Shadow Interest Rate

Jing Cynthia Wu

Notre Dame and NBER

Coauthors: Dora Xia (BIS) and Ji Zhang (Tsinghua)
ZLB: monetary policy

Before ZLB, policy rates are the tool for monetary policy and its research

- Central banks lower policy rates to stimulate aggregate demand
- Economists rely on them to study monetary policy

Policy rates at ZLB

- Japan, US, Europe
- Unconventional policy tools
  - large-scale asset purchases (QE)
  - lending facilities
  - forward guidance
  - negative interest rate policy
ZLB: economic models

Term structure models

- Benchmark Gaussian ATSM
  - ZLB: Yields are unconstrained in the model, but constrained in the data
- My papers: Wu and Xia (JMCB 2016), Wu and Xia (JAE forthcoming)
  - respect the ZLB
ZLB: economic models

Term structure models

- Benchmark Gaussian ATSM
  - ZLB: Yields are unconstrained in the model, but constrained in the data
- My papers: Wu and Xia (JMCB 2016), Wu and Xia (JAE forthcoming)
  - respect the ZLB

New Keynesian models

- Benchmark models: no unconventional monetary policy
- My papers: Wu and Zhang (JEDC 2019), Wu and Zhang (JIE 2019)
  - incorporate unconventional monetary policy
  - Key feature: tractable
Common theme: shadow rate

Black (1995)

\[ r_t = \max(s_t, r) \]

Sources: Board of Governors of the Federal Reserve System and Wu and Xia (2015)
Wu and Xia (JMCB 2016) shadow rate

Wu-Xia shadow rates for US, Euro area, and UK are available at
- Atlanta Fed
- Haver Analytics
- Thomson Reuters
- Bloomberg

Wu-Xia shadow rate has been discussed by
- **Policy makers:** then Governor Powell (2013), Altig (2014) of the Atlanta Fed, Hakkio and Kahn (2014) of the Kansas City Fed
Outline

1. Wu-Xia shadow rate: Wu and Xia (JMCB 2016)

2. Empirical evidence: Wu and Xia (JMCB 2016), Wu and Zhang (JEDC 2019)


Shadow rate

Black (1995):

\[ r_t = \max(s_t, r) \]

The shadow rate is affine

\[ s_t = \delta_0 + \delta_1' X_t \]

- \( X_t \): 3 factors
Bond pricing

Physical dynamics:

\[ X_{t+1} = \mu + \rho X_t + \sum \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I). \]

Risk-neutral \( Q \) dynamics:

\[ X_{t+1} = \mu^Q + \rho^Q X_t + \sum \varepsilon^Q_{t+1}, \quad \varepsilon^Q_{t+1} \sim N(0, I). \]

Pricing equation

\[ P_{nt} = \mathbb{E}^Q_t [\exp(-r_t)P_{n-1,t+1}] \]

Yield

\[ y_{nt} = -\frac{1}{n} \log(P_{nt}) \]

Forward rate from \( t + n \) to \( t + n + 1 \)

\[ f_{nt} = (n + 1)y_{n+1,t} - ny_{nt} \]
Forward rates

Our approximation

\[ f_{nt} \approx r + \bar{\sigma}_n \Phi \left( \frac{a_n + b'_n X_t - r}{\bar{\sigma}_n} \right) \]

where \( g(z) = z\Phi(z) + \phi(z) \).

Forward rate in GATSM

\[ f_{nt} = a_n + b'_n X_t. \]
Property of \( g(.) \)

\[
\begin{align*}
f_{nt}^{SR} &\approx r, \text{ at the ZLB} \\
&\approx a_n + b'_n X_t = f^{G}_{nt}, \text{ when interest rates are high}
\end{align*}
\]
State space form

State equation

\[ X_{t+1} = \mu + \rho X_t + \sum \varepsilon_{t+1}, \varepsilon_{t+1} \sim N(0, I) \]

Observation equation

\[ f_{nt}^o = r + \sigma_n^Q g \left( \frac{a_n + b'_n X_t - r}{\sigma_n^Q} \right) + \eta_{nt}, \eta_{nt} \sim N(0, \omega) \]

We apply extended Kalman filter for estimation
Model fit

Figure: Average forward curve in 2012

Log likelihood values

- SRTSM: 850; GATSM: 750
## Approximation error

Average absolute approximation error between 1990M1 and 2013M1 (in basis points)

