The Portfolio Channel of Capital Flows and Foreign Exchange Intervention in A Small Open Economy

Carlos Montoro
Banco Central de Reserva (BCRP)*

Marco Ortiz
Universidad del Pacífico (UP) †

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Abstract

In this paper we extend a new Keynesian small open economy model to include risk-averse FX dealers and FX intervention by the monetary authority. The former ingredients generate deviations from the uncovered interest parity (UIP) condition. More precisely, in this setup portfolio decisions of the dealers add endogenously a time variant risk-premium element to the traditional UIP that depends on FX intervention by the central bank and FX orders by foreign investors. We present closed form solutions for optimal FXI and monetary policy. We analyse the effectiveness of different strategies of FX intervention. Our findings are as follows: (i) FX intervention is a useful tool to improve welfare when financial markets are segmented; (ii) monetary and FX intervention policies present strong complementarities; (iii) fundamental shocks cause distortions through the portfolio balance channel and (iv) simple FX intervention rules effectiveness depends on the frequency of shocks hitting the economy. *Key words:* Exchange rate dynamics, Exchange rate intervention, Monetary policy.

JEL Classification: E4, E5, F3, G15.

*The views expressed in this paper are those of the authors and are not necessarily reflective of views at Banco Central de Reserva del Perú.

†Correspondence to: Marco Ortiz, Universidad del Pacífico, Jr. Sanchez Cerro 2050, Jesús María, Lima 11, Perú. Email: ma.ortizs@up.edu.pe. We thank Gerardo Herrera for invaluable research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the institutions they represent. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

1 Introduction

Interventions by central banks in foreign exchange (FX) markets have been common in many countries, and they have become even more frequent after the Great Financial Crisis (GFC), in both emerging market economies and some advanced economies.[1] These interventions have been particularly large during periods of capital inflows, when central banks bought foreign currency to prevent an appreciation of the domestic currency. Also, they have been recurrent during periods of financial stress and capital outflows, when central banks used their reserves to prevent sharp depreciations of their currencies. These FX interventions were sterilised in most cases, enabling central banks to keep short-term interest rates in line with policy rates.

Given the scale of interventions in FX markets by some central banks, it should be important for them to include this factor in their policy analysis frameworks. A variety of questions need to be addressed, such as: How does sterilised intervention affect the transmission mechanism of monetary policy? Which channels are at work? Are there benefits to intervention rules? What should be the optimal monetary policy design in the context of FX intervention? To analyse these questions we need an adequate framework of exchange rate determination in macroeconomic models.

There is substantial empirical evidence that traditional approaches of exchange rate determination (e.g., asset markets) fail to explain exchange rate movements in the short-run.[2] The literature shows that most exchange fluctuations at short- to medium-term horizons are related to order flows - the flow of transactions between market participants - as in the microstructure approach presented by Lyons (2006), and not to macroeconomic variables. However, in most of the models used for monetary policy analysis, the exchange rate is closely linked to macroeconomic fundamentals, as in the uncovered interest rate parity (UIP) condition. Such inconsistency between the model and real exchange rate determination in practice could lead in some cases to incorrect policy prescriptions such as the overestimation of the impact of fundamentals and the corresponding underestimation of the impact of *liquidity* trading. The latter include, *inter alia*, changes to the ownership of domestic currency instruments by non-residents, current account transactions such as trade in good and services, transfers in capital income, remittances, and tourism related flows which are not related to traditional macroeconomic fundamentals.

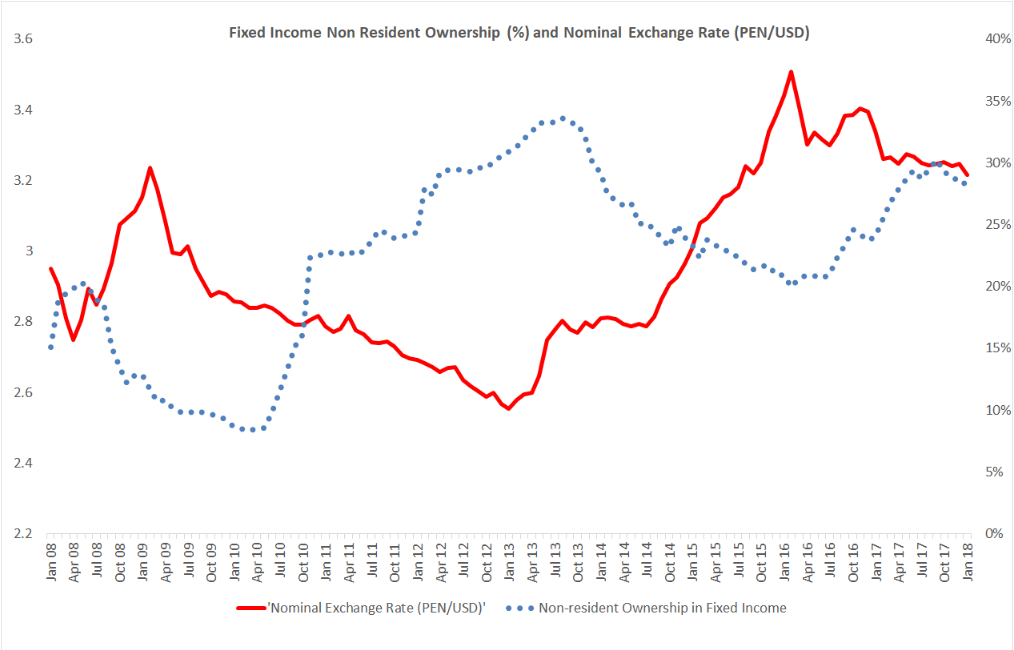
As an example Figure 1 presents the share of ownership of fixed income assets by non-residents for the case of Peru over the last decade. Foreign ownership increases during periods of domestic currency appreciation while the Central Bank intervenes purchasing dollars from the public. Moreover, the increase in foreign ownership has a negative correlation with the 10 year bond yields. These dynamics, namely the positive correlation between the exchange rate and interest rates, constitute a challenge for models in which the exchange rate is determined by the interest rate differential.

We present a model with segmented and incomplete financial markets. The presence of risk adverse financial intermediaries, who act as market-makers and absorb the changes in portfolio positions of the rest of agents, will generate deviations from the uncovered interest parity (UIP) condition. More precisely, dealers' portfolio decisions endogenously add a time-variant exchange rate risk premium element to the traditional UIP that will react to FX interventions by the central bank. In this setup, central bank FX intervention

[1] Domanski et al. (2016) reports that between 2009 and 2014 FX reserves rose from \$ 4 trillion to \$ 7 trillion and since then they decline by \$ 900 million. Notwithstanding significant fluctuations over the years, these shares are significantly higher now than they were a decade ago. Filardo et al. (2011) document how the central banks of Chile and Poland, which were inactive in the FX market for years, decided to resume FX interventions during the 2010-2011 period.

[2] See Meese and Rogoff (1983), Frankel and Rose (1995) and Cogley and Sargent (2005).

Figure 1: Foreign Ownership of Fixed Income Instruments in Peru and Exchange Rate



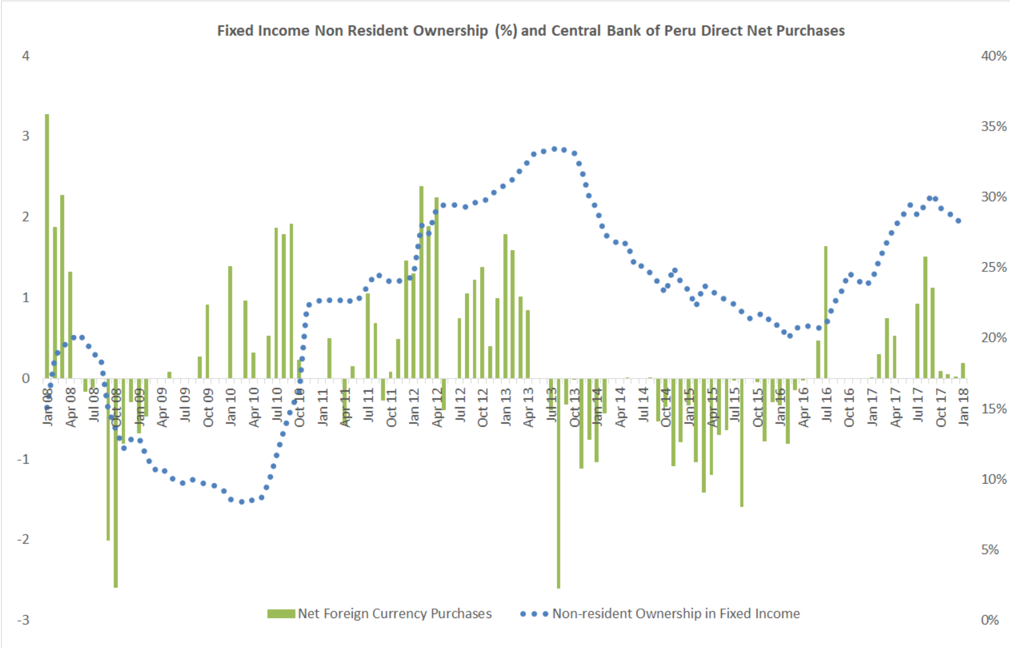
Source: Central Reserve Bank of Peru (BCRP).

can affect exchange rate determination through three channels: (i) the portfolio balance channel, (ii) the expectations channel and (iii) the volatility channel. In the first one, a sterilised intervention alters the value of the currency because it modifies the ratio between domestic and foreign assets held by the private sector; according to the second, the promise of future interventions impacts the current exchange rate. The final channel affects the risk-bearing capacity of financial intermediaries.

Findings. Our results show that FX intervention is a useful policy tool for central banks, particularly in small open economies with incomplete and segmented financial markets. The presence of frictions creates welfare losses that can be mitigated with FX interventions directly targeting them. Additionally, we show how monetary and FX intervention policies can either complement or hinder each other. A careful identification of shocks is key for effectively using both tools. We also compare the optimal policy with a set of policies commonly discussed in the literature. We document that the volatility reduction channel should not be overlooked, as it can improve welfare and reduce the impact of capital flow shocks. Finally we show how this framework relates to the closing small open economy devices proposed in Schmitt-Grohe and Uribe (2003).

Related Literature. Our paper follows the literature of FX determination in general equilibrium with imperfect financial markets. Our closest references are Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017) which show how the role of risk-adverse financiers create a risk-bearing channel that can explain the disconnection and exchange rate determination puzzles. Moreover, these frictions create a role for FX intervention. Cavallino (2019) study the optimal FX intervention policy in a continuous time SOE New Keynesian model, finding how this tool complements monetary policy. Gabaix and Maggiori (2015) and Cavallino (2019) work in an incomplete financial markets setup, in which financial intermediaries face a collateral constraint in the spirit of Kiyotaki and Moore (1997). The limit to their position in foreign currency is linked to a moment of the equilibrium distribution of the exchange rate in an ad-hoc manner. In this case, FX intervention affects the economy by increasing or decreasing the value of financiers collateral.

Figure 2: Foreign Ownership of Fixed Income Instruments in Peru and Exchange Rate



Source: Central Reserve Bank of Peru (BCRP).

In contrast, we emphasize the portfolio balance channel as in Itskhoki and Mukhin (2017), with two key differences. First, we focus in a small open econom. [3] Second, we assume that the equilibrium volatility of the exchange rate is affected by the central bank policy. We call this the 'volatility channel'. Thus, FX intervention will affect the risk-bearing capacity of financial intermediaries.

There are other notable papers studying the role of FX interventions in a DSGE setup. Chang (2019) presents a model in which imperfect substitution across assets denominated in different currencies occurs only when banks face a binding borrowing constraint. In this region of the state-space, FX interventions by the central bank become effective. Fanelli and Straub (2020) introduce convex costs to carry traders, reducing the central bank's incentives to curtail exchange rate volatility. Amador et al. (2020) study the use of exchange rate policies when the economy is at the zero lower bound. Benes et al. (2015) provides a framework for the joint analysis of hybrid inflation targeting (IT) regimes with FX interventions strategies (e.g., exchange rate corridors, pegged or crawling exchange rates, managed floats.), where the central bank can exercise control over the exchange rate as an instrument independent of monetary policy and the policy interest rate.[4] Their strategy consists of introducing imperfect substitutability between central bank securities - used for purposes of sterilization - and private sector bank loans in a model where banks hold local currency denominated assets and foreign currency liabilities. In a related work, which also assumes imperfect substitutability of assets, Vargas et al. (2013) find that sterilised FX interventions can have an effect on credit supply by changing the balance sheet composition of commercial banks.

[3]This is an important distinction since financial account transactions made by small open economies are mostly settled in hard currencies. Goldberg and Tille (2008) and Gopinath et al. (2020) document how the vast majority of international trade transactions is invoiced in hard currencies. Regarding financial instruments Maggiori et al. (2020) documents the pervasive preference of investor holdings for securities in hard currencies. Aizenman et al. (2020) documents how the reduction of the original sin has brough up the risk on/risk off dynamics associated with the dynamics of capital flow highlighted in our model. Thus the higher participation of foreign investors in domestic markets has exposed countries to larger capital outflows.

[4]Chamon et al. (2012) discusses the use of hybrid IT schemes in emerging market economies (EME). Authors recommend the use of a two-instrument IT framework as a way to reinforce its commitment to a low inflation rate.

Regarding the study of optimal FX policy in a micro-founded macroeconomic model our work is close to Cavallino (2019). Other important contributions are made by Fanelli and Straub (2020) and Amador et al. (2020).

Our paper also relates to the literature on the effectiveness of FX intervention. The empirical evidence in this front ranges widely and remains inconclusive. Reviews by Menkhoff (2012) and Chamon et al. (2012) suggest that interventions in some cases have a systematic impact on the rate of change in exchange rates, while in other cases they have been able to reduce exchange rate volatility. Intervention appears to be more effective when it is consistent with monetary policy (Amato et al. (2005), Kamil (2008)). This evidence suggests that the impact of FX interventions depend on the specific episode and instrument used. Clearly, the effectiveness of central bank intervention also needs to be evaluated against its policy goal.

Outline. The paper is organized as follows. Section 2 presents the model. Section 3 discusses the optimal policy under the Linear Quadratic Approach. Section 4 presents an analysis of simple FX intervention rules. Section 5 concludes. All proofs and details of the derivations are left for the appendix.

2 The Model

We present a model for a small open economy with nominal rigidities in line with the contributions from Obstfeld and Rogoff (1995), Chari et al. (2002a), Galí and Monacelli (2005), Christiano et al. (2005) and Devereux et al. (2006), among others. To maintain the concept of general equilibrium, we use a two-country framework taking the size of one of the domestic economy close to zero, such that the small (domestic) economy does not affect the large (foreign) economy.[5] The economy is composed by households, firms, financial intermediaries (FX dealers) and central bank who intervenes in FX markets.

2.1 FX Dealers

There are (a measure) m of symmetric risk-averse dealers who operate in the secondary bond market. Each dealer receives FX purchase and sale orders from households, the central bank and foreign investors, respectively. As in Stoll (1978), intermediaries will use their own portfolio to provide the immediacy service to the rest of the population, acting as market-makers. As in Gabaix and Maggiori (2015), the market will present imbalances, and the net position of dealers will be affected by the orders of the rest of agents. In compensation for providing this service, FX dealers will charge a risk premium priced in the equilibrium exchange rate.

The exchange rate S is defined as the price of foreign currency in terms of domestic currency, such that a decrease (increase) corresponds to an appreciation (depreciation) of the domestic currency. At the end of the period, any profits - either positive or negative- are transferred to the households. Dealers maximize a CARA utility of the return over investments in units of the domestic consumption good:

$$\max_{d^*_{t+1}} \mathbb{E}_t \left\{ -\frac{1}{\omega} \exp\left(-\omega \frac{\tilde{R}^*_{t+1}}{R_t} d^*_{t+1} \right) \right\} \tag{1}$$

[5]We acknowledge the general equilibrium perspective introduces a series of linear relationships among the foreign economy variables. The disadvantage of following this modelling strategy is that shocks to foreign variables will not be observed independently, as only combination of foreign variables will impact the domestic economy. This would not allow us to analyse the impact of shocks to foreign variables independently (and the impact would depend as well on the calibration of the foreign economy.)

The return on intermediaries' strategy is given by:

$$\tilde{R}^*_{t+1} = R^*_t \frac{S_{t+1}}{S_t} - R_t \tag{2}$$

where $\mathbb{E}_t$ is the rational expectations operator, $\omega \geq 0$ is the coefficient of absolute risk aversion and $\tilde{R}^*_{t+1}$ is the peso carry trade return on the portfolio. As in Itskhoki and Mukhin (2017), the open position absorbed by the dealers (d^*) will be an endogenous object as it will be derived from the domestic currency demand from households (via the financial account flows), foreign carry traders and the central bank FX intervention.

FX dealers will quote a price for each equilibrium position they have to absorb. Since trade against all agents occurs simultaneously, the portfolio equation can be utilized to get the exchange rate at which FX dealers are willing to mirror the position of the rest of agents.

Lemma 2.1. *The equilibrium condition in the financial market is given by:*

$$s_t = \mathbb{E}(s_{t+1}) + i^*_t - i_t + \frac{1}{2}\sigma^2 - \frac{\omega}{m}\sigma^2 D^*_{t+1} \tag{3}$$

where $i^*_t - i_t \equiv log(R^*_t/R_t)$, $\sigma^2 \equiv var_t(\Delta s_{t+1})$, *which is the volatility of the log nominal exchange rate, and* D^*_{t+1} *is the aggregate position of FX dealers.*

Notice that this equation holds exactly, without the need to approximate it around the non-stochastic steady-state.[6]

The problem with approximating (3) around the non-stochastic steady state is that variances is dropped out and the portfolio channel vanishes. To work around this problem the literature has followed two different avenues. The first one, present in Gabaix and Maggiori (2015) and Cavallino (2019) is to assume that the incentive compatibility constraint depends on a moment of the standard deviation of the exchange rate.[7]

This way the portfolio channel is preserved. Nonetheless, the risk bearing capacity of intermediaries remains exogenous.

The second avenue is presented in Itskhoki and Mukhin (2017) by assuming that as $\sigma^2 \downarrow 0$, ω increases proportionally ($\omega \to \infty$, leaving a risk premium term in the limit, and allowing for changes in financial intermediaries portfolios to affect the first order dynamics of the exchange rate.[8] By taking this limit, the impact of capital flows on the exchange rate will remain independent of FX intervention policy.

We assume that domestic and foreign households have portfolios composed entirely of assets in their respective currencies, while foreign demand for local currency bonds is left to a group of noise-traders who have an exogenous demand for domestic currency. Their position is given by:

$$N^*_{t+1} = n(e^{\psi_t} - 1) \tag{5}$$

where $\psi_t = \rho_\psi \psi_{t-1} + \sigma_\psi \varepsilon^\psi_t$; $\varepsilon^\psi_t \sim iid(0,1)$ and n represents the mass of noise traders. We assume noise traders follow a zero capital strategy.

These assumptions allow us to capture some of the stylized facts documented for small open economies:[9]

[6] To apply Itô's Lemma, the approximation point requires that the sum of period holdings returns forms a martingale and that the variance of returns are bounded.

[7] This is equivalent to write the modified UIP as:

$$s_t = E_t s_{t+1} + \tilde{i}^*_t - i_t - \frac{\omega}{m}\sigma^\nu \left(D^*_{t+1}\right) \tag{4}$$

and then take the limit as $\nu \downarrow 0$.

[8] Itskhoki and Mukhin (2017) relates this analysis to models with ambiguity aversion as in Hansen and Sargent (2011) and Hansen and Miao (2018)

[9] See Amador et al. (2020), Gopinath et al. (2020), Aizenman et al. (2020) and Maggiori et al. (2020)

- A large share of trade is invoiced in foreign currency.
- A large share of financial account transactions is denominated in foreign currency.
- The share of instruments denominated in local currency, create a 'risk-off' and 'risk-in' channel. Capital flow shocks are not necessarily driven by macroeconomic fundamentals.

2.2 FX Intervention

We model a central bank or government authority that intervenes in FX markets. The central bank will take an open position in foreign currency assets off-setting the position of FX intermediaries.

$$\Gamma_t^{cb} = \left(S_t R_{t-1}^* - S_{t-1} R_{t-1}\right) B_t^{cb,*}$$

where $B^{cb,*}$ represents the central bank's foreign currency position and B^{cb} are bonds issued (deposit certificates) for sterilization purposes. Profits and losses made by the central bank are transferred on next period to the households in the form of domestic bonds. The central bank operates in the same amount with all financial intermediaries. Central bank profits will have a return differential component and a valuation component. We assume all interventions are sterilized, thus $B_{t+1}^{cb} + S_t B_{t+1}^{cb,*} = 0$.

The rest of the model describes a standard small open economy model. See section A in the Appendix for details.

2.3 Central Bank

The central bank issues the domestic bonds and sets the nominal interest rates paid by these assets. The central bank can control the interest rate regardless of the FX intervention, that is we assume the central bank can always perform fully sterilised interventions.[10]

For simplicity, we assume the central bank follows a zero capital strategy:

$$B_{t+1}^{cb} + S_t B_{t+1}^{cb,*} = 0$$

where $B_t^{cb,*}$ represents the central bank's foreign reserves and B_t^{cb} are bond issued by the central bank (deposit certificates) used for sterilized FX interventions. . The Central Bank flow constraint is given by:

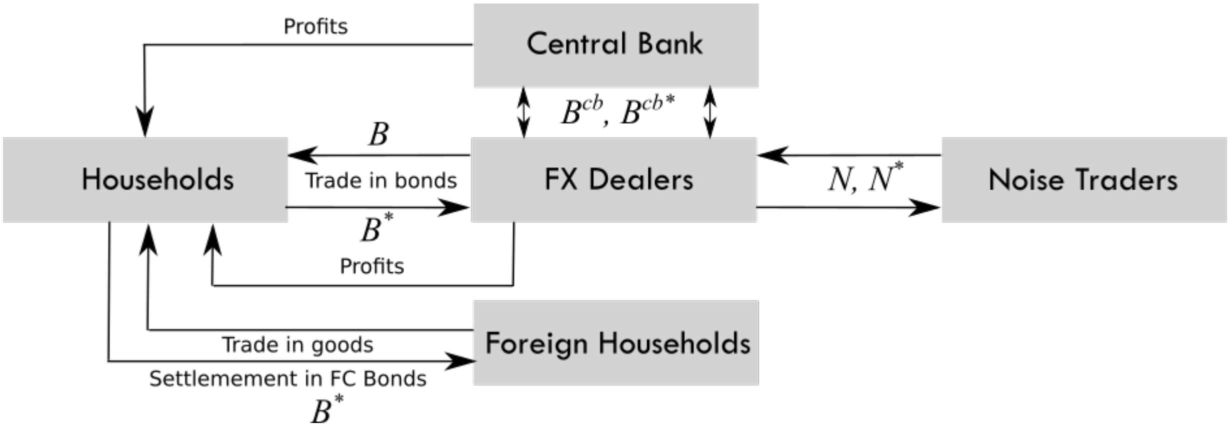
$$B_{t+1}^{cb} + S_{t+1} B_{t+1}^{cb,*} + P_t \Gamma_t^{cb} = (1 + i_t) B_t^{cb} + (1 + i_t^*) S_{t+1} B_t^{cb,*}$$

where Γ_t^{cb} are the Central Bank's transfers to the population. Central Bank transfers are given back to households in the form of domestic bonds to keep the Central Bank's net worth bounded.

Thus, the evolution of Central Bank asset composition will be a function of asset returns and sterilized intervention. When the Central Bank sells reserves it will do it against domestic bonds. Regarding monetary policy, we assume the central bank sets the interest rate to maximize welfare, conditional on a FX intervention policy.

[10] However, in practice sterilised interventions have limits. For example, the sale of foreign bonds by the central bank is limited by the level of foreign reserves. On the other hand, the sterilised purchase of foreign currency is limited by the availability of instruments to sterilise those purchases (e.g., given by the demand for central bank bonds or by the stock of treasury bills in the hands of the central bank). Also, limits to the financial losses generated by FX intervention can represent a constraint for intervention itself.

Figure 3: Basic Structure of the Model



Note: Figure describes the agents and the structure of markets in the baseline model.

2.4 Bonds Market clearing

The domestic bonds positions of households, noise traders, the central bank and intermediaries nets out:

$$B_{t+1} + N_{t+1} + D_{t+1} + B^{cb}_{t+1} = 0 \tag{6}$$

since noise traders hold zero-capital positions we obtain:

$$B_{t+1} + D_{t+1} + B^{cb}_{t+1} = S_t N^*_{t+1} \tag{7}$$

Equation (7) shows how the only way domestic agents (households, central bank and intermediaries) can hold a positive (negative) external position on domestic currency bonds is if carry traders hold a positive (negative) position of foreign currency bonds relative to the steady state. Although this looks like a strong assumption, in reality it constitutes a slight modification from the small open economy benchmark where it is normally assumed that all current account transactions are settled in foreign currency. Now the presence of non-resident investors in peso markets make the small open economy subject to capital inflows or outflows associated with the changing conditions of international markets.

