Shadow Interest Rate

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ZLB: monetary policy

Before ZLB, policy rates are the tool for monetary policy and its research

- Central banks lower policy rates to stimulate aggregate demand
- Economists rely on them to study monetary policy

Policy rates at ZLB

- Japan, US, Europe
- Unconventional policy tools
  - large-scale asset purchases (QE)
  - lending facilities
  - forward guidance
  - negative interest rate policy

ZLB: economic models

Term structure models

- Benchmark Gaussian ATSM
  - ZLB: Yields are unconstrained in the model, but constrained in the data
- My papers: Wu and Xia (JMCB 2016), Wu and Xia (JAE forthcoming)
  - respect the ZLB

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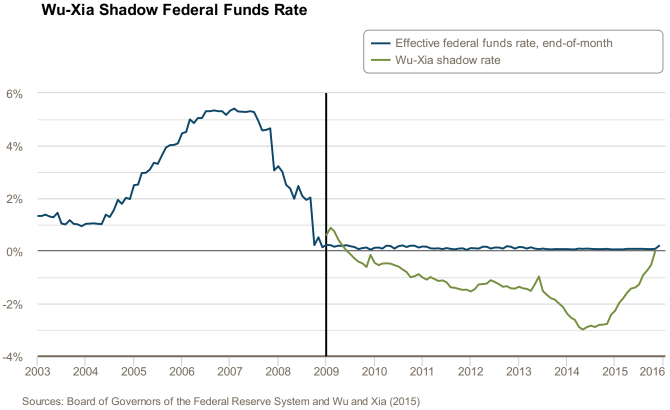
New Keynesian models

- Benchmark models: no unconventional monetary policy
- My papers: Wu and Zhang (JEDC 2019), Wu and Zhang (JIE 2019)
  - incorporate unconventional monetary policy
  - Key feature: tractable

Common theme: shadow rate

Black (1995)

$$r_t = max(s_t, \underline{r})$$



Wu and Xia (JMCB 2016) shadow rate

- Wu-Xia shadow rates for US, Euro area, and UK are available at
  - Atlanta Fed
  - Haver Analytics
  - Thomson Reuters
  - Bloomberg
- Wu-Xia shadow rate has been discussed by
  - Policy makers: then Governor Powell (2013), Altig (2014) of the Atlanta Fed, Hakkio and Kahn (2014) of the Kansas City Fed
  - Media:Wall Street Journal, Financial Times, The New York Times, Bloomberg news, Bloomberg Businessweek, Forbes, Business Insider, VOX

Outline

1. Wu-Xia shadow rate: Wu and Xia (JMCB 2016)
2. Empirical evidence: Wu and Xia (JMCB 2016), Wu and Zhang (JEDC 2019)
3. New Keynesian model: Wu and Zhang (JEDC 2019), Wu and Zhang (JIE 2019)
4. International evidence: Wu and Zhang (JIE 2019)

Shadow rate

Black (1995):

$$r_t = max(s_t, \underline{r})$$

The shadow rate is affine

$$s_t = \delta_0 + \delta_1' X_t$$

- X_t: 3 factors

Bond pricing

Physical dynamics:

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, I).$$

Risk-neutral $\mathbb{Q}$ dynamics:

$$X_{t+1} = \mu^{\mathbb{Q}} + \rho^{\mathbb{Q}} X_t + \Sigma \varepsilon^{\mathbb{Q}}_{t+1}, \quad \varepsilon^{\mathbb{Q}}_{t+1} \overset{\mathbb{Q}}{\sim} N(0, I).$$

Pricing equation

$$P_{nt} = \mathbb{E}^{\mathbb{Q}}_t[\exp(-r_t) P_{n-1,t+1}]$$

Yield

$$y_{nt} = -\frac{1}{n}\log(P_{nt})$$

Forward rate from $t+n$ to $t+n+1$

$$f_{nt} = (n+1) y_{n+1,t} - n y_{nt}$$

Forward rates

Our approximation

$$f_{nt} \approx \underline{r} + \tilde{\sigma}_n^{\mathbb{Q}} g\left(\frac{a_n + b_n' X_t - \underline{r}}{\tilde{\sigma}_n^{\mathbb{Q}}}\right)$$

