Corporate Legacy Debt, Inflation, and the Efficacy of Monetary Policy

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Abstract

While the recent rise in public debt has received much attention, the increase in corporate debt has received less. We show that high corporate debt levels may impede monetary policy transmission and make it qualitatively and quantitatively less effective in controlling inflation. When firms’ indebtedness is sufficiently high, the negative substitution effect on aggregate demand of higher nominal interest rates is offset or even dominated by an additional income effect on creditors and amplifies the cost channel of monetary policy. This mechanism is independent of standard financial and nominal frictions and aggravates the trade-off between inflation and output stabilization.

Keywords: Corporate indebtedness, debt, inflation, working capital, monetary transmission mechanism, income effect, Taylor principle,

JEL Codes: E31, E44, E52

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1 Introduction

The non-financial corporate debt-to-GDP ratio has risen globally since 2007. Corporate indebtedness in the Euro Area increased by almost 14% from an already high 93.3% in 2007. Sweden saw an increase of 26.8% from 125.2%, while in Canada the increase was almost 40%. The COVID-19 pandemic crisis has led to a further sharp buildup of corporate debt. US corporate indebtedness rose by 12.5% between December 2018 and December 2020, much more than its total increase in the decade leading up to the COVID-19 pandemic. Meanwhile, inflation has been rising. We show that corporate debt poses challenges for monetary policy in two ways. First, it introduces an additional income effect across heterogenous households that counteracts the traditional substitution effect and affects the overall effectiveness of interest rates. Second, it creates a more difficult trade-off between output and inflation stabilization, requiring a reassessment of policy priorities. Debt causes demand to offset the usual response of prices to monetary policy, and the elasticity of labor to increase with the policy rate. In response to monetary contractions, for example, these two effects make labor more scarce, amplifying the response of output and muting that of inflation.

While a growing literature on corporate indebtedness focuses on the implications for investment and aggregate demand (Gomes, Jermann and Schmid, 2016; Abraham, Cortina Lorente and Schmukler, 2020; Bräuning and Wang, 2020; Brunnermeier and Krishnamurthy, 2020; Jordà, Kornejew, Schularick and Taylor, 2020; Ottonello and Winberry, 2020), how debt may hamper the transmission mechanism of monetary policy and its effectiveness in controlling inflation has received less attention. In contrast, the buildup of public sector debt has led to concerns about the future path of inflation. In this paper, we assess how effective monetary policy may be in controlling inflation when there is a large amount of corporate legacy debt in the economy.

We first build a flexible price static model and obtain the closed-form solution to show how legacy corporate debt causes monetary policy to have a redistribution effect on income, impeding the effect on inflation through the adjustment of money balances. We then extend the static model to a dynamic setting and show that this effect impedes monetary policy transmission through nominal rigidities. The presence of debt necessitates the distinction between key savers and lenders of the macroeconomy. Our economy features two types of households, lender households, i.e., the bondholders, that

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1 This has also occurred in emerging economies: China, Chile, Brazil, and Turkey have all seen more than a 50% rise during this period. Hong Kong’s non-financial corporate debt-to-GDP ratio soared by over 77% to more than 200%.

2 Throughout the paper, the term ‘corporate debt’ is used to refer to the debt of non-financial corporations.

accumulate safe corporate debt to save, and owner households, the equity holders that own firms that, in turn, issue the corporate debt. The differentiation between these two types of households is consistent with Fisher’s (1910) narrative on the ‘enterpriser-borrower’ and the ‘creditor, the salaried man, or the laborer’. Firms face a working capital constraint along the lines of Christiano, Eichenbaum and Evans (2005) and Ravenna and Walsh (2006) in that they must borrow money to finance expenses for labor in advance of receiving income from production. In addition to working capital loans, firms owe longer-term corporate debt to the lender households.

Our result complements the literature on the cost channel of monetary policy by showing how corporate debt may intermediate the transmission mechanism from monetary policy to economic activity. We show that the income effect of corporate debt and the wealth distribution between heterogeneous households amplifies the impact of this cost channel. In particular, we show that the presence of corporate legacy debt hampers contractionary monetary policy from controlling inflation and that higher debt leads to a smaller fall in inflation after monetary contractions. This result depends on the income effect of corporate debt, which affects both aggregate demand and supply.

We show that the transmission of monetary policy to both aggregate demand and supply depends on corporate debt through an income effect that debt generates. As monetary contractions increase the financial costs of wage bills, there is downward pressure on aggregate demand: the usual substitution effect that pushes down prices. However, when legacy debt is high, firms need to spread the financial cost over a larger production scale and demand more labor, leading to upward pressure on aggregate demand. Put another way, in contrast to representative agent frameworks such as Ravenna and Walsh (2006), the cost channel in our model operates through both the IS and Phillips curves because our real marginal cost depends on the distribution of wealth. The income effect of debt of debt also increases the effective elasticity of labor supply. Following a monetary contraction, both the real wage and the price of corporate bonds decrease, leading to a deterioration of the lender households’ wealth. However, the bond price decreases less relative to real wages in a high-debt case than in a low-debt case. This is because the monetary contraction does not lead to a parallel shift in the term structure of the interest rate. Even though both the short rate for working capital and the long rate for bonds go up, the long rate increases less, and the term structure becomes flatter. Therefore, the negative impact on wealth in the high-debt scenario is less severe than in the low-debt scenario. Thus the effective elasticity of labor supply increases when legacy debt increases and holds even when we include a fixed coupon corporate bond in our robustness checks, consistent with empirical evidence in Ziliak

4see Mankiw and Zeldes (1991), Toda and Walsh (2020), and Doerr, Drechsel and Lee (2022) among many others, documenting and exploiting the heterogeneity in asset ownership along the wealth or income distribution.
When corporate debt is below a certain threshold, the traditional Taylor principle holds: raising the policy rate lowers current inflation. However, higher debt levels lead to smaller falls in prices, meaning that monetary policy becomes less effective in containing inflation. Traditionally, raising the policy rate lowers aggregate demand and causes prices to fall, i.e., the substitution effect that puts downward pressure on aggregate demand and inflation. However, via the income effect through nominal corporate debt, the aggregate demand curve shifts less to the left, and the aggregate supply curve becomes more elastic due to a higher effective labor supply elasticity, and it moves in the same direction as the aggregate demand curve. Therefore, in equilibrium, although output falls responsively, prices and inflation only respond mildly. In the extreme albeit unlikely case, the Taylor principle becomes inverted when the corporate debt level is above a threshold. Raising the policy rate increases inflation because income dominates the substitution effect. These results rely on the heterogeneous income effect across households and reinforce the importance of the heterogeneity of households and the relatively high pro-cyclicality of income and consumption expenditure of high-income and high-wealth households that own an overwhelmingly large share of stocks (see Parker, Vissing-Jorgensen, Blank and Hurst, 2010, for a deconstruction of the cyclical properties of household groups in the US). They also offer a new rationalization of why prices generally respond less than output to monetary disturbances and increase with the level of corporate debt in the economy. At extremely high corporate debt levels, prices may even rise after a monetary contraction.

We embed our model in a New Keynesian framework with heterogeneous households and nominal rigidities via the Calvo Pricing assumption to study the dynamic properties. We show that as the steady-state corporate debt-to-GDP ratio increases, the coefficient of monetary policy rate on the dynamic path of inflation declines, i.e., a weaker effect of monetary contractions in lowering inflation. The output gap reflects two distortions in the economy: the first arising from price rigidities and the second from market incompleteness affecting the distribution of wealth and hence aggregate demand and supply. The latter inefficiency means that inflation targeting should also account for the corporate debt dynamics. In a numerical exercise, we compare the responses of the economy when the steady-state corporate debt-to-GDP ratio is low (benchmark) and when it is high, describing the increase in corporate debt levels in the US over the last 15 years. This quantitative example considers a contractionary monetary shock and a positive consumption demand shock with a standard benchmark Taylor rule. The choice of a positive demand shock is motivated by considering how the recovery of

According to Ziliak and Kniesner (1999), their estimated labor supply elasticities rise with saving wealth so that the hours response to wage changes is about 40 percent larger for the wealthiest 25% men than for the poorest 25%. Cesarini et al. (2017) find winning a lottery prize reduces earnings with effects roughly constant over time.
demand post-pandemic could potentially affect the economy’s monetary profile and pose challenges to price stabilization. Model simulations shed light on the cyclicity of the consumption expenditure of wealthy stockholding households and those who do not hold stocks. We find that the consumption expenditure of owner households, the equity owners, tends to be highly procyclical, whereas the expenditure of the lender households, those who do not own shares, is much less cyclical. As the level of debt increases, the more procyclical owner households’ consumption becomes more procyclical, and lender households’ consumption more acyclical.

On the dynamic responses, after the monetary contraction, inflation falls on impact in both cases before rising to positive values. The subsequent rise in inflation is higher in the high debt case than in the benchmark case, suggesting that the higher corporate indebtedness is, the more challenging it is to rein in inflation. On the real side, output falls in both the high debt and the benchmark cases, but it falls more aggressively in the high debt case. Following a positive consumption demand shock inflation rises and output increases on impact. Notably, inflation is much higher in the high-debt case than in the benchmark case, and the subsequent drop in output and employment is also more severe in the high-debt case.

We conduct a counterfactual experiment where we consider a monetary authority that cares more about output stabilization than our benchmark Taylor rule. In this experiment, an output stabilization Taylor rule could bring output back up to the steady-state rather quickly, whereas our benchmark Taylor rule leads to greater and more persistent output and employment loss. Nevertheless, the output stabilization Taylor rule leads to a much higher inflationary profile. Thus, the path of interest rates that stabilizes the path of inflation may cause instability in output directly through instability in working capital which indirectly causes instability in the path of intertemporal debt. The overall takeaway from this experiment is that the trade-off between inflation stabilization and output stabilization becomes acute with a large volume of corporate debt in the economy.

We extend the dynamic model to include a fixed-coupon corporate bond so that after a monetary contraction, the drop in bond value also causes a deterioration in the lender working households’ wealth, which may raise the concern whether the effective labor supply elasticity would still be higher in the high debt scenario than in the low debt scenario. Our results still go through even after considering this effect. This is because the monetary contraction does not lead to a parallel shift in the term

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6For example, the FOMC’s ‘balanced approach’ of accommodative policy is more consistent with a Taylor rule that includes a much higher output coefficient (see Bernanke, 2015; Yellen, 2012).
7Ravenna and Walsh (2006) show that in the presence of the cost channel, the interest rate changes necessary to stabilize the output gap lead to inflation rate fluctuations. While building on the cost channel, our model shows that the intensity of the trade-off between output stabilization and inflation stabilization depends on the level of corporate debt and working capital loans.
structure of the interest rate. Even though both the short rate for working capital and the long rate for bonds go up, the long rate increases less, and the term structure becomes flatter. Overall, the negative impact on wealth is less severe in the higher debt case, and the income effect of debt on the supply side still prevails.

Related literature. There is a flourishing literature that focuses on (corporate) debt and its implications for inflation and monetary policy (see Gomes, Jermann and Schmid, 2016; Ottonello and Winberry, 2020; Mian, Straub and Sufi, 2021). Existing work has focused on the drag of debt on firm investment or aggregate demand or the impact of unexpected inflation on the real burden of debt, while less attention has been paid to how nominal debt could affect the efficacy of monetary policy in controlling inflation. Our work serves to fill this gap in the literature. Gomes, Jermann and Schmid (2016) investigate how lower-than-expected inflation creates a debt overhang, and the authors focus on the macroeconomic responses to inflation changes while Jungherr, Meier, Reinelt and Schott (2022) show that firms’ investment is more responsive to monetary policy with more maturing debt. The existing literature focuses the effect on investment while we ask how debt hampers the ability of the central bank to control inflation. Our model shows that nominal debt may be a source of inflation even when the central bank tries to use contractionary monetary policy to combat inflation.

Ottonello and Winberry (2020) and Mian, Straub and Sufi (2021) show that monetary policy is less powerful when the debt level is high or the distance to default is low. In Ottonello and Winberry (2020), the authors show that the investment of low debt firms or those with a high distance to default is more responsive to expansionary monetary shocks, while the investment of high debt firms with high default risks is less so; the concern there is not with contractionary monetary policy controlling inflation. Mian, Straub and Sufi (2021) focus on household debt, propose a theory of indebted demand and show that large household debt lowers aggregate demand and the natural rate of interest.8 Whereas both their paper and ours suggest that monetary policy has limited ammunition in the presence of large debt and the policy angles and mechanisms differ. In their paper, the policy angle is on accommodative monetary policy supporting aggregate demand, whereas, in our model, it is on the general ability of monetary policy to target inflation. Furthermore, the mechanism in Mian, Straub and Sufi (2021) works through the demand side where the assumption of non-homothetic preferences generates the property that large debt levels weigh negatively on aggregate demand. Our mechanism works through the income effect of nominal debt shifting the aggregate supply curve in addition to aggregate demand; the income effect of debt flattens the aggregate supply curve, which blunts contractionary monetary policy in lowering inflation. In this regard, we see our work as complementary.