The domestic economy as a whole will be able to finance their current account using both foreign currency assets and the exogenous demand of domestic currency assets coming from foreign carry traders. The remaining changes in their asset positions stemming from current account transactions are settled in foreign currency. We consider this is an improvement on the benchmark setup.

Now we can characterize the portfolio channel of capital flows. When carry traders increase their demand for domestic currency bonds, they will do it by selling foreign currency bonds to either financial intermediaries or the central bank, since we assume that domestic households do not hold foreign currency bonds. If the central bank does not intervene, the foreign currency holdings of financial intermediaries increase, and from equation (3), this generates a currency appreciation. Intermediaries will only hold this position if the risk premium increase to compensate them for their long position in foreign currency. Notice that the central bank can intervene against financial intermediaries, changing their portfolio and affecting the equilibrium exchange rate or offset shocks to carry traders' demand for local currency bonds.

A key aspect of the model is that dealers do not consider the Central Bank's portfolio as part of their own. Therefore agents will not undo the shifts in the Central Bank portfolio, a result present in a canonical

Modigliani-Miller model.[11] Dealers will demand a premium for shifts in the exposure generated by central bank FX interventions.[12]

Figure 4 shows the reaction of the exchange rate, current account and the position of FX dealers to a negative portfolio flow shock. Foreign noise traders change their portfolio away from local currency, this transaction will happen against the foreign currency holdings of FX dealers. In this transaction they will demand a compensation by selling FX at a higher exchange rate. The depreciation triggers an improvement in net exports, which replenishes the FX dealers' position.

As discussed by seminal contributions of Mendoza (1991) and Schmitt-Grohe and Uribe (2003), the equilibrium of the canonical small open economy model solved by local methods exhibit a random walk component. In particular, the marginal utility of wealth will follow a unit root process. For this reason, the literature considers a number of "closing devices". A property of this model is that the portfolio balance channel creates and endogenous risk premium that stabilizes the net foreign assets around its steady state level.

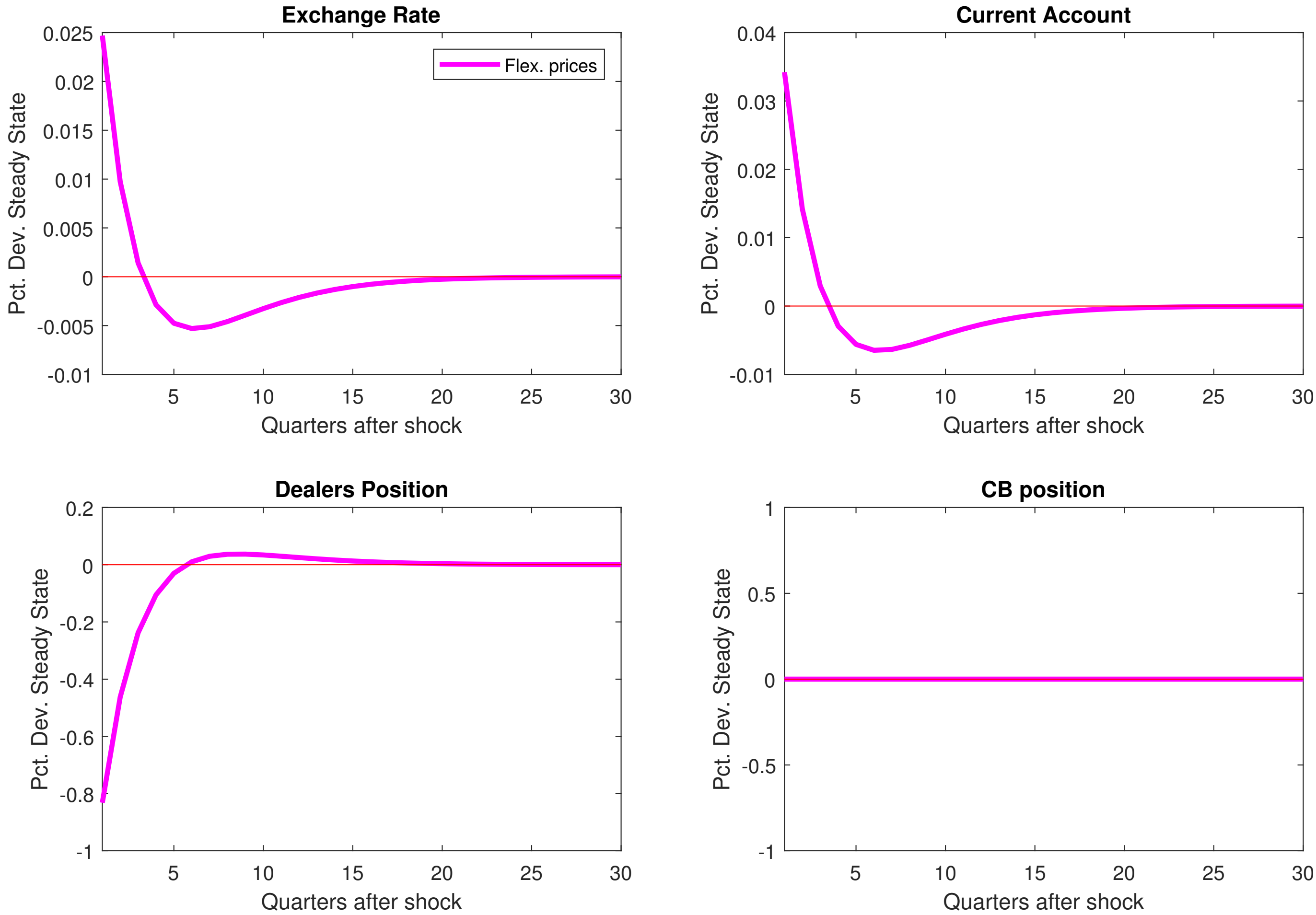
Proposition 2.1. *The endogenous risk premium that results form the FX dealers' problem makes the model equilibrium stationary.*

Proof: See C in the appendix.

[11]This result is related to a violation of the Ricardian equivalence. It is possible to understand it as an extension from the government wealth to the composition of the government's portfolio.

[12]A weaker restriction can be introduced into the model by assuming agents do not have full information regarding the portfolio held by the Central Bank. Thus, when the Central Bank takes dollars from domestic agents, agents are unable to offset the changes in their portfolio. For example, the Central Reserve Bank of Peru only reports the total value of their assets but not the specific way in which these assets are invested. This can be motivated by political constraints, as reporting where the Central Bank assets are invested could generate pressure by politicians to try to divert funds to specific assets in which they have a vested interest.

Figure 4: Reaction to a 1% portfolio shock



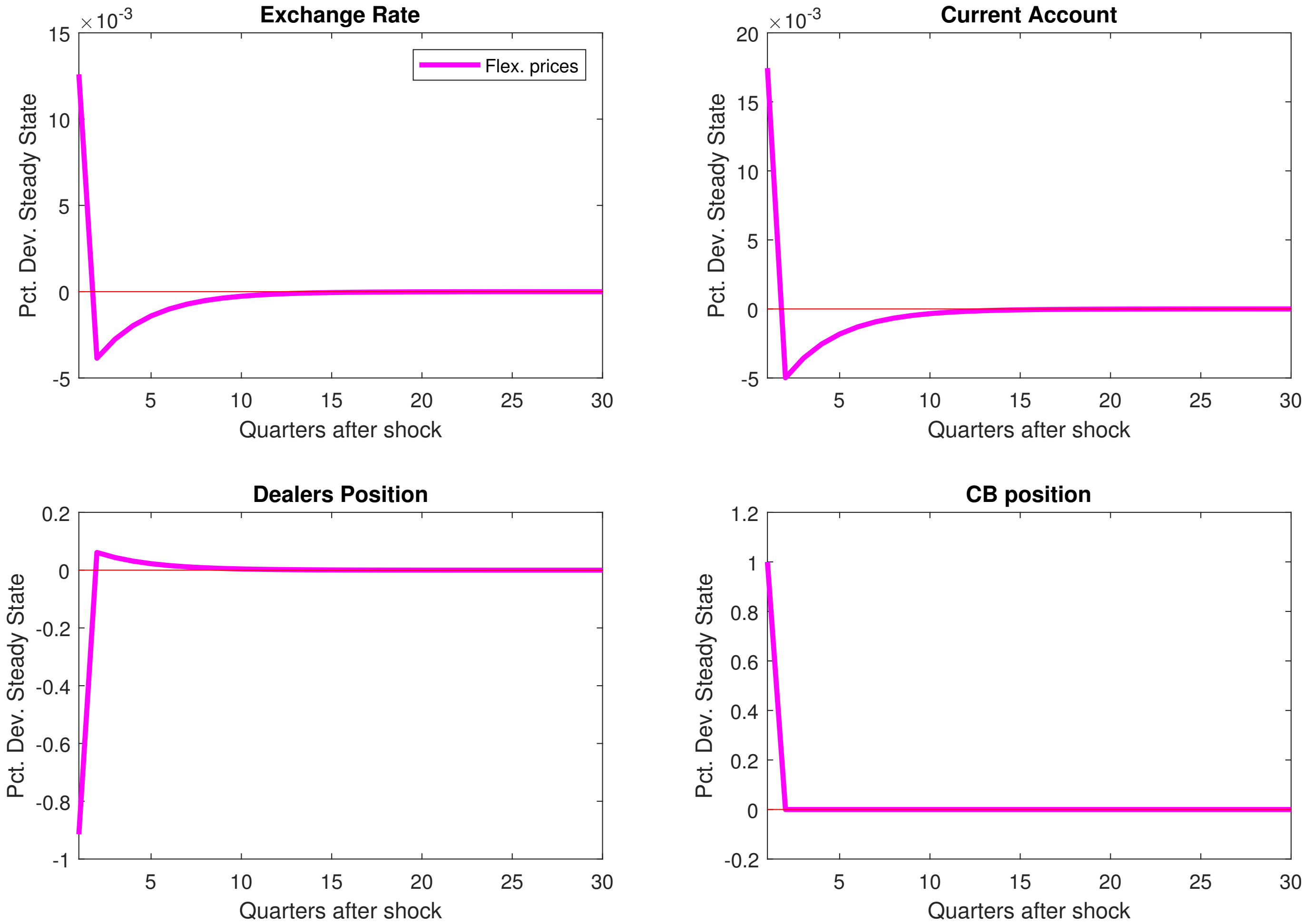
Source: Central Reserve Bank of Peru (BCRP).

The effectiveness of FX intervention opens the question of the optimal use of this instrument. In the next section we discuss the optimal FX policy.

3 Optimal Policy under the Linear Quadratic Approach

In this section we perform optimal policy exercises. As Benigno and Woodford (2012) discuss, the linear-quadratic approximation helps to gain insight into fully optimal policy. However, in order to obtain a closed solution, we use some simplifying assumptions. In particular, following Galí and Monacelli (2005), for the case of optimal monetary policy in a small open economy, and Cavallino (2019), for the case of foreign exchange interventions, we use a logarithmic utility in consumption and a unit elasticity of substitution parameter between H and F goods. We call this special case, the Cole-Obstfeld economy. Additionally, the linear-quadratic approximation will only provide the best rule for linear dynamics of the model. For these reasons we complement our findings in this section with optimal simple rules exercises, which will allow for a wider range of parameters and fully capture the second order dynamics of the economy.

Figure 5: Reaction to a 1% FX intervention shock.



3.1 Optimal FXI with Flexible Prices

In this section we discuss the optimal FXI policy following the linear-quadratic approach when prices are flexible. Following De Paoli (2009), the optimal policy will attempt to replicate the complete international financial markets. Following the literature we define the Backus-Smith wedge as:[13]

$$\Lambda_t = \frac{C_t}{Q_t C_t^*} \tag{8}$$

In section D.2 in the appendix we show that the loss function under flexible prices is given by:

$$\mathbb{W} \approx -\sum_{t=0}^{\infty} \beta^t U_t = -\frac{1}{2}(\Phi_1 + \Phi_2) \sum_{t=0}^{\infty} \beta^t \lambda_t^2 + \mathcal{O}(\|a\|^3) + t.i.p. \tag{9}$$

Where $\lambda_t \equiv \hat{\Lambda}_t$ represents the consumption wedge in deviations from its steady state value.

Identity (9) shows that welfare losses are a function of the quadratic sum of the consumption wedge terms. The next proposition characterizes the solution to the problem.

[13] Backus et al. (1992) showed that international capital markets completeness predicts a close correlation in consumption growth across countries. Backus and Smith (1993) found that when differences across consumption baskets are included, the ratio of marginal utilities should be corrected by relative prices, represented by the real exchange rate. Complete risk sharing should consider an increase in consumption growth in countries experiences relative drops in the real price of their consumption basket. The deviations from the Backus-Smith condition has motivated a vast empirical and theoretical literature. See Hess and Shin (2010), Benigno and Thoenissen (2005), Engel (1999), Chari et al. (2002b) and Tuesta and Selaive (2004).

Proposition 3.1. *The solution to the problem:*

$$\max \frac{1}{2}\mathbb{W} = -E_0(\Phi_1 + \Phi_2)\beta^t \sum_{t=0}^{\infty} \lambda_t^2 + \mathcal{O}(\|a\|^3) + t.i.p. \tag{10}$$

subject to:

$$b_{t+1} - \frac{1}{\beta} b_t = -(1-\gamma)\lambda_t \tag{11}$$

and:

$$\lambda_{t+1} - \lambda_t = \frac{\omega\sigma_e^2}{m}\left(-\bar{Y}b_{t+1} - \bar{Y}b_{t+1}^{cb} - n\psi_t\right) \tag{12}$$

Consists in setting:

$$b_{t+1}^{cb} = -\bar{Y}b_{t+1} + n\psi_t \tag{13}$$

Proof: *Appendix D.2.*

By offsetting the open position absorbed by the financial intermediaries the central bank restores the optimal allocation. The optimal allocation involves a constant level of labour, and a consumption that depends on foreign demand and productivity shocks.[14]

When portfolio shocks dominate the dynamics of the exchange rate, the central bank will exhibit a leaning-against-the-wind behaviour, though the policy does not focus in the exchange rate but in the inefficiencies caused by the market segmentation and limited risk absorption of intermediaries. The optimal rule under the LQ approximation will force the central bank to identify the 'non-fundamental' shocks in order to react in an optimal manner.[15]

3.2 Optimal Monetary and FX policies with Price Rigidities

In this section we study how the frictions introduced in the foreign exchange market interact with the objectives of a central bank under price rigidities. In section D.3 in the appendix we show that the loss function under sticky prices is given by:

$$\mathbb{W} \equiv -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left(\phi_\lambda \lambda_t^2 + \phi_\pi (\pi_t^H)^2 + \phi_y (y_t^H)^2\right) \tag{14}$$

where:

$$\phi_\lambda = \gamma(3(1-\gamma) + \gamma^2) \tag{15}$$

$$\phi_\pi = (1-\gamma)\varepsilon\frac{\theta}{(1-\beta\theta)(1-\theta)} \tag{16}$$

$$\phi_y = (1-\gamma)(1+\chi) \tag{17}$$

[14] The solution in (13) differs from Cavallino (2019) in two key aspects. First, we assume FX dealers are owned by domestic households. This means that wealth transfers created by FX intervention are offset with central bank profits and losses. Thus, the budget constraint of the households will be unaffected. This assumption is critical, since the welfare approximation entails a second order approximation to the households budget constraint. If transfers are made to foreign households, FX interventions will create shocks to households wealth, entailing a cost and becoming an additional term in the welfare approximation. The second difference stems from the fact that in Cavallino (2019) the portfolio of domestic households also varies with changes in the return dynamics. Thus FX intervention will affect the demand of domestic households for assets. Through these mechanisms FX intervention generate distortions in the economy that the central bank must consider when intervening.

[15] See Vitale (2006) and Vitale (2011) for a discussion of fundamental and non-fundamental shocks. Montoro and Ortiz (2020) present a setup with heterogeneous information and an uninformed central bank. The authors show that when the central bank affects the equilibrium exchange rate volatility, can change the signal extraction problem and reduce the deviations of the exchange rate from its fundamentals.

The last expressions show how the central bank will try to close the three gaps in the economy: the price dispersion gap, the growth of the gap and the Backus-Smith gap. The following proposition characterizes the solution to the central bank problem.

Proposition 3.2. *The solution to the problem:*

$$\max \mathbb{W} \equiv -\frac{1}{2}\sum_{t=0}^{\infty} \beta^t \left(\phi_\lambda \lambda_t^2 + \phi_\pi (\pi_t^H)^2 + \phi_y (y_t^H)^2\right) \tag{18}$$

subject to:

$$b_{t+1} - \frac{1}{\beta} b_t = -(1-\gamma)\lambda_t$$
$$y_{H,t} = (\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*})(\frac{\omega\sigma_e^2}{m}(-n_{t+1} - b_{t+1}^{cb} - b_{t+1})) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi_{t+1})$$
$$\pi_t^H = \beta E_t\left(\pi_{t+1}^H\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(-v + (1+\chi)y_{H,t} + (1-\gamma)\lambda_t - a_t\right)$$

is given by the the optimal FX intervention:

$$b_{t+1}^{cb} = \frac{m}{\omega\sigma_e^2}\phi_\pi(E_t\pi_{t+1}^H) + n\psi_t - b_{t+1} \tag{19}$$

and the monetary policy:

$$i_t = (1 - \kappa\gamma\frac{\phi_\pi}{\alpha})E_t\pi_{t+1}^H + \rho \tag{20}$$

where ρ is the steady state value of the domestic interest rate.
Proof: *Apprendix D.3.*

The result shows an interesting interaction between monetary and FXI policies. When the FXI policy is optimal, the central bank can use its monetary instrument to focus in the domestic front, however, the shocks that deviate prices from its optimal level will affect the Backus Smith wedge, and the FXI policy will have to correct the relative price distortion between domestic and foreign goods. Now the FXI policy reacts to porfolio shocks and domestic shocks that affect relative prices.

4 Optimal Simple Rules

In order to build intuition for the optimal policy outcomes, we compare them to those obtained under simple policy rules. These include simple FXI rules that react to the exchange rate, the real exchange rate and portfolio shocks respectively.

Following Galí and Monacelli (2005), we evaluate each policy by calculating the welfare loss, expressed in terms of the proportion of each period's consumption that a typical household in the home economy would need to give up in a deterministic world so that its welfare is equal to the expected conditional utility in the stochastic case. More precisely, we calculate ω_c that satisfies the following equation.

$$\mathbb{E}_t\left[\sum_{t=0}^{\infty}\beta^t \ln C_t - \frac{L_t^{1+\chi}}{1+\chi}\right] = \frac{1}{1-\beta}\left[\ln\left(1 - \frac{\omega_c}{10000}\right)C - \frac{L^{1+\chi}}{1+\chi}\right] \tag{21}$$

where variables without time subscripts denote their respective steady state values. We consider the measures for conditional and unconditional welfare.[16]

By contrasting these rules with the optimal linear-quadratic plans we can observe the welfare losses from following different rules and how welfare changes in more general setups.

4.1 FXI rules

We present three simple rules that are commonly discussed among policy makers. The first one takes into account the changes in the exchange rate, constituting a pure 'leaning-against-the-wind' (LAW) strategy.

$$B_{t+1}^{cb,*} = -\phi_s (S_t - S_{t-1}) \tag{22}$$

According to this rule, when the economy faces depreciation (appreciation) pressures on the domestic currency, the central bank sells (purchases) foreign bonds to prevent the exchange rate from fluctuating.

Under the second rule, the central bank reacts to misalignments of the real exchange rate to its steady state value. We call this strategy the 'real-exchange-rate stabilization' rule (RER-ST).

$$B_{t+1}^{cb,*} = -\phi_q Q_t \tag{23}$$

Finally we present a third rule, in which the central bank reacts directly to the noise shocks. We call this the 'portfolio-flows' rule (PF).

$$B_{t+1}^{cb,*} = -\phi_{n^*} N_{t+1}^* \tag{24}$$

In order to set the values of these rules we use the optimal simple rules command in Dynare to make a search for the parameter that maximizes welfare. In this manner we consider the maximum welfare the central bank can achieve following a particular family of rules. We perform a robustness exercise to understand how the effectiveness of these rules depends on the structure of the economy.

Calibration. Before presenting the results of the quantitative analysis, we discuss the calibration of the model. To be consistent with the special case for which the optimal policy is calculated we set $\gamma_c = 1$, to follow the Cole-Obstfeld case for which welfare measures were calculated.

Instead of calibrating the parameters to a particular economy, we set the parameters to values that are standard in the new open economy literature, as shown in Table 1. The discount factor β is fixed at 0.9975, which implies a real interest rate of 1% in the steady state. The labour supply elasticity is set at 1, within the values found in empirical studies.[17] The value for the elasticity of substitution between home and foreign goods is a controversial parameter. We follow previous studies in the DSGE literature, which consider values between 0.75 and 1.5.[18] The share of domestic tradable goods in the CPI is set to 0.85, implying a participation of imported final goods of 0.15 in the domestic CPI, in line with other studies for small open economies.[19] Regarding price stickiness, the assumption implies that firms keep their prices fixed for 4 quarters on average.

The coefficient of absolute risk aversion for dealers was set to 500 as in Bacchetta and Wincoop (2006). Finally, The standard deviation of all exogenous processes was set to 0.01 and the auto-correlation coefficient is set to 0.5 for the TFP and 0.7 for the portfolio capital flows.

[16] For the unconditional measure we calculate the constant that makes the measure equivalent to the unconditional and conditional ergodic means of the calculated welfare variable, starting from the steady state.

[17] See Chetty et al. (2011).

[18] See Rabanal and Tuesta (2010). Other authors in the trade literature find values for this elasticity around 5, see Lai and Trefler (2002).

[19] See Castillo et al. (2009).

Table 1: Baseline Calibration

Parameter	Value	Description
β	0.995	Consumers' time-preference parameter
χ	1	Labour supply elasticity
γ_c	1	Risk aversion parameter
$\varepsilon_H, \varepsilon_F$	1	Elast. of subst. btw. home and foreign goods
γ, γ^*	0.85	Share of domestic goods in consumption
θ_H	[0 , 0.5]	Domestic goods price rigidity
m	1	Size of FX dealer market
n	1	Size of Noise traders
ω	500	Absolute risk aversion parameter (dealers)
σ_A	0.03	S.D. of TFP shock
σ_ψ	0.01	S.D. of Portfolio shock

Note: We implicitly fix $\sigma = 1$ to keep the portfolio channel when uncertainty goes to zero. For a discussion see Itskhoki and Mukhin (2017).

Table 2: Standard Dev. and Welfare under Flex. Prices
$\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1, \theta_H = 0, \sigma_\psi = 0.01$

	No FXI	*OSR PF*	*OSR RER*	*OSR LAW*
$\sigma(Y)$	3.06	3.00	2.83	2.88
$\sigma(C)$	3.37	2.55	3.20	3.03
$\sigma(L)$	0.58	0.00	0.19	0.15
$\sigma(\lambda)$	9.74	0.00	3.11	2.53
$\sigma(Q)$	7.95	2.55	1.07	1.49
$Wunc.$	22.47	3.82	5.73	5.08
$Wcond$	22.31	3.81	5.70	5.06
$Rule\ Coef.$	0.00	1.00	1.12	0.82

Table 3: Standard Dev. and Welfare under Flex. Prices
$\gamma_c = 1.5, \varepsilon_H = \varepsilon_F = 3, \theta_H = 0, \sigma_\psi = 0.01$

	No FXI	*OSR PF*	*OSR RER*	*OSR LAW*
$\sigma(Y)$	2.63	2.52	2.40	2.43
$\sigma(C)$	2.60	2.43	2.61	2.56
$\sigma(L)$	0.89	0.48	0.66	0.62
$\sigma(\lambda)$	3.19	1.97	2.54	2.39
$\sigma(Q)$	1.64	0.46	0.57	0.52
$Wunc.$	9.32	6.09	7.47	7.08
$Wcond$	9.26	6.06	7.43	7.04
$Rule\ Coef.$	0.00	1.00	1.25	1.18

The results obtained are in line with the closed form optimal policy found under flexible prices. A full reaction to portfolio shocks obtains the stabilization of labour and controls the Backus-Smith wedge.
When households make use of the financial account to smooth their consumption, these capital flows enter the economy and affect the Backus-Smith wedge. Through the reaction to portfolio shocks the central bank is not capable to fully correct the distortions from segmented capital markets.
When we increase the elasticity of substitution between H and F goods, the productivity shocks will now create larger deviations in trade flows. The flows in the financial account affect the exchange rate, generating an inefficient towards the equilibrium. As such, rules that react to both TFP and portfolio shocks now deliver a welfare result closer to the portfolio flows rule.

