Optimal robust monetary policy in a small open emerging economy: the case of Mexico

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Abstract

We study for a benchmark small open emerging economy an optimal robust monetary policy à la Hansen and Sargent (2003) considering additive model uncertainty. The robust control approach supposes that economic agents are not able to assign probabilities to a set of all plausible models and rather focuses on the worst possible misspecification from a benchmark model. We calibrate our model for Mexico, using the estimation of Sidaoui and Ramos-Francia (2008). Our findings are threefold. First, conducting a global robust optimal monetary policy is limited since the departure from the benchmark model leads to multiple equilibria. Second, when model uncertainty arises only from the IS curve or the UIP condition, the space of unique solutions is expanded. In fact, when the central bank has a preference for robustness on the IS curve only, it should be more aggressive to demand and real exchange rate shocks but more conservative to cost-push shocks. On the other hand, when it has a preference for robustness only for the UIP, the central bank should be more aggressive to demand and cost-push shocks. Third, a sensitivity analysis suggests that conducting a global robust optimal monetary policy with the same misspecification in all equations is limited due to the persistence of inflation, the low exchange-rate pass-through and the need to anchor inflation expectations.

Keywords: Robust control, model uncertainty, optimal monetary policy, small open economy.

JEL Classification: C62, D83, D84, E52, E58.

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1 Introduction

Since the Global Financial Crisis (GFC), there has been increasing concerns about macroeconomic modeling and the need to design optimal policies (Stiglitz, 2017; Romer, 2016). In fact, during the last two economic crises, the monetary policy has been repeatedly put into question as for its efficiency and its ability to smooth the business cycle and to avoid major crisis. In this regard, the design of optimal policies contributes to stabilizing the economy at the lowest possible welfare cost; however, it depends on key model blocks and transmission mechanisms that are model-specific. Recently, the COVID-19 crisis has demonstrated that policymakers are facing increasing uncertainty about the economic environment. Such uncertainty makes it difficult to select a single model that is able to capture the main features prevailing in the economy and it reinforces the importance of being robust on economic modelling.

The goal of this paper is to study a robust optimal monetary policy for a small-open emerging economy. The robust approach à la Hansen and Sargent (2001, 2003, 2007) followed in this paper supposes that policymakers cannot assign probabilities to a set of all plausible models and rather, focus on the worst possible misspecification from a benchmark model. Particularly, we study additive uncertainty; thus, the worst possible misspecification refers to a situation in which the benchmark model does not incorporate key variables or blocks describing the economy to which policymakers seek to be robust. For example, fiscal policy is frequently absent in many macroeconomic models but could play a role for transmitting default concerns into long-term rates, increasing the risk premium and the exchange rate (André et al. 2021). Moreover, informality is a widespread phenomenon in emerging economies, which tends to generate labor market frictions, having implications on monetary policy (Alberola and Urrutia 2020).[1]

For that purpose, this paper presents a small open economy New Keynesian model focusing on optimal monetary policy.[2] The model is calibrated for Mexico as it is a representative small open emerging economy, with a GDP nine times smaller than its major trading partner, with a dynamic exporting sector, a floating exchange rate and an inflation-targeting regime

[1] Model uncertainty in advanced economies could be more focused on financial frictions given the higher credit-to-GDP ratios as well as their financial depth. Moreover, recent literature has focused on the effect of unconventional monetary policies (Sims et al. 2020) and the effective lower bound on short-term interest rates.

[2] For a canonical small open economy version of the New Keynesian model, see Galí and Monacelli (2005). We follow closely the model used in Ramos-Francia and Sidaoui (2008) which incorporate backward-looking components, but we depart from them since we consider an optimal monetary policy instead of a standard Taylor rule. As our choice of optimal monetary policy, we assume that the CB shares the preferences for inflation and output-gap stabilization with society, characterized by an intertemporal loss function.

(Elizondo and Carrillo 2015).[3] We study how optimal monetary policy should behave when considering model uncertainty, that is, when the model departs from its benchmark. Therefore, we analyze four different scenarios: (1) uncertainty arising from the entire model with the same misspecification in all equations; (2) concerns about only the specification of the New Keynesian Phillips curve; (3) uncertainty only when modelling the IS curve; and (4) uncertainty arising from the uncovered interest rate parity (UIP). Additionally, we explore the limits of a robust optimal monetary policy through a sensitivity analysis.

Our findings are threefold. First, conducting a global robust optimal monetary policy is limited since the departure from the benchmark model leads to multiple equilibria. Second, when model uncertainty arises only from the IS curve or the UIP condition, the space of unique solutions is expanded. In fact, when the central bank (CB) has a preference for robustness on the IS curve only, it should be more aggressive to demand and real exchange rate shocks but more conservative to cost-push shocks. On the other hand, when it has a preference for robustness only for the UIP, the CB should be more aggressive to demand and cost-push shocks. Third, the sensitivity analysis suggests that conducting a global robust optimal monetary policy with the same misspecification in all equations is limited due to the persistence of inflation, the low exchange-rate pass-through and the need to anchor inflation expectations.

This paper adds to the literature on robust optimal monetary policy in several ways. First, to the best of our knowledge this is the first paper that analyses a robust optimal monetary policy for Mexico. Second, we provide a qualitative guidance on how optimal monetary policy should behave when central bankers are concerned with global model uncertainty as well as concerned with each structural equation separately. That is, a reaction function is given for all three types of shocks considered in this paper. Third, we study which factors may be limiting the conduct of a robust monetary policy in small open emerging economies. In that regard, our findings highlight the effects of inflation persistency and confirm the great importance of anchoring inflation expectations for the conduct of optimal policy.

The remainder of the paper is as follows. Section 2 presents a selected related literature. Section 3 and 4 outlines the model and the benchmark monetary policy. Section 5 explores the optimal robust monetary policy and its consequences. Section 6 examines the limits of robust monetary policy. Section 7 offers a discussion and possible extensions. Finally, section 8 concludes.

[3]The model is deliberately stylized and does not intent to fully characterize the Mexican economy. In this regard, we provide a benchmark model to analyze an optimal robust monetary policy for a small open emerging economy. Further lines of research can develop more detailed DSGE models to address model uncertainty.

2 Related Literature

Given our assumptions, our paper relates to three important strands of literature on monetary policy: (1) optimal policy; (2) model uncertainty and the robust control approach and (3) the constraints on monetary policy for emerging economies. In this section, a selected literature review is discussed.

First, optimal monetary policy and its stabilizing properties have been a topic widely studied in the literature (Khan et al. 2003, Woodford 2004, Galí 2008, Corsetti et al. 2010, Benigno and Woodford 2012, among others). As pointed out by Woodford (2010), optimal policy can be robust to changes in the specification of price adjustment dynamics. However, as argued by Taylor (2010), simple monetary policy rules can be desirable in practice since they can provide robust outcomes within a wide variety of models, especially when considering model uncertainty. In fact, recent studies have highlighted the importance of the differences that may arise in the design of an optimal monetary policy for emerging economies due to their model-dependent nature, which motivates the need to conduct a robust policy.

For instance, optimal monetary policy response to exchange rate fluctuations is an important topic for emerging economies. Iyer (2016) finds that targeting the exchange rate is appropriate when most of the households are excluded from the financial markets as it directly stabilizes the import component of financial-excluded agents' consumption baskets, reducing macroeconomic volatility. Conversely, Hove et al. (2015) argue that in emerging market economies where commodity terms of trade shocks drive macroeconomic fluctuations, the CPI inflation targeting performs better than the exchange rate targeting, both in terms of improving social welfare and reducing macroeconomic volatility but at a cost of increased exchange rate volatility. In fact, as John Taylor puts it, "market conditions in emerging-market economies may require modifications of the typical policy rules that has been recommended for economies with more developed financial markets."

Second, regarding model uncertainty, the robust control approach à la Hansen and Sargent (2001, 2003, 2007) gives the tools to conduct a policy that would be robust to plausible deviations from the benchmark New Keynesian model. Without a faithful description to reality, central bankers are more inclined to base policy on principles, prevailing across different assumptions. In the sense of Hansen and Sargent, the CB is unable to formulate a probability distribution over the full set of realistic models, and thus may design a robust policy that can respond to the worst possible outcome within a pre-specified set of models (Leitemo and Söderström 2008b). Said in other words, the worst-case outcome is a situation where a malevolent agent (Nature) will choose model misspecification to be as damaging as possible, while the CB's policy rule and private agents' expectations reflect this misspecification.

Generally, optimal interest rate policy is more aggressive to deviations from the inflation target under the robust control approach in both closed and open economy (Leitemo and Söderström 2008a, b; André and Dai 2018). This approach derives from the seminal contribution of Brainard (1967), who was already considering the consequences of parameter uncertainty and advocates that the CB should be cautious in the sense that it should use less intensively each policy instrument following an attenuation principle.[4] Taking worst-case scenarios for the economy into consideration, the CB's response tends to be more hawkish when facing shocks in a closed economy (e.g., Giannoni and Woodford 2002, Onatski and Stock 2002, Giordani and Söderlind 2004, Leitemo and Söderström 2008a, Gonzalez and Rodriguez 2013). When the CB responds more aggressively to inflation, then robust monetary policy can prevent suboptimal outcomes; however, inflation persistence might occur under optimal robust control and rational expectations (RE) (Qin, Sidiropoulos and Spyromitros 2013). Appointing a hawkish central banker reduces the cost of uncertainty about potential output and uncertainty lies in the true degree of shock persistence (Tillmann 2009, 2014, Grasso and Traficante 2021), or when misspecification is present in the Phillips curve (Dai and Spyromitros 2010).

Finally, small open emerging economies could face more constraints on the design of monetary policy than advanced economies. Emerging economies, particularly in LatAm, have experienced balance of payments' crisis rooted on periods of fiscal dominance that leaded to hyperinflations, massive capital outflows and exchange rate depreciations (Kehoe and Nicolini 2022). Indeed, many governments in the region began to let the exchange rate to float and provide autonomy to CBs, who gradually adopted inflation-targeting regimes. As pointed out by Chiquiar and Ibarra (2020), a greater CB independence is associated with lower levels of inflation as well as lower inflation volatility. Although CBs have gained credibility since the adoption of inflation-targeting regimes, fiscal policy can still play a role in the design of monetary policy. Alberola et al. (2022) suggest that the effects of monetary policy on the exchange rate depends on the fiscal regime, since a contractionary monetary policy shock could lead to an exchange rate depreciation if debt is not backed by future fiscal surpluses.

[4]The attenuation principle is named the "conservatism principle" by Blinder (1998). According to this principle, the CB should be cautious given that the choice of a policy instrument can yield to more severe consequences than in the absence of parameter uncertainty. The current robust monetary policy literature has reversed the meaning of "cautious" so that "being cautious or precautionary" signifies "to do more". In other words, the CB wants to avoid worst outcomes by responding more aggressively to shocks (Söderström 2002, Gianonni 2007).

Moreover, the GFC has raised questions on how the monetary policy should react to increasing global financial imbalances. According to Tobal and Menna (2018), the prescription of the "leaning against the wind" should not be applied to emerging economies in the same way as it has usually applied for advanced economies. In fact, a policy that "leans against the wind" by raising the interest rate, reduces the output gap and the demand for credit but also attracts capital inflows, increasing the credit supply. Thus, a strong dependence of financial conditions on capital flows weakens the ability of monetary policy to "lean against the wind" in emerging economies.[5] De la Peña (2021) finds that following a "leaning against the wind" policy by augmenting a Taylor rule with an argument on credit growth is not an optimal policy; instead, the use of two separate tools that focus each on price and financial stability, following the Tinbergen rule principle. Additionally, recent literature on labor market frictions points out that the presence of high informality, a common feature on emerging economies, tends to make monetary policy less effective, increasing its sacrifice ratio (Castillo and Montoro 2010, Alberola and Urrutia 2021).

3 The Model

As our benchmark model, we extend a New Keynesian small open economy model based on Sidaoui and Ramos-Francia (2008) to analyze an optimal robust monetary policy.[6] For that matter, we first present in subsection (3.1) the model used in Sidaoui and Ramos-Francia (2008) but considering additive misspecification and then, in subsection (3.2), we show how the model should be modified to design an optimal robust policy. The model is formed by four main equations: i) a New-Keynesian Phillips Curve; ii) a dynamic IS curve; iii) an uncovered real interest rate parity, which anchors the real exchange rate and iv) a Taylor rule type for the nominal interest rate. Moreover, the model is enriched with backward looking components in each equation to better capture the inertia found in the data for inflation and the output gap laws of motion.[7]

[5] As we focus on Mexico, it is noteworthy that according to the most recent Triennial Central Bank survey by the Bank for International Settlements (2019), the Mexican peso is the second most traded emerging-market currency, only after the Chinese Renminbi but as mentioned by López-Noria and Busch (2021), increased uncertainty has implied greater exchange rate volatility since 1999. Also, trade policy uncertainty has negatively affected the Mexican economy through FDI flows (Cebreros et al. 2020).

