Optimal Bank Regulation In the Presence of Credit and Run Risk

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Motivation

- Financial intermediaries perform various socially useful functions
- Both assets and liabilities are critical to delivering these services
- However, the balance sheet structure can also be a source of fragility
- We present a model featuring these interactions, study the externalities emerging from intermediation and examine regulation to mitigate their effects

Our framework

We modify the classic Diamond-Dybvig model such that banks:

- Provide liquidity and monitoring services
- Are funded by deposits and equity
- Make risky loans, hold liquidity and are subject to limited liability
- Face endogenous run risk determined by a global game
  - Akin to Goldstein and Pauzner (2005), but with a trigger based on uncertain liquidation values for loans

The economy

t = 1

- Entrepreneurs (E) borrow to invest in long-term, illiquid and risky projects
- Savers (S) invest in demandable bank deposits
- Bankers (B) raise equity and deposits to invest in risky loans and liquid safe assets

t = 2

- Each saver learns whether she is impatient or patient
- B decides whether to recall and liquidate some loans to serve early withdrawals
- Due to sequential service, decision to withdraw depends on beliefs about others' actions and loan liquidation value $\xi \in U\left(\underline{\xi}, \overline{\xi}\right)$

t = 3

- Good productivity shock (A) with probability ω and 0 otherwise
- E privately learns the value of the shock and B decides whether to monitor
- Repayment (or default on loans and deposits in the bad state)

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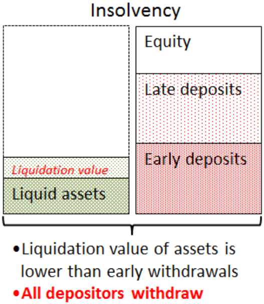
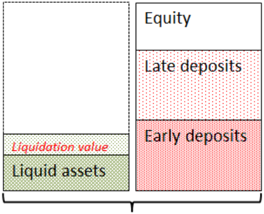


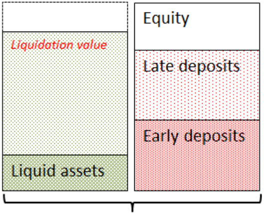
Date 2 possibilities

Insolvency



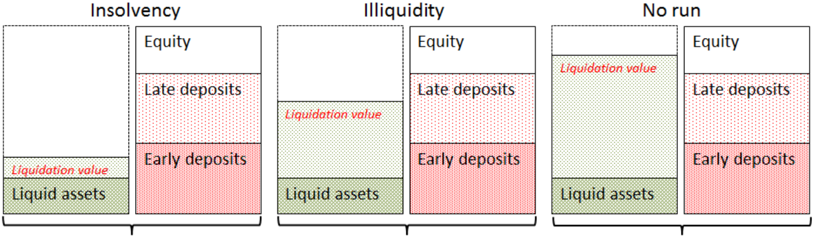
- Liquidation value of assets is lower than early withdrawals
- **All depositors withdraw**

No run



- Liquidation value is higher than total demandable deposits
- **Late depositors do not withdraw early**

Date 2 possibilities



Insolvency

- Liquidation value of assets is lower than early withdrawals
- **All depositors withdraw**

Illiquidity

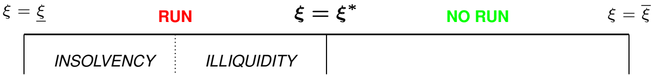
- Liquidation value is lower than total demandable deposits
- **A late depositor withdraws if she expects others to withdraw**

No run

- Liquidation value is higher than total demandable deposits
- **Late depositors do not withdraw early**

Date 2 actions by savers

- Savers get private noisy signals $x_i = \xi + \epsilon_i$, $\epsilon_i \sim U[-\epsilon, \epsilon]$ about ξ
- Unique run threshold ξ^*, which depends on bank's balance sheet



