_{j=0}^{N_H} \varphi_j^H I_{t-j}^{AH}$$

Where φ_j^H (the fraction of projects authorized in period $t-j$ that is outlaid in period t) satisfy $\sum_{j=0}^{N_H} \varphi_j^H = 1$ and $\varphi_j^H = \rho^{\varphi H} \varphi_{j-1}^H$.[4]

Therefore a representative housing producer chooses how much to authorize in new projects I_t^{AH} in order to maximize the discounted utility of its profits,

$$\sum_{i=0}^{\infty} r_{t,t+i} \left\{ Q_{t+i}^H \left[1 - \frac{\gamma_H}{2} \left(\frac{I_{t-N_H+i}^{AH}}{I_{t-N_H+i-1}^{AH}} - a \right)^2 \right] \xi_{t-N_H+i}^{ih} I_{t-N_H+i}^{AH} - P_{t+i} I_{t+i}^H \right\}$$

Where discounting is done according to patient households' preferences, who are the owners of the firms. From the first order condition a new relation can be obtained that relates the price of housing to the level of housing investment

$$\begin{aligned} E_t \sum_{j=0}^{N_H} r_{t,t+j} \varphi_j^H P_{t+j} &= E_t r_{t,t+N_H} Q_{t+N_H}^H \left\{ \left[1 - \frac{\gamma_H}{2} \left(\frac{I_t^{AH}}{I_{t-1}^{AH}} - a \right)^2 \right] - \gamma_H \left(\frac{I_t^{AH}}{I_{t-1}^{AH}} - a \right) \frac{I_t^{AH}}{I_{t-1}^{AH}} \right\} \xi_t^{ih} \\ &\quad + E_t r_{t,t+N_H+1} Q_{t+N_H+1}^H \left\{ \gamma_H \left(\frac{I_{t+1}^{AH}}{I_t^{AH}} - a \right) \left(\frac{I_{t+1}^{AH}}{I_t^{AH}} \right)^2 \xi_{t+1}^{ih} \right\} \end{aligned} \tag{35}$$

2.4.3 Final goods

A representative final goods firm demands composite home and foreign goods in the amounts X_t^H and X_t^F, respectively, and combines them according to the following technology:

$$Y_t^C = \left[\omega^{1/\eta} \left(X_t^H \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(X_t^F \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \tag{36}$$

where $\omega \in (0,1)$ is inversely related to the degree of home bias and $\eta > 0$ measures the substitutability between domestic and foreign goods. The selling price of this final good is denoted by P_t, while the prices of the domestic and foreign inputs are P_t^H and P_t^F, respectively. Subject to the technology constraint (36), the firm maximizes its profits over the inputs, taking prices as given:

$$\max_{X_t^H, X_t^F} P_t \left[\omega^{1/\eta} \left(X_t^H \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(X_t^F \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} - P_t^H X_t^H - P_t^F X_t^F$$

[4]Notice that $\rho^{\varphi H} > 1$ implies that expenditure for any authorized project is back-loaded (increasing over time), while the converse is true for $\rho^{\varphi H} < 1$.

The first-order conditions of this problem determine the optimal input demands:

$$X_t^H = \omega \left(\frac{P_t^H}{P_t}\right)^{-\eta} Y_t^C \tag{37}$$

$$X_t^F = (1-\omega) \left(\frac{P_t^F}{P_t}\right)^{-\eta} Y_t^C \tag{38}$$

Combining these optimality conditions and using that zero profits hold in equilibrium, we can write

$$P_t = \left[\omega \left(P_t^H\right)^{1-\eta} + (1-\omega)\left(P_t^F\right)^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{39}$$

2.4.4 Home composite goods

A representative home composite goods firm demands home goods of all varieties $j \in [0,1]$ in amounts X_{jt}^H and combines them according to the technology

$$Y_t^H = \left[\int_0^1 \left(X_{jt}^H\right)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H - 1}} \tag{40}$$

with $\epsilon_H > 0$. Let P_{jt}^H denote the price of the home good of variety j. Subject to the technology constraint (40), the firm maximizes its profits $\Pi_t^H = P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj$ over the input demands X_{jt}^H taking prices as given:

$$\max_{X_{jt}^H} P_t^H \left[\int_0^1 \left(X_{jt}^H\right)^{\frac{\epsilon_H - 1}{\epsilon_H}} dj\right]^{\frac{\epsilon_H}{\epsilon_H - 1}} - \int_0^1 P_{jt}^H X_{jt}^H dj$$

This implies the following first-order conditions for all j:

$$\partial X_{jt}^H : \; P_t^H \left(Y_t^H\right)^{1/\epsilon_H} \left(X_{jt}^H\right)^{-1/\epsilon_H} - P_{jt}^H = 0$$

such that the input demand functions are

$$X_{jt}^H = \left(\frac{P_{jt}^H}{P_t^H}\right)^{-\epsilon_H} Y_t^H \tag{41}$$

Substituting (41) into (40) yields the price of home composite goods:

$$P_t^H = \left[\int_0^1 \left(P_{jt}^H\right)^{1-\epsilon_H} dj\right]^{\frac{1}{1-\epsilon_H}} \tag{42}$$

2.4.5 Home goods of variety j

There is a continuum of j's firms, with measure one, that demand a domestic wholesale good X_t^Z and differentiate into home goods varieties Y_{jt}^H. To produce one unit of variety j, firms need one unit of input according to

$$\int_0^1 Y_{jt}^H dj = X_t^Z \tag{43}$$

The firm producing variety j satisfies the demand given by (41) but it has monopoly power for its variety. For varieties, the nominal marginal cost in terms of the composite good price is given by $P_t^H mc_{jt}^H$. Given that, every firm buys their input from the same wholesale market. It implies that all of them face the same nominal marginal costs

$$P_t^H mc_{jt}^H = P_t^H mc_t^H = P_t^Z \tag{44}$$

Given nominal marginal costs $P_t^H mc_{jt}^H$, firm j chooses its price P_{jt}^H to maximize profits. In setting prices, the firm faces a Calvo-type problem, whereby each period the firm can change its price optimally with probability $1-\theta_H$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_H \in [0,1]$ and $1-\kappa_H$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^H$ that maximizes the current market value of the profits generated until it can reoptimize again. [5] As the firms are owned by the households, profits are discounted using the households' stochastic discount factor for nominal payoffs, $r_{t,t+s}$. A reoptimizing firm, therefore, solves the following problem:

$$\max_{\tilde{P}_{jt}^H} \; E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left(P_{jt+s}^H - P_{t+s}^H mc_{jt+s}^H \right) Y_{jt+s}^H \quad \text{s.t.} \quad Y_{jt+s}^H = X_{jt+s}^H = \left(\frac{\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H}}{P_{t+s}^H} \right)^{-\epsilon_H} Y_{t+s}^H$$

which can be rewritten as

$$\max_{\tilde{P}_{jt}^H} \; E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \left[\left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{1-\epsilon_H} \left(P_{t+s}^H \right)^{\epsilon_H} - mc_{jt+s}^H \left(\tilde{P}_{jt}^H \Pi_{i=1}^s \pi_{t+i}^{I,H} \right)^{-\epsilon_H} \left(P_{t+s}^H \right)^{1+\epsilon_H} \right] Y_{t+s}^H$$

[5] Therefore, the following relation holds: $P_{jt+s}^H = \tilde{P}_{jt}^H \pi_{t+1}^{I,H} \dots \pi_{t+s}^{I,H}$, where $\pi_t^{I,H} = \left(\pi_{t-1}^H\right)^{\kappa_H} \left(\pi_t^T\right)^{1-\kappa_H}$, $\pi_t^H = P_t^H / P_{t-1}^H$, and π_t^T denotes the inflation target in period t.

The first-order conditions determining the optimal price $\tilde{P}_t^H$ can be written as follows:[6]

$$
\begin{aligned}
0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[(1-\epsilon_H) \left(\tilde{P}_t^H\right)^{-\epsilon_H} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(P_{t+s}^H\right)^{\epsilon_H} \\
&\qquad + \epsilon_H mc_{t+s}^H \left(\tilde{P}_t^H\right)^{-\epsilon_H - 1} \left(\Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(P_{t+s}^H\right)^{1+\epsilon_H} \Bigg] Y_{t+s}^H \\
\Leftrightarrow \quad 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \frac{\left(P_{t+s}^H\right)^{\epsilon_H}}{P_t^H} \\
&\qquad - mc_{t+s}^H \left(\tilde{P}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \frac{\left(P_{t+s}^H\right)^{1+\epsilon_H}}{P_t^H} \Bigg] Y_{t+s}^H \\
\Leftrightarrow \quad 0 &= E_t \sum_{s=0}^{\infty} \theta_H^s r_{t,t+s} \Bigg[\frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} \\
&\qquad - mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H} \Bigg] Y_{t+s}^H
\end{aligned}
$$

where the second step follows from multiplying both sides by $-\tilde{P}_t^H/(P_t^H \epsilon_H)$, and the third by defining $\tilde{p}_t^H = \tilde{P}_t^H/P_t^H$. The first-order condition can be rewritten in recursive form as follows, defining $F_t^{H_1}$ as

$$
\begin{aligned}
F_t^{H_1} &= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} Y_{t+s}^H \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{\epsilon_H} Y_{t+s+1}^H \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + \theta_H E_t \Bigg\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} \frac{\epsilon_H - 1}{\epsilon_H} \\
&\qquad \times \left(\tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H}\right)^{1-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{\epsilon_H} Y_{t+s+1}^H \Bigg\} \\
&= \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H\right)^{1-\epsilon_H} Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{1-\epsilon_H} \left(\pi_{t+1}^H\right)^{\epsilon_H} F_{t+1}^{H_1} \right\} \qquad (45)
\end{aligned}
$$

[6] Notice that the subscript j has been removed from $\tilde{P}_t^H$; this simplifies notation and underlines that the prices chosen by all firms j that reset prices optimally in a given period are equal as they face the same problem by (44).

and, analogously, $F_t^{H_2}$ as

$$
\begin{aligned}
F_t^{H_2} &= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=1}^{\infty} \theta_H^s r_{t,t+s} mc_{t+s}^H \left(\tilde{p}_t^H \Pi_{i=1}^s \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s}^H}{P_t^H}\right)^{1+\epsilon_H} Y_{t+s}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + E_t \sum_{s=0}^{\infty} \theta_H^{s+1} r_{t,t+s+1} mc_{t+s+1}^H \left(\tilde{p}_t^H \Pi_{i=1}^{s+1} \pi_{t+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_t^H}\right)^{1+\epsilon_H} Y_{t+s+1}^H \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} \sum_{s=0}^{\infty} \theta_H^s r_{t+1,t+s+1} mc_{t+s+1}^H \right. \\
&\qquad \left. \times \left(\tilde{p}_{t+1}^H \Pi_{i=1}^s \pi_{t+1+i}^{I,H}\right)^{-\epsilon_H} \left(\frac{P_{t+s+1}^H}{P_{t+1}^H}\right)^{1+\epsilon_H} Y_{t+s+1}^H \right\} \\
&= \left(\tilde{p}_t^H\right)^{-\epsilon_H} mc_t^H Y_t^H + \theta_H E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H}\right)^{-\epsilon_H} \left(\pi_{t+1}^H\right)^{1+\epsilon_H} F_{t+1}^{H_2} \right\} \qquad (46)
\end{aligned}
$$

such that

$$
F_t^{H_1} = F_t^{H_2} = F_t^H \qquad (47)
$$

Using (42), we have

$$
\begin{aligned}
1 &= \int_0^1 \left(\frac{P_{jt}^H}{P_t^H}\right)^{1-\epsilon_H} dj \\
&= (1-\theta_H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H \left(\frac{P_{t-1}^H \pi_t^{I,H}}{P_t^H}\right)^{1-\epsilon_H} \\
&= (1-\theta_H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{1-\epsilon_H} \qquad (48)
\end{aligned}
$$

The second equality above follows from the fact that, under Calvo pricing, the distribution of prices among firms not reoptimizing in period t corresponds to the distribution of aggregate prices in period $t-1$, though with total mass reduced to θ_H.

2.4.6 Wholesale Domestic Goods

There is a representative firm producing a homogeneous wholesale home good, combining capital and labor according to the following technology:

$$
Y_t^Z = z_t K_{t-1}^{\alpha} \left(A_t \widetilde{n}_t\right)^{1-\alpha} \qquad (49)
$$

with capital share $\alpha \in (0,1)$, an exogenous stationary technology shock z_t and a non-stationary technology A_t. Production of the wholesale good composite labor services $\widetilde{n}_t$ and capital K_{t-1}. Additionally, following Lechthaler and Snower (2010), the firm faces a quadratic adjustment costs of labor which is a function of parameter γ_n, and of aggregate wholesale domestic goods $\widetilde{Y}_t^Z$, which in equilibrium are equal to Y_t^Z and which the representative firm takes as given. In a first stage, the firm hires composite labor and rents capital to solve the following problem:

$$\min_{\widetilde{n}_{t+s},K_{t+s-1}} \sum_{s=0}^{\infty} r_{t,t+s} \left\{ W_{t+s}\widetilde{n}_{t+s} + \frac{\gamma_n}{2}\left(\frac{\widetilde{n}_{t+s}}{\widetilde{n}_{t+s-1}} - 1\right)^2 \widetilde{Y_{t+s}}^Z P_t^Z + R_t K_{t+s-1} \right\}$$

$$s.t. \quad Y_{t+s}^Z = X_{t+s}^Z = z_{t+s} K_{t+s-1}^{\alpha} \left(A_{t+s}\widetilde{n}_{t+s}\right)^{1-\alpha}$$

Then, the optimal capital and labor demands are given by:

$$\widetilde{n}_t = (1-\alpha)\left\{ \frac{mc_t^Z Y_t^Z}{W_t + \gamma_n\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}_t^Z P_t^Z - r_{t,t+1}\gamma_n \mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2}\right)\widetilde{Y}_{t+1}^Z P_{t+1}^Z} \right\} \tag{50}$$

$$K_{t-1} = \alpha\left(\frac{mc_t^Z}{R_t^k}\right) Y_t^Z \tag{51}$$

Where mc_t^Z is the lagrangian multiplier on the production function and $r_{t,t+1}$ the households' stochastic discount factor between periods t and $t+1$. The, combining both optimality conditions:

$$\frac{K_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) R_t^k}\left\{ W_t + \gamma_n\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}_t^Z P_t^Z - r_{t,t+1}\gamma_n \mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2}\right)\widetilde{Y}_{t+1}^Z P_{t+1}^Z \right\}$$

Substituting (50) and (51) into (49) we obtain an expression for the real marginal cost in units of the wholesale domestic good:

$$mc_t^Z = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \frac{\left(R_t^k\right)^{\alpha}}{z_t A_t^{1-\alpha}} \left\{ W_t + \gamma_n\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)\left(\frac{1}{\widetilde{n}_{t-1}}\right)\widetilde{Y}_t^Z P_t^Z \right.$$
$$\left. - r_{t,t+1}\gamma_n \mathbb{E}_t\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1\right)\left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2}\right)\widetilde{Y}_{t+1}^Z P_{t+1}^Z \right\}^{1-\alpha}$$

In a second stage, the wholesale firm maximize its profits from the production of Y_t^Z, which is sold as X_t^Z at P_t^Z. The problem is:

$$\max_{Y_t^Z} \left(P_t^Z - mc_t^Z\right) Y_t^Z$$

The first-order condition implies that

$$P_t^Z = mc_t^Z.$$

2.4.7 Foreign composite goods

As in the case of home composite goods, a representative foreign composite goods firm demands foreign goods of all varieties $j \in [0,1]$ in amounts X_{jt}^F and combines them according to the technology

$$Y_t^F = \left[\int_0^1 \left(X_{jt}^F\right)^{\frac{\epsilon_F - 1}{\epsilon_F}} dj\right]^{\frac{\epsilon_F}{\epsilon_F - 1}} \tag{52}$$

with $\epsilon_F > 0$. Let P_{jt}^F denote the price of the foreign good of variety j. Analogously to the case of home composite goods, profit maximization yields the input demand functions

$$X_{jt}^F = \left(\frac{P_{jt}^F}{P_t^F} \right)^{-\epsilon_F} Y_t^F \tag{53}$$

for all j, and substituting (53) into (52) yields the price of foreign composite goods:

$$P_t^F = \left[\int_0^1 \left(P_{jt}^F \right)^{1-\epsilon_F} dj \right]^{\frac{1}{1-\epsilon_F}} \tag{54}$$

2.4.8 Foreign goods of variety j

Importing firms buy an amount M_t of a homogeneous foreign good at the price $P_t^{M\star}$ abroad and convert this good into varieties Y_{jt}^F that are sold domestically, and where total imports are $\int_0^1 Y_{jt}^F dj$. We assume that the import price level $P_t^{M\star}$ cointegrates with the foreign producer price level $P_t^\star$, i.e., $P_t^{M\star} = P_t^\star \xi_t^m$, where ξ_t^m is a stationary exogenous process. The firm producing variety j satisfies the demand given by (53) but it has monopoly power for its variety. As it takes one unit of the foreign good to produce one unit of variety j, nominal marginal costs in terms of composite goods prices are

$$P_t^F mc_{jt}^F = P_t^F mc_t^F = S_t P_t^{M\star} = S_t P_t^\star \xi_t^m \tag{55}$$

Given marginal costs, the firm producing variety j chooses its price P_{jt}^F to maximize profits. In setting prices, the firm faces a Calvo-type problem similar to domestic firms, whereby each period the firm can change its price optimally with probability $1 - \theta_F$, and if it cannot optimally change its price, it indexes its previous price according to a weighted product of past and steady state inflation with weights $\kappa_F \in [0, 1]$ and $1 - \kappa_F$ respectively. A firm reoptimizing in period t will choose the price $\tilde{P}_{jt}^F$ that maximizes the current market value of the profits generated until it can reoptimize.[7] The solution to this problem is analogous to the case of domestic varieties, implying the first-order condition

$$F_t^{F_1} = F_t^{F_2} = F_t^F \tag{56}$$

where, defining $\tilde{p}_t^F = \tilde{P}_t^F / P_t^F$,

$$F_t^{F_1} = \frac{\epsilon_F - 1}{\epsilon_F} \left(\tilde{p}_t^F \right)^{1-\epsilon_F} Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\pi_{t+1}^F \right)^{\epsilon_F} F_{t+1}^{F_1} \right\}$$

and

$$F_t^{F_2} = \left(\tilde{p}_t^F \right)^{-\epsilon_F} mc_t^F Y_t^F + \theta_F E_t \left\{ r_{t,t+1} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\pi_{t+1}^F \right)^{1+\epsilon_F} F_{t+1}^{F_2} \right\}$$

[7] As in the home varieties case, the following relation holds: $P_{jt+s}^F = \tilde{P}_{jt}^F \pi_{t+1}^{I,F} \ldots \pi_{t+s}^{I,F}$, where $\pi_t^{I,F} = (\pi_{t-1}^F)^{\kappa_F} (\pi_t^T)^{1-\kappa_F}$, and, in turn, $\pi_t^F = P_t^F / P_{t-1}^F$.