4.1.1 Robustness - Flexible Prices

Now we study how the results found in the previous section change with the parameters of the model.
Frequency of Shocks: We normalize the sum of variances of the TFP and portfolio shocks and change their relative weight. As the importance of capital flow shocks increase, the central bank obtains a higher welfare gain with the rules that respond to the exchange rate. The reason is that when the dynamics of the exchange rate are dominated by capital flow shocks, the central bank does a better job reacting to the variable. At the other corner, when the dynamics of the economy is purely dominated by productivity shocks, the best response by the central bank is no FX intervention. Obtaining a zero parameter in the optimal rules.
Home bias: We run the model for different values of the home bias parameter. We find that as the economy becomes more closed, the welfare of the economy in the No FXI case increases. The reason is that FXI is more important the more open the economy is. In terms of the ranking of FXI rules, the portfolio shocks rules makes a better job. The reason is that distortions to the exchange rate generate a higher welfare loss when the relative price of foreign goods is more important in the households' basket.
FX Dealers Risk Aversion/Capacity: As FX dealers demand a higher compensation for absorbing and open position, the welfare of the economy worses. Since in the Cole-Obstfeld case the central bank can fully control the Backus-Smith wedge by reacting to offset portfolio shocks, this rule delivers the same results. In the case of the leaning agaist the wind, as the relative importance of portfolio shocks increases, reacting to changes in the exchange rate improves welfare relatively more.
Elasticity of Intertemporal Substitution: As the elasticity of intertemporal substitution decreases, shocks will affect households in a stronger manner. Also, households will be more prone to smooth consumption. Since this involves using the capital account, the portfolio channel will affect the economy in a stronger manner.

4.2 FXI Optimal Simple Rules with Sticky Prices

Now we introduce sticky prices to study how the results in the previous section change. We first calculate the results with the Cole-Obstfeld parameterization ($\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1$). We use the optimal simple rules to calculate the optimal parameter in the Taylor rule responding to PPI and CPI inflation and contrast the role of FXI reacting to portfolio shocks. Figures 14 and 15 present the results.

Under Calvo pricing the technology shock creates deviations in the consumption wedge. The central bank with FX intervention and monetary policy is capable of reduce the volatility. As in Galí and Monacelli (2005), the monetary policy rule reacting to PPI inflation is superior to the CPI inflation rule. Although the FXI rule stabilizes the economy, which is reflected in the reduced values for standard deviations, the no FX intervention generates higher welfare than the one in the steady state. The reason for this is related to the convexity in the production and terms of trade functions. A higher volatility of shocks generate an increase in the mean due to the second order terms. In our case the portfolio shock increases the average terms of trade and output, with a positive effect on households' welfare. As we turn the demand for foreign and home goods more elastic, this effect matters less and reacting to portfolio shocks brings a higher welfare relative to the other FX intervention strategies considered. (See Figures 18 and 19).

Tables **??** shows how the distortions stemming from the portfolio reduce welfare. Table **??**, compares the results under different intervention strategies. Although identifying and offsetting portfolio shocks is achieves the highest welfare, leaning against the wind improves welfare relative to the no FXI policy. In fact, the ordering across FX intervention strategies is a function of the frequency of shocks. When the dynamics of the exchange rate are driven by portfolio shocks, FXI rules that react to the exchange rate do a good job. However, as fundamental shocks, represented by the TFP shock, are more frequent, reacting to portfolio shocks dominate the other two strategies. (See Figures 20 and 21).

Regarding the impact of price stickyness, Figure 25 shows that it has a limited impact on the welfare. Thus, the FXI rules allows for monetary policy to tackle the domestic gap. Since we have only included TFP and portfolio shocks in our setup, the optimal PPI Taylor rule parameter reacts to the degree of price stickyness, leaving the overall welfare almost unaltered.

5 Conclusions

In this paper, we present a model to analyse the interaction between monetary policy and FX intervention by central banks, which also includes capital flow shocks in the determination of the exchange rate. We introduce a portfolio decision of risk-averse dealers, which adds an endogenous risk premium to the traditional uncovered interest rate condition. In this model, FX intervention affects the exchange rate through a portfolio-balance channel.

Our results illustrate that FX intervention has strong interactions with monetary policy as they complement each other. We complement closed forms for optimal FX and monetary policy with a set of optimal simple rules that allow for a more general setup. Here, the central bank optimal strategy will involve reacting to fundamental shocks as the financial account will affect the path of the exchange rate through the portfolio channel.

As it is frequently the case, the effectiveness of the central bank will depend on the nature of the shock and its ability to identify it in a timely manner. Likewise, rules reacting to the exchange rate, without taking into consideration the shock behind its movement, will be more effective when the prevalent shocks are financial ones; while in economies where the contrary occurs, FX intervention will be ineffective. Thus, exchange rate stabilization policies would be more effective in small open economies (lower home bias) subject to large portfolio shocks (relative to TFP shocks) and with a lower degree of financial development (lower risk-bearing capacity).

In terms of policy, the design of a FX intervention policy needs to consider more information than previously thought and relying on broad recipes for exchange rate stabilization could generate more instability, the exact opposite result to the one intended with the policy.

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Appendices

A Rest of the model

Households. There is a continuum of households who derive utility from consumption and leisure. Because of the segmented financial markets assumption, households will only consider the domestic interest rate in their maximization problem. The representative household maximizes:

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma_c}}{1-\gamma_c} - \frac{L_t^{1+\chi}}{1+\chi} \right] \tag{25}$$

Subject to:

$$B_{t+1} = R_{t-1} B_t + W_t L_t - P_t C_t + \Gamma_t^d + \Gamma_t^{cb} \tag{26}$$

where W_t is the nominal wage, P_t is the consumer price index, R_{t-1} is the domestic nominal interest rate, Γ_t^d are profits from financial intermediaries and Γ_t^{cb} are transfers from the central bank. Each household owns the same share of firms and dealer agencies in the home economy. Households only save in domestic currency bonds. Transactions between foreign and domestic markets occur in foreign currency bonds (hard currency). Domestic households offset their foreign currency bond position originated in the financial account transactions against financial intermediaries.

The consumption basket is given by:

$$C_t \equiv \left[(\gamma)^{1/\varepsilon_H} \left(C_t^H \right)^{\frac{\varepsilon_H - 1}{\varepsilon_H}} + (1-\gamma)^{1/\varepsilon_H} \left(C_t^M \right)^{\frac{\varepsilon_H - 1}{\varepsilon_H}} \right]^{\frac{\varepsilon_H}{\varepsilon_H - 1}}, \tag{27}$$

where supra-indexes H and F stand for home and foreign produced goods, respectively. The parameter γ regulates the home bias. Domestic prices are given by. Consumer price index, under these preference assumptions, is determined by the following condition:

$$P_t \equiv \left[\gamma \left(P_t^H \right)^{1-\varepsilon_H} + (1-\gamma) \left(P_t^F \right)^{1-\varepsilon_H} \right]^{\frac{1}{1-\varepsilon_H}} \tag{28}$$

where P_t^H and P_t^F denote the price level of the home-produced and imported goods, respectively. Home goods index is defined as follows:

$$P_t^H \equiv \left[\int_0^n P_t^H(z)^{1-\varepsilon} dz \right]^{\frac{1}{1-\varepsilon}} \tag{29}$$

Consumption decisions and the supply of labour. The condition characterizing the optimal allocation of domestic consumption is given by the following equation:

$$U_{C,t} = \beta E_t \left\{ U_{C,t+1} R_t \frac{P_t}{P_{t+1}} \right\} \tag{30}$$

The first-order conditions that determine the supply of labour are characterized by the following equation:

$$-\frac{U_{L,t}}{U_{C,t}} = \frac{W_t}{P_t} \tag{31}$$

where $\frac{W_t}{P_t}$ denotes real wages. In a competitive labour market, the marginal rate of substitution equals the real wage.

Demand for each type of good is given by:

$$C_t^H = \gamma \left(\frac{P_t^H}{P_t}\right)^{-\varepsilon_H} C_t \tag{32}$$

$$C_t^F = (1-\gamma) \left(\frac{P_t^F}{P_t}\right)^{-\varepsilon_H} C_t \tag{33}$$

Foreign economy follows a similar pattern for home goods demand:

$$C_t^{*,H} = (1-\gamma^*) \left(\frac{P_t^H}{S_t P_t^*}\right)^{-\varepsilon_F} C_t^* \tag{34}$$

We take the small economy assumption, thus domestic economy demand will not affect international prices.

$$P_t^* = P_t^F \tag{35}$$

Firms. A continuum of z of intermediate firms exists. These firms operate in a perfectly competitive market and use the following linear technology:

$$Y_t^{int}(z) = A_t L_t(z) \tag{36}$$

$L_t(z)$ is the amount of labour demand from households, A_t is the level of technology.
These firms take as given the real wage, W_t/P_t, paid to households and choose their labour demand by minimising costs given the technology. The corresponding first order condition of this problem is:

$$L_t(z) = \frac{MC_t(z)}{W_t/P_t} Y_t^{int}(z)$$

where $MC_t(z)$ represents the real marginal costs in terms of home prices. After replacing the labour demand in the production function, we can solve for the real marginal cost:

$$MC_t(z) = (1-\tau^H)\frac{W_t}{A_t} \tag{37}$$

Given that all intermediate firms face the same constant returns to scale technology, the real marginal cost for each intermediate firm z is the same, that is $MC_t(z) = MC_t$. Also, given these firms operate in perfect competition, the price of each intermediate good is equal to the marginal cost. Therefore, the relative price $P_t(z)/P_t$ is equal to the real marginal cost in terms of consumption unit (MC_t).
Price-Setting. Final goods producers purchase intermediate goods and transform them into differentiated final consumption goods. Therefore, the marginal costs of these firms equal the price of intermediate goods. These firms operate in a monopolistic competitive market, where each firm faces a downward-sloping demand function, given below. Furthermore, we assume that each period t final goods producers face an exogenous probability of changing prices given by $(1-\theta)$. Following Calvo (1983), we assume that this probability is independent of the last time the firm set prices and the previous price level. Thus, given a price fixed from period t, the present discounted value of the profits of firm z is given by:

$$\max_{\hat{P}^H} \sum_{k=0}^{\infty} \theta^k E_t \left[\mathcal{Q}_{t,t+k} \left(Y_{t+k}^H(j) \left\{ \hat{P}_t^H - (1-\tau^H) P_t MC_{t+k} \right\} \right) \right] \tag{38}$$

where $\mathcal{Q}_{t,t+k} = \beta^k \frac{U_{C,t+k}}{U_{C,t}}$ is the stochastic discount factor and $Y_{t,t+k}^H(j)$ is the demand for good j in $t+k$ conditioned to a fixed price from period t, given by:

$$Y_{t+k}^H(j) = \left(\frac{\hat{P}_t^H}{P_{t+k}^H}\right)^{-\varepsilon} Y_{t+k}^H \tag{39}$$

Regarding the subsidy τ^H, we the policy maker sets it constant to maximize the steady state welfare of the domestic economy. In section D.1 in the Appendix, we show this value is given by:

$$\tau^H = \frac{\varepsilon\gamma - \varepsilon + 1}{\varepsilon\gamma} \tag{40}$$

Each firm j chooses $\hat{P}_t^H(j)$ to maximise (38). The first order condition of this problem is:

$$\sum_{t=0}^{\infty} \theta^k E_t \left[\mathcal{Q}_{t,t+k} Y_{t+k}^H (P_{t+k}^H)^{\varepsilon} \left\{ (1-\varepsilon)(\hat{P}_t^H)^{-\varepsilon} + \varepsilon(1-\tau^H) P_{t+k} MC_{t+k} (\hat{P}_t^H)^{-\varepsilon-1} \right\} \right] = 0 \tag{41}$$

We define, $\tilde{p}_t^H \equiv \frac{\hat{P}_t^H}{P_t^H}$

$$\sum_{t=0}^{\infty} P_t C_t \left(\beta\theta\right)^k E_t \left[\left((1-\gamma^*) \frac{C_{t+k}^*}{C_{t+k}} \mathcal{S}_{t+k} + \gamma \mathcal{S}_{t+k}^{1-\gamma} \right) (X_{t,k}^H)^{-\varepsilon} \left\{ t_{t+k}^H \tilde{p}_t^H X_{t,k}^H - \frac{\varepsilon}{\varepsilon-1}(1-\tau^H) MC_{t+k} \right\} \right] = 0 \tag{42}$$

where: and:

$$X_{t,k}^H = X_{t+1,k-1}^H \frac{1}{\pi_{t+1}^H},\ k \geq 0. \tag{43}$$

Solving for $\tilde{p}^H$ yields:

$$\tilde{p}_t^H = \frac{\sum_{t=0}^{\infty} \left(\beta\theta\right)^k E_t \left[(X_{t,k}^H)^{-\varepsilon} \frac{\varepsilon}{\varepsilon-1} (1-\tau^H) MC_{t+k} \right]}{\sum_{t=0}^{\infty} \left(\beta\theta\right)^k E_t \left[(X_{t,k}^H)^{1-\varepsilon} t_{t+k}^H \right]} = \frac{K_t}{F_t} \tag{44}$$

Now we set the recursive equations:

$$K_t = \frac{\varepsilon}{\varepsilon-1}(1-\tau^H)\frac{MC_{t+k}}{t_{t+k}^H} + \beta\theta E_t \left(\frac{1}{\pi_{t+1}^H} \right)^{-\varepsilon} K_{t+1} \tag{45}$$

$$F_t = 1 + \beta\theta E_t \left(\frac{1}{\pi_{t+1}^H} \right)^{1-\varepsilon} K_{t+1} \tag{46}$$

The Calvo pricing assumption allows to write an expression for the home prices index:

$$P_t^H \equiv \left[\theta P_{H,t-1}^{1-\varepsilon} + (1-\theta) \left(\hat{P}_t^H \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{47}$$

Dividing by P_t^H:

$$1 \equiv \left[\theta \left(\pi_t^H \right)^{\varepsilon-1} + (1-\theta) \left(\tilde{p}_t^H \right)^{1-\varepsilon} \right] \tag{48}$$

Solving for $\tilde{p}_t^H$, we obtain:

$$\tilde{p}_t^H = \left[\frac{1-\theta \left(\pi_t^H \right)^{\varepsilon-1}}{1-\theta} \right]^{\frac{1}{1-\varepsilon}} \tag{49}$$

Regarding the subsidy, we assume the government can set a constant value for τ^H to fix the distortion caused by the assumption the market power in the steady-state. See appendix 2 for the derivation of the value. Domestic firms will sell at the same local currency price in both economies. This is also known as producer currency pricing (PCP).

From Yun (1996) the evolution of the price dispersion is given by:

$$Z_t = (1-\theta)\left[\frac{1-\theta\left(\pi_t^H\right)^{\varepsilon-1}}{1-\theta}\right]^{\frac{\varepsilon}{\varepsilon-1}} + \theta\left(\pi_t^H\right)^{\varepsilon} Z_{t-1} \tag{50}$$

Goods Market Clearing and Current Account. Domestic goods market clearing yields:

$$\begin{aligned} Y_t^H &= \gamma\left(\frac{P_t^H}{P_t}\right)^{-1} C_t + (1-\gamma)\left(\frac{P_t^H}{S_t P_t^*}\right)^{-1} C_t^* && (51) \\ &= \gamma\left(\mathcal{S}_t\right)^{1-\gamma} C_t + (1-\gamma)\mathcal{S}_t C_t^* && (52) \end{aligned}$$

For the current account, it is convenient to define net foreign assets as:

$$\mathcal{A}_t = S_{t-1} B_t^{cb,*} + S_{t-1} D_t^* - N_t \tag{53}$$

So, the current account is equivalent to:

$$CA_t = \mathcal{A}_{t+1} - \mathcal{A}_t \tag{54}$$

From the budget constraints in the model we obtain:

$$CA_t = NX_t + \left(\frac{S_t}{S_{t-1}} R_{t-1}^* - 1\right)\left(S_{t-1} B_t^{cb,*} + S_{t-1} D_t^*\right) - \left(R_{t-1} - 1\right) N_t \tag{55}$$

where

$$NX_t = P_t^H Y_t^H - P_t C_t \tag{56}$$

B Figures and Tables

B.1 Impulse Response Functions - Flexible Prices

Figure 6: Response to a 1% standard deviation shock to TFP (Optimal Simple Rules, $\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1$)

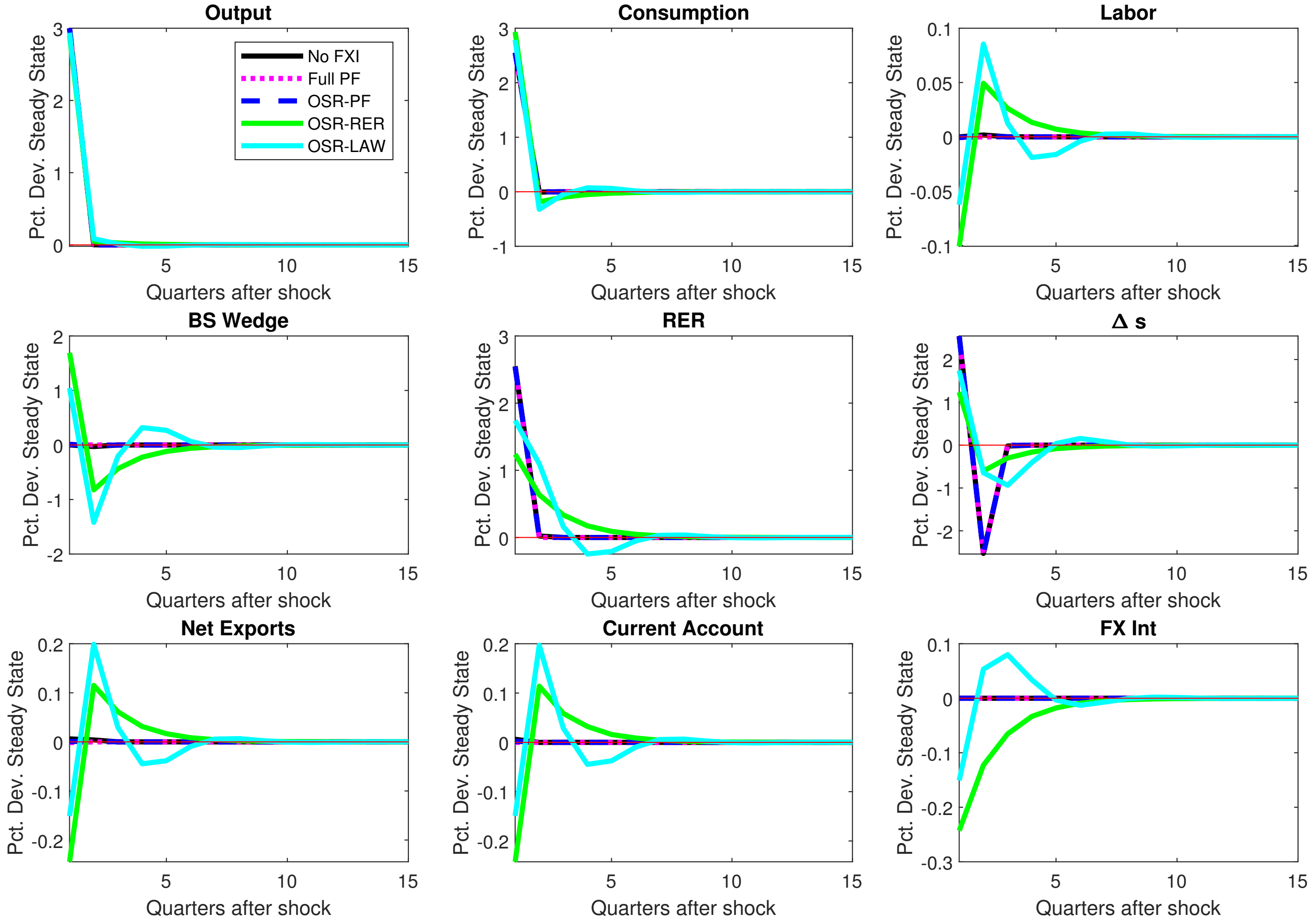


Figure 7: Response to a 1% standard deviation shock to Portfolio Shock (Optimal Simple Rules, $\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1$))

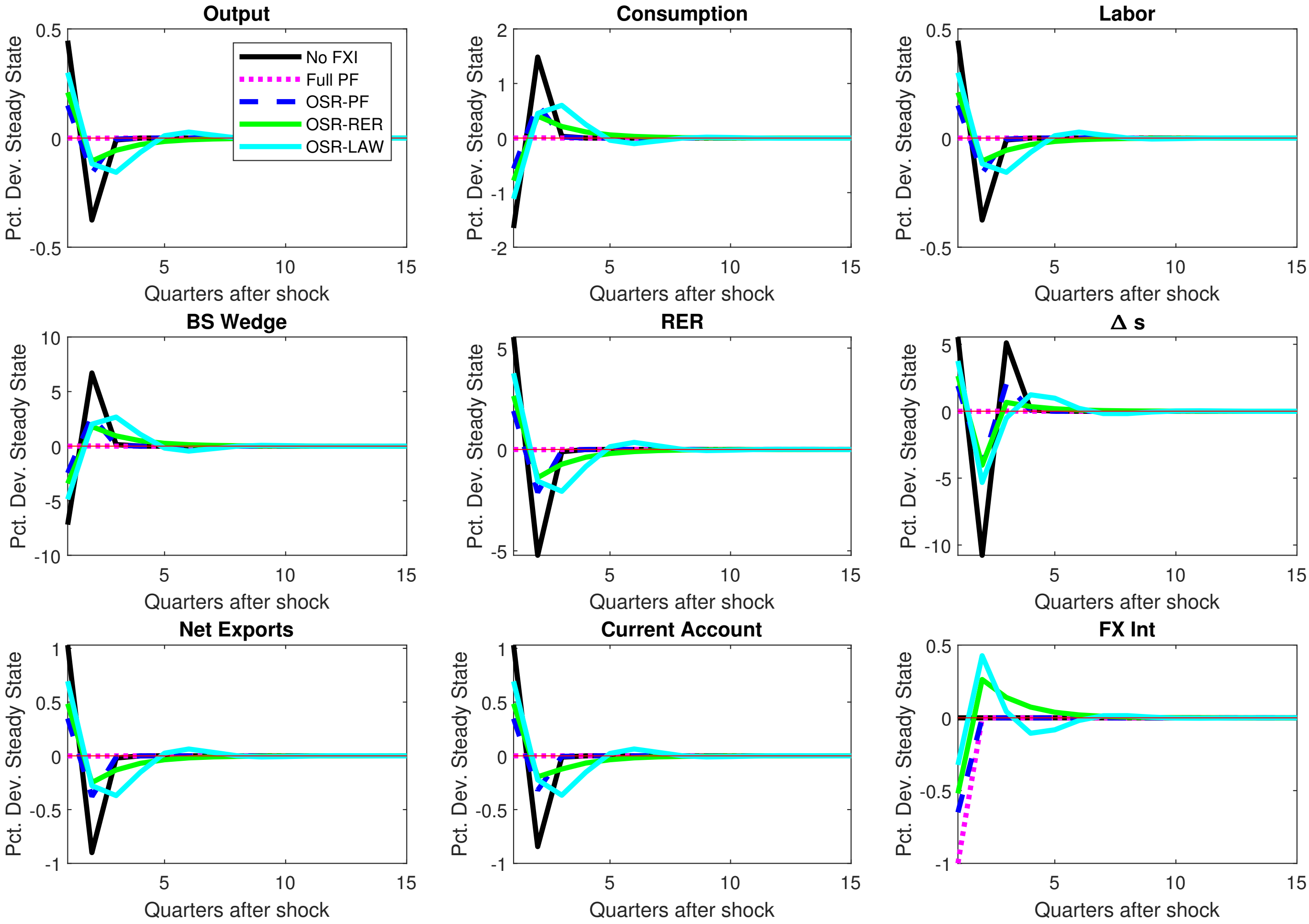


Figure 8: Response to a 1% standard deviation shock to TFP (Optimal Simple Rules, $\gamma_c = 1.4, \varepsilon_H = \varepsilon_F = 3$)