[6] Sidaoui and Ramos-Francia (2008) use the model to show how the monetary tranmission mechanism operates in Mexico and how has it evolved over time. For instance, the model is closely related to Roldán-Peña et al. (2017) and Galí and Monacelli (2005).

[7] In fact, several studies have incorporated backward-looking components in microfounded DSGE models by assuming consumption habit formation and information rigidities.

3.1 The Structural Equations

Following Sidaoui and Ramos-Francia (2008), the Phillips Curve describes core inflation dynamics:

$$\pi_t^c = a_1 \pi_{t-1}^c + a_2 E_t \pi_{t+1}^c + a_3 x_t + a_4 \left[\triangle e_t + \pi_t^{US}\right] + h_t^\pi + \varepsilon_t^\pi, \tag{1}$$

where core inflation is directly related to its own past value π_{t-1}^c and its expected value π_{t+1}^c; the output gap x_t, represents the log deviation of the flexible-price equilibrium level of domestic output from the steady-state output; the nominal exchange rate depreciation $\triangle e_t$ and US inflation π_t^{US}.[8] E_t is the conditional expectation operator; $\varepsilon_t^\pi \sim \mathcal{N}(0, \sigma_\pi^2)$ is an *i.i.d.* cost-push shock and the term h_t^π represents the additive misspecification in the Phillips curve being defined below by (8) and endogenously determined by the malevolent agent. On the other hand, since non-core inflation is not driven by cyclical conditions nor affected by monetary policy, it is assumed to follow an AR(1) process:

$$\pi_t^{nc} = \phi_1 \pi_{t-1}^{nc} + \varepsilon_t^{\pi^{nc}},$$

where ϕ_1 captures non-core inflation persistence and $\varepsilon_t^{\pi^{nc}} \sim \mathcal{N}(0, \sigma_{\pi^{nc}}^2)$ is an i.i.d. shock to non-core inflation. Headline inflation is constructed as a weighted average between core inflation and non-core inflation:

$$\pi_t = \omega \pi_t^c + (1\text{-}\omega)\pi_t^{nc},$$

where ω is the weight of the core inflation on headline inflation. The New Keynesian IS curve describes the output gap:

$$x_t = b_1 x_{t-1} + b_2 E_t x_{t+1} + b_3 r_{t-1} + b_4 x_t^{US} + b_5 q_t + h_t^x + \varepsilon_t^x \tag{2}$$

Therefore, the output gap evolves according to its own lagged value x_{t-1}, its expected component $E_t x_{t+1}$ and by the lagged ex-ante real interest rate r_{t-1}. The foreign output gap term x_t^{US} implies that when foreign income increases, so does home income, since exports demand is higher.[9] The real exchange rate q_t captures the effect of the terms of trade on the

[8] Differing from Galí and Monacelli (2005), the Phillips curve as in Sidaoui and Ramos-Francia (2008), Roldán-Peña et al. (2017) and Leitemo and Söderström (2008b) includes the real exchange rate.

[9] As we model a small open economy, we assume all foreign variables are exogenous and follow a VAR(1) process.

trade balance. A depreciation of the real exchange rate makes home consumption goods more attractive from foreign point of view, boosting net exports. $\varepsilon_t^x \sim \mathcal{N}(0, \sigma_x^2)$ is an *i.i.d.* demand shock and the term h_t^x denotes the misspecification in the IS equation, below defined by (8).

Real exchange dynamics evolve according to the Uncovered Real Interest Rate parity, which relates real interest rates differentials with the expected rate of real depreciation:

$$q_t = c_1 q_{t-1} + c_2 [E_t q_{t+1} + (r_t^{us} - r_t)] + h_t^q + \varepsilon_t^q, \tag{3}$$

where c_1 captures the inertia of the real exchange rate and c_2 captures the strength of the uncovered real interest rate parity. An increase in the relative monetary position $(r_t^{us} - r_t)$ depreciates the real exchange rate since it is more attractive to invest in foreign risk-free bonds. h_t^q denotes the misspecification in the UIP equation below defined by (8), and $\varepsilon_t^q \sim \mathcal{N}(0, \sigma_q^2)$ is an *i.i.d.* real exchange rate disturbance. A positive ε_t^q means that investors require a positive risk premium on domestic bonds compared to foreign bonds. To close the model, Sidaoui and Ramos-Francia (2008) assume a standard Taylor rule:

$$i_t = d_3 i_{t-1} + (1 - d_3) [d_1 (\pi_t - \pi^{target}) + d_2 x_t] + \varepsilon_t^i, \tag{4}$$

where d_3 captures the degree of interest rate smoothing, while d_1 and d_2 represent the elasticity of the inflation gap and the output gap to the interest rate, respectively. The nominal interest rate increases when inflation is above CB's target or when the output gap is positive, which may induce inflationary pressures. Finally, the Fisher equation defines the real ex-ante interest rate as the difference between the nominal interest rate and inflation expectations:

$$r_t = i_t - E_t \pi_{t+1}. \tag{5}$$

The nominal exchange rate depreciation, $\triangle e_t$, is defined as follows:

$$\triangle e_t = \triangle q_t + \left(\pi_t - \pi_t^{US}\right) \tag{6}$$

Importantly, the model takes into account the role of expectations endogenously and thus, is well-suited to shed light of the monetary transmission mechanism, which works as follows: a positive monetary shock that increases the nominal interest rate (4), increases the ex-ante real rate, given any value of inflation expectations (5). The latter appreciates the real exchange rate by the UIP (3), decreasing directly both, the output gap (2) and inflation (1). Since monetary policy affects the output gap with a lag, in the following time period the increase in the real interest rate put downward pressures on the output gap (2) and thus, to inflation and its expectations. The decrease of inflation expectations have two additional effects: it reinforces the above mentioned mechanism since it increases even more the ex-ante real rate (5) and, it directly affects inflation via its expectations (1).

Onward, we study robust optimal monetary policy, thus, we consider the worst-case model where the CB sets the interest rate to minimize its loss function while a fictitious malevolent agent in the sense of Hansen and Sargent (2007) selects the level of misspecification to maximize the CB's loss.[10] Such an agent represents the policy maker's biggest challenge about model misspecification.

The worst-case scenario is the outcome that the CB is most averse to, and against which the CB conducts robust policy. The model misspecification cannot arise independently of random noises that affect model equations and positively depend on the variance of such noises (Giordani and Söderlind 2004). This is due to the fact that if the noise variance in one equation was null, then the misspecification would be detected at once. Therefore, the larger the variance of the disturbance is, the larger the misspecification can go undetected.

3.2 Monetary Policy Objectives

The CB shares the preferences for inflation and output-gap stabilization with society, whose expected loss function is given by:[11]

$$L_t^s = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[(\pi_{t+i} - \pi^{target})^2 + \alpha x_{t+i}^2 \right], \tag{7}$$

where $\alpha > 0$ denotes the relative weight assigned to the objective of stabilizing the output gap. For simplicity, we assume that output-gap target is equal to zero. Without the overly ambitious output-gap target that is common in the Barro-Gordon framework, the discretionary monetary policy set with the aim of minimizing social loss (7) would avoid an average inflation bias when private agents form RE. Optimal robust monetary policy results from a sequential Nash game between the CB conducting robust policy to minimize the social loss and the malevolent agent (or Nature) who sets the level of model misspecification to maxi-

[10]An alternative approach is to consider the 'approximating model' (Hansen and Sargent 2007) postulating that while the policy rule and agents' expectations reflect the CB's focus on robustness, there is no model misspecification in the reference model that turns out to be correct.

[11]This type of objective function is commonly used to characterize inflation-targeting policy in small open economies. Woodford (2003) demonstrate that a welfare loss function based on a second-order approximation of the representative consumer's utility loss has a similar form.

mize the social loss.[12]

Given the model misspecification set by the malevolent agent, the CB designs the robust discretionary policy for the worst possible model within a given set of plausible models. The CB allocates a budget χ_j^2, $j = \pi, x, q$, to the malevolent agent, for the misspecification to be created in the Phillips curve, the IS equation and the UIP condition, respectively. This budget means that the misspecifications, h_t^j, are finite. The misspecifications, h_t^j, with $j = \pi, x, q$, monitored by the malevolent agent are subject to the following budget constraints:

$$E_t \sum_{i=0}^{+\infty} \beta^t \left(h_{t+i}^j \right)^2 \leq \chi_j^2, \; j = \pi, x, q. \tag{8}$$

In the absence of robust control, $\chi_j = 0$ for all j.

Under discretion, the CB designs a robust policy that takes into account not only different shocks affecting the economy but also model misspecification. The optimal robust monetary policy is obtained by solving the min-max problem:

$$\min_{\pi_t, x_t, q_t, r_t} \max_{h_t^j} L_t^{CB} = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[(\pi_{t+i} - \pi^{target})^2 + \alpha x_{t+i}^2 - \theta^\pi {h_{t+i}^\pi}^2 - \theta^x {h_{t+i}^x}^2 - \theta^q {h_{t+i}^q}^2 \right], \tag{9}$$

subject to the misspecified structural equations (1), (2), (3), and the malevolent agent's budget constraints (8). The FOC from this model are presented in Appendix A.1. The penalty parameter $\theta^j > 0$, with $j = \pi, x, q$, controls the CB's focus in favor for robustness. The higher θ^j are, the lower the focus in favor for a robust monetary policy. The misspecifications h_t^j, with $j = \pi, x, q$, are inversely proportional to θ^j. The absence of concern regarding robustness corresponds to the case where $\theta^j \to \infty$, implying that $h_t^j \to 0$. In the following, we assume for simplicity that the malevolent agent's budget constraints (8) are not binding.

4 Benchmark Monetary Policy

4.1 A Brief Description of the Transmission Mechanisms

The optimal monetary policy is a special case of the robust monetary policy, where the CB considers that his benchmark model is the realistic one, that is, the model captures the dy-

[12] Alternatively, the CB and the malevolent agent can play a Stackelberg game with the first acting as a Stackelberg leader. Notice that if the malevolent agent is the Stackelberg leader, the CB could adjust its policy according to the scenario designed by the malevolent agent (Hansen and Sargent, 2003). It results that the approach in terms of model misspecification would lose its interest.

namics of the true data generating process. Given that the CB considers his model as being correct, it chooses the penalty parameters $\theta^j \to \infty$ which means that the CB does not take into account the existence of misspecification from the malevolent agent. Therefore, in order to analyze how the CB modifies its behavior to avoid the worst possible outcomes when being robust, it is worth mentioning briefly how the optimal monetary policy looks like and how it differs from the case where the CB uses a Taylor Rule. We assume that α is fixed to 0.7 in the CB loss function, which implies a slightly bias towards inflation stabilization compared to the output gap.[13] We calibrate the model using the estimation of Sidaoui and Ramos-Francia (2008) for the Mexican economy between 2001 and 2006 on a monthly basis (see Table 1), period after the implementation of the inflation targeting.[14]

Phillips Curve				**IS Curve**				
π^c_{t-1}	$E_t\pi^c_{t+1}$	x_t	$\Delta e_t + \pi^{US}_t$	x_{t-1}	$E_t x_{t+1}$	r_{t-1}	x^{US}_t	q_t
a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4	b_5
0.333	0.664	0.013	0.003	0.312	0.569	–0.035	0.219	1.415

UIP		**Nominal Interest Rate**		
q_{t-1}	$E_t q_{t+1} + (r^{US}_t - r_t)$	$(\pi_t - \pi^{target})$	x_t	i_{t-1}
c_1	c_2	d_1	d_2	d_3
0.315	0.677	1.086	1.556	0.807

Table 1: Calibration.
Source: Sidaoui and Ramos-Francia (2008).

The considered period takes into account the structural change that Banco de México made since 1999 when for the first time, Banco de México set a medium-target for the inflation rate. In 2000, Banco de México defined core inflation, strengthened its communication strategy and most importantly, mentioned an explicit inflation target of 3 percent. At the beginning of 2001, where the sample begins, Banco de México formally adopted the inflation targeting regime. The 2001-2006 period represents how the economy adapted itself to the inflation targeting regime in Mexico. Furthermore, as Sidaoui and Ramos-Francia (2008) highlighted, those reforms have increased the flexibility of the economy to adjust to different shocks, explaining why we can study how introducing robustness in the policy making can improve social welfare in that context.

[13]Our results are robust to the cases in which the CB weights equally the inflation gap and the output gap, $\alpha = 1$, and the case in which the CB weights double the inflation gap than the output gap, $\alpha = 0.5$.