8They point out that household debt, not corporate debt, drives indebted demand.
Other salient examples of corporate indebtedness in the macroeconomy include but are not limited to Bernanke, Campbell, Friedman and Summers (1988), Bernanke, Campbell, Whited and Warshawsky (1990), Farhi and Tirole (2009), Bhamra, Fisher and Kuehn (2011), Occhino and Pescatori (2014, 2015), Greenwald (2019), Darmouni, Giesecke and Rodnyansky (2020), and Lakdawala and Moreland (2021). While our paper shares the similarity with many papers in this literature that inflation reduces the real burden of corporate debt, it differs because of our general equilibrium channel through legacy debt and heterogeneous households. Much of the empirical literature on corporate debt investigates the real consequences of corporate debt on investment, output, or tail risks (see, for example, Mian, Sufi and Verner, 2017; Jordà, Kornejew, Schularick and Taylor, 2020), but there is little work concerning how corporate debt affects the monetary transmission mechanism and whether it hampers the monetary authority’s ability to control inflation, for which our model provides testable implications. Nevertheless, our results echo a similar point in Schularick and Taylor (2012) that policymakers should watch money and credit carefully when implementing monetary policy rules.

Papers including Gomes, Haliassos and Ramadorai (2020) emphasize the skewed cross-sectional distribution of stock ownership, while Becker and Ivashina (2014), and Adrian, Colla and Song Shin (2013) among others, document the strong cyclicality of bank versus bond financing of corporate liabilities. This result suggests that the cyclicality of aggregate savings is important to understanding corporate leverage implications. Furthermore, we argue that the distinction between households that own equity and the lender/worker households that save, either through the banking system or through non-bank financial intermediaries, is important. First, empirical evidence suggests that the top rich invest relatively more in stocks (see Mankiw and Zeldes, 1991; Haliassos and Bertaut, 1995; Parker, 2001; Carroll, 2002; Vissing-Jorgensen, 2002; Campbell, 2006; Wachter and Yogo, 2010; Bucciol and Miniaci, 2011; Calvet and Sodini, 2014; Gårleanu and Panageas, 2015), and a significant proportion of safe corporate debt are held either by households directly, or through bank deposits, or in mutual funds, ETFs, life insurance, pension funds, which the ‘salaried creditors’ indirectly hold. As Campbell (2006) shows, low-wealth households hold large amounts of liquid or safe assets and do not participate in the risky stock markets. Second, Toda and Walsh (2020) also differentiates households as equity holders or bondholders. Based on their model, Toda and Walsh (2020) provide empirical evidence that suggests that the portfolio share of the 1% income earners in the United States concentrates in

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9The implication of high levels of corporate debt has also been studied extensively in the corporate finance literature focusing solely on optimal firm decisions (see Myers (1977) as a classic example), but usually in real models that do not focus on monetary policy transmission mechanisms. Earlier works such as Bernanke, Campbell, Friedman and Summers (1988) and Bernanke, Campbell, Whited and Warshawsky (1990) investigate the default risks of excessive corporate leverage and potential debt deflation rather than debt inflation, while Benjamin Friedman believed the more likely outcome due to corporate America’s post-1982 borrowing binge was inflation.
stocks and that when the income share of the top 1% rises, the subsequent 1-year excess stock market return falls on average. Toda and Walsh (2020) also shows that this finding is not specific to the US. Third, that the lender households supply labor and do not participate in the equity market is also consistent with Benzoni, Collin-Dufresne and Goldstein, 2007, who shows that zero equity allocations arise where labor income risks are highly correlated with stocks and produce results consistent with empirical observation.

Our paper complements the cost channel of monetary policy literature. Barth and Ramey (2001) and Ravenna and Walsh (2006) provide empirical evidence for the working capital channel. There is also a long list of credit-channel papers offering evidence on the cost channel of monetary policy via firms borrowing from banks (see, e.g., Kashyap, Stein and Wilcox, 1993; Kashyap, Lamont and Stein, 1994; Gertler and Gilchrist, 1994; Christiano, Eichenbaum and Evans, 2005; Ravenna and Walsh, 2006, and more recently Phaneuf, Sims and Victor, 2018; Ascari, Phaneuf and Sims, 2018; Grosse-Rueschkamp, Steffen and Streitz, 2019; Gomez, Landier, Sraer and Thesmar, 2021, non-exhaustive). The working capital cost channel provides important implications for the monetary transmission mechanism. For example, Ravenna and Walsh (2006) shows that the interest rate changes necessary to stabilize the output gap leads to inflation rate fluctuations due to the working capital cost channel. Jermann and Quadrini (2012), Bianchi (2016), and Bianchi and Mendoza (2018) model working capital-in-advance financing constraints. However, the cost channel in these papers is not operational as they assume zero interest on working capital loans. Our paper identifies another mechanism that interacts with working capital by showing that the intensity of the trade-off between output stabilization and inflation stabilization depends on the wealth distribution between heterogeneous households and the level of longer-term corporate indebtedness on top of working capital loans.

Similar to the Heterogeneous Agent New Keynesian (HANK) literature (for example, Kaplan, Moll and Violante, 2018; Auclert, 2019; Bayer, Lüticke, Pham-Dao and Tjaden, 2019; Hagedorn and Mitman, 2020; non-exhaustive), our model emphasizes the indirect general equilibrium effect of monetary policy with heterogeneous households. In contrast, in Representative Agent New Keynesian economies the direct effect (i.e., substitution effect) drives the transmission of interest rate changes to consumption. In typical HANK economies, labor income shocks combined with the coexistence of liquid and illiquid assets in financial portfolios are the basis for amplifying shocks through household heterogeneity. In our model, heterogeneity amplifies shocks because of the distribution of firm ownership and lending relationships across households. While HANK models emphasize

\footnote{Aron, Duca, Muellbauer, Murata and Murphy (2012) provide empirical evidence of the difference between the marginal propensity to consume liquid and illiquid assets.}
the consumption implications of policy in the presence of household heterogeneity, we complement this channel with the direct effect of the working capital channel on firm output dynamics. The two channels co-existing create a new trade-off that impedes the effective conduct of monetary policy.

More broadly, our paper connects with the classic literature on inside money in general equilibrium that dates back at least to Grandmont and Younes, 1972, 1973; Shapley and Shubik, 1977. In this literature, money is inside because it enters the economy issued against an offsetting loan, and the repayment of the loan guarantees money’s departure. The presence of non-Ricardian seigniorage transfers determines the equilibrium price level. As a result, money is non-neutral, even with flexible prices (see Dubey and Geanakoplos (2003); Tsomocos (2003); Bloise and Polemarchakis (2006); Goodhart, Sunirand and Tsomocos (2006)).

The next section provides some motivating facts, and Section 3 presents a static model and obtains closed-form solutions for equilibrium analysis. Section 4 extends the static model to a dynamic setting while Section 4.9 presents a quantitative example to illustrate the analytic results. Section 6 concludes.

2 Motivating facts

2.1 Rise of corporate debt

Table 1 documents the non-financial corporate indebtedness of both advanced and emerging economies in Q4 2007, Q4 2018, and Q4 2020. Two observations emerge. First, in the decade since the onset of the Global Financial Crisis leading up to the COVID-19 pandemic, there was a significant increase in non-financial corporate indebtedness across both advanced and emerging economies. Second, between Q4 2018 and Q4 2020, the rise in corporate debt has been even more pronounced, primarily due to the pandemic crisis.
Table 1: Indebtedness of non-financial corporations

<table>
<thead>
<tr>
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<th>Advanced Economies</th>
<th></th>
<th>Emerging Economies</th>
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<td></td>
<td>US</td>
<td>EA</td>
<td>SWE</td>
<td>CAN</td>
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<tr>
<td>Dec-07</td>
<td>70</td>
<td>93.3</td>
<td>125.2</td>
<td>81.7</td>
</tr>
<tr>
<td>Dec-18</td>
<td>75.2</td>
<td>106.2</td>
<td>158.8</td>
<td>114.3</td>
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<tr>
<td>Dec-20</td>
<td>84.6</td>
<td>115.1</td>
<td>175.3</td>
<td>132.4</td>
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</tbody>
</table>

Source: BIS and Goodhart and Pradhan (2020). Numbers express non-financial corporate debt as % of GDP.

### 2.2 Corporate debt, markup and inflation

Since 1980, the non-financial corporate debt-to-GDP ratio and the aggregate markup in the US have both been on the rise.\(^\text{11}\) Figure 1a plots the time series of corporate debt-to-GDP ratio and the aggregate markup in the US. Such a trend may raise concerns that given high debt and higher debt servicing costs, firms with market power may raise product prices to reduce the real burden of debt so that post-COVID-19 debt inflation becomes a possibility. Figure 1b shows the recent uptick in core inflation and documents trends of core inflation and non-financial corporate indebtedness in the US. The left-side graph of Figure 1b plots the quarterly time series of core inflation (i.e., CPI excluding food and energy) and the de-trended corporate leverage since 2000. To capture these features, the model set up in the next section features corporate debt holders and those who owe the debt and also follows the New Keynesian tradition by modeling non-competitive firms that charge markups.

\(^{11}\)De Loecker, Eeckhout and Unger (2020) find that the results hold across industries and sizes though higher in smaller firms. Moreover, Díez, Fan and Villegas-Sánchez (2021) provides comprehensive empirical evidence suggesting the global decline in competition.
Figure 1: Aggregate markup, corporate debt, and core inflation in the US

(a) Markup and non-financial corporate debt-to-GDP

Source: The markup data is from De Loecker et al. (2020). Data on corporate debt and GDP are from Board of Governors of the Federal Reserve System (US) and U.S. Bureau of Economic Analysis, retrieved from FRED, Federal Reserve Bank of St. Louis.

(b) Core inflation and de-trended corporate debt/GDP

Source: Board of Governors of the Federal Reserve System (US) and U.S. Bureau of Economic Analysis, retrieved from FRED, Federal Reserve Bank of St. Louis, and authors’ calculation.

3 Static Model

In this section, we present a stylized flexible price one-period general equilibrium with money to fix ideas on how corporate debt, in the presence of a working capital channel, generates an additional income effect of monetary policy. In Section 4, we extend the static model to a familiar New Keynesian dynamic setting with nominal rigidities to show the implications of this income effect on the trade-off between output and inflation stabilization.

Our thesis is that the accretion of corporate debt makes models that assume no such historical legacy inappropriate for assessing current conditions. That said, however, the introduction of history and time makes it more complicated to apply static one-period models. In particular, we assume that there are two types of households: the first is ‘owner households’ that own firms, which is in accord with the usual assumptions, or the ‘enterpriser-borrower’ à la Fisher (1910). But the innovation in our paper is that we assume that funds which ‘lender and worker households’ were required to contribute from previous periods own the historical debt issued by firms. This type of household is essentially Fisher’s ‘creditor, the salaried man, or the laborer’. These funds pay out a proportion of their accumulated returns from corporate debt, $D$, depending on past interest rates, $R$. Because it is a one-period static model, we assume that both owner and lender households seek to use all their available funds in this period for consumption. In the subsequent dynamic setting, we relax this assumption and model the saving decision of the lenders, where both the quantity and the price of
debt are endogenous.

For the rest, the underlying assumptions are more standard. The static model illustrates a one-period production economy with morning and evening sub-periods. A unit measure of firms produces different consumption goods, so firms possess market power. A central bank exists to supply liquidity against offsetting credits and sets the policy rate, which we take as the borrowing cost in the money market. Owner households are endowed with a monetary (fiat) endowment, and all private agents can borrow inside money against an offsetting credit from the money market should they wish. Lender households supply labor endogenously. There are two transaction moments in the period, which we term 'morning' and 'evening'. In the morning, firms borrow from the money market to obtain liquidity and pay wages, and this is the working capital financing-in-advance constraint that follows a long tradition in the literature on the cost channel of the monetary transmission mechanism (see Blinder, 1987; Farmer, 1984, 1988a,b; Fuerst, 1992; Christiano et al., 2005, 2015, non-exhaustive). Production then takes place. In the evening, firms sell all output. Households carry their wealth and income into the evening to purchase goods. Firms repay the debt that comes due in the evening.

3.1 Households

Owner households and lender households are indexed by \( h \in \{o, l\} \) respectively, and they demand a consumption bundle \( C^h \), given by

\[
C^h \equiv \left( \int_0^1 (c_j^h)^{1 - \frac{1}{\theta}} d\theta \right)^{\frac{\theta}{\theta - 1}},
\]

(1)

with \( c_j^h \) representing the quantity of goods variety \( j \) consumed by the household, and \( \theta > 1 \) being the elasticity of substitution between goods varieties. A lower \( \theta \) leads to a higher markup set by the firms.

The price index is given by

\[
P \equiv \left( \int \left(p_j\right)^{1-\theta} \right)^{\frac{1}{1-\theta}}.
\]

(2)

Owner households are shareholders of the firms, and the rest of the households are lenders to the firms. Each owner household has a monetary (fiat) endowment \( m^o \geq 0 \). We now outline the maximization program for the owner and lender households.
3.1.1 Owner Households

Owner households have a monetary endowment of $m^0$ and profits of $\Pi$ from all firms as income. They spend the income on consumption $c^o$. (3) represents their preferences,

$$U = c^o.$$ (3)

Initial cash balances are carried over till the evening without earning interest in the morning. In the evening, the owner household receives the firm’s profits and spends any unspent money on goods. Their flow constraint is (4),

$$pc^o = \Pi + m^o.$$ (4)

3.1.2 Lender Households

Lender households have wage income of $wL^l$, and net debt repayment $R(1 + ii)\psi D$ as income sources, where $w$ denotes the nominal wage, $L^l$ is the labor supply, $R(1 + ii)$ is the gross interest rate of the debt, $D$ is the total stock of debt firms owe to the Lender Households, and $\psi$ is the proportion of debt that comes due in the evening. We refer to $D$ as the legacy debt, and the repayment of debt principal $\psi D$ and its interest rate $R(1 + ii)$ will be made endogenous in the dynamic section of the model. We fix $R$ but allow the gross repayment to depend linearly on the current policy rate $i$. This assumption is to capture two possible specifications for the bond: fixed rate coupons (where $R$ is predetermined and $i = 0$) and floating rate coupons (where $i > 0$, implying that the coupon payment depends on the current policy rate). We show that the distinction between fixed rate and floating rate coupons does not affect our analysis in either the static nor dynamic case.