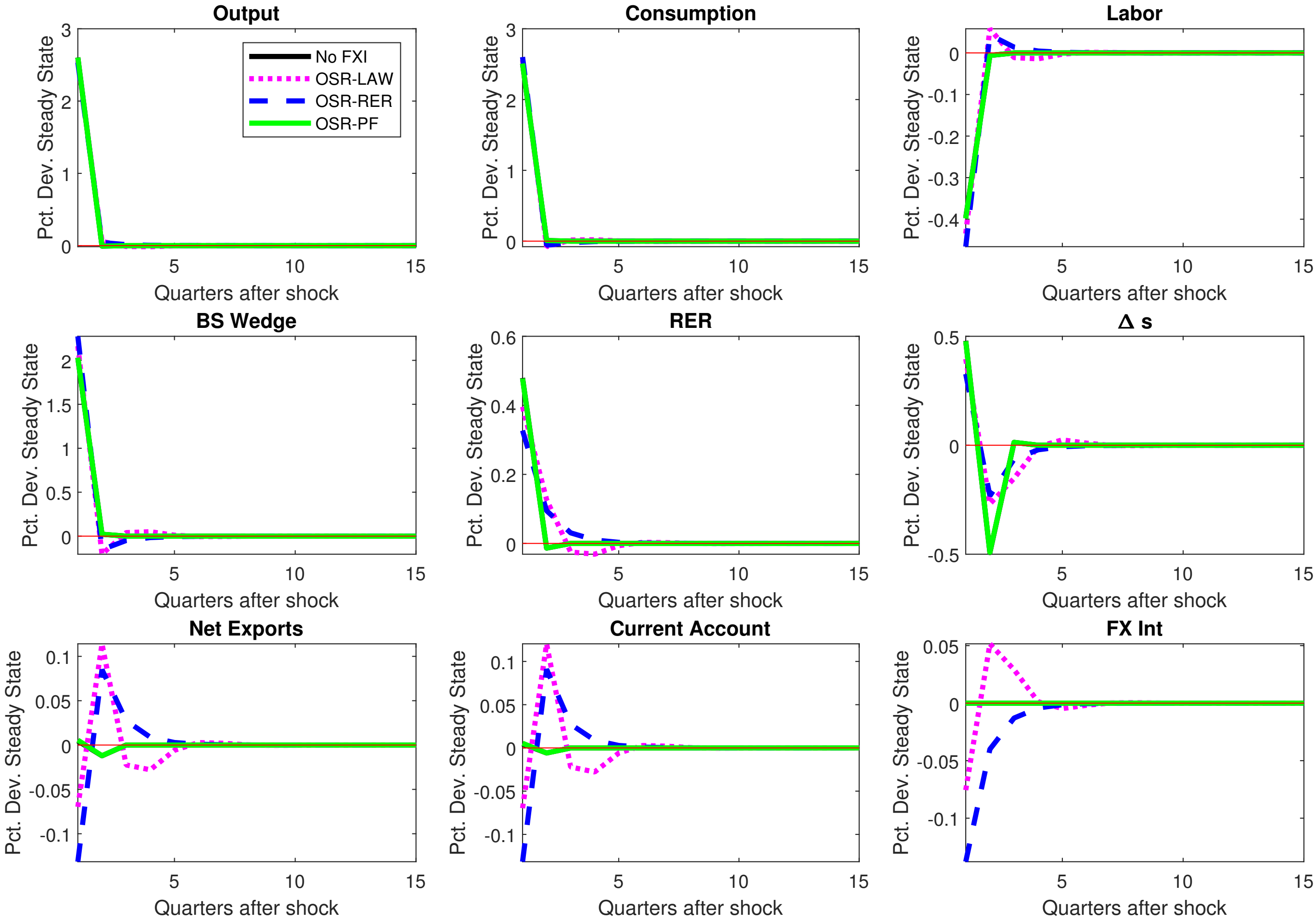
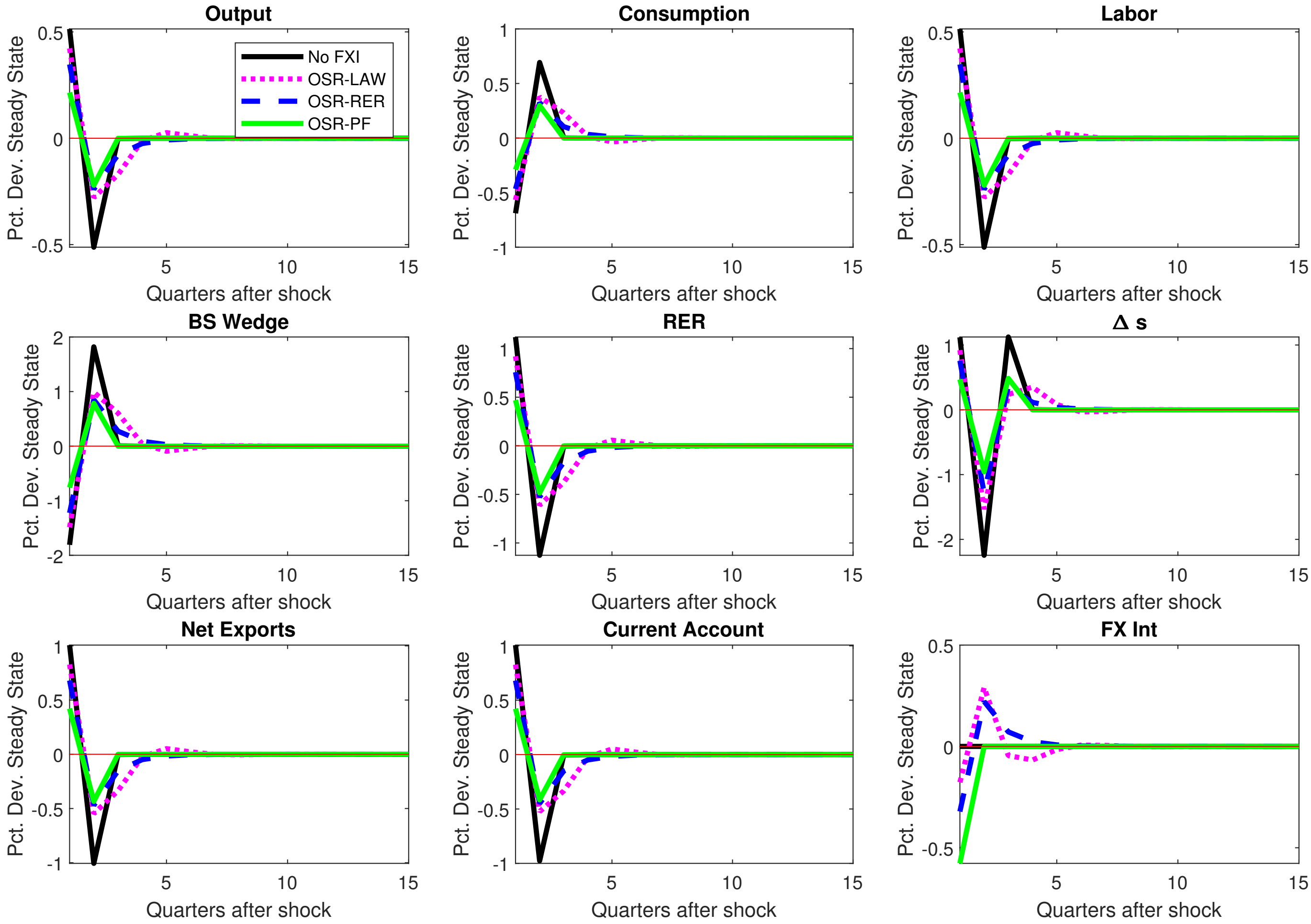


Figure 9: Response to a 1% standard deviation shock to Portfolio Shock (Optimal Simple Rules, $\gamma_c = 1.4, \varepsilon_H = \varepsilon_F = 3$))



B.2 Robustness Exercises - Flexible Prices

Figure 10: Robustness to Shocks Relative Frequency, $(\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1)$

Latex/G30CFLEX_SHOCKS-eps-converted-to.pdf

Figure 11: Robustness to Home Bias parameter, $(\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1)$

Latex/G3OCFLEX_HB-eps-converted-to.pdf

Figure 12: Robustness to FX Dealers Risk Aversion/Capacity parameter, $(\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1)$

Latex/G30CFLEX_M-eps-converted-to.pdf

Figure 13: Robustness to Elasticity of Intertemporal Substitution, $(\varepsilon_H = \varepsilon_F = 1)$

Latex/G3OCFLEX_EIS-eps-converted-to.pdf

B.3 Impulse Response Functions - Sticky Prices

Figure 14: Response to a 1% standard deviation shock to TFP (Optimal Simple Rules, $\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1$)

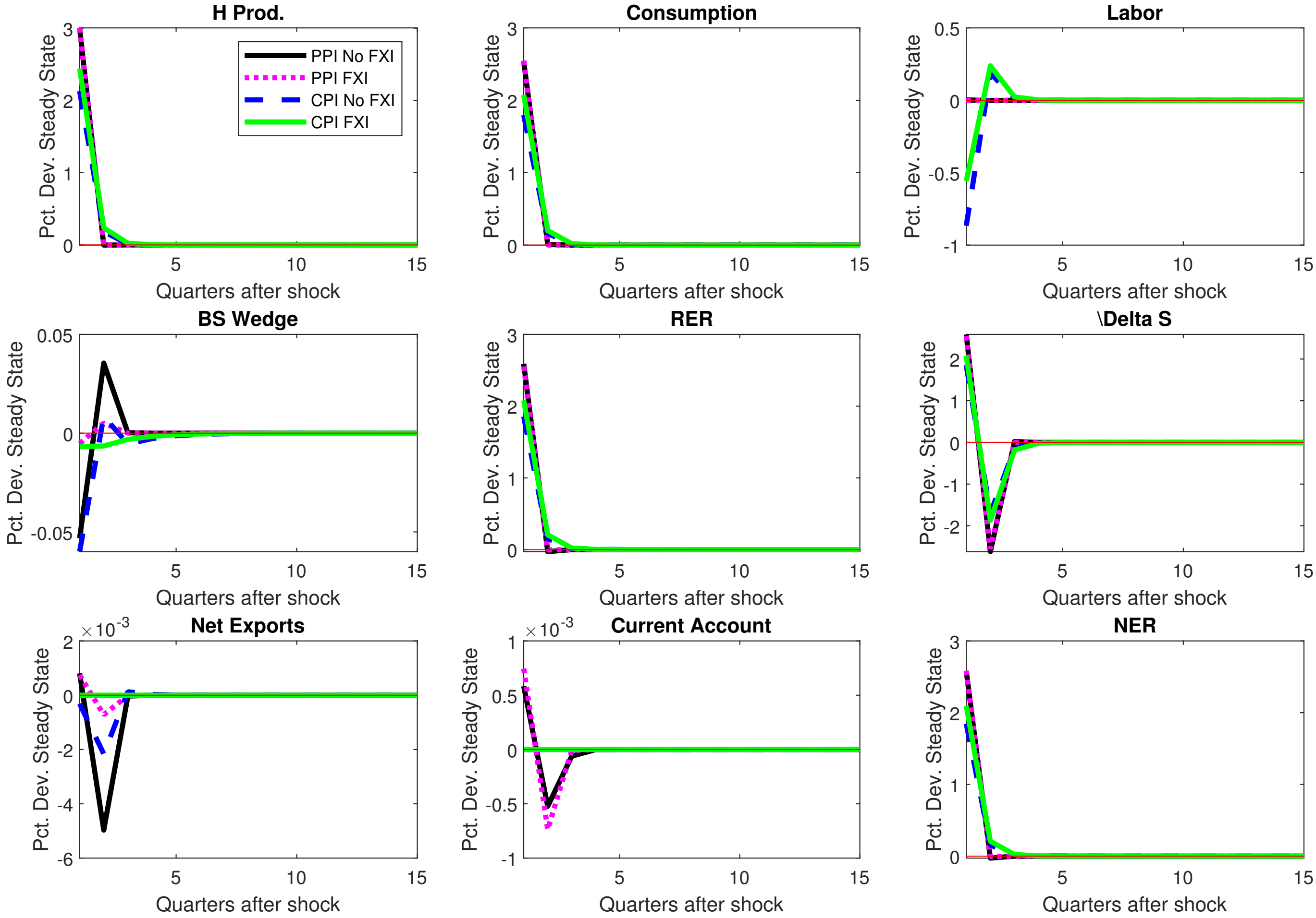


Figure 15: Response to a 1% standard deviation shock to Portfolio Shock (Optimal Simple Rules, $\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1$)

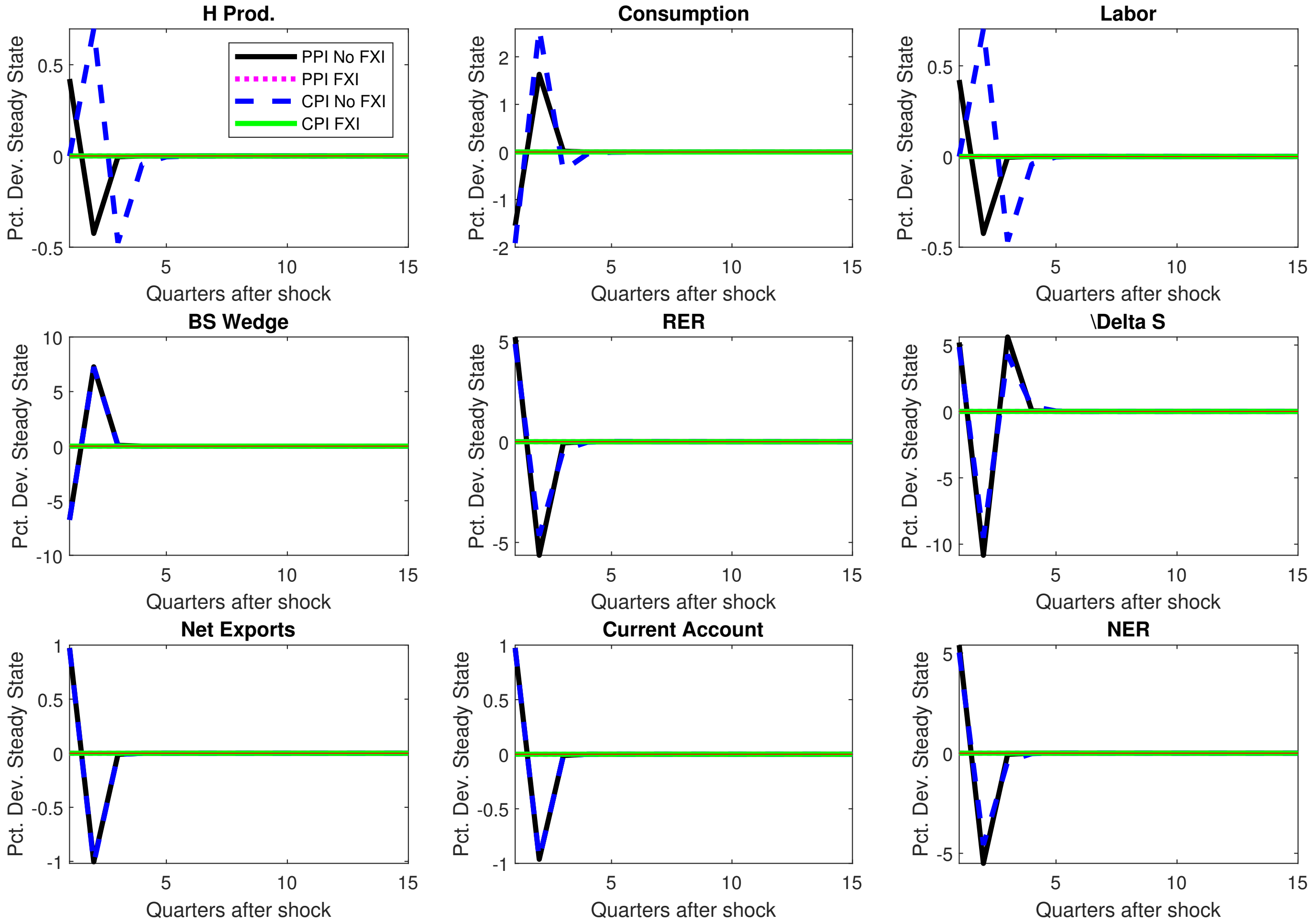


Figure 16: Response to a 1% standard deviation shock to TFP (FXI Rules, $\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1$)

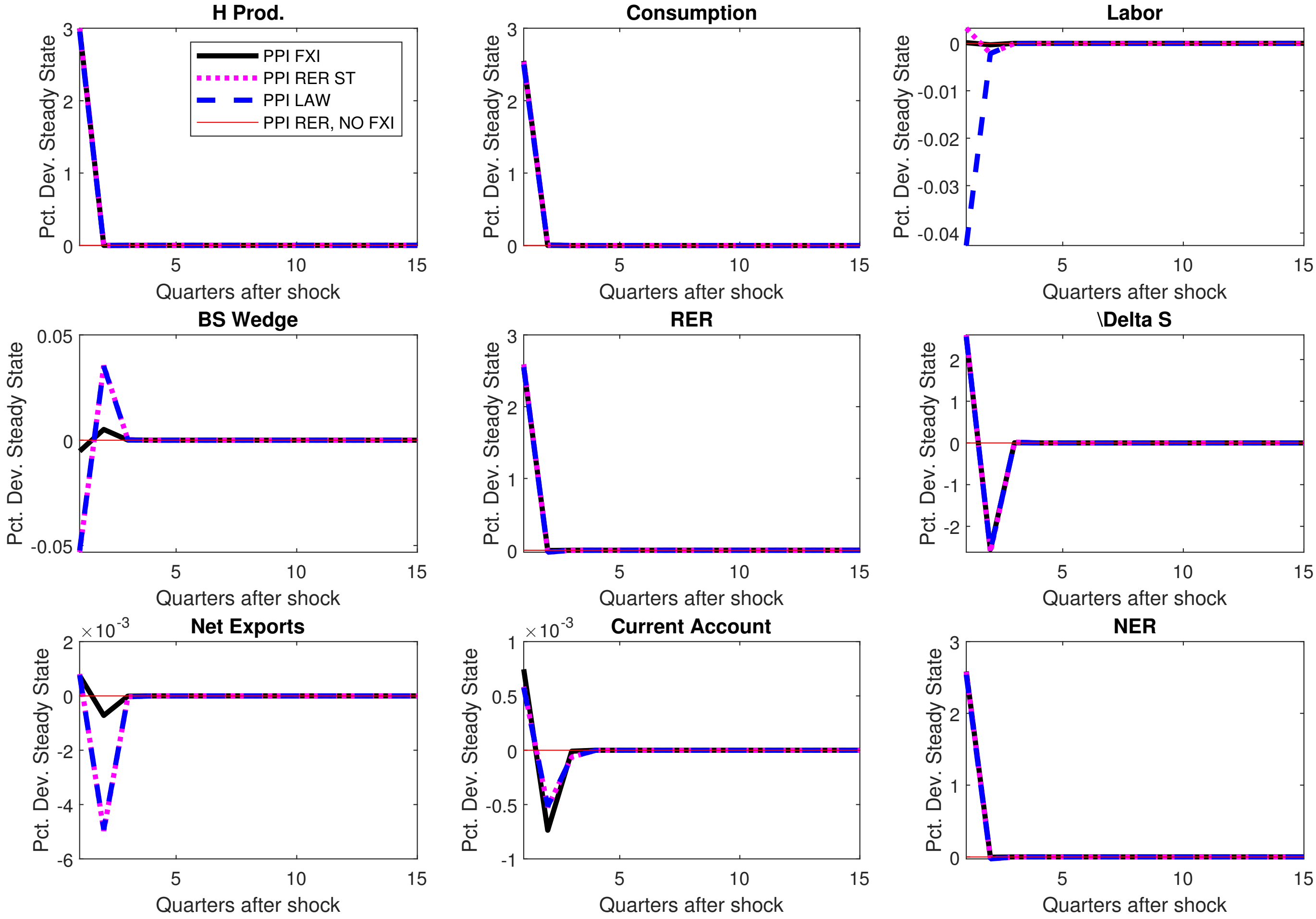
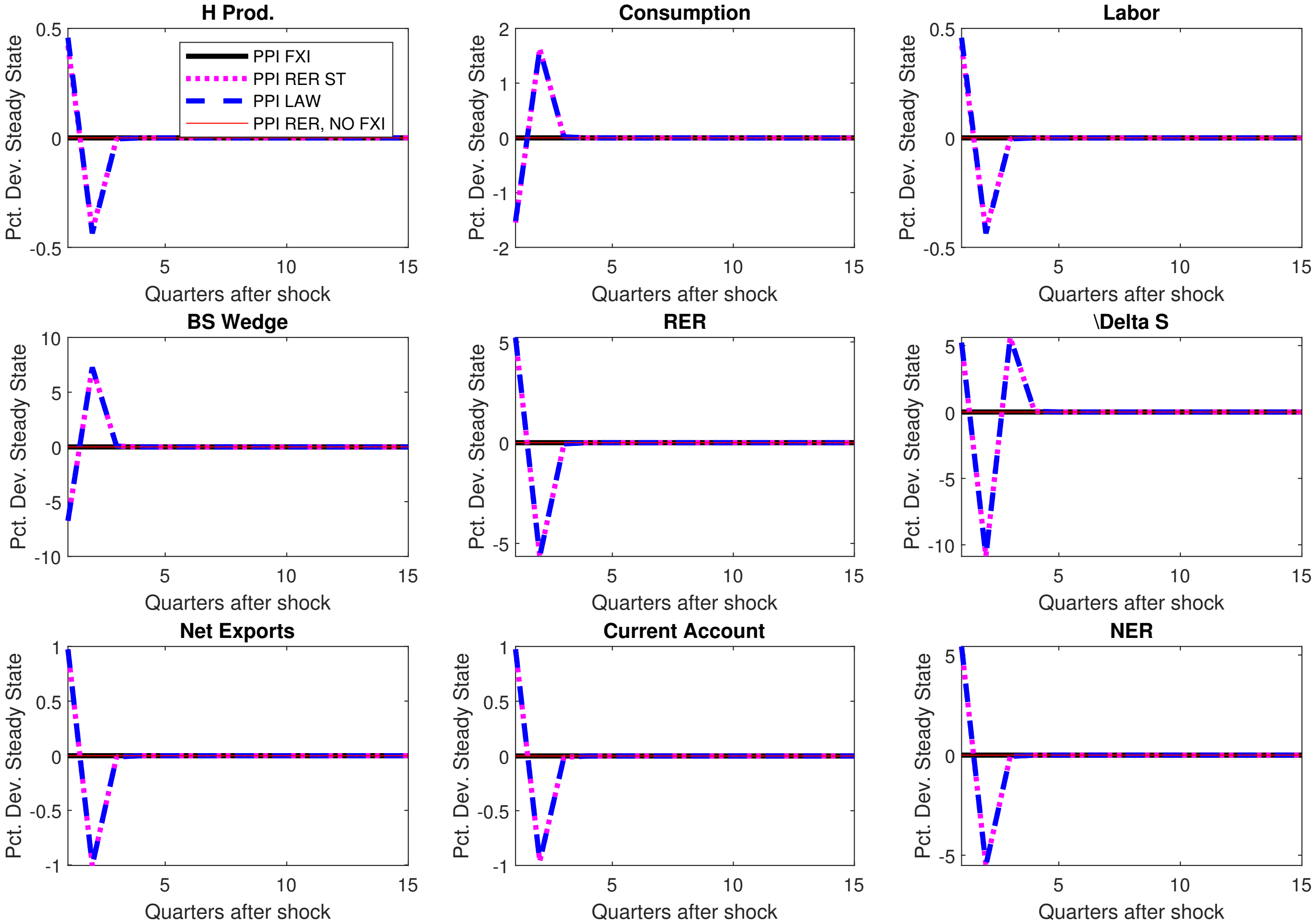


Figure 17: Response to a 1% standard deviation shock to Portfolio Shock (FXI Rules, $\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1$)



B.4 Robustness Exercises - Sticky Prices

Figure 18: Robustness to Shocks Relative Frequency, ($\gamma_c = 1, \varepsilon_H = \varepsilon_F = 1$, $\theta_H = 0.5$)

