Implementing monetary policy rules in the presence of inside money*

Balázs Világi[†], Balázs Vonnák[‡]

Preliminary version

August 5, 2022

Abstract

The majority of the New Keynesian DSGE literature assumes that the macroeconomic effects of monetary policy can be satisfactorily described by an interest rate rule without addressing the details of the money supply. We investigate whether this approach remains valid in the presence of inside money created by the banking system. To analyze this issue we present a framework based on the generalization of the *IS* and *LM* curves to dynamic general equilibrium models. We find that it is possible to implement a policy based on an interest rate rule even in the presence of inside money, although it requires a more complex toolkit of monetary policy implementation than it is assumed in models with only outside money.

*We are heavily indebted to András Kollarik for his substantial and detailed discussion of the paper and his suggestions. We would also like to thank Dániel Baksa, Gábor Horváth, Lóránt Kaszab, Pál Péter Kolozsi, Olivér Rácz and Ferenc Tóth for their comments and editing suggestions.

[†]Magyar Nemzeti Bank (Central Bank of Hungary) and John von Neumann University, MNB Institute - Sustainable Finance Research Center, e-mail: vilagib@mnb.hu.

[‡]Magyar Nemzeti Bank (Central Bank of Hungary) and John von Neumann University, MNB Institute - Sustainable Finance Research Center, e-mail: vonnakb@mnb.hu.

1 Introduction

Over the past two decades, New Keynesian DSGE models have become the main workhorse for macroeconomic analysis of monetary policy, consisting of a generalized, *dynamic IS curve*, an *interest rate rule* representing monetary policy, and a *pricing block* (often a Phillips curve), see for example Clarida, Galí and Gertler (1999) and Galí (2015). Details of the money supply mechanism do not usually appear explicitly in these models, as it is assumed that the quantity of money does not provide additional relevant information compared to the model blocks listed.

Central bankers, however, have never been fully convinced by the approach of academic researchers. Although they have not formally proved it, they still believe that ignoring the mechanism of money creation can lead to substantial errors in macroeconomic analysis. This doubt has intensified since the financial crisis of 2007-2008. Their main argument is that because the banking sector is also involved in money creation by supplying *inside money*, the relationship between the *outside money* issued by the central bank and the economy has become more indirect and complicated.

In contrast, the New Keynesian counter-argument can be intuitively summarized as that no matter how complicated the relationship between outside and inside money, the LM curve can always be "fitted" to the intersection of the IS curve and the interest rate rule, so its role is redundant from a macroeconomic point of view. In other words, the implementation of monetary policy may be more complicated in the presence of inside money, but the interest rate rule, the IS curve and the pricing block are still form a sufficient toolkit for macroeconomic analysis.

Since the above debate is mainly based on conjectures and non-formal partial analyses, the aim of our study is to create a simple formal macroeconomic model for examining whether the prevailing academic view that the consequences of banks' money creation are macroeconomically negligible can be justified or, on the contrary, neglecting inside money leads to fundamentally flawed results.

The starting point of our analysis is the observation that there are two main functions in the modern banking system: financial intermediation and the provision of transaction instruments for economic agents by issuing liquid liabilities. Banks' provision of transaction instruments is part of the money creation process and therefore part of the LM curve, and as Woodford (2010) showed, financial intermediation can be incorporated into the IS curve. In the modern banking system, these two functions are mixed when long-term loans are financed by liquid liabilities (deposits). Hence, a link is created between the IS and the LM curve that is absent when only outside money exists.[1] We show that in the case of inside money, for example, a monetary easing results in an increase in investment not only because of falling interest rates, but also because the growing stock of liquid deposits directly finances more investment loans. That is, a monetary easing shifts not only the LM but also the IS curve.

This observation is in strong contrast to the the standard New Keynesian approach that monetary policy affects the IS curve only through the nominal interest rate, and the information on the money supply is therefore redundant, and the inside money is only significant to the extent that the relationship between the outside money and the LM curve becomes more complicated. In our paper, we investigate whether, despite the previous argument being flawed, the New Keynesian approach can still provide valid macroeconomic analysis.

We examine the above issue in a simple *general equilibrium model*. The central element of our model is the banking block based on Piazzesi and Schneider (2018). The banking system provides investment loans, and its liability side consists of long-term and liquid deposits of households. Liquid deposits fulfill the function of a transaction instrument, i.e., money. An individual bank has an incentive to use as many liquid deposits as possible to fund investment loans, as they are cheaper.

At the same time, there is a risk associated with holding liquid deposits, since when a buyer withdraws his deposit from his bank during a transaction and it is transferred to the seller's bank at the end of the transaction, the movement of deposits must be accompanied by the movement of central bank reserves. Due to the resulting liquidity risk, banks are forced to cover part of their liquid deposits with central bank reserves. In the model, the money multiplier, i.e., the ratio of total money stock and central bank reserves, is the result of optimal liquidity management of banks.[2] Liquidity and lending decisions, i.e., the provision of transaction instruments and financial intermediation are interrelated in the banking system of the model, which creates a new, additional relationship between the IS and LM blocks.

In the above framework, it is not trivial whether the New Keynesian approach remains valid, which is based on the assumption that when a shock shifts the IS curve, it is "only" a monetary policy implementation issue to shift the LM curve to the new intersection of the IS curve and the interest rate

[1] As well known, if the money term in the utility function of households is non-separable then the IS and LM curves are also related. What we want to show is that even the separable utility function is not sufficient for the independence of the IS and LM curves in the presence of inside money.

[2] The way we model the banking system is in line with the view of central bankers, see Maclay, Radia, and Thomas (2014) from the Bank of England.

rule. However, as discussed, shifting the LM curve changes the position of the IS curve, too, thus, the size of the money supply is not merely redundant information.

At the same time, if the banking system is also involved in money creation, the central bank has other instruments at its disposal besides the stock of outside money: it can control the interest rate on central bank reserves and it can influence liquidity in the interbank market. We show that with these tools the central bank can not only shift the IS curve, but also influence its slope. As a consequence, it can compensate for the shift in the IS curve caused by the change in the money stock.

The implication of the above is that it is possible to implement a policy based on an interest rate rule and the IS curve even in the presence of inside money, however, it requires a more complex and sophisticated toolkit of monetary policy implementation than it is assumed in models with only outside money.

However, the validity of the above equivalence of the inside and outside money models is limited to a certain range of the shocks. Moreover, the policy toolkit required for the appropriate policy is based on a very detailed knowledge of the economy and it is so complicated that legitimate doubts may arise that it cannot be applied in practice. Because of that we take a less complicated approximation of the perfect policy rule and analyse its errors. However, we find that the error of the approximation is rather small for most shocks.

Since the financial crisis of 2007-2008, a lot of effort has been made in macroeconomics to incorporate the specifics of the financial system into the models,[3] however, most of these models typically focus on the role of the banking system in financial intermediation and neglect the role it plays in the mechanism of money supply, assuming that the total amount of money is equal to the outside money issued by the central bank.

On the other hand, since 2008 central bank studies on the role of the banking system in money creation have proliferated, see for example Maclay, Radia and Thomas (2014), Deutsche Bundesbank (2017) and Jordan (2018). Werner (2016) takes an even more radical view, totally disagreeing with the mainstream academic approach and claims that the process of money creation is not a negligible detail, and any analysis that omits it is fundamentally flawed. However, these studies focus on the description of the banking system and the macroeconomic context is only superficially considered, furthermore, they lack formal models.

[3]See, for example, Gertler and Kiyotaki (2015), Clerk et al. (2015) and Boissay, Collard and Smets (2016).

Although there are macroeconomic models in which the money creation of the banking system appears explicitly, these are rather exceptions. Goodfriend and MacCallum (2007) is an early example of a formal macroeconomic model with inside money. They study the implications of binding reserve requirements as a constraint on the deposit production by banks. Another early example is the Post Keynesian model of Godley and Lavoie (2007, *chapter 10*) where liquidity risk and the constraint for deposit creation arises from the tendency of households to convert their demand deposits into cash.

Jakab and Kumhoff (2019) add inside money to a standard DSGE model, and like Werner (2016), they question whether it makes sense to talk about financial intermediation in the case of bank money creation. However, their model completely lacks liquidity risk and the only constraint on banks' money creation is non-bank economic agents' demand for money.[4]

Rodríguez Mendizábal (2017) explicitly takes into account the liquidity risk management of banks in a static model and studies the benefits and costs of narrow banking. Rivero Lieva and Rodríguez Mendizábal (2019) considers the financial stability consequences of banks' money creation.

Piazzesi, Rogers and Schneider (2021) also studies interest rate rules in the presence of inside money. They compare the implications of different monetary policy regimes (floor system, corridor system) and investigate how the aggressiveness of the response to inflation affects the stability properties of the model. However, in their model there is no investment and corporate behaviour is not affected by bank lending.

The paper is structured as follows. In *Section 2* the model is presented. *Section 3* analyzes the adjustment of the IS and LM curves in response to exogenous shocks if monetary policy is passive. In *Section 4* we investigate whether it is possible to implement a monetary policy determined by the IS curve and the interest rate rule in the presence of inside money. *Section 5* discusses the case when instead of implementing perfectly the above policy, it is only approximated. Finally, *Section 6* concludes.

2 The model

In the model, households and firms are represented in a standard way. Investments are financed by the banking sector, the liabilities of which are stable long-term and liquid deposits, with the latter playing the role of money in

[4]In Világi and Vonnák (2022) we also study this issue using a model similar to the one presented in this paper, and we conclude that the existence of inside money does not invalidate the common macroeconomic wisdom that investments are linked to savings through the financial intermediation of banks.

the model. Households' savings portfolios include both stable and liquid deposits. Firms also hold liquid deposits for transaction purposes. The banking system has two types of assets: corporate loans and central bank reserves. Banks are actively involved in the money-creation process, as they hold more liquid deposits than central bank reserves.

Banks are subject to idiosyncratic liquidity shocks. If the outflow of liquid deposits from a given bank exceeds the amount of its central bank reserves, it has to borrow on the interbank market, which is relatively expensive. As a result, banks need to actively manage their liquidity risk, which is explicitly reflected in the model. The ratio of liquid deposits to the central bank reserve, i.e. the money multiplier, is determined in the model by liquidity management.

In addition to idiosyncratic liquidity shocks, the model also includes aggregate macroeconomic shocks.

Production takes place in three stages. First, an intermediate good is produced using physical capital, then intermediate goods producers competitively sell goods to retailers. Retailers use this intermediate good and labor to produce differentiated goods for the final good producing sector. Retailers' prices are sticky, they cannot adjust their prices within a given period after the realization of macroeconomic shocks. Finally, firms in the final good producing sector aggregate input goods and sell them for consumption and investment purposes. Final goods producers then bundle retail goods into final goods usable for consumption and capital.

Due to the presence sticky prices, monetary policy has real effect in the model.

The timing of the shocks and economic decisions within a given time period is the following: First, firms set prices and quantities on the basis of the expected values of macroeconomic shocks. Then the macroeconomic shocks are realized, the product, labor, loan and deposit markets open and monetary policy sets the relevant interest rate and the macroeconomic allocation decisions are made: since firms cannot readjust their prices, they react by adjusting labor input and the quantity of retail and final goods production. Then the idiosyncratic liquidity shocks are realized and the interbank market opens, where monetary policy is also active.

2.1 Components of aggregate demand

2.1.1 Households

Households' instantaneous utility function is given by

$$\mathcal{U}\left(c_t, n_t, D_t, \zeta_t\right) = \frac{c_t^{1-\nu}}{1-\nu} + \frac{\zeta_t \left(D_t/P_t\right)^{1-\nu}}{1-\nu} - \varphi n_t.$$

where c_t is consumption, D_t/P_t is real money holding, n_t is labor and ζ_t is a time varying preference parameter. The intertemporal budget constraint is:

$$P_t c_t + F_t^h + D_t = Y_t^h + (1 + i_{t-1})F_{t-1}^h + \left(1 + i_{t-1}^D\right) D_{t-1},$$

where Y_t^h is the income of households, F^h and D^h denote their time and demand deposits with i_t and i_t^D being the nominal interest rates paid on them. Households' income consists of the following components:

- Labor income received from the production sector, $W_t n_t$, where W_t is the nominal wage.
- Labor income received from the banking sector, $W_t^\kappa n_t^\kappa$. We assume that the banking sector uses different type of labor (n_t^κ) than the production sector which is also supplied by households. For simplicity, the disutility of this type of labor does not appear in $\mathcal{U}$. W_t^κ is the nominal wage of this type of labor.
- Profit income, $\Pi_t = \Pi_t^y + \Pi_t^z$, where Π_t^y is the profit of final good producers and retailers, and Π_t^z is the profit of intermediate good producers.
- Dividend from banks, $\mathcal{D}_t$.
- Lump-sum transfer from the government, $\mathcal{T}_t$.

Households' lifetime is infinite, however, due to bounded rationality, they have finite planning horizon, as in Woodford (2018), Lustenhouwer and Mavrotamis (2021) and Boutros (2022). At each date t households want to maximize their discounted utility of consumption and leisure over their planning horizon (T periods). Their income and price expectations are rational over the planning horizon. However, they do not have sufficient information and calculation capacity to forecast events after date $t+T$. Hence they use a rule-of-thumb: they assume that they will choose the steady-state deposit levels at date $T+1$. Furthermore, they assume that their date $T+1$ income will be consistent with the steady-state consumption level.

Formally, they solve the following finite horizon optimization problem:

$$\max_{\left\{c_{t+j}, n_{t+j}, F^h_{t+j}, D_{t+j}\right\}} \sum_{j=0}^{T+1} \mathrm{E}_t \left[\Gamma_{t+j} \mathcal{U}\left(c_{t+j}, n_{t+j}, D_{t+j}, \zeta_{t+j}\right)\right],$$

subject to the budget constraints,

$$P_{t+j} c_{t+j} + F^h_{t+j} + D_{t+j} = Y^h_{t+j} + (1 + i_{t+j-1}) F^h_{t+j-1} + \left(1 + i^D_{t+j-1}\right) D_{t+j-1},$$

for all $j = 0, \ldots, T+1$, and the terminal conditions

$$c_{t+T+1} = c, \qquad \frac{F^h_{t+T+1}}{P_{t+T+1}} = f^h, \qquad \frac{D_{t+T+1}}{P_{t+T+1}} = d,$$

where $\Gamma_t = \beta_t \Gamma_{t-1}$, $\Gamma_0 = 1$, $0 < \beta_t < 1$ is the time varying discount factor of households, c, f^h and d are the steady-state levels of real consumption, and real deposit holdings. Furthermore, as discussed, households' date $t+T+1$ income expectation is consistent with the above terminal conditions:

$$Y^h_{t+T+1} = P_{t+T+1}\left(c + d + f^h\right) - (1 + i_{t+T}) F^h_{t+T} - \left(1 + i^D_{t+T}\right) D^h_{t+T}. \tag{1}$$

The solution of the above optimization problem is derived in *Appendix A.2*.

2.1.2 Intermediate good producers

In the model, intermediate goods producers are the only ones who use physical capital (k), so investment demand can be derived from their behavior. They operate on a perfectly competitive market with the following technology:

$$z_{t+1} = A_{t+1}\left(k_t - \omega k_t^2\right), \tag{2}$$

where z_{t+1} denotes the intermediate good, and A_{t+1} is a time varying and ω is a constant productivity parameter.

Their initial wealth is zero, hence they need bank loan to buy the necessary capital for production. The capital fully depreciates after production. As a consequence, the intermediate good producer firms' demand for bank loan will be:

$$L_t = P_t k_t.$$

Intermediate good producer solve the following profit maximization problem,

$$\max_{k_t} P^z_{t+1} z_{t+1} - \left(1 + i^L_t\right) P_t k_t.$$

Since $z_{t+1} = A_{t+1}(k_t - \omega k_t^2)$, the first order condition is

$$P_{t+1}^z A_{t+1} - P_{t+1}^z A_{t+1} 2\omega k_t = \left(1 + i_t^L\right) P_t,$$

from which the demand for physical capital is

$$k_t = \frac{p_{t+1}^z A_{t+1} - \left(1 + r_t^L\right)}{2 p_{t+1}^z A_{t+1} \omega} \tag{3}$$

where r_t^L is the real loan rate, and $p_{t+1}^z \equiv P_{t+1}^z / P_{t+1}$ is the expected relative price of the intermediate good. It is shown in *section 2.2* that p_{t+1}^z is constant and function of the parameters of the production functions of final and retail goods.

Substituting formula (3) into the production function (2) yields the supply of z_{t+1} as a function of its (relative) price and the real loan rate. The aggregate profit of the sector:

$$\Pi_{t+1}^z = P_{t+1}^z z_{t+1} - \left(1 + i_t^L\right) P_t k_t. \tag{4}$$

2.1.3 The banking sector

The banking block of the model is inspired by Piazzesi and Schneider (2018). The main feature of their model is that the banks make decisions on their balance sheet facing liquidity risk and subject to a cost function which we will specify later. Banks can create liquid deposits, that is money, but this money creation is constrained by the costs of expanding their balance sheet as well as by the need to maintain a liquidity buffer for future liquidity shocks.

The banking system is formed by a continuum of homogenous banks owned by the households. Banks are operated by independent managers, whose decisions are not influenced by the owners. The task of the managers is to maximize the discounted net real cash flow (dividends) of households. Households take the cash flow stream as given by passively collecting positive cash flow from the banks and providing the necessary resources if the cash flow is negative.

Assets and liabilities of banks

The liability side of the banks' balance sheet consists of time deposits (F_t), demand deposits of households and firms ($D_t = D_t^h + D_t^z$), and loans from the interbank market (B_t^b). On the asset side they hold reserves (M_t), loans to the intermediate goods producing corporate sector (L_t) and loans to other banks (B_t^l). The net interbank position is denoted by B_t with positive value indicating a net lender position. Consequently, lending and borrowing can be written as $B_t^l = \max[0, B_t]$ and $B_t^b = \max[0, -B_t]$, respectively.

The interest rates paid on reserves, interbank loans, corporate loans, demand and time deposits are denoted by i_t^R, i_t^B, i_t^L, i_t^D and i_t, respectively.

At the beginning of date t the bank collects deposits F_t and D_t, provides loans to firms producing intermediate goods L_t and borrows reserves M_t from the central bank for expected future transactions with other banks. At this stage the interbank market is not open yet, and thus the balance sheet constraint is

$$M_t + L_t = D_t + F_t. \tag{5}$$

Idiosyncratic liquidity shocks

After having decided on their balance sheets, banks are hit by an idiosyncratic liquidity shock $\hat{\lambda}_t$. This idiosyncratic shock is different from the macroeconomic shocks discussed in *section 2.3*. The idiosyncratic liquidity shock aims to capture the real life fact that banks' customers often initiate payments to counterparties having account at another bank, so the payer's bank has to transfer the corresponding amount in reserves to the payee's bank account at the central bank. We assume that $\hat{\lambda}_t D_t$ has to be paid by the bank. If $\hat{\lambda}_t > 0$, the depositors withdraw part of their deposits. If $\hat{\lambda}_t < 0$, new deposits arrive to the bank. We assume that $\hat{\lambda}_t$ has a continuous distribution over the $[-\bar{\lambda}_t, \bar{\lambda}_t]$ interval described by the cumulative distribution function G and the corresponding probability density function g.

