Financial Fragility Modelling and Applications
Section II: Liquidity and default in an exchange economy[1]

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[1]DISCLAIMER: The views expressed here are my own and do not necessarily represent those of the St. Edmund Hall, U. of Oxford or the Saïd Business School. Based on "Liquidity and default in an exchange economy" (Martinez & Tsomocos, 2018).

The paper in a nutshell

The question:

What is the impact of financial frictions -liquidity and default- in financial stability?

What we do:

- We develop a stylized model of trade and intermediation that allows us to study financial stability under the presence of financial frictions.
- We provide theoretical and empirical evidence of the interplay of liquidity and default.
- This constitutes a framework that allows us to explain the effects of liquidity and default on welfare and financial stability.
- We further find that default and liquidity frictions are sufficient to explain price and activity trade-offs.

Context for the question

Need of a model which focus on liquidity effects on financial stability:

- The origin and consequences of liquidity and default risks lied at the heart of the 2007-2008 financial crisis (Chacko et al (2011)).
- The effects of liquidity on financial stability are ambiguous and certainly depend on timing (Adrian &Shin (2010, 2011)).
- Monetary policy (liquidity) can be used to address financial stability issues (Stein (2012)).
- The leading examples in the literature of financial frictions in dynamic stochastic models are Bernanke et. al. (1999) and Kiyotaki and Moore (2012).
- The literature has some remaining challenges:
    1. We need a better understanding of the connection between different sources of liquidity and default.
    2. The extension of these works to a DSGE framework is still in progress.

Model background

- Goodhart et. al. (2006): Introduces the cohabitation of liquidity constraints, agent heterogeneity and default.
- Dubey, Geanakoplos and Shubik (2005): Extends the model of general equilibrium with incomplete markets to allow for default and punishment.
- Clower (1967) and Svensson (1985): Models cash in advance constraint.
- Espinoza and Tsomocos (2010): Analyzes liquidity in a general equilibrium setting.
- de Walque et. al. (2010): Similar to ours, but it does not include financing restrictions, liquidity, default, and agent heterogeneity altogether.

Benchmark model

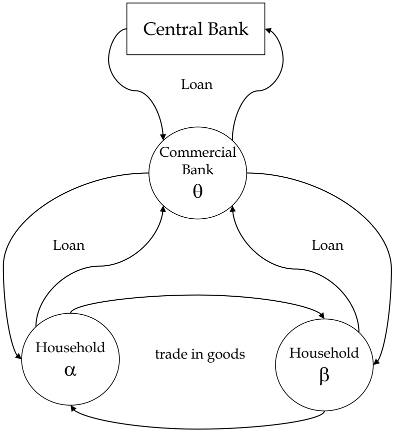
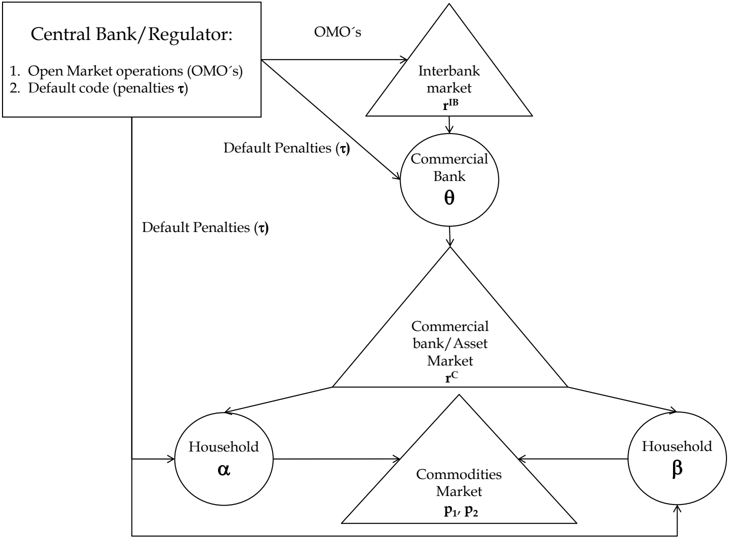


Figure: Nominal flows of the economy

Market structure of the model



Financial Frictions

Default

- Agents are allowed to default partially: They choose the fraction of outstanding debt they repay.
- We model the punishment in case of default with a non-pecuniary penalty proportional to the defaulted amount of credits (for households and commercial bank loans).
- Default choice trade-offs the benefit of defaulting (more consumption) and its cost (credit costs).

