Securitization in the Mortgage Market under General Equilibrium

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XXVI Meeting of the Central Bank Research Network
CEMLA

Views presented are of the author and do not necessarily represent those of the Bank of Spain and the Eurosystem.

November 9, 2021
Motivation

1. Dynamics of mortgage lending closely tied to securitization.
   • US credit cycle of 2000’s partly fueled by securitization.

2. Securitization: large source of liquidity to mortgage originators.
   • Large fraction of mortgage originators are liquidity constrained.

3. Evidence of information frictions along mortgage origination and securitization chain.
   • Private Segment of securitization market collapsed in 2008.

Yet, there is not much quantification of

• equilibrium connection between securitization and mortgage credit.
• aggregate effects of information frictions in this market.

→ This paper
What I do

Develop a quantitative GE model of financial intermediation.

• Endogenous securitization market.
• Main friction: private information (adverse selection).
• Exogenous shocks: borrower’s income and housing depreciation.

Quantify the role of information frictions during the Great Recession (GR).

Evaluate policy changes introduced after GR.

• Expansion of insurance on securities
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**Quantify** the role of information frictions during the Great Recession (GR).

**Evaluate** policy changes introduced after GR.
- Expansion of insurance on securities
Results

1. Model replicates 2/3 the dynamics of mortgage lending and securities issuance during the GR.

2. Information frictions account for 27% of contraction in mortgage lending
   - Elements: (i) Info frictions, (ii) high exposure to securitization, (iii) high concentration among mortgage originators.

3. Expanding insurance on securities can be welfare improving
   - $\Delta - \text{volatility}$ of mortgage lending and mortgage rate.
   - $\Delta + \text{borrower's default rate}$.
   - $\Delta + \text{cost of financing the policy by about 2 times}$.
   - Small welfare gains to borrowers, larger welfare gains for lenders.
Results

1. Model replicates $\frac{2}{3}$ the dynamics of mortgage lending and securities issuance during the GR.

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   - Elements: (i) Info frictions, (ii) high exposure to securitization, (iii) high concentration among mortgage originators.
   - Insight: X-section mortgage data informative about equilibrium in lending-securitization market.

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   - $\Delta^+\ cost$ of financing the policy by about 2 times.
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Related Literature

• Macro Models of Aggregate Fluctuations with Housing
  **Contribution:** quantify the role information frictions in aggregate dynamics.

• Information Frictions in Asset Markets
  **Contribution:** link dynamics of securitization market to primary credit market.

• Policy in the Securitization Market
  **Contribution:** study the role of GSEs policies in macro model with adverse selection.
Outline

I. Model
   • Environment
   • Main mechanism

II. Quantification
   • Calibration
   • Simulating the Great Recession
   • Decomposition exercise

III. Policy Evaluation
Part I. The Model
Model Overview

Credit Market

Borrower household

Lenders (heterogenous origination cost)

Securitization Market

Sellers (sell loans)

Pool loans into MBS

Buyers (buy MBS)

Government

Pay & default

Issue loans

Lump sum transfer

Insurance fee

Security insurance

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Model: borrowers

- Log-preferences over ND consumption $C_t$, and housing $H_t$.
- **Long-term mortgages** $B_t$ (geometrically declining payments $\phi$), **defaultable**, competitive price $q_t$.
- Borrowing constraint: $B_{t+1} \leq \pi H_{t+1}$
Model: borrowers

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- **Default** on mortgages:
  - aggregate across borrowers: continuous **default rate** $\lambda(\bar{\omega}_t)$
  - family member s.t. individual housing valuation shocks $\omega^i_t$.
  - default if $\omega^i_t < \bar{\omega}_t = f(B_t, H_t, q_t, \phi)$ endogenous threshold.
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- Exogenous aggregate shocks:
  - Income endowment: $Y_t \sim$ Markov process.
  - Housing valuation volatility: $\sigma_{\omega,t} \in \{\sigma^H_{\omega,t}, \sigma^L_{\omega,t}\} \sim$ Markov process.
Model: lenders

