Business Cycles when Consumers Learn by Shopping

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Abstract

Consumers rely on their shopping experiences to form beliefs about inflation. In other words, they *learn by shopping*. I introduce this empirical observation as an informational friction in the New Keynesian model and study its consequences for the transmission of aggregate shocks and the design of monetary policy. Learning by shopping anchors households' beliefs about inflation to its past, causing disagreement with firms over the value of the real wage. The discrepancy allows nominal shocks to have real effects, making the slope of the Phillips curve an endogenous function of the degree of anchoring and the monetary policy stance. In particular, a more hawkish monetary policy anchors households' inflation expectations, flattens the Phillips curve, and reduces the volatility and persistence of inflation. Such a policy also changes the drivers of business cycles: It amplifies the impact of aggregate demand shocks on output and makes supply shocks more inflationary.

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1 Introduction

The empirical evidence suggests that consumers have limited knowledge about inflation: Their perception of current inflation differs substantially from the inflation reflected in the CPI[1]. Moreover, providing consumers with information about inflation statistics has only partial and short-lived effects on their inflation expectations.[2] Instead of using public signals, the empirical evidence suggests that consumers rely on imperfect memories from their own shopping experiences to form beliefs about inflation.[3] In other words, they *learn by shopping*.

In this paper, I investigate the macroeconomic consequences of learning by shopping (henceforth LBS). To do so, I introduce this empirical observation as an informational friction on the households' block of a standard New Keynesian model: I assume they have incomplete information about inflation but acquire noisy signals of its value while shopping for the different goods in their consumption basket. Using this signal exclusively, they form beliefs about current and future inflation and make decisions conditional on those beliefs. I use the model to study analytically and quantitatively how this information friction affects the transmission of aggregate shocks and the design of monetary policy. Two results from this analysis stand out.

First, LBS amplifies the impact of exogenous shifts in aggregate demand (like those originating from monetary policy and consumer confidence shocks) on output. The friction affects households' perception of the real wage, leading them to disagree with firms about its value. This discrepancy allows nominal shocks to have real effects even when prices are flexible. LBS also distorts households' perception of their permanent income and the real interest rate, creating a confidence multiplier that amplifies the impact of aggregate demand shocks on consumption, employment, and output. Furthermore, nominal price rigidities exacerbate the effect of the informational friction non-linearly. Quantitatively, this amplification is substantial: under LBS, a contractionary shock to aggregate demand produces a fall in output *eight times larger* on impact than the corresponding response under full information.

Second, LBS makes the slope of the Phillips curve depend on the degree of anchoring of households' inflation expectations. But the degree of anchoring is endogenous and influenced by the strength with which the central bank responds to inflation. For this reason, a more hawkish monetary policy, like the one observed in the post-Volcker era, can simultaneously anchor households' inflation expectations, flatten the Phillips curve, and lower the volatility and persistence of inflation. The model suggests that such a policy also changed the drivers of the business cycle: It reduced the impact of supply shocks on output and amplified their inflationary effects. At the same time, it increased the importance of exogenous shifts in aggregate demand.

[1]See Jonung (1981), Detmeister, Lebow, and Peneva (2016), Arioli et al. (2017), Stanisławska (2019), for evidence from households in the E.U. and the U.S.

[2]See Cavallo, Cruces, and Perez-Truglia (2016) and Coibion, Gorodnichenko, and Weber (2019).

[3]See, among others, Cavallo, Cruces, and Perez-Truglia (2017), Angelico and Di Giacomo (2019), Mosquera-Tarrío (2019), Coibion, Gorodnichenko, and Weber (2020), D'Acunto, Malmendier, and Weber (2021b), D'Acunto, Malmendier, Ospina, and Weber (2021a), and Weber, Gorodnichenko, and Coibion (2022).

Framework. I extend the standard New Keynesian model to allow for a continuum of households with incomplete information about the price level. Shopping experiences provide households with a noisy and idiosyncratic signal about the price level, which they use to update their beliefs about its value and the inflation rate using Bayes rule. As a result, consumers in the model disagree in their perceived and expected levels of inflation—a feature consistent with the empirical evidence. Consequently, each household shares a different view on the purchasing power of their income and the real return of their financial assets. The heterogeneity in beliefs translates into heterogeneity in consumption, labor supply, and asset holdings across households.

Despite this large heterogeneity, I show that the dynamics of aggregate output and inflation admit a simple characterization: The aggregate demand in the model is described by a standard Euler equation augmented with the presence of an information wedge. This wedge captures the cross-sectional differences in households' beliefs about their permanent income and interest rates from the corresponding beliefs under full information. On the other hand, the supply side of this economy is described by a standard New Keynesian Phillips Curve (hereafter NKPC) augmented with a second information wedge that captures the disagreement between firms and households regarding wages. With the help of two auxiliary assumptions, the equilibrium of the model admits an almost closed-form solution. Equipped with this solution, I derive the paper's main results analytically and provide intuition for them.

LBS propagates nominal shocks. The disagreement between firms and households resulting from the information friction allows nominal shocks to have real effects, even when prices are flexible. Households condition their labor supply on the real wage they perceive. They observe their nominal wage but rely on the noisy information from their shopping experiences to learn about the price level. As a result, the real wage paid by firms doesn't necessarily coincide with its corresponding perception by households.

To illustrate the consequences of this information wedge, consider a positive shock to households discount factor (a standard proxy for exogenous fall in consumer spending). When firms have flexible prices, such a shock produces a fall in wages accompanied by a one-to-one reduction in the prices set by firms. Households observe the fall in nominal wages but, because of LBS, they only observe part of the accompanying reduction in the aggregate price level. Consequently, households perceive that their real wage has fallen and reduce their consumption and labor supply as a result. The previous mechanism bears a close resemblance to the original versions of the Phillips curve proposed by Phelps (1967) and Friedman (1968) and their subsequent formalization by Lucas (1972).[4]

In contrast to the previous theories, LBS simultaneously affects the demand block of the economy,

[4]In particular, Friedman (1968) noted that differences in the real wages perceived by firms and households produce comovement between output and inflation. Influenced by the work of Lucas (1972), he expanded this vision in his Nobel Prize lecture (Friedman, 1977) by attributing this wedge to information frictions on households. In the third section of this lecture, he notes: *"To workers, the situation is different: what matters to them is the purchasing power of wages not over the particular good they produce but over all goods in general. Roth they and their employers are likely to adjust more slowly their perception of prices in general - because it is more costly to acquire information about that - than their perception of the price of the particular good they produce. As a result, a rise in nominal wages may be perceived by workers as a rise in real wages and hence call forth an increased supply, at the same time that it is perceived by employers as a fall in real wages and hence calls forth an increased offer of jobs."*

introducing a second channel that amplifies the impact of nominal shocks: After the exogenous contraction in aggregate demand, households' income from dividends and wages falls in nominal terms. Because they only observe part of the reduction in the aggregate price level, they perceive this as a drop in their permanent income. In response, they reduce their consumption further, amplifying the initial effect of the shock on aggregate output.

Price stickiness amplifies the effects of LBS. Price stickiness allows demand shocks to shift firms' labor demand. Since LBS shifts households' labor supply, it operates as a multiplier that amplifies the real effects of nominal shocks in the standard New Keynesian model.

Perhaps surprisingly, the amplification is non-linear: the impact of a demand shock on output when both frictions are present can be larger than the sum of the corresponding impact when each friction is considered separately. The key behind this result is the endogenous nature of the degree of anchoring of households' inflation expectations. LBS makes their beliefs underreact to shocks to the inflation rate. In this sense, it anchors their beliefs about inflation.

But the degree of anchoring is an endogenous object in the model: It is directly related to the rate at which households learn about inflation from their shopping experiences. The volatility and persistence of inflation affect the informational content of households' this experiences, and both properties of inflation are themselves a function of the degree of anchoring. In equilibrium, an increase in price-stickiness reduces the volatility of inflation, reducing the information about aggregate shocks contained in households' shopping experiences. As a result, higher price rigidity makes household beliefs about inflation more anchored, exacerbating the propagation of demand shocks originating from the information friction.

Hawkish monetary policy flattens the Phillips curve. The model shows that the central bank can indirectly affect the aggregate supply of the economy through its ability to anchor households' beliefs about inflation. As in the standard New Keynesian model, a more hawkish policy stance reduces the volatility of inflation by flattening the slope of aggregate demand. With LBS, the lower inflation volatility also reduces the information about inflation and aggregate shocks contained in households' shopping experiences, increasing the degree of anchoring of their beliefs.

The previous result associates the flattening of the Phillips curve documented in the data to the more active monetary policy that followed Chairman's Volcker tenure at the Fed.[5] The model predicts that households' limited knowledge of inflation is a direct consequence of the success of this policy in stabilizing inflation. The resulting anchoring of inflation expectations, in turn, reduced the slope of the NKPC by decreasing the speed at which households learn without changing the sensitivity of actual inflation to marginal costs. Consistent with this prediction, the empirical evidence suggests there has been no change in the relationship between inflation and marginal costs over this period.[6]

[5] See, among others, Ball and Mazumder (2011), Coibion and Gorodnichenko (2015b), Coibion, Gorodnichenko, and Kamdar (2018a).

[6] See Del Negro, Lenza, Primiceri, and Tambalotti (2020), Barnichon and Mesters (2021), and Hazell, Herreño, Nakamura, and Steinsson (2020).

LBS mitigates the impact of supply shocks on output. While LBS amplifies the business cycle fluctuations triggered by demand shocks, it hinders the impact of supply shocks. Consider, for instance, an unexpected reduction in TFP: Such a shock results in a decrease in real wages, partially due to a sudden increase in the aggregate price level. With LBS, households' beliefs are anchored, so they perceive a more moderate spike in inflation. Consequently, their perception of real wages and permanent income falls less compared to the full information case, mitigating the negative impact of the shock on consumption and labor supply. However, it makes supply shocks more inflationary.

LBS as Rational Inattention to Inflation. I consider an extension where the noise in shopping experiences is the byproduct of households' rational inattention to inflation. Following the literature pioneered by Sims (2003), I allow households to choose the attention allocated to aggregate inflation by trading the benefits and costs of acquiring information about this variable, and model these costs as a linear function of Shannon's mutual information.

I show that the inattention to inflation produced by LBS has only second-order effects on households' welfare. This result is consistent with the observation by Cochrane (1989) that the costs of deviating from the permanent income decision rule are arbitrarily small for a consumer. As a result, small costs of acquiring information can make consumers largely inattentive to news about inflation. Like menu cost models, LBS is a form of "near-rational behavior", as coined by Akerlof and Yellen (1985), where second-order individual losses can have first-order effects on the aggregate economy.

Quantitative analysis. I conclude by studying the robustness and quantitative relevance of the earlier analytical results. To do so, I use a version of the model that includes the extensions mentioned above and allows learning to be persistent over time. I calibrate the model to match the behavior of core CPI inflation in the U.S. and discipline the magnitude of the information frictions using data on households' inflation expectations from the Michigan Survey of Consumers.

The quantitative exercise suggests that the amplification of demand shocks induced by LBS is substantial: an expansionary shock to aggregate demand produces an expansion in output that is *eight times larger* on impact than the corresponding response under full information. Following Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021), I also compare the predictions of a counterfactual dovish policy by the central bank with the behavior of macroeconomic variables observed during the pre-Volcker era. The exercise shows that such a policy change can quantitatively account for the reduction in the volatility and persistence of inflation, as well as the anchoring of inflation expectations that followed this period. However, the exercise suggests that the increase in anchoring also exacerbated the information frictions affecting households and, through this channel, the impact of demand shocks on the economy.

Related literature. This paper belongs to the literature studying the macroeconomic consequences of informational frictions on households. In this paper, learning by shopping gives rise to informational frictions that affect their beliefs about the the price level (or cost of living) and, consequently, their beliefs about inflation.

A strand of the literature has studied economies where households have incomplete information about the prices of all available consumption goods in the economy. Nevertheless, they do have full information about the overall price level of their own consumption basket (Lucas, 1973; Lorenzoni, 2009; Maćkowiak and Wiederholt, 2015; Chahrour and Gaballo (2020); Gaballo and Paciello, 2021; Angeletos and Lian 2018; 2021). As a result, the frictions in these papers do not directly affect households' perception of the price level or current inflation. In contrast, the model in this paper allows households to have incomplete information about both variables. I show that this friction alters the economy's supply side, allowing nominal shocks to have real effects while simultaneously amplifying their impact through a "confidence multiplier" akin to the one introduced by Angeletos and Lian (2021).

Close in this respect is the work of Mankiw and Reis (2006) and Wiederholt (2015), who also consider NK models where consumers make decisions while facing uncertainty about the current price level. However, both papers focus on different questions and consider environments where consumers update their information at exogenous and fixed intervals. Instead, this paper focuses on an economy where the rate at which households acquire information is an endogenous object in the model. As a result, the strength of the information friction depends on structural features of the economy, including the degree of price rigidity and the monetary policy stance.

Closely related is also the work by L'Huillier (2020), who studies consumer learning in a decentralized market for goods. In that setting, the information friction allows nominal shocks to have real effects by affecting households' perceptions of the relative price of consumption goods. In comparison, the non-neutrality in this paper operates through households' labor supply by affecting their perception of real wages, giving rise to a wedge in the labor market.

A distinctive feature of this paper is that it studies the interaction between two different frictions, LBS and price-stickiness, each of which allows the propagation of demand shocks on its own. A large part of the literature has focused on the role of information frictions as a substitute for nominal price rigidities (Ball, Mankiw, and Romer, 1988; Mankiw and Reis, 2002; Mackowiak and Wiederholt, 2009). This paper shows that both frictions can complement each other, and their interaction can amplify the propagation of aggregate demand shocks beyond what is possible when they are considered separately. Moreover, the same frictions attenuate the impact of supply-side shocks on the economy. This result suggests that when households learn by shopping, demand shocks arise as the most suitable candidates to drive the business cycle, consistent with the empirical evidence by Angeletos, Collard, and Dellas (2020).

This paper also contributes to the literature studying the relationship between inflation dynamics, expectation anchoring, and the flattening of the Phillips curve. A strand of this literature has employed models with incomplete information by firms to account for the changing relationship between inflation dynamics and economic activity (Afrouzi and Yang, 2021; Gallegos, 2021; L'Huillier, Phelan, and Zame, 2021). This paper shows both analytically and quantitatively how a change in the monetary policy stance (like the one observed in the post-Volcker era) can anchor households' inflation expectations and, through this channel, flatten the Phillips curve. Closely related is the literature studying inflation dynamics and expectation anchoring under learning (Marcet and Nicolini, 2003; Jørgensen and Lansing, 2021; Gáti, 2022). This literature assumes agents have full information while relaxing the assumption that agents have rational expectations. This paper studies expectation anchoring under the complementary assumption: households have rational expectations but incomplete information about aggregate inflation.

This paper also contributes to the rational inattention literature pioneered by Sims (2003) and surveyed in Mackowiak, Matejka, and Wiederholt (Forthcoming) by studying the interaction between rational inattention to aggregate inflation and price-stickiness.

Finally, this paper is motivated by the empirical literature documenting consumers' inattention to prices (surveyed by DellaVigna (2009), Anderson and Simester (2009), and Gabaix (2019)), the deviations of their beliefs about inflation from the FIRE assumption (Coibion and Gorodnichenko, 2012; Coibion and Gorodnichenko, 2015a; Coibion, Gorodnichenko, and Weber, 2019), and the impact of shopping and life experiences on those beliefs (Malmendier and Nagel, 2016; Kuchler and Zafar, 2019; Cavallo, Cruces, and Perez-Truglia, 2017; Coibion, Gorodnichenko, and Weber, 2019; D'Acunto, Malmendier, Ospina, and Weber, 2021a).[7] The model presented here provides a theoretical framework that researchers can use to incorporate the empirical findings from this literature in standard models used to study the transmission of macroeconomic shocks and the design of monetary policy.

Outline. This paper contains six sections, including this introduction. Section 2 sets up the model and characterizes its equilibrium. Section 3 studies the impact of LBS on the transmission of macroeconomic shocks and illustrates analytically the main results in this paper. Section 4 discusses the extensions of the model. Section 5 presents the quantitative exercise. Section 6 concludes. The appendix contains all proofs and the computational method used to solve the quantitative model.

2 Learning by Shopping in a New Keynesian Model

In this section, I present a New Keynesian model where consumers learn by shopping. The first part of this section sets up the model and discusses its key assumptions. The second part derives the NKPC and the aggregate Euler equation that characterize the supply and demand blocks of this model. The third part characterizes the equilibrium dynamics of output and inflation, as well as the equilibrium degree of anchoring of households' beliefs about inflation.

2.1 The model

Time is discrete and indexed by t. The model is inhabited by a continuum of households indexed by subscript $i \in [0,1]$. Every household supplies labor, saves, and consumes an infinite variety of goods. Each consumption variety is produced by a different firm indexed by subscript $j \in [0,1]$. Firms have market power and set the price of the variety they produce while facing nominal rigidities a la Calvo (1983).

[7] See also Coibion, Gorodnichenko, and Kamdar (2018a) for a survey of this literature.

There is a single aggregate shock affecting the discount factor of all households. This shock drives exogenous fluctuations in aggregate demand and is the only source of aggregate uncertainty in the model.[8] There are also three auxiliary shocks affecting the discount rate, the wage, and the return on savings faced by each household. However, these shocks are i.i.d. across time and also across households, so they have no direct effect on the behavior of aggregate variables. As discussed below, the only role of these auxiliary shocks is to add "noise" to the information set of each household, allowing the introduction of information frictions in the model. In the absence of these frictions, the equations characterizing the dynamics of output and inflation are identical to those found in Chapter 3 of Galí (2015).

Households. The problem of household i in period t is to maximize:

$$\mathrm{E}_{i,t} \sum_{k=0}^{\infty} \beta^k U\left(C_{i,t+k}, N_{i,t+k}; Z_{i,t+k}\right), \tag{1}$$

where $\beta \in (0,1)$ is the households' discount factor, and $\mathrm{E}_{i,t}[\cdot] \equiv \mathbb{E}[\cdot|\mathcal{I}_{i,t}]$ denotes the expectation operator conditional on the information set of the household at the beginning of period t. This information set is denoted as $\mathcal{I}_{i,t}$ and is described in detail below. The period utility function $U(\cdot)$ depends on the household's consumption index $C_{i,t}$, the labor supplied $N_{i,t}$, and a preference shifter $Z_{i,t}$. The later captures exogenous shifts in the household's discount factor. I assume that the per-period utility function $U(\cdot)$ takes the form

$$U\left(C_{i,t}, N_{i,t}; Z_{i,t}\right) = Z_{i,t}\left\{\frac{C_{i,t}^{1-\sigma}-1}{1-\sigma} - \frac{N_{i,t}^{1+\varphi}}{1+\varphi}\right\}, \tag{2}$$

and that households' consumption index is a CES bundle given by:

$$C_{i,t} = \left(\int_0^1 C_{i,j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where $\varepsilon > 1$ denotes the elasticity of substitution across goods, and $C_{i,j,t}$ denotes the consumption of variety j by household i. The preference shifter $Z_{i,t}$ is given by

$$\begin{gathered} \log Z_{i,t} = \rho_z \log Z_{i,t} + \eta_t + \xi^z_{i,t}, \\ \eta_t \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_{AD}\right), \quad \xi^z_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \zeta^2_x\right), \end{gathered} \tag{4}$$

with $\rho_z \in [0,1)$. The shock η_t generates correlated desire across households to spend. A positive value of η_t increases (nominal) aggregate spending in the current period. On the other hand, the shock $\xi^z_{i,t}$ produces idiosyncratic variations in the discount rate of each household. The only purpose

[8] This shock is commonly used in the literature as proxy for unexpected shifts in aggregate demand (AD). As discussed below, this shock has no real effects when prices are flexible and households have full information. In section 4, I introduce TFP shocks as a second source of aggregate uncertainty.

of this shock is to prevent households from observing directly η_t by observing $Z_{i,t}$.

The maximization of (1) is subject to the following sequence of budget constraints in every period:

$$\int_0^1 P_{j,t}C_{i,j,t}dj + B_{i,t} = R_{i,t-1}B_{i,t-1} + W_{i,t}N_{i,t} + D_{i,t}, \tag{5}$$

where $W_{i,t}$ denotes the nominal wage rate faced by household i, $P_{j,t}$ is the price of consumption variety j, $B_{i,t}$ denotes the quantity of nominally riskless one-period bonds purchased by this household in period t, $R_{i,t}$ is the gross nominal interest rate between $t-1$ and t faced by the household, and $D_{i,t}$ denotes the dividends it receives from firm ownership.

Let W_t denote the nominal wage rate payed by firms, and let R_t denote the nominal interest rate on bonds set by the central bank. I assume that the corresponding nominal wage and interest rate faced by household i are given by $W_{i,t} = W_t e^{\xi^w_{i,t}}$ and $R_{i,t} = R_t e^{\xi^r_{i,t}}$, respectively. The shocks $\xi^w_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \zeta_x^2\right)$, $\xi^r_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \zeta_x^2\right)$, and $\xi^z_{i,t}$ in (4) are i.i.d. across households and time, and are also independent of the aggregate shock η_t.

This type of auxiliary shock is standard in the information frictions literature.[9] They can be alternatively microfounded as the result of idiosyncratic income risk, market segmentation, intermediation costs, perceptual noise, or rational inattention. For the results in this paper, the particular microfoundation is not crucial, as their only role in the model is to add "noise" to the market signals available to households. I assume all households and firms hold rational expectations, so this noise is necessary to preserve the absence of common knowledge in the model (Grossman and Stiglitz, 1980)).

Households' information set. The price index of the consumption bundle (3) is given by:

$$P_t = \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}. \tag{6}$$

I assume that households do not observe the aggregate price index (6) directly, but acquire noisy information about this variable from their own shopping experiences. To introduce this friction formally, I assume that the problem of households in each period takes place in two consecutive stages: A *shopping stage* and a *paying stage*.[10]

In the *shopping stage*, each household receives a set of noisy and private signals about the price of each consumption variety. I denote signal about price $P_{j,t}$ received by household i as $S_{i,j,t}$ and assume that its given by:

$$\log S_{i,j,t} = \log P_{j,t} + \epsilon_{i,t}, \quad \epsilon_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_\epsilon^2\right), \tag{7}$$

where $\epsilon_{i,t}$ is i.i.d. across households and time, and also uncorrelated with other shocks in the economy.[11]

[9] See, for instance, Lorenzoni (2009), Hellwig and Veldkamp (2009), Nimark (2014), Angeletos and Lian (2018), and Angeletos and Lian (2021)

[10] To abbreviate, we can call this the "*Shopping and Paying in A New Keynesian Model*", or SPANK.

[11] For the analytical results in this paper, it is not necessary to take a stand on the nature of the noise in signals (7). One can

During the *shopping stage*, household i also observes the nominal wage $W_{i,t}$ and interest rate $R_{i,t}$ it faces, the nominal dividends received from firm ownership, and the preference shock $Z_{i,t}$. Using this information, the household forms beliefs about using Bayes rule and chooses the labor supply $N_{i,t}$ and the consumption of varieties $C_{i,j,t}$, both of which are delivered in the following stage.

During the *paying stage*, households receive the consumption varieties ordered in the previous stage and supply labor as planned. They also observe the value of their expenditures, denoted as $M_{i,t} \equiv \int_0^1 P_{j,t} C_{i,j,t} dj$. With this additional information, each household adjusts its bond holdings $B_{i,t}$ to make sure that their budget constraint binds.

Consequently, the information set of household i at the beginning of every period contains the history of wages, interest rates, and preference shocks faced. It also includes the history of signals about the price of each consumption variety and the total expenditures, bond holdings, and dividends observed at the end of the previous period. Formally, the information set of household i at the beginning of period t is given by:

$$\mathcal{I}_{i,t} = \mathcal{I}_{i,t-1} \cup \{W_{i,t}, R_{i,t}, Z_{i,t}, D_{i,t}\} \cup \{M_{i,t-1}, B_{i,t-1}\} \cup \{S_{i,j,t}\}_{j \in [0,1]} . \tag{8}$$

The problem of household i in period t is to choose the labor supply $N_{j,t}$ and the consumption of each variety $C_{i,j,t}$ to maximize (1) conditional on its private information (8), subject to the budget constraint (5). At the end of every period, the household adjusts its bond holdings $B_{i,t}$ to make sure that (5) binds.[12]

The information structure implied by (7) and (8) relaxes the assumption that consumers have full information about the price index P_t and, consequently, about the inflation rate $\pi_t \equiv \log P_t - \log P_{t-1}$. The idiosyncratic nature of households' shopping experiences gives rise to dispersion in beliefs about *current* and *future* inflation. Full information about π_t is nested as a special case where $\sigma_\epsilon^2 \to 0$. In this case, households have *common knowledge* about π_t and share the same belief about the current, past, and future value of inflation.

This information structure has two other implications which will help keep the analysis tractable: First, households have common knowledge about *past* aggregate outcomes. The reason is that, during the *paying stage*, households observe total expenditures $M_{i,t}$, as well as all other variables affecting their income. Using their budget constraint (5), they can use this information to infer perfectly P_{t-1} at the beginning of period t. Since households have rational expectations, they can infer the aggregate state from this variable and, as a result, the value of all aggregate outcomes in the past.[13] Second,

think of this noise as a modeling device to incorporate incomplete information about the price level, allowing the model to replicate two salient features of the data: The large disagreement in households' beliefs about current and future inflation and their reliance on their shopping experiences to form these beliefs. In Section 4, I discuss an extension of the model where the noise $\epsilon_{i,t}$ is microfounded as resulting from rational inattention to aggregate inflation. In this microfoundation, $\epsilon_{i,t}$ is the byproduct of households' optimal allocation of attention to inflation, and the variance of $\epsilon_{i,t}$ is endogenous and chosen by households to trade the costs and benefits of acquiring information about this variable.

[12] This restriction on the timing of households decisions is similar in spirit to the one introduced by Rotemberg and Woodford (1997).

[13] In the quantitative model used in Section 5, I will let households consume only a subset of all available goods every period. As a result, they will not be able to infer the past price level from their own expenditures, relaxing the assumption of common knowledge about past outcomes.

households can infer perfectly the relative price of each good. To see why, notice that the shock $\epsilon_{i,t}$ is common across all the shopping signals a household receives. For this reason, the noise in signals will only affect the ability of each household to observe the aggregate price level but not their ability to observe the relative price of each good.[14]

Learning by Shopping. I introduce the following assumption to simplify the characterization of households' beliefs.

Assumption 1. *The variance of the auxiliary shocks ζ_x^2 is such that $\sigma_\epsilon^2/\zeta_x^2 \to 0$.*

To form beliefs about inflation, households can use all the signals acquired during their shopping experiences and all other signals available in their information set. Assumption 1 limits the informational content of these additional signals. As a result, households will form beliefs about inflation relying exclusively on the signals acquired during their shopping experiences.[15] For this reason, I refer to the informational friction introduced in this paper as *learning by shopping* (LBS).