Money

- Introduced by a cash-in-advance (liquidity) transaction technology.
- Modelled as inside money (enters the system accompanied by an offsetting obligation $\rightarrow$ exits the system with accrued interest and net of default)

Financial Frictions

Liquidity

- There are two main sources of liquidity:
  1. Through the injections by the Central Bank (OMO's)
  2. There is a fraction (λ^{α} and λ^{β}) of the goods traded every period that can be used immediately as a mean of payment.

- Liquidity in goods is modeled in three main cases:
  1. No liquidity
  2. Partial asymmetric liquidity
  3. Partial symmetric liquidity

- We interpret this parameter as the **speed of liquidation**.

Speed of liquidation

Business sector	Turnover 2013Q2-2014Q3	Speed (*)
Services	29,5	0,082
Energy	15	0,042
Technology	10,8	0,030
Basic Materials	5,3	0,015
Home Improvement Industry	4,2	0,012
Capital Goods	3,7	0,010

(*)Speed is measured as the Average 2013Q2-2014Q3 of the turnover ratio, on a daily basis.
Source: CSI Market.

Other elements

- Model the production sector in a reduced fashion. We present an endowment economy (i.e. 1 factor and constant returns to scale).
- Assume a stochastic AR(1) process for the commodity endowments.
- The only way to smooth consumption is through commodity trade.
- Inflation has a non-trivial form because of the presence of 2 goods (we use Laspeyres).

Timing of the model

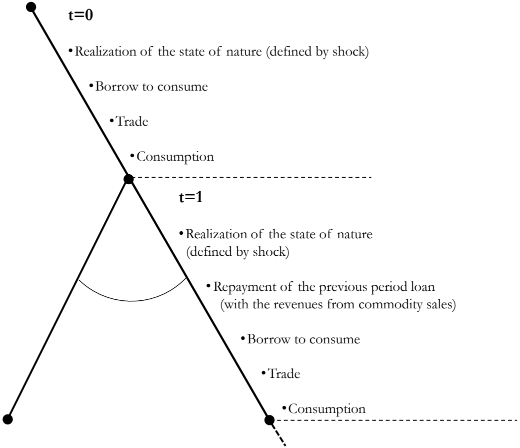


Figure: Timing of events.

Household $h \in \{\alpha, \beta\}$ optimization problem

$$\max_{\mu_t^h, b_{j_h,t}^h, \upsilon_t^h, q_{i_h,t}^h} U^\alpha = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ u\left(e_{i_h,t}^h - q_{i_h,t}^h\right) + u\left(\frac{b_{j_h,t}^h}{p_{j_h,t}}\right) - \frac{\tau^h}{\mathbb{P}_t} Max[0, (1-\upsilon_t^h)\mu_{t-1}^h] \right\}$$

s.t.

$$\upsilon_t^h \mu_{t-1}^h \leq p_{i_h,t-1} q_{i_h,t-1}^h \cdot \left(1 - \lambda_{t-1}^h\right) \qquad \left(\eta_{1,t}^h\right) \tag{1}$$

Loan repayment $\leq$ Previous period illiquid sales of commodities.

$$b_{j_h,t}^h \leq \lambda_t^h \cdot p_{i_h,t} q_{i_h,t}^h + \frac{\mu_t^h}{1+r_t^c} + \frac{1-\phi}{2}\Pi_t^\theta \qquad \left(\eta_{2,t}^h\right) \tag{2}$$

Money spent $\leq$ Liquid portion of sales of commodities + Loan taken from the commercial bank + Bank dividends.