Holding reserves is costly because the interest paid on it is less than the interest on corporate loans. Therefore, banks may hold less reserves than what would cover outflows. Those banks that do not have enough reserves to make interbank payments have to borrow on the interbank market. If we define the reserve ratio as $\lambda_t = M_t/D_t$, a bank must borrow on the interbank market if $M_t < \hat{\lambda}_t D_t$ or $\lambda_t < \hat{\lambda}_t$. That is, a bank with liquidity shock $\hat{\lambda}_t$ borrows at least $B^b\left(\hat{\lambda}_t\right)$ amount,[5]

$$B^b\left(\hat{\lambda}_t\right) \geq \hat{\lambda}_t D_t - M_t. \tag{6}$$

It is assumed that a bank with net deposit outflow does not pay interest on $\hat{\lambda}_t D_t$. On the other hand, it receives the interest paid by the central bank on

[5]The variables $B_t(\hat{\lambda}_t)$, $B_t^l(\hat{\lambda}_t)$, $B_t^b(\hat{\lambda}_t)$ are functions of $\hat{\lambda}_t$. Whenever it is necessary to avoid confusion, we will explicitly indicate this in the notation, and the letters without the $(\hat{\lambda}_t)$ extension will represent the related aggregate variables. However, to simplify notation, when it does not result in confusion we will omit the term $(\hat{\lambda}_t)$ even in the case of individual, non-aggregate variables.

$B^b\left(\hat{\lambda}_t\right)$.

A bank does not have extra liquidity need if $M_t \geq \hat{\lambda}_t D_t$, that is, if $\lambda_t \geq \hat{\lambda}_t$. In this case the bank can lend part of its excess liquidity on the interbank market

$$B^l\left(\hat{\lambda}_t\right) \leq M_t - \hat{\lambda}_t D_t. \tag{7}$$

A bank with net deposit inflow has to pay interest on $-\hat{\lambda}_t D_t$ and it will lose the interest paid by the central bank on $B^l\left(\hat{\lambda}_t\right)$.

Operation cost of banking

The main focus of the banking block in our model is representing liquidity risk management and considering its consequences. On the other hand, we do not want to provide a deeper understanding of other aspects of banking behavior. Therefore, following Cúrdia and Woodford (2016) and Piazzesi and Schneider (2018), we simply posit a reduced-form intermediation technology represented by a cost function to capture the operation of banks.

Specifically, we assume the banks' have the following real cost function:

$$\begin{aligned} \kappa_t \;&=\; \tau^L \frac{L_t}{P_t} + \phi^L \left(\frac{L_t}{P_t}\right)^2 + \tau^F \frac{F_t}{P_t} + \tau^D \frac{D_t}{P_t} \\ &-\; \phi^{DL} \frac{D_t L_t}{P_t^2} + \phi^B \frac{\left(B^l/P_t\right)^2}{\left(M_t - \hat{\lambda}_t D_t\right)/P_t}. \end{aligned} \tag{8}$$

The first four terms on the right hand side represent the operation cost of collecting deposits and providing loans, including the marketing cost.

The term $-\phi^{DL} D_t L_t / P_t^2$, ($\phi^{DL} > 0$) can have two interpretations:

- First, as in Piazzesi and Schneider (2018), it takes resources to convince the owners of demand deposits that their claims are satisfied on demand at any time. Moreover, we assume that convincing depositors is cheaper if the bank owns more assets to back the commitments, especially if those assets are relatively safe. Corporate loans are not immune to uncertainty and are therefore not considered to be safe assets, but we assume that bankruptcy losses do not jeopardize the ability of banks to repay their deposits. According to this interpretation having more assets, that is, more L_t, reduces the cost of deposit creation:

$$\left(\tau^D - \phi^{DL} \frac{L_t}{P_t}\right) \frac{D_t}{P_t}.$$

- Just the other way around, according to the second interpretation more demand deposits reduce the cost of lending:

$$\left(\tau^{L} - \phi^{DL}\frac{D_t}{P_t}\right)\frac{L_t}{P_t} + \phi^{L}\left(\frac{L_t}{P_t}\right)^{2}.$$

  This approach can be justified by the following line of thought. Beyond liquidity risk, banks have to manage their solvency risk as well. This can be captured by the *value-at-risk* approach of banks to keep the probability of default within reasonable limits, as in the models in *chapters 2* and *3* of Shin (2010). Taking *value-at-risk* decisions into account implies that not only the marginal cost of funding, but also the average cost of liabilities determine lending since *ceteris paribus* smaller repayment reduces the probability of default. Therefore, more cheap funding by demand deposits facilitates corporate lending since the *value-at-risk* constraint becomes looser and the higher leverage is allowed.

  However, this mechanism does not appear explicitly in our model. Instead of representing the above mechanism in detail we capture this feature by a shortcut, that is, by assuming that D_t reduces the cost of L_t.

The final term represents the cost of interbank lending. Since the interbank market is a standardized and organized market, this cost is not proportional to the magnitude of lending. Rather, this term wants to capture the phenomenon that it is easier to lend overnight if the bank has abundant liquidity, and it is more difficult if the bank's liquidity is scarce. This term is positive if the bank is a net lender on the interbank market and zero if she is a net borrower. We assume that borrowing has no operating costs because interbank borrowing is a coercive decision, if the outflow of deposits is large enough and a bank wants to avoid bankruptcy, it must do so, in which case no sophisticated liquidity management considerations are required. However, it is exactly the sophisticated liquidity management that we assume to be costly. Although the denominator can take negative values for banks that do not have enough reserves to make the necessary transfer payments, the whole term cannot go below zero, because if a bank is net borrower on the market, the numerator will be zero. As a consequence of this type of cost, banks with excess liquidity will hold reserves even if the interest on reserves is lower than the interbank lending rate.

It is assumed the κ_t is the real cost paid for a special good which produced by linear technology from the final good (also used for consumption and investments) and from labor:

$$\begin{aligned} \kappa_t &= p_t^\kappa q_t^\kappa, \\ q_t^\kappa &= y_t^\kappa + n_t^\kappa, \end{aligned}$$

where q_t^κ denotes the quantity of the special good, p_t^κ is its relative price, y_t^κ is quantity of final good used for the production of q_t^κ. This simple linear technology implies that the real wage for n_t^κ is constant, $w_t^\kappa = W_t^\kappa / P_t = 1$, furthermore $p_t^\kappa = 1$. To simplify the analysis, we assume that in equilibrium $q_t^\kappa = n_t^\kappa$, only the labor input is used by the banking sector.[6]

Behavior of the banking system

At date t banks collect the principal and interest on their assets and pay the principal and interest on their liabilities. The banks' net income is the dividend which is transferred to the household sector:

$$\begin{aligned} \mathcal{D}_t &= \left(1+i_{t-1}^L\right) L_{t-1} + \left(1+i_{t-1}^R\right)\left(M_{t-1} - \hat{\lambda}_{t-1} D_{t-1} - B_{t-1}\right) \\ &+ \left(1+i_{t-1}^B\right) B_{t-1} - (1+i_{t-1}) F_{t-1} - (1+i_{t-1}^D)\left(1-\hat{\lambda}_{t-1}\right) D_{t-1} - P_t \kappa_t. \end{aligned} \tag{9}$$

Individual banks take as given the interest rates i_t^R, i_t^L, i_t, i_t^D, i_t^B and the price level P_t. The problem of a bank is to maximize the discounted value of the real dividends paid to the households (recall, that banks are owned by households), subject to the constraints:

$$\begin{aligned} &M_t + L_t = D_t + F_t, \\ &M_t - \hat{\lambda}_t D_t \geq B_t, \end{aligned}$$

The first constraint represents the balance sheet of the bank, the second liquidity constraint is derived from equations (6) and (7).

To get closed form solutions we assume that $\hat{\lambda}_t$ is drawn from a uniform distribution over the interval $[-\bar{\lambda}_t, \bar{\lambda}_t]$. Here we characterize the most important properties of the solution of an individual banks' optimization problem. For more details see *Appendix A.4*.

First of all, the optimal solution does not depend on the absolute level of the interest rate, only relative interest rate matters, that is, the spreads between the different interest rates and the interest rate on time deposits,

$$\Delta_t^R \equiv \frac{1+i_t^R}{1+i_t}, \quad \Delta_t^L \equiv \frac{1+i_t^L}{1+i_t}, \quad \Delta_t^B \equiv \frac{1+i_t^B}{1+i_t}, \quad \Delta_t^D \equiv \frac{1+i_t^D}{1+i_t}.$$

[6]This assumption ensures that the equilibrium income of households and thus the IS and LM curves derived in *section 2.1.5* are independent of κ_t. However, relaxing this assumption does not change the result significantly, see Világi and Vonnák (2022).

The first order condition determining reserve holding can be expressed in the following way,

$$\Delta_t^R + \frac{\delta_t}{2\bar{\lambda}_t} 2\phi^B \rho_t + \frac{\varsigma_t}{2\bar{\lambda}_t} \phi^B \rho_t^2 = 1 + \tau^F, \tag{10}$$

where $\delta_t \equiv \max\left[0, \bar{\lambda}_t - \lambda_t\right]$ and $\varsigma_t \equiv \max\left[2\bar{\lambda}_t, \bar{\lambda}_t + \lambda_t\right]$, furthermore, in equilibrium

$$\rho_t = \frac{B_t^b - B_t^{CB}}{TL_t} < 1.$$

The variable ρ_t represents the tightness of liquidity on the interbank market, where $B^b = \delta_t^2 / \left(4\bar{\lambda}_t\right)$ is the demand on the interbank market, B_t^{CB} is the liquidity supplied by the central bank, and $TL_t = \varsigma_t^2 / \left(4\bar{\lambda}_t\right)$ is the total liquidity stock of potential lenders.

The first order condition with respect to corporate loans is the following:

$$\Delta_t^L = 1 + \bar{\tau}^L + 2\phi^L \frac{L_t}{P_t} - \phi^{DL} \frac{D_t}{P_t}, \tag{11}$$

where $\bar{\tau}^L \equiv \tau^F + \tau^L$.

Considering the liability side, the first order condition determining demand deposits (D_t):

$$\Delta_t^D = 1 - \bar{\tau}^D + \phi^{DL} \frac{L_t}{P_t} - \left(2\phi^B \rho_t - \phi^B \rho_t^2\right) \frac{\delta_t \varsigma_t}{4\bar{\lambda}_t}, \tag{12}$$

where $\bar{\tau}^D \equiv \tau^D - \tau^F$.

Using equation (9) it is easy to show that the aggregate dividend of the banking sector is given by

$$\begin{aligned} \mathcal{D}_t &= \left(1 + i_{t-1}^L\right) L_{t-1} + \left(1 + i_{t-1}^R\right) M_{t-1} - (1 + i_{t-1}) F_{t-1} - (1 + i_{t-1}^D) D_{t-1} \\ &+ \left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB} - P_t \kappa_t, \end{aligned} \tag{13}$$

since

$$\int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} \hat{\lambda} D_t \, \mathrm{d}\hat{\lambda} = 0, \qquad \int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} B_t\left(\hat{\lambda}\right) \mathrm{d}\hat{\lambda} = B_t^l - B_t^b = -B_t^{CB}.$$

2.1.4 The government and the central bank

Monetary policy

The central bank has three instruments. First, the central bank sets the interest rate paid on reserves (i^R) which determines Δ_t^R.

Second, the central bank lends on the interbank market after the realization of the liquidity shock (B_t^{CB}) in order to control the interest rate on the market (i^B) which determines Δ_t^B.

Finally, it determines the total quantity of reserves (M_t) available for banks at the beginning of date t. In practice, the aggregate quantity of reserves is often controlled by open market operations. In our model this option is not available since there is no government debt. Instead, we assume that the central bank lends to commercial banks before the realization of the liquidity shock. From the point of view of an individual bank, households ex ante lending F_t^h and central bank ex ante lending $F_t^{CB} = M_t$ are perfect substitutes, thus total time deposits (F_t) are the sum of the two.[7]

The central bank can influence the magnitude of the money multiplier ($1/\lambda_t$) by Δ_t^R and Δ_t^B, furthermore, for a given value of the multiplier, it can control the quantity of inside money stock (D_t) by M_t. In *Appendix A.4* it is shown that

$$\rho_t = \frac{\Delta_t^B - \Delta_t^R}{2\phi^B}, \tag{14}$$

and

$$\lambda_t = \bar{m}_t \bar{\lambda}_t, \tag{15}$$

where

$$\bar{m}_t \equiv \frac{\phi^B \rho_t (2+\rho_t) - 2\left(1+\tau^F - \Delta_t^R\right)}{\phi^B \rho_t (2-\rho_t)} > 0.$$

Furthermore, by using the definition of λ_t,

$$D_t = \frac{M_t}{\lambda_t}.$$

At the beginning of each period all economic agents have uniform and time independent[8] expectations for the values of macroeconomics shocks, denoted by ξ^0. Taking into account ξ^0, monetary policy announces the expected values of its instruments, and they are chosen in such a way that $P_t = P_{t-1}$ (the details are in *Appendix A.5* and *A.6*). If the realized values of shocks coincide with the expectations then monetary policy sets the announced values of the intsruments.

However, if the realized values of shocks, ξ_t^*, are different from the expectations, the central bank adjusts its instruments to implement the nominal

[7]Perfect substitution implies that lending from households and from the central bank have the same operation cost, see *Section 2.1.3*. This is just a simplifying assumption.

[8]For simplicity we assume that the macroeconomic shocks are temporary.

interest rate level determined by the following standard interest rate rule:

$$\frac{1+i_t}{1+r_t^0} = \left(\frac{y_t}{y_t^0}\right)^{\psi_y} \left(\frac{P_t}{P_{t-1}}\right)^{\psi_\pi},$$

where y_t^0 and r_t^0 flexible-price real output and real interest rate. (The flexible-price allocation is described in details in *Appendix A.6*). Since in this model retail prices cannot be adjusted within a time period even after the realization of macroeconomic shocks, P_t remains unchanged, and the inflation rate is zero. Thus the above interest rate rule is simplified to the following form:

$$\frac{1+i_t}{1+r_t^0} = \left(\frac{y_t}{y_t^0}\right)^{\psi_y}. \tag{16}$$

Consolidated budget constraint

The central bank's profit at date t has two components. First, the difference between the revenue on lending at the beginning of a time period and the interest paid on reserves: $(1+i_{t-1})F_{t-1}^{CB} - \left(1+i_{t-1}^R\right) M_{t-1}$. Second, the difference between the revenue and expenditure related to B_{t-1}^{CB}. Recall that after the settlement of interbank payments B_{t-1}^{CB} is held as a reserve, hence the central bank has to pay the reserve rate on it, hence this component of the profit is given by $\left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB}$. Since $M_{t-1} = F_{t-1}^{CB}$, the central banks' profit:

$$\left(i_{t-1} - i_{t-1}^R\right) M_{t-1} + \left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB}.$$

The central bank pays the profit to the central government. We assume that there is no government consumption and the government's budget is always balanced, the central bank's profit is transferred to the household sector:

$$\mathcal{T}_t = \left(i_{t-1} - i_{t-1}^R\right) M_{t-1} + \left(i_{t-1}^B - i_{t-1}^R\right) B_{t-1}^{CB}. \tag{17}$$

2.1.5 IS and LM curves

Since prices are rigid in the model within a time period, aggregate demand determines the evolution of macroeconomic allocations in the short run. In this section, we introduce the IS and LM curves to describe the behavior of aggregate demand. These curves are similar to the textbook IS-LM curves in terms of their economic content, but unlike them they are not static, as they are derived from a dynamic equilibrium model and contain expectations in addition to the present variables.

Our goal with the introduction of the IS and LM curves is to make the adjustment of the economy to unexpected shocks at date t transparent and understandable. These curves are suitable tools to clarify the role of inside money in the adjustment process and to point out the essential difference between the macroeconomic effects of inside and outside money.

Similar to Woodford (2010), the IS curve is derived from the savings functions of households and retailers' investment demand, as well as the supply of intermediation of banks, while the LM curve is derived from households' demand for transaction instruments and banks' need for liquid liabilities. So the behavior of the banking system affects both curves.

From a macroeconomic point of view, this is just the significance of inside money: as we will see later, in the case of outside money, banks do not affect the LM curve, but inside money creates an additional link between the two curves through the banking system.

First, consider the intermediate good producers' demand for capital. Recall that monetary policy ensures that expected future inflation rates are always zero. As a consequence, nominal and real interest rate coincide, thus $r_t^L = i_t^L$. Furthermore, as shown in *section 2.2*, the relative price of the intermediate good $p^z = a^z/\vartheta$. Taking all this into account, equation (3) takes the following form:

$$k_t = \frac{\frac{a^z}{\vartheta}A_{t+1} - \left(1 + i_t^L\right)}{2\frac{a^z}{\vartheta}A_{t+1}\omega}.$$

Substituting formula $L_t/P_t = k_t$ into the corporate loan rate spread equation (11)yields

$$1 + i_t^L = (1 + i_t)\Delta_t^L = (1 + i_t)\left[1 + \bar{\tau}^L + 2\phi^L k_t - \phi^{DL}\frac{D_t}{P_t}\right]. \tag{18}$$

Combining the above two equations provides an expression for k_t:

$$k_t = \frac{\frac{a^z}{\vartheta}A_{t+1} - (1 + i_t)\left(1 + \bar{\tau}^L - \phi^{DL}\frac{D_t}{P_t}\right)}{2\frac{a^z}{\vartheta}A_{t+1}\omega + 2\left(1 + i_t\right)\chi_t\phi^L}. \tag{19}$$

As the above equation reveals, monetary policy can influence the level of capital stock through two channels: by the nominal interest rate i_t and the quantity of demand deposits D_t (see *section 2.1.4*). The presence of D_t in the above expression is a consequence of the fact that in the modern banking system the function of financial intermediation is inseparable from the function of providing transaction instruments.

Equation (12) can be expressed as

$$\Delta_t^D = 1 - \bar{\tau}^D + \phi^{DL}k_t - \Psi_t, \tag{20}$$

where

$$\Psi_t \equiv \phi^B \rho_t (2 - \rho_t) \frac{\delta_t \varsigma_t}{4\bar{\lambda}_t} = \phi^B \rho_t (2 - \rho_t) \frac{\bar{\lambda}_t^2 - \lambda_t^2}{4\bar{\lambda}_t}.$$

The spread Δ_t^D is also determined by monetary policy. Not only k_t is a function of monetary policy but also ρ_t, δ_t, and ς_t and thus Ψ_t, as discussed in *section 2.1.4*.

