Nestedness and systemic risk in financial networks

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Abstract

In this paper, we to explore the relationship between node nestedness contribution and network stability in financial networks. We rely on data from the Brazilian interbank market. For each bank in the network, we computed the individual nestedness contribution (INC), beside two measures of systemic risk: systemic impact (SI) and systemic vulnerability (SV). The INC is computed considering the different roles played by the banks: lender and borrower. We found banks with a higher INC would cause more damage to the network if they were hit by a shock – i.e, they have a higher SI. However, they are not necessarily those with a higher SV – i.e., more vulnerable to shocks on the network. A positive correlation between INC and vulnerability is observed only when the lenders' INC is considered.

Keywords: nestedness, complex networks, systemic risk, financial networks

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1 Introduction

Nestedness is a hierarchical structure commonly observed in complex networks. In a perfectly nested network, the neighbors of a node also interact with the nodes with a higher topological measure – usually, the degree. The nodes with many (few) counterparties are called *generalists* (*specialists*). Specialists interact mostly with generalists and interactions among specialists are unusual (Bascompte et al., 2003).

A simple illustration of a perfectly nested network is depicted in Figure 1. We portray a bank-firm credit network. Each row (column) corresponds to a firm (bank). Banks (firms) are labeled as B1,..., B6 (F1,..., F7). A colored square represents a loan extended by the bank in the corresponding column to the firm in the corresponding row. The two types of nodes – banks and firms – are ranked in descending order according to the degree (firms from top to bottom, banks from left to right). The banks connected to a given firm are also connected to firms with a higher degree. For instance, bank 3 is connected to firm 5 (the blue square in the figure) and it is also connected to firms above firm 5 (with a higher degree). Similarly, firms connected to a given bank are also connected to banks with a higher degree (e.g., firm 5 is connected to banks on the left of bank 3). The more generalist (specialist) banks correspond to the columns located on the left (right) of the figure. Similarly, the more generalist (specialist) firms correspond to the rows located at the top (bottom) of the figure.

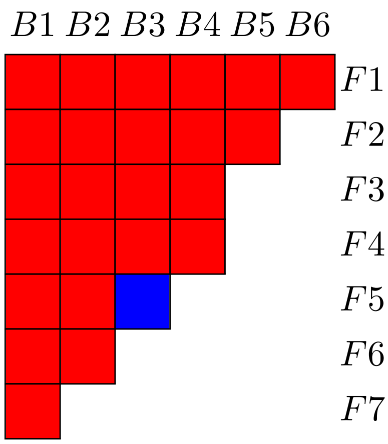

Figure 1: *Example of a perfectly nested bank-firm credit network. The connection between bank 3 and firm 5 is represented by the blue square. Bank 3 is connected to the firms above firm 5, as they have a degree higher than that of firm 5. Similarly, firm 5 is connected to the banks on the left of bank 3, i.e., those with a degree higher than that of bank 3.*

Nestedness is closely related to some network topological properties. Some studies (Abramson et al., 2011; Jonhson et al., 2013) confirmed that nestedness is significantly correlated with disassortativity. Lee et al. (2016) point out that nestedness is a generalization of the core-periphery structure. Payrató-Borras et al. (2019) propose that the most heterogeneous networks in terms of degree distribution are also the most nested ones. Moreover, nestedness also correlates with properties not captured by the topological structure of the network. Nestedness minimizes competition and allows for the coexistence of a higher number of species in ecological networks (Bastolla et al., 2009). Bustos et al. (2012) show nestedness in industrial ecosystems is quite stable, and hence it predicts the appearance and disappearance of individual industries in each location. The nestedness of world trade networks plays an important role in the prediction of countries' growth trajectories (Cristelli et al., 2017; Tacchella et al., 2012).

Saavedra et al. (2011) follow a slightly different approach, in the sense their focus is on how the nodes' contribution to the network nestedness – rather than the nestedness itself – is related to network properties. Assessing an ensemble of flowering plant/insect pollinator networks and a network of designer and contractor firms in the New York City garment industry, they reached two main conclusions. First, the removal of a strong contributor to network nestedness tends to decrease overall network persistence more than the removal of a weak contributor. Second, strong contributors to nestedness are the nodes most vulnerable to extinction.

