Optimal inflation in small open economies: the role of asymmetric nominal rigidities

Luis Rodrigo Arnabal* Javier García-Cicco †

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Abstract

This paper characterizes optimal inflation target in small open economies with nominal rigidities that are potentially asymmetric. While with symmetric nominal rigidities (as in standard New Keynesian models) a zero inflation level is desirable, with asymmetric rigidities having positive inflation becomes desirable. To understand this trade-off we make use of a small open economy model with downward nominal wage rigidities (through a linex adjustment cost function), indexation and price dollarization of international prices (for export and imports). We derive quantitatively relevant intervals for long-term inflation in this context, calibraring the model for the Uruguayan economy.

JEL *classification*: E4 ; E5.
Keywords: Optimal inflation; Downward wage rigidity; Small open economies; Indexation.

*Banco Central del Uruguay. Address: Diagonal Fabini 777, Montevideo, Uruguay. E-mail: larnabal@bcu.gub.uy

†Universidad de San Andrés. Address: E-mail: jgcicco@udesa.edu.ar

1 Introduction

In order to avoid the opportunity cost of holding money in an economy with flexible prices, inflation must be equal to minus the nominal interest rate, a theoretical result known in the literature as the Friedman rule. This result is at odds with the monetary policy followed by most central banks today who target positive values of inflation (Schmitt-Grohé and Uribe, 2010). A branch of monetary analysis has been dedicated to quantifying what the most desirable long-term inflation target is. In this literature, a number of frictions and/or externalities that reduce the welfare of agents are considered and whose impact could be reduced with a positive inflation rate.[1] Indeed, in the presence of price frictions or externalities, a positive inflation can help to accelerate the adjustment of real prices, an argument introduced by Tobin (1972), who argued that a positive inflation is necessary to "grease the wheels" of the economy. The literature tries to quantify this trade-off and arrive at an optimal value (or range of values) for average inflation. For example, the meta analysis performed by Diercks (2019) documents 440 different contributions that aim to quantify the optimal level of inflation, among which less than 20 address what should be the optimal inflation in the context of a small open economy.

Recently, some studies have explored the role of downward wage rigidities for monetary policy analysis as well as the impact of the dollar as the dominant international currency in determining the price of tradable goods. The role of these characteristics, which are empirically relevant for most emerging economies, have not yet been integrated into the analysis of the optimal inflation target. The aim of this project is to fill this gap in the literature by incorporating these features into a small open economy model to study optimal inflation.

Downward nominal wage rigidites (DNWR) have been integrated in different ways into the standard New Keynesian framework to study optimal monetary policy (*e.g.*, with menu costs as in Fagan and Messina (2009), by considering nominal wage reductions extremely costly as in Benigno and Antonio Ricci (2011) or by imposing a constraint on the nominal wage as a function of its previous value (Schmitt-Grohé and Uribe, 2016, 2022)).[2] We model nominal wage adjustment costs with a LINEX function as in Kim and Ruge-Murcia (2009, 2011), in order to account for the fact that decreasing nominal wages is more costly than increasing them.[3] The former contribution finds that when the central bank follows a simple targeting policy such as the Taylor rule, the optimal inflation rate for the US economy should be 0.75%, while the latter, incorporating a demand for money into the model finds that optimal inflation should be around 1%. Using the same functional specifications for adjustment costs and adding labor searching frictions into the model, Abo-Zaid (2013) concludes that the optimal annual inflation rate for the US should be around 1.81% with no money demand and 0.86% without it.[4] More recently, the analysis of Mineyama (2022) adds heterogeneity in labor productivity, introducing in this way additional

[1]An argument to target a positive inflation that has gained popularity recently that we do not address in this work is the existence of a zero lower bound of nominal interest rates (*e.g.,* Billi et al., 2008; Coibion et al., 2012; Blanco, 2021; Amano and Gnocchi, 2020).

[2]For a more detailed discussion on this issue, see Evans (2020).

[3]On the asymmetry in the distribution of nominal wage changes see, for instance, Dickens et al. (2007).

[4]The study of Carlsson and Westermark (2016) considers the interaction between nominal (temporally fixed) wage rigidities and search and matching frictions. Their calibration on the US economy results in a Ramsey optimal inflation rate of 1.15%.

inefficiencies in the allocation of labor across sectors. Considering that wage adjustment costs are represented by a fixed cost and a linear cost proportional to the size of wage changes, the calibration of the model for the US economy results in an inflation rate of 2%.

The other main relevant feature in emerging economies is the role that the US dollar has as a dominant currency in the pricing of tradable goods. Several studies have analyzed the effect of dominant currency pricing for optimal monetary policy (*e.g.,* Corsetti et al., 2007; Devereux et al., 2007; Goldberg and Tille, 2009; Casas et al., 2017; Egorov and Mukhin, 2020). Their main finding in terms of the optimal monetary policy is that emergent economies should aim to stabilize domestic prices.

This paper aims to contribute to the literature that studies optimal inflation by extending the traditional New Keynesian framework to incorporate empirically relevant features for emerging economies that have not yet been considered. We do so by building a small open economy model with downward nominal wage rigidites, wage indexation and dollar currency pricing (for exports and imports) to try to establish quantitatively relevant ranges for long-term inflation. We calibrate our model for the Uruguayan economy, which is a small open economy where the aforementioned features are very relevant. The key assumption is that nominal wages are downward rigid (as in Kim and Ruge-Murcia (2009, 2011)): the decision on wages is subject to asymmetric adjustment costs where wage cuts are much more costly than wage increases. Given this configuration, reaching situations where it would be desirable to lower nominal wages is socially costly. Thus, the greater the probability that these circumstances will occur, the more desirable it is to have a higher average inflation rate, as this will minimize the probability of occurrence of these socially costly events.

The following section describes the baseline model, its parameterization and discusses the different trade-offs that arise in this model in choosing an optimal inflation target. Section 3 presents the main results of our analysis. Section 4 concludes (TBA).

2 Baseline Model

The setup is one of a cashless small open economy with incomplete financial markets, under rational expectations. There are several goods: home, imported and final goods. The final consumption good is composed of home and imported goods. The home good can also be exported. Additionally, there exist an endowment of commodities that is fully exported. The markets for final goods and labor have a monopolistic-competitive structure, where prices and wages are subject to adjustment costs. Prices of home goods are sticky in domestic currency when consumed in the domestic market, while they are sticky in dollars when they are to be exported (in line with the dominant-currency-pricing literature). Imported goods also face price adjustment costs. Households derive utility from consumption and leisure. They also have access to international bonds and domestic treasuries. Monetary policy is implemented by a Taylor-type rule. The rest of this section describes the different agents in the model, the aggregation and market-clearing conditions, the policy rule, the calibration of the model and provides intuition regarding the optimal inflation target.

