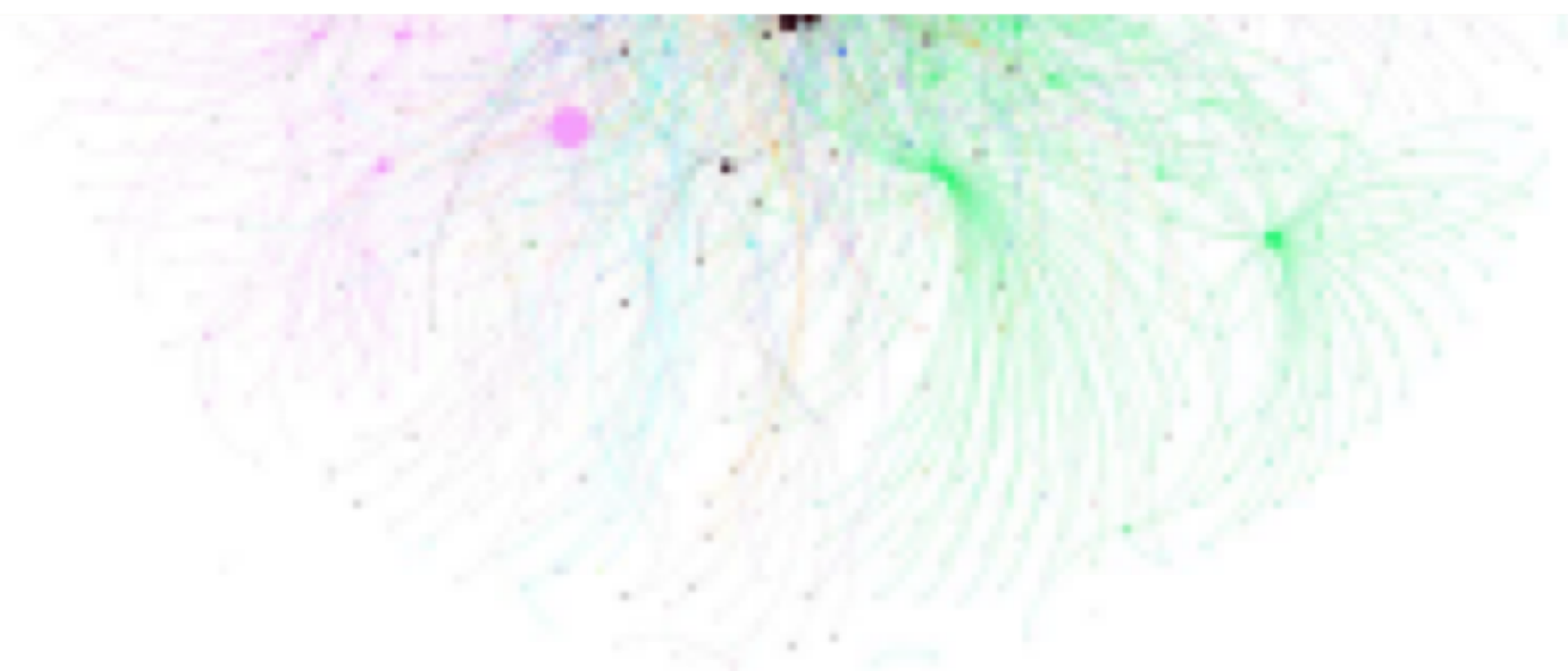


Financial networks and financial stability



Sources: ECB supervisory data and ECB calculations.

Notes: The node size captures the weighted in-degree of interconnectedness. The node colour represents the country of origin. The thickness of the links reflects the value of the exposures in € trillions. The colour of the links refers to the country of the target, thus also capturing the borrower's perspective.

Modelling systemic events

This special feature uses simulations to examine how different contagion channels might lead to a systemic crisis, using recent real data. We do this by modelling the impact of losses that banks could face on their capital ratios.

Learning goals

Part 1

- Explain 3 main channels of financial contagion:
 - Default cascades,
 - Funding contagion / liquidity hoarding
 - Fire sales externality
- Compute by hand:
 - Fictitious default algorithm of Eisenberg & Noe (2001)
 - DebtRank algorithm of Bardoscia, Battiston, Caccioli et Cardarelli (2015)

Learning goals

Part 2

- Give typical characteristics of large networks
- Construct financial network data from balance sheet data and large exposures
- Compute measures of financial contagion:
 - System level: systemic risk, expected systemic loss
 - Bank level: systemic importance, vulnerability
- Explain what is a multilayer network and why it is important for assessing systemic risk
 - Poledna et al. (JFS, 2015)

Financial networks and systemic risk

Financial network: set of

- Nodes: financial institutions (banks)
- Links between banks: *exposures, common assets, from transactions, funding*

Financial contagion:

- Spread of a shock from one bank to other banks through the financial network

Systemic risk:

- Risk that financial stress in one bank leads to financial stress in the whole financial sector

Two approaches to financial networks

Asset price approach

Publicly traded stock market
prices of banks

Network estimated from time
series dependencies

Examples: SRISK, CoVAR

Key papers:

- Diebold & Yilmaz (2009)
- Billio et al. (2012)
- Brownlees & Engle (2016)
- Adrian-Brunnermeier (2016)

Balance sheet approach

Private data on assets and
liabilities of banks

Network is (partly) known

Systemic risk is estimated from
assumptions on contagion
mechanism

Key papers:

- Eisenberg & Noe (2001)
- Cifuentes et al. (2005)
- Gai et al. (2011)