Firms. Firms are price takers in the input market and use a linear technology of production $Y_{j,t} = N_{j,t}$, where $N_{j,t}$ denotes the demand of labor by firm j in period t. The problem of firm j is to choose the price of its own variety $P_{j,t}$ to maximize the present value of its dividends, given by

$$\mathbb{E}_t \sum_{k=0}^{\infty} \Lambda_{t,k} \left(\frac{P_{j,t+k}}{P_{t+k}} - \frac{W_{t+k}}{P_{t+k}} \right) \mathcal{C}_{j,t+k}, \tag{9}$$

where $\mathcal{C}_{j,t} \equiv \int_0^1 C_{i,j,t} di$ is the demand for variety j across all households, $\mathbb{E}_t [\cdot]$ denotes the full information expectation operator, and $\Lambda_{t,k}$ is a stochastic discount factor.

Every household in the economy has equal ownership of each firm, and their profits are redistributed accordingly. It follows that the stochastic discount factor used by every firm is an equally-weighted average of the stochastic discount factor of each household, which is given by:

$$\Lambda_{i,t,k} \equiv \beta^k \left(C_{i,t+k}/C_{i,t} \right)^{-\sigma} \left(Z_{i,t+k}/Z_{i,t} \right).$$

Finally, I assume that firms face nominal rigidities that prevent them from adjusting prices in every period. Specifically, I adopt the formalism proposed by Calvo (1983) and assume that each firm can reset its price only with probability $1-\theta$. This probability is exogenous, common across

[14] I will show this formally when characterizing the beliefs of households in the next section. Intuitively, each household can pool all signals $S_{i,j,t}$ in its information set to construct a noisy signal $S_{i,t}$ of the aggregate price level. Using this new signal, the household can eliminate the common noise in the signals about relative prices. See Gabaix (2014) for a model where inattention to prices alters the relative price perceived by households and the consequences of this form of bounded rationality.

[15] Households are Bayesian. In the log-linear approximation studied in the next sections, their beliefs about P_t are given by a weighted average of their shopping signals and the remaining variables in their information set. As $\sigma_\epsilon^2/\zeta_x^2 \to 0$, the weight assigned to these additional signals converges to zero. This allows keeping the characterization of households' beliefs simple. Since ζ_x^2 is a free parameter in the model, one can always choose a region of the parameter space where this assumption holds approximately. Alternativelly, one can simply state Assumption 1 as $\mathrm{E}_{i,t}\left[P_t|\mathcal{I}_{i,t}\right] = \mathrm{E}_{i,t}\left[P_t|\left\{S_{i,j,t}\right\}_{j\in[0,1]}\right]$.

firms, and independent from the time elapsed since the last time the price was adjusted. It follows that a fraction θ of firms keeps their prices unchanged in any period, and the average duration of a price is given by $\frac{1}{1-\theta}$.

Firms' information set. To isolate the role of LBS, I assume that firms face no informational frictions. They can observe the value of aggregate productivity and their marginal costs. Firms also understand that consumers form beliefs based on private signals. However, they don't observe consumers' signals or beliefs about prices directly. For this reason, firms cannot discriminate prices across customers or commit to holding a specific price for multiple periods.

Government. The central bank issues bonds B_t at zero net supply, and sets the interest rate $i_t \equiv \log R_t + \log \beta$ following a standard Taylor rule of the form:

$$i_t = \phi_\pi \pi_t, \tag{10}$$

where $\phi_\pi > 0$ measures the strength with which the central bank responds to deviations of the inflation rate from its target.

Equilibrium definition. In this paper, I focus on an equilibrium where agents hold rational expectations, make decisions contingent on their private information, and prices adjust to clear all markets.

Formally, an equilibrium of this economy is defined by a set of stochastic processes for the average wage rate W_t, the interest rate R_t, the price of each variety $\{P_{j,t}\}_{j\in[0,1]}$, the labor supply and bond holdings of each household, $\{N_{i,t}, B_{i,t}\}_{i\in[0,1]}$, and the consumption of each variety by each household $\{C_{i,j,t}\}_{(i,j)\in[0,1]^2}$ such that:

1. Every household $i \in [0,1]$ maximizes its expected utility (1) conditional on its own information set (8) and budget constraint (5).

2. Every firm $j \in [0,1]$ maximizes the present value of its expected profits (9).

3. The interest rate follows the central bank rule (9).

4. Agents have rational expectations.

5. The goods and labor markets clear.

By Walras law, the last condition also implies clearance of the bonds market.

2.2 Equilibrium characterization

To keep the analysis tractable, I will work with a log-linear approximation of the model around a neighborhood of its non-stochastic steady-state with zero inflation. In what follows, I denote the log-deviation of a variable from its steady-state value in lower case.[16]

[16]The only exceptions are the price level $p_t \equiv \log P_t$, the nominal interest rate $i_t \equiv \log R_t + \log \beta$, and the real interest rate, denoted as r_t.

Beliefs about inflation. I start by characterizing households beliefs about inflation. To begin, note that each household can construct a noisy signal of P_t by averaging across $S_{i,j,t}$. Let

$$S_{i,t} \equiv \exp\left\{\int_0^1 \log S_{i,j,t} dj\right\} \tag{11}$$

denote this average signal. Using $S_{i,t}$, each household can construct a second set of demeaned signals $S^R_{i,j,t} \equiv \log\left(S_{i,j,t}/S_{i,t}\right)$ that are exactly equal to the (log) relative price $P^R_{j,t}$. As a result, the relative price of each variety is included in $\mathcal{I}_{i,t}$.

Furthermore, households have common knowledge about past aggregate outcomes, so the past price level p_{t-1} and the past aggregate shock η_{t-1} are also included in $\mathcal{I}_{i,t}$. Using this observation, together with equations (6), (7) and (11), we conclude that each household has access to a noisy private signal about the inflation rate of the form:

$$\pi^*_{i,t} = \pi_t + \epsilon_{i,t}, \quad \epsilon_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_\epsilon\right), \tag{12}$$

where $\pi^*_{i,t} \equiv \log S_{j,t} - \log P_{t-1}$. The next step is to find the belief about inflation of each household. To do this, start by noting that Assumption 1, and the observation that households have common knowledge about past outcomes, imply that $\mathrm{E}_{i,t}\pi_t = \mathrm{E}\left[\pi_t|\pi^*_{i,t}, \pi_{t-1}\right]$. Next, recall that both $\epsilon_{i,t}$ and η_t are Gaussian random variables. This implies that, up to a first-order approximation, the inflation rate π_t is also a Gaussian random variable. Consequently, we can use 12 and a well-known regression lemma for bivariate normal random variables to express the beliefs about inflation of each household, conditional on their own information set, as

$$\mathrm{E}_{i,t}\pi_t = \mathbb{E}_{t-1}\pi_t + \underbrace{\frac{\mathrm{Cov}\left[\pi_t, \pi^*_{i,t}|\pi_{t-1}\right]}{\mathrm{Var}\left[\pi^*_{i,t}|\pi_{t-1}\right]}}_{1-\psi_\pi}\left(\pi^*_{i,t} - \mathbb{E}_{t-1}\pi_t\right). \tag{13}$$

In what follows, I refer to ψ_π as the *degree of anchoring* of households beliefs about inflation. In other words, $\psi_\pi \in [0,1]$ is defined as one minus the Kalman gain coefficient of their filtering problem. This parameter measures the sensitivity of households' inflation perceptions to aggregate inflation. In the limit of full information, $\sigma^2_\epsilon \to 0$ so $\psi_\pi \to 0$, and the beliefs of households respond one-to-one to the movements in the inflation rate. As σ^2_ϵ increases, ψ_π increases and approaches to one. A higher value of ψ_π makes the beliefs of households underreact more to news about current inflation. In this sense, ψ_π measures how *anchored* are the beliefs of households to their past.[17]

[17] This interpretation of *anchoring* is consistent with the definition used by Bernanke (2007), Mishkin (2007), Jørgensen and Lansing (2021), and Hazell, Herreño, Nakamura, and Steinsson (2020) . Since a higher value of ψ_π implies that households have less knowledge of current inflation, this term can be also interpreted as the degree of household *inattention* to aggregate inflation.

Using (12), the degree of anchoring can be expressed as:

$$\psi_\pi = 1 - \frac{\mathrm{Var}\left[\pi_t|\pi_{t-1}\right]}{\mathrm{Var}\left[\pi_t|\pi_{t-1}\right] + \sigma_\epsilon^2}. \tag{14}$$

We can thus use (13) and the fact that $\mathbb{E}\left[\epsilon_{i,t}\right] = 0$ to get:

$$\overline{\mathrm{E}}_t \pi_t = \psi_\pi \mathbb{E}_{t-1}\pi_t + (1-\psi_\pi)\,\pi_t, \tag{15}$$

with $\overline{\mathrm{E}}_t \equiv \int_0^1 \mathrm{E}_{i,t}\left[\cdot\right] di$ denoting the average belief across households.

Individual demand for varieties, labor supply and Euler equation. I now turn attention to the problem of each household. Let $\widehat{P}_{i,t} \equiv \mathrm{E}_{i,t} P_t$ denote the belief of household i about the aggregate price level, conditional on $\mathcal{I}_{i,t}$. The first order conditions of the problem of household i are:

$$C_{i,j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\varepsilon} C_{i,t}, \quad N_{i,t}^{\varphi} C_{i,t}^{\sigma} = \mathcal{D}_{i,t}^{w} \frac{W_{i,t}}{\widehat{P}_{i,t}}, \quad 1 = \beta \mathrm{E}_{i,t}\left[R_{i,t} \frac{\mathcal{D}_{i,t+1}^{w}}{\mathcal{D}_{i,t}^{w}} \left(\frac{C_{i,t+1}}{C_{i,t}}\right)^{-\sigma} \frac{Z_{i,t+1}}{Z_{i,t}} \frac{\widehat{P}_{i,t}}{\widehat{P}_{i,t+1}}\right]. \tag{16}$$

The detailed derivation of the previous expressions is given in Appendix A. The first condition in (16) is the standard demand under CES preferences, as expected from the observation that households have full information about the relative price of each variety. The second and third equation are similar to the standard labor supply and Euler equation under full information. There are only two differences with respect to these counterparts: First, households have private information so they condition their decisions to their own information set. Second, both equations are affected by the presence of an additional term $\mathcal{D}_{i,t}^{w}$. This term captures a wedge in expectations that arises due to Jensen's inequality. In Appendix (A), I show that, up to a first-order approximation, the term $\mathcal{D}_{i,t}^{w}$ is equal to zero. Consequently, a log-linear approximation of the previous two equations yields:

$$\varphi n_{i,t} + \sigma c_{i,t} = w_{i,t} - \mathrm{E}_{i,t} p_t, \tag{17}$$

$$c_{i,t} = \mathrm{E}_{i,t} c_{i,t+1} - \frac{1}{\sigma}\left(i_{i,t} - \mathrm{E}_{i,t}\pi_{i,t+1} + \mathrm{E}_{i,t} z_{i,t+1} - z_{i,t}\right). \tag{18}$$

We can see that the standard labor supply and Euler equation of the NK model also hold in this model at the household level, after conditioning on their private information.

New Keynesian Phillips Curve. Integrating the first condition in (16) across consumers, we can express the aggregate demand for variety j as

$$\mathcal{C}_{j,t} \equiv \int_0^1 C_{i,j,t} dj = \left(P_{j,t}/P_t\right)^{-\varepsilon} C_t,$$

with $C_t \equiv \int_0^1 C_{i,t} di$. It follows that the problem of the firm in this setting is isomorphic to the problem of the firm when households have full information about P_t. From the first order conditions of this problem, we obtain the following NKPC:[18]

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \lambda^{-1} mc_t, \tag{19}$$

where

$$\lambda \equiv \frac{\theta}{(1-\theta)(1-\beta\theta)} \tag{20}$$

measures the inverse of the response of inflation to real marginal costs, defined as $mc_t \equiv w_t - p_t$. The parameter λ is increasing in the degree of price stickiness. Flexible prices are nested as the special case where $\theta = 0$, in which case $\lambda = 0$.

Now, let $y_t \equiv \int_0^1 y_{j,t} dj$ and $n_t \equiv \int_0^1 n_{i,t} di$ denote, respectively, the aggregate output and labor supply of this economy. The production technology of firms implies that $y_t = n_t$. We can thus integrate (17) across households and use the market clearing condition $c_t = y_t$ to derive the following expression for the aggregate labor supply:

$$(\varphi + \sigma) y_t = w_t - p_t + \nu_t^p \tag{21}$$

where $\nu_t^p \equiv p_t - \overline{\mathrm{E}}_t p_t$ denotes the average *perception error* about the price level across households. Equation (21) resembles the standard aggregate labor supply of a model with full information, but is augmented by the presence of ν_t^p, reflecting an information wedge produced by LBS. Equation (21) shows that LBS creates a labor wedge driven by the differences between the average wage perceived by households and the real wage, which coincides with the wage perceived by firms. Using (21) to replace the real marginal costs in (19), we arrive to the following expression characterizing the aggregate supply of this economy:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \alpha_{PC}^* y_t - \lambda^{-1} \nu_t^p \tag{22}$$

where $\alpha_{PC}^* \equiv (\sigma + \varphi) / \lambda$ is the slope of the Phillips curve in the full-information case. Equation (22) reveals how this information friction augments the standard NKPC with the average perception error about the price level across households. In the spirit of Friedman (1977), this term captures the differences in the perception of real wages between firms and households. To simplify the NKPC further, we can use the observation that p_{t-1} is part of the information set of all households. As a result, the perception error ν_t^p is equal to the perception error about the current inflation rate $\nu_t^\pi \equiv \pi_t - \overline{\mathrm{E}}_t \pi_t$. We can thus replace ν_t^π in (22) to write the NKPC as:

$$\pi_t = \frac{\lambda}{1+\lambda} \beta \mathbb{E}_t \pi_{t+1} + \left(\frac{\sigma + \varphi}{1+\lambda} \right) y_t + \frac{1}{1+\lambda} \overline{\mathrm{E}}_t \pi_t. \tag{23}$$

To conclude, we can use (15) and reorder terms to arrive to the following result.

[18]The solution of this problem is well known (see, for instance, Chapter 3 in Galí (2015)), so I skip the details of the derivation of this curve. Notice however that my definition of λ is different from the standard presentation. In particular, I take the inverse of the coefficient defined in textbook presentations to make this parameter increasing in the degree of price stickiness. This will simplify the notation in the following results.

Proposition 1. *(**NKPC**) The aggregate supply of this economy is characterized by*

$$\pi_t = (1 - \Psi_\pi)\, \beta \mathbb{E}_t \pi_{t+1} + \Psi_\pi \mathbb{E}_{t-1} \pi_t + \alpha_{PC} y_t, \tag{24}$$

where $\Psi_\pi \equiv \psi_\pi / (\lambda + \psi_\pi)$, ψ_π is the equilibrium degree of anchoring defined in (14), λ is the degree of price stickiness given by (20), and

$$\alpha_{PC} \equiv \frac{\sigma + \varphi}{\lambda + \psi_\pi} \tag{25}$$

is the slope of the Phillips curve.

Proof. See Appendix B.3. □

The above proposition shows that LBS affects the supply side of this economy in two ways.

First, LBS affects the comovement between inflation and output, as reflected by the presence of ψ_π in the slope of the NKPC given by (25). In particular, this slope is positive even when prices are flexible ($\lambda = 0$), suggesting that the aggregate demand shock η_t can have real effects in this case. I will verify this conjecture formally in the next section.

Second, LBS induces persistence in the behavior of inflation by making it a weighted average of current and past expectations of this variable. The NKPC (24) resembles the one obtained when firms face sticky-information a la Mankiw and Reis (2002). The weight on past forecasts is given by parameter Ψ_π, which is increasing in ψ_π. It follows that the dynamic properties of inflation will vary with the degree of anchoring, which is, in turn, an endogenous object in the model.

Aggregate Euler equation. Derivation of the aggregate Euler equation of this economy is complicated by the fact that the Law of Iterated Expectations does not hold for the average expectations across households. Following Angeletos and Lian (2018) and Angeletos and Lian (2021), we can use the budget constraint (5), together with individual Euler equation (18) to express the consumption of each household as a function of its expectations about current and future income and interest rates. Using this *beauty-contest* representation of individual consumption, we arrive to the following result.

Proposition 2. *(**Euler equation**) The aggregate demand of this economy is characterized by*

$$y_t = -\frac{1}{\sigma}\left(i_t - \mathbb{E}_t \pi_{t+1} + \mathbb{E}_t z_{t+1} - z_t\right) + \mathbb{E}_t y_{t+1} + \mathcal{X}_t + \beta \mathbb{E}_t \mathcal{X}_{t+1}, \tag{26}$$

where $\mathcal{X}_t \equiv \mathcal{H}_t + \mathcal{R}_t$ is the sum of two information wedges given by

$$\mathcal{H}_t \equiv \chi v_t^p - \left(\frac{1-\beta}{\beta}\right) \mathbb{E}_t \sum_{k=1}^{\infty} \beta^k v^y_{t+k|t}, \tag{27}$$

and

$$\mathcal{R}_t \equiv -\sigma^{-1} \mathbb{E}_t \left\{ v^\pi_{t+1|t} + \sum_{k=1}^{\infty} \beta^k \left\{ v^\pi_{t+k+1|t} - \phi_\pi v^\pi_{t+k|t} \right\} \right\}, \tag{28}$$

where $\nu^{\pi}_{t+k|t} \equiv \pi_{t+k} - \overline{\mathrm{E}}_t \pi_{t+k}$ and $\nu^{y}_{t+k|t} \equiv y_{t+k} - \overline{\mathrm{E}}_t y_{t+k}$ denote, respectivelly, the average forecast error of inflation in $t+k$ across households, and $\nu^{p}_{t} \equiv p_t - \overline{\mathrm{E}}_t p_t$ denotes the average perception error about the price level across households. Finally,

$$\chi \equiv \left(\frac{1-\beta}{\beta}\right)\left(\frac{\mathcal{M}\varphi}{\mathcal{M}\varphi + \sigma}\right), \tag{29}$$

where $\mathcal{M} \equiv \varepsilon / (\varepsilon - 1)$ denotes the firms markup in the non-stochastic steady-state.

Proof. See Appendix B.2. □

Equation (26) is similar to the standard dynamic IS equation but is augmented by two information wedges that are a byproduct of LBS.[19]

The first wedge, $\mathcal{H}_t$, captures the effect of LBS on households' perception of their *human wealth*, defined as the present value of the purchasing power of their wage and dividend income. LBS makes households' perception of their human wealth differ from the corresponding wealth deflated using p_t. Equation (27) shows that the cross-sectional average of these differences makes aggregate consumption deviate from its full information counterpart. The differences, in turn, are proportional to the present value of the average perception error about the price level, ν^p_t, and its strength depends on the coefficient χ.

The second wedge, $\mathcal{R}_t$, captures the effect of LBS on households' perception of their *non-human wealth*, defined as the present value of the real return of their assets, denoted as $r_{i,t} \equiv i_{i,t} - \pi_{t+1}$. The information friction makes households misperceive the current inflation rate. Equation (28) shows that this misperception creates a wedge in aggregate demand by generating dispersion on beliefs about current and future real returns. This wedge is proportional to the effect of inattention on households' forecasts about inflation and the nominal interest rate.[20]

2.3 Equilibrium dynamics and degree of anchoring

The degree of anchoring (14), together with the NKPC (21), and the aggregate Euler equation (26), characterize the dynamics of output and inflation in the model. I now solve the model by deriving explicit expressions for the stochastic process of inflation and output in equilibrium. I then use these expressions to characterize the existence and uniqueness of the equilibrium level of ψ_π.

To do so, I start by conjecturing that both the inflation rate and the output gap follow ARMA processes of the form:

$$\pi_t = \rho_z \pi_{t-1} + \theta^{\pi}_0 \eta_t - \theta^{\pi}_1 \rho_z \eta_{t-1}, \tag{30}$$

$$y_t = \rho_z y_{t-1} + \theta^{y}_0 \eta_t - \theta^{y}_1 \rho_z \eta_{t-1}, \tag{31}$$

where $\left\{\theta^{\pi}_0, \theta^{\pi}_1, \theta^{y}_0, \theta^{y}_1\right\}$ are coefficients to be determined next.

[19] As discussed in the introduction, this representation is similar to the one derived by Angeletos and Lian (2021), with the main differences arising from the nature of the information friction and the fact that I allow learning to be potentially persistent over time.

[20] To see why this is the case, note that households have rational expectations and know that the central bank sets the interest rate following (10). As a result, their interest rate forecasts are consistent with this rule, so the errors forecasting the inflation rate will produce proportional errors forecasting the interest rate.

Information wedges. We can use the conjecture (30) to find a simple expression for the information wedges $\mathcal{H}_t$ and $\mathcal{R}_t$. Using the observation that both η_{t-1} and π_{t-1} are part of households information set, we can express the difference between households' average expectations and their full-information counterparts as

$$\mathbb{E}_t \nu^x_{t+k|t} = \begin{cases} \theta^x_0 \psi_\pi \eta_t & ; \quad k = 0 \\ \rho^k_z \psi_\pi \left(\theta^x_0 - \theta^x_1\right) \eta_t & ; \quad k \geq 1, \end{cases} \tag{32}$$

for $x \in \{\pi, y\}$. In particular, households perception error about current inflation is proportional to the degree of anchoring:

$$\nu^\pi_t = \psi_\pi \theta^\pi_0 \eta_t.$$

We can thus use (32) in (27) to express the first information wedge as:

$$\mathcal{H}_t = \left[\chi \theta^\pi_0 - \rho_z \left(\theta^y_0 - \theta^y_1\right) \left(\frac{1-\beta}{1-\beta\rho_z}\right)\right] \psi_\pi \eta_t.$$

Simmilarly, we can express the wedge $\mathcal{R}_t$ as:

$$\mathcal{R}_t = \frac{1}{\sigma} \left[\rho_z \left(\theta^\pi_0 - \theta^\pi_1\right) \left(\frac{\beta\phi_\pi - 1}{1-\beta\rho_z}\right)\right] \psi_\pi \eta_t.$$

Collecting the previous results, we can write the information wedge in the aggregate Euler equation (26) as:

$$\mathcal{X}_t = \left(\chi \theta^\pi_0 + \left(\frac{\rho_z}{1-\beta\rho_z}\right) \left[\sigma^{-1} \left(\theta^\pi_0 - \theta^\pi_1\right) \left(\beta\phi_\pi - 1\right) - \left(\theta^y_0 - \theta^y_1\right) \left(1-\beta\right)\right]\right) \psi_\pi \eta_t, \tag{33}$$

which in turn implies $\mathbb{E}_t \mathcal{X}_{t+1} = 0$.

Equilibrium dynamics. Equation (33) defines the information gap as a function of the undetermined coefficients $\{\theta^\pi_0, \theta^\pi_1, \theta^y_0, \theta^y_1\}$. Using the Taylor rule (10), to replace i_t in (26), we can write the Euler equation of this economy as a function on inflation, the output gap and the shock η_t only. We can use the resulting expression, together with conjectures (30) and (31), and the NKPC (24), to solve for the undetermined coefficients. After some manipulation, it can be shown that the initial conjecture is verified when

$$\theta^\pi_0 = \left(\frac{\lambda + (1+\varphi/\sigma)\phi_\pi + \rho_z \left((1+\varphi/\sigma)\left(\frac{\beta\phi_\pi - 1}{1-\beta\rho_z}\right) - \lambda(1-\beta)\right)\psi_\pi}{\lambda + (1+\varphi/\sigma)\phi_\pi + (1 - \chi(\sigma+\varphi))\psi_\pi}\right) \left(\theta^\pi_0 - \theta^\pi_1\right), \tag{34}$$

$$\theta^y_0 = \left(\frac{\psi_\pi + \lambda}{1+\varphi/\sigma}\right) \theta^\pi_0 - \frac{\beta\rho_z}{\alpha^*_{PC}} \left(\theta^\pi_0 - \theta^\pi_1\right), \tag{35}$$

togheter with

$$\theta_0^\pi - \theta_1^\pi = \left(\sigma\left(\frac{1-\beta\rho_z}{\alpha_{PC}^*}\right) + \left(\frac{\phi_\pi - \rho_z}{1-\rho_z}\right)\right)^{-1},$$
$$\theta_0^y - \theta_1^y = \frac{1}{\sigma}\left[1 - \left(\frac{\phi_\pi - \rho_z}{1-\rho_z}\right)(\theta_0^\pi - \theta_1^\pi)\right].$$

Equilibrium degree of anchoring. Given the value of the previous coefficients, equations (30) and (31) characterize the equilibrium dynamics of inflation and aggregate output conditional on a value of ψ_π. This value is itself a function of π_t, as shown by (14). To complete the characterization of the equilibrium, all that is left is to show that this fixed-point problem has a solution. The following proposition provides to conditions that are sufficient to guarantee the existence and uniqueness of an equilibrium in this economy.

Proposition 3. *(**Degree of anchoring**) An equilibrium $\psi_\pi \in [0,1]$ exists and is given by the solution of*

$$1 - \psi_\pi = \frac{\sigma_{AD}^2}{\sigma_\epsilon^2}\left(\theta_0^\pi\right)^2\psi_\pi, \tag{36}$$

where θ_0^π is given by (34). Moreover, if $(\sigma+\varphi)\chi < 1$ and

$$\phi_\pi > \beta^{-1} + \left(\frac{\lambda(1-\beta)\left(\beta^{-1}-\rho_z\right)}{1+\varphi/\sigma}\right), \tag{37}$$

the equilibrium ψ_π is unique.

Proof. See Appendix B.3. □

The above proposition shows the conditions that guarantee the existence and uniqueness of the equilibrium of this model. The definition of coefficient χ in (29) suggests that, for reasonable values of β, σ and φ, the condition $(\sigma+\varphi)\chi < 1$ is met.[21] The proof of Proposition 3 shows that, when shocks are i.i.d. ($\rho_z = 0$), this condition alone is sufficient for the uniqueness of the equilibrium. When shocks are persistent ($\rho_z > 0$), this condition alone is no longer sufficient. However, a stronger version of the *Taylor principle*, defined by condition (37), is sufficient to guarantee existence of the equilibrium for any degree of shock persistence.

Intuitively, LBS results in slow adjustment of households' perception of the real interest rate. Incomplete awareness of the macroeconomic conditions results in an incomplete passthrough of movements in the interest rate set by the central bank to households' expectations. To stabilize inflation after an expansionary demand shock, households should expect an increase in the real interest rate. To achieve this, the central bank needs to compensate for the incomplete passthrough with a stronger response of the interest rate, compared to the full information case.