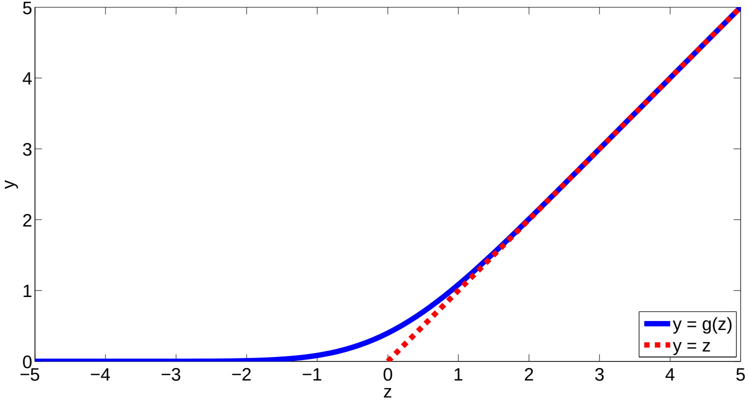
where $g(z) = z\Phi(z) + \phi(z)$.

Details

Forward rate in GATSM

$$f_{nt} = a_n + b_n' X_t.$$

Property of $g(.)$



$$f_{nt}^{SR} \begin{cases} \approx \underline{r}, \text{ at the ZLB} \\ \approx a_n + b_n' X_t = f_{nt}^G, \text{ when interest rates are high} \end{cases}$$

State space form

State equation

$$X_{t+1} = \mu + \rho X_t + \Sigma \varepsilon_{t+1}, \varepsilon_{t+1} \sim N(0, I)$$

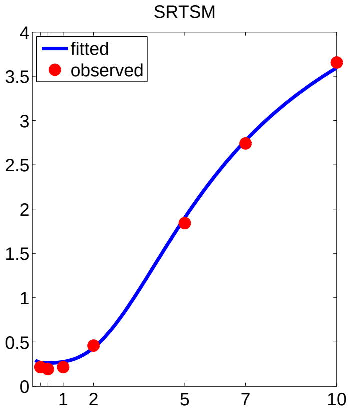
Observation equation

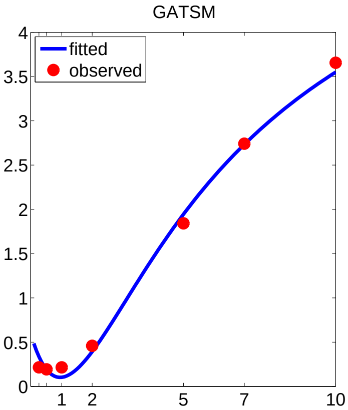
$$f^o_{nt} = \underline{r} + \sigma^{\mathbb{Q}}_n g\left(\frac{a_n + b'_n X_t - \underline{r}}{\sigma^{\mathbb{Q}}_n}\right) + \eta_{nt}, \eta_{nt} \sim N(0, \omega)$$

We apply extended Kalman filter for estimation

Model fit

Figure: Average forward curve in 2012





Log likelihood values

- SRTSM: 850; GATSM: 750

Approximation error

Average absolute approximation error between 1990M1 and 2013M1 (in basis points)

	3M	6M	1Y	2Y	5Y	7Y	10Y
forward rate error	0.01	0.02	0.04	0.13	0.69	1.14	2.29
forward rate level	346	357	384	435	551	600	636
yield error	0.00	0.01	0.01	0.04	0.24	0.42	0.78