Bank balance sheet

Bank i

Assets

Outside assets

e_i

- Liquid assets
 - Cash, gov bonds

- Illiquid assets

- Loans to firms/consumers

In-network assets

P_{ji}

Liabilities

Outside liabilities

b_i

- Deposits

In-network liabilities

P_{ji}

Equity

- Capital + reserves

w_i



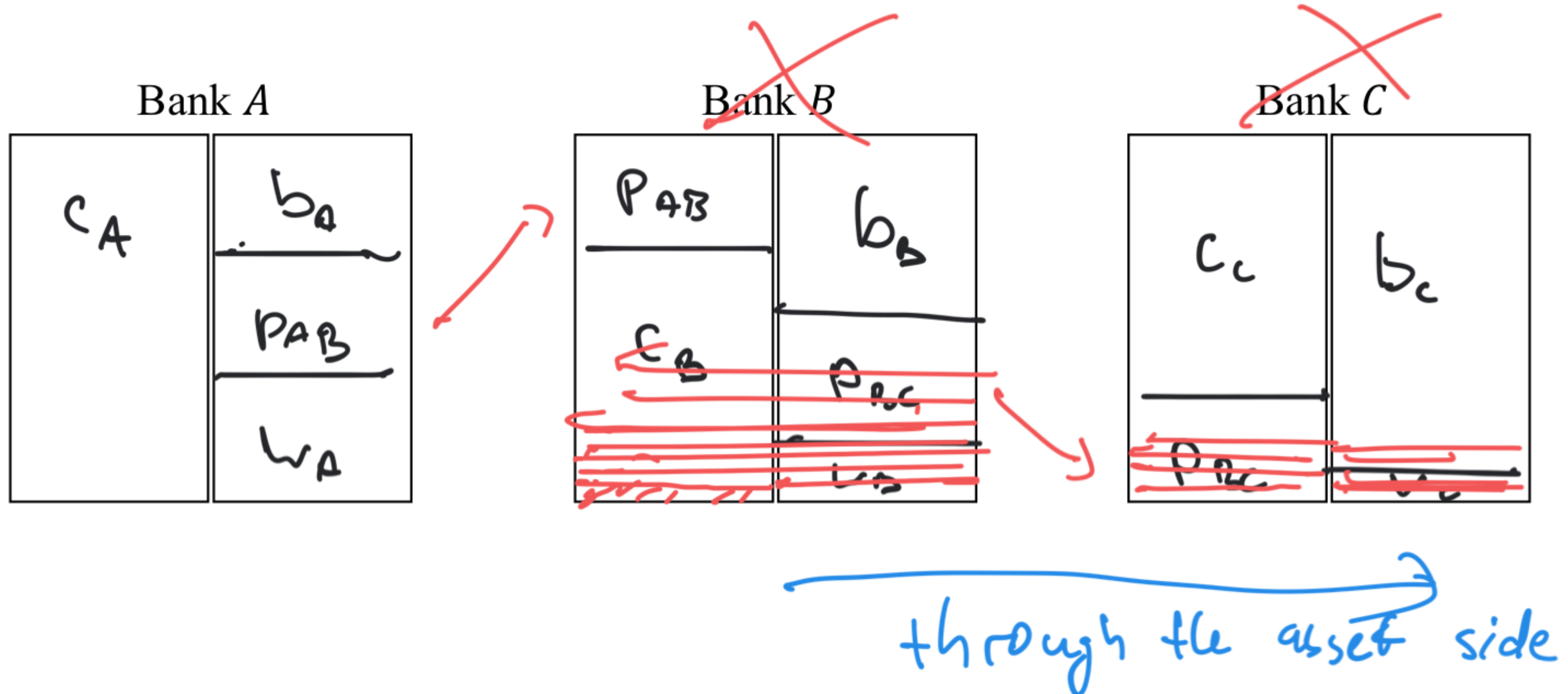
Contagion channels

Types of contagion

- Default cascades
- Funding contagion / liquidity hoarding
- Fire sales externality

Default cascade

Shock leads to default (if $w_i < 0$)



Default cascade

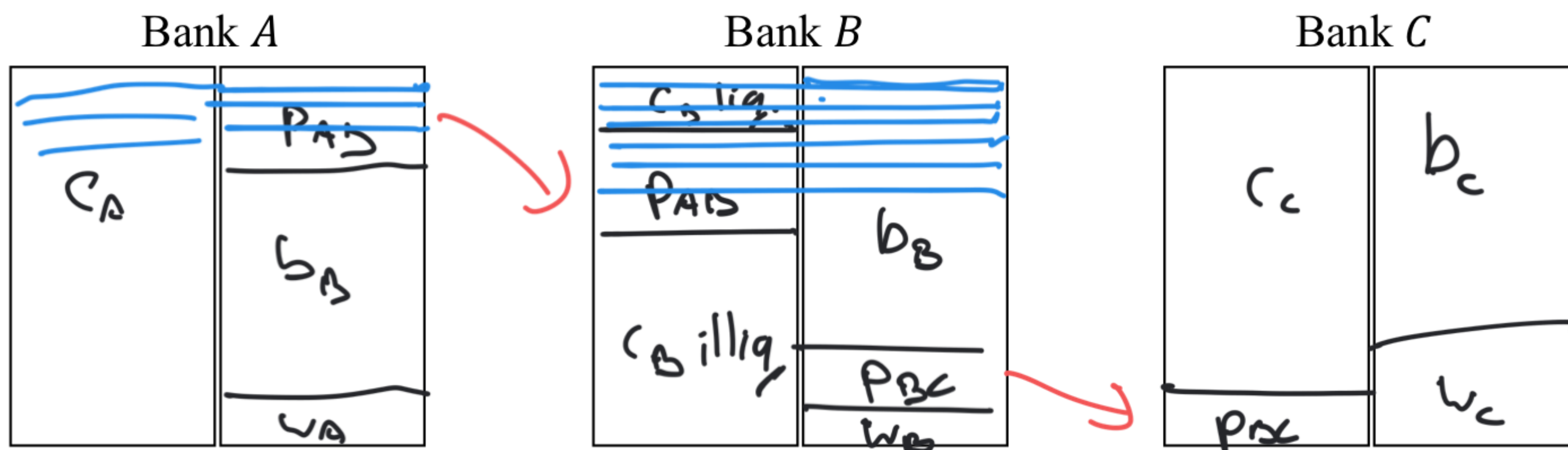
Shock is transmitted through asset side

Default cascade mechanism gets amplified by

- Bankruptcy cost
- Incorporating default risk in the asset values
- Fire sales externality

Funding contagion

Shock to external funding b_B



← liability side

Funding contagion

Shock is transmitted through liability side

Net worth is not directly affected (no defaults)

But mechanism gets amplified by:

- Liquidity hoarding: Bank B converts its remaining loan to A into cash
- Sales of illiquid assets (fire sales)

Fire sales externality

Shock on asset price

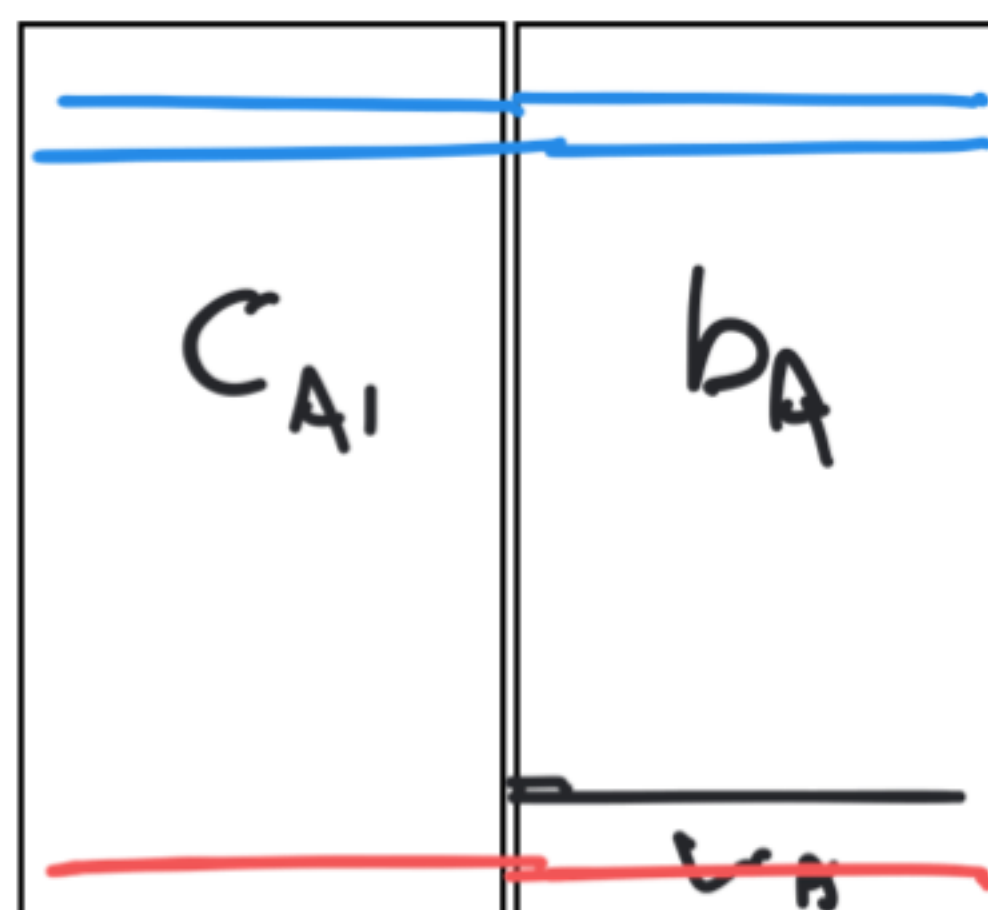
Assumptions:

