Bubbly Firm Dynamics and Aggregate Fluctuations

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Abstract

We study the interaction between asset bubbles and firm dynamics, and their implications for business cycles. We build a firm dynamics model where the value of a firm might contain a bubble component. Consequently, a new transmission mechanism of bubble arises: bubbles incentivize new firms to enter and existing firms to stay. This mechanism results into an increase in the aggregate production. The model predicts the overshooting of the entry rate following an expansionary bubble shock. Empirically, we identify bubble shocks as the shocks that maximize the forecast error variance decomposition of the price-fundamental differential. Our empirical findings validate the predictions of the model.

Keywords: Rational Bubble, Firm Dynamics, Heterogeneous Firms, Bubble Shock Identification, SVAR, Medium-run Restriction
JEL Classification: E32, E44, O40

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1 Introduction

Large swings in asset prices and their consequences for the aggregate economy draw renewed attention from macroeconomists. Jordà *et al.* (2015), Schularick and Taylor (2012) provide evidence that links the boom-bust of asset bubbles with business cycles.Farhi and Tirole (2012), Martin and Ventura (2012, 2016) and Miao and Wang (2018) explore this connection in theoretical models that feature rational bubbles. Galí (2014) studies the implication for the design of the optimal monetary policy. However, there has been almost no work on the firm-level implications of bubbles. At the micro level, bubbles, when attached to firms' value, may affect firms' choices such as their entry and exit decisions, which in turn might exert aggregate effects that are absent in a model without firm-level dynamism.

In this paper, we study the interaction between asset bubble and firm dynamics, and their implications for business cycles, both empirically and theoretically. We emphasize a new transmission channel of bubble shocks through firms' creations and exit decisions that are currently not explored in the literature. To this end, we build a model with heterogeneous firms that make entry and exit decisions, and importantly we relax the No-Ponzi game condition. Consequently, the rational asset bubble arises.

Bubbles affect the selection of firms. On the one hand, for bubbly incumbents, exiting the market incurs the loss of bubbles. Therefore, bubbles make it less likely for firms to exit. On the other hand, for potential entrants, bubbles act as a subsidy to firm entry. As a result, the total number of production units and the aggregate output increase after a positive bubble shock.[1]

One matrix that has attracted considerable attention in the firm dynamics literature is the entry rate. A decrease in the entry rate is often interpreted as the decline in firm dynamism. Our model predicts that a positive bubble shock leads to an increase in the entry rate in the short-run, followed by a persistent decline. We label this phenomenon as the overshooting of the entry rate. This pattern originates from the fact that although both the number of new firms and the number of existing firms increases, the effect of bubble shocks on new firms' entry decisions are dominating in the short run. In the medium run, however, the rising share of bubbly firms suppresses the exit rate, and consequently,

[1] Olivier (2000) model R&D and capital investment, which can be interpreted as firm entry or something alike. But the lack of firm heterogeneity removes the selection effect that changes the distribution of entrants. More importantly, the previous literature does not take into account endogenous firm exit, which plays a crucial role in our model.

gradually boosts the number of firms resulting in a declining share of new firms.

We compare bubbly firms' characteristics to their bubble-less counterpart in the steady-state. Everything else equal, a bubbly firm features lower exit rate, capital, and productivity across the entire life cycle as compared to the corresponding variables of a bubble-less firm. With an asset bubble, firms with lower productivity become more likely to survive. As a result, bubbly firms are, on average, less productive, and thus accumulate less capital.

We provide a set of empirical findings that are novel to the literature, and more importantly, they validate the predictions of our model. First, we identify bubble shocks in a Vector Autoregressive (VAR) model. We find that a positive asset bubble shock, which raises the difference between the value of an asset and its fundamental component—the price-fundamental differential, is expansionary. It affects productivity, real economic activities, and firm entry and exit in a significant and persistent fashion. Interestingly, following a positive bubble shock, the firm entry rate increases in the short run and declines persistently afterward — the overshooting of the entry rate. Our identification strategy relies on the assumption that, once controlled for productivity shocks, and other selected structural shocks such as credit supply, monetary policy and fiscal policy shocks, bubble shocks maximize the movements in the forecast error variance decomposition (FEVD) of the price-fundamental differential in the subsequent periods.

1.1 Literature Review

The paper contributes to a growing literature on asset bubbles. In his seminal work, Tirole (1985) views bubbles as assets without intrinsic value, or pyramid schemes. He argues that bubbles crowd out capital stock and lower output. By incorporating financial frictions, Caballero and Krishnamurthy (2006), Farhi and Tirole (2012), Martin and Ventura (2012, 2016), Miao and Wang (2018) , Bengui and Phan (2018), Ikeda and Phan (2019), Biswas *et al.* (2020) and Asriyan *et al.* (2020) suggest that bubbles relax financial constraints, and therefore, crowd in capital stock.[2] Using models without financial frictions, Olivier (2000), Queiros (2019), and Vuillemey and Wasmer (2020) show that bubbles can also boost economic activity by acting as a subsidy. Our framework contributes to this literature by allowing for firm heterogeneity and firms' entry and exit decisions. We emphasize an alternative transmission channel of asset bubble through firms' endogenous

[2]See also Miao *et al.* (2015) and Larin (2019) for the quantitative importance of bubble shocks as the source of business cycles.

entry and exit decisions. The selection effect of bubbles is hitherto neglected in the literature as the existing models generally abstract from firm heterogeneity and exit decisions. Furthermore, our model suggests that bubble shocks affect the transmission of a productivity shock. The last but not the least, by allowing for firm heterogeneity in asset bubble, we are able to discuss the effects of an asset bubble on firm-level variables.

Our paper is closely related to a growing literature on heterogenous firms and firm dynamics. Recent works by Khan and Thomas (2013), Senga *et al.* (2017), Arellano *et al.* (2019), and Ottonello and Winberry (2018) study firm dynamics and business cycle propagations with imperfect financial markets. Jaimovich and Floetotto (2008), Bilbiie *et al.* (2012), Lee and Mukoyama (2015), Sedláček and Sterk (2017) show that firm entry and exit account for important features of business cycles. Our model is related to Khan and Thomas (2008), Bachmann and Bayer (2014), Clementi and Palazzo (2016), Bloom *et al.* (2018), and Winberry (2020), in the sense that we introduce capital adjustment costs as the major frictions faced by firms. With respect to this literature, our contribution is to allow for rational bubbles. In our model, the size of bubbles is an extra dimension of firm heterogeneity. Bubbles interact with the selection mechanism of firms and directly affect firm entry and exit. Introducing bubble shocks — a potentially important source of fluctuations in the asset market and the aggregate production, we show that bubble shocks have significant impact on the distribution of firms.

The empirical literature on the identification of bubble shocks is limited. This is partly due to the reason that bubbles are difficult to measure. The empirical literature on asset bubbles has mainly focused measuring the size of bubble or testing the existence of bubble. Queirós (2017) applies Campbell and Shiller (1988)'s method to construct measures of the price-fundamental differential at the sectoral level. Jordà *et al.* (2015), Schularick and Taylor (2012) provide evidence that link the boom bust of asset bubbles with business cycles. One exception is Gilchrist *et al.* (2005). The authors use the recursive identification (short-run restrictions) to show that a shock to forecast dispersion, which leads to an increase in the bubble within the context of their model, generates a pronounced increase in investment. We contribute to this empirical line of research by identifying bubble shocks using the medium-run restriction, pioneered by Uhlig (2003, 2004), that are widely used to identify news shocks (Barsky and Sims 2011) and other structural shocks, see e.g., Zeev and Pappa (2017), Ben Zeev *et al.* (2017), and Levchenko and Pandalai-Nayar (2020). Our assumption is consistent with Miao *et al.* (2015)'s finding that bubble shocks explain most of the stock market fluctuations using an estimated DSGE model.

The remainder of the paper is organized as follows. Section 2 provides empirical evidence on the effects of a bubble shock. Section 3 introduces the model. Section 4 presents the predictions of our model. Section 5 concludes.

2 Empirical Analysis

2.1 Description of data

We begin the section with a brief overview of the data we use throughout this paper. The Business Dynamics Statistics (BDS) provide information about firms' entry and exit rates at annual frequency from 1977 to 2016. We interpolate them into quarterly frequency, using a simple linear interpolation. The stock price, dividend and earning of the SP500, and the cyclically adjusted price earnings ration (CAPE) are taken from Shiller (2015), which are updated and made available on the author's website. We take the Gilchrist and Zakrajsek (2012) excess bond premium (EBP henceforth) updated by Favara *et al.* (2016). We construct the time series of monetary shocks following Gertler and Karadi (2015), and fiscal expenditure shocks following Blanchard and Perotti (2002)'s identification strategy. We include Fernald (2014)'s utility adjusted TFP as a measure of TFP. The remaining variables are taken from FRED: the real gross domestic product per capita (real GDP), the gross private domestic Investment, and the civilian unemployment rate that are seasonally adjusted, 10-Year treasury rate and GDP deflator. All nominal variables are rescaled by GDP deflator to obtain their real values.

2.2 The effects of a bubble shock

Before moving to the discussion of our identification strategy, we begin with a simple asset pricing equation that is nested in our model. Let P_t denote the value of a representative infinite-lived asset that yields a stream of dividend $\{D_t\}$. To anticipate what is coming later, in our model firms make endogenous entry and exit decisions. Therefore, there is no single firm-level asset that is infinite-lived with a probability one. We interpret this representative asset as a portfolio of top 500 firms' stock prices: the exact composition of this portfolio changes overtime without affecting the representativeness of this asset. The value (price) of such an asset is the sum of a fundamental component (F_t) and a bubble

(B_t) component:

$$P_t = F_t + B_t,$$

The fundamental component is the net present value of future dividends:

$$F_t \equiv E_t \left\{ \sum_{h=1}^{\infty} \Big(\prod_{j=0}^{h-1} (1/R_{t+j}) \Big) D_{t+j} \right\}.$$

Log-linearize this equation leads to:

$$f_t = c + \sum_{h=0}^{\infty} \Lambda^h \big[(1-\Lambda) E_t\{d_{t+h+1}\} - E_t\{r_{t+h}\} \big], \tag{1}$$

where a variable in lower case denote the log of the original variable, $\Lambda \equiv \frac{\Gamma}{R} < 1$, with Γ and R denote growth rate of dividend and real interest rate in steady state.

Note that f_t is not directly observed in the data. In fact, this is the main challenge behind the construction of an unconditional measure of bubble or the price-fundamental differential. One stream of the literature attempts to construct such an unconditional measure based on a VAR, see e.g., Campbell and Shiller (1988).

