Nonlinear Unemployment Effects of the Inflation Tax

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The views expressed are those of the authors and do not reflect the official position of the Federal Reserve System or the Board of Governors.
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Recent policy discussions about raising the inflation target or adopting average inflation targeting to avoid the “ZLB”.

Negative level effects of inflation tax on output are well known.

Empirical literature on threshold effects of inflation on growth:

We explore the possibility of nonlinear and state-dependent effects of the inflation tax on unemployment, output and welfare.

We answer this question empirically and quantitatively.
What we do

We document three novel stylized facts about inflation and unemployment in OECD countries:

1. A positive long-run relationship between anticipated inflation and unemployment.
2. A positive correlation between anticipated inflation and unemployment volatility.
3. The long-run inflation-unemployment relationship is stronger when unemployment is higher.

We show that these correlations arise in a standard monetary search model with two shocks – productivity and monetary – and frictions in labor and goods markets.
What we do

- Inflation tax lowers the surplus from a worker-firm match, in turn making it sensitive to productivity shocks or to further increases in inflation.

- We calibrate the model to match the US postwar labor market and monetary data and show that it is consistent with observed cross-country correlations.

- The model implies that the welfare cost of inflation is nonlinear in the level of inflation and is amplified by the presence of aggregate shocks.
Related macro literature


Empirical evidence

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Data

- Quarterly panel data on 35 OECD countries (Main Economic Indicators database).

- Data on long-term nominal interest rates (10y government bonds) and unemployment rates (harmonized).

- We use the long-term nominal interest rate as a proxy for anticipated inflation.

- We focus on the trend (low frequency) component of each series (HP filter and 5y moving averages).
Stylized fact n° 1

- Is there a long-run relationship between unemployment and anticipated inflation?

- Regress trend unemployment on the trend long-term interest rate.

- Pooled OLS regression:

  \[ \bar{u}_{jt} = \alpha + \beta \bar{r}_{jt} + \varepsilon_{jt}, \]

- Fixed-effects panel regression:

  \[ \bar{u}_{jt} = \alpha + \beta \bar{r}_{jt} + \gamma_j + \delta_t + \varepsilon_{jt}, \]
### Stylized fact n° 1

<table>
<thead>
<tr>
<th></th>
<th>Trend unemployment (HP filter)</th>
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<tbody>
<tr>
<td></td>
<td>(1)</td>
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<td>(0.618)</td>
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<td>(1.366)</td>
<td>(1.209)</td>
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<td>0.301***</td>
<td>0.727**</td>
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<td>(0.091)</td>
<td>(0.062)</td>
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<td>4,007</td>
<td>4,007</td>
<td>4,007</td>
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<tr>
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<td>0.140</td>
<td>0.121</td>
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<td>Time fixed effects</td>
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*p<0.1; **p<0.05; ***p<0.01

*In log
### Stylized fact n° 1

**Trend unemployment (5y moving average)**

<table>
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<td></td>
<td>(0.597)</td>
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<td>(1.302)</td>
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*p<0.1; **p<0.05; ***p<0.01
Is there a relationship between anticipated inflation and unemployment volatility?

Regress unemployment volatility on trend long-term interest rate.

Unemployment volatility is measured as the standard deviation of cyclical log unemployment over a 5y moving window.

Pooled OLS regression:

\[ \sigma_{u_{jt}} = \alpha + \beta \bar{l}_{jt} + \varepsilon_{jt}, \]

Fixed effects panel regression:

\[ \sigma_{u_{jt}} = \alpha + \beta \bar{l}_{jt} + \gamma_j + \delta_t + \varepsilon_{jt}, \]
### Stylized fact n° 2

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<td>(0.002)</td>
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*p<0.1; **p<0.05; ***p<0.01
Stylized fact n° 2

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*p<0.1; **p<0.05; ***p<0.01
Stylized fact n° 3

- Does the long-run inflation-unemployment relationship vary in the level of unemployment?

- Regress trend unemployment on trend long-term interest rate for different quantiles of unemployment.

- Quantile regression approximates the conditional quantile function at quantile $q$ by a linear relationship.