2.1 Households

Households seek to maximize,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\log \left(c_t - \phi_C \frac{\tilde{c}_{t-1}}{a_{t-1}} \right) - \Xi^h \frac{(h_t)^{1+\varphi}}{1+\varphi} \right] \right\} \tag{1}$$

subject to the constraint,

$$P_t c_t + S_t B_t^{H^*} + B_t^T + T_t \leq \widetilde{W}_t h_t + S_t \frac{B_{t-1}^{H^*}}{a_{t-1}} R_{t-1}^* + \frac{B_{t-1}^T}{a_{t-1}} R_{t-1} + \Omega_t.$$

Here, $\beta \in (0,1)$ is the subjective discount factor, $\phi_C \in (0,1)$ captures habits in consumption, while $\Xi^h, \varphi > 0$ are parameters describing the dis-utility of labor. The growth rate of permanent (non-stationary) productivity shock is denoted by a_t,[5] c_t denotes consumption, $\tilde{c}_t$ is average consumption (in equilibrium $c_t = \tilde{c}_t$) h_t are hours worked, $B_t^{H^*}$ are holdings of foreign bonds (with interest rate R_t^*), B_t^T are holdings of domestic treasuries (with rate R_t), T_t are lump-sum transfers, S_t is the nominal exchange rate, P_t is the price of final consumption goods, $\widetilde{W}_t$ is the nominal wage obtained by the representative household and Ω_t denotes profits from the ownership of all firms in the economy.[6]

Letting $\beta^t \frac{\lambda_t}{P_t}$ denote the Lagrange multiplier associated with the resource constraint, the optimality conditions are

$$\lambda_t = \frac{1}{c_t - \phi_C \frac{c_{t-1}}{a_{t-1}}}, \quad \widetilde{w}_t \lambda_t = \psi(h_t)^{\varphi}, \quad \lambda_t = \frac{\beta}{a_t} R_t^* E_t \left\{ \frac{\pi_{t+1}^S \lambda_{t+1}}{\pi_{t+1}} \right\}, \quad \lambda_t = \frac{\beta}{a_t} R_t E_t \left\{ \frac{\lambda_{t+1}}{\pi_{t+1}} \right\},$$

where $\widetilde{w}_t = \frac{\widetilde{W}_t}{P_t}$, $\pi_t \equiv \frac{P_t}{P_{t-1}}$, and $\pi_t^S \equiv \frac{S_t}{S_{t-1}}$.

The first equation links the Lagrange multiplier with the marginal utility of consumption, while the second represents the inter-temporal trade-off characterizing labor supply. The third and fourth characterize the inter-temporal trade-offs related to the choices of foreign and domestic bonds. For future references, let $\chi_{t,t+\tau} \equiv \frac{\beta^\tau}{a_t} \frac{\lambda_{t+\tau}}{\lambda_t} \frac{P_t}{P_{t+\tau}}$ be the stochastic discount factor for claims in domestic currency, τ periods ahead. Moreover, P_t will represent the numeraire, used to compute relative prices.

2.2 Labor markets and wage setting

Household supply labor services to a continuum of intermediaries $i \in [0,1]$, which in turn supply labor to firms. Households are indifferent between working in any of these markets and there are no differences in the quality of labor provided in each of them. The total number of hours allocated to the different labor markets (h_{it}) must satisfy the constraint $h_t = \int_0^1 h_{it} \mathrm{d}i$.

[5] We assume the model features a long-run stochastic real trend, in the form of a labor-augmenting total factor productivity (TFP) shock, inducing a balanced-growth path. Accordingly, the variables appearing in this description have already been detrended.

[6] Throughout, uppercase letters denote nominal variables in levels, while lowercase letters indicate real variables (detrended if required, as previously discussed), relative prices, or rates of change. Variables without time subscript denote non-stochastic steady-state values. Finally, we use the notation $\widehat{x}_t \equiv \ln(x_t/x)$ for a generic variable x_t.

Firm's labor demand h_t^d is a CES combination of workers from each of these labor markets

$$h_t^d = \left[\int_0^1 h_{it}^{1-\frac{1}{\epsilon_W}} \mathrm{d}i\right]^{\frac{\epsilon_W}{\epsilon_W-1}}$$

where $\epsilon_W > 1$ is the elasticity of substitution across labor markets. Letting $W_{i,t}$ denote the nominal wage charged by intermediary i, and W_t the wage payed by firms, the demand for each labor variety is,

$$h_{it} = [W_{i,t}/W_t]^{-\epsilon_W} h_t^d.$$

Given this, the following is the relationship between the final wage W_t and those for each variety: $(W_t)^{1-\epsilon_W} = \int_0^1 W_{i,t}^{1-\epsilon_W} \mathrm{d}i$.

Taking the demand as given, the intermediary i hires labor from households paying wages $\widetilde{W}_t$ which, in addition to W_t and h_t^d, is taken as given. It's choice then boils down to choosing wages to maximize the net present value of profits, subject to an adjustment costs given by

$$\Phi_W\left(\frac{\pi_t^W}{\pi_t^{I,W}}\right) W_t h_t^d,$$

where $\Phi_W(\cdot)$ is a convex function satisfying $\Phi_W(1) = 0$.

In addition, $\pi_t^{I,W}$ is the rate at which wages can change in period t without generating adjustment costs; capturing indexation (discussed below).

Using the factor $\chi_{t,t+h}$, the part of discounted profits relevant for the choice of $W_{i,t}$ is,

$$h_t^d\left[\left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_W}\left(W_{i,t} - \widetilde{W}_t\right) - \Phi_W\left(\frac{\pi_t^W}{\pi_t^{I,W}}\right) W_t\right] - E_t\left\{\chi_{t,t+1} W_{t+1} h_{t+1}^d \Phi_W\left(\frac{\pi_{t+1}^W}{\pi_{t+1}^{I,W}}\right)\right\}.$$

This yields the optimality condition

$$\left[(1-\epsilon_W)\left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_W} + \epsilon_W\left(\frac{W_{i,t}}{W_t}\right)^{-\epsilon_W-1}\frac{\widetilde{W}_t}{W_t} - \Phi_W'\left(\frac{\pi_t^W}{\pi_t^{I,W}}\right)\frac{W_t}{W_{i,t-1}\pi_t^{I,W}}\right] h_t^d + \ldots$$
$$+ E_t\left\{\chi_{t,t+1} W_{t+1} h_{t+1}^d \Phi_W'\left(\frac{\pi_{t+1}^W}{\pi_{t+1}^{I,W}}\right)\frac{W_{i,t+1}}{(W_{i,t})^2 \pi_{t+1}^{I,W}}\right\} = 0.$$

Letting $\pi_t^W \equiv \frac{W_t}{W_{t-1}}$ and $w_t \equiv \frac{W_t}{P_t}$, focusing in a symmetric equilibrium the optimality condition is

$$(\epsilon_W - 1) = \epsilon_W \frac{\widetilde{w}_t}{w_t} - \Phi_W'\left(\frac{\pi_t^W}{\pi_t^{I,W}}\right)\frac{\pi_t^W}{\pi_t^{I,W}} + E_t\left\{\chi_{t,t+1}\frac{h_{t+1}^d}{h_t^d}\Phi_W'\left(\frac{\pi_{t+1}^W}{\pi_{t+1}^{I,W}}\right)\frac{(\pi_{t+1}^W)^2}{\pi_{t+1}^{I,W}}\right\}.$$

This is the non-linear version of the wage Phillips curve in this model.