[14]Chiquiar et. al. (2007) found that after 2001 and the adoption of the inflation targeting regime in Mexico, inflation dynamics seemed to become a stationary process.

In the following, we study the transmission mechanisms of a cost-push shock, a demand shock, and a real exchange rate depreciation shock.

4.2 Cost-Push Shock

After a shock that increases exogenously the inflation rate, when the CB follows the Taylor Rule (yellow lines in Figure 1), the CB raises the nominal interest rate. However, due to the high interest rate persistence, the increase is not enough to offset the higher inflation expectations, reducing the ex-ante real interest rate at the impact. The latter depreciates the real exchange rate due to the UIP, boosting aggregate demand via higher exports. While the inflation shock dissipates and the CB maintains a restrictive stance, inflation expectations decreases, inducing an increase in the real interest rate and an exchange rate appreciation. This mechanism generates a mild economic slowdown, facilitating the convergence of inflation to its steady state.

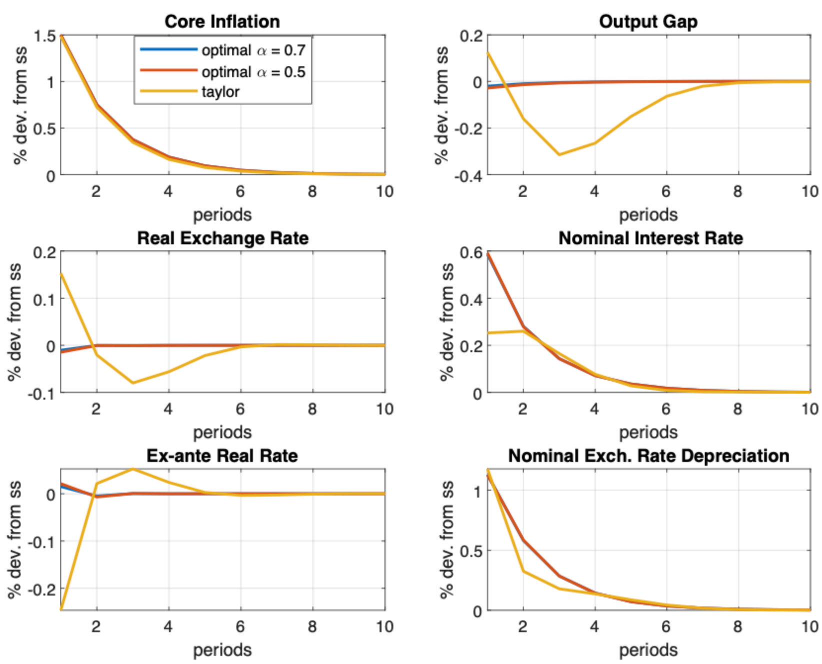

Figure 1: Impulse Response Functions: Cost-Push Shock.

On the other hand, when the CB implements an optimal monetary policy (Figure 1, orange lines), the CB rises more the nominal interest rate, compared to the previous case, in such a way that the real interest rate is restrictive at impact. This allows the real exchange rate to appreciate and the output gap to decrease marginally, which fuels a decrease in inflation expectations through the Phillips curve. Since the cost-push shock is transitory, the CB maintains its nominal interest rate such that the monetary policy is neutral. Noteworthy, while implementing an optimal monetary policy, the CB stabilizes output gap fluctuations but leads to a higher interest rate volatility compared to the Taylor rule case.

4.3 Demand Shock

When monetary policy is set by the Taylor rule (see Figure 2, yellow lines), the exogenous increase in the aggregate demand raises inflation marginally, given the small elasticity of the output gap to inflation. The CB raises the policy rate and thus, the real interest rate. The higher real interest rate appreciates the exchange rate, making exports relatively more expensive, reducing the aggregate demand and therefore, the inflationary pressures.

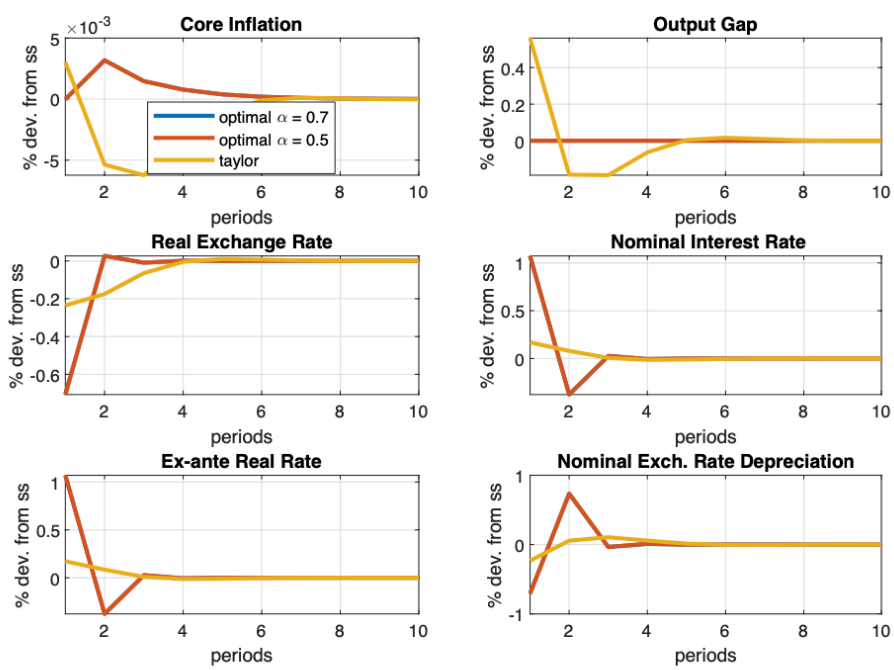

Figure 2: Impulse Response Functions: Demand Shock.

In the case of optimal policy (see Figure 2, orange lines), the CB raises the nominal rate and the real ex-ante interest rate significantly, such that on impact, it appreciates the real exchange rate enough to stabilize inflation and output gap. However, since the real rate acts with a lag on the output gap (see equation (2)), the CB changes its monetary policy from a restrictive to an expansionary stance to avoid an economic slowdown. After this monetary policy switch, the CB does not intervene further in the economy, i.e. maintains its interest rate at the neutral level, allowing the output gap and inflation to remain at their steady state. It is worth mentioning that, after a demand shock, optimal monetary policy successfully stabilizes the economy.

4.4 Exchange-Rate Shock

After a shock that depreciates the real exchange rate, from Figure 3, the aggregate demand increases through higher exports and the inflation rate raises on impact. However, notice that both the increase in the output gap and the nominal exchange rate depreciation are close to 1% but inflation increases marginally, around to 0.01%. This highlights the fact that, not only the elasticity of the output gap to inflation is small, but also the exchange rate pass-through. When the CB conducts its monetary policy with the Taylor rule, it raises the policy rate to increase the ex-ante real rate, reversing the initial depreciation of the exchange rate. Given that the interest rate is highly persistent, both inflation and the output gap undershoot for four periods before converging to their steady state.

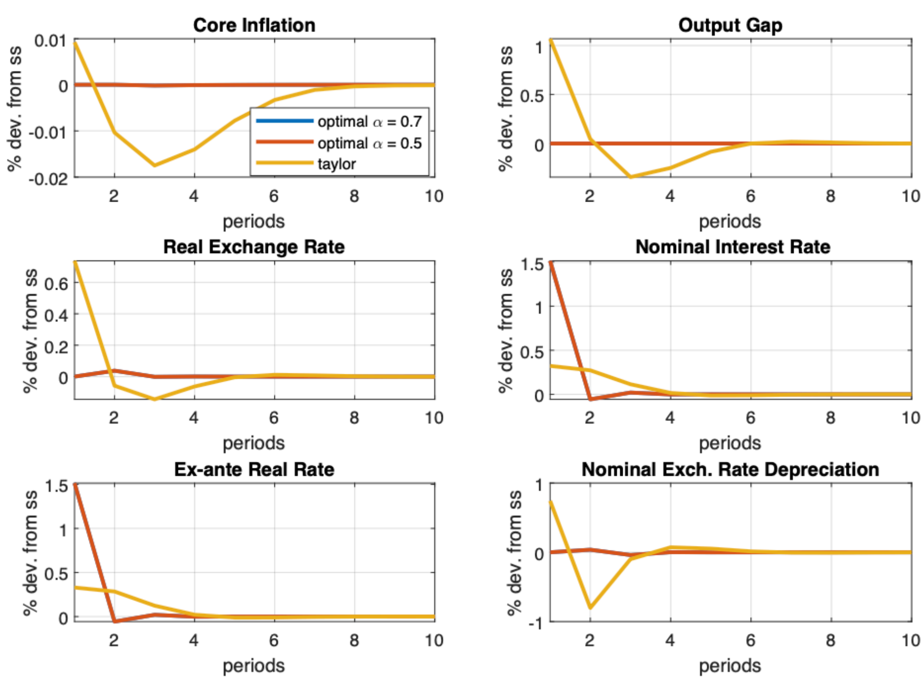

Figure 3: Impulse Response Functions: Exchange-Rate Shock.

On the other hand, when the CB implements an optimal monetary policy, the CB raises the nominal interest rate to allow the ex-ante real interest rate to raise and offset the depreciation shock. That is, monetary policy is adjusted sufficiently to neutralize the real exchange rate shock and thus prevent the output gap from widening and inflation from rising. Later on, the CB maintains a neutral monetary stance. To sum up, the optimal monetary policy response is more aggressive under the optimal rule than under the Taylor rule given the loss function we adopted and, as with the demand shock, optimal policy successfully stabilizes the economy when facing an exchange rate shock.

5 Optimal Robust Monetary Policy

This section is divided in two parts. In the first one, we consider that the CB has a preference for conducting a global robust policy. We analyze and compare the reduced form solution of the model with respect to the non-robust monetary policy to identify how the malevolent agent introduces the worst possible misspecification to the economy and how the CB reacts to avoid worst outcomes. In the second part of this section, we examine the case where the CB has a preference for robustness in only one structural relation of its benchmark model at a time, that is, we evaluate the effect of conducting a robust policy on every equation of the model when isolated.

5.1 Worst-case model and global CB's preference for robustness

In the worst-case model, we assume that the CB chooses a value of θ^j with $j = \pi, x, q$ in a way that the parameter reveals the highest preference of the CB to conduct a robust policy, which allows the model to have a unique solution. In order to analyze the reaction function of the CB and how the misspecifications are defined, we look computationally for the threshold value of θ^j. For that matter, we start with an arbitrary large value of theta and we solve the model using perturbation techniques with Dynare. Then, we decrease marginally the value of θ^j and verify if the model have a unique and stable solution. We repeat that process until we find the minimum value of theta that guarantees uniqueness. Once we obtain θ^j, we compare the reduced form solution with respect to the non-robust monetary policy. We find that the threshold value for the globally robust case is $\theta^j = 933$, for the calibration used in Table 1. The following Table 2 shows how the malevolent agent may introduce misspecification in the reduced form, meaning that the malevolent agent introduces the misspecifications in the economy depending on the state variables and the exogenous shocks.

Coefficients	Inflation	Output Gap	Real Exchange Rate
r_{t-1}	0	0	0
π^c_{t-1}	0.0004	0	0
x_{t-1}	0	0	0
q_{t-1}	0	0	0
$\varepsilon_t^{\pi^c}$	0.0011	0	0
ε_t^x	0	0	0
ε_t^q	0	0	0

Table 2: Misspecification in the global robust case $\theta^j = \theta, \forall j = \pi, x, q$.

Table 2 shows that, given the CB's focus on robustness, the malevolent agent can hardly introduce distortions into the economy. However, misspecifications that can be introduced are only present in the Phillips curve. This may be explained by the fact that, as pointed out in the impulse response functions, the cost-push shock makes policy trade-offs harder for the CB and cannot be perfectly offset; in contrast, misspecifications from the exchange rate and output gap behave in the same way as demand shocks, and can be neutralized by the CB. Therefore, despite the fact that the CB has a focus on robustness for each equation of its model simultaneously, the malevolent agent optimally decides to introduce distortions in the Phillips curve. Those misspecifications depend positively on the lagged inflation and the cost-push shock, which makes it more complex for the CB to identify if its model is correctly specified when the inflation is high and persistent.

Given that the CB conducts a robust monetary policy, it changes its reaction function to avoid additional losses. The following Table shows the reduced form of the model when the CB focuses on global robustness and non-robust monetary policy. In Table 3, for each endogenous variable (inflation, output gap, real exchange rate and nominal interest rate), Columns 3 and 4 indicate the model coefficients of the solution associated with the exogenous variables. Column 3 shows the coefficients when the CB does not conduct a robust policy while Column 4 represents the opposite scenario. Column 5 illustrates in which direction the parameter is moving when the monetary policy is robust, compared to the benchmark case when the monetary policy is not robust.