- Pooled quantile regression:

$$ Q_q(\bar{u}_{jt} | \bar{l}_{jt}) = \alpha_q + \beta_q \bar{l}_{jt} + \varepsilon_{qjt}. $$
Stylized fact n° 3

Figure: Quantile regressions of $\bar{u}$ on $\bar{\iota}$ (HP filter).
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Model overview

- Standard monetary search model (Berentsen et al., 2011).
- Labor market frictions give rise to equilibrium unemployment (Pissarides, 2000).
- Goods market frictions generate a transaction demand for money (Lagos and Wright, 2005).
- Stochastic productivity $y_t$ and nominal interest rate $\nu_t$. 
Environment

- Discrete time. Infinitely lived agents. Discounting factor $\beta$.

- Unit measure of workers, either employed ($e$) or unemployed ($u$).

- Large number of firms with free entry.

- 3 sequential markets take place in each period:
  - Decentralized labor market (LM);
  - Decentralized goods markets (DM);
  - Centralized goods market (CM).

- Two perishable goods: CM good $y$ (numeraire) and DM good $x$. 
Environment

- Aggregate state: $\Omega_t = \{n_t, y_t, \iota_t\}$

- Productivity and monetary shocks are realized at the beginning of the CM.

- Fisher equation: $\iota_t = (1 + \pi_t) / \beta - 1$ where $\pi_t$ is inflation.

- Fiat money supply $M_t$ grows stochastically via lump-sum transfers $T(\iota_t)$ in the CM.
Preferences and technology

Worker preferences:

$$\sum_{t=0}^{\infty} \beta^t (u(x_t) + c_t)$$

where $c_t = \text{CM good}$, $x_t = \text{DM good}$.

Firm hires worker to produce quantity $y$ of CM goods.

Firm can produce $x$ units of DM goods on-demand at cost $C(x)$. 
Labor Market (LM)

- Random search and matching between vacancies and unemployed workers.

- LM tightness: \( \theta_t = \frac{v_t}{1 - n_t} \)

- Matching probabilities: \( f(\theta_t) = \theta_t q(\theta_t) \)

- Exogenous job separation at rate \( \delta \).

- Law of motion for employment:

\[
    n_{t+1} = (1 - \delta) n_t + f(\theta_t) (1 - n_t)
\]
Decentralized Goods Market (DM)

- Random matching between buyers/workers and firms.
- Buyers’ matching probability: $\alpha(n_t)$.
- Sellers’ matching probability: $\frac{\alpha(n_t)}{n_t}$.
- Informational frictions require the use of liquid assets for immediate settlement.
- Price setting: proportional bargaining (Kalai, 1977).
Centralized Goods and Settlement Market (CM)

- Frictionless Walrasian market.
- Firms liquidate inventories, pay wage $w_t$ and distribute profits.
- Households consume $x_t$ and decide on money holdings $z_{t+1}$.
- Central banks distributes lump-sum transfers $T(\nu_t)$. 
Timeline

LM
- Matching
  + wage bargaining
  + job separation
- Production

DM
- Matching
  + terms of trade bargaining
- Unsold inventories carried

CM
- Inventory liquidation
  + vacancy posting
  + money holdings
- Next period
Unemployment and inflation tax

- Inflation matters for firm-worker match surplus through the DM.

- An increase in anticipated inflation increases unemployment:
  \[
  \frac{dx}{dl} < 0 \Rightarrow \frac{d\theta}{dl} < 0 \Rightarrow \frac{du}{dl} > 0
  \]
Nonlinear inflation effects

What about the nonlinear effects of inflation?

Following Ljungqvist and Sargent (2017), we have:

\[
\varepsilon_{\theta,y} = \left(1 - \frac{1}{\epsilon_{\gamma,\theta} \mathcal{O} - b \mathcal{P} \varepsilon_{\mathcal{P},n\varepsilon_{n,\theta}}}\right)^{-1} \frac{1}{\epsilon_{\gamma,\theta} \mathcal{O} - b \mathcal{O}} y,
\]

and

\[
\varepsilon_{\theta,t} = \left(1 - \frac{1}{\epsilon_{\gamma,\theta} \mathcal{O} - b \mathcal{P} \varepsilon_{\mathcal{P},n\varepsilon_{n,\theta}}}\right)^{-1} \frac{1}{\epsilon_{\gamma,\theta} \mathcal{O} - b \mathcal{P}} \varepsilon_{\mathcal{P},t},
\]

Higher trend inflation amplifies unemployment responsiveness to both productivity and monetary shocks.