- Log-preferences over ND consumption (dividends).
- Only income: borrowers payments $\phi b_t^i$.
- Only equity: portfolio of outstanding loans $(1 - \phi) b_t^i$. 
Model: lenders

- Log-preferences over ND consumption (dividends).
- Only income: **borrowers payments** $\phi b_t^j$.
- Only equity: portfolio of **outstanding loans** $(1 - \phi)b_t^j$.
- Lending technology:
  - every period draws **lending cost** $z_t^j \sim \text{i.i.d}$ (idiosyncratic risk).
  - lender issues **new loans** $n_t^j$ at gross cost $n_t^jz_t^j$. (heterogeneity).
Model: lenders

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- Only income: **borrowers payments** \( \phi b^j_t \).
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Lending technology:
- every period draws lending cost \( z^j_t \sim \text{i.i.d} \) (idiosyncratic risk).
- lender issues **new loans** \( n^j_t \) at gross cost \( n^j_t z^j_t \). (heterogeneity).

Securitization market à la Kurlat(2013):
- Lender can **sell outstanding loans** and/or **buy securities**.
- Assumption 1: trade is anonymous.
- Assumption 2: trade is non-exclusive, competitive (pooling) price \( p_t \).
Lender’s timeline

- **Aggregate default rate** $\lambda_t(\bar{\omega})$ affects all lenders equally.
- **Private information**: lender privately identifies defaulting loans $\lambda_t(\bar{\omega}) b^j_t$.
- Defaulting loans do not accumulate to the next period.
Lender’s timeline

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- **Private information**: lender privately identifies defaulting loans \( \lambda_t(\bar{\omega})b_t^j \).
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Lender’s Budget Constraint
Model Properties
The Role of the Securitization Market

**Complete Information**: defaulting loans are identified by everyone.

Securitization allows for:

i. **Financial specialization**: lenders become originators and security investors.

ii. **Lower intermediation costs**, mortgage rate under securitization is lower than without it: \( r(q)^{SM} \leq r(q)^{\text{without SM}} \).
Securitization Market + Private Information

**Private Information:** defaulting loans are identified only by owner.

i. Private info + anonymity + pooling market leads to an **adverse selection problem**.
   - All lenders sell their defaulting loans $s_B$.
   - Only high-z cost lenders sell their non-defaulting (good) loans $s_G$.
Securitization Market + Private Information

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i. Private info + anonymity + pooling market leads to an **adverse selection problem**.
   - All lenders sell their defaulting loans $s_B$.
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ii. Buying securities becomes less profitable: buyers face an **adverse selection discount** $\mu$.

$$\mu = \frac{S_B}{S_B + S_G}$$

$\mu$: fraction of defaulting loans traded.
Securitization Market + Private Information

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$\mu$: fraction of defaulting loans traded.

iii. Holders: some lenders remain with their illiquid portfolio of good loans.
Government Policy

- **Subsidy** $\tau$ (insurance) to **buyers** of securities: $p(1 - \tau)$.
- **Tax loan originators** ($\tilde{q} = q + \gamma$) and **borrowers** to finance the subsidy.
Main Mechanism
Main mechanism: securitization market

Consider an increase in $\sigma_\omega \rightarrow \Delta^+ \lambda(\bar{\omega})$, then:

In the securitization market

- $\Delta^+ \mu$ fraction of defaulting loans traded.
- $\Delta^- D$ lower demand of securities.
- $\Delta^- p$ lower price of securities.
Main mechanism: securitization market

Consider an increase in $\sigma_\omega \rightarrow \Delta^+ \lambda(\bar{\omega})$, then:

Model allows for crash of securitization market:

- There is no positive price that clears the market, $p \not> 0$
- **All lenders operate** with their technology $n^j z^j$.
- Same as **model without securitization**.
Main mechanism: primary market

In the credit market, consider an increase in $\sigma_\omega \to \Delta^+ \lambda(\bar{\omega})$, can lead to:

- $\Delta^- \text{ liquid resources for lending.}$
- $\Delta^- N \text{ aggregate lending.}$
- $\Delta^+ r(q): \text{ higher lending rate.}$

$\bullet$ Distribution $F(z)$ determines the magnitude of the effect on prices.
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   • Calibration
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   • Decomposition exercise

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Part II. Quantitative Analysis
## Calibration

### Benchmark calibration: 1990-2006

#### Lenders

<table>
<thead>
<tr>
<th>Param</th>
<th>Value</th>
<th>Target moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^L$</td>
<td>0.985</td>
<td>interest rate 1Y T-bill (risk free, pp)</td>
<td>1.6</td>
<td>1.7</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.21</td>
<td>maturity of mortgage bond index</td>
<td>4.0</td>
<td>4.0</td>
</tr>
</tbody>
</table>

**$F(z)$ Beta($\alpha$, $\beta$)**

- **lending distribution $\Theta(n)$ in HMDA data**

<table>
<thead>
<tr>
<th>Param</th>
<th>Value</th>
<th>Target moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>4.20</td>
<td>market share top 25% originators</td>
<td>95.7</td>
<td>95.9</td>
</tr>
<tr>
<td>$\beta$</td>
<td>2.25</td>
<td>loan issuance volume top-10/bot-90</td>
<td>9.3</td>
<td>9.2</td>
</tr>
<tr>
<td>$l_c$</td>
<td>0.63</td>
<td>mortgage rate 30Y FRM real, %</td>
<td>5.0</td>
<td>5.1</td>
</tr>
</tbody>
</table>

#### Government

<table>
<thead>
<tr>
<th>Param</th>
<th>Value</th>
<th>Target moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.007</td>
<td>Guarantee fee GSEs (bps)</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.69 $\mu$</td>
<td>GSEs market share of RMBS issuance</td>
<td>69.0</td>
<td>69.0</td>
</tr>
</tbody>
</table>
## Non-targeted Moments

Benchmark calibration: 1990-2006

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>average sales of loans, fraction of portfolio. (pp)</td>
<td>61.8</td>
<td>73.9</td>
</tr>
<tr>
<td>average mortgage spread (bps)</td>
<td>178</td>
<td>329</td>
</tr>
</tbody>
</table>

### Correlations

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>volume lending &amp; sec-issuance</td>
<td>0.86</td>
<td>0.90</td>
</tr>
<tr>
<td>log-lending &amp; default</td>
<td>-0.71</td>
<td>-0.81</td>
</tr>
<tr>
<td>log-security issuance &amp; default</td>
<td>-0.68</td>
<td>-0.85</td>
</tr>
<tr>
<td>borrower’s income &amp; default</td>
<td>-0.37</td>
<td>-0.41</td>
</tr>
</tbody>
</table>

### Distribution of lending $\Theta(n)$

<table>
<thead>
<tr>
<th></th>
<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>0.002</td>
<td>0.008</td>
<td>0.030</td>
<td>0.959</td>
</tr>
<tr>
<td>Model</td>
<td>0.006</td>
<td>0.007</td>
<td>0.030</td>
<td>0.957</td>
</tr>
</tbody>
</table>
Simulating the Great Recession
The Great Recession. Exogenous Processes

- Income shock, \( Y \): cyclical component of GDP.
- Housing valuation shock, \( \sigma^2_\omega \): matches model’s default rates to the data.
From 2008 to 2013 the model replicates:

- 2/3 of the contraction in mortgage lending.
- total contraction in MBS issuance.
- X-section mortgage data informative about equilibrium in lending-securitization market.
Quantifying Information Frictions
Table 1: Average contribution (pp), 08-13

<table>
<thead>
<tr>
<th>Volume of issuance</th>
<th>priv. info</th>
<th>$\sigma^2_\omega$</th>
<th>$Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Credit Market</td>
<td>43</td>
<td>52</td>
<td>5</td>
</tr>
<tr>
<td>Securitization Market</td>
<td>46</td>
<td>50</td>
<td>4</td>
</tr>
</tbody>
</table>