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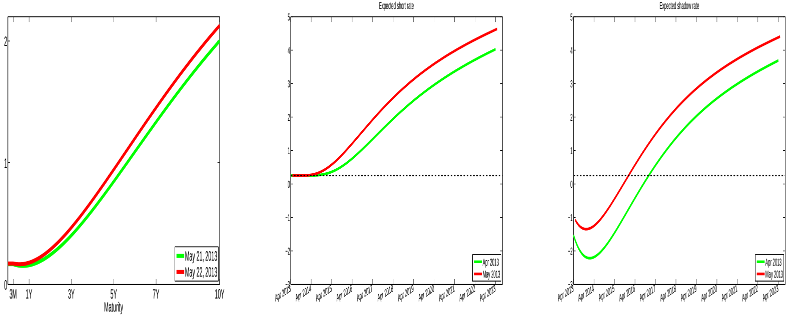
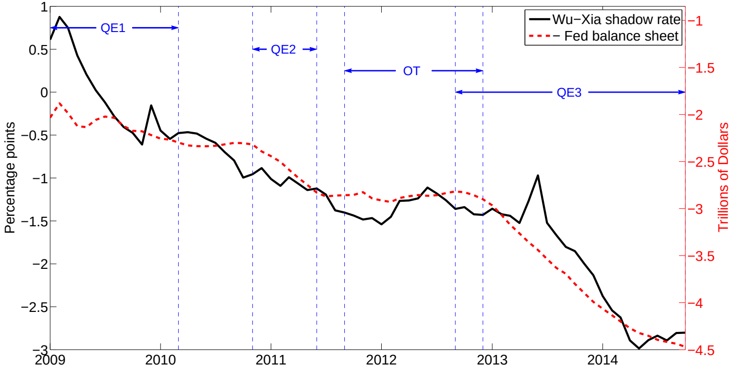
Evidence 1: taper tantrum



- May 22, 2013: Bernanke told Congress Fed may decrease the size of QE

Shift in shadow rate summarizes this effect

Evidence 2: shadow rate and Fed's balance sheet



Correlation

- QE1 - QE3: -0.94

Evidence 3: structural break test in VAR

Wu and Xia (JMCB 2016): structural break test

- $p = 0.29$ for s_t
- $p = 0.0007$ for EFFR

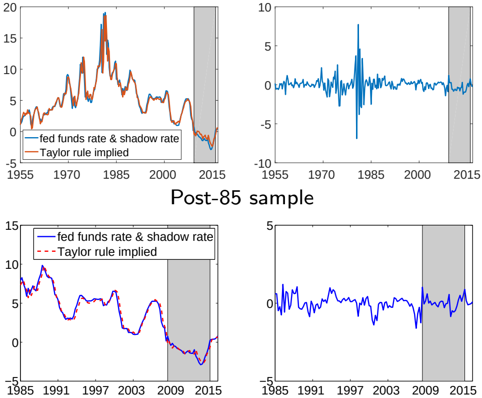
▸ model details ▸ robustness

Evidence 4: shadow rate Taylor rule

$$s_t = \beta_0 + \beta_1 s_{t-1} + \beta_2(y_t - y_t^n) + \beta_3 \pi_t + \varepsilon_t$$

Full sample

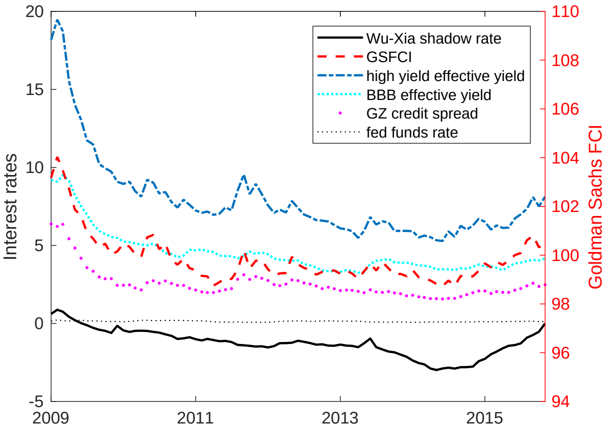
Post-85 sample



No structural break

- F statistics: 0.48 & 1.42
- Critical values: 2.64 & 2.68

Evidence 5: shadow rate and private rates



- private rates are the relevant rates for agents and the economy
- correlation with SR: 0.8
- private rate $= s_t + rp$