Table 3: Comparison between effects of global robust and non-robust monetary policy.

Equation	Coefficients	$\theta \to \infty$	$\theta = 933$	Δ
Column number	2	3	4	5
Inflation		Non-Robust Policy	Global Robust Policy	
	r_{t-1}	0	0	-
	π^c_{t-1}	0.50015	0.50127	$\uparrow$
	x_{t-1}	0	0	-
	q_{t-1}	-0.00451	-0.00452	$\downarrow$
	$\varepsilon^{\pi^c}_t$	1.50196	1.50532	$\uparrow$
	ε^x_t	-0.00001	0	$\uparrow$
	ε^q_t	0	0	-
Output Gap				
	r_{t-1}	0	0	-
	π^c_{t-1}	-0.00689	-0.00690	$\downarrow$
	x_{t-1}	0	0	-
	q_{t-1}	0.00006	0.00006	-
	$\varepsilon^{\pi^c}_t$	-0.02068	-0.02073	$\downarrow$
	ε^x_t	-0.00001	-0.00001	-
	ε^q_t	0	0	-
Real Exchange Rate				
	r_{t-1}	0.02474	0.02474	-
	π^c_{t-1}	-0.00348	-0.00349	$\downarrow$
	x_{t-1}	-0.22049	-0.22049	-
	q_{t-1}	0.00003	0.00003	-
	$\varepsilon^{\pi^c}_t$	-0.01046	-0.01047	$\downarrow$
	ε^x_t	-0.70670	-0.70670	-
	ε^q_t	0	0	-
Nominal Interest Rate				
	r_{t-1}	-0.03755	-0.03755	-
	π^c_{t-1}	0.19517	0.19603	$\uparrow$
	x_{t-1}	0.33470	0.33470	-
	q_{t-1}	0.47533	0.47532	$\downarrow$

Table 3: Comparison between effects of global robust and non-robust monetary policy.

Equation	Coefficients	$\theta \to \infty$	$\theta = 933$	Δ
Column number	2	3	4	5
	$\varepsilon_t^{\pi^c}$	0.58610	0.58868	$\uparrow$
	ε_t^x	1.07274	1.07275	$\uparrow$
	ε_t^q	1.51457	1.51457	-

The results display that even if the solution of the model is quantitatively different, the changes are small, since the malevolent agent introduces few distortions. Regarding the core inflation, when the CB has a preference for robustness, inflation results to be more persistent over time and becomes more sensitive to cost-push shocks while it does not react to demand shocks. It may be explained by the fact that, in the worst-case scenario, shocks have a stronger effects on inflation dynamics on impact and on its persistence. Simultaneously, the output gap and the real exchange rate react stronger (in absolute value) to inflation persistence and cost-push shocks. Therefore, when the CB has a preference for robustness, after a cost-push shock, inflation increases more whereas output gap and the real exchange rate decrease more compared to the case where the CB does not focus on robustness. This higher sensitivity of variables to shocks raises the CB losses, everything else being equal.

However, the CB reacts so that it avoids the worst-case scenario to happen. Indeed, if the CB has a preference for robustness, and is aware that cost-push shocks are worse than those correctly specified in its model, then, the CB responds more aggressively to inflation in the occurrence of a cost-push shock, by raising its policy interest rate for a longer period when observing higher persistence of inflation. Note that the CB also reacts more aggressively to demand shocks. It is worth mentioning that the IRFs do not change compared to the case where the CB does not conduct a robust policy (optimal policy), since the malevolent agent is unable to introduce relevant misspecification. This result confirms that the CB is unable to have a preference for robustness since little changes from its benchmark model may lead to explosive, unstable or multiple solutions.

Proposition 1: *When the CB has a global preference for robustness, the threshold for conducting a robust policy is limited, given that it generates otherwise unstable or multiple* *solutions. The worst misspecification lies on the Phillips Curve and the CB reacts more aggressively to cost-push shocks and output gap shocks with respect to the non-robust policy.*

The above discussed subsection shows the consequences of a global robust monetary policy. However, in a more realistic scenario, the CB may have more information about some characteristics of the economy because of their easier observation. For instance, the CB may know more of the evolution of inflation but not so much of the evolution of the real exchange rate or viceversa. For that matter, it is crucial to study what could be the worst-case scenario if the CB has a focus of robustness for one equation at a time.

5.2 Impact of different types of CB's preferences for robustness

Performing simulations with the above mentioned algorithm, we find that the CB can be robust facing misspecification only present in the output gap equation or in the uncovered interest rate parity condition but we do not find the same for the Phillips curve. For instance, the threshold values are $\theta^x = 0.002, \theta^q = \infty, \theta^\pi = \infty$; $\theta^q = 0.002, \theta^x = \infty, \theta^\pi = \infty$ and $\theta^\pi = 933, \theta^x = \infty, \theta^q = \infty$, respectively. Note that this last threshold value corresponds to the threshold value corresponding to a preference for global robustness. The following Table shows the reduced form misspecifications in the case where the CB only cares about robustness in one equation of the model at the time and conducts the maximal level of robust policy possible allowing for stable solutions.

Coefficients	Inflation through h^π	Output Gap through h^x	Real Exchange Rate through h^q
Threshold values	$\theta^\pi = 933$	$\theta^x = 0.002$	$\theta^q = 0.002$
r_{t-1}	0	-0.0001	0
π^c_{t-1}	0.0004	-0.1826	0.0047
x_{t-1}	0	0.0005	0
q_{t-1}	0	0.0016	0
$\varepsilon^{\pi^c}_t$	0.0011	-0.5484	0.0141
ε^x_t	0	0.0016	0
ε^q_t	0	0	0

Table 4: Misspecifications (Each column corresponds to one misspecification considered in the model one at a time)

Table 4 represents, for each column, the misspecification for each corresponding equation when the CB has a preference for robustness when misspecifications occur in the same equation only. For instance, the second column shows the inflation misspecifications (in reduced form) introduced by the malevolent agent when the CB has a preference for robustness only in the Phillips Curve; the third column shows the reduced form of the misspecifications in the output gap when the CB has preference for robustness only in the output gap and, so on. That is, each column exhibit how does the malevolent agent behaves to maximize the CB's loss function when the latter is concerned only in one equation of the model at the time.

When the CB only considers inflation misspecification, two points stand out. The first is that the malevolent agent introduces the same level of distortions as when the CB wants to be globally robust of its entire model. The second is that optimally, the malevolent agent only distorts inflation when there is a cost-push shock and with lagged inflation, which increases the CB's cost of the stabilization trade-off by amplifying the initial effect of a cost-push shock.

When the CB only focuses on robustness of the output gap, it introduces negative misspecification in the face of cost-push shocks and positive misspecification in the face of a demand shock. Additionally, misspecifications depend negatively on lagged inflation and positively on lagged real exchange rate. In the event of a cost-push shock, optimal monetary policy manages to stabilize the output gap while inflation fades over time (see Figure 1). A misspecification impacting negatively the output gap complicates the CB's optimization problem, because, on one hand, it would imply inflation to be above its steady state and the output gap below its steady state, and since the elasticity of the output gap on inflation is small, inflationary pressures remain. In the event of a demand shock, the malevolent agent would simply amplify the CB's problem by introducing additional positive distortions to the output gap.

Finally, when the CB only considers misspecifications in the UIP condition, the malevolent agent introduces positive distortions in the face of a cost-push shock and to lagged inflation. This is because inflation converges gradually to its steady state while the CB can effectively stabilize the output gap -recall the cost-push shock in the non-robust case (Figure 1). Thus, a depreciation of the real exchange rate would widen the output gap and furthermore raise inflation, worsening the CB's loss.

Given that the malevolent agent introduces the above mentioned distortions, the CB optimally changes its monetary policy reaction to avoid the worst-case scenarios. The following table shows the reduced forms of the model, where endogenous variables are expressed in terms of the exogenous ones. We consider 4 cases: the absence of robustness focus, the robustness preference for the Phillips curve, for the output gap and for the real exchange rate one at the time.

First, it highlights that the model solution when the CB is robust to misspecification in the

Phillips curve is the same as when the CB has a preference for robustness for all equations (global robustness). This suggests that the CB's ability to be robust is constrained by the law of motion for inflation. Second, when the CB has a preference for robustness only for the output gap, the interest rate reacts more strongly to output gap shocks and real exchange rate shocks. On the other hand, the interest rate is significantly less sensitive to inflation shocks since the effect of the shocks goes from 0.58610 to 0.19611 in Table 5. This implies that, in the face of a cost-push shock, since the malevolent agent introduces negative shocks into the output gap, the CB does not increase the interest rate in the same proportion with respect to the non-robust case to avoid amplifying this misspecification. Additionally, the interest rate is more sensitive to the lagged output gap and less sensitive to lagged inflation. Finally, when the CB is robust to misspecification in the real exchange rate, it reacts more aggressively to output gap shocks and inflation shocks as well as to the lagged value of these variables, while it is less sensitive to the lagged component of the real exchange rate because it has greater uncertainty about how it evolves over time.

Table 5: Misspecification (Each column corresponds to one misspecification considered one at a time, and the evolution is referring to the baseline model where $\theta \to \infty$)

		No robustness	Robustness for one equation					
Column number	2	3	4	5	6	7	8	9
Equation	Coeff. for	$\theta^j \to \infty$	$\theta^\pi = 933$	Δ	$\theta^x = 0.002$	Δ	$\theta^q = 0.002$	Δ
Inflation								
*	r_{t-1}	0	0	-	0	-	0	-
*	π^c_{t-1}	0.50015	0.50127	$\uparrow$	0.50015	-	0.50015	-
*	x_{t-1}	0	0	-	0	-	0	-
*	q_{t-1}	-0.00451	-0.00452	$\downarrow$	-0.00451	-	-0.00451	-
*	$\varepsilon^{\pi^c}_t$	1.50196	1.50532	$\uparrow$	1.50197	$\uparrow$	1.50196	-
*	ε^x_t	-0.00001	0	$\uparrow$	-0.00001	-	-0.00001	-
*	ε^q_t	0	0	-	0	-	0	-
Output gap								
*	r_{t-1}	0	0	-	0	-	0	-
*	π^c_{t-1}	-0.00689	-0.00690	$\downarrow$	-0.00689	-	-0.00689	-

Table is continued on next page

Table 5: Misspecification (Each column corresponds to one misspecification considered one at a time, and the evolution is referring to the baseline model where $\theta \to \infty$)

		No robustness	Robustness for one equation					
Column number	2	3	4	5	6	7	8	9
Equation	Coeff. for	$\theta^j \to \infty$	$\theta^\pi = 933$	Δ	$\theta^x = 0.002$	Δ	$\theta^q = 0.002$	Δ
Output gap								
*	x_{t-1}	0	0	-	0	-	0	-
*	q_{t-1}	0.00006	0.00006	-	0.00006	-	0.00006	-
*	$\varepsilon_t^{\pi^c}$	-0.02068	-0.02073	↓	-0.02068	-	-0.02068	-
*	ε_t^x	-0.00001	-0.00001	-	-0.00001	-	-0.00001	-
*	ε_t^q	0	0	-	0	-	0	-
Real exchange rate								
*	r_{t-1}	0.02474	0.02474	-	0.02477	↑	0.02474	-
*	π_{t-1}^c	-0.00348	-0.00349	↓	0.12557	↑	-0.00348	-
*	x_{t-1}	-0.22049	-0.22049	-	-0.22084	↓	-0.22049	-
*	q_{t-1}	0.00003	0.00003	-	-0.00113	↓	0.00003	-
*	$\varepsilon_t^{\pi^c}$	-0.01046	-0.01047	↓	0.37710	↑	-0.01046	-
*	ε_t^x	-0.70670	-0.70670	-	-0.70782	↓	-0.70670	-
*	ε_t^q	0	0	-	0.00002	↑	0	-
Nominal interest rate								
*	r_{t-1}	-0.03755	-0.03755	-	-0.03764	↓	-0.03755	-
*	π_{t-1}^c	0.19517	0.19603	↑	0.06531	↓	0.20227	↑
*	x_{t-1}	0.33470	0.33470	-	0.33550	↑	0.33473	↑
*	q_{t-1}	0.47533	0.47532	↓	0.47651	↑	0.47527	↓
*	$\varepsilon_t^{\pi^c}$	0.58610	0.58868	↑	0.19611	↓	0.60741	↑
*	ε_t^x	1.07274	1.07275	↑	1.07532	↑	1.07284	↑
*	ε_t^q	1.51457	1.51457	-	1.51460	↑	1.51457	-
*								

Noteworthy, our results suggest that the CB reacts differently depending on the source of misspecification and the type of robustness preference implemented in the model. If the CB has a preference for robustness on the Phillips curve or the UIP condition, then it should conduct a more aggressive policy towards cost-push shocks or demand shocks. However, if the CB has a preference for robustness in the IS equation, the best response for the CB to avoid the worst case scenario is to conduct a more aggressive policy towards real exchange rate shocks and output gap shocks but less aggressive towards cost-push shocks.