- Information frictions account for about 45% of predicted contraction.
Quantifying Information Frictions: shock decomposition

Table 2: Average contribution (pp), 08-13

<table>
<thead>
<tr>
<th>Volume of issuance</th>
<th>priv. info</th>
<th>$\sigma^2$</th>
<th>Y</th>
</tr>
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<td>46</td>
<td>50</td>
<td>4</td>
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</table>

Mortgage lending contraction during Great Recession

- This paper:
  - Information frictions (45%), housing dynamics (50%), income (5%).

- Kaplan, Mitman, Violante (QJE, 2020).
  - Decomposition: house price (50%), households' beliefs (50%).
Part III. Evaluating Policy Changes
Policy: expanding insurance on securities

GSEs effectively took on the entire MBS market after 2012.

<table>
<thead>
<tr>
<th>Description</th>
<th>Benchmark</th>
<th>$\Delta^+(\tau, \gamma)$</th>
<th>$\Delta$ Model</th>
<th>$\Delta$ Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Primary Market</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mortgage spread, avg (bps)</td>
<td>330</td>
<td>290</td>
<td>$\Delta^-$</td>
<td>$\Delta^-$</td>
</tr>
<tr>
<td>Mortgage spread, std (pp)</td>
<td>6.2</td>
<td>4.7</td>
<td>$\Delta^-$</td>
<td>$\Delta^-$</td>
</tr>
<tr>
<td>Hhs default (pp)</td>
<td>2.7</td>
<td>3.0</td>
<td>$\Delta^+$</td>
<td>$\Delta^+$</td>
</tr>
<tr>
<td><strong>Securitization Market</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of loans traded %</td>
<td>74.0</td>
<td>100</td>
<td>$\Delta^+$</td>
<td>$\Delta^+$</td>
</tr>
<tr>
<td>Prob. market collapse (pp)</td>
<td>5.9</td>
<td>0.0</td>
<td>$\Delta^-$</td>
<td></td>
</tr>
<tr>
<td><strong>Gov. Policy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Costs of policy (pp), $\tau$</td>
<td>6.5</td>
<td>11.3</td>
<td>$\Delta^+$</td>
<td></td>
</tr>
<tr>
<td>Gov deficit/Y</td>
<td>0.8</td>
<td>2.7</td>
<td>$\Delta^+$</td>
<td>$\Delta^+$</td>
</tr>
</tbody>
</table>

1. higher insurance stabilizes price of securities and mortgage spread.
2. default rates increase due to higher indebtedness of households.
   housing wealth accumulation increases by 6%.
3. Cost of policy doubles $\rightarrow$ higher taxes
Table 3: Welfare effects: policy changes after Great Recession

<table>
<thead>
<tr>
<th>Description</th>
<th>$\Delta^+ (\tau, \gamma)$</th>
<th>Decomposition</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta^+ \tau$</td>
<td>$\Delta^+ \gamma$</td>
</tr>
<tr>
<td>$\Delta%$ Borrower welfare</td>
<td>0.06</td>
<td>-0.16</td>
</tr>
<tr>
<td>$\Delta%$ Non-durable cons.</td>
<td>-0.15</td>
<td>-0.69</td>
</tr>
<tr>
<td>$\Delta%$ Housing good cons.</td>
<td>0.55</td>
<td>2.63</td>
</tr>
<tr>
<td>$\Delta%$ Lenders’ welfare</td>
<td>1.3</td>
<td>3.01</td>
</tr>
</tbody>
</table>
Main Takeaways

- **Information frictions** can account for **large fluctuations** in mortgage lending

For the Great Recession:
- 45% of contraction in MBS issuance.
- 27% of contraction in mortgage lending.

- **Expanding insurance** on securities can be **welfare improving**
  - Provides **stabilization** at a **high cost**.
  - lower mortgage rates,
  - higher default,
  - **higher taxes** to households.
Thanks!!
Model. Formal results.