(5) represents Lender households’ preferences, and they choose consumption and the supply of labor,

$$U = \log(c^l) - L.$$ (5)

In the morning, the lender households obtain their labor income and carry the money till the evening

$$\dot{m}^l = wL^l.$$ (6)

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12 This specification was simple enough to incorporate meaningful substitution between consumption and leisure and still permit analytic results. Nevertheless, in the New Keynesian extension in the next section, we use more standard preferences.
In the evening, they purchase goods from their income from corporate bonds and their income from labor.

Their effective flow budget constraint is (7)

\[ P e^t = wL^t + \psi R(1 + \iota)D. \]  

(7)

### 3.2 Firms

Owner households own a unit measure of firms. Firm \( j \) produces good \( j \) according to a linear production function, where \( y_j \) is firm \( j \)'s output, \( l_j \) is the labor it demands, and \( A \) denotes technology.

\[ y_j = A l_j. \]  

(8)

Let \( b_j \) be the amount of liquidity the firm obtains from the money market by borrowing, and \( i \) be the monetary policy rate. Equation (9) is the liquidity constraint firm \( j \) faces in the morning. It states that firm \( j \) uses the money market \( b_j \) to pay for wages, essentially the working capital financing constraint. Equation (10) states that at the end of the period, the firm uses the sales proceeds to pay back money market credit \( b_j(1 + i) \), repay the legacy debt plus interest due \( (R(1 + \iota)\psi D) \) and distribute profits \( \pi_j \). As we assume strictly positive interest rates, each constraint binds.

Firm \( j \) maximizes profits \( \pi_j \) from the perspective of owner households by choosing labor \( l_j \) and money market liquidity \( b_j \) and by setting the price of its variety of goods \( p_j \) monopolistically.

The morning constraint is

\[ w l_j = b_j, \]  

(9)

the evening constraint is

\[ \pi_j + R(1 + \iota)\psi D + b_j(1 + i) = p_j y_j. \]  

(10)

and the flow budget constraint is:

\[ \pi_j + (1 + i)w l_j + R(1 + \iota)\psi D = p_j y_j. \]  

(11)
3.3 Equilibrium

We define equilibrium as an allocation of resources and positive prices, given a positive monetary policy rate and monetary endowment, and legacy debt such that

(i) firms set prices while taking into account the price impact on demand,
(ii) agents maximize subject to their budget and liquidity constraints,
(iii) goods market, labor market, and money market clear, and expectations are rational.

We now characterize the equilibrium to show that the combination of legacy debt and working capital can provide clear monetary transmission mechanisms, even when allowing prices to adjust. To start with, Lemma 1 below summarises how real wage and the effective labor supply elasticity respond after a contractionary monetary policy shock (see Appendix A for the proof).

Lemma 1.

1. Contractionary monetary policy reduces real wages.
2. Given the price level, the effective labor supply elasticity with respect to real wages is increasing on the real value of legacy debt and decreasing on the real value of working capital.

The above lemma shows that real wages fall in response to a contractionary monetary policy shock. Furthermore, the markup, $\sigma$ interacts with the policy rate positively. Through the working capital channel alone, the fall in real wages is unambiguous, in contrast to canonical sticky wage models. Furthermore, Lemma 1 implies that the effective labor supply elasticity in our model depends not only on preferences but also on the state of the economy through legacy debt and the working capital used in the economy. The elasticity of labor is given by

$$\epsilon_L = \frac{\psi R(1 + \iota i)D}{bP}.$$ (12)

In contrast, in Christiano et al. (1997), the labor supply elasticity only depends on the parameter for leisure in preferences, and their model’s empirical performance depends sensitively on this parameter.

3.4 Distribution of Income and Aggregate Demand

Aggregate profits $\Pi$ can be derived from (11) as

$$\frac{\Pi}{P} = Y - (1 + i)\tilde{w}L - \psi \frac{R(1 + \iota i)D}{P}.$$ (13)
We obtain the income, and hence demand, from the owner household by substituting (70) and (67). The equilibrium expression for income, $\frac{m^o}{P} + \frac{\Pi}{P}$, can be represented as

$$\frac{m^o}{P} + Y - \frac{A}{\sigma} + i\psi \frac{R(1 + \iota)D}{P}.$$  \hspace{1cm} (14)

From this, raising interest rates increases demand from owner households because raising interest rates lowers the demand for labor. As a result, the wage bill for the firm decreases, which puts upward pressure on profits.

We can combine the owner and lender households’ budget constraints ($\frac{1}{P}(R(1 + \iota)\psi D + wL + \Pi + m)$) to obtain the expression (15) for aggregate demand in equilibrium,\(^\text{13}\)

$$\frac{m}{P} + Y + i \left\{ \psi \frac{R(1 + \iota)D}{P} - \frac{A}{\sigma(1 + \iota)} \right\}. \hspace{1cm} (15)$$

From (15) we can see two effects of monetary policy. Contractionary monetary policy that increases $i$ may increase or decrease aggregate demand depending on how large legacy debt is. On the one hand, higher interest rates increase the financing cost of labor, and the firm demands less labor. As a result, real wages decrease, causing downward pressure on aggregate demand. This is the usual substitution effect. On the other hand, legacy debt renders labor supply more elastic (see Lemma 1) so that the increase in $i$ causes the decrease in wage expenditure to dominate the increase in financing costs. Thus, faced with the fixed cost of the legacy debt, firms need to spread the fixed cost over a larger production scale and demand more labor, which leads to an upward pressure on aggregate demand. This is the income effect through legacy debt. We collect the insights so far in the following proposition.

**Proposition 1.** In equilibrium, the response of aggregate demand to contractionary monetary policy (increasing $i$) depends positively on legacy debt.

The income effect of monetary policy crucially depends on legacy debt and heterogeneous households. This can also be seen through the supply of labor which depends on the distribution of income (and hence demand) through legacy debt ($L = 1 - \psi \frac{R(1 + \iota)D}{\sigma(1 + \iota)A}$). With a representative household, the income effect disappears even when legacy debt is present, and contractionary monetary policy always decreases aggregate demand. To see this, we compare the model with the outcome if we had a $m = m^o$ denotes the aggregate monetary endowment of households.
representative agent combining owner and lender households. Aggregate income would become

\[ \tilde{w}L + \psi \frac{R(1 + \iota i)D}{P} + \frac{m}{\bar{P}} + \Pi, \]

(16)

and substituting in aggregate profits, aggregate demand becomes

\[ \frac{m}{\bar{P}} + Y - i \frac{A}{\sigma(1 + \iota)}. \]

(17)

Comparing (15) and (17), given a price level, raising interest rates has the sole effect of reducing aggregate demand in the representative agent case. This is because in the representative agent case, as income distribution does not matter, the increase in financing costs exactly offsets the upward pressure on profits from lower wage expenditure, and hence, the income effect is no longer present. Note that even though prices are flexible (and there is no price dispersion), monetary policy is non-neutral because of the distributional effect on equity holders and debt holders.\(^{14}\)

Building on the above analysis, we derive the closed-form solution for the price level and allocation in Appendix B. The steps to obtain the closed-form solution leads to the following corollary.

**Corollary 1. In equilibrium, nominal and real profits fall when nominal interest rates rise.**

Even though the rise of nominal interest rates reduces wage expenditure, it also causes revenue to go down due to the drop in labor supply. In equilibrium, firm profits unambiguously fall when nominal interest rates rise, and vice versa, which is consistent with the empirical facts documented in Christiano, Eichenbaum and Evans (2005) and Christiano, Eichenbaum and Evans (1997).

We now characterize the transmission mechanism of monetary policy and state the central result in the following proposition (see the proof in Appendix C).

**Proposition 2. In equilibrium**

1. when legacy debt is sufficiently low \((iR\psi D(1 + \iota i) + (1 + \iota)\iota R\psi D < b)\),

   (a) the standard Taylor principle applies,

   (b) the higher debt is, the less effective is raising interest rates in lowering current inflation;

2. when legacy debt is sufficiently high \((iR\psi D(1 + \iota i) + (1 + i)\iota R\psi D > b)\),

   (a) the Taylor principle is inverted - raising interest rates increases current inflation,
(b) as debt increases, inflation responds increasingly positively to raising interest rates.

Proposition 2 states that the transmission of monetary policy depends on how monetary policy affects the effective cost of legacy debt. The first term, $iR\psi D(1 + \iota i)$, represents the liquidity cost of repaying legacy debt, while the second term, $(1 + i)iR\psi D$, represents the higher repayment costs through floating rate coupons. The effectiveness of monetary policy is determined by how these two costs relate to the quantity of working capital available (b). The quantity of outstanding debt in the economy determines the importance of these costs relative to the working capital available. The standard Taylor principle holds ($\epsilon_{\pi i} < 0$) iff $iR\psi D(1 + \iota i) + (1 + i)iR\psi D < b$. When this is the case, higher legacy debt implies higher labor supply elasticity and a flatter $AS$ curve and when nominal rates rise, current inflation falls less. In other words, prices become less responsive than output following a monetary disturbance because the associated fall in wages creates a large reduction in labor supplied, with the resultant labor shortage lifting wages and cutting labor demand. When $iR\psi D(1 + \iota i) + (1 + i)iR\psi D > b$ the Taylor principle is inverted and $\epsilon_{\pi i} > 0$. That is, if debt is extremely high relative to working capital liquidity, raising interest rates raises the rate of inflation.\(^{15}\)

To reinforce this intuition, we use an aggregate supply $AS$ and aggregate demand $AD$ diagram for the goods market to illustrate a low and high debt scenario with a rise in the policy rate. For this $AS$-$AD$ diagram, we have factored in the clearing of the labor market and money market, but not the goods market ($P, y$); therefore, we can express the $AS$ and $AD$ as functions of output $y$, price level $P$, and exogenous parameters $m, i, D, \sigma, A, \psi, R$. The aggregate demand is expressed in (15). As can be seen in (15), with the rise in $i$, the substitution effect shifts the $AD$ curve to the left, but the income effect through debt offsets the shift; thus, the high debt scenario sees the $AD$ shift less to the left than the low debt case. To obtain the $AS$ curve, we combine the producer’s optimality condition for labor demand (67), the labor supply curve (70), and the production function, and we get

\[
y = A - \sigma(1 + i)\psi \frac{R(1 + \iota i)D}{P}.
\]

As seen in (18), an increase in $i$ reduces aggregate supply, and a higher debt renders the $AS$ curve more elastic.

\(^{15}\)This is an extreme case because, in reality, $\psi$ is extremely low so that $iR\psi D(1 + \iota i) + (1 + i)iR\psi D > b$ is unlikely to hold most of the time. Indeed, when we calibrate our dynamic model with data, this condition does not hold.
Figure 2: AS-AD diagram: a rise in policy rate

The left diagram (a) illustrates a low debt scenario. The right diagram (b) illustrates a high debt scenario. Equilibrium $e$ is the equilibrium before the rise in the policy rate, and equilibrium $e^*$ is the equilibrium after the rise in the policy rate. The vertical line at $A$ is the output when there is no debt in the economy.

Figure 2 displays the AS-AD diagram to qualitatively show the equilibrium changes when the central bank raises interest rates. The left diagram (a) illustrates a low debt case, and the right (b) shows a high debt case. In the low debt case, the rise in the policy rate significantly reduces inflation, whereas, in the high debt case, the rise in the policy rate only moderately reduces inflation, but output falls more responsively. This is because the high debt case shifts the $AD$ to the left less, and the $AS$ curve also becomes more elastic due to the income effect through debt. Indeed, if the debt level is exceptionally high, the rise in the policy rate would even increase inflation, as proved in the inverted Taylor principle case in Proposition 2.2.

4 Dynamic Model

We now show that the intuition and mechanisms illustrated in the static model hold in an environment based on the canonical New Keynesian framework with nominal rigidities (via Calvo pricing) and an endogenous monetary policy rule (Taylor rule). The dynamic version distinguishes wholesale producers from intermediate goods producers. Wholesale producers are price-takers and can access short-term financing from the money market. Intermediate goods producers are static price-setters with market power. We assume a steady-state level of legacy debt which wholesale firms choose to roll over at prevailing interest rates. Wholesale firms solve a dynamic problem by maximizing the discounted value of real profits, valued at the owner household marginal utility. We also replace the
monetary endowment of households with central bank open market operations in the bond market.

