State-Dependent Central Bank Communication with Heterogeneous Beliefs

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Abstract

This paper studies the optimal disclosure strategy of a Sender who wants to influence heterogeneous Receivers' expectations by providing public information. I introduce heterogeneous priors in an otherwise standard Bayesian persuasion model à la Gentzkow and Kamenica (2011) and characterize the dependence of optimal disclosure on the heterogeneity of beliefs. I show that heterogeneity matters in two ways: (i) it is optimal to send moderating signals, which implies sending signals with positive error probabilities in both states, and constitutes a non-trivial departure from the homogeneous beliefs case; (ii) higher dispersion in beliefs leads the information authority to send signals with lower error probabilities. I apply my framework to a central bank communication problem in which the policy maker communicates about aggregate conditions to influence firms' investment decisions. I empirically validate the model's predictions by showing that the FOMC unemployment rate forecasts are systematically biased in opposite directions in recessions and expansions. Also in line with the model's predictions, the forecast biases are decreasing in the degree of private sector disagreement for each state.

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1 Introduction

Central banks provide a substantial amount of information about their monetary policy decisions, as well as publish ample data and analyses on the state of the economy.[1] One of the reasons behind such information provision is that communication can help coordinate the expectations of forward looking agents about both the state of the economy and future interest rates, thereby influencing their consumption and investment decisions. This ability to influence expectations through information provision is particularly of assistance when central banks' objectives are not completely aligned with those of the private sector, so that central banks want to bias beliefs upwards. In good times, it can foster investment which is inefficiently low by emphasizing the strength of the economy. In bad times, it can avoid adding more gloom to the economy by increasing expectations through an encouraging depiction of the economy. However, existing models of central bank communication abstract from these strategic motives to communicate differently in good and bad times and thus cannot provide guidance on the state-dependence of disclosure.

In this paper, I introduce heterogeneous priors in a model of Bayesian persuasion to study how to persuade rational agents by controlling their informational environment as a function of the state, and how the heterogeneity in beliefs influences this communication strategy.[2] More precisely, I characterize the optimal public signaling strategy of a Sender who chooses a distribution of signals as a function of the state to provide information to Receivers with dispersed beliefs in order to influence their actions. I show that heterogeneity generates a non-trivial departure from the homogeneous beliefs case or the single receiver case of Gentzkow and Kamenica (2011). Heterogeneity in beliefs matters in two ways for the optimal disclosure. First, the Sender should send moderating signals. This means that he should send signals with positive error probabilities in both states, being sometimes unduly optimistic in bad states, and pessimistic in good states. This is in contrast to the one-receiver case where good times were revealed systematically with zero error. Secondly, the more disagreement there is among agents, the lower the probability of sending erroneous signals (i.e., less pessimism or optimism). I apply my framework to a central bank communication problem, in which a policy maker communicates to firms about aggregate conditions to shape

[1] As recently as 1994, the Federal Open Market Committee (FOMC) did not to communicate immediately its policy decision. The push for greater transparency has led to the publication of detailed, standardized minutes and monetary policy statements, as well as Summary of Economic Projections (SEP) since 2010, and systematic press conferences after each FOMC meeting. There has also been an increase in speeches and comments by the Federal Reserve System's presidents.

[2] In this paper, the heterogeneity in beliefs is distinct from the dogmatic priors in which agents agree to disagree. Agents have heterogeneous priors, they are all bayesian and all update their beliefs through Bayes' rule.

their investment decisions. I show that the two aforementioned conclusions also hold in this application, for a cost of investment low enough. I test one piece of communication published by the Federal Reserve, the Federal Open Market Committee (FOMC henceforth) forecasts, and show that their behavior is consistent with the type of strategic bias predicted by the model.

The premise of this paper is the presence of strategic motives to communicate differently to the private sector in good times and bad times. When the economy is booming, revealing the facts would tend to help, while revealing a dampening of economic activity would not. Similarly, it may not be optimal to reveal with full transparency the state of the economy if the economy is hit by an inefficient or temporary cost-push shock. For example, Christine Lagarde, in a recent press conference, on September 9th, 2021 talked inflation down by emphasizing its temporality:

> "*Inflation increased to 3.0 per cent in August [...] This temporary upswing in inflation [...] The new staff projections foresee annual inflation at 2.2 per cent in 2021, 1.7 per cent in 2022 and 1.5 per cent in 2023.*"

These strategic motives are in fact of practical relevance for policy makers, who have also repeatedly raised concerns about communication during downturns or financial crises.[3] Ben Bernanke, former Chairman of the Federal Reserve, admitted in Bernanke (2015) that at the onset of the Great Recession,

> "*I wasn't willing to use the r-word in public at that point, even though the risk of a downturn was clearly significant. [...] I didn't want to add unnecessarily to the prevailing gloom by talking down the economy.*"

Indeed, full disclosure of a downturn may lead to further deterioration of firms' and consumers' confidence, which may in turn worsen both economic outlook and welfare. For instance, firms making investment decisions will maintain or increase investments only if

[3] Examples related to the financial crisis and regarding communication to the financial markets include Bernanke (2015), who was especially worried about fire sales externalities: "*I knew that immediate transparency in this instance would pose a serious problem during any future financial panic*" (referring to transparency about banks borrowing at the discount window). Former Federal Reserve Chairman Alan Greenspan, referring to the 9/11 episode and probabilities of recessions, mentioned that *"yet on the record [he] took a less pessimistic stance because if [he] had fully expressed what [he] thought the probabilities were, [he]'d have scared the markets half to death."* (2007). More recently, speaking about the 2007 financial crisis, Andrew Haldane from the Bank of England admitted that *"it is not always and everywhere the case that greater openness and transparency is a good thing. Had we been fully open and fully transparent about what was going on during the financial crisis, it would, let me tell you, have been a lot, lot worse. That would have been [like] shouting "fire" in the theatre. And however bad it was, it would have caused an even greater hemorrhaging in confidence, and even greater collateral damage for savers and borrowers than we ever saw."* (Financial Times, 2017).

they are confident in the economic outlook, expecting high demand for their products. When the returns to investment depend endogenously on aggregate investment and firms do not internalize that, this generates incentives for the monetary authority to induce more investment: when fundamentals are low, low investment would generate even lower aggregate welfare, and it would be beneficial to distort the beliefs towards a more positive outlook.[4] Policy makers would benefit, in terms of increased welfare, from designing the summary of economic developments or forecasting error probabilities as a function of the state of the economy in order to persuade agents. In practice, how does this translate into the communication stance a central bank should adopt during recessions? Should it release the same information during good times as during financial crises[5]? How does the counter-cyclicality of disagreement interact with optimal communication? In the framework I develop, the communication strategy is a distribution of signals as a function of the states. Therefore, it allows me to speak meaningfully about central bank communication in good and bad times. The heterogeneity in beliefs featured in my model will also enable me to describe how the optimal communication should vary with the private sector's disagreement.

I build a simple and abstract strategic communication game between a Sender and multiple Receivers that relies on three key features: (i) the incentives of the Sender and Receivers are misaligned; (ii) the Sender can systematically distort the beliefs of the agents by committing to a signaling strategy, and (iii) Receivers hold different priors over the state. The first two features are standard in the Bayesian persuasion literature. The misalignment in incentives is needed for the Sender to want to communicate strategically. With misaligned incentives, the Sender wants to influence Receivers' beliefs such that they take his preferred action more often than if he did not reveal any information, or if he consistently revealed truthfully the states. If incentives were aligned, the Sender would just need to reveal both states with zero error for Receivers to take his preferred action. The second assumption, the commitment to a signaling strategy, is the key assumption in the Bayesian persuasion literature.[6] It is a commitment to a set of probabilities conditional on a state that the Sender makes before observing the state, and which agents observe along with a signal realization. This means the Sender cannot conceal or distort the information once he has applied the

[4]This is an investment coordination externality. For instance, firms do not internalize that by investing, they buy goods from other firms, firms which in turn invest by buying goods from others.

[5]Norges Bank has been seen by the market as being too pessimistic in good times, even when inflation was above 2.5 %: "Having been excessively pessimistic about the economic outlook, the Norges Bank has largely ignored its inflation target" (Econotimes, 2016)

[6]Bayesian persuasion can be seen as a communication protocol in a similar way to cheap talk (for instance Crawford and Sobel (1982)), but relative to these models, Bayesian persuasion endows the Sender with more commitment power. Gentzkow and Kamenica (2017) has shown that this assumption generates similar equilibrium outcomes as in a model where the Sender publicly chooses how much information he will privately observe and then strategically decides how much of this private information to reveal.

probability and a signal is realized. This allows me to abstract from incentive compatibility issues. The key feature I add to the theoretical framework is heterogeneous priors. I will show that this one deviation from the standard persuasion framework yields non-trivial implications for optimal communication.

The mere presence of heterogeneity incentivizes the Sender to choose moderating signals. This means he should be sometimes unduly optimistic in bad states (sending a good signal in a bad state with positive probability) and pessimistic in good states (similarly, sending a bad signal in a good state with positive probability), unlike the one-receiver case of Gentzkow and Kamenica (2011) in which good times required truthful disclosure. With a unique Receiver, the Sender only needs to bring this one Receiver's belief about the good state above a threshold for him to take his preferred action. He does so by always revealing the truth in good times and sending a good signal in a bad state with positive probability. He does not want to set that probability to be too high. Indeed, while it increases the probability that a good signal realizes, conditional on such a signal, agents' belief shifts by less (since they know that probability). The multiplicity of receivers with heterogeneous beliefs implies that there are agents whose beliefs are below the threshold and need to be shifted, as well as some agents with beliefs already above the threshold who are already taking the correct action. If all receivers were to have beliefs above the threshold, the last thing the Sender would want to do is generate information that distorts their beliefs. In the presence of beliefs both below and above the threshold, the Sender needs to shift the beliefs below the threshold above while maintaining those already above, regardless of the signal realization. Always reporting good times with zero probability of error would make Receivers assign a zero probability on the good state upon seeing a signal of a bad state. Indeed, if the Sender is always sending good signals in a good state, observing a bad signal could only mean that it was generated from a bad state. This means that he "loses" those who would take the favorable action without information. This trade-off originates from the fact that he sends the same signal to Receivers above and below the threshold, and is therefore absent in models of heterogeneous priors with private signaling such as Arieli and Babichenko (2016).

The second main result is that the extent of heterogeneity matters. Higher dispersion in beliefs leads the Sender to decrease his optimism in bad states and his pessimism in good states (i.e., decreasing the probability of sending an erroneous signal in a given state). Higher dispersion in beliefs implies that there are more agents with extreme beliefs whose beliefs need to be shifted by a lot. This generates the need for a more powerful disclosure strategy, and the strategy is more powerful if good signals in bad times are less frequent.

Interestingly, because agents know the conditional probabilities, this is somewhat akin to a reputation effect but in a static setting.[7] I extend the static setting to a dynamic version in which the Sender observes the state every period and sends a signal, therefore removing the commitment device. Receivers have beliefs about the Sender's communication strategy, which update each period after observing the signal and the state. I show that this is enough to sustain an equilibrium where the Sender does not always send the signal that corresponds to the state of the world, nor does he always send a signal opposite of the state.

An important application of my model is to strategic central bank communication. As above-mentioned, in presence of externalities such as investment coordination externalities, central banks have misaligned incentives with the private sector, and would want to shape their beliefs in good states to influence their actions. Similarly, in bad times, policy makers have incentives to avoid a drop in expectations, consumption and investment by not revealing the full extent of the crisis. Turbulent times such as recessions also feature higher disagreement among the private sector, as can be seen, for instance, in the Survey of Professional Forecasters (SPF). Given its features, my framework easily maps to the aforementioned policy makers' problem, and can consequently be used to guide welfare analysis. I carry out an application where a policy maker communicates to firms about aggregate conditions to shape their investment decisions, and the misalignment in incentives stems from coordination externalities. I demonstrate that a central bank would want to send moderating signals to the firms if the fixed costs of investment are low enough.

I document the potential strategic behavior of the Federal Reserve by showing that the model's predictions are borne out in the data. While Kawamura et al. (2019) document the strategic tone in the Bank of Japan's minutes over the business cycle, I focus on quantitative information published by the Fed, the FOMC forecasts.[8] In the static model, the communication strategy is a set of error probabilities, which specifies the statistical relationship between the state and the signal. Forecast errors therefore correspond well with the idea of sending signals with positive error probabilities. In this context, sending positive signals in bad states implies positive unemployment rate forecast errors in bad times. Comparing the distribution of FOMC forecast errors in recessions and expansions and in periods of low and high disagreement, I find that the policy makers tend to systematically underestimate (overestimate) unemployment when releasing forecasts during a recession (expansion), but less so if private sector is disagreement is high. Given that the FOMC

[7]In the Bayesian persuasion literature, the commitment assumption is justified as being obtained through repeated interactions (where then reputation will have a role). I refer to Best and Quigley (2017) for a detailed discussion.

[8]He highlights the Bank of Japan's strategic obfuscating strategy by showing that more ambiguous words are used in the Monthly Report of Recent Economic and Financial Development when the leading index of the economy is lower.

forecasting behavior validates the predictions of the model, deviating systematically from the private sector's forecasting behavior, this provides evidence in favor of the existence of strategic biases in FOMC forecasts.

The remainder of the paper proceeds as follows. I connect this paper to the relevant literature in section 2. The static model of persuasion is presented in section 3. The optimal information strategy is derived in section 4. Section 5 extends the analysis to a dynamic setting. Section 6 applies the framework to a central bank communication problem, while section 7 concludes. Proofs are provided in the Appendix.