<table>
<thead>
<tr>
<th></th>
<th>3M</th>
<th>6M</th>
<th>1Y</th>
<th>2Y</th>
<th>5Y</th>
<th>7Y</th>
<th>10Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>forward rate error</td>
<td>0.01</td>
<td>0.02</td>
<td>0.04</td>
<td>0.13</td>
<td>0.69</td>
<td>1.14</td>
<td>2.29</td>
</tr>
<tr>
<td>forward rate level</td>
<td>346</td>
<td>357</td>
<td>384</td>
<td>435</td>
<td>551</td>
<td>600</td>
<td>636</td>
</tr>
<tr>
<td>yield error</td>
<td>0.00</td>
<td>0.01</td>
<td>0.01</td>
<td>0.04</td>
<td>0.24</td>
<td>0.42</td>
<td>0.78</td>
</tr>
</tbody>
</table>
Outline

1. Wu-Xia shadow rate: Wu and Xia (JMCB 2016)

2. Empirical evidence: Wu and Xia (JMCB 2016), Wu and Zhang (JEDC 2019)


Evidence 1: taper tantrum

- May 22, 2013: Bernanke told Congress Fed may decrease the size of QE

Shift in shadow rate summarizes this effect
Evidence 2: shadow rate and Fed’s balance sheet

Correlation

▶ QE1 - QE3: -0.94
Evidence 3: structural break test in VAR

Wu and Xia (JMCB 2016): structural break test

- $p = 0.29$ for $s_t$
- $p = 0.0007$ for EFFR
Evidence 4: shadow rate Taylor rule

$$s_t = \beta_0 + \beta_1 s_{t-1} + \beta_2 (y_t - y_t^n) + \beta_3 \pi_t + \varepsilon_t$$

**Full sample**

![Graph showing fed funds rate and shadow rate for the full sample with Taylor rule implied.]

**Post-85 sample**

![Graph showing fed funds rate and shadow rate for the post-85 sample with Taylor rule implied.]

**No structural break**

- $F$ statistics: 0.48 & 1.42
- Critical values: 2.64 & 2.68
Evidence 5: shadow rate and private rates

- private rates are the relevant rates for agents and the economy
- correlation with SR: 0.8
- private rate = $s_t + rp$
Summary

Shadow rate summarizes unconventional monetary policy
  ▶ Taper tantrum
  ▶ Fed’s balance sheet

There is no structural break in
  ▶ VAR
  ▶ shadow rate Taylor rule

Private rates
  ▶ are the relevant interest rates for economic agents
  ▶ respond to unconventional monetary policy
  ▶ the shadow rate is a sensible summary
Outline

1. Wu-Xia shadow rate: Wu and Xia (JMCB 2016)

2. Empirical evidence: Wu and Xia (JMCB 2016), Wu and Zhang (JEDC 2019)


Shadow rate New Keynesian model

Definition 1

The shadow rate New Keynesian model consists of the shadow rate IS curve

\[ y_t = -\frac{1}{\sigma} (s_t - \mathbb{E}_t \pi_{t+1} - s) + \mathbb{E}_t y_{t+1}, \]

New Keynesian Phillips curve

\[ \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa (y_t - y^n_t), \]

and shadow rate Taylor rule

\[ s_t = \phi_s s_{t-1} + (1 - \phi_s) [\phi_y (y_t - y^n_t) + \phi_\pi \pi_t + s]. \]
## Outline

1. Wu-Xia shadow rate: Wu and Xia (JMCB 2016)
2. Empirical evidence: Wu and Xia (JMCB 2016), Wu and Zhang (JEDC 2019)
Taylor rule

Definition

- $r_t$: observed policy rate
- $s_t$: Taylor rule implied

Cases

- Normal times: $r_t = s_t$
- ELB:
  \[ r_t = \lambda s_t \]
  
  - $\lambda = 0$: Standard model
  - $\lambda = 1$: UMP behaves the same as normal times; Wu and Zhang (2017)
  - $0 < \lambda < 1$: partially active UMP
  - $\lambda > 1$: hyper active UMP
VAR: how large is \( \lambda \)?

Model implication for a negative TFP shock

![Graph showing the impact of ELB and ELB + UMP on output after a negative TFP shock. The graph illustrates the response of output (y) in percentage terms to different monetary policy regimes over time.]
VAR: how large is $\lambda$?