Using (54), we further have

$$1 = (1-\theta_F)\left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F\left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{1-\epsilon_F} \tag{57}$$

2.4.9 Wages

Recall that demand for productive labor is satisfied by perfectly competitive packing firms that demand all varieties $i \in [0,1]$ of labor services in amounts $n_t(i)$ and combine them in order to produce composite labor services $\widetilde{n}_t$. The production function, variety i demand, and aggregate nominal wage are respectively given by:

$$\widetilde{n}_t = \left[\int_0^1 n_t(i)^{\frac{\epsilon_W - 1}{\epsilon_W}}\, di\right]^{\frac{\epsilon_W}{\epsilon_W - 1}}, \qquad \epsilon_W > 0. \tag{58}$$

$$n_t(i) = \left(\frac{W_t(i)}{W_t}\right)^{-\epsilon_W}\widetilde{n}_t \tag{59}$$

$$W_t = \left[\int_0^1 W_t(i)^{1-\epsilon_W}\, di\right]^{\frac{1}{1-\epsilon_W}}. \tag{60}$$

Regarding the supply of differentiated labor, as in Erceg et al. (2010), there is a continuum of monopolistically competitive unions indexed by $i \in [0,1]$, which act as wage setters for the differentiated labor services supplied by households. These unions allocate labor demand uniformly across patient and impatient households, so $n_t^P(i) = n_t^I(i)$ and $n_t^P(i) + n_t^I(i) = n_t(i)\ \forall i, t$, with $n_t^P(i) = \wp_U n_t^U(i) + (1-\wp_U)\, n_t^R(i)$, which also holds for the aggregate n_t^P, n_t^I and n_t.

The union supplying variety i satisfies the demand given by (59) but it has monopoly power for its variety. Wage setting is subject to a Calvo-type problem, whereby each period a union can set its nominal wage optimally with probability $1-\theta_W$. The wages of unions that cannot optimally adjust, are indexed to a weighted average of past and steady state productivity and inflation, with a gross growth rate of

$$\pi_t^{I,W} \equiv a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W}$$

Where $\Gamma_{t,s}^W = \Pi_{i=1}^s \pi_{t+i}^{I,W}$ is the growth of indexed wages s periods ahead of t. A union reoptimizing in period t chooses the wage $\widetilde{W}_t$ (equal for patient and impatient households) that maximizes the households' discounted lifetime utility. This union weights the benefits of wage income by considering the agents' marginal utility of consumption –which will usually differ between patient and impatient households– and weighs each household equally by considering a lagrangian multiplier of $\lambda_t^W = \left(\lambda_t^P + \lambda_t^I\right)/2$, with $\lambda_t^P = \wp_U \lambda_t^U + (1-\wp_U)\,\lambda_t^R$. We assume, for the sake of simplicity, that $\beta_W = (\beta_P + \beta_I)/2$ with $\beta_P = \wp_U \beta_U + (1-\wp_U)\,\beta_R$, and $\Theta_t = \left(\Theta_t^P + \Theta_t^I\right)/2$ with $\Theta_t^P = \wp_U \Theta_t^U + (1-\wp_U)\,\Theta_t^R$.

All things considered, taking the aggregate nominal wage as given, the union i's maximization problem can be expressed as

$$\max_{\widetilde{W}_t(i)} E_t \sum_{s=0}^{\infty} (\beta_U \theta_W)^s \varrho_{t+s} \left(\frac{\lambda_{t+s}^U A_{t+s}^{-\sigma}}{P_{t+s}} \widetilde{W}_t \Gamma_{t,s}^W n_{t+s}(i) - \Theta_{t+s} (A_{t+s})^{1-\sigma} \xi_{t+s}^n \frac{n_{t+s}(i)^{1+\varphi}}{1+\varphi} \right),$$
$$\text{s.t.} \quad n_{t+s}(i) = \left(\frac{\widetilde{W}_t \Gamma_{t,s}^W}{W_{t+s}} \right)^{-\epsilon_W} \widetilde{n}_{t+s},$$

Which, after some derivation, results in the FOCs in a recursive formulation:

$$f_t^{W1} = \tilde{w}_t^{1-\epsilon_W} \left(\frac{\epsilon_W - 1}{\epsilon_W} \right) \widetilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W - 1} f_{t+1}^{W1} \right\}$$

$$f_t^{W2} = \tilde{w}_t^{-\epsilon_W(1+\varphi)} mc_t^W \widetilde{n}_t + \beta_U \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}}{\varrho_t} \frac{\lambda_{t+1}^U}{\lambda_t^U} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W(1+\varphi)} f_{t+1}^{W2} \right\}$$

Where $f_t^{W1} = f_t^{W2} = f_t^W$ are the LHS and RHS of the FOC respectively, $mc_t^W = -{}^{(U_n/U_C)}/_{(W_t/A_t P_t)} = {}^{\xi_t^n (\widetilde{n}_t)^{\varphi}}/_{\lambda_t^U} (\frac{A_t P_t}{W_t}) \Theta_t$, is the gap with the efficient allocation when wages are flexible[8], $\pi_{t+1}^W = {}^{W_{t+1}}/_{W_t}$, $\pi_{t+1}^{\widetilde{W}} = {}^{\widetilde{W}_{t+1}}/_{\widetilde{W}_t}$ and $\tilde{w}_t = \tilde{W}_t / W_t$.

Further, let $\Psi^W(t)$ denote the set of labor markets in which wages are not reoptimized in period t. By (60), the aggregate wage index W_t evolves as follows:

$$\begin{aligned} (W_t)^{1-\epsilon_W} = \int_0^1 W_t(i)^{1-\epsilon_W} di &= (1-\theta_W) \left(\widetilde{W}_t \right)^{1-\epsilon_W} + \int_{\Psi^W(t)} \left[W_{t-1}(i) \pi_t^{I,W} \right]^{1-\epsilon_W} di, \\ &= (1-\theta_W) \left(\widetilde{W}_t \right)^{1-\epsilon_W} + \theta_W \left[W_{t-1} \pi_t^{I,W} \right]^{1-\epsilon_W}, \end{aligned}$$

or, dividing both sides by $(W_t)^{1-\epsilon_W}$:

$$1 = (1-\theta_W) \tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W} \right)^{1-\epsilon_W}.$$

The third equality above follows from the fact that the distribution of wages that are not reoptimized in period t corresponds to the distribution of effective wages in period $t-1$, though with total mass reduced to θ_W.

Finally, the clearing condition for the labor market is

$$n_t = \int_0^1 n_t(i) \, di = \widetilde{n}_t \int_0^1 \left(\frac{W_t(i)}{W_t} \right)^{-\epsilon_W} di = \widetilde{n}_t \Xi_t^W,$$

Where Ξ_t^W is a wage dispersion term that satisfies

$$\Xi_t^W = (1-\theta_W) \tilde{w}_t^{-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W} \right)^{-\epsilon_W} \Xi_{t-1}^W.$$

[8] U_n and U_C are the first derivatives of the utility function with respect to labor and consumption respectively.

2.4.10 Commodities

We assume the country receives an exogenous and stochastic endowment of commodities Y_t^{Co}. Moreover, these commodities are not consumed domestically but entirely exported. Therefore, the entire production is sold at a given international price $P_t^{Co\star}$, which is assumed to evolve exogenously. We further assume that the government receives a share $\chi \in [0,1]$ of this income and the remaining share goes to foreign agents.

2.5 Fiscal and monetary policy

The government consumes an exogenous stream of final goods G_t, pays through an insurance agency IA_t for deposits and bonds defaulted by banks, levies lump-sum taxes on patient households T_t^P, and issues one-period bonds BS_t^G and long-term bonds BL_t^G. Hence, the government satisfies the following period-by-period constraint:

$$T_t - BS_t^G - Q_t^{BL} BL_t^G + \chi S_t P_t^{Co\star} Y_t^{Co} = P_t G_t - R_{t-1} BS_{t-1}^G - R_t^{BL} Q_t^{BL} BL_{t-1}^G + IA_t \tag{61}$$

where

$$T_t = \alpha^T GDPN_t + \epsilon_t \left(BS_{SS}^G - BS_t^G + Q_{SS}^{BL} BL_{SS}^G - Q_t^{BL} BL_t^G \right) \tag{62}$$

and

$$IA_t = \gamma_D PD_t^D R_{t-1}^D D_{t-1}^F + \gamma_{BH} PD_t^H R_t^{BB} Q_t^{BB} BB_{t-1}^{Pr} \tag{63}$$

As in Chen et al. (2012), we assume that the government control the supply of long-term bonds according to a simple rule given by an exogenous AR(1) process on BL_t^G. In turn, monetary policy is carried out according to a Taylor-type rule of the form

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\,\pi_t + \alpha_E \mathbb{E}_t \{\pi_{t+4}\}}{\pi_t^T} \right)^{\alpha_\pi} \left(\frac{GDP_t / GDP_{t-1}}{a} \right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \tag{64}$$

where $\alpha_R \in [0,1)$, $\alpha_\pi > 1$, $\alpha_y \geq 0$, $\alpha_E \in [0,1]$ and where π_t^T is an exogenous inflation target and e_t^m an i.i.d. shock that captures deviations from the rule.[9]

2.6 Rest of the world

Foreign agents demand home composite goods and buy the domestic commodity production. There are no transaction costs or other barriers to trade. The structure of the foreign economy is identical to the domestic economy, but the domestic economy is assumed to be small relative to the foreign economy. The latter implies that the foreign producer price level $P_t^\star$ is identical to the foreign consumption-based price index. Further, let $P_t^{H\star}$ denote the price of home composite goods expressed in foreign currency. Given full tradability and competitive export pricing, the law of one price holds separately for home composite goods and the commodity good, i.e. $P_t^H = S_t P_t^{H\star}$ and $P_t^{Co} = S_t P_t^{Co\star}$. That is, domestic and foreign prices of both goods are identical when expressed

[9] We do not need a time-varying target, so we will set it to a constant.

in the same currency. Due to local currency pricing, a weak form of the law of one price holds for foreign composite goods, i.e., $P_t^F mc_t^F = S_t P_t^\star \xi_t^m$ from (55). The real exchange rate rer_t therefore satisfies

$$rer_t = \frac{S_t P_t^\star}{P_t} = \frac{P_t^F}{P_t} \frac{mc_t^F}{\xi_t^m} \tag{65}$$

We also have the following relation

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{66}$$

where $\pi_t^s = S_t / S_{t-1}$. Foreign demand for the home composite good $X_t^{H\star}$ is given by

$$X_t^{H\star} = \left(\frac{P_t^H}{S_t P_t^\star} \right)^{-\eta^\star} Y_t^\star \tag{67}$$

with $\eta^\star > 0$ and where $Y_t^\star$ denotes foreign aggregate demand or GDP. Both $Y_t^\star$ and $\pi_t^\star$ evolve exogenously. The relevant foreign nominal interest rate is composed by an exogenous risk-free world interest rate R_t^W plus a country premium that decreases with the economy's net foreign asset position (expressed as a ratio of nominal GDP):

$$R_t^\star = R_t^W \exp \left\{ -\frac{\phi^\star}{100} \left(\frac{S_t B_t^\star}{GDPN_t} - \bar{b} \right) \right\} \xi_t^R z_t^R \tag{68}$$

with $\phi^\star > 0$ and where ξ_t^R is an exogenous shock to the country premium.

2.7 Aggregation and Market Clearing

2.7.1 Aggregation across patient households

Aggregate variables add up the per-capita amounts from unrestricted and restricted patient households, according to their respective mass $\wp_U$ and $1 - \wp_U$:

$$C_t^P = \wp_U C_t^U + (1 - \wp_U) C_t^R$$

$$H_t^P = \wp_U H_t^U + (1 - \wp_U) H_t^R$$

$$n_t^P = \wp_U n_t^U + (1 - \wp_U) n_t^R$$

$$n_t^U = n_t^R$$

$$D_t^{Tot} = \wp_U D_t^U$$

$$B_t^{*,Tot} = \wp_U B_t^{\star,U}$$

$$BS_t^{Pr} = \wp_U BS_t^U$$

$$BL_t^{Pr} = \wp_U BL_t^U + (1 - \wp_U) BL_t^R$$

$$BB_t^{Pr} = \wp_U BB_t^U$$

2.7.2 Goods market clearing

In the market for the final good, the clearing condition is

$$Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \Upsilon_t/P_t \tag{69}$$

where Υ_t includes final goods used in default costs: the resources lost by households recovering deposits at failed banks, the resources lost by the banks to recover the proceeds from defaulted bank loans by the recovery of deposits by the deposit insurance agency and the cost of adjusting labor.

$$\Upsilon_t = \begin{array}{c} \gamma_D PD_t^B R_{t-1}^D D_{t-1}^{Tot} + \gamma_D PD_t^B Q_t^{BB} R_t^{BB} BB_{t-1}^{Pr} + \mu_e G_e\left(\bar{\omega}_t^e\right) R_t^e Q_{t-1}^K K_{t-1} + \mu_I G_I\left(\bar{\omega}_t^I\right) R_t^H Q_{t-1}^H H_{t-1}^I \\ + \mu_H G_H\left(\bar{\omega}_t^H\right) \tilde{R}_t^H Q_{t-1}^L L_{t-1}^H + \mu_F G_F\left(\bar{\omega}_t^F\right) \tilde{R}_t^F L_{t-1}^F \\ + \frac{\gamma_n}{2}\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)^2 Y_t^Z + Q_t^L (L_t^H - \kappa L_{t-1}^H)\left[\frac{\gamma_L}{2}\left(\frac{L_t^H - \kappa L_{t-1}^H}{L_{t-1}^H - \kappa L_{t-2}^H} - \bar{a}\right)^2\right] \end{array}$$

In the market for the home and foreign composite goods we have, respectively,

$$Y_t^H = X_t^H + X_t^{H\star} \tag{70}$$

and

$$Y_t^F = X_t^F \tag{71}$$

while in the market for home and foreign varieties we have, respectively,

$$Y_{jt}^H = X_{jt}^H \tag{72}$$

and

$$Y_{jt}^F = X_{jt}^F \tag{73}$$

for all j.

In the market for the wholesale domestic good, we have

$$Y_t^Z = X_t^Z \tag{74}$$

Finally, in the market for housing, demand from both households must equal supply from housing producers:

$$H_t = H_t^P + H_t^I \tag{75}$$

2.7.3 Factor market clearing

In the market for labor, the clearing conditions are:

$$n_t^P + n_t^I = n_t = \widetilde{n}_t \Xi_t^W \tag{76}$$

$$n_t^P = n_t^I = \frac{n_t}{2} \tag{77}$$

Combining (51) and (50), the capital-labor ratio satisfies:

$$\frac{K_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) R_t^k} \left\{ W_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) Y_t^Z P_t^Z - r_{t,t+1} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) Y_{t+1}^Z P_{t+1}^Z \right\} \tag{78}$$

2.7.4 Deposits clearing

Bank F takes deposits, and its demand must equal the supply from unrestricted households:

$$D_t^F = D_t^{Tot} \tag{79}$$

2.7.5 Domestic bonds clearing

The aggregate net holding of participating agents in bond markets are in zero net supply:

$$BL_t^{Pr} + BL_t^{CB} + BL_t^G = 0 \tag{80}$$

$$BS_t^{Pr} + BS_t^G = 0 \tag{81}$$

Where BL_t^{CB} is an exogenous process that represents the long-term government bond purchases done by the Central Bank.

2.7.6 No-arbitrage condition in bond markets

The no-arbitrage condition implies the following relation between short and long-tem interest rates:

$$R_t \left(\frac{1+\zeta_t^L}{R_t^{BL} - \kappa_B} \right) = \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} \left(\frac{R_{t+1}^{BL}}{R_{t+1}^{BL} - \kappa_B} \right) A_{t+1}^{-\sigma} \right\} \left(\mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^{UP}}{\pi_{t+1}} A_{t+1}^{-\sigma} \right\} \right)^{-1}$$

which can be further rearranged (up to a first order) by using the definition of R_t^{BL}

$$R_t \left(1 + \zeta_t^L\right) \quad \approx \quad \mathbb{E}_t \left\{ \left(\frac{Q_{t+1}^{BL}}{Q_t^{BL}} R_{t+1}^{BL} \right) \right\} \tag{82}$$

2.7.7 Inflation and relative prices

The following holds for $j = H, F$:

$$p_t^j = \frac{P_t^j}{P_t}$$

and, also,

$$\frac{p_t^j}{p_{t-1}^j} = \frac{\pi_t^j}{\pi_t}$$

2.7.8 Aggregate supply

Using the productions of different varieties of home goods (43)

$$\int_0^1 Y_{jt}^H dj = X_t^Z$$

Integrating (72) over j and using (41) then yields aggregate output of home goods as

$$\int_0^1 Y_{jt}^H dj = \int_0^1 X_{jt}^H dj = Y_t^H \int_0^1 \left(p_{jt}^H\right)^{-\epsilon_H} dj$$

or, combining the previous two equations,

$$Y_t^H \Xi_t^H = X_t^Z$$

where Ξ_t^H is a price dispersion term satisfying

$$\begin{aligned} \Xi_t^H &= \int_0^1 \left(\frac{P_{jt}^H}{P_t^H}\right)^{-\epsilon_H} dj \\ &= (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H \end{aligned}$$

2.7.9 Aggregate demand

Aggregate demand or GDP is defined as the sum of domestic absorption and the trade balance. Domestic absorption is equal to $Y_t^C = C_t^P + C_t^I + I_t + I_t^H + G_t + \Upsilon_t$. The nominal trade balance is defined as

$$TB_t = P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \tag{83}$$

Integrating (73) over j and using (53) allows us to write imports as

$$M_t = \int_0^1 Y_{jt}^F dj = \int_0^1 X_{jt}^F dj = Y_t^F \int_0^1 \left(\frac{P_{jt}^F}{P_t^F}\right)^{-\epsilon_F} dj = Y_t^F \Xi_t^F$$

where Ξ_t^F is a price dispersion term satisfying

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F$$

We then define real GDP as

$$GDP_t = Y_t^{NoCo} + Y_t^{Co}$$

where non-mining GDP, Y_t^{NoCo}, is given by

$$Y_t^{NoCo} = C_t^P + C_t^I + I_t + I_t^H + G_t + X_t^{H\star} - M_t$$

and nominal GDP is defined as

$$GDPN_t = P_t\left(C_t^P + C_t^I + I_t + I_t^H + G_t\right) + TB_t \tag{84}$$

Note that by combining (84) with the zero profit condition in the final goods sector, i.e., $P_t Y_t^C = P_t^H X_t^H + P_t^F X_t^F$, and using the market clearing conditions for final and composite goods, (69)-(70), GDP is seen to be equal to total value added (useful for the steady state):

$$\begin{aligned} GDPN_t &= P_t Y_t^C - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H X_t^H + P_t^F X_t^F - \Upsilon_t + P_t^H X_t^{H\star} + S_t P_t^{Co\star} Y_t^{Co} - S_t P_t^{M\star} M_t \\ &= P_t^H Y_t^H + S_t P_t^{Co\star} Y_t^{Co} + P_t^F X_t^F - S_t P_t^{M\star} M_t - \Upsilon_t \end{aligned}$$

2.7.10 Balance of payments

Aggregate nominal profits, dividends, rents and taxes are given by

$$\begin{aligned}
\Psi_t &= \underbrace{P_t Y_t^C - P_t^H X_t^H - P_t^F X_t^F}_{\Pi_t^C} + \underbrace{P_t^H Y_t^H - \int_0^1 P_{jt}^H X_{jt}^H dj}_{\Pi_t^H} + \underbrace{P_t^F Y_t^F - \int_0^1 P_{jt}^F X_{jt}^F dj}_{\Pi_t^F} \\
&\quad + \underbrace{\int_0^1 Y_{jt}^H \left(P_{jt}^H - P_t^Z\right) dj}_{\int_0^1 \Pi_{jt}^H dj} + \underbrace{\int_0^1 \left(P_{jt}^F Y_{jt}^F - S_t P_t^{M\star} Y_{jt}^F\right) dj}_{\int_0^1 \Pi_{jt}^F dj} \\
&\quad + \underbrace{Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) - P_t I_t}_{\Pi_t^I} + \underbrace{Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) - P_t I_t^H}_{\Pi_t^{I^H}} + \underbrace{\left(P_t^Z - mc_t^Z\right) Y_t^Z}_{\Pi_t^Z} \\
&\quad + \underbrace{\zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U}_{\Pi_t^F} + C_t^e + C_t^b + S_t REN_t^* - T_t \\
&= P_t \left(C_t + G_t\right) + \Upsilon_t + P_t^H X_t^{H\star} - S_t P_t^{M\star} M_t - W_t n_t - R_t^k K_{t-1} + Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) \\
&\quad + Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U \\
&= P_t \left(C_t + G_t\right) + \Upsilon_t + TB_t - S_t P_t^{Co\star} Y_t^{Co} - W_t n_t - R_t^k K_{t-1} + Q_t^K \left(K_t - (1-\delta_K) K_{t-1}\right) \\
&\quad + Q_t^H \left(H_t - (1-\delta_H) H_{t-1}\right) + C_t^e + C_t^b + S_t REN_t^* - T_t + \zeta_t^L \left(\frac{1}{R_t^{BL} - \kappa_B}\right) BL_t^U
\end{aligned}$$

Where the second equality uses the market clearing conditions (69)-(81), and the third equality uses the definition of the trade balance, (83). Substituting out Ψ_t in the households' budget constraint (5) and using the government's budget constraint (61) to substitute out taxes T_t shows that the net foreign asset position evolves according to

$$S_t B_t^\star = S_t B_{t-1}^\star R_{t-1}^\star + TB_t + S_t REN_t^* - (1-\chi) S_t P_t^{Co\star} Y_t^{Co}$$

3 Parameterization strategy and estimation results

The model parameters are calibrated and estimated. The calibrated parameters include those characterizing model dynamics for which we have a data counterpart, those drawn from related studies, and those chosen to match the Chilean economy's sample averages or long-run ratios. In particular, we follow closely the calibration strategy from Garcia et al. (2019) and Clerc et al. (2014), as the models described there form the basis of this paper's framework. We estimate the non-calibrated parameters using Bayesian techniques as discussed below.