To better understand the effects of the CB's preference for robustness for each structural equation at a time, Figure 4 shows the impulse response functions for misspecifications and the main variables in the model to demand, real exchange rate and cost-push shocks.

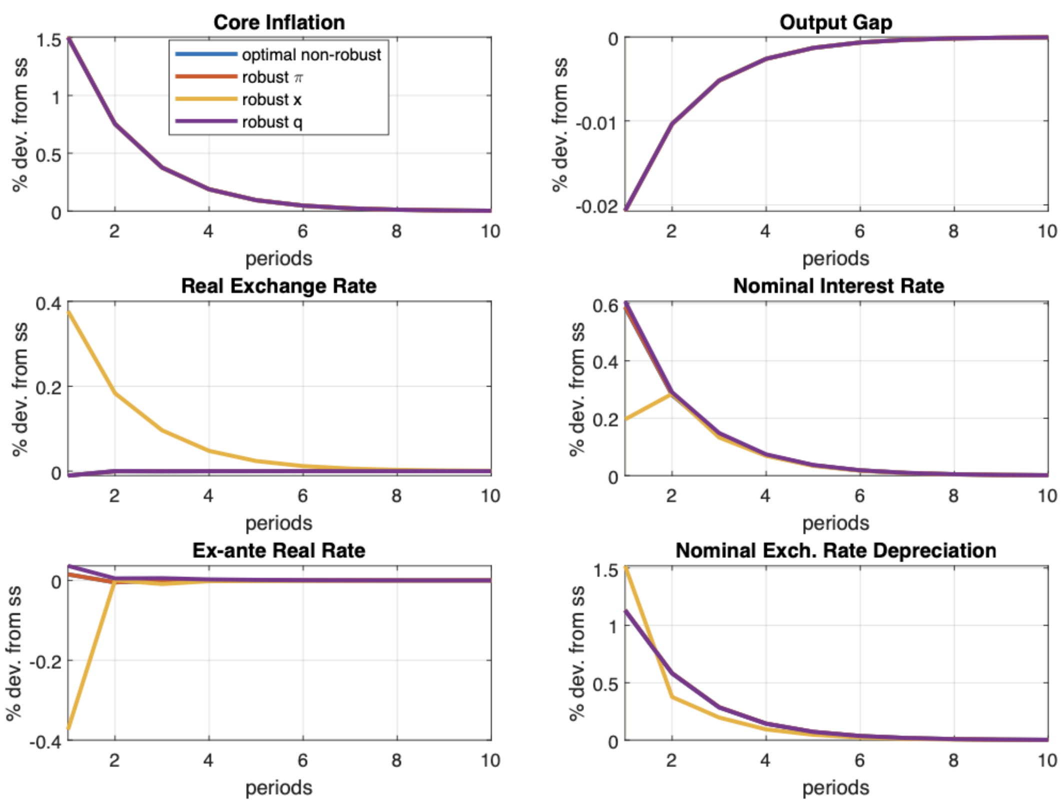

Figure 4: IRF for a cost-push shock when the CB has a preference for robustness one equation at a time

As pointed out in Table 4, misspecifications react to demand and cost-push shocks. However, the only case when the malevolent agent can introduce quantitatively relevant misspecifications is in the event of a cost-push shock and simultaneously, when the CB has a preference for robustness for the output gap only (see the orange line in the left panel, Figure 5 below).[15] That is, the malevolent agent recognizes that the major CB's trade-off is on the Phillips curve and has incentives to distort it since this trade-off may represent the main source of losses. In this case, in the event of a cost-push shock and the CB's preference for robustness for the output gap, the malevolent agent introduces a negative misspecification term close to 0.5% to the output gap, generating a wider gap relative to the non-robust scenario. Given that the CB wants to avoid this worst-case outcome, it raises the nominal interest rate but lower than the non-robust case. The latter generates an accommodative monetary policy stance (i.e, the ex-ante real interest rate is lower than the neutral rate), depreciating the real exchange rate, which neutralizes the misspecification. Notably, the CB is able to offset the worst-case outcome since inflation and the output gap behave in the same way as in the optimal case but the policy response is remarkably different. The rest of the impulse response functions (relative to the non-robust policy) are quantitative similar since the misspecification introduced by the malevolent agent are close to zero.[16]

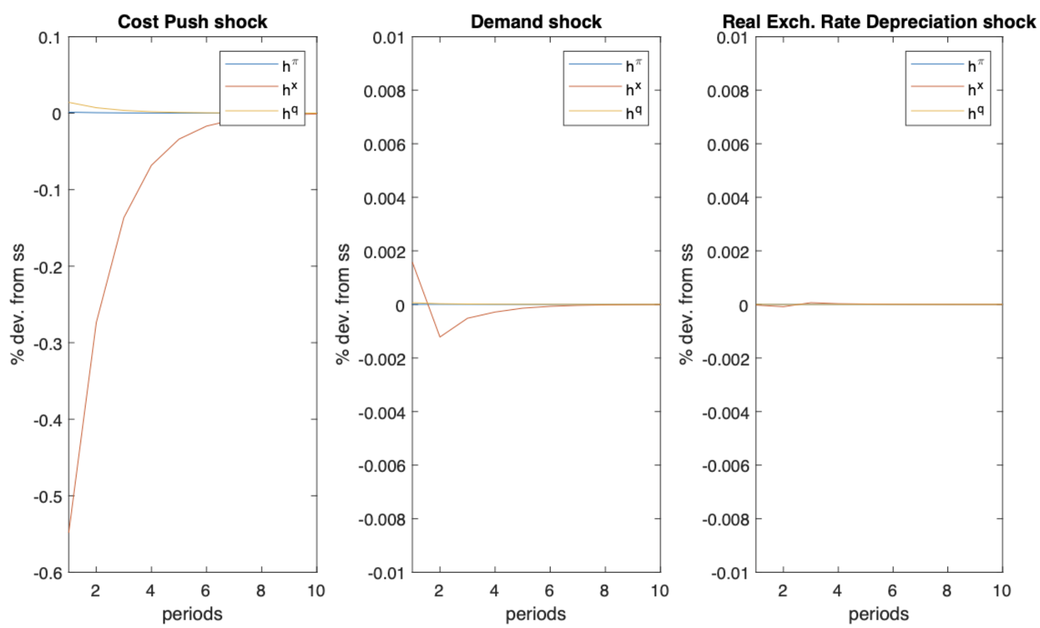

Figure 5: Comparison of IRFs for misspecifications

[15]In the rest of the shocks and CB's preference for robustness (see Appendix A.2), misspecifications increase or decrease lower than 0.002% given a unit shock (see Figure 5).

[16]The rest of the impulse responses functions (demand and exchange rate shocks) when the CB has a preference for robustness in each equation of the model separately are located in the Appendix A.2.

Proposition 2: *When the CB has a preference for robustness in the IS equation or in the UIP condition, then the policy can be robust. On one hand, when the CB has a preference for robustness for the output gap only, it should be more aggressive to demand and exchange rate shocks while being more conservative to cost-push shocks. On the other hand, when the CB has a preference for robustness for the UIP only, the CB should be more aggressive to demand and cost-push shocks.*

6 Limits of a Robust Monetary Policy

The previous propositions show how the CB may optimally behave in different situations: when the CB does not have any preference for robustness i.e. $\theta^j \rightarrow \infty$, when the CB has a global preference for robustness (robust in every equation of the model) or when the CB only has a preference for robustness for one equation in particular. Importantly, when the CB has a global preference for robustness, we found that the threshold value granting a stable equilibrium is when $\theta^j = 933$ for $j = \pi, x, q$. Note that, when the CB has a preference for robustness for the Phillips curve only, the threshold value for a stable equilibrium is as well $\theta^\pi = 933$. However, when the CB conducts a robust policy with respect to the output gap or the UIP condition, the threshold can be very low ($\theta^x = \theta^q = 0.002$), allowing a quite robust policy. This suggests that the limit to conduct a robust monetary policy is imposed by the Phillips Curve.

In this section, we perform a sensitivity analysis to identify why the Phillips curve is limiting the ability to conduct a global robust monetary policy. Thus, for each element of the Phillips curve (lagged inflation, inflation expectations, output gap and imported inflation), we identify first, for which values of the parameter of interest the model have a stable solution and, second, which is the threshold value for the global CB preference for robustness.

6.1 Inflation Persistence

We start with the parameter of inflation persistence in the Phillips Curve (1), a_1. From the sensitivity analysis we find that the current parameter is at its upper limit: for values greater than the current one ($a_1 = 0.333$), the model has no unique stable solution. Additionally, for smaller values of a_1, the model quickly approaches to cases where the CB can be very robust. In fact, a decrease from its current value to $a_1 = 0.3$ allows to reduce the robustness threshold from $\theta^j = 933$ to $\theta^j = 25$ for $j = \pi, x, q$, and for $a_1 \leq 0.2$, the CB can implement a fully global robust monetary policy. It is worth mentioning what is the effect of inflation inertia on optimal robust monetary policy.

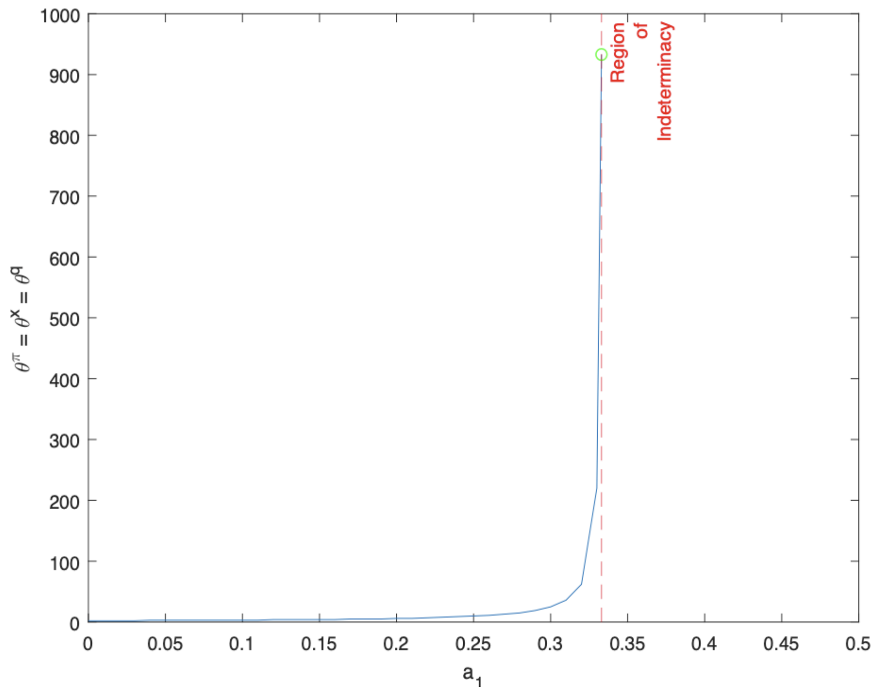

Figure 6: Robustness thresholds for different values for the coefficient of inflation inertia

The more persistent inflation is, the greater the effort of monetary policy is required to change its trend and induce that it converges to the steady state. That is, inertia imposes an additional challenge to the conduct of optimal monetary policy. As it is well known, under purely forward-looking DSGE models, conducting an optimal monetary policy results from the trade-off between stabilizing inflation and the output gap, known as a “leaning against the wind” monetary policy stance, where when inflation is above its target, the CB should contract demand below capacity by higher interest rates, and vice-versa when inflation is below its target.[17] However, when inflation is persistent, the CB faces an additional intertemporal trade-off: monetary policy actions will have also effects in the future, thus policy also depends on forecasts of future variables, explaining the further need to anchor inflation expectations.

[17]For instance, see Clarida, Gali and Gertler (1999) or Woodford (2000) as a reference of the canonical purely forward-looking DSGE model.