Bank θ optimization problem

$$\max_{\Pi_t^\theta, \mu_t^\theta, l_t^\theta, \upsilon_t^\theta} U^\theta = E_0 \sum_{t=0}^{\infty} \hat{\beta}^t \left\{ u\left(\Pi_t^\theta\right) - \frac{\tau^\theta}{\mathbb{P}_t} Max[0, (1-\upsilon_t^\theta)\mu_{t-1}^\theta] \right\}$$

s.t.

$$\Pi_t^\theta = \frac{R_t l_{t-1}^\theta \left(1 + r_{t-1}^c\right)}{\mathbb{P}_t} - \upsilon_t^\theta \frac{\mu_{t-1}^\theta}{\mathbb{P}_t} \qquad \left(\eta_{1,t}^\theta\right) \tag{3}$$

Utility = Expected loan repayment - Repayment to Central Bank.

$$l_t^\theta \leq \frac{\mu_t^\theta}{1 + r_t^{IB}} + e_t^\theta \qquad \left(\eta_{2,t}^\theta\right) \tag{4}$$

Credit extension $\leq$ Loan taken from Central Bank + Equity.

$$e_t^\theta = \phi \Pi_t^\theta \tag{5}$$

Equity = Retained earnings.

Closing the model

Rational Expectations

Commercial bank expected repayment rate:

$$R_t = \begin{cases} \frac{\upsilon_t^\alpha \mu_{t-1}^\alpha + \upsilon_t^\beta \mu_{t-1}^\beta}{\mu_{t-1}^\alpha + \mu_{t-1}^\beta}, & \text{if } \mu_{t-1}^\alpha + \mu_{t-1}^\beta > 0; \\ \textbf{Arbitrary}, & \text{if } \mu_{t-1}^\alpha + \mu_{t-1}^\beta = 0. \end{cases}$$

Market Clearing Conditions

Commodity Market

$$b_{1,t}^\beta = p_{1,t} q_{1,t}^\alpha$$
$$b_{2,t}^\alpha = p_{2,t} q_{2,t}^\beta$$

Consumer Loans Market

$$1 + r_t^c = \frac{\mu_t^\alpha + \mu_t^\beta}{l_t^\theta}$$

REPO Market

$$1 + r_t^{IB} = \frac{\mu_t^\theta}{M_t}$$

Equilibrium - short run

Given the previous definitions we are able to define the **FSMLD** (financial stability with money, liquidity and default equilibrium), for the short as well as the long run. In our case in the long run the economy converges to its steady state.

In our model, $(\Sigma^\alpha, \Sigma^\beta, \Sigma^\theta, \kappa)$ is a short run **FSMLD** iff:

(i) All agents optimize given their budget sets:
   (a) $\Sigma^h \in \text{Argmax}_{\Sigma^h \in B^h(\kappa)} U\left(C^h\right)$, for h $\in \{\alpha, \beta\}$ and $\forall t \in T$.
   (b) $\Sigma^\theta \in \text{Argmax}_{\Sigma^\theta \in B^\theta(\kappa)} U\left(\Pi^\theta\right)$, $\forall t \in T$.

(ii) All markets clear.

(iii) Expectations are rational.

Proposition 1: Money non-neutrality

This proposition implies that if there is a non-zero monetary operation by the Central Bank (i.e. $M_t \neq M_t' \Rightarrow r_t^c \neq r_t^{c\prime}$, from market clearing conditions), monetary policy is not neutral in the short-run. Therefore it affects the consumption and consequently real variables.

Suppose that for $\alpha, \beta \in H$, $b_t^h > 0$, for $l \in L$, $\lambda_t^h \in [0, 1)$ and some state of nature defined by the set of shocks at t. We have that at a **FSMLD**,

$$r_t^c \leq r_t^{c\prime}, \text{ and } \lambda_t^\alpha \geq \lambda_t^{\alpha\prime} \Rightarrow \boldsymbol{q}_{\mathbf{1},t}^\alpha \geq \boldsymbol{q}_{\mathbf{1},t}^{\alpha\prime}$$

Note that by symmetry the proposition holds also for household β.

Proposition 1: Implications (Edgeworth 2-agents 2-good case).

We have two extreme cases to analyze:

- If $\lambda_t^\alpha = 1$, we have no liquidity restrictions. Therefore there are no incentives to borrow money and monetary policy is neutral.

$$\frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{1,t}^{\alpha}} = \frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{2,t}^{\alpha}} \frac{p_{1,t}}{p_{2,t}} \tag{6}$$

  It should be noted that **in this case default does not make sense**, since there is nothing to default on. Thus, if there is full liquidity we are at the standard Edgeworth 2-agents 2-good case.