The purpose of this paper is to explore the relationship between node nestedness contribution and network stability in financial networks.[1] Using quarterly information from March 2012 through December 2015 of the Brazilian interbank market, we apply the methodology developed by Saavedra et al. (2011) to compute the individual nestedness contribution (INC) of banks. The INC of a given node is computed by comparing the nestedness of the network when the interactions of this node are randomized. Keeping the same number of connections, the original links are deleted and new connections are created. The average nestedness of the randomized network is computed by performing such randomization as many times as possible. The INC of the node is given by comparing the average nestedness of the randomized network to that of the original network. If the average nestedness increases (decreases) when the node links are randomized, its INC is positive (negative).

We innovate in this study by computing the INC according to the role played by the bank in the interbank network – borrower or lender. To obtain the lending INC of a given bank, we randomize only its outgoing links – the loans granted by the bank – and keep its incoming links — the loans received by the bank -– fixed. The borrowing INC is computed similarly, through the opposite operation.

After computing the INC of the nodes, we assess the correlation between INC and two systemic risk measures presented in Alexandre et al. (2021): the systemic impact (SI) and the systemic vulnerability (SV) of the banks. While the former refers to the loss caused by a shock in the bank to the whole system, the latter measures the loss suffered by the bank in case of a shock in the system. In order to compute both SI and SV, we consider different levels of shock. Note, according to Saavedra et al. (2011) findings, we expect to find a positive relationship between the INC and both SI and SV.

Our main conclusions are the following: i) INC correlates positively to SI. Thus, nodes that contribute to the nestedness of the network are those that would cause more damage to the network if they were hit by a shock, and ii) nodes with higher INC are not necessarily the most vulnerable to

[1] Despite nested networks having been discovered (Patterson and Atmar, 1986) and mainly studied in ecology (Bascompte and Jordano, 2013), nestedness has also been reported in financial (König et al., 2014), as well as in other economic networks (De Benedictis and Tajoli, 2011; Saavedra et al., 2009; Tacchella et al., 2012).

shocks on the network. A positive correlation between INC and vulnerability is observed only when the lenders' INC is considered. Therefore, the findings of Saavedra et al. (2011) are only partially corroborated by this study.

We extend the analysis performed by Saavedra et al. (2011) in at least three ways. First, this is the first study to apply the methodology developed in Saavedra et al. (2011) to financial networks. Second, we assess the relationship between nestedness contribution and network stability considering partial shocks. In Saavedra et al. (2011), shocks are complete – i.e., nodes are removed. Here, we consider also the case in which nodes lost a fraction of their resources. Third, we disentangle the INC according to the role played by the node. Specifically, we compute the lending INC and the borrowing INC of the banks. Finally, this study is related to the literature on the role of topological features in identifying systemically important banks (Alexandre et al., 2021; Ghanbari et al., 2018; Kuzubas et al., 2014; Martinez-Jaramillo et al., 2014).

This paper proceeds as follows. Sections 2 and 3 discuss, respectively, the data set and methodological issues. In Section 4, we bring the results concerning the correlation between INC and systemic risk. Finally, final considerations are presented in Section 5.

2 The data set

Using several unique Brazilian databases which comprises supervisory and accounting data, we extract quarterly information from March 2012 through December 2015 (16 periods) and build the bank-bank (interbank) network.

The interbank network comprises all types of unsecured financial instruments registered in the Central Bank of Brazil (BCB). Credit, capital, foreign exchange operations, and money markets are among the main types of financial instruments. Different custodian institutions register and control these operations: Cetip[2] (private securities), the BCB's Credit Risk Bureau System – SCR[3] (credit-based operations), and the BM&FBOVESPA[4] (swaps and options operations).

[2]Cetip is a depositary of mainly private fixed income, state and city public securities, and other securities. As a central securities depositary, Cetip processes the issue, redemption, and custody of securities, as well as, when applicable, the payment of interest and other events related to them. The institutions eligible to participate in Cetip include commercial banks, multiple banks, savings banks, investment banks, development banks, brokerage companies, securities distribution companies, goods and future contracts brokerage companies, leasing companies, institutional investors, non-financial companies (including investment funds and private pension companies) and foreign investors.

[3]SCR is a very thorough data set that records every single credit operation within the Brazilian financial system worth 200BRL or above. Up to June 30th, 2016, this lower limit was 1,000BRL. Therefore, all the data we are assessing have been retrieved under this rule. SCR details, among other things, the identification of the bank, the client, the loan's time to maturity and the parcel that is overdue, modality of loan, credit origin (earmarked or non-earmarked), interest rate, and risk classification of the operation and the client.