2.3 Final consumption

At the wholesale level, a set of competitive firms supply the final consumption good using the following technology:

$$y_t^C = \left[\omega^{1/\eta}\left(c_t^H\right)^{1-1/\eta} + (1-\omega)^{1/\eta}\left(c_t^F\right)^{1-1/\eta}\right]^{\frac{\eta}{\eta-1}}. \tag{2}$$

These good are sold at price P_t. Nominal profits are given by $P_t y_t^C - P_t^H c_t^H - P_t^F c_t^F$, leading to the following demands:

$$c_t^F = (1-\omega)\left(p_t^F\right)^{-\eta} y_t^c, \qquad c_t^H = \omega\left(p_t^H\right)^{-\eta} y_t^c,$$

with $p_t^F \equiv P_t^F/P_t$ and $p_t^H \equiv P_t^H/P_t$.

2.4 Home goods

They are produced competitively using labor (h_t^d) according to the production function

$$y_t^H = z_t a_t h_t^d,$$

where z_t is a temporary productivity shock. Profit maximization generates the following input demand,

$$\tilde{p}_t^H z_t a_t = w_t,$$

where $\tilde{p}_t^H$ is the wholesale price of these goods (in relative terms of the numeraire).

These goods are sold at the retail level domestically and abroad by two different monopolistic-competitive structures: firms supplying to domestic consumers set prices in domestic currency units, while those selling abroad choose prices in foreign currency units, each of them facing price-adjustment costs similar to those described for wages. This is in line with the dominant-currency pricing literature (*e.g.*. Gopinath et al., 2020) documenting that international price of tradables is generally denominated in a few dominant currencies. This in turn will limit the expenditure-switching channel, by preventing real depreciation to have a direct effect into export's demand.

Let mc_t^H and $mc_t^{H^*}$ be the real marginal costs for both type of monopolists (expressed in terms of their own goods price). These should satisfy

$$p_t^H mc_t^H = \tilde{p}_t^H, \quad rer_t p_t^{H^*} mc_t^{H^*} = \tilde{p}_t^H,$$

with $p_t^{H^*} = \frac{P_t^{H^*}}{P_t^*}$ and $rer_t \equiv \frac{S_t P^{H^*}}{P_t}$. Notice that, in the last equation rer_t is included, as exporters are assumed to set prices in foreign currency.

The optimality conditions for prices chosen by monopolists in each group lead to the following non-linear Phillips curves:

$$(\epsilon_H - 1) = \epsilon_H mc_t^H - \Phi_H'\left(\frac{\pi_t^H}{\pi_t^{I,H}}\right)\frac{\pi_t^H}{\pi_t^{I,H}} + E_t\left\{\chi_{t,t+1}\frac{(y_{t+1}^H - c_{t+1}^{H^*})}{(y_t^H - c_t^{H^*})}\Phi_H'\left(\frac{\pi_{t+1}^H}{\pi_{t+1}^{I,H}}\right)\frac{(\pi_{t+1}^H)^2}{\pi_{t+1}^{I,H}}\right\},$$

$$(\epsilon_{H^*}-1)=\epsilon_{H^*}mc_t^{H^*}-\Phi'_{H^*}\left(\frac{\pi_t^{H^*}}{\pi_t^{I,H^*}}\right)\frac{\pi_t^{H^*}}{\pi_t^{I,H^*}}+E_t\left\{\chi_{t,t+1}\frac{c_{t+1}^{H^*}}{c_t^{H^*}}\Phi'_{H^*}\left(\frac{\pi_{t+1}^{H^*}}{\pi_{t+1}^{I,H^*}}\right)\frac{(\pi_{t+1}^{H^*})^2}{\pi_{t+1}^{I,H^*}}\right\},$$

where $c_t^{H^*}$ denotes exports of home goods.

2.5 Imported goods

Imported goods are sold domestically by two sets of firms. One of them includes competitive firms charging a domestic prices equal to the foreign price multiplied by the exchange rate (*i.e.*, these goods have perfect pass-through). Thus, for them the relevant relative price is rer_t. The other group of firms buy imported goods from abroad and sell them domestically in a monopolistic-competitive structure, subject to price adjustment costs. Let $p_t^{Fs}=\frac{P_t^{Fs}}{P_t}$ be the (relative) price of these goods domestically (in local currency) and mc_t^F the real marginal costs, these two are related by:

$$rer_t=p_t^{Fs}mc_t^F.$$

Given this marginal cost, the optimality condition for these prices is analogous to those previously described for wages and home goods, *i.e.*,

$$(\epsilon_F-1)=\epsilon_F mc_t^F-\Phi'_F\left(\frac{\pi_t^{Fs}}{\pi_t^{I,F}}\right)\frac{\pi_t^{Fs}}{\pi_t^{I,F}}+E_t\left\{\chi_{t,t+1}\frac{c_{t+1}^F}{c_t^F}\Phi'_F\left(\frac{\pi_{t+1}^{Fs}}{\pi_{t+1}^{I,F}}\right)\frac{(\pi_{t+1}^{Fs})^2}{\pi_{t+1}^{I,F}}\right\},$$

where $\pi_t^{Fs}=\frac{P_t^{Fs}}{P_{t-1}^{Fs}}$.

Finally, if α_{Fs} denotes the fraction of imported goods whose prices are subject to adjustment costs, the following determines the overall domestic import price:

$$p_t^F=\left(p_t^{Fs}\right)^{\alpha_{Fs}}(rer_t)^{1-\alpha_{Fs}}.$$

2.6 Commodities

At any time t there is an endowment of commodities y_t^{Co} that is fully exported at an exogenous international price $P_t^{Co^*}$ (in dollars).

2.7 Fiscal and monetary policy

The consolidated balance sheet of the government is given by:

$$P_tg_t=S_t\left(B_t^{T^*}-R_t^*B_{t-1}^{*,T}\right)+\left(B_t^T-R_{t-1}B_{t-1}^T\right)+T_t,$$

where g_t is an exogenous process.

In this setup, T_t adjusts to satisfy this constraint (fiscal policy is passive) and thus Ricardian equivalence holds (only the path of g_t matters for equilibrium determination). In turn, the monetary authority sets a Taylor-type rule for interest rates:

$$\left(\frac{R_t}{\bar{R}}\right) = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\alpha_R} \left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi} \left(\frac{\Delta y_t^H}{a}\right)^{\alpha_{y^*}} \left(\frac{\pi_t^s}{\bar{\pi}^s}\right)^{\alpha_{\pi^s}}\right]^{1-\alpha_R} e_t^{MP}, \tag{3}$$

where $\Delta y_t^H \equiv \frac{y_t^H a_{t-1}}{y_{t-1}^H}$ is the growth rate of non-commodity's GDP and e_t^{MP} is an i.i.d. policy shock.