[21] To fix ideas, consider values of $\beta = 0.99$, $\sigma = 2$, $\varphi = 4$ and $\varepsilon = 6$, which are standard in the business cycle literature. In this case, $(\sigma+\varphi)\chi \approx 0.05$.

3 The Macroeconomic Implications of Learning by Shopping

In this section, I use the conditions characterizing the dynamics of the model to show analytically the mechanisms through which LBS affects the transmission of aggregate shocks. I start by showing how the degree of anchoring changes with the structural parameters of the model. I then show how LBS propagates the impact of demand shocks on output, even when prices are flexible. Next, I show how price stickiness amplifies this propagation. Finally, I conclude by showing how the monetary policy stance affects the slope of the Phillips curve and discuss the relationship of this result with the findings of the empirical literature.

3.1 Endogenous anchoring

Equations (30), (31) and (36) characterize the equilibrium dynamics of output, inflation and beliefs of this economy. I now use these equations to analyze the impact of inattention to aggregate inflation on the transmission of aggregate shocks. To do so, it will be important to know how ψ_π changes with the structure of the economy. The following proposition shows that the same conditions that guarantee the uniqueness of the equilibrium imply that ψ_π is increasing in the degree of price stickiness θ, and the response of monetary policy to inflation ϕ_π.

Proposition 4. *(**Endogenous degree of anchoring**) Assume $(\sigma + \varphi)\,\chi < 1$ and condition (37) holds. Then*

$$\frac{\partial \psi_\pi}{\partial \theta} > 0, \qquad \frac{\partial \psi_\pi}{\partial \phi_\pi} > 0.$$

Proof. See Appendix B.4. □

The intuition behind this result is straightforward. An increase in either θ or ϕ_π reduces the volatility of inflation. The former makes prices more rigid directly, while the latter "flattens" the economy's aggregate demand curve through the effect of interest rates on households savings decisions. For a given level of noise in signals σ_ϵ^2, the lower volatility of inflation results in a reduction of the informational content of the signals received by households. Consequently, they put less weight on these signals, as implied by (36).

In what follows, I will assume that the two conditions guaranteeing the existence and uniqueness of the equilibrium hold.

3.2 The propagation of demand shocks

To isolate the effects of the information friction, I will temporarily shut down nominal rigidities in the model by setting $\theta = 0$. As discussed in the introduction, LBS introduces simultaneously three channels that propagate the effect of AD shocks. The first channel allows this shock to have real effects by distorting households' perception of wages, introducing a wedge in the labor market. The second channel amplifies the first by distorting households' perception of current income and, consequently, aggregate consumption. Finally, the third channel introduces further amplification when shocks are persistent by through the effect of the information friction on households' expectations of future interest rates and aggregate demand.

To dissect the effect of each channel, I will start by considering the case where AD shocks are i.i.d. over time ($\rho_z = 0$). In this case, the aggregate supply side of this economy, given by the NKPC (24), takes a very simple form:

$$\pi_t = \alpha_{PC} y_t, \tag{38}$$

with $\alpha_{PC} = (\sigma + \varphi) / \psi_\pi$. Notice that the aggregate supply is upward sloping as long as $\psi_\pi > 0$. On the other hand, the aggregate demand side of the economy simplifies to:

$$\pi_t = -\alpha_{AD} \left(y_t - \sigma^{-1} z_t \right), \tag{39}$$

with $\alpha_{AD} \equiv 1/ \left(\sigma^{-1}\phi_\pi - \chi\psi_\pi\right)$ denoting the slope of the aggregate demand curve. When $\rho_z = 0$, the information wedge $\mathcal{R}_t$ is turned off, and only $\mathcal{H}_t$ affects the aggregate demand, as reflected by the second term in the denominator of α_{AD}. In the special case where $\varphi = 0$, the coefficient χ disappears and the demand side of this economy is unaffected by the LBS assumption. Finally, note that, under flexible prices, the labor supply is still given by (21), but the labor demand is flat since firms keep their markups constant, so $p_t = w_t$.

Non-neutrality: The labor market wedge. To understand the first channel through which LBS propagates demand shocks, it is helpful to represent on a diagram the labor market of this economy under flexible prices, as given by equations (19) and (21). The first diagram of Figure 1 shows the effect of the aggregate demand shock in the labor market of this economy.

Point ***A*** in the plot corresponds to the initial equilibrium before the shock. Suppose there is an unexpected contraction in the aggregate demand in the economy. Proposition 5 shows that, under full information ($\psi_\pi = 0$), this contraction is fully absorbed by the inflation rate. Wages and prices fall proportionally due to firms' desire to keep markups constant, and the real wage and the labor supplied by households remain unchanged. After the shock, the equilibrium remains at point ***A***.

Suppose now that households' inflation beliefs are anchored due to LBS ($\psi_\pi > 0$). In this case, households observe the reduction in wages that follows the demand shock. But the inflation perceptions are anchored, so they observe only part of the fall in the price level. As a result, households perceive a reduction in the real wage even though it remains constant after the shock. As illustrated in the first panel, the perception error $\nu_t^p \equiv p_t - \widehat{p}_t$ acts as a wedge that shifts the labor supply and moves the economy to a new equilibrium with lower output at the same real wage, as indicated by point **B**.While the real wage remains constant, prices and wages fall with the demand shock, but less so than in the full information case.

Amplification (i): Misperception of income. The previous analysis of the labor market offers an incomplete view of the total effect of LBS. This information friction also affects households' perception of their human wealth. The perception error ν_t^p enters as a wedge in the aggregate Euler equation (26) and its effect on aggregate demand is captured by the presence of parameter χ in equation (39).

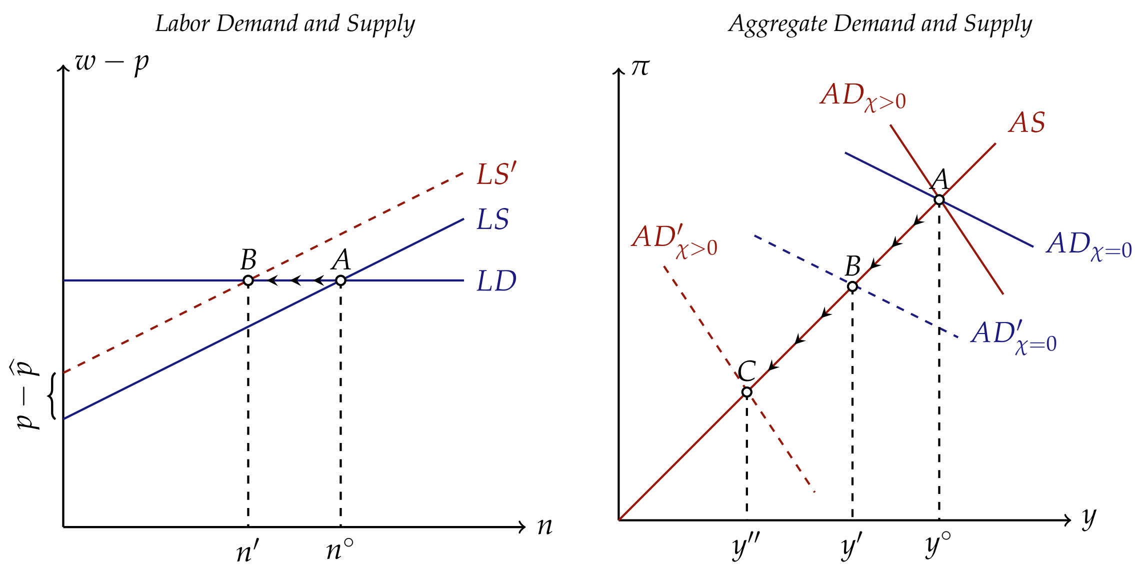


Figure 1: Propagation and amplification of a contractionary demand shock

Notes: The figure illustrates how *learning by shopping* propagates and amplifies the impact of a contractionary aggregate demand shock. The left panel shows how the differences in the perception of the real wage between households and firms produces a fall labor supply and output after the shock. The right panel shows how the intial effect is amplified by a fall in permanent income households' perceive.

To visualize the amplification coming from this channel, the second diagram of Figure 1 plots the aggregate demand and supply of this economy, as given by (39) and (38). The aggregate supply has a positive slope, as implied by the previous analysis of the labor market.

When $\chi = 0$, the aggregate demand is equal to its full information counterpart, as illustrated by the blue downward sloping line. In this case, the contractionary shock to aggregate demand shifts the AD curve and moves the equilibrium from point $\boldsymbol{A}$ to point $\boldsymbol{B}$ as a consequence of households' perceiving an increase in real wages.

When $\chi > 0$, the aggregate demand curve is more sensitive to the shock, as shown by (39). As a result, the same shock displaces the aggregate demand curve further. This additional amplification results from the fall in households' perception of their permanent income: they observe the reduction in the present value of their wage and dividend income after the shock but only observe part of the reduction in the aggregate price level. In response, households reduce their consumption further, amplifying the initial effect of the shock. Consequently, the new equilibrium, indicated by point **C** in the graph, features lower output and inflation than when this channel is muted.

Amplification (ii): Discounting of Future Interest Rates. When $\rho_z > 0$, a third channel due to LBS affects the transmission of aggregate demand shocks: Households over-estimate the current response of the real interest rate, while simultaneously under-estimating future adjustments in this variable. After an expansionary AD shock, the central bank's increases the nominal interest rate. To stabilize inflation, this response must be accompanied by an increase in the real interest rate. Households observe the current nominal rate but, because of the information friction, they underestimate the future path of interest rates. As a result, they overestimate the increase in the real interest rate, but also under-estimate the future reduction in the real interest rate. The first effect mitigates the effect of demand shocks on impact, while the second effect amplifies this impact.[22] The total effect of this channel is captured by the information wedge $\mathcal{R}_t$ defined in (28).

To summarize, the following proposition characterizes how LBS allows the propagation of demand shocks when prices are flexible.

Proposition 5. *(**Propagation of demand shocks**) Assume that $\theta = 0$. The response of inflation and output to an aggregate demand shock in t is given by:*

$$\frac{\partial \pi_t}{\partial \eta_t} = \Delta_\pi > 0, \qquad \frac{\partial y_t}{\partial \eta_t} = \frac{\psi_\pi}{\sigma + \varphi}\Delta_\pi > 0$$

with

$$\Delta_\pi \equiv \left(\frac{\phi_\pi + \left(\frac{\beta\rho_z}{1-\beta\rho_z}\right)\left(\phi_\pi - \beta^{-1}\right)\psi_\pi}{\phi_\pi + (1+\varphi/\sigma)^{-1}\left(1 - \chi\left(\sigma+\varphi\right)\right)\psi_\pi} \right)\Omega_\pi, \tag{40}$$

and $\Omega_\pi = (1-\rho_z)\,/\,(\phi_\pi - \rho_z)$ denoting the response of inflation under full-information.

Proof. See Appendix B.5. □

3.3 LBS amplifies the effects of nominal rigidities

The previous discussion highlighted how LBS allows nominal shocks to have real effects, even under flexible prices. In particular, an AD shock creates a wedge between households' and firms' perceptions of the real wage. This wedge shifts the labor supply curve of the economy, allowing this shock to affect equilibrium employment and output.

This mechanism differs from the one underlying the standard NK model under full information. In this model, an AD shock shifts households' nominal spending, but nominal rigidities prevent prices from adjusting one-to-one. As a result, the shift in nominal expenditures translates into a change in real spending. An increase in production must accompany this change to clear the goods market. Consequently, the AD shock manifests itself as a shift in firms' demand for labor, as illustrated in the left panel of Figure 2.

For a fixed degree of anchoring ψ_π and price-stickiness λ, both mechanisms operate independently. With nominal rigidities and LBS, an AD shock simultaneously shifts the labor demand and supply of the economy, as illustrated in the right panel of Figure 2. It follows that LBS acts as a multiplier on the real effects of nominal rigidities. The following proposition formalizes this observation.

Proposition 6. *(**The LBS multiplier**) Let $[\partial y_t/\partial \eta_t]^{SP}$ denote the response of output to a demand shock under full information and sticky prices ($\sigma^2_\epsilon > 0, \theta = 0$). Let $[\partial y_t/\partial \eta_t]^{LBS+SP}$ denote the response when there both LBS and sticky prices are present ($\sigma^2_\epsilon > 0, \theta > 0$). The response of output to an aggregate demand shock in t is given by:*

$$\left[\frac{\partial y_t}{\partial \eta_t}\right]^{LBS+SP} = \left[\frac{\partial y_t}{\partial \eta_t}\right]^{SP} \times \Psi_{LBS},$$

[22] This mechanism is related to the "discounting the GE adjustment in real interest rates" in Angeletos and Lian (2021).

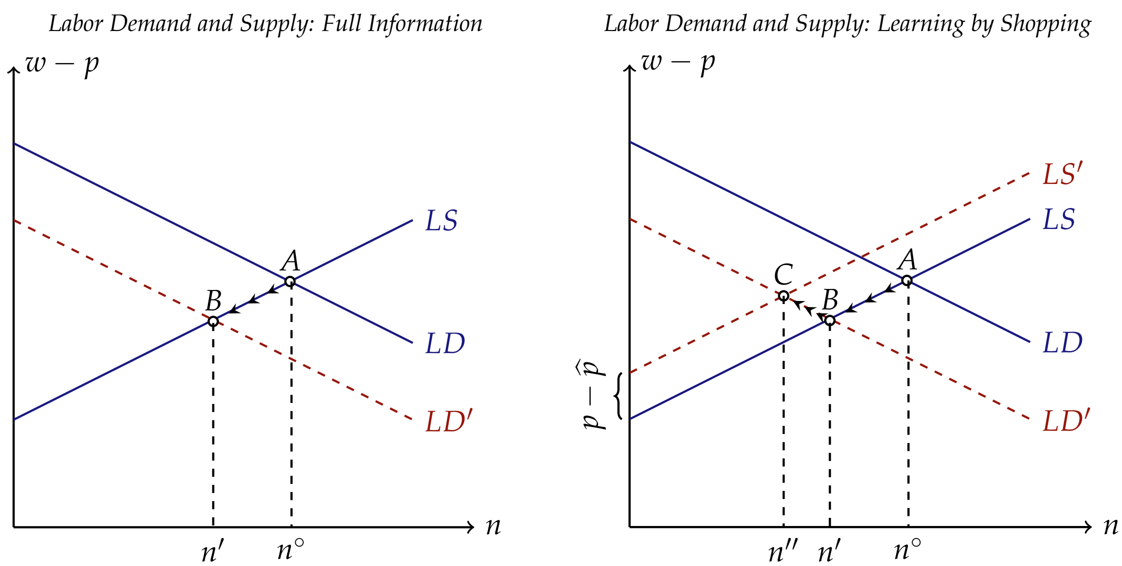


Figure 2: The interaction of LBS and price-stickiness

Notes: The figure illustrates the effect of a contractionary AD shock in the labor market when prices are sticky. The left panel illustrates the shift in firms' demand for labor resulting from this shock. In this case, equilibrium employment falls from n° to n', which is the new equilibrium employment under full information. The right panel shows how, under *learning by shopping*, the same shock simultaneously shifts the labor supplied by households, amplifying the impact of the shock and further reducing the equilibrium level of employment to n''.

with

$$\Psi_{LBS} \equiv \left(\frac{\psi_\pi + \lambda}{\lambda\left(1-\beta\rho_z\right)}\right)\left(\frac{\Lambda + \Theta\rho_z\psi_\pi}{\Lambda + \left(1-\left(\sigma+\varphi\right)\chi\right)\psi_\pi}\right) - \left(\frac{\beta\rho_z}{1-\beta\rho_z}\right) > 1, \tag{41}$$

where $\Lambda \equiv \lambda + \left(1+\varphi/\sigma\right)\phi_\pi$ *and* $\Theta \equiv \left(1+\varphi/\sigma\right)\left(\frac{\beta\rho_z}{1-\beta\rho_z}\right)\left(\phi_\pi - \beta^{-1}\right) - \lambda\left(1-\beta\right)$.

Proof. See Appendix B.6. □

Since each friction affects a different side of the labor market, it is not surprising that their interaction allows AD shocks to have significant effects on economic activity. Perhaps more surprising is the observation that the impact of a demand shock when both frictions are present can be larger than the sum of the corresponding impact when each friction is considered in isolation. The following proposition establishes the conditions under which this non-linear amplification can arise in the special case of an iid shock.

Proposition 7. *(**The interaction of LBS and sticky prices**) Assume that* $\rho_z = 0$. *Let* $\left[\partial y_t/\partial z_t\right]^{SP}$ *denote the response of output to a demand shock under full information and sticky prices* ($\sigma_\epsilon^2 > 0$, $\theta = 0$)*. Let* $\left[\partial y_t/\partial z_t\right]^{LBS}$ *denote the corresponding response under LBS and flexible prices* ($\sigma_\epsilon^2 > 0$, $\theta = 0$)*. Let* $\left[\partial y_t/\partial z_t\right]^{LBS+SP}$ *denote the response when there both LBS and sticky prices are present* ($\sigma_\epsilon^2 > 0, \theta > 0$)*. If*

$$\frac{\psi_\pi^{LBS+SP}}{\psi_\pi^{LBS}} - 1 > \left(\frac{\sigma}{\sigma+\varphi}\right)\frac{\lambda}{\phi_\pi} \tag{42}$$

then

$$\left[\partial y_t/\partial z_t\right]^{LBS+SP} > \left[\partial y_t/\partial z_t\right]^{LBS} + \left[\partial y_t/\partial z_t\right]^{SP}$$

Proof. See Appendix B.7. □

To understand this result, it is important to recall that the equilibrium conditions for output and inflation are linear, but the model is not. The degree of anchoring ψ_π affecting the slope of the labor supply is an endogenous object in the model that increases with the degree of price stickiness, as shown in Proposition 4. Intuitively, when prices become more rigid, the volatility of inflation falls. This, in turn, reduces the information about aggregate inflation contained in households' shopping experiences, slowing the rate at which they learn about inflation. Proposition 7 shows that when the relative slope of labor demand and supply is small enough, the introduction of LBS propagates the demand shocks beyond what is possible by each friction considered independently.

3.4 The flattening of the Phillips curve

The previous results illustrated how the endogenous nature of the degree of anchoring ψ_π amplified the effect of nominal rigidities. I now show that it also has important implications for the design of monetary policy.

To begin, consider a change in the conduct of monetary policy to a more hawkish stance. Such a change is usually modeled as an increase in the Taylor rule coefficient ϕ_π. As illustrated in the left panel of Figure 3, the policy change "flattens" the aggregate demand of this economy and mitigates the impact of AD shocks on output. Importantly, under full information, the increase in ϕ_π has no effects on the supply side of the economy. The following proposition shows that when households *learn by shopping*, this is no longer the case.

Proposition 8. *(**Monetary policy and the NKPC**) An increase in the response to inflation ϕ_π by the central bank flattens the slope of the Phillips curve.*

Proof. See Appendix B.8. □

The intuition behind this result is simple: By reducing the sensitivity of consumption to aggregate demand shocks, an increase in ϕ_π reduces the volatility of the inflation rate. In equilibrium, the lower volatility of inflation reduces the informational content in households' shopping experiences, increasing the degree of anchoring ψ_π. This, in turn, exacerbates the non-neutrality of nominal shocks produced by the information friction, which manifests itself as a "flatter" Phillips curve. Importantly, this flattening occurs without any change in nominal rigidities or the relationship between inflation and marginal costs.

An important corollary of this result is that, by increasing the degree of anchoring and flattening the Phillips curve, a more hawkish monetary policy stance can actually amplify the impact of aggregate demand shocks on output, as illustrated in the right panel of Figure 3. This result is in stark contrast with the prediction of the NK model under full information, where an increase in ϕ_π reduces the impact of nominal shocks, including monetary policy shocks. On the other hand, the

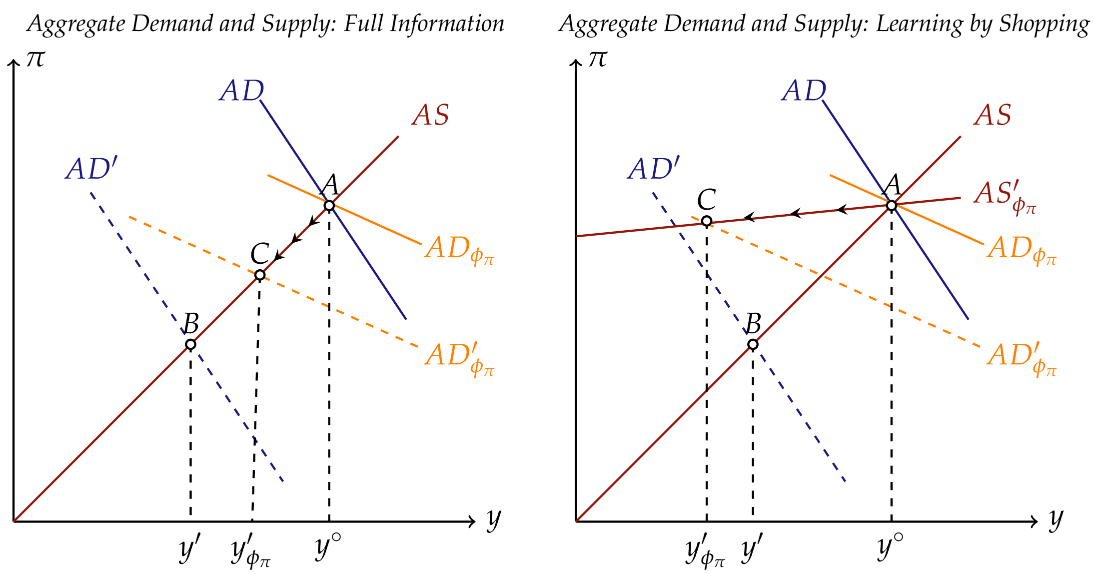


Figure 3: Flattening of the Phillips curve after a change to a more hawkish monetary policy stance

Notes: The figure illustrates the effect of a contractionary AD shock in the model before and after a change to a more hawkish monetary policy stance, represented by an increase in the Taylor rule coefficient ϕ_π. The left panel illustrates the effect of this policy change under full information. Before this policy, the AD shock produces a fall in output from y° to y'. The policy change flattens the AD curve and reduces the impact of AD shocks. The same shock now reduces output to $y'_{\phi_\pi} < y'$. The right panel shows how, under *learning by shopping*, the policy change also changes the slope of the AS curve. The flattening of the Phillips curve in this example counteracts the flattening of the AD curve, amplifying the impact of the AD shock. In this case, the shock produces a fall in output from y° to $y'_{\phi_\pi} > y'$.

model under LBS predicts that this policy change reduces the volatility of inflation while simultaneously anchoring households' beliefs about inflation. As a result, doing so can increase the impact of monetary policy surprises on output.

In other words, anchoring households' inflation expectations in this model gives the central bank further room to stimulate the economy during recessions. However, it also increases the impact of other demand shocks on the economy. The degree of this amplification will depend on the specific calibration of the model. I will revisit this question in the quantitative exercise in Section 5.

Relationship with the empirical evidence. We can use the model to interpret several empirical results regarding the flattening of the Phillips curve. To do so, recall that the NKPC of this model is given by:

$$\pi_t = (1 - \Psi_\pi)\,\beta\mathbb{E}_t\pi_{t+1} + \Psi_\pi\mathbb{E}_{t-1}\pi_t + \left(\frac{\sigma+\varphi}{\lambda+\psi_\pi}\right)y_t. \tag{43}$$

With full information ($\psi_\pi = 0$), the parameter λ serves as a sufficient statistic to characterize the comovement between inflation and output induced by aggregate demand shocks. The slope of the NKPC, in this case, depends only on λ, which is a function of the degree of price-stickiness but is independent of the monetary policy stance.

With LBS, a second term appears in the Phillips curve. This term captures the impact of the differences in perception of wages between households and firms. This information wedge acts as an endogenous source of fluctuations in the firms' desired markup. The parameter λ ceases to be a sufficient statistic of the slope of the NKPC because LBS also induces positive comovement between inflation and output. This can be clearly observed by the slope of the NKPC in (43), which is now a function of ψ_π.

But ψ_π is an increasing function of ϕ_π, as shown by Proposition 4. For this reason, a change to a more hawkish monetary policy stance flattens the slope of the Phillips curve by increasing the degree of anchoring of households' beliefs about inflation.

We can use the previous results to interpret several findings of the empirical literature estimating the NKPC. The first empirical result relates to the flattening of the slope of the NKPC. Several researchers have observed that the correlation of inflation and different measures of the output gap has fallen over time, with the fall starting at some point in the 80's.[23] The timing of the flattening of the Phillips curve coincides with the change in the way monetary policy was conducted after Paul Volcker was appointed Chairman of the Board of Governors of the Federal Reserve System (Clarida, Gali, and Gertler, 2000). In the model, such a change in policy is captured by an increase of the response of the central bank to the inflation rate, as measured by ϕ_π.

Consider now an econometrician that estimates α_{PC} using some measures of inflation, the output gap, and the expectations of fully informed agents (for instance, those of professional forecasters who presumably know the current inflation rate). The model predicts that this econometrician would be estimating a specification like (43), where the slope α_{PC} is endogenous and changes with the monetary policy stance. This econometrician will observe that, after the policy change, the slope α_{PC} has become flatter, consistent with the empirical evidence for the U.S. The model suggests that the estimated flattening is a consequence of the change in the conduct of monetary policy during this period, which would be reflected in the lower correlation between output and inflation observed in reduced-form specifications.

Recent work has also estimated the slope of the Phillips curve exploiting regional variation to control for the confounding effect of aggregate variables, including the long-run inflation expectations and the response of monetary policy to demand shocks (see, e.g., McLeay and Tenreyro, 2019; Fitzgerald, Jones, Kulish, and Nicolini (2020); Hazell, Herreño, Nakamura, and Steinsson (2020)). These authors find that the slope of the Phillips curve is small and has remained constant in the last decades. Their evidence is consistent with the finding that the response of inflation to variations in marginal costs has not changed over time (Del Negro, Lenza, Primiceri, and Tambalotti, 2020; Barnichon and Mesters, 2021). To interpret the results of these authors, recall from Section 2 that we can express the NKPC of this model as:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \alpha^*_{PC} y_t - \lambda^{-1} \nu^p_t$$

This equation suggests that that the estimation strategy used by the aforementioned authors controls for the effect of the average perception error ν^p_t on inflation in a specification of the NKPC like

[23] See, for instance, Ball and Mazumder (2011), Kiley (2015), Blanchard (2016), Stock and Watson (2019), Höynck (2020), and Barnichon and Mesters (2020).

the previous one. Consequently, their empirical strategy delivers consistent estimates of the full-information slope α^*_{PC}, which is a function of of the response of inflation to marginal costs, as measured by λ^{-1}. Through the lens of this model, the findings of these authors suggest that the degree of price stickiness in the economy has not changed over time.