In *Appendix A.2* it is shown that real savings is given by the following function,

$$s_t = \frac{\mathcal{B}_t^T - 1}{\mathcal{B}_t^T} y_t - \frac{\mathcal{Y}_t^T}{\mathcal{B}_t^T},$$

where

$$\begin{aligned} \mathcal{B}_t^T &= \sum_{j=0}^{T} \left(1 + \eta \left(1 - \Delta_{t+j}^D \right)^{1-\sigma} \right) \beta^{j\sigma} \left(R_t^{t+j} \right)^{\sigma - 1} \\ &+ \beta^{(T+1)\sigma} \left(R_t^{t+T+1} \right)^{\sigma-1}, && (21) \\ \mathcal{Y}_t^T &= \sum_{j=1}^{T} \frac{y_{t+j}^h}{R_t^{t+j}} - \frac{f+d}{R_t^{t+T+1}}, && (22) \end{aligned}$$

where

$$R_t^t = 1, \qquad R_t^{t+j} = (1+i_t)(1+i_{t+1}) \cdots (1+i_{t+j-1}),\ j > 0,$$

furthermore, y_t is the real GDP, $y_{t+j}^h = Y_{t+j}^h / P_{t+j}^h$ and in equilibrium

$$y_{t+j}^h = y_{t+j} - (1 + i_{t+j-1}) \left[k_{t+j-1} - \left(1 - \Delta_{t+j-1}^d \right) d_{t+j-1} \right].$$

The aggregate balance sheet of the banking system:

$$M_t + L_t = F_t + D_t = F_t^h + F_t^{CB} + D_t.$$

Since

$$L_t = P_t k_t, \qquad M_t = F_t^{CB},$$

the balance sheet equation can be expressed as

$$P_t k_t = L_t = F_t^h + D_t^h = P_t s_t,$$

that is,

$$k_t = s_t, \tag{23}$$

the real savings of households is equal to the capital stock (which is quite clear intuitively since k_t is equal to investments in this model).

Combining the above saving function with equation (23) yields the *IS curve*:

$$y_t = \frac{1}{\mathcal{B}_t^T - 1}\left(\mathcal{B}_t^T k_t + \mathcal{Y}_t^T\right). \tag{24}$$

Households' demand for real money is derived in *Appendix A.2*. Rearranging it yields the *LM curve*:

$$y_t = \frac{\left(1-\Delta_t^D\right)^\sigma \mathcal{B}_t^T}{\eta_t}\frac{D_t}{P_t} - \mathcal{Y}_t^T. \tag{25}$$

The right hand side of the IS and LM curves are functions of i_t, since both $\mathcal{B}_t^T$ and $\mathcal{Y}_t^T$, the expected discounted future income stream of households, are also functions of it. Furthermore, monetary policy can also influence the curves through k_t, Δ_t^D and D_t.

It is important to emphasize that the money stock (D_t) influences both curves in our model. This is in sharp contrast with the approaches of most textbooks or New Keynesian models (see Clarida, Galí and Gertler (1999), Galí (2015)) that the IS curve is not affected directly by the money stock.

We will also apply the following notations for the equations (24) and (25):

$$y_t = y^{IS}\left(i_t, k_t, \Delta_t^D, \xi_t^{LM}, \xi_t^{IS}\right), \tag{26}$$

$$y_t = y^{LM}\left(i_t, \frac{D_t}{P_t}, k_t, \Delta_t^D, \xi_t^{LM}, \xi_t^{IS}\right), \tag{27}$$

where $\xi_t^{LM} = \left[\bar{\lambda}_t, \eta_t\right]$ and $\xi_t^{IS} = [\beta_t, A_{t+1}]$. The above formulas emphasize that the IS and LM curves are functions mapping i_t to the real output, y_t. and they also depend on monetary policy represented by ρ_t, λ_t, D_t.

The IS and LM curves define two equations for y_t and i_t. We denote the solution, that is, the intersection of the two curves by

$$y_t^{islm} = y^{islm}\left(\frac{D_t}{P_t}, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right), \tag{28}$$

$$i_t^{islm} = i^{islm}\left(\frac{D_t}{P_t}, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right). \tag{29}$$

The above formulas reveal that the equilibrium values of y_t and i_t are functions of monetary policy and the exogenous shocks. Then using the equilibrium value of i_t^{islm}, one can calculate the equilibrium values i_t^L, k_t, Δ_t^D by equations (18)–(20).

2.1.6 The outside-money version of the model

As benchmark for comparison, we provide the t*outside-money version* of the model. In this version instead of the liquid deposits of the banking system, M_t the outside-money issued by the central bank plays the role of transaction instruments.

Households

Households' instantaneous utility function is similar in both versions of the model, the only difference is that Mt/P represent households' real money holding:

$$\mathcal{U}(c_t, n_t, M_t, \zeta_t) = \frac{c_t^{1-\nu}}{1-\nu} + \frac{\zeta_t (M_t/P_t)^{1-\nu}}{1-\nu} - \varphi n_t.$$

Households' has two instruments of saving, time deposits (F_t^h), and the outside-money. As a consequence the intertemporal budget constraint becomes

$$P_t c_t + F_t^h + M_t = Y_t^h + (1 + i_{t-1}) F_{t-1}^h + M_{t-1},$$

where it is assumed that the interest paid on outside-money is zero. In *Appendix A.2* it is shown that the saving function is given by

$$s_t = \frac{\mathcal{B}_t^{T,om} - 1}{\mathcal{B}_t^{T,om}} y_t - \frac{\mathcal{Y}_t^{T,om}}{\mathcal{B}_t^{T,om}},$$

where

$$\mathcal{B}_t^{T,om} = \sum_{j=0}^{T+1} \beta^{j\sigma} \left(R_t^{t+j}\right)^{\sigma-1}, \tag{30}$$

$$\mathcal{Y}_t^{T,om} = \sum_{j=1}^{T} \frac{y_{t+j}^h}{R_t^{t+j}} - \frac{f^h}{R_t^{t+T+1}}. \tag{31}$$

The banking sector

In the outside-money version financial intermediation is the only function of banks, and they do not play any role in providing transaction instruments. Banks collect time deposits and supply corporate loans. Since demand deposits are missing from the balance sheet, banks do not have liquidity risk, hence they do not need central bank reserves and interbank loans. Their cost function becomes

$$\kappa_t = \bar{\kappa} + \tau^L \frac{L_t}{P_t} + \phi^L \left(\frac{L_t}{P_t}\right)^2 + \tau^F \frac{F_t}{P_t}.$$

At each date t banks solve the following optimization problem:

$$\max_{L_t, F_t^h} \mathrm{E}_t \left[\bar{\beta}_t \frac{\mathcal{D}_{t+1}}{P_{t+1}} + \frac{\mathcal{D}_t}{P_t} \right]$$

subject to the balance sheet constraints,

$$L_t = F_t^h.$$

where

$$\mathcal{D}_t = \left(1 + i_{t-1}^L\right) L_{t-1} - (1 + i_{t-1}) F_{t-1}^h.$$

The solution of the above problem is characterized by the following condition:

$$\Delta_t^L = 1 + \bar{\tau}^L + 2\phi^L \frac{L_t}{P_t}.$$

The government and the central bank

As the interbank market is missing from the outside-money version, monetary policy has only one instrument, M_t. It is assumed that money supply is controlled by direct lump-sum transfers (or taxes) to households (i.e. by helicopter money). Again, we assume that there is no government consumption and the government's budget is always balanced, hence the transfer paid to (tax levied on) households is the following:

$$\mathcal{T}_t = M_t - M_{t-1}.$$

IS and LM curves

It is straightforward to show that the capital stock is determined by the following formula:

$$k_t^{om} = \frac{\frac{a^z}{\vartheta} A_{t+1} - (1 + i_t)\left(1 + \bar{\tau}^L\right)}{2\frac{a^z}{\vartheta} A_{t+1}\omega + 2(1 + i_t)\chi_t \phi^L}.$$

Unlike in equation (19), the capital stock is independent of the money stock, since financial intermediation is independent of provision of transaction instruments.

In this case the IS and LM curves have the following forms:

$$y_t = \frac{1}{\mathcal{B}_t^{T,om} - 1} \left(\mathcal{B}_t^{T,om} k_t^{om} + \mathcal{Y}_t^{T,om} \right). \tag{32}$$

$$y_t = \frac{\left(1 - \Delta_t^M\right)^{\sigma} \mathcal{B}_t^{T,om}}{\eta_t} \frac{M_t}{P_t} - \mathcal{Y}_t^{T,om}, \tag{33}$$

where $\Delta_t^M = 1/(1+i_t)$ since the interest rate paid on M_t is zero. Unlike in the general case, here the money stock does not affect directly the IS curve, since it is missing from k_t^{om}, $\mathcal{B}_t^{T,om}$, $\mathcal{Y}_t^{T,om}$.

We will also apply the following notations for equations (32) and (33):

$$y_t = \tilde{y}^{IS}\left(i_t, \eta_t, \xi_t^{IS}\right), \tag{34}$$

$$y_t = \tilde{y}^{LM}\left(i_t, \frac{M_t}{P_t}, \eta_t, \xi_t^{IS}\right). \tag{35}$$

The above formulas emphasize that the outside-money IS and LM curves are independent of $\bar{\lambda}_t$, λ_t and ρ_t, because $M_t = D_t$ and, as a consequence, the interbank market does not exist. Furthermore, equation (34) reveals that the outside-money IS curve does not depend on M_t either.

Figure 1 displays and compares the IS and LM curves in the general and the outside-money cases. In the general case Δ_t^D is decreasing in i_t, and, consequently, the term $\left(1-\Delta_t^D\right)^\sigma \left(1+\mathcal{B}_t^T\right)\eta_t$ in the LM curve is increasing in i_t. On the other hand, its impact on $\mathcal{B}_t$ is negligible. Hence, the LM curve, as a function of i_t, is flatter in the general case than in the outside-money case.

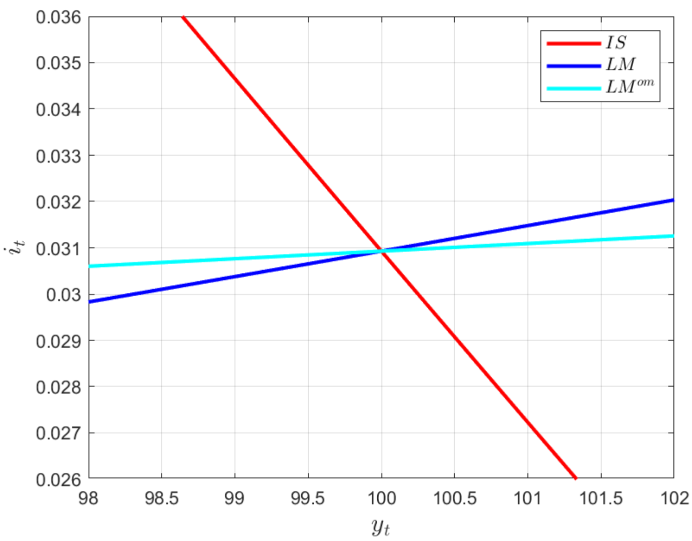

Figure 1: The IS and LM curves in the outside money and the general versions of the model. Note that the IS curves almost perfectly coincide in the two versions if monetary policy instruments are consistent with the steady state of the model.

2.2 Aggregate supply

In this section, we discuss, on the one hand, how the level of production is determined at flexible prices, on the other hand, how supply adapts to unexpected shocks in the presence of sticky prices.

The retail goods $(y(j))$ are not perfect substitutes and are produced by infinitely many firms indexed by $j \in [0,1]$ acting on a market described by the concept of the Dixit-Stiglitz type monopolistic competition. Retail goods are produced using intermediate goods (z) and labor (n) with a quasi linear technology

$$y_t(j) = a^n \left(n_t(j)\right)^{1-\alpha} + a^z z_t(j),$$

where a^n, $a^z > 0$ and $0 < \alpha < 1$.

The final good y is produced on a competitive market by a representative firm using infinitely many retail goods and a CES production technology:

$$y_t = \left[\int_0^1 y_t(j)^{\frac{\theta-1}{\theta}} \, \mathrm{d}j\right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$.

Because of perfect competition, the price of the final good is the CES average of the input prices:

$$P_t = \left[\int_0^1 P_t(j)^{1-\theta} \, \mathrm{d}j\right]^{\frac{1}{1-\theta}}.$$

Due to the assumption of perfect competition and the constant-return-to-scale technology, the final goods producer earns zero profit.

It can be shown easily that demand for the jth retail good is a function of its relative price and total output:

$$y_t(j) = \left(\frac{P_t}{P_t(j)}\right)^{\theta} y_t,$$

which implies that retailers operate on a *Dixit-Stiglitz* type *monopolistically competitive* market.

As discussed, some parameters of the model are driven by macroeconomics shocks, the vector of these shocks is denoted by ξ_t. Economic agents have uniform expectation for the shocks, at the beginning of date t the expected value of the shocks is ξ_t^0. At the beginning of period t the central bank announces its monetary policy and retailers set their prices. The aggregate price index becomes P_t, and wages and the price of the intermediate input good are also chosen. The prices and wages set at the beginning of the period are market clearing conditional on ξ_t^0 and the announced monetary policy. The allocation of goods consistent with the market clearing prices is the *flexible price* allocation described in details in *Appendix A.6*.

As shown in *Appendix A.3* the demand for labor is

$$n_t = \left(\frac{P_t^z}{W_t} \frac{a^n(1-\alpha)}{a^z} \right)^{\frac{1}{\alpha}},$$

and the demand for intermediate goods is

$$z_t = \frac{y_t - a^n n_t^{1-\alpha}}{a^z}, \tag{36}$$

where P_t^z is the price of the intermediate good, W_t is nominal wage.

Hence the cost function becomes

$$\mathcal{C}(W_t, P_t^z, y_t) = W_t n_t + \frac{P_t^z}{a^z}\left(y_t - a^n n_t^{1-\alpha}\right).$$

Since labor demand does not depend on the output, the marginal cost function is simply

$$\mathcal{MC}_t = \frac{P_t^z}{a^z}.$$

Observe that all retailers face the same marginal cost. Profit maximization in the Dixit-Stiglitz type monopolistic competition model implies the following price formula:

$$P_t = \vartheta \mathcal{MC}_t = \vartheta \frac{P_t^z}{a^z},$$

where

$$\vartheta = \frac{\theta}{\theta - 1} > 0$$

is the markup. Since the marginal cost is the same for all firms, prices and production quantities will also be uniform.

The above formula implies that the relative price of the intermediate goods is constant, that is

$$p_t^z = \frac{P_t^z}{P_t} = \frac{a^z}{\vartheta},$$

As a consequence, the flexible price labor demand is a function of the real wage,

$$n_t^0 = \left(\frac{a^n(1-\alpha)}{\vartheta w_t^0} \right)^{\frac{1}{\alpha}}, \tag{37}$$

where $w_t^0 = W_t^0 / P_t$ and W_t^0 is the nominal wage set at the beginning of the period.

After setting the prices and wages the shocks, denoted by ξ_t^*, are realized. Of course, ξ_t^* is not necessarily equal to ξ_t^0. Retailers cannot adjust their price after the shocks, however, wages can be adjusted.

We also assume that firms cannot adjust the quantity of z_t set at the beginning of the period, they can adjust only labor. Hence labor demand and the real marginal cost after the realization of ξ_t^* become

$$n_t = \left(\frac{y_t - a^z z_t}{a^n}\right)^{\frac{1}{1-\alpha}}, \tag{38}$$

$$mc(w_t, y_t) = w_t \frac{\partial n_t}{\partial y_t} = w_t \frac{(a^n)^{\frac{1}{\alpha-1}} y_t^{\frac{\alpha}{1-\alpha}}}{1-\alpha}. \tag{39}$$

The representative final good producer firm earns zero profit due to the constant-return-to-scale technology:

$$P_t y_t = \int_0^1 P_t(j) y_t(j) \, \mathrm{d}j$$

As a consequence, the aggregate profit of the final and retail goods sectors can be expressed as

$$\Pi_t^y = P_t y_t - W_t n_t - P_t^z z_t, \tag{40}$$

since the profit of final goods production is zero.

2.3 Exogenous shocks

As discussed, there are two types of shocks in the model: macroeconomic shocks, ξ_t, and an idiosyncratic liquidity shock $\hat{\lambda}_t$.

The vector of macroeconomic shocks consists of the following variables:

$$\xi_t = \left[\bar{\lambda}_t, \eta_t, \beta_t, A_{t+1}\right],$$

where $\bar{\lambda}_t$ is the upper limit of the liquidity shock's absolute value, $\eta_t \equiv \zeta_t^{\frac{1}{\nu}}$ is a parameter of households' money demand, β_t is the discount factor of households, A_{t+1} is the productivity factor in the intermediate goods producing sector. In this paper, only temporary shocks are considered, lasting only over one time period.

We will also apply the following notations:

$$\xi_t^{LM} = \left[\bar{\lambda}_t, \eta_t\right], \qquad \xi_t^{IS} = \left[\beta_t, A_{t+1}\right].$$

The shocks in the ξ_t^{LM} vector have an effect primarily on the LM curve, while those within the ξ_t^{IS} have an effect primarily on the IS curve.

The timing of the shocks and economic decisions is the following:

- First, firms set prices and the quantity of the intermediate good on the basis of ξ_t^0 and the announced values of monetary policy instruments. The chosen prices and quantity are the market clearing ones, if ξ_t^0 coincides with the realized value of the shocks.

- Then the macroeconomics shocks are realized (ξ_t^*) and the product, labor, loan and deposit market open, and allocation decisions are made.

    - If $\xi_t^0 = \xi_t^*$ then monetary policy maintains the pre-announced values of its instruments and the flexible price allocation is realized.

    - If $\xi_t^0 \neq \xi_t^*$ due to the rigid prices firm can only adjust quantities and monetary policy also reacts to the shocks, sets i_t by adjusting M_t, Δ_t^R and Δ_t^B.

- Then the liquidity shocks are realized and the interbank market opens.

3 The effect of shocks with unchanged monetary policy

Although the main objective of our paper is to understand how interest rate rule based monetary policy works in the presence of inside money, it is worth breaking it down into two steps. First, we look at what happens as a result of macroeconomic shocks if monetary policy does not respond.[9] In the second step, we examine the impact of the monetary policy response.

Our starting point is the steady-state allocation consistent with ξ^0 (discussed in details in *Appendix A.5*). We examine how *unexpected changes* in exogenous variables (ξ_t^*) shift the IS and LM curves and thus change the economic allocation, and compare the adjustment processes in the inside-money and outside-money versions.

To keep the analysis simple and transparent, as in Eggertsson and Krugman (2012), we analyze the effects of unexpected *temporary shocks*. That is, at date t, it is assumed, that $\xi_t = \xi_t^* \neq \xi^0$, however $\xi_{t+j} = \xi^0$, for all $j > 0$. Therefore, from time period $t + 1$ onwards the behavior of the economy can be described by the flexible price allocation.

We restrict our analysis to cases where moderate shocks hit the economy. In the case of shocks that cause an excessive recession, the nominal interest rate reaches the zero lower bound. This problem is not addressed in our model. In the case of shocks that cause excessively large expansion, the

[9] Recall that this means unchanged M_t, Δ_t^R and Δ_t^B.

economy runs into capacity constraints, which has a strong inflationary effect and the assumption of rigid prices is no longer plausible.

In *Appendix A.1* we discuss how we calibrated the parameter values for the following simulations.

LM shocks

First, we consider the effect of the unexpected change of the size of liquidity shocks, captured by the $\bar{\lambda}$ parameter. Note that $\bar{\lambda}$ does not appear in the formulas of the IS and LM curves of the outside money version. As a consequence, the $\bar{\lambda}$ shock does not have any impact on these curves.