4.1 Owner Households

Owner households own both wholesale and intermediate goods firms, and they maximize their expected inter-temporal utility

\[ U^o = \sum_t E_t \beta^t \exp(\epsilon^d_t) \log(c^o_t), \]  

(19)

where \( \epsilon^d_t \) is a normally distributed demand shock\(^{16}\). Preferences are subject to their flow budget constraint written in real terms as follows:

\[ c^o + k' = \tilde{\pi}_W + \tilde{\pi}_k + \int_j \tilde{\pi}_j, \]  

(20)

where \( \tilde{\pi}_W \) are profits from wholesale producers, \( \tilde{\pi}_j \) are profits from intermediate goods producers each period. Optimality with respect to capital gives

\[ \frac{1}{c^o} = \beta E \frac{1}{c^o} (\tilde{r}_k^t). \]  

(21)

4.2 Lender Households

Similar to owner households, lender households maximize

\[ U^l = \sum_t E_t \beta^t \left\{ \exp(\epsilon^d_t) \log(c^l_t) - \frac{\kappa}{2} l^2 \right\}. \]  

(22)

and are subject to the budget constraint written in real terms

\[ \tilde{q} d' + \frac{\phi d}{2} \tilde{q} (\tilde{d} - \bar{d})^2 + c^l = \tilde{w} l + \frac{\tilde{d}}{1 + \eta}, \]  

(23)

where \( \bar{d} \) is the steady-state value of debt and \( \frac{\phi d}{2} \tilde{q} (\tilde{d} - \bar{d})^2 \) is a quadratic adjustment cost for debt, and \( \eta \) is the net rate of inflation\(^{17}\). The optimality condition with respect to labor is

\[ \frac{\tilde{w}}{c^l} = \kappa l, \]  

(24)

\(^{16}\)We suppress notation for this for brevity and reintroduce it in the quantitative simulation. Nevertheless, the shock should appear wherever the marginal utility of households appears, including in the forward-looking equations of the firms.

\(^{17}\)In the robustness check section, we also include a fixed coupon corporate bond to generate a deterioration in lenders’ non-labor income wealth after a monetary contraction, and our key results also go through.
while the optimality condition with respect to debt is

$$\frac{\tilde{q}}{d'} (1 + \phi_d(d' - \bar{d})) = \beta \mathbb{E}_{t} \frac{1}{c_{t+1}} \frac{1}{1 + \eta'}.$$  \hfill (25)

### 4.3 Wholesale Firms

Wholesale firms are price takers and maximize the present discounted value of real value profits valued at the owner’s marginal utility

$$\sum_{t} \beta^t \mathbb{E}_{t} \frac{1}{c_{t+1}} \tilde{\pi}_{W,t}. \hfill (26)$$

They have a production function with capital $k$ and labor $l$ being the inputs and $A$ being productivity:

$$y_W = A k^\alpha l^{1-\alpha}. \hfill (27)$$

Capital is rented from the owner households, while labor is rented from the lenders. As in the static model, firms face a morning budget constraint and an evening one. In equilibrium, these can be represented as the working capital and the flow budget constraints, respectively. The nominal working capital constraint is represented by eq (28), and the end-period nominal constraint is represented by eq (29).

$$wl = b \hfill (28)$$

$$\pi_W + r_k k + d_W + b(1 + i) = p_W y_W + q d_W', \hfill (29)$$

where $p_W$ is the nominal value of a unit of wholesale goods, and $b$ is the money wholesale firms borrow from the short-term money market at a nominal interest rate $i$. $d_W'$ is the nominal value of inter-temporal bonds sold at a price $q$, and which is repaid one period in the future. Define the real value of short-term borrowing as $\tilde{b} = \frac{b}{P_{t}},$ the real value of inter-temporal bonds as $\tilde{d}_W = \frac{d_W}{P_{t}}$, and recall that inflation is given by $1 + \eta = \frac{P_{t+1}}{P_t}$. With this, we obtain the real flow budget constraints as follows:

$$\tilde{wl} = \tilde{b}, \hfill (30)$$

$$\tilde{\pi}_W + \tilde{r}_k k + \frac{1}{1 + \eta} \tilde{d}_W + \tilde{b}(1 + i) = \tilde{p}_W y_W + \tilde{q} d_W'. \hfill (31)$$
Optimality with respect to debt gives
\[ \hat{q} \left( \frac{1}{c^o} \right) = \beta E \left( \frac{1}{c^o} \right) \left( \frac{1}{1 + \eta} \right). \] (32)

Optimality conditions with respect to capital and labor are
\[ \tilde{r}_k = \alpha \tilde{p}_W y_W / k, \] (33)
\[ \tilde{w} = \frac{1}{1 + i} (1 - \alpha) \tilde{p}_W y_W / l. \] (34)

Using these optimality conditions, we obtain the expression for the price of wholesale goods,
\[ \tilde{p}_W = \frac{1}{1 - \alpha} \alpha \tilde{w} \left( (1 + i) \tilde{w} \right)^{1-\alpha} \left( r_k \right)^{\alpha}. \] (35)

4.4 Intermediate Goods Firms

Intermediate goods firms purchase goods from wholesale firms and have a simple linear production function. They each have differentiated goods and sell that to the consumer, setting the price of the goods they sell. The marginal cost of each firm is $\tilde{p}_W$.

The nominal flow budget constraint summarizes these constraints:
\[ \tilde{\pi}_j = \frac{1}{P} \left\{ p_j y_j - \tilde{p}_W y_j \right\} \] (36)

Substituting in the demand function $y_j = \left( \frac{p_j}{p} \right)^{-\theta} y$,
\[ \tilde{\pi}_j = \left( \frac{p_j}{p} \right)^{1-\theta} y - \tilde{p}_W \left( \frac{p_j}{p} \right)^{-\theta} y \] (37)

where $\tilde{p}_W$ is the real marginal cost.

Let $\phi$ be the probability that an intermediate goods firm does not change its price each period.

Using the above, we obtain the following expression for the price of the firms that re-set their price each period
\[ p^*_j = \sigma \frac{X_1}{X_2} \] (38)
where $\sigma = \frac{\theta}{\theta - 1}$ and

$$X_1 = \frac{1}{c^o} \tilde{p}_W P^\theta y + \phi \beta \mathbb{E} X'_1$$

$$X_2 = \frac{1}{c^o} P^{\theta - 1} y + \phi \beta \mathbb{E} X'_2.$$  \hspace{1cm} (39) \hspace{1cm} (40)

With flexible prices, it follows that

$$\bar{p}_j = \sigma P \tilde{p}_W.$$  \hspace{1cm} (41)

Finally, aggregate profits of this sector are

$$\tilde{\pi} = \int_0^1 \tilde{\pi}_j \, dj = y \int_0^1 \left\{ \left( \frac{p_j}{p} \right)^{1-\theta} - \tilde{p}_W \left( \frac{p_j}{p} \right)^{-\theta} \right\} \, dj$$

$$= y - \tilde{p}_W \nu y,$$  \hspace{1cm} (42) \hspace{1cm} (43)

where $\nu$ is price dispersion.

### 4.5 Final Goods Firm

The final goods firm’s problem is the same as in the standard literature. Each period a perfectly competitive, representative final goods firm produces the final consumption good, $y$. The firm produces the final good by combining a continuum of intermediate goods, indexed by $j \in (0, 1)$, using the technology

$$y = \left( \int_0^1 y_j^{1-\frac{\theta}{\sigma - 1}} \right)^{\frac{\theta}{\sigma - 1}} \, dj.$$  \hspace{1cm} (44)

Optimality implies

$$y_j = \left( \frac{p_j}{p} \right)^{-\theta} y,$$  \hspace{1cm} (45)
and

\[ P = \left[ \int_0^1 p_j^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}}, \quad (46) \]

and note that integration of (45) using the production function of the intermediate goods firm gives

\[ y_W = \nu y = \int_0^1 \left( \frac{p_j}{p} \right)^{-\theta} y \, dj. \quad (47) \]

### 4.6 Monetary Policy

The monetary authority sets the short-term interest rate of the money market according to a Taylor rule. It also trades inter-temporal bonds in its regular open market operation. Let the overline symbol denote the steady-state real value, let \( \rho_y, \rho_i, \rho_\eta \) be the Taylor rule coefficients, and the Taylor rule is specified as follows:

\[
\frac{1 + i}{1 + \bar{i}} = \left( \frac{y}{\bar{y}} \right)^{\rho_y} \left( \frac{1 + i - 1}{1 + \bar{i}} \right)^{\rho_i} \left( \frac{1 + \eta}{1 + \bar{\eta}} \right)^{\rho_\eta} e^{\epsilon_i}, \quad (48)
\]

where \( \epsilon_i \) is a Normally distributed shock.

A meaningful trade-off between inflation and output stabilization requires a real rigidity in the canonical New Keynesian model (Blanchard and Galí, 2007 call this the absence of the ‘divine coincidence’).\(^{18}\) What should be the appropriate output target is also unclear (Woodford, 2001, Garín, Lester and Sims, 2016). We include the log deviation of output from its trend in the Taylor rule. We do this because the nominal interest rate enters as a direct working capital financing cost and because of the additional transmission mechanism we obtain through corporate debt. These reasons imply that monetary policy can meaningfully target overall output fluctuations and not only its deviation from the flexible price equilibrium.

Given the nominal interest rate specified by the Taylor rule, the monetary authority supplies money on demand in the money market, \( \tilde{M} \). We interpret these activities as discount window actions. In addition, the monetary authority commits to trade a constant real amount of inter-temporal bonds \( \tilde{\mu} \), and we interpret the trading of inter-temporal bonds as open market operations. These actions result in a public flow balance equation.

\(^{18}\)Ravenna and Walsh (2006) show that the cost channel via working capital loans alters the trade-off between inflation and output stabilization. We show that the intensity of this trade-off depends on the quantity of corporate debt in the economy and that the mechanism hinges on the income effect through corporate debt, which reinforces the cost channel via working capital loans. The higher the level of corporate debt is, the more difficult this trade-off becomes.
\( \tilde{M}i + \frac{\tilde{\mu}}{1+\eta} - \tilde{q}\tilde{q}' = 0 \). \hspace{1cm} (49)

The monetary policy rule gives the interest rate \( i \), and the central bank supplies \( \tilde{M} \) to clear the money market.

### 4.7 Market Clearing and Equilibrium

Below we summarise the market clearing conditions for final goods, the money market, and the inter-temporal bond market:

- The market clearing condition for final goods is

\[ Y = C^o + C^l + K' + \frac{\phi_M}{2} q(\tilde{D}' - \bar{D})^2. \] \hspace{1cm} (50)

- The money market clearing condition is

\[ \tilde{B} = \tilde{M}. \] \hspace{1cm} (51)

- The inter-temporal bond market clearing condition is

\[ \tilde{D}'_W = \tilde{D}' + \tilde{\mu}'. \] \hspace{1cm} (52)

Note that the upper case variables coincide with the aggregate value of the population share. In the quantitative simulations, we calibrate our economy such that the population share of the owner households is smaller than that of the workers. We assume each household type is of unit measure and use the lowercase variables to denote aggregate quantities.

In addition, the labor market, capital rental market, and the wholesale goods market clears. For the sake of brevity, we have assumed markets clear in the problem description in the previous sections. Equilibrium is defined as a sequence of quantities and prices, given the monetary policy rule, and the real quantity of inter-temporal bonds traded by the monetary authority \( (\tilde{\mu}) \), such that

1. the monetary authority supplies real money balances on demand \( (\tilde{M} = \tilde{b}) \),
2. intermediate goods firms set prices while taking into account the price impact on demand,
(iii) agents maximize subject to their budget and liquidity constraints,

(iv) goods market, labor market, capital market, corporate bond market, and money market clear,
and expectations are rational.

Summing up the flow of funds constraint of the economy, we note that the interest payment of the monetary market equals the trading cost in the open market operation, i.e., \( i\hat{b} = q\hat{\mu}' - \frac{\hat{\mu}}{1+\eta} \). Let \( m \equiv q\hat{\mu}' - \frac{\hat{\mu}}{1+\eta} \), it follows that \( \hat{M} = \frac{\hat{m}}{1} \), and variable \( \hat{M} \) refers to the real value of money balance. Appendix D - G presents the system of equations that summarise equilibrium together with the closed-form solution for the steady-state and linearised dynamic equations. Proposition 3 characterizes the real effects of money and legacy debt in the steady-state.

**Proposition 3.** In the steady-state,

a More legacy debt decreases real money balance and output;

b An increase in the nominal interest rate reduces real money balance, but the reduction is weaker the higher legacy debt is;

c Changing the nominal interest rate exerts real effects in the steady-state when debt \( \bar{d} \neq 0 \), but is neutral when debt \( \bar{d} = 0 \).

This result arises because corporate debt affects both aggregate demand (through the distribution of household income) and aggregate supply (through the level of inputs of the firm). The nominal interest rate is neutral when corporate debt is zero because we allow for a Ricardian seigniorage transfer \( m \) each period. A non-Ricardian seigniorage transfer, on the other hand, determines the price level and makes monetary policy non-neutral in the steady state even with zero debt (see Nakajima and Polemarchakis (2005)).