Feedback effects through goods market frictions.
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Calibration

- Model is set to a monthly frequency.

- We calibrate the model to match post-war US data (January 1948 to December 2019).

- We match both monthly and quarterly moments.

- Model is solved globally and calibrated using Simulated Method of Moments.
Calibration - Stochastic processes

- Labor productivity shock:
  \[ \log y_{t+1} = (1 - \rho_y) \log \bar{y} + \rho_y \log y_t + \varepsilon_{y,t+1} \]

- Nominal interest rate shock:
  - We decompose the shock into a trend and cycle components:
    \[ \nu_t = \bar{\nu}_t + \hat{\nu}_t \]
  - The cyclical component is modeled as stationary AR1 process:
    \[ \hat{\nu}_{t+1} = \rho_{\hat{\nu}} \hat{\nu}_t + \varepsilon_{\hat{\nu},t+1} \]
  - The trend component is modeled as a very persistent Markov chain with 5 states (transition probabilities estimated using ML).
Calibration and Numerical Results

Calibration - Functional forms

- LM matching function: \( f(\theta) = \theta q(\theta) = \frac{\theta}{(1 + \theta x)^{1/x}} \) (Den Haan et al., 2000).

- DM matching function: \( \alpha(n) = \zeta \frac{n}{n+1} \)

- DM utility: \( u(x) = A \frac{x^{1-\gamma}}{1-\gamma} \)

- DM cost: \( C(x) = x \)
## Calibration - External parameters

<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Source</th>
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</thead>
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<td>$\beta$</td>
<td>Discount factor</td>
<td>0.998</td>
<td>Data</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Job separation probability</td>
<td>0.025</td>
<td>Data</td>
</tr>
<tr>
<td>$\bar{y}$</td>
<td>Average labor productivity</td>
<td>1.00</td>
<td>Normalization</td>
</tr>
<tr>
<td>$\rho_{\hat{\varepsilon}}$</td>
<td>Autocorr. of interest rate shocks</td>
<td>0.939</td>
<td>Data</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon_{\hat{\varepsilon}}}$</td>
<td>SD of interest rate shocks</td>
<td>0.0001</td>
<td>Data</td>
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</table>
Calibration - Simulated Method of Moments

- Vector of 10 parameters $\Theta$.

- Vectors of 10 moments in the data $\mu$ and model $\mu_s(\Theta)$.
  - Model moments averaged over $S = 1'000$ simulations of length $T = 1'000$.
  - Burn first 133 observations to match length of data (867 months).

- Minimize the distance $G(\Theta) = \mu - \frac{1}{S} \sum_{s=1}^{S} \mu_s(\Theta)$:

$$\hat{\Theta} = \arg \min_{\Theta} G(\Theta)^T W^{-1} G(\Theta)$$

where $W$ is a weighting matrix.
Calibration - Data

- **Labor market data:**
  - Unemployment rate: CPS, civilian population under 16.
  - Job vacancy rate: Barnichon data and JOLTS.
  - Job separation rate: constructed using short-term unemployment.
  - Job finding rate: constructed using short-term unemployment.
  - Labor productivity: BLS non-farm real output per person.
  - Real wage: labor productivity $\times$ BLS labor income share.

- **Monetary data:**
  - Monetary aggregate: M1+MMDA (Rasche, 1987; Lucas and Nicolini, 2015).
  - Interest rate: Moody’s AAA long-term corporate bond index.
  - Nominal GDP.
  - CPI inflation.
  - Markup: data from De Loecker et al. (2020).
Figure: Measuring money demand: M1 v. M1+MMDA
## Calibration - Results