2 Related literature

My paper contributes to the literature on two fronts. Within the Bayesian persuasion literature, my model extends the standard one-receiver Bayesian persuasion framework by Gentzkow and Kamenica (2011) and Rayo and Segal (2010) to several receivers with heterogeneous priors. There are two classes of multiple-receiver extensions with analysis close to the standard framework.[9] The first extension is multiple receivers with common priors, which means receivers share the same posterior and the analysis can proceed as in the single receiver case.[10] The other case is when receivers have heterogeneous priors but the information provider can send private signals, as in Arieli and Babichenko (2016). In this case, the Sender faces a set of separate single-receiver problems. A distinguishing feature of my model is that the Sender communicates through a public signal, which generates a trade-off between shifting the beliefs of the receivers that are below the threshold, while maintaining the beliefs of those who were already above the threshold.

My model is static, and the Sender is assumed to commit to a disclosure strategy. An alternative to that is that he has to maintain a reputation for how he communicates. Such an alternative would require adding time to the basic model. Past behavior will influence current opportunities when the state evolves over time. Ely (2017) and Renault, Solan and Vieille (2017) study dynamic information provision in which the state evolves according to a Markov process, the Receiver chooses each period an action that maximizes his contemporaneous utility, and the Sender commits to a distribution of signals as a function of the history and current state. My dynamic extension shares the first two features, but I investigate whether reputation is enough to obtain similar information provision as in the static case with commitment.

The Bayesian persuasion literature is particularly helpful in improving policy-making

[9] Bergemann and Morris (2016) and Kamenica (2018) provide an informative review of the literature and the various extensions that have been considered.

[10] See for instance Cheng et al. (2015).

institutions through optimal information design, such as for financial sector stress tests, as in Inostroza and Pavan (2017) or Goldstein and Leitner (2018). My paper is to my knowledge the first to bring Bayesian persuasion to central bank communication about the state of the economy. This framework enables me to characterize communication as a function of the state. A large part of the existing literature has been concerned with the optimal degree of transparency where the strategic motives were between the use of private and public information.[11] In this paper, the strategic motives are across states. This allows me to characterize disclosure rules that are *contingent* on the state. This therefore complements the work on the optimal signal precision of for instance Morris and Shin (2002), Amato, Morris and Shin (2002) or Hellwig (2005)[12], optimal precision in the presence of learning externalities as in Morris and Shin (2005), Amador and Weill (2010), Gaballo (2016), or Morris and Shin (2018), the work on the optimal number of signals of Chahrour (2014) or on the timing of the signal as in Reis (2013). The state dependent communication in this paper would be akin to choosing the noise of the Gaussian signaling according to the state of the economy in the Gaussian set up of these aforementioned papers. Here, the central bank will choose the probability that a signal realizes in a *given state* (and thus there can be a different probability that a signal realizes) - a non state dependent policy would just be choosing a noise in the signal (probability distribution of a signal) in the Gaussian set up or probability of a signal in the present set up.

Disagreement about the interpretation of a forward guidance announcement has been shown to matter for the effectiveness of forward guidance, as in Andrade et al. (2019). In their paper, the central bank communicates about the length of a low interest rate period, but there is a fraction of optimistic agents who interpret the announcement as monetary easing and another fraction of pessimistic agents see it as stemming from bad fundamentals. This affects the efficiency of the forward guidance policy. My paper considers how the disagreement about the state of the economy, not the announcement, interacts with the optimal communication policy.

3 A Bayesian persuasion model with heterogeneous priors

I start by presenting a simple strategic communication game in which one Sender, who has misaligned incentives with a continuum of Receivers, communicates to them about the

[11] Geraats (2002), Winkler (2000) and Jensen (2002) are early references that describe the various dimensions of central bank communication problems including optimal transparency.

[12] Blinder et al. (2008) provides a survey of the earlier literature. Others include Svensson (2006), Morris, Shin and Tong (2006), Morris and Shin (2007), Tamura (2016), and Wiederholt (2016). Angeletos and Pavan (2007*a*) and Angeletos and Pavan (2007*b*) have considered the social value of information (precision of the signal) depending on the shock hitting the economy.

state of the world to shape their beliefs. Receivers have heterogeneous beliefs about the state. A special case of this model, homogeneous beliefs will therefore converge to the one-receiver case and nests Gentzkow and Kamenica (2011). The Sender can influence their beliefs, hence their actions, by committing to information provision as a function of the fundamental state.

3.1 Preferences

The economy consists of a continuum of Receivers, indexed by $i \in [0,1]$ and a single Sender. Receivers choose an action $a_i \in \{0,1\}$, given their information. Receiver i's utility is decreasing in the distance between his action and the state of the economy, so that

$$u_i(a_i, w) = -(a_i - w)^2, \tag{3.1}$$

where $\omega \in \Omega = \{0,1\}$ is the state. If Receiver i had perfect knowledge about the state, his optimal action would be to align his action with it such that $a_i = \omega$. With imperfect information, it will depend on his beliefs. He will take action $a_i = 1$ if he thinks that the probability of state $\omega = 1$, denoted $p(\omega = 1)$, is above a certain threshold.

The Sender's payoff $\mathcal{V}$ does not depend on the state but only the total number of Receivers who take action $a_i = 1$,

$$\mathcal{V}(\mathbf{a}) = \int_0^1 a_i d_i, \tag{3.2}$$

where $\mathbf{a}$ is the action profile over all Receivers. Note that $\mathcal{V}(a) = \int_0^1 a_i d_i \neq \int_0^1 u_i(a_i,\omega) d_i$, indicating that incentives are misaligned. This is the multiple-receiver version of Gentzkow and Kamenica (2011)'s misalignment in incentives, but there are many possible sources that could generate such misalignment, such as externalities not internalized by agents.[13] I remain agnostic about the source of misalignment and defer the discussion about the source of externalities in the application to central bank communication. Because of this misalignment, the Sender has incentives to send a signal to influence the maximum number of Receivers to take action $a_i = 1$. To do so, he will choose a disclosure policy to shape their posterior beliefs.

[13] Suppose that agents' payoff depend on the aggregate action, which they do not internalize when deciding on their action. For instance, $U_i = \frac{-1}{2}(a_i - \omega)^2 + \overline{a}$ where $\overline{a} = \int_0^1 a_i d_i$. The optimal action from the receiver's perspective is $a_i = E_i(\omega)$. If the planner maximizes welfare, his payoff would be $\mathcal{V} = \int_0^1 U_i d_i = \int_0^1 \frac{-1}{2}(a_i - \omega)^2 + a_i di$, and the optimal action $a_i = 1 + E(\omega)$. Given the action can only be either 0 or 1, the best attainable option is $a_i = 1$, therefore maximizing the fraction of agents taking action 1 is equivalent to this payoff, and all the results will carry through.

3.2 Communication game

Before the realization of the state, the Sender (S) chooses his disclosure policy, specifying a distribution of signals as a function of the state of the world. It consists of a pair of probabilities of a signal realization conditional on the state. Uncertainty is resolved, then the Sender applies the probabilities to send a signal. Once Receivers (R) have observed the conditional probabilities and a signal, they choose their actions. Figure 1 summarizes the sequence of events in the model.

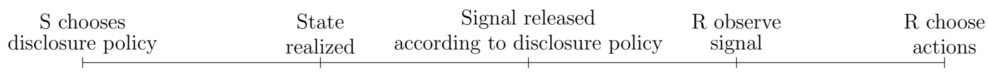


Figure 1. Timeline of the communication game.

Sender's disclosure policy. The Sender does not know the state when choosing his communication strategy. He has a prior γ on the state realization $\omega = 1$, such that $\gamma = p(\omega = 1)$. To shape the beliefs of the agents, he chooses and commits to the conditional probability distribution of a signal for each of the state. Let $\mathcal{S}$ be the set of possible signals to the agents, denoted by $\mathcal{S} = \{0, 1\}$. I define the disclosure policy $\pi : \omega \to \Delta(\mathcal{S})$, as a mapping from the state to a distribution over a signal realization. It specifies, for each fundamental ω, a probability distribution over the information disclosed to the agents, and can be represented by a pair of error probabilities (ϵ_1, ϵ_0) where $\epsilon_1 = \pi(s = 0|\omega = 1)$ and $\epsilon_0 = \pi(s = 1|\omega = 0)$. The disclosure rule $\pi(s|\omega)$ is observed by the Receivers.

Receivers' beliefs and actions. Receivers have heterogeneous prior beliefs about the state. Each Receiver has a prior probability λ_i that the state is $\omega = 1$. Their priors are uniformly distributed, such that $\lambda_i \sim U(\alpha, 1-\alpha)$. U is symmetric around $\frac{1}{2}$, with $\alpha \in (0, \frac{1}{2})$. The priors' variance σ^2 determines the degree of heterogeneity or disagreement among the receivers in this economy and is pinned down by the length of the support $1 - 2\alpha$, since $\sigma^2 = \frac{(1-2\alpha)^2}{12}$. This heterogeneity in beliefs could reflect differences in private information, the priors capturing all other sources of information.[14] The higher the variance, the higher the dispersion, meaning the more receivers have diverging private information. Receivers are

[14]While private information is a potential source of heterogeneous beliefs, I do not take a stand on the source of heterogeneity but rather take it as given. I refer the reader to Morris (1994, 1995) and Van den Steen (2010) who provide a good discussion of other potential sources of heterogeneous priors. Players are Bayes rational, but initially disagree on the likelihood of the state. This disagreement remains possible because of a lack of evidence, or data that would allow them to reach a consensus. The private information considered is as a source of heterogeneous beliefs however does not add any layer of uncertainty as Kolotilin et al. (2017) who allows for the receiver to send a signal about his type (and allows for private persuasion, which I do not consider here)

Bayesians, and upon receiving a signal, update their beliefs . For a given prior λ_i, Receiver i's posterior belief about the state being $\omega = 1$ conditional on observing the signal $s = 1$ is

$$p_i(\omega = 1|s = 1) = \frac{(1-\epsilon_1)\lambda_i}{(1-\epsilon_1)\lambda_i + \epsilon_0(1-\lambda_i)}, \tag{3.3}$$

while if he observes $s = 0$, he believes the probability of state 1 to be

$$p_i(\omega = 1|s = 0) = \frac{\epsilon_1\lambda_i}{(1-\epsilon_0)(1-\lambda_i) + \epsilon_1\lambda_i}. \tag{3.4}$$

Given his prior belief λ_i and a signal realization, Receiver i will choose $a_i = 1$ if his expected payoff is higher than the expected payoff upon choosing $a_i = 0$,

$$\mathbb{E}_i[u(1,\omega)|s] \geq \mathbb{E}_i[u(0,\omega)|s]. \tag{3.5}$$

This translates into a threshold on his posterior belief $p(\omega = 1|s) \geq \frac{1}{2}$. I describe the Sender's optimization problem and the equilibrium in the next section.

4 Optimal state-dependent communication

In this section, I solve for the optimal disclosure strategy of the Sender. I show that the optimal disclosure is (i) informative if the Sender believes the good state to be more likely than the other, (ii) optimistic in bad states and pessimistic in good states and (iii) the extent of the truthfulness depends on disagreement.

Given the Receivers's threshold strategy and the distribution of their priors, the Sender chooses the conditional probabilities of a signal realization in a given state to maximize his expected total payoff. This payoff will depend on both the realized signal and the fraction of agents with posteriors (given that signal) above the threshold.

4.1 Optimal disclosure strategy

The Sender will choose the disclosure strategy characterized by the error probabilities ϵ_1 and ϵ_0 to maximize his expected payoff as given in equation 3.1, taking as given the fraction of agents above the threshold. The Sender's objective function $\mathcal{V}$ can therefore be rewritten as

$$\mathbb{E}(\mathcal{V}) = \sum_{s\in\{0,1\}} \sum_{\omega\in\Omega} p(s) \int_0^1 \mathbb{1}_{p_i(\omega=1|s)\geq\frac{1}{2}} g(\lambda_i) d\lambda_i, \tag{4.1}$$

where $p(s)$ is the ex-ante probability of the Sender sending a signal s given his prior γ about the state. It is defined as follows: $p(s = 1) = \gamma(1 - \epsilon_1) + (1 - \gamma)\epsilon_0$ and $p(s = 0) = (1 - \gamma)\epsilon_1 + (1 - \gamma)(1 - \epsilon_0)$. $g(\lambda_i)$ is the probability distribution of the priors. Without loss of generality, I assume the disclosure strategy satisfied $\epsilon_0 + \epsilon_1 \leq 1$, such that $p(s = \omega) \geq \frac{1}{2}$. This is a labelling normalization so that $p(\omega = 1|s = 1) > p(\omega = 1|s = 0)$. This means the Sender wants the Receivers to take action 1 when they receive $s = 1$, such that a signal is an action recommendation.[15]

The relevant equilibrium concept is the Perfect Bayesian equilibrium. We can think of choice of the disclosure policy π (equivalently, the error probabilities (ϵ_1, ϵ_0)) as inducing a distribution of posteriors. Therefore, taking as given how his disclosure strategy influences the distribution of the Receivers' posteriors, the Sender chooses the pair (ϵ_1, ϵ_0) to maximize equation 4.1. His choice of error probabilities influences his expected payoff by changing the distribution of signal realizations (i.e., the probability that signal 1 or 0 is realized), as well as the distribution of the posteriors. This affects the fraction of agents whose beliefs are above the threshold.