[4]BM&FBOVESPA is a privately-owned company that was created in 2008 through the integration of the Sao Paulo Stock Exchange (Bolsa de Valores de Sao Paulo) and the Brazilian Mercantile & Futures Exchange (Bolsa de Mercadorias e Futuros). As Brazil's main intermediary for capital market transactions the company develops, implements and provides systems for trading equities, equity derivatives, fixed income securities, federal government bonds, financial derivatives, spot FX, and agricultural commodities. On March 30th, 2017, BM&FBOVESPA and Cetip merged into a new company named B3.

We compute the net financial exposures taking into account financial conglomerates or individual financial institutions that do not belong to conglomerates (classified as "b1", "b2", or "b4" in the BCB's classification system[5]), removing intra-conglomerate exposures. We exclude institutions with negative equity. Financial institutions' equity was retrieved from `https://www3.bcb.gov.br/ifdata`. Some statistics of the interbank network are presented in Table 1.

Table 1: *Summary statistics of the interbank network.*

Quarter-year	N. of banks	Density	Avg. weighted degree*	Avg. net worth*
01-2012	128	0.0843	2747.6	3516.2
02-2012	128	0.0850	2940.7	3598.1
03-2012	130	0.0825	3142.7	3620.7
04-2012	130	0.0802	3257.2	3690.7
01-2013	130	0.0823	3604.7	3609.1
02-2013	128	0.0837	3401.2	3610.6
03-2013	127	0.0796	3474.4	3728.4
04-2013	127	0.0777	3557.1	3840.9
01-2014	130	0.0773	3551.3	3724.5
02-2014	130	0.0773	3433.9	3830.6
03-2014	130	0.0781	3756.4	3908.8
04-2014	129	0.0732	3970.7	3878.8
01-2015	129	0.0757	3966.3	3943.7
02-2015	130	0.0743	3819.8	4071.2
03-2015	128	0.0781	4023.9	4127.6
04-2015	126	0.0792	4111.5	4181.9

*: in BRL million.

3 Methodology

3.1 Measuring nestedness and INC

In this paper, we quantify nestedness using the NODF (Almeida-Neto et al., 2008).[6] The nestedness of the network N is defined by the following equation:

$$N = \frac{\sum_{i<j}^{C} M_{ij} + \sum_{i<j}^{R} M_{ij}}{\left[\frac{C(C-1)}{2}\right] + \left[\frac{R(R-1)}{2}\right]}. \tag{1}$$

In Eq. 1 above, C (R) is the number of nodes of the type displayed in columns (rows). Note that these numbers can be different in bivariate networks, but will necessarily be equal in univariate networks. For every pair of nodes i and j, $M_{ij} = 0$ if $k_i = k_j$, and $M_{ij} = n_{ij}/min(k_i, k_j)$ otherwise,

[5]See `https://www.bcb.gov.br/content/estabilidadefinanceira/scr/scr.data/metodologia.pdf`.

[6]There is not a consensus on how nestedness should properly be quantified. For this reason, there are other metrics to measure nestedness being used, such as the *spectral radius* (Staniczenko et al., 2013). To more details, see, for instance, Payrató-Borràs et al. (2020) and Mariani et al. (2019), Section 3.1.

where k_i is the number of interactions of node i, and n_{ij} is the number of interactions in common between i and j. N varies between 0 and 1, where 1 designates a perfectly nested network.

The INC is quantified following the methodology developed by Saavedra et al. (2011). The INC of node i is given by the following equation:

$$c_i = \frac{(N - \langle N_i^* \rangle)}{\sigma_{N_i^*}}, \tag{2}$$

where N is the network's observed nestedness, $\langle N_i^* \rangle$ is the average nestedness across an ensemble of random replicates within which the interactions of node i have been randomized, and $\sigma_{N_i^*}$ is the standard deviation of N_i^*. We randomize the interactions of a node following the null model specified in Bascompte et al. (2003), generating 1,000 random replicates. The randomization of the interactions of a given node i works as follows: we cancel some link between i and another node, and then we connect i with another node with which i does not have a connection. Node i is connected to another node j with probability[7]