3.1 Calibration

Table 1 presents the values of the parameters related to the real sector of the economy that are either chosen from previous studies in the relevant literature or chosen in order to match exogenous steady state moments. The value of the parameters α, α_E, β_U, β_R, χ, ϵ_F, ϵ_H, ϵ_W, ω and π^T are taken from Garcia et al. (2019). We assume that the housing capital depreciation rate, δ_H is equal to the productive capital depreciation rate, δ_K, whose value is taken from **?**. The value for β_I is taken from Clerc et al. (2014).

Table 1: Calibration, Real Sector

Parameter	Description	Value	Source
α	Capital share in production function	0.34	Garcia et al. (2019)
α_E	Expected Inflation weight in Taylor Rule	0.50	Garcia et al. (2019)
α^{BSG}	Short-term govt. bonds as percentage of GDP	-0.40	Data: 2009-2019
α^{BLG}	Long-term govt. bonds as percentage of GDP	-4.50	Data: 2009-2019
β_U, β_R	Patient HH Utility Discount Factors	0.99997	Garcia et al. (2019)
β_I	Impatient Utility HH Discount Factor	0.98	Clerc et al. (2014)
δ_K	Capital Annual depreciation rate	0.01	**?**
δ_H	Housing Annual Depreciation rate	0.01	Same as capital depreciation
ϵ_F	Elasticity of substitution among foreign varieties	11	Garcia et al. (2019)
ϵ_H	Elasticity of substitution among home varieties	11	Garcia et al. (2019)
ϵ_W	Elasticity of substitution among types of workers	11	Garcia et al. (2019)
ϵ_τ	Convergence speed towards SS Gov debt	0.10	Normalization
N_H	Time-to-build periods in housing goods	6	CBC's 2018S2 Financial report
κ	Coupon discount in housing loans	0.975	10 years duration of loan contract
κ_{BL}	Coupon discount in long term government bonds	0.975	10 years bond duration
κ_{BB}	Coupon discount in long term banking bonds	0.95	5 years bond duration
π^T	Annual inflation target of 3%	$1.03^{1/4}$	Garcia et al. (2019)
$\rho_{\varphi h}$	Spending profile for long term housing investment	1	Even investment distribution
σ	Log Utility	1	Garcia et al. (2019)
υ	Strength of households wealth effect	0	No wealth effect
χ	Government share in commodity sector	0.33	Garcia et al. (2019)
ω	Home bias in domestic demand	0.79	Garcia et al. (2019)
ω_U	Fraction of unrestricted patient households	0.70	Chen et al. (2012)
ω_{BL}	Ratio of long term assets to short assets	0.822	Chen et al. (2012)

The parameters that set the steady state value of short term and long term government bonds as a percentage of GDP, α^{BSG} and α^{BLG}, respectively, were calculated from data obtained from DCV[10]. The value used for the time that takes a house to be built, N_H is taken from the second semester of 2018 IEF.[11] The parameters that determine the coupons' geometric decline of the long term housing debt, κ, and government bonds, κ_{BL}, are set so their duration is 10 years. The duration of the bank bonds, κ_{BB}, is set to 5 years.

For the housing investment sector, we set the time to built duration, defined by the parameter N_H, to 6 quarters in order to match the average length of construction projects, and assume an even investment spending profile for housing capital, consistent with a value of 1 for $\rho_{\varphi h}$. Following Garcia et al. (2019), we set the value of the parameter that determines the strength of the wealth effect, υ, to 0, to avoid undesired dynamics in the labor market.

For the calibration of the parameters related to the financial sector, shown in Table 2, the values of χ_b, χ_e, γ_{bh}, γ_d, μ_e, μ_F, μ_H and μ_I come from Clerc et al. (2014). The values for the parameters related to bank capital requirements, ϕ_F and ϕ_H, are set as the ratio between the average level of TIER I capital of over the risk weighted assets of the banking system from the year 2000 to the year 2020. In particular, we calculate 4.3% excess of TIER

[10]DCV is an entity that processes and registers transfer operations that take place in several exchange markets.
[11]IEF stands for Financial Stability Report published twice a year by the Central Bank of Chile.

I capital in addition to legal 9.75%. For corporate banks we assume 100% weight in corporate loans, while for housing bank we assume 60% weight in housing loans.

Table 2: Calibration, Financial Sector

Parameter	Description	Value	Source
χ_b	Banks dividend policy	0.04	Clerc et al. (2015)
χ_e	Entrepreneurs dividend policy	0.05	Clerc et al. (2015)
γ_{bh}	Household cost bank bonds default	0.10	Clerc et al. (2015)
γ_d	Cost of recovering defaulted bank deposits	0.10	Clerc et al. (2015)
ϕ_F	Bank Capital Requirement (RWA)	0.14	Data (2000-2022)
ϕ_H	Bank Capital Requirement (RWA)	0.10	Data (2000-2022)

3.2 Estimation and Results

We compute the model solution by a second order approximation around the deterministic steady state. However, the parameters whose values are not calibrated are estimated using Bayesian methods with linear approximation. The data for the estimation, described in Table 3, includes 25 macroeconomic and financial variables from between 2001Q3 and 2019Q3. Data for the real Chilean sector is obtained from the Central Bank of Chile's National Accounts database, while prices and labor statistics are obtained from the National Statistics Institute (INE). Finally, local financial data is obtained from the Financial Markets Committee (CMF), and foreign data is obtained from Bloomberg. Variables regarding the real sector are log-differentiated with respect to the previous quarter. All variables are demeaned. Our estimation strategy also includes i.i.d. measurement errors for all local observables with the exception of the interest rate. The variance of the measurement errors is calibrated to 10% of the variance of the corresponding observable.

Table 3: Observable Data

Non Financial		Financial	
$\Delta \log Y_t^{NoCo}$	Non mining real GDP	R_t^L	Comercial Loans interest Rate
$\Delta \log Y_t^{Co}$	Copper real GDP	R_t^I	Housing Loans Interest Rate
$\Delta \log C_t$	Total Consumption	R_t^D	Nominal Interest Rate on Deposits
$\Delta \log G_t$	Goverment Consumption	R_t^{LG}	10 Year BCP Rate
$\Delta \log I_t^K$	Real Capital Investment	$\Delta \log L_t$	Housing and Corporate Loan
$\Delta \log I_t^H$	Real Housing Investment	ROE_t	Banks ROE
$TB_t/GDPN_t$	Trade Balance-GDP Ratio	R_t^*	LIBOR
$\Delta \log N_t$	Total Employment	Ξ_t^R	EMBI Chile
$\Delta \log WN_t$	Nominal Cost of labor	rer_t	Real Exchange Rate
π_t	Core CPI	R_t	Nominal MPR
$\Delta \log y_t^*$	Real External GDP		
π_t^*	Foreign Price Index		
π_t^M	Imports Deflactor		
π_t^{Co*}	Nominal Copper Price		
π_t^H	Housing Price Index		

Sources: INE, BCCh, CMF and Bloomberg.

The posterior estimates are obtained using full information (Bayesian) maximum likelihood estimation. To facilitate optimization, following Christiano et al. (2011), we scale some of the parameters for the shocks' standard deviations to have a similar posterior order of magnitude. We choose the type of priors according to the related literature from distributions that have supported distributions consistent with the theoretical values expected for the parameters. In columns three, four and five of Table (4) we show the chosen prior distributions and prior distribution moments of the estimated values of the deep parameters. The sixth and seventh columns of the same table show the posterior mean and the 95% interval of the estimation. On the other hand, on Table 5 we show the estimation priors and results of the parameters related to shock variables. For all autocorrelation coefficient we use a beta distribution while for the standard deviation we use a inverse gamma distribution.

Table 4: Estimation, Deep Parameters

Parameter	**Description**	**Prior**			**Posterior**		
		Dist	**Mean**	**St Dev**	**Mean**	**95%**	**Inter**
α_π	Inflation weight in Taylor Rule	N	1.70	0.10	1.92	[1.76	2.08]
α_R	Lagged interest rate weight in Taylor Rule	β	0.85	0.03	0.77	[0.74	0.81]
α_W	Weight on past productivity on wage indexation	β	0.25	0.08	0.17	[0.04	0.29]
α_y	Output weight in Taylor Rule	N	0.13	0.08	0.13	[0.01	0.25]
η	Elasticity of subst. home and foreign goods	γ	1.00	0.25	0.97	[0.71	1.23]
$\eta_{\hat{C}}$	Elasticity of subst. consumption and housing goods	γ	1.00	0.25	0.12	[0.05	0.19]
η^*	Foreign demand elasticity of substitution	γ	0.25	0.08	0.19	[0.07	0.30]
γ_H	Housing investment adjustment cost parameter	γ	3.00	0.25	2.98	[2.48	3.49]
γ_K	Capital investment adjustment cost parameter	γ	3.00	0.25	2.95	[2.46	3.43]
γ_n	Labor adjustment cost parameter	γ	3.00	0.25	1.80	[1.46	2.13]
γ_L	Housing debt cost parameter	γ	0.1	0.09	0.29	[0.11	0.47]
κ_F	Weight on past inflation on foreign good indexation	β	0.50	0.08	0.67	[0.55	0.79]
κ_H	Weight on past inflation on home good indexation	β	0.50	0.08	0.76	[0.66	0.86]
κ_W	Weight on past inflation on wages indexation	β	0.85	0.03	0.85	[0.79	0.90]
ϕ^*	Country premium elasticity to NFA position	γ^{-1}	1.00	Inf	0.34	[0.16	0.52]
ϕ_c	Habit formation in good consumption	β	0.85	0.03	0.89	[0.86	0.92]
ϕ_{hh}	Habit formation in housing consumption	β	0.85	0.03	0.81	[0.75	0.86]
θ_F	Calvo param. foreign goods producers	β	0.50	0.08	0.72	[0.68	0.75]
θ_H	Calvo param. domestic goods producers	β	0.50	0.03	0.82	[0.80	0.84]
θ_W	Calvo param. wage setters	β	0.50	0.08	0.58	[0.51	0.65]
φ	Inverse Frisch elasticty	γ	7.50	1.50	8.37	[5.84	10.9]
μ_e	Monitoring cost of corporate loan default	β	0.30	0.05	0.45	[0.36	0.54]
μ_F	Monitoring cost of F bank default	β	0.30	0.05	0.37	[0.26	0.47]
μ_H	Monitoring cost of H bank default	β	0.30	0.05	0.30	[0.20	0.40]
μ_i	Monitoring cost of housing loan default	β	0.30	0.05	0.23	[0.14	0.32]
η_{ζ_L}	Term premium elasticity to relative bond liquidity	γ	0.15	0.03	0.14	[0.08	0.20]

Notes.— Reported posterior means and standard deviations are based on full information maximum likelihood estimation and Laplace approximation.

Table 5: Estimation, exogenous variables AR1 processes

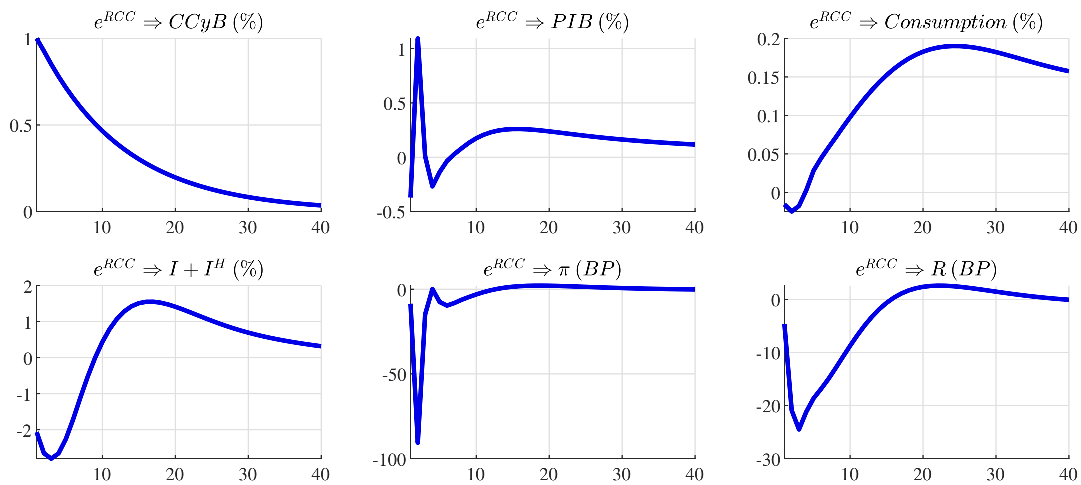
Shock process	**A.R**	**Prior**		**Posterior**		**S.D.**	**Prior**		**Posterior**	
		Mean	**S.D**	**Mean**	**90% HPD**		**Mean**	**S.D**	**Mean**	**90% HPD**
Non stat. productivity	ρ_a	0.25	0.08	0.37	[0.20 0.55]	$100 \times \sigma_a$	0.50	Inf	0.38	[0.26 0.51]
Monetary Policy	ρ_{e^m}	0.15	0.08	0.26	[0.06 0.46]	$1000 \times \sigma_{e^m}$	0.50	Inf	1.4	[1.03 1.77]
Government spending	ρ_g	0.75	0.08	0.75	[0.62 0.88]	$100 \times \sigma_g$	0.50	Inf	1.77	[1.46 2.09]
Copper price	$\rho_{p^{co}}$	0.75	0.08	0.89	[0.84 0.94]	$100 \times \sigma_{p^{co}}$	0.50	Inf	1.10	[0.90 1.30]
Foreign inflation	ρ_{π^*}	0.75	0.08	0.44	[0.37 0.52]	$100 \times \sigma_{\pi^*}$	0.50	Inf	2.20	[1.79 2.62]
Foreign interest rate	ρ_{R^W}	0.75	0.08	0.89	[0.84 0.94]	$1000 \times \sigma_{R^W}$	0.50	Inf	1.10	[0.84 1.36]
Entrepreneurs risk	ρ_{σ^e}	0.75	0.08	0.96	[0.93 0.99]	$100 \times \sigma_{\sigma^e}$	0.50	Inf	2.42	[1.77 3.07]
Corporate bank risk	ρ_{σ^F}	0.75	0.08	0.70	[0.56 0.85]	$10 \times \sigma_{\sigma^F}$	0.50	Inf	1.02	[0.46 1.59]
Housing bank risk	ρ_{σ^H}	0.75	0.08	0.77	[0.61 0.92]	$10 \times \sigma_{\sigma^H}$	0.50	Inf	0.23	[0.04 0.42]
Housing valuation risk	ρ_{σ^I}	0.75	0.08	0.92	[0.86 0.98]	$10 \times \sigma_{\sigma^I}$	0.50	Inf	5.39	[1.56 9.22]
Current consumption prefs.	ρ_{ϱ}	0.75	0.08	0.38	[0.28 0.49]	$10 \times \sigma_{\varrho}$	0.50	Inf	3.35	[1.78 4.91]
Housing consumption prefs	ρ_{ξ^h}	0.75	0.08	0.93	[0.90 0.95]	$10 \times \sigma_{\xi^h}$	0.50	Inf	1.42	[0.66 2.18]
Investment mg. eff.(K)	ρ_{ξ^i}	0.75	0.08	0.57	[0.42 0.72]	$10 \times \sigma_{\xi^I}$	0.50	Inf	0.69	[0.41 0.96]
Investment mg. eff.(H)	$\rho_{\xi^{ih}}$	0.75	0.08	0.88	[0.78 0.98]	$10 \times \sigma_{\xi^{ih}}$	0.50	Inf	1.75	[0.89 2.61]
Import prices	ρ_{ξ^m}	0.75	0.08	0.85	[0.76 0.93]	$100 \times \sigma_{\xi^m}$	0.50	Inf	2.56	[1.93 3.19]
Labor disutility	ρ_{ξ^n}	0.75	0.08	0.75	[0.60 0.89]	$10 \times \sigma_{\xi^n}$	0.50	Inf	3.86	[1.38 6.34]
Country premium	ρ_{ξ^R}	0.75	0.08	0.84	[0.75 0.92]	$1000 \times \sigma_{\xi^R}$	0.50	Inf	0.65	[0.50 0.79]
Banker dividend	$\rho_{\xi^{\chi_b}}$	0.75	0.08	0.82	[0.72 0.93]	$10 \times \sigma_{\xi^{\chi_b}}$	0.50	Inf	2.56	[1.93 3.19]
Entrepreneur dividend	$\rho_{\xi^{\chi_e}}$	0.75	0.08	0.45	[0.34 0.56]	$10 \times \sigma_{\xi^{\chi_e}}$	0.50	Inf	2.02	[1.53 2.51]
Banker required return	$\rho_{\xi^{roe}}$	0.75	0.08	0.83	[0.74 0.92]	$10 \times \sigma_{\xi^{roe}}$	0.50	Inf	0.37	[0.26 0.48]
Foreign demand	$\rho_{\xi^{y*}}$	0.85	0.08	0.90	[0.79 1.02]	$100 \times \sigma_{\xi^{y*}}$	0.50	Inf	0.24	[0.04 0.44]
Mining productivity	$\rho_{\xi^{yco}}$	0.85	0.08	0.80	[0.63 0.97]	$100 \times \sigma_{\xi^{yco}}$	0.50	Inf	3.23	[2.63 3.82]
Stat. productivity	ρ_z	0.85	0.08	0.84	[0.76 0.93]	$100 \times \sigma_z$	0.50	Inf	1.22	[0.91 1.53]
UIP shock	ρ_{ζ^u}	0.75	0.08	0.96	[0.93 0.98]	$1000 \times \sigma_{z_\tau}$	0.50	Inf	1.64	[0.76 2.52]
Liquidity costs	ρ_{ϵ^L}	0.75	0.05	0.76	[0.66 0.86]	$100 \times \sigma_{\epsilon^L}$	0.50	Inf	0.09	[0.02 0.17]

Notes.— Reported posterior means and standard deviations are based on full information maximum likelihood estimation and Laplace approximation. All of the autocorrelation parameters were estimated assuming a beta distribution while the standard deviation parameters were estimated using an inverse gamma distribution.