- If $\lambda_t^\alpha = 0$, we have that monetary policy is not neutral. This is the usual cash-in-advance setting,

$$\frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{1,t}^{\alpha}} = \frac{1}{1 + r_t^c} \frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{2,t}^{\alpha}} \frac{p_{1,t}}{p_{2,t}} \tag{7}$$

Proposition 2: Fisher effect

Suppose that for $h \in \{\alpha, \beta\}$, $b_t^h > 0$, for $l \in L$, $\lambda_t^h \in [0,1)$ and some state of nature defined by the set of shocks at t. We have that at a **FSMLD**, for agent h, we have,

$$\left(\frac{1}{1-\lambda_t^h} \left(\frac{\frac{\partial u\left(c_{\mathbf{1},t}^h, c_{\mathbf{2},t}^h\right)}{\partial c_{\mathbf{1},t}^h}}{\frac{\partial u\left(c_{\mathbf{1},t}^h, c_{l,t}^h\right)}{\partial c_{\mathbf{2},t}^h}} \frac{p_{\mathbf{2},t}}{p_{\mathbf{1},t}} - \lambda_t^h \right) \right)^{-1} = (1 + r_t^c) \tag{8}$$

Taking logarithms and interpreting loosely, this proposition indicates that nominal **interest rates are approximately equal to real interest rates plus expected inflation and risk premium, which depends on <u>liquidity</u> and default**. Fisher effect explains how nominal prices are linked directly to consumption.

Proposition 3: Quantity theory of money

Assume no money is carried over. In an interior **FSMLD** equilibrium, $\forall t \in T$

$$\left(1-\lambda_t^{\alpha}\right) p_{1,t} q_{1,t}^{\alpha} + \left(1-\lambda_t^{\beta}\right) p_{2,t} q_{2,t}^{\beta} = M_t \tag{9}$$

Thus, the model possesses a non-trivial quantity theory of money, where prices and quantities are determined simultaneously.
Fisher's (1911) quantity theory of money proposition states,

$$P_t Q_t = M_t V_t \tag{10}$$

It implies that **money supply has a direct, proportional relationship with the price level**, where P_t stands for the price index, Q_t is an index of the real value of final expenditures, M_t is the total amount of money in circulation every period, and V_t is the average velocity of money in the market.

Proposition 4: On the verge condition

Suppose that for $\alpha, \beta \in H$, $b_t^h > 0$, for $l \in L$, $\lambda_t^h \in [0, 1)$ and some state of nature defined by the set of shocks at t. We have that at a **FSMLD**, the on-the-verge condition for default penalties, for agents α, β and bank θ, respectively, is given by

$$\frac{1}{1+r_t^c} \frac{\partial u\left(c_{1,t}^{\alpha}, c_{2,t}^{\alpha}\right)}{\partial c_{2,t}^{\alpha}} \frac{1}{p_{2,t}} = \beta \mathbb{E}_t \left(\frac{\tau^{\alpha}}{\mathbb{P}_{t+1}}\right) \tag{11}$$

$$\frac{1}{1+r_t^c} \frac{\partial u\left(c_{1,t}^{\beta}, c_{2,t}^{\beta}\right)}{\partial c_{1,t}^{\beta}} \frac{1}{p_{1,t}} = \beta \mathbb{E}_t \left(\frac{\tau^{\beta}}{\mathbb{P}_{t+1}}\right) \tag{12}$$

$$\frac{\partial u\left(\Pi_t^{\theta}\right)}{\partial \Pi_t^{\theta}} = \tau^{\theta} \left(1 - \phi \hat{\beta} \mathbb{E}_t \frac{R_{t+1}^{\theta}(1+r_t^c)}{\mathbb{P}_{t+1}})\right) \tag{13}$$

These conditions imply that the optimal amount of default is defined when the marginal utility of defaulting equals the marginal dis-utility.