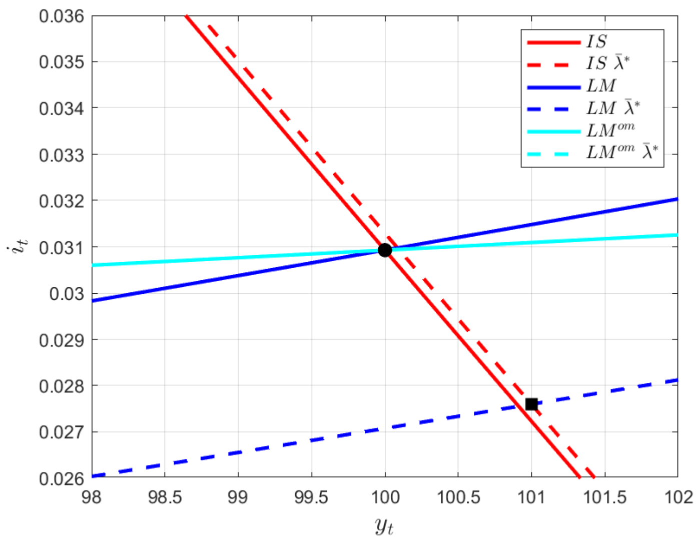

Figure 2: The effect of the decrease in the liquidity shocks on the IS and LM curves

In the general case $\lambda_t^* = \bar{m}_t \bar{\lambda}_t^*$ and $\bar{m}_t > 0$ (see equation (15)) and $D_t^* = M_t/\lambda_t^*$. As a consequence, a decrease of $\bar{\lambda}_t$ results in an increase of the money supply, therefore the LM curve shifts downward. Since $\phi^{DL} > 0$, the capital stock k_t increases when the liquidity shocks become smaller. This implies that the IS curve shifts upward, see *Figure 2*.

The next experiment is a decrease in money demand, that is, in η_t. Although the variable η_t appears in $\mathcal{B}_t^T$ and $\mathcal{B}_t^{T,om}$, and shifts the IS curves in both cases, its impact is negligible with our parameter choice.

Obviously, the main effect of a negative money demand shock is shifting the LM curve. In both cases decreasing money demand shifts the LM curve

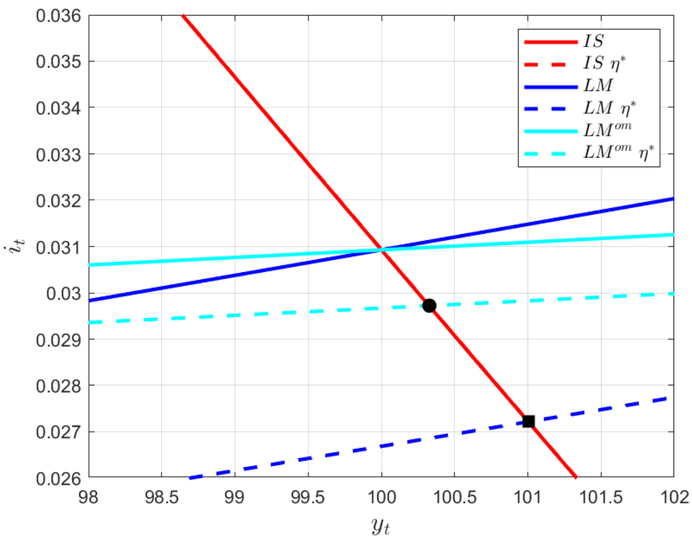

Figure 3: The effect of the decrease in money demand on the IS and LM curves

downward. The effect will be expansionary in both cases, but in the inside-money case the expansion is larger, see *Figure 3*.

IS shocks

Our next shock is an unexpected change in the discount factor (β_t), which affects the propensity to save, and thus, has direct impact on the IS curve. Smaller β_t implies less saving in period t.

Formally, if β_t decreases, $\mathcal{B}_t^T$ and $\mathcal{B}_t^{T,om}$ decrease as well (see equations (21) and (30)), which shifts the IS curve upward. The change of $\mathcal{B}_t^T$ and $\mathcal{B}_t^{T,om}$ will shift the LM curves upward, too. The overall effect is expansionary in both cases.

A_{t+1} is the productivity in the intermediate good sector. If A_{t+1} increases, firms invest more and k_t increases which shifts the IS curve to the right. As equation (20) reveals, an increase in k_t results in an increase in Δ_t^D. As a consequence, in the inside-money version the LM curve is shifted upward. The overall effect results in a slight decrease in output. This is in sharp contrast with the outside-money version, since k_t does not affect Δ^M, hence in this case the overall effect of the productivity shock will be expansionary, see *Figure 5*.

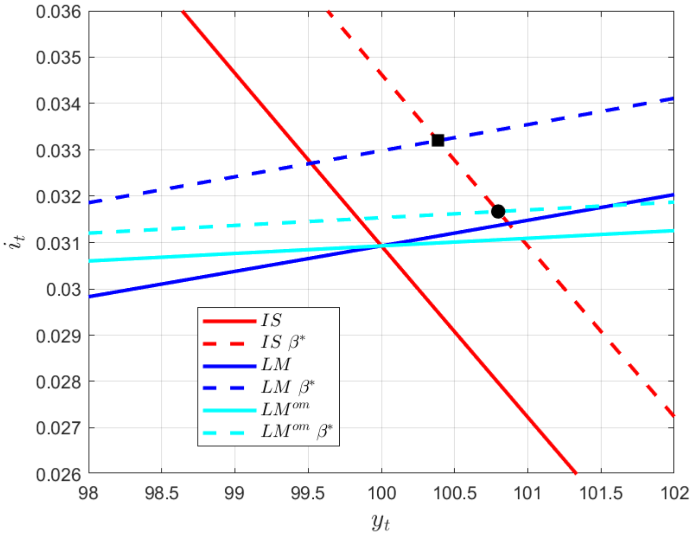

Figure 4: The effect of the decrease of the discount factor on the IS and LM curves

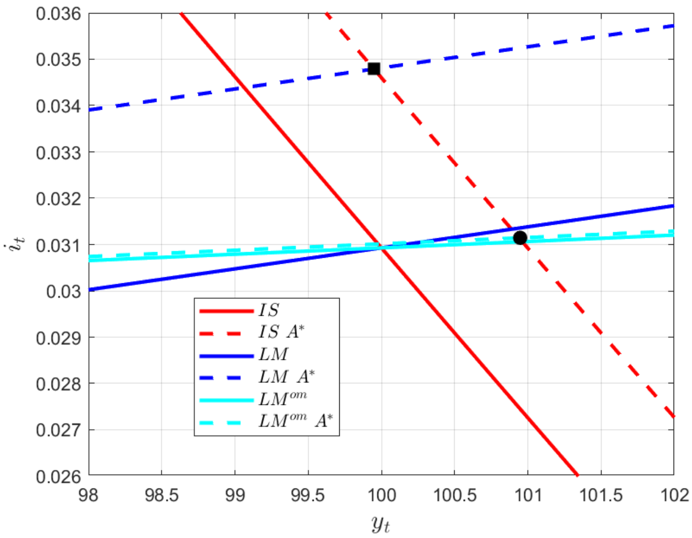

Figure 5: The effect of an increase in productivity on the IS and LM curves

Summary

Using the IS and LM curves, we compared how the economy responds to macroeconomic shocks in the presence of outside and inside money. Since the curves in the two cases differ already in the no-shock baseline scenario and also react differently to the shocks, it is not surprising that we obtain numerically different results in the model versions. On the other hand, if we focus on the direction of change in real output, except for the productivity shock, the results will not differ qualitatively: if the effect of a shock is expansive for inside money, it will remain expansive for outside money.

4 Implementation of an interest rate rule

In the previous section, we have shown how the presence of inside money modifies the macroeconomic effects of exogenous shocks if monetary policy does not respond to them. At the same time, it may rightly be argued that all of this is not really important, as if monetary policy responds to shocks, these differences may disappear or become insignificant. The objective of monetary policy is to facilitate the achievement of some certain macroeconomic goals in response to exogenous shocks, and the central bank controls its monetary policy instruments to achieve these objectives. In the case of inside money, monetary policy must be implemented differently in order to achieve the same macroeconomic goal compared to the case of outside money, but the exact mechanism of implementation is not necessarily interesting from a macroeconomic point of view.

Of course, this line of reasoning is based on the implicit assumption that the same macroeconomic objective can always be achieved with monetary policy, regardless of the role of the banking system in the process of money creation. In this section, we will investigate the validity of this assumption.

We assume that the monetary policy behavior required to achieve the above objectives can be described by the interest rate rule (16). The rule is a relationship between the output and an interest rate, and it can be represented by an increasing curve in the output-interest rate space. The actual output-interest rate combination desirable to monetary policy is given by the intersection of the IS and the interest-rate-rule curves.

We introduce the following notation for the output and interest rate determined by the interest rate rule and the IS curve:

$$\begin{aligned} y_t^{ir} &= y^{ir}\left(D_t, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right), & (41) \\ i_t^{ir} &= i^{ir}\left(D_t, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS}\right). & (42) \end{aligned}$$

Since the IS curve depends on the shocks and D_t, λ_t, ρ_t (recall equation (26)), y_t^{ir} and i_t^{ir} also depend on these variables.

4.1 LM shocks

As pointed out by Poole (1970), an important advantage of conducting monetary policy on the basis of an interest rate rule is that it stabilizes the real output in the presence of fluctuations in the supply and demand of money. This is illustrated by *Figure 6* in the case of outside money.

In the figure, the LM curve shifts because money supply increases from M_t^0 to M_t^* due to an exogenous shock. If monetary policy did not react, the equilibrium output would be the intersection of the IS and LM curves, so the output would increase. If, on the other hand, monetary policy reacts in line with the interest rate rule, then it must implement the allocation (y_t^0, i_t^0) given by the intersection of the unchanged IS and the interest-rate-rule curves.

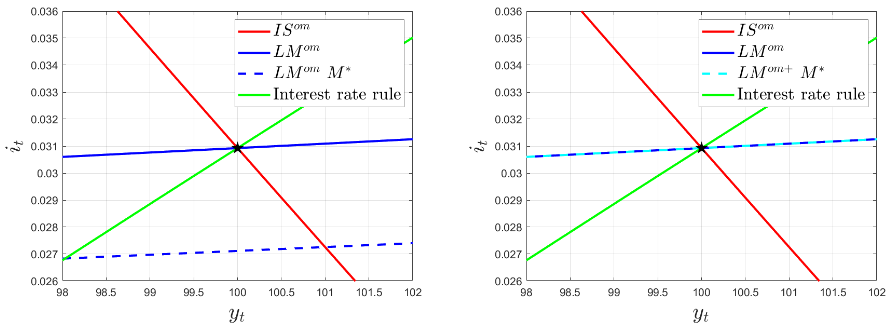

Figure 6: Money supply shock in the outside-money version of the model

If we denote the reaction of monetary policy by M_t^+, the *post-reaction* money supply becomes $M_t^* + M_t^+$. Monetary policy must choose M_t^+ so that the LM curve returns exactly to its original position.[10] Using the notation introduced in equation (35), we can express the formal condition for implementing the interest rate rule:

$$y_t^0 = \tilde{y}^{LM}\left(i_t^0, M_t^* + M_t^+, \eta_t^0, \xi_t^{IS0}\right).$$

The above condition is obviously satisfied if

$$M_t^+ = -\left(M_t^* - M_t^0\right),$$

that is, if monetary policy reduces the money supply by exactly as much as it increased as a result of the exogenous shock.

[10]Suppose monetary policy controls the amount of money, but it can only do so with some stochastic error (ϵ_t^M) After the error is realized ($M_t^* = M_t^0 + \epsilon_t^M$), it tries to correct it ($M_t^* + M_t^+$).

The example above illustrates why it is advisable to follow an interest rate rule in the presence of inside money in the case of an LM curve shocks (i.e. money market shocks). This is because by doing so the turbulence of the money market does not cause unnecessary fluctuations in the real economy, and the effects of the shocks can be completely eliminated.

In what follows, we examine whether the above implementation is feasible in the presence of inside money in the case of shocks to money supply or demand. However, the problem is now more complicated, since in contrast to the case of outside money, the IS curve also reacts to changes in the amount of money. That is, when monetary policy pushes the LM curve back to its original position, the IS curve is also shifted, and it is not certain that it will eventually return to its original position.

Although the task of monetary policy is more complicated in the case of inside money, it has more instruments at its disposal: beyond the money stock, it can also control λ_t and ρ_t, see *section2.1.4.* (Moreover, as shown in *Appendix A.4*, monetary policy can unambiguously be represented by D_t, λ_t and ρ_t instead of M_t, Δ_t^R and Δ_t^B.) As equation (20) reveals, monetary policy can influence Δ_t^D, too. Furthermore, it is also able to act on $\mathcal{B}_t^T$ and $\mathcal{Y}_t^T$ via Δ_t^D, as shown in equations (21), (22). As a consequence, monetary policy can offset the effect of changing money supply on the IS curve by the change of $\mathcal{B}_t^T$ and $\mathcal{Y}_t^T$, see equation (24).

Let D_t^+ denote how much monetary policy changes the money supply in response to shocks, and λ_t^+ and ρ_t^+ the values of the variables in question as determined by the monetary policy response. Then, in order to stabilize output, the monetary policy response must meet the following conditions:

$$
\begin{aligned}
y_t^0 &= y^{IS}\left(i_t^0, D_t^* + D_t^+, \lambda_t^+, \rho_t^+, \xi_t^{LM*}, \xi_t^{IS0}\right), \\
y_t^0 &= y^{LM}\left(i_t^0, D_t^* + D_t^+, \lambda_t^+, \rho_t^+, \xi_t^{LM*}, \xi_t^{IS0}\right)
\end{aligned}
$$

where we applied the notations introduced in equations (26) and (27). As discussed in *Appendix A.7*, there exist D_t^+, λ_t^+, ρ_t^+ which satisfy the above conditions. Generally, the solution only makes economic sense if $0 < \lambda_t^+ \leq \bar{\lambda}_t$ and $0 < \rho_t^+ < 1$. However, there is no guarantee that an economically meaningful solution will be found for shocks of any magnitude. If not, the monetary policy that would stabilize the output cannot be implemented.[11]

[11] If we take the interest rate rule strictly, in the case of inside money, monetary policy should not push back the curves to the starting point. This is because in the presence of inside money, the shocks of the money market shift the IS curve as well, and instead of the starting point, the LM curve should be pushed to the intersection of the IS and the interest-rate-rule curve. At the same time, this point is very close to the starting point, and if the starting point is targeted, we retain the useful feature seen in the case of outside

Generally it is not the case that all monetary policies that can be implemented in the case of outside money can also be implemented in the case of inside money. In the following we examine in what range of shocks $\bar{\lambda}_t^*$ and η_t^* the interest rate rule can be implemented, or, in other words, the output can be fully stabilized. Recall that y_t^{islm} is the output determined by the intersection of the IS and LM curves, see equation (28), that is, the output level that the shocks would cause without a monetary policy response. As discussed in the previous section, we limit our attention to moderate shocks, that is, where the effect on output in the absence of monetary reaction is less than 5 percent.

We also exclude policies which require unrealistically low values of i^R for their implementation: the smallest possible value of i^R we consider is -1%.[12]

First, consider the exogenous change of $\bar{\lambda}_t$, which can be interpreted as a money supply shock in the inside money case. As discussed in *section 2.1.4*, a decrease of $\bar{\lambda}_t$ results in a decrease of λ_t and an increase in the money supply, D_t. It is easy to show that in this case, if λ_t^+ and ρ_t^+ are chosen in such a way that they restore the pre-shocks value of Ψ_t in equation (20), and

$$D_t^+ = -\left(D_t^* - D_t^0\right),$$

then the money supply shock can be eliminated and the output remains equal to y_t^0.

Figure 7 displays the range $[0.6463, 0.84]$ around the baseline value ($\bar{\lambda}^0 = 0.7464$) where the deviation of y_t^{islm} from y_t^0 is no more than 5 percent (see the left panel). The right panel reveals that over the whole range $0 < \lambda_t^+ < \bar{\lambda}_t$ and $0 < \rho_t < 1$, that is, the interest rate can be implemented, and the output can be stabilized at y_t^0.

The next shock we investigate is a shock to the households' money demand. Contrary to the $\bar{\lambda}$ case, it is not always possible to neutralize the effect of the shock within the range that would result in less than 5 percent change in the output without the response of monetary policy. The baseline value of η_t is 0.0241. As it turns out, monetary policy can fully offset shocks that are within the range of $[0.0216, 0.0257]$, which would correspond to a change in output between -3.35 and 5 percent without monetary policy reaction (see *Figure 8*).

---

money that turbulences in the money market do not cause real economic fluctuations at all.

[12]As discussed, there is no cash in our model, so the zero lower bound on nominal interest rates does not appear explicitly. However, we want to avoid examining cases that are irrelevant in practice. Therefore, we exclude from our analysis the cases where the interest paid on the central bank reserve is unrealistically low.

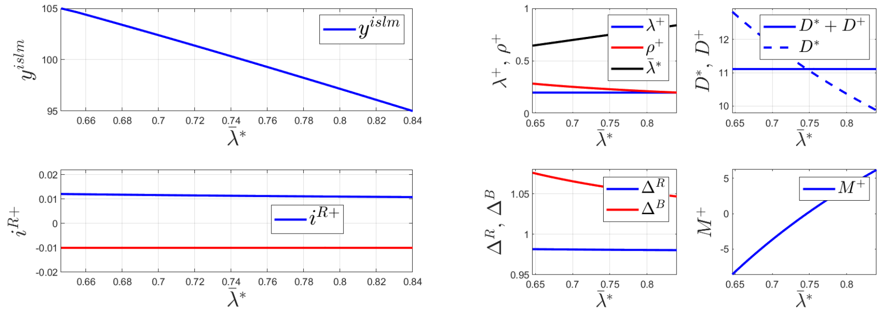

Figure 7: Implementation of the interest rate rule – money supply shock

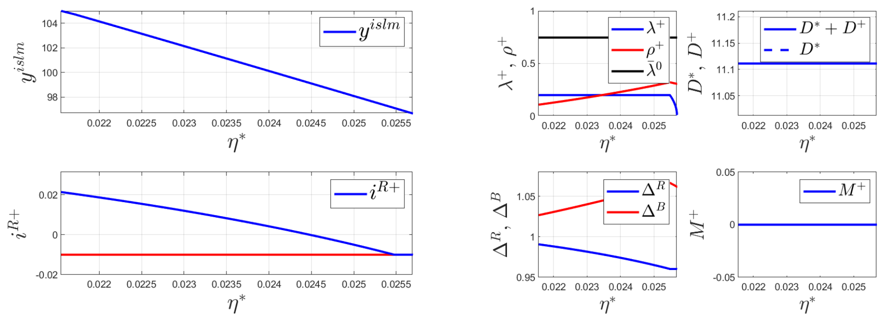

Figure 8: Implementation of the interest rate rule – households' money demand shock

The figure also reveals that the appropriate policy in this case leaves the money supply unchanged, that is, $D_t^+ = 0$, and the positions of the IS and the LM curves are adjusted only by λ_t^+ and ρ_t^+.

4.2 IS shocks

As an illustrative example, let us first examine how the monetary policy based on the interest rate rule can be implemented in the outside money version of the model in the presence of households' discount factor shock. In this case, monetary policy should achieve the output determined by the intersection of the moving IS curve and the interest rate rule (see *Figure 9*). This is possible by shifting the LM curve to this point by changing the money supply.