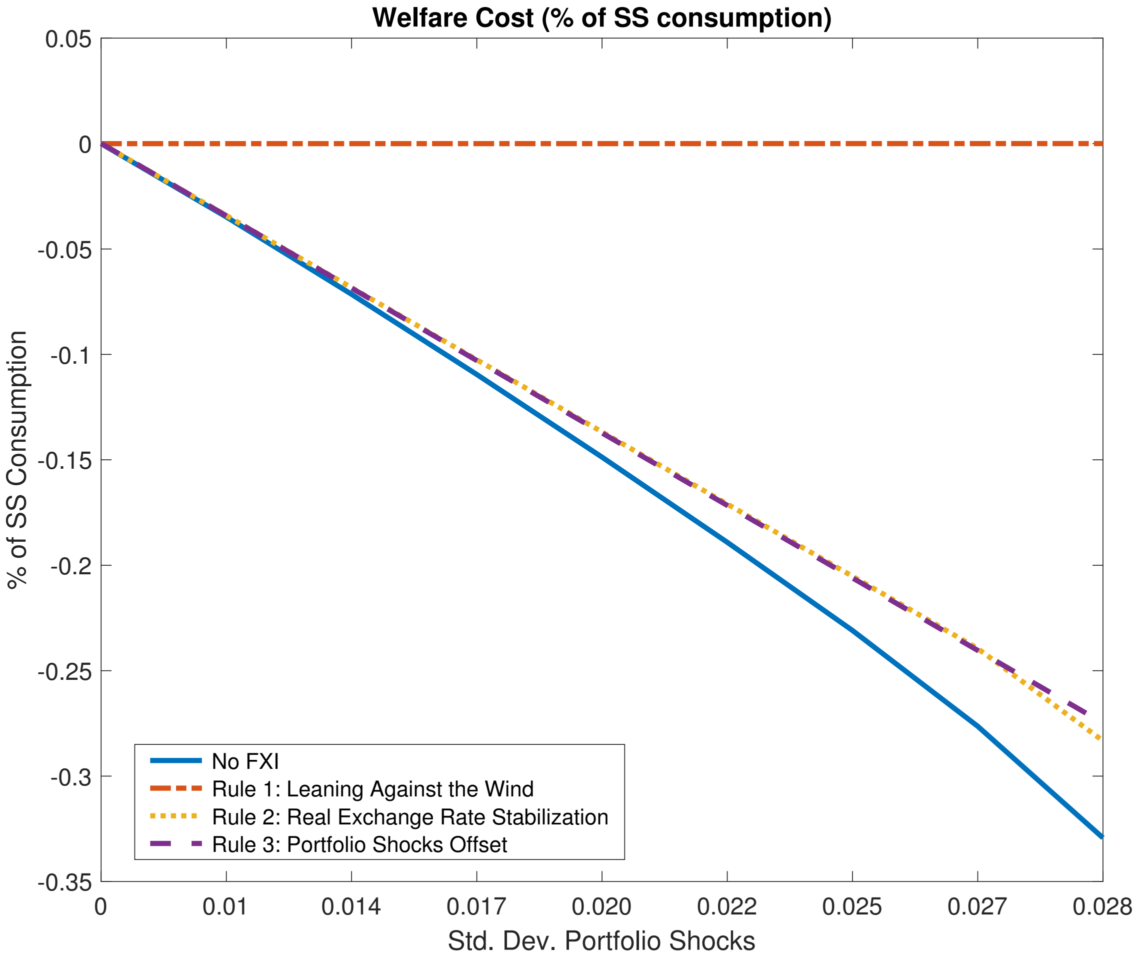


Figure 19: Robustness to Shocks Relative Frequency, ($\gamma_c = 1.4, \varepsilon_H = \varepsilon_F = 3$, $\theta_H = 0.5$)

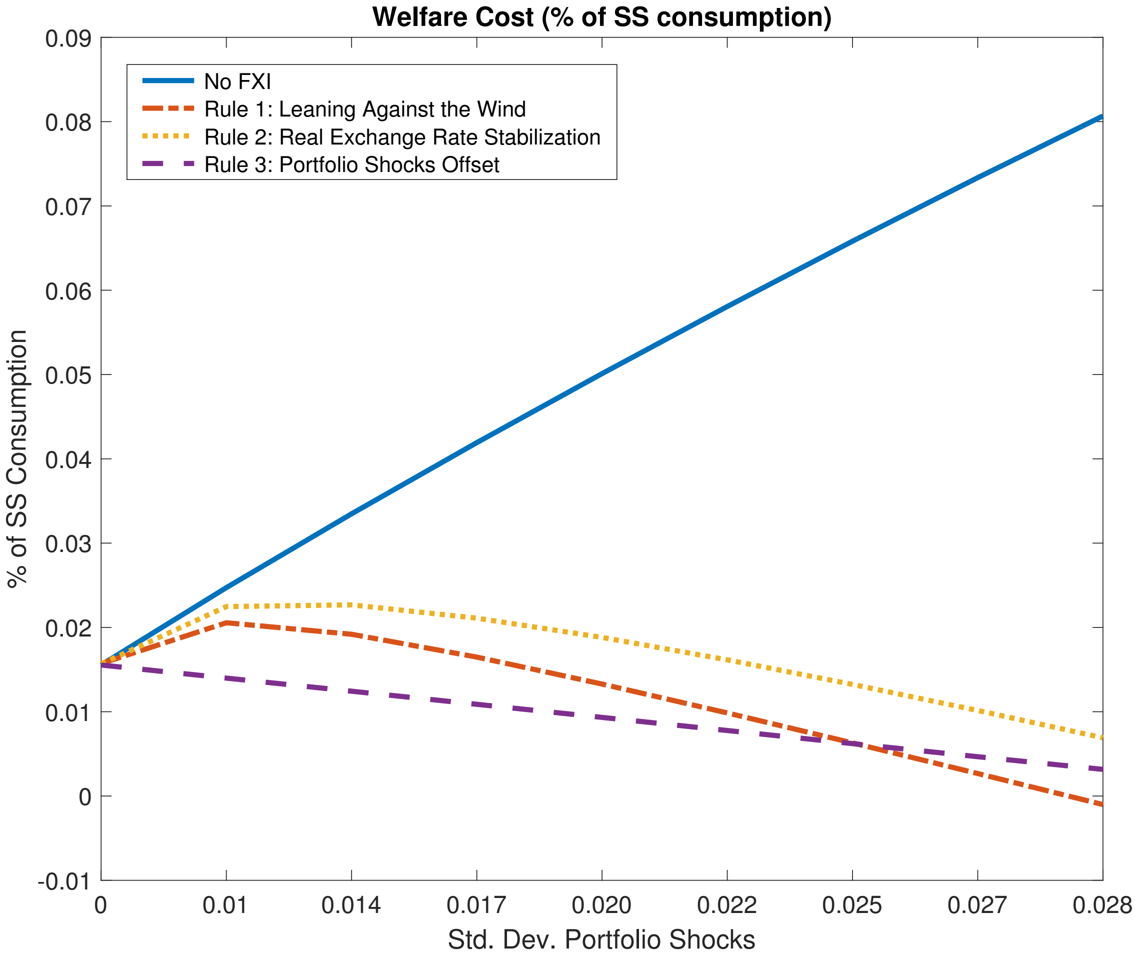


Figure 20: Robustness to Home/Foreign Elasticity of Demand, ($\gamma_c = 1$, $\theta_H = 0.5$)

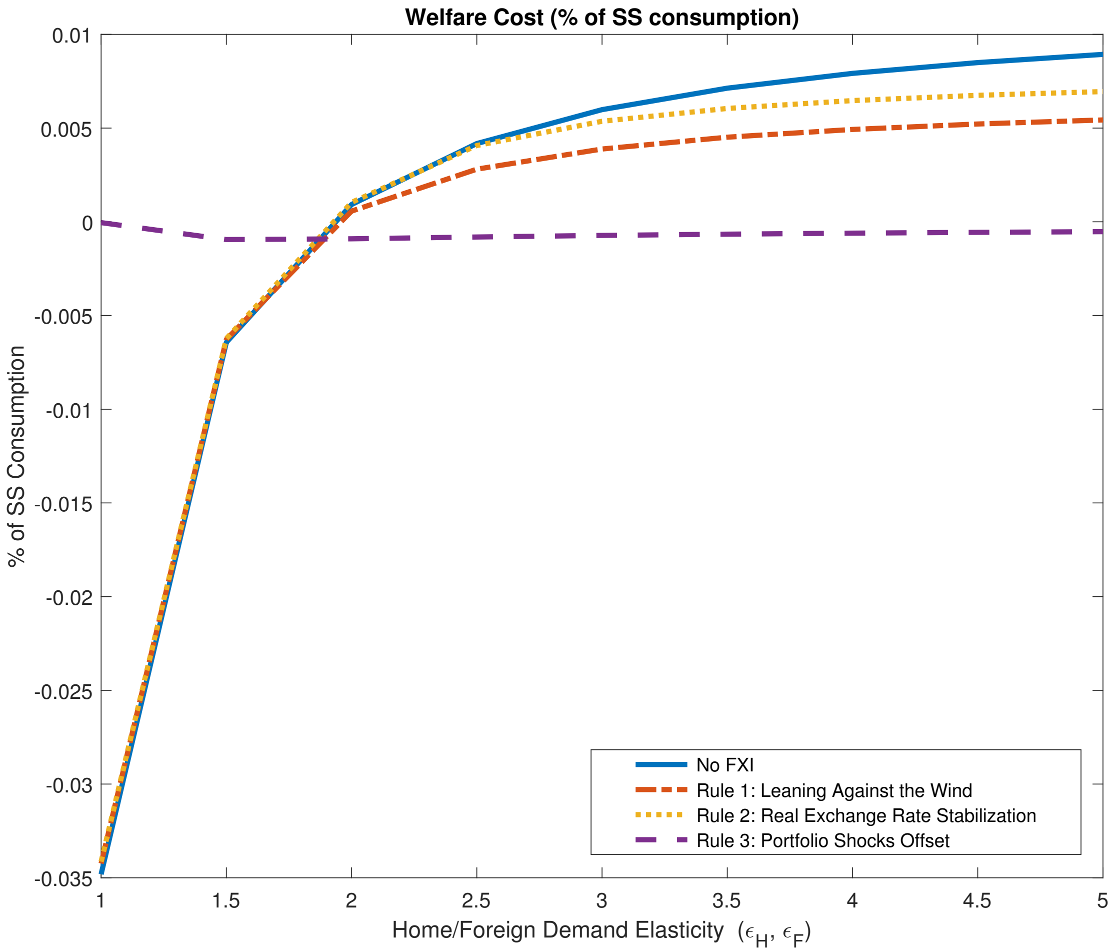


Figure 21: Robustness to Home/Foreign Elasticity of Demand, ($\gamma_c = 1.4$, $\theta_H = 0.5$)

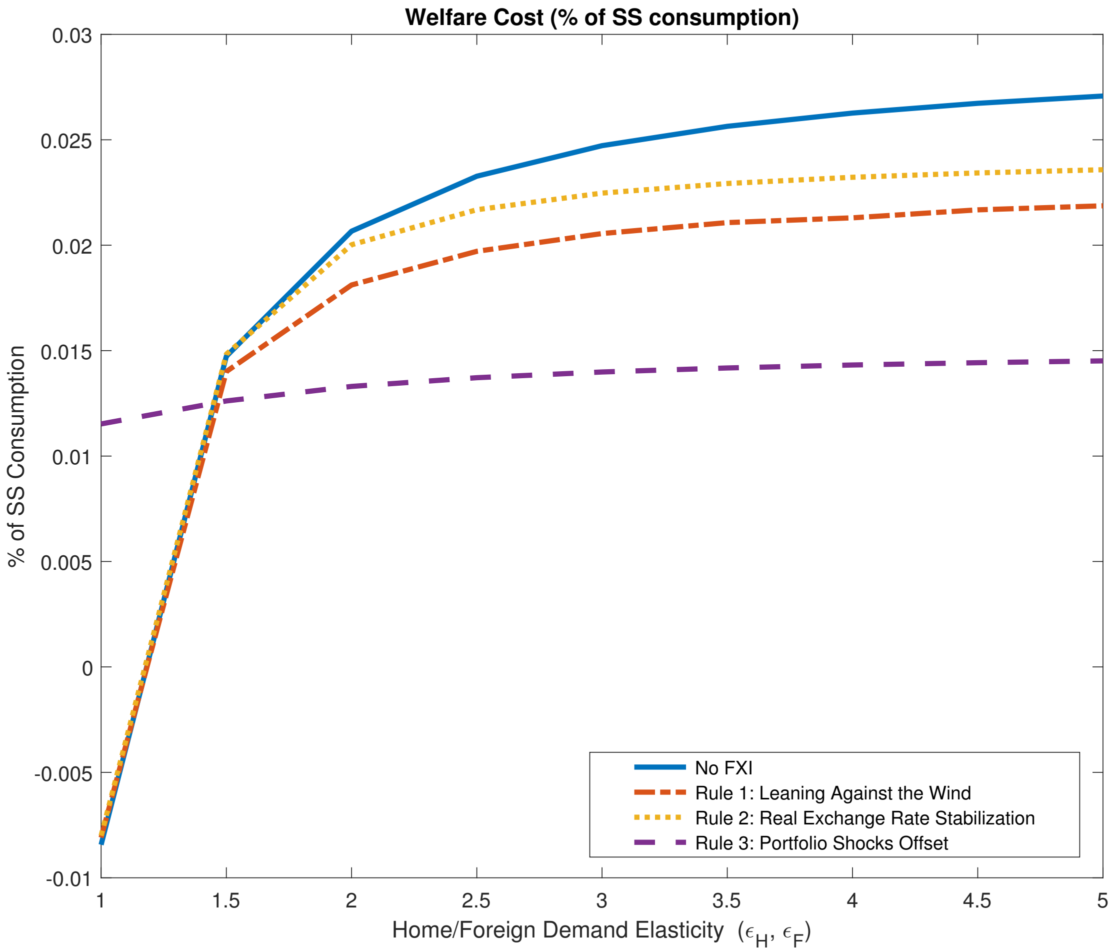


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Figure 22: Robustness to Home Bias Parameter, ($\gamma_c = 1.4, \varepsilon_H = \varepsilon_F = 3,\ \theta_H = 0.5$)

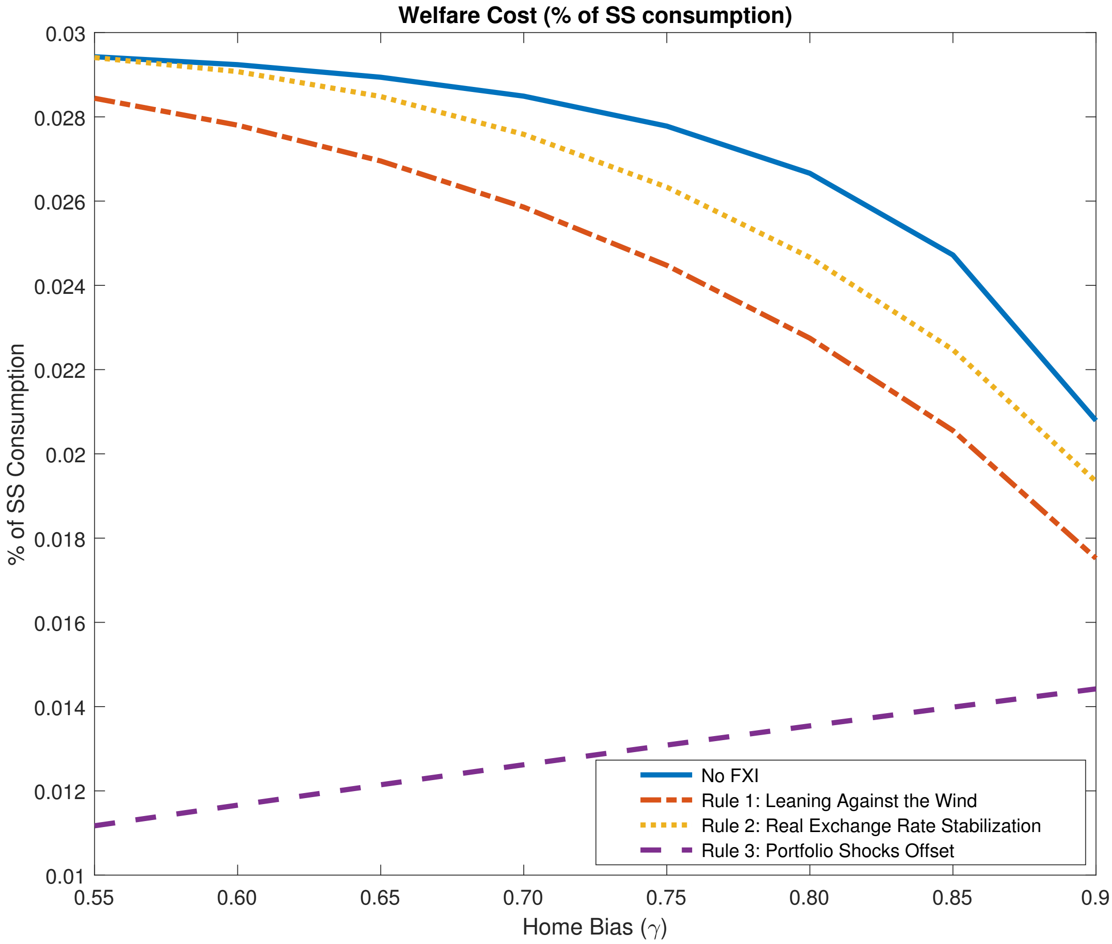


Figure 23: Robustness to FX Dealers Risk Aversion/Capacity parameter, ($\gamma_c = 1.4, \varepsilon_H = \varepsilon_F = 3$, $\theta_H = 0.5$)

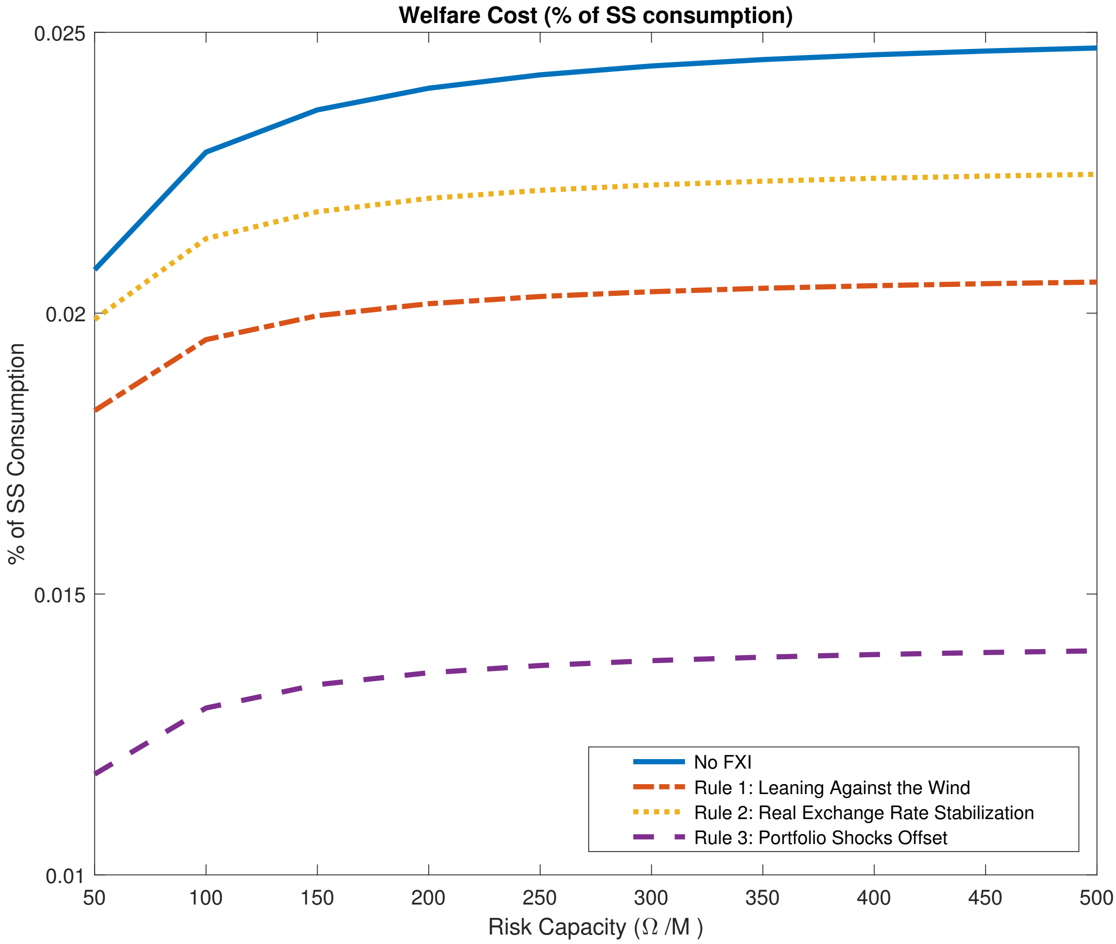


Figure 24: Robustness to Elasticity of Intertemporal Substitution, ($\varepsilon_H = \varepsilon_F = 3$, $\theta_H = 0.5$)

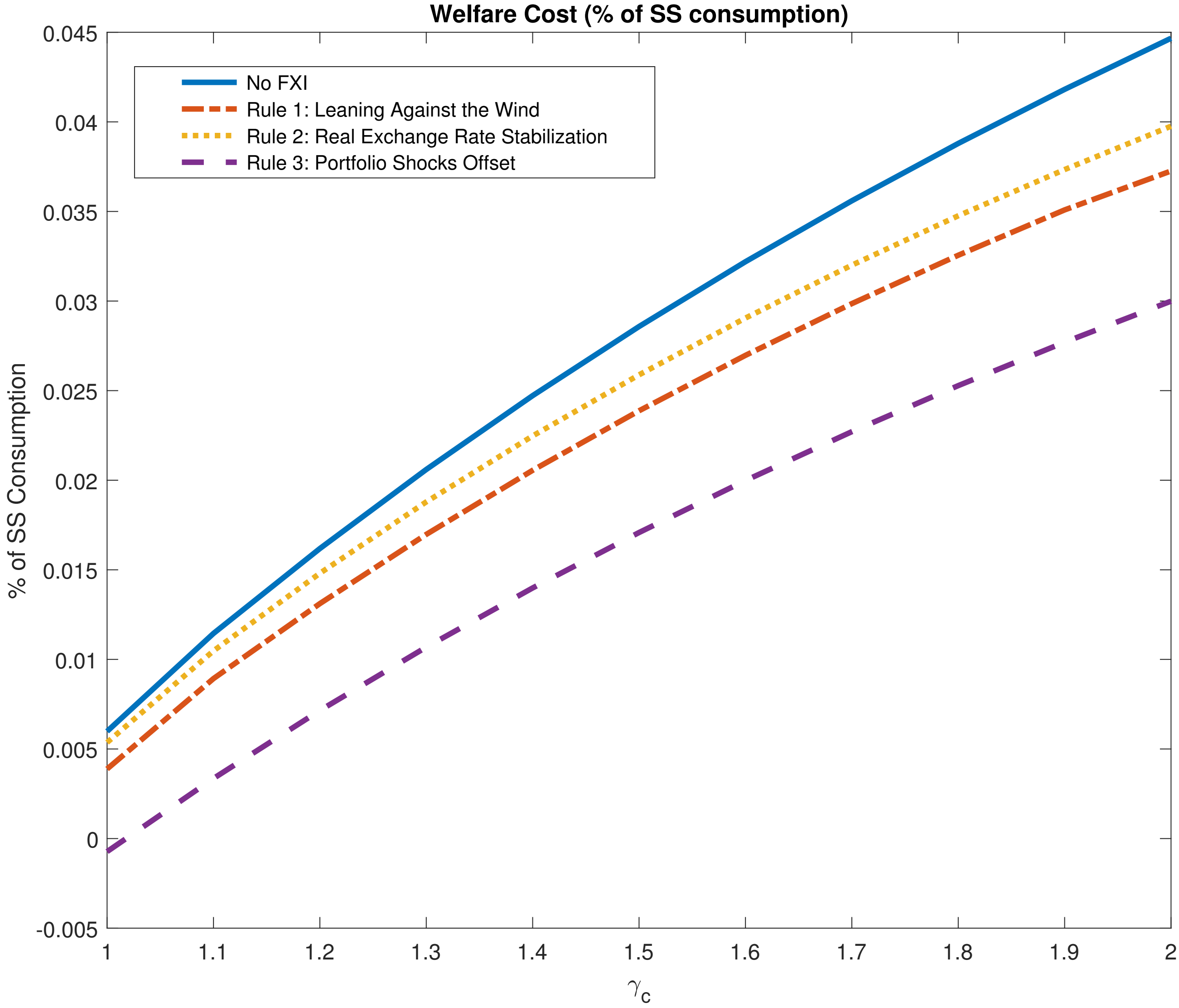
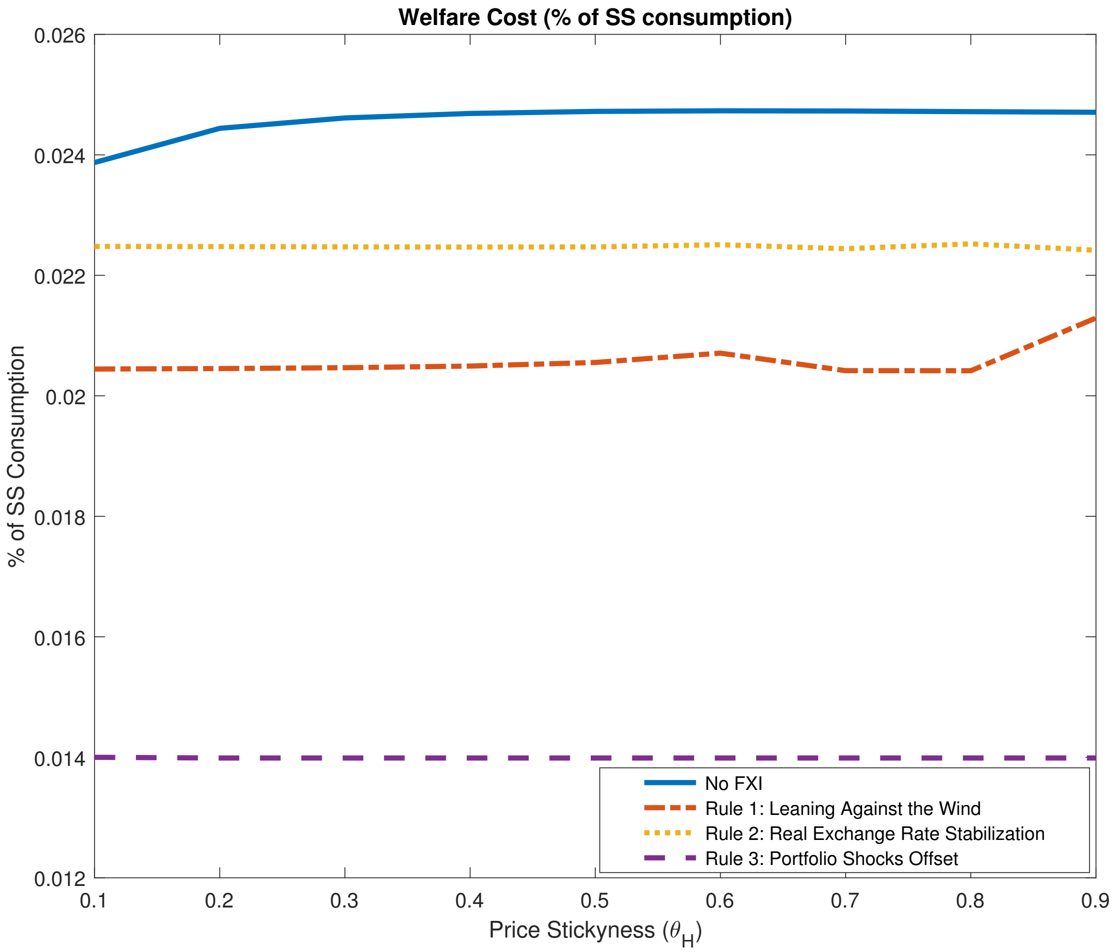