Summary

Shadow rate summarizes unconventional monetary policy

- Taper tantrum
- Fed's balance sheet

There is no structural break in

- VAR
- shadow rate Taylor rule

Private rates

- are the relevant interest rates for economic agents
- respond to unconventional monetary policy
- the shadow rate is a sensible summary

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Shadow rate New Keynesian model

Definition 1

The shadow rate New Keynesian model consists of the shadow rate IS curve

$$y_t = -\frac{1}{\sigma}(\mathbf{s_t} - \mathbb{E}_t \pi_{t+1} - s) + \mathbb{E}_t y_{t+1},$$

New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa(y_t - y_t^n),$$

and shadow rate Taylor rule

$$s_t = \phi_s s_{t-1} + (1 - \phi_s)\left[\phi_y (y_t - y_t^n) + \phi_\pi \pi_t + s\right].$$

QE

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Taylor rule

Definition

- r_t: observed policy rate
- s_t: Taylor rule implied

Cases

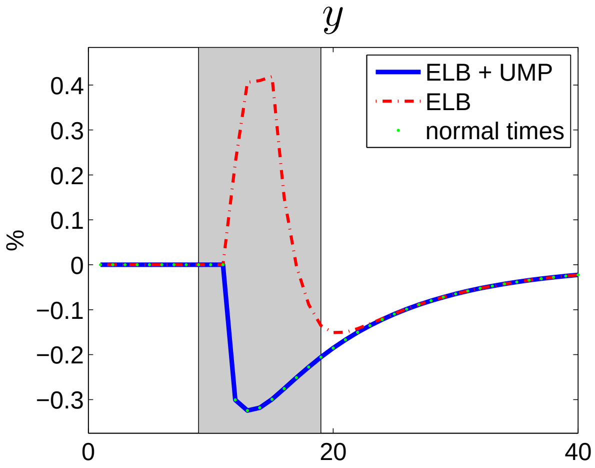
- Normal times: $r_t = s_t$
- ELB:

$$r_t = \lambda s_t$$

  - $\lambda = 0$: Standard model
  - $\lambda = 1$: UMP behaves the same as normal times; Wu and Zhang (2017)
  - $0 < \lambda < 1$: partially active UMP
  - $\lambda > 1$: hyper active UMP

VAR: how large is λ?

Model implication for a negative TFP shock



VAR: how large is λ?

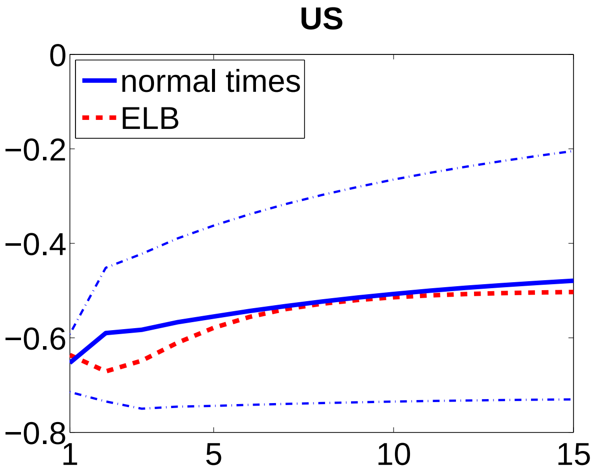
- 2 variables: growth rate of labor productivity and per-capita hours
  similar to Debortoli, Galí, and Gambetti (2016)
- Identification: Cholesky decomposition
- quarterly VAR(1)