Formally, M_t^+ must be chosen so that the following condition is met,

$$\tilde{y}_t^{ir*} = y^{LM,om}\left(\tilde{\imath}_t^{ir*}, M_t + M_t^+, \eta_t, \xi_t^{IS*}\right),$$

where $\tilde{y}_t^{ir*}$ and $\tilde{\imath}_t^{ir*}$ are determined by the IS and the interest-rate-rule curves.

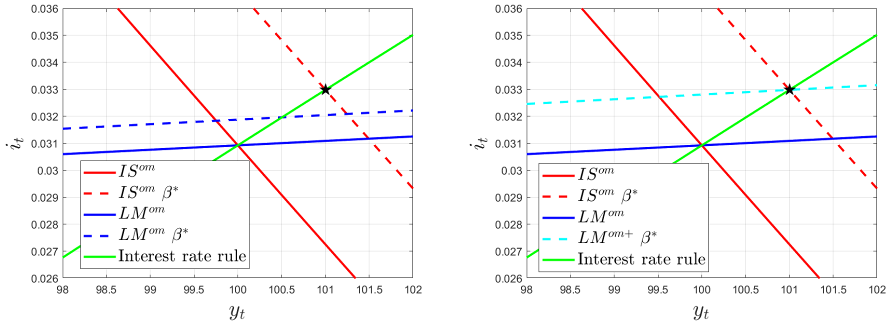

Figure 9: Households' discount factor shock in the outside-money version of the model

In the case of inside money, the problem is similar to that of the previous section: if the LM curve is shifted to the desired point by changing the money supply, then the IS curve will move away from the intersection point. This can still be handled by changing monetary policy to affect λ_t and ρ_t. Formally, the following conditions must be met:

$$
\begin{aligned}
y_t^{ir*} &= y^{IS}\left(i_t^{ir*}, D_t + D_t^+, \lambda_t^+, \rho_t^+, \xi_t^{LM0}, \xi_t^{IS*}\right), \\
y_t^{ir*} &= y^{LM}\left(i_t^{ir*}, D_t + D_t^+, \lambda_t^+, \rho_t^+, \xi_t^{LM0}, \xi_t^{IS*}\right),
\end{aligned}
$$

where, using the notations introduced in equations (41) (42), y_t^{ir*} and i_t^{ir*} are defined as

$$
\begin{aligned}
y_t^{ir*} &= y^{ir}\left(D_t, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS*}\right), \\
i_t^{ir*} &= i^{ir}\left(D_t, \lambda_t, \rho_t, \xi_t^{LM}, \xi_t^{IS*}\right),
\end{aligned}
$$

that is they are determined by the intersection of the post shock IS curve and interest rate rule.

In the following, we examine the size of the shocks for which the monetary policy defined by the interest rate rule can be implemented in the manner defined by the above equations, and we measure the output effect of the shocks by the change of y_t^{ir*} relative to y_t^0. Again, we focus on the range of shocks when $|y_t^{ir} - y_t^0|$ is no more than 5 percent, and exclude policies which require too low values of Δ^R to implement.

First, we investigate shocks to the discount factor (β_t). Its baseline value equals to 0.97. Although, in finite time horizon it is not necessary to assume that $\beta_t < 1$, we use this widespread assumption, hence the highest value of β_t we consider is 0.999. In a wide range of β_t the interest rate rule can be implemented. *Figure 10* displays the range of $[0.8052, 0.999]$ where at the

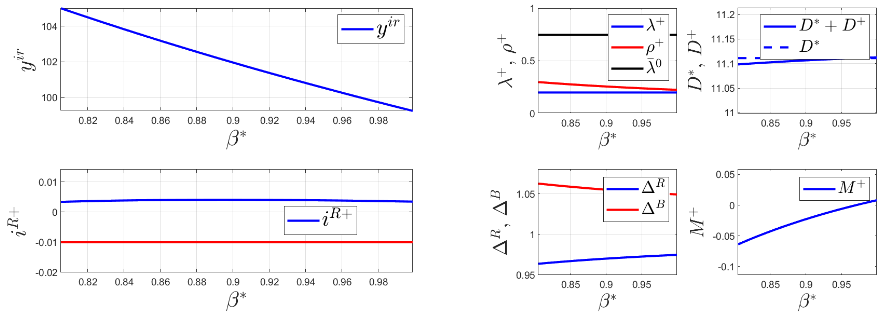

Figure 10: Implementation of the interest rate rule – households' discount factor shock

lower limit the output is higher by 5 percent than its baseline value, as the left panel reveals.

In the case of the productivity shock the central bank can implement the interest rate rule over the full range of shocks that cause output fluctuations of up to 5 percent, that is over the interval $[1.3201, 1.4063]$ (baseline value is 1.3633).

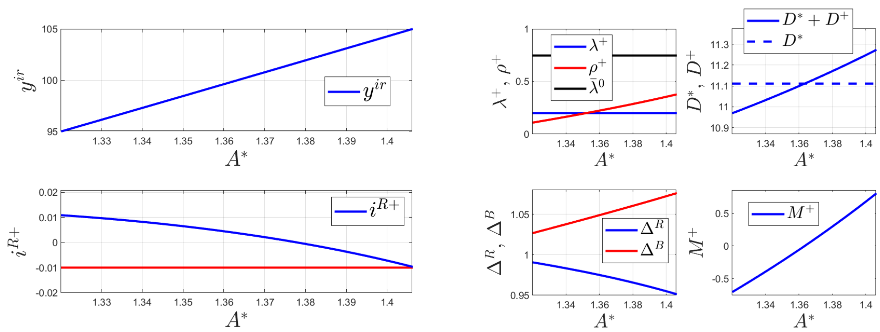

Figure 11: Implementation of the interest rate rule – productivity shock

4.3 Summary

In this section, we have demonstrated that the interest rate rule can be implemented for inside money as well, but requires a more sophisticated monetary policy than for outside money. It needs all the three instruments of monetary policy used in a coordinated way.

In the case of money supply, discount factor shocks and productivity shocks, the above implementation is possible for a fairly wide range of shocks.

In the case of money demand shock, the interest rate rule can only be implemented in a narrower range of possible values of shocks. In this case, the problem is that for large enough shocks, unrealistically low values of i_t^R (interest paid on reserves) would be required for implementation.

5 Approximation of the interest rate rule

In the previous section, we saw that it is possible to implement a monetary policy based on an interest rate rule even in the case of inside money, but we have also shown that this is only true for a limited range of shocks.

But there is another problem with the implementation. In order to be perfectly able to implement the interest rate rule, the central bank must know the exact structure of the economy and the numerical values of the parameters, and on this basis it must coordinate the control of its three instruments with extreme precision. In reality, central banks do not have such an accurate knowledge of the economy and cannot conduct such a sophisticated monetary policy.

Therefore, in this section, we examine the consequences of the limited ability to implement the interest rate rule. Specifically, we assume that the central bank responds to shocks only with the money supply, and as a result, it can only approximately stabilize the output in the presence of LM shocks and reach the (y_t^{ir*}, i_t^{ir*}) allocation in the presence of IS shocks.

Let us denote the central bank's post shock reaction in money supply by D_t^a. Assume that the central bank chooses this so that the IS, LM, and interest-rate-rule curves intersect each other at the same point, but this point does not necessarily match the (y_t^0, i_t^0) or (y_t^{ir*}, i_t^{ir*}) allocations. That is, D_t^a is chosen in such a way that the resulting (y_t^a, i_t^a) allocation satisfies the following conditions:

$$
\begin{aligned}
y_t^a &= y^{ir}\left(D_t^* + D_t^a, \lambda_t^*, \rho_t, \xi_t^{LM*}, \xi_t^{IS*}\right), \\
i_t^a &= i^{ir}\left(D_t^* + D_t^a, \lambda_t^*, \rho_t, \xi_t^{LM*}, \xi_t^{IS*}\right), \\
y_t^a &= y^{LM}\left(i_t^a, D_t^* + D_t^a, \lambda_t^*, \rho_t, \xi_t^{LM*}, \xi_t^{IS*}\right).
\end{aligned}
$$

The first two conditions guarantee that (y_t^a, i_t^a) is at the intersection of the IS and the interest-rate-rule curves. The third assures that the LM curve is also on this point.

5.1 LM shocks

Figure 12 displays the approximate implementation of the interest rate rule for the money supply shock. The left panel shows the shift of the IS and LM curves as a result of the shock without monetary policy response. The symbol '★' represents the (y_t^0, i_t^0) allocation that monetary policy would achieve if the interest rate rule were perfectly implemented.

The right panel shows the shift of the IS and LM curves as a result of the monetary policy response. The symbol '♦' represents the (y_t^a, i_t^a) allocation. The figure reveals that the two allocations are very close to each other, so in the case of a money supply shock, the simpler monetary policy closely approximates the results of the sophisticated one.

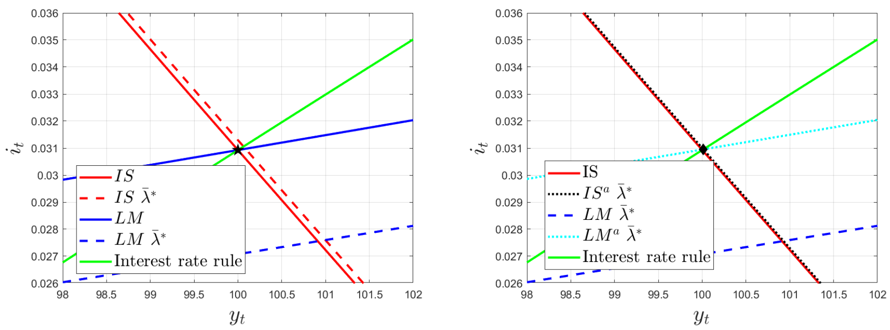

Figure 12: Approximate implementation of the interest rate rule – money supply shock

This is confirmed by *Table 1*. The table shows the fluctuation of output for different sizes of the shock if there is no monetary policy reaction, and the extent to which the approximate implementation of the interest rate rule will stabilize output. It is clear that the stabilization is quite successful: even in the case of shocks capable of causing 5 percent change in output, the approximation deviates from y_t^0 at most by 0.07 percent.

Table 1:

$\bar{\lambda}_t^*$	0.8400	0.8030	0.7654	0.7272	0.6879	0.6463
y_t^{islm}	95	97	99	101	103	105
y_t^0	100	100	100	100	100	100
y_t^a	99.94	99.96	99.99	100.01	100.04	100.07

Figure 13 displays the approximate implementation of the interest rate rule in the case of households' money demand shock. It can be seen visually that the error of the approximation is now larger than in the previous case.

This is confirmed in *Table 2*. Even in the case of shocks that could potentially cause 5 percent output fluctuations, monetary policy allows only

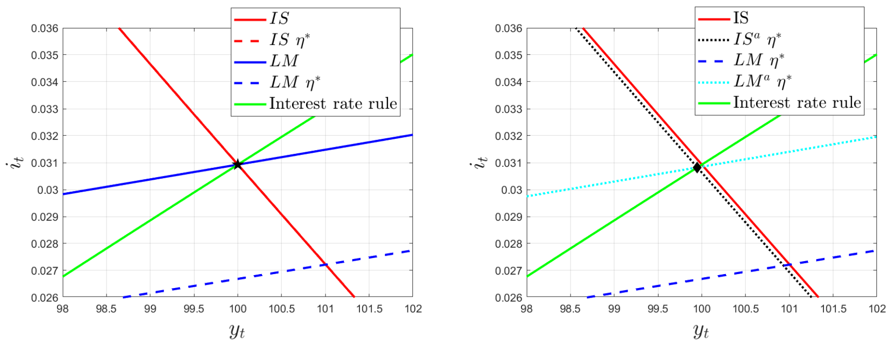

Figure 13: Approximate implementation of the interest rate rule – households' money demand shock

Table 2:

η_t^*	0.0265	0.0255	0.0245	0.0236	0.0226	0.0216
y_t^{islm}	95	97	99	101	103	105
y_t^0	100	100	100	100	100	100
y_t^a	100.26	100.15	100.05	99.95	99.85	99.74

around 0.25 percent fluctuations. So it can neutralize around 95 percent of the output impact of the shock. Although this is an order of magnitude larger fluctuation than in the previous case, it is still a fairly successful stabilization of the economy.

5.2 IS shocks

Figure 14 displays the approximate implementation of the interest rate rule for the discount factor shock. As can be seen, similarly to the money supply shock, the approximation is almost perfect in this case as well, which is also confirmed by *Table 3*

Table 3:

β_t^*	0.999	0.9333	0.8658	0.8052
y_t^{ir}	99.25	101	103	105
y_t^a	99.23	101.02	103.06	105.11

In contrast to the previous case, as *Figure 15* reveals, in the case of a pro-

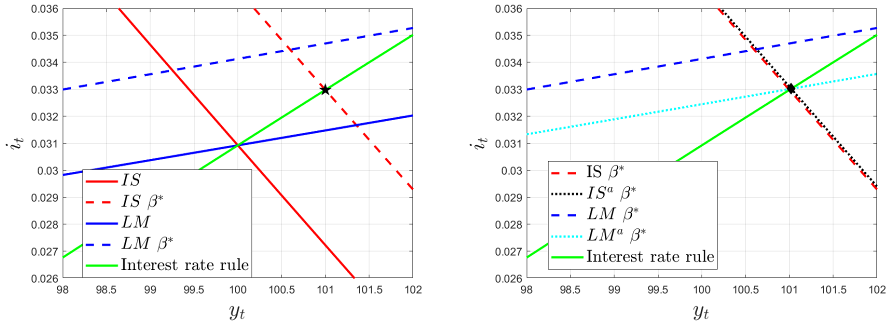

Figure 14: Approximate implementation of the interest rate rule – households' discount factor shock

ductivity shock, the error of approximation is no longer negligible, although it is still not very large. The figure also shows that in the case of an approximate implementation, the shock causes more fluctuation in output than in the case of a perfect implementation.

Table 4 also demonstrates that the approximate implementation amplifies the output effect of shocks by increasing it by about 1.06 times.

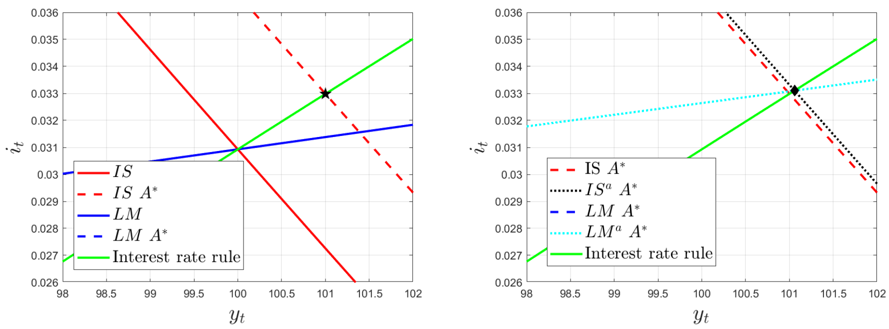

Figure 15: Approximate implementation of the interest rate rule – productivity shock

Table 4:

A_t^*	1.3201	1.3374	1.3547	1.3719	1.389	1.4062
y_t^{ir}	95	97	99	101	103	105
y_t^a	94.75	96.84	98.95	101.06	103.18	105.34

5.3 Summary

In this section we considered what happens when the central bank has limited ability to pursue sophisticated monetary policy and controls only the supply of reserves.

We found that, of course, it is not possible to perfectly implement the monetary policy rule in this case, only to approximate it, but the error of the approximation does not seem significant from a practical point of view.

6 Conclusions

We generalized the traditional IS and LM curves to dynamic general equilibrium models to examine the macroeconomic consequences of banks' creation of inside money. We used a simple dynamic model to study the problem, however, our framework based on the generalized IS and LM curves can be applied in more complex general equilibrium models, too.

The starting point of our analysis was the observation that financial intermediation and the provision of transaction instruments cannot be separated in the modern banking system, they are inherently mixed. The close connection of the two function creates a link between the IS and LM curves since the financial intermediation function is part of the relationship between savings and investment, or, translated into the language of modeling, of the IS block of macroeconomics models, while the provision of transaction instruments is part of the LM block. Hence, unlike in models only with outside money, changing the money supply affects both the IS and LM curves. Moreover, this is true not only for monetary policy, but also for all exogenous shocks. In models with only outside money, one can imagine exogenous shocks which shift either the IS curve only or the LM curve only. However, adding inside money to the model creates a new link between the IS and LM curves, and it is no longer possible to affect the two curves separately.

First, we studied the impact of exogenous macroeconomic shocks in the case of passive monetary policy. Due to the above additional relationship between the two curves, there is always quantitative difference between the impact of shocks in a model version with only outside money and the version with inside money. However, despite the quantitative differences, the results are qualitatively similar in the two model versions.

Then we examined whether the approach of the New Keynesian literature is valid, namely, whether the macroeconomic effects of monetary policy can be satisfactorily described by an interest rate rule and the IS block of the model without addressing the details of the money supply. We have shown that despite the complexity of the creation of inside money, it is possible to implement perfectly a monetary policy based on the IS curve and an interest rate rule, although it requires a more complex toolkit of monetary policy implementation than assumed in models with only outside money.

However, the above equivalence result is valid only in certain limited ranges of the shocks. That is why, in addition to the perfect implementation of a policy based on the interest rate rule, we also examined its approximation and we have found that the error of the approximation is rather small for most shocks.

This paper has demonstrated that a framework based on the generalized IS and LM curves is suitable for investigating problems where the details of the money creation process of the banking system matter. We have shown that the approach of the New Keynesian macroeconomics to examine the effects of monetary policy using the IS block and the interest rate rule of the model, abstracted from money creation, is justified.

In our paper, we examined the role of inside money under normal circumstances when the economy is not hit by extreme shocks and the nominal interest rate does not reach its zero lower bound. A natural extension of this research could be to use the framework of generalized IS and LM curves to examine situations where the nominal interest rate has reached its lower bound, the economy is in liquidity trap, and the abundance of liquidity makes monetary policy ineffective. The applied framework is also suitable for analyzing issues related to the money creation process such as unconventional monetary policies or central bank digital currency.

References

[1] Arciero, L., R. Heijmans, R. Heuver, M. Massarenti, C. Picillo and F. Vacirca, (2016). How to Measure the Unsecured Money Market: The Eurosystem's Implementation and Validation Using TARGET2 Data, *International Journal of Central Banking*, 12(1), 247-280.

[2] Bigio, S. and P.O. Weill (2016). A Theory of Bank Balance Sheets, UCLA.

[3] Boissay, F., F. Collard and F. Smets (2016). Booms and Banking Crises. *Journal of Political Economy*, 124(2), 489-538.

[4] Calice, P. and N. Zhou (2018). Benchmarking Costs of Financial Intermediation around the World, Policy Research Working Paper Series 8478, The World Bank.

[5] Clarida, Galí and Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective, *Journal of Economic Literature*, 37(4), 1661-1707.

[6] Clerc, L., A. Derviz, C. Mendicino, S. Moyen, K. Nikolove, L. Stracca, J. Suarez and A.P. Vardoulakis (2015). Capital Regulation in a Macroeconomic Model with Three Layers of Default, *International Journal of Central Banking*, 11(3), 9-63.

[7] Cúrdia, V. and M. Woodford (2016). Credit Frictions and Optimal Monetary Policy, *Journal of Monetary Policy*, 84, 30-65.