We derive analytically the effect of introducing inflation persistence in the model. For that, we compare the implicit targeting rule (10) when we suppress every lagged component of the model to the targeting rule where we only introduce inflation persistence (11).[18]

$$\pi_t \frac{\omega\left(a_4+a_3 b_5\right)}{1-\omega a_4}-\beta E_t \pi_{t+1} \frac{\omega\left(a_4 c_2+a_3 b_3\right)}{c_2\left(1-\omega a_4\right)} = -\alpha b_5 x_t+\alpha \beta \frac{b_3}{c_2} E_t x_{t+1} \tag{10}$$

$$\begin{aligned}\pi_t \frac{\omega\left(a_4+a_3 b_5\right) c_2}{\Theta}-\beta E_t \pi_{t+1} \frac{\omega\left(a_4 c_2+a_3 b_3\right)}{\Theta} &= -\alpha \frac{b_5 c_2\left(1-\omega a_4\right)}{\Theta} x_t+\alpha \beta \frac{a_1 b_5 c_2+b_3\left(1-\omega a_4\right)}{\Theta} E_t x_{t+1} \\ &\quad -\alpha \beta^2 \frac{a_1 b_3}{\Theta} E_t x_{t+2}\end{aligned} \tag{11}$$

where, $\Theta=\left(a_4 c_2+a_3 b_3\right)\left(1-\omega a_4\right)-c_2 a_1\left(a_4+a_3 b_5\right)$. From the targeting rules, we identify that introducing inflation persistence complicates the CB's intertemporal trade-off by adding one more period in the targeting rule.[19] When including inflation persistence, the CB's targeting rule is even more complex by comparing equations (10) and (11). Indeed, the CB does not take into account two periods to stabilize the economy, but now three periods. Due to the presence of inflation persistence, the consequences of shocks last more due to the transmission mechanisms of monetary policy, so the CB must incorporate this persistence into the design of its monetary policy. For instance, if we assume that the output gap and inflation are above their steady state (due to a demand shock, for example), the CB would raise the interest rate to offset the inflationary pressures and the output gap. When stabilizing the economy in the current period, the CB would have to raise sufficiently the policy rate to yield an appreciation of the real exchange rate, therefore contracting aggregate demand and reducing inflation through the the exchange rate pass-through and expectations channels. However, the interest rate has a lagged effect on the output gap, which means that in the previous case, by stabilizing economy in the current period, this would further contract the output gap and inflation in the next period. Additionally, due to the inflation persistence, the inflation undershooting could keep on many periods before allowing the inflation to converge to its steady state

[18]The equation (10) comes from combining the first order conditions of the optimal robust problem and setting $a_1=b_1=c_1=0$ while the equation (11) is by setting only $b_1=c_1=0$.

[19]In this model without any type of persistence , the targeting rule is different from the "classical" static leaning against the wind rule (equation 10). Indeed, it also includes terms with expectations for inflation and output gap. This is because the interest rate acts with lag. On impact, the ex-ante real rate does not affect the gap directly but does affect the real exchange rate and it is until the next period that the real interest rate has a direct effect on the output gap. Therefore, at the shock impact, the CB can stabilize the economy through the channel of the exchange rate and by that of expectations, since the direct channel of the interest rate on consumption and investment decisions is given with a lag. This lag in the transmission mechanism of monetary policy explains the emergence of intertemporal trade-off even without the presence of inertia, as the CB needs to consider that its policy will have consequences both in the present and in the future.

level, which would increase the CB's losses. Therefore, for a higher inflation persistence, the steeper the trade-off condition gets for the CB, reducing the solutions space to conduct a robust policy, meaning that the introduction of any misspecification may lead to unstable solutions.

6.2 Inflation Expectations

In the same way than for the inflation persistence, the coefficient $a_2 = 0.664$, related to inflation expectations in the Phillips curve (1) is in its upper limit to ensure a stable and unique solution. Everything else being equal, for values of $a_2 > 0.664$, the model does not feature a unique solution (see Figure 7).

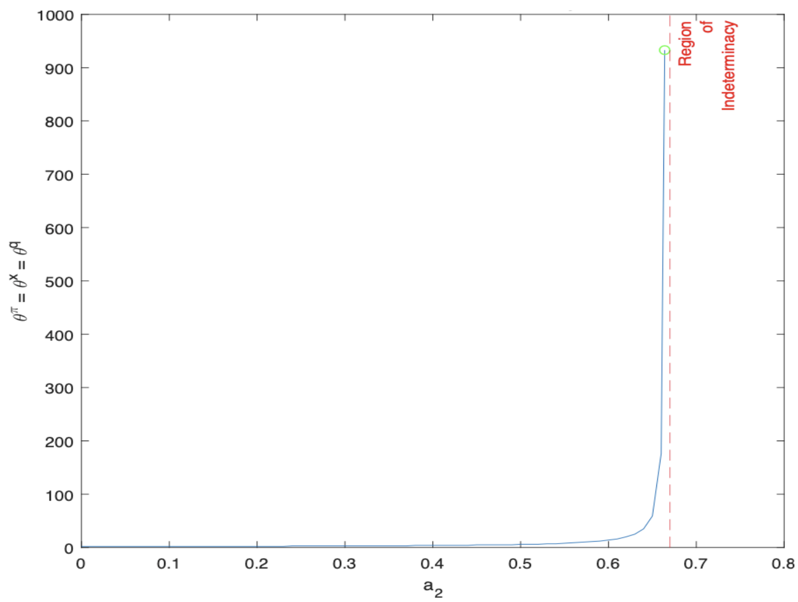

Figure 7: Robustness thresholds for different values for the coefficient of expected inflation

Additionally, the CB can increase its preference for robustness when the inflation expectations have less impact on current inflation in the Phillips curve. That is, the more the Phillips curve becomes contemporaneous, the greater the ability of the CB to implement a robust monetary policy. For values of $a_2 \leq 0.5$, the CB can implement its highest preference for robustness.

It is worth mentioning that a greater relative importance of the inflation expectations channel implies that the CB must better anchor inflation expectations to stabilize current inflation. For instance, in the face of a shock that increases inflation expectations, the CB has to be more aggressive to inflation in order to move current inflation and thus, better anchoring inflation expectations which ensures a less volatile path for future inflation. If this channel is strengthened while the other transmission mechanisms remain unchanged, in a context of low interest rate-output gap elasticity and low exchange rate depreciation pass-through to inflation, a greater importance of this channel complicates the conduct of a robust monetary policy.

However, a strengthening of the expectations channel might be caused by a structural change in the economy, possibly related with an increased CB credibility (better communication, transparency, autonomy, among others). This structural change that strengthens the expectations channel could also reduce the persistence of inflation given that private agents may have better information and understanding on the conduct of monetary policy.[20]

Figure 8 shows how the robustness threshold value changes when the expectations channel gets stronger (an increase in a_2 from (1)), and inflation becomes less persistent (a decrease in a_1), simultaneously.[21] Interestingly, Figure 8 exhibits a positive relationship between the ability of the CB to conduct a robust monetary policy and an inflation, both, more forward-looking and less backward-looking at the same time. In fact, the threshold value decreases from $\theta^j = 933$ in the baseline scenario (last bar) to $\theta^j = 693$ (first bar) when $a_1 = 0.033$ and $a_2 = 0.964$.

[20] Ramos-Francia and Torres (2008) found when estimating a New-Keynesian hybrid Phillips Curve that the backward-looking component has decreased over time while the forward-looking component has strengthened. Those propositions are in line with more recent estimates using an ampler sample and a semi-structural model (Banxico, 2016)).

[21] In this graph, each bar represents a pair $\{a_1, a_2\}$ such that, from the baseline values, a_2 increases by 0.02 and a_1 decreases by 0.02, simultaneously. Then, we look numerically for the threshold value θ^j for $j = \pi, x, q$, that guarantees a unique and stable solution.

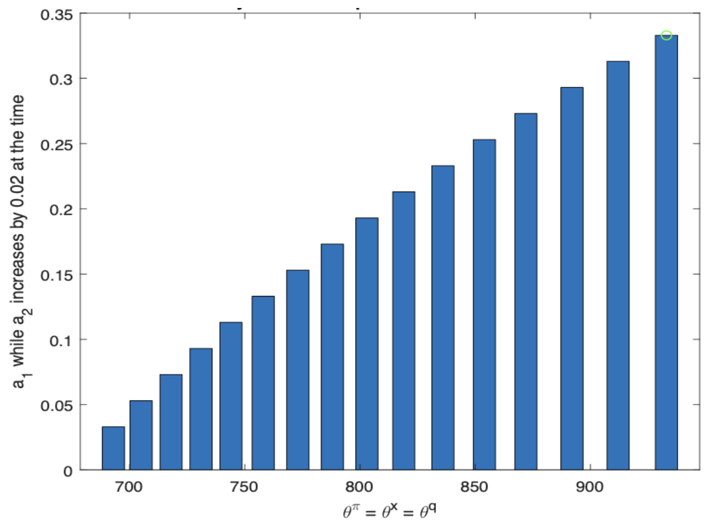

Figure 8: Robustness thresholds when the inflation persistence gets lower and inflation expectations coefficient increases

6.3 Output Gap

When the Phillips curve gets steeper, meaning that inflation is more sensitive to the output gap, the CB's ability to be robust increases by being able to consider larger misspecifications from its reference model (see Figure 9). Figure 9 shows that in the baseline calibration, the effect of the output gap on inflation is very small ($a_3 = 0.013$). Furthermore, the closer to zero a_3 gets, the smaller the solutions space for stability and uniqueness when conducting a robust monetary policy and, conversely, an increase in a_3 allows the CB to have a greater space for robustness.

This result lays from the fact that, the greater the effect of the output gap on inflation, the stronger the transmission mechanisms of aggregate demand. Note that, even though the interest rate have a direct effect on the output gap with a one-period lag, the CB is always able to influence the exchange rate and thus, the output gap indirectly through the relation in the IS curve (2). This mechanism allows the CB to have a greater control over inflation (i.e, a smoother trade-off between stabilizing the output gap and inflation) and therefore, allowing an easier stabilization of inflation when the model is misspecified.

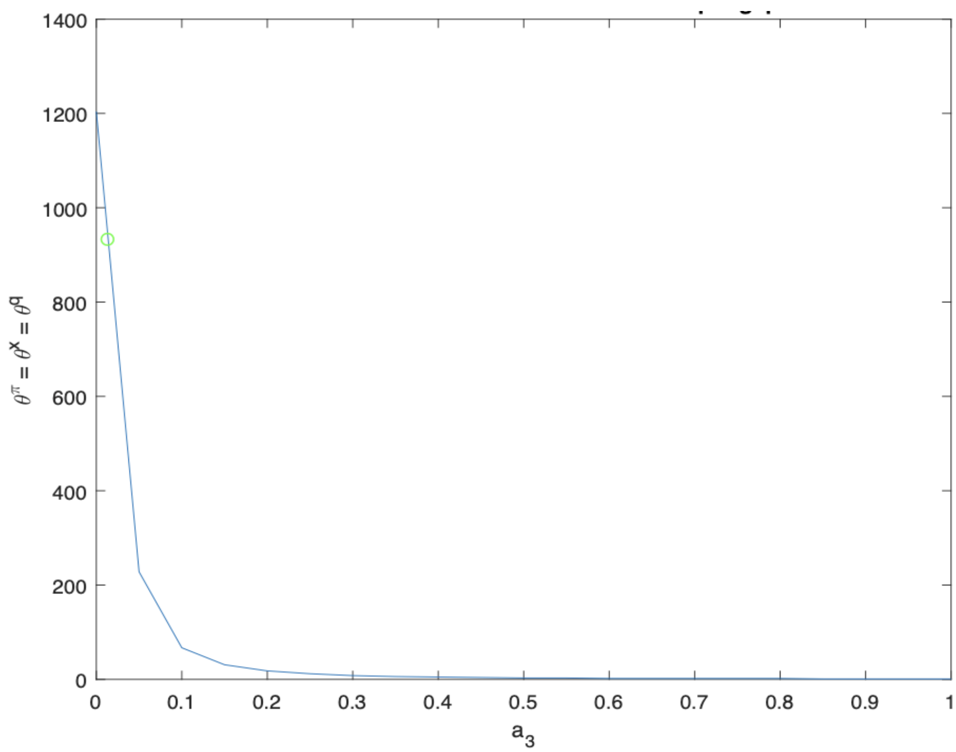

Figure 9: Robustness thresholds for different values for the coefficient of output gap

6.4 Exchange Rate Depreciation

With respect to the sensitivity analysis of the effect of the exchange rate depreciation to inflation, we find that a marginal change in the baseline parameter where, initially $a_4 = 0.003$, yields to unstable solutions. Moreover, by performing the sensitivity analysis on the whole interval for the parameter $a_4 \in [0, 1]$, only a value higher than 0.96 ensures a region of stable solutions. Interestingly, from Figure 10, values of $a_4 \geq 0.96$, increase the exchange-rate pass-through to inflation but allow a wider space for conducting robust monetary policy. Despite the latter, a lower exchange-rate pass-through produces less inflation volatility in the face of current account imbalances or external adjustments, which makes it easier to conduct monetary policy. However, when there is a larger exchange-rate pass-through, it increases the space of stable solutions in the model so that the CB can set a more robust monetary policy. In fact, monetary policy impacts output gap directly with a one-period lag but the channels that allow stabilizing the economy at the impact of shocks are the exchange rate channel as well as the expectations one, explaining why a higher pass-through allows the CB to conduct a wider robust monetary policy.

