Rational Sentiments and Financial Frictions

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Models with financial frictions: shortcomings

### Large macro literature featuring financial frictions


### Problem 1: reproducing the severity and suddenness of financial crises

⇒ Add systemic bank runs

Gertler and Kiyotaki [2015]; Gertler et al. [2020]; Mendo [2020]

### Problem 2: generate booms that are prone to bust

⇒ Add non-rational beliefs

Krishnamurthy and Li [2020]; Maxted [2020]

### This paper:

- this class of economies has unstudied equilibria (sunspot equilibria)
- sunspots help alleviate issues with these models, e.g., Problems 1&2
Model
A very common macro-finance setting

- All agents have log utility over consumption.

- Production is linear in capital, with *experts* more productive than *households* \((a_e > a_h)\).

- Capital is freely traded at price \(q_t\) and grows evolves as

  \[
  \frac{dK_t}{K_t} = g dt + \sigma dZ_t^{(1)}
  \]

  \(dZ_t^{(1)}\) fundamental shock

- **Financial friction:** producers cannot issue equity, but can borrow/lend freely in riskless bonds at rate \(r_t\).
  - no credit constraints
  - all results generalize to partial but limited equity issuance

- **Information structure:** extrinsic uncertainty \(dZ^{(2)}\)
Capital price and return

Capital price $q$

\[
\frac{dq_t}{q_t} = \mu_{q,t} dt + \sigma_{q,t}^{(1)} dZ_t^{(1)} + \sigma_{q,t}^{(2)} dZ_t^{(2)}
\]

amplification of fundamentals

sunspot fluctuations

Volatility of capital returns $|\sigma_R|^2$

\[
\sigma_{R,t} := \sigma \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \sigma_{q,t}
\]
Equilibrium

- **Price-output relation:**
  \[ \rho q = a_e \kappa + a_h (1 - \kappa) \]  
  (from goods market)
  where \( \kappa \) is experts’ capital share.

- **Risk-balance condition:**
  \[ \frac{a_e - a_h}{q} = \frac{\kappa - \eta}{\eta(1 - \eta)} |\sigma_R|^2 \]  
  (optimal portfolios when \( \kappa < 1 \))
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- **Risk premium:**
  \[ \mu_q - r + \sigma \sigma_q \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -(\rho + g) + \left( \frac{\kappa^2}{\eta} + \frac{(1 - \kappa)^2}{1 - \eta} \right) |\sigma_R|^2 \]

- **Wealth share dynamics:**
  \[ d\eta_t = \mu_{\eta,t} dt + \sigma_{\eta,t} \cdot dZ_t \]  
  given \( \eta_0 \)
  \[ \mu_\eta = \mu_\eta(\eta, \kappa, |\sigma_R|^2), \quad \sigma_\eta = (\kappa - \eta)\sigma_R \]
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**Equilibrium:** Given \( \eta_0 \in (0, 1) \), an *equilibrium* consists of processes \((\eta_t, q_t, \kappa_t, r_t)_{t \geq 0}\) such that equations above hold for all \( t \geq 0 \).
Types of equilibria

\[
\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \sigma_q \equiv 0
\]

Markov in \( \eta \)

\[
\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \sigma_q \neq 0
\]

Non-Markov in \( \eta \)

Sentiment-driven BSE (S-BSE)

Fundamental Equilibrium (FE)

Wealth-driven BSE (W-BSE)
Types of equilibria

Usual solution path: imposing a Markov solution in $\eta$ (i.e., $q = q(\eta)$)

- Extra conditions: $dq$ consistent with $d\eta$ (Ito’s Lemma)

$$q\sigma_q = q'\sigma_\eta, \quad q\mu_q = q'\mu_\eta + 0.5q^{''}|\sigma_\eta|^2$$
Types of equilibria

![Diagram showing types of equilibria]

- **Markov in $\eta$**
  - Fundamental Equilibrium (FE)
  - Wealth-driven BSE (W-BSE)

- **Non-Markov in $\eta$**
  - Sentiment-driven BSE (S-BSE)

**Usual solution path:** imposing a Markov solution in $\eta$ (i.e., $q = q(\eta)$)

- **FE:** widely studied (e.g., Brunnermeier and Sannikov [2016])
- **W-BSE:** inconsistent w/ fundamental shocks ($\sigma > 0$
  - w/o fundamental shocks ($\sigma = 0$), there exist a W-BSE but it strongly resembles a FE with small $\sigma$.