But the previous empirical findings do not rule out the possibility that the slope of the NKPC, as specified in (43), has flattened over time. Proposition 8 shows that the comovement between inflation and output can fall, even if the degree of price stickiness λ is constant, as a result of the anchoring of households' inflation perceptions. This result offers a way to reconcile the conflicting evidence regarding the estimation of the Phillips curve.

The second result relates to the fit of estimated NKPCs. Many authors have estimated equations similar to (43) using different proxies for the expectations of economic agents. A common finding in this literature is that the expectations of households allow the estimated model to fit the data better, explaining puzzles like the missing disinflation after the great recession (see, e.g., Coibion and Gorodnichenko, 2015b; Coibion, Gorodnichenko, and Kamdar, 2018a; Jørgensen and Lansing (2021)). To interpret this finding, recall from Section (2) that we can also express the NKPC in this model as:

$$\pi_t = \frac{\lambda}{1+\lambda}\beta\mathbb{E}_t\pi_{t+1} + \left(\frac{\sigma+\varphi}{1+\lambda}\right)y_t + \frac{1}{1+\lambda}\overline{\mathrm{E}}_t\pi_t.$$

This equation suggests that the average belief about inflation across households should be included in the econometric specifications of the NKPC. Moreover, the data suggests a very close relationship between households' perceptions of current inflation and their expectations about future inflation[24]. If households answer expectations surveys by reporting their current perception, the addition of their expectations to econometric specification acts as a proxy of the missing term $\overline{\mathrm{E}}_t\pi_t$ in the right-hand side of the previous specification.

Furthermore, LBS implies that the average beliefs of inflation across households are persistent over time, which in turn induces endogenous and time-varying persistence in inflation, as captured by the lagged expectations in (43). The persistent behavior of inflation and the fall in this persistence in the last decades is also a well-documented empirical fact (see, for instance, Gali and Gertler (1999) and Gallegos (2021)). Taken together, the previous observations may explain why Phillips curves fit better the data when the expectations of households are used in their estimation.

[24]Using special questionnaires introduced in this survey, Axelrod et al. (2018) find that one-third of respondents report the same perception of inflation as their reported expectation, and one-sixth reports expectations that deviate from their perception by less than one percentage point. Similar evidence is provided by Jonung (1981) for a cross-section of swedish households, Armantier et al. (2016) for a cross-section of households in the NY FED Survey of Consumers Expectations and Coibion et al. (2018b), Candia et al. (2021) for firms in New Zeland and the U.S. Note that this is consistent with households perceiving that the 12-month inflation rate follows a random walk. As shown by Atkeson and Ohanian (2001) and Stock and Watson (2007), this is indeed a good approximation of the data generating process of this variable.

4 Extensions: Technology Shocks and Rational Inattention

In the first part of this section, I discuss the impact of TFP shocks when consumers learn by shopping. I show that the same forces propagating demand shocks work to attenuate the impact of technology shocks on output. In the second part, I provide a microfoundation of LBS as the result of households' rational inattention to aggregate inflation. In this extension, the variance of the noise in households' signals becomes an endogenous function of the structural parameters of the model. Both extensions will be included in the model used to study LBS quantitativelly in the next section.

4.1 The attenuation of technology shocks

I now turn attention to the transmission of technology shocks. I now assume that production function of firms is given by

$$y_t = a_t + n_t,$$

with a_t denoting aggregate TFP, which is exogenous and given by

$$a_t = \rho_a a_{t-1} + \eta_t^{AS}; \quad \eta_t^{AS} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma_{AS}^2\right).$$

The conditions characterizing the equilibrium are similar to the ones derived in Section 2. In particular, the aggregate Euler equation (26) remains unchanged, and the NKPC is still given by (24), after replacing the output gap y_t with the natural output gap, defined as

$$\widetilde{y}_t \equiv y_t - \widetilde{a}_t,$$

with $\widetilde{a}_t \equiv (1+\varphi)/(\sigma+\varphi)\, a_t$. Moreover, equation (36), which defines implicitly the equilibrium value of ψ_π, remains unchanged if we redefine the signal to noise ratio as $q = \frac{\sigma_{AD}^2 + \sigma_{AS}^2}{\sigma_\epsilon^2}$. Finally, if we set $\rho_a = \rho_z$, the characterization of the equilibrium is identical to the one provided when only aggregate demand shocks are present.[25] To illustrate how LBS alters the transmission of technology shocks on output, I will focus now on the case where shocks are i.i.d. ($\rho_a = \rho_z = 0$) and prices are flexible ($\theta = 0$). In this case, the aggregate demand and supply blocks of the model simplify to

$$\pi_t = -\alpha_{AD}\left(y_t - \widetilde{z}_t\right), \tag{44}$$

$$\pi_t = \alpha_{PC}\left(y_t - \widetilde{a}_t\right), \tag{45}$$

with $\widetilde{z}_t \equiv \sigma^{-1} z_t$, $\alpha_{AD} \equiv 1/\left(\sigma^{-1}\phi_\pi - \chi\psi_\pi\right)$ and $\alpha_{PC} = (\sigma+\varphi)/\psi_\pi$. Notice that these equations are similar to the ones derived in the previous section to illustrate the propagation of demand shocks, except for the presence of $\widetilde{a}_t$ in the supply block of the model. Technology shocks now act as a source of exogenous shifts in aggregate supply.

Using (44) and (45), we can derive the following expressions for the equilibrium value of output

[25] If $\rho_a \neq \rho_z$, the only difference is that the process has now an AR(2) component.

and inflation:

$$y_t = \Delta_y \widetilde{z}_t + \left(1 - \Delta_y\right) \widetilde{a}_t, \tag{46}$$

$$\pi_t = \Delta_\pi \left(\widetilde{z}_t - \widetilde{a}_t\right), \tag{47}$$

where $\Delta_y \equiv \alpha_{AD} / \left(\alpha_{AD} + \alpha_{PC}\right)$ denotes the response of output to (normalized) aggregate demand shocks $\widetilde{z}_t$, and $\Delta_\pi = \alpha_{PC}\Delta_y$ is the response of inflation to the reduced form shock $u_t = \widetilde{z}_t - \widetilde{a}_t$. Collecting the previous expressions, we arrive to the following result.

Proposition 9. *(**Attenuation of technology shocks**) Let $\left[\partial y_t / \partial a_t\right]^{FI}$ denote the equilibrium response of output to a technology shock under full information and let $\left[\partial y_t / \partial a_t\right]^{LBS}$ denote the response under LBS. Then:*

$$\left[\frac{\partial y_t}{\partial a_t}\right]^{LBS} = \left(1 - \Delta_y\right)\left(\frac{1+\varphi}{\sigma+\varphi}\right) \leq \left[\frac{\partial y_t}{\partial a_t}\right]^{FI},$$

with

$$\Delta_y \equiv \frac{\sigma^{-1}\psi_\pi}{\left(\sigma+\varphi\right)\sigma^{-1}\phi_\pi + \left(1 - \left(\sigma+\varphi\right)\chi\right)\psi_\pi} \tag{48}$$

Proof. See Appendix B.9. □

To understand this result, it is useful again to plot the labor supply and demand of this economy. The first diagram of Figure 4 shows the effect of a positive productivity shock on the labor market of this economy.

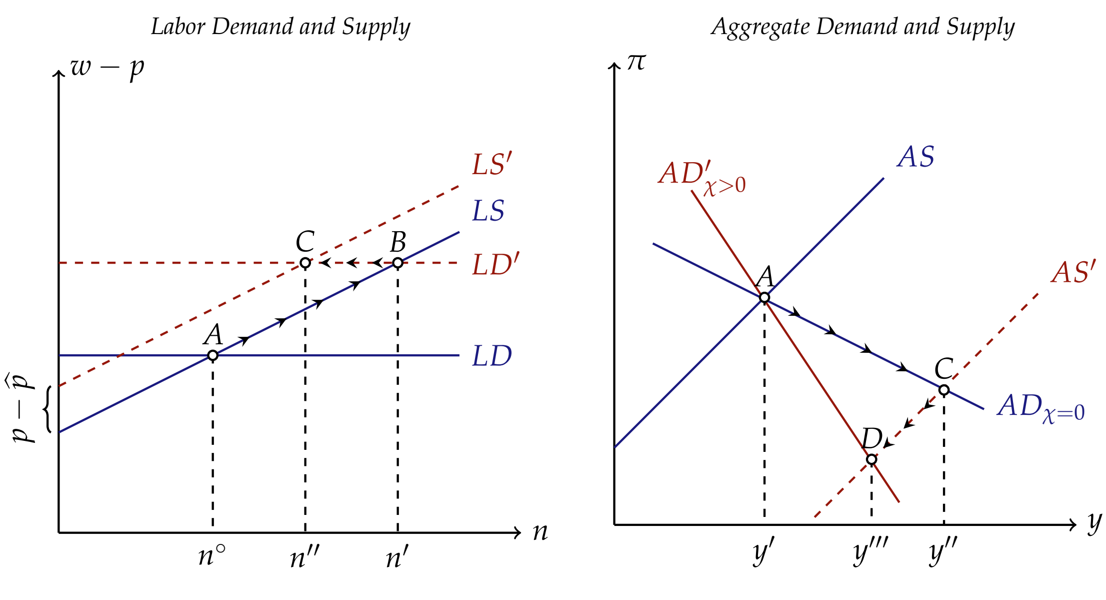


Figure 4: Learning by shopping and the attenuation of technology shocks

Notes: The figure illustrates how LBS attenuates the impact of a positive TFP shock. The left panel shows the the reduced impact of the shock in the labor supply due to the differences in the perceived real wage between households and firms. The right panel shows the further attenuation arising from households' underreaction to the increase in permanent income produced by the shock.

The equilibrium before the shock is highlighted by point ***A***. The increase in aggregate productivity allows firms to produce at a lower (nominal) marginal cost. Firms want to keep their markups constant, so they reduce prices proportionally, leading to an increase in the real wage. This effect is captured by the upward shift of the labor demand curve.

Under full information ($\psi_\pi = 0$), the labor supply curve remains at the initial position, so the increase in TFP pushes the economy to a new equilibrium ***B*** featuring higher output and higher real wages. If households' beliefs about inflation are anchored ($\psi_\pi > 0$), the reduction in prices perceived by households is lower in magnitude than the reduction in prices by firms. As a result, households perceive a more moderate increase in real wages and consume less in response. The information friction creates a wedge in labor demand that shifts the labor supply, offsetting part of the increase in output due to the rise in productivity. The equilibrium with LBS, indicated by point ***C***, features higher real wages but an output level that lies between the initial output level y and the full information level y'.

The second diagram of Figure 4 shows the effect of a technology shock on the aggregate demand and supply of this economy. As indicated in the previous paragraph, the productivity shock moves the equilibrium from point **A** to point **C**. When the information wedge in aggregate demand is present ($\chi > 0$), there is a second round of attenuation: The slope of aggregate demand is now steeper, the increase in output is further mitigated, and the technology shock is largely deflationary. This additional attenuation comes from the fact that households underreact to the increase in permanent income from the technology shock, since they don't fully observe the fall in the aggregate price level that follows this shock. The final equilibrium, indicated by point D, features a more modest increase in output than what would be observed under full information.

4.2 Learning by Shopping as Rational Inattention to Aggregate Prices

An underlying assumption in the analysis made so far is that the variance of the noise in signals σ_ϵ^2 is constant and exogenously given. I now relax this assumption by allowing households to choose the precision of their signals. Following the Rational Inattention literature pioneered by Sims (2003), I assume households decide the amount of attention they allocate to aggregate inflation by trading the costs of acquiring information about this variable with the cost of ignoring this information. I now describe the two components of this problem.

The costs of ignoring inflation. Inattention to aggregate inflation results in consumption, savings and labor supply decisions that differ from those that the household would take under full information. It follows that an agent that ignores inflation achieves a lower welfare (1), compared to a fully attentive agent.

To derive an expression for the welfare costs incurred by household i from ignoring aggregate inflation, I replace the budget constraint (5) in the objective function (1). A log-quadratic approximation of the household's objective function around the non-stochastic steady-state yields the following result.

Proposition 10. *(**The costs of inattention to inflation**) The welfare cost for household i from having in-*

complete information about P_t is given by:

$$\mathcal{IC}_\pi = -\frac{1}{2}C^{1-\sigma}\mathrm{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{\sigma\left(c_{i,t}-c_{i,t}^*\right)^2+\mathcal{M}^{-1}\varphi\left(n_{i,t}-n_{i,t}^*\right)^2\right\}, \tag{49}$$

where $c_{i,t}-c_{i,t}^*$ and $n_{i,t}-n_{i,t}^*$ are the deviations of household's consumption and labor from their full-information counterparts. These deviations are given by

$$c_{i,t}-c_{i,t}^* = -\frac{1}{\sigma}\beta\left\{\nu_{i,t+1}^{\pi}+\sum_{k=1}^{\infty}\beta^k\left\{\nu_{i,t+k+1}^{\pi}-\phi_\pi\nu_{i,t+k}^{\pi}\right\}\right\} \tag{50}$$
$$+\beta\left\{\chi\nu_{i,t}^{p}-\left(\frac{1-\beta}{\beta}\right)\mathbb{E}_t\sum_{k=1}^{\infty}\beta^k\nu_{i,t+k|t}^{y}\right\},$$

$$n_{i,t}-n_{i,t}^* = \frac{1}{\varphi}\nu_{i,t}^{p}-\frac{\sigma}{\varphi}\left(c_{i,t}-c_{i,t}^*\right). \tag{51}$$

Proof. See Appendix B.10. □

The above proposition shows that the private costs of ignoring inflation are proportional to the magnitude of the misperception about the price and the forecast errors about future inflation. The information gaps translate into sub-optimal consumption and labor supply decisions. The deviations (50) and (51) closely resemble the information wedges affecting the aggregate Euler equation (26) and the aggregate labor supply (26). The resemblance is not a coincidence, as these wedges are the result of aggregating this microeconomic friction across households.

Importantly, Proposition 10 shows that these deviations have second-order effects on the welfare of each household. Nevertheless, they can have first-order effects on the behavior of macroeconomic variables, as illustrated by the presence of the aggregate information wedge in (26). As a result, small costs of acquiring information at the private level can have first-order effects on the behavior of output and inflation, as illustrated by the results in the previous sections. Akerlof and Yellen (1985) observed that this is also the case with menu cost models. The two frictions represent forms of near-rationality where individual agents face second-order losses from deviating from the frictionless behavior. Still, the small deviations at the individual level can give rise to comovement between output and inflation.

The costs of acquiring information. In absence of any constraint on information acquisition, households would choose to observe inflation with infinite precision. Following the Rational Inattention literature, I assume that the utility costs of acquiring information are linear in Shannon's mutual information function. Formally, let $\boldsymbol{p}^T \equiv \{p_t\}_{t=0}^T$ and $\boldsymbol{s}_i^T \equiv \{s_{i,t}\}_{t=0}^T$ denote the history of the aggregate price and the signals received by household i up to period T. Let $H\left(\boldsymbol{p}^T\right)$ and $H\left(\boldsymbol{p}^T|\boldsymbol{s}_i^T\right)$ denote the entropy and conditional entropy of $\boldsymbol{p}^T$ and $\boldsymbol{s}_i^T$. I assume that the agent's flow cost of information at time t is given by $\omega\mathbb{I}\left(\boldsymbol{p}^T,\boldsymbol{s}^T\right)$, where

$$\mathbb{I}\left(\boldsymbol{p}^T,\boldsymbol{s}_i^T\right)\equiv H\left(\boldsymbol{p}^T\right)-H\left(\boldsymbol{p}^T|\boldsymbol{s}_i^T\right), \tag{52}$$

is the mutual information between of $\boldsymbol{p}^T$ and $\boldsymbol{s}^T$, and $\omega > 0$ is the marginal cost of a unit of information.[26] Intuitively, mutual information measures the reduction in uncertainty about aggregate prices $\boldsymbol{p}^T$ from observing $\boldsymbol{s}^T$. The cost $\omega > 0$ can be interpreted as an opportunity cost, measured in utility terms, of devoting attention to tracking inflation. This will be a crucial parameter in the quantitative model used in the next section.

The attention problem of the household. We are now in position to state the attention problem of the household. In period $t = -1$, before choosing consumption, each household chooses the precision of the signals that it receives in the following periods. In each period $t \geq 0$, the expectation of current and future prices is formed conditional on the sequence of all signals that the household has received up to that point in time.

Formally, let $\widetilde{\omega} \equiv 2\omega / C^{1-\sigma}$. The problem of the household is to choose σ_ϵ^2 to maximize:

$$-\frac{1}{2} C^{1-\sigma} \mathrm{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left\{ \sigma \left(c_{i,t} - c_{i,t}^* \right)^2 + \mathcal{M}^{-1} \varphi \left(n_{i,t} - n_{i,t}^* \right)^2 + \widetilde{\omega} \mathbb{I} \left(\boldsymbol{p}^T, \boldsymbol{s}^T \right) \right\}, \tag{53}$$

subject to the signal structure (7) and equations (50) and (51) defining $c_{i,t} - c_{i,t}^*$ and $n_{i,t} - n_{i,t}^*$.

Solving this problem is only possible using numerical methods, as will be the case in the next section. But we can gain some intuition by studying the closed-form solution that results when the aggregate shocks are i.i.d.To do so, recall that, conditional on a value of σ_ϵ^2, the equilibrium inflation is given by equation (47). Moreover, both inflation and signals are Normal random variables that follow i.i.d. processes, so mutual information (52) takes a simple form:[27]

$$\mathbb{I} \left(\pi_t, \pi_{i,t}^* \right) = \frac{1}{2} \log \left(1 + \frac{\mathrm{Var}\left[\pi_t \right]}{\sigma_\epsilon^2} \right).$$

We can see that mutual information is increasing in the signal-to-noise ratio of the signals. Notice also that the agent is atomistic and takes the variance of inflation $\mathrm{Var}\left[\pi_t\right]$ as given. The fact that inflation follows an i.i.d. process implies that the deviations (50) and (51) simplify to

$$c_{i,t} - c_{i,t}^* = \beta \chi \nu_{i,t}^p,$$

$$n_{i,t} - n_{i,t}^* = \frac{1}{\varphi} \left(1 - \sigma \beta \chi \right) \nu_{i,t}^p$$

We can thus rewrite the information acquisition problem (53) as

$$\min_{\sigma_\epsilon^2} \quad \Omega \mathrm{E}_{i,-1} \left[\left(\nu_{i,t}^p \right)^2 \right] + \widetilde{\omega} \log \left(1 + \frac{\mathrm{Var}\left[\pi_t \right]}{\sigma_\epsilon^2} \right) \tag{54}$$

[26] See Cover and Thomas (2012) for a comprehensive introduction to information theory.

[27] Here I use the natural logarithm to express information units in nats, as opposed to bits, in which case, the logarithm has base 2.

where the parameter Ω is given by

$$\Omega \equiv C^{1-\sigma}\left(\sigma\left(\beta\chi\right)^2 + \mathcal{M}^{-1}\frac{1}{\varphi}\left(1-\sigma\beta\chi\right)^2\right). \tag{55}$$

This parameter summarizes the costs from sub-optimal attention to inflation. Finally, we can use the well-known regression lemma for the distribution of bivariate normal variables to get:

$$\mathrm{E}_{i,-1}\left[\left(\nu_{i,t}^{p}\right)^2\right] = \mathrm{Var}_{i,t}\left[\pi_t|\pi_{i,t}^{*}\right] = \mathrm{Var}\left[\pi_t\right] - \frac{\mathrm{Var}\left[\pi_t\right]}{\mathrm{Var}\left[\pi_t\right]+\sigma_\epsilon^2}$$

We can thus take first order conditions of (54) and solve for σ_ϵ^2 to arrive to the following result.

Proposition 11. *(**Optimal attention to inflation**) The degree of anchoring ψ_π^* of a rationally inattentive household is given by*

$$\psi_\pi^* = \max\left\{\min\left\{\frac{\widetilde{\omega}}{\Omega},1\right\},0\right\}.$$

Proof. See Appendix B.11. □

Proposition 11 shows that the optimal level of inattention ψ_π^* is common across households, increasing in the costs of acquiring information ω, and decreasing in the utility costs of ignoring inflation Ω. These costs, defined in (55), reflect the impact how the suboptimal choice of consumption and labor due to LBS impacts the utility of the household. The optimal choice of attention σ_ϵ^2 in this simple setting requires households to keep a constant signal-to-noise ratio $\mathrm{Var}\left[\pi_t\right]/\sigma_\epsilon^2$.

5 Learning by Shopping: A Quantitative Analysis

In this section, I explore the quantitative relevance of LBS. To do so, I calibrate an extended version of the model to the U.S. data and use it to study the dynamic response of inflation and output to aggregate shocks. I conclude by studying the impact of a change in the monetary policy stance in this quantitative setting.

5.1 Quantitative model

The quantitative model used in this section incorporates the two extensions discussed in Section 4: I allow the presence of shocks to both aggregate demand and TFP and assume these can be persistent over time. I also assume that households are rationally inattentive and choose σ_ϵ^2 to maximize (53) subject to the information flow constraint (52).

I also relax the assumption that households have common knowledge about past aggregate outcomes, allowing learning to be persistent over time. To do so, I assume that the total expenditures $M_{i,t}$ and dividends $D_{i,t}$ are subject to an auxiliary noise shocks similar to those affecting households wages. This noise prevents households from inferring the past price level at the beginning of each period.[28] For this reason, their prior beliefs no longer coincide with the past aggregate price. As a result, households also disagree about past outcomes.

Finally, I introduce the following assumption on the initial information set of households.

Assumption 2. *The initial information set, $\mathcal{I}_{i,-1}$, contains an infinite history of signals.*

The above assumption is common in the rational inattention literature.[29] It allows me to abstract from purely deterministic transitional dynamics in the conditional second moments of beliefs. This guarantees that the Kalman gain coefficients characterizing the learning process of households are constant over time. Nevertheless, the Kalman gains will still be endogenous objects determined in equilibrium.

With these additional features, the model can only be solved numerically. In Appendix C, I provide a computational algorithm to do so. Using this algorithm, I now study the behavior of the model quantitativelly by calibrating it to the U.S. data.

5.2 Calibration

Most of the parameters in the model can be calibrated using values commonly found in the business cycle literature. The only non-standard parameter is the cost of acquiring information ω in (52). This cost determines the magnitude of the information friction and plays a crucial role in this model. Unfortunately, there is no direct counterpart of ω in the data. However, there is a direct relationship between ω and the Kalman gain coefficients associated with households' filtering problem. To see this, note that the beliefs of households are now given by:

$$\widehat{p}_{i,t-h|t} = \widehat{p}_{i,t-h|t-1} + \kappa_h \left(p_t - \widehat{p}_{i,t|t-1}\right) + \kappa_h \epsilon_{i,t}, \quad h = 0, 1, \ldots, \tag{56}$$

where $\widehat{p}_{i,t|s} \equiv \mathrm{E}_{i,s} p_{i,t}$, and $\kappa_h \in [0,1]$ is a Kalman gain determined in equilibrium. Now, let $\widehat{\pi}^{YoY}_{t|s} \equiv \int_0^1 \left\{\widehat{p}_{i,t|s} - \widehat{p}_{i,t-12|s}\right\} di$ denote the average belief across households about year-on-year inflation, conditional on information up to s. Using (56) when $h \in \{0, 12\}$ and averaging across households, we obtain:

$$\widehat{\pi}^{YoY}_{t|t} = (1 - \psi_\pi)\, \pi^{YoY}_t + \psi_\pi \widehat{\pi}^{YoY}_{t|t-1} + u_t, \tag{57}$$

where $u_t \equiv (1 - \psi_\pi) \int_0^1 \left\{s_{i,t-12} - \widehat{p}_{i,t-12|t-1}\right\} di$ is a residual term proportional to the signals acquired in the previous year, and $\psi_\pi \equiv 1 - (\kappa_0 - \kappa_{12})$ is now the *degree of anchoring of year-on-year inflation* (assuming a monthly frequency).

As shown in the previous section, the cost of acquiring information ω affects the precision of signals σ^2_ϵ and, through this channel, the value of the Kalman gains κ_h. We can thus use data on average

[28] The shocks plays a similar role to the auxiliary shocks introduced in Section 2. One can think about M_t as the credit card bill and interpret these shocks as unexpected fees and charges in the credit card bill that prevent each household from inferring the aggregate price level P_t from just looking at its credit card bill.

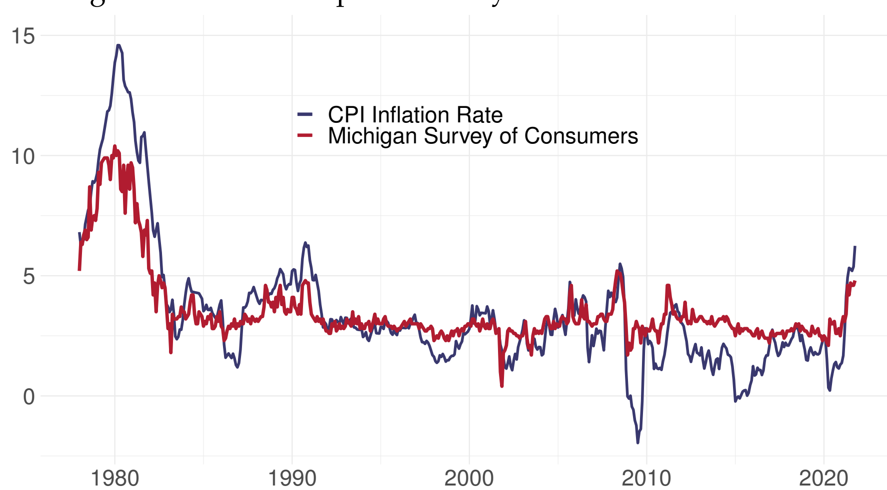
[29] See, for instance, Woodford (2009), Mackowiak and Wiederholt (2009), and Maćkowiak and Wiederholt (2015). The assumption also provides a useful benchmark to compare the model with models where firms are inattentive.

inflation beliefs across households to estimate ψ_π using (57), and then calibrate ω to target the resulting estimate. Doing so (57) requires data on both inflation perceptions and inflation expectations. Unfortunately, such a dataset is not available for the U.S.

However, recent evidence by Axelrod, Lebow, and Peneva (2018) suggests that the inflation expectations reported by participants of the Michigan Survey are very similar to their perceptions about current inflation.[30] This suggests that the measures of expectations available in this survey are good proxies of households perceptions about inflation. Under this interpretation, we can estimate (57) using the data shown in Figure 5, by replacing $\widehat{\pi}^{YoY}_{t|t}$ with $\widehat{\pi}^{YoY}_{t+1|t}$.