- 2 variables: growth rate of labor productivity and per-capita hours similar to Debortoli, Galí, and Gambetti (2016)
- Identification: Cholesky decomposition
- quarterly VAR(1)

Benefits of the VAR

- Simple and robust
- Does not depend on any one shadow rate
IRF of output to a productivity shock: US

Conclusion: $\lambda \approx 1$
IRF of output to a productivity shock: Euro area

Conclusion: $0 < \lambda < 1$
IRF of output to a productivity shock: UK

Conclusion: $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$
Taylor rule: quantify $\lambda$

Why Taylor rule? It gives us a quantitative value of $\lambda$

Basic idea: compare what has been done at the ELB with the interest rate implied by the historical Taylor rule
United States

- pre-ELB: 1985Q2 - 2007Q4, ELB: 2009Q1 - 2015Q4
- Simple method: 1.02
- Iterative method: 1.12

Vary pre-ELB sample: \( t_0 \in \{1982Q1 : 1990Q1\}, \ t_1 \in \{2003Q1 : 2008Q4\} \)

Median (std): simple 1.03 (0.065), iterative 1.19 (0.45)
Euro area and UK

- Euro area: simple 0.998(0.031), iterative 0.63(1.07)
- UK: simple 0.98(0.10), iterative 0.39(4.10)

Again, we conclude $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$
Empirically, we find

\[ \lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0 \]
\[ b_t^G = b^G - \frac{S_t}{\pi} \]

**THE FED'S BOND HOLDINGS AT THE TIME IN QUESTION**

**THE FED'S BOND HOLDINGS DURING NORMAL TIMES**

**CALCULATION OF HOW THE SHADOW RATE RESPONDS WHEN THE FED ADJUSTS ITS HOLDINGS**

**SHADOW FED FUNDS RATE**

**FED FUNDS RATE MONITOR 5000**

**FED FUNDS RATE MONITOR 5000**

**THE ECONOMY**

**WORKING JUST FINE**

**OUT OF ORDER**

**INSERT SHADOW RATE**

**THE FED'S BOND HOLDINGS**

**THE TIME IN QUESTION**

**NORMAL TIMES**

**ADJUSTS ITS HOLDINGS**

**SHADOW FED FUNDS RATE**

**FED FUNDS RATE MONITOR 5000**

**FED FUNDS RATE MONITOR 5000**

**THE ECONOMY**

**WORKING JUST FINE**

**OUT OF ORDER**

**INSERT SHADOW RATE**
Shadow rate

FAVAR

Replace the fed funds rate with $s_t$ in Bernanke, Boivin, and Eliasz (2005)

$$Y_t^m = a_m + b_x x_t^m + b_s s_t + \eta_t^m, \quad \eta_t^m \sim N(0, \Omega)$$

$Y_t^m$: 97 economic variables from 1960 to 2013

$x_t^m$: 3 underlying macro factors

Factor dynamics:

$$x_t^m = \mu^x + \rho^{xx} X_{t-1}^m + u_t^x$$

$$+ \mathbb{1}_{(t<\text{December 2007})} \rho_1^{xs} S_{t-1} + \mathbb{1}_{(\text{December 2007} \leq t \leq \text{June 2009})} \rho_2^{xs} S_{t-1} + \mathbb{1}_{(t>\text{June 2009})} \rho_3^{xs} S_{t-1}$$

$\mathbb{1}$: monthly VAR(13)

Null hypothesis

$$H_0: \rho_1^{xs} = \rho_3^{xs}$$
## Robustness

<table>
<thead>
<tr>
<th>Method</th>
<th>$p$-value for $\rho_1^{xs} = \rho_3^{xs}$</th>
<th>$p$-value for $\rho_1^{sx} = \rho_3^{sx}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>0.29</td>
<td>1.00</td>
</tr>
<tr>
<td>A1 estimate $r$</td>
<td>0.18</td>
<td>1.00</td>
</tr>
<tr>
<td>A2 2-factor SRTSM</td>
<td>0.13</td>
<td>0.97</td>
</tr>
<tr>
<td>A3 Fama-Bliss</td>
<td>0.38</td>
<td>1.00</td>
</tr>
<tr>
<td>A4 5-factor FAVAR</td>
<td>0.70</td>
<td>1.00</td>
</tr>
<tr>
<td>A5 6-lag FAVAR</td>
<td>0.09</td>
<td>0.98</td>
</tr>
<tr>
<td>A5 7-lag FAVAR</td>
<td>0.19</td>
<td>0.97</td>
</tr>
<tr>
<td>A5 12-lag FAVAR</td>
<td>0.22</td>
<td>1.00</td>
</tr>
</tbody>
</table>
Samples

pre-ELB and ELB samples

- US: 1985Q2 - 2007Q4 and 2009Q1 - 2015Q4
- Euro area: 1999Q1 - 2009Q1 and 2009Q2 - 2017Q4
- UK: 1993Q1 - 2009Q1 and 2009Q2 - 2017Q4