To understand this trade-off, it is helpful to define the marginal receiver, who is the receiver who has a prior such that, given a signal s and the pair (ϵ_1, ϵ_0), his posterior $p(\omega = 1|s = 1) = \frac{1}{2}$. Given the monotonicity of the posteriors in the priors, any agent with posterior beliefs upon receiving a signal s that is higher than the marginal receiver will also automatically take the desired action. These marginal receivers allow us to identify the fraction of agents who take action $a_i = 1$ upon observing $s = 1$ or $s = 0$. Any receiver with belief above $m_{s=1} = \frac{\epsilon_0}{1-\epsilon_1+\epsilon_0}$ is going to take action 1 after seeing $s = 1$. Similarly, any receiver with belief above $m_{s=0} = \frac{1-\epsilon_0}{1+\epsilon_1-\epsilon_0}$ will take action $a_i = 1$ upon receiving either signal. This allows us to rewrite the Sender's problem as a function of the fraction of agents persuaded, such as to emphasize the role of (ϵ_1, ϵ_0) on the probability of signal realizations $s = 1$ and $s = 0$, as well as on the fraction of agents above the threshold given that signal. The objective of the Sender is rewritten as maximizing

$$\begin{aligned}\mathcal{V} =& \frac{1}{1-2\alpha}\left([\gamma(1-\epsilon_1) + (1-\gamma)\epsilon_0]\left[1 - \alpha - \frac{\epsilon_0}{1-\epsilon_1+\epsilon_0}\right]\right) \\ &+ \frac{1}{1-2\alpha}\left([1 - \gamma(1-\epsilon_1) - (1-\gamma)\epsilon_0]\left[1 - \alpha - \frac{1-\epsilon_0}{1+\epsilon_1-\epsilon_0}\right]\right)\end{aligned} \tag{4.2}$$

[15] Gentzkow and Kamenica (2011) refer to this class of signals as straightforward, where the equilibrium action follows the signal realization.

subject to

$$\epsilon_0 + \epsilon_1 \leq 1 \tag{4.3}$$

$$\alpha \leq \frac{\epsilon_0}{1 - \epsilon_1 + \epsilon_0} \leq 1 - \alpha \tag{4.4}$$

$$\alpha \leq \frac{1 - \epsilon_0}{1 + \epsilon_1 - \epsilon_0} \leq 1 - \alpha \tag{4.5}$$

$$0 \leq \epsilon_1 \leq 1 \tag{4.6}$$

$$0 \leq \epsilon_0 \leq 1. \tag{4.7}$$

The left term in equation 4.2 represents the expected probability of sending signal 1, multiplied by the fraction of agents taking action 1 upon signal realization $s = 1$, while the term on the right is the expected probability of sending the signal $s = 0$ multiplied by the fraction of agents whose posteriors are above the threshold upon seeing $s = 0$, therefore taking action $a_i = 1$. It is clear that a higher ϵ_1 increases the probability that $s = 1$ is realized, at the expense of $s = 0$, and affects the fraction of agents whose posterior is above the threshold. The first inequality is the labelling normalization such that $p(\omega = 1|s = 1) > p(\omega = 1|s = 0)$. Equations 4.4 and 4.5 are restricting the marginal receivers' priors to lay on the support of the prior distribution. The objective function is continuous by continuity of the uniform probability density function, and the set is compact; therefore, a maximum exists by the Weierstrass theorem.[16] In what follows, I will denote the optimal disclosure strategy as the pair $(\epsilon_0^*, \epsilon_1^*)$

Without communication, only half of the Receivers take $a_i = 1$, those with priors above the threshold. Insofar as the Sender wants as many Receivers as possible to take action $a_i = 1$, he could achieve a better outcome through strategic disclosure. From equation 4.2, he can achieve that by shifting as many Receivers above $\frac{1}{2}$ if $s = 1$, while keeping as many above $\frac{1}{2}$ (those whose priors were already above the threshold) if $s = 0$. His choice of ϵ_1 and ϵ_0 impacts both the probability of each signal realization as well as the fraction of agents above the threshold upon either signal realization. He will therefore face two trade-offs. A higher ϵ_0 increases the probability of the signal realized being $s = 1$, at the cost of a lower fraction of Receivers' beliefs shifted above $\frac{1}{2}$. However, the higher ϵ_0, the higher the fraction of Receivers whose beliefs he can keep above $\frac{1}{2}$ when $s = 0$. The optimal strategy equates the marginal expected benefit of increasing ϵ_0, which is the increased probability of $s = 1$ and the higher fraction of agents with beliefs kept above $\frac{1}{2}$ if $s = 0$, with the marginal cost of doing so, which is a lower fraction of agents whose beliefs are pushed above $\frac{1}{2}$ upon $s = 1$. A similar intuition holds when choosing ϵ_1.

[16]See Weierstrass (1988) for full characterization of the theorem and proof.

4.2 Optimal signaling strategy

A given disclosure strategy's advantage may differ across states, therefore a Sender committing to a disclosure strategy will take into account his own belief about the state before choosing his strategy. Indeed, the marginal benefit of increasing ϵ_0 or ϵ_1, which is the increase probability of signal realization $s = 1$, $p(s = 1) = \gamma(1 - \epsilon_1) + (1 - \gamma)\epsilon_0$, depends on the Sender's prior. I show that the optimal signal will depend therefore depends on how likely the Sender believes the good state is.

Proposition 1. *(Informativeness) For $\gamma \in [0, \frac{1}{2})$, the Sender chooses a signaling strategy that is completely uninformative about the state, $1 - \epsilon_1^* = \epsilon_0^*$. For $\gamma \in (\frac{1}{2}, 1]$, the signaling strategy is informative, $1 - \epsilon_1^* > \epsilon_0^*$.*

The proof of proposition 1 is provided in Appendix A, and relies on the Kuhn Tucker conditions obtained using equations 4.2 to 4.7. To develop the intuition, consider the extreme case of $\gamma = 0$. If $s = 1$ is realized, the Sender can convince between at least half and up to the entirety of the Receivers, while if $s = 0$, he can only have at most half of the Receivers above the threshold. Therefore, he would gain more by increasing the probability that $s = 1$. He can only increase the probability of $s = 1$ by setting ϵ_0, which he would want higher than $1 - \epsilon_1$, given he believes $1 - \epsilon_1$ will never be applied. The best he can do is set $\epsilon_0 = 1 - \epsilon_1$, which is the highest feasible possible option given the constraint 4.3. He would therefore not want to generate information and sober up the Receivers who, from his perspective, are mistaken in a favorable direction. When $\gamma > \frac{1}{2}$, he gets a higher increase in $p(s = 1)$ by having ϵ_1 low, such that he will set $1 - \epsilon_1 > \epsilon_0$.

I have shown in proposition 1 that the signaling strategy is informative if the Sender believes the good states to be *a priori* more likely. Despite being informative, the optimal strategy does not imply full disclosure. In fact, the Sender should adopt moderating signals in both states.

Proposition 2. *(Moderating signals) For $\alpha \in (0, \frac{1}{2})$ and $\gamma \in (\frac{1}{2}, 1)$, the optimal signaling strategy features positive error probabilities for both states, i.e $0 < \epsilon_1^*, \epsilon_0^* < 1$.*

Proof. The Proposition is proven by contradiction. Proposition 1 stated that if $\gamma > \frac{1}{2}$, it is optimal to set $\epsilon_0 \neq 1 - \epsilon_1$. This implies that $1 - \epsilon_1 > \epsilon_0$. If $\epsilon_0 = 1$, then $1 - \epsilon_1 > 1$ too, but $\epsilon_0 = 1 - \epsilon_1$ is not optimal. Similar arguments hold for ϵ_1. □

There are two noteworthy implications of the moderating signals. First, this means that as long as the Sender is uncertain about the state, even if he thinks the good state to be very likely, it is never optimal to choose zero error probability in both states (i.e., setting ϵ_1 and

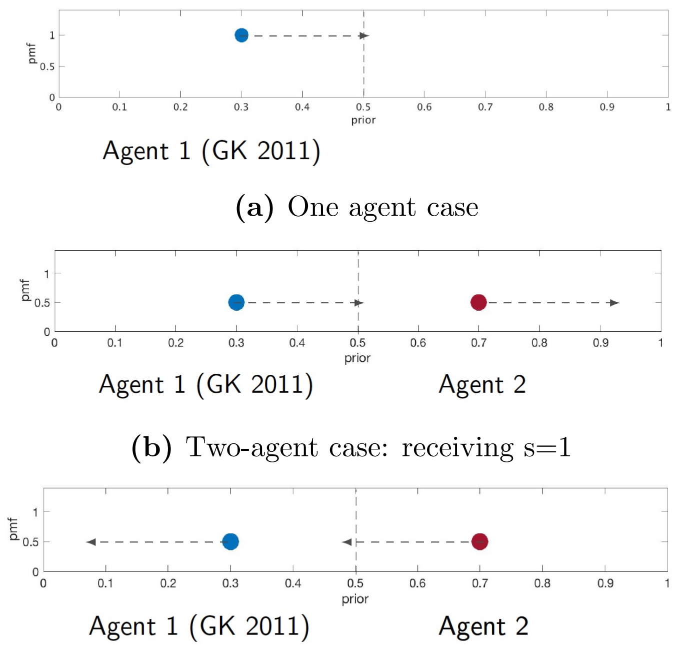
ϵ_0 to 0), or unit error probability in bad states (ϵ_0 to 1). The latter fact is true because both conditional probabilities are communicated to the agents. As a consequence, the strategy in bad times matters for the efficiency of the strategy in good times since they both appear in the posterior belief about a good state $p(\omega = 1|s = 1)$. Therefore, even if the Sender is convinced the probability of the realization $s = 1$ only depends on $1-\epsilon_1$, he will not set ϵ_0 to 1 as agents would think the signal realization $s = 1$ is very likely to have been generated from a bad state, and the posterior beliefs would not shift much. The former is because as long as some uncertainty remains, full disclosure would make him lose the optimistic agents whose beliefs are above the threshold. Full disclosure would allow him to convince all agents when $s = 1$, but as long as there is a positive probability that $\omega = 0$ (and therefore $s = 0$) can realize, he can achieve better by convincing even the lowest belief agent by setting $\epsilon_1, \epsilon_0 > 0$ and not losing all agents above the threshold.

Most importantly, the optimality of moderating signals implies that the Sender should be optimistic in bad states, as well as pessimistic in good states. The pessimism in good states is a result that departs non-trivially from the homogeneous beliefs case or one-receiver case of Gentzkow and Kamenica (2011). The intuition can be drawn from a two-receiver example (see figures 2a to 2c). In the one-receiver case, the Sender wanted to shift the belief of a single Receiver above the threshold (see panel 2a). To do so, it was optimal to fully disclose good states and partially disclose bad states in order to shift the posterior in equation 3.3 above the threshold. When there is a second Receiver (denoted Receiver 2), the Sender also wants to shift Receiver 1 above the threshold if $s = 1$, while keeping Receiver 2 above the threshold regardless of the signal realization. If he were to set $\epsilon_1 = 0$, then, upon seeing $s = 0$, Receiver 2's posterior $p(\omega = 1|s = 0)$ would be 0 (as in panel 2b). The trade-off the Sender faces is that, setting ϵ_1 high increases the posterior of Receiver 1, bringing it towards the threshold when $s = 1$, at the cost of lowering Receiver 2's posterior towards, if not below the threshold. A similar intuition holds for ϵ_0. This trade-off is absent in the homogeneous beliefs case.

Interestingly, even though the incentives of the Sender were not symmetric, as the Sender would want all agents to believe it is a good state so they take action $a_i = 1$, the optimal disclosure is symmetric across states, with $\epsilon_0 = \epsilon_1$. This is in contrast with the homogeneous beliefs' case, where full transparency in good states, and positive error probability in bad states implied asymmetric disclosure.

4.3 Optimal signaling strategy and receivers' disagreement

I have shown in the previous section that heterogeneity therefore matters significantly for communication in good times. It also impacts the extent to which the Sender will be



(a) One agent case

(b) Two-agent case: receiving s=1

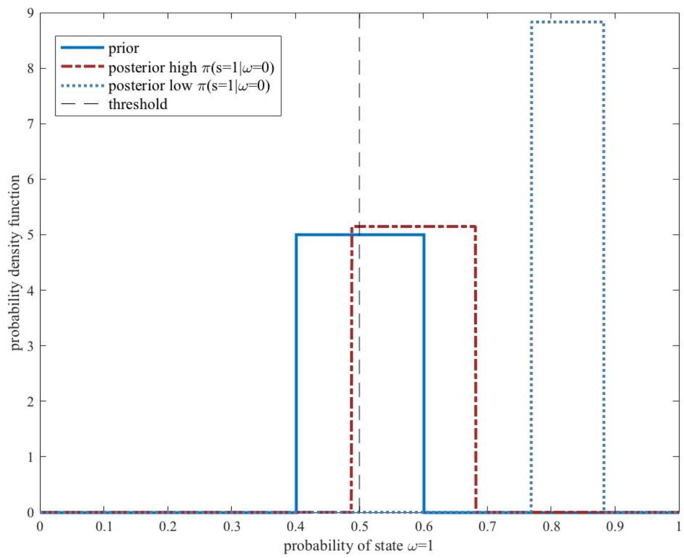
(c) Two-agent case: receiving s=0

Figure 2. Illustration of beliefs shifting with one-agent and two-agent cases

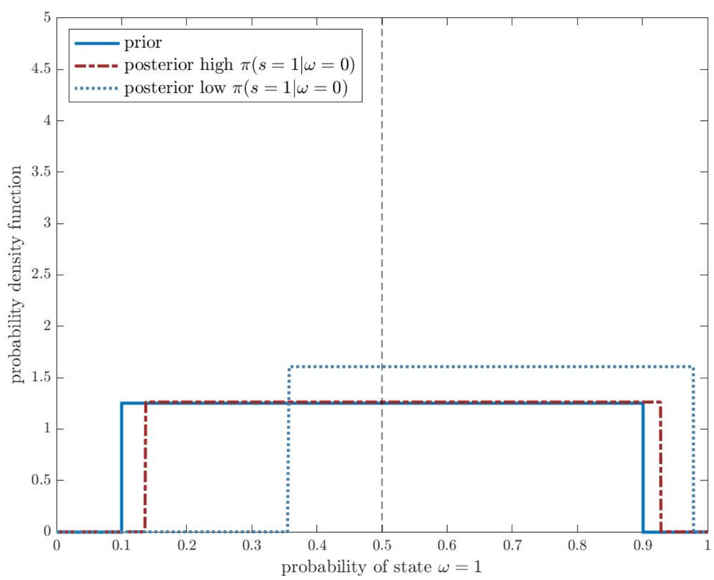
optimistic in bad states and pessimistic in good states, which I show in this section. The disagreement will incentivize the Sender to send a more powerful signal to convince the most pessimistic agents. Therefore higher disagreement generates a more truthful disclosure strategy.