[8] Deutsche Bundesbank (2017). The role of banks, non-banks and the central bank in the money creation process, Monthly Report, April 2017.

[9] Eggertsson, G.B and P.R. Krugman (2012). Debt, Deleveraging and the Liquidity Trap: a Fisher-Minsky-Koo Approach, *Quarterly Journal of Economics*, 127(3), 1469-1513.

[10] Galí, J. (2015). *Monetary Policy, Inflation and the Business Cycle – An Introduction to the New Keynesian Framework and its Applications*, Princeton University Press.

[11] Gertler, M. and N. Kiyotaki (2015). Banking, Liquidity, and Bank Runs in an Infinite Horizon Economy, *American Economic Review*, 105(7), 2011–2043.

[12] Godley, W. and M. Lavoie (2007).*Monetary Economics – An Integrated Approach to Credit, Money, Income, Production and Wealth*, Palgrave Macmillan.

[13] Goodfriend, M. and B. McCallum (2007). Banking and interest rates in monetary policy analysis: A quantitative exploration, *Journal of Monetary Economics* 54, 1480-1507.

[14] Jakab, Z. and M. Kumhof (2019). Banks Are Not Intermediaries of Loanable Funds — Facts, Theory and Evidence. Bank of England Staff Working Paper No. 761.

[15] Jordan, T.J. (2018). How Money is Created by the Central Bank and the Banking System, Swiss National Bank.

[16] Lagos, R. (2006). Inside and outside money, Federal Reserve Bank of Minneapolis Research Department Staff Report 374.

[17] Maclay, M., A. Radia and R. Thomas (2014). Money Creation in the Modern Economy, Bank of England Quarterly Bulletin, 2014 Q1.

[18] Piazzesi, M. and M. Schneider (2018). Payments, Credit and Asset Prices, BIS WP 734.

[19] Piazzesi, M., C. Rogers and M. Schneider (2021). Money and banking in a New Keynesian model, manuscript, Stanford University.

[20] Poole, W. (1970). Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model, *Quarterly Journal of Economics*, 84(2), May, 197-216.

[21] Rivero Leiva, and H, Rodríguez Mendizábal (2019). Self-fulfilling runs and endogenous liquidity creation, *Journal of Financial Stability*, 45, 1-15.

[22] Rodríguez Mendizábal, H. (2017). Narrow banking with modern depository institutions: Is there a reason to panic?

[23] Shin, H.Y. (2010). *Risk and Liquidity*, Oxford University Press.

[24] Tobin, J. (1963). Commercial banks as creators of 'money', Cowles Foundation Discussion Papers No. 159.

[25] Világi, B. and B. Vonnák (2022). A Simple Framework for Analyzing the Macroeconomics Effects of Inside Money, MNB (Central Bank of Hungary) Working Papers, 2022/3.

[26] Werner, R. (2016). A lost century in economics: Three theories of banking and the conclusive evidence, *International Review of Financial Analysis* 46, 361–379.

[27] Woodford, M. (2010). Financial Intermediation and Macroeconomic Analysis, *Journal of Economic Perspectives*, 24(4), Fall, 21–44.

A Appendix

A.1 Parameter values

The parameter values of the model are chosen in such a way to match the most important stylized facts of the banking system and the aggregate economy.

Table 5 displays the steady-state values of balance sheet items of the banking system ($P = 1$). The items represented in the table imply that

- The steady-state value of the money multiplier $D/M = 5$, see the online dataset of Maclay, Radia and Thomas (2016).
- The ratio of stable and liquid liabilities $F^h/D = 1$ which is in line with Bigio and Weil (2016, page 5) who claim that demand deposit correspond to 50-60% of banks' liabilities. The empirical share of time deposits is significantly smaller, only 10-20%. However, in our model F_t^h represents all other types of stable liabilities, hence it is not the exact theoretical counterpart of time deposits in reality. That is why we choose higher share for F^h.

Table 5: Balance sheet of the banking system – baseline values

Assets	Liabilities
$M = 2.22$	$F^{CB} = 2.22$
$L = 20$	$F^h = 8.89$
	$D = 11.11$

In our model there is no explicit lower bound for nominal interest rate (there is no cash in the model), however, we still want to avoid zero nominal interest rates in the simulations. Therefore we choose a relatively high baseline value for the nominal interest rate: $i = 0.0309$. We choose β to be consistent with the baseline interest rates, that is, $\beta = 1/(1 + i)$, since the expected inflation rate is zero. Since this is a stylized model, there is no clear interpretation of the length of time periods, therefore, it should not be inferred from the magnitude of households' discount factor either.

$\Delta^R = \Delta^D = 0.97$ implying $i^R = i^D = 0$. $\Delta^L = 1.08$, that is, we assume 8% premium on risky corporate loans which is roughly consistent with the equity premium literature. (In our model there are no equities, corporate loans represent all types of risky assets.)

We assume that the volume of loans and deposits have very moderate direct impact on the above spreads: one percent increase of loans/deposits induce 10 basis points increase in the loan rate/deposit rate spread, that is $\phi^L = 0.0019$ and $\phi^D = 0.0039$. We assume that the cost function parameter capturing the cross effect of D_t and L_t is weaker than the parameters of the direct effects, that is, $\phi^{DL} = 0.0015$. If we also assume that one percent increase in the loan stock implies on average 0.43 percent increase in demand deposits, these parameters are consistent with Calice and Zhou (2018), who estimated the effect of gross loans on the net interest margin from bank-level panel data on more than 14,000 commercial banks in 160 countries for the period 2005-2014.

The baseline consumption is 80%, the baseline investment is 20%, the baseline cost of intermediation is 1.25% of the real GDP.

The following tables display the values of the parameters and the steady-state values of the exogenous variable used in the model.

Table 6: Parameter values of the model

Name	a^n	a^z	α	θ	ϑ	ω	ν	σ	φ
Value	49.70	1	0.67	6	1.2	0.0005	2	0.5	0.001

Name	τ^F	$\bar{\tau}^L$	ϕ^L	$\bar{\tau}^D$	ϕ^{DL}	ϕ^B
Value	0.004	-0.0033	0.0025	0.0482	0.0015	0.1656

Table 7: Steady-state values of the exogenous shocks

Name	$\bar{\lambda}$	η	β	A
Value	0.7464	0.0147	0.97	1.8178

A.2 The solution to the households' problem

In this paper we consider the effects of unexpected temporary shocks at date t. We assume that households do not expect further shocks from date $t+1$

onward. Hence after the realization of the shocks at date t they solve a deterministic problem.

The Lagrangian of the household's finite horizon optimization problem:

$$\mathcal{L} = \sum_{j=0}^{T+1} \Gamma_{t+j} \left(\frac{c_{t+j}^{1-\nu}}{1-\nu} + \frac{\zeta_{t+j} \left(D_{t+j}/P_{t+j}\right)^{1-\nu}}{1-\nu} - \varphi n_{t+j} \right)$$
$$+ \sum_{j=0}^{T+1} \upsilon_{t+j} \left(W_{t+j} n_{t+j} + \Pi_{t+j} + \mathcal{D}_{t+j} + \mathcal{T}_{t+j} + (1+i_{t+j-1}) F_{t+j-1}^{h} + \left(1 + i_{t+j-1}^{D}\right) D_{t+j-1} \right)$$
$$- \sum_{j=0}^{T+1} \upsilon_{t+j} \left(P_{t+j} c_{t+j} + D_{t+j} + F_{t+j}^{h} \right).$$

The first order conditions with respect to c_{t+j}:

$$\Gamma_{t+j} c_{t+j}^{-\nu} = P_{t+j} \upsilon_{t+j};$$

with respect to n_t:

$$\Gamma_{t+j} \varphi = W_{t+j} \upsilon_{t+j};$$

with respect to F_{t+j}^{h}:

$$\upsilon_{t+j} = (1 + i_{t+j}) \upsilon_{t+j+1};$$

and with respect to D_{t+j}:

$$\Gamma_{t+j} \zeta_{t+j} \left(d_{t+j}\right)^{-\nu} = P_{t+j} \upsilon_{t+j},$$

where $d_{t+j} \equiv D_{t+j}/P_{t+j}$.

Combining the first order conditions one can easily derive the Euler equation

$$c_{t+j}^{-\nu} = \beta_{t+j} \left(1 + r_{t+j}\right) c_{t+j+1}^{-\nu},$$

where

$$1 + r_{t+j} \equiv \frac{1 + i_{t+j}}{P_{t+j}/P_{t+j+1}}$$

is the real interest rate. The Euler equation can also be expressed as

$$c_{t+j+1} = \beta_t^{\sigma} \left(1 + r_{t+j}\right)^{\sigma} c_{t+j}, \tag{43}$$

where $\sigma \equiv \nu^{-1}$. From the first order conditions with respect to c_t and n_t one obtains the labor supply:

$$\varphi c_{t+j}^{\nu} = \frac{W_{t+j}}{P_{t+j}} \equiv w_{t+j}. \tag{44}$$

Consider the first order condition with respect to D_t and divide both side by υ_t:

$$\frac{\zeta_{t+j}\left(d_{t+j}\right)^{-\nu}}{\upsilon_{t+j}} = P_{t+j}\left[1 - \frac{\upsilon_{t+j+1}}{\upsilon_{t+j}}\left(1 + i^D_{t+j}\right)\right]$$

Using the first order conditions with respect to F^h_{t+j} and c_{t+j}, this can be rewritten as

$$\frac{\zeta_{t+j}\left(d_{t+j}\right)^{-\nu}}{c^{-\nu}_{t+j}} = 1 - \Delta^D_{t+j},$$

where $\Delta^D_{t+j} \equiv (1 + i^D_{t+j})/(1 + i_{t+j})$. Rearranging it yields the money demand at date 1:

$$d_{t+j} = \frac{\eta_{t+j}}{\left(1 - \Delta^D_{t+j}\right)^{\sigma}} c_{t+j}. \tag{45}$$

Recall that

$$Y^h_t = W_t n_t + W^\kappa_t n^\kappa_t + \Pi_t + \mathcal{D}_t + \mathcal{T}_t,$$

and define

$$y^h_t \equiv \frac{Y^h_t}{P_t}.$$

Then the budget constraints can be written in real terms as follows:

$$\frac{f^h_{t+j}}{R^{t+j}_t} = \frac{y^h_{t+j} - c_{t+h} - d_{t+j}}{R^{t+j}_t} + \frac{f^h_{t+j-1}}{R^{t+j-1}_t} + \frac{\Delta^D_{t+j-1} d_{t+j-1}}{R^{t+j-1}_t},$$

for all $j = 0, \cdots, T$, and

$$\frac{f^h + d}{R^{t+T+1}_t} = \frac{y^h_{t+T+1} - c_{t+T+1}}{R^{t+T+1}_t} + \frac{f^h_{t+T}}{R^{t+T}_t} + \frac{\Delta^D_{t+T} d_{t+T}}{R^{t+T}_t},$$

where $f^h_{t+j} = F^h_{t+j}/P_{t+j}$, $d_{t+j} = D_{t+j}/P_{t+j}$, $\Delta^D_{t+j} = (1 + i^D_{t+j})/(1 + i_t)$ and

$$\begin{aligned} R^{t-1}_t &= (1 + r_{t-1})^{-1}, \qquad R^t_t = 1, \\ R^{t+j}_t &= (1 + r_t)(1 + r_{t+1}) \cdots (1 + r_{t+j-1}),\ j > 0. \end{aligned}$$

Applying recursive substitutions yields the following present-value budget constraint:

$$\sum_{j=0}^{T} \frac{c_{t+j} + \left(1 - \Delta^D_{t+j}\right) d_{t+j}}{R^{t+j}_t} + \frac{c_{T+1}}{R^{t+T+1}_t} = \bar{y}^h_t + \sum_{j=1}^{T+1} \frac{y^h_{t+j}}{R^{t+j}_t} - \frac{f + d}{R^{t+T+1}_t},$$

where

$$\bar{y}^h_t = y^h_t + (1 + r_{t-1}) f^h_{t-1} + (1 + r_{t-1}) \Delta^D_{t-1} d_{t-1}.$$

Substituting the Euler equation (43) and the money demand equation (45) into the above formula yields

$$\left[\sum_{j=0}^{T}\left(1+\eta\left(1-\Delta_{t+j}^{D}\right)^{1-\sigma}\right)\beta^{j\sigma}\left(R_t^{t+j}\right)^{\sigma-1}+\beta^{(T+1)\sigma}\left(R_t^{T+1}\right)^{\sigma-1}\right]c_t =$$

$$\bar{y}_t^h+\sum_{j=1}^{T+1}\frac{y_{t+j}^h}{R_t^{t+j}}-\frac{f+d}{R_t^{t+T}}.$$

After rearranging, we obtain the following consumption function:

$$c_t=\frac{\bar{y}_t^h+\mathcal{Y}_t^T}{\mathcal{B}_t^T},$$

where

$$\mathcal{B}_t^T=\sum_{j=0}^{T}\left(1+\eta\left(1-\Delta_{t+j}^{D}\right)^{1-\sigma}\right)\beta^{j\sigma}\left(R_t^{t+j}\right)^{\sigma-1}+\beta^{(T+1)\sigma}\left(R_t^{t+T+1}\right)^{\sigma-1},$$

and

$$\mathcal{Y}_t^T=\sum_{j=1}^{T}\frac{y_{t+j}^h}{R_t^{t+j}}-\frac{f+d}{R_t^{t+T}}.$$

Consequently, the real savings function becomes

$$s_t=\bar{y}_t^h-c_t=\frac{\left(\mathcal{B}_t^T-1\right)\bar{y}_t^h-\mathcal{Y}_t^T}{\mathcal{B}_t^T},$$

By combining equations (4), (13), (17), (40) and condition $P_t\kappa_t=W_t^{\kappa}n_t^{\kappa}$ one can show that

$$y_t^h=y_t-(1+r_{t-1})f_{t-1}^h+(1+r_{t-1})\Delta_{t-1}^{D}d_{t-1}.$$

As a consequence $\bar{y}_t^h=y_t$ and the consumption function can be expressed as

$$c_t=\frac{y_t+\mathcal{Y}_t^T}{\mathcal{B}_t^T}, \tag{46}$$

and the real savings function as

$$s_t=\frac{\left(\mathcal{B}_t^T-1\right)y_t-\mathcal{Y}_t^T}{\mathcal{B}_t^T}. \tag{47}$$

Combine equations (45) and (46) to get a formula for real money demand,

$$d_t = \frac{\eta_t}{(1-\Delta_t^D)^\sigma} \frac{y_t + \mathcal{Y}_t^T}{\mathcal{B}_t^T},$$

and the demand for time deposits is given by

$$f_t^h = s_t - d_t.$$

In the outside-money version the first order conditions have the same form. As a consequence, money demand is the following:

$$m_t = \frac{\eta_t}{(1-\Delta_t^M)^\sigma} c_t, \tag{48}$$

where $m_t = M_t/P_t$, and the spread $\Delta_t^M = 1/(1+i_t)$ since the interest rate paid on M_t is zero.

To find the consumption function, first, define

$$\hat{y}_t^h \equiv w_t n_t + w_t^\kappa n_t^\kappa + \frac{\Pi_t + \mathcal{D}_t}{P_t},$$

thus

$$y_t^h = \hat{y}_t^h + \frac{T_t}{P_t}.$$

As a consequence, the intertemporal budget constraints become

$$c_t + f_t^h + m_t = \hat{y}_t^h + \frac{\mathcal{T}_t}{P_t} + (1+r_{t-1}) f_{t-1}^h + m_{t-1},$$

In equilibrium $\mathcal{T}_t = M_t - M_{t-1}$, thus

$$c_t + f_t^h = \hat{y}_t^h + (1+r_{t-1}) f_{t-1}^h.$$

Furthermore, equations (4), (13), (17), (40) and condition $P_t \kappa_t = W_t^\kappa n_t^\kappa$ imply that

$$\hat{y}_t^h + (1+r_{t-1}) f_{t-1}^h = y_t.$$

Thus the series of budget constraints can expressed as,

$$\begin{aligned} \frac{f_{t+j}}{R_t^{t+j}} &= \frac{\hat{y}_{t+j}^h - c_{t+j}}{R_t^{t+j}} + \frac{f_{t+j-1}}{R_t^{t+j-1}}, \ j = 0, \cdots, T-1, \\ \frac{f}{R_t^{t+T+1}} &= \frac{\hat{y}_{t+T}^h - c_{t+T}}{R_t^{t+T+1}} + \frac{f_{t+T-1}}{R_t^{t+T}}. \end{aligned}$$

Applying recursive substitutions yields

$$\sum_{j=0}^{T} \frac{c_{t+j}}{R_t^{t+j}} = y_t + \sum_{j=1}^{T} \frac{y_{t+j}^h}{R_t^{t+j}} - \frac{f}{R_t^{t+T+1}}.$$

Substituting the Euler equation (43) and the money demand equation (48) into the above formula results in

$$\left[\sum_{j=0}^{T} \beta^{j\sigma} \left(R_t^{t+j}\right)^{\sigma-1}\right] c_t = \bar{y}_t^h + \sum_{j=1}^{T} \frac{y_{t+j}^h}{R_t^{t+j}} - \frac{f}{R_t^{t+T+1}}.$$

One can obtain the consumption function by rearranging the above formula:

$$c_t = \frac{\bar{y}_t^h + Y_t^{om,T}}{\mathcal{B}_t^{om,T}} = \frac{y_t + Y_t^{om,T}}{\mathcal{B}_t^{om,T}},$$

where

$$\mathcal{B}_t^{om,T} = \sum_{j=0}^{T} \beta^{j\sigma} \left(R_t^{t+j}\right)^{\sigma-1}.$$

and

$$Y_t^{om,T} = \sum_{j=1}^{T} \frac{y_{t+j}^h}{R_t^{t+j}} - \frac{f}{R_t^{t+T+1}}.$$

A.3 The cost minimization problem of retailers

The cost minimization problem of an input producer is the following:

$$\min_{n_t, z_t} W_t n_t + P_t^z z_t,$$

subject to

$$a^n n_t^{1-\alpha} + a^z z_t \geq y_t,$$

where W_t is the nominal wage. Here we dropped the (j) index to simplify the notation.

The Lagrangian of the cost minimization:

$$\mathcal{L} = W_t n_t + P_t^z z_t + \upsilon \left(y_t - a^n n_t^{1-\alpha} - a^z z_t\right),$$

where υ is the multiplier.