The rule determines the evolution of the policy rate (relative to it's steady state value $\bar{R}$), as a function of deviations of inflation from the target ($\bar{\pi}$), GDP growth from the long run trend (a), the nominal exchange rate from its steady state value ($\bar{\pi}^S$) and past interest rates. All steady state values $\bar{R}$, $\bar{\pi}$, $\bar{\pi}^S$ and a are consistent with each other (*i.e.*, as implied by equilibrium relationships). They are a function of the chosen inflation target $\bar{\pi}$ and parameters such as the discount factor β, productivity trend a and foreign average inflation π^*. The calibration of the parameters describing the rule will be discussed below.

2.8 Rest of the world

The domestic economy interacts with the rest of the world through several channels. The interest rate is given by:

$$R_t^* = R_t^W \exp\left\{\phi_B\left(\bar{b} - b_t^*\right)\right\}\xi_t^*, \tag{4}$$

where R_t^W denotes the world interest rate, the term $\exp\left\{\phi_B\left(\bar{b} - b_t^*\right)\right\}$ is a debt-elastic premium (with $b_t^* \equiv (B_t^{H^*} - B_t^{T^*})/P_t^*$ denoting aggregate net-foreign assets in real terms) which serves as the "closing device" (see Schmitt-Grohé and Uribe, 2003), $\phi_B, \bar{b}$ are parameters and ξ_t^* is a risk-premium shock that captures deviations from the interest rate parity. In the baseline model, ϕ_B is calibrated to a small but positive number, while $\bar{b}$ is pinned down by the averge trade-balance to output ratio.

Finally, the foreign demand for exports is determined by:

$$c_t^{H^*} = \left(p_t^{H^*}\right)^{\eta^*} y_t^*.$$

Overall, there following are exogenous processes determined in the rest of the world: foreign inflation $\pi_t^* = \frac{P_t^*}{P_{t-1}^*}$, the relative price of commodities $p_t^{Co^*} = \frac{P_t^{Co^*}}{P_t^*}$, the world interest rate R_t^W, the risk premium ξ_t^* and foreign output y_t^*.

2.9 Aggregation and market clearing

Market clearing conditions have to be satisfied in all markets, *i.e.*,

$$y_t^H = c_t^H + c_t^{H^*} + g_t, \quad y_t^H = z_t a_t h_t.$$

Moreover, we assume that nominal adjustment costs are paid in final consumption units, such that:

$$y_t^C = c_t + \Phi_P^H\left(\frac{\pi_t^H}{\pi_t^{I,H}}\right)p_t^H\left[y_t^H - c_t^{H^*}\right] + \Phi_P^{H^*}\left(\frac{\pi_t^{H^*}}{\pi_t^{I,H^*}}\right)p_t^{H^*}rer_t c_t^{H^*} + \Phi_F^{H^*}\left(\frac{\pi_t^{Fs}}{\pi_t^{I,F}}\right)p_t^{Fs}c_t^F + \Phi_W\left(\frac{\pi_t^W}{\pi_t^{I,W}}\right)w_t h_t.$$

In equilibrium, $h_t^d = h_t$. Real GDP is defined as:

$$gdp_t = c_t + g_t + c_t^{H*} + y_t^{Co} - c_t^F,$$

In addition, the following relate inflation rates with relative prices:

$$\frac{p_t^H}{p_{t-1}^H} = \frac{\pi_t^H}{\pi_t}, \qquad \frac{rer_t}{rer_{t-1}} = \frac{\pi_t^s \pi_t^*}{\pi_t}, \qquad \frac{p_t^{F^*}}{p_{t-1}^{F^*}} = \frac{\pi_t^{F^*}}{\pi_t^*}, \qquad \frac{p_t^{H^*}}{p_{t-1}^{H^*}} = \frac{\pi_t^{H^*}}{\pi_t^*}, \qquad \frac{w_t}{w_{t-1}} = \frac{\pi_t^W}{\pi_t a_{t-1}}.$$

The evolution of net foreign assets follows from the resource constraints of households, firms and the government:

$$rer_t b_t^* + tb_t = rer_t * \frac{b_{t-1}^*}{a_{t-1}\pi_t^* R_{t-1}^*},$$

where the trade-balance in real terms tb_t is given by:

$$tb_t \equiv rer_t p_t^{Co^*} y_t^{Co} + p_t^H c_t^H - p_t^F c_t^F.$$

2.10 Functional forms

Here we describe the functional forms used for adjustment costs as well as assumptions regarding indexation process. Firms choosing prices face a quadratic adjustment cost as in Rotemberg (1982):

$$\Phi_j\left(\frac{\pi_t^j}{\pi_t^{I,j}}\right) = \frac{\phi_j}{2}\left[\frac{\pi_t^j}{\pi_t^{I,j}} - 1\right]^2, \tag{5}$$

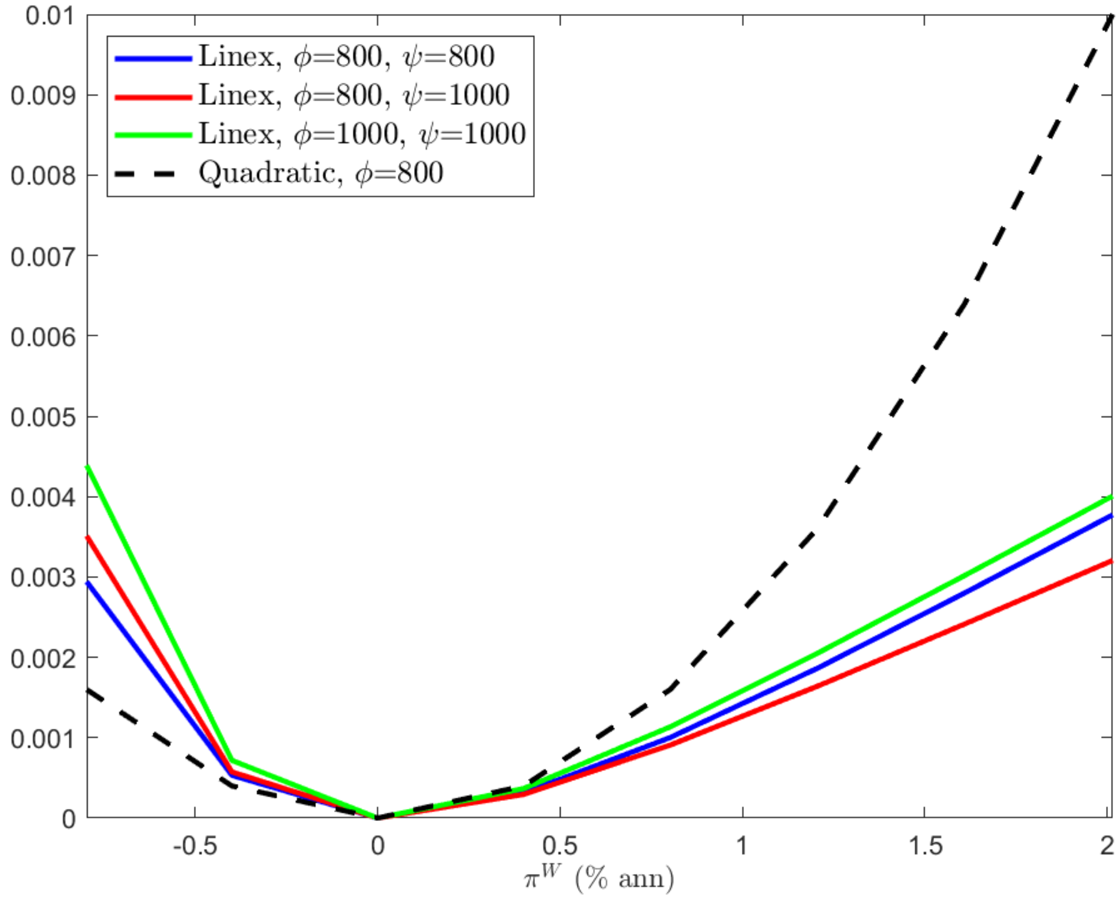
for $j = H, H^*, F$. The parameter ϕ_j determines the degree of price stickiness (if $\phi_j = 0$, prices are flexible in sector j).