Figure 25: Robustness to Calvo Pricing Parameter, ($\gamma_c = 1.4$,$\varepsilon_H = \varepsilon_F = 3$)



C Proof of Propositions

Proof of Lemma 2.1 Following Campbell and Viceira (2002) and Itskhoki and Mukhin (2017), we can write the discounted nominal return in domestic currency as:

$$\begin{aligned}\frac{\tilde{R}^*_{t+1}}{R_t} &= \frac{S_{t+1}R^*_t}{R_tS_t} - 1\\ &= \exp\left(\epsilon_{t+1} - 1\right)\end{aligned}$$

where:

$$\epsilon_{t+1} \equiv i^*_t - i_t + \Delta s_{t+1} = \log\left(R^*_t/R_t\right) + \Delta log S_{t+1}$$

$i_t \equiv logR_t$, $i^*_t \equiv logR^*_t$ and $\Delta\log S_{t+1} \equiv \log\left(\frac{S_{t+1}}{S_t}\right)$. In continuous time and assuming ϵ_{t+1} follows a normal diffusion process:

$$dX_t = \alpha_t dt + \sigma dZ_t$$

where Z_t is a Wiener or Brownian motion process, where the drift and difussions are given by:

$$\alpha_t = \mathbb{E}_t\epsilon_{t+1} = i^*_t - i_t + \mathbb{E}_t\Delta s_{t+1}$$

and:

$$\sigma^2 = \sigma^2_{\Delta s_{t+1}}$$

where $\sigma^2_{\Delta s_{t+1}}$ is the time-invariant conditional variance of the variation of the exchange rate. In line with Merton (1992), we approximate the period return by the variation in the diffusion process. This allows us to rewrite the problem in terms of dX_t:

$$\max_{d^*}\mathbb{E}_t\left\{-\frac{1}{\omega}\exp\left(-\omega\exp\left(dX_t\right)d^*\right)\right\}$$

Itô's lemma allows us to rewrite the objective function as:

$$\mathbb{E}\left\{-\frac{1}{\omega}exp\left(-\omega\left(dX_t + \frac{1}{2}(dX_t)^2\right)\right)d^*\right\}$$

since the period return follows a normal distribution, we can use the properties of the log-normal distribution to obtain the reformulate maximization problem as:

$$\max_{d^*}\left\{-\frac{1}{\omega}\exp\left(-\omega\left(\alpha_t + \frac{1}{2}\sigma^2\right)d^* + \frac{\omega^2\sigma^2}{2}(d^*)^2\right)dt\right\}$$

The solution to the problem yields:

$$d^* = \frac{\alpha_t + \frac{1}{2}\sigma^2}{\omega\sigma^2}$$

substituting for α_t and σ we obtain:

$$d^* = \frac{i^*_t - i_t + \mathbb{E}(s_{t+1}) - s_t + \frac{1}{2}\sigma^2}{\omega\sigma^2}$$

Aggregating across dealers, we can obtain the modified UIP equation:

$$s_t = \mathbb{E}(s_{t+1}) + i_t^* - i_t + \frac{1}{2}\sigma^2 - \frac{\omega}{n}\sigma^2 D_{t+1}^* \tag{57}$$

Proof of Proposition 2.1. The proof follows two steps. First, reduce the dimensionality of the system of equations of log-linearized the model. Second, we solve for the eigenvalues of the dynamic system.
Without loss of generality, we assume $\rho_a = \rho_n = 0$. Also, we assume the central bank does not intervene, $b_t^* = 0$ and that foreign monetary policy is constant, thus $i^* = 0$, $\forall t$. Additionally, since we are interested in the dynamics of d_{t+1}^* we exclude the auxiliary variable λ_t from the system. We obtain:

$$\begin{aligned}
E_t s_{t+1} &= s_t - \frac{1}{\left(\left[1 - \frac{[(\gamma-1)(1+\chi\phi_c)-\chi\phi_{c^*}]}{\gamma(1+\chi\phi_c)}\right](2\gamma-1)\right)} \left[\frac{(1+\chi)}{(1+\chi\phi_c)}\varepsilon_t^A - \frac{\omega}{m}\sigma^2 \hat{d}_{t+1}^*\right] \\
\hat{d}_{t+1}^* &= \left[\phi_{c^*} + (1-\phi_c)\left(\frac{\chi}{1+\chi\phi_c}\right)\right]\frac{2\gamma-1}{\gamma\phi_{sy}}\phi_H s_t + \\
&\quad - (1-\phi_c)\frac{(1+\chi)}{(1+\chi\phi_c)\phi_{sy}}\phi_H \varepsilon_t^A + \left(\frac{1}{\beta}\right)\left(\hat{d}_t^* + \psi_{t-1}\right) - \psi_t
\end{aligned}$$

We assume the economy starts at $\psi_{t-1} = 0$. This yields the following system of equations expressed in matrix form:

$$\begin{pmatrix} 1 & \frac{\omega\sigma^2}{m\Omega_1} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \mathbb{E}_t s_{t+1} \\ d_{t+1}^* \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \Omega_2 & \frac{1}{\beta} \end{pmatrix}\begin{pmatrix} s_t \\ d_t^* \end{pmatrix} + \tag{58}$$

$$+ \begin{pmatrix} -\frac{(1+\chi)}{(1+\chi\phi_c)\Omega_1} & 0 \\ -\frac{(1-\phi_c)(1+\chi)}{(1+\chi\phi_c)\phi_{sy}}\phi_H & -1 \end{pmatrix}\begin{pmatrix} \varepsilon_t^A \\ \psi_t \end{pmatrix} \tag{59}$$

where:

$$\Omega_1 \equiv \left(\left[1 - \frac{[(\gamma-1)(1+\chi\phi_c) - \chi\phi_{c^*}]}{\gamma(1+\chi\phi_c)}\right](2\gamma-1)\right) \tag{60}$$

$$\Omega_2 \equiv \left[\phi_{c^*} + (1-\phi_c)\left(\frac{\chi}{1+\chi\phi_c}\right)\right]\frac{2\gamma-1}{\gamma\phi_{sy}}\phi_H \tag{61}$$

Now we diagonalize the dynamic system to obtain:

$$\mathbb{E}_t x_{t+1} = B x_t + C\begin{pmatrix} \varepsilon_t^A \\ \psi_t \end{pmatrix}$$

where:

$$B = \begin{pmatrix} 1 & \frac{\omega\sigma^2}{m\Omega_1} \\ 0 & 1 \end{pmatrix}^{-1}\begin{pmatrix} 1 & 0 \\ \Omega_2 & \frac{1}{\beta} \end{pmatrix} = \begin{pmatrix} 1 & -\frac{\omega\sigma^2}{m\Omega_1} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ \Omega_2 & \bar{R} \end{pmatrix} = \begin{pmatrix} 1 - \frac{\omega\sigma^2}{m\Omega_1}\Omega_2 & \frac{\omega\sigma^2}{m\Omega_1}\bar{R} \\ \Omega_2 & \bar{R} \end{pmatrix}$$

Notice that $\bar{R} = 1/\beta > 1$. For the eigenvalues of B we solve:

$$det\begin{pmatrix} 1 - \frac{\omega\sigma^2}{m\Omega_1}\Omega_2 - \mu & \frac{\omega\sigma^2}{m\Omega_1}\bar{R} \\ \Omega_2 & \bar{R} - \mu \end{pmatrix} = 0$$

So:

$$\mu^2 - (1 - \frac{\omega\sigma^2}{m\Omega_1}\Omega_2 + \bar{R})\mu + \bar{R} = 0$$

Thus, the eigenvalues are given by:

$$\mu_{1,2} = \frac{1 - \frac{\omega\sigma^2}{m\Omega_1}\Omega_2 + \bar{R} \pm \sqrt{\left(1 - \frac{\omega\sigma^2}{m\Omega_1}\Omega_2 + \bar{R}\right)^2 - 4\bar{R}}}{2\bar{R}} \tag{62}$$

If the portfolio balance were not present, such that $\frac{\omega}{m}\sigma^2 = 0$,

$$\mu_{1,2}^{NP} = \frac{1 + \bar{R} \pm \sqrt{\left(1 + \bar{R}\right)^2 - 4\bar{R}}}{2\bar{R}} \tag{63}$$

Where $\mu_1 + \mu_2 = 1 + \frac{\omega\sigma^2}{m\Omega_1}\Omega_2 + 1/\beta$ and $\mu_1 \times \mu_2 = 1/\beta$. Where we have used $\bar{R} = 1/\beta$.
Equation (62) shows that without the endogenous risk premium resulting from the FX dealers' problem, the dynamics of the model will present a unit root, ruling out convergence to the steady-state, since the eigenvalues become: $\mu_1^{NP} = 1$ and $\mu_2^{NP} = \bar{R}^* > 1$,
When this channel is present and we allow for parameter combinations such that $\Omega_2/\Omega_1 > 0$, we can obtain a unique stable equilibrium for the model, such that:

$$0 < \mu_1 \leq 1 < \frac{1}{\beta} \leq \mu_2.$$

yielding stable dynamics.

D Welfare

D.1 Optimal τ^H

We assume the policy maker can set a fiscal tool in the shape of a fixed subsidy to labor that maximizes the steady-state welfare. Using the steady-state values for C and L we obtain:

$$\bar{\mathbb{W}} = \frac{1}{1-\beta}V \tag{64}$$

$$= \frac{1}{1-\beta}\left(\ln \mathcal{S}^{\gamma-1}L - \frac{L^{1+\chi}}{1+\chi}\right) \tag{65}$$

$$= \frac{1}{1-\beta}\left[\ln\left(\left(\frac{(1-\gamma)}{(1-\gamma^*)}\frac{1}{\bar{C}^*}\right)^{\gamma-1}\left(\frac{1}{(1-\tau^H)}\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{\gamma}{1+\chi}}\right) - \frac{1}{1+\chi}\left(\frac{1}{(1-\tau^H)}\frac{\varepsilon-1}{\varepsilon}\right)\right] \tag{66}$$

Now we take the first order condition with respect to τ^H

$$\frac{\partial \mathbb{W}}{\partial \tau^H} = \frac{1}{1-\beta}\left[\frac{\left(\frac{(1-\gamma)}{(1-\gamma^*)}\frac{1}{\bar{C}^*}\right)^{\gamma-1}\frac{\gamma}{1+\chi}\left(\frac{1}{(1-\tau^H)}\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{\gamma}{1+\chi}-1}\frac{\varepsilon-1}{\varepsilon}\frac{1}{(1-\tau^H)^2}}{\left[\left(\frac{(1-\gamma)}{(1-\gamma^*)}\frac{1}{\bar{C}^*}\right)^{\gamma-1}\left(\frac{1}{(1-\tau^H)}\frac{\varepsilon-1}{\varepsilon}\right)^{\frac{\gamma}{1+\chi}}\right]} - \frac{1}{1+\chi}\frac{\varepsilon-1}{\varepsilon}\frac{1}{(1-\tau^H)^2}\right] \tag{67}$$

$$= \frac{1}{1-\beta}\left[\frac{\frac{\gamma}{1+\chi}}{(1-\tau^H)} - \frac{1}{1+\chi}\frac{\varepsilon-1}{\varepsilon}\frac{1}{(1-\tau^H)^2}\right] \tag{68}$$

$$= \frac{1}{1-\beta}\frac{1}{(1+\chi)(1-\tau^H)}\left[\gamma - \frac{\varepsilon-1}{\varepsilon}\frac{1}{(1-\tau^H)}\right] \tag{69}$$

$$\tag{70}$$

Thus:

$$1 - \tau^H = \frac{\varepsilon - 1}{\varepsilon} \frac{1}{\gamma} \tag{71}$$

and:

$$\tau^H = \frac{\varepsilon\gamma - \varepsilon + 1}{\varepsilon\gamma} \tag{72}$$

And:

$$\frac{\partial \tau^H}{\partial \gamma} < 0. \tag{73}$$

As the economy opens up (lower γ), the incentives for terms of trade manipulation increase. From here we can recover the steady state value for labour. Replacing the optimal subsidy into

D.2 Welfare under Flexible Prices

From the definition of the Backus-Smith wedge we have:

$$C_t = \Lambda_t \mathcal{S}_t^{\gamma} C_t^* \tag{74}$$

The home goods market clearing condition in (51):

$$\mathcal{S}_t = \frac{Y_t^H}{(1 - \gamma^* + \gamma\Lambda_t)C_t^*} \tag{75}$$

The optimal consumption-leisure decision in (31):

$$\frac{W_t}{P_t} = L^{\chi} C_t \tag{76}$$

From the technology:

$$L_t = \frac{Y_t^H}{A_t} \tag{77}$$

Finally, by correcting the market power using the optimal subsidy we obtain:

$$P_t^H Y_t^H = \frac{\varepsilon}{\varepsilon - 1}(1 - \tau_H) W_t L_t \tag{78}$$

Combining (76), (77) and (78), we obtain:

$$\frac{P_t^H Y_t^H}{P_t L_t} = \frac{\varepsilon}{\varepsilon - 1}(1 - \tau_H) \left(\frac{Y_t^H}{A_t}\right)^{\chi} C_t \tag{79}$$

We replace (74) into the last equation:

$$\frac{P_t^H Y_t^H}{P_t \Lambda_t \mathcal{S}_t^{\gamma} C_t^*} = \frac{\varepsilon}{\varepsilon - 1}(1 - \tau_H) \left(\frac{Y_t^H}{A_t}\right)^{1+\chi}$$

Which yields:

$$Y_t^H = \left(A_t^{-(1+\chi)} \frac{\varepsilon}{\varepsilon - 1}(1 - \tau_H) \Lambda_t \mathcal{S}_t C_t^*\right)^{-\frac{1}{\chi}} \tag{80}$$

Replacing (75) into the last expression:

$$
\begin{aligned}
Y_{H,t} &= \left(A_t^{-(1+\chi)}\frac{\varepsilon}{\varepsilon-1}(1-\tau_H)\Lambda_t(\frac{Y_{H,t}}{(1-\gamma^*+\gamma\Lambda_t)C_t^*})C_t^*\right)^{-\frac{1}{\varphi}} \\
Y_{H,t} &= \qquad (81) \\
Y_{H,t} &= \left(A_t^{-(1+\chi)}\frac{\varepsilon}{\varepsilon-1}(1-\tau_H)\Lambda_t(\frac{1}{(1-\gamma^*+\gamma\Lambda_t)})\right)^{-\frac{1}{\chi}} \\
Y_{H,t}^{1+\frac{1}{\chi}} &= \left(A_t^{-(1+\chi)}\frac{\varepsilon}{\varepsilon-1}(1-\tau_H)\Lambda_t(\frac{1}{(1-\gamma^*+\gamma\Lambda_t)})\right)^{-\frac{1}{\chi}} \\
Y_{H,t}^{1+\frac{1}{\chi}} &= \left(A_t^{-(1+\chi)}\frac{\frac{\varepsilon}{\varepsilon-1}(1-\tau_H)\Lambda_t}{1-\gamma^*+\gamma\Lambda_t}\right)^{-\frac{1}{\chi}} \\
Y_{H,t}^{\frac{1+\chi}{\chi}} &= \left(A_t^{-(1+\chi)}\frac{\frac{\varepsilon}{\varepsilon-1}(1-\tau_H)\Lambda_t}{1-\gamma^*+\gamma\Lambda_t}\right)^{-\frac{1}{\chi}} \qquad (82)
\end{aligned}
$$

$$
Y_{H,t}^{\frac{1+\chi}{\chi}} = \left(A_t^{-(1+\chi)}\frac{\frac{\varepsilon}{\varepsilon-1}(1-\tau_H)\Lambda_t}{1-\gamma^*+\gamma\Lambda_t}\right)^{-\frac{1}{\chi}} \tag{83}
$$

$$
Y_{H,t} = A_t\left(\frac{1-\gamma^*+\gamma\Lambda_t}{\frac{\varepsilon}{\varepsilon-1}(1-\tau_H)\Lambda_t}\right)^{\frac{1}{1+\chi}} \tag{84}
$$

Now we replace (83) into (75):

$$
\begin{aligned}
\mathcal{S}_t &= \frac{(A_t^{-(1+\chi)}\Lambda_t\mathcal{S}_tC_t^*)^{-\frac{1}{\chi}}}{(1-\gamma^*+\gamma\Lambda_t)C_t^*} \\
\mathcal{S}_t &= \frac{A_t^{\frac{1+\chi}{\chi}}\Lambda_t^{-\frac{1}{\chi}}\mathcal{S}_t^{-\frac{1}{\chi}}(C_t^*)^{-(1+\frac{1}{\chi})}}{1-\gamma^*+\gamma\Lambda_t} \\
\mathcal{S}_t^{\frac{1+\chi}{\chi}} &= \frac{A_t^{\frac{1+\chi}{\chi}}\Lambda_t^{-\frac{1}{\chi}}(C_t^*)^{-(\frac{1+\chi}{\chi})}}{1-\gamma^*+\gamma\Lambda_t} \\
\mathcal{S}_t &= \left(\frac{A_t^{\frac{1+\chi}{\chi}}\Lambda_t^{-\frac{1}{\chi}}(C_t^*)^{-(\frac{1+\chi}{\chi})}}{1-\gamma^*+\gamma\Lambda_t}\right)^{\frac{\chi}{1+\chi}} \\
\mathcal{S}_t &= \frac{A_t\Lambda_t^{-\frac{1}{1+\chi}}(C_t^*)^{-1}}{(1-\gamma^*+\gamma\Lambda_t)^{\frac{\chi}{1+\chi}}} \qquad (85)
\end{aligned}
$$

$$
\mathcal{S}_t = \frac{A_t\Lambda_t^{-\frac{1}{1+\chi}}(C_t^*)^{-1}}{(1-\gamma^*+\gamma\Lambda_t)^{\frac{\chi}{1+\chi}}} \tag{86}
$$

Now, we obtain an expression for C_t by substituting (86) into (74):

$$
C_t = (C_t^*)^{1-\gamma}A_t^{\gamma}\Lambda_t^{1-\frac{\gamma}{1+\chi}}\frac{1}{(1-\gamma^*+\gamma\Lambda_t)^{\frac{\gamma\chi}{1+\chi}}} \tag{87}
$$

And we get an expression for L_t by using (77) and (83):

$$L_t = \frac{(A_t^{-(1+\chi)} \frac{\Lambda_t}{1-\gamma^*+\gamma\Lambda_t})^{-\frac{1}{1+\chi}}}{A_t}$$
$$L_t = \left(\frac{\Lambda_t}{1-\gamma^*+\gamma\Lambda_t}\right)^{-\frac{1}{1+\chi}}$$
$$L_t^{1+\chi} = \left(\frac{\Lambda_t}{1-\gamma^*+\gamma\Lambda_t}\right)^{-1} \tag{88}$$

$$L_t^{1+\chi} = \left(\frac{\Lambda_t}{1-\gamma^*+\gamma\Lambda_t}\right)^{-1} \tag{89}$$

Now we replace both expression into the utility function in (25):

$$U_t = lnC_t - \frac{1}{1+\chi}L_t^{1+\chi}$$
$$U_t = \left[(1-\gamma)lnC_t^* + \gamma lnA_t + (1-\frac{\gamma}{1+\chi})ln\Lambda_t - \frac{\gamma\chi}{1+\chi}ln(1-\gamma^*+\gamma\Lambda_t)\right] - \left[\left(\frac{\Lambda_t}{1-\gamma^*+\gamma\Lambda_t}\right)^{-1}\right]$$
$$U_t = \left[\left(1-\frac{\gamma}{1+\chi}\right)ln\Lambda_t - \frac{\gamma\chi}{1+\chi}ln(1-\gamma^*+\gamma\Lambda_t)\right] - \left[\frac{1-\gamma^*+\gamma\Lambda_t}{\Lambda_t}\right] + ...t.i.p$$
$$U_t = \left(1-\frac{\gamma}{1+\chi}\right)ln\Lambda_t - \frac{\gamma\chi}{1+\chi}ln(1-\gamma^*+\gamma\Lambda_t) - \frac{1-\gamma^*}{\Lambda_t} + ...t.i.p \tag{90}$$

$$U_t = \left(1-\frac{\gamma}{1+\chi}\right)ln\Lambda_t - \frac{\gamma\chi}{1+\chi}ln(1-\gamma^*+\gamma\Lambda_t) - \frac{1-\gamma^*}{\gamma\Lambda_t} + ...t.i.p \tag{91}$$

We follow Benigno and Woodford (2012) and derive optimal policy by taking a second order approximation to the last expression. Define $\hat{X}_t$ as:

$$\frac{X_t - \bar{X}}{\bar{X}} = \hat{X}_t + \frac{1}{2}\hat{X}_t^2 + \mathcal{O}(\|a\|^3)$$

Using this notation, the second order approximation of (91) yields:

$$U_t = \left(1-\frac{\gamma}{1+\chi}\right)(\tilde{\Lambda}_t - \tilde{\Lambda}_t^2) - \frac{\chi\gamma}{1+\chi}(\gamma\tilde{\Lambda}_t - \gamma^2\tilde{\Lambda}_t^2) - (1-\gamma^*)(1-\tilde{\Lambda}_t+\tilde{\Lambda}_t^2) + ...t.i.p$$

Grouping the terms and assuming $\gamma^* = \gamma$:

$$U_t = \left(2-\frac{\gamma}{1+\chi}-\frac{\chi\gamma^2}{1+\chi}-\gamma\right)\tilde{\Lambda}_t - \frac{\chi\gamma^3}{1+\chi}\tilde{\Lambda}_t^2 - (1-\gamma)(\tilde{\Lambda}_t^2) - \left(1-\frac{\gamma}{1+\chi}\right)\tilde{\Lambda}_t^2 + ...t.i.p$$
$$U_t = \left(2-\frac{\gamma}{1+\chi}-\frac{\chi\gamma^2}{1+\chi}-\gamma\right)\tilde{\Lambda}_t - \left(2-\frac{\gamma}{1+\chi}+\frac{\chi\gamma^3}{1+\chi}-\gamma\right)\tilde{\Lambda}_t^2 + ...t.i.p$$

$$U_t = \left(2-\frac{\gamma}{1+\chi}-\frac{\chi\gamma^2}{1+\chi}-\gamma\right)\tilde{\Lambda}_t - \left(2-\frac{\gamma}{1+\chi}+\frac{\chi\gamma^3}{1+\chi}-\gamma\right)\tilde{\Lambda}_t^2 + t.i.p. \tag{92}$$