4 Results

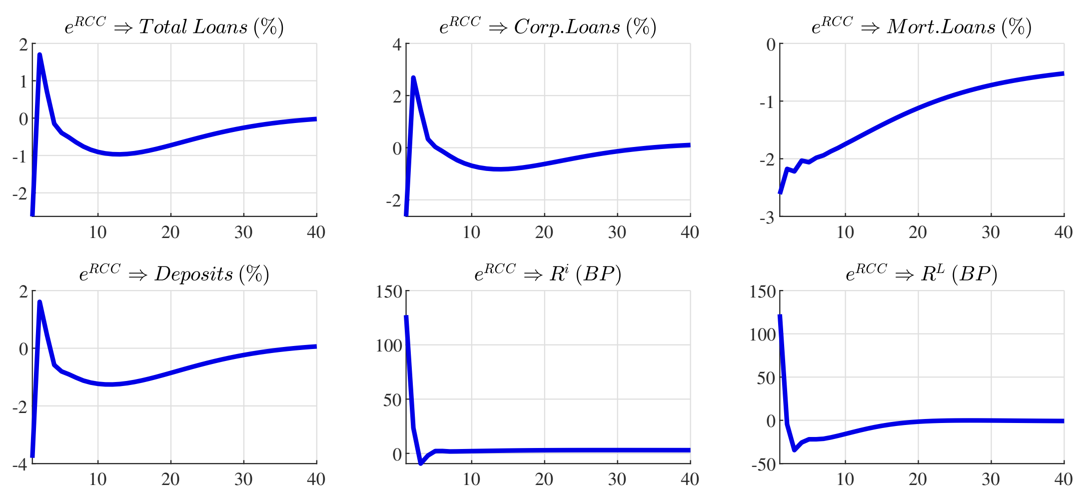
4.1 Macro-financial implications of the CCyB

Figure 2: Response of real variables to a CCyB Shock



Notes.— The figure shows the the impulse response to a CCyB shock ($e_{req,0} = 1\%$) that follows the AR(1) process $CCyB_{t+1} = 0.9175 \cdot CCyB_t + e_{req,t}$ as shown in the upper left figure. The vertical axis of the figures corresponds to percentage deviations from steady state (%) and quarterly basis points (BP).

Figure 3: Response of financial variables to a CCyB Shock



Notes.— The figure shows the the impulse response to a CCyB shock ($e_{req,0} = 1\%$) that follows the AR(1) process $CCyB_{t+1} = 0.9175 \cdot CCyB_t + e_{req,t}$. The vertical axis of the figures corresponds to percentage deviations from steady state (%) and quarterly basis points (BP).

4.2 Optimal Simple Implementable Rule (SIR)

In this subsection we perform a welfare analysis to find optimal SIRs. In doing so, we define a SIR as a CCyB rule of the form $CCyB_t(\theta) = X_t\theta$, where X_t is a matrix of time-varying variables that will trigger the CCyB mechanism in our model, and θ is a vector of weights on these variables. To find the optimal SIR we perform a welfare analysis in the spirit of Carrillo et al. (2021) and define the welfare of the economy, $\mathbf{W}(\theta)$, as function of the parameters θ using the equation (2) as follows

$$\mathbf{W}(\theta) = \sum_{i}^{I,U,R} \mathbb{E}_{i,0} \left\{ \sum_{t=1}^{\infty} \beta_i^t \varrho_t \left[\frac{1}{1-\sigma} \left(\hat{C}_t^i(\theta) \right)^{1-\sigma} - \Theta_t^i(\theta) A_t^{1-\sigma} \xi_t^n \frac{\left(n_t^i(\theta) \right)^{1+\varphi}}{1+\varphi} \right] \right\} \tag{85}$$

To compare the welfare gains or losses resulting from activating the CCyB rule, we use as a baseline welfare, $\mathbf{W}^{base}$, the welfare of the economy without a CCyB rule, i.e., with $\theta = 0$. To define $\mathbf{W}^{base}$ we use the perpetuity of welfare in stochastic steady state, obtaining following the expression

$$\mathbf{W}^{base} \equiv \mathbf{W}(\theta|\theta=0) = \sum_{i}^{I,U,R} \frac{1}{1-\beta_i} \left[\frac{1}{1-\sigma} \left(\hat{C}_{ss}^i(\theta|\theta=0) \right)^{1-\sigma} - \Theta_{ss}^i(\theta|\theta=0) A_{ss}^{1-\sigma} \frac{\left(n_{ss}^i(\theta|\theta=0) \right)^{1+\varphi}}{1+\varphi} \right] \tag{86}$$

To obtain a meaningful interpretation of the changes in welfare after implementing a SIR, the economy's gains or losses are expressed in consumption units. To do so, we define a *consumption equivalent*, C^e, as the permanent change in consumption that equals the welfare of the economy with a SIR, $\mathbf{W}(\theta)$, and the welfare of the economy without a SIR, $\mathbf{W}^{base}$. In other words, it is the level of permanent consumption required to offset the welfare gains/losses from implementing a certain SIR. From (1) we know that the consumption $\hat{C}_t^i$ is an aggregate goods, composed of housing and consumer final goods, so we define the *consumption equivalent* as the consumption C^e that solves

$$\mathbf{W}(\theta) = \mathbf{W}(C^e|\theta=0) \tag{87}$$

$$= \sum_{i}^{I,U,R} \frac{1}{1-\beta_i} \left[\frac{1}{1-\sigma} \left(\hat{C}_{ss}^i(C^e|\theta=0) \right)^{1-\sigma} - \Theta_{ss}^i(C^e|\theta=0) A_{ss}^{1-\sigma} \frac{\left(n_{ss}^i(C^e|\theta=0) \right)^{1+\varphi}}{1+\varphi} \right] \tag{88}$$

with

$$\hat{C}_{ss}^i(C^e|\theta=0) = \left[\left(1-o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(C_{ss}^i(1-\phi_c)(1+C^e) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(H_{ss}^i(1-\phi_{hh}) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{89}$$

$$\Theta_{ss}^i(C^e|\theta=0) = \tilde{\chi}_{ss}^i(C^e|\theta=0) A_{ss}^{\sigma} \left(\hat{C}_{ss}^i(C^e|\theta=0) \right)^{-\sigma} \tag{90}$$

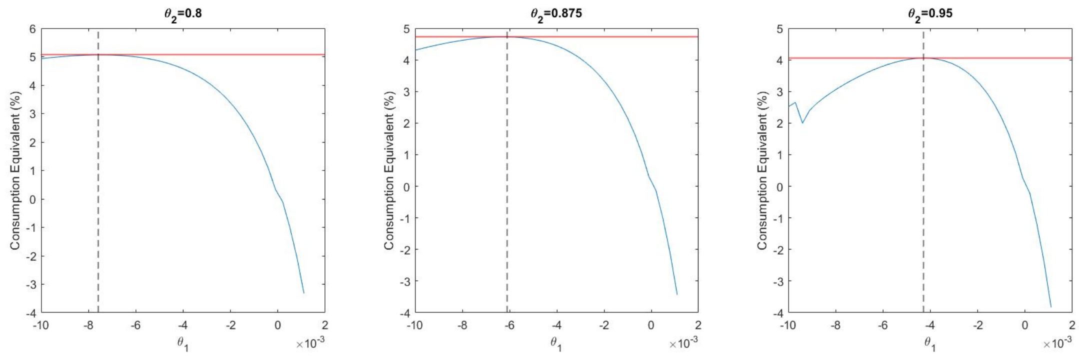
$$\left(\tilde{\chi}_{ss}^i(C^e|\theta=0) \right)^{v} = A_{ss}^{-\sigma v} \left(\hat{C} \left(\hat{C}_{ss}^i(C^e|\theta=0) \right) \right)^{\sigma v} \tag{91}$$

Therefore, when $C^e > 0$ there is a welfare gain from implementing the SIR with respect to the baseline scenario. To keep the analysis simple, we will perform our analysis using SIRs of only two factors, i.e., $\theta = (\theta_1, \theta_2)$ and $CCyB_t = \theta_1 X_{1,t} + \theta_2 X_{2,t}$, where the variables $(X_{1,t}, X_{2,t})$ will define different implementation rules. In order to find optimal SIRs we look for the weights (θ_1, θ_2) that maximize the welfare for a given set of variables $(X_{1,t}, X_{2,t})$. First, the Figure 4 shows how the welfare defined in (88) changes when implementing an SIR of the type

$$CCyB_t = \theta_1 \log \left(\frac{\Delta(R_L, R)_t}{\Delta(R_L, R)_{tss}} \right) + \theta_2 \cdot CCyB_{t-1} \tag{92}$$

with $\Delta(R_L, R)_t$ is the rate spread between the commercial rate and the risk-free rate and for different values for θ_1 and θ_2.

Figure 4: Consumption Equivalente for rule (92)



Notes.— This figure shows the consumption equivalent for different values of θ_1 and θ_2.

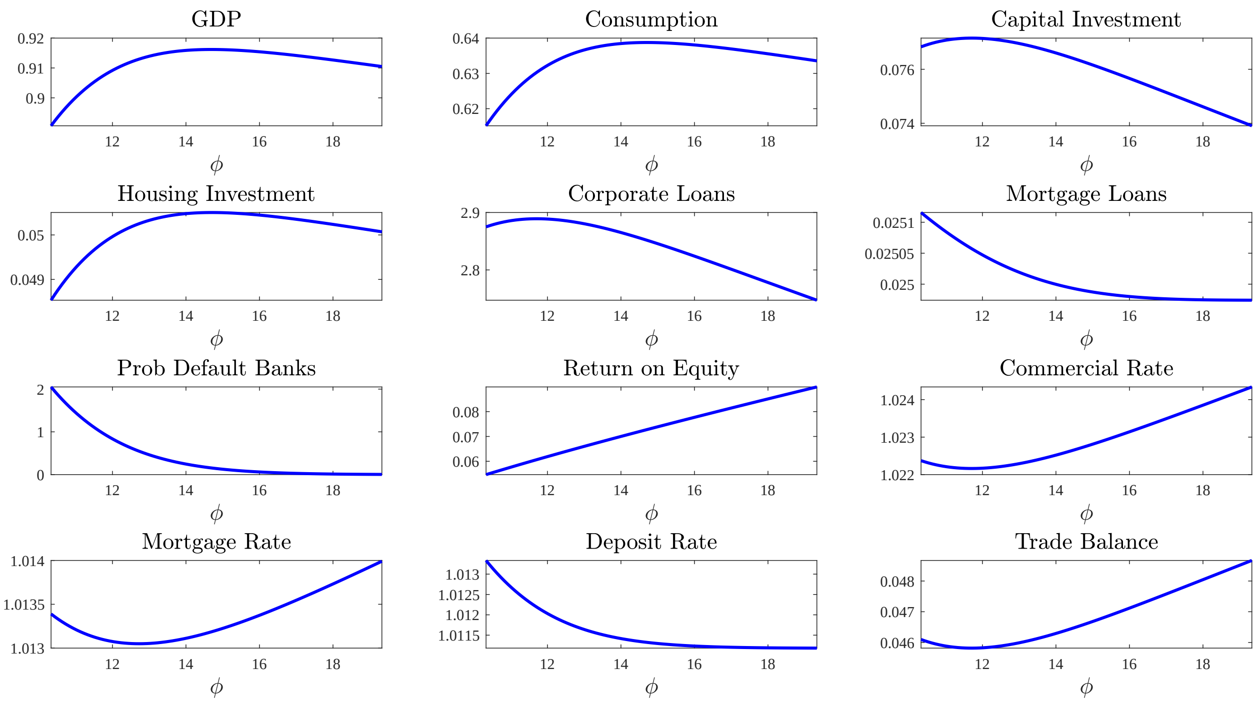
We can see that there are non-negligible welfare gains from using this SIFR. In the first place, we can note the counter-cyclical nature of the rule by releasing CCyB when the rate spread increases (negative sign of θ_1), and also, the welfare gains obtained are higher when the capital adjustment can be carried out more quickly (less θ_2) which is in line with active capital management by banks.

TO BE COMPLETED

4.3 Optimal SIR with ELB

To be completed

Figure 5: Steady state for different capital requirements



Note: The figure shows the steady state for different levels of basic capital.

5 Conclusion

In this paper we have evaluated the welfare implications of introducing a countercyclical buffer rule which is simple and implementable. We do so by building a macro-banking model with two inefficiencies: nominal rigidities and financial frictions. This gives room for monetary and financial policies to be desirable. We use our model to study the functional form of a SIR for financial policy. Further, we argue that the countercyclical nature of the CCyB and its institutional design (zero lower bound) imply a rationale for a neutral positive level of the buffer.

To be completed

References

Andres, J., J. D. López-Salido, and E. Nelson (2004). Tobin's imperfect asset substitution in optimizing general equilibrium. *Journal of Money, Credit and Banking*, 665–690. 4

Basel Committee on Banking Supervision, B. (2022). Buffer usability and cyclicality in the basel framework. Technical report, Bank for International Settlements. 3

Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). The financial accelerator in a quantitative business cycle framework. Volume 1, part C of *Handbook of Macroeconomics*, Chapter 21, pp. 1341–1393. Elsevier. 2

Calani, M., B. García, T. Gómez, M. González, S. Guarda, M. Paillacar, et al. (2022). A macro financial model for the chilean economy. Technical report, Central Bank of Chile. 2

Carrillo, J. A., E. G. Mendoza, V. Nuguer, and J. Roldán-Peña (2021). Tight money-tight credit: coordination failure in the conduct of monetary and financial policies. *American Economic Journal: Macroeconomics 13*(3), 37–73. 2, 3, 39

Chen, H., V. Curdia, and A. Ferrero (2012). The Macroeconomic Effects of Large-scale Asset Purchase Programmes. *The Economic Journal 122*(564), F289—-F315. 4, 6, 27, 34

Christiano, L. J., M. Trabandt, and K. Walentin (2011). Introducing financial frictions and unemployment into a small open economy model. *Journal of Economic Dynamics and Control 35*(12), 1999–2041. 35

Clerc, L., A. Derviz, C. Mendicino, S. Moyen, K. Nikolov, L. Stracca, J. Suarez, and A. Vardoulakis (2014). Capital regulation in a macroeconomic model with three layers of default. 2, 4, 8, 11, 14, 16, 33, 34

Financial Stability Institute, F. (2019). The capital buffers in basel iii – executive summary. Technical report, Bank for International Settlements. 2

Galí, J., F. Smets, and R. Wouters (2012). Unemployment in an estimated new Keynesian model. *NBER Macroeconomics Annual 26*(1), 329–360. 5

Garcia, B., S. Guarda, M. Kirchner, and R. Tranamil (2019, may). XMas: An extended model for analysis and simulations. *Central Bank of Chile Working Paper* (833). 4, 33, 34

Kydland, F. E. and E. C. Prescott (1982). Time to build and aggregate fluctuations. *Econometrica 50*(6), 1345–1370. 17

Lechthaler, W. and D. J. Snower (2011). Quadratic labor adjustment costs, business cycle dynamics and optimal monetary policy. *Kiel Working Paper 1453.* 4

Malherbe, F. (2020). Optimal capital requirements over the business and financial cycles. *American Economic Journal: Macroeconomics 12*(3), 139–174. 3

Mendicino, C., K. Nikolov, J. Suarez, and D. Supera (2018). Optimal dynamic capital requirements. *Journal of Money, Credit and Banking 50*(6), 1271–1297. 2

Mendicino, C., K. Nikolov, J. Suarez, and D. Supera (2020). Bank capital in the short and in the long run. *Journal of Monetary Economics 115*, 64–79. 2

Uribe, M. and V. Z. Yue (2006). Country spreads and emerging countries: Who drives whom? *Journal of International Economics 69*(1), 6–36. Emerging Markets. 17

Van den Heuvel, S. J. (2008). The welfare cost of bank capital requirements. *Journal of Monetary Economics 55*(2), 298–320. 2

Vayanos, D. and J.-L. Vila (2009). A preferred-habitat model of the term structure of interest. *NBER Working Paper 15487*. 4

Woodford, M. (2001). Fiscal Requirements for Price Stability. Working Paper 8072, National Bureau of Economic Research. 2, 4, 5

ONLINE APPENDIX

A Stationary Equilibrium Conditions

In the model described in the previous sections, real variables in uppercase contain a unit root in equilibrium due to the presence of the non-stationary productivity vector A_t. Uppercase nominal variables contain an additional unit root given by the non-stationarity of the price level. In this section we show the stationary version of the model, where we define $a_t = A_t/A_{t-1}$, and all lowercase variables denote the stationary counterpart of the original variables, obtained by dividing them by its co-integration vector(A_t or P_t).

The rational expectations equilibrium of the stationary version of the model is then the set of sequences for the endogenous variables such that for a given set of initial values and exogenous processes the following conditions are satisfied:

A.1 Patient Households

A.1.1 Unrestricted (U)

$$\hat{c}_t^U = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(c_t^U - \phi_c \frac{c_{t-1}^U}{a_t}\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(\xi_t^h\left(\frac{h_{t-1}^U}{a_t} - \phi_{hh}\frac{h_{t-2}^U}{a_t a_{t-1}}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{1}$$

$$\lambda_t^U = \left(\hat{c}_t^U\right)^{-\sigma}\left(\frac{(1-o_{\hat{C}})\,\hat{c}_t^U}{\left(c_t^U - \phi_c\frac{c_{t-1}^U}{a_t}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}} \tag{2}$$

$$\varrho_t\lambda_t^U q_t^H = \beta_U \mathbb{E}_t\varrho_{t+1}\left\{\left(\hat{c}_{t+1}^U a_{t+1}\right)^{-\sigma}\xi_{t+1}^h\left(\frac{o_{\hat{C}}\hat{c}_{t+1}^U a_{t+1}}{\xi_{t+1}^h\left(h_t^U - \phi_{hh}\frac{h_{t-1}^U}{a_t}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}} + (1-\delta_H)\,\lambda_{t+1}^U a_{t+1}^{-\sigma} q_{t+1}^H\right\} \tag{3}$$

$$\varrho_t\lambda_t^U = \beta_U R_t \mathbb{E}_t\left\{\frac{\varrho_{t+1}\lambda_{t+1}^U}{\pi_{t+1}}a_{t+1}^{-\sigma}\right\} \tag{4}$$

$$\varrho_t\lambda_t^U = \beta_U \mathbb{E}_t\left\{\frac{\tilde{R}_{t+1}^D}{\pi_{t+1}}\varrho_{t+1}\lambda_{t+1}^U a_{t+1}^{-\sigma}\right\} \tag{5}$$

$$\varrho_t\lambda_t^U = \beta_U R_t^\star \mathbb{E}_t\left\{\frac{\varrho_{t+1}\lambda_{t+1}^U\pi_{t+1}^s}{\pi_{t+1}}a_{t+1}^{-\sigma}\right\} \tag{6}$$

$$\varrho_t\lambda_t^U\left(1+\zeta_t^L\right)q_t^{BL} = \beta_U\mathbb{E}_t\left\{\varrho_{t+1}\lambda_{t+1}^U a_{t+1}^{-\sigma}R_{t+1}^{BL}q_{t+1}^{BL}\right\} \tag{7}$$

$$\varrho_t\lambda_t^U\left(1+\zeta_t^L\right)q_t^{BB} = \beta_U\mathbb{E}_t\left\{\varrho_{t+1}\lambda_{t+1}^U a_{t+1}^{-\sigma}\tilde{R}_{t+1}^{BB}q_{t+1}^{BB}\right\} \tag{8}$$