Table 1 shows the results of this exercise using monthly data and the year-on-year CPI inflation rate as a proxy for π^{YoY}_t.[31] The results of specifications 1 and 2 shows that the model has a good fit to the data. Moreover, when the constant term is dropped, we cannot reject the null hypothesis that the sum of the coefficients associated to π^{YoY}_t and $\widehat{\pi}^{YoY}_{t|t-1}$ is equal to one. This is that we would expect when one of the variables is a distributed lag of the other, so that they are cointegrated. Specifications 3-6 show that the value of this coefficient has not been stable over time. In the period preceding Volcker's tenure as Fed Chairman, the anchoring coefficient was almost half the size of the coefficient in the post-Volcker period.[32] This is consistent with the prediction of the model that the degree of anchoring is endogenous and depends on the conduct of monetary policy.

Figure 5: Inflation Expectations by Households in the U.S.



Notes: The figure shows the average expectation about future inflation held by participants of the *Survey of Consumers* conducted by the University of Michigan. The red line shows the average belief about how prices will change in the following 12 months. The blue line shows the 12-month CPI inflation rate provided by the U.S. Bureau of Labor Statistics.

[30] As discussed in the third part of Section 3, several authors have found similar findings in other countries and surverys, and also on the firms' inflation expectations.

[31] A similar econometric specification is used in Carroll (2003) to estimate the relationship between households' expectations and those of professional forecasters.

[32] The results are similar if we instead split the sample in 1990m01, which is a break commonly used in the literature to estimate the slope of the Phillips curve.

Table 1: Inattention to Inflation in the Michigan Survey of Consumers

Estimating Equation: $\hat{\pi}_t^{YoY} = \beta_0 + \beta_1 \pi_t^{YoY} + \beta_2 \hat{\pi}_{t-1}^{YoY} + \epsilon_t$						
Equation	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\psi}_\pi$	**Sample**	R^2
1	0.315	0.106	0.804			0.94
	(0.092)	(0.022)	(0.046)		1978M01 - 2019M12	
2		0.047	0.941	0.833		0.98
		(0.015)	(0.016)	(0.042)		
3	0.495	0.166	0.721			0.83
	(0.538)	(0.052)	(0.099)		1978M01 - 1982M12	
4		0.166	0.775	0.466		0.98
		(0.051)	(0.080)	(0.145)		
5	0.689	0.086	0.695			0.70
	(0.118)	(0.018)	(0.046)		1983M01 - 2019M12	
6		0.052	0.946	0.847		0.98
		(0.016)	(0.016)	(0.044)		

Notes: This table shows the estimated degree of anchoring. $\hat{\pi}_t^{YoY}$ is the period-t mean of the Michigan survey measure of households inflation expectations over the next 12 months. π_t^{YoY} is the CPI inflation rate between period t and $t-12$. Standard errors (shown in parentheses) are corrected for heteroskedasticity and autocorrelation following a Newey-West ((1987)) procedure with twelve lags. The level of anchoring at quarterly frequency is computed as $\hat{\psi}_\pi = \hat{\beta}_2^3$ and its std. erros is calculated using the Delta method.

Parameter values. The baseline calibration of the model is summarized in Table 2 . I assume each period is a quarter and set the discount factor β to 0.99, so that the steady-state real risk-free rate is 4 percent. I set the inverse of elasticity of intertemporal substitution σ to 2, consistent with the baseline estimates by Crump, Eusepi, Tambalotti, and Topa (2015).[33] I set the inverse of the Frisch elasticity of labor supply φ to 4, following Chetty, Guren, Manoli, and Weber (2011). I also set the elasticity of substitution across varieties to $\varepsilon = 6$, the Calvo index of price rigidities θ to 0.75 (consistent with an average price duration of one year), and the inflation coefficient in the Taylor rule ϕ_π to 1.5. Finally, I fix $\rho_{TFP} = \rho_{AD} = \rho$ to make sure that differences in the response of AD and TFP shocks are not driven by differences in their persistence. I then calibrate simultaneously parameters $\left(\rho, \sigma_{AD}^2, \sigma_{TFP}^2, \omega\right)$ to match four moments of the data for the post-Volcker period: 1) The correlation and 2) variance of quarterly Core CPI inflation observed, 2) the share of variance in output explained by non-technology shocks estimated in Galí and Gambetti (2009), and 3) a value of ψ_π of 0.85, in line with the estimated values of specification 6 in Table 1.[34] The value of ω necessary to match the desired calibration implies that the costs of acquiring information $\omega\mathbb{I}(\cdot)$ are equivalent to 0.2% of the steady-state level of consumption of each household. These costs are small, in line with the predictions from Proposition 10 and the observation by Cochrane (1989) that the costs of deviating from the permanent income decision rule are arbitrarily small for a consumer..

[33]Note that these authors estimate an Euler equation by individual that corresponds directly to equation (18) in this model.

[34]Matching the behavior of inflation is particularly important in this exercise as this determines households' costs and benefits of ignoring inflation.

Table 2: Model Calibration

Parameter	Value	Description	Source / Target
Assigned			
β	0.99	Discount factor	quarterly frequency
σ	2	Inv. elasticity of intertemporal subs.	Crump et al. (2015)
φ	4	Inv. Frisch elasticity of labor supply	Chetty et al. (2011)
ε	6	Elasticity of substitution	avg. price markup of 20%
θ	0.75	1 - Prob. of adjusting prices	avg. price duration of 4 quarters
ϕ_π	1.5	Interest rate rule coefficient	Taylor (1993)
Calibrated			
ρ	0.93	Persistence of shocks	$\text{Corr}\left[\pi_t, \pi_{t-1}\right] = 0.79$
σ_{TFP}	0.85×10^{-3}	Std. Dev. TFP shock	$\text{SD}\left[y_t \vert z_t\right] / \text{SD}\left[y_t\right] = 0.70$
σ_{AD}	3.81×10^{-3}	Std. Dev. AD shock	$\text{SD}\left[\pi_t, \pi_{t-1}\right] = 0.79$
ω	1.35×10^{-3}	Information cost	$\psi_\pi = 0.85$

Notes: The table presents the baseline parameters for the quantitative model. The first panel shows the value of the parameters assigned based on values commonly found in the literature. The second panel shows the value of four parameters calibrated jointly to match different moments in the data.

5.3 Dynamic response to aggregate shocks

Aggregate demand shock. I start by studying the dynamic response of aggregate variables in the model to an expansionary shock in demand. Figure 6 shows the response to a one standard deviation shock under three different scenarios.The first scenario, in blue, shows the response when price stickiness is the only friction present ($\omega = 0$, $\theta > 0$). The shock produces comovement between output, inflation, and employment, as is the usual case with this nominal rigidity. Notice that the inflation perceived by households is identical to the actual inflation rate in this case.

The second scenario, in red, shows the response when LBS is the only friction present in the model ($\omega > 0$, $\theta = 0$). Consistent with the results from Proposition 5, the information friction also produces comovement between inflation, employment and output. In contrast to the scenario with price stickiness, the dynamics of output and employment show additional persistence and a hump-shaped response to the shock, consistent with the analytical results from the previous sections.

The third scenario, in yellow, shows the response when both sticky prices and LBS are present. The interaction of the two frictions amplifies the response of output and employment dramatically: the response of output on impact is approximately *8 times larger* than the corresponding response under full information. This suggests that the propagation and amplification induced by LBS is substantial in quantitative settings, allowing small demand shocks to produce large fluctuations in output.

Notice also that the response of output on impact in this scenario is arppoximatelly 2.5 times larger than the sum of the impact response under the other two scenarios. This non-linearity is consistent with the results of Propistion 7. The quantitative exercise shows that this interaction also adds additional persistence to the response of output: after ten periods, the response of output to this shocks is still larger than corresponding response in the two alternative scenarios.

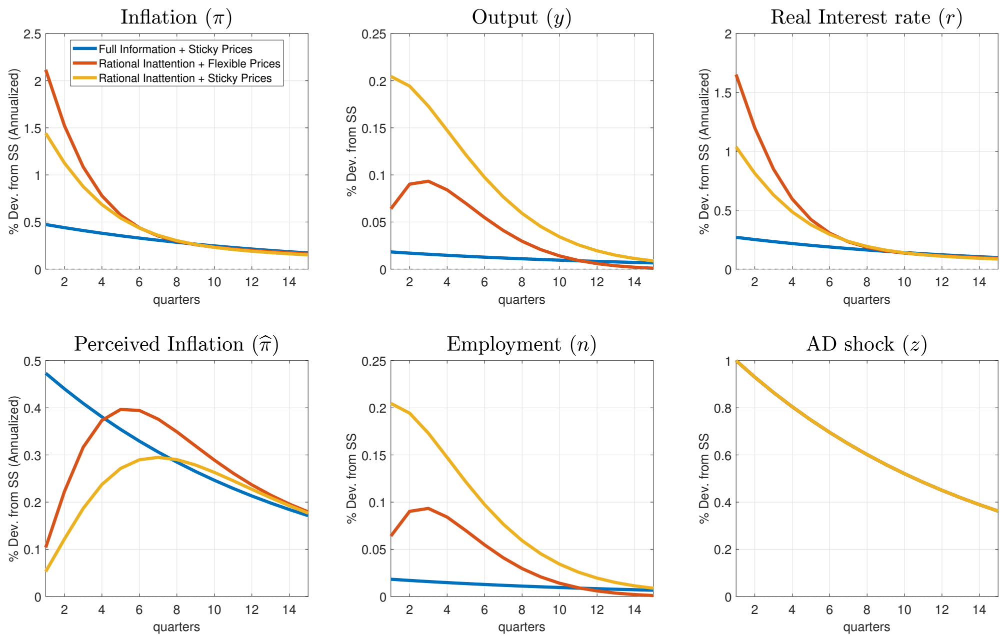


Figure 6: Dynamic Responses to an Aggregate Demand Shock

Notes: The figure plots the impulse responses to a one standard deviation expansionary shock in aggregate demand under the baseline calibration. The blue line shows the response when price stickiness is the only friction present. The red line shows the corresponding response when *learning by shopping* is the only friction present. The yellow line shows the response when both frictions are present.

Technology shock. Figure 7 shows the response of a one standard deviation positive shock to TFP under the three different scenarios considered before. We can see that LBS attenuates the response of output, but the attenuation is much larger when this information friction interacts with price stickiness. The figure shows how this mitigation comes from the lower perceived inflation, which makes households perceive a real wage that is lower than the actual wage. The effect of this inattention is observed in the amplification of the negative response of employment to this shock. This result suggests that technology shocks are even less likely to generate positive comovement between employment and output when consumers learn about inflation by shopping.

5.4 The effect of a change in the monetary policy stance

I now use the calibrated model to analyze the impact of a change in the monetary policy stance. Proposition 4 shows that, in the more stylized model, the degree of anchoring ψ_π changes with ϕ_π. Consider now a change of monetary policy to a more hawkish stance, reflected as an increase in the value of ϕ_π. This policy change flattens the aggregate demand by making the interest rate more sensitive to variations in inflation. In the absence of information frictions, this policy unambiguously reduces the volatility of inflation produced by demand shocks. With LBS, the reduced volatility in inflation increases ψ_π, which in turn increases the persistence of inflation, as well as the propagation of demand shocks. I now explore the extent to which a change in the monetary policy stance can affect the dynamics of inflation and the propagation of aggregate shocks quantitatively.

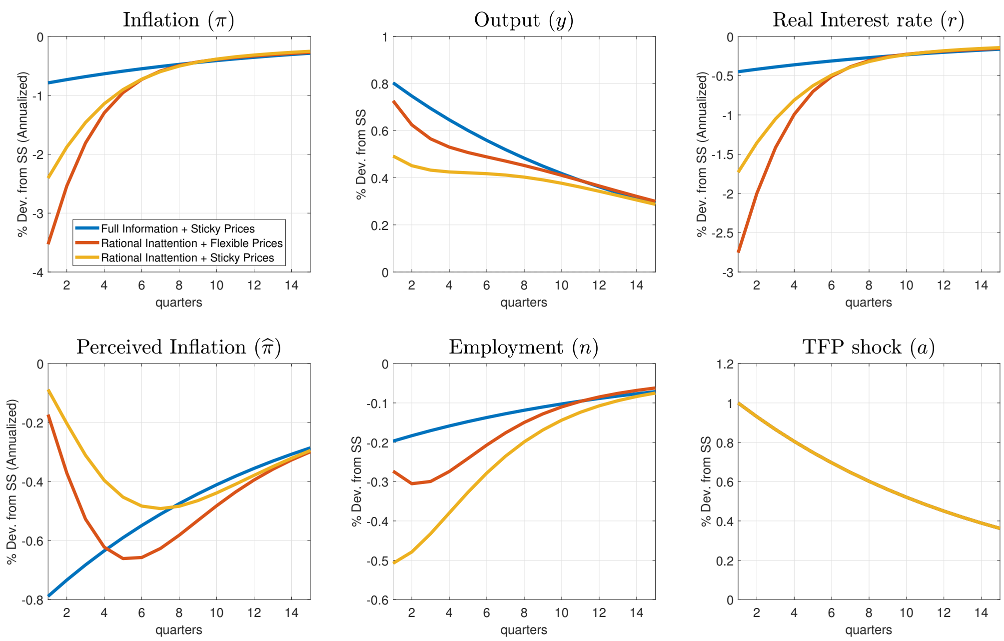


Figure 7: Dynamic Responses to a Technology Shock

Notes: The figure plots the impulse responses to a one standard deviation expansionary shock in TFP under the baseline calibration. The blue line shows the response when price stickiness is the only friction present. The red line shows the corresponding response when *learning by shopping* is the only friction present. The yellow line shows the response when both frictions are present.

Following Maćkowiak and Wiederholt (2015) and Afrouzi and Yang (2021), I lower the coefficient of ϕ_π from 1.5 to a value close to β^{-1}, and compare the IRF's and the second moments of inflation and output with those observed in the pre-Volcker period. This allows us to test some of the theoretical predictions from the last sections and see if they can match the U.S. experience from the last decades. I will also consider two alternative counterfactuals that will disentangle the effect of having endogenous information acquisition in the model. Table 3 shows the second moments that results of this exercise, and Figures 8 and 9 show the IRFs in response to an aggregate demand and technology shock in each scenario.

Column (2) shows that the calibrated model predicts an increase in volatility and persistence of inflation after the policy change. This prediction is consistent with the higher volatility and persistence in Core CPI inflation observed during the pre-Volcker era, as shown in Column (3). Such a policy leads to an unanchoring of households' inflation perceptions, but its magnitude is larger than what is suggested by the estimates from Table 1. Column (2) shows that the model also predicts that the share of volatility of GDP explained by aggregate demand shocks decreases under a more dovish policy. This is a result of the amplification of demand shocks produced by inattention to inflation.[35]

To gain further insight on the impact of having incomplete information in the model, Column (4) shows the corresponding moments when only nominal rigidities are present. The results show that

[35]This observation may seem at odds with the evidence of a lower contribution of demand shocks to fluctuations in output by Galí and Gambetti (2009). One possible explanation is that the change in policy considered here is larger than the change that actually took place during these periods, as shown by the "overshooting" of the inflation anchor, and that the sample used for this calibration includes an additional decade of observations after the Great Recession.

Table 3: Moments Implied by the Model Under Different Calibrations

	Full Sample	Pre-Volcker ($\phi_\pi = 1$)		Post-Volcker ($\phi_\pi = 1.5$)			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
		Endog.		Full.	Exog.	Endog.	
Moment	Data	Info.	Data	Info.	Info.	Info.	Data
$\text{SD}\,(\pi_t)$	0.65	0.83	0.88	0.12	0.18	0.24	0.24
$\text{Corr}\,(\pi_t, \pi_{t-1})$	0.87	0.93	0.85	0.92	0.77	0.79	0.79
ψ_π	0.83	0.23	0.46	0	0.57	0.85	0.85
$\text{SD}\,[y_t \mid z_t]\,/\text{SD}\,[y_t]$	-	0.62	0.76	0.09	0.49	0.70	0.70
$\text{SD}\,[y_t \mid z_t]^{Post}\,/\text{SD}\,[y_t \mid z_t]^{Pre}$			-	0.15	0.81	1.23	0.59
$\text{SD}\,[y_t \mid a_t]^{Post}\,/\text{SD}\,[y_t \mid a_t]^{Pre}$			-	1.27	1.14	0.98	0.83

Notes: The table presents moments of the data and simulated series from the model under four counterfactual scenarios. Column (1) displays the moments of the data for the full sample. Column (2) and (3) show the moments implied by a more dovish monetary policy and compares them with the moments in the data for the Pre-Volcker era. Column (4) shows the corresponding moments when housheolds have full information about inflation. Column (5) shows the moments implied by the model when information is exogenos and fixed to its value in the Pre-volcker era. Column (6) shows the moments implied by the baselina calibration, and Column (7) shows the corresponding moments for the Post-Volcker era.

a model without LBS has a hard time rationalizing the fall in persistence of inflation observed after an increase in ϕ_π. It also predicts a strong reduction in the volatility of inflation and the contribution of demand shocks that goes beyond what is observed in the data.

To highlight the importance of taking into account the endogenous response of households to changes in policy, consider an scenario where the value of σ_ϵ^2 is fixed to the value implied by the counterfactual exercise of column (2). We can interpret this scenario as an experiment where a policy maker in the pre-Volcker era tries to predict the effects of an increase in ϕ_π. Column (5) shows that the policy maker using a model with exogenous information would correctly predict the fall in the volatility and persistence of inflation, as well as part of the anchoring of beliefs. Moreover, the exercise would predict that the change in policy would result in a response of output to demand shocks that is larger but short-lived, as shown in Figure 8.

But this model would give an incomplete picture of the effects of the policy. The success in reducing the volatility of inflation lowers the incentives to learn about inflation. Households rationally choose to ignore inflation even more after the change in policy, producing a second round of re-anchoring of their beliefs. As shown in Figure 8, this re-anchoring amplifies the persistence in output from demand shocks and mitigates even further the impact of technology shocks.

This exercise suggests that inattention to inflation is a sign of success by the central bank on its mission of stabilizing inflation. However, it also suggests that this success has unintended consequences: The anchoring of household inflation expectations exacerbates the effect of information frictions in the economy. As a result, the impact of technology shocks on output is mitigated, further enhancing the role of demand shocks in driving the business cycle.

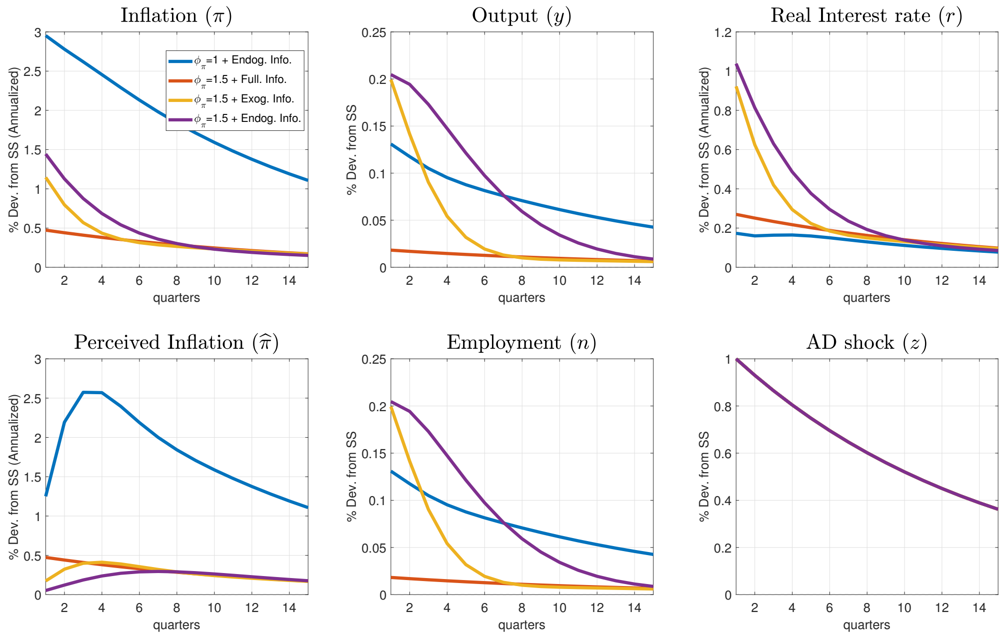


Figure 8: Dynamic Response to an Aggregate Demand Shock under Different Policy Scenarios

Notes: The figure plots the impulse responses to a one standard deviation expansionary shock in aggregate demand under different scenarios. The blue line shows the response when $\phi_\pi = 1.0$ and all other parameters remain as in the baseline calibration. The red line shows the response when $\phi_\pi = 1.5$ and price stickiness is the only friction present. The yellow line shows the response when $\phi_\pi = 1.5$ and both price-stickiness and LBS are present, but information is exogenous, and attention is fixed to its value of the first scenario. The purple line shows the corresponding response when $\phi_\pi = 1.5$, both frictions are present, and agents choose the attention to inflation.

6 Concluding Remarks

Since Lucas (1973), a large part of the business cycle literature has viewed informational frictions as a substitute to nominal price rigidities. The information friction introduced in this paper challenges that view. Learning by shopping propagates and amplifies demand shocks by itself, but it can also coexist with nominal rigidities in price-setting. Moreover, the interaction of both frictions gives rise to business cycles dominated by exogenous shifts in aggregate demand. The results in this paper also suggest that central banks can indirectly affect the strength of this information friction and, through this channel, the supply side of the economy. Consequently, monitoring households' beliefs about current and future inflation can be crucial to stabilizing economic activity.

To conclude, let me suggest future research avenues. The results of this paper offer new insights into the role of monetary policy. They show that stabilizing inflation can also alter the transmission of technology and non-technology shocks into the economy. Several questions automatically follow: What is the optimal monetary policy in this environment? Should central banks target some measure of households beliefs? Are policies designed to inform the general audience about the inflation rate desirable?

A second interesting avenue of future work is to study the impact of oil shocks when consumers learn by shopping. As argued forcefully by Coibion and Gorodnichenko (2015b), energy prices are the main driver of fluctuations in households' inflation expectations in the short term. Moreover, Blanchard and Galí (2007) show that the impact of oil shocks on economic activity has decreased

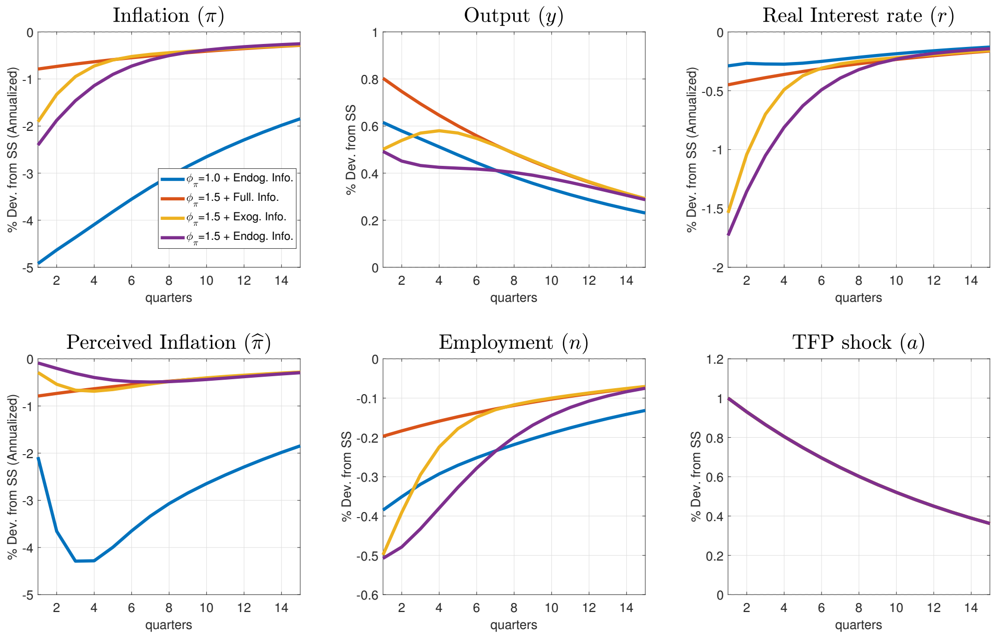


Figure 9: Dynamic Response to a Technology Shock under Different Policy Scenarios

Notes: The figure plots the impulse responses to a one standard deviation expansionary shock in TFP under different scenarios. The blue line shows the response when $\phi_\pi = 1.0$ and all other parameters remain as in the baseline calibration. The red line shows the response when $\phi_\pi = 1.5$ and price stickiness is the only friction present. The yellow line shows the response when $\phi_\pi = 1.5$ and both price stickiness and *learning by shopping* are present, but information is exogenous, and attention is fixed to its value of the first scenario. The purple line shows the corresponding response when $\phi_\pi = 1.5$, both frictions are present, and agents choose the attention to inflation.

over time. Can these shocks produce exogenous differences in households and firms' perceptions that feedback in the inflation rate? Has better monetary policy contributed to reducing the impact of oil shocks by anchoring inflation expectations? The framework presented in this paper provides a starting point to answer these questions.

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A First Order Conditions of Households' Problem

In this appendix I provide a detailed derivation of the first order conditions characterizing household's i problem. Without loss of generality, I assume $Z_{i,t} = 1$ and $D_{i,t} = 0$ in the following derivations, and drop the subscript i to keep the notation simple.

Problem Description

For completeness, I state again the problem faced by each household. The problem of household i in period t is to choose consumption of each variety, $C_{j,t}$, and employment, N_t, to maximize:

$$\mathrm{E}_t \sum_{k=0}^{\infty} \beta^k \left\{ \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right\}, \tag{A.58}$$

where C_t is a consumption index of the form

$$C_t = \left[\int_0^1 C_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{A.59}$$

with $P_{j,t}$ denoting the price of variety j and $P_t \equiv \left(\int_0^1 P_{j,t}^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}}$ denoting the price index associated to this consumption basket. Maximization of (A.58) is subject to the following budget constraint:

$$M_t + B_t = W_t N_t + R_{t-1} B_{t-1}, \tag{A.60}$$

where $M_t \equiv \int_0^1 P_{j,t} C_{j,t} dj$ denotes the household's total expenditures. The information set of the household in period t includes the nominal wage and interest rate W_t and R_t, as well as the expenditures from the previous period M_{t-1} and the initial bond holdings B_{t-1}. It also includes a set of noisy signals that allows the household to observe perfectly the relative price of each consumption variety, $P_{j,t}^R$, but not the aggregate price level P_t.