$$p_{ij} = \frac{1}{2}\left(\frac{k_i}{C} + \frac{k_j}{R}\right), \tag{3}$$

supposing i is a node of the type displayed in columns (if i is a row-type node, k_i and k_j are divided by R and C, respectively, in Eq. 3). We innovate in the computation of the INC by considering the different roles a node can play in a network. In bivariate networks, nodes play only one role. For instance, in a bank-firm credit network, banks are always lenders and firms, borrowers. However, in univariate, directed networks, our innovation can be quite useful. For example, in interbank networks, all nodes are of the same type (banks), but a given node i can be a lender, a borrower, or both. We will compute the lending INC INC_L and the borrowing INC INC_B. The former is obtained by randomizing only its outgoing links, which represent loans granted by i, and keeping its incoming links – loans received by i – fixed. The latter is computed similarly, through the opposite operation. Finally, we compute for each node its total INC $INC_T = INC_B + INC_L$. Observe INC_T will be equal to INC_B (INC_L) if i acts only as borrower (lender) in the interbank market.

3.2 Systemic risk

Saavedra et al. (2011) show the nodes with higher INC are those whose removal leads to a decrease in network persistence, as well as are the more vulnerable to extinction. That is, shocks in strong contributors cause more damage to the whole network, and shocks in the network affect mostly the strong contributors. To test this hypothesis, we compute the *systemic impact* and *systemic vulnerability* – SI and SV, respectively (Alexandre et al., 2021) – for the banks participating in the

[7] See Saavedra et al. (2011), esp. Figure 1 and Methods, for details.

Brazilian interbank market. We take into consideration various levels of the initial shock.

Both SI and SV are computed following the *differential DebtRank* methodology (Bardoscia et al., 2015).[8] The exposure network of the interbank market is represented by $\mathbf{A} \in N \times N$, where N is the number of banks and A_{ij} is the asset invested by i in j. At period 0, we impose an exogenous shock on FI j, reducing its equity by a fraction of ζ. It will cause a subsequent loss $L_{ij}(1)$ to its creditors, indexed by i, equal to $\mathbf{A}_{ij}\zeta$. At period 2, j's creditors will propagate this loss to their creditors in a similar fashion, and so on. Formally, we have

$$L_{ij}(t) = min\left(A_{ij}, L_{ij}(t-1) + \mathbf{A}_{ij}\frac{[L_j(t-1) - L_j(t-2)]}{E_j}\right), \tag{4}$$

$$L_i(t) = min\left(E_i, L_i(t-1) + \sum_j \mathbf{A}_{ij}\frac{[L_j(t-1) - L_j(t-2)]}{E_j}\right), \tag{5}$$

in which $t \geq 0$ and E_j is financial institution (FI) j's equity. Thus, when an FI j suffers an additional loss equal to fraction ζ of its equity, it will impose a loss to its creditors that corresponds to ζ times their exposures towards j. Observe equity positions as well as the exposure network are time-invariant, i.e., they are taken as exogenous. The propagation considers stress differentials rather than stress absolute values (hence the methodology's name) to avoid double-counting.

Observe L_{ij} cannot be greater than A_{ij}. It means that j cannot impose to i a loss greater than i's exposures towards j. When $L_{ij} = A_{ij}$, j stops imposing losses on i. Moreover, L_i cannot be greater than E_i, i.e., i's losses cannot be greater than its equity. When $L_i = E_i$, i stops propagating losses to other FIs.

The system converges after a sufficiently large number of periods $T \gg 1$. Then we have the final matrix of losses $\mathbf{L}^{j,\zeta} \in N \times 1$, where $L_j^{i,\zeta}$ is the total loss suffered by agent j after an initial shock of size ζ on agent i. After repeating this process for the other FIs, we compute our two measures of SR. The *systemic impact* (SI) of bank i is defined as

$$SI_{i\zeta} = \frac{\sum_j \left[L_j^{i,\zeta} - L_j^{i,\zeta}(0)\right]}{\sum_j E_j}, \tag{6}$$

where $L_j^{i,\zeta}(0) = \zeta E_j$ if $j = i$ and 0 otherwise. The *systemic vulnerability* (SV) is represented by the following equation:

[8] The rest of this subsection strictly follows Alexandre et al. (2021).

$$SV_{i\zeta} = \frac{1}{N}\sum_{j}\frac{L_i^{j,\zeta} - L_i^{j,\zeta}(0)}{E_i}. \tag{7}$$

Therefore, $SI_{i\zeta}$ measures the fraction of the aggregate FIs' equity which is lost as a consequence of an initial shock of size ζ at FI i's equity. On the other hand, $SV_{i\zeta}$ refers to the average i's equity loss when the other FIs are reduced by ζ.