- Banks would like to keep their leverage ratio constant
 - Bank regulation, Internal risk management
- Assets are illiquid: sale leads to drop in price
- Balance sheet assets are valued at mark-to-market

Fire sales externality

Shock on asset price

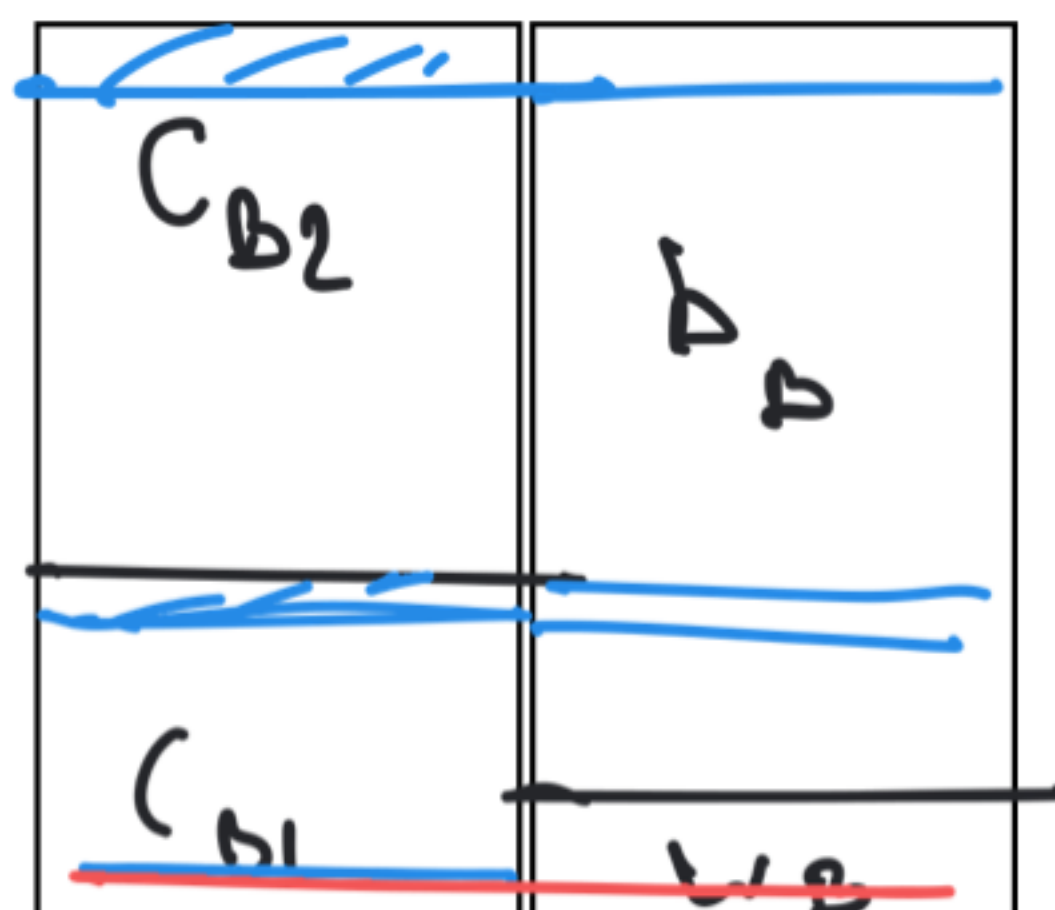
Bank A



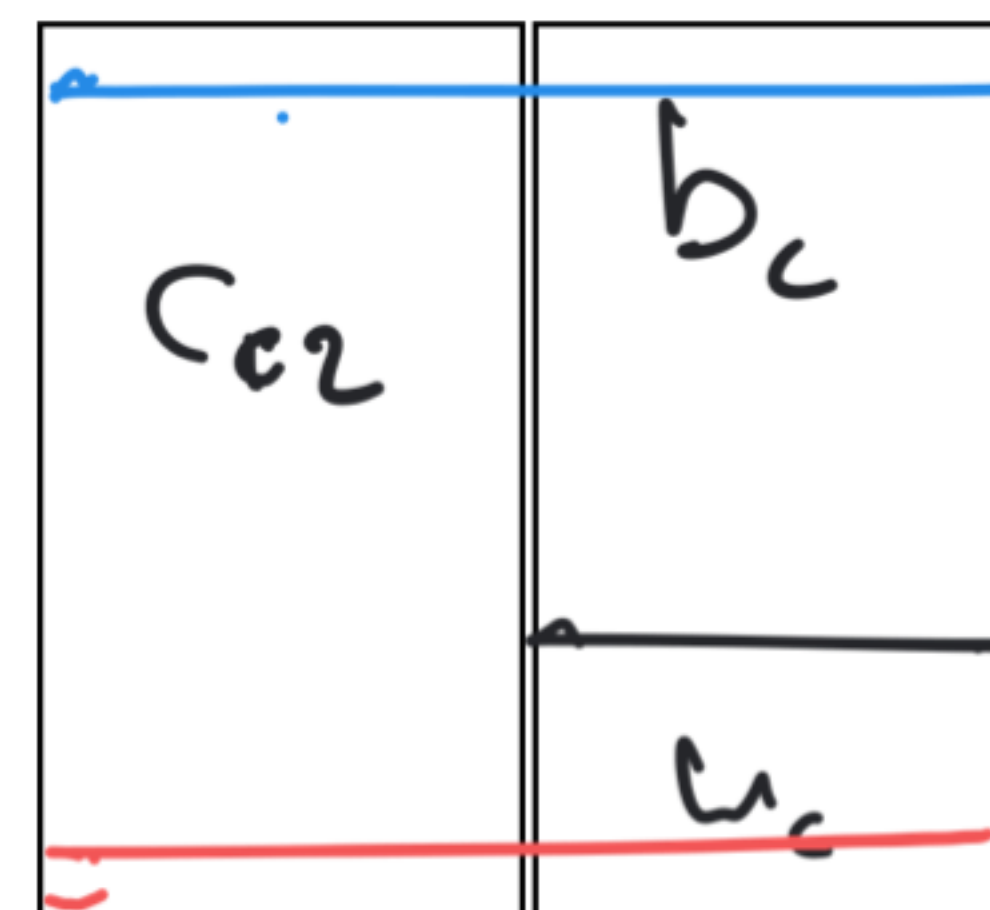
Asset 1

$P_1 \downarrow$

Bank B



Bank C



Asset 2

$P_2 \downarrow$

Contagion channels

Types of contagion

- Default cascades
- Funding contagion / liquidity hoarding
- Fire sales externality
- Other?