### Table: SMM calibrated parameters

<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Moment</th>
<th>Frequency</th>
<th>Data</th>
<th>Model</th>
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<td>$\kappa$</td>
<td>Vacancy cost</td>
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<td>Average $\theta$</td>
<td>Monthly</td>
<td>0.634</td>
<td>0.634</td>
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<td>$b$</td>
<td>Flow value of unemployment</td>
<td>0.990</td>
<td>Unemployment volatility</td>
<td>Quarterly</td>
<td>0.138</td>
<td>0.138</td>
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<td>$\chi$</td>
<td>Parameter of the LM matching fun.</td>
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<td>Average JFP</td>
<td>Monthly</td>
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<td>0.430</td>
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<td>$\xi$</td>
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<td>Elast. of wage to labor prod.</td>
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<td>0.470</td>
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<td>$\rho_y$</td>
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<td>Autocorr. of labor productivity</td>
<td>Quarterly</td>
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<td>0.760</td>
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<td>$\sigma_y$</td>
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<td>SD of labor productivity</td>
<td>Quarterly</td>
<td>0.013</td>
<td>0.013</td>
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<td>$A$</td>
<td>Level parameter of DM utility</td>
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<td>Average money demand</td>
<td>Quarterly</td>
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<td>Average price markup</td>
<td>Monthly</td>
<td>36.00%</td>
<td>36.00%</td>
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Policy Functions

(a) $\theta(\Omega)$ in $(\bar{\ell}, y)$ space

(b) $q(\Omega)$ in $(\bar{\ell}, u)$ space

Figure: Policy functions of the calibrated model.
Steady State Elasticities

(a) Elasticity of $\theta$ wrt. $y$, $\varepsilon_{\theta,y}$

(b) Semi-elasticity of $\theta$ wrt. $\nu$, $\varepsilon_{\theta,\nu}$

Figure: Steady state elasticities of $\theta$ in the calibrated model.
## Business Cycle Statistics

<table>
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<tr>
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<th>$v$</th>
<th>$\theta$</th>
<th>$\phi$</th>
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<td><strong>Quarterly US data, 1948-2019</strong></td>
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<td>Standard deviation</td>
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<td>Correlation matrix</td>
<td></td>
<td>$\theta$</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$\phi$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Model simulations</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.137</td>
<td></td>
<td>0.740</td>
<td>0.013</td>
</tr>
<tr>
<td>Autocorrelation</td>
<td>0.843</td>
<td>0.431</td>
<td>0.636</td>
<td>0.760</td>
</tr>
<tr>
<td></td>
<td>$u$</td>
<td></td>
<td>-0.559</td>
<td>-0.792</td>
</tr>
<tr>
<td></td>
<td>$v$</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Correlation matrix</td>
<td></td>
<td>$\theta$</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>$\phi$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure: Linear and Quantile regressions of trend $u$ on $v$ using simulated data
### Unemployment Volatility Regression

<table>
<thead>
<tr>
<th></th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant</strong></td>
<td>0.031***</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td><strong>Trend long-term rate (HP filter)</strong></td>
<td>0.013***</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>269,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>$R^2$</strong></td>
<td>0.182</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* $p < 0.1$; ** $p < 0.05$; *** $p < 0.01$
State-dependent reaction to shocks.

Generalized impulse response function (Koop et al., 1996):

\[ \text{GIRF}_Y(k, \varepsilon_t, \Omega_t) = \mathbb{E}[Y_{t+k} | \varepsilon_t, \Omega_t = \omega_t] - \mathbb{E}[Y_{t+k} | \Omega_t = \omega_t], \]

where \( \omega_t \) is the state of economy at the beginning of period \( t \).
Generalized Impulse Response Functions

Figure: GIRFs following a negative productivity shock
Generalized Impulse Response Functions

(a) Unemployment

(b) LM tightness $\theta$

(c) DM consumption $x$

(d) Total output

Figure: Average GIRFs following a negative productivity shock conditional on trend inflation.
Welfare Cost of Inflation

1. Simulate the model with cyclical shocks under different levels of trend inflation.

2. Compute average welfare for each trend inflation level:

\[ \mathcal{W}(\Omega_t) = \alpha(n_t)[u(x_t) - c(x_t)] + n_t y_t + (1 - n_t)b - \kappa v_t/\beta. \]