S's Optimization problem

$$\mathbb{U}_S = U\left(e_S - D\right) + \overbrace{\int_{\underline{\xi}}^{\xi^*} \theta \cdot D(1 + r_2^D)\frac{d\xi}{\Delta_\xi}}^{\text{run}} + \overbrace{\int_{\xi^*}^{\overline{\xi}} \delta \cdot D(1 + r_2^D)\frac{d\xi}{\Delta_\xi}}^{\text{no run,impatient}}$$

$$+ \underbrace{\int_{\xi^*}^{\overline{\xi}} (1-\delta) \cdot \omega \cdot D(1 + r_3^D)\frac{d\xi}{\Delta_\xi}}_{\text{no run,patient}} + \underbrace{\int_{\xi^*}^{\overline{\xi}} V\left(D\right)\frac{d\xi}{\Delta_\xi}}_{\text{transaction services}}$$

- Quasi-linear preferences for consumption and additional utility from transactions services of deposits
- θ is the (endogenous) probability of being repaid in a run
- δ is the (exogenous) probability of being impatient

Optimization wrt D yields a **Deposit Supply** schedule, $DS(D, r_2^D, r_3^D, \theta, \xi^*) = 0$

- Because each S is small, she takes ξ^* and θ as given

DS details

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E's Optimization problem

$$\mathbb{U}_E = \int_{\xi^*}^{\overline{\xi}} \{\omega \cdot [\overbrace{A \cdot (1-y) \cdot l}^{\text{realized output}} - \overbrace{(1-y) \cdot l \cdot (1+r^l)}^{\text{loan obligation}}] - \overbrace{c(l)}^{\text{cost}}\} \frac{d\xi}{\Delta_\xi}$$

where:

- E has a linear production function, but incurs a convex (effort) cost
- y is the (endogenous) fraction of loans recalled and $y = 1$ in a run
- E is protected by limited liability and defaults in the bad state

Optimization wrt l yields a **Loan Demand** schedule, $LD(r^l, l, y, \xi^*) = 0$

- Because each E is small, she takes ξ^* and y as given

▸ LD details

E's Optimization problem

$$\mathbb{U}_E = \int_{\xi^*}^{\overline{\xi}} \{\omega \cdot [\overbrace{A \cdot (1-y) \cdot I}^{\text{realized output}} - \overbrace{(1-y) \cdot I \cdot (1+r^l)}^{\text{loan obligation}}] - \overbrace{c(I)}^{\text{cost}}\} \frac{d\xi}{\Delta_\xi}$$

where:

- E has a linear production function, but incurs a convex (effort) cost
- y is the (endogenous) fraction of loans recalled and $y = 1$ in a run
- E is protected by limited liability and defaults in the bad state

Optimization wrt I yields a **Loan Demand** schedule, $LD(r^l, I, y, \xi^*) = 0$

- Because each E is small, she takes ξ^* and y as given

▸ *LD* details

B's Optimization problem

$$\mathbb{U}_B = U(e_B - E) + \int_{\xi^*}^{\overline{\xi}} \{\omega \cdot [\ \underbrace{(1-y)\cdot I}_{\text{outstanding loans}} \cdot \underbrace{(1+r^I)}_{\text{loan rate}} - \underbrace{(1-\delta)\cdot D}_{\text{patient deposits}} \cdot \underbrace{(1+r_3^D)}_{\text{deposit rate}}] - \underbrace{X}_{\substack{\text{monit.}\\ \text{cost}}}\} \frac{d\xi}{\Delta_\xi}$$

- At t=1 the balance sheet constraint is:

$$BS: \quad I + LIQ = D + E$$

- In a run, the probability of being repaid is:

$$\theta = \frac{LIQ + \xi \cdot I}{D \cdot (1 + r_2^D)}$$

- Absent a run, it liquidates $y \in (0,1)$ of its loans to pay early withdrawals:

$$y = \frac{\delta \cdot D \cdot (1 + r_2^D) - LIQ}{\xi \cdot I}$$

Monitoring

- The productivity shock is privately revealed to E
- B needs to expend resources to learn it
- Given that dividends are increasing in ξ, B monitors if

$$\overbrace{\omega\left[\underbrace{\frac{\xi^* I-\delta D(1+r_2^D)+LIQ}{\xi^*}(1+r^l)}_{\text{revenue from outstanding loans}}-\underbrace{(1-\delta)D(1+r_3^D)}_{\text{deposit repayments due}}\right]}^{\text{net expected benefit from monitoring}}-\overbrace{X}^{\substack{\text{monitoring}\\ \text{cost}}}\geq 0$$