### 4.8 Dynamic Properties

In this section, we study the effects of legacy debt on the dynamic properties of the model and on the monetary transmission mechanism away from the steady-state. Using (161) and (162) from Appendix G,

\[
\hat{x}_1 - \hat{x}_2 = (1 - \phi\beta)\hat{p}_W + \phi\beta(1 + \eta') + \phi\beta(\hat{x}'_1 - \hat{x}'_2),
\]

and then using (159) and (160) from Appendix G we obtain the ‘naïve’ Phillips curve\(^{19}\):

\(^{19}\)‘Naïve’ because we do not present it in terms of the output gap.
\[
(1 + \eta) = \frac{(1 - \phi)(1 - \phi \beta)}{\phi} \hat{p}_W + \beta (1 + \eta').
\] (54)

where the marginal cost is given by\(^{20}\)

\[
\hat{p}_W = -\frac{(1 + \eta) + \bar{q} \bar{q}' - \bar{d}}{1 - \bar{q}} - \frac{(1 + \bar{i})}{((1 + \bar{i}) - 1)} \left\{ 1 - \frac{(1 + \bar{i})(1 - \alpha)\bar{d}(1 - \bar{q})}{2(\bar{w} + \bar{d}(1 - \bar{q}))} \right\} - \bar{A} - \alpha \bar{k} - \frac{(1 - \alpha)\bar{d}\{\bar{q} \bar{d}' - \bar{d}\}}{2(\bar{w} + \bar{d}(1 - \bar{q}))}.
\] (55)

As the steady-state level of legacy debt increases, the absolute value of the coefficient of interest rates on the path of inflation declines, i.e., changes in interest rates has a smaller negative effect on inflation.

The following proposition summarizes this result

**Proposition 4.** Given monetary policy, as the steady-state debt level increases, the effectiveness of interest rates on the path of inflation declines.

Here we show that the lack of ‘divine coincidence’ depends, in part, on the level of legacy debt. We can see this by taking the labour first order condition and the working capital constraint, (153), (151) and for analytical convenience set \(\phi_d = 0\),

\[
\hat{p}_W = \hat{b} + (1 + \bar{i}) - \hat{y}
\]

\[
= (1 + \bar{i}) + \bar{w} + \hat{i} - \hat{y}
\]

Hence the ‘naïve’ Phillips curve is

\[
(1 + \eta) = \frac{(1 - \phi)(1 - \phi \beta)}{\phi} \hat{p}_W + \beta (1 + \eta').
\] (58)

The expression above is identical to the standard expression in the presence of a cost channel with the standard real marginal cost supplemented with the liquidity cost. However, in Ravenna and Walsh (2006), the real marginal cost can be expressed in terms of output because the labor supply decision of households depends only on wages and aggregate output. With heterogeneous households, the real marginal cost also depends on the wealth distribution (i.e., corporate debt holdings), as seen in Equation 55.

Our ‘naïve’ IS curve can be obtained by combining in the individual Euler (149) and labour supply

\(^{20}\text{The derivation is in Appendix H.}\)
condition (148) with the marginal product of labour (153) and the definition of firm output (150), (and \( \phi = 0 \))

\[
\hat{q} + (1 + i) - \hat{p}_W - \hat{y}(1 - \frac{2}{1 - \alpha}) - 2\frac{\hat{A} + \alpha \hat{k}}{1 - \alpha} \\
= (1 + i)' - \hat{p}_W' - \hat{y}'(1 - \frac{2}{1 - \alpha}) - 2\frac{\hat{A}' + \alpha \hat{k}'}{1 - \alpha} - (1 + \eta)'.
\]

(59)

Here we can see that the level and dynamics of corporate debt will affect aggregate demand through the real marginal cost, \( \hat{p}_W \). Note that a standard IS or Phillips curve using a measure of the output gap, i.e., the difference between a flexible price economy and a sticky price one, would not affect our core result. The dynamics of debt affect both aggregate demand and price setting behavior, meaning that the output gap would reflect two distortions in the economy: the first arising from pricing rigidities and the second from market incompleteness affecting the distribution of wealth and hence aggregate demand and supply. The latter inefficiency means that inflation targeting should also account for debt dynamics. Putting this together, the path of interest rates that stabilizes the path of inflation may cause instability in output directly through instability in working capital which indirectly causes instability in the path of intertemporal debt.

### 4.9 Quantitative Example

We now present our simulation, calibrated to the US. We take the population share of the owners to be 10% (and the worker-lenders to be 90%) to match known distributions in financial asset holdings, in particular, equity (see Toda and Walsh, 2020 and Campbell, 2006, for example). Other than corporate leverage, we appeal to standard calibrated parameters from recent literature (see Table 2). The model period is one quarter, and we set the discount factor \( \beta \) to 0.99, the same as in Ottonello and Winberry (2020). We set the markup parameter to 1.25, which is at the low end of the estimated markup in De Loecker, Eeckhout and Unger (2020) but at the high end of the value conventionally used in the New Keynesian literature. In the monetary policy rule, we set the response to inflation to 1.5 and the smoothing parameter to 0.5 (similar to Gomes, Jermann and Schmid (2016)). Following Christiano, Trabandt and Walentin (2010), we set the output coefficient to 0.2 as our benchmark.

A crucial calibration in this economy is the value of the corporate debt-to-GDP ratio at the steady-state, i.e., the steady-state leverage. This parameter matters for the wealth distribution of the ‘enterpriser-borrower’ and the ‘salaried creditor’. We set the benchmark leverage to a 75% corporate debt-to-GDP ratio at the steady-state and high leverage as 100%. In our numerical illustrations, we compare the macroeconomic responses between the benchmark and high debt cases. We base our
choice of leverage on the ratio of corporate debt to quarterly revenue of non-financial corporate businesses from 2001 to 2022. We find it fluctuates between 3 and 4 (or 75% and 100% on an annualized basis) and has been trending up in the recent decade, consistent with corporate debt-to-GDP ratios in various economies documented in Section 2.1. Furthermore, the total non-financial business debt in the US stands at a historically high level of around 130% of GDP in 2020 (see Jordà, Kornejew, Schularick and Taylor, 2020 and Federal Reserve Board Financial Accounts of the United States 2020). In Appendix E, we report the steady-state values in Table 4.

Table 2: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$A$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$i$</th>
<th>$\sigma$</th>
<th>$\kappa$</th>
<th>$\phi$</th>
<th>$\phi_d$</th>
<th>$\rho_y$</th>
<th>$\rho_l$</th>
<th>$\rho_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>100</td>
<td>0.33</td>
<td>0.99</td>
<td>0.01</td>
<td>1.25</td>
<td>0.1</td>
<td>0.7</td>
<td>0.001</td>
<td>0.2</td>
<td>1.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

We simulate the model with two shocks, a positive shock to interest rates and a positive demand shock. We assume the former has no persistence while the latter has a persistence of 0.9. A consumption demand shock gives us an insight into the policy response in a post-pandemic recovery.

The model simulation sheds light on the cyclicality of the consumption expenditure of the households that own large shares of equity and those that do not. Table 3 presents the correlation matrix of key variables with output. The consumption expenditure of owner households, that is, the equity owners, tends to be highly pro-cyclical, whereas the expenditure of the lender households, those who do not own shares, is much less cyclical. Moreover, both working capital and labor income appear highly pro-cyclical. As the debt level increases, the more pro-cyclical owner households’ consumption appears, and the more acyclical lender households’ consumption expenditure becomes. This result connects with the literature on the high sensitivity of consumption growth of wealthy stockholders to the stock market and aggregate fluctuations. For example, Malloy, Moskowitz and Vissing-Jørgensen (2009) finds higher sensitivity of the consumption growth of wealthy stockholders to both the stock market and to aggregate consumption growth, and Parker and Vissing-Jorgensen (2009) show that consumption growth of high-consumption and high-income households are significantly more exposed to aggregate fluctuations, among others (see Mankiw and Zeldes, 1991; Parker, 2001).

Table 3: Cyclical properties: correlations with output

<table>
<thead>
<tr>
<th>$y$ (BMK lev)</th>
<th>$c^o$</th>
<th>$c^l$</th>
<th>$b$</th>
<th>$l$</th>
<th>$d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.73</td>
<td>0.38</td>
<td>0.96</td>
<td>0.93</td>
<td>-0.76</td>
<td></td>
</tr>
</tbody>
</table>

$y$ (High lev) | 0.88 | 0.20 | 0.99 | 0.97 | -0.86 |

BMK lev refers to the benchmark leverage of 75% (annual), or $b/y = 3$. High lev refers to the high debt leverage of 100% (annual), or $b/y = 4$. $c^o$ is the consumption of owner households, $c^l$ is the consumption of lender households, $b$ is working capital in real terms, $l$ is labor, $d$ is debt in real terms, and $y$ is real output.

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4.9.1 The Effect of Monetary Contractions

The tightening monetary policy shock we introduce is 0.025 standard deviations of the nominal policy rate, which leads to an endogenous increase in the policy rate of around one percentage point. Figure 3 shows the dynamic responses to the monetary contraction shock, where the blue line represents benchmark leverage, or corporate debt-to-output ratio, of 75%, while the red line represents high debt leverage of 100%. In both cases, inflation falls on impact after a monetary contraction before rising to the positive realm. The subsequent rise in inflation is higher in the high debt case than in the benchmark case, suggesting that the higher corporate indebtedness is, the more challenging it is to rein in inflation. On the real side, output falls in both the high debt and benchmark cases. However, output responds much more aggressively in the high debt case because corporate debt triggers the income effect of rising interest rates, causing the labor supply to become more elastic. Consequently, the AS curve is more elastic in the high debt case than in the low debt case. The positive shock to the nominal interest rates dampens both aggregate demand and aggregate supply, and with a more elastic AS curve, inflation, although it falls on impact, can even increase slightly after a monetary contraction (see Proposition 2.2).

Our impulse responses for real wages and labor confirm that Lemma 1 also holds on the dynamic path (that the effective elasticity of labor supply depends on legacy debt). A monetary contraction increases the borrowing cost of financing the working capital, driving down real wages. Although the price of corporate bonds also falls, it falls less than the real wages. With a high effective labor supply elasticity, wage decreases drive down labor supply significantly. As seen in the high debt case, labor decreases more than in the benchmark case. Moreover, corporate profits fall after a monetary contraction.
4.9.2 Output Stabilisation Taylor Rule

We now compare how legacy debt affects output-inflation stabilization trade-offs and show that the trade-off between inflation stabilization and output stabilization becomes more acute with a large volume of corporate debt in the economy. With high levels of corporate debt, if the monetary authority is more concerned about output and employment stabilization, the inflationary pressure then is high; if the monetary authority is strictly sticking to its price stability mandate, it could bring down inflation on impact but at the cost of hurting output and employment persistently.

Figure 4 shows different Taylor rule coefficients (in which we set the output coefficient to 0.2 or 0.9 and the inflation coefficient remains at 1.5) in a high leverage regime (100% debt to GDP). We consider the counterfactual experiment between a monetary authority who cares more about output stabilization than our benchmark Taylor rule. To model this, we increase the Taylor rule output coefficient to 0.9, which is among the high range estimated in the literature (see, e.g., Clarida, Gali and Gertler, 2000) and suggested by policymakers (see Bernanke, 2015; Yellen, 2012).\footnote{As Bernanke (2015) pointed out that 'in principle, the relative weights on the output gap and inflation should depend on, among other things, the extent to which policymakers are willing to accept greater variability in inflation in exchange for greater stability in output'. Moreover, according to Bernanke (2015), the FOMC pays closer attention to variants of}
In Figure 4 the solid line corresponds to the benchmark Taylor rule $\rho_y = 0.2$ and the dashed line corresponds to the output stabilisation Taylor rule $\rho_y = 0.9$. Compared with the benchmark Taylor rule, the output stabilization Taylor rule ($\rho_y = 0.9$) brings output back up to the steady-state within seven quarters, whereas with the benchmark Taylor rule, the loss of output is greater and much more persistent. Furthermore, the benchmark Taylor rule also sees more persistent loss in employment and business profits than the output stabilization Taylor rule. Nevertheless, the output stabilization Taylor rule leads to a much higher inflationary profile.

![Figure 4: Tightening shock to nominal policy rate $i$ with or without output stabilisation.](image)

Red solid line is the benchmark Taylor rule ($\rho_y = 0.2$) and the dashed black line is the output stabilisation Taylor rule ($\rho_y = 0.9$). y-axis is % change and x-axis is the number of periods. Other than inflation and policy rate, all variables are in real terms.

### 4.9.3 The Effect of a Positive Demand Shock

We now study a positive demand shock of 0.05 standard deviation and an autoregressive coefficient of 0.9. Figure 5 demonstrates the dynamic responses with the positive demand shock and our benchmark Taylor rule. Unsurprisingly, inflation rises when demand picks up, and output increases on impact. The monetary authority responds by tightening monetary policy and increasing the policy rate. As the policy rate increases, the cost channel of monetary policy starts to dampen aggregate supply, and with the income effect of debt, the aggregate supply curve shifts inward and becomes more elastic, the Taylor rule that include the higher output coefficient, and Janet Yellen has also suggested that the FOMC’s ‘balanced approach is more consistent with an output coefficient of 1.
leading to a subsequent drop in output. Notably, inflation is much higher in the high debt case than in the benchmark case, and the subsequent drop in output is more severe in the high debt case than in the benchmark case, for reasons already explained. Relatedly, employment in the high debt case falls, but it holds well in the benchmark case, suggesting that a high level of corporate debt increases the effective labor supply elasticity.

![Figure 5: A positive consumption demand shock.](image)

Blue line is 75% leverage and red line is 100% leverage. y-axis is % change and x-axis is the number of periods. Other than inflation and policy rate, all variables are in real terms.

5 Robustness Check

Monetary contractions lead to a reduction in both real wages and corporate bond prices. One may be concerned that if lenders hold fixed coupon bonds whose market value is negatively affected by the rate hike but not compensated by the rising interest payment, lenders’ wealth may be more adversely affected in the high debt case than in the low debt case. In that scenario, would the effective labor elasticity still turn out higher in the high debt case, and our results go through? In this robustness check, we added a two-period fixed coupon bond whose steady-state quantity is four times as much as the floating rate bond. This scenario captures a noticeable potential decrease in lender working households’ non-labor income wealth after monetary contractions.