$\Rightarrow$ No interesting new dynamics if equilibrium is Markov in $\eta$!
Fundamental equilibrium and W-BSE

$q$ from W-BSE (with $\sigma = 0$)

$\kappa = 1$ border

$q$ from FE (with $\sigma = 0.1$)

$max_{\eta} |\sigma_R(\eta)| = 0.198$
Beyond wealth: sentiment-driven BSE (S-BSE)
Beyond wealth: sentiment-driven BSE (S-BSE)

Theorem (Existence of S-BSEs):
Under mild parametric restrictions, there exists an S-BSE in which \((\eta_t, q_t)_{t \geq 0}\) remains in \(\mathcal{D} := \{(\eta, q) : 0 < \eta < 1 \text{ and } \eta a_e + (1 - \eta) a_h < q \bar{\rho}(\eta) \leq a_e\}\) almost-surely and possesses a non-degenerate stationary distribution.
Static indeterminacy mechanism

**Price-output:**  \( \rho q = a_e \kappa + a_h (1 - \kappa) \)

**Risk-balance:**  \( \frac{a_e - a_h}{q} = \frac{\kappa - \eta}{\eta(1 - \eta)} |\sigma_R|^2 \)
Static indeterminacy mechanism

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Dynamic stability mechanism

- Static indeterminacy is compatible with equilibrium only if it does not lead to violations of equilibrium conditions in the future (i.e., \((\eta_t, q_t)_{t \geq 0}\) remain in triangle \(\mathcal{D}\)).

- Only the risk premium is pinned down, not \(\mu_q\) and \(r\) separately,

\[
\mu_q - r + \sigma \sigma_q \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = - (\rho + g) + \left( \frac{\kappa^2}{\eta} + \frac{(1 - \kappa)^2}{1 - \eta} \right) |\sigma_R|^2
\]

Hence, we use the degree of freedom to choose \(\mu_q\) to ensure stochastic stability.

- Choice of \(\mu_q\) is straightforward. For example, \(\mu_q \to \infty\) if \(q\) falls too low, and \(\mu_q \to -\infty\) if \(q\) rises too high.

- Stability requirements translate to boundary conditions.
Sentiment-driven BSE (S-BSE)
Two indeterminacies in S-BSEs

**Corollary (Decoupling)**
The economy can be arbitrarily coupled or decoupled from fundamentals in the following sense. Let $\gamma(\eta, q) \in [0, 1]$ be any $C^1$ function. An equilibrium exists such that when $\kappa < 1$, a fraction $\gamma(\eta, q)$ of return variance $|\sigma_R|^2$ is due to the fundamental shock.

**Corollary (Drift indeterminacy)**
The economy can feature any degree of persistence or transience in the following sense. Let $m(\eta, q)$ be any $C^1$ function. An equilibrium exists with $P[\mu_{q,t} = m(\eta_t, q_t) | \kappa_t < 1]$ arbitrarily close to one. Furthermore, the inefficiency probability $P[\kappa_t < 1]$ can take any value between zero and one.
Resolving puzzles with sentiment
Explicit construction with sentiment variable

- Let $s_t$ be a pure sunspot that is irrelevant to economic fundamentals and loads on only the second shock
  \[ ds_t = \mu_{s,t} dt + \sigma_{s,t} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot dZ_t, \quad s_t \in S \]

- Auxiliary sunspot state variable $x_t \in X$ that may only affect the drift $\mu_{s,t}$ (flexibility due to indeterminacy corollary).

**Definition** A Markov S-BSE in states $(\eta, s, x) \in (0, 1) \times S \times X$ consists of functions $(q, \kappa, r, \sigma_\eta, \mu_\eta, \sigma_s) : (0, 1) \times S \mapsto \mathbb{R}$, and $\mu_s : (0, 1) \times S \times X \mapsto \mathbb{R}$ such that the process $(\eta_t, q(\eta_t, s_t), \kappa(\eta_t, s_t), r(\eta_t, s_t))_{t \geq 0}$ is a S-BSE.

- We allow $(\mu_s, \sigma_s)$ to depend on $\eta$.
  - Why? It’s sensible to use asset prices directly in forecasting.
  - Novel construction: fix $q(\eta, s)$, recover the $\sigma_s$ process that justifies it, then set $\mu_s$ to ensure stability.
Example equilibrium construction

- Fundamental equilibrium with $\sigma > 0$
- Terrible equilibrium where $\kappa \approx \eta$

Price-volatility relation
Example equilibrium construction

Fundamental equilibrium with $\sigma > 0$
Example equilibrium construction

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Example equilibrium construction

Terrible equilibrium where $\kappa \approx \eta$

Fundamental equilibrium with $\sigma > 0$

Price-volatility relation
The IRFs labeled \( \eta \) shock are responses to a decrease in \( \eta \) from \( \eta_0^- = 0.5 \) to \( \eta_0^+ = 0.2 \), holding \( s_0 \) fixed at 0.1.