- If B does not monitor, E will report the bad shock and default$\rightarrow$ implications for global game

Run threshold determination

- Global games in Diamond-Dybvig due to Goldstein-Pauzner (2005)
  - Incentives to run depend on deposit contract $\rightarrow$ important for welfare analysis
- We extend GP to allow for limited liability and uncertain liquidation value:
  - Obtain endogenously upper dominance region, but uniqueness is harder to show
- Utility differential between waiting and withdrawing for different conjectured level of withdrawals, λ, as a function of ξ

$$\nu(\xi,\lambda) = \begin{cases} \omega D(1+r_3^D) - D(1+r_2^D) & \text{if} \quad \hat{\lambda}(\xi) \geq \lambda \geq \delta \qquad \textit{Partial run with monitoring} \\ -D(1+r_2^D) & \text{if} \quad \theta(\xi) \geq \lambda \geq \hat{\lambda}(\xi) \qquad \textit{Partial run no monitoring} \\ -(LIQ + \xi \cdot I)/\lambda & \text{if} \quad 1 \geq \lambda \geq \theta(\xi) \qquad \textit{Full run} \end{cases}$$

- $\hat{\lambda}$ is the maximum level of withdrawals below which B has incentives to monitor

▸ $\hat{\lambda}$ derivation

Run threshold determination ctd.

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- One-sided strategic complementarities: $\nu(\xi,\lambda)$ is increasing in λ in run region
  - In a full run, the margin gain from running is lower as more people opt to run
  - Goldstein-Pauzner deal with this issue and establish uniqueness
- Perverse state monotonicity: $\nu(\xi,\lambda)$ is *decreasing* in ξ in run region, but length of regions also moves
  - In a full run, the expected return is higher for a strong bank than a weak bank
  - Not an issue in Goldstein-Pauzner because of fixed liquidation value

Existence and Uniqueness

- As $\epsilon \to 0$, ξ^* is given by $GG(\xi^*) = \int_\delta^1 \nu(\xi^*, \lambda) d\lambda = 0$

$$\underbrace{\int_\delta^{\hat{\lambda}(\xi^*)} \left[\omega D(1+r_3^D) - D(1+r_2^D)\right] d\lambda}_{\substack{\text{Partial run} \\ \text{with monitoring}}} - \underbrace{\int_{\hat{\lambda}(\xi^*)}^{\theta(\xi^*)} D(1+r_2^D) d\lambda}_{\substack{\text{Partial run} \\ \text{no monitoring}}} - \underbrace{\int_{\theta(\xi^*)}^1 \frac{LIQ + \xi^* I}{\lambda} d\lambda}_{\text{Full run}} = 0$$

- Does a unique ξ^* exist? — (focus on limiting noise; detailed proof for $\epsilon > 0$)
- Existence: GG is continuous and there exist thresholds $\underline{\xi} < \xi_{LD} < \xi_{UD} < \bar{\xi}$ such that $GG(\xi) < 0$ for $\xi < \xi_{LD}$ and $GG(\xi) > 0$ for $\xi > \xi_{UD}$
- Typical uniqueness proof requires that $dGG/d\xi > 0$ *everywhere*

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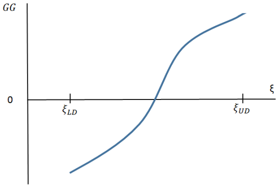
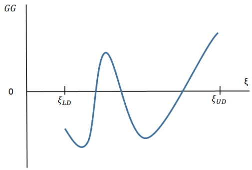


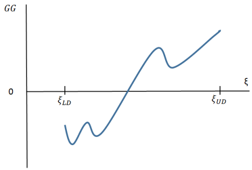
Uniqueness proof

- But, in our case $\frac{dGG}{d\xi} = \overbrace{\omega D(1+r_3^D)\frac{d\hat{\lambda}}{d\xi}}^{\substack{\text{Incentive to wait}\\\text{More monitoring}}} - \overbrace{\int_\theta^1 \frac{I}{\lambda} d\lambda}^{\substack{\text{Incentive to run}\\\text{Higher recovery}}} \overset{?}{\lessgtr} 0$, because $\frac{d\hat{\lambda}}{d\xi} > 0$