Proposition 3. *(Truth-enhancing disagreement) For $\gamma > \frac{1}{2}$, the error probabilities $(\epsilon_0^*, \epsilon_1^*)$ are decreasing with the variance of receivers' priors.*

In particular, one can show that $\epsilon_0 = \epsilon_1 = \alpha$ (due to specific choice of the uniform distribution where the trade-off is linear in terms of marginal agents lost and gained). The proof of proposition 3 is developed in Appendix B. This result originates from one main trade-off. Recall that the higher the probability of sending a good signal in a bad state, the higher the *ex-ante* probability of the signal realized being $s = 1$, at the cost of a lower weight in the posterior, therefore shifting fewer agents above the threshold when $s = 1$. Indeed, the posterior $p(\omega = 1|s = 1)$ is decreasing with ϵ_0. With heterogeneity, the same trade-off applies, except that, with a higher variance, there are more Receivers in the tails of the distribution. This means there are more agents far away from the threshold whose beliefs need to be shifted by a lot, as illustrated in figures 3a and 3b. To shift these agents' beliefs more, the Sender needs agents to put more weight on his signal. Therefore, he needs to set the probability of sending a good signal in a bad state to be lower. This is of course at the cost of keeping fewer Receivers above the threshold when $s = 0$, so the Sender loses one for one. Setting the conditional probabilities such as $p(s = 1) > p(s = 0)$, the expected cost of losing the Receivers when $s = 0$ will be lower than the expected gain of convincing more when $s = 1$. In the general case, for instance with the normal distribution, this trade-off can be summarized with the elasticity of the probability density function (pddf): the elasticity of the pdf decreases with dispersion, therefore it becomes harder to shift beliefs far away. Also note that with non-symmetric distributions, this trade-off will be distorted and it will matter where the mass of agents is in either case.



(a) Trade-off with low disagreement



(b) Trade-off with high disagreement

Figure 3. Posterior beliefs for different disclosure policies and different dispersion in beliefs. Note: plotted here for $\pi(s = 1|\omega = 1) = 1$, with $\pi(s = 1|\omega = 0) = 0.2$ (dotted blue line), $\pi(s = 1|\omega = 0) = 0.7$ (dashed red line).

5 A persuasion model with reputation dynamics

The analysis above treated the conditional probabilities ϵ_0 and ϵ_1 to which the Sender was committing to, and which was known by the agents. Here, I study in a dynamic setting how the learning by Receivers about these probabilities over time generates a reputation effect. This reputation effect will be sufficient to incentivize the Sender to not be completely untruthful every period, even when he does not commit to a disclosure strategy.

5.1 The dynamic disclosure choice

Time is discrete and indexed by $t = 0, 1, 2, ...$ The state space, signal space and preferences are unchanged from the static model of section 3, with $\Omega = \{0, 1\}$, $s \in \{0, 1\}$ and payoffs as defined by equations 3.1 and 3.2. The key difference is that now the state evolves according to a Markov probability transition matrix where q_{ij} denotes the probability of going from state i to j, with $i, j \in \{0, 1\}$. Each period, the Sender observes the state before choosing a signal, but the Receivers do not until after having taken their actions.[17] The Sender knows the true transition probabilities of the state q_{ij}.

Receivers' beliefs and their evolution. Receivers have heterogeneous priors on the good state $\lambda_{i,t} = p_i(\omega = 1) \sim Beta(\alpha_t, \beta_t)$, where $\alpha_t = \alpha + n_{1,t}$ and $\beta_t = \beta + n_{0,t}$. α and β are the initial distribution hyperparameters, while $n_{1,t}$ and $n_{0,t}$ are the number of occurrences of good and bad states.[18] Over time, agents learn about the true transition matrix of the states. Receivers also hold priors on the disclosure strategy of the Sender, defined by $p_{0,t} = p(s_t = 1|\omega_t = 0)$ and $p_{1,t} = p(s_t = 0|\omega_t = 1)$. They are beliefs on the error probabilities of the Sender sending a signal which is the opposite of the realized state. The priors about the disclosure strategy will evolve with the realizations of signals and states. I assume that their priors are identical, and given that they all observe the same signal, posterior beliefs about the disclosure strategy will also be the same for all agents. Each period, agents will observe the state after their actions are taken and update their beliefs about these conditional probabilities. Upon observing a bad state and a good signal $(s = 1, \omega = 0)$, they realize the signal was untruthful; if they observe $(s = 1, \omega = 1)$, they build trust in the Sender. I denote $n^L_{0,t}$ and $n^L_{1,t}$ the number of times Receivers have observed the pairs $(s = 1, \omega = 0)$ and $(s = 0, \omega = 1)$, respectively, up until time t (before receiving

[17] Note that, compared to Bizzotto, Rüdiger and Vigier (2018), and Ely (2017), who also consider dynamic Bayesian persuasion games, in my setting there is no commitment at the stage game given the Sender observes the state and is free to choose either signal.

[18] The Beta distribution is used here because it is a conjugate prior for the binomial outcomes.

the signal at time t). Their priors at time t are thus given by

$$p_{0,t} = \frac{n^L_{0,t}}{n_{0,t}} \tag{5.1}$$

and

$$p_{1,t} = \frac{n^L_{1,t}}{n_{1,t}}. \tag{5.2}$$

Sender's and Receivers' problems. The Sender will choose the signaling strategy to maximize the discounted sum of his expected utility,

$$\mathcal{V} = \mathbb{E}\left(\sum_{t=0}^{\infty} \beta^t \int_0^1 a_{i,t} di\right). \tag{5.3}$$

Upon receipt of a signal, Receivers take an action $a_i \in \{0,1\}$ to maximize their contemporaneous utility, who define these agents as myopic, as they do not take into account their past actions' influence on the contemporaneous utility.[19]. They will follow a threshold strategy as in the static model, which the Sender takes into account when deciding on his disclosure strategy.

5.2 Optimal dynamic disclosure choice

I can express the Sender's problem recursively as a Bellman equation with state variables $n_{0,t}$, $n_{1,t}$, $n^L_{0,t}$, $n^L_{1,t}$ (history of past signal and state realizations) and ω_t (current state realization). Each period, upon learning about the state, he chooses his disclosure strategy to maximize the sum of the within-period objective function H where $H = \int a_i di$ and the discounted expected continuation value,

$$\begin{aligned} \mathcal{V}(\omega_t, n^L_{0,t}, n^L_{1,t}, n_{0,t}, n_{1,t}) = \max_{\pi(s_t|\omega_t)\in[0,1]} \{ & H(\omega_t, n^L_{0,t}, n^L_{1,t}, n_{0,t}, n_{1,t}, \pi(s_t)) \\ & + \delta E[\mathcal{V}(\omega_{t+1}, n^L_{0,t+1}, n^L_{1,t+1}, n_{0,t+1}, n_{1,t+1})|\pi(s_t)]\}. \end{aligned} \tag{5.4}$$

The parameter $\delta \in (0,1)$ is the rate at which the sender discounts the future. I denote $H(\omega_t, n^L_{0,t}, n^L_{1,t}, n_{0,t}, n_{1,t}, \pi(s_t))$ the period t expected utility of the Sender, which depends on the state ω_t, past states and signal realizations $n_{0,t}$, $n_{1,t}$, $n^L_{0,t}$, $n^L_{1,t}$.

Even though the numbers of state realizations $n_{0,t}, n_{1,t}$ are in the state space, this does not prevent $\mathcal{V}$ from being bounded insofar as they are needed here to compute the prior

[19]In this, I follow Renault, Solan and Vieille (2017) This can be interpreted as short-lived agents who pass along their knowledge to the other agents next period.

beliefs $p_{0,t}$, $p_{1,t}$, $p_{0,t+1}$ and $p_{1,t+1}$. Consequently, one can show that a fixed point exists.

Proposition 4. *(Contraction mapping) The Bellman equation (5.4) is a contraction mapping.*

Proof. In the appendix C. □

Proposition 4 can be proven by showing the Blackwell sufficient conditions for a contraction mapping are satisfied (see for instance Theorem 3.3 in Stokey and Lucas (1989)). By the Principle of Optimality, if the Bellman equation is a contraction mapping, this implies that there is a unique solution to the optimal signaling strategy. I can derive the solution by iterating through the value function over the discrete beliefs space. A disclosure policy in the dynamic setting is a rule $\pi(\omega_t, n^L_{0,t}, n^L_{1,t}, n_{0,t}, n_{1,t})$ that maps the Sender's prior history of signals and state realizations into a probability distribution over signals.

While I did not restrict the Sender's choice to a pure strategy, it is never optimal to randomize between signals. His optimal choice will be either $s = 1$ or $s = 0$ with probability 1. The value function ultimately only depends on the beliefs of the agents, which depends on signal realizations. He therefore does not gain from randomization.

The dynamic choice introduces a new trade-off. While the Sender can increase his period utility by sending a good signal in a bad state, he now needs to consider the impact of his choice on his continuation value through the weight that agents will put on his signal next period. Indeed, the continuation value $\mathcal{V}(\omega_{t+1}, n^L_{0,t+1}, n^L_{1,t+1}, n_{0,t+1}, n_{1,t+1})$ depends on Receivers' beliefs in the next period, $p_{0,t+1}$ and $p_{1,t+1}$, which get updated after seeing both signal and state at the end of period t. If he sends $s = 1$ when he observes the bad state, he knows the agents will revise $p_{0,t}$ upwards, and he will have less power in shifting their beliefs next period.

Because of the cost in terms of continuation value, the Sender will not always choose to send $s = 1$ when $\omega = 0$ and vice versa for $\omega = 1$. The optimal choice will depend on the Receivers' beliefs $p_{0,t}$ and $p_{1,t}$ (i.e., the Sender's past actions). In particular, as shown in figure 4, when the Sender has built a reputation for making errors in bad states such that $p_{0,t}$ gets too high, it is optimal to reveal truthfully a bad state to build reputation. The reputation trade-off here is reminiscent of the static version's trade-off but is at play intertemporally. Even when the Sender observes the state $\omega = 0$ and therefore cares about impacting $p_{0,t}$ next period, the current belief $p_{1,t}$ also enters into his optimal choice, because the pair determines the fraction of agents he can convince today.

This means that commitment is not needed to obtain moderating signals and reputation concerns are enough when there are interactions over time. Because agents form and update their beliefs about the conditional probabilities of a signal in a given state, this prevents the

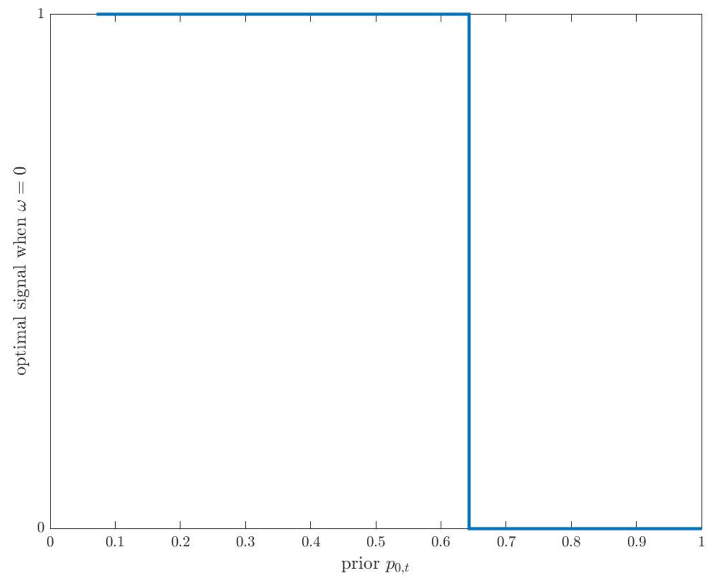


Figure 4. Policy function (signal s when $\omega = 0$), as a function of $p_{0,t} = \frac{n^L_{0,t}}{n_{0,t}}$. The policy function is plotted for a fixed number of pairs ($s = 0, \omega = 1$), with $p_{1,t} = \frac{n^L_{1,t}}{n_{1,t}} = \frac{1}{8}$, $\delta = 0.8$ and $\alpha = \beta = 2$.

Sender from always choosing an untruthful signal. These results provide an alternative way of obtaining persuasion in the long run, complementing the work of Best and Quigley (2017). In a repeated cheap talk game between a long-lived Sender and short-lived Receivers observing past histories of reports and states, the authors have showed that one could introduce a "Cup and Coin" mechanism to condition the punishment of a deviating signal on the realization of a random variable distinct from the state.

6 Strategic central bank communication

The previous sections developed an abstract framework of the state dependence of the optimal disclosure strategy and its relationship to disagreement among agents. The framework developed in this paper can help understand the strategic incentives policy makers face when wanting to avoid "adding unnecessarily to the prevailing gloom by talking down the economy" during bad states, as emphasized in Bernanke (2015). Therefore, in this section, I apply the framework to a more concrete example in which a central bank communicates to firms making investment decision and misalignment in incentives stems from investment externalities. I will subsequently test for the optimal behavior of one piece of communication published by the Fed, the FOMC forecasts, and show that unemployment rate forecasts are strategically biased in the directions predicted by my model.