Our results still go through. Take the policy experiment of a positive consumption demand shock...
as an example (we leave the numerical results of a contractionary monetary policy shock and output stabilization Taylor rule in the appendix to save space). As the economy experiences a positive consumption demand shock - again, we have the post-pandemic economy rebound in mind as the context - the monetary policy rate increases as an endogenous response. Both real wages and bond prices go down after the monetary contraction. However, the bond price-to-wage ratio increases, and in particular, it increases more in the high debt case than in the low debt case. In a high-debt scenario, the bond price decreases less relative to the real wage than in a low-debt scenario. This result suggests that even though the short and long rates increase after the monetary contraction, the long rate increases to a lesser degree, and the term structure becomes flatter. Therefore, the negative impact on wealth in the high-debt scenario is less severe than in the low-debt scenario, resulting in the effective labor elasticity increasing on legacy debt. Hence, when the corporate debt level is high, monetary contraction is less effective in controlling inflation.

Figure 6: A positive consumption demand shock (with fixed coupon bonds).

Blue line is 75% leverage and red line is 100% leverage. y-axis is % change and x-axis is the number of periods. Other than inflation and policy rate, all variables are in real terms.

6 Conclusion

We have presented a general equilibrium model to study the effect of corporate indebtedness on the monetary transmission mechanism. We highlight that high corporate debt levels render contractionary monetary policy less effective in controlling inflation so long as the monetary authorities strive to prevent large-scale bankruptcies and a recession from developing. While Irving Fisher’s narrative is
that booms and busts are caused by changes in the relative wealth of the ‘enterpriser-borrower’ and the ‘creditor, the salaried man, or the laborer’, our focal point is on the impact of such wealth distribution on the efficacy of monetary policy in controlling inflation.

We kept the static and dynamic model simple to derive results intuitively. In the dynamic model we derive the Phillips curve augmented with corporate debt and show that effectiveness of interest rates along the path of inflation declines as the steady-state debt level increases. This debt mechanism provides an explanation of the slope dynamics of the Phillips curve (on the insensitivity of inflation to unemployment, see, e.g., Blanchard, 2016; Gilchrist, Schoenle, Sim and Zakrajšek, 2017; Hazell, Herreno, Nakamura and Steinsson, 2022, non-exhaustive.). Then a quantitative example is given to illustrate that the key results hold on the dynamic path away from the steady-state. We acknowledge that the attempt to write a tractable model to unpack the main mechanism and logic of debt inflation unavoidably leaves out many other features (such as financial frictions and labor market frictions) from the dynamic model to evaluate the quantitative implications fully. Nevertheless, our result that monetary policy effectiveness depends on corporate debt levels adds support to the argument in papers including Curdia and Woodford (2010), Schularick and Taylor (2012), Jordà, Schularick and Taylor (2013), and Jungherr, Meier, Reinelt and Schott (2022) that monetary policy should be conducted taking into account financial market conditions and that credit and money deserve to be watched carefully when implementing monetary policy rules.

The mechanism of our central result is independent of standard financial and nominal frictions. The fundamental channel is the income effect of longer-term corporate legacy debt interacting with the cost channel of short-term working capital loans. Our result complements the literature on the cost channel of monetary policy by showing how corporate debt may intermediate the transmission mechanism from monetary policy to economic activity. We show that the income effect of corporate debt and the wealth distribution between heterogeneous households amplifies the cost channel of monetary policy. In contrast to representative agent frameworks, because our real marginal cost depends on the distribution of wealth, the cost channel operates through both the IS and Phillips curves. Thus, the monetary authority faces a much more difficult trade-off between inflation stabilization and output stabilization when there is a large volume of corporate debt in the economy and large-scale bankruptcies are not imminent.

Our model assumes that no firm goes (or expects to go) into bankruptcy. There are several reasons for making this assumption. The first is that policies have been so expansionary, liquid savings among consumers so high, and the labor market so tight, that the vast majority of firms now would not expect that they might become bankrupt. Up to bankruptcy, the prior argument about labor usage going
down by less in a legacy debt case remains true. In the event of bankruptcy, labor demand in such a firm would suddenly fall to zero and would make the normal assumption of a representative firm particularly challenging. The second is that, with the possibility of bankruptcy, the basic problem of contractionary monetary policy in a world with high corporate debt is that a small increase in interest rates may not restore inflation to target, while a larger increase in interest rates might cause such large bankruptcies as to bring about a recession. At the moment, the monetary authorities seem reluctant to implement a sufficiently contractionary monetary policy that might lead to large-scale bankruptcies. So long as they are more fearful of bankruptcies and recession than continuing inflation, a model with no bankruptcy seems appropriate. Under these circumstances, our work suggests that monetary policy will not effectively reduce inflation gently toward a soft landing. Consequently, central banks ultimately have to choose between generating a recession with significant bankruptcies or accepting continuing stagflation. We believe this is also a policy conundrum for post-crisis scenarios such as the present high-debt zombie firms staying afloat with imminent firm defaults at record lows (see, e.g., Acharya, Crosignani, Eisert and Steffen 2021 and Caballero et al. (2008)). However, even though modeling the macroeconomic system with bankruptcy is more difficult, it could be useful for future research to examine the monetary transmission mechanism when large-scale bankruptcies become possible.
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Appendix

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A Proof of Lemma 1

Note that households’ optimisation gives \( \int_j c_j^h = \int_j \left( \frac{p_j}{P} \right)^{-\theta} C^h \), goods market clearing gives \( c_j^o + c_j^l = c_j = y_j \) and hence \( \int_j y_j = Y \int_j \left( \frac{p_j}{P} \right)^{-\theta} \) where \( Y \) is the aggregate bundle of goods produced. The aggregate goods market clearing is \( c^o + c^l = Y \).

Substituting in the demand function \( y_j = \left( \frac{p_j}{P} \right)^{-\theta} Y \) and \( l_j = \frac{1}{A} \left( \frac{p_j}{P} \right)^{-\theta} Y \) into (11):

\[
\pi_j = (p_j)^{1-\theta} Y - R(1+i)\psi D - (1+i)(wp_j^{-\theta}P^\theta Y). 
\]  

(60)

We now break the firm’s problem into one, minimizing cost and setting the price, which will help us illustrate the working capital channel.

Cost Minimisation

From 11, Firms solve

\[
\begin{align*}
\text{min}_{l_j} (1+i)wl_j \\
\text{s.t.} A l_j & \geq \left( \frac{p_j}{P} \right)^{-\theta} Y.
\end{align*}
\]  

(61)

The solution to this satisfies

\[
m^c_j = \frac{(1+i)w}{A} \hat{w},
\]  

(62)

where \( m^c_j \) is the real marginal cost and \( \hat{w} \) is the real wage. This is the expression for the working capital channel of Christiano et al. (2005). We show below that debt and household heterogeneity amplifies the working capital channel.

Price Setting

Take the first-order condition for optimal profits with respect to price and substitute 62:

\[
\begin{align*}
0 &= (1-\theta)(p_j)^{-\theta} P^\theta Y - (1+i)(-\theta w(p_j)^{-1-\theta} P^\theta l_j), \\
0 &= (1-\theta)A - (1+i)(-\theta w(p_j)^{-1}) \\
p_j &= \sigma Pm^c_j
\end{align*}
\]  

(63)

(64)

(65)
Where $\sigma = \frac{\theta}{\theta - 1}$ is the markup, where a higher value of $\sigma$ means greater market power. This shows that the real marginal cost is constant and equal to the inverse of $\sigma$ in this example. Monetary policy affects the wage rate, labor supply, and the corresponding effect on income distribution across household types. Although a direct effect of an increase in the monetary policy rate is to increase marginal cost via the financial cost of working capital (as seen in 62), the increase in the monetary policy rate reduces the real wage. This leads to an indirect effect pushing down marginal cost. These two effects cancel out in general equilibrium, t. As we shall shortly prove, even in this case, when monetary policy does not affect real marginal cost, prices can respond much less than output to monetary disturbances, simply owing to the income effect through corporate debt.

**Aggregate prices**

Use $p_j = P$, and substitute $l_j = L$,

$$0 = (1 - \theta)Y + (1 + i)(\theta \tilde{w}L),$$

(66)

equivalent to

$$\tilde{w} = \frac{A}{\sigma (1 + i)},$$

(67)

**Labour Supply**

The optimality conditions for the Lender Households’ labor supply gives

$$\tilde{w} = c_L$$

(68)

$$= \tilde{w}L + \psi \frac{R(1 + i)D}{P}$$

(69)

$$\tilde{w}L = \tilde{w} - \psi \frac{R(1 + i)D}{P}.$$

(70)

The above equation shows that debt flattens the labor supply curve and supports the high effective labor supply elasticity emphasized in the cost channel of monetary policy literature.\(^{22}\) This high elasticity may dampen the response of prices in the presence of monetary disturbances, even though output remains responsive. Given the price level, the elasticity of labor supplied $\epsilon_L$ is

\(^{22}\)See Barth and Ramey (2001) for the aggregate and industry-level evidence on the strength of monetary disturbances as a cost shock.
\[ \epsilon_L = \frac{\partial L}{\partial \tilde{w}} = \frac{R(1 + \imath \iota) \psi D}{P \tilde{w} L} = \frac{\psi R(1 + \imath \iota) D}{b P}. \] (71)

**B Proof of Corollary 1**

To derive the closed-form solution for the price level, we simply equate Aggregate Demand and Supply and obtain (72):

\[ P = \frac{m + iR(1 + \imath \iota) \psi D}{\frac{1}{\sigma} \frac{1}{1+i} A}. \] (72)

To obtain the closed-form solution for allocation, we combine all flow of funds constraints of households (4) and (7) and of the firms (11). This leads to (73), showing that when the working capital liquidity that was injected in the morning exits the economy, the net interest payment of the working capital liquidity \( b_i \) equates the aggregate monetary endowment \( m \) - an outstanding liability of the monetary-fiscal authority, which becomes monetary authority’s seigniorage profits. In the dynamic model, nominal seigniorage profits are transferred to the next period.\(^{23}\)

\[ b_i = m. \] (73)

The total money lent by the monetary-fiscal authority (inside money) is given by \( M = \frac{m}{\iota} \). This is because the seigniorage profits of the monetary-fiscal authority is \( m \), and the total money supply is \( M + m \), the inside money plus outside money. Substituting \( b = wL \) and (67) into (73), we obtain

\[ L = \frac{m}{iP} \left( \frac{A}{\sigma(1 + i)} \right)^{-1}. \] (74)

Combine the above equation with (72) and \( Y = AL \), we have the closed-form solution for output:

\[ Y = \frac{A}{1 + iR(1 + \imath \iota) \psi D m}. \] (75)

We obtain nominal profits from 13

\(^{23}\) Fiscal policy is non-Ricardian. The monetary transfer is a government liability that is recovered through seigniorage profits at a unique price level (as in the Fiscal Theory of the Price Level). See Drèze and Polemarchakis (2000), Buiter (2002), and Dubey and Geanakoplos (2003) among others.
It follows that \( \partial \Pi / \partial i = -i^{-2}m(\sigma - 1) - R\psi D \). Since \( \sigma > 1 \), \( \partial \Pi / \partial i < 0 \).

Moreover, given that we have obtained the close form for the price level (72), the expression for real profits \( \tilde{\Pi} \) is as follows:

\[
\tilde{\Pi} = \sigma - 1 \sigma mA - R(1 + \iota i)\psi D \left(1 - \frac{1}{1+i}\right) \frac{A}{\sigma}.
\] (77)

As can be seen from the previous equation, real profits decrease when \( i \) increases.

### C Proof of Proposition 2

Let \( \epsilon_{P_i} \) be the elasticity of the price level with respect to the monetary policy rate. We use (72) to derive \( \epsilon_{P_i} \). First, the price level can be rearranged as

\[
P = m + (1 + i)(1 - \iota R\psi D + \iota R\psi D(1 + i)^2 - (1 - \iota R\psi D - \iota R\psi D(1 + i) \frac{1}{\sigma} A - \frac{1}{\sigma} \frac{1}{1+i} A).
\] (78)

The direct response of the price level to inflation is

\[
\frac{\partial P}{\partial (1+i)} = -\frac{P}{i(1+i)} + \frac{P}{m + iR(1 + \iota i)\psi D}.
\] (79)

Finally, the elasticity is given by

\[
\frac{\partial P}{P} = \frac{\partial P}{i(1+i)} = \frac{iR\psi D(1 + \iota i) + (1 + i)iR\psi D - b}{m + iR(1 + \iota i)\psi D}.
\] (80)

The first term in the numerator is the direct liquidity cost incurred through higher policy rates, while the second term is the direct effect of monetary policy on the repayment of outstanding debt. Therefore, \( \epsilon_{P_i} < 0 \) (the standard Taylor principle) holds \textit{iff} \( iR\psi D(1 + \iota i) + (1 + i)iR\psi D < b \)\(^{24}\). Otherwise, the Taylor principle is inverted and \( \epsilon_{P_i} > 0 \). If debt is extremely high relative to working

\(^{24}\)In terms of primitives, the condition can be written as \( iR\psi D(1 + \iota i) + (1 + i)iR\psi D < \frac{m}{i} \).

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capital requirements, raising interest rates raises the inflation rate.