Bad case



Good case



- Trick: Realize that *GG* does not need to be strictly increasing everywhere, but only at candidate solutions
- We show there are no solutions where $\{GG(\xi^*) = 0 \text{ and } dGG/d\xi|_{\xi=\xi^*} \leq 0\}$
- Hence, the run threshold is **unique**

Details

Uniqueness proof

- But, in our case $\frac{dGG}{d\xi} = \overbrace{\omega D(1+r_3^D)\frac{d\hat{\lambda}}{d\xi}}^{\text{Incentive to wait, More monitoring}} - \overbrace{\int_\theta^1 \frac{I}{\lambda} d\lambda}^{\text{Incentive to run, Higher recovery}} \overset{?}{\lessgtr} 0$, because $\frac{d\hat{\lambda}}{d\xi} > 0$

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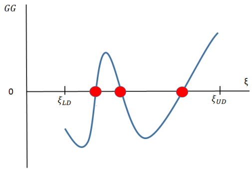

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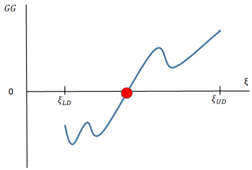

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Details

Private Equilibrium

- B chooses *I*, *LIQ*, *D* and *E* to maximize her utility while *internalizing* how these choices affect:
  - the run threshold via *GG*
  - the deposit rates that S demand via *DS*
  - the loan rates that E are willing to accept via *LD*
- Balance sheet constraint eliminates one choice variable $\rightarrow$ three (free) choices:
  - The asset mix that trades off loans and liquid assets
  - The liability mix that trades off equity and deposits
  - The overall scale of the balance sheet

Optimality conditions

Social Planner and Externalities

- Savers and Entrepreneurs are atomistic and take (ξ^*, θ, y) as given
- Consider a social planner with the following welfare function

$$\mathbb{U}_{sp} = \mathbb{U}_B + w_S \mathbb{U}_S + w_E \mathbb{U}_E$$

- If the planner respects the DS and LD constraints $\mathbb{U}_S$ and $\mathbb{U}_E$ can be replaced by

$$\mathbb{U}_S^* = U(e_s - D) + U'(e_S - D)D + \int_{\xi^*}^{\bar{\xi}} [V(D) - V'(D)D]/\Delta_\xi$$

$$\mathbb{U}_E^* = \int_{\xi^*}^{\bar{\xi}} [c'(I)I - c(I)]/\Delta_\xi$$

- Recall S and E take ξ^* as given, but planner will explicitly account how their actions affect ξ^* and, thus, their welfare

Social Planner and Externalities ctd.

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Trade-offs for the Planner

- Trade-off 1: Planner trades off more deposits versus higher run risk when trying to help savers
- Trade-off 2: Planner trades off more investment versus higher run risk when trying to help entrepreneurs

Example	PE	SP for weights (w_E, w_S)		
		(0.0,0.2)	(0.1,0.1)	(0.2,0.0)
I	0.862	0.785	0.873	0.906
LIQ_1	0.052	0.221	0.060	0.000
D	0.875	0.962	0.894	0.867
E	0.038	0.044	0.039	0.038
Run prob.	0.407	0.386	0.403	0.408
Capital ratio	0.044	0.049	0.045	0.042
Liquidity ratio	0.060	0.281	0.069	0.000
$\%\Delta\mathbb{U}_E$	-	-1.66%	0.33%	1.19%
$\%\Delta\mathbb{U}_S$	-	3.63%	0.71%	-0.30%
$\%\Delta\mathbb{U}_B$	-	-0.44%	-0.05%	-0.09%

Capital ratio$= E/I$; Liquidity ratio$= LIQ/I$

- More liquid asset mix and more stable capital structure when S is favored
- More liquidity and/or capital reduce run probability
- More loans at the expense of liquidity when E is favored
- Yet, higher investment is not incompatible with more stable banking – both E and S gain
- B *loses*: already internalizes what matters to her – but total welfare is higher

Implementing the planner's solution

- The three intermediation margins differ between the private and social solutions
- One solution is to use taxes on, for example, I, LIQ and D to correct for the distorted intermediation margins
- Instead, we examine how regulation can decentralize the planner's solution
- It can be shown analytically that capital and liquidity regulations reduce the probability of runs (abstracting from GE effects)

▸ Partial effect of regulation on run prob.