<table>
<thead>
<tr>
<th>Annual inflation rate</th>
<th>Implied interest rate</th>
<th>Flow welfare level</th>
<th>Difference with FR</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.75%</td>
<td>0.00%</td>
<td>1.084</td>
<td>-</td>
</tr>
<tr>
<td>0.00%</td>
<td>2.82%</td>
<td>1.080</td>
<td>-0.37%</td>
</tr>
<tr>
<td>5.00%</td>
<td>7.97%</td>
<td>1.061</td>
<td>-2.13%</td>
</tr>
<tr>
<td>10.00%</td>
<td>13.11%</td>
<td>1.035</td>
<td>-4.52%</td>
</tr>
</tbody>
</table>

Table: Welfare cost of inflation in baseline economy
Welfare Cost of Inflation

Figure: Nonlinearity of the welfare cost of trend inflation.
Figure: Contribution of aggregate uncertainty to the cost of inflation.
Evidence of a positive and nonlinear long-run relationship between anticipated inflation and unemployment.

A standard monetary search model with productivity and interest rate shocks can replicate these facts.

The nonlinear unemployment effects amplify substantially the welfare cost of inflation.

The business cycle is not invariant to the long-run inflation target.
### Stylized fact n° 1

<table>
<thead>
<tr>
<th></th>
<th>Trend log unemployment (HP filter)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.707***</td>
<td>1.755***</td>
<td>1.555***</td>
<td>1.705***</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.050)</td>
<td>(0.175)</td>
<td>(0.114)</td>
</tr>
<tr>
<td>Trend long-term rate (HP filter)</td>
<td>0.039***</td>
<td>0.031***</td>
<td>0.067**</td>
<td>0.039*</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.009)</td>
<td>(0.033)</td>
<td>(0.021)</td>
</tr>
<tr>
<td>Observations</td>
<td>4,007</td>
<td>4,007</td>
<td>4,007</td>
<td>4,007</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.072</td>
<td>0.090</td>
<td>0.072</td>
<td>0.024</td>
</tr>
<tr>
<td>F-Statistic</td>
<td>312.93***</td>
<td>395.14***</td>
<td>291.58***</td>
<td>92.49***</td>
</tr>
<tr>
<td>Country fixed effects</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Time fixed effects</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Clustered errors (country level)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*p<0.1; **p<0.05; ***p<0.01
## Stylized fact n° 1

<table>
<thead>
<tr>
<th></th>
<th>Trend log unemployment (5y moving average)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Constant</td>
<td>1.728***</td>
</tr>
<tr>
<td></td>
<td>(0.082)</td>
</tr>
<tr>
<td>Trend long-term rate</td>
<td>0.041***</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
</tr>
<tr>
<td>Observations</td>
<td>3,262</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.075</td>
</tr>
<tr>
<td>F-Statistic</td>
<td>263.02***</td>
</tr>
<tr>
<td>Country fixed effects</td>
<td>No</td>
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<tr>
<td>Time fixed effects</td>
<td>No</td>
</tr>
<tr>
<td>Clustered errors (country level)</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*p<0.1; **p<0.05; ***p<0.01
Stylized fact n°2

**Figure:** Quantile regression of $u$ on $i$ (5y average)
Stylized fact n°2

Figure: Quantile regression of log $u$ on $i$ (HP filter)
Stylized fact n°2

**Figure:** Quantile regression of log $u$ on $i$ (5y average)
### Stylized fact n° 3

**Unemployment volatility (5y moving window SD)**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.390***</td>
<td>0.354***</td>
<td>0.222*</td>
<td>-0.024</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.053)</td>
<td>(0.122)</td>
<td>(0.166)</td>
</tr>
<tr>
<td>Trend long-term rate (HP filter)</td>
<td>0.046***</td>
<td>0.053***</td>
<td>0.079***</td>
<td>0.128***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.010)</td>
<td>(0.028)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>Observations</td>
<td>3,616</td>
<td>3,616</td>
<td>3,616</td>
<td>3,616</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.077</td>
<td>0.132</td>
<td>0.090</td>
<td>0.135</td>
</tr>
<tr>
<td>F-Statistic</td>
<td>301.46***</td>
<td>544.71***</td>
<td>333.41***</td>
<td>519.35***</td>
</tr>
<tr>
<td>Country fixed effects</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Time fixed effects</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Clustered errors (country level)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*p<0.1; **p<0.05; ***p<0.01
## Stylized fact n° 3