As described in the main text, the household chooses consumption and employment in advance using noisy signals about prices, and adjusts it's bond holdings at the end of the period to make sure A.60 binds. We can thus solve the problem of the household in two stages. In the first stage, the household chooses the consumption level $C_{j,t}$ that minimize expected expenditures, for a given level of consumption C_t. In the second stage, the household chooses C_t and N_t to maximize (A.58), conditional on it's information set. At the end of the period, the household adjusts the B_t to make sure it's budget constraint (A.60) binds.

Consumption varieties. The expenditure minimization problem of each household in any period t can be written as

$$\min_{C_{j,t}} \quad \mathrm{E}_t \left[P_t \int_0^1 P_{j,t}^R C_{j,t} d \right], \qquad \text{s.t.} \quad C_t = \left[\int_0^1 C_{j,t}^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

The first order condition of this problem yields:

$$\mathrm{E}_t \left[P_t P_{j,t}^R - \widetilde{\Lambda}_t \left(C_t / C_{j,t} \right)^{\frac{1}{\varepsilon}} \right] = 0,$$

where $\widetilde{\Lambda}_t$ is the Lagrange multiplier associated to this problem. Denote $\widehat{P}_t \equiv \mathrm{E}_t P_t$ as the belief of the household about the price level conditional on its own information set . Using the fact that $P_{j,t}^R$ is part of this information set, we can rewrite the first order condition of the household as:

$$C_{j,t} = \left(\widehat{P}_t P_{j,t}^R / \widetilde{\Lambda}_t \right)^{-\varepsilon} C_t. \tag{A.61}$$

Using this condition to replace $C_{j,t}$ in (A.59), and using the fact that $\int_0^1 \left(P_{j,t}^R \right)^{1-\varepsilon} dj = 1$, we can show that $\widetilde{\Lambda}_t = \widehat{P}_t$. Using this expression to replace $\widetilde{\Lambda}_t$ back in (A.61), we can express the optimal consumption of each variety as

$$C_{j,t} = \left(P_{j,t}^R \right)^{-\varepsilon} C_t \tag{A.62}$$

Conditional on this behavior, we can express the total expenditures M_t in the budget constraint (A.60)

$$\int_0^1 P_{j,t} C_{j,t} dj = P_t C_t. \tag{A.63}$$

Consumption and labor supply Using (A.63), we can rewrite the budget constraint (A.60) as

$$P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1}. \tag{A.64}$$

We can use the previous expression to rewrite the problem of the household in recursive form:

$$\begin{aligned} \nu\left(B_{t-1}\right) = \max_{C_t, N_t} & \quad \left\{U\left(C_t, N_t\right) + \beta \mathrm{E}_t\left[\nu\left(B_t\right)\right]\right\} \\ \text{s.t.} & \quad P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1} \end{aligned}$$

The first order conditions of this problem yield:

$$C_t^{-\sigma} = \mathrm{E}_t\left[P_t \Lambda_t\right], \tag{A.65}$$

$$N_t^{\varphi} = \mathrm{E}_t\left[W_t \Lambda_t\right], \tag{A.66}$$

$$0 = \mathrm{E}_t\left[\beta \nu'\left(B_t\right) - \Lambda_t\right], \tag{A.67}$$

where Λ_t is the Lagrange multiplier associated to this problem. Now, define $\widehat{P}_t \equiv \mathrm{E}_t P_t$. We can combine (A.65) and (A.66) as

$$N_t^{\varphi} C_t^{\sigma} = \mathcal{D}_t^w \frac{W_t}{\widehat{P}_t} \tag{A.68}$$

with

$$\mathcal{D}_t^w \equiv \frac{\mathrm{E}_t\left[\widehat{P}_t \Lambda_t\right]}{\mathrm{E}_t\left[P_t \Lambda_t\right]}.$$

Note that, up to a first order approximation, $\log \mathcal{D}_t^w \approx 0$. We can thus take logs of (A.68) and subtract the corresponding expression evaluated at the non-stochastic steady-state to get :

$$\varphi n_t + \sigma c_t = w_t - \widehat{p}_t, \tag{A.69}$$

with $\widehat{p}_t \equiv \mathrm{E}_t p_t$. This corresponds to the individual labor supply obtained in the main text. Now, the envelope condition of the household's problem yields:

$$\nu'\left(B_{t-1}\right) = \beta \mathrm{E}_t\left[\nu'\left(B_t\right)\right] R_{t-1}, \tag{A.70}$$

where I have used the fact that Q_t is part of the household's information set. Using (A.66) and (A.67), we can rewrite (A.70) as

$$\nu'\left(B_{t-1}\right) = \left(\frac{N_t^{\varphi}}{W_t}\right) R_{t-1}. \tag{A.71}$$

Using the previous expression in (A.67) yields:

$$R_t \frac{N_t^{\varphi}}{W_t} = \beta \mathrm{E}_t\left[\left(\frac{N_{t+1}^{\varphi}}{W_{t+1}}\right)\right]$$

Finally, using (A.68), we can rewrite the previous expression as:

$$R_t^{-1} = \beta \mathrm{E}_t\left[\frac{\mathcal{D}_{t+1}^w}{\mathcal{D}_t^w}\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{\widehat{P}_t}{\widehat{P}_{t+1}}\right] \tag{A.72}$$

Taking a log-linear approximation of the previous expression yields

$$c_t = \mathrm{E}_t c_{t+1} - \frac{1}{\sigma}\left(i_t - \left(\widehat{p}_{t+1} - \widehat{p}_t\right)\right), \tag{A.73}$$

which corresponds to the Euler equation of each household in the main text.

B Proofs

This appendix contains the proofs of the propositions in the main text.

B.1 Proof of Proposition 1

Follows directly from the derivations in the main text.

B.2 Proof of Proposition 2

I start by deriving an expression characterizing individual consumption as a beauty contest from the households' budget constraint and first order conditions. I then characterize the aggregate demand of this economy as a function of the information wedges defined in the proposition. I conclude by characterizing the information wedges as a function of misperception of the price level and the inflation rate[36]. In what follows, I assume assume that both aggregate demand and TFP shocks are present as in Section 4 and 5.

Consumption as a beauty contest

Define $r^Z_{i,t+1} \equiv r_{i,t+1} + z_{i,t+1} - z_{i,t}$, where $r_{i,t+1} \equiv i_{i,t} - \pi_{t+1}$ is the real interest rate. The labor supply and Euler equation of household i (18) and (17) can be expressed as:

$$w_{i,t} - \mathrm{E}_{i,t} p_t = \sigma c_{i,t} + \varphi n_{i,t}, \tag{B.74}$$

$$c_{i,t} = \mathrm{E}_{i,t} c_{i,t+1} - \frac{1}{\sigma}\mathrm{E}_{i,t} r^Z_{i,t+1}. \tag{B.75}$$

The aggregate labor supply can be expressed as

$$w_t - \overline{\mathrm{E}}_t p_t = (\varphi + \sigma)\, c_t - \varphi a_t, \tag{B.76}$$

where $\overline{\mathrm{E}}_t p_t \equiv \int_0^1 \mathrm{E}_{i,t} p_t di$. Log-linearizing the end-of period budget constraint (5) gives

$$c_{i,t} + b^R_{i,t} = \beta^{-1} b^R_{i,t-1} + \omega_W \left(w^R_{i,t} + n_{i,t}\right) + \omega_D d^R_{i,t}, \tag{B.77}$$

where the superscript R denotes the variable deflated by the price level p_t. The constants $\omega_W = \frac{WN}{PC}$ and $\omega_D \equiv \frac{D}{PC}$ denote steady-state ratios[37]. Define $\nu^p_{i,t} \equiv p_t - \mathrm{E}_{i,t} p_t$. Using (B.74), we can rewrite (B.77) as

$$\left(1 + \frac{\sigma}{\varphi}\omega_W\right) c_{i,t} + b^R_{i,t} = \beta^{-1} b^R_{i,t-1} + e^R_{i,t},$$

with

$$e^R_{i,t} \equiv \omega_W \left(1 + \frac{1}{\varphi}\right) w^R_{i,t} + \frac{\omega_W}{\varphi} \nu^p_{i,t|t} + \omega_D d^R_{i,t}. \tag{B.78}$$

[36] All variables in lower case denote log-deviations from steady-state, except for the price level $p_t \equiv \log P_t$, the nominal interest rate $i_t = \log R_t + \log \beta$, and the bond holdings, which are written as $b_{i,t} = B_{i,t}/C$ and $b^R_{i,t} = B_{i,t}/\left(P_t C\right)$, where C denotes the steady-state level of consumption. This redefinition takes care of the issue that $B = 0$ in the non-stochastic steady-state, and is standard in the literature (see, for instance, Woodford (2011), Angeletos and Lian (2018), and Angeletos and Lian (2021)).

[37] Notice that $\omega_W = \mathcal{M}^{-1}$, and $\omega_W + \omega_D = 1$.

Solving for $b^R_{i,t-1}$, iterating forward, using the transversality condition, and taking expectations yields

$$b^R_{i,t-1} + \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} e^R_{i,t+k} = \left(\frac{\varphi + \sigma \omega_W}{\varphi} \right) \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} c_{i,t+k}. \tag{B.79}$$

The next step is to use the Euler equation of the household to rewrite (B.79). Iterating (B.75) forward and using the fact that the law of iterated expectations holds, conditional on the household information set, we have

$$c_{i,t} = -\frac{1}{\sigma} \sum_{h=0}^{\infty} \mathrm{E}_{i,t} r^Z_{i,t+h+1}. \tag{B.80}$$

Multiplying this equation by β^k in different periods and adding the respective equations yields:

$$\sum_{k=0}^{\infty} \beta^k E_{i,t} c_{i,t+k} = -\frac{1}{\sigma} \mathrm{E}_{i,t} \left[\sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \beta^k r^Z_{i,t+h+k+1} \right]. \tag{B.81}$$

Now, notice that

$$\begin{aligned}
\sum_{k=0}^{\infty} \sum_{h=0}^{\infty} \beta^k r^Z_{i,t+h+k+1} &= \left(\frac{1-\beta}{1-\beta} \right) r^Z_{i,t+1} + \frac{1-\beta^2}{1-\beta} r^Z_{i,t+2} + \frac{1-\beta^3}{1-\beta} r^Z_{i,t+3} + \\
&= \sum_{k=0}^{\infty} \left(\frac{1-\beta^{k+1}}{1-\beta} \right) r^Z_{i,t+k+1} \\
&= \frac{1}{1-\beta} \left(\sum_{k=0}^{\infty} r^Z_{i,t+k+1} - \beta \sum_{k=0}^{\infty} \beta^k r^Z_{i,t+k+1} \right).
\end{aligned}$$

Consequently, we can use the previous expression back in (B.81) and use (B.80) to get

$$\begin{aligned}
\sum_{h=0}^{\infty} \beta^h E_{i,t} c_{i,t+h} &= -\frac{1}{\sigma} \left(\frac{1}{1-\beta} \right) \left(\sum_{k=0}^{\infty} \mathrm{E}_{i,t} r^Z_{i,t+k+1} - \beta \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right) \\
&= \left(\frac{1}{1-\beta} \right) \left(\left\{ -\frac{1}{\sigma} \sum_{k=0}^{\infty} \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right\} + \frac{1}{\sigma} \beta \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right) \\
&= \left(\frac{1}{1-\beta} \right) \left(c_{i,t} + \frac{1}{\sigma} \beta \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right).
\end{aligned} \tag{B.82}$$

Plugging this expression in (B.79) and solving for $c_{i,t}$, we get:

$$c_{i,t} = -\frac{1}{\sigma} \beta \left\{ \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right\} + (1-\beta) \left(\frac{\varphi}{\varphi + \sigma \omega_W} \right) \sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} e^R_{i,t+k} + (1-\beta) \left(\frac{\varphi}{\varphi + \sigma \omega_W} \right) b^R_{i,t-1} \tag{B.83}$$

Integrating this expression across households and using the market clearing condition for bonds yields:

$$c_t = -\frac{1}{\sigma} \beta \int_0^1 \left[\sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} r^Z_{i,t+k+1} \right] di + (1-\beta) \left(\frac{\varphi}{\varphi + \sigma \omega_W} \right) \int_0^1 \left[\sum_{k=0}^{\infty} \beta^k \mathrm{E}_{i,t} e^R_{i,t+k} \right] di. \tag{B.84}$$

The next step is to express the second term in brackets as a function of aggregate consumption. To do so, start by observing that, up to a first-order approximation, the (real) dividends of each firm are given by:

$$d^R_{j,t} = y_{j,t} + \left(\frac{1}{1-\omega_W} \right) p^R_{j,t} - \left(\frac{\omega_W}{1-\omega_W} \right) \left(w^R_{j,t} - a_t \right).$$

Integrating across firms and using the market clearing condition in the goods market yields:

$$d^R_{i,t} \equiv \int d^R_{i,j,t} dj = c_t - \left(\frac{\omega_W}{1-\omega_W} \right) \left(w^R_t - a_t \right).$$

Replacing this expression in (B.78), we have:

$$
\begin{aligned}
e^R_{i,t+k} =& \omega_W\left(1+\frac{1}{\varphi}\right)w^R_{i,t+k}+\frac{\omega_W}{\varphi}\nu^p_{i,t+k|t+k}+(1-\omega_W)\,d^R_{i,t+k} \\
=& \frac{\omega_W}{\varphi}\left(w^R_{i,t+k}+\nu^p_{i,t+k|t+k}\right)+(1-\omega_W)\,c_{t+k}+\omega_W\left(w^R_{i,t+k}-w^R_{t+k}\right)+\omega_W a_{t+k} \\
=& \frac{\omega_W}{\varphi}\left(w_{i,t+k}-\widehat{p}_{i,t+k|t+k}\right)+(1-\omega_W)\,c_{t+k}+\omega_W\left(w^R_{i,t+k}-w^R_{t+k}\right)+\omega_W a_{t+k}.
\end{aligned}
\tag{B.85}
$$

Start by considering $k \geq 1$. Households understand that their differences in nominal wages and dividends are unpredictable. They hold rational expectations and can use (B.85) and the aggregate labor supply (B.74) to get:

$$
\begin{aligned}
\mathrm{E}_{i,t}e^R_{i,t+k} &= \frac{\omega_W}{\varphi}\mathrm{E}_{i,t}\left[w_{t+k}-\overline{\mathrm{E}}_t p_{t+k}\right]+(1-\omega_W)\,\mathrm{E}_{i,t}c_{t+k}+\omega_W\mathrm{E}_{i,t}a_{t+k} \\
&= \frac{\omega_W}{\varphi}\mathrm{E}_{i,t}\left[(\varphi+\sigma)\,c_{t+k}-\varphi a_{t+k}\right]+(1-\omega_W)\,\mathrm{E}_{i,t}c_{t+k}+\omega_W\mathrm{E}_{i,t}a_{t+k} \\
&= \left(\frac{\omega_W}{\varphi}\,(\varphi+\sigma)+(1-\omega_W)\right)\mathrm{E}_{i,t}\left[c_{t+k}\right]-\omega_W\mathrm{E}_{i,t}a_{t+k}+\omega_W\mathrm{E}_{i,t}a_{t+k} \\
&= \left(\frac{\varphi+\sigma\omega_W}{\varphi}\right)\mathrm{E}_{i,t}c_{t+k}.
\end{aligned}
\tag{B.86}
$$

Following similar steps, it is easy to show that:

$$
\int_0^1 e^R_{i,t}di = \left(\frac{\varphi+\sigma\omega_W}{\varphi}\right)c_t \tag{B.87}
$$

Plugging these results back into (B.84), we get

$$
c_t = -\frac{1}{\sigma}\beta\sum_{k=0}^{\infty}\beta^k\int_0^1 \mathrm{E}_{i,t}r^Z_{i,t+k+1}di+(1-\beta)\left[\sum_{k=0}^{\infty}\beta^k\int_0^1\mathrm{E}_{i,t}c_{t+k}di\right]. \tag{B.88}
$$

This equation characterizes aggregate consumption as a beauty contest, in the spirit of Angeletos and Lian (2018).

Aggregate demand as a function of information wedges

Start by writing the first term in (B.88) as

$$
\begin{aligned}
\sum_{k=0}^{\infty}\beta^k\int_0^1\mathrm{E}_{i,t}r^Z_{i,t+k+1}di &= \sum_{k=0}^{\infty}\beta^k\int_0^1\mathrm{E}_{i,t}\left\{r_{i,t+k+1}+z_{i,t+k+1}-z_{i,t+k}\right\}di \\
&= \sum_{k=0}^{\infty}\beta^k\int_0^1\mathrm{E}_{i,t}r_{i,t+k+1}di+\sum_{k=0}^{\infty}\beta^k\int_0^1\mathrm{E}_{i,t}\left\{z_{i,t+k+1}-z_{i,t+k}\right\}di \\
&= \sum_{k=0}^{\infty}\beta^k\left\{\int_0^1\mathrm{E}_{i,t}r_{i,t+k+1}-\mathbb{E}_t r_{t+k+1}\right\}di+\sum_{k=0}^{\infty}\beta^k\mathbb{E}_t r^Z_{t+k+1},
\end{aligned}
$$

where $\mathbb{E}_t$ is the full information operator. Now, rewrite the second term in (B.88) as

$$
\begin{aligned}
\sum_{k=1}^{\infty}\beta^k\int_0^1\mathrm{E}_{i,t}c_{t+k}di &= \sum_{k=1}^{\infty}\beta^k\int_0^1\mathrm{E}_{i,t}c_{t+k}di-\sum_{k=1}^{\infty}\beta^k\mathbb{E}_t c_{t+k}+\sum_{k=1}^{\infty}\beta^k\mathbb{E}_t c_{t+k} \\
&= \sum_{k=1}^{\infty}\beta^k\int_0^1\left\{\mathrm{E}_{i,t}c_{t+k}-\mathbb{E}_t c_{t+k}\right\}di+\sum_{k=1}^{\infty}\beta^k\mathbb{E}_t c_{t+k}.
\end{aligned}
$$

Using the previous expressions in equation (B.88) and taking expectations, we get:

$$c_t = -\frac{1}{\sigma}\beta \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t r^Z_{t+k+1} + (1-\beta) \sum_{k=0}^{\infty} \beta^k \mathbb{E}_t c_{t+k} + \beta \mathcal{X}_t, \tag{B.89}$$

where

$$\begin{aligned}
\mathcal{X}_t &\equiv \mathcal{H}_t + \mathcal{R}_t, \\
\mathcal{H}_t &\equiv \left(\frac{1-\beta}{\beta}\right) \sum_{k=0}^{\infty} \beta^k \int_0^1 \left\{\mathrm{E}_{i,t} c_{t+k} - \mathbb{E}_t c_{t+k}\right\} di, \\
\mathcal{R}_t &\equiv -\frac{1}{\sigma} \sum_{k=0}^{\infty} \beta^k \int_0^1 \left\{\mathrm{E}_{i,t} r_{i,t+k} - \mathbb{E}_t r_{t+k}\right\} di.
\end{aligned}$$

All that is left is to write this expression in recursive form. To do so, start by taking out c_t from the RHS of (B.89) and solve c_t to get

$$c_t = -\frac{1}{\sigma} \sum_{k=1}^{\infty} \beta^{k-1} \mathbb{E}_t r^Z_{t+k} + \left(\frac{1-\beta}{\beta}\right) \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t c_{t+k} + \mathcal{X}_t. \tag{B.90}$$

Writing this equation in $t+1$ and taking expectations in t yields

$$\mathbb{E}_t c_{t+1} = -\frac{1}{\sigma} \sum_{k=1}^{\infty} \beta^{k-1} \mathbb{E}_t r^Z_{t+k+1} + \left(\frac{1-\beta}{\beta}\right) \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t c_{t+k+1} + \mathbb{E}_t \mathcal{X}_{t+1}.$$

Using this expression back in (B.90), we get

$$\begin{aligned}
c_t &= -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + (1-\beta)\,\mathbb{E}_t c_{t+1} + \left\{-\frac{1}{\sigma} \sum_{k=2}^{\infty} \beta^{k-1} \mathbb{E}_t r^Z_{t+k} + \left(\frac{1-\beta}{\beta}\right) \sum_{k=2}^{\infty} \beta^k \mathbb{E}_t c_{t+k}\right\} + \mathcal{X}_t \\
&= -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + (1-\beta)\,\mathbb{E}_t c_{t+1} + \beta\left\{-\frac{1}{\sigma} \sum_{k=1}^{\infty} \beta^{k-1} \mathbb{E}_t r^Z_{t+k+1} + \left(\frac{1-\beta}{\beta}\right) \sum_{k=1}^{\infty} \beta^k \mathbb{E}_t c_{t+k+1}\right\} + \mathcal{X}_t \\
&= -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + (1-\beta)\,\mathbb{E}_t c_{t+1} + \beta\left\{\mathbb{E}_t c_{t+1} - \mathbb{E}_t \mathcal{X}_{t+1}\right\} + \mathcal{X}_t \\
&= -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + \mathbb{E}_t c_{t+1} + \mathcal{X}_t - \beta \mathbb{E}_t \mathcal{X}_{t+1}.
\end{aligned}$$

Finally, replacing c_t by y_t using market clearing, we get

$$y_t = -\frac{1}{\sigma}\mathbb{E}_t r^Z_{t+1} + \mathbb{E}_t y_{t+1} + \mathcal{X}_t - \beta \mathbb{E}_t \mathcal{X}_{t+1}. \tag{B.91}$$

Information wedges as a function of price perceptions

Start by considering the information wedge $\mathcal{H}_t$. Using the market clearing condition $y_t = c_t$, we can express it as

$$\mathcal{H}_t \equiv \left(\frac{1-\beta}{\beta}\right)\left(\int_0^1 \left\{\mathrm{E}_{i,t} y_t - y_t\right\} di + \sum_{k=1}^{\infty} \beta^k \int_0^1 \left\{\mathrm{E}_{i,t} y_{t+k} - \mathbb{E}_t y_{t+k}\right\} di\right).$$

Notice that equation (B.87) implies:

$$\mathrm{E}_{i,t} y_t = \left(\frac{\varphi}{\varphi + \sigma\omega_W}\right) \mathrm{E}_{i,t} e^R_{i,t}.$$

Now, let $e_{i,t} = e^R_{i,t} + p_t$ denote the nominal part of households income from labor and dividends, which is part of households information set $\mathcal{I}_{i,t}$. It follows that:

$$\int_0^1 \left\{\mathrm{E}_{i,t} y_t - y_t\right\} di = \left(\frac{\varphi}{\varphi + \sigma\omega_W}\right)\left(\int_0^1 \mathrm{E}_{i,t} e_{i,t} di - \int_0^1 \mathrm{E}_{i,t} p_t di - e_t + p_t\right) = \left(\frac{\varphi}{\varphi + \sigma\omega_W}\right) v^p_t, \tag{B.92}$$

with $\nu_t^p \equiv p_t - \overline{\mathrm{E}}_t p_t$. Using this result, we can rewrite the information wedge $\mathcal{H}_t$ as:

$$\mathcal{H}_t = \chi \nu_t^p - \left(\frac{1-\beta}{\beta}\right) \mathbb{E}_t \sum_{k=1}^{\infty} \beta^k \nu_{t+k|t}^y$$

where $\nu_{t+k,t}^y \equiv y_{t+k} - \overline{\mathrm{E}}_t y_{t+k}$ and

$$\chi \equiv \left(\frac{1-\beta}{\beta}\right)\left(\frac{\varphi}{\varphi + \sigma\omega_W}\right) = \left(\frac{1-\beta}{\beta}\right)\left(\frac{\mathcal{M}\varphi}{\mathcal{M}\varphi + \sigma}\right).$$

Next, consider the the wedge on the real interest rate $\mathcal{R}_t$. We have

$$\begin{aligned}\int_0^1 \left\{\mathrm{E}_{i,t} r_{i,t+k+1} - \mathbb{E}_t r_{t+k+1}\right\} di &= \int_0^1 \left\{\mathrm{E}_{i,t}\left\{i_{i,t+k} - \pi_{t+k+1}\right\} - \mathbb{E}_t\left\{i_{t+k} - \pi_{t+k+1}\right\}\right\} di \\ &= \int_0^1 \mathrm{E}_{i,t} i_{i,t+k} di - \int_0^1 \mathrm{E}_{i,t}\pi_{t+k+1} di - \mathbb{E}_t i_{t+k} + \mathbb{E}_t \pi_{t+k+1}.\end{aligned}$$

For $k = 0$, we can use the observation that the interest rate faced by each household is part of their information set to get:

$$\int_0^1 \left\{\mathrm{E}_{i,t} r_{i,t} - \mathbb{E}_t r_{t+k+1}\right\} di = -\mathrm{E}_{i,t}\pi_{t+1} + \mathbb{E}_t \pi_{t+1} = \mathbb{E}_t \nu_{t+1|t}^{\pi}.$$

For $k > 0$, we can use the monetary policy rule, which is common knowledge across households, to get

$$\begin{aligned}\int_0^1 \left\{\mathrm{E}_{i,t} r_{i,t+k+1} - \mathbb{E}_t r_{t+k+1}\right\} di &= \phi_\pi\left(\mathrm{E}_{i,t}\pi_{t+k} - \pi_{t+k}\right) - \left(\mathrm{E}_{i,t}\pi_{t+k+1} - \mathbb{E}_t \pi_{t+k+1}\right) \\ &= \mathbb{E}_t\left[\nu_{t+k+1|t}^{\pi} - \phi_\pi \nu_{t+k|t}^{\pi}\right]. \end{aligned} \tag{B.93}$$

We can thus express the information wedge $\mathcal{R}_t$ as:

$$\mathcal{R}_t = -\frac{1}{\sigma}\mathbb{E}_t\left[\nu_{t+1|t}^{\pi} + \sum_{k=1}^{\infty}\beta^k\left\{\nu_{t+k+1|t}^{\pi} - \phi_\pi \nu_{t+k|t}^{\pi}\right\}\right].$$

This concludes the proof of the proposition.