DEFAULT CASCADES

Eisenberg & Noe (2001)

Define:

- Payment that bank i owes to j : \bar{p}_{ij}
- Payment that bank i makes to j : p_{ij}

Similar distinction for external assets and liabilities

$$\bar{b}_i, \bar{c}_i \quad b_{ij}, c_i$$

We are interested in solving for: $p_{ij} \quad \forall ij$

Eisenberg & Noe (2001)

Assumptions:

- External assets are always paid out: $c_i = \bar{c}_i$
- If bank is solvent: $w_i \geq 0$
 - Bank pays what it owes $p_{ij} = \bar{p}_{ij}$ $b_i = \bar{b}_i$
- If bank defaults: $w_i < 0$
 - Bank pays out all its assets $\bar{c}_i + \sum_k p_{ki}$
 - Assets are divided equally among all its creditors (equal seniority)

Eisenberg & Noe (2001)

Define:

- Total payment that bank i owes: $\bar{p}_i = \bar{b}_i + \sum_j \bar{p}_{ij}$
- Total payment that bank i pays: $p_i = b_i + \sum_j p_{ij}$
- Share of bank i 's total payments to bank j :

$$\alpha_{ij} = \frac{\bar{p}_{ij}}{\bar{p}_i}$$

Then

$$p_{ij} = \alpha_{ij} p_i$$

$$p_i = \begin{cases} \bar{p}_i & \text{if } \bar{p}_i \leq \bar{c}_i + \sum_k p_{ki} \\ \bar{c}_i + \sum_k p_{ki} & \text{otherwise} \end{cases}$$

Eisenberg & Noe (2001)

Payments clear if: Solve $p_i \forall i$

$$p_i = \min \left\{ \bar{p}_i, c_i + \sum_k \alpha_{ki} p_k \right\} \text{ for all } i$$

Eisenberg & Noe (2001)

Eisenberg & Noe (2001) show that there exist a generically unique payment vector \vec{p} that clears the system of payment equations

If all banks default, then

Computation clearing payment vector

Fictitious default algorithm:

Start with everyone paying what they owe

In each period t

- Check which banks default
- Solvent banks pay what they owe
- Solve the system of price equations for the defaulting banks

Go back to 2 until no further banks default

Exercise: Fictitious default algorithm

Round 1

~~Bank A~~

c_A	10 3	b_A	5
		p_{AB}	3
		p_{AC}	1
		w_A	1
Total		Total	

$5/3$
 $4/3$
 $1/3$

Bank B

c_B	4	b_B	4
p_{AB}	3	p_{BC}	2
		w_B	1
Total	7	Total	7

Bank C

c_C	3.8	b_C	6
p_{AC}	1	w_C	0.8
p_{BC}	2		
Total		Total	

~~Bank A~~ round 2

c_A	3	b_A	5/3
		p_{AB}	1
		p_{AC}	1/3
		w_A	
Total	3	Total	3

~~Bank B~~

c_B	4	b_B	10/3
p_{AB}	1	p_{BC}	5/3
		w_B	
Total	5	Total	5

Bank C

c_C	3.8	b_C	6
p_{AC}	1/3	w_C	2/5
p_{BC}	2		
Total		Total	

round 3

~~Bank C~~

c_C	3.8
p_{AC}	1/3
p_{BC}	5/3
	5.8

$$p = \begin{pmatrix} 3 \\ 5.8 \\ 5.8 \end{pmatrix}$$

DebtRank (Bardoscia et al., 2015)

Idea:

- The market value of bank A's interbank debt may drop **before** bank A defaults.
- If assets are value at mark-to-market, then a shock of A leads to a loss at other banks that own debt issued by A

DebtRank:

- Market value of debt issued by A decreases **proportionally** to decrease in equity of A

DebtRank algorithm

Define

- p_{ij} : the “market value” of debt issued by i and owned by j in round t
- Similarly b_i, c_i

Algorithm

- $t = 0$: Initiate balance sheets $\forall i : c_i(0) = \bar{c}_i, p_{ij}(0) = \bar{p}_{ij}$
- $t = 1$: Apply shocks to banks $\forall i : s_i$

$$c_i(1) = \bar{c}_i - s_i$$

$$w_i(1) = \max(0, w_i(0) - s_i)$$

default solvent

DebtRank algorithm

- $t = 0$: Initiate balance sheets
- $t = 1$: Apply shocks to banks
- $t \geq 2$: Revalue interbank assets proportional to drop in debt issuer's equity