A.1.2 Restricted (R)

$$\hat{c}_t^R = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(c_t^R - \phi_c \frac{c_{t-1}^R}{a_t}\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(\xi_t^h\left(\frac{h_{t-1}^R}{a_t} - \phi_{hh}\frac{h_{t-2}^R}{a_t a_{t-1}}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{9}$$

$$\lambda_t^R = \left(\hat{c}_t^R\right)^{-\sigma}\left(\frac{(1-o_{\hat{C}})\,\hat{c}_t^R}{\left(c_t^R - \phi_c\frac{c_{t-1}^R}{a_t}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}} \tag{10}$$

$$\varrho_t\lambda_t^R q_t^H = \beta_R \mathbb{E}_t\varrho_{t+1}\left\{\left(\hat{c}_{t+1}^R a_{t+1}\right)^{-\sigma}\left(\frac{o_{\hat{C}}\hat{c}_{t+1}^R a_{t+1}}{\xi_{t+1}^h\left(h_t^R - \phi_{hh}\frac{h_{t-1}^R}{a_t}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}\xi_{t+1}^h + (1-\delta_H)\,\lambda_{t+1}^R a_{t+1}^{-\sigma} q_{t+1}^H\right\} \tag{11}$$

$$\varrho_t\lambda_t^R q_t^{BL} = \beta_R\mathbb{E}_t\left\{\varrho_{t+1}\lambda_{t+1}^R q_{t+1}^{BL}R_{t+1}^{BL}a_{t+1}^{-\sigma}\right\} \tag{12}$$

$$q_t^{BL}bl_t^R + c_t^R + q_t^H h_t^R = q_t^{BL}R_t^{BL}\frac{bl_{t-1}^R}{a_t} + w_t n_t^R + q_t^H(1-\delta_H)\frac{h_{t-1}^R}{a_t} \tag{13}$$

A.2 Impatient Households

$$\frac{R_t^H}{\pi_t} = \frac{q_t^H\left(1-\delta_H\right)}{q_{t-1}^H} \tag{14}$$

$$\hat{c}_t^I = \left[\left(1-o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}}\left(c_t^I - \phi_c \frac{c_{t-1}^I}{a_t}\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}}\left(\xi_t^h\left(\frac{h_{t-1}^I}{a_t} - \phi_{hh}\frac{h_{t-2}^I}{a_t a_{t-1}}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}} \tag{15}$$

$$\lambda_t^I = \left(\hat{c}_t^I\right)^{-\sigma}\left(\frac{\left(1-o_{\hat{C}}\right)\hat{c}_t^I}{\left(c_t^I - \phi_c\frac{c_{t-1}^I}{a_t}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}} \tag{16}$$

$$\hat{q}_t^L = \left(1 - \frac{\kappa l_{t-1}^H}{l_t^H a_t}\right) q_t^L + \left(\frac{\kappa l_{t-1}^H}{l_t^H a_t}\right)\hat{q}_{t-1}^L \tag{17}$$

$$\bar{\omega}_t^I = \frac{\hat{R}_t^I \hat{q}_t^L l_{t-1}^H}{R_t^H q_{t-1}^H h_{t-1}^I}\pi_t \tag{18}$$

$$R_t^I = \frac{1}{q_t^L} + \kappa \tag{19}$$

$$\hat{R}_t^I = \frac{1+\kappa\hat{q}_{t-1}^L}{\hat{q}_t^L} \tag{20}$$

$$\varrho_t \lambda_t^I q_t^H = \mathbb{E}_t\left\{\begin{array}{c} \beta_I \varrho_{t+1}\left(\left(\hat{c}_{t+1}^I a_{t+1}\right)^{-\sigma}\left(\frac{o_{\hat{C}}\hat{c}_{t+1}^I a_{t+1}}{\xi_{t+1}^h\left(h_t^I - \phi_{hh}\frac{h_{t-1}^I}{a_t}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}\xi_{t+1}^h + \lambda_{t+1}^I a_{t+1}^{-\sigma}\left[1-\Gamma_I\left(\bar{\omega}_{t+1}^I\right)\right]\frac{R_{t+1}^H}{\pi_{t+1}}q_t^H\right) \\ +\varrho_t\lambda_t^H\left[1-\Gamma_H\left(\bar{\omega}_{t+1}^H\right)\right]\left[\Gamma_I\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I\left(\bar{\omega}_{t+1}^I\right)\right] R_{t+1}^H q_t^H \end{array}\right\} \tag{21}$$

$$\beta_I = \mathbb{E}_t\left\{\frac{\varrho_t\lambda_t^H\pi_{t+1}}{\varrho_{t+1}\lambda_{t+1}^I a_{t+1}^{-\sigma}}\left[1-\Gamma_H\left(\bar{\omega}_{t+1}^H\right)\right]\frac{\left[\Gamma_I'\left(\bar{\omega}_{t+1}^I\right) - \mu_I G_I'\left(\bar{\omega}_{t+1}^I\right)\right]}{\Gamma_I'\left(\bar{\omega}_{t+1}^I\right)}\right\} \tag{22}$$

$$c_t^I + q_t^H h_t^I - q_t^L\left(l_t^H - \frac{\kappa l_{t-1}^H}{a_t}\right)\left[1 - \frac{\gamma_L}{2}(\nabla\tilde{l}_t - \bar{a})^2\right] - \frac{\kappa l_{t-1}^H \hat{q}_{t-1}^L}{a_t} = \frac{w_t n_t}{2} + \left[1-\Gamma_I\left(\bar{\omega}_t^I\right)\right]\frac{R_t^H q_{t-1}^H h_{t-1}^I}{a_t\pi_t} \tag{23}$$

$$\varrho_t q_t^L\left\{\lambda_t^I\left[1-\frac{\gamma_L}{2}(\nabla\tilde{l}_t - \bar{a})^2\right] - \lambda_t^I\nabla\tilde{l}_t\gamma_L(\nabla\tilde{l}_t - \bar{a}) - \lambda_t^H\rho_{t+1}^H\phi_H\right\} = \ldots$$
$$\ldots\ \beta_I\mathbb{E}_t\left\{\varrho_{t+1}\lambda_{t+1}^I a_{t+1}^{-\sigma}\left[\kappa q_{t+1}^L\left[1-\frac{\gamma_L}{2}(\nabla\tilde{l}_{t+1}-\bar{a})^2\right] - q_{t+1}^L\nabla\tilde{l}_{t+1}\gamma_L(\nabla\tilde{l}_{t+1}-\bar{a})(\nabla\tilde{l}_{t+1}+\kappa) - \kappa\hat{q}_t^L\right]\right\} \tag{24}$$

$$PD_t^I = F_I\left(\bar{\omega}_t^I\right) \tag{25}$$

A.3 Entrepreneurs

$$q_t^K k_t = n_t^e + l_t^F \tag{26}$$

$$\frac{R_t^e}{\pi_t} = \frac{r_t^k + (1-\delta_K)\,q_t^K}{q_{t-1}^K} \tag{27}$$

$$\bar{\omega}_t^e = \frac{R_{t-1}^L l_{t-1}^F}{R_t^e q_{t-1}^K k_{t-1}} \tag{28}$$

$$c_t^e = \chi_e \xi_t^{\chi_e}\psi_t^e \tag{29}$$

$$n_t^e = \left(1-\chi_e\xi_t^{\chi_e}\right)\psi_t^e \tag{30}$$

$$\psi_t^e a_t \pi_t = \left[1-\Gamma_e\left(\bar{\omega}_t^e\right)\right] R_t^e q_{t-1}^K k_{t-1} \tag{31}$$

$$\left(1-\Gamma_{t+1}^e\right) = \lambda_t^e\left(\frac{\rho_{t+1}^F\phi_t^F}{R_{t+1}^e} - \left(1-\Gamma_{t+1}^F\right)\left[\Gamma_{t+1}^e - \mu^e G_{t+1}^e\right]\right) \tag{32}$$

$$\Gamma_{t+1}^{e'} = \lambda_t^e\left(1-\Gamma_{t+1}^F\right)\left[\Gamma_{t+1}^{e'} - \mu^e G_{t+1}^{e'}\right] \tag{33}$$

$$PD_t^e = F_e\left(\bar{\omega}_t^e\right) \tag{34}$$

A.4 Bankers and Banking System

$$\mathbb{E}\left[\rho^F_{t+1}\right] = \xi^{b,roe}_t \mathbb{E}\left[\tilde{\rho}^H_{t+1}\right] \tag{35}$$

$$c^b_t = \xi^{\chi_b}_t \chi_b \psi^b_t \tag{36}$$

$$n^b_t = \left(1 - \xi^{\chi_b}_t \chi_b\right) \psi^b_t \tag{37}$$

$$\psi^b_t a_t \pi_t = \rho^F_t e^F_{t-1} + \tilde{\rho}^H_t e^H_{t-1} \tag{38}$$

$$n^b_t = e^F_t + e^H_t \tag{39}$$

$$PD^D_t = \frac{Q^{BB}_{t-1} BB_{t-1} PD^H_t + d^{Tot}_{t-1} PD^F_t}{Q^{BB}_{t-1} BB_{t-1} + d^{Tot}_{t-1}} \tag{40}$$

A.5 F Banks

$$d^F_t + e^F_t = l^F_t \tag{41}$$

$$\bar{\omega}^F_t = (1 - \phi_F) \frac{R^D_{t-1}}{\tilde{R}^F_t} \tag{42}$$

$$e^F_t = \phi_F l^F_t \tag{43}$$

$$\rho^F_t = \left[1 - \Gamma_F\left(\bar{\omega}^F_t\right)\right] \frac{\tilde{R}^F_t}{\phi_F} \tag{44}$$

$$\tilde{R}^F_t = \left[\Gamma_e\left(\bar{\omega}^e_t\right) - \mu_e G_e\left(\bar{\omega}^e_t\right)\right] \frac{R^e_t q^K_{t-1} k_{t-1}}{l^F_{t-1}} \tag{45}$$

$$PD^F_t = F_F\left(\bar{\omega}^F_t\right) \tag{46}$$

A.6 H Banks

$$\tilde{\rho}^H_t = (1 - \kappa)\rho^H_t + \kappa \mathbb{E}\left[\tilde{\rho}^H_{t+1}\right] \tag{47}$$

$$\bar{\omega}^H_t = (1 - \phi_H) \frac{R^{BB}_t q^{\widehat{BB}}_t}{\widetilde{R}^H_t q^{\widehat{BB}}_{t-1}} \pi_t \tag{48}$$

$$q^{BB}_t bb^{Pr}_t + e^H_t = q^L_t l^H_t \tag{49}$$

$$e^H_t = \phi_H q^L_t l^H_t \tag{50}$$

$$\rho^H_t = \left[1 - \Gamma_H\left(\bar{\omega}^H_t\right)\right] \frac{\tilde{R}^H_t}{\phi_H} \tag{51}$$

$$\tilde{R}^H_t = \left[\Gamma_I\left(\bar{\omega}^I_t\right) - \mu_I G_I\left(\bar{\omega}^I_t\right)\right] \frac{R^H_t q^H_{t-1} h^I_{t-1}}{q^L_{t-1} l^H_{t-1}} \tag{52}$$

$$PD^H_t = F_H\left(\bar{\omega}^H_t\right) \tag{53}$$

A.7 Capital and Housing Goods

$$k_t = (1 - \delta_K)\frac{k_{t-1}}{a_t} + \left[1 - \frac{\gamma_K}{2}\left(\frac{i_t}{i_{t-1}} a_t - a\right)^2\right] \xi^i_t i_t \tag{54}$$

$$\begin{aligned} 1 \quad &= \quad q^K_t \left[1 - \frac{\gamma_K}{2}\left(\frac{i_t}{i_{t-1}} a_t - a\right)^2 - \gamma_K\left(\frac{i_t}{i_{t-1}} a_t - a\right)\frac{i_t}{i_{t-1}} a_t\right] \xi^i_t \\ &+ \quad \beta_P \mathbb{E}_t \left\{\frac{\varrho_{t+1}\lambda^P_{t+1}}{\varrho_t \lambda^P_t} a^{-\sigma}_{t+1} q^K_{t+1} \gamma_K \left(\frac{i_{t+1}}{i_t} a_{t+1} - a\right)\left(\frac{i_{t+1}}{i_t} a_{t+1}\right)^2 \xi^i_{t+1}\right\} \end{aligned} \tag{55}$$

$$h_t = (1 - \delta_H)\frac{h_{t-1}}{a_t} + \left[1 - \frac{\gamma_H}{2}\left(\frac{i^{AH}_{t-N_H}}{i^{AH}_{t-N_H-1}} a_t - a\right)^2\right] \xi^{ih}_{t-N_H} \frac{i^{AH}_{t-N_H}}{\prod_{i=0}^{N_H-1} a_{t-j}} \tag{56}$$

$$
\begin{aligned}
0 &= E_t \sum_{j=0}^{N_H} \beta_P^j \varrho_{t+j} \lambda_{t+j}^P \varphi_j^H \prod_{i=j+1}^{N_H} \left(a_{t+i}^{\sigma}\right) \\
&- E_t \beta_P^{N_H} \varrho_{t+N_H} \lambda_{t+N_H}^P q_{t+N_H}^H \left\{ \left[1 - \frac{\gamma_H}{2} \left(\frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right)^2 \right] - \gamma_H \left(\frac{i_t^{AH}}{i_{t-1}^{AH}} a_t - a \right) \frac{i_t^{AH}}{i_{t-1}^{AH}} a_t \right\} \xi_t^{ih} \\
&- E_t \beta_P^{N_H+1} \varrho_{t+N_H+1} \lambda_{t+N_H+1}^P q_{t+N_H+1}^H a_{t+N_H+1}^{-\sigma} \left\{ \gamma_H \left(\frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} - a \right) \left(\frac{i_{t+1}^{AH}}{i_t^{AH}} a_{t+1} \right)^2 \xi_{t+1}^{ih} \right\}
\end{aligned} \tag{57}
$$

$$
i_t^H = \sum_{j=0}^{N_H} \varphi_j^H \frac{i_{t-j}^{AH}}{\prod_{i=0}^{j-1} a_{t-j}} \tag{58}
$$

A.8 Final Goods

$$
y_t^C = \left[\omega^{1/\eta} \left(x_t^H \right)^{1-1/\eta} + (1-\omega)^{1/\eta} \left(x_t^F \right)^{1-1/\eta} \right]^{\frac{\eta}{\eta-1}} \tag{59}
$$

$$
x_t^F = (1-\omega) \left(p_t^F \right)^{-\eta} y_t^C \tag{60}
$$

$$
x_t^H = \omega \left(p_t^H \right)^{-\eta} y_t^C \tag{61}
$$

A.9 Home Goods

$$
f_t^H = \frac{\epsilon_H - 1}{\epsilon_H} \left(\tilde{p}_t^H \right)^{1-\epsilon_H} y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{1-\epsilon_H} \left(\pi_{t+1}^H \right)^{\epsilon_H} f_{t+1}^H \right\} \tag{62}
$$

$$
f_t^H = \left(\tilde{p}_t^H \right)^{-\epsilon_H} mc_t^H y_t^H + \beta_U \theta_H \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^H \pi_{t+1}^{I,H}}{\tilde{p}_{t+1}^H} \right)^{-\epsilon_H} \left(\pi_{t+1}^H \right)^{1+\epsilon_H} f_{t+1}^H \right\} \tag{63}
$$

$$
1 = (1-\theta_H) \left(\tilde{p}_t^H \right)^{1-\epsilon_H} + \theta_H \left(\frac{\pi_t^{I,H}}{\pi_t^H} \right)^{1-\epsilon_H} \tag{64}
$$

$$
\pi_t^{I,H} = \left(\pi_{t-1}^H \right)^{\kappa_H} \left(\pi^T \right)^{1-\kappa_H} \tag{65}
$$

$$
mc_t^H = \frac{p_t^Z}{p_t^H} \tag{66}
$$

A.10 Wholesale Domestic Goods

$$
\begin{aligned}
mc_t^Z = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(r_t^k)^\alpha}{z_t} \Bigg\{ & w_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) y_t^Z p_t^Z \\
& - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \Bigg\}^{1-\alpha}
\end{aligned} \tag{67}
$$

$$
\begin{aligned}
\frac{k_{t-1}}{\widetilde{n}_t} = \frac{\alpha}{(1-\alpha) r_t^k} \Bigg\{ & w_t + \gamma_n \left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1 \right) \left(\frac{1}{\widetilde{n}_{t-1}} \right) y_t^Z p_t^Z \\
& - \beta_U \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P} \gamma_n \mathbb{E}_t \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t} - 1 \right) \left(\frac{\widetilde{n}_{t+1}}{\widetilde{n}_t^2} \right) y_{t+1}^Z p_{t+1}^Z \Bigg\} a_t
\end{aligned} \tag{68}
$$

$$
p_t^Z = mc_t^Z \tag{69}
$$

A.11 Foreign Goods

$$
p_t^F mc_t^F = rer_t \xi_t^m \tag{70}
$$

$$
f_t^F = \frac{\epsilon_F - 1}{\epsilon_F} \left(\tilde{p}_t^F \right)^{1-\epsilon_F} y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{1-\epsilon_F} \left(\pi_{t+1}^F \right)^{\epsilon_F} f_{t+1}^F \right\} \tag{71}
$$

$$f_t^F = \left(\tilde{p}_t^F\right)^{-\epsilon_F} mc_t^F y_t^F + \beta_U \theta_F \mathbb{E}_t \left\{ \frac{\varrho_{t+1} \lambda_{t+1}^P a_{t+1}^{1-\sigma}}{\varrho_t \lambda_t^P \pi_{t+1}} \left(\frac{\tilde{p}_t^F \pi_{t+1}^{I,F}}{\tilde{p}_{t+1}^F} \right)^{-\epsilon_F} \left(\pi_{t+1}^F\right)^{1+\epsilon_F} f_{t+1}^F \right\} \tag{72}$$

$$1 = (1-\theta_F)\left(\tilde{p}_t^F\right)^{1-\epsilon_F} + \theta_F \left(\frac{\pi_t^{I,F}}{\pi_t^F} \right)^{1-\epsilon_F} \tag{73}$$

$$\pi_t^{I,F} = \left(\pi_{t-1}^F\right)^{\kappa_F} \left(\pi^T\right)^{1-\kappa_F} \tag{74}$$

A.12 Wages

$$\lambda_t^W = \frac{\lambda_t^P + \lambda_t^I}{2} \tag{75}$$

$$\lambda_t^P = \wp_U \lambda_t^U + (1-\wp_U)\lambda_t^R \tag{76}$$

$$\Theta_t = \frac{\left(\wp_U \Theta_t^U + (1-\wp_U)\Theta_t^R\right) + \Theta_t^I}{2} \tag{77}$$

$$mc_t^W = \Theta_t \frac{\xi_t^n \left(\tilde{n}_t\right)^{\varphi}}{\lambda_t^U w_t} \tag{78}$$

$$\Theta_t^i = \tilde{\chi}_t^i \left(\hat{c}_t^i\right)^{-\sigma} \quad \forall \quad i = \{U, R, I\} \tag{79}$$

$$\tilde{\chi}_t^i = \left(\tilde{\chi}_{t-1}^i\right)^{1-v} \left(\hat{c}_t^i\right)^{\sigma v} \quad \forall \quad i = \{U, R, I\} \tag{80}$$

$$\begin{aligned} f_t^W =& \left(\frac{\epsilon_W - 1}{\epsilon_W}\right) \tilde{w}_t^{1-\epsilon_W} \tilde{n}_t \\ &+ \left(\frac{\left(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}\right) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}\lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W - 1} f_{t+1}^W \right\} \end{aligned} \tag{81}$$

$$\begin{aligned} f_t^W =& \tilde{w}_t^{-\epsilon_W(1+\varphi)} mc_t^W \tilde{n}_t \\ &+ \left(\frac{\left(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}\right) + \beta_I}{2} \right) \theta_W \mathbb{E}_t \left\{ a_{t+1}^{-\sigma} \frac{\varrho_{t+1}\lambda_{t+1}^W}{\varrho_t \lambda_t^W} \frac{\pi_{t+1}^W}{\pi_{t+1}} \left(\frac{\pi_{t+1}^{\widetilde{W}}}{\pi_{t+1}^{I,W}} \right)^{\epsilon_W(1+\varphi)} f_{t+1}^W \right\} \end{aligned} \tag{82}$$

$$1 = (1-\theta_W)\tilde{w}_t^{1-\epsilon_W} + \theta_W \left(\frac{\pi_t^{I,W}}{\pi_t^W} \right)^{1-\epsilon_W} \tag{83}$$

$$\pi_t^{I,W} = a_{t-1}^{\alpha_W} a^{1-\alpha_W} \pi_{t-1}^{\kappa_W} \pi^{1-\kappa_W} \tag{84}$$