B.3 Proof of Proposition 3

To begin, notice that the equilibrium process for inflation (30) and the observation that households have common knowledge about π_{t-1} and η_{t-1} implies that

$$\mathrm{Var}\left[\pi_t | \pi_{t-1}, \eta_{t-1}\right] = \left(\theta_0^{\pi}\right)^2 \sigma_{AD}^2,$$

which in turn implies that $q_\pi = q\left(\theta_0^{\pi}\right)^2$. Consequently, we can use (14) to express ψ_π as

$$\psi_\pi = \frac{1}{q\left(\theta_0^{\pi}\right)^2 + 1}$$

Reorganizing terms, we get the following expression that defines implicitly the equilibrium value of ψ_π:

$$1 - \psi_\pi = q\left(\theta_0^{\pi}\right)^2 \psi_\pi.$$

Now, notice that the LHS of this equation is decreasing in ψ_π, is equal to 1 when $\psi_\pi = 0$ and equal to 0 when $\psi_\pi = 1$. The RHS is equal to 0 when $\psi_\pi = 0$ and equal to some positive constant when $\psi_\pi = 1$. Continuity of the RHS guarantees the existence of a solution of this equation. To prove its uniqueness, it is sufficient to show that the RHS is always increasing in ψ_π. To do so, start by observing that

$$\frac{\partial RHS}{\partial \psi_\pi} = q\left(\theta_0^2 + 2\theta_0 \frac{\partial \theta_0}{\partial \psi_\pi}\psi_\pi\right) = q\theta_0^2\left(1 + 2\frac{\psi_\pi}{\theta_0}\frac{\partial \theta_0}{\partial \psi_\pi}\right).$$

So the RHS is increasing if the term in parenthesis is positive. Using (34), it follows that:

$$\frac{\psi_\pi}{\theta_0}\frac{\partial\theta_0}{\partial\psi_\pi} = \frac{\psi_\pi\rho_z\left\{(1+\varphi/\sigma)\left(\frac{\beta\phi_\pi-1}{1-\beta\rho_z}\right)-\lambda(1-\beta)\right\}}{\lambda+(1+\varphi/\sigma)\phi_\pi+\psi_\pi\rho_z\left\{(1+\varphi/\sigma)\left(\frac{\beta\phi_\pi-1}{1-\beta\rho_z}\right)-\lambda(1-\beta)\right\}}. \\ - \frac{(1-(\sigma+\varphi)\chi)\psi_\pi}{\lambda+(1+\varphi/\sigma)\phi_\pi+(1-(\sigma+\varphi)\chi)\psi_\pi}$$

Consequently, the equilibrium is unique if

$$1+2\frac{\psi_\pi}{\theta_0}\frac{\partial\theta_0}{\partial\psi_\pi} = \left(\frac{\lambda+(1+\varphi/\sigma)\phi_\pi-(1-(\sigma+\varphi)\chi)\psi_\pi}{\lambda+(1+\varphi/\sigma)\phi_\pi+(1-(\sigma+\varphi)\chi)\psi_\pi}\right) \\ + \left(\frac{2\psi_\pi\rho_z\left\{(1+\varphi/\sigma)\left(\frac{\beta\phi_\pi-1}{1-\beta\rho_z}\right)-\lambda(1-\beta)\right\}}{\lambda+(1+\varphi/\sigma)\phi_\pi+\psi_\pi\rho_z\left\{(1+\varphi/\sigma)\left(\frac{\beta\phi_\pi-1}{1-\beta\rho_z}\right)-\lambda(1-\beta)\right\}}\right) > 0.$$

The condition that $(\sigma+\varphi)\chi<1$ guarantees the first term in parenthesis is always positive. If $\phi_\pi > \beta^{-1}+\left(\frac{\lambda(1-\beta)(\beta^{-1}-\rho_z)}{1+\varphi/\sigma}\right)$, the second term is always positive. It follows that these two conditions are sufficient to guarantee the uniqueness of the equilibrium degree of anchoring ψ_π.

B.4 Proof of Proposition 4

From Proposition 3, we can define the equilibrium level of anchoring implicitly as the root of the following equation:

$$F(\theta,\phi_\pi,\psi_\pi(\theta)) = RHS+\psi_\pi-1,$$

with $RHS \equiv q\left(\theta_0^\pi\right)^2\psi_\pi$. Note that the sign of $\partial\psi_\pi/\partial\theta$ is the same as that of $\partial\psi_\pi/\partial\lambda$, so I can focus on the later. Taking the partial derivative of $F(\cdot)$, with respect to ψ_π and using the results of Proposition 3, we get:

$$\frac{\partial F(\cdot)}{\partial\psi_\pi} = \frac{\partial RHS}{\partial\psi_\pi}+1>0.$$

Now, taking the partial derivative of $F(\cdot)$, with respect to λ and ϕ_π yields

$$\frac{\partial F(\cdot)}{\partial\phi_\pi} = 2q\psi_\pi\theta_0^\pi\left(\frac{\partial\theta_0^\pi}{\partial\lambda}\right), \qquad \frac{\partial F(\cdot)}{\partial\lambda} = 2q\psi_\pi\theta_0^\pi\left(\frac{\partial\theta_0^\pi}{\partial\phi_\pi}\right).$$

Using the Implicit Function Theorem, it follows that:

$$\frac{\partial\psi_\pi}{\partial\lambda} = -\frac{\partial\theta_0^\pi}{\partial\lambda}\left(\frac{2q\psi_\pi\theta_0^\pi}{\partial F(\cdot)/\partial\psi_\pi}\right), \qquad \frac{\partial\psi_\pi}{\partial\phi_\pi} = -\frac{\partial\theta_0^\pi}{\partial\phi_\pi}\left(\frac{2q\psi_\pi\theta_0^\pi}{\partial F(\cdot)/\partial\phi_\pi}\right).$$

It follows that the sign of $\partial\psi_\pi/\partial\lambda$ and $\partial\psi_\pi/\partial\phi_\pi$ is equal to opposite sign of $\partial\theta_0^\pi/\partial\lambda$ and $\partial\theta_0^\pi/\partial\phi_\pi$. To find these derivatives, start by writing rewriting θ_0^π as

$$\theta_0^\pi = W\left(\frac{Z+Y\psi_\pi}{Z+X\psi_\pi}\right)\left(\frac{1}{Z-D\rho_z}\right)$$

with

$$\begin{aligned}
Z &\equiv \lambda + (1+\varphi/\sigma)\,\phi_\pi \\
Y &\equiv \rho_z \left\{ (1+\varphi/\sigma)\left(\frac{\beta\phi_\pi - 1}{1-\beta\rho_z}\right) - \lambda\,(1-\beta) \right\} \\
X &\equiv 1 - \chi\,(\sigma+\varphi) \\
W &\equiv \left(1+\frac{\varphi}{\sigma}\right)(1-\rho_z) \\
D &\equiv \lambda + \lambda\beta\,(1-\rho_z) + \sigma^{-1}\,(\sigma+\varphi)
\end{aligned}$$

Note that the assumptions guaranteeing the uniqueness of the equilibrium also guarantee that all the previous constants are positive. Now, the derivative of θ_0^π w.r.t. ϕ_π is given by:

$$\begin{aligned}
\frac{\partial\theta_0^\pi}{\partial\phi_\pi} &= \left(\frac{W}{(Z+X\psi_\pi)^2\,(Z-D\rho_z)^2}\right) = \\
&\times \left\{ \underbrace{\left(\frac{\partial Z}{\partial\phi_\pi} + \frac{\partial Y}{\partial\phi_\pi}\psi_\pi\right)(Z+X\psi_\pi)(Z-D\rho_z) - (Z+Y\psi_\pi)\left(2Z\frac{\partial Z}{\partial\phi_\pi} + X\psi_\pi\frac{\partial Z}{\partial\phi_\pi} - D\rho_z\frac{\partial Z}{\partial\phi_\pi}\right)}_{AUX_2} \right\}.
\end{aligned}$$

The term inside braces determines the sign of this derivative. This term can be simplified as:

$$\begin{aligned}
AUX_1 = &\left(\frac{\partial Z}{\partial\phi_\pi}\right)\{(Z+X\psi_\pi)(Z-D\rho_z) - (Z+Y\psi_\pi)(Z+X\psi_\pi) - (Z+Y\psi_\pi)(Z-D\rho_z)\} \\
&+ \frac{\partial Y}{\partial\phi_\pi}\psi_\pi\,(Z+X\psi_\pi)(Z-D\rho_z)
\end{aligned}$$

Notice that

$$\frac{\partial Y}{\partial\phi_\pi} = \sigma^{-1}\,(\sigma+\varphi)\left(\frac{\beta\rho_z}{1-\beta\rho_z}\right) = \left(\frac{\partial Z}{\partial\phi_\pi}\right)\left(\frac{\beta\rho_z}{1-\beta\rho_z}\right)$$

So the sign of AUX_2 is equal to the sign of

$$\begin{aligned}
AUX_1' = &(Z+X\psi_\pi)(Z-D\rho_z) - (Z+Y\psi_\pi)(Z+X\psi_\pi) \\
&- (Z+Y\psi_\pi)(Z-D\rho_z) + \left(\frac{\beta\rho_z}{1-\beta\rho_z}\right)\psi_\pi\,(Z+X\psi_\pi)(Z-D\rho_z)
\end{aligned}$$

After some simplifications, we get

$$\begin{aligned}
AUX_1' &= (Z+X\psi_\pi)(Z-D\rho_z)\left(\frac{1-\beta\rho_z\,(1-\psi_\pi)}{1-\beta\rho_z}\right) - (Z+Y\psi_\pi)(Z+X\psi_\pi) - (Z+Y\psi_\pi)(Z-D\rho_z) \\
&< (Z+X\psi_\pi)(Z-D\rho_z) - (Z+Y\psi_\pi)(Z+X\psi_\pi) - (Z+Y\psi_\pi)(Z-D\rho_z) \\
&= -\left(Z^2 + X\psi_\pi D\rho_z + Y\psi_\pi\,((Z+X\psi_\pi) + (Z-D\rho_z))\right) < 0
\end{aligned}$$

It follows that $\partial\theta_0^\pi/\partial\phi_\pi < 0$, so $\partial\psi_\pi/\partial\phi_\pi > 0$.

Now, to find the derivative with respect to λ, rewrite θ_0^π as

$$\theta_0^\pi = \left(\frac{1+\varphi/\sigma}{C+Q}\right)\left(\frac{Z+Y\psi_\pi}{Z+X\psi_\pi}\right)$$

with $C \equiv \lambda\,(1-\beta\rho_z)$ and $Q \equiv (1+\varphi/\sigma)\left(\frac{\phi_\pi-\rho_z}{1-\rho_z}\right)$.

Furthermore:

$$\begin{aligned}
\frac{\partial Z}{\partial \lambda} &= 1 > 0 \\
\frac{\partial Z}{\partial \phi_\pi} &= \sigma^{-1}(\sigma+\varphi) > 0 \\
\frac{\partial Y}{\partial \lambda} &= -\rho_z(1-\beta) < 0 \\
\frac{\partial Y}{\partial \phi_\pi} &= \sigma^{-1}(\sigma+\varphi)\left(\frac{\beta\rho_z}{1-\beta\rho_z}\right) > 0.
\end{aligned}$$

We can thus express the derivative of θ_0^π w.r.t. λ as:

$$\begin{aligned}
\frac{\partial \theta_0^\pi}{\partial \lambda} =& \left(\frac{W}{(Z+X\psi_\pi)^2(Z-D\rho_z)^2}\right) \\
&\times \left\{ \underbrace{\left(\frac{\partial Z}{\partial \lambda}+\frac{\partial Y}{\partial \lambda}\psi_\pi\right)(Z+X\psi_\pi)(C+Q)-(Z+Y\psi_\pi)\left(\left(\frac{\partial C}{\partial \lambda}Z+C\frac{\partial Z}{\partial \lambda}\right)+Q\frac{\partial Z}{\partial \lambda}+X\psi_\pi\frac{\partial C}{\partial \lambda}\right)}_{AUX_2} \right\}.
\end{aligned}$$

The sign of this derivative depends on the sign of the term in braces. Replacing the derivatives and using the observation that $(1-\rho_z(1-\beta)\psi_\pi) < 1$ and $X < Y$, we have:

$$\begin{aligned}
AUX_2 &= (1-\rho_z(1-\beta)\psi_\pi)(Z+X\psi_\pi)(C+Q)-(Z+Y\psi_\pi)((1-\beta\rho_z)Z+C+Q+X\psi_\pi(1-\beta\rho_z)) \\
&< (Z+Y\psi_\pi)(C+Q)-(Z+Y\psi_\pi)((1-\beta\rho_z)Z+C+Q+X\psi_\pi(1-\beta\rho_z)) \\
&= -(1-\beta\rho_z)(Z+Y\psi_\pi)(Z+X\psi_\pi) < 0.
\end{aligned}$$

Consequently, $\partial\theta_0^\pi/\partial\lambda < 0$, so $\partial\psi_\pi/\partial\lambda > 0$, implying that $\partial\psi_\pi/\partial\theta > 0$.

B.5 Proof of Proposition 5

Follows directly from equations (30) and (31).

B.6 Proof of Proposition 6

Follows directly from equations (34) and (35). The observation that $\Psi_{LBS} > 1$ follows from condition (37) guaranteeing the uniqueness of the equilibrium.

B.7 Proof of Proposition 7

Let $\Phi \equiv (\sigma+\varphi)\sigma^{-1}\phi_\pi$ and $X \equiv (1-(\sigma+\varphi)\chi)$. Equation (31) implies that the response considered in each scenario is given by

$$\begin{aligned}
\left[\frac{\partial y_t}{\partial z_t}\right]^{LBS} &= \sigma^{-1}\left(\frac{\psi_\pi^{LBS}}{\Phi+X\psi_\pi^{LBS}}\right), \\
\left[\frac{\partial y_t}{\partial z_t}\right]^{SP} &= \sigma^{-1}\left(\frac{\lambda}{\lambda+\Phi}\right), \\
\left[\frac{\partial y_t}{\partial z_t}\right]^{LBS+SP} &= \sigma^{-1}\left(\frac{\lambda+\psi_\pi^{LBS+SP}}{\lambda+\Phi+X\psi_\pi^{LBS+SP}}\right),
\end{aligned}$$

where ψ_π^{LBS} and ψ_π^{LBS+SP} is the degree of anchoring in the corresponding scenario. Amplification is obtained when

$$\mathcal{A} \equiv \sigma^{-1}\left(\left\{\underbrace{\frac{\psi_\pi^{LBS+SP}}{\lambda+\Phi+X\psi_\pi^{LBS+SP}}-\frac{\psi_\pi^{LBS}}{\Phi+X\psi_\pi^{LBS}}}_{B}\right\}-\left\{\underbrace{\frac{\lambda}{\lambda+\Phi}-\frac{\lambda}{\lambda+\Phi+X\psi_\pi^{LBS+SP}}}_{C}\right\}\right)>0.$$

Notice that C is always positive. It follows that amplification is only possible if B is also positive. For this to be the case, we must have

$$\frac{\psi_\pi^{LBS+SP}}{\psi_\pi^{LBS}}>1+\frac{\lambda}{\Phi}.$$

Now, B can be simplified

$$\begin{aligned}
B \equiv \frac{\psi_\pi^{LBS+SP}}{\lambda+\Phi+X\psi_\pi^{LBS+SP}}-\frac{\psi_\pi^{LBS}}{\Phi+X\psi_\pi^{LBS}} &= \frac{\psi_\pi^{LBS+SP}\left(\Phi+X\psi_\pi^{LBS}\right)-\psi_\pi^{LBS}\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)}{\left(\lambda+\Phi+X\psi_\pi^{LS+SP}\right)\left(\Phi+X\psi_\pi^{LBS}\right)} \\
&= \frac{\psi_\pi^{LBS+SP}\left(\Phi+X\psi_\pi^{LBS}\right)-\psi_\pi^{LBS}\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)}{\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)\left(\Phi+X\psi_\pi^{LBS}\right)} \\
&= \frac{\psi_\pi^{LBS+SP}\Phi-\psi_\pi^{LBS}\left(\lambda+\Phi\right)}{\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)\left(\Phi+X\psi_\pi^{LBS}\right)}.
\end{aligned}$$

And C can be simplified as

$$\begin{aligned}
C \equiv \frac{\lambda}{\lambda+\Phi}-\frac{\lambda}{\lambda+\Phi+X\psi_\pi^{LBS+SP}} &= \lambda\left\{\frac{\lambda+\Phi+X\psi_\pi^{LBS+SP}-\lambda-\Phi}{(\lambda+\Phi)\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)}\right\} \\
&= \psi_\pi^{LBS+SP}\left(\frac{\lambda X}{(\lambda+\Phi)\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)}\right)
\end{aligned}$$

We can thus rewrite $\mathcal{A}$ as

$$\begin{aligned}
\mathcal{A} &= \sigma^{-1}\left(\psi_\pi^{LBS+SP}\left(\frac{\lambda X}{(\lambda+\Phi)\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)}\right)-\frac{\psi_\pi^{LBS+SP}\Phi-\psi_\pi^{LBS}\left(\lambda+\Phi\right)}{\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)\left(\Phi+X\psi_\pi^{LBS}\right)}\right) \\
&= \frac{\sigma^{-1}\psi_\pi^{LBS}}{\left(\lambda+\Phi+X\psi_\pi^{LBS+SP}\right)}\left\{\frac{\psi_\pi^{LBS+SP}}{\psi_\pi^{LBS}}\left[\frac{\lambda X}{\lambda+\Phi}-\frac{\Phi}{\Phi+X\psi_\pi^{LBS}}\right]+\frac{\lambda+\Phi}{\Phi+X\psi_\pi^{LBS}}\right\}
\end{aligned}$$

It follows that a sufficient condition for $\mathcal{A}>0$ is

$$\frac{\psi_\pi^{LBS+SP}}{\psi_\pi^{LBS}}\left[\frac{\lambda X}{\lambda+\Phi}-\frac{\Phi}{\Phi+X\psi_\pi^{LBS}}\right]+\frac{\lambda+\Phi}{\Phi+X\psi_\pi^{LBS}}>0.$$

We can rewrite this as

$$\frac{\psi_\pi^{LBS+SP}}{\psi_\pi^{LBS}}\left(\frac{\lambda X}{\lambda+\Phi}\right)+\frac{\lambda+\Phi}{\Phi+X\psi_\pi^{LBS}}>\left(\frac{\psi_\pi^{LBS+SP}}{\psi_\pi^{LBS}}\right)\frac{\Phi}{\Phi+X\psi_\pi^{LBS}}.$$

The necessary condition for amplification implies that the previous equation holds whenever

$$\frac{\psi_\pi^{LBS+SP}}{\psi_\pi^{LBS}}\left(\frac{\lambda X}{\lambda+\Phi}\right)+\frac{\lambda+\Phi}{\Phi+X\psi_\pi^{LBS}}>\left(1+\frac{\lambda}{\Phi}\right)\frac{\Phi}{\Phi+X\psi_\pi^{LBS}},$$

or, equivalently

$$\frac{\psi_{\pi}^{LBS+SP}}{\psi_{\pi}^{LBS}}\left(\frac{\lambda X}{\lambda+\Phi}\right)>0.$$

The assumption that $(\sigma+\varphi)\chi<1$ implies that $X>0$, and Proposition 4 implies that $\psi_{\pi}^{LBS+SP}>\psi_{\pi}^{LBS}$. It follows that $\mathcal{A}>0$.

B.8 Proof of Proposition 8

From the definition of α_{AS} in (25), we have

$$\frac{\partial\alpha_{PC}}{\partial\phi_{\pi}}=-\frac{\alpha_{PC}^{2}}{\sigma+\varphi}\frac{\partial\psi_{\pi}}{\partial\phi_{\pi}}<0,$$

where the inequality follows from the results in Proposition 4.

B.9 Proof of Proposition 9

Follows directly from (46).

B.10 Proof of Proposition 10

Start by replacing the demand for varieties in (16) in the definition of total expenditures in the budget constraint (5). Using this expression, we can express the consumption level of each household as:

$$C_{i,t}=R_{i,t}B_{i,t-1}^{R}+W_{i,t}^{R}N_{i,t}+D_{i,t}^{R}-B_{i,t}^{R},$$

where $B_{i,t}^{R}\equiv B_{i,t}/P_{t}$, $W_{i,t}^{R}\equiv W_{i,t}/P_{t}$, and $D_{i,t}^{R}\equiv D_{i,t}/P_{t}$, and $R_{t}=Q_{i,t}^{-1}P_{t-1}/P_{t}$. denotes the real interest rate. Substituting this expression in (2), we can express the period utility of the household as:

$$Z_{i,t}\left\{\frac{1}{1-\sigma}\left(R_{i,t}B_{i,t-1}^{R}+W_{i,t}^{R}N_{i,t}+D_{i,t}^{R}-B_{i,t}^{R}\right)^{1-\sigma}-\frac{N_{t}^{1+\varphi}}{1+\varphi}-\frac{1}{1-\sigma}\right\}.$$

Rewrite the expression in brackets as:

$$\frac{1}{1-\sigma}C^{1-\sigma}\left(\beta^{-1}e^{r_{i,t}}b_{i,t-1}^{R}+\omega_{W}e^{w_{i,t}^{R}+n_{i,t}}+\omega_{D}e^{d_{i,t}^{R}}-b_{i,t}^{R}\right)^{1-\sigma}-N^{1+\varphi}\frac{e^{(1+\varphi)n_{i,t}}}{1+\varphi}-\frac{1}{1-\sigma},$$

where the notation is the same used in the proof of in the proof of Proposition 2.[38] Multiplying this expression by β^{t}, summing over all $t=0,1,\ldots$ and taking expectation conditional on information in $t=-1$, we can rewrite the objective (1) as:

$$\begin{aligned}\mathcal{W}\left(\boldsymbol{x}_{i,t};\boldsymbol{y}_{i,t}\right)=\mathrm{E}_{i,-1}\sum_{t=0}^{\infty}\beta^{t}Z_{i,t}\Bigg\{&\frac{1}{1-\sigma}C^{1-\sigma}\left(\beta^{-1}e^{r_{i,t}}b_{i,t-1}^{R}+\omega_{W}e^{w_{i,t}^{R}+n_{i,t}}+\omega_{D}e^{d_{i,t}^{R}}-b_{i,t}^{R}\right)^{1-\sigma}\\&-\omega_{W}C^{1-\sigma}\frac{e^{(1+\varphi)n_{i,t}}}{1+\varphi}-\frac{1}{1-\sigma}\Bigg\}\end{aligned}$$

where $\boldsymbol{x}_{i,t}\equiv\left(b_{i,t}^{R},n_{i,t}\right)'$ is a vectors of choice variables, and $\boldsymbol{y}_{i,t}\equiv\left(r_{i,t-1},w_{i,t}^{R},d_{i,t}^{R},z_{i,t}\right)$, is a vector of variables and prices taken as given by the household.

Now, let $\boldsymbol{x}_{i,t}^{*}$ denote the optimal action of household i under full information and assume for simplicity that $b_{i,-1}^{*}=b_{-1}$. Under some regularity conditions that guarantee that $\boldsymbol{x}_{i,t}-\boldsymbol{x}_{i,t}^{*}$ has finite second moments,[39] we can take a quadratic

[38]Notice that the labor supply in (16) implies that $N^{1+\varphi}=\omega_{W}C^{1-\sigma}$.

[39]See Proposition 2 in the Online Appendix of Maćkowiak and Wiederholt (2015) for details.

approximation of $\mathcal{W}(\cdot)$ around the origin to derive the following expression of the expected loss in utility for any action $\boldsymbol{x}_{i,t} \neq \boldsymbol{x}^*_{i,t}$:

$$\begin{aligned}\mathcal{IC}_\pi\left(\boldsymbol{x}_{i,t}\right) &\equiv \mathcal{W}\left(\boldsymbol{x}_{i,t};\boldsymbol{y}_{i,t}\right) - \mathcal{W}\left(\boldsymbol{x}^*_{i,t};\boldsymbol{y}_{i,t}\right) \\ &\approx \mathrm{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{\frac{1}{2}\left(\boldsymbol{x}_{i,t}-\boldsymbol{x}^*_{i,t}\right)^T\boldsymbol{\mathcal{H}}_0\left(\boldsymbol{x}_{i,t}-\boldsymbol{x}^*_{i,t}\right)+\left(\boldsymbol{x}_t-\boldsymbol{x}^*_{i,t}\right)^T\boldsymbol{\mathcal{H}}_1\left(\boldsymbol{x}_{i,t+1}-\boldsymbol{x}^*_{i,t+1}\right)\right\}+t.i.p.\end{aligned} \tag{B.94}$$

where the matrices of derivatives $\boldsymbol{\mathcal{H}}_0$ and $\boldsymbol{\mathcal{H}}_1$ are given by:

$$\boldsymbol{\mathcal{H}}_0 = -C^{1-\sigma}\begin{bmatrix}\sigma\left(1+\beta^{-1}\right) & -\sigma\omega_W \\ -\sigma\omega_W & \omega_W\left(\varphi+\sigma\omega_W\right)\end{bmatrix},$$

$$\boldsymbol{\mathcal{H}}_1 = C^{1-\sigma}\begin{bmatrix}\sigma & -\sigma\omega_W \\ 0 & 0\end{bmatrix}.$$

At this stage, we can follow the same steps in Proposition 2 of the Online Appendix of Maćkowiak and Wiederholt (2015) to rewrite $\mathcal{IC}_\pi$ as a function of $\widetilde{c}_{i,t} \equiv c_{i,t} - c^*_{i,t}$ and $\widetilde{n}_{i,t} \equiv n_{i,t} - n^*_{i,t}$. First, note that (B.77) implies the optimal actions $\boldsymbol{x}^*_t$ under full information satisfy

$$c^*_{i,t} = \beta^{-1}b^*_{i,t-1} - b^*_{i,t} + \omega_W\left(n^*_{i,t} + w^R_{i,t}\right) + \omega_D d^R_{i,t}.$$

Consequently, we can express bond holdings deviations $\widetilde{b}_{i,t} = b_{i,t} - b^*_{i,t}$ as

$$\widetilde{b}_{i,t} = \beta^{-1}\widetilde{b}_{i,t-1} + \omega_W\widetilde{n}_{i,t} - \widetilde{c}_{i,t}$$