Identification Strategy Our identification strategy relies on the assumption that, once controlled for productivity shocks, both the unexpected ones and anticipated ones (news shocks), and selected structural shocks such as credit supply and monetary policy shocks, the shock that maximizes the forecast error variance decomposition of the price-fundamental differential ($p_t - f_t$) in the subsequent periods is a bubble shock. This identification strategy, now labeled as the medium run restriction, is pioneered by Uhlig (2003, 2004) and made popular by Barsky and Sims (2011). It is worth to note that controlling for productivity and other demand shocks merely serve to reduce the likelihood that the identified bubble shocks are confounded with those alternative shocks. We do not claim, and we do not believe in any of those shocks that we control for contribute more to the variations in the forecast error variance decomposition of the price-fundamental differential than bubble shocks do.

Implementation In the baseline, we consider a VAR that consists the following variables: TFP, real GDP (y_t), real dividend (d_t), real stock price (p_t), real interest rate (r_t) and the firm entry (en_t) or exit rate (ex_t). Let $Y_t \equiv [TFP_t, y_t, d_t, p_t, r_t, en_t]'$, the reduced form representation of our VAR model is:

$$Y_t = B(L)Y_t + U_t \tag{2}$$

where L is the lag operator, and B(L) is the matrix of lag order polynomials, and U_t includes the vector of reduced form residuals, which are linear combinations of structural shocks:

$$U_t = B_0^{-1}\zeta_t, \tag{3}$$

where ζ_t denotes a vector of structural shocks normalized to have unit variances, $(B_0^{-1})'B_0^{-1} = \Sigma_u$, where Σ_u is the covariance matrix of the reduced form residuals. It is trivial to get an unbiased estimate $\hat{\Sigma}_u$. However, the identification issue arises because there are more parameters to be estimated in B_0^{-1} than the number of knowns contained in $\hat{\Sigma}_u$. Therefore, structural assumptions are required to overcome this identification problem. One way to understand how to impose structural assumptions is to rewrite B_0^{-1} as:

$$B_0^{-1} = AQ, \tag{4}$$

where A is a lower triangular matrix with $AA' = \Sigma_u$, e.g., the Cholesky decomposition of $\hat{\Sigma}_u$ is a natural candidate for $\hat{A}$, and Q is a orthonormal matrix such that $QQ' = I$. Solving the identification problem boils down to find the orthonormal matrix Q such that identification assumptions are satisfied.

We are interested in one shock, labeled as a bubble shock, hence it is sufficient to identify one column of B_0^{-1} associated with the bubble shock. To do so, we rely on the medium-run restriction. As explained above, the medium-run restriction is problematic if we fail to control for other shocks that might be, albeit unlikely, the main driver of the price-fundamental differential. In the baseline, we control for both unanticipated and anticipated productivity shocks that are, arguably, the main drivers of the business cycle.

To control for unexpected TFP shocks, following Levchenko and Pandalai-Nayar (2020), we include Fernald (2014)'s measure of utility adjusted TFP in our VAR. The first difference of this variable is widely used in the empirical literature as a measure of unexpected productivity shock, see e.g., Garín, Lester and Sims (2019) and Loria, Matthes and Zhang

(2019) for recent applications, and Ramey (2016) for a survey.[3] We assume that unanticipated shocks are the only shocks that affect TFP contemporaneously. With TFP ordered the first, a simple Cholasky decomposition of $\hat{\Sigma}_u$ gives us a lower triangular matrix $\hat{A}$ whose first column is associated with the TFP shocks:

$$B_0^{-1}\zeta_t = \underbrace{\begin{bmatrix} * & 0 & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 & 0 \\ * & * & * & 0 & 0 & 0 \\ * & * & * & * & 0 & 0 \\ * & * & * & * & * & 0 \\ * & * & * & * & * & * \end{bmatrix}}_{\hat{A}} \begin{bmatrix} \epsilon_t^{TFP} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \\ u_{5,t} \\ u_{6,t} \end{bmatrix} \tag{5}$$

In addition, we control for anticipated productivity shocks, i.e., news shocks. Those shocks are identified as a linear combination (Q_1) of remaining reduced form residuals, $[u_{2,t}, ..., u_{6,t}]'$, such that it contributes the most to the cumulative sum of the square of the forecast error of TFP, i.e., $\sum_{h=0}^{H_1}(FE_{t+h|t}^{tfp})^2$. With this Q_1, the second shock is identified as news shocks:

$$B_0^{-1}\zeta_t = \hat{A}Q_1 \begin{bmatrix} \epsilon_t^{TFP} & \epsilon_t^{news} & u_{3,t} & u_{4,t} & u_{5,t} & u_{6,t} \end{bmatrix}'. \tag{6}$$

In appendix B, we explain the procedure to derive Q_1.

The last identification assumption assumes that, once controlled for unanticipated and anticipated news shocks, bubble shocks are the ones that maximize the forecast error variance decomposition of price-fundamental differential. That is, bubble shocks are identified as a linear combination (Q_2) of remaining reduced form residuals, $[u_{3,t}, u_{4,t}, u_{5,t}, u_{6,t}]'$, such that it contributes the most to the cumulative sum of the square of the forecast error of $p_t - f_t$, i.e., $\sum_{h=0}^{H_2}(FE_{t+h|t}^{p/f})^2$, where f_t is defined as in (1). With such an orthonormal matrix Q_2 in place, the third shock is identified as the bubble shock:

$$B_0^{-1}\zeta_t = \hat{A}Q_1Q_2 \begin{bmatrix} \epsilon_t^{TFP} & \epsilon_t^{news} & \epsilon_t^{b} & u_{4,t} & u_{5,t} & u_{6,t} \end{bmatrix}'. \tag{7}$$

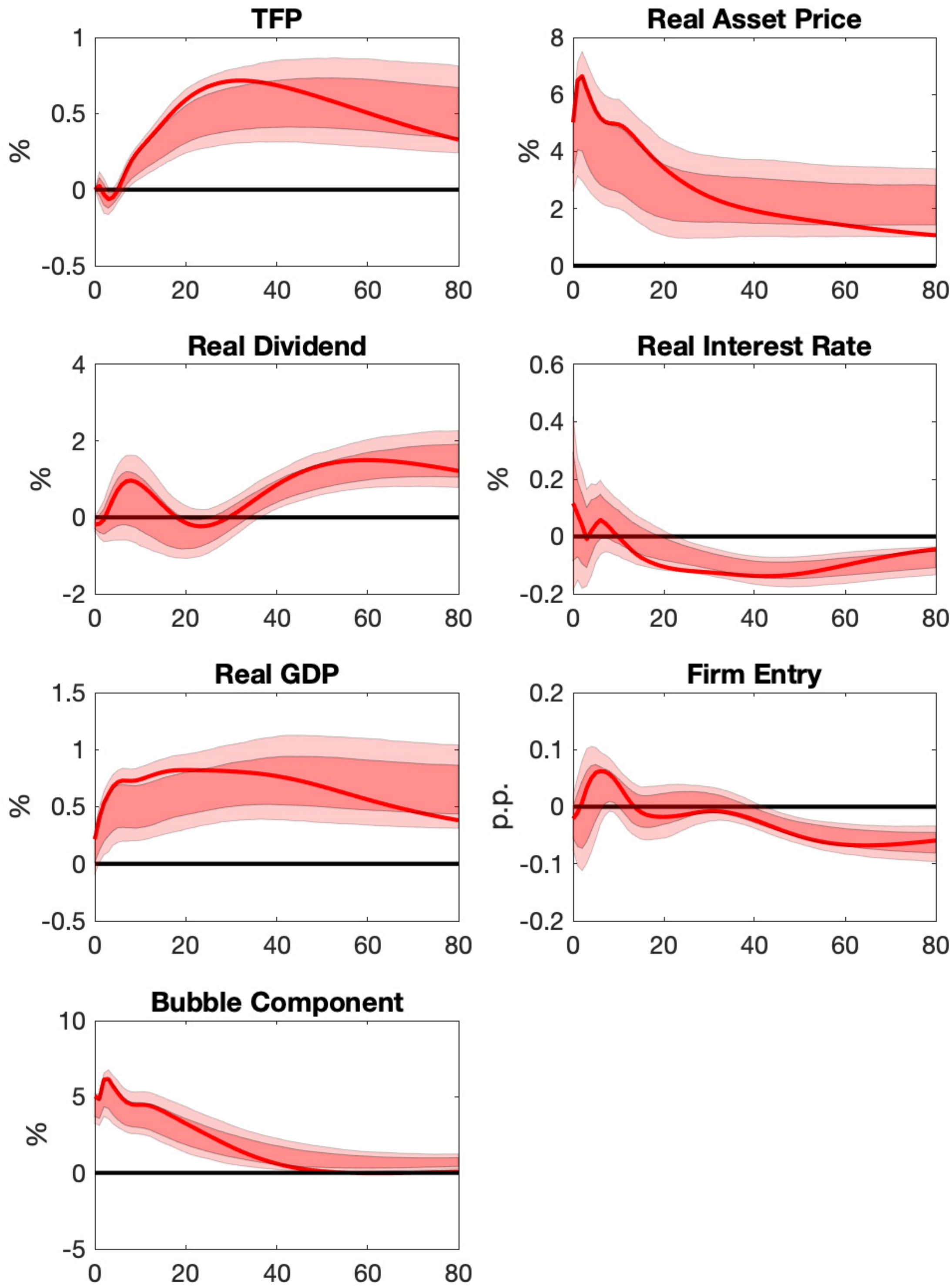
See appendix B, for the formal procedure to derive Q_2.

[3] An alternative popular approach to identify productivity shock replies on long-run restriction (Blanchard and Quah 1989 and Gali 1999), this is not suitable as we show both empirically and theoretically that bubble shocks have very persistent effect on productivity.

We now discuss the relevance of the identification assumptions made above. First, we have assumed that no other shocks affect TFP contemporaneously apart from unanticipated productivity shocks. A failure of this assumption, i.e., if there is another shock that affect TFP contemporaneously, this shock would be captured by our unanticipated productivity shocks. However, this is not an issue given that our goal is to identify the third shock: bubble shock. The unanticipated productivity shocks serve merely as a "control" variable. As long as bubble shocks do not affect TFP contemporaneously, which is the case in our model, this assumption neither leads to an over-prediction nor an under-prediction of bubble shocks. Second, news shocks are assumed to be the ones that maximize forecast error variance decomposition of future TFP. As we have mentioned earlier, and we shall discuss in detail through the lens of our theoretical model, bubble shocks affect productivity persistently, i.e., bubble shocks affect future TFP in a similar fashion as traditional news shocks do. To state the problem differently, the second assumption that aims to capture news shocks might partly capture bubble shocks. Thus, controlling for news shocks put us in a disadvantage since, intuitively, there is less bubble shocks left to be captured by our third shock. As a result, our identification strategy might under predict the importance of bubble shocks. Similarly, without controlling for news shocks, our identified bubble shocks would be confounding with the former. In the empirical application, we take the intermediate case, and control for movement in TFP that are anticipated at a maximum of three years in advance. Third, we assume that, apart from unanticipated TFP shocks and news shocks, bubble shocks contribute the most to the variation in price-fundamental differential in the future. This assumption is consistent with Miao *et al.* (2015)'s finding that bubble shocks explain most of the stock market fluctuations using an estimated DSGE model. Nevertheless, we conduct robustness checks by controlling for more shocks: credit supply, monetary policy and fiscal policy shocks separately.