The first order conditions with respect to labor and intermediate goods are

$$W_t = \upsilon a^n (1-\alpha) n_t^{-\alpha}$$

$$P_t^z = \upsilon a^z.$$

Eliminating υ yields the demand for labor,

$$n_t = \left(\frac{P_t^z}{W_t} \frac{a^n(1-\alpha)}{a^z} \right)^{\frac{1}{\alpha}},$$

and by substituting it into the production function we get the demand for intermediate goods:

$$z_t = \frac{y_t - a^n n_t^{1-\alpha}}{a^z}.$$

Hence the cost function:

$$\mathcal{C}(W_t, P_t^z, y_t) = W_t n_t + \frac{P_t^z}{a^z} \left(y_t - a^n n_t^{1-\alpha} \right).$$

Since labor demand does not depend on the output, the marginal cost function is simply

$$\mathcal{MC}_t = \frac{P_t^z}{a^z}.$$

A.4 The solution to the banks' problem

Banks solve the following problem:

$$\max_{x_t, B_t} \mathrm{E}_t \left[\bar{\beta}_t \frac{\mathcal{D}_{t+1}}{P_{t+1}} + \frac{\mathcal{D}_t}{P_t} \right]$$

subject to

$$\begin{aligned} M_t - \hat{\lambda}_t D_t &\geq B_t, \\ M_t + L_t &= D_t + F_t, \end{aligned}$$

and

$$x_t \geq 0,$$

where $x_t = L_t, M_t, D_t, F_t$ and

$$\begin{aligned} \mathcal{D}_t &= \left(1 + i_{t-1}^L\right) L_{t-1} + \left(1 + i_{t-1}^R\right) \left(M_{t-1} - \hat{\lambda}_{t-1} D_{t-1} - B_{t-1} \right) + \left(1 + i_{t-1}^B\right) B_{t-1} \\ &- (1 + i_{t-1}) F_{t-1} - (1 + i_{t-1}^D) \left(1 - \hat{\lambda}_{t-1} \right) D_{t-1} - P_t \kappa_t, \\ \mathcal{D}_{t+1} &= \left(1 + i_t^L\right) L_t + \left(1 + i_t^R\right) \left(M_t - \hat{\lambda}_t D_t - B_t \right) + \left(1 + i_t^B\right) B_t \\ &- (1 + i_t) F_t - (1 + i^D) \left(1 - \hat{\lambda}_t \right) D_t - P_{t+1} \kappa_{t+1}, \end{aligned}$$

and

$$\bar{\beta}_t = \beta_t \frac{c_{t+1}^{-\nu}}{{}^{-\nu}c_t} = \frac{1}{1+r_t}.$$

Multiplying the objective function by a positive constant does not alter the results, therefor we can multiply it by P_t

$$\frac{1}{1+r_t}\frac{P_t}{P_{t+1}}\mathcal{D}_{t+1} + \mathcal{D}_t = \frac{1}{1+i_t}\mathcal{D}_{t+1} + \mathcal{D}_t.$$

Date $t-1$ variables do not constraint the date t decisions. Hence they can be treated as constants from the point of view of optimization. Hence all date $t-1$ terms can be omitted from the objective function. On the other hand, date t decisions do not have any impact on κ_{t+1}, therefore we can omit those terms as well.

Expressing F_t from the balance sheet constraint and substituting into the modified objective function yields

$$\frac{1+i_t^L}{1+i_t}L_t + \frac{1+i_t^R}{1+i_t}\left(M_t - \hat{\lambda}_t D_t - B_t\left(\hat{\lambda}_t\right)\right) + \frac{1+i_t^B}{1+i_t}B_t\left(\hat{\lambda}_t\right)$$
$$-\frac{1+i_t^D}{1+i_t}\left(1-\hat{\lambda}_t\right)D_t + D_t - M_t - L_t - P_t\kappa_t.$$

We can form the Lagrangian of the optimization problem by the above expression and the liquidity constraint:

$$\begin{aligned}
\mathcal{L}\left(\hat{\lambda}_t\right) &= \Delta_t^L L_t + \Delta_t^R\left(M_t - \hat{\lambda}_t D_t - B_t\left(\hat{\lambda}_t\right)\right) + \Delta_t^B B_t\left(\hat{\lambda}_t\right) \\
&- \Delta_t^D\left(1-\hat{\lambda}_t\right)D_t + D_t - M_t - L_t - P_t\kappa_t + \\
&+ \mu_t\left(\hat{\lambda}_t\right)\left(M_t - \hat{\lambda}_t D_t - B_t\left(\hat{\lambda}_t\right)\right),
\end{aligned}$$

where

$$\Delta_t^L \equiv \frac{1+i_t^L}{1+i_t}, \quad \Delta_t^R \equiv \frac{1+i_t^R}{1+i_t}, \quad \Delta_t^B \equiv \frac{1+i_t^B}{1+i_t}, \quad \Delta_t^D \equiv \frac{1+i_t^D}{1+i_t}.$$

The Lagrangian is a function of the liquidity shock $\hat{\lambda}_t$ since the variable B_t and the multiplier μ_t are also functions of it. The expected Lagrangian can be calculated as

$$\mathrm{E}_1\left[\mathcal{L}\right] = \int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} \mathcal{L}\left(\hat{\lambda}\right) g\left(\hat{\lambda}\right) \mathrm{d}\hat{\lambda}.$$

where $g(\hat{\lambda}) = 1/(2\bar{\lambda})$ is the density function of the uniform distribution on the $[-\bar{\lambda}, \bar{\lambda}]$ interval.

First order conditions

The variables L_t, M_t, D_t, F_t are independent of $\hat{\lambda}_t$, which is the result of the timing of decisions, since they are determined prior to the realization of $\hat{\lambda}_t$. On the other hand, when B_t and μ_t are determined, $\hat{\lambda}_t$ is already observed. Thus, while the first order conditions for interbank lending have to be met for all possible realizations of the liquidity shock, for the other choice variables only in expectation.

Formally, the first order condition with respect to B_t:

$$\frac{\partial \mathcal{L}\left(\hat{\lambda}_t\right)}{\partial B_t\left(\hat{\lambda}_t\right)} = 0, \text{ for all } \hat{\lambda}_t \in [-\bar{\lambda}_t, \bar{\lambda}_t].$$

The first order conditions with respect to L_t, M_t, D_t and F_t:

$$\frac{\partial \mathrm{E}_1\left[\mathcal{L}\right]}{\partial x_t} \leq 0,$$

where $x_t = L_t$, M_t, D_t, F_t. The inequalities in the above conditions are due to the non-negativity constraints. A *strict inequality* implies that $x_t = 0$.

To find the solution beyond the first order conditions one also needs the constraints and the complementary slackness condition:

$$\mu_t\left(\hat{\lambda}_t\right)\left(M_t - \hat{\lambda}_t D_t - B_t\left(\hat{\lambda}_t\right)\right) = 0, \text{ for all } \hat{\lambda}_t \in [-\bar{\lambda}_t, \bar{\lambda}_t].$$

That is, a positive μ_t implies a binding constraint, $M_t = \hat{\lambda}_t D_t + B_t$. On the other hand, if $M_t > \hat{\lambda}_t D_t + B_t$ then $\mu_t = 0$.

To derive the first order condition with respect to B_t, first calculate the marginal cost of B_t. Using equation (8) one can obtain

$$\frac{\partial P_t \kappa_t}{\partial B_t\left(\hat{\lambda}_t\right)} = 2\phi^B \rho_t\left(\hat{\lambda}_t\right),$$

where

$$\rho_t\left(\hat{\lambda}_t\right) = \frac{B_t^l\left(\hat{\lambda}_t\right)}{M_t - \hat{\lambda}_t D_t}.$$

Therefore the first order condition:

$$\Delta_t^B - \Delta_t^R - 2\phi^B \rho_t\left(\hat{\lambda}_t\right) - \mu_t = 0 \quad \text{for all} \quad \hat{\lambda}_t \in [-\bar{\lambda}_t, \bar{\lambda}_t].$$

First, consider the case when the bank has enough reserves to meet the interbank payment obligations due to the liquidity shock, if any. This is the case when $\hat{\lambda}_t \in [-\bar{\lambda}_t, \bar{\lambda}]$. These banks are potential lenders on the interbank market. We assume symmetric solution, that is,

$$\rho_t\left(\hat{\lambda}_t\right) = \frac{B_t^l\left(\hat{\lambda}_t\right)}{M_t - \hat{\lambda}_t D_t} = \rho_t < 1 \quad \text{for all} \quad \hat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t]. \tag{49}$$

Later we will show that such a symmetric solution is consistent with an equilibrium on the interbank market. Since $\rho_t < 1$

$$B_t^l\left(\hat{\lambda}_t\right) < M_t - \hat{\lambda}_t D_t \quad \text{for all} \quad \hat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t].$$

Then the complementary slackness condition implies that $\mu\left(\hat{\lambda}_t\right) = 0$ for all $\hat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t]$. Therefore the first order condition for the lenders becomes

$$\Delta_t^B = \Delta_t^R + 2\phi^B \rho_t \quad \text{for all} \quad \hat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t]. \tag{50}$$

Now, consider the case when the bank has to borrow on the interbank market, because its reserves are not sufficient to cover the deposit outflow, that is when $M_t < \hat{\lambda}_t D_t$ or, equivalently $\hat{\lambda}_t \in (\lambda_t, \bar{\lambda}_t]$. For such a bank $B_t^l = 0$, thus $\rho_t\left(\hat{\lambda}_t\right) = 0$ and, consequently, the first order condition becomes

$$\Delta_t^B = \Delta_t^R + \mu_t \quad \text{for all} \quad \hat{\lambda}_t \in (-\lambda_t, \bar{\lambda}_t]. \tag{51}$$

Since $\mu_t = \Delta^B - \Delta_t^R > 0$, the liquidity constraint will bind and

$$B_t^b = \hat{\lambda}_t D_t - M_t. \tag{52}$$

The first order condition with respect to M_t is

$$\int_{-\bar{\lambda}}^{\bar{\lambda}} \left(\Delta_t^R - 1 - \tau^F + \mu_t + \phi^B \rho_t^2\right) g\left(\hat{\lambda}\right) \mathrm{d}\hat{\lambda} \leq 0.$$

Since we have just shown that if $\hat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t]$ then $\mu_t = 0$, $\rho_t > 0$, and if $\hat{\lambda}_t \in (\lambda_t, \bar{\lambda}_t]$ then $\mu_t > 0$, $\rho_t = 0$, and both are independent of $\hat{\lambda}$ inside these two intervals, the above integral can be decomposed as

$$\left(\Delta_t^R - 1 - \tau^F\right) \int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} g\left(\hat{\lambda}\right) \mathrm{d}\hat{\lambda} + \mu_t \int_{\lambda_t}^{\bar{\lambda}_t} g\left(\hat{\lambda}\right) \mathrm{d}\hat{\lambda} + \phi^B \rho_t^2 \int_{-\bar{\lambda}_t}^{\lambda_t} g\left(\hat{\lambda}\right) \mathrm{d}\hat{\lambda} \leq 0.$$

Since

$$\int_{\lambda_t}^{\bar{\lambda}_t} g\left(\hat{\lambda}\right) \mathrm{d}\hat{\lambda} = 1 - G(\lambda_t), \qquad \int_{-\bar{\lambda}_t}^{\lambda_t} g\left(\hat{\lambda}\right) \mathrm{d}\hat{\lambda} = G(\lambda_t),$$

and focusing only on solutions in which M_t is positive, the above condition simplifies to the following equation:

$$\Delta_t^R + \mu_t \left[1 - G(\lambda_t)\right] + \phi^B \rho_t^2 G(\lambda_t) = 1 + \tau^F. \tag{53}$$

Since we assume that $\hat{\lambda}_t$ is drawn from a uniform distribution over the interval $[-\bar{\lambda}_t, \bar{\lambda}_t]$, as we will show later, the above formula has the following closed form solution:

$$\Delta_t^R + \frac{\delta_t}{2\bar{\lambda}_t}\mu_t + \frac{\varsigma_t}{2\bar{\lambda}_t}\phi^B \rho_t^2 = 1 + \tau^F, \tag{54}$$

where $\delta_t \equiv \max\left[0, \bar{\lambda}_t - \lambda_t\right]$ and $\varsigma_t \equiv \max\left[2\bar{\lambda}_t, \bar{\lambda}_t + \lambda_t\right]$.

The first order condition with respect to L_t is

$$\int_{-\bar{\lambda}_t}^{\bar{\lambda}} \left(\Delta_t^L - 1 - \tau^F - \tau^L - 2\phi^L \frac{L_t}{P_t} + \phi^{DL}\frac{D_t}{P_t}\right) g\left(\hat{\lambda}\right) \, \mathrm{d}\hat{\lambda} \leq 0.$$

Since all terms inside the integral are independent of $\hat{\lambda}_t$, in equilibrium with non-zero lending the previous expression simplifies to

$$\Delta_t^L = 1 + \bar{\tau}^L + 2\phi^L \frac{L_t}{P_t} - \phi^{DL}\frac{D_t}{P_t},$$

where $\bar{\tau}^L \equiv \tau^F + \tau^L$.

The first order condition with respect to D_t is

$$\int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} \left(\Delta_t^D - 1 - \tau^F + \tau^D - \phi^{DL}\frac{L_t}{P_t}\right) g\left(\hat{\lambda}\right) \, \mathrm{d}\hat{\lambda} +$$
$$\int_{-\bar{\lambda}_t}^{\bar{\lambda}_t} \hat{\lambda}\left(\mu_t + \phi^B \rho_t^2 + \Delta_t^R - \Delta_t^D\right) g\left(\hat{\lambda}\right) \, \mathrm{d}\hat{\lambda} \geq 0.$$

Taken into account that if $\hat{\lambda}_t \in [-\bar{\lambda}_t, \lambda_t]$ then $\mu_t = 0$, $\rho_t > 0$, and if $\hat{\lambda}_t \in (\lambda_t, \bar{\lambda}_t]$ then $\mu_t > 0$, $\rho_t = 0$, in equilibrium with positive demand deposit the above condition can be rewritten as

$$\Delta_t^D + \tau^D + 2\phi^D \frac{D_t}{P_t} - \phi^{DL}\frac{L_t}{P_t} + \mu_t \int_{\lambda_t}^{\bar{\lambda}_t} \hat{\lambda} g\left(\hat{\lambda}\right) \, \mathrm{d}\hat{\lambda}$$
$$= 1 + \tau^F - \phi^B \rho_t^2 \int_{-\bar{\lambda}_t}^{\lambda_t} \hat{\lambda}_t g\left(\hat{\lambda}\right) \, \mathrm{d}\hat{\lambda}. \tag{55}$$

assuming uniform distribution implies that the above equation becomes

$$\Delta_t^D = 1 - \bar{\tau}^D + \phi^{DL}\frac{L_t}{P_t} - \left(\mu_t - \phi^B \rho_t^2\right)\frac{\delta_t \varsigma_t}{4\bar{\lambda}_t}, \tag{56}$$

where $\bar{\tau}^D \equiv \tau^D - \tau^F$.

Finally, time deposits (F_t) are determined by the balance sheet constraint:

$$F_t = M_t + L_t - D_t.$$

Equilibrium on the interbank market

Since the banking sector is homogeneous and banks are similar before the liquidity shock, in equilibrium all banks choose the same M_t, L_t, D_t, F_t and, as a consequence, the same reserve ratio λ_t. Since there is a continuum of banks in the model, the cross sectional distribution of $\hat{\lambda}_t$ can be described by the probability distribution of $\hat{\lambda}_t$.

The interbank equilibrium condition is

$$B_t^l + B_t^{CB} = B_t^b, \tag{57}$$

where $0 \leq B_t^{CB} \leq B_t^b$ is central bank lending on the interbank market and

$$\begin{aligned} B_t^l &= \int_{-\bar{\lambda}_t}^{\lambda_t} B_t^l\left(\hat{\lambda}\right) \frac{1}{2\bar{\lambda}_t} \mathrm{d}\,\hat{\lambda}, \\ B_t^b &= \int_{\lambda_t}^{\bar{\lambda}_t} B_t^b\left(\hat{\lambda}\right) \frac{1}{2\bar{\lambda}_t} \mathrm{d}\,\hat{\lambda}, \end{aligned}$$

where $1/\left(2\bar{\lambda}_t\right)$ is the uniform probability density function. Equation (52) implies that

$$B_t^b = \int_{\lambda_t}^{\bar{\lambda}_t} \left(\hat{\lambda} - \lambda_t\right) D_t \frac{1}{2\bar{\lambda}_t} \mathrm{d}\,\hat{\lambda}. \tag{58}$$

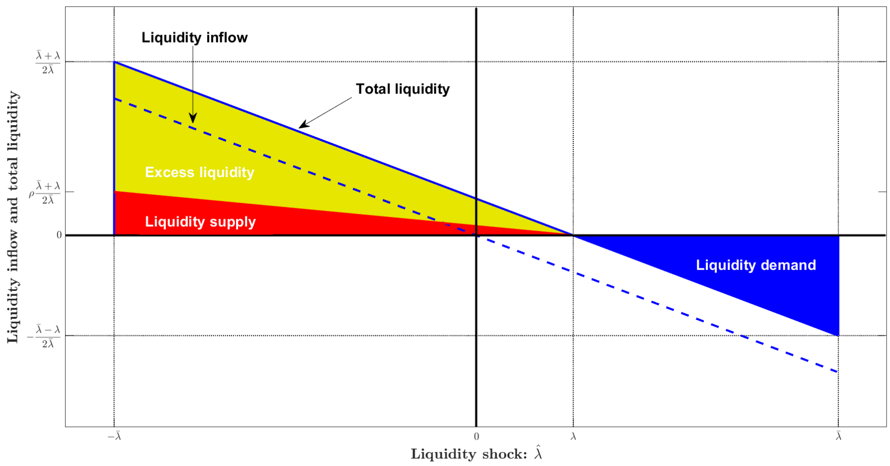

Figure 16: Liquidity demand and supply

In *Figure A.4* the blue triangle represents B_t^b/D_t, the total demand for liquidity per unit of demand deposit on the interbank market. Since the area of the triangle is equal to $\delta_t^2/\left(4\bar{\lambda}_t\right)$,

$$B_t^b = \frac{\delta_t^2}{4\bar{\lambda}_t} D_t. \tag{59}$$

In *Figure A.4* the inflow per unit of demand deposit as a function of $\hat{\lambda}_t$ is represented by the dashed line. However, the total liquidity of potential lenders ($\hat{\lambda}_t < \lambda_t$) is greater than the aggregate liquidity inflow, since they can lend their reserves plus the $\hat{\lambda}_t D_t$. The total liquidity per unit of demand deposit is represented by sum of the yellow and red triangle, its area is equal to $\varsigma_t^2/\left(4\bar{\lambda}_t\right)$. Hence the total excess liquidity (TL_t) is clearly greater than the market demand for liquidity,

$$TL_t = \frac{\varsigma_t^2}{4\bar{\lambda}_t} D_t > \frac{\delta_t^2}{4\bar{\lambda}_t} D_t = B_t^b.$$

We assumed that each lender supplies the same ρ_t fraction of its available liquidity as equation (49) indicates. As a consequence, $\rho_t < 1$. The total liquidity supply is equal to ρ_t times the total liquidity, that is,

$$B_t^l = \rho_t \int_{\lambda_t}^{\bar{\lambda}_t} \left(\hat{\lambda} - \lambda_t\right) D_t \frac{1}{2\bar{\lambda}_t} \mathrm{d}\,\hat{\lambda} = \rho_t TL_t. \tag{60}$$

Now we can express equilibrium condition (57) as

$$\rho_t TL_t + B_t^{CB} = B_t^b.$$

Rearranging it yields

$$\rho_t = \frac{B_t^b - B_t^{CB}}{TL_t} < 1,$$

since $TL_t > B_t^b$ and $0 \leq B_t^{CB} \leq B_t^b$.