- Are these tools complements or substitutes?

Implementation example – $w_E = 0.1$, $w_S = 0.1$

	PE	CR	CR&LR	SP
I	0.862	0.861	0.858	0.873
LIQ_1	0.052	0.055	0.059	0.060
D	0.875	0.877	0.879	0.894
E	0.038	0.039	0.039	0.039
Run prob.	0.407	0.406	0.406	0.403
Cap.ratio	0.044	0.045	0.045	0.045
Liq.ratio	0.060	0.063	0.069	0.069
$\%\Delta\mathbb{U}_E$	-	-0.03%	-0.10%	0.33%
$\%\Delta\mathbb{U}_S$	-	0.04%	0.12%	0.71%
$\%\Delta\mathbb{U}_B$	-	-0.00%	-0.00%	-0.05%

$CR = E/I$; $LR = LIQ/I$

- Tightening CR increases E and reduces run risk
- But, results in lower I
- Tightening LR too, reduces I and run risk further
- The two are not redundant
- Third tool needed to encourage intermediation – e.g. tax subsidy on D

Takeaways from regulatory tools

- Other tools that work are a liquidity coverage ratio, a net-stable funding ratio, reserve requirements, a leverage ratio
- But, at minimum the regulator needs a tool to manage capital, a tool to manage liquidity, and a tool to manage the scale of intermediation
- The distortions in the three intermediation margins are not *collinear*
- Liquidity tools can be combined with capital tools (and vice versa), but not with each other

Conclusions

- Presented a model of fragile financial intermediation where a bank offers liquidity and monitoring services
- Studied the externalities from intermediation and derived optimal regulation to address them
- Proposed a new proof for uniqueness in incomplete information bank-run models

Back-up slides

Deposit Supply

$$\mathbb{U}_S = U\left(e_S - D\right) + \int_{\underline{\xi}}^{\xi^*} \theta \cdot D(1 + r_2^D)\frac{d\xi}{\Delta_\xi} + \int_{\xi^*}^{\overline{\xi}} \delta \cdot D(1 + r_2^D)\frac{d\xi}{\Delta_\xi}$$
$$+ \int_{\xi^*}^{\overline{\xi}} (1 - \delta) \cdot \omega \cdot D(1 + r_3^D)\frac{d\xi}{\Delta_\xi} + \int_{\xi^*}^{\overline{\xi}} V\left(D\right) \frac{d\xi}{\Delta_\xi}$$

- Taking θ and ξ^* as given, optimization wrt to D yields the following DS schedule

$$-U'(e_S - D) + (1 + r_2^D)\int_{\underline{\xi}}^{\xi^*} \theta \frac{d\xi}{\Delta_\xi} + \left[\delta(1 + r_2^D) + (1 - \delta)\omega(1 + r_3^D) + V'(D)\right]\int_{\xi^*}^{\overline{\xi}} \frac{d\xi}{\Delta_\xi} = 0$$

Back to Savers

Loan Demand

$$\mathbb{U}_E = \int_{\xi^*}^{\overline{\xi}} \{\omega \cdot [\overbrace{A \cdot (1-y) \cdot l}^{\text{realized output}} - \overbrace{(1-y) \cdot l \cdot (1+r^l)}^{\text{loan obligation}}] - \overbrace{c(l)}^{\text{cost}}\} \frac{d\xi}{\Delta_\xi}$$

- Taking y and ξ^* as given, optimization wrt to l yields the following LD schedule

$$\int_{\xi^*}^{\overline{\xi}} \{\omega \cdot [A - (1+r^l)] \cdot (1-y) \cdot l - c'(l)\} \frac{d\xi}{\Delta_\xi} = 0$$