### Unemployment volatility (5y moving window SD)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.588***</td>
<td>0.523***</td>
<td>-0.234</td>
<td>-0.740**</td>
</tr>
<tr>
<td></td>
<td>(0.142)</td>
<td>(0.129)</td>
<td>(0.288)</td>
<td>(0.357)</td>
</tr>
<tr>
<td>Trend long-term rate</td>
<td>0.098***</td>
<td>0.110***</td>
<td>0.256***</td>
<td>0.354***</td>
</tr>
<tr>
<td>(5y average)</td>
<td>(0.031)</td>
<td>(0.025)</td>
<td>(0.065)</td>
<td>(0.069)</td>
</tr>
<tr>
<td>Observations</td>
<td>2,882</td>
<td>2,882</td>
<td>2,882</td>
<td>2,882</td>
</tr>
<tr>
<td>( R^2 )</td>
<td>0.079</td>
<td>0.139</td>
<td>0.196</td>
<td>0.216</td>
</tr>
<tr>
<td>F-Statistic</td>
<td>248.16***</td>
<td>460.15***</td>
<td>650.05***</td>
<td>721.46***</td>
</tr>
<tr>
<td>Country fixed effects</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Time fixed effects</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Clustered errors (country level)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

*p < 0.1; **p < 0.05; ***p < 0.01
LM - Workers

- Employed worker with liquid assets $z$:

$$V_{LM}^e (z_t, \Omega_t) = (1 - \delta) \ V_{DM}^e (z_t, w_t, \Omega_t) + \delta \ V_{DM}^u (z_t, \Omega_t)$$

- Unemployed worker with liquid assets $z$:

$$V_{LM}^u (z_t, \Omega_t) = f(\theta_t) \ V_{DM}^e (z_t, w_t, \Omega_t) + (1 - f(\theta)) \ V_{DM}^u (z_t, \Omega_t)$$
Firm with a worker:

\[ J_{LM}^e (\Omega_t) = (1 - \delta) J_{DM}^e (w_t, \Omega_t) \]

Firm without a worker:

\[ J_{LM}^v (\Omega_t) = q (\theta_t) J_{DM}^e (w_t, \Omega_t) \]
DM - Workers

- Employed worker with liquid assets $z_t$:

\[
V_{DM}^e (z_t, w_t, \Omega_t) = \alpha (n_{t+1}) \left[ u (x(z_t)) + \mathbb{E} V_{CM}^e (z_t - d(z_t) + T(\nu_t) + w_t, \Omega_{t+1}) \right]
\]

- Unemployed worker with liquid assets $z_t$:

\[
V_{DM}^u (z_t, \Omega_t) = \alpha (n_{t+1}) \left[ u (x(z_t)) + \mathbb{E} V_{CM}^u (z_t - d(z_t) + T(\nu_t) + b, \Omega_{t+1}) \right]
\]
Firm with a worker produces and sells its output, getting

\[
J_{DM}^e(w_t, \Omega_t) = \frac{\alpha(n_{t+1})}{n_{t+1}} E J_{CM}^e (y_t - C(x_t; y_t) + d_t, w_t, \Omega_{t+1}) + \left(1 - \frac{\alpha(n_{t+1})}{n_{t+1}}\right) E J_{CM}^e (y_t, w_t, \Omega_{t+1})
\]
CM - Workers

- Worker with employment status $j \in \{e, u\}$ and liquid assets $z$:

$$V^j_{CM} (z_t, \Omega_{t+1}) = \max_{c_t, z_{t+1}} c_t + \beta V^j_{LM} (z_{t+1}, \Omega_{t+1})$$

subject to

$$c_t + (1 + \pi_t) z_{t+1} = z_t$$
CM - Firms

- Firm with a worker sells its inventories $o$ and pays the wage $w$:
  \[ J_{CM}^e (o_t, w_t, \Omega_{t+1}) = o_t - w_t + \beta J_{LM}^e (\Omega_{t+1}) \]