Results Figure 1 reports the baseline results. Following Kilian (1998), we construct standard errors from 2000 bias-corrected bootstraps. Both the 90% and the 68% confidence bands are included. A positive bubble shock is expansionary: it has persistent positive impacts on real asset price, real dividend, real GDP and TFP. Interestingly, firms' entry rate overshoots: it increases in the short-run, and eventually decreases persistently. Figure 5 plots the estimation results replacing firms' entry rate in the baseline by firms' exit rate. A positive bubble shock leads to a persistent decline in firms' exit rate.

Figure 1: **Impacts of a Bubble Shock on Macro Variables: Baseline**



Controlling for other Shocks In the baseline, we constructed bubble shocks controlling for unanticipated and anticipated TFP shocks. But other shocks might also affect the price-fundamental differential. An omitted shock that contributes substantially to the FEVD of the price-fundamental differential might be confounded with our bubble shocks. Even though there is no theoretical support for this argument, we do address this concern in this section. We control for, separately, the following shocks: Gilchrist and Zakrajsek (2012)'s excess bond premium, monetary shocks identified in a Proxy-VAR using high frequency identified monetary surprises as instrument (Gertler and Karadi 2015), and fiscal expenditure shocks following Blanchard and Perotti (2002)'s identification strategy.

The expansionary effects of bubble shocks on TFP, real GDP and the overshooting pattern of firms' entry rate survive when we control for credit supply shocks (Figure 6), government expenditure shocks (Figure 7), and monetary policy shocks (Figure 8)

3 Model

Our model builds upon a standard firm dynamics model à la Hopenhayn (1992) and Clementi and Palazzo (2016). We extend the standard model by introducing a firm-level bubble component, which raises the equity price of a firm above its net present value of dividends. Bubbles influence the selection mechanism of firms: since firm exit incurs the loss of bubbles, bubbles discourage incumbents to exit. At the end of this section, we show that the aggregate bubble can remain stationary even though idiosyncratic bubbles are explosive.

3.1 Firms

Time is discrete and the horizon is infinite. Firms produce a homogeneous good in a perfectly competitive market. The production technology displays decreasing returns to scale

$$y_{jt} = A_t \varphi_{jt} k_{jt}^{\alpha} \tag{8}$$

with $\alpha \in (0,1)$. A_t denotes a common productivity component that is identical across firms. φ_{jt} is the an idiosyncratic productivity shock. k_{jt} denotes capital stock which is predetermined. The idiosyncratic productivity shock φ_{jt} follows a Markov process

$$\log \varphi_{jt+1} = \rho \log \varphi_{jt} + \varepsilon_{jt+1}, \tag{9}$$

where $\rho \in (0,1)$, $\varepsilon_{jt} \sim N\left(0,\sigma^2\right)$ $\forall t$, $\forall j$.

The start-of-period value of a firm equals

$$V(\lambda,\mu,k) = y(\lambda,k) - c^f + p\max\left\{V^c(\lambda,\mu,k), V^x(k)\right\} + (1-p)V^x(k), \tag{10}$$

where λ represents aggregate state variable, μ represents idiosyncratic state variables other than capital stock k, c^f denotes a fixed operation cost, and p denotes the probability of drawing an idiosyncratic exogenous death shock which forces firm to exit.[4] We assume that, after production, the owners of firms decide whether or not to exit from the market. As for the firms without exogenous death, the owners compare their value of continuation V^c with the value of exit V^x.If firms exit, they disinvest all the capital stock and the owners get the scrap value. Therefore the value of exit is equal to

$$V^x(\mu) = (1-\delta)k - g(k,0),$$

where δ denotes the depreciation rate during production, $g(k,k')$ denotes the cost of adjusting capital stock from k to k'. The adjustment cost follows a standard functional form

$$g\left(k,k'\right) = c_0 1\left\{k \neq k'\right\}k + c_1\left(\frac{k'-(1-\delta)k}{k}\right)^2 k,$$

where $c_0 \in (0,1)$, $c_1 \in (0,1)$. Firms stay for future production and make investment if and only if $V^c(\lambda,\mu,k) \geq V^x(k)$. If a firm remains, the owners can trade the shares in a frictionless market and decide how much to invest for future production. The continuation value of a firm is equal to

$$V^c(\lambda,\mu,k) = \max_{k'}\left\{(1-\delta)k - k' - g\left(k,k'\right) + \int \Lambda(\lambda) V\left(\lambda',\mu',k'\right) dJ\left(\lambda',\mu'|\lambda,\mu\right)\right\}, \tag{11}$$

where $\Lambda(\lambda)$ denotes the stochastic discount factor, and $J(\lambda',\mu'|\lambda,\mu)$ denotes the transition probability of λ and μ. The optimal size of future capital k' can be represented by function $k^*(\lambda,\mu,k)$. As we will discuss later, what μ includes depends on whether we impose no-bubble condition.

[4]We suppress the subscripts to reduce notation, when writing functions of state variables.

3.1.1 No-bubble Condition and the Fundamental Component of a Firm

The conventional solution to the firm optimization problem described by (10) and (11) rules out the existence of bubbles. In the absence of bubbles, the value of a firm is equal to its net present value of cash inflows, which we refer to as the fundamental component of firm value. In the literature, the value of a firm is typically an interchangeable concept of the fundamental component of the firm's value. However, as we will show, the two concepts are not necessarily equivalent. The fundamental component of firm value is equal to In the absence of bubbles, firms' optimization problem can be reformulated into:

$$F^c(\lambda,\mu,k) = \max_{k'}\left\{(1-\delta)k - k' - g(k,k') + \int \Lambda(\lambda')V(\lambda',\mu',k')\,dJ(\lambda',\mu'|\lambda,\mu)\right\}, \quad (12)$$

$$V(\lambda,\mu,k) = y(\lambda,k) - c^f + p\max\{F^c(\lambda,\mu,k), V^x(k)\} + (1-p)V^x(k), \quad (13)$$

$$\lim_{i\to\infty} p^i \int_{\Phi^{(1)}} \Lambda^{(1)} \cdots \int_{\Phi^{(i)}} \Lambda^{(i)} F^c\left(\lambda^{(i)},\mu^{(i)},k^{(i)}\right) dJ\left(\lambda^{(i)},\mu^{(i)}|\lambda^{(i-1)},\mu^{(i-1)}\right) \cdots dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right) = 0. \quad (14)$$

where $\lambda^{(i)}$ and $\mu^{(i)}$ represent the aggregate and idiosyncratic states after i periods, $k^{(i)} = k^*\left(\lambda^{(i-1)},\mu^{(i-1)},k^{(i-1)}\right)$, $\Lambda^{(i)} \equiv \Lambda\left(\lambda^{(i)}\right)$, and $\Phi^{(i)} \equiv \left\{\left(\lambda^{(i)},\mu^{(i)}\right)|V^c\left(\lambda^{(i)},\mu^{(i)},k^{(i)}\right) \geq V^x\left(k^{(i)}\right)\right\}$. $F^c(\lambda,\mu,k)$ represents the fundamental component of continuation value. Clearly (12) and (13) are rewritten from (10) and (11) when we impose $V^c(\lambda,\mu,k) = F^c(\lambda,\mu,k)$. Next we show that, when combine with (14), (12) and (13) imply that $F^c(\lambda,\mu,k)$ is equal to the net present value of cash inflows when firms continue to produce.

Equation (12) can be recursively expanded into

$$\begin{aligned}
F^c(\lambda,\mu,k) = {} & (1-\delta)k - k^{(1)} - g\left(k,k^{(1)}\right) + \int \Lambda^{(1)}\left[y\left(\lambda^{(1)},k^{(1)}\right) - c^f\right] dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right) \\
& + p\int_{\Psi^{(1)}} \Lambda^{(1)} V^x\left(k^{(1)}\right) dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right) + (1-p)\int \Lambda^{(1)} V^x\left(k^{(1)}\right) dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right) \\
& + p\int_{\Phi^{(1)}} \Lambda^{(1)} F^c\left(\lambda^{(1)},\mu^{(1)},k^{(1)}\right) dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right), \quad (15)
\end{aligned}$$

where $\Psi^{(i)} \equiv \left\{\left(\lambda^{(i)},\mu^{(i)}\right)|V^c\left(\lambda^{(i)},\mu^{(i)},k^{(i)}\right) < V^x\left(k^{(i)}\right)\right\}$. We define cash inflow $\pi(\lambda,\mu,k)$

as:

$$\pi\left(\lambda,\mu,k\right)=\left(1-\delta\right)k-k^{(1)}-g\left(k,k^{(1)}\right)+\int\Lambda^{(1)}\left[y\left(\lambda^{(1)},k^{(1)}\right)-c^{f}\right]dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right)$$
$$+p\int_{\Psi^{(1)}}\Lambda^{(1)}V^{x}\left(k^{(1)}\right)dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right)+\left(1-p\right)\int\Lambda^{(1)}V^{x}\left(k^{(1)}\right)dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right).$$

Cash inflow $\pi\left(\lambda,\mu,k\right)$ is the sum of net capital increment $\left(1-\delta\right)k-k^{(1)}-g\left(k,k^{(1)}\right)$, plus the discounted dividend in the next period. By repeatedly expanding the last term on the right hand side of (15), the fundamental component $F^{c}\left(\lambda,\mu,k\right)$ can be expressed as

$$F^{c}\left(\lambda,\mu,k\right)=\pi\left(\lambda,\mu,k\right)+p\int_{\Phi\left(k^{(1)}\right)}\Lambda^{(1)}\pi\left(\lambda^{(1)},\mu^{(1)},k^{(1)}\right)dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right) \tag{16}$$
$$+p^{2}\int_{\Phi^{(1)}}\Lambda^{(1)}\int_{\Phi^{(2)}}\Lambda^{(2)}\pi\left(\lambda^{(2)},\mu^{(2)},k^{(2)}\right)dJ\left(\lambda^{(2)},\mu^{(2)}|\lambda^{(1)},\mu^{(1)}\right)dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right)+\cdots$$
$$+\lim_{i\to\infty}p^{i}\int_{\Phi^{(1)}}\Lambda^{(1)}\cdots\int_{\Phi^{(i)}}\Lambda^{(i)}\pi\left(\lambda^{(i)},\mu^{(i)},k^{(i)}\right)dJ\left(\lambda^{(i)},\mu^{(i)}|\lambda^{(i-1)},\mu^{(i-1)}\right)\cdots dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right)$$
$$+\lim_{i\to\infty}p^{i}\int_{\Phi^{(1)}}\Lambda^{(1)}\cdots\int_{\Phi^{(i)}}\Lambda^{(i)}F^{c}\left(\lambda^{(i)},\mu^{(i)},k^{(i)}\right)dJ\left(\lambda^{(i)},\mu^{(i)}|\lambda^{(i-1)},\mu^{(i-1)}\right)\cdots dJ\left(\lambda^{(1)},\mu^{(1)}|\lambda,\mu\right).$$

Equation (14) implies that the last term of (16) is equal to zero. Therefore the fundamental component $F^{c}\left(\lambda,\mu,k\right)$ is equal to the net present value of cash inflows upon continuation. When we impose $V^{c}\left(\lambda,\mu,k\right)=F^{c}\left(\lambda,\mu,k\right)$, the firm-level states are fully summarized by φ and k, since the cash inflows depend only on φ and k, and firms only consider their fundamental components when solving the optimization problem. In this scenario, μ is equivalent to idiosyncratic productivity shock φ.