As we are interested only in the losses caused by the contagion, we remove the initial shock from the computation of the SR measures. Observe we also compute $SI_{i\zeta}$ for the FI that suffered the initial shock. Due to network cyclicality, a shock propagated by a given FI can hit it back. For the same reason, we include the loss imposed by an FI on itself in the calculation of $SV_{i\zeta}$.

4 Nestedness and systemic risk

Both SI and SV are computed for each node. We vary the level of the initial shock ζ within the interval (0.1,1] with step 0.1. Finally, we compute the correlation between INC (INC_T, INC_B, and INC_L) and systemic risk (SV and SI).

Considering the total INC, we did not observe the correlation between INC and vulnerability found by Saavedra et al. (2011) (Figure 2, left panel). The correlation between total INC and SV is negative; moreover, it is not significantly different from zero for all levels of the initial shock ζ. However, as in Saavedra et al. (2011), the INC is positively correlated to SI (Figure 2, right panel). Therefore, the nodes that contribute the most to the nestedness of the network are also those that would cause more damage to the network in case of suffering a shock. Also, this correlation is nonlinear concerning ζ.

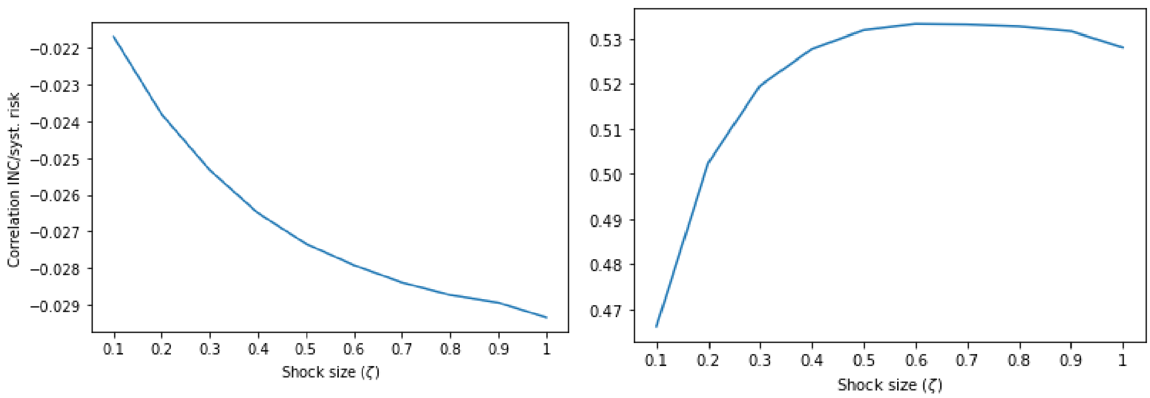

Figure 2: *Correlation between INC_T and SV (left) and SI (right). Except for the left panel, the correlation is statistically different from zero for all levels of ζ (p-value < 10^{-100}).*

Decomposing the INC of the nodes considering both their roles – lender and borrower –, we find that, while INC_L and node vulnerability are positively correlated (Figure 3, left panel), this correlation is negative in the case of INC_B (Figure 4, left panel). In both cases, the absolute value of the correlation increases with the size of the initial shock. Both INC_L and INC_B are positively correlated to SI (Figures 3 and 4, right panel). While in the latter case the correlation increases with ζ, in the former one this relationship is nonlinear, represented by an inverted U-shaped curve.