A.13 Fiscal Policy

$$\begin{aligned} \tau_t + R_{t-1}\frac{bs_{t-1}^G}{a_t \pi_t} + q_t^{BL} R_t^{BL} bl_{t-1}^G \frac{1}{a_t} + \chi s_t p_t^{Co\star} y_t^{Co} =& g_t + bs_t^G + q_t^{BL} bl_t^G + \gamma_D \frac{PD_t^D R_{t-1}^D d_{t-1}^F}{a_t \pi_t} \\ &+ \gamma_{BH} \frac{PD_t^H R_t^{BB} q_t^{BB} bb_{t-1}^{Pr}}{a_t} \end{aligned} \tag{85}$$

$$\tau_t = \alpha^T gdpn_t + \epsilon_t \left(bs^G - bs_t^G + q^{BL} bl^G - q_t^{BL} bl_t^G \right) \tag{86}$$

A.14 Monetary Policy and Rest of the World

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\alpha_R} \left[\left(\frac{(1-\alpha_E)\pi_t + \alpha_E \mathbb{E}_t\{\pi_{t+4}\}}{\pi_t^T} \right)^{\alpha_\pi} \left(\frac{gdp_t}{gdp_{t-1}} \right)^{\alpha_y} \right]^{1-\alpha_R} e_t^m \tag{87}$$

$$\frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^\star}{\pi_t} \tag{88}$$

$$R_t^\star = R_t^W \exp\left\{ \frac{-\phi^\star}{100} \left(\frac{rer_t b_t^\star}{gdpn_t} - \frac{rer b^\star}{gdpn} \right) \right\} \xi_t^R z_t^\tau \tag{89}$$

$$x_t^{H\star} = \left(\frac{p_t^H}{rer_t} \right)^{-\eta^\star} y_t^\star \tag{90}$$

A.15 Aggregation and Market Clearing

$$y_t^C = c_t^P + c_t^I + i_t^K + i_t^H + g_t + \upsilon_t \tag{91}$$

$$c_t^P = \wp_U c_t^U + (1-\wp_U)\, c_t^R \tag{92}$$

$$\begin{aligned}\upsilon_t a_t \pi_t =& \gamma_D PD_t^D R_{t-1}^D d_{t-1}^F + \gamma_{BH} PD_t^H R_t^{BB} q_t^{BB} bb_{t-1}^{Pr} + \mu_e G_e\left(\bar{\omega}_t^e\right) R_t^e q_{t-1}^K k_{t-1} + \mu_I G_I\left(\bar{\omega}_t^I\right) R_t^H q_{t-1}^H h_{t-1}^I \\ &+ \mu_H G_H\left(\bar{\omega}_t^H\right) \tilde{R}_t^H l_{t-1}^H q_{t-1}^L + \mu_F G_F\left(\bar{\omega}_t^F\right) \tilde{R}_t^F l_{t-1}^F + \frac{\gamma_n}{2}\left(\frac{\widetilde{n}_t}{\widetilde{n}_{t-1}} - 1\right)^2 y_t^Z p_t^Z\end{aligned} \tag{93}$$

$$y_t^H = x_t^H + x_t^{H\star} \tag{94}$$

$$y_t^F = x_t^F \tag{95}$$

$$h_t = h_t^P + h_t^I \tag{96}$$

$$h_t^P = \wp_U h_t^U + (1-\wp_U)\, h_t^R \tag{97}$$

$$bl_t^{Pr} = \wp_U bl_t^U + (1-\wp_U)\, bl_t^R \tag{98}$$

$$bs_t^{Pr} = \wp_U bs_t^U \tag{99}$$

$$bb_t^{Tot} = \wp_U bb_t^U \tag{100}$$

$$b_t^{*Tot} = \wp_U b_t^{*U} \tag{101}$$

$$bl_t^{Pr} + bl_t^{CB} + bl_t^G = 0 \tag{102}$$

$$bs_t^{Pr} + bs_t^G = 0 \tag{103}$$

$$d_t^F = \wp_U d_t^U \tag{104}$$

$$\zeta_t^L = \left(\frac{q_t^{BL} bl_t^U + q_t^{BB} bb_t^U}{bs_t^U + rer_t b_t^{\star,U} + d_t^U}\right)^{\eta_\zeta} \epsilon_t^{L,S} \tag{105}$$

$$\tilde{R}_t^D = R_{t-1}^D\left(1 - \gamma_D PD_t^D\right) \tag{106}$$

$$\tilde{R}_t^{BB} = R_t^{BB}\left(1 - \gamma_{BH} PD_t^H\right) \tag{107}$$

$$R_t^{BL} = \frac{1}{q_t^{BL}} + \kappa_{BL} \tag{108}$$

$$R_t^{BB} = \frac{1}{q_t^{BB}} + \kappa_{BB} \tag{109}$$

$$R_t^{Nom,BL} = R_t^{BL}\pi_t \tag{110}$$

$$\frac{p_t^H}{p_{t-1}^H} = \frac{\pi_t^H}{\pi_t} \tag{111}$$

$$\frac{p_t^F}{p_{t-1}^F} = \frac{\pi_t^F}{\pi_t} \tag{112}$$

$$\pi_t^W = \frac{w_t}{w_{t-1}} a_t \pi_t \tag{113}$$

$$\pi_t^{\widetilde{W}} = \frac{\widetilde{w}_t}{\widetilde{w}_{t-1}} \pi_t^W \tag{114}$$

$$y_t^H \Xi_t^H = x_t^Z \tag{115}$$

$$y_t^Z = z_t\left(\frac{k_{t-1}}{a_t}\right)^\alpha \widetilde{n}_t^{1-\alpha} \tag{116}$$

$$y_t^Z = x_t^Z \tag{117}$$

$$\Xi_t^H = (1-\theta_H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \theta_H\left(\frac{\pi_t^{I,H}}{\pi_t^H}\right)^{-\epsilon_H} \Xi_{t-1}^H \tag{118}$$

$$m_t = y_t^F \Xi_t^F \tag{119}$$

$$\Xi_t^F = (1-\theta_F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \theta_F\left(\frac{\pi_t^{I,F}}{\pi_t^F}\right)^{-\epsilon_F} \Xi_{t-1}^F \tag{120}$$

$$n_t = \widetilde{n}_t \Xi_t^W \tag{121}$$

$$\Xi_t^W = (1-\theta_W)\tilde{w}_t^{-\epsilon_W} + \theta_W\left(\frac{\pi_t^{I,W}}{\pi_t^W}\right)^{-\epsilon_W} \Xi_{t-1}^W \tag{122}$$

$$n_t = n_t^P + n_t^I \tag{123}$$

$$n_t^P = n_t^I \tag{124}$$

$$n_t^P = \wp_U n_t^U + (1-\wp_U)\, n_t^R \tag{125}$$

$$n_t^U = n_t^R \tag{126}$$

$$gdp_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + x_t^{H\star} + y_t^{Co} - m_t \tag{127}$$
$$gdpn_t = c_t^P + c_t^I + i_t^K + i_t^H + g_t + tb_t \tag{128}$$
$$tb_t = p_t^H x_t^{H\star} + rer_t p_t^{Co\star} y_t^{Co} - rer_t \xi_t^m m_t \tag{129}$$
$$rer_t b_t^\star = \frac{rer_t}{a_t \pi_t^\star} b_{t-1}^\star R_{t-1}^\star + tb_t + rer_t ren^* - (1-\chi)\, rer_t p_t^{Co\star} y_t^{Co} \tag{130}$$

The exogenous processes are:

$$\log(z_t/z) = \rho_z \log(z_{t-1}/z) + u_t^z$$
$$\log(a_t/a) = \rho_a \log(a_{t-1}/a) + u_t^a$$
$$\log(\xi_t^n/\xi^n) = \rho_{\xi^n} \log(\xi_{t-1}^n/\xi^n) + u_t^{\xi^n}$$
$$\log(\xi_t^h/\xi^h) = \rho_{\xi^h} \log(\xi_{t-1}^h/\xi^h) + u_t^{\xi^h}$$
$$\log(\xi_t^i/\xi^i) = \rho_{\xi^i} \log(\xi_{t-1}^i/\xi^i) + u_t^{\xi^i}$$
$$\log(\xi_t^{ih}/\xi^{ih}) = \rho_{\xi^{ih}} \log(\xi_{t-1}^{ih}/\xi^{ih}) + u_t^{\xi^{ih}}$$
$$\log(\xi_t^R/\xi^R) = \rho_{\xi^R} \log(\xi_{t-1}^R/\xi^R) + u_t^{\xi^R}$$
$$\log(e_t^m/e^m) = \rho_{e^m} \log(e_{t-1}^m/e^m) + u_t^{e^m}$$
$$\log(g_t/g) = \rho_g \log(g_{t-1}/g) + u_t^g$$
$$\log(y_t^{Co}/y^{Co}) = \rho_{y^{Co}} \log(y_{t-1}^{Co}/y^{Co}) + u_t^{y^{Co}}$$
$$\log(\pi_t^\star/\pi^\star) = \rho_{\pi^\star} \log(\pi_{t-1}^\star/\pi^\star) + u_t^{\pi^\star}$$
$$\log(R_t^W/R^W) = \rho_{R^W} \log(R_{t-1}^W/R^W) + u_t^{R^W}$$
$$\log(y_t^\star/y^\star) = \rho_{y^\star} \log(y_{t-1}^\star/y^\star) + u_t^{y^\star}$$
$$\log(p_t^{Co\star}/p^{Co\star}) = \rho_{p^{Co\star}} \log(p_{t-1}^{Co\star}/p^{Co\star}) + u_t^{p^{Co\star}}$$
$$\log(\xi_t^m/\xi^m) = \rho_{\xi^m} \log(\xi_{t-1}^m/\xi^m) + u_t^{\xi^m}$$
$$\log(\sigma_t^I/\sigma^I) = \rho_{\sigma^I} \log(\sigma_{t-1}^I/\sigma^I) + u_t^{\sigma^I}$$
$$\log(\sigma_t^e/\sigma^e) = \rho_{\sigma^e} \log(\sigma_{t-1}^e/\sigma^e) + u_t^{\sigma^e}$$
$$\log(\sigma_t^F/\sigma^F) = \rho_{\sigma^F} \log(\sigma_{t-1}^F/\sigma^F) + u_t^{\sigma^F}$$
$$\log(\sigma_t^H/\sigma^H) = \rho_{\sigma^H} \log(\sigma_{t-1}^H/\sigma^H) + u_t^{\sigma^H}$$
$$\log(\epsilon_t^{L,S}/\epsilon^{L,S}) = \rho_{\epsilon^{L,S}} \log(\epsilon_{t-1}^{L,S}/\epsilon^{L,S}) + u_t^{\epsilon^{L,S}}$$
$$\log(bl_t^G/bl^G) = \rho_{bl^G} \log(bl_{t-1}^G/bl^G) + u_t^{bl^G}$$
$$\log(bl_t^{CB}/bl^{CB}) = \rho_{bl^{CB}} \log(bl_{t-1}^{CB}/bl^{CB}) + u_t^{bl^{CB}}$$
$$\log(\varrho_t/\varrho) = \rho_\varrho \log(\varrho_{t-1}/\varrho) + u_t^\varrho$$
$$\log(\xi_t^{\chi b}/\xi^{\chi b}) = \rho_\xi^{\chi b} \log(\xi_{t-1}^{\chi b}/\xi^{\chi b}) + u_t^{\xi^{\chi b}}$$
$$\log(\xi_t^{\chi e}/\xi^{\chi e}) = \rho_\xi^{\chi e} \log(\xi_{t-1}^{\chi e}/\xi^{\chi e}) + u_t^{\xi^{\chi e}}$$
$$\log(\xi_t^{roe}/\xi^{roe}) = \rho_\xi^{roe} \log(\xi_{t-1}^{roe}/\xi^{roe}) + u_t^{\xi^{roe}}$$
$$\log(z_t^\tau/z^\tau) = \rho_{z^\tau} \log(z_{t-1}^\tau/z^\tau) + u_t^{z^\tau}$$

Donde todas las perturbaciones u_t^j se distribuyen normalmente con media cero y σ^j desviación estándar: $u_t^j \sim \mathcal{N}(0, (\sigma^j)^2)$

B Steady State Computation

In this section we show how to compute the steady state for a given value of most of the parameters and all exogenous variables in the long run, except for:

$$R^W,\ \pi^\star,\ \sigma^F,\ \sigma^H,\ \sigma^e,\ \sigma^I,\ g,\ y^{Co},\ y^\star,\ o_{\hat{C}},\ ren^*,\ \xi^n.$$

that are determined endogenously by imposing values for the steady state of the following endogenous variables:

$$\pi^s,\ \xi^i = 1,\ \xi^R,\ R^D,\ PD^F = PD^H,\ n,\ R^{nom,BL},\ R^{nom,I},\ R^L,\ p^H,\ r^{h,k} = {}^{q^H h}\!/\!_{q^K k},\ s^g = g/gdpn,\ s^{Co} = p^{Co\star} y^{Co} rer/gdpn,$$
$$s^{tb} = tb/gdpn,\ s^{b*} = b^* rer/gdpn,\ \alpha_{BLG} = \frac{bl^G * q^{BL}}{gdpn},\ \alpha_{SG} = \frac{bs^G}{gdpn}$$

Start with (4), (5), (6), (87) (88) and (89):

$$R = \frac{\pi a^\sigma}{\beta_U}; \quad \tilde{R}^D = R; \quad R^\star = \frac{R}{\pi^s}; \quad \pi = \pi^T; \quad \pi^\star = \frac{\pi}{\pi^s}; \quad R^W = \frac{R^\star}{\xi^R}$$

From (65), (74) and (111), (112):

$$\pi^{I,H} = \pi^{I,F} = \pi^H = \pi^F = \pi$$

From (84), (113) and (114) :

$$\pi^{I,W} = \pi^W = \pi^{\widetilde{W}} = a\pi$$

From (64), (73), (83), (62),(63), (71),(72), (81), (82), (118), (120) and (122):

$$\tilde{p}^H = \tilde{p}^F = \widetilde{w} = 1$$
$$mc^H = \frac{\epsilon_H - 1}{\epsilon_H}$$
$$mc^F = \frac{\epsilon_F - 1}{\epsilon_F}$$
$$mc^W = \frac{\epsilon_W - 1}{\epsilon_W}$$
$$\Xi^H = \Xi^F = \Xi^W = 1$$

From (55) and (57):

$$q^K = 1/\xi^i$$
$$q^H = \frac{a^{N_H \sigma} \varphi_0^H}{\beta_{UP}^{N_H} \xi^{ih}} \left(\frac{1 - \left(\frac{\beta_{UP} \rho^{\varphi H}}{a^\sigma} \right)^{N_H + 1}}{1 - \frac{\beta_P \rho^{\varphi H}}{a^\sigma}} \right)$$

From (14) and (121):

$$R^H = \pi (1 - \delta_H)$$
$$\widetilde{n} = n$$

From (35), (37), (38), (39) and (47):

$$\rho^H = \tilde{\rho}^H = \rho^F = \frac{a\pi}{1 - \chi_b}$$

From (40), (106), R^D and using $PD^F = PD^H$

$$PD^D = \frac{1}{\gamma_D} \left(1 - \frac{\tilde{R}^D}{R^D} \right) = PD^H = PD^F$$

From (12)

$$R^{BL} = \frac{R^{Nom,BL}}{\pi}$$
$$\beta_{RP} = \frac{a^\sigma}{R^{BL}}$$

From (17), (19) and (20)

$$R^I = \frac{R^{Nom,I}}{\pi}$$
$$\hat{R}^I = R^I$$
$$\hat{q}^L = \frac{1}{\hat{R}^I - \kappa_L}$$

$$q^L = \hat{q}^L$$

From (7) and (8)

$$\tilde{R}^{BB} = R^{BL}$$

From (107)

$$R^{BB} = \frac{\tilde{R}^{BB}}{1 - \gamma_D PD^H}$$

From (109)

$$q^{BB} = \frac{1}{R^{BB} - \kappa_{BB}}$$

From (108)

$$q^{BL} = \frac{1}{R^{BL} - \kappa_B}$$

$$\Delta l = a$$

Numerical solution for $\bar{\omega}^F$ and σ^F using (42), (44) and (46)

$$\bar{\omega}^F - \left[1 - \Gamma_F \left(\bar{\omega}^F, \sigma^F\right)\right] \left(\frac{1 - \phi_F}{\phi_F}\right) \frac{R^D}{\tilde{\rho}^F} = 0$$

$$PD^F - F_F \left(\bar{\omega}^F, \sigma^F\right) = 0$$

Numerical solution for $\bar{\omega}^H$ and σ^H using (48), (51) and (53)

$$\bar{\omega}^H - \left[1 - \Gamma_H \left(\bar{\omega}^H, \sigma^H\right)\right] \left(\frac{1 - \phi_H}{\phi_H}\right) \frac{R^{BB}}{\rho^H} \pi = 0$$

$$PD^H - F_H \left(\bar{\omega}^H, \sigma^H\right) = 0$$

Then, from (44) and (51):

$$\tilde{R}^F = \frac{\phi_F \rho^F}{1 - \Gamma_F \left(\bar{\omega}^F, \sigma^F\right)}$$

$$\tilde{R}^H = \frac{\phi_H \rho^H}{1 - \Gamma_H \left(\bar{\omega}^H, \sigma^H\right)}$$

Numerical solution for $\bar{\omega}^e$ and σ^e: Use (33) in (32), then use (44), (45), (26) and (31). Later combine (28) and (45) to obtain

$$\frac{\Gamma_e' \left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e' \left(\bar{\omega}^e, \sigma^e\right)}{\Gamma_e' \left(\bar{\omega}^e, \sigma^e\right)} - \frac{\left(1 - \chi_e\right) \tilde{R}^F}{a\pi} = 0$$

$$R^L - \frac{\tilde{R}^F \bar{\omega}^e}{\Gamma_e \left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e \left(\bar{\omega}^e, \sigma^e\right)} = 0$$

From (34):

$$PD^e = F_e \left(\bar{\omega}^e\right)$$

Numerical solution for $\bar{\omega}^I$ and σ^I: use (51) and (24) in (22). Also, use (52) in (18)

$$\frac{\Gamma_I' \left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I' \left(\bar{\omega}^I, \sigma^I\right)}{\Gamma_I' \left(\bar{\omega}^I, \sigma^I\right)} - \frac{\beta_I \tilde{R}^H}{a^\sigma \pi} = 0$$

$$R^I - \frac{\tilde{R}^H \bar{\omega}^I}{\pi \left[\Gamma_I \left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I \left(\bar{\omega}^I, \sigma^I\right)\right]} = 0$$

From (25):

$$PD^I = F_I \left(\bar{\omega}^I\right)$$

From (30), (26), (31) and (45):

$$R^e = \frac{\tilde{R}^F a\pi}{a\pi \left[\Gamma_e \left(\bar{\omega}^e\right) - \mu_e G_e \left(\bar{\omega}^e\right)\right] + \left[1 - \Gamma_e \left(\bar{\omega}^e\right)\right] \left(1 - \chi_e\right) \tilde{R}^F}$$