$$p_{ij}(t) = p_{ij}(0) \frac{w_i(t-1)}{w_i(0)}$$

- Update equity

$$w_i(t) = \max(0, w_i(0) - s_i - \sum_k (p_{ki}(0) - p_{ki}(t)))$$

default solved

Repeat until convergence

Example DebtRank

Round 0: initial situation

Bank A

c_A	10	b_A	4
		p_{AB}	4
		w_A	2
Total		Total	

Bank B

c_B	12	b_B	4
		p_{BC}	8
		w_B	4
Total		Total	

Bank C

c_C	4	b_C	10
		w_C	2
Total		Total	

~~Round 1: shock to bank A~~

Bank A

c_A	9	b_A	4
		p_{AB}	4
		w_A	1
Total		Total	

Bank B

c_B	12	b_B	4
		p_{BC}	8
		w_B	2
Total		Total	

~~Bank C~~

c_C	4	b_C	10
		w_C	0
Total		Total	

Bank A

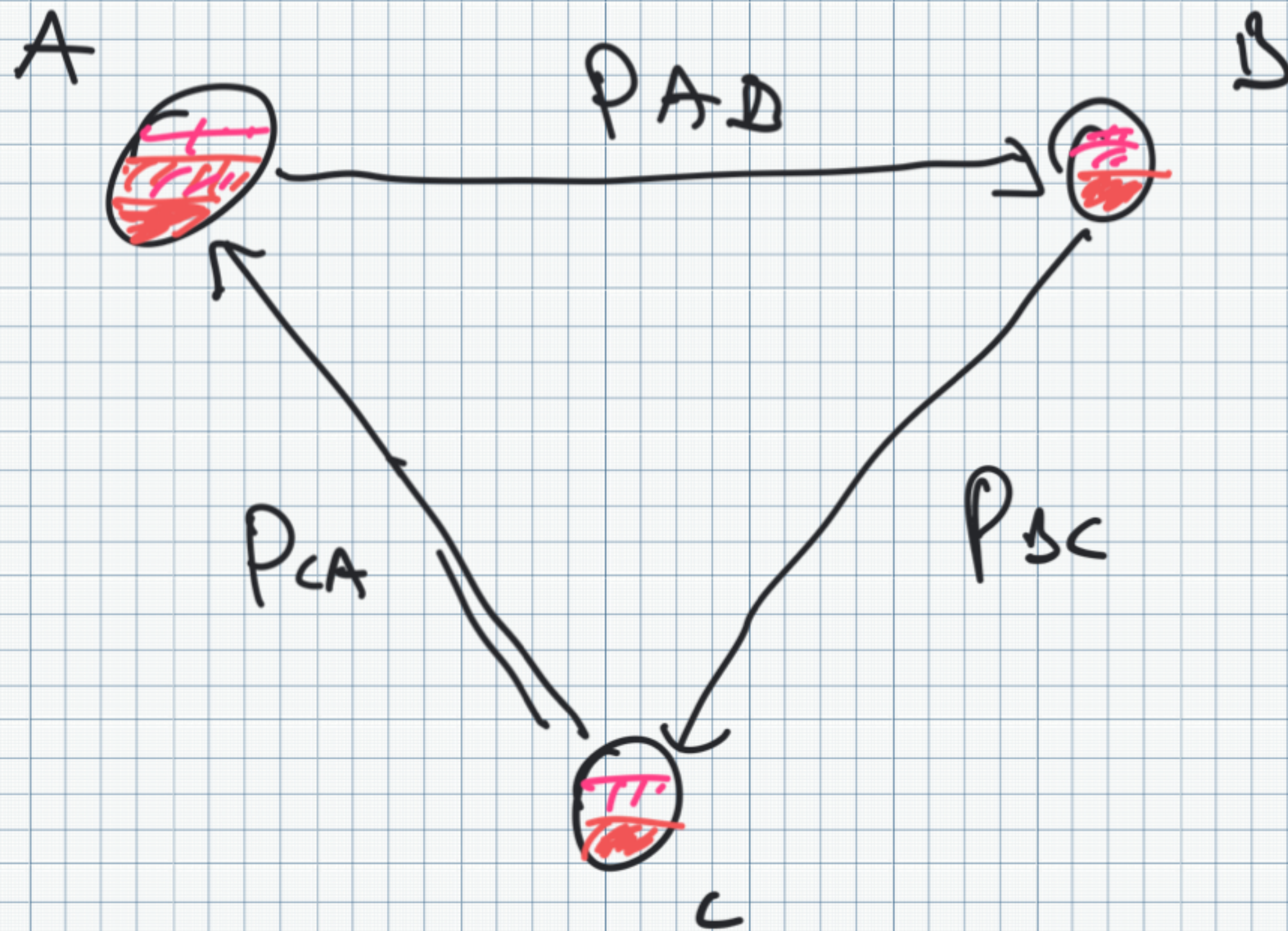
Bank B

Bank C

DebtRank

Two versions of DebtRank

- New: Bardoscia et al. (2015)
$$p_{ij}(t) = p_{ij}(0) \frac{w_i(t-1)}{w_i(0)}$$
- Old: Battiston et al. (2012)
$$p_{ij}(t) = \begin{cases} p_{ij}(0) \frac{w_i(t-1)}{w_i(0)} & \text{if } i \text{ is hit for first time} \\ p_{ij}(t-1) & \text{otherwise } i \text{ is already hit} \end{cases}$$
 if $w_i(s) = w_i(0) \forall s < t-1$





DebtRank

Difference between old and new version

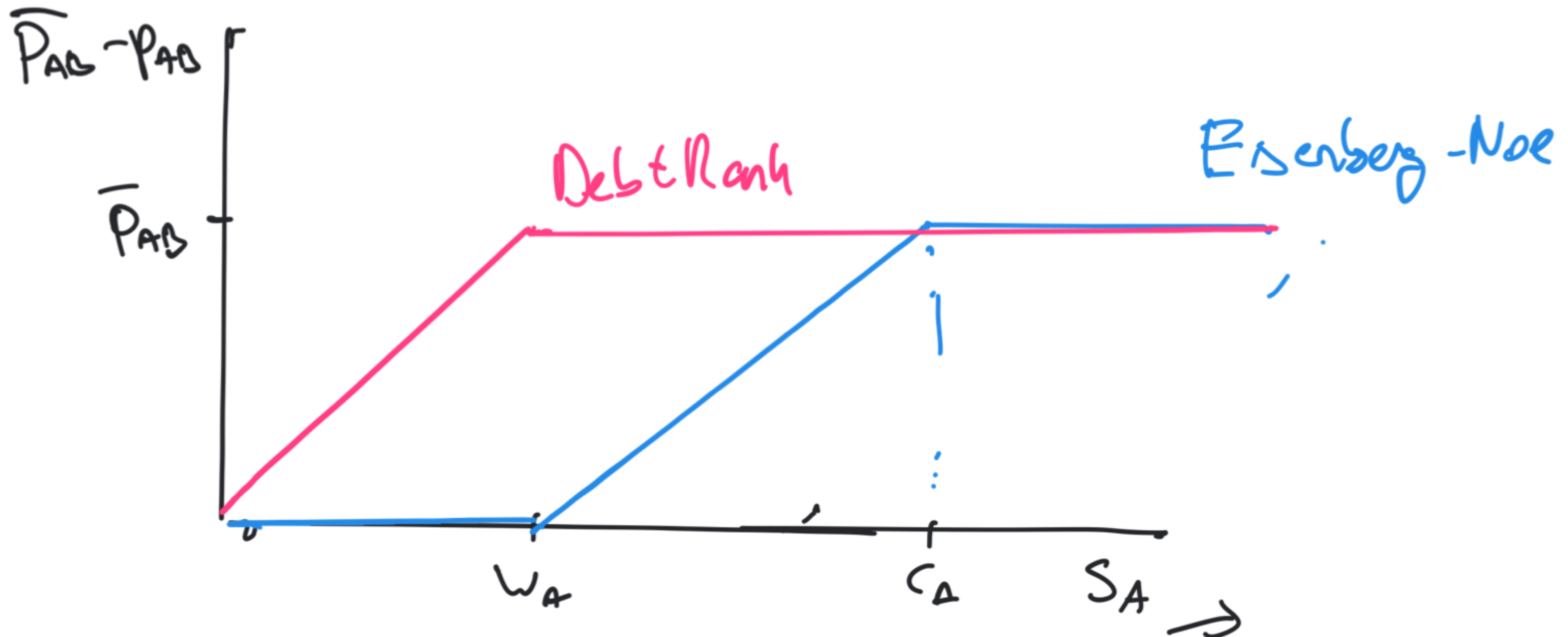
Comparison DebtRank vs Eisenberg-Noe

Bank A

c_A $-s_A$	b_A
	p_{AB}
	w_A

Bank B

c_B	b_B
p_{AB}	w_B



Standard default cascade: (Furfine, 2003)

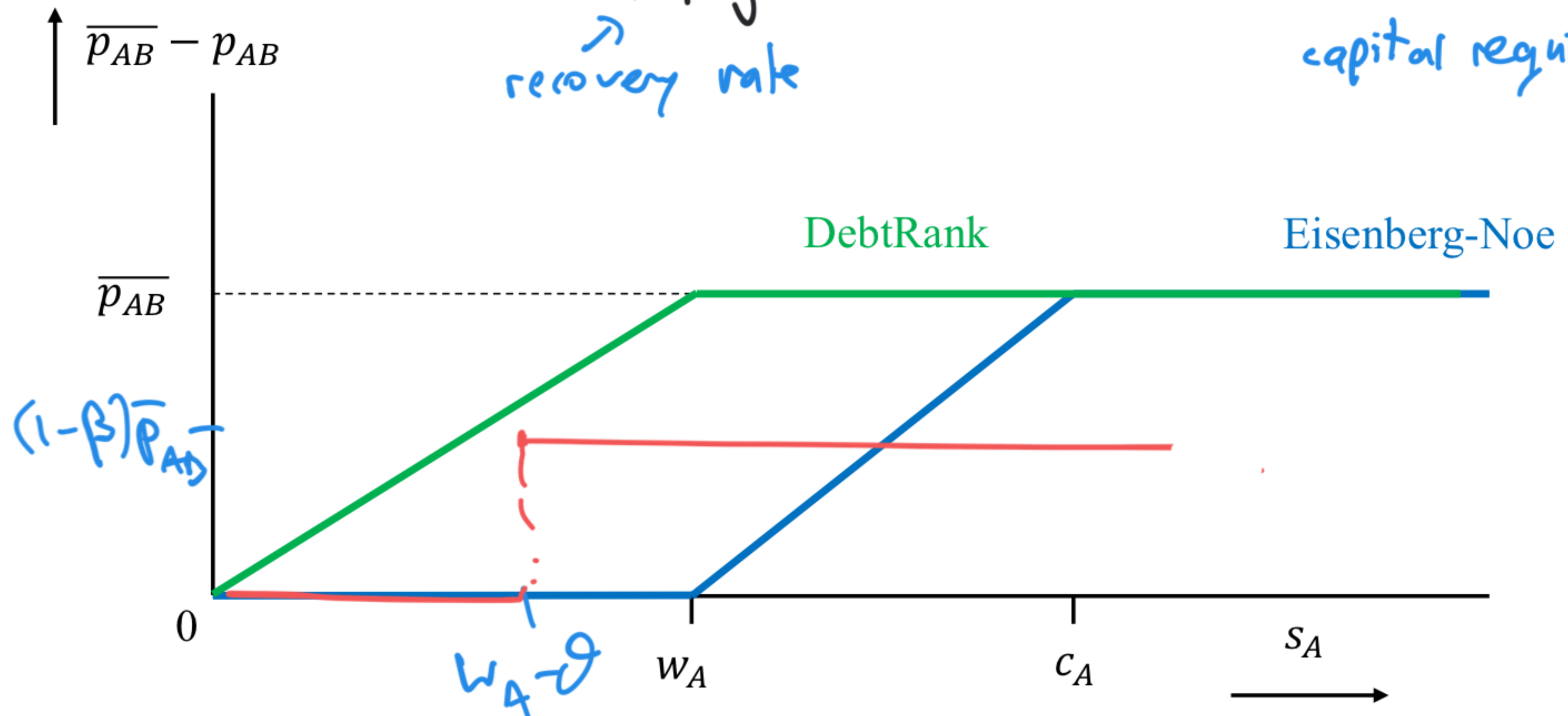
$$p_{ij}(t) = \begin{cases} \bar{p}_{ij} & \text{if } w_i \geq \vartheta \\ \beta \bar{p}_{ij} & \text{if } w_i < \vartheta \end{cases}$$