Now, we define $\hat{\Lambda}_t$ as

$$\tilde{\Lambda}_t = \hat{\Lambda}_t + \frac{1}{2}\hat{\Lambda}_t^2 + \mathcal{O}(\|a\|^3)$$

Using this notation we can express U_t as:

$$U_t = \Phi_1\hat{\Lambda}_t - \Phi_2\hat{\Lambda}_t^2 + \mathcal{O}(\|a\|^3) + t.i.p. \tag{93}$$

where:

$$\Phi_1 \equiv (1-\gamma)\frac{(2+\chi(2+\gamma))}{1+\chi}$$
$$\Phi_2 \equiv \frac{\left((1-\gamma)\left(1-\frac{\chi\gamma}{2}\right)+\chi\gamma^3\right)}{1+\chi}$$

If $0 \leq \gamma \leq 1$, then $\Phi_1 \geq 0$ and $\Phi_2 \geq 0$. Notice that in then natural equilibrium $\Lambda_t^n = 1$. Then, we can express (93) as deviations from natural equilibrium:

$$U_t = \Phi_1 \lambda_t - \Phi_2 \lambda_t^2 + \mathcal{O}(\|a\|^3) + t.i.p. \tag{94}$$

To approximate linear part, we start with the definition of Λ_t. We use the Euler equations of the home and foreign economy:

$$\frac{1}{C_t} = \beta E_t R_t \frac{P_t}{P_{t+1}} \frac{1}{C_{t+1}}$$
$$\frac{1}{C_t^*} = \beta E_t R_t^* \frac{P_t^*}{P_{t+1}^*} \frac{1}{C_{t+1}^*}$$

Taking the ratio, we obtain:

$$\frac{C_t^*}{C_t} = \frac{R_t}{R_t^*} E_t \frac{\frac{P_t}{P_{t+1}}}{\frac{P_t^* S_t/S_t}{P_{t+1}^* S_{t+1}/S_{t+1}}} \frac{C_{t+1}^*}{C_{t+1}}$$
$$\frac{C_t^*}{C_t} = \frac{R_t}{R_t^*} E_t \frac{1}{\frac{Q_t/S_t}{Q_{t+1}/S_{t+1}}} \frac{C_{t+1}^*}{C_{t+1}}$$
$$\frac{Q_t C_t^*}{C_t} = \frac{R_t}{R_t^* \frac{S_{t+1}}{S_t}} E_t \frac{Q_{t+1} C_{t+1}^*}{C_{t+1}}$$
$$\frac{1}{\Lambda_t} = \frac{R_t}{R_t^* E_t \frac{S_{t+1}}{S_t} \Lambda_{t+1}}$$

Log-linearizing the last expression:

$$E_t \lambda_{t+1} - \lambda_t = i_t - i_t^* - E_t \Delta s_{t+1} \tag{95}$$

Using the consolidated budget constraint from households (26):

$$B_{t+1} = P_t^H Y_t - P_t C_t + R_{t-1} B_t + \left(R_{t-1} - R_{t-1}^* \frac{S_t}{S_{t-1}}\right)(B_t^{cb} + D_t)$$

From the domestic bond market clearing: $D_t = -N_t - B_t - B_t^{cb}$. We can rewrite the budget constraint as:

$$B_{t+1} = P_t^H Y_t - P_t C_t + R_{t-1} B_t + \left(R_{t-1} - R_{t-1}^* \frac{S_t}{S_{t-1}}\right)(-B_t - N_t) \tag{96}$$
$$B_{t+1} = P_t^H Y_t - P_t C_t + R_{t-1}^* \frac{S_t}{S_{t-1}} B_t + \left(R_{t-1} - R_{t-1}^* \frac{S_t}{S_{t-1}}\right)(-N_t) \tag{97}$$

We take the approximation to equation (96):

$$b_{t+1} = (\gamma - 1)\tilde{\Lambda}_t \bar{C}^* + \frac{1}{\beta} b_t \tag{98}$$

We begin at $t = 0$.

$$
\begin{aligned}
&b_1 = (\gamma - 1)\tilde{\Lambda}_0 \bar{C}^* \\
&\beta b_2 - b_1 = \beta(\gamma - 1)\tilde{\Lambda}_1 \bar{C}^* \\
&\beta b_2 - (\gamma - 1)\tilde{\Lambda}_0 \bar{C}^* = \beta(\gamma - 1)\tilde{\Lambda}_1 \bar{C}^* - (\gamma - 1)\tilde{\Lambda}_0 \bar{C}^* \\
&\beta b_2 = (\gamma - 1)\tilde{\Lambda}_1 \beta \bar{C}^* + (\gamma - 1)\tilde{\Lambda}_0 \bar{C}^*
\end{aligned}
$$

Next period yields:

$$
\beta^2 b_3 = (\gamma - 1)\bar{C}^* \left(\tilde{\Lambda}_2 \beta^2 + \tilde{\Lambda}_1 \beta + \tilde{\Lambda}_0 \right)
$$

Solving forward:

$$
0 = -\sum_{t=0}^{\infty} (1 - \gamma)\bar{C}^* \beta^t \left(\tilde{\Lambda}_t \right) \tag{99}
$$

or:

$$
\sum_{t=0}^{\infty} (1 - \gamma)\bar{C}^* \beta^t \lambda_t = -\sum_{t=0}^{\infty} (1 - \gamma)\bar{C}^* \beta^t \lambda_t^2 \tag{100}
$$

We replace the last expression in (93) and obtain:

$$
\begin{aligned}
\sum_{t=0}^{\infty} \beta^t U_t &= \sum_{t=0}^{\infty} \Phi_1 \beta^t \lambda_t - \sum_{t=0}^{\infty} \Phi_2 \beta^t \lambda_t^2 + \mathcal{O}(\|a\|^3) + t.i.p. \\
&= \frac{\Phi_1}{\bar{C}^*(1 - \gamma)} \left(-\sum_{t=0}^{\infty} (1 - \gamma)\bar{C}^* \beta^t \lambda_t^2 \right) - \Phi_2 \sum_{t=0}^{\infty} \beta^t \lambda_t^2 + \mathcal{O}(\|a\|^3) + t.i.p.
\end{aligned}
$$

Assuming $\bar{C}^* = 1$, then:

$$
\mathbb{W} \equiv \sum_{t=0}^{\infty} \beta^t U_t = -(\Phi_1 + \Phi_2) \sum_{t=0}^{\infty} \beta^t \lambda_t^2 + \mathcal{O}(\|a\|^3) + t.i.p. \tag{101}
$$

The constrained central planner maximizes (101) subject to

$$
b_{t+1} - \frac{1}{\beta} b_t = -(1 - \gamma)\lambda_t \tag{102}
$$

and

$$
\lambda_{t+1} - \lambda_t = \frac{\omega \sigma_e^2}{m} \left(-\bar{Y} b_{t+1} - \bar{Y} b_{t+1}^{cb} - n\psi_t \right) \tag{103}
$$

Now we obtain the optimal policy. First, we build the Lagrangian:

$$
\begin{aligned}
\mathcal{L}_t = &\max_{\{\lambda_t, b_{t+1}, b_{t+1}^{cb}\}} E_t[\ldots + \beta^t \{(1 - \gamma)\frac{1}{2}\lambda_t^2 [1 + \gamma + \gamma^2] \\
&+ \mu_t^{BG}(b_{t+1} - \frac{b_t}{\beta} - \lambda_t) + \mu_t^{EU}(\lambda_{t+1} - \lambda_t - \frac{\omega \sigma_e^2}{m}(-\bar{Y} b_{t+1} - \bar{Y} b_{t+1}^{cb} + n\psi_t)\}]
\end{aligned}
$$

First order conditions are given by:

$$\lambda_t : E_t\{(1-\gamma)\lambda_t[1+\gamma+\gamma^2] - \mu_t^{BG} + \mu_t^{EU}(-1) + \beta^{-1}\mu_{t-1}^{EU}\} = 0 \tag{104}$$

$$b_{t+1}^{cb} : E_t\{\mu_t^{EU}\frac{\sigma_e^2\omega}{m}\bar{Y}\} = 0 \tag{105}$$

$$b_{t+1} : \mu_t^{BG} - E_t\mu_{t+1}^{BG} + \mu_t^{EU}(-1)\frac{\sigma_e^2\omega}{m}(-\bar{Y}) = 0 \tag{106}$$

From equation (105):

$$E_t\mu_{t+1}^{EU} = \mu_t^{EU} = 0$$

Replacing the result in (106):

$$\mu_t^{BG} = E_t\mu_{t+1}^{BG} \tag{107}$$

Replacing in (104) the optimal plan must implement:

$$\lambda_t = E_t\lambda_{t+1} \tag{108}$$

A solution for (108) is $\lambda_t = \bar{\lambda}$ for all t. We can pick $\bar{\lambda} = 0$, which yields:

$$b_{t+1} = \frac{1}{\beta}b_t \tag{109}$$

$$-\bar{Y}b_{t+1} - \bar{Y}b_{t+1}^{cb} + n\psi_t = 0 \tag{110}$$

From the last equation, **the optimal FX policy** is given by:

$$b_{t+1}^{cb} = -\bar{Y}b_{t+1} + n\psi_t \tag{111}$$

The central bank can implement the constrained central planner optimal allocation by offsetting the excess position of the financial intermediaries.

With this, we implement the optimal allocation that consists in a constant level of labour, and a consumption that depends on foreign demand and productivity shocks.

D.3 Welfare under Price Rigidities

The first term in the utility function follows the derivations from the previous section. For the second term we start by taking a second order approximation:

$$\frac{L_t^{1+\chi}}{1+\chi} = \frac{\bar{L}^{1+\chi}}{1+\chi} + \bar{L}^{1+\chi}\left[\tilde{L}_t + \frac{1}{2}(1+\chi)\tilde{L}_t^2\right] + \mathcal{O}(\|a\|^3) \tag{112}$$

The next step is to use the labor market clearing:

$$L_t = \left(\frac{Y_t}{A_t}\right)\int_0^1 \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\varepsilon} dj \tag{113}$$

we have:

$$l_t = y_t^H - a_t + z_t \tag{114}$$

The price dispersion is given by:

$$z_t \equiv \log \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \tag{115}$$

Let $\hat{p}_t^H(j) \equiv p_t^H(j) - p_t^H$. Now:

$$\left(\frac{P_t^H(j)}{P_t^H}\right)^{1-\varepsilon} = exp[(1-\varepsilon)\hat{p}_t^H(j)] \tag{116}$$

$$= 1 + (1-\varepsilon)\hat{p}_t^H(j) + \frac{(1-\varepsilon)^2}{2}\hat{p}_t^H(j)^2 + \wr(\|a\|^3) \tag{117}$$

From the definition of P_t^H:

$$1 = \int_0^1 \left(\frac{P_t^H(j)}{P_t^H}\right)^{1-\varepsilon} dj \tag{118}$$

Integration over (116):

$$E_j\hat{p}_t^H(j) = \frac{(\varepsilon-1)}{2}E_j\left[(\hat{p}_t^H(j))^2\right] + \mathcal{O}(\|a\|^3) \tag{119}$$

Taking a second order approximation to $\left(\frac{P_t^H(j)}{P_t^H}\right)^{-\varepsilon}$:

$$\left(\frac{P_t^H(j)}{P_t^H}\right)^{-\varepsilon} = exp[-\varepsilon\hat{p}_t^H(j)] \tag{120}$$

$$= 1 - \varepsilon\hat{p}_t^H(j) + \frac{\varepsilon^2}{2}\left(\hat{p}_t^H(j)\right)^2 + \mathcal{O}(\|a\|^3) \tag{121}$$

We take the integral and obtain:

$$\int_0^1 \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\varepsilon} di = 1 - \varepsilon E_j[\hat{p}_t^H(j)] + \frac{\varepsilon^2}{2}E_j\left[\left(\hat{p}_t^H(j)\right)^2\right] + \mathcal{O}(\|a\|^3) \tag{122}$$

From (119):

$$\int_0^1 \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\varepsilon} dj = 1 - \frac{\varepsilon(\varepsilon-1)}{2}E_j\left[\left(\hat{p}_t^H(i)\right)^2\right] + \frac{\varepsilon^2}{2}E_j\left[\left(\hat{p}_t^H(j)\right)^2\right] + \mathcal{O}(\|a\|^3) \tag{123}$$

$$= 1 + \frac{\varepsilon}{2}E_j\left[\left(\hat{p}_t^H(j)\right)^2\right] \tag{124}$$

which yields:

$$\int_0^1 \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\varepsilon} di = 1 + \frac{\varepsilon}{2}var_j\left[p_t^H(j)\right] \tag{125}$$

This allows us to rewrite the second order approximation to the disutility of labor as:

$$\frac{L_t^{1+\chi}}{1+\chi} = \frac{\bar{L}^{1+\chi}}{1+\chi} + \bar{L}^{1+\chi}\left[y_t^H - a_t + z_t + \frac{1}{2}(1+\chi)(y_t^H - a_t)^2\right] + \mathcal{O}(\|a\|^3) \tag{126}$$

since the higher order terms of z are part of the approximation error. Under the optimal subsidy scheme, the optimality condition $\bar{L}_t^{1+\chi} = \gamma$ (see ...). Thus:

$$lnC_t = ln\Lambda_t + \gamma lnY_t^H - \gamma ln(1-\gamma^* + \gamma\Lambda_t) + t.i.p.$$

$$lnC_t = (1-\gamma^2)\lambda_t - \frac{1}{2}(1-\gamma)\gamma^2\lambda_t^2 + \gamma lnY_t^H + t.i.p.$$

$$U(C_t, L_t) = (1-\gamma^2)\lambda_t - \frac{1}{2}(1-\gamma)\gamma^2\lambda_t^2 + \gamma\hat{Y}_t^H - \bar{L}^{1+\chi}\left[\hat{Y}_t^H + z_t + \frac{1}{2}(1+\chi)(\hat{Y}_t^H)^2\right] + \mathcal{O}(\|a\|^3) \tag{127}$$

$$U(C_t, L_t) = (1-\gamma^2)\lambda_t - \frac{1}{2}(1-\gamma)\gamma^2\lambda_t^2 + \gamma\hat{Y}_t^H - \gamma\left[\hat{Y}_t^H + z_t + \frac{1}{2}(1+\chi)(\hat{Y}_t^H)^2\right] + t.i.p + \mathcal{O}(\|a\|^3) \tag{128}$$

$$U(C_t, L_t) = (1-\gamma^2)\lambda_t - \frac{1}{2}(1-\gamma)\gamma^2\lambda_t^2 - \gamma z_t - \gamma\frac{(1+\chi)}{2}(\hat{Y}_t^H)^2 + t.i.p + \mathcal{O}(\|a\|^3) \tag{129}$$

Calvo pricing implies that each period, the distribution of prices consists on θ times the distribution of prices in the previous period plus an atom size of $1-\theta$ at $p_t^{H,*}$ the price. Letting:

$$\tilde{P}^H \equiv E_i \log p_t^H(i) \tag{130}$$

one observes from this recursive characterization of the distribution of prices at date t that:

$$\begin{aligned}
\tilde{P}_t^H - \tilde{P}_{t-1}^H &= E_i\left[\log p_t^H(i) - \tilde{P}_{t-1}^H\right] && (131)\\
&= \theta E_i\left[\log p_t^H(i) - \tilde{P}_{t-1}^H\right] + (1-\theta)\left[\log p_t^{H,*} - \tilde{P}_{t-1}^H\right] && (132)\\
&= (1-\theta)\left[\log p_t^{H,*} - \tilde{P}_{t-1}^H\right] && (133)
\end{aligned}$$

Similar reasoning about the dispersion measure yields:

$$\begin{aligned}
\Delta_t^H &= var_i\left[\log p_t^H(i) - \tilde{P}_{t-1}^H\right] && (134)\\
&= E_i\left\{[\log p_t^H(i) - \tilde{P}_{t-1}^H]^2\right\} - \left(E_i \log p_t^H(i) - \tilde{P}_{t-1}^H\right)^2 && (135)\\
&= \theta E_i\left\{[\log p_{t-1}^H(i) - \tilde{P}_{t-1}^H]^2\right\} + (1-\theta)\left(\log p_t^{H,*} - \tilde{P}_{t-1}^H\right)^2 - \left(\tilde{P}_t^H - \tilde{P}_{t-1}^H\right)^2 && (136)\\
&= \theta\Delta_{t-1}^H + \frac{(1-\theta)}{(1-\theta)^2}\left(\tilde{P}_t^H - \tilde{P}_{t-1}^H\right)^2 - \left(\tilde{P}_t^H - \tilde{P}_{t-1}^H\right)^2 && (137)\\
&= \theta\Delta_{t-1}^H + \frac{\theta}{1-\theta}\left(\tilde{P}_t^H - \tilde{P}_{t-1}^H\right)^2 && (138)
\end{aligned}$$

Finally,$\tilde{P}_t^H$ can be related to the Dixit-Stiglitz price index through a linear approximation:

$$\tilde{P}_t^H = logP_t^H + \mathcal{O}\left(\|\Delta_{t-1}^{1/2}, \varphi, \tilde{\xi}\|^2\right) \tag{139}$$

In specific we have:

$$P_t^H = \left(\int_0^1 P_t^H(i)^{1-\varepsilon}di\right)^{\frac{1}{1-\varepsilon}} \tag{140}$$

We first express it in terms of $\log p_t^H(i)$:

$$P_t^H = \left(\int_0^1 exp((1-\varepsilon)\tilde{p}_t^H(i))di\right)^{\frac{1}{1-\varepsilon}} \tag{141}$$

The linear approximation yields:

$$\begin{aligned}
\log P_t^H &= \frac{1}{1-\varepsilon}\log\left(\int_0^1 \exp((1-\varepsilon)\tilde{p}_t^H(i))di\right)\\
&\approx \left[\frac{1}{1-\varepsilon}\frac{1}{\int_0^1 exp((1-\varepsilon)\tilde{p}_t^H(i))di} \times (1-\varepsilon)exp((1-\varepsilon)\tilde{p}_t^H(i))\right]\Bigg|_{ss}\int_0^1 \tilde{p}_t^H(i)di\\
&\approx E_i \log p_t^H(i) = \tilde{P}_t^H
\end{aligned}$$

Finally we obtain:

$$\Delta_t^H = \theta\Delta_{t-1}^H + \frac{\theta}{1-\theta}\left(\pi_t^H\right)^2 + \mathcal{O}\left(\|\Delta_{t-1}^{1/2}, \varphi, \tilde{\xi}\|^2\right)$$

Assuming $\Delta_{-1} = 0$:

$$\mathbb{W} \equiv \sum_{t=0}^{\infty} \beta^t \left[(1-\gamma^2)\hat{\Lambda}_t - \frac{1}{2}(1-\gamma)\gamma^2\hat{\Lambda}_t^2 - \gamma\frac{\varepsilon}{2}\frac{\theta}{(1-\beta\theta)(1-\theta)}(\pi_t^H)^2 - \gamma\frac{(1+\chi)}{2}(\hat{Y}_t^H)^2\right] + t.i.p$$

We still have the linear term of the wedge, which we must take out. Here we must use the budget constraint, though it is a bit problematic. We define:

$$\begin{aligned} nx_t &= P_t^H Y_t^H - P_t C_t && (142)\\ &= \left(P_t^H Y_t^H - P_t C_t\right)\frac{P_t^F C_t^*}{P_t^F C_t^*} && (143)\\ &= \left(\frac{P_t^H Y_t^H}{P_t^F C_t^*} - \frac{P_t C_t}{P_t^F C_t^*}\right) P_t^F C_t^* && (144)\\ &= \left(\frac{Y_t^H}{S C_t^*} - \frac{C_t}{Q_t C_t^*}\right) P_t^F C_t^* && (145) \end{aligned}$$

Now we use:

$$\begin{aligned} \frac{C_t}{Q_t C_t^*} &= \Lambda_t && (146)\\ \frac{Y_{H,t}}{S_t C_t^*} &= 1-\gamma+\gamma\Lambda_t && (147) \end{aligned}$$

Thus, (142) becomes:

$$\begin{aligned} nx_t &= (1-\gamma+\gamma\Lambda_t-\Lambda_t)P_t^F C_t^* && (148)\\ -b_0 &= \sum_{t=0}^{\infty}\beta^t(1-\gamma+\gamma\Lambda_t-\Lambda_t)P_t^F C_t^* && (149)\\ \sum_{t=0}^{\infty}\beta^t &- (1-\gamma)(\Lambda_t-1)P_t^F C_t^* = 0 && (150)\\ \sum_{t=0}^{\infty}\beta^t&(-(1-\gamma)\bar{C}^*\bar{P}^F(\Lambda_t-1)+0+\ldots) = 0 && (151)\\ \sum_{t=0}^{\infty}\beta^t&\left(\frac{\Lambda_t-1}{1}\right) = 0 && (152)\\ \sum_{t=0}^{\infty}\beta^t&\left(\tilde{\Lambda}_t\right) = 0 && (153)\\ \sum_{t=0}^{\infty}\beta^t&\left(\hat{\Lambda}_t+\frac{1}{2}\hat{\Lambda}_t^2\right) = 0 && (154)\\ -\sum_{t=0}^{\infty}\beta^t\left(\hat{\Lambda}_t\right) &= \sum_{t=0}^{\infty}\beta^t\left(\frac{1}{2}\hat{\Lambda}_t^2\right) && (155) \end{aligned}$$

$$\mathbb{W} \equiv \sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2}\left((1-\gamma^2)+(1-\gamma)\gamma^2\right)\hat{\Lambda}_t^2 - \gamma\frac{\varepsilon}{2}\frac{\theta}{(1-\beta\theta)(1-\theta)}(\pi_t^H)^2 - \gamma\frac{(1+\chi)}{2}(\hat{Y}_t^H)^2\right] + t.i.p$$
$$\mathbb{W} \equiv \sum_{t=0}^{\infty}\beta^t\left[-\frac{1}{2}\left(1-\gamma^3\right)\hat{\Lambda}_t^2 - \gamma\frac{\varepsilon}{2}\frac{\theta}{(1-\beta\theta)(1-\theta)}(\pi_t^H)^2 - \gamma\frac{(1+\chi)}{2}(\hat{Y}_t^H)^2\right] + t.i.p$$

Finally, we obtain:

$$\mathbb{W} \equiv -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left(\phi_\lambda \hat{\Lambda}_t^2 + \phi_\pi (\pi_t^H)^2 + \phi_y (\hat{Y}_t^H)^2\right) \tag{156}$$

where:

$$\phi_\lambda = 1 - \gamma^3 \tag{157}$$

$$\phi_\pi = \frac{\gamma\varepsilon\theta}{(1-\beta\theta)(1-\theta)} \tag{158}$$

$$\phi_y = \gamma(1+\chi) \tag{159}$$

The loss function in presence of price frictions is a function of quadratic terms on the Backus-Smith wedge, producer inflation and output.

$$\mathbb{W} \equiv \frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left(\phi_\lambda \lambda_t^2 + \phi_\pi (\pi_t^H)^2 + \phi_y (y_t^H)^2\right) \tag{160}$$

Now, we set the optimal problem of for the central planner:

$$\max_{\hat{b}^{cb,*}, i_t} \mathbb{W} \equiv -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left(\phi_\lambda \lambda_t^2 + \phi_\pi (\pi_t^H)^2 + \phi_y (y_t^H)^2\right) \tag{161}$$

subject to the following constraints given by the model dynamics:

$$\begin{aligned}
&b_{t+1} - \frac{1}{\beta}b_t = -(1-\gamma)\lambda_t \\
&E_t\lambda_{t+1} - \lambda_t = i_t - i_t^* - E_t dep_{t+1} \\
&y_t^H = \phi_c\left((1-\gamma)\hat{\mathcal{S}}_t + c_t\right) + \phi_{c^*}\left(\hat{\mathcal{S}}_t + c_t^*\right) \\
&\pi_t^H = \beta E_t\left(\pi_{t+1}^H\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(\lambda_t + \gamma\hat{\mathcal{S}}_t + c_t^* + \chi y_t^H - (1+\chi)a_t\right) \\
&b_{t+1} = -n_{t+1} - b_{t+1}^{cb} + \frac{m}{\omega\sigma_e^2}E_t\left[i_t^* + dep_{t+1} - i_t\right]
\end{aligned}$$

We still need to take away the $\hat{\mathcal{S}}_t$.
Log-linearizing:

$$e^{ln\hat{\mathcal{S}}_t} = \frac{e^{lnY_t^H}}{\left[\gamma e^{ln\Lambda_t} + 1 - \gamma^*\right] e^{lnC_t^*}} \tag{162}$$