The impact of monetary policy on the interbank market

Rearranging equation (50) results in

$$\rho_t = \frac{\Delta_t^B - \Delta_t^R}{2\phi^B}.$$

Combining equations (50) and (51) provides

$$\mu_t = 2\phi^B \rho_t. \tag{61}$$

Then substituting the above expression into equation (54):

$$\Delta_t^R + \frac{\delta_t}{2\bar{\lambda}_t} 2\phi^B \rho_t + \frac{\varsigma_t}{2\bar{\lambda}_t} \phi^B \rho_t^2 = 1 + \tau^F,$$

that is,

$$\frac{\bar{\lambda}_t - \lambda_t}{2\bar{\lambda}_t} 2\phi^B \rho_t + \frac{\bar{\lambda}_t + \lambda_t}{2\bar{\lambda}_t} \phi^B \rho_t^2 = 1 + \tau^F - \Delta_t^R.$$

Rearranging it yields a solution for λ_t,

$$\lambda_t = \bar{m}_t \bar{\lambda}_t,$$

where

$$\bar{m}_t \equiv \frac{\phi^B \rho_t (2 + \rho_t) - 2\left(1 + \tau^F - \Delta_t^R\right)}{\phi^B \rho_t (2 - \rho_t)} > 0.$$

Using the definition of λ_t yields

$$D_t = \frac{M_t}{\lambda_t}.$$

Using formula (61) equation (56) can also be expressed in a simpler way:

$$\Delta_t^D = 1 - \bar{\tau}^D + \phi^{DL} \frac{L_t}{P_t} - \left(2\phi^B \rho_t - \phi^B \rho_t^2\right) \frac{\delta_t \varsigma_t}{4\bar{\lambda}_t}.$$

As shown, M_t, Δ_t^R and Δ_t^B clearly defines ρ_t, λ_t and D_t However, this is also true the other way round:

$$\begin{aligned}
\Delta_t^R &= 1 + \tau^F - \frac{\bar{\lambda}_t - \lambda_t}{2\bar{\lambda}_t} 2\phi^B \rho_t - \frac{\bar{\lambda}_t + \lambda_t}{2\bar{\lambda}_t} \phi^B \rho_t^2, \\
\Delta_t^B &= 2\phi^B \rho_t + \Delta_t^R, \\
M_t &= \lambda_t D_t.
\end{aligned}$$

That is, $(M_t, \Delta_t^R, \Delta_t^B)$ and (ρ_t, λ_t, D_t) mutually unambiguously determine each other. As a consequence, it is possible to represent monetary policy by ρ_t, λ_t and D_t as well.

Implications of the uniform distribution

In this section the closed form expressions are derived for the probabilistic terms in equations (53), (55) and (58) assuming that $\hat{\lambda}_t$ can be described by a uniform distribution. Its cumulative distribution function is

$$\begin{aligned}
G\left(\hat{\lambda}\right) &= \frac{\hat{\lambda} + \bar{\lambda}_t}{2\bar{\lambda}_t}, \text{ if } \hat{\lambda} \in [-\bar{\lambda}_t, \bar{\lambda}_t], \\
&= 0 \text{ if } \hat{\lambda} < -\bar{\lambda}_t, \\
&= 1 \text{ if } \bar{\lambda}_t < \hat{\lambda}.
\end{aligned}$$

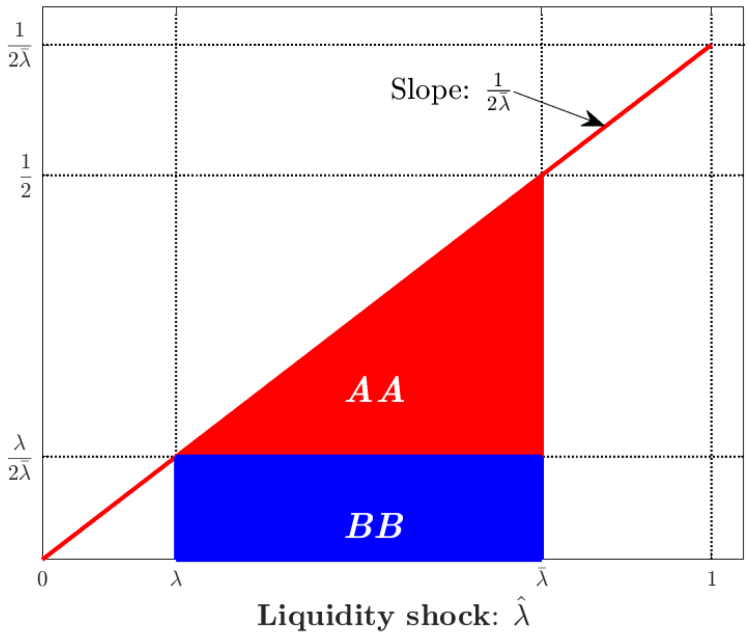

Figure 17: Uniform distribution

The probability density function:

$$\begin{aligned} g\left(\hat{\lambda}\right) &= \frac{1}{2\bar{\lambda}_t}, \text{ if } \hat{\lambda} \in [-\bar{\lambda}_t, \bar{\lambda}_t], \\ &= 0, \text{ otherwise.} \end{aligned}$$

Using the definition of G, the terms in equation (53) become

$$G(\lambda_t) = \frac{\varsigma_t}{2\bar{\lambda}_t}, \qquad 1 - G(\lambda_t) = \frac{\delta_t}{2\bar{\lambda}_t},$$

where $\varsigma_t \equiv \max\left[2\bar{\lambda}, \bar{\lambda}_t + \lambda_t\right]$ and $\delta_t \equiv \max\left[0, \bar{\lambda}_t - \lambda_t\right]$.

The term

$$\int_{\lambda_t}^{\bar{\lambda}_t} \left(\hat{\lambda} - \lambda_t\right) \frac{1}{2\bar{\lambda}_t} \,\mathrm{d}\hat{\lambda}.$$

in equation (58) is equal to the red shaded area AA in *Figure 26*, that is,

$$AA = \frac{1}{2}(\bar{\lambda}_t - \lambda_t)\left(\frac{1}{2} - \frac{\lambda_t}{2\bar{\lambda}_t}\right) = \frac{\delta_t}{4}\left(1 - \frac{\lambda_t}{\bar{\lambda}_t}\right) = \frac{\delta_t^2}{4\bar{\lambda}_t}.$$

Observe that the term

$$\int_{\lambda_t}^{\bar{\lambda}_t} \hat{\lambda} \frac{1}{2\bar{\lambda}_t} \,\mathrm{d}\hat{\lambda}.$$

in equation (55) is equal to the sum of areas AA and BB in *Figure 26*,

$$BB = (\bar{\lambda}_t - \lambda_t)\frac{\lambda_t}{2\bar{\lambda}_t} = \frac{\delta_t \lambda_t}{2\bar{\lambda}_t}.$$

Hence,

$$AA + BB = \frac{\delta_t}{2\bar{\lambda}_t}\left[\frac{\delta_t}{2} + \lambda_t\right] = \frac{\delta_t \varsigma_t}{4\bar{\lambda}_t}.$$

Furthermore, consider the term

$$\int_{-\bar{\lambda}_t}^{\lambda_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda}.$$

in equation (55). First observe that

$$\int_{-\lambda_t}^{0} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda} = -\int_{0}^{\lambda_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda}.$$

As a consequence,

$$\int_{-\bar{\lambda}_t}^{\lambda_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda} = \int_{-\bar{\lambda}_t}^{-\lambda_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda}. = -\int_{\lambda_t}^{\bar{\lambda}_t} \hat{\lambda}\frac{1}{2\bar{\lambda}_t}\,\mathrm{d}\hat{\lambda} = -\frac{\delta_t}{2\bar{\lambda}_t}\left[\frac{\delta_t}{2} + \lambda_t\right] = -\frac{\delta_t \varsigma_t}{4\bar{\lambda}_t}.$$

A.5 The steady state

Suppose that for some ξ, Δ^R and Δ^B, $\xi_t = \xi^0 = \xi$, $\Delta_t^R = \Delta^{R0} = \Delta^R$ and $\Delta_t^B = \Delta^{B0} = \Delta^B$ for all t. Then equations (2), (19), (20), (36), (37), (44), (45), (46), equilibrium condition $y_t = c_t + k_t$, and formula $y_t^h = y_t - (1 + r_{t-1})f_{t-1}^h - (1 + r_{t-1})\Delta_{t-1}^D d_{t-1}$ determine the steady-state values of the following endogenous variables: k, z, n, y, y^h, c, d, w, r, Δ^D (we denote the steady-state values by omitting time indeces).

The above system of equations defines the steady of real variables, the price level and the nominal variables are determined by M set by monetary policy. Δ^R and Δ^B determine the steady state value of λ by equations (14) and (15). Then formula

$$d = \frac{D}{P} = \frac{M/\lambda}{P}.$$

provides the price level.

A.6 Flexible price allocation and expectations

In section (2.1.5) the IS and LM curves are derived to calculate the equilibrium outcome of the economy in the presence of sticky prices. However, variables $\mathcal{Y}_t^T$ and $\mathcal{B}_t^T$ contains expectations for future values of some endogenous variables. This section discusses how the households form their expectations in boundedly rational way.

As discussed, we restrict our analysis to temporary shocks, and it is assumed that after the realization of macroeconomic shocks at date t, from date $t+1$ the economy returns to the flexible price allocation. As a consequence, households expectations are based on the flexible price values of the variables.

To calculate the flexible price allocation at date $t+j$, $j \leq 1$, for a given value of k_{t+j-1}, one has to evaluate equations (2), (19), (20), (36), (37), (44), (45), (46), at date $t+j$, and take in account the equilibrium condition

$$y_{t+j} = c_{t+j} + k_{t+j}.$$

The above eight equations determine the value of the following vector of endogenous variable,

$$\chi_t \equiv [k_t, z_t, n_t, y_t, c_t, d_t, w_t, r_t, \Delta_t^D],$$

if the expectations are known.

Vector χ_t contains real variables, the price level and the nominal variables depends on M_t set by monetary policy. For given Δ^{R0} and Δ^{B0}, equations (14) and (15) determine λ_t. Since $D_t = M_t/\lambda_t$, the price level is given by $P_t = D_t/d_t$.

We can calculate the necessary expectations by backward induction: we start the calculations at the end of forecast horizon, at date $t+T$, then we recursively calculate the allocations and expectations for dates $t+T-1, t+T-2, \ldots, t+1$.

Since households' optimization problem is time consistent, the optimal consumption and saving plan decided at date t coincides with the outcome of optimal decision at date $t+T$. Consider the two-period optimization problem of date T:

$$\max_{\left\{c_{t+j}, n_{t+j}, F^h_{t+j}, D_{t+j}\right\}} \sum_{j=t+T}^{t+T+1} \mathrm{E}_t \left[\Gamma_{t+j} \mathcal{U}\left(c_{t+j}, n_{t+j}, D_{t+j}, \zeta_{t+j}\right)\right],$$

subject to the following budget constraints,

$$\begin{aligned} &c_{t+T} + f^h_{t+T} + d_{t+T} = \bar{y}_{t+T} = y_{t+T}, \\ &c + f^h + d = y^h_{t+T+1} + (1 + r_{t+T+1})\left(f^h_{t+T} + \Delta^D_{t+T} d_{t+T}\right), \end{aligned}$$

and the terminal conditions,

$$f_t^h = f^h, \qquad d_t = d.$$

The present value budget constraint derived for the above problem:

$$c_{t+T} + \left(1 - \Delta^D_{t+T}\right) d_{t+T} = y_{t+T} + \frac{y^h_{t+T+1} - f - d}{1 + r_{t+T}}.$$

As assumed, households' income expectations for date $t+T+1$ is given by formula (1), expressing it in real terms:

$$y^h_{t+T+1} = c + d + f^h - (1 + r_{t+T})\left(f^h_{t+T} + \Delta^D_{t+T} d_{t+T}\right).$$

Equation (23) implies $k = s = f^h + d$ and $k_{t+T} = k_{t+T} = f^h_{t+T} + d_{t+T}$, furthermore, $y = c + k$, as a consequence,

$$y^h_{t+T+1} = y - (1 + r_{t+T})\left[k_{t+T} - \left(1 - \Delta^D_{t+T}\right) d_{t+T}\right].$$

Substituting the Euler equation (43) and the deposit demand equation (45) into the above expression yields

$$c_{t+T} = \frac{y_{t+T} + \mathcal{Y}^0_{t+T}}{\mathcal{B}^0_{t+T}},$$

where

$$\begin{aligned}
\mathcal{Y}^0_{t+T} &= \frac{y - (1 + r_{t+T})\left[k_{t+T} - \left(1 - \Delta^D_{t+T}\right) d_{t+T}\right] - f^h - d}{1 + r_{t+T}}, \\
\mathcal{B}^0_{t+T} &= 1 + \eta\left(1 - \Delta^D_{t+T}\right)^{1-\sigma} + \beta^\sigma (1 + r_{t+T})^{\sigma - 1}.
\end{aligned}$$

For a given value of k_{t+T-1}, combining the above consumption demand formula with equations (36), (37), (2), (19), (20), (44), (45) and equilibrium condition $y_{t+T} = c_{t+T} + k_{t+T}$ provides

$$\chi_{t+T} \equiv [k_{t+T}, z_{t+T}, n_{t+T}, y_{t+T}, c_{t+T}, d_{t+T}, w_{t+T}, r_{t+T}, \Delta^D_{t+T}].$$

Obviously all components of χ_{t+T} are functions of k_{t+T-1}. As a consequence, $\mathcal{Y}^0_{t+T}(k_{t+T-1})$ and $\mathcal{B}^0_{t+T}(k_{t+T-1})$ are also functions of it.

Similarly one can show that household's optimization at date $t+T-1$ yields the following consumption demand:

$$c_{t+T-1} = \frac{y_{t+T-1} + \mathcal{Y}^1_{t+T-1}}{\mathcal{B}^1_{t+T-1}},$$

where

$$
\begin{aligned}
\mathcal{Y}^1_{t+T-1} &= \frac{y_{t+T} - (1+r_{t+T-1})\left[k_{t+T-1} - \left(1-\Delta^D_{t+T-1}\right) d_{t+T-1}\right] + \mathcal{Y}^0_{t+T}(k_{t+T-1})}{1+r_{t+T-1}}, \\
\mathcal{B}^1_{t+T-1} &= 1 + \eta\left(1-\Delta^D_{t+T-1}\right)^{1-\sigma} + \beta^\sigma (1+r_{t+T-1})^{\sigma-1}\mathcal{B}^0_{t+T}(k_{t+T-1}).
\end{aligned}
$$

Again, for given value of k_{t+T-2}, combining the above formula with equations (36), (37), (2), (19), (20), (44), (45) and equilibrium condition $y_{t+T-1} = c_{t+T-1} + k_{t+T-1}$ provides χ_{t+T-1} and $\mathcal{Y}^1_{t+T-1}(k_{t+T-2})$ and $\mathcal{B}^1_{t+T-1}(k_{t+T-2})$.

Following the iteration for any $0 \le j < T$ one can calculate

$$
c_{t+j} = \frac{y_{t+j} + \mathcal{Y}^{T-j}_{t+j}}{\mathcal{B}^{T-j}_{t+j}},
$$

where

$$
\begin{aligned}
\mathcal{Y}^{T-j}_{t+j} &= \frac{y_{t+j+1} - (1+r_{t+j})\left[k_{t+j} - \left(1-\Delta^D_{t+j}\right) d_{t+j}\right] + \mathcal{Y}^{T-j-1}_{t+j+1}(k_{t+j})}{1+r_{t+j}}, \\
\mathcal{B}^{T-j}_{t+j} &= 1 + \eta\left(1-\Delta^D_{t+j}\right)^{1-\sigma} + \beta^\sigma (1+r_{t+j})^{\sigma-1}\mathcal{B}^{T-j-1}_{t+j+1}(k_{t+j}).
\end{aligned}
$$

A.7 Implementation of an interest rate rule

As discussed in *section 4*, if monetary policy wants to implement an interest rate rule, its instruments have to satisfy the following conditions:

$$
\begin{aligned}
y^\star_t &= y^{IS}\left(i^\star_t, D^*_t + D^+_t, \lambda^+_t, \rho^+_t, \xi^{LM*}_t, \xi^{IS*}_t\right), \\
y^\star_t &= y^{LM}\left(i^\star_t, D^*_t + D^+_t, \lambda^+_t, \rho^+_t, \xi^{LM*}_t, \xi^{IS*}_t\right),
\end{aligned}
$$

where

$$
\begin{aligned}
y^\star_t &= y^{ir}\left(D^0_t, \lambda^0_t, \rho^0_t, \xi^{LM0}_t, \xi^{IS*}_t\right), \\
i^\star_t &= i^{ir}\left(D^0_t, \lambda^0_t, \rho^0_t, \xi^{LM0}_t, \xi^{IS*}_t\right),
\end{aligned}
$$

if $\xi^{IS0} \neq \xi^{IS*}$, and

$$
y^\star_t = y^0_t, \qquad i^\star_t = i^0_t,
$$

if $\xi^{IS0} = \xi^{IS*}$.

Observe that λ^+_t and ρ^+_t influences the IS and LM curves only via Ψ_t in equation (20). Slightly changing our notation, express the above two conditions as functions of Ψ^+_t:

$$
\begin{aligned}
y^\star_t &= y^{IS}\left(i^\star_t, D_t + D^+_t, \Psi^+_t, \xi^{LM*}_t, \xi^{IS*}_t\right), \\
y^\star_t &= y^{LM}\left(i^\star_t, D_t + D^+_t, \Psi^+_t, \xi^{LM*}_t, \xi^{IS*}_t\right).
\end{aligned}
$$

The above two equations provide a solution for D_t^+ and Ψ_t^+.

For a given Ψ_t^+ one can find λ_t^+ and ρ_t^+ to satisfy

$$\Psi_t^+ = \phi^B \rho_t^+ (2 - \rho_t^+) \frac{\bar{\lambda}_t^2 - \left(\lambda_t^+\right)^2}{4\bar{\lambda}_t}. \tag{62}$$

Of course, the solution of the above equation is not unique. Monetary policy should choose a pair of $\left(\lambda_t^+, \rho_t^+\right)$ such that

$$0 < \lambda_t^+ \leq \bar{\lambda}, \qquad 0 < \rho_t^+ < 1. \tag{63}$$

Specifically, in *section 4*, the following simple procedure was used: we set $\lambda_t = \lambda$ (the steady-state value) and chose ρ_t^+ to satisfy equation (62). Over the range of shocks investigated, the procedure used always provided solutions that satisfied conditions (63).

As discussed, in addition to the above, one more constraint has been taken into account:

$$i_t^{R+} = \Delta_t^{R+}(1 + i_t^\star) - 1 \geq -0.01, \tag{64}$$

as discussed in *Appendix A.4*,

$$\Delta_t^{R+} = 1 + \tau^F - \frac{\bar{\lambda}_t - \lambda_t^+}{2\bar{\lambda}_t} 2\phi^B \rho_t^+ - \frac{\bar{\lambda}_t + \lambda_t^+}{2\bar{\lambda}_t} \phi^B \left(\rho_t^+\right)^2.$$

When we applied the above procedure condition (64) was binding in the case of shock η. When the constraint was binding instead of setting $\lambda_t = \lambda$, we tried several values of λ_t over a fine grid in the range $(0, \bar{\lambda}_t]$, and chose a pair of $\left(\lambda_t^+, \rho_t^+\right)$ satisfying both conditions (63) and (64). As discussed in *section 4*, there was a certain range of the shock η where it was impossible to find a solution satisfying both constraints.