$$w_i \geq \vartheta$$

$$w_i < \vartheta$$

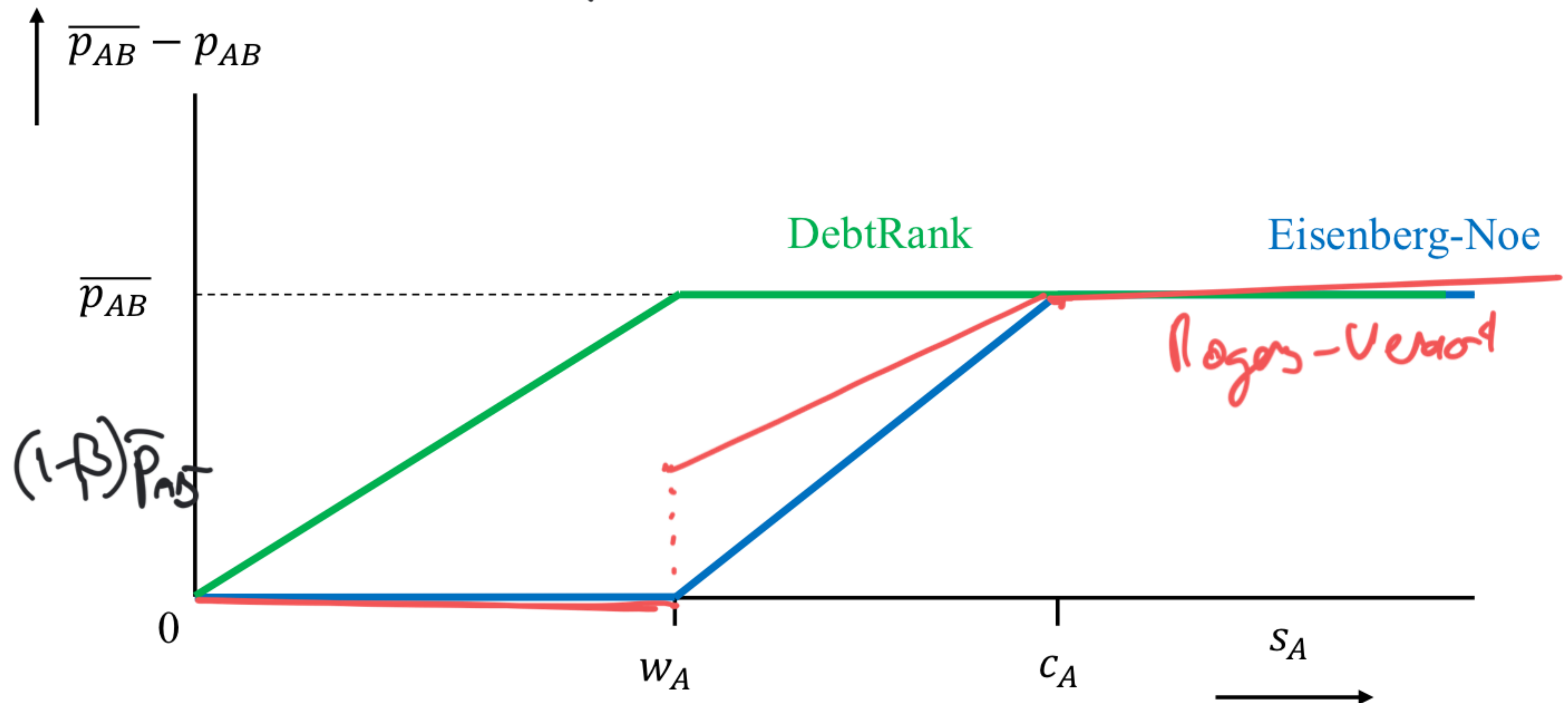
← minimum capital requirement

recovery rate

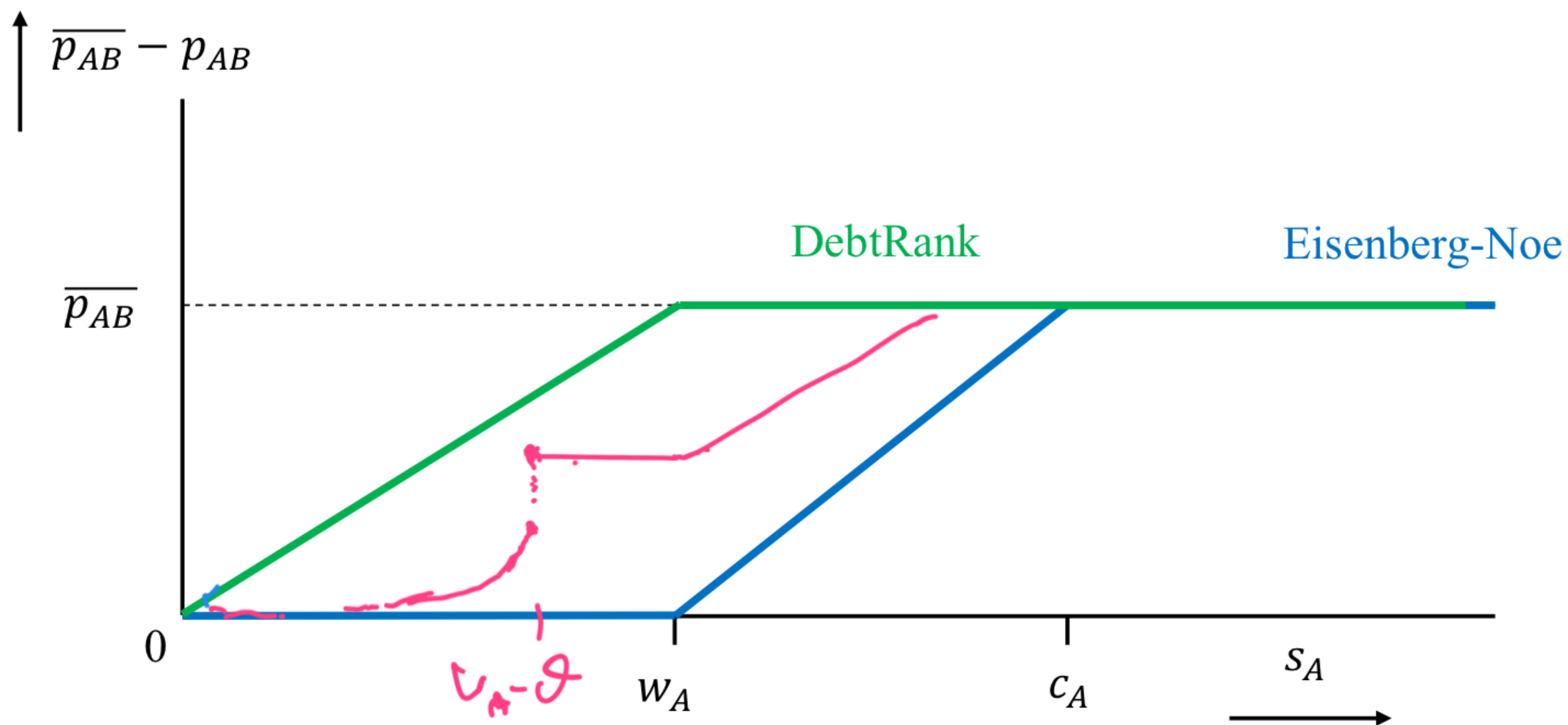


Rogers & Veraart (2013)

$$p_i(t) = \begin{cases} \bar{p}_i & \text{if } \bar{p}_i \leq \bar{c}_i + \sum_k p_{ki} \\ \beta(\bar{c}_i + \sum_k p_{ki}) & \text{otherwise} \end{cases}$$

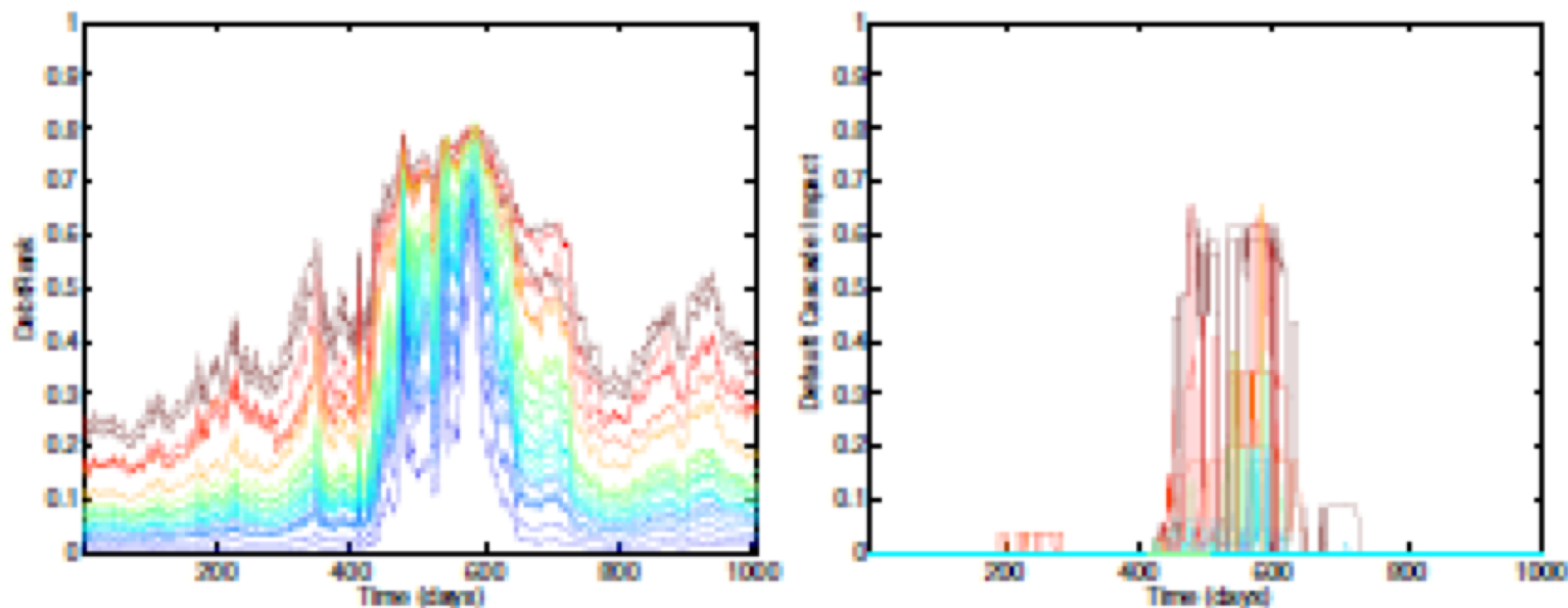


Comparison DebtRank vs Eisenberg-Noe



DebtRank vs Furfine

Source: Battiston et al. 2012, Supplementary Information Figure 16



Comparison DebtRank vs Eisenberg-Noe

DebtRank

- ‘Agent-based’
- Dynamic process
- Upperbound on contagion
- Contagion before default
- Always volatile

Eisenberg-Noe

- Accounting identities
- Fixed point clearing vector
- Lowerbound on contagion
- Contagion only after default
- No contagion in quiet periods

Constructing network data

Financial contagion analysis requires **complete** network data on interbank exposures

Usually information on network data is **incomplete**

- Only for large exposures
- Only for certain registered type of transactions
- Only for banks within own jurisdiction

What to do then?

Constructing network data

Predict network using:

- Bank balance sheet reports:
 - Aggregate interbank assets and liabilities often available

$$\forall i \quad \underline{a_i} = \sum_k p_{ki}$$

$$\underline{b_i} = \sum_j p_{ij}$$

- Available (incomplete) network data
 - Large exposures
- General information on financial network structure

Financial network structure

Common characteristics of financial networks:

- Relatively few links: low density
- Large inequality in # of links among banks
 - Degree distribution has a fat tail
- Short path lengths
- Core-periphery structure:
 - Few core banks with many links and dense core
 - Many peripheral banks with few links, only to core banks

$$\Phi = \begin{bmatrix} 0 & 40 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} 300 \\ 240 \end{matrix}$$

Estimation

Gandy & Veraart (2017) propose a Bayesian approach

- Prior: information on
 - Aggregate interbank assets and liabilities
 - Known links
 - Some random network model (Erdős-Renyi, scale-free, etc.)
- Posterior: distribution of potential networks

Networks can be sampled from the posterior using Gibbs-sampling

- Package in R: systemicrisk

Evaluating systemic risk

Typical simulation

- If network data is incomplete, randomly draw K financial networks from posterior distribution

For each financial network:

- For each bank i
 - Let bank i default (large random shock)
 - Run some contagion algorithm
 - Measure contagion effect on system

Measuring contagion

- # additional default triggered by i 's default
 - Direct contagion after 1 round
 - Indirect contagion in further rounds
- Welfare loss triggered by i 's default
 - Loss of total asset value
 - Direct and indirect contagion