We obtain:

$$\hat{\mathcal{S}}_t = y_t^H - c_t^* - \gamma\frac{1}{[\gamma\bar{\Lambda} + 1 - \gamma^*]}\lambda_t \tag{163}$$

with $\bar{\Lambda} = 1$, we obtain:

$$\hat{\mathcal{S}}_t = y_t^H - c_t^* - \frac{\gamma}{[1+\gamma-\gamma^*]}\lambda_t \tag{164}$$

with $\gamma = \gamma^*$,

$$\hat{\mathcal{S}}_t = y_t^H - c_t^* - \gamma\lambda_t \tag{165}$$

Additional equations: From (1) and (2)

$$C_t = \Lambda_t \mathcal{S}_t^{\gamma} C_t^*$$
$$\mathcal{S}_t = \frac{Y_{H,t}}{(1-\gamma^* + \gamma\Lambda_t)C_t^*}$$

$$C_t = \Lambda_t \left(\frac{Y_{H,t}}{(1-\gamma^* + \gamma\Lambda_t)C_t^*} \right)^{\gamma} C_t^*$$
$$c_t = \lambda_t + \gamma \left(y_t^H - c_t^* - \frac{\gamma}{[1+\gamma-\gamma^*]}\lambda_t \right) + c_t^*$$
$$c_t = \left(1 - \frac{\gamma^2}{1+\gamma-\gamma^*}\right)\lambda_t + \gamma y_{H,t} + (1-\gamma)c_t^*$$

From Euler Equation

$$E_t C_{t+1} = \beta E_t \frac{1+i_t}{\frac{P_{t+1}}{P_t}} C_t$$
$$E_t c_{t+1} = ln\beta + ln(1+i_t) - E_t ln(1+\pi_{t+1}) + c_t$$
$$E_t c_{t+1} = i_t - \rho - \pi_{t+1} + c_t$$
$$c_t = E_t c_{t+1} - (i_t - \rho - E_t\pi_{t+1})$$
$$\left(1 - \frac{\gamma^2}{1+\gamma-\gamma^*}\right)\lambda_t + \gamma y_{H,t} + (1-\gamma)c_t^* = E_t \left(\left(1 - \frac{\gamma^2}{1+\gamma-\gamma^*}\right)\lambda_{t+1} + \gamma y_{H,t+1} + (1-\gamma)c_{t+1}^* \right)$$
$$\gamma y_{H,t} = -(1-\gamma)(c_t^* - E_t c_{t+1}^*) - \left(1 - \frac{\gamma^2}{1+\gamma-\gamma^*}\right)(\lambda_t - E_t\lambda_{t+1}) + E_t \gamma y_{H,t+1}$$
$$\gamma y_{H,t} = (1-\gamma)(E_t c_{t+1}^* - c_t^*) + \left(1 - \frac{\gamma^2}{1+\gamma-\gamma^*}\right)(E_t\lambda_{t+1} - \lambda_t) + E_t \gamma y_{H,t+1}$$
$$y_{H,t} = \frac{(1-\gamma)}{\gamma}(E_t c_{t+1}^* - c_t^*) + \frac{1}{\gamma}\left(1 - \frac{\gamma^2}{1+\gamma-\gamma^*}\right)(E_t\lambda_{t+1} - \lambda_t) + E_t y_{H,t+1}$$
$$y_{H,t} = \frac{(1-\gamma)}{\gamma}(E_t c_{t+1}^* - c_t^*) + \left(\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*}\right)(E_t\lambda_{t+1} - \lambda_t) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi_{t+1})$$

To reduce last expression, I will define $c_t^* = 0$

$$y_{H,t} = \left(\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*}\right)(E_t\lambda_{t+1} - \lambda_t) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi_{t+1}) \tag{166}$$

The marginal cost and $\mathcal{S}_t^{1-\gamma} = \frac{P_{F,t}^{1-\gamma}}{P_{H,t}^{1-\gamma}} = \frac{P_{F,t}^{1-\gamma}P_{H,t}^{\gamma}}{P_{H,t}^{1-\gamma}P_{H,t}^{\gamma}} = \frac{P_t}{P_{H,t}}$ and $\mathcal{S}_t^{1-\gamma}C_t = \frac{\Lambda_t}{(1-\gamma^*+\gamma\Lambda_t}Y_{H,t}$

$$mc_t = -v + (w_t - p_{H,t}) - a_t$$
$$mc_t = -v + w_t - p_t + (p_t - p_{H,t}) - a_t$$
$$mc_t = -v + \chi l_t + c_t + (p_t - p_{H,t}) - a_t$$
$$mc_t = -v + \chi y_{H,t} + (c_t + (1-\gamma)\tilde{\mathcal{S}}_t) - a_t$$
$$mc_t = -v + \chi y_{H,t} + (\lambda_t - \gamma\lambda_t + y_{H,t}) - a_t$$
$$mc_t = -v + (1+\chi) y_{H,t} + (1-\gamma)\lambda_t - a_t$$

Then inflations is:

$$\pi_t^H = \beta E_t\left(\pi_{t+1}^H\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}(mc_t)$$
$$\pi_t^H = \beta E_t\left(\pi_{t+1}^H\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}(-v + (1+\chi)y_{H,t} + (1-\gamma)\lambda_t - a_t)$$

The constrained central planner will choose the combination of the nominal interest rate (i_T) and the amount of sterilized FX interventions b_t^* to maximize welfare. For this we use:

So:

We can express the dynamics as:

$$\begin{aligned}
&b_{t+1} - \frac{1}{\beta} b_t = -(1-\gamma)\lambda_t \\
&E_t\lambda_{t+1} - \lambda_t = i_t - i_t^* - E_t dep_{t+1} \\
&\frac{m}{\omega\sigma_e^2} E_t\left[i_t^* + dep_{t+1} - i_t\right] = -n_{t+1} - b_{t+1}^{cb} - b_{t+1} \\
&y_{H,t} = \left(\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*}\right)(E_t\lambda_{t+1} - \lambda_t) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi_{t+1}) \\
&\pi_t^H = \beta E_t\left(\pi_{t+1}^H\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(-v + (1+\chi)y_{H,t} + (1-\gamma)\lambda_t - a_t\right) \\
&b_{t+1} = -n_{t+1} - b_{t+1}^{cb} + \frac{m}{\omega\sigma_e^2} E_t\left[i_t^* + dep_{t+1} - i_t\right]
\end{aligned}$$

We can reduce the system using:

$$E_t\left[i_t^* + dep_{t+1} - i_t\right] = \frac{\omega\sigma_e^2}{m}(-n_{t+1} - b_{t+1}^{cb} - b_{t+1})$$

Replacing in the system:

$$\begin{aligned}
&b_{t+1} - \frac{1}{\beta} b_t = -(1-\gamma)\lambda_t \\
&E_t\lambda_{t+1} - \lambda_t = \frac{\omega\sigma_e^2}{m}(-n_{t+1} - b_{t+1}^{cb} - b_{t+1}) \\
&y_{H,t} = (\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*})(E_t\lambda_{t+1} - \lambda_t) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi_{t+1}) \\
&\pi_t^H = \beta E_t\left(\pi_{t+1}^H\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(-v + (1+\chi)y_{H,t} + (1-\gamma)\lambda_t - a_t\right)
\end{aligned}$$

Even more with

$$E_t\lambda_{t+1} - \lambda_t = \frac{\omega\sigma_e^2}{m}(-n_{t+1} - b_{t+1}^{cb} - b_{t+1})$$

Thus, the system is defined by the next three equations:

$$\begin{aligned}
&b_{t+1} - \frac{1}{\beta} b_t = -(1-\gamma)\lambda_t \\
&y_{H,t} = (\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*})(\frac{\omega\sigma_e^2}{m}(-n_{t+1} - b_{t+1}^{cb} - b_{t+1})) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi_{t+1}) \\
&\pi_t^H = \beta E_t\left(\pi_{t+1}^H\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(-v + (1+\chi)y_{H,t} + (1-\gamma)\lambda_t - a_t\right)
\end{aligned}$$

We build the Lagrangian:

$$\begin{aligned}
\max_{b_{t+1}^{cb}, b_{t+1}, y_t, \pi_t^H} \mathcal{L} \equiv \sum_{t=0}^{\infty} \beta^t E_t \Bigg(&-\frac{1}{2}(\phi_\lambda \lambda_t^2 + \phi_\pi (\pi_t^H)^2 + \phi_y \left(y_t^H\right)^2) + \mu_{1,t}\left(-(1-\gamma)\lambda_t - b_{t+1} + \frac{1}{\beta} b_t\right) + \dots \\
&+ \mu_{2,t}\left((\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*})(\frac{\omega\sigma_e^2}{m}(-n_{t+1} - b_{t+1}^{cb} - b_{t+1})) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi_{t+1}) - y_{H,t}\right) + \dots \\
&+ \mu_{3,t}\left(\beta E_t\left(\pi_{t+1}^H\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(-v + (1+\chi)y_{H,t} + (1-\gamma)\lambda_t - a_t\right) - \pi_t^H\right)\Bigg)
\end{aligned} \tag{167}$$

FOC yield:

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = -\phi_\lambda \lambda_t - \mu_{1,t}(1-\gamma) + \mu_{3,t}\frac{(1-\theta)(1-\beta\theta)}{\theta}(1-\gamma) = 0 \tag{168}$$
$$\frac{\partial \mathcal{L}}{\partial b_{t+1}} = -\mu_{1,t} + E_t\mu_{1,t+1} - (\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*})\frac{\omega\sigma_e^2}{m}\mu_{2,t} = 0 \tag{169}$$
$$\frac{\partial \mathcal{L}}{\partial b^{cb}_{t+1}} = -\mu_{2,t}(\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*})\frac{\omega\sigma_e^2}{m} = 0 \tag{170}$$
$$\frac{\partial \mathcal{L}}{\partial y^H_t} = -\phi_y y^H_t + \frac{1}{\beta}\mu_{2,t-1} - \mu_{2,t} + \mu_{3,t}\frac{(1-\theta)(1-\beta\theta)}{\theta}(1+\chi) = 0 \tag{171}$$
$$\frac{\partial \mathcal{L}}{\partial \pi^H_t} = -\phi_\pi \pi^H_t + \frac{1}{\gamma}\mu_{2,t-1} + \mu_{3,t-1} - \mu_{3,t} = 0 \tag{172}$$

Now, from (170) and (169) we obtain:

$$\mu_{1,t} = E_t(\mu_{1,t+1}) \tag{173}$$

Ruling out explosive dynamics for $\mu_{1,t}$ we can obtain:

$$\mu_{1,t} = 0 = \mu_{2,t},\ \forall t \tag{174}$$

This means we can use the FX policy to eliminate the constraints on the capital flows affecting the valuation and risk-sharing inefficiencies.
Now, the system becomes:

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = -\phi_\lambda \lambda_t + \mu_{3,t}\frac{(1-\theta)(1-\beta\theta)}{\theta}(1-\gamma) = 0 \tag{175}$$
$$\frac{\partial \mathcal{L}}{\partial y^H_t} = -\phi_y y^H_t + \mu_{3,t}\frac{(1-\theta)(1-\beta\theta)}{\theta}(1+\chi) = 0 \tag{176}$$
$$\frac{\partial \mathcal{L}}{\partial \pi^H_t} = -\phi_\pi \pi^H_t + \mu_{3,t-1} - \mu_{3,t} = 0 \tag{177}$$

From (175) and (176)

$$\begin{aligned}
\frac{1-\gamma}{1+\chi} &= \frac{\phi_\lambda \lambda_t}{\phi_y y_{H,t}} \\
\phi_y(1-\gamma)y_{H,t} &= \phi_\lambda(1+\chi)\lambda_t \\
\lambda_t &= \frac{\phi_y(1-\gamma)}{\phi_\lambda(1+\chi)}y_{H,t}
\end{aligned}$$

From (177), we obtain:

$$\begin{aligned}
\phi_\pi \pi^H_t &= \frac{\phi_y(1-\gamma)}{\phi_\lambda(1+\chi)}(y_{H,t-1} - y_{H,t}) \\
\frac{\phi_\pi}{\frac{\phi_y(1-\gamma)}{\phi_\lambda(1+\chi)}}\pi^H_t &= (y_{H,t-1} - y_{H,t}) \\
\frac{\phi_\pi}{\alpha}\pi^H_t &= y_{H,t-1} - y_{H,t} \\
y_{H,t} &= y_{H,t-1} - \frac{\phi_\pi}{\alpha}\pi^H_t
\end{aligned}$$

Where $\alpha = \frac{\phi_y(1-\gamma)}{\phi_\lambda(1+\chi)}$. Starting from $t = 0$

$$
\begin{aligned}
y_{H,t} &= -\frac{\phi_\pi}{\alpha} \sum_{t=0}^{\infty} \pi^H_{t-j} \\
y_{H,t} &= -\frac{\phi_\pi}{\alpha} p^H_t
\end{aligned}
$$

Since $p^H_t = lnP^H_t$. Then, in the IS equation:

$$
\begin{aligned}
& y_{H,t} = \left(\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*}\right)(E_t\lambda_{t+1} - \lambda_t) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi^H_{t+1}) \\
& y_{H,t} = \left(\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*}\right)(E_t\alpha y_{H,t} - \alpha y_{H,t+1}) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi^H_{t+1}) \\
& y_{H,t} = \left(\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*}\right)\alpha(E_t y_{H,t+1} - y_{H,t}) + E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi^H_{t+1}) \\
& (1 + (\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*})\alpha) y_{H,t} = \left(\left(\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*}\right)\alpha + 1\right) E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi^H_{t+1} \\
& \kappa y_{H,t} = \kappa E_t y_{H,t+1} - \frac{1}{\gamma}(i_t - \rho - E_t\pi^H_{t+1}) \\
& \kappa\gamma y_{H,t} = \kappa\gamma E_t y_{H,t+1} + \rho + E_t\pi^H_{t+1} \\
& i_t = \kappa\gamma(E_t y_{H,t+1} - y_{H,t}) + \rho + E_t\pi^H_{t+1} \\
& i_t = \kappa\gamma(E_t - \frac{\phi_\pi}{\alpha} p^H_{t+1} - (-\frac{\phi_\pi}{\alpha} p^H_{t+1})) + \rho + E_t\pi^H_{t+1} \\
& i_t = -\kappa\gamma\frac{\phi_\pi}{\alpha} E_t\pi^H_{t+1} + \rho + E_t\pi^H_{t+1} \\
& i_t = (1 - \kappa\gamma\frac{\phi_\pi}{\alpha}) E_t\pi^H_{t+1} + \rho
\end{aligned}
$$

Where $\kappa = (1 + (\frac{1}{\gamma} - \frac{\gamma}{1+\gamma-\gamma^*})\alpha)$. The Phillips curve:

$$
\begin{aligned}
\pi^H_t &= \beta E_t\left(\pi^H_{t+1}\right) + \frac{(1-\theta)(1-\beta\theta)}{\theta}(-v + (1+\chi)y_{H,t} + (1-\gamma)\lambda_t - a_t) \\
\beta E_t\left(\pi^H_{t+1}\right) &= \pi^H_t - \frac{(1-\theta)(1-\beta\theta)}{\theta}(-v + (1+\chi)y_{H,t} + (1-\gamma)\alpha y_{H,t} - a_t) \\
\beta E_t\left(\pi^H_{t+1}\right) &= \pi^H_t - \frac{(1-\theta)(1-\beta\theta)}{\theta}(-v + (1+\chi+(1-\gamma)\alpha)y_{H,t} - a_t) \\
\beta E_t\left(\pi^H_{t+1}\right) &= \pi^H_t - \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(-v - (1+\chi+(1-\gamma)\alpha)\frac{\phi_\pi}{\alpha} p_t - a_t\right)
\end{aligned}
$$

Then the optimal monetary policy:

$$
\begin{aligned}
i_t &= \left(1 - \kappa\gamma\frac{\phi_\pi}{\alpha}\right) E_t\pi^H_{t+1} + \rho \\
i_t &= (1 - \kappa\gamma\frac{\phi_\pi}{\alpha})\frac{1}{\beta}\left(\pi^H_t - \frac{(1-\theta)(1-\beta\theta)}{\theta}\left(-v - (1+\chi+(1-\gamma)\alpha)\frac{\phi_\pi}{\alpha} p_t - a_t\right)\right) + \rho
\end{aligned}
$$

The optimal FX intervention is:

$$
\begin{aligned}
& E_t\lambda_{t+1} - \lambda_t = \frac{\omega\sigma^2_e}{m}(-n_{t+1} - b^{cb}_{t+1} - b_{t+1}) \\
& \frac{m}{\omega\sigma^2_e}\alpha(E_t y_{H,t+1} - y_{H,t}) = (-n_{t+1} - b^{cb}_{t+1} - b_{t+1}) \\
& b^{cb}_{t+1} = -\frac{m}{\omega\sigma^2_e}\alpha(E_t y_{H,t+1} - y_{H,t}) - n_{t+1} - b_{t+1}
\end{aligned}
$$

or The optimal FX intervention is:

$$b^{cb}_{t+1} = -\frac{m}{\omega\sigma_e^2}\alpha\left(E_t(-\frac{\phi_\pi}{\alpha}p_{H,t+1}) - (-\frac{\phi_\pi}{\alpha}p_{H,t})\right) - n_{t+1} - b_{t+1}$$
$$b^{cb}_{t+1} = \frac{m}{\omega\sigma_e^2}\phi_\pi(E_t\pi_{H,t+1}) - n_{t+1} - b_{t+1}$$

Where

$$\alpha = \frac{\phi_y(1-\gamma)}{\phi_\lambda(1+\chi)}$$

The last expression shows how the central bank will try to close the three gaps in the economy: the price dispersion gap, the output gap and the international risk sharing gap.

E Non-linear Model - Flexible Prices

Aggregate demand (y_t)

$$Y_t^H = \gamma\left(\mathcal{S}_t\right)^{1-\gamma}C_t + (1-\gamma)\mathcal{S}_t C_t^* \tag{178}$$

Real exchange rate (rer_t)

$$Q_t = \frac{S_t P_t^*}{P_t} \tag{179}$$

Euler equation (c_t)

$$C_t^{-\gamma} = \beta E_t\left(C_{t+1}^{-\gamma}\frac{1+i_t}{1+\pi_{t+1}}\right) \tag{180}$$

Exports $\left(c_t^{H,*}\right)$

$$C_t^{H,*} = (1-\gamma^*)\left(\frac{P_t^H}{P_t^F}\right)^{-1} C_t^* \tag{181}$$

Price Level (p_t)

$$P_t = \left(P_t^H\right)^\gamma\left(P_t^F\right)^{1-\gamma} \tag{182}$$

Terms of trade $\hat{\mathcal{S}}_t$

$$\hat{\mathcal{S}}_t = \frac{P_t^F}{P_t^H} \tag{183}$$

Labour supply (w_t)

$$L_t^\chi C_t = w_t \tag{184}$$

Domestic home goods demand $\left(C_t^H\right)$

$$C_t^H = \gamma\left(t_t^H\right)^{-1}C_t \tag{185}$$

Domestic foreign goods demand $\left(C_t^F\right)$

$$C_t^F = (1-\gamma)\left(t_t^F\right)^{-1}C_t \tag{186}$$

Modified UIP (s_t)

$$S_t = E_t S_{t+1}\frac{(1+i_t^*)}{1+i_t}\left(1+\frac{\omega}{m}\sigma^2 d_{t+1}^*\right) \tag{187}$$

Home goods supply (y_t^H)

$$Y_t^H = \frac{A_t L_t}{Z_t} \tag{188}$$

Labour demand (l_t)

$$\frac{P_t^H}{P_t} = \frac{\epsilon}{\epsilon - 1} MC_t^H \tag{189}$$

Marginal cost $\left(mc_t^H\right)$

$$MC_t^H = (1 - \tau^H)\frac{w_t}{A_t} \tag{190}$$

Current account LHS (CA_t)

$$\frac{CA_t}{\bar{Y}} = S_t B_{t+1}^{cb,*} + S_t D_{t+1}^* - N_{t+1} - S_{t-1} B_t^{cb,*} - S_{t-1} D_t^* + N_t \tag{191}$$

Current account RHS (CA_t)

$$\frac{CA_t}{\bar{Y}} = NX_t + \left(\frac{S_t}{S_{t-1}} R_{t-1}^* - 1\right)\left(S_{t-1} B_t^{cb,*} + S_{t-1} D_t^*\right) - (R_{t-1} - 1) N_t \tag{192}$$

Net exports (NX_t)

$$NX_t = P_t^H Y_t^H - P_t C_t \tag{193}$$

Domestic goods inflation π_t^H

$$\Pi_t^H = 1 \tag{194}$$

CPI inflation π_t

$$\Pi_t = 1 \tag{195}$$

Price dispersion z_t

$$Z_t = 1 \tag{196}$$

Domestic goods relative price t^H

$$t_t^H = \frac{P_t^H}{P_t} \tag{197}$$

Foreign goods relative price t^F

$$t_t^F = \frac{P_t^F}{P_t} \tag{198}$$

Foreign output (C_t^*):

$$C_t^* = \bar{C}^* \tag{199}$$

Portfolio shocks (ψ_t)

$$n_t^* = \rho_\psi n_{t-1}^* + \sigma_\psi \varepsilon_t^\psi \tag{200}$$

Productivity shocks (a_t):

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a \tag{201}$$

F Non-linear Model - Sticky Prices

Aggregate demand (y_t)

$$Y_t^H = \gamma \left(\mathcal{S}_t\right)^{1-\gamma} C_t + (1 - \gamma)\mathcal{S}_t C_t^* \tag{202}$$

Real exchange rate (rer_t)

$$Q_t = \frac{S_t P_t^*}{P_t} \tag{203}$$

Euler equation (c_t)

$$C_t^{-\gamma} = \beta E_t \left(C_{t+1}^{-\gamma} \frac{1+i_t}{1+\pi_{t+1}} \right) \tag{204}$$

Exports $\left(c_t^{H,*}\right)$

$$C_t^{H,*} = (1-\gamma^*) \left(\frac{P_t^H}{P_t^F} \right)^{-1} C_t^* \tag{205}$$

Price Level (p_t)

$$P_t = \left(P_t^H\right)^{\gamma} \left(P_t^F\right)^{1-\gamma} \tag{206}$$

Terms of trade $\hat{\mathcal{S}}_t$

$$\hat{\mathcal{S}}_t = \frac{P_t^F}{P_t^H} \tag{207}$$

Labour supply (w_t)

$$L_t^{\chi} C_t = w_t \tag{208}$$

Domestic home goods demand $\left(C_t^H\right)$

$$C_t^H = \gamma \left(t_t^H\right)^{-1} C_t \tag{209}$$

Domestic foreign goods demand $\left(C_t^F\right)$

$$C_t^F = (1-\gamma) \left(t_t^F\right)^{-1} C_t \tag{210}$$

Modified UIP (s_t)

$$S_t = E_t S_{t+1} \frac{(1+i_t^*)}{1+i_t} \left(1 + \frac{\omega}{m} \sigma^2 d_{t+1}^*\right) \tag{211}$$

Home goods supply (y_t^H)

$$Y_t^H = \frac{A_t L_t}{Z_t} \tag{212}$$

Labour demand (l_t)

$$\frac{P_t^H}{P_t} = \frac{\epsilon}{\epsilon - 1} M C_t^H \tag{213}$$

Marginal cost $\left(mc_t^H\right)$

$$MC_t^H = (1-\tau^H) \frac{w_t}{A_t} \tag{214}$$

Current account LHS (CA_t)

$$\frac{CA_t}{\bar{Y}} = S_t B_{t+1}^{cb,*} + S_t D_{t+1}^* - N_{t+1} - S_{t-1} B_t^{cb,*} - S_{t-1} D_t^* + N_t \tag{215}$$

Current account RHS (CA_t)

$$\frac{CA_t}{\bar{Y}} = NX_t + \left(\frac{S_t}{S_{t-1}} R_{t-1}^* - 1 \right) \left(S_{t-1} B_t^{cb,*} + S_{t-1} D_t^* \right) - (R_{t-1} - 1) N_t \tag{216}$$

Net exports (NX_t)

$$NX_t = P_t^H Y_t^H - P_t C_t \tag{217}$$

Domestic goods inflation π_t^H

$$\Pi_t^H = 1 \tag{218}$$

CPI inflation π_t

$$\Pi_t = 1 \tag{219}$$

Price dispersion z_t

$$Z_t = 1 \tag{220}$$

Domestic goods relative price t^H

$$t_t^H = \frac{P_t^H}{P_t} \tag{221}$$

Foreign goods relative price t^F

$$t_t^F = \frac{P_t^F}{P_t} \tag{222}$$

Foreign output (C_t^*):

$$C_t^* = \bar{C}^* \tag{223}$$

Portfolio shocks (ψ_t)

$$n_t^* = \rho_\psi n_{t-1}^* + \sigma_\psi \varepsilon_t^\psi \tag{224}$$

Productivity shocks (a_t):

$$a_t = \rho_a a_{t-1} + \sigma_a \varepsilon_t^a \tag{225}$$