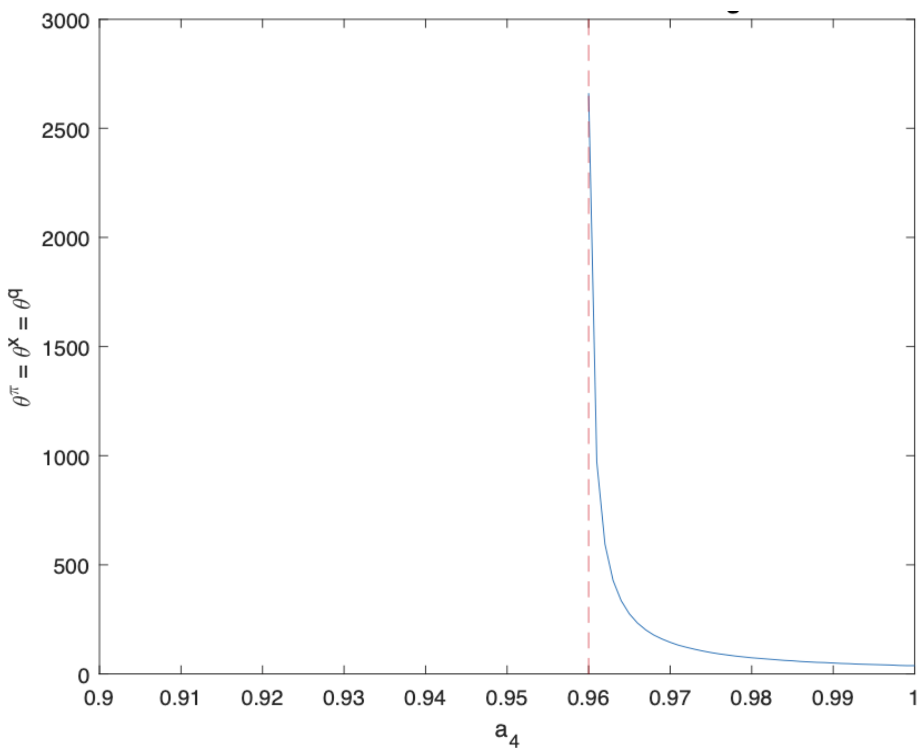

Figure 10: Robustness thresholds for different values for the coefficient of foreign inflation

In that sense, although a higher exchange-rate pass-through to prices could complicate the conduct of monetary policy in practice by generating a greater inflation and interest rate volatility, it also strengthens the exchange rate channel, broadening the ability to counteract misspecifications. Indeed, if the reference model is incorrect, a higher exchange-rate pass-through allows the CB to offset those errors through changes in the policy interest rate in such a way that the exchange rate appreciates or depreciates, depending on the type of shocks.

Finally, two elements stand out from the sensitivity analysis on the elements of the Phillips Curve. The first one is that the ability to be robust depends on the intensity of the transmission mechanisms, where a strengthening of the aggregate demand channel or of the exchange rate channel could imply a greater possibility to conduct a robust monetary policy. The weakness of the aggregate demand channel in Mexico has multiple dimensions. In the literature for instance, Alberola and Urrutia (2019) suggest that high informality in Mexico decreases the effectiveness of monetary policy and increases the CB's sacrifice ratio. However, informality could also play a role in decreasing inflation volatility in the face of certain exogenous shocks (Alberola and Urrutia 2019, Medina 2019). On the other hand, as mentioned by Ramos-Francia and Sidaoui (2008), the low financial depth in the country could explain the low sensitivity of credit demand to interest rate movements. In this sense, in an environment of weak demand channel, a greater pass-through of exchange rate depreciation to prices could strengthen this transmission mechanism and thus increase the possibility for the CB to have a higher preference for robustness , but it would be at the cost of an increase in inflation volatility, complicating the conduct of monetary policy. In fact, recent studies indicate that the CB credibility in emerging economies with inflation targeting regimes has managed to reduce the pass-through from the exchange rate to prices (IMF, 2018), so a strengthening of this channel seems unlikely to happen. Indeed, the evidence for the Mexican economy suggest that the exchange rate pass-through to consumer prices is low (Cortés, 2013, Kochen and Sámano 2016) and has decreased since 2001 (Capistrán et al. 2012).

The second element to highlight from the sensitivity analysis is the key role played by the expectations channel. As previously mentioned, the limited possibility of implementing a robust monetary policy lies in the fact that the exchange rate and the aggregate demand channels are weak, while the expectations channel plays a key role in stabilizing inflation. A strengthening of this channel, everything being equal, makes it harder for the CB to conduct of a robust monetary policy. Indeed, the need for anchoring inflation expectations becomes more critical, but its achievement requires clear actions capable of modifying the law of motion for inflation and therefore, its expectations. However, a strengthening of the expectations channel possibly implemented through improved communication, transparency and better public understanding of the CB's actions could decrease inflation persistence, which effectively increases the ability to conduct a robust monetary policy.

Proposition 3: *Five facts arise from the sensitivity analysis: i) the less backward-looking and ii) the less forward-looking inflation is, the larger the space to be robust; iii) the stronger the effect of the output gap to inflation and iv) the higher the exchange rate pass-through to inflation, the wider the space to be robust; v) simultaneously, the more forward-looking and the less backward-looking inflation is, the bigger the space to be robust.*

7 Discussion

So far we have analyzed how to design an optimal robust monetary policy in a small open economy such as Mexico. To do so, we have studied a small-scale semi-structural model estimated for Mexico using a sample with monthly data from 2001 to 2006, as presented by Sidaoui and Ramos-Francia (2008). The propositions suggest that, for this model, the CB's ability to implement a robust monetary policy is limited. That is, the CB cannot consider large deviations from its benchmark model, since it has no longer a unique and stable solution. In this particular case, when the CB has a high preference for robustness considering all equations, the worst that could happen would be to misspecify the Phillips curve, as this is the source of the greatest social loss in the model since it generates the trade-off between stabilizing inflation and the output gap.

Our propositions are in line with the literature that focus on representing different types of inflation persistence. In particular, a few recent studies deal with the design of optimal policy conditional on belief structures that are based on learning (Gaspar, Smets and Vestin 2006, 2010, Molnár and Santoro 2014, André and Dai 2017, 2018, Eusepi, Giannoni, and Preston 2018, Mele, Molnár and Santoro 2019). These studies examine the effect of learning algorithm on the design of optimal policy compared to the rational expectation assumption, and provide insights on the constraints that non-rational belief structures place on optimal monetary policy. Indeed, the backward looking algorithm used to represent learning induce a form of inflation persistence.

While our propositions are of interest, it is crucial to handle carefully some interpretations. First, the results so far arise from using a model for a small open economy with estimates for Mexico, which implies that the transmission mechanisms are not necessarily comparable with those of other similar emerging economies, although this could be the case for economies that, like Mexico, are widely open and depend on commodities that, in the face of real exchange rate depreciations, expand their exports on the one hand and increase inflationary pressures on the other. This is worth mentioning, as the intensity and direction of the transmission mechanisms are what generate CB reactions in the worst case. For example, Söderstrom and Leitemo (2008) also find that CB behavior depends on the type of robustness; however, in their model the exchange rate expands exports, but contracts inflation since it applies to the Swedish economy, presenting advanced economy features. Thus, they find that the CB should be more permissive with real exchange rate shocks when it is robust to misspecification of the output gap or inflation. While when the CB is robust to exchange rate misspecification, the CB's response is less aggressive to inflation or output gap shocks, which clearly contrasts with our results. This difference can be explained by the fact that the exchange-rate pass-through to inflation in the Swedish economy is negative for the sample they studied, whereas for the Mexican economy, the exchange-rate pass-through is positive.

Second, our baseline calibration comes from the estimation that considers a sample for the period 2001 to 2006. Although the estimation considers the period after the implementation of the inflation targeting regime adopted by the Bank of Mexico in 2001, it is well known that the success of this regime depends on the CB credibility and the wide communication about inflation targeting so that private agents know its existence (Svensson 2007, Bernanke et al. 2018, Clinton et. al. 2015). However, it is natural to think of this process of communication and public awareness as an ongoing process, such that the reputation and credibility of the CB may increase over time. Indeed, a higher CB credibility with its inflation target decreases inflation persistence and volatility, given that economic agents are more interested in the expected trajectory of inflation to determine their prices and wages rather than in past inflation (Capistrán and Ramos-Francia 2009, Sidaoui and Ramos-Francia, 2008). Given the results of this paper, one natural extension would be to study how the degree of communication and transparency of a CB can affect the level of robustness of monetary policy. Intuitively, we assume that with a better communication and higher transparency, the space with stable solutions increases for robust monetary policy

Additionally, another extension to this work would be to verify those results by estimating the model on a larger time period. In this paper, we showed that the robust monetary policy in Mexico between 2001 and 2006 is limited due to the role played by the Phillips curve. Indeed, a high inflation inertia and the presence of a low sensitivity between the output gap and inflation explains why robustness is harder to implement since it reduces the space for stable solutions. Therefore, as future lines of research, it is of interest to estimate the model with a larger sample to capture, on one hand, whether the transmission mechanisms have changed recently and whether they have modified the ability to implement a robust monetary policy. In the same vein, although the model considered in this paper represents the functioning of a small open economy, the emphasis is put on analyzing how the CB behavior would change in the face of domestic shocks when the CB has a preference for robustness. One way to continue our work would be to carefully estimate the external sector and identify the effect of robustness on the CB's reaction to external shocks.

Furthermore, we find that when the CB has a preference for robustness regarding the IS equation, it would increase the interest rate by a smaller proportion in the face of a supply shock than in the non-robust case. This is due to the fact that, in the worst-case scenario, the worst that could happen to the CB is that the output gap contracts greater than in the non-robust case. However, note that this result is due to the presence of robustness exclusively in the IS equation. For instance, recently, during the COVID-19 crisis, uncertainty was widely spread when modeling laws of motion for inflation, the output gap and the exchange rate, so analyzing COVID-19 crisis is more akin to generalized uncertainty throughout the model. In this case, the conduct of a robust monetary policy corresponds to a more aggressive reaction to inflation and output gap shocks. Additionally, the model assumes that the shocks are temporary and do not affect potential output. However, the economy could be recently being affected by shocks that influence the economy's potential output, so the output gap could be both underestimated or overestimated. We leave this study for future research.

This paper focuses on the worst-case model, meaning that the malevolent agent chooses model misspecification to be as damaging as possible while the CB's policy rule and private agents' expectations reflect this misspecification. An interesting extension is to examine the case where the CB uses the robust control approach to design the policy interest rate rule but the economy functions according to an approximating model as in Leitemo and Söderström (2008b). Since only the interest rate rule is disturbed to account of model misspecification while the true model of the economy remains undisturbed, it seems that the economy has a smaller risk of being destabilized, meaning that the CB could have higher focus in favor for model misspecification than in the worst-case model.

An alternative research approach to robustness would be to consider multiplicative Knightian uncertainty, implying that the uncertainty is located in one or more specific parameters of the model, and the true values of these parameters are bounded between minimum and maximum plausible values (Giannoni 2002, 2007, Onatski and Stock 2002, Tetlow and von zur Muehlen 2004, Tetlow 2019). Numerical simulations show that under parameter uncertainty, the robust interest rate rule generally responds more strongly to changes in inflation and the output gap, with greater inertia than in the absence of such uncertainty, invalidating thus the Brainard attenuation principle. The CB is less cautious in the sense of Brainard (1967) but more cautious in the sense commonly used in the robust control literature by conducting a policy that is more aggressive towards inflation, as it is more averse to worst-case scenarios.

8 Conclusion

This paper studies an optimal robust monetary policy for a small open emerging economy using a stylized New Keynesian model with additive uncertainty à la Hansen and Sargent. Particularly, we focus on Mexico , being a representative small open emerging economy with a floating exchange rate, an inflation-targeting regime, and a dynamic exporting sector.

We analyze four different scenarios of model uncertainty: (1) uncertainty arising from the entire model with the same misspecification in all equations; (2) concerns about only the specification of the New Keynesian Phillips curve; (3) uncertainty only when modelling the IS curve; and (4) uncertainty arising from the uncovered interest rate parity (UIP).

Our findings suggest that conducting a global robust optimal monetary policy is limited since the departure from the benchmark model leads to multiple equilibria. Second, when model uncertainty arises only from the IS curve or the UIP condition, the space of unique solutions is expanded. Indeed, when the central bank has a preference for robustness on the IS curve only, it should be more aggressive to demand and real exchange rate shocks but more conservative to cost-push shocks. Conversely, when the central bank has a preference for robustness only for the UIP, it should be more aggressive to demand and cost-push shocks. As our results suggest that the ability of the central bank to implement is restricted by the Phillips curve, we explore the limits of a robust optimal monetary policy through a sensitivity analysis.