Benefits of the VAR

- Simple and robust
- Does not depend on any one shadow rate

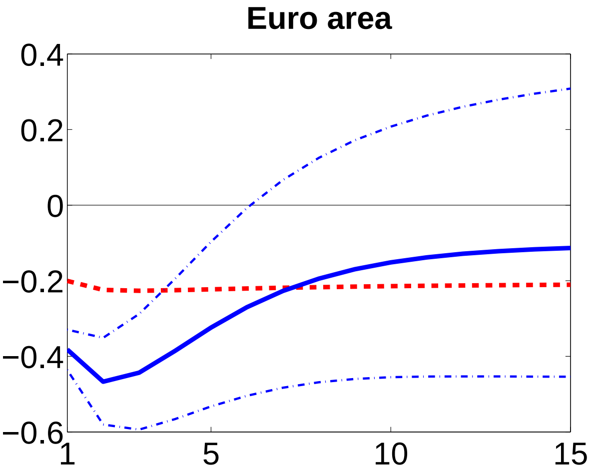
Samples

IRF of output to a productivity shock: US



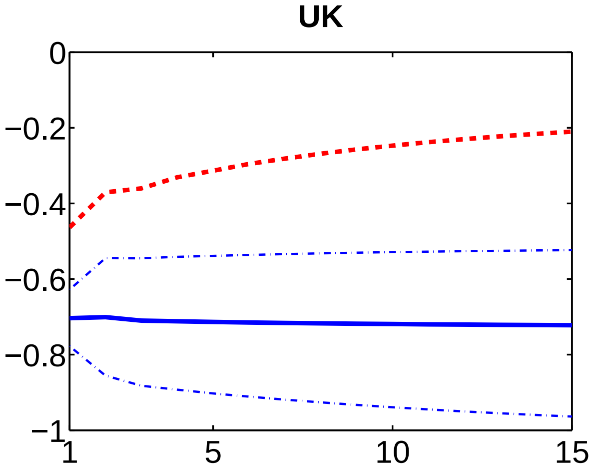
Conclusion: $\lambda \approx 1$

IRF of output to a productivity shock: Euro area



Conclusion: $0 < \lambda < 1$

IRF of output to a productivity shock: UK



Conclusion: $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$

Taylor rule: quantify λ

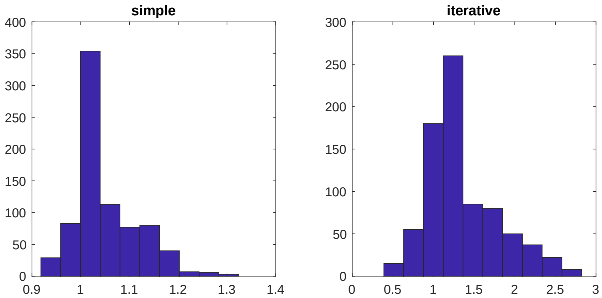
Why Taylor rule? It gives us a quantitative value of λ

Basic idea: compare what has been done at the ELB with the interest rate implied by the historical Taylor rule

United States

- pre-ELB: 1985Q2 - 2007Q4, ELB: 2009Q1 - 2015Q4
- Simple method: 1.02
- Iterative method: 1.12

Vary pre-ELB sample: $t_0 \in \{1982Q1 : 1990Q1\}$, $t_1 \in \{2003Q1 : 2008Q4\}$



Median (std): simple 1.03 (0.065), iterative 1.19 (0.45)