Back to Entrepreneurs

Derivation of $\hat{\lambda}$

- $\hat{\lambda}(\xi)$ is the level of withdrawals at which the banker is indifferent between monitoring E's projects or not when the liquidation value is ξ

$$\omega\left[\frac{\xi I-\hat{\lambda}(\xi)D(1+r_2^D)+LIQ}{\xi}(1+r^I)-(1-\hat{\lambda}(\xi))D(1+r_3^D)\right]-X=0$$

$$\Rightarrow\hat{\lambda}(\xi)=\frac{(\xi I+LIQ)(1+r^I)-\xi(D(1+r_3^D+X/\omega)}{D[(1+r_2^D)(1+r^I)-\xi(1+r_3^D)]}$$

- Because the incentives to monitor are decreasing in λ, we get that $\hat{\lambda}>\delta$

- Also, $\partial\hat{\lambda}(\xi)/\partial I>0$, $\partial\hat{\lambda}(\xi)/\partial LIQ>0$, $\partial\hat{\lambda}(\xi)/\partial D<0$, $\partial\hat{\lambda}(\xi)/\partial r^I>0$, $\partial\hat{\lambda}(\xi)/\partial r_2^D<0$, $\partial\hat{\lambda}(\xi)/\partial r_3^D<0$

Back to Global Game

Uniqueness proof details

- At any candidate solution ξ', $GG(\xi') = 0$ yields the following necessary condition:

$$-\int_{\theta}^{1} \frac{I}{\lambda} d\lambda = \frac{1}{\xi'} \left[\int_{\theta}^{1} \frac{LIQ}{\lambda} d\lambda + \int_{\delta}^{\theta} D(1 + r_2^D) d\lambda - \int_{\delta}^{\hat{\lambda}} \omega D(1 + r_3^D) d\lambda \right]$$

- Evaluating the derivative $dGG/d\xi$ at $\xi = \xi'$ and substituting in the above necessary condition yields:

$$\left.\frac{dGG}{d\xi}\right|_{\xi=\xi'} = \overbrace{\frac{1}{\xi'} \left[\int_{\theta}^{1} \frac{LIQ}{\lambda} d\lambda + \int_{\delta}^{\theta} D(1 + r_2^D) d\lambda \right]}^{>0} + \omega D(1 + r_3^D) \left[\frac{d\hat{\lambda}(\xi')}{d\xi} - \frac{\hat{\lambda} - \delta}{\xi'} \right]$$

- After some algebra

$$\frac{d\hat{\lambda}(\xi')}{d\xi} - \frac{\hat{\lambda} - \delta}{\xi'} = \frac{(\hat{\lambda} - \delta)\xi' D(1 + r_3^D) + (\delta D(1 + r_2^D) - LIQ)(1 + r^I)}{\xi' D[(1 + r_2^D)(1 + r^I) - \xi'(1 + r_3^D)]} > 0$$

since $\hat{\lambda} > \delta$ to provide monitoring incentives and $\delta D(1 + r_2^D) - LIQ > 0$ from lower dominance

Back to Global Game

Private Optimality Conditions

- Denote by ψ_{BS}, ψ_{GG}, ψ_{DS}, and ψ_{LD} the Lagrange multipliers on the balance sheet, global game, deposit supply, and loan demand constraints, respectively
- The first-order conditions of B for choices $\mathcal{C} \in \{I, LIQ, D, E, \xi^*, r^l, r_2^D, r_3^D\}$ are:

$$\frac{d\mathbb{U}_B}{d\mathcal{C}} + \psi_{BS}\frac{dBS}{d\mathcal{C}} + \psi_{GG}\frac{dGG}{d\mathcal{C}} + \psi_{DS}\frac{dDS}{d\mathcal{C}} + \psi_{LD}\frac{dLD}{d\mathcal{C}} = 0$$

- From the foc with respect to r_3^D we obtain

$$\psi_{DS} = -\left(\frac{d\mathbb{U}_B}{dr_3^D} + \psi_{GG}\frac{dGG}{dr_3^D}\right)\frac{dDS}{dr_3^D}^{-1}$$

- From the foc with respect to r^l we obtain

$$\psi_{LD} = -\left(\frac{d\mathbb{U}_B}{dr^l} + \psi_{GG}\frac{dGG}{dr^l}\right)\frac{dLD}{dr^l}^{-1}$$

Private Optimality Conditions ctd.