Environment

- Borrower Recursive Problem
- Lender Recursive Problem
- Aggregate states
- Recursive Competitive Equilibrium

Properties

- Characterization
- Mechanism
Main mechanism: model + data

- High concentration (data): **small mass** of (low cost) lenders originate most loans.
  → benefit: **low cost intermediation**.
Main mechanism: model + data

- **High concentration (data):** small mass of (low cost) lenders originate most loans.
  - → benefit: **low cost intermediation.**

- **Large liquidity benefits** of accessing securitization market.
  - → cons: **higher fragility.**
Main mechanism: model + data

- **High concentration (data): small mass** of (low cost) lenders originate most loans.
  → **benefit:** low cost intermediation.

- **Large liquidity benefits** of accessing securitization market.
  → **cons:** higher fragility.
Borrower’s problem

\[ V^{B,j}(b, h; X) = \max_{\{c, n, h', \nu(\omega^j)\}} u(c, h) + \beta^B \mathbb{E}_{X'} |X V^B(b', h'; X') \]
Borrower’s problem

\[ V^{B,j}(b, h; X) = \max_{\{c, n, h', \nu(\omega^j)\}} \left\{ u(c, h) + \beta^B \mathbb{E}_{X^t|X} V^B(b', h'; X') \right\} \]

\[
c + p_h \psi(h') - \omega^j p_h h \nu(\omega^j) \leq y + qn - \phi b \nu(\omega^j) - T^B
\]
\[
b' = (1 - \phi) b \nu(\omega^j) + n
\]
\[
b' \leq \pi p_h h'
\]

given \( b_0, h_0 \).

- income: stochastic endowment \( y \) and new debt \( n \).
- housing adjustment costs: \( \psi(h') = h' + \frac{\nu}{2} (h' - \bar{h})^2 \).
Borrower’s problem

\[ V^{B,j}(b, h; X) = \max_{\{c, n, h', \iota(\omega^j)\}} \left\{ u(c, h) + \beta^B \mathbb{E}_{X'|X} V^B(b', h'; X') \right\} \]

\[
\begin{align*}
  c + p_h \psi(h') - \omega^j p_h \iota(\omega^j) & \leq y + qn - \phi b \iota(\omega^j) - T^B \\
b' & = (1 - \phi) b \iota(\omega^j) + n \\
b' & \leq \pi p_h h' \\
given \quad b_0, h_0.
\end{align*}
\]

- \( \omega^j \sim G_\omega \): idiosyncratic housing valuation shock
  as in Elenev, Landvoigt, Van Nieuwerburgh (JME, 2016).
- default: each borrower decides whether to repay \( b \)

\[
\iota(\omega^j) = \begin{cases} 
  0 & \omega^j < \tilde{\omega} \\
  1 & \omega^j \geq \tilde{\omega}
\end{cases}
\]

- after default decision, family of borrower jointly chooses \( \{c, n, h'\} \).
Borrower’s problem

\[ V^{B,j} (b, h; X) = \max_{\{c,n,h',\iota(\omega^j)\}} u(c, h) + \beta^B \mathbb{E}_{X'|X} V^{B} (b', h'; X') \]

\[ c + p_h \psi(h') - \omega^j p_h \iota(\omega^j) \leq y + q n - \phi b \iota(\omega^j) - T^B \]

\[ b' = (1 - \phi) b \iota(\omega^j) + n \]

\[ b' \leq \pi p_h h' \]

\[ \text{given } b_0, h_0. \]

• \( \omega^j \sim G_\omega \): idiosyncratic housing valuation shock.