It is straightforward that $\epsilon_{p_i}$ is higher when $D$ is larger. Hence the negative response of inflation is increasingly muted and eventually becomes positive as the size of legacy debt increases.

D Equilibrium Equations

Based on the analysis so far, the equilibrium of the dynamic economy is characterized by the following system of equations consisting of optimality conditions, market-clearing conditions, and the monetary policy rule:
\[ c^o + k' = \tilde{\pi} W + \tilde{r} k + \tilde{\pi}, \quad (81) \]

\[ \frac{1}{c^o} = \beta E \frac{1}{c^o} (r'_k), \quad (82) \]

\[ \tilde{q} \tilde{d}' + \frac{\phi_d}{2} \tilde{q} (\tilde{d}' - \tilde{d})^2 + c' = \tilde{w} l + \frac{\tilde{d}}{1+\eta}, \quad (83) \]

\[ \frac{\tilde{w}}{c} = kl, \quad (84) \]

\[ \frac{\tilde{q}}{c} (1 + \phi_d (\tilde{d}' - \tilde{d})) = \beta E \frac{1}{c^o} \frac{1}{1+\eta'}, \quad (85) \]

\[ y_W = A \rho^a k^{1-\alpha}, \quad (86) \]

\[ \tilde{w} = \beta E \frac{1}{c^o} (r'_k), \quad (87) \]

\[ \tilde{w} l = \tilde{b}, \quad (88) \]

\[ \tilde{\pi} W + \tilde{r} k + \frac{1}{1+\eta} \tilde{d} W + \tilde{w} l (1+i) = \tilde{p} W y_W + \tilde{q} \tilde{d} W, \quad (89) \]

\[ \tilde{w} = \frac{1}{1+i} (1 - \alpha) \tilde{p} W y/l, \quad (90) \]

\[ \frac{\tilde{q}}{c^o} = \beta E \frac{1}{c^o} \frac{1}{1+\eta'}, \quad (91) \]

\[ \tilde{\pi} = y - \nu y \tilde{p} W, \quad (92) \]

\[ y_W = \nu y, \quad (93) \]

\[ \tilde{p} W = \frac{1}{A} (1+i) \tilde{w} \frac{1}{1+\eta} (r'_k)^{\alpha}, \quad (94) \]

\[ (1+\eta) = [(1-\phi) (1+\eta^\#)^{1-\theta} + \phi] \frac{1}{1+\eta^\#}, \quad (95) \]

\[ (1+\eta^\#) = \frac{\theta}{\theta - 1} (1+\eta) \frac{x_1}{x_2}, \quad (96) \]

\[ x_1 = \frac{1}{c^o} \tilde{p} W y + \phi \beta E (1+\eta')^{\theta-1} x_1', \quad (97) \]

\[ x_2 = \frac{1}{c^o} y + \phi \beta E (1+\eta')^{\theta-1} x_2', \quad (98) \]

\[ \nu = (1-\phi) (1+\eta^\#)^{-\theta} (1+\eta)^{\theta} + (1+\eta)^{\theta} \phi \nu_{-1}, \quad (99) \]

\[ \frac{1+i}{1+i} = \frac{y}{y} \frac{1+i}{1+i} \frac{1+\eta}{1+\eta}, \quad (100) \]

\[ \tilde{d} W = \tilde{d} + \tilde{\mu}, \quad (101) \]

\[ y = c^o + c' + k' + \frac{\phi_d}{2} \tilde{q} (\tilde{d}' - \tilde{d})^2. \quad (102) \]
E  steady-state

The equations required to solve the zero-inflation steady-state are:

\[ \bar{c}^o + \bar{k} = \bar{\pi}_W + \bar{r}_k \bar{k} + \bar{\pi}, \]  \hspace{1cm} (103)

\[ 1 = \beta \bar{r}_k, \]  \hspace{1cm} (104)

\[ \bar{q} \bar{d} + \bar{c}^d = \bar{w} \bar{l} + \bar{d}, \]  \hspace{1cm} (105)

\[ \frac{\bar{w}}{\bar{c}^d} = \kappa \bar{\delta}, \]  \hspace{1cm} (106)

\[ \bar{y}_W = A \bar{k}^\alpha \bar{l}^{1-\alpha}, \]  \hspace{1cm} (107)

\[ \bar{w} \bar{l} = \bar{b}, \]  \hspace{1cm} (108)

\[ \bar{\pi}_W + \bar{r}_k \bar{k} + \bar{d}_W + \bar{w} \bar{l}(1 + \bar{i}) = \bar{p}_W \bar{y}_W + \bar{q} \bar{d}_W, \]  \hspace{1cm} (109)

\[ \frac{\alpha}{1 - \alpha} (1 + \bar{i}) \bar{w} \bar{l} = \bar{r}_k \bar{k}, \]  \hspace{1cm} (110)

\[ \bar{q} = \beta, \]  \hspace{1cm} (111)

\[ \bar{\pi} = \bar{y} - \nu \bar{y} \bar{p}_W, \]  \hspace{1cm} (112)

\[ \bar{y}_W = \nu \bar{y}, \]  \hspace{1cm} (113)

\[ \bar{p}_W = \frac{1}{A} \left( \frac{1 + i}{1 - \alpha} \right)^{1-\alpha} \left( \frac{\bar{r}_k}{\alpha} \right)^{\alpha}, \]  \hspace{1cm} (114)

\[ 1 = \frac{\theta}{\theta - 1} \frac{x_1}{x_2}, \]  \hspace{1cm} (115)

\[ \bar{x}_1 = \frac{1}{\bar{\pi}_W} \bar{p}_W \bar{y}, \]  \hspace{1cm} (116)

\[ \bar{x}_2 = \frac{1}{\bar{\pi}_W} \bar{y}, \]  \hspace{1cm} (117)

\[ \nu = 1 \]  \hspace{1cm} (118)

\[ \bar{d}_W = \bar{d} + \bar{\mu}, \]  \hspace{1cm} (119)

\[ \eta = 0 \]  \hspace{1cm} (120)

\[ \bar{y} = \bar{c}^o + \bar{c}^d + \bar{k}. \]  \hspace{1cm} (121)

F  Proof of Proposition 3

Aggregate demand at the steady-state is \( \bar{c}^o + \bar{k} + \bar{c}^d \). Substitute in households’ and firms’ flow of funds constraints into aggregate demand for output, with the market-clearing condition for final output
\( \bar{y} = c^o + \bar{d} + \bar{k}, \) we obtain the following:

\[
\bar{y} = c^o + \bar{k} + \bar{d} = -\bar{w}\bar{l} + \bar{y} + \bar{m}, \tag{122}
\]

\[
\bar{w}\bar{l} = \frac{\bar{m}}{i}, \tag{123}
\]

\[
= M. \tag{124}
\]

From the marginal cost of the firm we get that \( \bar{p}_W = \frac{1}{\sigma} \) in the steady-state (simply by combining (115), (116), and (117)). Combine (114), (104), and \( \bar{p}_W = \frac{1}{\sigma} \), we obtain the analytic expression for real wage at the steady-state (125). We can see that contractionary monetary policy reduces real wages in the steady-state.

\[
\bar{w} = \frac{1}{1 + i} \left\{ \frac{A(\beta\alpha)^\alpha(1 - \alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}}, \tag{125}
\]

To obtain the closed-form solution for labor in the steady-state, we combine (125) and (124):

\[
\bar{l} = \frac{M(1 + \bar{i})}{\left\{ \frac{A(\beta\alpha)^\alpha(1 - \alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}}}. \tag{126}
\]

Combine the lenders’ first order condition for labour (106) and their budget constraint (105):

\[
\bar{w} = \kappa\bar{l}(\bar{w}\bar{l} + \bar{d}(1 - \bar{q})), \tag{127}
\]

\[
\bar{l} = \frac{\bar{w}}{\kappa(\bar{m} + \bar{d}(1 - \bar{q}))}. \tag{128}
\]

Now we use the steady-state equations to prove Proposition 3. We combine (110), (104), and (125):

\[
\frac{\bar{k}}{\bar{l}} = \beta \frac{\alpha}{1 - \alpha} (1 + \bar{l}) \bar{w} \tag{129}
\]

\[
= \frac{\beta\alpha}{1 - \alpha} \left\{ \frac{A(\beta\alpha)^\alpha(1 - \alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}} \tag{130}
\]

\[
= \left\{ \frac{\beta\alpha}{1 - \alpha} \right\}^{1-\alpha} A(\beta\alpha)^\alpha(1 - \alpha)^{1-\alpha} \tag{131}
\]

\[
= \left\{ \frac{A\beta\alpha}{\sigma} \right\}^{\frac{1}{1-\alpha}}. \tag{132}
\]
and so the steady-state level of output is

\[ \bar{y} = A \left( \frac{\bar{k}}{l} \right)^{\alpha} \bar{l} \]

\[ = A \left\{ \frac{A \beta \alpha}{\sigma} \right\}^{1-\alpha} \bar{l} \]

\[ = A \left\{ \frac{A \beta \alpha}{\sigma} \right\}^{1-\alpha} \frac{\bar{M}(1 + \bar{i})}{\left\{ \frac{A(\beta \alpha)^{\alpha} (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{1-\alpha}} \tag{133} \]

\[ = \frac{\sigma}{1 - \alpha} \bar{M}(1 + \bar{i}) \tag{134} \]

\[ = \frac{\sigma}{1 - \alpha} \bar{M}(1 + \bar{i}) \tag{135} \]

This is independent of household preferences. Keeping \( \bar{i} \) unchanged, the ratio of real money balance to output is constant. We can now solve for the steady-state real money balance. Note that (127) can be re-expressed as follows:

\[(\bar{w})^2 = \kappa \bar{M}(\bar{M} + \bar{d}(1 - \bar{q})) \tag{136} \]

\[\frac{1}{(1 + \bar{i})^2} \left\{ \frac{A(\beta \alpha)^{\alpha} (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{2}{1-\alpha}} = \kappa \bar{M}(\bar{M} + \bar{d}(1 - \bar{q})) \tag{137} \]

\[\kappa = \frac{1}{(1 + \bar{i})^2} \left\{ \frac{A(\beta \alpha)^{\alpha} (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{2}{1-\alpha}} \tag{138} \]

Suppose that \( \bar{d} = 0 \). In this case, \( \bar{M} = \kappa^{-5} \frac{1}{1+\bar{i}} \left\{ \frac{A(\beta \alpha)^{\alpha} (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}} \) and the nominal interest rate has an inverse relationship with the steady-state level of money balance. As legacy debt \( \bar{d} \) increases, the steady-state level of money decreases. Furthermore, as the nominal interest rate increases, due to the legacy debt, money balances decrease to a lesser degree.

Note that when \( \bar{d} = 0 \), \( \bar{y} = \frac{\sigma}{1 - \alpha} \kappa^{-5} \left\{ \frac{A(\beta \alpha)^{\alpha} (1-\alpha)^{1-\alpha}}{\sigma} \right\}^{\frac{1}{1-\alpha}} \), so money is neutral in the steady-state. When \( \bar{d} \neq 0 \), money is non-neutral in the steady-state.
It is convenient to denote legacy debt in terms of leverage: \( \text{lev} = \frac{\bar{d}}{\bar{y}} \).

\[
\frac{1}{(1 + i)^2} \left\{ \frac{A(\beta \alpha)^{\alpha}(1 - \alpha)^{1 - \alpha}}{\sigma} \right\}^\frac{1}{1 - \alpha} = \kappa \frac{\bar{M}}{1 + i} \left( \frac{\bar{M}}{1 + i} + \bar{y} \text{lev}(1 - \bar{q}) \right) \tag{139}
\]

\[
= \kappa \bar{M} \left( \frac{\bar{M}}{1 + i} + \frac{\sigma}{1 - \alpha} \bar{M} \text{lev}(1 - \bar{q}) \right) \tag{140}
\]

\[
= \kappa \left( \frac{\bar{M}}{1 + i} \right)^2 \left( 1 + \frac{\sigma}{1 - \alpha} (1 + \bar{i}) \text{lev}(1 - \bar{q}) \right) \tag{141}
\]

\[
\left\{ \frac{A(\beta \alpha)^{\alpha}(1 - \alpha)^{1 - \alpha}}{\sigma} \right\}^\frac{2}{1 - \alpha} = \kappa \left( \bar{M} \right)^2 \left( 1 + \frac{\sigma}{1 - \alpha} (1 + \bar{i}) \text{lev}(1 - \beta) \right) \tag{142}
\]

\[
\bar{M} = \left\{ \frac{A(\beta \alpha)^{\alpha}(1 - \alpha)^{1 - \alpha}}{\sigma} \right\}^\frac{1}{1 - \alpha} \left( \frac{\kappa}{1 + \frac{\sigma}{1 - \alpha} (1 + \bar{i}) \text{lev}(1 - \beta)} \right)^\frac{1}{2} \tag{143}
\]

\[
\bar{M} = \left\{ \frac{A(\beta \alpha)^{\alpha}(1 - \alpha)^{1 - \alpha}}{\sigma} \right\}^\frac{1}{1 - \alpha} \left( \kappa \left[ 1 + \frac{\sigma}{1 - \alpha} (1 + \bar{i}) \text{lev}(1 - \beta) \right] \right)^\frac{1}{2} \tag{144}
\]

The expression above implies that as leverage increases, the quantity of real money balance decreases.

Given our parameterisation in Table 2, below Table 4 displays the model steady-state values with quantity variables normalised by output.