- From the foc with resect to ξ^*, and using ψ_{DS} and ψ_{LD}, we obtain

$$\psi_{GG} = -\frac{\frac{d\mathbb{U}_B}{d\xi^*} - \frac{d\mathbb{U}_B}{dr_3^D}\frac{dDS}{dr_3^D}^{-1}\frac{dDS}{d\xi^*} - \frac{d\mathbb{U}_B}{dr^I}\frac{dDS}{dr^I}^{-1}\frac{dLD}{d\xi^*}}{\frac{dGG}{d\xi^*} - \frac{dGG}{dr_3^D}\frac{dDS}{dr_3^D}^{-1}\frac{dDS}{d\xi^*} - \frac{dGG}{dr^I}\frac{dLD}{dr^I}^{-1}\frac{dLD}{d\xi^*}}$$

- From the foc with respect to E we obtain the shadow cost of equity

$$\psi_{BS} = -d\mathbb{U}_B/dE = U'(e_B - E)$$

- Note that the shadow cost of equity is increasing in the amount of equity raised
- Given the balance sheet constraint $E = I + LIQ - D$ and, thus, all Lagrange multiplier can be expressed as functions of I, LIQ and D
- ξ^*, r^I and r_3^D are also implicit functions of I, LIQ and D via constraints GG, DS and LD

Private Optimality Conditions ctd.

- Hence, there are three free choices for B
- One choice regards the asset mix which is described by combining the focs wrt I and LIQ

$$\frac{d\mathbb{U}_B}{dI} - \frac{d\mathbb{U}_B}{dLIQ} + \psi_{GG}\left(\frac{dGG}{dI} - \frac{dGG}{dLIQ}\right) + \psi_{DS}\left(\frac{dDS}{dI} - \frac{dDS}{dLIQ}\right) + \psi_{LD}\left(\frac{dLD}{dI} - \frac{dLD}{dLIQ}\right) = 0$$

- Another choice regards the liability mix which is described by the foc wrt to D

$$\frac{d\mathbb{U}_B}{dD} + U'(e_B - E) + \psi_{GG}\frac{dGG}{dD} + \psi_{DS}\frac{dDS}{dD} + \psi_{LD}\frac{dLD}{dD} = 0$$

- The last choice regards the overall scale of the bank, which is described by the foc wrt I given the other two choices

$$\frac{d\mathbb{U}_B}{dI} + U'(e_B - E) + \psi_{GG}\frac{dGG}{dI} + \psi_{DS}\frac{dDS}{dI} + \psi_{LD}\frac{dLD}{dI} = 0$$

▸ Optimality conditions

Partial effect of regulation on run risk

- We compute the partial derivatives of run risk with respect to capital and liquidity
- Partial effects keeping the loan rate, the deposits rates and cost of equity constant
- The problem is not scale invariant so we normalize by the size of the balance sheet and partial the partial derivative with respect to:
  1. A leverage ratio: $k = E/(I + LIQ)$
  2. A liquidity ratio: $\ell = LIQ/(I + LIQ)$
- The effect on the fundamental run probability, $q_f = (\xi_{LD} - \underline{\xi})/\Delta_\xi$, is captured by the derivative of the lower dominance threshold, $\partial\xi_{LD}/\partial T$, $T \in \{k, \ell\}$, where

$$\xi_{LD} = \frac{\delta(1-k)(1+r_2^D) - \ell}{1-\ell}$$

- The effect of the total run probability, $q = (\xi^* - \underline{\xi})/\Delta_\xi$, is captured by the implicit derivative of the run threshold ξ^*,

$$\frac{\partial\xi^*}{\partial T} = -\frac{\partial GG/\partial T}{\partial GG/\partial\xi^*}$$

Partial effect of regulation on fundamental run probability

- Increasing capital reduces the probability of fundamental runs

$$\frac{\partial \xi_{LD}}{\partial k} = -\frac{\delta(1 + r_2^D)}{1 - \ell} < 0$$