The IRFs labeled \( s \) shock are responses to an increase in \( s \) from \( s_0^- = 0.1 \) to \( s_0^+ = 0.9 \), holding \( \eta_0 \) fixed at 0.5.

These shock sizes are chosen such that the initial response of \( q \) are approximately equal.
Proposition (Arbitrary volatility)
Given a target variance $\Sigma^* > 0$ and under mild parameter restrictions, there exists a Markov S-BSE with stationary average return variance exceeding the target, i.e., $\mathbb{E}[|\sigma_R|^2] > \Sigma^*$.

Proposition (Volatility decoupling)
In the Markov S-BSEs constructed both the fraction of return volatility due to sentiments $|\begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \sigma_R|/|\sigma_R|$ and total return volatility $|\sigma_R|$ increase with $s$. 
Booms predict crises

- Following some models of extrapolative beliefs [Barberis et al., 2015, Maxted, 2020], define an exponentially-declining weighted average of sentiment shocks:

\[ x_t := x_0 + \sigma_x \int_0^t e^{-\beta x(t-u)} dZ_u^{(2)}. \]

Assume the drift of \( s \) depends on \( x \) via

\[ \mu_{s,t} = b_x x_t + \hat{\mu}_s(s_t) \quad \text{with} \quad b_x \leq 0. \]

the term \( \hat{\mu}_s \) is designed to prevent non-stationarity in \( s_t \).

- After a series of good sentiment shocks \( (dZ_t^{(2)} < 0) \), \( s_t \) and \( x_t \) will be low (boom times), but this buoys \( \mu_{s,t} \) and shifts conditional distributions of \( s_{t+h} \) to the right (future busts).
Booms predict crises

- The IRFs labeled “η shock” are responses to an increase in η from \( \eta_0^- = 0.5 \) to \( \eta_0 = 0.7 \), holding \( s_0 \) fixed at 0.4.
- The IRFs labeled “s shock” are responses to a decrease in \( s \) from \( s_0^- = 0.4 \) to \( s_0 = 0.1 \), holding \( \eta_0 \) fixed at 0.5.
- These shock sizes are chosen such that the initial response of \( q \) are approximately equal.
Behavior around financial crises

- Crises are defined as the bottom 3rd percentile of month-to-month log output declines.
- Conditions are improving up to 2 years before the crisis, with risk premia below average and *declining*.
- The crisis emerges suddenly and features spikes in all variables.
- These dynamics cannot be produced in the non-sunspot equilibria of the model.
Sentiment-based jumps

- Consider a broader class of solutions for the baseline model where capital price can also respond to an extrinsic jump shock, i.e.,

\[
\frac{dq_t}{q_{t-}} = \mu_{q,t-} dt + \sigma_{q,t-} \cdot dZ_t - \ell_{q,t-} dJ_t,
\]

where \( J \) is a Poisson process with intensity \( \lambda \).

- The risk-balance condition

\[
\frac{a_e - a_h}{q} = \frac{\kappa - \eta}{\eta(1 - \eta)} \left( |\sigma_R|^2 + \frac{\lambda \ell_q^2}{(1 - \frac{\kappa}{\eta} \ell_q)(1 - \frac{1 - \kappa}{1 - \eta} \ell_q)} \right)
\]

disciplines overall risk but not the split between Brownian and Poisson shocks. Additional degree of freedom.

- Chosen jump sizes for exercise

\[
\ell_q = \begin{cases} 
0.95 \ell_q^{\text{max}}, & \text{if } \kappa > 0.9 \text{ and } 0.9 \ell_q^{\text{max}} > 0.2 \\
0, & \text{otherwise},
\end{cases}
\]
**Sentiment-based jumps: behavior around crises**

- Crises: bottom 3rd percentile of month-to-month log output declines.
- Crises tend to arrive after a sequence of positive fundamental shocks.
- In the years before the crisis, asset prices are high, and both volatility and risk premia are below their usual level.
- Crises arrive suddenly—with only a few months “warning” in terms of rising volatility and risk premia—and generate large movements in observables, because simulated crises often coincide with realizations of a jump.
Conclusion

- Macroeconomic models with financial frictions inherently permit sunspot volatility. These models are extremely common, so this phenomenon cannot be ignored.

- Fully-rational notion of “sentiments” can be a powerful input into macro-finance dynamics. Unbounded amplification, sharp volatility spikes, and sentiment-driven boom-bust cycles are among the possibilities.

- Our results suggest a modicum of caution. Numerical techniques used to solve DSGE models with financial frictions implicitly select an equilibrium, without any explicit justification. A deeper analysis of refinements still remains to be done.

- **Policy?**
  - Deposit insurance less effective, because run-like behavior can be an asset-side phenomenon.
  - Capital requirements, bailouts, etc, are likely less effective when volatility is decoupled from balance sheets.