3.1.2 Bubbles

We now turn to a more general form of solutions

$$V^{c}\left(\lambda,\mu,k\right)=F^{c}\left(\lambda,\mu,k\right)+B, \tag{17}$$

where $F^{c}\left(\lambda,\mu,k\right)$ is characterized by Equation (16) and subject to condition (14). B represents the deviation of the continuation value from its fundamental component. We name it bubbles. The solution we discussed previously is a special case when $B=0$. Following the literature, we model B as a Ponzi-game following

$$B = \int \Lambda\left(\lambda'\right) B' dJ\left(\left(\lambda', \mu' | \lambda, \mu\right)\right). \tag{18}$$

Equation (18) implies that bubbles yields the same return as fundamentals. It can also be infered from Equation (18) that bubbles yields the same return as fundamentals. We can conclude that the existence of this bubble does not violate the optimaility of investors, in other words, the bubble is rational.

We assume that future bubble B' is stochastic and subject to investor sentiment shocks. In the next period, if a firm decides to continue, its bubble component evolves according to

$$B' = \begin{cases} 0, & \text{with } 1 - p^b \\ \left(\Lambda\left(\lambda'\right) \cdot p^b \cdot p \cdot p^s\left(\lambda, \mu, k'\right)\right)^{-1} B, & \text{with } p^b \end{cases} \tag{19}$$

where probability $p^s\left(\lambda, \mu, k'\right)$ is defined as

$$p^s\left(\lambda, \mu, k'\right) \equiv \int_{\Phi'} dJ\left(\lambda', \mu' | \lambda, \mu\right) \tag{20}$$

where $\Phi' \equiv \left\{\left(\lambda', \mu'\right) | V^c\left(\lambda', \mu', k'\right) \geq V^x\left(k'\right)\right\}$. With probability $1 - p^b$, investors become pessimistic about a firm, and its bubble component crashes, i.e., $B' = 0$. With probability p^b, the bubble component rolls over. Besides, bubbles become zero if firms exit. If we substitute B' in Equation (18) by Equation (19), we get

$$\begin{aligned} B &= \int_{\Phi'} \Lambda\left(\lambda'\right) \cdot p \cdot p^b \cdot \left(\Lambda\left(\lambda'\right) \cdot p^b \cdot p \cdot p^s\left(\lambda, \mu, k'\right)\right)^{-1} B dJ\left(\lambda', \mu' | \lambda, \mu\right) \\ &= \left(p^s\left(\lambda, \mu, k'\right)\right)^{-1} B \int_{\Phi'} dJ\left(\lambda', \mu' | \lambda, \mu\right) \\ &= B \end{aligned}$$

Obviously, (19) guarantees (18) to hold. It is rather straightforward to verify that Equations (17) and (19) solve (10) and (11), as long as $k' = k^*\left(\lambda, \mu, k\right)$. The process ensures that bubbles are consistent with investor optimality and rational expectation, while maintaining the model's tractability. Bubbles follow a "backward looking" process as we can pin down the size of future bubbles (if bubble crashes do not take place) given the current bubble size and discount factor. The "backward" feature of the process facilitates our analysis since we can keep track of the dynamics of bubbles on a given route.[5] A key feature of (19) is idiosyncratic bubble crashes, hence our model allows for the coexistence of bubbly and bubble-less firms. Equation (19) also implies that the growth rate of bubbles is increasing in the likelihood of bubble crash and firm exit. Intuitively, bubble have to grow fast enough to compensate for the possible loss if firms are likely to exit, or if bubbles are likely to crash. There are indeed infinite possible processes of bubbles that are consistent with investor rationality, yet we believe our approach is the simplest baseline to incorporate idiosyncratic bubble crashes.[6]

There are four features that are worth discussing at this point. Firstly, throughout our analysis, we take as given that there exist unique $V^c(\lambda, \mu, k)$, $k^*(\lambda, \mu, k)$, and $p^s(\lambda, \mu, k')$. Nonetheless, their existence and uniqueness are not analytically guaranteed. In our numerical exercise, $V^c(\lambda, \mu, k)$, $k^*(\lambda, \mu, k)$, and $p^s(\lambda, \mu, k')$ are solved iteratively, and we find that the algorithm converges to the same solution given different initial guesses. The details of the numerical method can be found in Appendix .

Secondly, firms exit if and only if $F^c(\lambda, \mu, k) + B \geq V^x(k)$. Whether to continue is dependent on the size of bubbles. The decision in turn affects the cash inflow and thus the fundamental component of a firm. Now idiosyncratic states μ of a firm include the size of its bubble B, in addition to φ.[7]

Secondly, idiosyncratic states μ of a firm include the size of its bubble B, in addition to φ, if $B > 0$. Firms exit if and only if $F^c(\lambda, \mu, k) + B < V^x(k)$. If $F^c(\lambda, \mu, k) + B$ is dependent on the value of B, then firm exit decision, cash inflow, and fundamental component $F^c(\lambda, \mu, k)$ are all dependent on B. Therefore B is a state variable. If instead, $F^c(\lambda, \mu, k) + B$ is independent on B, then the fundamental component can be represented by a function $F^c(\lambda, \mu, k) = f^c(\lambda, \varphi) - B$. This is obviously self-contradicting.

[5]See Gali (2016), and Martin and Ventura (2016) for the examples of backward-looking bubbles. Both papers point out that there are usually multiple possible process of bubbles that are consistent with investor rationality. On the contrary, Farhi and Tirole (2012), Miao and Wang (2019) characterize the size of bubbles through forward-looking problems. Dong et al. discuss the differences between the two approaches.

[6]For example, consider the following alternative for (19)

$$B' = \begin{cases} 0, & \text{with } 1 - p_1^b - p_2^b \\ a\left(\Lambda(\lambda') \cdot p_1^b \cdot p \cdot p^s(\lambda, \mu, k')\right)^{-1} B & \text{with } p_1^b \\ (1-a)\left(\Lambda(\lambda') \cdot p_2^b \cdot p \cdot p^s(\lambda, \mu, k')\right)^{-1} B, & \text{with } p_2^b \end{cases}$$

How to choose *the* process is still an open question in this field, as it is pointed out by Martin and Ventura (2018).

[7]Suppose that $B > 0$ while μ does not include B. Then exit decision is independent to the size of bubbles

Thirdly, the existing literature of rational bubbles focuses on aggregate bubble shocks that shift the entire economy between a bubble state and a bubble-less state.[8] In our model, aggregate bubble shocks are captured by changes in p^b. A decrease in p^b implies a lower likelihood to roll over the bubble component. If p^b drops to zero, the entire economy shifts to a bubble-less state.

Last but not least, bubbles have to be sustainable, i.e., the aggragate amount of bubbles must be stationary relative to the aggregate output. The economy would eventually run out of its resources if bubbles persistently outgrow its output, and thereafter such an equilibrium does not exist. In Section 3.3 we discuss the condition under which a bubbly equilibrium is sustainable.

3.2 Households and Firm Entry

The economy is populated with T generations of risk-neutral individuals, which survive for T periods before getting replaced by a new cohort of size M_t. The size of new cohorts increase over time at the rate g

$$M_{t+1} = (1+g)\, M_t \tag{21}$$

The representative individual of a new cohort maximized its lifetime utility

$$U_t = E_t \sum_{i=1}^{T} \beta^{i-1} C_{t+i-1} \tag{22}$$

where $\beta \in (0,1)$ is the discount factor, and C_t denotes the concumption of the homogeneous goods. Accordingly, the lifetime utility of an idividual of any age equals its discounted sum of consumption in its remaining life.

The representative individual from a new cohort is endowed with opportunities to create new firms. We assume every individual has a unit continuum of projects, which has the potential to become a firm that produces in subsequent periods. We call these projects "potential entrants". Each potential entrant has its own realization of productivity and bubble. The productivity of a potential entrant is drawn from the distribution

$$\varphi_t \sim \log N\left(\mu_0, \sigma_0^2\right)$$

[8]For instance, see Martin and Ventura (2012), and Miao and Wang (2018).

Again, with probability p^b, investors are optimistic about a potential entrant. If investors are optimistic, potential entrants receive an initial bubble B_0 prior to their entry.

Analogous to incumbents, potential entrants can only issue tradable equity if they decide to produce in the subsequent period, i.e., to enter the market. Entering the market incurs an entry cost c^e. Potential entrants enter the market if and only if the value of entry is no less than the entry cost

$$V^e(\lambda, \mu) \geq c^e \tag{23}$$

where the value of entry follows

$$V^e(\lambda, \mu) = V^c(\lambda, \mu, 0) \tag{24}$$

where 0 means entrants start with zero capital stock. Following (17), the value of entry can be decomposed into the sum of the fundamental component and the bubble component, which is equal to B_0 among the bubbly potential entrants.

We assume that the households own all the firms in the economy. Even though households live for a finite horizon, the firms they create can operate and be traded indefinitely. Shares are traded in a frictionless market. The supply of each firm's share is normalized to one. Individuals of age larger than one face the following budget constraint

$$C + \int \left\{ V^c(\lambda, \mu, k) + \left[k' - (1-\delta)k + g\left(k, k'\right)\right] \right\} \theta'\left(d\left(\lambda \times \mu \times k\right)\right) = \int V(\lambda, \mu, k)\, \theta\left(d\left(\lambda \times \mu \times k\right)\right),$$

where $\theta(\lambda, \mu, k)$ stands for the number of shares for the firms with state (λ, μ, k). Similarly, individuals of age 1 face the following budget constraint

$$C + \int \left\{ V^c(\lambda, \mu, k) + \left[k' - (1-\delta)k + g\left(k, k'\right)\right] \right\} \theta'\left(d\left(\lambda \times \mu \times k\right)\right) = \int \max\left\{V^e(\lambda, \mu) - c^e, 0\right\} \Gamma(d\mu),$$

where $\Gamma(\mu)$ represents the distribution of μ, i.e., B and φ, for potential entrants.