• default: each borrower decides whether to repay \( b \)

\[ \iota(\omega^j) = \begin{cases} 
0 & \omega^j < \bar{\omega} \\
1 & \omega^j \geq \bar{\omega} 
\end{cases} \]

• after default decision, family chooses \( \{c, n, h'\} \).
Borrower’s Problem

- Recursive problem of the family

\[ V^B(B, H; X) = \max_{\{C, N, H\}} u(C, H) + \beta^B \mathbb{E}_{X'|X} V(B', H'; X') \]

\[ C + p_h \psi(H') - (1 - \lambda(\bar{w})) \mathbb{E}_{\omega > \bar{w}} p_h H = Y + qN - (1 - \lambda(\bar{w})) \phi B + T^B \]

\[ B' = (1 - \phi)(1 - \lambda(\bar{w})) B + N \]

\[ B' \leq \pi p_h H' \]

where \( \lambda(\bar{w}_t) = G_\omega(\bar{w}_t; \chi) \) default rate at the optimal cutoff \( \bar{w}_t \).

\[ \bar{w}_t = \frac{B_t}{p_{h,t} H_t}(\phi + (1 - \phi) q_t) \]

- Assume \( G_\omega(\chi_1, \chi_2) \) is a Gamma Distribution.
Lender’s Recursive Problem

\[
V^L(z^j, b^j; X) = \max_{\{c, b', n, d, s_B, s_G\}} \log c^j + \beta^L E_{z', X'} V^L(z'^j, b'^j; X')
\]

\[
(1 - \lambda(\bar{\omega}))\phi b^j + p(s'_G + s'_B) \leq c^j + n^j z^j(q + \gamma) + pd^j(1 - \tau)
\]

\[
b'^j = (1 - \lambda(\bar{\omega}))(1 - \phi)b^j - s'_G + n^j + (1 - \mu)d^j
\]

\[
s'_G \in [0, (1 - \phi)(1 - \lambda)b^j]
\]

\[
s'_B \in [0, (1 - \phi)\lambda b^j]
\]

\[
d^j \geq 0, \quad n^j \geq 0.
\]
Aggregate states

• Aggregate states
  \[ X = \{ B, H, \Gamma; \sigma_\omega, y \} \]

• Endogenous states
  • \( B \), aggregate stock of debt
  • \( H \), aggregate housing stock
  • \( \Gamma(z, b) \), joint distribution across lenders

• Exogenous states
  • \( y \), borrower’s income endowment
  • \( \sigma_\omega \), volatility of housing valuation shock
  • \( \{ \sigma_\omega, y \} \sim \) joint stochastic process, first order Markov
Calibration: borrowers

Benchmark calibration: 1990-2006

<table>
<thead>
<tr>
<th>Param</th>
<th>Value</th>
<th>Target moment</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta^B$</td>
<td>0.97</td>
<td>cons. ndur &amp; serv to DPI, $C/Y$</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.13</td>
<td>cons. ndur &amp; serv to real estate, $C/H$</td>
<td>0.40</td>
<td>0.40</td>
</tr>
<tr>
<td>$\pi$</td>
<td>0.43</td>
<td>mortgage debt to real estate, $B/H$</td>
<td>0.43</td>
<td>0.43</td>
</tr>
<tr>
<td>$\nu$</td>
<td>2.0</td>
<td>residential real estate investment, $I/H$</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$\mu_\omega$</td>
<td>0.975</td>
<td>residential housing depreciation.</td>
<td>0.03</td>
<td>0.03</td>
</tr>
<tr>
<td>$\sigma^L_\omega$</td>
<td>0.057</td>
<td>RM default 30 dd+ (pp), normal times</td>
<td>2.18</td>
<td>2.74</td>
</tr>
<tr>
<td>$\sigma^H_\omega$</td>
<td>0.175</td>
<td>RM default 30 dd+ (pp), crisis times</td>
<td>8.64</td>
<td>8.14</td>
</tr>
</tbody>
</table>

- Exogenous processes $\{y, \sigma^2_\omega\}$ joint Markov

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std</th>
<th>$\rho$</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y_{cy}$</td>
<td>1.00</td>
<td>0.01</td>
<td>0.69</td>
<td>cyclical component of household’s DPI</td>
</tr>
<tr>
<td>$\sigma^2_\omega$</td>
<td>0.074</td>
<td>0.04</td>
<td>0.66</td>
<td>2-state Markov chain, ELV(2016). $\sigma^2_\omega \in (\sigma^L_\omega, \sigma^H_\omega) = (0.057, 0.175)$</td>
</tr>
</tbody>
</table>