Instead, in order to capture asymmetric wage-adjustment costs, we specify a Linex function, following Kim and Ruge-Murcia (2009, 2011):

$$\Phi_W\left(\frac{\pi_t^W}{\pi_t^{I,W}}\right) = \frac{\phi_W}{\psi_W^2}\left[e^{-\psi_W\left(\frac{\pi_t^W}{\pi_t^{I,W}}-1\right)} + \psi_W\left(\frac{\pi_t^W}{\pi_t^{I,W}} - 1\right) - 1\right]. \tag{6}$$

Here, Φ_W captures the overall stickiness, while ψ_W determines the degree of asymmetry in the function (if $\psi_W \to 0$, the function converges to a quadratic equation as in (5)). Figure 1 plots the Linex function for alternative parameter values, comparing them also with the quadratic-adjustment cost case.

Figure 1: Linex Adjustment Cost Function



Finally, for all prices and wages featuring nominal rigidities we allow for the possibility of indexation. In particular, for price we consider:

$$\pi_t^{I,j} = [\pi_{t-1}]^{\mu_j} ,$$

for $j = H, H^*, F$. Here μ_j determines the degree of indexation to past inflation. The structure for wages is slightly different:

$$\pi_t^{I,W} = [\pi_{t-1}]^{\mu_W} a_{t-1}^{\mu_{Wa}},$$

The parameter μ_{Wa} is included because, according to the model, nominal wage do not only increase in the long run due to inflation but also because of long-run productivity growth.

2.11 Parameterization

The selection of parameter values follows heavily the estimation of a DSGE model for Uruguay performed by Basal et al. (2016). Here we discuss three aspects that are specially relevant for the analysis in this paper. First, in terms of shocks, we focus only on those that we can directly estimate from the data, for they are assumed to be strictly exogenous in the model. These are all the external shocks previously mentioned: foreign inflation π_t^*, the relative price of commodities $p_t^{Co^*} = \frac{P_t^{Co*}}{P_t^*}$, the world interest rate R_t^W, the risk premium ξ_t^* (approximated by the EMBI index for Uruguay) and foreign output y_t^*. In addition, government expenditures, g_t and output from the commodity sector, y^{Co} are also estimated from the data. All of these are specified as AR(1) processes and their parameters are chosen as in Basal et al. (2016).

Second, the baseline parameters for the Taylor rule are $\alpha_R = 0.74$, $\alpha_\pi = 1.6$, $\alpha_{y^*} = 0.4$, $\alpha_{\pi^s} = 0$; again following the estimation in Basal et al. (2016).

Finally, parameters related to price and wage rigidties are chosen to minimize the distance between the following moments generated by the model and those observed in the data:

- Variables: Core inflation (IPX-N), Imported goods inflation (IPX-T, excluding Meat and Dairy products), Nominal wages.

- Moments: Variance, auto-correlations of order 1 and 2.

Table 1 describes the calibration of nominal rigidity parameters used in the baseline benchmark.

Table 1: Calibration of Nominal Rigidity parameters

ϕ_H	μ_H	ϕ_{H^*}	μ_{H^*}	ϕ_F	μ_F	ϕ_W	ψ_W	μ_W	μ_{Wa}
800	0.3	600	0	700	0.9	2000	2300	0.6	1

2.12 Discussion about optimal inflation target

The choice of the optimal level for the inflation target $\bar{\pi}$ is related to the presence of nominal rigidities (otherwise money is neutral in this model, so alternative values for $\bar{\pi}$ would produce the same level of welfare). In the absence of indexation, either price and wage adjustment-cost functions are minimized at zero inflation. In particular, as foreign inflation is assumed to be positive, achieving zero domestic imported inflation would require a negative nominal depreciation rate. Besides this observation, in steady state all these can be achieved simultaneously.

But in an stochastic equilibrium there are several trade-offs. First, full price and wage stability cannot be achieved at the same time. Second, concentrating only on prices and not wages, there is also a trade-off because both home and imported goods are sticky and therefore using the nominal exchange rate to minimize minimize distortions coming from home-price stickiness will exacerbate inefficiencies from imported prices rigidities. Thus, optimal policy will require a positive target in a stochastic equilibrium to balance these trade-offs.

In this context, welfare is affected by overall volatility (which required working with a second-order of approximation to equilibrium conditions). In addition, if there are asymmetries (like those induced by the Linex adjustment costs) it is desirable to reduce the chance of reaching situations in which the welfare costs are relatively higher; *i.e.*, those featuring negative inflation. This also provides a reason to target a positive inflation.

Finally, it is worth noticing that overall volatility is not only influenced by size of the shocks. Endogenous propagation mechanisms as well as monetary policy can also contribute to overall volatility. Thus, a priori, the question of optimal inflation target cannot be separated from the issue of how monetary policy is conducted whenever there are deviations from the target. For instance, it is expected that under a more dovish Taylor rule, it is optimal to target a higher level of inflation, as a more dovish policy increases overall volatility.

All these intuitive analysis will we studied quantitatively in the next section, using the baseline model and alternative parameterizations.