- A firm without a worker decides whether to post a vacancy at cost $\kappa$:
  \[ J_{CM}^v (\Omega_{t+1}) = \max \{ 0, -\kappa + \beta J_{LM}^v (\Omega_{t+1}) \} \]
**DM bargaining**

- Kalai (1977) bargaining solution:

\[
\max_{x_t, d_t} u(x_t) - d_t
\]

subject to

\[
u(x_t) - d_t = \frac{\varphi}{1 - \varphi} \left[ d_t - C(x_t; y_t) \right]; \quad d_t \leq z_t
\]

- Solution is a pair \((x_t, d_t)\) that satisfies

\[
x_t = \min \left\{ x^*(y_t), g^{-1}(z_t; y_t) \right\}
\]

\[
d_t = \min \left\{ g(x^*(y_t); y_t), z_t \right\}
\]

where

\[
g(x_t; y_t) = (1 - \varphi) u(x_t) + \varphi C(x_t; y_t)
\]

and \(x^*(y_t)\) solves

\[
u'(x_t) - C_x(x_t; y_t) = 0
\]
Optimal choice of real balances

Given the bargaining solution we have

\[ \frac{\partial V_{LM}^j}{\partial z_t} = 1 + \alpha (n_{t+1}) \max \left\{ 0, \frac{u'(x_t)}{g'(x_t; y_t)} - 1 \right\} \]

In the CM, the first-order condition for \( z \) is

\[ 1 + \nu_t = \frac{\partial V_{LM}^j}{\partial z_{t+1}} \]

Combining the above, we get

\[ u'(x_t) = \left( 1 + \frac{\nu_t}{\alpha (n_{t+1})} \right) g'(x_t; y_t) \]

and \( z_t = g(x_t; y_t) \).
LM bargaining

- Worker’s surplus from being employed at wage $w$ is

$$S^e_{DM}(w_t, \Omega_t) \equiv V^e_{DM}(z_t, w_t, \Omega_t) - V^u_{DM}(z_t, \Omega_t) = w_t - b + \beta \mathbb{E} (1 - \delta - f(\theta (\Omega_{t+1}))) S^e_{DM}(w_{t+1}, \Omega_{t+1})$$

- The firm’s surplus from having a worker at wage $w$ is

$$J^e_{DM}(w_t, \Omega_t) = \mathcal{O}(\Omega_t) - w_t + \beta (1 - \delta) \mathbb{E} J^e_{DM}(w_{t+1}, \Omega_{t+1})$$

where $\mathcal{O}(\Omega_t) = y_t + \frac{\alpha(n_{t+1})}{n_{t+1}} (d_t - C(x_t; y_t))$
LM bargaining

- The surplus from an employment match is
  \[ S(\Omega_t) = S_{DM}^e(w_t, \Omega_t) + J_{DM}^e(w_t, \Omega_t) \]

- Wage \( w_t = w(\Omega_t) \) is determined by Nash bargaining such that
  \[ S_{DM}^e(w_t, \Omega_t) = \xi S(\Omega_t) \]

- Wage equation:
  \[ w(\Omega_t) = \xi \Omega(\Omega_t) + (1 - \xi) b + E \xi \kappa \theta(\Omega_{t+1}) \]
Job surplus and free entry

- Recursive formulation:

\[ S(\Omega_t) = O(\Omega_t) - b + \beta \mathbb{E}(1 - \delta - \xi f(\theta(\Omega_{t+1}))) S(\Omega_{t+1}) \]

- \( \theta_t = \theta(\Omega_t) \) is determined by the free entry condition

\[ \kappa = \beta q(\theta_t)(1 - \xi) S(\Omega_t) \]
The equilibrium consists of functions $x(\Omega), O(\Omega), S(\Omega), \theta(\Omega), w(\Omega), n_{t+1}(\Omega)$ such that:

1. $x(\Omega)$ solves the optimal choice of real balances.
2. Output $O(\Omega)$ is given by DM bargaining solution.
3. Surplus from a job match $S(\Omega)$ satisfies its Bellman equation.
4. Free entry condition determines $\theta(\Omega)$.
5. The wage $w(\Omega)$ satisfies LM bargaining solution.
6. Employment $n_{t+1}(\Omega)$ is given by its law of motion.