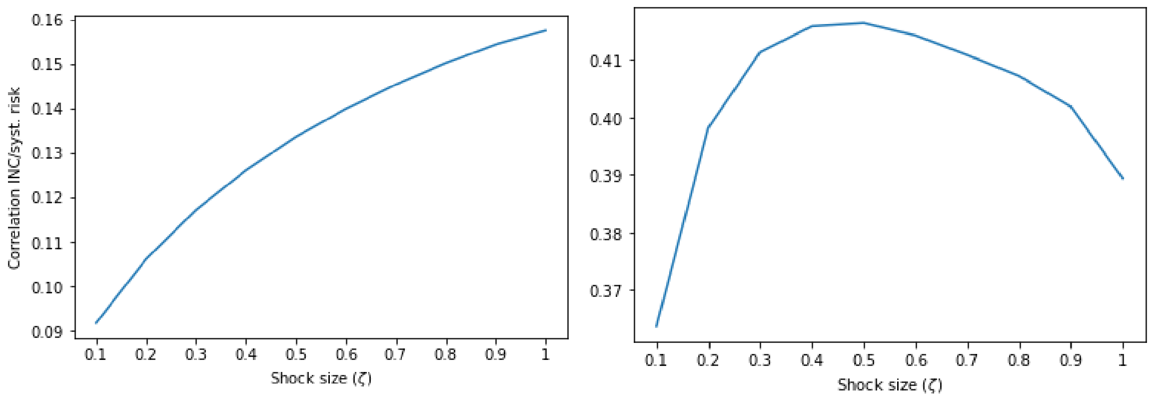

Figure 3: *Correlation between INC_L and SV (left) and SI (right). In both panels, the correlation is statistically different from zero for all levels of ζ (p-value $< 10^{-4}$).*

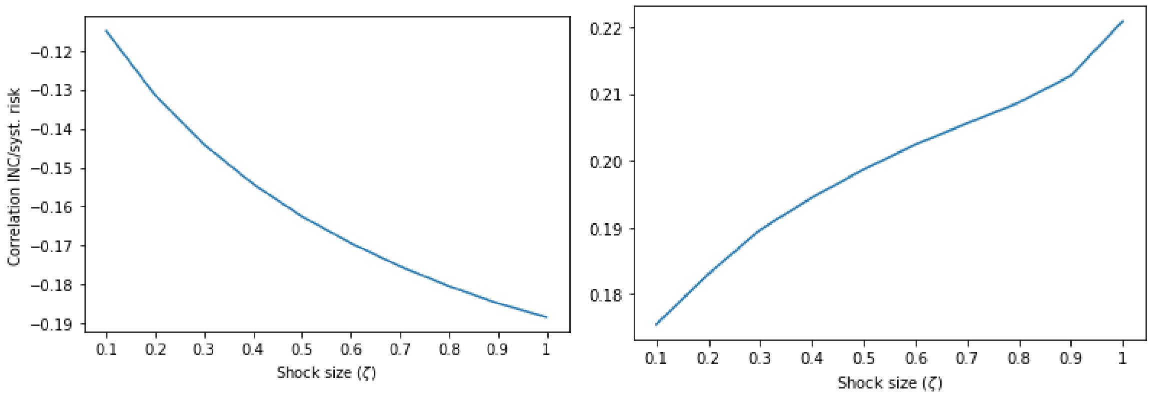

Figure 4: *Correlation between INC_B and SV (left) and SI (right). In both panels, the correlation is statistically different from zero for all levels of ζ (p-value $< 10^{-6}$).*

5 Final considerations

In this study, we assessed the correlation between nestedness and systemic risk of the Brazilian interbank market. Considering the nestedness of the network as measured by the NODF, we calculated the individual nestedness contribution (INC) of the banks, which is a measure of the bank contribution to the network nestedness. The INC was computed separately for the different roles played by banks in interbank markets, lender and borrower.

We assessed the relationship between INC and systemic risk. We computed the correlation between the INC and the systemic impact (SI) – the loss caused in the network by a shock on the node – and systemic vulnerability (SV) – the loss suffered by the node due to a shock in the network – in the interbank network. The INC is positively correlated to the SI. Thus, nodes that contribute the most to the nestedness of the network are those that would cause more damage to the network if they were hit by a shock. The correlation between the total INC and SV is not significantly different from zero. However, while the lending INC is positively correlated to SV, the correlation between the borrowing INC and SV is negative. It means that nodes with a higher lending (borrowing) INC are more (less) vulnerable to shocks on the network. Furthermore, the absolute value of this correlation increases with the size of the initial shock.

This study contributes to the literature on identifying systemically relevant banks through the analysis of the topological features of the financial network. We show the INC is correlated to the systemic importance of banks. Shocks on banks with higher INC would cause a higher loss in the whole system. Moreover, shocks on the system would cause more damage to banks with a greater lending INC, while banks with a higher borrowing INC would be less impacted by such shocks. A natural follow-up study of this paper would investigate the INC as a driver of the systemic importance of the banks, in a model including other explanatory variables.

References

Abramson, G., Trejo Soto, C. A., and Ona, L. (2011). The role of asymmetric interactions on the effect of habitat destruction in mutualistic networks. *PLoS One*, 6(6):e21028.