A larger effect of the output gap on inflation or a larger pass-through of exchange rate depreciation on inflation increases the possibility of being robust, while a lower inflation persistence or a lower weight of inflation expectations on inflation increases the ability to implement a robust monetary policy. In fact, our findings highlight the effects of inflation persistency and confirm the great importance of anchoring inflation expectations for the conduct of optimal policy. As we provide a benchmark model to analyze an optimal robust monetary policy for Mexico, further lines of research could develop more detailed DSGE models and employ different alternatives to account for model misspecification.

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A APPENDIX

A.1 Model appendix

This appendix derives the model used in this paper to analyze a robust optimal monetary policy for a small open emerging economy. For that purpose, we describe the optimization problem faced by the CB and derive the first order conditions.

A.1.1 Optimal Robust Monetary Policy

The model used in this paper is defined by the three main structural equations as in Sidaoui and Ramos-Francia (2008) but including additive misspecification. That is, an endogenous variable (misspecification) is added to the New Keynesian Phillips Curve (1), to the dynamic IS curve (2) and to the Uncovered Interest rate Parity (3). Additionally, non-core inflation dynamics are considered as an AR(1) process while headline inflation, the nominal exchange depreciation and the ex-ante real interest rate are defined. Therefore, the model is formed by the following equations.

$$\pi_t^c = a_1\pi_{t-1}^c + a_2 E_t\pi_{t+1}^c + a_3 x_t + a_4\left[\triangle e_t + \pi_t^{US}\right] + h_t^\pi + \varepsilon_t^\pi, \tag{A.1}$$

$$\pi_t^{nc} = \phi_1\pi_{t-1}^{nc} + \varepsilon_t^{\pi^{nc}},$$

$$\pi_t = \omega\pi_t^c + (1\text{-}\omega)\pi_t^{nc},$$

$$x_t = b_1 x_{t-1} + b_2 E_t x_{t+1} + b_3 r_{t-1} + b_4 x_t^{US} + b_5 q_t + h_t^x + \varepsilon_t^x, \tag{A.2}$$

$$q_t = c_1 q_{t-1} + c_2\left[E_t q_{t+1} + (r_t^{us} - r_t)\right] + h_t^q + \varepsilon_t^q, \tag{A.3}$$

$$r_t = i_t - E_t\pi_{t+1}, \tag{A.4}$$

$$\triangle e_t = \triangle q_t + \left(\pi_t - \pi_t^{US}\right). \tag{A.5}$$

To design the robust policy, the CB takes into account a certain degree of model misspecification by minimizing its objective function in the worst possible model within a given set of plausible models. Depending on its preference for robustness, the CB allocates a budget χ_j^2 to the malevolent agent, which is used to create misspecification every structural equation. The budget constraints are:

$$E_t \sum_{i=0}^{+\infty} \beta^t \left(h_{t+i}^j \right)^2 \leq \chi_j^2,\ j = \pi, x, q. \tag{A.6}$$

Following Hansen and Sargent (2003) the robust monetary policy is obtained by solving the minmax problem:

$$\min_{\pi_t, x_t, q_t, r_t} \max_{h_t^j} L_t^{CB} = \frac{1}{2} E_t \sum_{i=0}^{+\infty} \beta^i \left[(\pi_{t+i} - \pi^{target})^2 + \alpha x_{t+i}^2 \right], \tag{A.7}$$

subject to the misspecified model (A.1)—(A.3) and the malevolent agent's budget constraints (A.6). The CB sets the interest rate to minimize the value of its intertemporal loss function, while the malevolent agent sets its controls to maximize the CB's loss, given the constraints on misspecification. The Lagrangian for this problem is given by:

$$\begin{aligned}
L(.) \quad = \quad & \tfrac{1}{2} \mathbf{E_t} \sum_{i=0}^{+\infty} \beta^i (\pi_{t+i}^2 + \alpha x_{t+i}^2 - \theta^\pi (h_t^\pi)^2 - \theta^x (h_t^x)^2 - \theta^q (h_t^q)^2) \\
& -\lambda_{1,t} [\tfrac{1}{\omega} \pi_t - \tfrac{(1-\omega)}{\omega} \pi_t^{nc} - \tfrac{a_1}{\omega} \pi_{t-1} + \tfrac{a_1(1-\omega)}{\omega} \pi_{t-1}^{nc} - \tfrac{a_2}{\omega} E_t \pi_{t+1} \\
& + \tfrac{a_2(1-\omega)}{\omega} E_t \pi_{t+1}^{nc} - a_3 x_t - a_4 \quad q_t + a_4 q_{t-1} - a_4 \pi_t - h_t^\pi - \epsilon_t^\pi] \\
& -\lambda_{2,t} [x_t - b_1 x_{t-1} - b_2 E_t x_{t+1} - b_3 r_{t-1} - b_4 x_t^{US} - b_5 q_t - h_t^x - \epsilon_t^x] \\
& -\lambda_{3,t} [q_t - c_1 q_{t-1} - c_2 E_t q_{t+1} - c_2 r_t^{US} + c_2 r_t - h_t^q - \epsilon_t^q]\}
\end{aligned}$$

where $\lambda_{1,t}, \lambda_{2,t}$ and $\lambda_{3,t}$ are the Lagrange multipliers on the constraints. Therefore, the FOC's are the following:

$$
\begin{aligned}
\frac{\partial L(.)}{\partial \pi_t} = 0 &\Rightarrow \quad \pi_t - \lambda_{1,t}\frac{1}{\omega} + \beta\lambda_{1,t+1}\frac{a_1}{\omega} + \lambda_{1,t}a_4 = 0 \\
\frac{\partial L(.)}{\partial x_t} = 0 &\Rightarrow \quad \alpha x_t + \lambda_{1,t}a_3 - \lambda_{2,t} + \beta\lambda_{2,t+1}b_1 = 0 \\
\frac{\partial L(.)}{\partial q_t} = 0 &\Rightarrow \quad \lambda_{1,t}a_4 - \beta\lambda_{1,t+1}a_4 + \lambda_{2,t}b_5 - \lambda_{3,t} + \beta\lambda_{3,t+1}c_1 = 0 \\
\frac{\partial L(.)}{\partial r_t} = 0 &\Rightarrow \quad \beta\lambda_{2,t+1}b_3 - \lambda_{3,t}c_2 = 0 \\
\frac{\partial L(.)}{\partial h_t^{\pi}} = 0 &\Rightarrow \quad -\theta^x h_t^x + \lambda_{1,t} = 0 \\
\frac{\partial L(.)}{\partial h_t^{x}} = 0 &\Rightarrow \quad -\theta^\pi h_t^\pi + \lambda_{2,t} = 0 \\
\frac{\partial L(.)}{\partial h_t^{q}} = 0 &\Rightarrow \quad -\theta^q h_t^q + \lambda_{3,t} = 0
\end{aligned}
\tag{A.8}
$$

Combining the FOC's (A.8) and defining $\Theta = (a_4c_2 + a_3b_3)(1-\omega a_4) - c_2a_1(a_4 + a_3b_5)$, we derive the targeting rule:

$$\pi_t\frac{\omega(a_4+a_3b_5)c_2}{\Theta} - \beta E_t\pi_{t+1}\frac{\omega(a_4c_2+a_3b_3)}{\Theta} = -\alpha x_t\frac{b_5c_2(1-\omega a_4)}{\Theta} + \alpha\beta E_t x_{t+1}\left(\frac{a_1b_5c_2+b_3(1-\omega a_4)}{\Theta}\right) - \alpha\beta^2 E_t x_{t+2}\frac{a_1b_3}{\Theta}.$$

A.2 IRFs when the CB has preferences for robustness on output gap or on the real exchange rate

Following subsection 5.2., we present the two others IRFs when comparing each sort of CB's preference for robutsness: when there is no preference for robustness, or the CB has a preference for robustness in the IS equation or in the UIP condition.

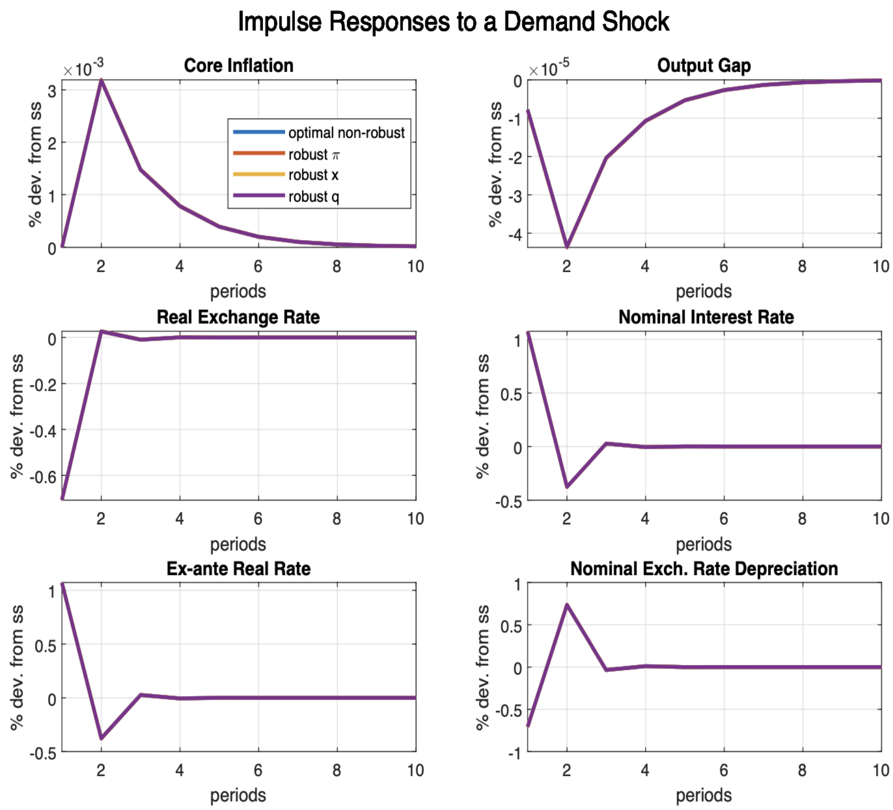

Figure 11: IRFs to an Output Gap shock comparing each preference for robustness

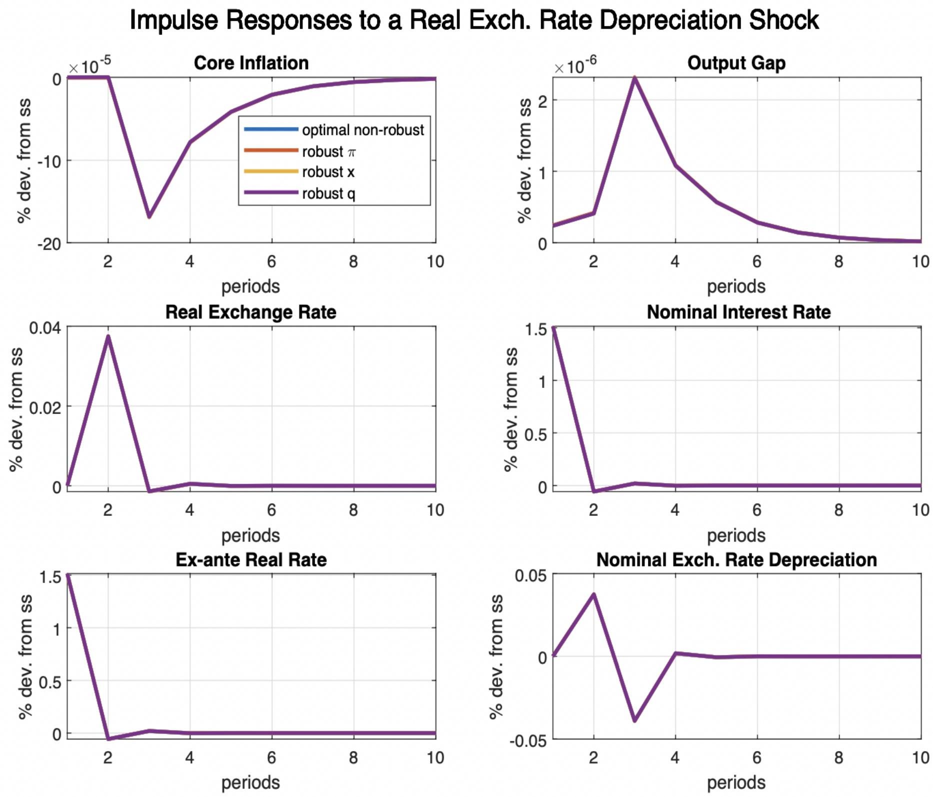

Figure 12: IRFs to a Real Exchange-Rate Depreciation shock comparing each preference for robustness