6.1 Central bank communication in presence of coordination externalities

Firms. Consider a continuum of firms making a binary investment decision (for instance choosing capacity utilization or adding a production plant) $u_i \in \{0, 1\}$.[20] Choosing investment incurs a fixed cost $c > 0$. The returns to investment depend on a fundamental θ that follows a binary process, where $\theta \in \{\theta_L, \theta_H\}$. The fundamental can represent the state of aggregate demand. Therefore, this means that returns to investment are high when aggregate demand is high, and a firm would like to invest or expand its capacity when it expects demand to be high. Firms choose their investment to maximize their profits, which are defined as

$$\Pi_i = \left(\theta + \int_0^1 u_j dj\right) u_i - cu_i. \tag{6.1}$$

Their profits exhibit a coordination externality, in which the total returns to investment are endogenous to the aggregate action. Firms do not take into account the impact of their action on the total returns when making their investment decision. As in the general framework, firms will hold different priors on the probability of high returns, with $\lambda_i \sim U(\alpha, 1-\alpha)$, and will decide to invest if they believe the high return to be likely enough.

Central bank. The monetary authority has a utilitarian welfare function and chooses a communication strategy, which is a probability distribution of signals as a function of the state, to maximize aggregate payoffs as defined above, such that

$$\mathcal{V} = \int_0^1 \left(\left(\theta + \int_0^1 u_j dj\right) u_i - cu_i\right) d_i. \tag{6.2}$$

Because the policy maker takes the coordination externality into account, he will want to induce firms to choose $u_i = 1$ whenever $c \in (0, 1)$. He will do so by committing to a communication policy before knowing the state.

We can think of the communication policy of the central bank as choosing and committing to a data generating process, i.e., specifying the statistical relationship between the state $\theta \in \{\theta_L, \theta_H\}$ and the data (signal $s \in \{\theta_L, \theta_H\}$). One example could be choosing the forecasting framework as a function of the state.

Optimal state-dependent communication. The policy maker knows firms follow a

[20] The capacity decision to adjust production over the business cycle can act as a TFP shifter where $Y = Au_i e^{\theta + \int_0^1 u_j d_j}$, where θ is the fundamental productivity, and A is a scaling factor.

threshold strategy and wants to shift their posteriors such that

$$p(\theta = \theta_H | s) \geq c \tag{6.3}$$

by committing to a set a error probabilities (ϵ_0, ϵ_1) before knowing the state. Solving for his optimal disclosure yields the following proposition:

Proposition 5. *As long as the distribution of priors is symmetric around $c \in (0,1)$, the moderating signals and truth-enhancing disagreement properties of the static model of Bayesian persuasion will hold.*

Proof. For $c = 0.5$, this is a problem identical to the one in section 3, so the proof is identical. For other values of $c \in (0,1)$, this follows from deriving the Karush-Kuhn-Tucker conditions as in the static part, but for an objective function where the marginal Receivers are $m_{s=1} = \frac{\epsilon_0 c}{(1-\epsilon_1)(1-c)+\epsilon_0 c}$ and $m_{s=0} = \frac{(1-\epsilon_0)c}{\epsilon_1(1-c)+(1-\epsilon_0)c}$. Algebra similar to that used in the proofs of section 3 applies. For $c > 1$, the aggregate cost is higher than the benefit of more investment. □

Proposition 5 implies that investment externalities are a source of misalignment in incentives that is sufficient to generate strategic communication, when investment cost is low enough. The model replicates the two main predictions of the abstract framework of section 3. Indeed, in this application, the policy maker would want to send signals with positive error probabilities in both states. The higher the dispersion in firms' beliefs, the lower he will set these probabilities. The only difference from the static setting is that the cost c affects the marginal cost of increasing ϵ_1 or ϵ_0. As a consequence, the pair $(\epsilon_1^*, \epsilon_0^*)$ will still be positive, but the symmetry of the disclosure policy $\epsilon_1^* = \epsilon_0^*$ breaks.

Discussion: There are alternative payoffs functions that yield similar results. One could adapt the set up to a more familiar loss function as a deviation from a target, where for instance, $\mathcal{V} = -(\int_0^1 a_i - a^T)^2$. One could think of this as the central bank as targeting nominal output, which possibility was discussed in the November 2011 FOMC meeting, and where the agents' action is producing or not. If the nominal target is high enough, all results will go through. Similar results would hold if a central bank suffers from an inflation bias, with a payoff function of $\mathcal{V} = -(\int_0^1 a_i - \theta - b)^2$ with $b > 0$, the inflation bias being high enough, and again letting a_i be the action of producing or not.

We now know from the theoretical model that central banks should send moderating signals in both states, meaning being sometimes unduly optimistic in bad states and pessimistic in good states. Moreover, from the model, we should expect lower error probabilities when there is higher disagreement among the private sector. I now turn to the analysis of one piece of communication that central banks publish in practice to investigate how the choice of the statistical relationship between the state and the data operates in practice.

6.2 Strategic bias of FOMC forecasts

One medium through which central banks communicate about aggregate conditions is public forecasts. Given that they act as a focal point for beliefs, (Hubert (2014) and Fujiwara (2005)), they could be used strategically. In this section, I investigate whether FOMC members' forecasts validate the model's predictions. This means testing whether the Fed displays the kind of strategic communication bias that my model predicts, as the quotes I have referred to earlier would suggest: Bernanke (2015) wanted to avoid "adding unnecessarily to the prevailing gloom by talking down the economy", Andrew Haldane emphasized in an interview that "had [they] been fully open and fully transparent about what was going on during the financial crisis [...], it would have been a lot, lot worse."[21] If the FOMC forecasts were to be published strategically, my model predicts that they should be systematically biased in opposite directions in expansions and recessions and exhibit biases that are decreasing in the disagreement among market participants. Therefore, I test whether FOMC forecasts exhibit such behavior and contrast it with the behavior of private forecasts.

6.2.1 Data

I use forecasts on GDP, inflation and unemployment rates published by the FOMC. These forecasts can be seen as a signal about the state of the economy that central banks send to the agents in the economy. To determine whether this signal is optimistic or pessimistic, I gather the corresponding estimates of realized GDP, inflation, and unemployment rate to compute forecast errors.

The FOMC forecasts are taken from Monetary Policy Reports (MPR), published bi-annually in the Monetary Policy Reports to Congress in February and July, and can be found in the Summary of Economic Projections (SEP) of the precedent FOMC meetings (which corresponds to the January and June/July SEP). They have the advantage of being

[21] The corresponding Financial Times article in which the Andrew Haldane's interview was recorded is Jackson (2017). Regarding the strategic biases, while not answering these questions, Ellison and Sargent (2012) mention the possibility that FOMC members might use their forecasts strategically, and in particular when interacting with other FOMC members

published without a lag, unlike the Greenbook forecasts. I collect the top and bottom points of the central tendency of the 12 Presidents' forecasts[22] for inflation (Q4/Q4 change), unemployment rate (Q4) and real GDP (Q4/Q4 change) from 1979 to 2016.[23]

Outcome variables are the initial estimates of GDP and inflation for the fourth quarter that are released late January of the next year in the Survey of Current Business (SCB). For the unemployment rate, I collect the data from both the Greenbook and Monthly Labor Report.[24] The sample ranges from 1979 to 2016, as the FOMC forecast data.

The variable of interest, the forecast bias, is constructed as the difference between the policy makers' forecasts (midpoint of the central tendency) and the corresponding outcome variable

$$e_{t|t+k} = y_{t+k} - f_{t|t+k},$$

where k stands for the forecast horizon. Table 1 of appendix D provides summary statistics.

The private sector beliefs about economic conditions are taken from both the Survey of Professional Forecasters (SPF) and the Michigan Survey of Consumers (MSC). These surveys covers different types of agents and I use the MSC for robustness checks. From the SPF, I collect data on respondents' probability of recession for horizons from the current quarter up to the next four quarters (recess1 to recess5) from 1979 to 2016. From the MSC, I gather the index of current economic conditions, the index of consumer expectations and the index of consumer sentiment, for the equivalent period. I measure disagreement as the cross-sectional variance of individual responses at time t. From the release dates of the SPF, I can use the latest data on disagreement that was available to forecasters before each MPR.[25]

6.2.2 Are FOMC forecasts strategically biased?

The two main predictions of the static model were to send moderating signals, but with lower probability of error when disagreement is higher. Applied to forecasts, which are

[22]Individual forecasts for the 12 members are released with a ten-year lag. While most of the SEP indicate the median forecasts, this could not be found for the 2012-2014 period, which makes. the central tendency measures preferable to the median.

[23]Up until 2004, the first report only contained forecasts for the current year, while the second report had forecasts for both current and next year.

[24]This corresponds to the February publication.

[25]The SPF is usually released in the middle to late January for the first quarter, while late August for the third quarter. The timing pre-1990 is unknown by the San Francisco Fed. As explained in their codebook, the San Francisco Fed does not know with certainty the timing of the releases pre 1990Q3 as the survey was then a product of the ASA/NBER but they think it was similar to their own schedule. Whenever contemporaneous survey data was not available prior to the MPR at the time that policy makers were making their forecasts, I use a one-quarter lag. From 1991-onwards, time frame for which I have the schedule for the release of the MSC, I proceed in a similar manner. For the periods for which I do not have the schedule, I follow the releasing trends the post-1991 as release dates only vary by a few days, but I also conduct robustness checks to verify it does not affect the results.

the signals in this context, this means forecasts should be biased in opposite directions in expansions and recessions, and exhibit less bias during periods of higher disagreement. Take, for instance, unemployment rate forecasts. If the model holds, we should therefore observe somewhat positive average forecast errors during recessions, which corresponds to sending a good signal in a bad state, since the policy maker would want to publish an unemployment rate that is lower than the realized value. We should observe the converse in expansions. These forecast errors correspond to ϵ_1 and ϵ_0 of the static model in section 3. These ϵ_1 and ϵ_0 are decreasing with disagreement, so we should observe correspondingly lower biases when there is high disagreement in the SPF.

The gist of the empirical test is to check whether the Fed follows four different strategies in recessions and expansions with more or less disagreement, corresponding to $\epsilon_1^H, \epsilon_1^L, \epsilon_0^H$ and ϵ_0^L in the model. The data is split into these four categories, as identified by the NBER indicators of recessions, and where disagreement in a state is measured by the cross section of responses in the SPF. I define time t to be a period of high disagreement (in a recession) if the standard deviation of responses at t is higher than the mean standard deviation of responses over all the recessions, and similarly for expansions. If the Fed follows four distinct strategies, then the data must be drawn from four different distributions. I test for different distributions by testing for different means. I estimate the mean forecast errors distribution with Bayesian methods. To estimate the means' distribution, I fix the prior on both the mean and standard deviation to an uninformative prior, Jeffreys (1946)'s prior. Assuming forecasts errors are i.i.d and drawn from four different normal distributions, I obtain the following formula for the posterior marginal distribution of the mean,

$$f(\mu_S|X) = \frac{\Gamma(\frac{n}{2})}{\Gamma(\frac{n-1}{2})(\pi C)^{\frac{1}{2}}}[1 + \frac{(\mu_S - \overline{x})^2}{C}]^{\frac{n}{2}}. \tag{6.4}$$

Define periods of expansion with low disagreement $E - LD$, expansion with high disagreement $E - HD$, and similarly for recessions $R - LD$ and $R - HD$. S represents the type of period when the forecasts are made, with $S \in \{E - LD, E - HD, R - HD, R - LD\}$. $\overline{x} = \frac{\sum x_i}{n}$ is the sample mean and $C = \frac{1}{n}\sum(x_i - \overline{x})^2$.

To understand the logic of the empirical exercise, recall that the model delivered two main predictions, which implied forecast biases of opposite signs in good and bad times, along with lower biases in periods of high disagreement. I investigate whether central banks communicate as the model predicts, which would translate in four different forecasting strategies (posterior distributions of forecast errors). I then ask whether these patterns could be truly stemming from strategic behavior, or are just inherent to forecasting, with agents without any strategic incentives making similar errors as predicted by the model.

First, I find that the Federal Reserve's forecasting behavior is consistent with the model's recommendations with regards to communicating about unemployment. Figures 5 to 7 represent the marginal posterior distributions of the mean forecast errors of the FOMC during expansions and recessions with low and high disagreement for output, inflation and unemployment. When communicating their forecasts about the unemployment rate in a recession, policy makers tend to be more optimistic, which translates into positive forecast errors. By contrast, in an expansion, I observe a negative bias. Secondly, as illustrated by the shift in the distribution of the mean in periods of high disagreement towards zero, FOMC forecasts are less biased during periods of high disagreement for both states, which is in line with the model's predictions. This holds regardless of whether the observed variable is measured by the initial or final estimates. The Fed GDP forecasts does exhibit different biases in recessions and expansions, but they do not with disagreement. The absence of difference in distributions for the inflation is not unsurprising insofar as these forecasts are mainly used to anchor expectations around the inflation target.

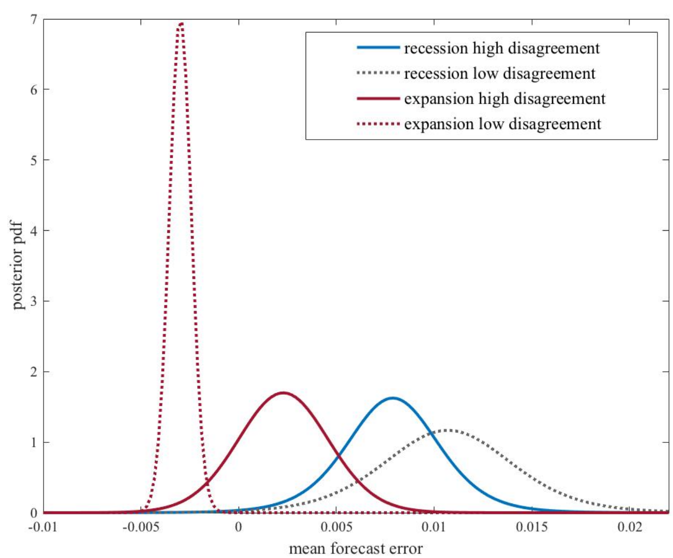


Figure 5. Posterior distribution of unemployment rate forecast errors (FOMC). Marginal posterior distribution of the mean forecast errors, estimated using Jeffreys' prior and under the assumption of i.i.d normal data. Sample 1979-2016 with all forecast horizons. Disagreement as measured by the SPF expectations over the next 3 quarters.