**Table 4: steady-state values**

<table>
<thead>
<tr>
<th></th>
<th>(c^y/\bar{y})</th>
<th>(c^\bar{y}/\bar{y})</th>
<th>(k/\bar{y})</th>
<th>(b/\bar{y})</th>
<th>(\bar{\pi}/\bar{y})</th>
<th>(d/\bar{y})</th>
<th>(\bar{q})</th>
<th>(\bar{r}_k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMK lev</td>
<td>0.178</td>
<td>0.558</td>
<td>0.264</td>
<td>0.587</td>
<td>0.175</td>
<td>3</td>
<td>0.990</td>
<td>1.01</td>
</tr>
<tr>
<td>High lev</td>
<td>0.168</td>
<td>0.568</td>
<td>0.264</td>
<td>0.587</td>
<td>0.165</td>
<td>4</td>
<td>0.990</td>
<td>1.01</td>
</tr>
</tbody>
</table>

**BMK lev** refers to the benchmark leverage of 75% (annual), or \(b/\bar{y} = 3\). **High lev** refers to the high debt leverage of 100% (annual), or \(b/\bar{y} = 4\).
G Dynamic Equations

\[ \ddot{c} \dot{c} + \ddot{k} \dot{k} = \ddot{\pi}_W \dot{\pi}_W + \ddot{r}_k \dot{r}_k (\dot{r}_k + \dot{\hat{k}}) + \ddot{\hat{\pi}}, \]  
(145)

\[ \ddot{c} = \dot{c}_k', \]  
(146)

\[ \bar{q} \ddot{d} (\dot{q} + \dot{d}') + \phi_d \bar{q} \ddot{d} + c' \ddot{c} = \ddot{\omega} (\dot{w} + \dot{l}) + \ddot{d} (\dot{d} - (1 + \eta)), \]  
(147)

\[ \dot{w} - \dot{c}' = \dot{l}, \]  
(148)

\[ \dot{q} - \ddot{c}' + \phi_d \ddot{d} = -(\ddot{c} + (1 + \eta)) \]  
(149)

\[ \dot{\bar{y}}_W = \ddot{A} + \alpha \dot{k} + (1 - \alpha) \dot{l}, \]  
(150)

\[ \dot{\bar{w}} + \dot{l} = \bar{b}, \]  
(151)

\[ \ddot{\bar{q}} \ddot{W} + k \ddot{r}_k (\dot{r}_k + \dot{k}) + \ddot{d}_W (\dot{d}_W - (1 + \eta)) + \ddot{\omega} (\dot{1} + i) (\dot{1} + i) \]  
(152)

\[ \dot{\bar{q}} = -\ddot{\pi}_W (\ddot{\pi}_W + \ddot{\bar{y}}_W) + \ddot{\bar{q}} \ddot{d}_W (\dot{q} + \dot{d}_W), \]  
(153)

\[ \dot{\bar{r}}_k = \ddot{\bar{p}}_W + \dot{\bar{y}} - \dot{k}, \]  
(154)

\[ \dot{\bar{q}} - \ddot{c} = -\ddot{c}' - (1 + \eta'), \]  
(155)

\[ \ddot{\bar{y}} = \ddot{\bar{y}} - \nu \ddot{\bar{p}}_W (\dot{\bar{y}} + \ddot{\bar{p}}_W), \]  
(156)

\[ \ddot{\bar{W}} = \ddot{\bar{y}} + \dot{\bar{y}}, \]  
(157)

\[ \ddot{\bar{W}} = -\ddot{A} + (1 - \alpha) (1 + i) + (1 - \alpha) \dot{w} + \alpha \dot{r}_k, \]  
(158)

\[ (1 + \eta) = (1 - \phi) (1 + \eta'), \]  
(159)

\[ (1 + \eta') = (1 + \eta) + \dot{x}_1 - \dot{x}_2 \]  
(160)

\[ \dot{x}_1 = (1 - \phi \beta) (-\ddot{c} + \ddot{\bar{p}}_W + \dot{\bar{y}}) + \theta \phi \beta (1 + \eta') + \phi \beta \dot{x}_1, \]  
(161)

\[ \dot{x}_2 = (1 - \phi \beta) \ddot{\bar{p}} + \ddot{\bar{y}} + \phi \beta ((\theta - 1) (1 + \eta') + \dot{x}_2), \]  
(162)

\[ \dot{\nu} = 0 \]  
(163)

\[ (1 + i) = \rho_i (1 + i) + \rho_y \dot{y} + \rho_l (1 + \eta) + \epsilon_i, \]  
(164)

\[ \ddot{d}_W \dot{d}_W = \ddot{d}_W, \]  
(165)

\[ \dddot{\bar{y}} = \dddot{c} \dddot{c} + \dddot{d} + \dddot{k} + \phi_d \dddot{q} \dddot{d}, \]  
(166)
H  Proof of Proposition 4

Recall the public balance equation (49). After substituting the working-capital constraint, and the constant purchases of intertemporal bonds, this becomes

\[ \tilde{w}l + \tilde{\mu}\left(\frac{1}{1 + \eta} - \tilde{q}\right) = 0, \]  \hspace{1cm} (167)

When we linearise, this becomes

\[ \tilde{\mu}(\tilde{q}\hat{q} + (1 + \eta)) = \tilde{w}\tilde{l}((1 + i) - 1)(\tilde{w} + \hat{l}) + \tilde{w}\tilde{l}(1 + i)(1 + i). \]  \hspace{1cm} (168)

Simplifying

\[ \hat{w} + \hat{l} = \frac{\tilde{\mu}(\tilde{q}\hat{q} + (1 + \eta)) - \tilde{w}\tilde{l}(1 + i)(1 + i)}{\tilde{w}\tilde{l}((1 + i) - 1)}, \]  \hspace{1cm} (169)

where

\[ \tilde{w}\tilde{l} = \tilde{\mu} - \frac{1}{i}. \]  \hspace{1cm} (170)

We can now solve for labour supply from (147) and (148)

\[ \hat{l} = \frac{1}{2\hat{d}} \left\{ \tilde{q}\tilde{d}(\tilde{q} + \hat{d}') + \phi_{\lambda}\tilde{d}\hat{d}' + (\tilde{c} - \tilde{w}\tilde{l})(\tilde{w} + \hat{l}) - \tilde{d}(\hat{d} - (1 + \eta)) \right\}. \]  \hspace{1cm} (171)

With this in hand, we can obtain an expression for output 150:

\[ \hat{y}_W = \hat{A} + \alpha\hat{k} + (1 - \alpha) \frac{1}{2\hat{d}'} \left\{ \tilde{q}\tilde{d}(\tilde{q} + \hat{d}') + \phi_{\lambda}\tilde{d}\hat{d}' + (\tilde{c} - \tilde{w}\tilde{l})(\tilde{w} + \hat{l}) - \tilde{d}(\hat{d} - (1 + \eta)) \right\}. \]  \hspace{1cm} (172)
Take 153, and for analytical convenience set $\phi_d = 0$,

$$\hat{p}_W = \hat{i} + \hat{w} + (\hat{i} + i) - \hat{y}$$

(173)

$$= \hat{i} + \hat{w} + (\hat{i} + i) - \hat{A} - \alpha \hat{k}$$

$$- \left(1 - \alpha \right) \frac{1}{2 \hat{c}^d} \left\{ \bar{q} \hat{d}(\hat{q} + \hat{d}') + \phi_d \bar{q} \hat{d}' + (\hat{c}^d - \bar{w} \hat{I})(\hat{w} + \hat{i}) - \bar{d}(\hat{d} - (\hat{i} + \eta)) \right\}$$

(174)

$$= \left(\hat{i} + \hat{w} \right) \left\{ 1 - \left(1 - \alpha \right) \frac{1}{2 \hat{c}^d} (\hat{c}^d - \bar{w} \hat{I}) \right\} + (\hat{i} + i) - \hat{A} - \alpha \hat{k}$$

$$- \left(1 - \alpha \right) \frac{\hat{d}}{2 \hat{c}^d} \left\{ \bar{q} (\hat{q} + \hat{d}') - \hat{d} \right\} - \left(1 - \alpha \right) \frac{\bar{d}}{2 \hat{c}^d} (1 + \eta)$$

(175)

$$= \frac{\bar{\mu} (\bar{q} \hat{q} + (1 + \eta)) - \bar{w} \bar{l} (1 + i) (1 + \eta)}{\bar{w} \bar{l} ((1 + i) - 1)} \left\{ 1 - \left(1 - \alpha \right) \frac{1}{2 \hat{c}^d} (\hat{c}^d - \bar{w} \hat{I}) \right\} + (\hat{i} + i) - \hat{A} - \alpha \hat{k}$$

$$- \left(1 - \alpha \right) \frac{\hat{d}}{2 \hat{c}^d} \left\{ \bar{q} (\hat{q} + \hat{d}') - \hat{d} \right\} - \left(1 - \alpha \right) \frac{\bar{d}}{2 \hat{c}^d} (1 + \eta)$$

(176)

$$= \left(1 + \eta \right) \left\{ \frac{\bar{\mu} \left\{ 1 - \left(1 - \alpha \right) \frac{1}{2 \hat{c}^d} \bar{d} (1 - \bar{q}) \right\}}{\bar{w} \bar{l} ((1 + i) - 1)} - \left(1 - \alpha \right) \frac{\bar{d}}{2 \hat{c}^d} \right\}$$

$$+ \bar{q} \hat{q} \left\{ \frac{\bar{\mu} \left\{ 1 - \left(1 - \alpha \right) \frac{1}{2 \hat{c}^d} \bar{d} (1 - \bar{q}) \right\}}{\bar{w} \bar{l} ((1 + i) - 1)} - \left(1 - \alpha \right) \frac{\bar{d}}{2 \hat{c}^d} \right\}$$

$$+ (\hat{i} + i) \left\{ 1 - 1 + i \left\{ 1 - \left(1 - \alpha \right) \frac{1}{2 \hat{c}^d} \bar{d} (1 - \bar{q}) \right\} \right\} - \hat{A} - \alpha \hat{k} - \left(1 - \alpha \right) \frac{\bar{d}}{2 \hat{c}^d} \left\{ \bar{q} \hat{d}' - \hat{d} \right\}$$

(177)

where $\hat{c}^d = \bar{w} \hat{I} + \bar{d} (1 - \bar{q})$. Consider the coefficient in front of $(1 + i)$

$$\left\{ 1 - (1 + i) \frac{1 - (1 - \alpha) \frac{1}{2 \hat{c}^d} \bar{d} (1 - \bar{q})}{((1 + i) - 1)} \right\} = \frac{-1}{((1 + i) - 1)} \left\{ 1 - (1 + i) (1 - \alpha) \frac{1}{2 \hat{c}^d} \bar{d} (1 - \bar{q}) \right\}$$

(178)

As $(1 + i) (1 - \alpha) \frac{d (1 - q)}{2 \hat{c}^d} < 1$ holds, it follows that higher steady-state levels of legacy debt, $\bar{d}$, makes the coefficient of $(1 + i)$ closer to 0 in absolute value.

Similarly, we can simplify the expression in front of the inflation term, $(1 + \eta)$, and bond price term $\bar{q} \hat{q}$,

$$\bar{\mu} \left\{ 1 - \left(1 - \alpha \right) \frac{1}{2 \hat{c}^d} \bar{d} (1 - \bar{q}) \right\} - \left(1 - \alpha \right) \frac{\bar{d}}{2 \hat{c}^d} = \frac{\left\{ 1 - \left(1 - \alpha \right) \frac{1}{2 \hat{c}^d} \bar{d} (1 - \bar{q}) \right\}}{\bar{q} - 1} - \left(1 - \alpha \right) \frac{\bar{d}}{2 \hat{c}^d}$$

(179)

$$= - \frac{1}{1 - \bar{q}}.$$

(180)

This allows us to obtain the following expression for the marginal cost.
\[
\hat{p}_W = -\frac{(1 + \eta) + \bar{q}\hat{q}}{1 - \bar{q}} - \frac{(1 + \hat{i})(1 - \alpha)\bar{d}(1 - \bar{q})}{((1 + \hat{i}) - 1)} \left\{ 1 - \frac{(1 + \hat{i})(1 - \alpha)\bar{d}(1 - \bar{q})}{2(\bar{w}l + \bar{d}(1 - \bar{q}))} \right\} - \hat{A} - \alpha \hat{k} - \frac{(1 - \alpha)\bar{d} \{ \hat{q}d' - d \}}{2(\bar{w}l + \bar{d}(1 - \bar{q}))}, \\
\]

(181)

To summarise, higher steady-state legacy debt reduces the direct effect of interest rates on marginal cost and increases the sensitivity of changes in debt.

I Dynamic Responses of Robustness Checks

![Graph showing dynamic responses of various variables to tightening shock in nominal policy rate.](image)

**Figure 7:** Tightening shock to nominal policy rate \(i\) (with fixed coupon bonds).

*Blue line is 75% leverage and red line is 100% leverage. y-axis is % change and x-axis is the number of periods. Other than inflation and policy rate, all variables are in real terms.*
Figure 8: Tightening shock to nominal policy rate $i$ with or without output stabilisation (with fixed coupon bonds).

Red solid line is the benchmark Taylor rule ($\rho_y = 0.2$) and the dashed black line is the output stabilisation Taylor rule ($\rho_y = 0.9$). Y-axis is % change and x-axis is the number of periods. Other than inflation and policy rate, all variables are in real terms.