Iterating this expression backwards, we can rewrite it recursively as

$$\widetilde{b}_{i,t} = \Delta^N_{i,t} - \Delta^C_{i,t}$$

with $\Delta^C_{i,t} = \widetilde{c}_{i,t} + \beta^{-1}\Delta^C_{i,t-1}$, $\Delta^N_{i,t} = \omega_W\widetilde{n}_{i,t} + \beta^{-1}\Delta^N_{i,t-1}$ and $\Delta^C_{i,-1} = \Delta^N_{i,-1} = 0$. Using these expressions, and after some manipulation, we can express (B.94) as:

$$\begin{aligned}C^{\sigma-1}\mathcal{IC}_\pi =& \frac{1}{2}\left(\boldsymbol{x}_{i,t}-\boldsymbol{x}^*_{i,t}\right)^T\boldsymbol{\mathcal{H}}_0\left(\boldsymbol{x}_{i,t}-\boldsymbol{x}^*_{i,t}\right)+\left(\boldsymbol{x}_{i,t}-\boldsymbol{x}^*_{i,t}\right)^T\boldsymbol{\mathcal{H}}_1\left(\boldsymbol{x}_{i,t+1}-\boldsymbol{x}^*_{i,t+1}\right) \\ =& -\left\{\frac{\sigma}{2}\left(1+\beta^{-1}\right)\widetilde{b}^2_{i,t}+\frac{\omega_W}{2}\left(\varphi+\sigma\omega_W\right)\widetilde{n}^2_{i,t}-\sigma\omega_W\widetilde{b}_{i,t}\widetilde{n}_{i,t}+\sigma\widetilde{b}_{i,t}\widetilde{b}_{i,t+1}-\sigma\omega_W\widetilde{b}_{i,t}\widetilde{n}_{i,t+1}\right\} \\ =& -\frac{\sigma}{2}\left(1+\beta^{-1}\right)\widetilde{b}^2_{i,t}-\frac{\omega_W}{2}\varphi\widetilde{n}^2_{i,t}-\frac{1}{2}\sigma\left(\Delta^N_{i,t}\right)^2+\sigma\widetilde{b}_{i,t}\Delta^N_{i,t}-\sigma\widetilde{b}_{i,t}\widetilde{b}_{i,t+1}+\sigma\widetilde{b}_{i,t}\Delta^N_{i,t+1} \\ =& -\frac{\gamma}{2}\widetilde{c}^2_{i,t}-\frac{\omega_W\varphi}{2}\widetilde{n}^2_{i,t}+\frac{\gamma}{2}\widetilde{\Omega}_{i,t},\end{aligned}$$

with

$$\begin{aligned}\widetilde{\Omega}_{i,t} =& \beta^{-1}\left(\left(\Delta^C_{i,t}\right)^2-\beta^{-1}\left(\Delta^C_{i,t-1}\right)^2\right)+\beta^{-1}\left(\left(\Delta^N_{i,t}\right)^2-\beta^{-1}\left(\Delta^N_{i,t-1}\right)^2\right) \\ &+\left(\Delta^N_{i,t}\Delta^C_{i,t+1}-\beta^{-1}\Delta^N_{i,t-1}\Delta^C_{i,t}\right)-\left(\Delta^C_{i,t}\widetilde{c}_{i,t+1}-\beta^{-1}\Delta^C_{i,t-1}\widetilde{c}_{i,t}\right).\end{aligned}$$

Now, note that:

$$\widetilde{\Omega}_{i,0}+\beta\widetilde{\Omega}_{i,1}=\left(\Delta^C_{i,1}\right)^2+\left(\Delta^N_{i,1}\right)^2+\beta\Delta^N_{i,1}\Delta^C_{i,2}-\Delta^N_{i,0}\Delta^C_{i,1}-\beta\Delta^C_{i,1}\widetilde{c}_{i,2}$$
$$\widetilde{\Omega}_{i,0}+\beta\widetilde{\Omega}_{i,1}+\beta^2\widetilde{\Omega}_{i,2}=\beta\left(\Delta^C_{i,2}\right)^2+\beta\left(\Delta^N_{i,2}\right)^2+\beta^2\Delta^N_{i,2}\Delta^C_{i,3}-\beta^2\Delta^C_{i,2}\widetilde{c}_{i,3}$$
$$\vdots$$
$$\widetilde{\Omega}_{i,0}+\beta\widetilde{\Omega}_{i,1}+\ldots+\beta^T\widetilde{\Omega}_{i,T}=\beta^{T-1}\left(\Delta^C_{i,2}\right)^2+\beta^{T-1}\left(\Delta^N_{i,2}\right)^2+\beta^T\Delta^N_{i,2}\Delta^C_{i,3}-\beta^T\Delta^C_{i,2}\widetilde{c}_{i,3}.$$

It follows that:

$$\begin{aligned}\mathrm{E}_{-1}\sum_{t=0}^{\infty}\beta^t\widetilde{\Omega}_{i,t}&=\lim_{T\to\infty}\beta^{T-1}\mathrm{E}_{i,-1}\left[\left(\Delta^C_{i,2}\right)^2\right]+\lim_{T\to\infty}\beta^{T-1}\mathrm{E}_{i,-1}\left(\Delta^N_{i,2}\right)^2\\&+\lim_{T\to\infty}\beta^T\mathrm{E}_{i,-1}\left[\Delta^N_{i,2}\Delta^C_{i,3}\right]-\lim_{T\to\infty}\beta^T\mathrm{E}_{i,-1}\left[\Delta^C_{i,2}\widetilde{c}_{i,3}\right]\\&=0\end{aligned}$$

Consequently, the first part of (B.94) simplifies to

$$\mathcal{IC}_\pi=-\frac{1}{2}C^{1-\sigma}\mathrm{E}_{i,-1}\sum_{t=0}^{\infty}\beta^t\left\{\sigma\widetilde{c}^2_{i,t}+\omega_W\varphi\widetilde{n}^2_{i,t}\right\}$$

The last step is to express $\widetilde{c}_t$ and $\widetilde{n}_t$ as a function of the information wedges. To do this, recall that equation (B.83) relates the value of current consumption for a particular household with the the prices it faces, as well as expectations about the future value of those prices. Using (B.83) and (B.85), we can express the deviations of real income from their full information counterpart as:

$$c_{i,t}-c^*_{i,t}=-\frac{1}{\sigma}\beta\sum_{k=0}^{\infty}\beta^k\left(\mathrm{E}_{i,t}r^Z_{i,t+k+1}-\mathbb{E}_t r^Z_{i,t+k}\right)+\beta\chi\sum_{k=0}^{\infty}\beta^k\left[\mathrm{E}_{i,t}e^R_{i,t+k}-\mathbb{E}_t e^R_{i,t+k}\right].$$

Now, define $\nu^x_{i,t+k,t}\equiv x_{t+k}-\mathrm{E}_{i,t}x_{t+k}$ for $x\in\{p,\pi,y\}$. Using (B.92) and (B.93), we can express each discounted sum as:

$$\sum_{k=0}^{\infty}\beta^k\left[\mathrm{E}_{i,t}e^R_{i,t+k}-\mathbb{E}_t e^R_{i,t+k}\right]=\chi\nu^p_{i,t}-\left(\frac{1-\beta}{\beta}\right)\mathbb{E}_t\sum_{k=1}^{\infty}\beta^k\nu^y_{i,t+k|t}$$
$$\sum_{k=0}^{\infty}\beta^k\left(\mathrm{E}_{i,t}r^Z_{i,t+k+1}-\mathbb{E}_t r^Z_{i,t+k}\right)=\nu^\pi_{i,t+1|t}+\mathbb{E}_t\sum_{k=1}^{\infty}\beta^k\left\{\nu^\pi_{i,t+k+1|t}-\phi_\pi\nu^\pi_{i,t+k|t}\right\}.$$

It follows that the deviations of consumption of household i from its full information benchmark can be written as:

$$c_{i,t}-c^*_{i,t}=-\frac{1}{\sigma}\beta\left\{\nu^\pi_{i,t+1|t}+\mathbb{E}_t\sum_{k=1}^{\infty}\beta^k\left\{\nu^\pi_{i,t+k+1|t}-\phi_\pi\nu^\pi_{i,t+k|t}\right\}\right\}+\beta\left\{\chi\nu^p_{i,t}-\left(\frac{1-\beta}{\beta}\right)\mathbb{E}_t\sum_{k=1}^{\infty}\beta^k\nu^y_{i,t+k,t}\right\}$$

Now, using the optimality condition of labor supply (17), and the observation that the nominal wage in every period is part of the household's information set, we can express the deviation of household's i labor supply from it's full-information level as

$$n_{i,t}-n^*_{i,t}=\frac{1}{\varphi}\nu^p_{i,t|t}-\frac{\sigma}{\varphi}\left(c_{i,t}-c^*_{i,t}\right).$$

Putting the previous results together, we have that household's i cost of not paying attention to inflation as:

$$\mathcal{IC}_\pi=-\frac{1}{2}C^{1-\sigma}\mathrm{E}_{-1}\sum_{t=0}^{\infty}\beta^t\left\{\sigma\left(c_{i,t}-c^*_{i,t}\right)^2+\mathcal{M}^{-1}\varphi\left(n_{i,t}-n^*_{i,t}\right)^2\right\},$$

with

$$c_{i,t} - c^*_{i,t} =, -\frac{1}{\sigma}\beta \left\{ v^{\pi}_{i,t+1|t} + \mathbb{E}_t \sum_{k=1}^{\infty} \beta^k \left\{ v^{\pi}_{i,t+k+1|t} - \phi_\pi v^{\pi}_{i,t+k|t} \right\} \right\} + \beta \left\{ \chi v^p_{i,t} - \left(\frac{1-\beta}{\beta} \right) \mathbb{E}_t \sum_{k=1}^{\infty} \beta^k v^y_{i,t+k|t} \right\}$$

and

$$n_{i,t} - n^*_{i,t} = \frac{1}{\varphi} v^p_{i,t|t} - \frac{\sigma}{\varphi} \left(c_{i,t} - c^*_{i,t} \right).$$

B.11 Proof of Proposition 11

Following the discussion in the main text, the optimal attention problem (54) can be written as

$$\min_{\sigma^2_\epsilon} \quad \Omega \mathrm{Var}\left[\pi_t\right] \left(1 - \frac{1}{\mathrm{Var}\left[\pi_t\right] + \sigma^2_\epsilon} \right) + \omega \log \left(1 + \frac{\mathrm{Var}\left[\pi_t\right]}{\sigma^2_\epsilon} \right).$$

Define $q \equiv \mathrm{Var}\left[\pi_t\right] / \sigma^2_\epsilon$ as the signal-to-noise ratio implied by households choice of σ^2_ϵ. Since the household is atomistic, it takes $\mathrm{Var}\left[\pi_t\right]$ as given. It follows that choosing σ^2_ϵ is equivalent to choosing q, and we can restate the inattention problem as

$$\min_q \quad -\Omega \frac{q}{q+1} + \widetilde{\omega} \log\left(1+q\right).$$

Taking first order conditions and solving for q yields

$$q = \max \left\{ \frac{\Omega}{\omega} - 1, 0 \right\}.$$

Now, equation (14) implies

$$1 - \psi_\pi = \frac{q}{1+q}$$

Replacing q by the optimal choice of the household yields the expression in the main text.

C Quantitative Model and Solution Method

In this section I present the equations characterizing the quantitative model used in Section 5 and the computational algorithm used to solve it. To begin, I present the algorithm to compute the solution for a given level of σ^2_ε. I build on this algorithm to solve the problem under rational inattention to the aggregate price level.

C.1 The model

C.1.1 Equilibrium inflation and output

Recall that each household has access to a noisy signal about the aggregate price level of the form

$$s_{i,t} = p_{i,t} + \epsilon_{i,t}; \qquad \epsilon_{i,t} \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_\varepsilon\right).$$

Given a precision of signals, the equilibrium levels of output and inflation satisfy the following supply and demand relationships:

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa_{PC} \left(y_t - \left(\left(1+\varphi\right) / \left(\sigma + \varphi\right) \right) a_t \right) - \lambda^{-1} \nu_t \tag{C.95}$$

$$y_t = -\frac{1}{\sigma} \left(\phi \pi_t - \mathbb{E}_t \pi_{t+1} + z_{t+1} - z_t \right) + \mathbb{E}_t y_{t+1} + \mathcal{X}_t + \beta \mathbb{E}_t \mathcal{X}_{t+1} \tag{C.96}$$

where

$$\mathcal{H}_t = \chi \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k v^p_{t+k},$$

$$\mathcal{R}_t = -\sigma^{-1}\mathbb{E}_t \left\{ v^{\pi}_{t+1|t} + \sum_{k=1}^{\infty} \beta^k \left\{ v^{\pi}_{t+k+1|t} - \phi_\pi v^{\pi}_{t+k|t} \right\} \right\},$$

$$\chi \equiv \left(\frac{1-\beta}{\beta}\right)\left(\frac{\mathcal{M}\varphi}{\mathcal{M}\varphi+\sigma}\right),$$

with $v^p_t \equiv \int_0^1 \{p_t - \mathrm{E}_{i,t} p_t\}\, di$, $v^{\pi}_{t+k+1|t} \equiv \int_0^1 \{\pi_{t+k+1-}\mathrm{E}_{i,t}\pi_{t+k+1}\}\, di$, and $z_t$and a_t denoting the aggregate demand and technology shocks, which are given by:

$$z_t = \rho z_{t-1} + \eta^{AD}_t; \quad \eta^{AD}_t \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_{AD}\right)$$

$$a_t = \rho a_{t-1} + \eta^{AS}_t; \quad \eta^{AS}_t \overset{iid}{\sim} \mathcal{N}\left(0, \sigma^2_{AS}\right)$$

C.1.2 Beliefs

To compute the solution of this model note that, by Wold's representation theorem, any equilibrium π_t has finite second moments allows for an $MA(\infty)$ representation of both variables. Following the Box and Jenkings approach, we assume that this representation can be approximated by a $ARMA$ process of the form

$$(1 - \phi^{\pi}_1 L - \ldots - \phi^{\pi}_r L^r)\, \pi_t = \left(1 - \theta^{\pi}_1 L - \ldots - \theta^{\pi}_q L^q\right)\left(\psi^{AS}_\pi \eta^{AS}_t + \psi^{AD}_\pi \eta^{AD}_t\right)$$

Since shocks hitting the economy are causal, we can invert this polynomial to get a $AR(\infty)$ representation for π_t. We can then approximate numerically the law of motion of π_t to an arbitrary degree of accuracy by a finite-lag $AR(H)$ process. Let $\boldsymbol{\pi}_t = \left(\pi_t, \pi_{t-1}, \ldots, \pi_{t-(H-1)}\right)'$ represent a vectors stacking current and $H-1$ lags of the π_t, and denote as $\boldsymbol{e}_i$ is the i-th column of the identity matrix. We can write the reduced-form $AR(H)$ of π_t in state-space form as[40]

$$\boldsymbol{\pi}_t = \boldsymbol{\Phi}_\pi \boldsymbol{\pi}_{t-1} + \boldsymbol{e}_1 \left(\underbrace{\psi^{AS}_\pi \eta^{AS}_t + \psi^{AD}_\pi \eta^{AD}_t}_{\eta_t} \right). \tag{C.97}$$

The $H \times H$ matrix $\boldsymbol{\Phi}_\pi$, together with the impact coefficients $\left(\psi^{AS}_\pi, \psi^{AD}_\pi\right)$, summarize the behavior of inflation and are equilibrium objects to be determined[41]. We can use (C.97) to derive an $AR(H)$ process for p_t of the form

$$\boldsymbol{p}_t = \boldsymbol{\Phi}_A \boldsymbol{p}_{t-1} + \boldsymbol{e}_1 \eta_t, \tag{C.98}$$

with $\boldsymbol{p}_t = (p_t, p_{t-1}, \ldots, p_{t-H})'$. Since, $\mathrm{E}_{i,t}\left[\varepsilon_{i,t}\varepsilon_{k,t}\right] = 0$ for all $i \in [0,1]$ and $k \neq i$, we can characterize the beliefs about each relative price for each household independently using (7) and (C.98). Using Assumptions 1 and 2, and standard Kalman filter formulas yields:[42]

$$\widehat{\boldsymbol{p}}_{i,t|t} = \widehat{\boldsymbol{p}}_{i,t|t-1} + \boldsymbol{K}_A \boldsymbol{e}'_1 \left(\boldsymbol{p}_{j,t} - \widehat{\boldsymbol{p}}_{i,t|t-1}\right) + \boldsymbol{K}_A \boldsymbol{e}'_1 \varepsilon_{i,t}, \tag{C.99}$$

with $\widehat{\boldsymbol{p}}_{i,t|s} = \mathrm{E}_{i,s}\left[\boldsymbol{p}_t\right]$. The Kalman gain vector $\boldsymbol{K}_A$ is a $H \times 1$ vector given by

$$\boldsymbol{K}_A = \left(\frac{1}{\widehat{\boldsymbol{\Sigma}}_A[1,1] + \sigma^2_\epsilon}\right)\widehat{\boldsymbol{\Sigma}}_A \boldsymbol{e}_1,$$

where $\boldsymbol{\Sigma}_A[1,1]$ denotes the $[1,1]$ element of the covariance matrix $\widehat{\boldsymbol{\Sigma}}_A \equiv \mathrm{Var}_{i,t-1}\left[\boldsymbol{p}_t\right]$. This matrix can be found by solving

[40] See Chapter 3 in Hamilton (1994).

[41] Stability of the process implies that all eigenvalues of $\boldsymbol{\Phi}_\pi$. Notice, however, that $\boldsymbol{\Phi}_A$ may have an eigenvalue equal to 1.

[42] See Durbin and Koopman (2012).

the following Algebraic Riccati equation:

$$\widehat{\boldsymbol{\Sigma}}_A = \boldsymbol{\Phi}_A \widehat{\boldsymbol{\Sigma}}_A \boldsymbol{\Phi}_A' - \left(\widehat{\boldsymbol{\Sigma}}_A [1,1] + \sigma_\epsilon^2\right)^{-1} \boldsymbol{\Phi}_A \widehat{\boldsymbol{\Sigma}}_A \boldsymbol{e}\boldsymbol{e}_1' \widehat{\boldsymbol{\Sigma}}_A \boldsymbol{\Phi}_A' + \sigma_\eta^2 \boldsymbol{e}_1 \boldsymbol{e}_1',$$

with $\sigma_\eta^2 \equiv \text{Var}[\eta_t]$. Note that this matrix is constant and common across households as consequence of Assumption 2. Now let $\widehat{\boldsymbol{\pi}}_{t|s} \equiv \int_0^1 \mathrm{E}\left[\boldsymbol{p}_t - \boldsymbol{p}_{t-1} | \mathcal{I}_{i,s}\right] di$ denote the average belief across households about the inflation rate. Let $\boldsymbol{L}_H$ denote the $H \times H$ shift matrix

$$\boldsymbol{L}_H \equiv \begin{bmatrix} 0 & 0 & & \cdots & 0 \\ 1 & 0 & & & \vdots \\ \vdots & 1 & \ddots & & \\ & & \ddots & 0 & 0 \\ 0 & & \cdots & 1 & 0 \end{bmatrix}$$

and let $\boldsymbol{D}_H \equiv \boldsymbol{I}_H - \boldsymbol{L}_H$. Premultiplying both sides of (C.99) by $\boldsymbol{D}_H$ yields

$$\widehat{\boldsymbol{\pi}}_{i,t|t} = \widehat{\boldsymbol{\pi}}_{i,t|t-1} + \boldsymbol{K}_\pi \boldsymbol{e}_1' \left(\boldsymbol{p}_{j,t} - \widehat{\boldsymbol{p}}_{i,t|t-1}\right) + \boldsymbol{K}_\pi \boldsymbol{e}_1' \varepsilon_{i,t}, \tag{C.100}$$

with $\boldsymbol{K}_\pi \equiv \boldsymbol{D}_H \boldsymbol{K}_A$ denoting a $H \times 1$ vector of Kalman gains for inflation beliefs. Moreover, let $\nu_{i,t,s}^p \equiv p_{j,t,} - \widehat{p}_{i,j,t|s}$ denote each household forecast error about aggregate price in period t, conditional on her own information set up to period s. Writing (C.98) one period ahead and subtracting the corresponding forecast by the household using (C.99) yields

$$\boldsymbol{\nu}_{i,t+1|t}^p = \boldsymbol{\Phi}_A \boldsymbol{\nu}_{i,t|t}^p + \boldsymbol{e}_1 \eta_{t+1}$$

Subtracting $\boldsymbol{p}_t$ from (C.98) and manipulating terms, we get

$$\boldsymbol{\nu}_{i,t|t}^p = \left(\boldsymbol{I}_H - \boldsymbol{K}_A \boldsymbol{e}_1'\right) \boldsymbol{\nu}_{i,t|t-1}^p - \boldsymbol{K}_A \boldsymbol{e}_1' \varepsilon_{i,t}$$

Putting these two expressions together, we arrive to

$$\boldsymbol{\nu}_{i,t|t}^p = \Psi_A \boldsymbol{\nu}_{i,t-1|t-1}^p + \boldsymbol{\delta}_A \eta_t - \boldsymbol{K}_A \boldsymbol{e}_1' \varepsilon_{i,t}$$

with $\Psi_A \equiv \left(\boldsymbol{I}_H - \boldsymbol{K}_A \boldsymbol{e}_1'\right) \boldsymbol{\Phi}_A$ and $\boldsymbol{\delta}_A \equiv (\boldsymbol{e}_1 - \boldsymbol{K}_A)$. Notice that this implies that forecast errors are a combination of aggregate shocks an household-specific idiosyncratic noise:

$$\boldsymbol{\nu}_{i,t+1|t}^p = \boldsymbol{\Phi}_A \Psi_A \boldsymbol{\Phi}_A^{-1} \boldsymbol{\nu}_{i,t|t-1}^p + \boldsymbol{e}_1 \eta_{t+1} - \boldsymbol{\Phi}_A \boldsymbol{K}_A \boldsymbol{e}_1' \varepsilon_{i,t} \tag{C.101}$$

Following similar steps, we can derive an analogous representation for the forecast errors about each relative price. For the inflation rate, recall that equation (C.98) has an associated representation for the inflation rate

$$\boldsymbol{\pi}_{t+1} = \boldsymbol{\Phi}_\pi \boldsymbol{\pi}_t + \boldsymbol{e}_1 \eta_{t+1}$$

Subtracting the household forecast of the aggregate inflation rate yields

$$\boldsymbol{\nu}_{i,t+1|t}^\pi = \boldsymbol{\Phi}_\pi \boldsymbol{\nu}_{i,t|t}^\pi + \boldsymbol{e}_1 \eta_{t+1}$$

Subtracting $\boldsymbol{\pi}_t$ from (C.100) yields

$$\boldsymbol{\nu}_{i,t|t}^\pi = \boldsymbol{\nu}_{i,t|t-1}^\pi - \boldsymbol{K}_\pi \boldsymbol{e}_1' \boldsymbol{\nu}_{i,t|t-1}^p - \boldsymbol{K}_\pi \boldsymbol{e}_1' \varepsilon_{i,t}$$

Replacing in the previous equation and using the results for $\boldsymbol{\nu}_{i,t|t}$, we get

$$\boldsymbol{\nu}_{i,t|t}^\pi = \boldsymbol{\Phi}_\pi \boldsymbol{\nu}_{i,t-1|t-1}^\pi + \boldsymbol{\delta}_\pi \eta_t - \boldsymbol{\Upsilon}_\pi \boldsymbol{\nu}_{i,t-1|t-1}^p - \boldsymbol{K}_\pi \boldsymbol{e}_1' \varepsilon_{i,t} \tag{C.102}$$

with $\delta_\pi \equiv (\boldsymbol{e}_1 - \boldsymbol{K}_\pi)$ and $\mathbf{Y}_\pi = \boldsymbol{K}_\pi \boldsymbol{e}'_{H1} \boldsymbol{\Phi}_A$.

The previous expressions imply that households perception and forecast errors display persistence over time, from the perspective of a fully informed agent that observes these errors externally. These beliefs are dispersed due to the idiosyncratic shopping experiences of each household. But the average belief still displays persistence over time due to learning. Let $\boldsymbol{x}_t \equiv \int_0^1 \boldsymbol{x}_{i,t} di$ denote the average belief across households of a vector of variables $\boldsymbol{x}_t$. Equations (C.99) and (C.100) imply that the average belief about the aggregate price level and the inflation rate follow

$$\widehat{\boldsymbol{p}}_{t|t} = \widehat{\boldsymbol{p}}_{t|t-1} + \boldsymbol{K}_A \boldsymbol{e}'_1 \left(\boldsymbol{p}_t - \widehat{\boldsymbol{p}}_{t|t-1} \right)$$

$$\widehat{\boldsymbol{\pi}}_{t|t} = \widehat{\boldsymbol{\pi}}_{t|t-1} + \boldsymbol{K}_\pi \boldsymbol{e}'_1 \left(\boldsymbol{p}_t - \widehat{\boldsymbol{p}}_{t|t-1} \right)$$

Furthermore, we can integrate equations (C.102), (C.101), across households to get the following expressions for the average perception error about inflation and the price level:

$$\boldsymbol{\nu}^\pi_{t|t} = \boldsymbol{\Phi}_\pi \boldsymbol{\nu}^\pi_{t-1|t-1} + \delta_\pi \eta_t - \boldsymbol{K}_\pi \boldsymbol{e}'_1 \boldsymbol{\Phi}_A \boldsymbol{\nu}^p_{t-1|t-1} \tag{C.103}$$

$$\boldsymbol{\nu}_{t|t} = \Psi_A \boldsymbol{\nu}_{t-1|t-1} + \delta_A \eta_t \tag{C.104}$$

To conclude, notice that this characterization implies

$$\mathbb{E}_t \boldsymbol{\nu}^p_{t+k|t} = \boldsymbol{\Phi}^k_A \boldsymbol{\nu}^p_{t|t}$$

$$\mathbb{E}_t \boldsymbol{\nu}^p_{t+k|t+k} = \Psi^k_A \boldsymbol{\nu}^p_{t|t}$$

Similarly

$$\mathbb{E}_t \boldsymbol{\nu}^\pi_{t+k|t} = \boldsymbol{\Phi}^k_\pi \boldsymbol{\nu}^\pi_{t|t}$$

$$\mathbb{E}_t \boldsymbol{\nu}^\pi_{t+k|t+k} = \boldsymbol{\Phi}^k_\pi \boldsymbol{\nu}^\pi_{t|t} - \boldsymbol{\Pi}_k \boldsymbol{\nu}^p_{t|t}$$

with $\boldsymbol{\Pi}_k = \sum_{j=1}^k \boldsymbol{\Phi}^{k-j}_\pi \mathbf{Y}_\pi \boldsymbol{\Psi}^{j-1}_A$.

C.2 Computational method

Given this guess about the law of motion of households beliefs, I solve the model under exogenous information using the following algorithm:

1. Guess $\Theta \equiv \left(\boldsymbol{\Phi}_\pi, \psi^{AS}_\pi, \psi^{AD}_\pi \right)$ using the corresponding solution under full-information.
2. Use (C.103) and (C.104) to express y_t and π_t in (C.95) and (C.96) as a function of $\boldsymbol{\nu}_{t|t}$, $\widehat{\boldsymbol{\pi}}_{t|t}$, and the exogenous shocks only.
3. Find the ARMA process associated to the SS representation implied by (C.103) and (C.104) and the expressions obtained in the previous step (See Chapter 12 in Brockwell and Davis (2009) for an algorithm to do so).
4. Find the AR process representation of the previous ARMA process, truncated to H lags.
5. Update Θ based on the previous AR representation and go back to step (2) until convergence.

To compute the solution of the model under rational inattention, I use the previous algorithm to find the law of motion of inflation, conditional on a value of guess off σ^2_ϵ. Conditional on this law of motion, I update the guess of σ^2_ϵ by solving the rational inattention problem of a household. This is done by numerically maximizing (53) subject to (52). I then iterate over these steps until convergence in the value of σ^2_ϵ is achieved.