My results suggest that the FOMC unemployment forecasts are released strategically and in a state-dependent manner, replicating exactly the predictions of the model. This suggests that the Fed exhibit upward and downward biases relative to the business cycles. I verify

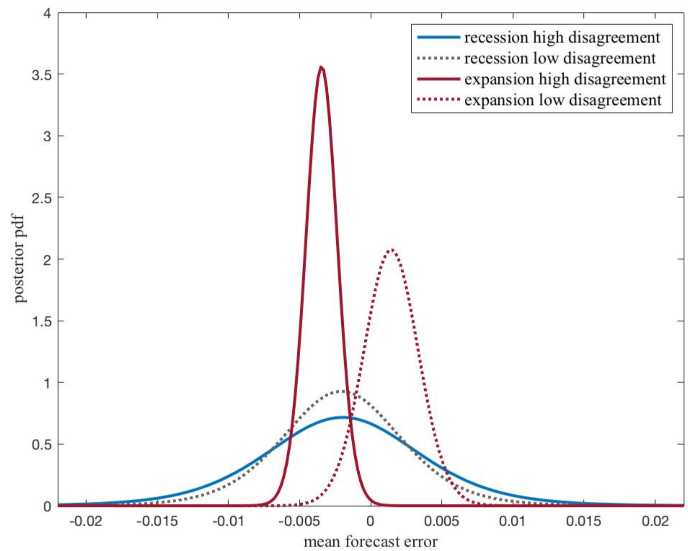


Figure 6. Posterior distribution of inflation forecast errors (FOMC). Marginal posterior distribution of the mean forecast errors, estimated using Jeffreys' prior and under the assumption of i.i.d normal data. Sample 1979-2016 with all forecast horizons. Disagreement as measured by the SPF expectations over the next 3 quarters.

whether this behavior carries on with the Greenbook forecasts (figure 8). Looking at the difference between the FOMC forecasts and their Greenbook counterparts, I find that FOMC forecasts for unemployment are systematically lower (higher) than those of the Greenbook in recession (expansion). However, the magnitude of that difference is quite small. This small difference can be rationalized by the fact that both entities coordinate to some extent to prevent any loss of credibility of the FOMC, but committee members can introduce some strategic biases. Most importantly, figure 9 indicates that the FOMC forecasts display a significantly distinct behavior from the private forecasts. This provides support for the interpretation of strategic behavior. Indeed, this means that even if this type of errors were inherent to forecasting, the FOMC forecasts are systematically more biased in opposite directions and according to disagreement than the private sector. For instance, the use of the past observations to forecast a given variable such as in a AR(1) process could generate opposite biases in recessions and expansions as observed in figure 5. The fact that FOMC and SPF forecasts still seem behave differently in recession and expansion allows for the hypothesis of strategic behavior to hold regardless of any systematic errors due to forecasting techniques.

A similar exercise for the European Central Bank reveals that the forecasts made by the ECB Staff and Eurosystem are not consistent with the predictions of the model. The availability of unemployment rate forecasts renders the comparison across recessions and expansions impossible, but even within expansions, biases are higher in periods of high disagreement contrarily to the optimal strategy. GDP forecast errors are systematically negatively biased, indicating a consistent optimism regarding output. Regarding its main policy objectives, the inflation target, there is no state dependency which can be observed, but forecasts biases do move negatively with the level of disagreement, as recommended by the theoretical results.

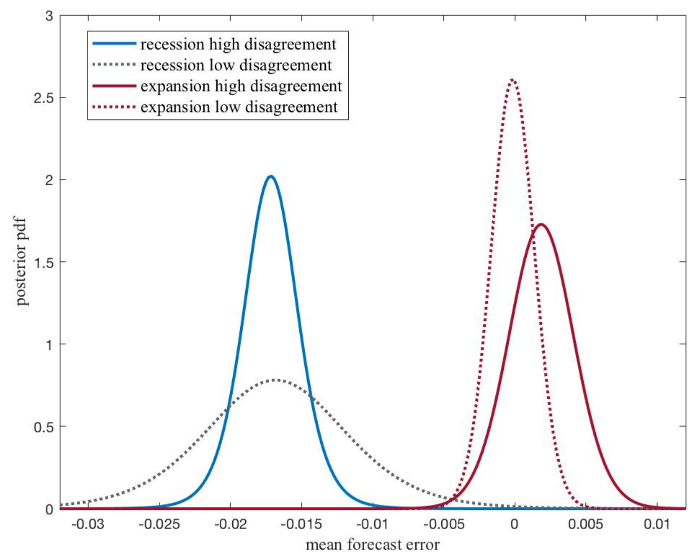


Figure 7. Posterior distribution of GDP forecast errors (FOMC). Posterior distribution of the mean forecast errors, estimated using Jeffreys' prior and under the assumption of i.i.d normal data. Sample 1979-2016 with all forecast horizons. Disagreement as measured by the SPF expectations over the next 3 quarters.

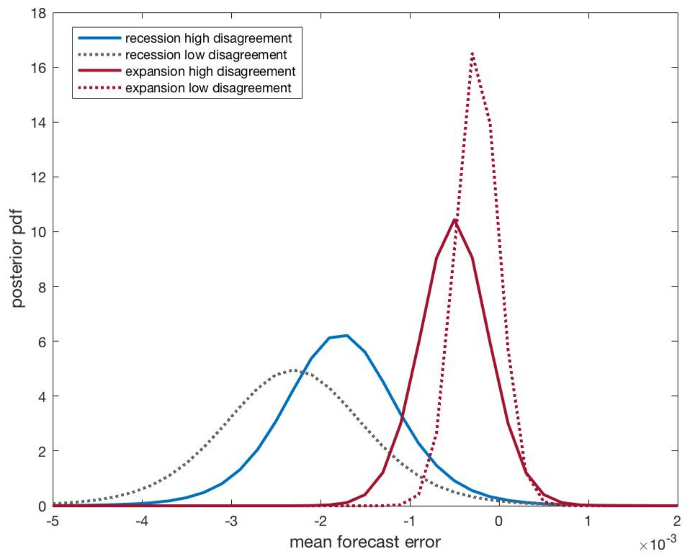


Figure 8. Posterior distribution of unemployment rate forecast difference between FOMC and Greenbook forecasts. Posterior distribution of the mean forecasts, estimated using Jeffreys' prior and under the assumption of i.i.d normal data. Sample 1979-2016 with all forecast horizons. Disagreement as measured by the SPF expectations over the next 3 quarters.

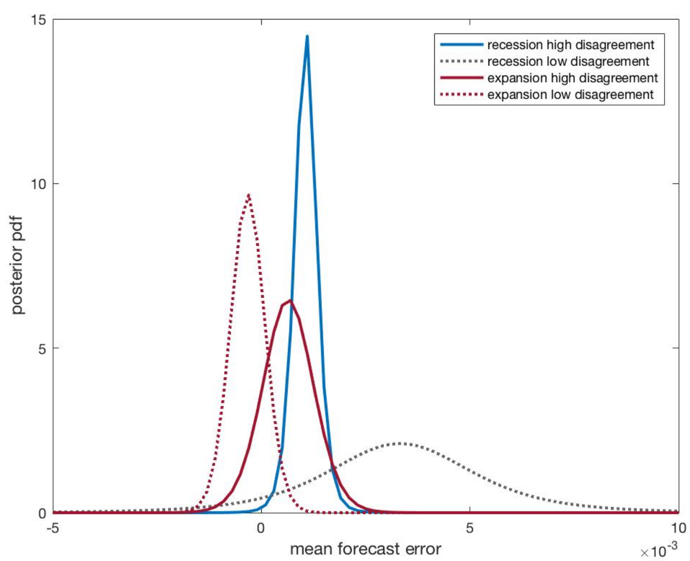


Figure 9. Posterior distribution of unemployment rate forecast difference between FOMC and SPF forecasts. Posterior distribution of the mean forecasts, estimated using Jeffreys' prior and under the assumption of i.i.d normal data. Sample 1979-2016 with forecast horizon Q4. Disagreement as measured by the SPF expectations over the next 3 quarters.

7 Conclusion

In this paper, I developed a Bayesian persuasion model where receivers have heterogeneous priors, and which allowed for strategic motives to communicate differently across states. Given its features, my model is particularly applicable to strategic central bank communication over the business cycle. For instance, central bank may face strategic motives to avoid adding more gloom to the economy by communicating strategically. The commitment to a set of conditional probabilities maps into communication strategies in different states of the world and allows me to speak meaningfully about state-dependent communication.

The focus of the paper is to study the optimal disclosure strategy for inducing a desired action and the variation of this disclosure strategy with the dispersion in beliefs. I prove that heterogeneous beliefs matter for the optimal signaling mechanism in two ways. First, it is optimal to send moderating signals, meaning sending signals with positive error probabilities in both state. This result contrasts with the one-receiver case (or homogeneous beliefs), where good times required truthful disclosure. This is because with heterogeneous beliefs, there are Receivers who already take the correct action and the Sender does not want to distort their belief about the good state too much. The limiting case of no heterogeneity in my model nests the model of Gentzkow and Kamenica (2011). Secondly, the higher the dispersion in beliefs, the lower these error probabilities are.

The results above are from a static model. An alternative to commitment to a disclosure strategy is reputation that is built over time. I extend the framework to a dynamic setting in which agents form beliefs about these error probabilities. I show that reputation concerns are enough to generate the main results of moderating signals.

My model generates several policy implications, such as how it should be tailored as a function of the business cycle and how it should vary with the degree of disagreement among the private sector. I have shown that the behavior of the FOMC unemployment rate forecasts validate the model's predictions. Using Bayesian estimation techniques, I showed that as in the model, FOMC forecasts of the unemployment rate are systematically biased in opposite directions in recessions and expansions, and the less so the higher the disagreement in the Survey of Professional Forecasters. Most importantly, this behavior contrasting from the private forecasts, I interpret such behavior as potentially stemming from strategic motives, ruling out that this is a behavior entirely inherent to forecasting techniques.

The model I have developed in this paper could be included in workhorse macroeconomic models, where on top on production and investment decisions, firms make binary capacity utilization choice, or in models in which monetary authorities also choose an instrument which has signaling effect that can interact with the state-dependent disclosure I have characterized in this paper.

References

Amador, Manuel, and Pierre-Olivier Weill. 2010. "Learning from prices: Public communication and welfare." *Journal of Political Economy*, 118(5): 866–907.

Amato, Jeffery D., Stephen Morris, and Hyun Song Shin. 2002. "Communication and Monetary Policy." *Oxford Review of Economic Policy*, 18(4): 495–503.

Andrade, Philippe, Gaetano Gaballo, Eric Mengus, and Benot Mojon. 2019. "Forward Guidance and Heterogeneous Beliefs." *American Economic Journal: Macroeconomics*, 11(3): 1–29.

Angeletos, George-Marios, and Alessandro Pavan. 2007*a*. "Efficient use of information and social value of information." *Econometrica*, 75(4): 1103–1142.

Angeletos, George-Marios, and Alessandro Pavan. 2007*b*. "Socially optimal coordination: Characterization and policy implications." *Journal of the European Economic Association*, 5(2-3): 585–593.

Arieli, Itai, and Yakov Babichenko. 2016. "Private bayesian persuasion." *Mimeo.*

Bergemann, Dirk, and Stephen Morris. 2016. "Information design, Bayesian persuasion, and Bayes correlated equilibrium." *American Economic Review*, 106(5): 586–91.

Bernanke, Ben S. 2015. *The courage to act: A memoir of a crisis and its aftermath.* WW Norton & Company.

Best, J., and D. Quigley. 2017. "Persuasion for the Long Run." Oxford Univ. Working paper.

Bizzotto, Jacopo, Jesper Rüdiger, and Adrien Vigier. 2018. "Dynamic persuasion with outside information." *Available at SSRN 3154313*.

Blinder, Alan S., Michael Ehrmann, Marcel Fratzscher, Jakob De Haan, and David-Jan Jansen. 2008. "Central Bank Communication and Monetary Policy: A Survey of Theory and Evidence." National Bureau of Economic Research Working Paper 13932.

Chahrour, Ryan. 2014. "Public Communication and Information Acquisition." *American Economic Journal: Macroeconomics*, 6(3): 73–101.

Cheng, Yu, Ho Yee Cheung, Shaddin Dughmi, Ehsan Emamjomeh-Zadeh, Li Han, and Shang-Hua Teng. 2015. "Mixture selection, mechanism design, and signaling." 1426–1445, IEEE.

Crawford, Vincent P., and Joel Sobel. 1982. "Strategic Information Transmission." *Econometrica*, 50(6): 1431–1451.

Ellison, Martin, and Thomas J. Sargent. 2012. "A DEFENSE OF THE FOMC." *International Economic Review*, 53(4): 1047–1065.

Ely, Jeffrey C. 2017. "Beeps." *American Economic Review*, 107(1): 31–53.

Fujiwara, Ippei. 2005. "Is the central bank's publication of economic forecasts influential?" *Economics Letters*, 89(3): 255–261.

Gaballo, Gaetano. 2016. "Rational Inattention to News: The Perils of Forward Guidance." *American Economic Journal: Macroeconomics*, 8(1): 42–97.

Gentzkow, Matthew, and Emir Kamenica. 2011. "Bayesian Persuasion." *American Economic Review*, 101(6): 2590–2615.

Gentzkow, Matthew, and Emir Kamenica. 2017. "Disclosure of endogenous information." *Economic Theory Bulletin*, 5(1): 47–56.

Geraats, Petra M. 2002. "Central Bank Transparency." *The Economic Journal*, 112(483): F532–F565.