The household optimization problem described above underlies Equation (11) and the stochastic discount factor $\Lambda(\lambda')$. An interior solution implies that $\Lambda(\lambda') = \beta$. However, a corner solution implies that $\Lambda(\lambda') < \beta$, and all individuals of age less than T consume nothing. Only the individuals of age T sell off their shares and consume all the wealth they own. Note that in equilibrium it is impossible to have $\Lambda(\lambda') > \beta$, otherwise no shares are purchased and the economy cannot exist anymore. Throughout the paper, we exclude the corner solution from our analysis and impose $\Lambda(\lambda') = \beta$ along the equilibrium paths we study. When conducting numerical analysis, we can compare the aggregate consumption with the wealth of the oldest cohort. If the consumption is higher than the wealth of the oldest cohort, other cohorts must consume positive amount of goods, and thus $\Lambda(\lambda') = \beta$ is valid.

3.3 Bubbly Balanced-growth-path

A dynamic equilibrium consists of value functions, decision rules, price functions, and the measure of firms, which satisfy the optimality conditions of firms and households, and clear all the markets. Besides, in equilibrium, the law of motion of firm measures is consistent with the decision rules of investment, entry, and exit. We introduce a formal definition of an equilibrium in Appendix . Here we focus our discussion on the dynamics along balanced-growth-paths (henceforth BGP). Along a BGP, the relative composition of firms remain stable, while the total amount of firms increase at rate g. In Section , We calibrate the BGP and study the transition dynamics around it.

Since all shares in the economy are traded and owned by risk-neutral individuals who maximize their lifetime utility (22), the stochastic discount factor $\Lambda(\lambda) = \beta$. From (19), it is obvious that $B' > \beta^{-1}B$ if a bubble rolls over. Hence at the firm level, there exists no upper bound for the size of bubbles. However, the aggregate bubble has to be bounded relative to the aggregate output for the equilibrium to be sustainable. It is worth noting that, in the presence of firm entry and exit, the firm-level explosiveness of bubbles does not necessarily lead to the explosiveness of aggragate bubble, since every period firms with large bubbles may exit the market and get replaced by new firms with smaller bubbles. Next we show that, if $\beta(1+g)$>1, a bubbly BGP is sustainable.

In the equity market, the total amount of bubbles B_A is equal to

$$B_A = B_I + B_N,$$

where B_I denotes the bubble of continuing incumbents, B_N denotes the bubble of potential entrants which decide to enter. According to (19), the bubble of continuing incumbents in the next period is equal to

$$\begin{aligned} B_I' &= \int \left[\left(\beta \cdot p^b \cdot p \cdot p^s(\lambda, \mu, k') \right)^{-1} B \right] \cdot \left[p^b \cdot p \cdot p^s(\lambda, \mu, k') \right] \eta\left(d\left(\varphi \times B \times k' \right) \right) \\ &= \beta^{-1} \int B \eta\left(d\left(\varphi \times B \times k' \right) \right), \end{aligned} \tag{25}$$

where $\eta(\varphi, B, k')$ represents the distribution of the incumbents and potential entrants that decide to produce in the following period. The right-hand-side of (25) equals $\beta^{-1}\hat{B}$. Therefore the law of motion for the aggregate bubble is

$$B'_A = \beta^{-1} B_A + p^{b'} B_0 M'. \tag{26}$$

From (26) we know that the aggregate bubble of incumbents grow at the rate of investment return. The bubble-output ratio follows

$$\frac{B_A}{Y'} = \beta^{-1} \frac{B_A}{Y} \frac{Y}{Y'} + \frac{p^{b'} B_0 M'}{Y'}. \tag{27}$$

On a BGP the measure of entrants M and aggregate output Y both grow at rate g. p^b remains fixed. We can thus rewrite (21) into

$$b' = [\beta(1+g)]^{-1} b + b_0,$$

where b denotes $\frac{B_A}{Y}$, b_0 denotes $\frac{p^b B_0 M_0}{Y_0}$. Bubble-output ratio follows a stationary process as long as $\beta(1+g)$>1. Indeed the ratio stays at a constant $\left(1 - \beta(1+g)^{-1}\right)^{-1} b_0$. Insofar as the ratio is bounded, we can assume that bubbles do not completely absorb the resources in the economy.

We have shown that bubble-output ratio stays at a constant on a BGP. At the firm-level, bubbles grow faster than the rate of investment return because of the likelihood of bubble crash and firm exit. In aggregate, however, due to firm exits and idiosyncratic bubble crashes, the aggregate bubble from continuing incumbents grow at the same rate of investment return. A BGP is sustainable as long as the growth rate of output exceeds the rate of investment return.

4 Results

Calibration Table 1 reports parameter values used in our simulation exercises. The model is in annual frequency. Panel A reports parameters that are pre-assigned. Following Hennessy and Whited (2007), we set α that governs the degree of decreasing return to scale to 0.65. The process for the idiosyncratic shock is estimated by Imrohoroglu and Tüzel (2014). We choose the exogenous death probability, $1 - p$, to equals to the exit rate

Panel A: Fixed Parameters

Parameter	Description	Value
α	Decreasing returns to scale	0.65
ρ	Idiosy. shock persistence	0.7
σ	Idiosy. shock volatility	0.3764
$1-p$	Prob. of a death shock	0.04
δ	Depreciation rate	0.1
β	Discount factor	0.98
g	Growth rate	2.42%

Panel B: Estimated Parameters

Parameter	Description	Value
μ_0	Average productivity of new entrants	2.278
σ_0	Std of productivity of new entrants	0.01
b_0	Initial bubble component	84.48
c_f	Fixed cost of production	9.25
c_0	Fixed adjustment cost	10^{-5}
c_1	Variable adjustment cost	0.021
c_e	Entry cost	67.26
p_b	Surviving probability of a bubble	0.919

Table 1: Parameters

of the left censored firms (the oldest corhort) in 2016 according to the BDS. The depreciation rate δ is equal to 10%. The growth rate, g, is equal to 2.6%, the average growth rate o the real GDP. The discount factor is set to be 0.98, which corresponds to a real interest rate smaller than g to ensure the stability of the aggregate bubble.

Heterogeneity between the lifecycle of Bubbly and bubble-less Firms Figure 2 shows the evolution of the exit rate, productivity and capital of an average firm by age. We plot the evolution both for an average bubbly and an average bubble-less firm, the later labels those firms whose asset prices exceed their fundamental values. For both groups of firms, their capital and productivity grow as the firms ages. As a result, older firms are less likely to exit. Everything else equal, a bubbly firm features a lower exit rate, a capital of smaller size, and a lower productivity across the entire life cycle as compared to the respective variables of an average bubble-less firm. With an asset bubble, firms have less incentive to exit, and firms with lower productivities have more incentive to enter. Consequently, bubbly firms are on average less productive who later on accumulate less

Moment	Data	Model
Average entry rate	0.104	0.117
Share of two-year-old establishments	0.07	0.09
Exit rate of one-year-old firms	0.243	0.105
Exit rate of three-year-old firms	0.158	0.091
Shiller's CAPE	20.6	20.6
Investment inaction rate	0.081	0.085
Average investment rate	0.122	0.170
Standard deviation of investment rate	0.337	0.180

Table 2: Calibration Targets and Model Fit

capitals.