From (27):

$$r^K = q^K \left[\frac{R^e}{\pi} - \left(1 - \delta_K\right)\right]$$

From (66) and (69):

$$p^Z = p^H mc^H$$
$$mc^Z = p^Z$$

From (67), (68), (116), (117) and (54) :

$$w = \left[\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha} mc^Z z}{(r^k)^{\alpha}}\right]^{\frac{1}{1-\alpha}}$$
$$k = \frac{\alpha}{1-\alpha}\widetilde{n}\frac{w}{r^k}a$$
$$y^Z = z\left(\frac{k}{a}\right)^{\alpha}\tilde{n}^{1-\alpha}$$
$$x^Z = y^Z$$
$$i = k\left[\frac{1-(1-\delta_K)/a}{\xi^i}\right]$$

Also, from (115)

$$y^H = \frac{x^Z}{\Xi^H}$$

From (26), (29), (30), (31) and (33):

$$\psi^e = \left[1-\Gamma_e\left(\bar{\omega}^e\right)\right]\frac{R^e q^K k}{a\pi}$$
$$n^e = \left(1-\chi_e\xi^{\chi_e}\right)\psi^e$$
$$c^e = \chi_e\xi^{\chi_e}\psi^e$$
$$\lambda^e = \frac{\Gamma^{e'}\left(\bar{\omega}^e\right)}{\left(1-\Gamma^F\left(\bar{\omega}^F\right)\right)\left[\Gamma^{e'}\left(\bar{\omega}^e\right)-\mu^e G^{e'}\left(\bar{\omega}^e\right)\right]}$$
$$l^F = q^K k - n^e$$

From (43), (41) and (104):

$$e^F = \phi_F l^F$$
$$d^F = l^F - e^F$$
$$d^U = d^F/\wp_U$$

From $r^{h,k} = {}^{q^H h}/_{q^K k}$, (56) to (58):

$$h = \frac{r^{h,k} q^K k}{q^H}$$
$$i^{AH} = \frac{h a^{N_H}}{\xi^{ih}}\left[1-\left(\frac{1-\delta_H}{a}\right)\right]$$
$$i^H = i^{AH}\varphi_0^H\left[\frac{1-\left(\frac{\rho^{\varphi H}}{a}\right)^{N_H+1}}{1-\frac{\rho^{\varphi H}}{a}}\right]$$

From (59), (60) and (61):

$$p^F = \left[\frac{1-\omega(p^H)^{1-\eta}}{1-\omega}\right]^{\frac{1}{1-\eta}}$$

From (70):

$$rer = mc^F p^F/\xi^m$$

Numerical solution for l_h iterating over the following equation up until $\Delta^l \approx 0$ (see Appendix B.1)

$$\Delta^l = gdpn - (c^P + c^I + i + i^H + s^g gdpn + s^{tb} gdpn)$$

From (18):

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$

from (50):

$$e^H = \phi_H q^L l^H$$

From (36), (37), (39) and (49):

$$n^b = e^F + e^H$$

$$\psi^b = \frac{n^b}{1-\chi_b \xi^{\chi_b}}$$
$$c^b = \chi_b \xi^{\chi_b} \psi^b$$
$$bb^{Tot} = (1-\phi_H) \frac{q^L l^H}{q^{BB}}$$

Then, from (93):

$$\upsilon = \frac{1}{a\pi} \begin{pmatrix} \gamma_D PD^D R^D d^F + +\gamma_{BB} PD^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e (\bar{\omega}^e) R^e q^K k \\ +\mu_I G_I (\bar{\omega}^I) R^H q^H h^I + \mu_H G_H (\bar{\omega}^H) \tilde{R}^H q^L l^H + \mu_F G_F (\bar{\omega}^F) \tilde{R}^F l^F \end{pmatrix}$$

From (128), (91), (129), (59), (60), (61), (119), (94) and (95):

$$gdpn = \frac{p^H y^H + (p^F)^{-\eta} (p^F - rer \xi^m \Xi^F)(1-\omega) \upsilon - \upsilon}{1 - s^{Co} - (1-s^{tb})(p^F)^{-\eta} (p^F - rer \xi^m \Xi^F)(1-\omega)}$$

From their definitions:

$$tb = s^{tb} gdpn$$
$$g = s^g gdpn$$
$$y^{Co} = \frac{s^{Co} gdpn}{p^{Co\star} rer}$$
$$b^{*Tot} = \frac{s^{b*} gdpn}{rer}$$

From (60), (61),(90), (91), (94), (95) , (119) and (128):

$$y^C = gdpn + \upsilon - tb$$
$$x^F = (1-\omega)(p^F)^{-\eta} y^C$$
$$x^H = \omega (p^H)^{-\eta} y^C$$
$$x^{H\star} = y^H - x^H$$
$$y^\star = x^{H\star} \left(\frac{p^H}{rer}\right)^{\eta\star}$$
$$y^F = x^F$$
$$m = y^F \Xi^F$$

From (96):

$$h^P = h - h^I$$

From (23):

$$c^I = \frac{wn}{2} + q^H h^I \left[(1-\Gamma_I) \frac{R^H}{a\pi} - 1\right] + q^L l^H$$

From (21) and (16):

$$o_{\hat{C}} = \left\{ (a)^{-\sigma \eta_{\hat{C}}} (\xi^h)^{\eta_{\hat{C}}-1} \left(\frac{ac^I \left(1-\frac{\phi_c}{a}\right)}{h^I \left(1-\frac{\phi_{hh}}{a}\right)} \right) \left(\frac{1}{\beta_I} \left[q^H - (\Gamma_I - \mu_I G_I) \frac{R^H q^H}{\tilde{R}^H} \right] - a^{-\sigma} (1-\Gamma_I) \frac{R^H}{\pi} q^H \right)^{-\eta_{\hat{C}}} + 1 \right\}^{-1}$$

Then from (15) we can compute

$$\hat{c}^I = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^I \left(1-\frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\frac{\xi^h h^I}{a} \left(1-\frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (16):

$$\lambda^I = \left\{ \left(\hat{c}^I\right)^{-\sigma} \right\} \left(\frac{(1-o_{\hat{C}}) \hat{c}^I}{c^I \left(1-\frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (21) and (22)

$$\lambda^H = \frac{\lambda^I}{\rho^H \phi_H}$$

Use ratios $\alpha_{BLG} = \frac{bl^G q^{BL}}{gdpn}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG} \frac{gdpn}{q^{BL}}$$
$$bs^G = \alpha_{BSG} gdpn$$

Then from (102) and (103), and normalizing $bl^{CB} = 1$

$$bl^{Pr} = -bl^{G}$$
$$bs^{Pr} = -bs^{G}$$

We can solve for bond holdings of the unrestricted households Also, from (99), (100) and (101)

$$bs^{U} = \frac{bs^{Pr}}{\wp^{U}}$$
$$b^{*U} = \frac{b^{*Tot}}{\wp^{U}}$$
$$bb^{U} = \frac{bb^{tot}}{\wp^{U}}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^{u} = \frac{\omega_{BL} * (bs^{u} + rer * b^{*U} + d^{U}) - bb^{U} q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^{R} = \frac{bl^{Pr} - \wp_{U} bl^{U}}{1 - \wp_{U}}$$

From 102

$$bl^{CB} = 1$$

Next, we solve for h^R, c^R, $\hat{c}^R$, λ^R. From (10) and (11) and the restricted household budget constraint (13)

$$h^{R} = \frac{q^{BL} bl^{R} \left(\frac{R^{BL}}{a} - 1\right) + \frac{wn}{2}}{q^{H} - \frac{q^{H}}{a}(1-\delta_{H}) + aux_{1}}$$

with aux_1

$$aux_{1} = (a)^{\sigma\eta_{\hat{C}}-1} (\xi^{h})^{1-\eta_{\hat{C}}} \left(\frac{q^{H}}{\beta_{P}} - (1-\delta_{H})\, a^{-\sigma} q^{H}\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1-\frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1-\frac{\phi_{c}}{a}\right)}$$

and

$$c^{R} = h^{R} aux_{1}$$

From (9):

$$\hat{c}^{R} = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^{R}\left(1-\frac{\phi_{c}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^{h} \frac{h^{R}}{a}\left(1-\frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (10):

$$\lambda^{R} = \left\{ \left(\hat{c}^{R}\right)^{-\sigma} \right\} \left(\frac{(1-o_{\hat{C}})\, \hat{c}^{R}}{c^{R}\left(1-\frac{\phi_{c}}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

Also, from (97) we get

$$h^{U} = \frac{h^{P} - (1-\wp_{U}) h^{R}}{\wp_{U}}$$

which together with (2) and (3) lets us solve for c^U

$$c^{U} = h^{U} (a)^{\sigma\eta_{\hat{C}}-1} (\xi^{h})^{1-\eta_{\hat{C}}} \left(\frac{q^{H}}{\beta_{P}} - (1-\delta_{H})\, a^{-\sigma} q^{H}\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1-\frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1-\frac{\phi_{c}}{a}\right)}$$

From (1) we solve for $\hat{c}^U$

$$\hat{c}^{U} = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^{U}\left(1-\frac{\phi_{c}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^{h} \frac{h^{U}}{a}\left(1-\frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

and from (2) we obtain λ^U

$$\lambda^{U} = \left(\hat{c}^{U}\right)^{-\sigma} \left(\frac{(1-o_{\hat{C}})\, \hat{c}^{U}}{c^{U}\left(1-\frac{\phi_{c}}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + (1 - \wp_U)\,\lambda^R$$

From (92):

$$c^P = \wp_U c^U + (1 - \wp_U)\,c^R$$
$$c = c^p + c^i$$

From (123), (124), (125), (126)

$$n^P = \frac{n}{2} = n^I = n^U = n^R$$

From (79), (80) and (77):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^\sigma$$
$$\Theta^U = 1$$
$$\tilde{\chi}^I = \left(\hat{c}^I\right)^\sigma$$
$$\Theta^I = 1$$
$$\tilde{\chi}^R = \left(\hat{c}^R\right)^\sigma$$
$$\Theta^R = \tilde{\chi}^R \left(\hat{c}^R\right)^{-\sigma}$$

$$\Theta = \frac{\left(\wp_U \Theta^U + (1 - \wp_U)\,\Theta^R\right) + \Theta^I}{2} = 1$$

From (75) and (78):

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}, \qquad \xi^n = \frac{mc^W \lambda^W w}{\Theta \widetilde{n}^\varphi}$$

From (127) and (130):

$$gdp = c + i + i^h + g + x^{H\star} + y^{Co} - m$$
$$ren^* = b^\star \left(1 - \frac{R^\star}{a\pi^\star}\right) - \frac{tb}{rer} + (1 - \chi)\,p^{Co\star} y^{Co}$$

From (7) and (105)

$$\epsilon^{L,S} = \beta_U R^{BL} a^{-\sigma} - 1$$

From (105) :

$$\zeta^L = \epsilon^{L,S}$$

From (85):

$$\tau = g + dia - bs^G \left(\frac{R}{a\pi} - 1\right) - q^{BL} bl^G \left(\frac{R^{BL}}{a} - 1\right) - \chi rer p^{Co\star} y^{Co}$$

From (86):

$$\alpha^T = \frac{\tau}{gdpn}$$

Finally, from (63), (72) and (82):

$$f^H = \frac{(\tilde{p}^H)^{-\epsilon_H} y^H mc^H}{1 - \beta_{UP}\theta_H a^{1-\sigma}}, \quad f^F = \frac{(\tilde{p}^F)^{-\epsilon_F} y^F mc^F}{1 - \beta_{UP}\theta_F a^{1-\sigma}}, \quad f^W = \frac{\tilde{w}^{-\epsilon_W(1+\varphi)} mc^W \widetilde{n}}{1 - \left(\frac{\left(\omega_{UP}\beta^{UP} + (1-\omega_{UP})\beta^{RP}\right) + \beta_I}{2}\right)\theta_W a^{1-\sigma}}$$

B.1 Numerical solution for l^H

First, guess l^H. Then, from (18) solve for h^I:

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$

From (50) and (49):

$$bb^{Tot} = (1 - \phi_H) \frac{q^L l^H}{q^{BB}}$$

Then, from (93):

$$\upsilon = \frac{1}{a\pi} \left(\begin{array}{c} \gamma_D PD^D R^D d^F + +\gamma_{BB} PD^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e \left(\bar{\omega}^e\right) R^e q^K k \\ +\mu_I G_I \left(\bar{\omega}^I\right) R^H q^H h^I + \mu_H G_H \left(\bar{\omega}^H\right) \tilde{R}^H q^L l^H + \mu_F G_F \left(\bar{\omega}^F\right) \tilde{R}^F l^F \end{array} \right)$$

From (128), (91), (129), (59), (60), (61), (119), (94) and (95):

$$gdpn = \frac{p^H y^H + \left(p^F\right)^{-\eta} \left(p^F - rer\xi^m \Xi^F\right)(1-\omega)\upsilon - \upsilon}{1 - s^{Co} - \left(1 - s^{tb}\right)\left(p^F\right)^{-\eta}\left(p^F - rer\xi^m \Xi^F\right)(1-\omega)}$$

From (96):

$$h^P = h - h^I$$

From (23):

From (21) and (16):

$$o_{\hat{C}} = \left\{ (a)^{-\sigma\eta_{\hat{C}}} \left(\xi^h\right)^{\eta_{\hat{C}}-1} \left(\frac{ac^I\left(1 - \frac{\phi_c}{a}\right)}{h^I\left(1 - \frac{\phi_{hh}}{a}\right)} \right) \left(\frac{1}{\beta_I}\left[q^H - \left(\Gamma_I - \mu_I G_I\right)\frac{R^H q^H}{\tilde{R}^H}\right] - a^{-\sigma}\left(1 - \Gamma_I\right)\frac{R^H}{\pi} q^H \right)^{-\eta_{\hat{C}}} + 1 \right\}^{-1}$$

Use ratios $\alpha_{BLG} = \frac{bl^G}{gdpn q^{BL}}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG} \frac{gdpn}{q^{BL}}$$

$$bs^G = \alpha_{BSG} gdpn$$

Then from (102) and (103), and normalizing $bl^{CB} = 0$

$$bl^{Pr} = -bl^G$$

$$bs^{Pr} = -bs^G$$

Also, from (99) and (100)

$$bs^U = \frac{bs^{Pr}}{\wp^U}, \qquad bb^U = \frac{bb^{tot}}{\wp^U}$$

Use ratio $s^{b*} = b^* rer/gdpn$, and (101)

$$b^{*Tot} = s^{b*} * gdpn/rer, \qquad b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * \left(bs^u + rer * b^{*U} + d^U\right) - bb^U q^{BB}}{q_{BL}}$$

which using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1 - \wp_U}$$

From (10) and (11) and the restricted household budget constraint (13)

$$h^R = \frac{q^{BL} bl^R \left(\frac{R^{BL}}{a} - 1\right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1 - \delta_H) + aux_1}$$

with aux_1

$$aux_1 = (a)^{\sigma\eta_{\hat{C}}-1}\left(\xi^h\right)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1 - \delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{\left(1 - o_{\hat{C}}\right)\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$

and

$$c^R = h^R aux_1$$

Also, from (97) we get

$$h^U = \frac{h^P - (1 - \wp_U)h^R}{\wp_U}$$

which together with (2) and (3) lets us solve for c^U

$$c^U = h^U (a)^{\sigma\eta_{\hat{C}}-1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H \right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}}) \left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}} \left(1 - \frac{\phi_c}{a}\right)}$$

From (92):

$$c^P = \wp_U c^U + (1 - \wp_U)\, c^R$$

Then, the following equation must hold:

$$gdpn = c^P + c^I + i + i^H + s^g gdpn + s^{tb} gdpn$$

If it does not, update guess of l^H and repeat.

C Steady state for capital requirements comparative statics

For a given value of capital requirements ϕ_f, ϕ_h we use estimated and calibrated parameters: related to real sector α, α^{BSG}, α^{BLG}, β_U, β_R, β_I, δ_K, δ_K, ϵ_F, ϵ_H, ϵ_W, N_H, κ, κ_{BL}, κ_{BB}, σ, χ, ω, ω_U, ω_{BL}, η, η^*, $\eta_{\hat{C}}$, θ_F, θ_H, θ_W, η_{ζ_L}; financial sector : χ_b, χ_e, γ_d, γ_{bh}, μ_e, μ_f, μ_h, μ_i, σ^e, σ^F, σ^H, σ^I, ξ^{χ_e}, ξ^{χ_b}; preference parameters and external sector parameters: $O_{\hat{C}}$, ϕ_c, ϕ_{hh}, $\rho^{\varphi H}$, φ, φ_0^H, a, bl^{cb}, $\epsilon^{L,S}$, g, n, $r^{h,k}$, π^T, p^{Co}, π^*, R^W, ξ^h, ξ^i, ξ^{ih}, ξ^m, ξ^n, ξ^R, y^*, y^{Co}, z, bl^G, bs^G, b^{*Tot} to compute the steady state of the model consistent with capital requirements different from that of the 2001-2019 period

Consider ϕ^F and ϕ^H total capital requirements including regulatory minimum capital, voluntary buffers and the neutral level (if any) for the CCyB requirement.

$$\phi^F = (\phi^F_{Reg} + \phi^F_{Vol} + CCyB)$$
$$\phi^H = 0.6(\phi^H_{Reg} + \phi^H_{Vol} + CCyB)$$

Use (4), (5), (6), (87) (88) and (89):

$$\pi = \pi^T; \quad R = \frac{\pi a^\sigma}{\beta_U}; \quad \tilde{R}^D = R; \quad \pi^s = \frac{\pi}{\pi^\star}; \quad R^\star = \frac{R}{\pi^s}; \quad R^W = \frac{R^\star}{\xi^R}$$

From (65), (74) and (111), (112):

$$\pi^H = \pi^F = \pi^{I,H} = \pi^{I,F} = \pi$$

From (84), (113) and (114):

$$\pi^W = \pi^{\widetilde{W}} = \pi^{I,W} = a\pi$$

From (62),(63),(64), (71),(72),(73), (81), (82), (83), (118), (120) and (122):

$$\tilde{p}^H = \tilde{p}^F = \widetilde{w} = 1$$
$$mc^H = \frac{\epsilon_H - 1}{\epsilon_H}$$
$$mc^F = \frac{\epsilon_F - 1}{\epsilon_F}$$
$$mc^W = \frac{\epsilon_W - 1}{\epsilon_W}$$
$$\Xi^H = \Xi^F = \Xi^W = 1$$

From (55) and (57):

$$q^K = 1/\xi^i; \quad \nabla l = a$$
$$q^H = \frac{a^{N_H \sigma}\varphi_0^H}{\beta_{UP}^{N_H}\xi^{ih}} \left(\frac{1 - \left(\frac{\beta_{UP}\rho^{\varphi H}}{a^\sigma}\right)^{N_H+1}}{1 - \frac{\beta_P \rho^{\varphi H}}{a^\sigma}} \right)$$

From (14) and (121):

$$R^H = \pi(1 - \delta_H)$$
$$\widetilde{n} = n$$

From (35), (37), (38), (39) and (47):

$$\rho^H = \tilde{\rho}^H = \rho^F = \frac{a\pi}{1 - \chi_b}$$

From (12) and (110)

$$R^{BL} = \frac{a^\sigma}{\beta_{RP}}$$
$$R^{Nom,BL} = R^{BL}\pi$$

From (7) and (8)

$$\tilde{R}^{BB} = R^{BL}$$

From (108)

$$q^{BL} = \frac{1}{R^{BL} - \kappa_B}$$

Given σ^F and the previous result for $\tilde{R}^D$, use a numerical solution for $\bar{\omega}^F$ and R^D using (42), (44) and (106)

$$\bar{\omega}^F - \left[1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)\right]\left(\frac{1 - \phi_F}{\phi_F}\right)\frac{R^D}{\tilde{\rho}^F} = 0$$
$$PD^F - \frac{1}{\gamma_D}\left(1 - \frac{\tilde{R}^D}{R^D}\right) = 0$$