3 Results

The main goal is to characterize the value of $\bar{\pi}$ that maximizes the unconditional welfare of the representative household. Results below are presented in terms of welfare-equivalent consumption relative to the value $\bar{\pi}^{opt}$ that maximizes welfare. In particular, for a given model configuration/parameterization we compute $\Lambda(\bar{\pi})$ such that:

$$Wel\left[c(\bar{\pi}^{opt}), h_t(\bar{\pi}^{opt})\right] = Wel\left[\left(1 + \frac{\Lambda(\bar{\pi})}{100}\right) c_t(\bar{\pi}), h_t(\bar{\pi})\right]$$

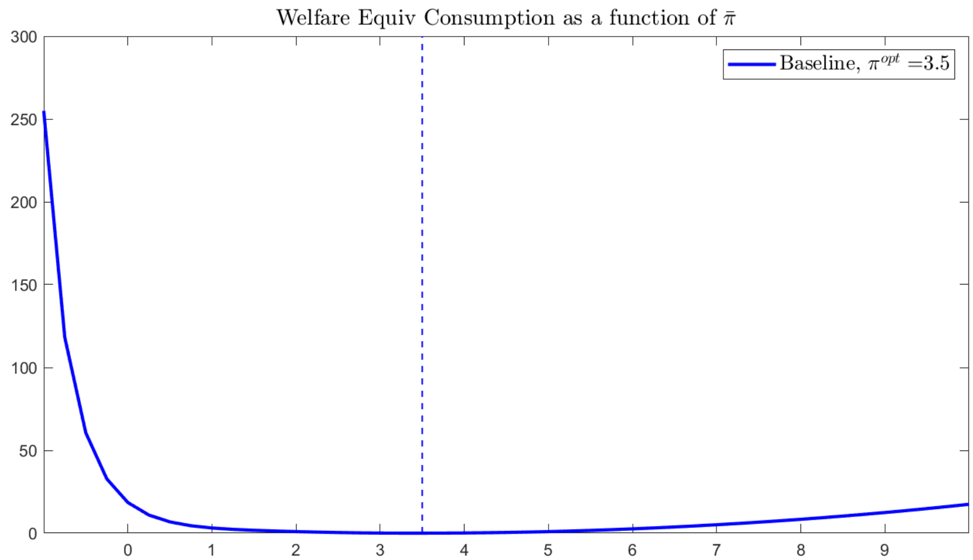
, where $Wel\left[c_t(\bar{\pi}), h_t(\bar{\pi})\right]$ denotes the unconditional expectation of the life-time utility (1), while $c_t(\bar{\pi}), h_t(\bar{\pi})$ denote the state-contingent allocations of, respectively, consumption and labor obtained when the target inflation equals an arbitrary value $\bar{\pi}$.

In other words, $\Lambda(\bar{\pi})$ measures the per-period consumption compensation (in percentage terms) that would make the representative household indifferent between living in world with an arbitrary level of inflation $\bar{\pi}$, relative to that under the optimal target $\bar{\pi}^{opt}$. These are obtained from a second-order of approximation of the equilibrium conditions, using a pruning method (Andreasen et al., 2018) to numerically compute unconditional expectations, implemented in Dynare.

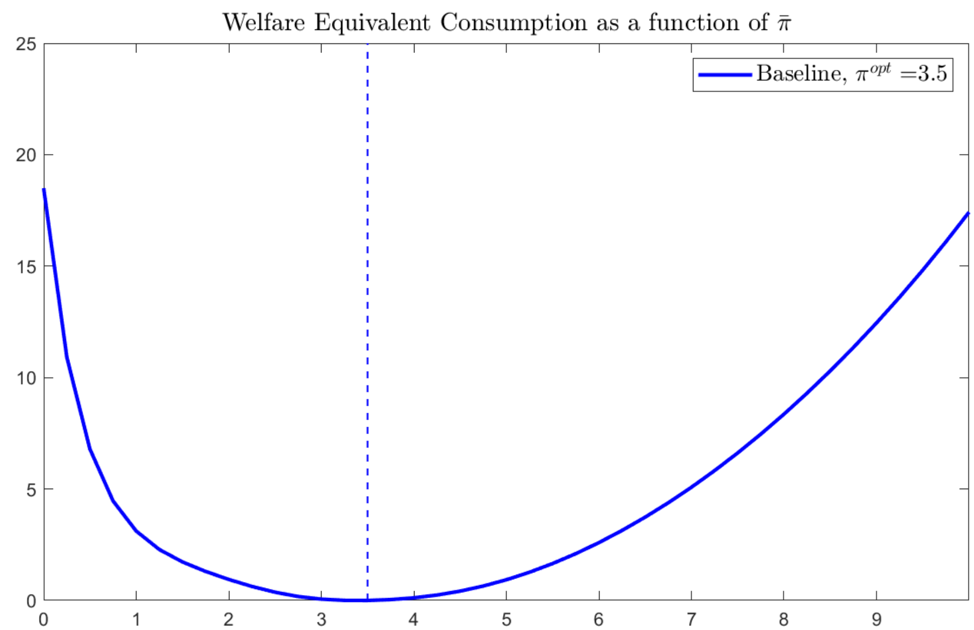
Panel A of Figure 2, displays the welfare equivalent consumption for a range of values for $\bar{\pi}$ going from -1% to 10% (in what follows, inflation targets are expressed in annualized terms). As it can be seen, the welfare cost of having a negative target is significantly larger than under the positive values considered in this range. This is in part due to the presence of asymmetric wage adjustment costs that induce larger welfare costs for negative inflation realizations. Given this, in what follows we focus only on positive values for the inflation target, as described in Panel B.

Figure 2: Welfare Evaluation. Baseline Model

A. Including values for $\bar{\pi} < 0$



B. Only $\bar{\pi} \geq 0$



We can see that the optimal inflation target for the baseline parameterization is 3.5% in annual terms. Moreover, targets in the 2 to 5% range induce a relatively mild welfare cost relative to the optimal, smaller than 1% of per-period consumption. However, the cost increases more than proportionally for values further away from the optimal $\bar{\pi}^{opt}$. As expected, welfare costs increase faster for targets smaller than $\bar{\pi}^{opt}$ compared to those under larger target values.

As discussed before, quantitatively the optimal level of inflation depends on the overall volatility of the shocks hitting the economy. Figure 3 compares the baseline results with those obtained by either doubling or halving the volatility of shocks to exogenous variables (Σ denotes the overall scale of shocks' standard deviations). We can see that if volatilities are doubled, we obtain an optimal target of 6%, while the target is just 1.25% if shocks are half as volatile than in the benchmark. While these results are qualitatively expected given the previous discussion, quantitatively we see that the optimal target does not change proportionally with the overall shock volatility.