Euro area and UK

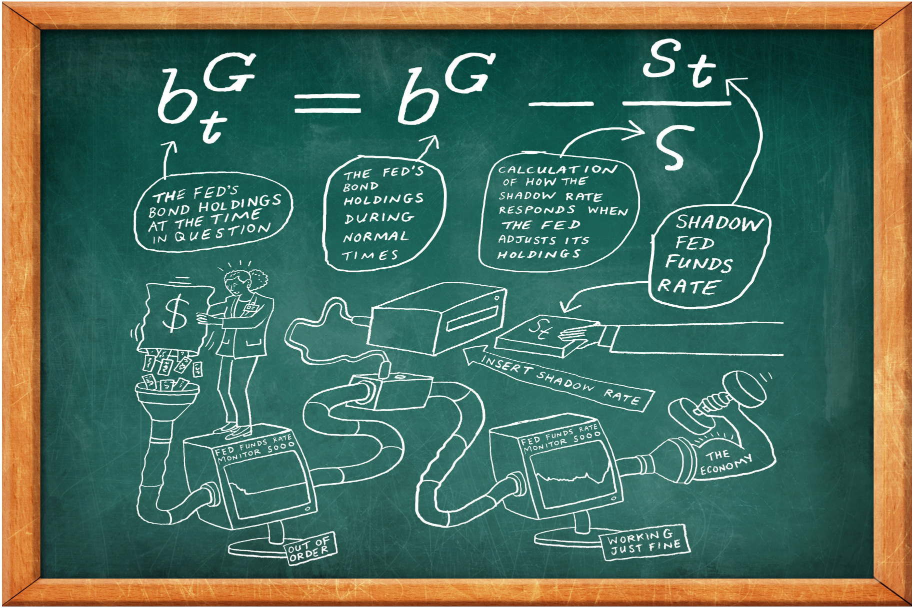
- Euro area: simple 0.998(0.031), iterative 0.63(1.07)
- UK: simple 0.98(0.10), iterative 0.39(4.10)

Again, we conclude $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$

Conclusion

Empirically, we find

- $\lambda_{US} \approx 1 > \lambda_{Euro} > \lambda_{UK} > 0$



$b_t^G = b^G - \frac{s_t}{s}$
THE FED'S BOND HOLDINGS AT THE TIME IN QUESTION
THE FED'S BOND HOLDINGS DURING NORMAL TIMES
CALCULATION OF HOW THE SHADOW RATE RESPONDS WHEN THE FED ADJUSTS ITS HOLDINGS
SHADOW FED FUNDS RATE
INSERT SHADOW RATE
FED FUNDS RATE MONITOR 5000
OUT OF ORDER
FED FUNDS RATE MONITOR 5000
WORKING JUST FINE
THE ECONOMY

FAVAR

Replace the fed funds rate with s_t in Bernanke, Boivin, and Eliasz (2005)

$$Y_t^m = a_m + b_x x_t^m + b_s s_t + \eta_t^m, \quad \eta_t^m \sim N(0, \Omega)$$

- Y_t^m: 97 economic variables from 1960 to 2013
- x_t^m: 3 underlying macro factors

Factor dynamics:

$$\begin{aligned} x_t^m &= \mu^x + \rho^{xx} X_{t-1}^m + u_t^x \\ &+ \mathbb{1}_{(t<\text{December 2007})} \rho_1^{xs} S_{t-1} + \mathbb{1}_{(\text{December 2007} \leq t \leq \text{June 2009})} \rho_2^{xs} S_{t-1} + \mathbb{1}_{(t>\text{June 2009})} \rho_3^{xs} S_{t-1} \end{aligned}$$

- monthly VAR(13)

Null hypothesis

$$H_0 : \rho_1^{xs} = \rho_3^{xs}$$

back

Robustness

		p-value for $\rho_1^{xs} = \rho_3^{xs}$	p-value for $\rho_1^{sx} = \rho_3^{sx}$
	Baseline	0.29	1.00
A1	estimate $\underline{r}$	0.18	1.00
A2	2-factor SRTSM	0.13	0.97
A3	Fama-Bliss	0.38	1.00
A4	5-factor FAVAR	0.70	1.00
A5	6-lag FAVAR	0.09	0.98
	7-lag FAVAR	0.19	0.97
	12-lag FAVAR	0.22	1.00

back

Samples

pre-ELB and ELB samples

- US: 1985Q2 - 2007Q4 and 2009Q1 - 2015Q4
- Euro area: 1999Q1 - 2009Q1 and 2009Q2 - 2017Q4
- UK: 1993Q1 - 2009Q1 and 2009Q2 - 2017Q4

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