And, from (44)

$$\tilde{R}^F = \frac{\phi_F \rho^F}{1 - \Gamma_F\left(\bar{\omega}^F, \sigma^F\right)}$$

Next, given σ^H and previous results for $\tilde{R}^{BB}$, use (48), (51) and (107) to find $\bar{\omega}^H$ and R^{BB} numerically,

$$\bar{\omega}^H - \left[1 - \Gamma_H\left(\bar{\omega}^H, \sigma^H\right)\right]\left(\frac{1-\phi_H}{\phi_H}\right)\frac{R^{BB}}{\rho^H}\pi = 0$$

$$\tilde{R}^{BB} = R^{BB}\left(1 - \gamma_{BH} PD^H\right)$$

Then, from (48), (53) and (109):

$$\tilde{R}^H = \frac{\phi_H \rho^H}{1 - \Gamma_H\left(\bar{\omega}^H, \sigma^H\right)}$$

$$PD^H = F_H\left(\bar{\omega}^H, \sigma^H\right)$$

$$q^{BB} = \frac{1}{R^{BB} - \kappa_{BB}}$$

Use (33) in (32), then use (44), (45), (26) and (31) to solve for $\bar{\omega}^e$

$$\frac{\Gamma_e'\left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e'\left(\bar{\omega}^e, \sigma^e\right)}{\Gamma_e'\left(\bar{\omega}^e, \sigma^e\right)} - \frac{\left(1-\chi_e\right)\tilde{R}^F}{a\pi} = 0$$

Then, from (34):

$$PD^e = F_e\left(\bar{\omega}^e\right)$$

Combine (28) and (45) to obtain

$$R^L = \frac{\tilde{R}^F \bar{\omega}^e}{\Gamma_e\left(\bar{\omega}^e, \sigma^e\right) - \mu_e G_e\left(\bar{\omega}^e, \sigma^e\right)}$$

Go back to (33) in (32) to obtain

$$\lambda^e = \frac{\Gamma^{e'}\left(\bar{\omega}^e\right)}{\left(1 - \Gamma^F\left(\bar{\omega}^F\right)\right)\left[\Gamma^{e'}\left(\bar{\omega}^e\right) - \mu^e G^{e'}\left(\bar{\omega}^e\right)\right]}$$

$$R^e = \left\{\frac{\left[1 - \Gamma_e\left(\bar{\omega}^e\right)\right]}{\lambda^e} + \left[1 - \Gamma_F(\bar{\omega}^F)\right]\left[\Gamma_e\left(\bar{\omega}^e\right) - \mu_e G_e\left(\bar{\omega}^e\right)\right]\right\}^{-1}\rho^F \phi_F$$

From (27):

$$r^K = q^K\left[\frac{R^e}{\pi} - \left(1 - \delta_K\right)\right]$$

Numerical solution for $\bar{\omega}^I$ using (51) and (22)

$$\frac{\Gamma_I'\left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I'\left(\bar{\omega}^I, \sigma^I\right)}{\Gamma_I'\left(\bar{\omega}^I, \sigma^I\right)} - \frac{\beta_I \tilde{R}^H}{a^\sigma \pi} = 0$$

From (25):

$$PD^I = F_I\left(\bar{\omega}^I\right)$$

From (18) and (52)

$$\hat{R}^I = \frac{\tilde{R}^H \bar{\omega}^I}{\pi\left[\Gamma_I\left(\bar{\omega}^I, \sigma^I\right) - \mu_I G_I\left(\bar{\omega}^I, \sigma^I\right)\right]}$$

and from (17), (19) and (20)

$$\hat{q}^L = \frac{1}{\hat{R}^I - \kappa_L}$$

$$q^L = \hat{q}^L$$

$$\hat{R}^I = R^I$$

From (20)

$$R^{Nom,I} = R^I \pi$$

Using the normalization $p^H = 1$, and from (66) and (69):

$$p^Z = p^H mc^H$$

$$mc^Z = p^Z$$

From (67), (68), (116), (117) and (54) :

$$w = \left[\frac{\alpha^{\alpha} (1-\alpha)^{1-\alpha} mc^Z z}{(r^k)^{\alpha}} \right]^{\frac{1}{1-\alpha}}$$
$$k = \frac{\alpha}{1-\alpha} \tilde{n} \frac{w}{r^k} a$$
$$y^Z = z \left(\frac{k}{a} \right)^{\alpha} \tilde{n}^{1-\alpha}$$
$$x^Z = y^Z$$
$$i = k \left[\frac{1-(1-\delta_K)/a}{\xi^i} \right]$$

Also, from (115)

$$y^H = \frac{x^Z}{\Xi^H}$$

From (26), (29), (30), (31) and (33):

$$\psi^e = [1-\Gamma_e(\bar{\omega}^e)] \frac{R^e q^K k}{a\pi}$$
$$n^e = (1-\chi_e \xi^{\chi_e}) \psi^e$$
$$c^e = \chi_e \xi^{\chi_e} \psi^e$$
$$l^F = q^K k - n^e$$

From (43), (41) and (104):

$$e^F = \phi_F l^F$$
$$d^F = l^F - e^F$$
$$d^U = d^F / \wp_U$$

From (59), (60) and (61):

$$p^F = \left[\frac{1-\omega(p^H)^{1-\eta}}{1-\omega} \right]^{\frac{1}{1-\eta}}$$

From (70):

$$rer = mc^F p^F / \xi^m$$

Next, we can find l^H, h^I, c^I solving the three equation system by (18), (23) and (21)

$$h^I = \frac{R^I q^L l^H}{\bar{\omega}^I R^H q^H}$$
$$c^I = \frac{wn}{2} + q^H h^I \left[(1-\Gamma_I) \frac{R^H}{a\pi} - 1 \right] + q^L l^H$$
$$\Delta^l = o_{\hat{C}} - \left\{ (a)^{-\sigma\eta_{\hat{C}}} (\xi^h)^{\eta_{\hat{C}}-1} \left(\frac{ac^I \left(1-\frac{\phi_c}{a}\right)}{h^I \left(1-\frac{\phi_{hh}}{a}\right)} \right) \left(\frac{1}{\beta_I} \left[q^H - (\Gamma_I - \mu_I G_I) \frac{R^H q^H}{\tilde{R}^H} \right] - a^{-\sigma} (1-\Gamma_I) \frac{R^H}{\pi} q^H \right)^{-\eta_{\hat{C}}} + 1 \right\}^{-1}$$

Then from (15) we can compute

$$\hat{c}^I = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^I \left(1-\frac{\phi_c}{a}\right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\frac{\xi^h h^I}{a} \left(1-\frac{\phi_{hh}}{a}\right) \right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

and from (16) and (24), respectively:

$$\lambda^I = \left\{ \left(\hat{c}^I\right)^{-\sigma} \right\} \left(\frac{(1-o_{\hat{C}})\, \hat{c}^I}{c^I \left(1-\frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}} ; \quad \lambda^H = \frac{\lambda^I}{\rho^H \phi^H}$$

Also, from (50):

$$e^H = \phi_H q^L l^H$$

From (39), (37), (36), and (49):

$$n^b = e^F + e^H$$
$$\psi^b = \frac{n^b}{1-\chi_b \xi^{\chi_b}}$$
$$c^b = \chi_b \xi^{\chi_b} \psi^b$$
$$bb^{Tot} = \frac{q^L l^H - e^H}{q^{BB}}$$

From (40)

$$PD^D = \frac{q^{BB} bb^{Tot} PD^H + d^F PD^F}{q^{BB} bb^{Tot} + d^F}$$

From (90), (94), (61), (60) (95) and (119)

$$x^{H\star} = \frac{y^\star}{\left(\frac{p^H}{rer}\right)^{\eta\star}}$$
$$x^H = y^H - x^{H\star}$$
$$y^C = \frac{x^H}{\omega (p^H)^{-\eta}}$$
$$x^F = (1-\omega)(p^F)^{-\eta} y^C$$
$$y^F = x^F$$
$$m = y^F \Xi^F$$

From (129)

$$tb = p^H x^{H\star} + p^{Co\star} y^{Co} rer - m \xi^m rer$$

From (93):

$$\upsilon = \frac{1}{a\pi} \begin{pmatrix} \gamma_D PD^D R^D d^F + +\gamma_{BB} PD^H R^{BB} q^{BB} bb^{Tot} + \mu_e G_e\left(\bar{\omega}^e\right) R^e q^K k \\ +\mu_I G_I\left(\bar{\omega}^I\right) R^H q^H h^I + \mu_H G_H\left(\bar{\omega}^H\right) \tilde{R}^H q^L l^H + \mu_F G_F\left(\bar{\omega}^F\right) \tilde{R}^F l^F \end{pmatrix}$$

Combine (91) and (128)

$$gdpn = y^C - \upsilon + tb$$

From their definitions:

$$s^g = \frac{g}{gdpn}$$
$$s^{Co} = \frac{y^{Co} p^{Co\star} rer}{gdpn}$$
$$s^{tb} = \frac{tb}{gdpn}$$

Supply of soverign debt instruments is inelastic, thus use ratios $\alpha_{BLG} = \frac{bl^G}{gdpn q^{BL}}$ and $\alpha_{SG} = \frac{bs^G}{gdpn}$

$$bl^G = \alpha_{BLG} \frac{gdpn}{q^{BL}}$$
$$bs^G = \alpha_{BSG} gdpn$$

From (102) and (103)

$$bl^{Pr} = -bl^G$$
$$bs^{Pr} = -bs^G$$
$$bs^U = \frac{bs^{Pr}}{\wp^U}$$
$$bb^U = \frac{bb^{tot}}{\wp^U}$$

Also, from (123), (124), (125), (126)

$$n^P = \frac{n}{2} = n^I = n^U = n^R$$

Next, we implement a numerical search for s^{b*} and $r^{h,k}$ (see Appendix C.1) using (78) and (128)

$$\xi^n = \frac{mc^W \lambda^W w}{\Theta \widetilde{n}^\varphi}$$
$$gdpn = c^P + c^I + i^K + i^H + g + tb$$

Then from its definition, we have

$$b^{*,Tot} = \frac{s^{b*} gdpn}{rer}$$

From (130)

$$ren^{*} = b^{*Tot}\left(1 - \frac{R^{\star}}{a\pi^{\star}}\right) - \frac{tb}{rer} + (1-\chi)\, p^{Co\star} y^{Co}$$

From $r^{h,k} = {}^{q^H h}/_{q^K k}$, (56) to (58):

$$h = \frac{r^{h,k} q^K k}{q^H}$$
$$i^{AH} = \frac{h a^{N_H}}{\xi^{ih}}\left[1 - \left(\frac{1-\delta_H}{a}\right)\right]$$
$$i^{H} = i^{AH}\varphi_0^H\left[\frac{1-\left(\frac{\rho^{\varphi H}}{a}\right)^{N_H+1}}{1-\frac{\rho^{\varphi H}}{a}}\right]$$

From (96)

$$h^P = h - h^I$$

From (101)

$$b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1-\wp_U}$$

From (102)

$$bl^{CB} = 1$$

From (10) and (11) and the restricted household budget constraint (13)

$$aux_1 = (a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}}\left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1-\frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1-\frac{\phi_c}{a}\right)}$$

$$h^R = \frac{q^{BL} bl^R\left(\frac{R^{BL}}{a}-1\right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1-\delta_H) + aux_1}$$
$$c^R = h^R aux_1$$

From (9):

$$\hat{c}^R = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(c^R\left(1-\frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}}\left(\xi^h\frac{h^R}{a}\left(1-\frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (10):

$$\lambda^R = \left\{\left(\hat{c}^R\right)^{-\sigma}\right\}\left(\frac{(1-o_{\hat{C}})\,\hat{c}^R}{c^R\left(1-\frac{\phi_c}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (97)

$$h^U = \frac{h^P - (1-\wp_U) h^R}{\wp_U}$$

From (2) and (3)

$$c^U = h^U (a)^{\sigma\eta_{\hat{C}}-1}(\xi^h)^{1-\eta_{\hat{C}}}\left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1-\frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1-\frac{\phi_c}{a}\right)}$$

From (1)

$$\hat{c}^U = \left[\left(1 - o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(c^U \left(1 - \frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + \left(o_{\hat{C}}\right)^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^U}{a} \left(1 - \frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} \right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (2)

$$\lambda^U = \left(\hat{c}^U\right)^{-\sigma} \left(\frac{\left(1 - o_{\hat{C}}\right) \hat{c}^U}{c^U \left(1 - \frac{\phi_c}{a}\right)} \right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + (1 - \wp_U) \lambda^R$$

From (92):

$$c^P = \wp_U c^U + (1 - \wp_U) c^R; \qquad c = c^p + c^i$$

From (79) and (80):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^{\sigma}$$

$$\Theta^U = \tilde{\chi}^U \left(\hat{c}^U\right)^{-\sigma}$$

$$\tilde{\chi}^I = \left(\hat{c}^I\right)^{\sigma}$$

$$\Theta^I = \tilde{\chi}^I \left(\hat{c}^I\right)^{-\sigma}$$

$$\tilde{\chi}^R = \left(\hat{c}^R\right)^{\sigma}$$

$$\Theta^R = \tilde{\chi}^R \left(\hat{c}^R\right)^{-\sigma}$$

$$\Theta = \frac{\left(\omega_{UP} \Theta^U + (1 - \omega_U) \Theta^R\right) + \Theta^I}{2} = 1$$

From (75)

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}$$

From (7) and (105)

$$\epsilon^{L,S} = \beta_U R^{BL} a^{-\sigma} - 1$$

From (105) :

$$\zeta^L = \epsilon^{L,S}$$

From (85):

$$\tau = g + dia - bs^G \left(\frac{R}{a\pi} - 1\right) - q^{BL} bl^G \left(\frac{R^{BL}}{a} - 1\right) - \chi rerp^{Co\star} y^{Co}$$

From (86):

$$\alpha^T = \frac{\tau}{gdpn}$$

Finally, from (63), (72) and (82):

$$f^H = \frac{(\tilde{p}^H)^{-\epsilon_H} y^H mc^H}{1 - \beta_{UP} \theta_H a^{1-\sigma}}, \quad f^F = \frac{(\tilde{p}^F)^{-\epsilon_F} y^F mc^F}{1 - \beta_{UP} \theta_F a^{1-\sigma}}, \quad f^W = \frac{\tilde{w}^{-\epsilon_W (1+\varphi)} mc^W \tilde{n}}{1 - \left(\frac{\left(\omega_{UP} \beta^{UP} + (1-\omega_{UP}) \beta^{RP}\right) + \beta_I}{2} \right) \theta_W a^{1-\sigma}}$$

C.1 Numerical solution for $(s^{b*},\, r^{h,k})$

Iterate on $(s^{b*},\, r^{h,k})$ until $\Delta \approx 0$

$$\Delta = \begin{bmatrix} \xi^n - \frac{mc^W \lambda^W w}{\Theta \tilde{n}^{\varphi}} \\ -gdpn + c^P + c^I + i^K + i^H + g + tb \end{bmatrix}$$

For each guess of $(s^{b*},\, r^{h,k})$ we have

$$b^{*,Tot} = \frac{s^{b*} gdpn}{rer}$$

From $r^{h,k} = {}^{q^H h}/_{q^K k}$, (56) to (58):

$$h = \frac{r^{h,k} q^K k}{q^H}$$

$$i^{AH} = \frac{h a^{N_H}}{\xi^{ih}} \left[1 - \left(\frac{1-\delta_H}{a}\right)\right]$$

$$i^H = i^{AH} \varphi_0^H \left[\frac{1 - \left(\frac{\rho^{\varphi H}}{a}\right)^{N_H+1}}{1 - \frac{\rho^{\varphi H}}{a}}\right]$$

From (96)

$$h^P = h - h^I$$

From (101)

$$b^{*U} = \frac{b^{*Tot}}{\wp^U}$$

Then using the (exogenously given) ratio of long to short term instruments held by the unrestricted patient household, ω_{BL}

$$bl^u = \frac{\omega_{BL} * (bs^u + rer * b^{*U} + d^U) - bb^U q^{BB}}{q_{BL}}$$

We can then, using (98) results in long term bonds held by the restricted household of

$$bl^R = \frac{bl^{Pr} - \wp_U bl^U}{1 - \wp_U}$$

From (102)

$$bl^{CB} = 1$$

From (10) and (11) and the restricted household budget constraint (13)

$$aux_1 = (a)^{\sigma\eta_{\hat{C}}-1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$

$$h^R = \frac{q^{BL} bl^R \left(\frac{R^{BL}}{a} - 1\right) + \frac{wn}{2}}{q^H - \frac{q^H}{a}(1-\delta_H) + aux_1}$$

$$c^R = h^R aux_1$$

From (9):

$$\hat{c}^R = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^R\left(1 - \frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^R}{a}\left(1 - \frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (10):

$$\lambda^R = \left\{\left(\hat{c}^R\right)^{-\sigma}\right\} \left(\frac{(1-o_{\hat{C}})\,\hat{c}^R}{c^R\left(1 - \frac{\phi_c}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (97)

$$h^U = \frac{h^P - (1-\wp_U) h^R}{\wp_U}$$

From (2) and (3)

$$c^U = h^U (a)^{\sigma\eta_{\hat{C}}-1} (\xi^h)^{1-\eta_{\hat{C}}} \left(\frac{q^H}{\beta_P} - (1-\delta_H)\, a^{-\sigma} q^H\right)^{\eta_{\hat{C}}} \frac{(1-o_{\hat{C}})\left(1 - \frac{\phi_{hh}}{a}\right)}{o_{\hat{C}}\left(1 - \frac{\phi_c}{a}\right)}$$

From (1)

$$\hat{c}^U = \left[(1-o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(c^U\left(1 - \frac{\phi_c}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}} + (o_{\hat{C}})^{\frac{1}{\eta_{\hat{C}}}} \left(\xi^h \frac{h^U}{a}\left(1 - \frac{\phi_{hh}}{a}\right)\right)^{\frac{\eta_{\hat{C}}-1}{\eta_{\hat{C}}}}\right]^{\frac{\eta_{\hat{C}}}{\eta_{\hat{C}}-1}}$$

From (2)

$$\lambda^U = \left(\hat{c}^U\right)^{-\sigma}\left(\frac{\left(1-o_{\hat{C}}\right)\hat{c}^U}{c^U\left(1-\frac{\phi_c}{a}\right)}\right)^{\frac{1}{\eta_{\hat{C}}}}$$

From (76):

$$\lambda^P = \wp_U \lambda^U + \left(1-\wp_U\right)\lambda^R$$

From (92):

$$c^P = \wp_U c^U + \left(1-\wp_U\right)c^R; \qquad c = c^p + c^i$$

From (79) and (80):

$$\tilde{\chi}^U = \left(\hat{c}^U\right)^{\sigma}$$
$$\Theta^U = \tilde{\chi}^U\left(\hat{c}^U\right)^{-\sigma}$$
$$\tilde{\chi}^I = \left(\hat{c}^I\right)^{\sigma}$$
$$\Theta^I = \tilde{\chi}^I\left(\hat{c}^I\right)^{-\sigma}$$
$$\tilde{\chi}^R = \left(\hat{c}^R\right)^{\sigma}$$
$$\Theta^R = \tilde{\chi}^R\left(\hat{c}^R\right)^{-\sigma}$$
$$\Theta = \frac{\left(\omega_{UP}\Theta^U + \left(1-\omega_U\right)\Theta^R\right)+\Theta^I}{2} = 1$$

From (75)

$$\lambda^W = \frac{\lambda^P + \lambda^I}{2}$$

Check if $\Delta = 0$

$$\Delta = \left[\begin{array}{l} \xi^n - \frac{mc^W\lambda^W w}{\Theta\tilde{n}^{\varphi}} \\ -gdpn + c^P + c^I + i^K + i^H + g + tb \end{array}\right]$$