Figure 3: Welfare Evaluation. The Role Of Overall Volatility

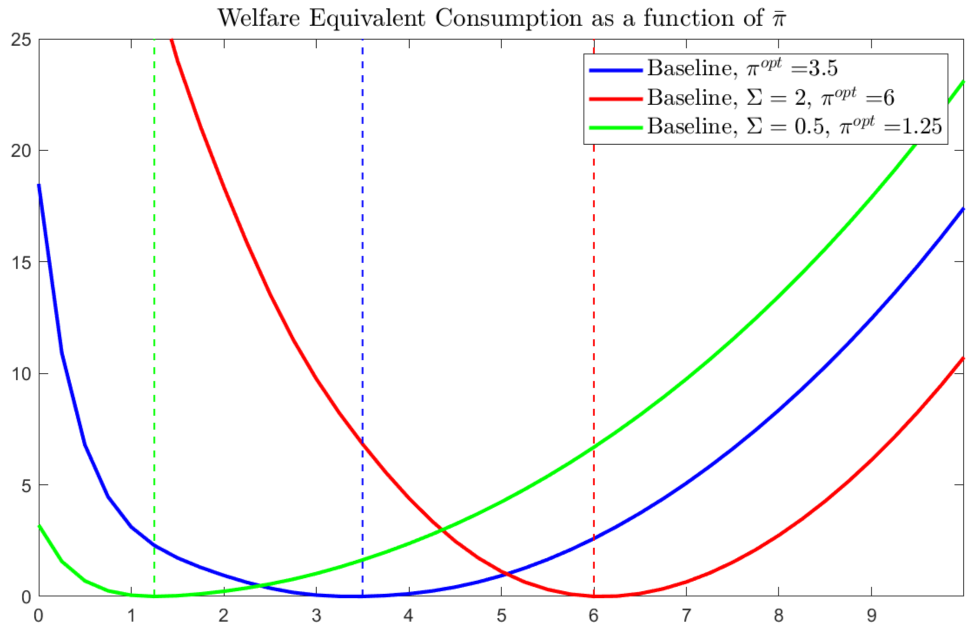
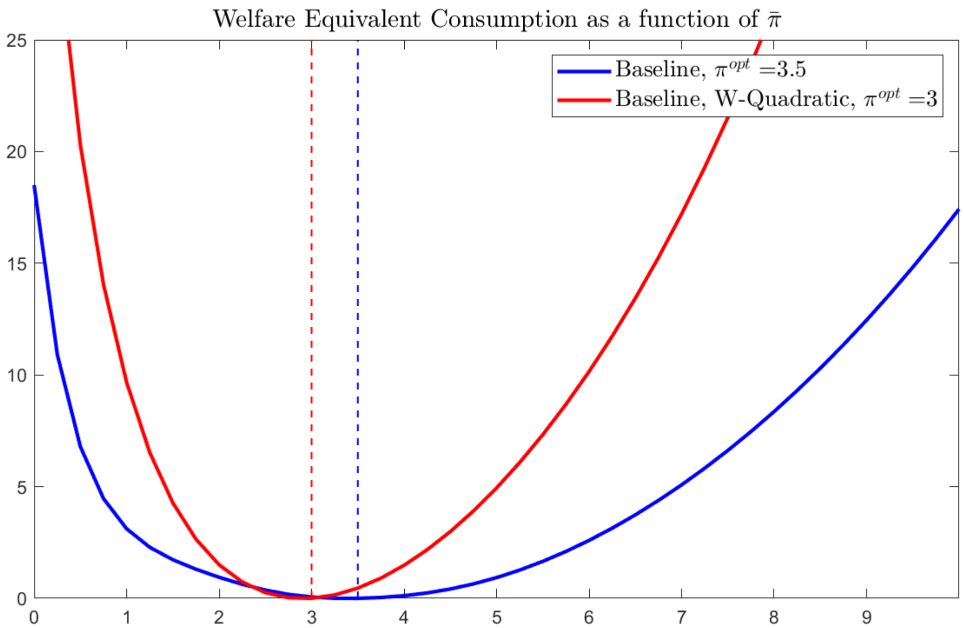


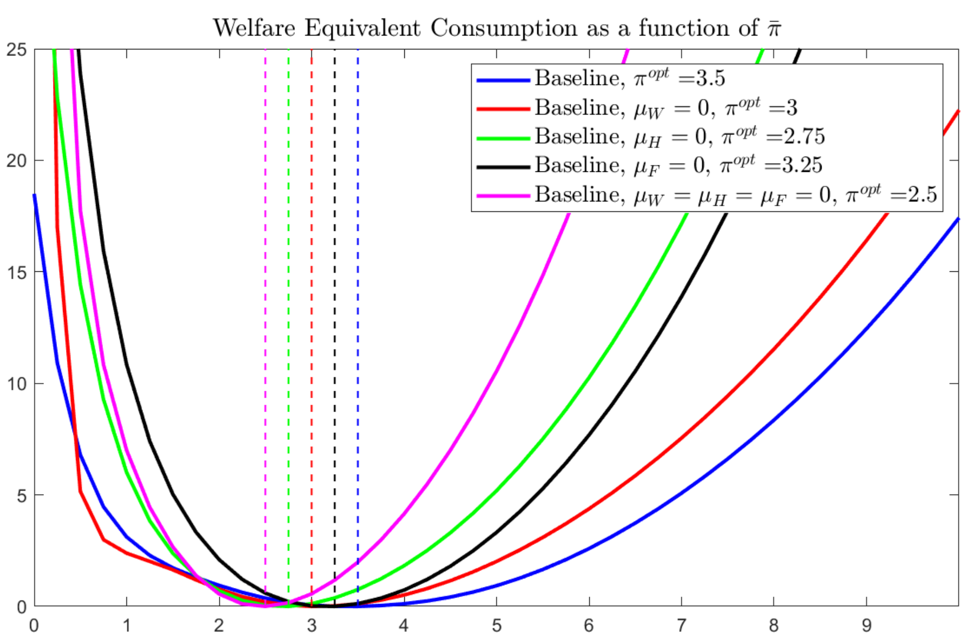
Figure 4 evaluates how the baseline results depend on the assumption of asymmetric wage adjustment, by comparing results with an alternative in which wages rigidities are generated by quadratic adjustment costs (assuming the same value for ϕ_W as in the baseline). As expected, the optimal inflation target is smaller (3% vs 3.5% in the baseline). But as we can see, the optimal value is still positive, as the trade-off from having several nominal rigidities is still present; requiring a non-zero target even under symmetric adjustment costs.

Figure 4: Welfare Evaluation. Quadratic vs. Linex Wages



In Figure 5 we explore the role of indexation to past inflation in all domestic prices and wages. We do so by setting the indexation parameter $\mu_j = o$, for $j = W, H, F$, one at the time, including also an alternative that sets all these parameters to zero simultaneously. As can be seen, eliminating indexation reduces the optimal inflation target, with the largest difference in the optimal inflation target generated by setting to zero indexation in domestic Home-good prices.

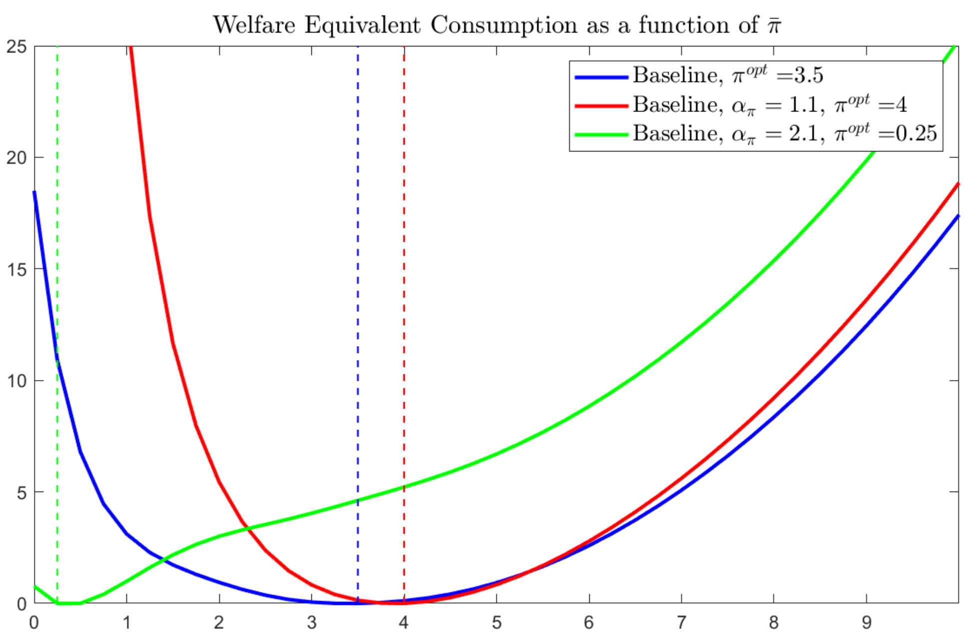
Figure 5: Welfare Evaluation. The Role of Indexation