Figure 2: Firms' life cycles

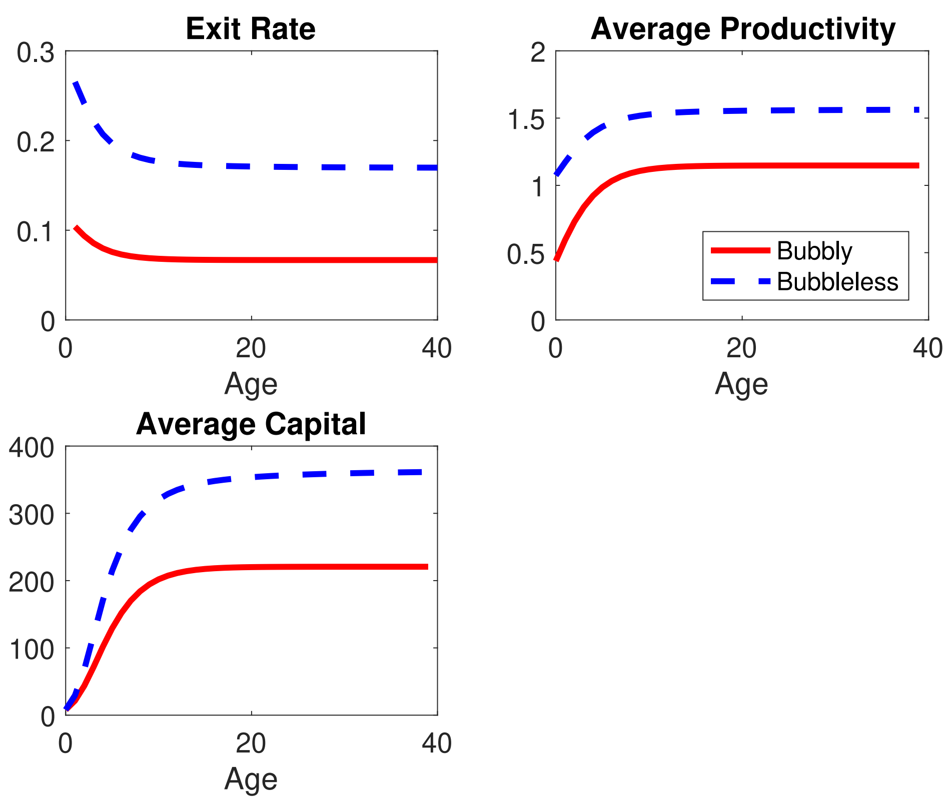
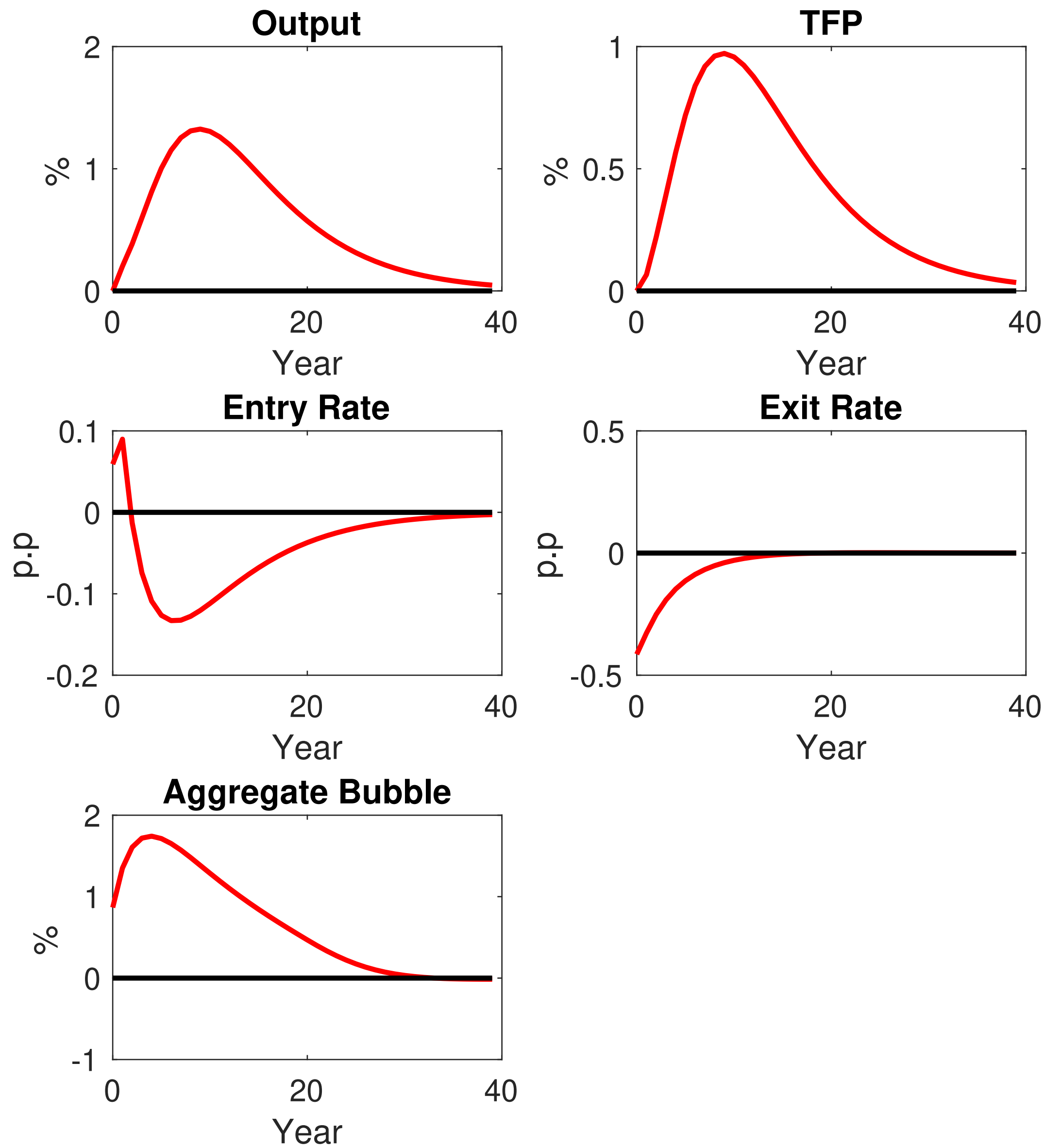


Figure 3: **Impulse Responses to a Bubble Shock**



Bubble shocks as the source of business cycle fluctuations We now analyze the impact of a bubble shock. To this end, we feed in an exogenous evolution of p^b starting from the stationary steady-state. The impulse response functions are then computed under the perfect foresight assumption. Figure 3 plots the result. A positive bubble shock leads to a persistent increase in the aggregate output. This is entirely driven by the increase in the number of production units in the economy. [9] The increased total number of firms, albeit not plotted, is resulted from the decline of exit rate, and the increased number of entrants. Firms have more incentive to enter the market and are less likely to exit if the aggregate asset bubble in the economy is bigger. Interestingly, the entry rate exhibits an overshooting pattern. This is due to the fact that the number of entrants jumps on impact and the total number of firms accumulates slowly in response to an expansionary bubble shock. The aggregate TFP, measured as the aggregate output divided the aggregate capital by the power of α[10], is improved after a positive bubble shock thanks to the decreasing return to scale and the the fact that those marginal firms who decided to stay or just entered because of the raised asset bubble are smaller in size.

5 Conclusion

We have demonstrated, both in the data and in a model with firm dynamics, that following a positive bubble shock the aggregate output and TFP increase, and the average exit rate declines. More interestingly, the popular measure of firm dynamism — the entry rate overshoots in the short run followed by a persistent decline. Our model suggests that the key mechanism that generates those findings is through the effect of an asset bubble on firms' endogenous entry and exit decisions. Asset bubble motivates new firms to enter and existing firms to stay, therefore, increases the total number of production units. The later leads to a boom.

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[9]In our model, the increase in the number of firms has limited crowding-out effect on the production of existing firms since the real interest rate is constant. In a richer model, the real interest rate would likely to increase after a bubble shock that raises number of firms. Consequently, it crowds-out the investment of other firms. However, this channel is inconsistent with our empirical finding that the real interest rate decreases after an expansionary bubble shock.

[10]In log, $tfp = y_t - \alpha k_t$.