Goldstein, Itay, and Yaron Leitner. 2018. "Stress tests and information disclosure." *Journal of Economic Theory*, 177: 34–69.

Hellwig, Christian. 2005. "Heterogeneous information and the welfare effects of public information disclosures." UCLA mimeo.

Hubert, Paul. 2014. "FOMC Forecasts as a Focal Point for Private Expectations." *Journal of Money, Credit and Banking*, 46(7): 1381–1420.

Inostroza, Nicolas, and Alessandro Pavan. 2017. "Persuasion in global games with application to stress testing." Northwestern. Univ. Working paper.

Jackson, Gavin. 2017. "https://www.ft.com/content/bfdbceb4-aa9d-11e7-ab55-27219df83c97."

Jeffreys, Harold. 1946. "An invariant form for the prior probability in estimation problems." *Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 186(1007): 453–461.

Jensen, Henrik. 2002. "Optimal Degrees of Transparency in Monetary Policymaking." *Scandinavian Journal of Economics*, 104(3): 399–422.

Kamenica, Emir. 2018. "Bayesian persuasion and information design." *Annual Review of Economics*, 11.

Kawamura, Kohei, Yohei Kobashi, Masato Shizume, and Kozo Ueda. 2019. "Strategic central bank communication: Discourse analysis of the Bank of Japans Monthly Report." *Journal of Economic Dynamics and Control*, 100: 230 – 250.

Kolotilin, Anton, Tymofiy Mylovanov, Andriy Zapechelnyuk, and Ming Li. 2017. "Persuasion of a privately informed receiver." *Econometrica*, 85(6): 1949–1964.

Morris, Stephen. 1994. "Trade with Heterogeneous Prior Beliefs and Asymmetric Information." *Econometrica*, 62(6): 1327–1347.

Morris, Stephen. 1995. "The Common Prior Assumption in Economic Theory." *Economics and Philosophy*, 11(02): 227–253.

Morris, Stephen, and Hyun Song Shin. 2002. "Social Value of Public Information." *American Economic Review*, 92(5): 1521–1534.

Morris, Stephen, and Hyun Song Shin. 2005. "Central bank transparency and the signal value of prices." *Brookings Papers on Economic Activity*, 2005(2): 1–66.

Morris, Stephen, and Hyun Song Shin. 2007. "Optimal communication." *Journal of the European Economic Association*, 5(2-3): 594–602.

Morris, Stephen, and Hyun Song Shin. 2018. "Central bank forward guidance and the signal value of market prices." Bank for International Settlements.

Morris, Stephen, Hyun Song Shin, and Hui Tong. 2006. "Social Value of Public Information: Morris and Shin (2002) Is Actually Pro-Transparency, Not Con: Reply." *The American Economic Review*, 96(1): 453–455.

Rayo, Luis, and Ilya Segal. 2010. "Optimal Information Disclosure." *Journal of Political Economy*, 118(5): 949–987.

Reis, Ricardo. 2013. "Central Bank Design." *Journal of Economic Perspectives*, 27(4): 17–44.

Renault, Jérôme, Eilon Solan, and Nicolas Vieille. 2017. "Optimal dynamic information provision." *Games and Economic Behavior*, 104: 329–349.

Stokey, Nancy, and Robert Lucas. 1989. *Recursive methods in economic dynamics.* Harvard University Press.

Svensson, Lars E. O. 2006. "Social Value of Public Information: Comment: Morris and Shin (2002) Is Actually Pro-Transparency, Not Con." *The American Economic Review*, 96(1): 448–452.

Tamura, W. 2016. "Optimal Monetary Policy and Transparency under Informational Frictions." *Journal of Money, Credit and Banking*, 48(6): 1293–1314.

Van den Steen, Eric. 2010. "On the origin of shared beliefs (and corporate culture)." *The RAND Journal of Economics*, 41(4): 617–648.

Weierstrass, Karl. 1988. "On continuous functions of a real argument which have no value of the latter a definite differential quotient." In *Select chapters from function gauges.* , ed. Springer, 190–193. Springer.

Wiederholt, Mirko. 2016. "Optimal Central Bank Communication." Society for Economic Dynamics 2016 Meeting Papers 1234.

Winkler, Bernhard. 2000. "Which kind of transparency? On the need for clarity in monetary policy-making." European Central Bank Working Paper Series 0026.

Appendix A Proof of proposition 1

The proof relies on the Karush-Kuhn-Tucker (KKT henceforth) conditions. Recall the maximization problem from section 3:

$$\mathcal{V} = \frac{1}{1-2\alpha}\{[\gamma(1-\epsilon_1)+(1-\gamma)\epsilon_0][1-\alpha-\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0}]+[1-\gamma(1-\epsilon_1)-(1-\gamma)\epsilon_0][1-\alpha-\frac{1-\epsilon_0}{1-\epsilon_0+\epsilon_1}]\}$$

subject to

$$\alpha \leq \frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} \leq 1-\alpha \quad (\theta_1, \theta_2)$$

$$\alpha \leq \frac{1-\epsilon_0}{1-\epsilon_0+\epsilon_1} \leq 1-\alpha \quad (\theta_3, \theta_4)$$

$$\epsilon_0 + \epsilon_1 \leq 1 \quad (\theta_5),$$

where the θs are the Lagrange multipliers associated with the constraints, and where I ignore the bound in [0,1] first, but check that the solutions satisfy these conditions. I describe the KKT conditions below.

- Stationarity

$$\frac{\partial \mathcal{V}}{\partial \epsilon_1} = \gamma(\frac{\epsilon_0}{1+\epsilon_0-\epsilon_1} - \frac{1-\epsilon_0}{1-\epsilon_0+\epsilon_1}) + [\gamma(1-\epsilon_1)+(1-\gamma)\epsilon_0]\frac{\epsilon_0}{(1+\epsilon_0-\epsilon_1)^2}$$
$$- [\gamma\epsilon_1 + (1-\gamma)(1-\epsilon_0)]\frac{\epsilon_0}{(1-\epsilon_1+\epsilon_0)^2} + \theta_1\alpha - \theta_2(1-\alpha) - \theta_3\alpha + \theta_4(1-\alpha) - \theta_5 \leq 0 \quad \text{(A.1)}$$

$$\frac{\partial \mathcal{V}}{\partial \epsilon_0} = (1-\gamma)[\frac{1-\epsilon_0}{1-\epsilon_0+\epsilon_1} - \frac{\epsilon_0}{1+\epsilon_0-\epsilon_1}] - [\gamma(1-\epsilon_1)+(1-\gamma)\epsilon_0]\frac{1-\epsilon_1}{(1-\epsilon_1+\epsilon_0)^2}$$
$$+ \frac{(\gamma\epsilon_1 + (1-\gamma)(1-\epsilon_0)\epsilon_1}{(1-\epsilon_0+\epsilon_1)^2} + \theta_1(1-\alpha) - \theta_2(\alpha) - \theta_3(1-\alpha) + \theta_4(\alpha) - \theta_5 \leq 0 \quad \text{(A.2)}$$

- Complementary slackness

$$\theta_1[(\alpha-1)\epsilon_0 + \alpha\epsilon_1] = 0, \quad \theta_1 \geq 0 \quad \text{(A.3)}$$

$$\theta_2[\alpha\epsilon_0 - (1-\alpha)(1-\epsilon_1)] = 0, \quad \theta_2 \geq 0 \quad \text{(A.4)}$$

$$\theta_3[(1-\alpha)\epsilon_0 + \alpha - 1 + \alpha\epsilon_1] = 0, \quad \theta_3, \geq 0 \quad \text{(A.5)}$$

$$\theta_4[(-\alpha)\epsilon_0 - (1-\alpha)\epsilon_1 + \alpha] = 0, \quad \theta_4, \geq 0 \tag{A.6}$$

$$\theta_5[\epsilon_0 + \epsilon_1 - 1] = 0, \quad \theta_5 \geq 0 \tag{A.7}$$

There are 10 different cases to consider for $\alpha < \frac{1}{2}$, which I develop below. I try all possible solutions and show that the solution such that $\epsilon_0 = 1 - \epsilon_1$ yields a higher value for the objective function than all the other potential solutions and is therefore an optimal strategy when $\gamma < \frac{1}{2}$.

- **Case 1:** $\epsilon_0 = 1 - \epsilon_1$

This yields a payoff function for the receiver of

$$\mathcal{V}_1 = \frac{1}{1-2\alpha}[1 - \alpha - \frac{1}{2}].$$

- **Case 2:** $1 - \epsilon_1 > \epsilon_0$, $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = \alpha$, $\frac{1-\epsilon_0}{1+\epsilon_1-\epsilon_0} = \alpha$

Both equations cannot hold at the same time for $\alpha < \frac{1}{2}$, so this case is not feasible unless we consider the limit case of receivers with homogeneous beliefs at $\frac{1}{2}$.

- **Case 3:** $1 - \epsilon_1 > \epsilon_0$, $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = \alpha$, $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = 1 - \alpha$

This implies that $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} < 1-\alpha$ and $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} > \alpha$. Rearranging these equations gives: $\epsilon_0 = \frac{\alpha(1-\epsilon_1)}{1-\alpha}$ and $1 - \epsilon_1 = \frac{(1-\alpha)\epsilon_0}{\alpha}$. Plugging in the objective function, I get that

$$\mathcal{V}_3 = \frac{1}{1-2\alpha}[\gamma(1-\alpha) + (1-\gamma)\alpha]\frac{1}{1-2\alpha}.$$

- **Case 4:** $1 - \epsilon_1 > \epsilon_0$, $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = \alpha$, other constraints are not binding.

I only know that $\epsilon_0 = \frac{\alpha(1-\epsilon_1)}{1-\alpha}$. Plugging that in the objective function, I get that

$$\mathcal{V}_4 = \frac{1}{1-2\alpha}\{[\gamma(1-\alpha) + (1-\gamma)\alpha]\frac{(1-\epsilon_1)}{1-\alpha}[\frac{1-2\alpha+\alpha\epsilon_1}{1-2\alpha+\epsilon_1} - \alpha] + [1 - \alpha - \frac{1-2\alpha+\alpha\epsilon_1}{1-2\alpha+\epsilon_1}]\}.$$

- **Case 5:** $1 - \epsilon_1 > \epsilon_0$, but no other binding ($\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = 0$)

I use the two stationarity conditions A.1 and A.2, which, once rearranged, yield:

$$\frac{\partial \mathcal{V}}{\partial \epsilon_1} = \frac{\epsilon_0(\epsilon_0 + 2\gamma(1-\epsilon_1))}{(1-\epsilon_1+\epsilon_0)^2} - \frac{(1-\epsilon_0)^2 + (1-\epsilon_0)2\gamma\epsilon_1}{(1-\epsilon_0+\epsilon_1)^2} = 0$$

$$\frac{\partial \mathcal{V}}{\partial \epsilon_0} = \frac{\gamma(1-\epsilon_1)^2 + (1-\gamma)\epsilon_0(2(1-\epsilon_1)+\epsilon_0)}{(1-\epsilon_1+\epsilon_0)} + \frac{\gamma\epsilon_1^2 + (1-\epsilon_0)(1-\gamma)(1-\epsilon_0+2\epsilon_1)}{(1-\epsilon_0+\epsilon_1)^2} = 0$$

The only solutions to this system of equations are such that $1-\epsilon_1 = \epsilon_0$, which violates the constraint that $1-\epsilon_1 > \epsilon_0$. Therefore, this is not a plausible solution.

- **Case 6:** $1-\epsilon_1 > \epsilon_0$, $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = 1-\alpha$

Given that $1-\epsilon_1 > \epsilon_0$, this would imply, after rearranging $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = 1-\alpha$ that $1-\alpha < \alpha$, which cannot be true since $\alpha < \frac{1}{2}$ for our uniform distribution symmetric around $\frac{1}{2}$. Therefore this solution is not feasible.

- **Case 7:** $1-\epsilon_1 > \epsilon_0$, $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = 1-\alpha$

- **Case 8:** $1-\epsilon_1 > \epsilon_0$, $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = 1-\alpha$, $\frac{1-\epsilon_0}{1+\epsilon_1-\epsilon_0} = \alpha$

After rearranging both equalities, these can only hold if $1-\alpha = \alpha$, which cannot be true if there is to be some heterogeneity among agents.

- **Case 9:** $1-\epsilon_1 > \epsilon_0$, $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = 1-\alpha$, $\frac{1-\epsilon_0}{1+\epsilon_1-\epsilon_0} = 1-\alpha$

Rearranging the equation, we get that $1-\epsilon_0 = \frac{\alpha}{1-\alpha}\epsilon_1$. Combined with $1-\epsilon_1 > \epsilon_0$, this implies that $\alpha > 1-\alpha$, which can't be true since the Uniform distribution is symmetric around $\frac{1}{2}$, $\alpha < \frac{1}{2}$.