In principle, there are two forces at play when eliminating indexation. One the one hand, inflation becomes less persistent and therefore less volatile without indexation, which in turn reduces the need to have a relatively larger target. On the other hand, a reduction in inflation may induce non-zero adjustment costs, as long as inflation falls below the required indexation. This in turn implies that a larger target might be required. Overall, the results in Figure 5 show that the first force seems to dominate.

As mentioned in the last section, a non-trivial interaction is expected between the optimal inflation target and the way monetary policy reacts to deviations relative to the target. Figure 6 compares the baseline Taylor rule, with an inflation-reaction parameter $\alpha_\pi = 1.6$, with two alternatives, one with a lower value ($\alpha_\pi = 1.1$) and another with a larger coefficient ($\alpha_\pi = 2$).

Figure 6: Welfare Evaluation. The Role of α_π



The optimal target is decreasing in the aggressiveness of the Taylor rule to deviations of inflation from the target. As a more hawkish rule will reduce overall inflation volatility, the probability of facing situation with reductions in inflation (which are relatively more socially costly) is diminished and therefore the economy can afford a target closer to zero. We also see that, quantitatively, the impact of different degrees of inflation reaction on the optimal target is asymmetric: a more hawkish rule induces a larger reduction in the optimal target than the increase that is observed with a similar reduction in α_π.

Figure 7: Welfare Evaluation. The Role of α_y

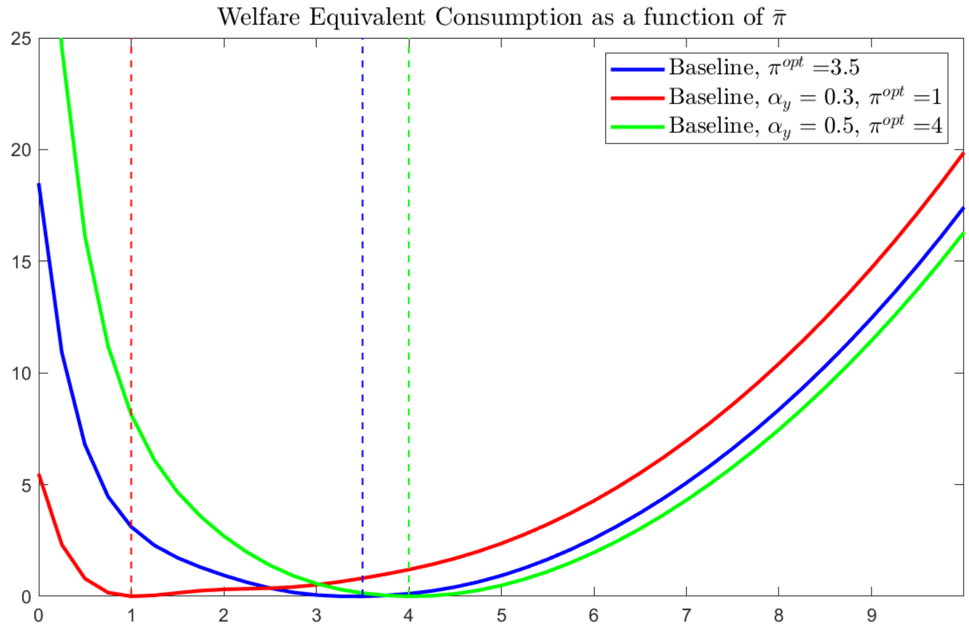
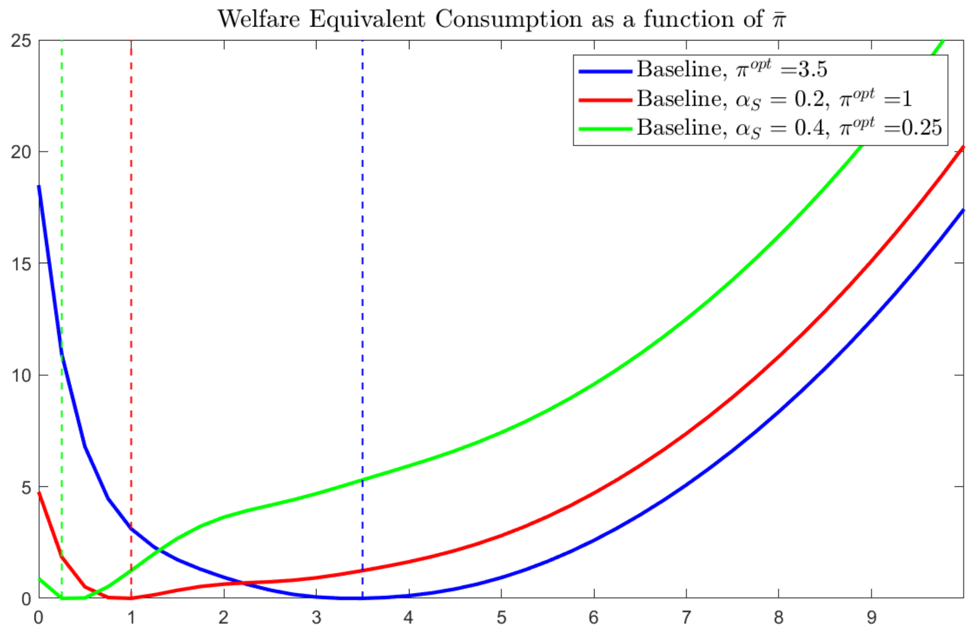


Figure 7 displays the comparison in terms of the Taylor-rule reaction to output-growth deviations from trend growth (α_y, equal to 0.4 in the baseline). Here we see a similar result than what we obtained for α_π. In this case, a larger α_y represents a central bank relatively less concern with inflation. As such, inflation is more volatile and therefore a larger target is optimally required to reduce the chance of facing situations with negative inflation.

Finally, Figure 8 explores the potential role for an exchange rate concern in the Taylor Rule (α_S, which is set to zero in the baseline). As we can see, a reaction to nominal depreciations different from the one consistent with the inflation target plays a qualitatively similar role than increasing the concern for inflation. For such a rule will limit exchange rate fluctuations, which in turn reduces overall inflation and the inefficiencies associated with imported price stickiness, requiring a lower inflation target to maximize welfare.

Figure 8: Welfare Evaluation. The Role of α_S



4 Conclusions

TBA

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