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A Figures

Figure 4: Robustness Check I: Real Earning

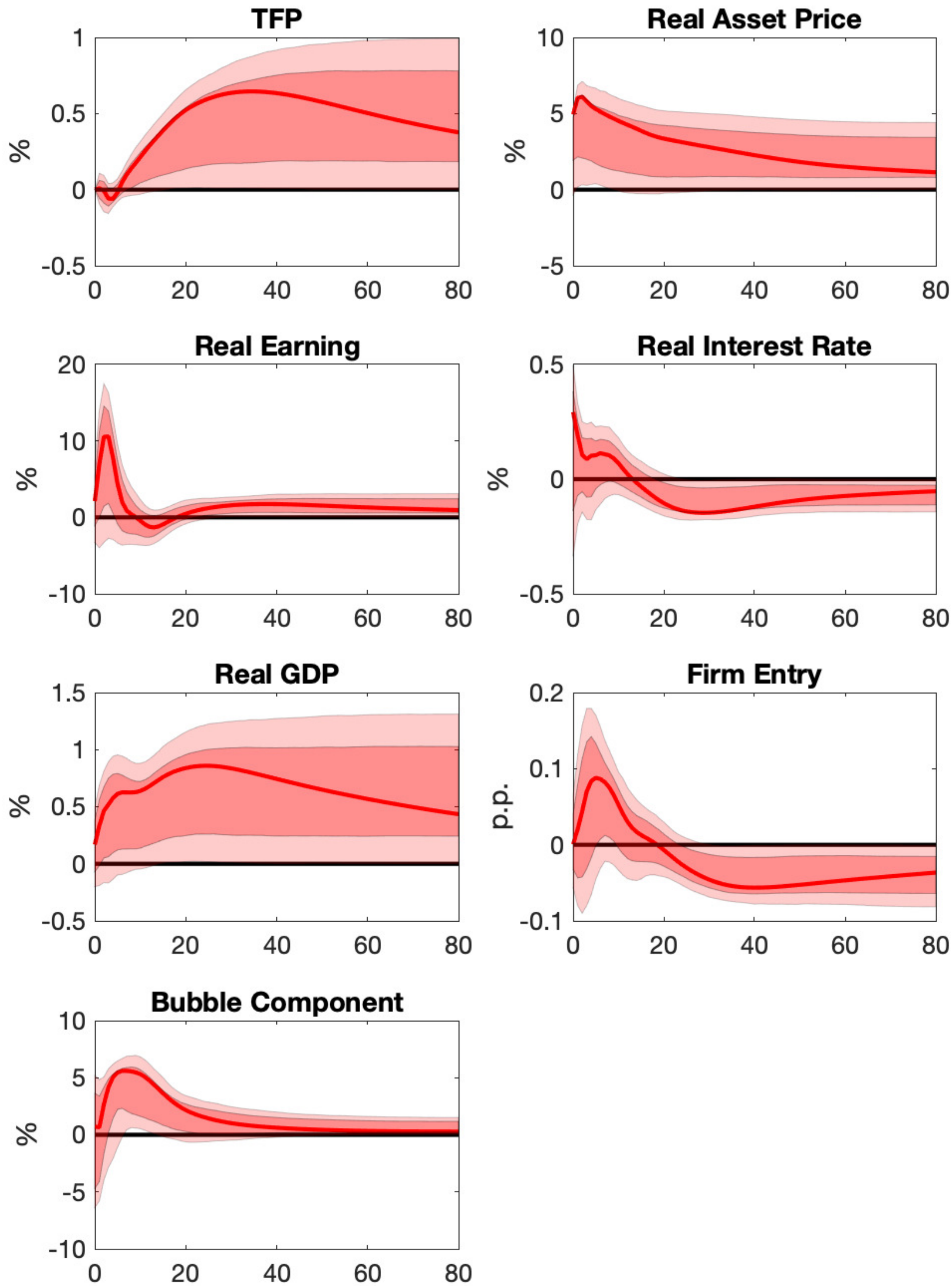


Figure 5: Impacts of a Bubble Shock on Firm Exit

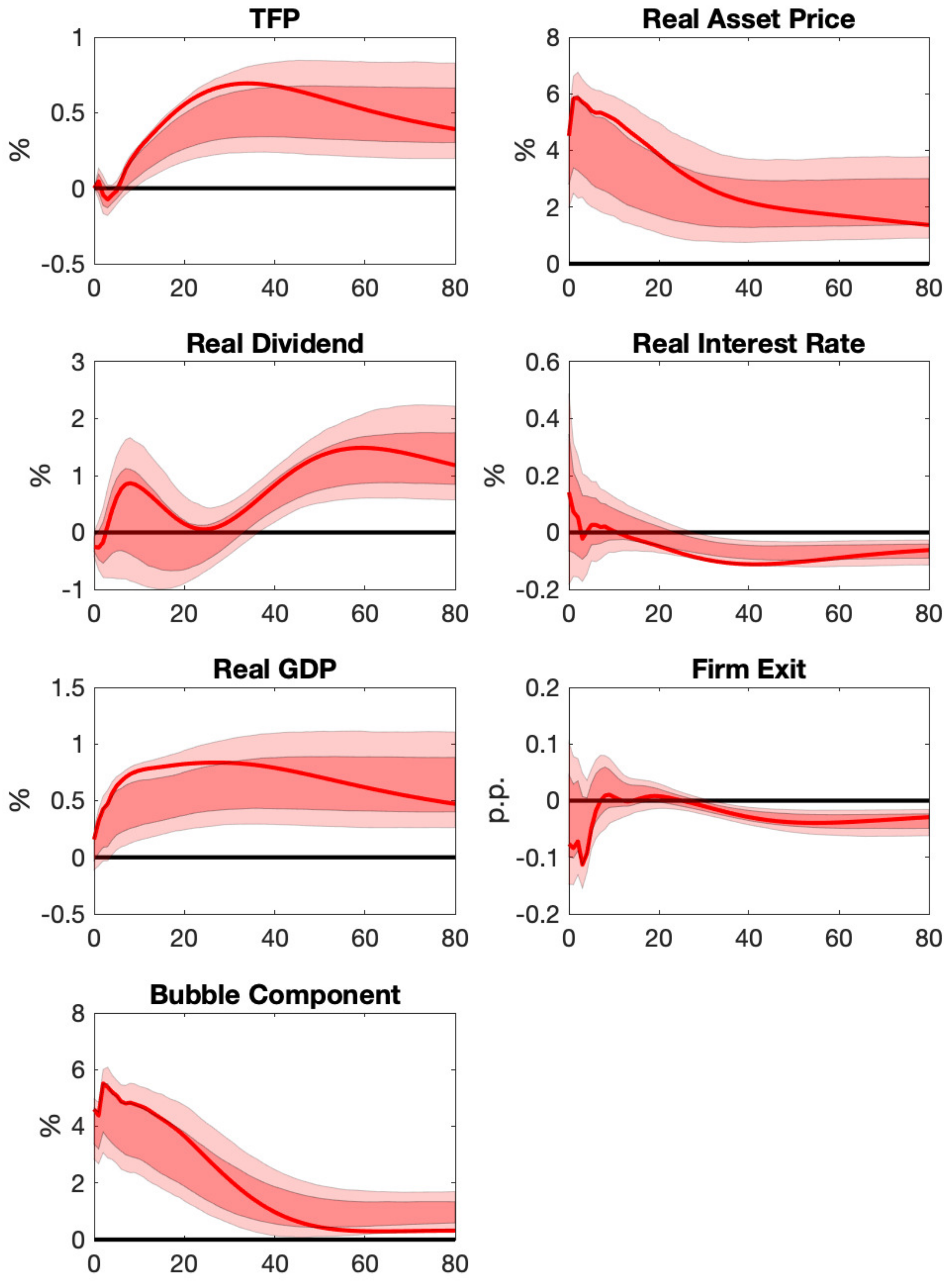


Figure 6: **Robustness Check II: Control for Credit Supply Shocks**

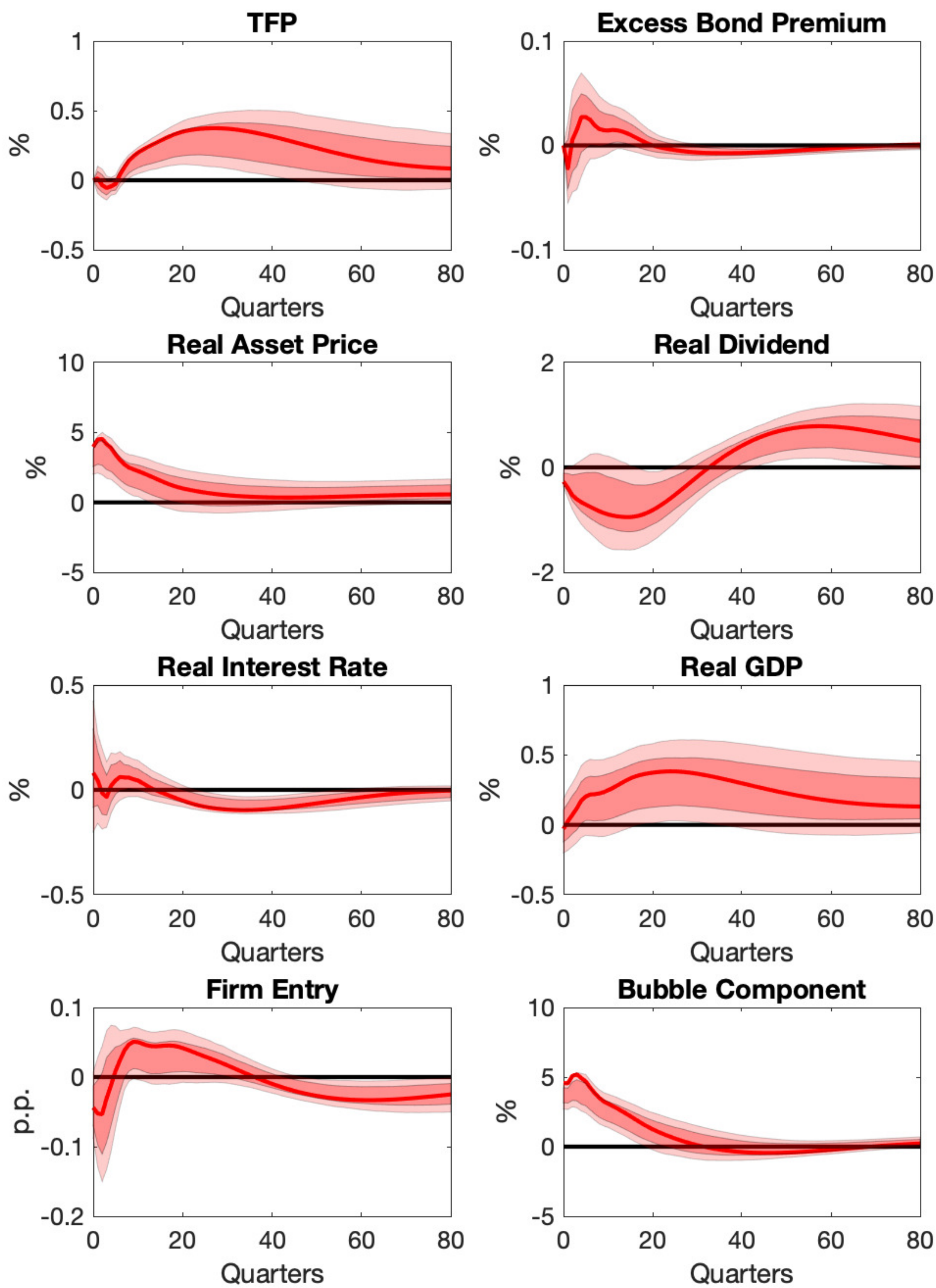


Figure 7: **Robustness Check III: Control for Fiscal Policy shocks**

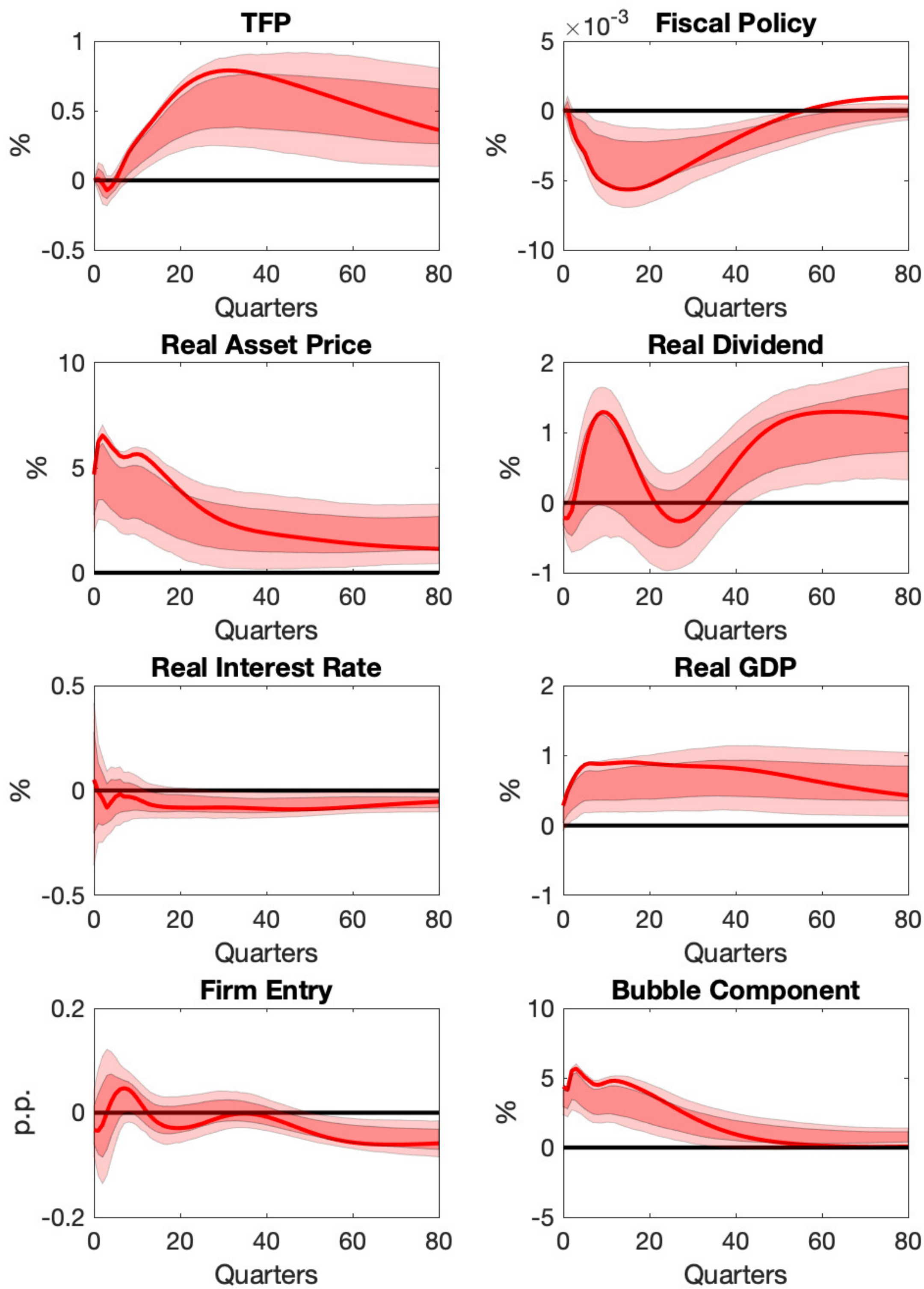
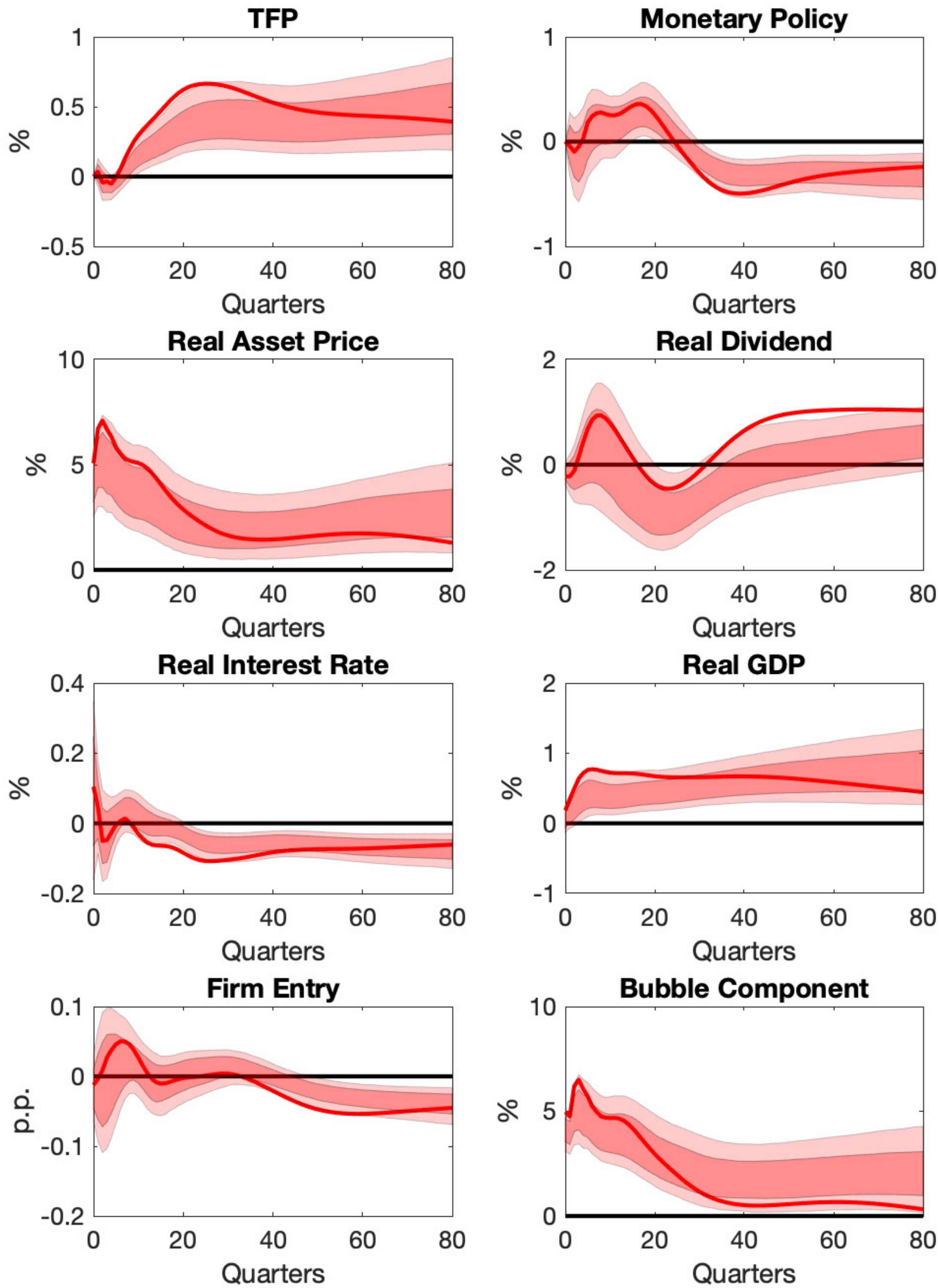