$\text{corr}(Y_{cy}, \sigma^2_\omega) = -0.35$
Recursive Competitive Equilibrium

A RCE given gov policy \( \{\tau, \gamma, T^B\} \) consists of prices \( \{q(X), p(X)\} \); adverse selection discount \( \{\mu(X)\} \); a law of motion \( \Gamma'(X) \); and transition density \( \Pi(X'|X) \); and policy functions \( \{ C, N, B', H' \}^B \) and \( \{ c^j, n^j, d^j, s^j_G, s^j_B \}_{j \in J}^L \) s.t.:

1. Borrowers and lenders optimize.
2. \( q(X) \) clears the **primary mortgage market**
   \[
   N(q; X) = \int n(q, p; X) d\Gamma.
   \]
3. Whenever \( p(X) > 0 \) the **securitization market** clears
   \[
   D(p, q; X) = S(p, q; X),
   \]
4. Government balances budget every period
   \[
   \gamma N(X) + T^B = \tau p D(X).
   \]
5. Resource constraint holds
   \[
   C^B + C^L + H' - \mu \omega (1 - \lambda(\bar{\omega})) H = Y + q \int (z - 1)n \ d\Gamma.
   \]
Characterization: lenders’ trading decisions \( \{s_B, s_G, d, n\} \)

- For any \( p > 0 \) all lenders sell their defaulting loans
  \[
  s_B = \lambda(\bar{\omega})(1 - \phi)b
  \]

- Lenders self-classify into three groups
  - Sellers: \( z < \hat{z} \) \( \{s_G > 0, d = 0, n > 0\} \)
  - Buyers: \( z > \hat{z}\frac{1 - \tau}{1 - \mu} \) \( \{s_G = 0, d > 0, n = 0\} \)
  - Holders: \( z \in [\hat{z}, \hat{z}\frac{1 - \tau}{1 - \mu}] \) \( \{s_G = 0, d = 0, n > 0\} \)

- Holders have limited access to liquidity from securitization.
Lender’s timeline

\[(z_t, b_t) \xrightarrow{\lambda_t} \text{Agg. default} \quad \xrightarrow{\text{Maturity}} \quad \text{Decisions}\]

\[
(1 - \lambda_t) b_t \\
\lambda_t b_t \quad \rightarrow \quad \lambda_t (1 - \phi) b_t \quad \text{(outstanding-defaulting)}
\]

\[
(1 - \lambda_t) \phi b_t \quad \text{(mortgage payments)}
\]

\[
(1 - \lambda_t)(1 - \phi) b_t \quad \text{(outstanding-good)}
\]

\[
c_t > 0 \quad \text{(pay dividends)}
\]

\[
n_t \geq 0 \quad \text{(issue new loans)}
\]

\[
d_t \geq 0 \quad \text{(purchase securities)}
\]

\[
S_{G,t} \in [0, (1 - \lambda_t)(1 - \phi)b_t]
\]

\[
S_{B,t} \in [0, \lambda_t(1 - \phi)b_t]
\]
Lender’s timeline

- **Lender’s budget constraint:**

\[
(1 - \lambda(\bar{\omega})) \phi b_t^j + p(s_G^j + s_B^j) \geq c^j + n^j z^j(q + \gamma) + pd^j(1 - \tau)
\]

Cash inflows: borrower’s payments + loan sales.
Lender’s timeline

- **Lender’s budget constraint:**

\[
(1 - \lambda(\bar{\omega})) \phi b^j + p(s^j_G + s^j_B) \geq c^j + n^j z^j(q + \gamma) + pd^j(1 - \tau)
\]

Cash outflows: **dividend payments + new lending + security purchases.**