- **Case 10:** $1-\epsilon_1 > \epsilon_0$, $\frac{1-\epsilon_0}{1+\epsilon_1-\epsilon_0} = 1-\alpha$

This implies $\epsilon_0 = \frac{\alpha-(1-\alpha)\epsilon_1}{\alpha}$, which, plugged in the objective function yields

$$\mathcal{V}_{10} = [\gamma(1-\epsilon_1) + (1-\gamma)\frac{(\alpha-(1-\alpha)\epsilon_1)}{\alpha}](1-\alpha-\frac{\alpha-(1-\alpha)\epsilon_1}{2\alpha-\epsilon_1}).$$

Summarizing, I have shown that for $\gamma < \frac{1}{2}$,

- case 1: feasible solution
- case 2: not feasible
- case 3: feasible solution
- case 4: feasible solution
- case 5: not feasible
- case 6: not feasible

- case 7: not feasible
- case 8: not feasible
- case 9: not feasible
- case 10: feasible solution

To characterize the optimal solution, I need to compare the value of the objective function at the feasible solutions and show that $\mathcal{V}_1 > \mathcal{V}_3$, $\mathcal{V}_1 > \mathcal{V}_4$ and $\mathcal{V}_1 > \mathcal{V}_{10}$. I prove by contradiction for each case aforementioned. First, if $V_3 > V_1$, this implies that

$$[1-\alpha-\frac{1}{2}] > [\gamma(1-\alpha)+(1-\gamma)\alpha].$$

Rearranging the terms, this would imply that $\gamma > \frac{\frac{1}{2}-\alpha}{1-2\alpha} = \frac{1}{2}$, which cannot be true given that we have $\gamma < \frac{1}{2}$. Now, assume that $\mathcal{V}_4 > \mathcal{V}_1$. After rearranging and simplifying both equations, one gets that this holds if $\gamma > \frac{1}{2}$, which cannot given our assumption that $\gamma < \frac{1}{2}$. Finally, assume $\mathcal{V}_{10} > \mathcal{V}_1$, this implies that

$$[\gamma(1-\epsilon_1)+\frac{(1-\gamma)(\alpha-(1-\alpha)\epsilon_1)}{\alpha}][1-\alpha-\frac{\alpha-(1-\alpha)\epsilon_1}{2\alpha-\epsilon_1}] > \frac{1-2\alpha}{2},$$

which after rearranging is true if $\gamma > \frac{1}{2}$, a contradiction.

Therefore, case 1, meaning $\epsilon_0 = 1-\epsilon_1$ is optimal when $\gamma < \frac{1}{2}$. Conversely, whenever $\gamma > \frac{1}{2}$, this means that one or several of the solutions in cases 3, 4 or 10, which all exhibit $\epsilon_0 < 1-\epsilon_1$, are optimal. In any case, this shows that $\epsilon_0 < 1-\epsilon_1$ when $\gamma > \frac{1}{2}$. □

Appendix B Proof of proposition 3

The proof relies on the Kuhn Tucker conditions as derived above in appendix A. Proposition 1 states that when $\gamma > \frac{1}{2}$, $\epsilon_0 < 1-\epsilon_1$. Therefore, I need to compare $\mathcal{V}_3$, $\mathcal{V}_4$ and $\mathcal{V}_{10}$ to determine which candidate solution is the optimal one.

- $\mathcal{V}_3$ and $\mathcal{V}_4$

I prove by contradiction. If $\mathcal{V}_4 > \mathcal{V}_3$, then

$$[\gamma(1-\alpha)+(1-\gamma)\alpha](1-2\alpha) < [\gamma(1-\alpha)+(1-\gamma)\alpha]\frac{\epsilon_0(1-2\alpha)}{\alpha}+[1-\gamma(1-\alpha)+(1-\gamma)\alpha](1-\alpha-\frac{\alpha(1-\epsilon_0)}{2\alpha-\epsilon_0})\frac{\epsilon_0}{\alpha}.$$

After rearranging, we get that

$$\gamma 2(\alpha-\epsilon_1)\ (\alpha-\epsilon_1).$$

Recall that the constraints in case 4 imply that $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = \alpha$ and $\alpha < \frac{1-\epsilon_0}{1+\epsilon_1-\epsilon_0}$, which implies that $\epsilon_1 > \alpha$. Simplifying the above equation, we get that $\mathcal{V}_4 > \mathcal{V}_3$ only if $\gamma < \frac{1}{2}$, which can't be true. Therefore $\mathcal{V}_3 > \mathcal{V}_4$.

- $\mathcal{V}_3$ and $\mathcal{V}_{10}$

Suppose $\mathcal{V}_3 < \mathcal{V}_{10}$. This implies that

$$[\gamma(1-\epsilon_1) + (1-\gamma)\frac{(\alpha-(1-\alpha)\epsilon_1)}{\alpha}(1-\alpha-\frac{\alpha-(1-\alpha)\epsilon_1}{2\alpha-\epsilon_1}] > [\gamma(1-\alpha)+(1-\gamma)\alpha](1-2\alpha),$$

which, after simplifying, reduces to

$$\gamma(1-2\alpha)(\frac{2\alpha-2\epsilon_1)}{2\alpha-\epsilon_1} < \frac{\alpha-\epsilon_1+2\alpha\epsilon_1-2\alpha^2}{2\alpha-\epsilon_1}.$$

Recall that case 10 entails that $\frac{1-\epsilon_0}{1-\epsilon_0+\epsilon_1} = 1-\alpha$ and $\alpha < \frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} < 1-\alpha$, which after rearranging implies $\alpha > \epsilon_1$. We can simplify the inequality above to get that this holds if $\gamma < \frac{1}{2}$, which can't be. Therefore $\mathcal{V}_3 > \mathcal{V}_{10}$

To summarize, I have shown that the optimal solution is case 3 such that $\frac{1-\epsilon_0}{1-\epsilon_0+\epsilon_1} = 1-\alpha$ and $\frac{\epsilon_0}{1-\epsilon_1+\epsilon_0} = \alpha$. This means $\epsilon_1 = \epsilon_0 = \alpha$. The variance of the truncated uniform distribution, symmetric around $\frac{1}{2}$ is $\frac{(1-2\alpha)^2}{12}$. So when I decrease α, I increase the variance, which decreases both ϵ_1 and ϵ_0.

Appendix C Proof of proposition 4

I want to show that the operator T is such that

$$Tv(\omega_t, n_{0,t}, n_{1,t}, n_{0,t}^L, n_{1,t}^L) = \max_{\pi(s_t|\omega_t)\in[0,1]} \{H(\omega_t, n_{0,t}, n_{1,t}, n_{0,t}^L, n_{1,t}^L, \pi(s_t|\omega_t)) + \delta E[v(\omega_{t+1}, n_{0,t+1}, n_{1,t+1}, n_{0,t+1}^L, n_{1,t+1}^L)|\pi(s_t|\omega_t)]\}$$

satisfies Blackwell's sufficient conditions for a contraction mapping (see Stokey and Lucas (1989)). T is bounded since the sender's utility cannot be lower than zero (if he does not convince anybody) and cannot be larger than one (if he convinces the unit mass of receivers, he gets 1). T describes a mapping from (0,1) to $(0, \frac{1}{1-\delta})$, therefore T describes a map of the space of bounded and continuous functions $B(X)$ onto itself (following from the Theorem of the Maximum). From Blackwell's Theorem, a mapping $T : B(X) \to B(X)$ is a contraction mapping with contractive constant β if these two conditions are met:

- Monotonicity: If $f, g \in B(X)$ and $f(x) \leq g(x)$ for all $x \in X$ implies that $Tf(x) \leq Tg(x)$ for all $x \in X$.

- Discounting: For $a \in \text{Re}_+$ there exists a β such that for all $f \in B(X)$ and all $x \in X$ $T(f + a)(x) \leq Tf(x) + \beta a$.

To verify that these two conditions hold, first denote $\pi(s|\omega)^f = \arg\max f$ and $\pi(s|\omega)^g = \arg\max g$

$$
\begin{aligned}
Tf(\omega, n_0, n_1, n_0^L, n_1^L) &= \{H(\omega, n_0, n_1, n_0^L, n_1^L, \pi(s|\omega)^f) + \delta f(\omega', n_0', n_1', n_0^L, n_1^L, \pi(s|\omega)^f)\} \\
&\leq \{H(\omega, n_0, n_1, n_0^L, n_1^L, \pi(s|\omega)f) + \delta g(\omega', n_0', n_1', n_0^L, n_1^L, \pi(s|\omega)^f)\} \\
&\leq \max_{\pi(s_t|\omega_t)\in[0,1]} \{H(\omega_t, n_{0,t}, n_{1,t}, n_{0,t}^L, n_{1,t}^L, \pi(s_t|\omega_t)) \\
&+ \delta E[g(\omega_{t+1}, n_{0,t+1}, n_{1,t+1}, n_{0,t+1}^L, n_{1,t+1}^L)|\pi(s_t|\omega_t)]\} \\
&= Tg(\omega, n_0, n_1, n_0^L, n_1^L).
\end{aligned}
$$

The first equality comes from the definition of the Bellman equation, and the second line follows from the condition that $f(x) \leq g(x)$ for all $x \in X$. The third line follows from the fact that if the sender maximizes the value function taking $g(\omega, p_0, p_1)$ as given, they cannot do worse than the choice implied by $\Phi_f(\omega, p_0, p_1)$. Secondly, to show the discounting conditions, I use

$$
\begin{aligned}
T(f + a)(x) &= \max_{\pi(s_t|\omega_t)\in[0,1]} \{H(\omega_t, n_{0,t}, n_{1,t}, n_{0,t}^L, n_{1,t}^L, \pi(s_t|\omega_t)) \\
&+ \delta E[(v(\omega_{t+1}, n_{0,t+1}, n_{1,t+1}, n_{0,t+1}^L, n_{1,t+1}^L) + a)|\pi(s_t|\omega_t)]\} \\
&= Tf(x) + \delta a.
\end{aligned}
$$

The Bellman equation satisfies both conditions, and is thus a contraction mapping. This implies that there exists a unique fixed point that can be found by function iteration.

Appendix D Summary Statistics

Table 1. Forecast errors summary statistics

	mean	min	max	std	N
GDP	-0.2440	-3.7348	3.7165	1.3776	111
UR	0.0698	-1.6500	4.9500	0.9805	111
Inflation	-0.0605	-2.9453	6.0072	1.2814	111

Appendix E ECB forecasts

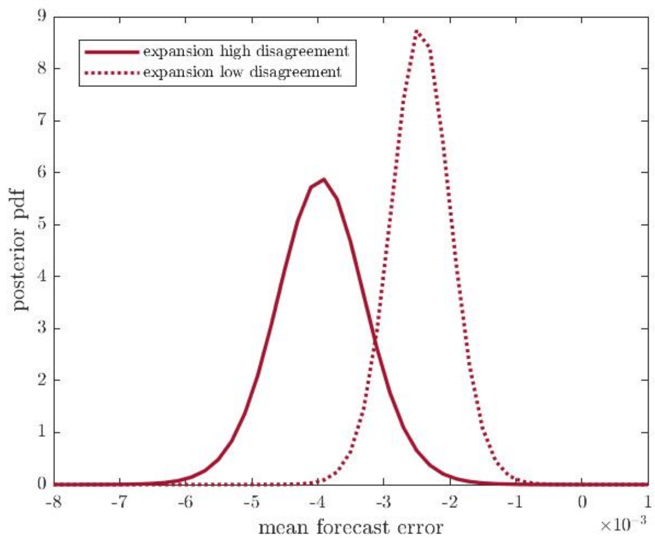


Figure 10. Posterior distribution of ECB's unemployment rate forecast errors. Posterior distribution of the mean forecast errors, estimated using Jeffreys' prior and under the assumption of i.i.d normal data. Sample 2013-2019 with forecast horizon next year. Disagreement as measured by the SPF expectations over the next 3 quarters. Annual percentage change.

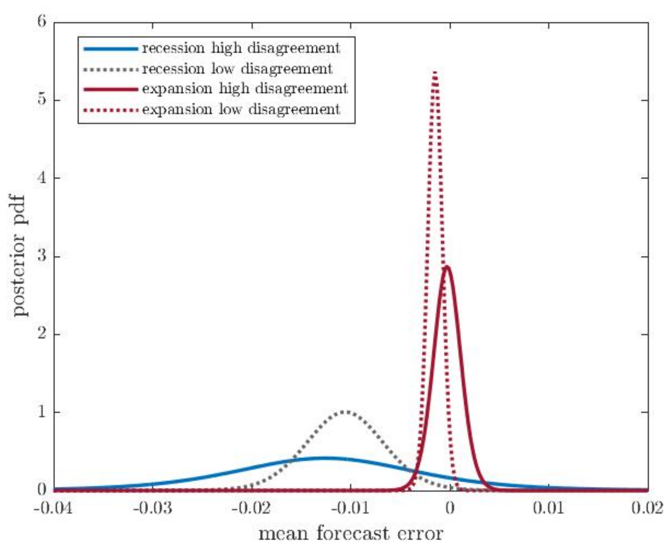


Figure 11. Posterior distribution of ECB's GDP forecast errors. Posterior distribution of the mean forecast errors, estimated using Jeffreys' prior and under the assumption of i.i.d normal data. Sample 2013-2019 with forecast horizon next year. Disagreement as measured by the SPF expectations over the next 3 quarters. Annual percentage change.

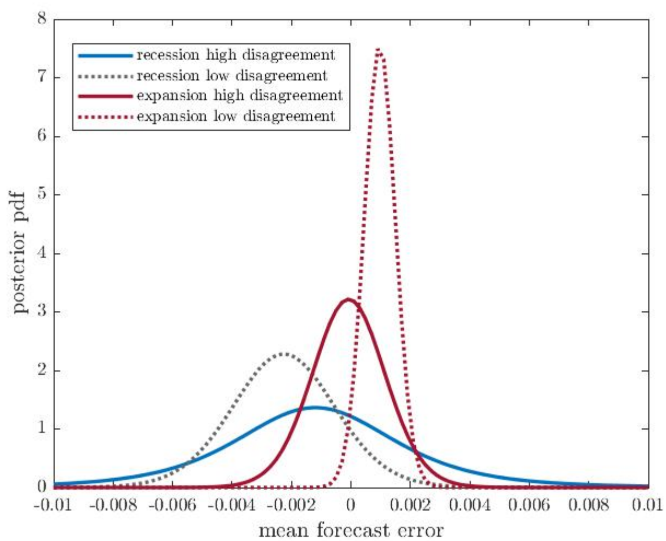


Figure 12. Posterior distribution of ECB's inflation forecast errors. Posterior distribution of the mean forecast errors, estimated using Jeffreys' prior and under the assumption of i.i.d normal data. Sample 2013-2019 with forecast horizon next year. Disagreement as measured by the SPF expectations over the next 3 quarters. Annual percentage change.