Alexandre, M., Silva, T. C., Connaughton, C., and Rodrigues, F. A. (2021). The drivers of systemic risk in financial networks: a data-driven machine learning analysis. *Chaos, Solitons & Fractals*, 153:111588.

Almeida-Neto, M., Guimaraes, P., Guimaraes Jr, P. R., Loyola, R. D., and Ulrich, W. (2008). A consistent metric for nestedness analysis in ecological systems: reconciling concept and measurement. *Oikos*, 117(8):1227–1239.

Bardoscia, M., Battiston, S., Caccioli, F., and Caldarelli, G. (2015). Debtrank: A microscopic foundation for shock propagation. *PloS One*, 10(6):e0130406.

Bascompte, J. and Jordano, P. (2013). *Mutualistic networks*. Princeton University Press.

Bascompte, J., Jordano, P., Melián, C. J., and Olesen, J. M. (2003). The nested assembly of plant–animal mutualistic networks. *Proceedings of the National Academy of Sciences*, 100(16):9383–9387.

Bastolla, U., Fortuna, M. A., Pascual-García, A., Ferrera, A., Luque, B., and Bascompte, J. (2009). The architecture of mutualistic networks minimizes competition and increases biodiversity. *Nature*, 458(7241):1018–1020.

Bustos, S., Gomez, C., Hausmann, R., and Hidalgo, C. A. (2012). The dynamics of nestedness predicts the evolution of industrial ecosystems. *PloS one*, 7(11):e49393.

Cristelli, M. C. A., Tacchella, A., Cader, M. Z., Roster, K. I., and Pietronero, L. (2017). On the predictability of growth. *World Bank Policy Research Working Paper*, (8117).

De Benedictis, L. and Tajoli, L. (2011). The world trade network. *The World Economy*, 34(8):1417–1454.

Ghanbari, R., Jalili, M., and Yu, X. (2018). Correlation of cascade failures and centrality measures in complex networks. *Future Generation Computer Systems*, 83:390–400.

Jonhson, S., Domínguez-García, V., and Muñoz, M. A. (2013). Factors determining nestedness in complex networks. *PloS One*, 8(9):e74025.

König, M. D., Tessone, C. J., and Zenou, Y. (2014). Nestedness in networks: A theoretical model and some applications. *Theoretical Economics*, 9(3):695–752.

Kuzubas, T. U., Omercikoglu, I., and Saltoglu, B. (2014). Network centrality measures and systemic risk: An application to the Turkish financial crisis. *Physica A*, 405:203–215.

Lee, S. H. et al. (2016). Network nestedness as generalized core-periphery structures. *Physical Review E*, 93(2):022306.

Mariani, M. S., Ren, Z.-M., Bascompte, J., and Tessone, C. J. (2019). Nestedness in complex networks: observation, emergence, and implications. *Physics Reports*, 813:1–90.

Martinez-Jaramillo, S., Alexandrova-Kabadjova, B., Bravo-Benitez, B., and Solórzano-Margain, J. P. (2014). An empirical study of the Mexican banking system's network and its implications for systemic risk. *Journal of Economic Dynamics & Control*, 40:242–265.

Patterson, B. D. and Atmar, W. (1986). Nested subsets and the structure of insular mammalian faunas and archipelagos. *Biological journal of the Linnean society*, 28(1-2):65–82.

Payrató-Borras, C., Hernández, L., and Moreno, Y. (2019). Breaking the spell of nestedness: The entropic origin of nestedness in mutualistic systems. *Physical Review X*, 9(3):031024.

Payrató-Borràs, C., Hernández, L., and Moreno, Y. (2020). Measuring nestedness: A comparative study of the performance of different metrics. *Ecology and evolution*, 10(21):11906–11921.

Saavedra, S., Reed-Tsochas, F., and Uzzi, B. (2009). A simple model of bipartite cooperation for ecological and organizational networks. *Nature*, 457(7228):463–466.

Saavedra, S., Stouffer, D. B., Uzzi, B., and Bascompte, J. (2011). Strong contributors to network persistence are the most vulnerable to extinction. *Nature*, 478(7368):233–235.

Staniczenko, P. P., Kopp, J. C., and Allesina, S. (2013). The ghost of nestedness in ecological networks. *Nature communications*, 4(1):1–6.

Tacchella, A., Cristelli, M., Caldarelli, G., Gabrielli, A., and Pietronero, L. (2012). A new metrics for countries' fitness and products' complexity. *Scientific reports*, 2(1):1–7.