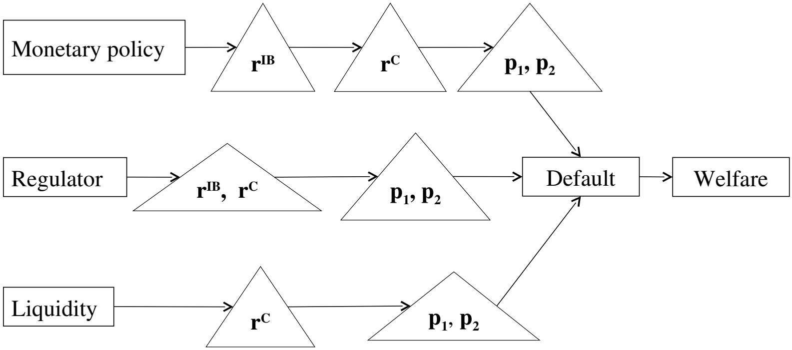
Calibration

Concept	Parameter	Basic model	$\lambda^\alpha = 0.5, \lambda^\beta = 0.2$	$\lambda^\alpha = 0.5, \lambda^\beta = 0.5$	Source
Penalties	τ^α	0.3525	0.22915	0.17625	ours
	τ^β	0.3525	0.22915	0.17625	ours
	τ^θ	8	8	8	ours
Allocations	$\bar{e}_1^\alpha$	2	2	2	ours
	$\bar{e}_2^\beta$	2	2	2	ours
Impatience	β	0.97	0.97	0.97	King and Rebelo (1999)
	$\hat{\beta}$	0.98	0.98	0.98	King and Rebelo (1999)
Persistence	ρ^{CB}	0.5	0.5	0.5	Christiano et al. (1998)
	ρ_e^α	0.5	0.5	0.5	Christiano et al. (1998)
	ρ_e^β	0.5	0.5	0.5	Christiano et al. (1998)
	ρ_e^θ	0.5	0.5	0.5	Christiano et al. (1998)
Money	$\bar{\eta}^{CB}$	1	1	1	ours
	$\bar{M}$	5	5	5	ours

Steady State

Concept	Variable	Basic model	$\lambda^\alpha = 0.5$ $\lambda^\beta = 0.2$	$\lambda^\alpha = 0.5$ $\lambda^\beta = 0.5$	Full liquidity -
Prices	p_1	2.817	4.248	5.523	0.524
	p_2	2.817	4.299	5.523	0.524
	r^c	0.079	0.079	0.079	-
	r^{IB}	0.024	0.024	0.024	-
Repayments	R	0.949	0.949	0.949	-
	υ^α	0.949	0.956	0.949	-
	υ^β	0.949	0.945	0.949	-
	υ^θ	0.977	0.977	0.977	-
Credit	μ^α	2.857	2.181	2.857	-
	μ^β	2.857	3.531	2.857	-
	μ^θ	5.119	5.119	5.119	-
	l^θ	5.294	5.294	5.294	-
Trade	b_2^α	2.710	4.169	5.420	0.524
	b_1^β	2.710	4.169	5.420	0.524
	q_1^α	0.962	0.981	0.981	1.000
	q_2^β	0.962	0.970	0.981	1.000
Profits	Π^θ	0.420	0.420	0.420	-

Transmission mechanism



Money, liquidity shocks and the Phillips curve

- We relate price variations and trade (as a proxy for output) in the presence of a positive monetary shock and market liquidity shock.
- The default and liquidity interplay allows us to obtain a version of the Phillips curve.
- Where there are no liquidity constraints trade does not depend on inflation.

Money, liquidity shocks and the Phillips curve (I)

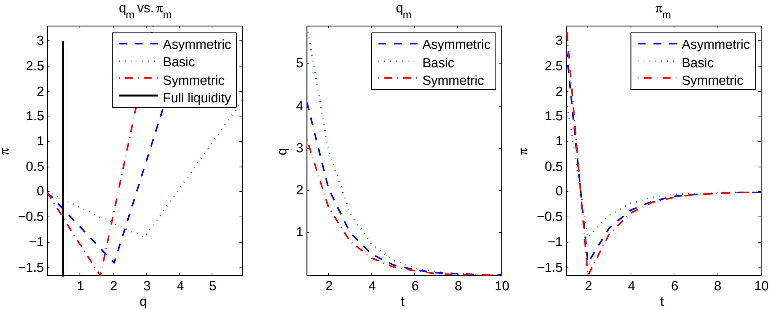


Figure: A version of the Phillips curve. Positive monetary shock (5%)

Money, liquidity shocks and the Phillips curve (II)