- Increasing liquidity reduces the probability of fundamental runs for $\ell < \bar{\ell} \equiv 1 - \delta(1 - k)(1 + r_2^D)$

$$\frac{\partial \xi_{LD}}{\partial \ell} = \frac{\delta(1 - k)(1 + r_2^D) - (1 - \ell)}{(1 - \ell)^2} < 0 \text{ for } \ell < \bar{\ell}$$

- $\ell < \bar{\ell}$ requires $\delta(1 - k)(1 + r_2^D) - (1 - \ell) < 0$, which is very intuitive

- The condition says that loans in the balance sheet are higher than the expected deposit withdrawals, hence there is maturity transformation

Partial effect of regulation on total run probability

- From uniqueness proof, $\partial GG/\partial\xi^* > 0$, so suffices to sign $\partial GG/\partial T$
- The global game condition GG can be written in terms of k and ℓ as:

$$GG: \quad \int_{\delta}^{\hat{\lambda}} \omega(1-k)(1+r_3^D)d\lambda - \int_{\delta}^{\theta^*}(1-k)(1+r_3^D) - \int_{\theta^*}^{1} \frac{\xi^*(1-\ell)+\ell}{\lambda}d\lambda = 0,$$

where $\hat{\lambda} = \frac{(\xi^*(1-\ell)+\ell)(1+r^I)-\xi^*((1-k)(1+r_3^D)+X/(\omega(I+LIQ)))}{(1-k)[(1+r_2^D)(1+r^I)-\xi^*(1+r_3^D)]}$

- k affects the payoff differential in a partial run as well as the range that monitoring occurs, $\hat{\lambda}-\delta$, via its effect on bank profitability
- ℓ affects the payoff differential in a full run as well as the range that monitoring occurs, $\hat{\lambda}-\delta$, via its effect on bank profitability

Partial effect of regulation on total run probability – Capital

- Trade-off from increasing capital: Monitoring more probable versus lower payoff given monitoring

$$\frac{\partial GG}{\partial k} = \underbrace{\frac{\partial \hat{\lambda}}{\partial k}\omega(1-k)(1+r_3^D)}_{\text{More monitoring}} - \underbrace{(\hat{\lambda}-\delta)[\omega(1+r_3^D)-(1+r_2^D)]}_{\substack{\text{Lower payoff}\\ \text{given monitoring}}} + \underbrace{(\theta^*-\hat{\lambda})(1+r_2^D)}_{\substack{\text{'Higher' payoff}\\ \text{absent monitoring}}}$$

- Overall, increasing capital reduces the total probability of runs

$$\frac{\partial GG}{\partial k} = \left[\frac{\xi^*(1+r_3^D)}{(1+r_2^D)(1+r^I)-\xi^*(1+r_3^D)} + \delta\right]\omega(1+r_3^D) + (\theta^*-\delta)(1+r_2^D) > 0$$

$$\Rightarrow \frac{\partial \xi^*}{\partial k} < 0$$

Partial effect of regulation on total run probability – Liquidity

- Trade-off from increasing capital: Monitoring more probable versus higher incentives to join full run

$$\frac{\partial GG}{\partial \ell} = \underbrace{\frac{\partial \hat{\lambda}}{\partial \ell}\omega(1-k)(1+r_3^D)}_{\text{More monitoring}} - \underbrace{\int_{\theta^*}^{1} \frac{1-\xi^*}{\lambda} d\lambda}_{\substack{\text{Higher payoff} \\ \text{in full run}}}$$

- Overall, increasing liquidity reduces the total probability of runs (but not always)

$$\frac{\partial GG}{\partial \ell} = (1-\xi^*)\left[\frac{\omega(1+r_3^D)(1+r^I)}{(1+r_2^D)(1+r^I)-\xi^*(1+r_3^D)} + \log\theta^*\right]$$

$$\Rightarrow \frac{\partial \xi^*}{\partial \ell} < 0$$

for $\delta > e^{-1}$, since $\theta^* > \delta$ and $\omega(1+r_3^D) > (1+r_2^D)$

or $\ell > \bar{\ell} \equiv (e^{-1}(1-k)(1+r_2^D) - \xi^*)/(1-\xi^*)$; true for high enough ξ^*

Back to Implementation