Figure 8: **Robustness Check IV: Control for Monetary Policy shocks**



B Medium run restriction

The staring point is to obtain impulse responses to reduced form residuals. Those can be obtained by considering our VAR(p) in companion form:

$$\mathbb{Y}_t = \mathbb{A}\mathbb{Y}_{t-1} + \mathbb{U}_t,$$

where

$$\mathbb{Y}_t \equiv \begin{bmatrix} Y_t \\ \vdots \\ Y_{t-p+1} \end{bmatrix}, \mathbb{A} \equiv \begin{bmatrix} B_1 & B_2 & \dots & B_{p-1} & B_p \\ I_K & 0 & & 0 & 0 \\ 0 & I_K & & 0 & 0 \\ \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & I_K & 0 \end{bmatrix}, \mathbb{U}_t \equiv \begin{bmatrix} U_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

where K is the number of variables. Solve this equation forward:

$$\mathbb{Y}_{t+h} = \mathbb{A}^{h+1}\mathbb{Y}_{t-1} + \sum_{j=0}^{h} \mathbb{A}^j \mathbb{U}_{t+h-j},$$

Multiply this equation by $\mathbb{J} \equiv [I_K, 0_{K\times K(p-1)}]$ yields:

$$\begin{aligned} Y_{t+h} &= \mathbb{J}\mathbb{A}^{h+1}\mathbb{Y}_{t-1} + \sum_{j=0}^{h} \mathbb{J}\mathbb{A}^j \mathbb{U}_{t+h-j} \\ &= \mathbb{J}\mathbb{A}^{h+1}\mathbb{Y}_{t-1} + \sum_{j=0}^{h} \mathbb{J}\mathbb{A}^j \mathbb{J}'\mathbb{J}\mathbb{U}_{t+h-j} \\ &= \mathbb{J}\mathbb{A}^{h+1}\mathbb{Y}_{t-1} + \sum_{j=0}^{h} \mathbb{J}\mathbb{A}^j \mathbb{J}' U_{t+h-j}. \end{aligned}$$

Therefore, the response of the variable $j = 1, \dots, K$ to a reduced form residual u_{kt} that occurred h periods ago, is given by:

$$\Phi_h \equiv [\phi_{jk,h}] \equiv \mathbb{J}\mathbb{A}^h\mathbb{J}'.$$

The h-step ahead forecast error is:

$$Y_{t+h} - Y_{t+h|t} = \sum_{i=0}^{h-1} \Phi_i U_{t+h-i}.$$

Hence the MSFE at horizon h is:

$$MSFE_h = \sum_{i=0}^{h-1} \Phi_i \Sigma_u \Phi_i'.$$

Identifying News Shocks By imposing structural assumptions through an orthonormal matrix Q, the structural impulse response is given by $\Phi_h AQ$. And recall that A is the lower triangular matrix resulting from the Cholesky decomposition of Σ_u. With TFP ordered the first, the first shock is identified as unanticipated productivity shock under the assumptions that no other shocks can affect TFP contemporaneously. News shocks are identified as the linear combination (Q_1) of remaining reduced form residuals that contribute the most to the MSFE of TFP at horizons up to H_1.

$$Q_1 = \text{argmax} \frac{\boldsymbol{e_i}'(\sum_{h=0}^{H_1} \Phi_h \hat{A} Q_1 \boldsymbol{e_j} \boldsymbol{e_j}' Q_1' \hat{A}' \Phi_h')\boldsymbol{e_i}}{\boldsymbol{e_i'}(\sum_{h=0}^{H_1} MSFE_h)\boldsymbol{e_i}}, \tag{28}$$

s.t.

$$Q_1 \equiv \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & Q_1(2,2) & & Q_1(2,p) \\ \vdots & & \ddots & \\ 0 & Q_1(K,2) & & Q_1(K,p) \end{bmatrix}$$
$$Q_1 Q_1' = 1$$

$\boldsymbol{e_i}$ denotes selection vectors with one in the ith place and zeros elsewhere. In our empirical application, we set $i = 1$ and $j = 2$ to label the second shock as the one that maximize forecast error variance of the first variable. Note that the first column and the first raw of raw of Q_1 are specified in this way to select non-productivity shocks at the same time guarantee its orthogonality.

Identifying Bubble Shocks Once we have identified the first two shocks, we identify the bubble shocks as the linear combination (Q_2) of the remaining residuals that contribute the most to the MSFE of $p_t - f_t$ at horizons up to H_2.

The response of $\sum_{i=0}^{\infty} \Lambda^i E_t(Y_{t+i})$ to a shock that occurred h periods ago, upon a successful selection of the matrix Q_2, is given by:

$$\widetilde{\Phi}_{h,0} = \mathbb{J}\mathbb{A}^{h-1}(1-\Lambda\mathbb{A})^{-1}\mathbb{J}' A Q_1 Q_2$$

The response of $\sum_{i=0}^{\infty} \Lambda^i (1-\Lambda) E_t(Y_{t+i+1})$ to a shock that occurred h periods ago is given by:

$$\widetilde{\Phi}_{h,1} = (1-\Lambda)\Lambda\mathbb{J}\mathbb{A}^{h}(1-\Lambda\mathbb{A})^{-1}\mathbb{J}' A Q_1 Q_2$$

Hence, the response of the variable f_t to a shock that occurred h periods ago, is given by:

$$\widetilde{\Phi}_h^f = \widetilde{\Phi}_{h,1}(d) - \widetilde{\Phi}_{h,0}(r),$$

where $\widetilde{\Phi}_{h,1}(d)$ and $\widetilde{\Phi}_{h,0}(r)$ selects, respectively, the vector of response associated with d_t and r_t. The reponse of $p_t - f_t$ to a shock that occurred h periods ago, is given by:

$$\widetilde{\Phi}_h^{p/f} = \Phi_h(p) - \widetilde{\Phi}_h^f.$$

Therefore, the MSFE of $p_t - f_t$ at horizon h is given by:

$$MSFE_h^{p/f} = \sum_{i=0}^{h-1} \widetilde{\Phi}_i^{p/f} \Sigma_u (\widetilde{\Phi}_i^{p/f})'.$$

We identify the bubble shocks as the linear combination (Q_2) of the remaining residuals that contribute the most to the MSFE of $p_t - f_t$ at horizons up to H_2. Formally:

$$Q_2 = \text{argmax} \frac{\sum_{h=0}^{H_2} \widetilde{\Phi}_h^{p/f} \boldsymbol{e_j} \boldsymbol{e_j}' (\widetilde{\Phi}_h^{p/f})'}{\sum_{h=0}^{H_1} MSFE_h^{p/f}}, \tag{29}$$

s.t.

$$Q_2 \equiv \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & Q_2(3,3) & & Q_2(3,p) \\ \vdots & & & \ddots & \\ 0 & 0 & Q_2(K,3) & & Q_2(K,p) \end{bmatrix}$$

$$Q_2 Q_2' = 1.$$

Controlling for other Shocks Our baseline identification strategy can be easily extend to control for more shocks. In our empirical application, as robustness checks, we control for credit supply, monetary policy and government expenditure shocks. To do so, we include each of those variables into our VAR, separately to avoid the curse of dimensionality, and order it the second. Our bubble shocks are then identified analogically to the procedure described above: as the linear combination of reduced form residuals, excluding unanticipated and anticipated TFP shocks and the additional shock that we control for, that maximize the FEVD of the price fundamental differential.