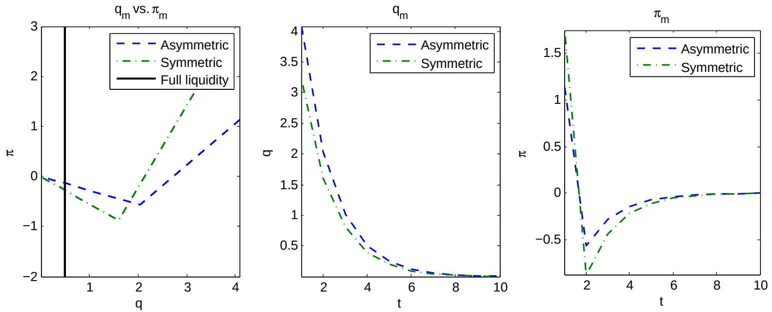


Figure: A version of the Phillips curve. Positive market liquidity shock (5%)

Financial stability analysis

- According to Goodhart et al. (2006), financial stability can be roughly approximated by bank profitability and repayment rates.
- We assess the relative importance of financial stability and price stability in the presence of a monetary and a liquidity shock.
- In the presence of a monetary shock the performance of the commercial bank is worse than the previous case.
- The simulations results indicate that financial stability is relatively more affected than price stability.

Financial stability analysis

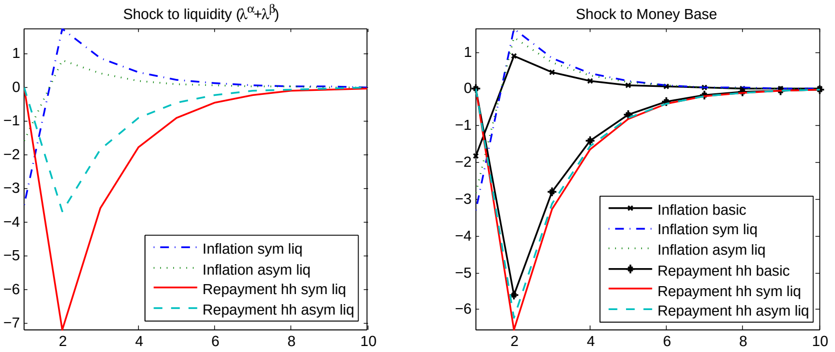


Figure: Simulation plot, $\Delta^{-}5\%\lambda^{\alpha} + \lambda^{\beta}$, $\Delta^{-}5\%$ M. Repayment rate is in percentage variations wrt steady state levels and inflation is the gross rate.

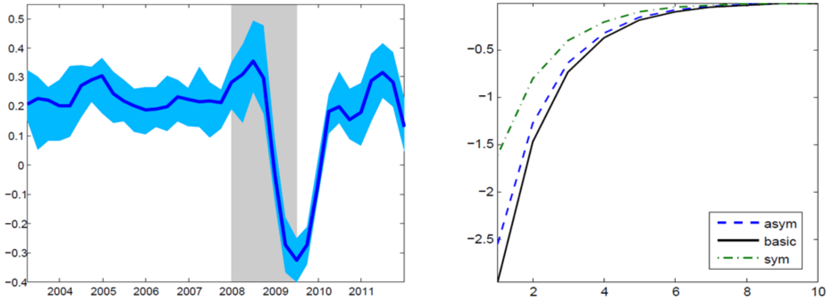
Welfare analysis

- We don't model production nor nominal frictions, so all of the other welfare effects are determined through trade.
- A reduction in monetary base decreases trade by a higher magnitude than the benefits obtained from price control.
- There is a trade off between price targeting and welfare.

Welfare analysis

(a) Exports more imports yearly changes on quarterly data, emerging and developing countries.

(b) Shock to money base on trade (q_2^β). % variations wrt steady state levels.



Source: IFS database, IMF.

Source: ours.

Figure: International trade and trade simulations ($\Delta^{-}5\%$ M).

Conclusions

- We have developed a model that is capable of addressing issues of financial stability.
- We generalize the Edgeworth-box example by including liquidity restrictions.
- Our results suggest that liquidity and default in equilibrium should be studied contemporaneously.
- The presence of financial frictions underlines the importance of studying the impact of shocks on the behaviour of short to medium run of financial variables and welfare.
- We derive a relationship between prices and activity (trade) with the inclusion of default but without nominal frictions.
- Possible extensions are: estimate and calibrate to a particular case, to model liquidity of assets as a result of the endogenous interaction with the liquidity of goods or as the response of asymmetric information on the quality of the goods or assets. It also can be extended to address further questions.