Systemic importance
of i

Measuring contagion

Suppose we have N simulations, one for each bank

- Systemic Risk at system level

→ $SR =$ average # of default in simulation

→ $SR = \sum_i \hat{p}_i R_i$

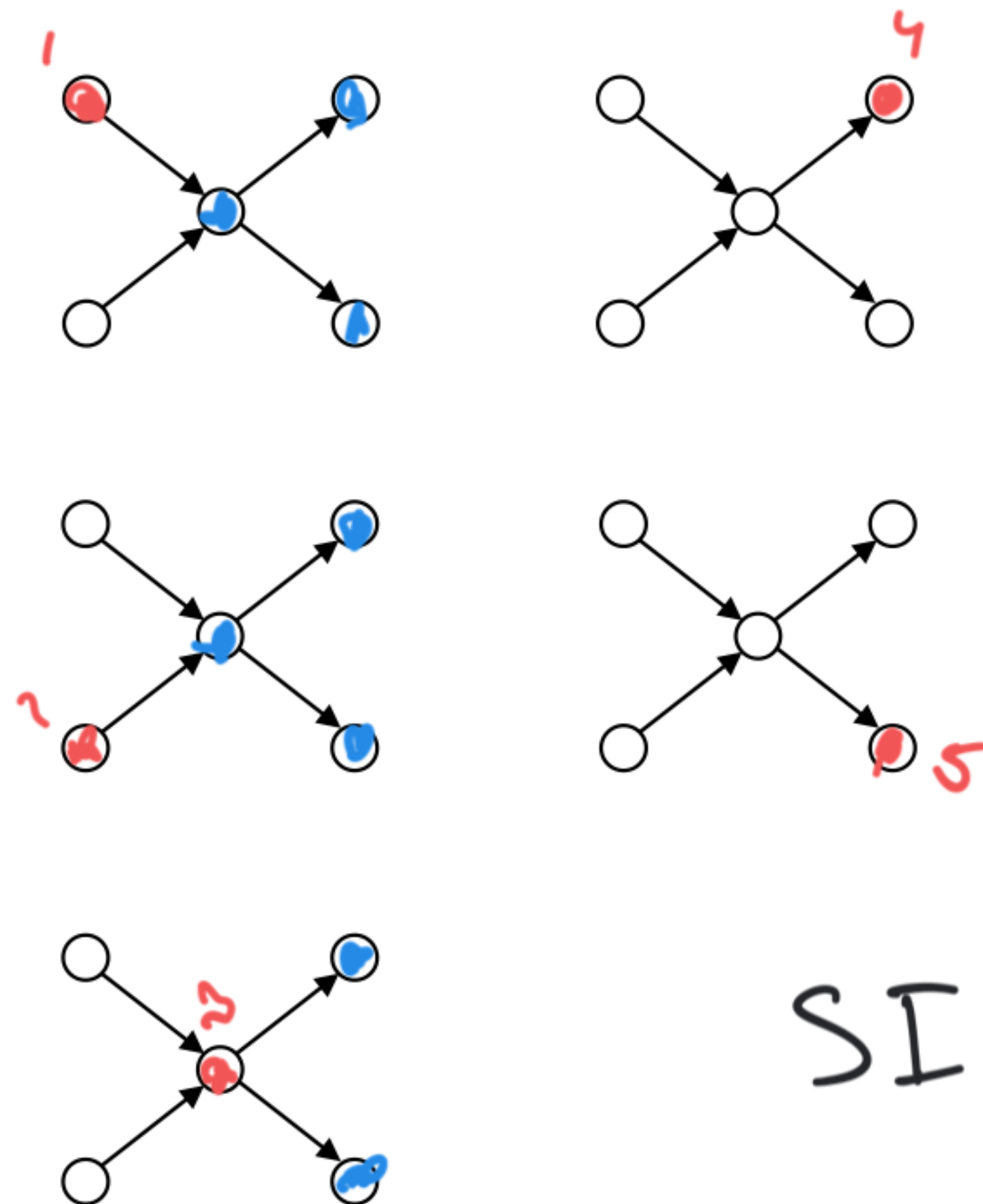
Bank level

idiosyncratic
prob. of default

← Welfare loss due to its default

- Systemic importance
- Vulnerability
 - # simulations in which i defaults
 - Average loss incurred by i in simulations

Systemic importance and vulnerability



- Most systemically important:

1, 2

- Most vulnerable: 4, 5

$$SI = SR(g) - SR(\underbrace{g \setminus \{i\}}_{\text{network with } i \text{ being 'saved'}})$$

systemic risk

network with
i being 'saved'

The multi-layer network nature of financial systemic risk and its implications

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Martínez-Jaramillo² Marco van der Leij³ Stefan Thurner^{1,4}

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the Banco de México, De Nederlandsche Bank or any of the authors' affiliations.

Course CEMLA Financial Stability
19 September 2019

CRAIG H. FURFINE

Interbank Exposures: Quantifying the Risk of Contagion

This paper examines the degree to which the failure of one bank would cause the subsequent collapse of other banks. Using unique data on interbank payment flows, the magnitude of bilateral federal funds exposures is quantified. These exposures are used to simulate the impact of various failure scenarios, and the risk of contagion is found to be economically small.

THIS PAPER QUANTIFIES contagion risk present in the U.S. banking system. Unlike previous studies that infer contagion indirectly by identifying common characteristics of banks that are affected by some event (e.g., third-world debt crisis, large bank failure), this study estimates contagion directly by examining data containing the complete universe of federal funds transactions across banks. Using such data allows for straightforward simulation exercises that demonstrate the degree of contagion that might arise from these exposures.

The cost of this direct approach to measuring contagion is clear. The data analyzed only incorporate federal funds transactions. Because of severe data limitations, other exposures among banks cannot be examined on a bilateral basis. As a result of this, an obvious criticism of the results that follow is that other exposures may actually be much higher or may be distributed in a particularly contagion-enhancing way. While it will be argued that the federal funds exposures used in this paper make up a substantial fraction of unsecured interbank credit exposures, one must realize that the conclusions reached are conditional on the set of exposures being examined. That is, the estimates of contagion reported here are accurate, yet potentially conservative.

Despite this caveat, the approach employed in this paper to measure contagion has at least three important advantages. First and foremost, the data measure exposures bilaterally. That is, each bank's exposure to every other bank is known. This

The views expressed are those of the author and do not necessarily reflect those of the Federal Reserve Bank of Chicago or the Federal Reserve system. The author appreciates the helpful comments of Allen Berger (the editor) and two anonymous referees.

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Furfine (2003), *Interbank Exposures: Quantifying the Risk of Contagion*, JMCB

“This paper examines the degree to which the failure of one bank would cause the subsequent collapse of other banks. Using unique data on interbank payment flows [in the U.S.], the magnitude of bilateral federal funds exposures is quantified. These exposures are used to simulate the impact of various failure scenarios, and the **risk of contagion is found to be economically small.**”

What was missing?

Contagion mechanism

- Eisenberg-Noe fictitious default algorithm using book value (no behavior)
- Underestimation of contagion
- Late developments: new mechanisms, e.g. DebtRank

Data

- Only one type of exposure: uncovered interbank loans in the U.S.
- Banks have **different types of exposures: Financial multilayer (multiplex) network**
- How does the multilayer nature of financial networks matter for estimating systemic risk?

Introduction

In this paper we

- consider a data set of different exposures between banks in Mexico
- analyze individual layers and the combined multilayer network
- using systemic risk measures based on DebtRank

Introduction

We find that

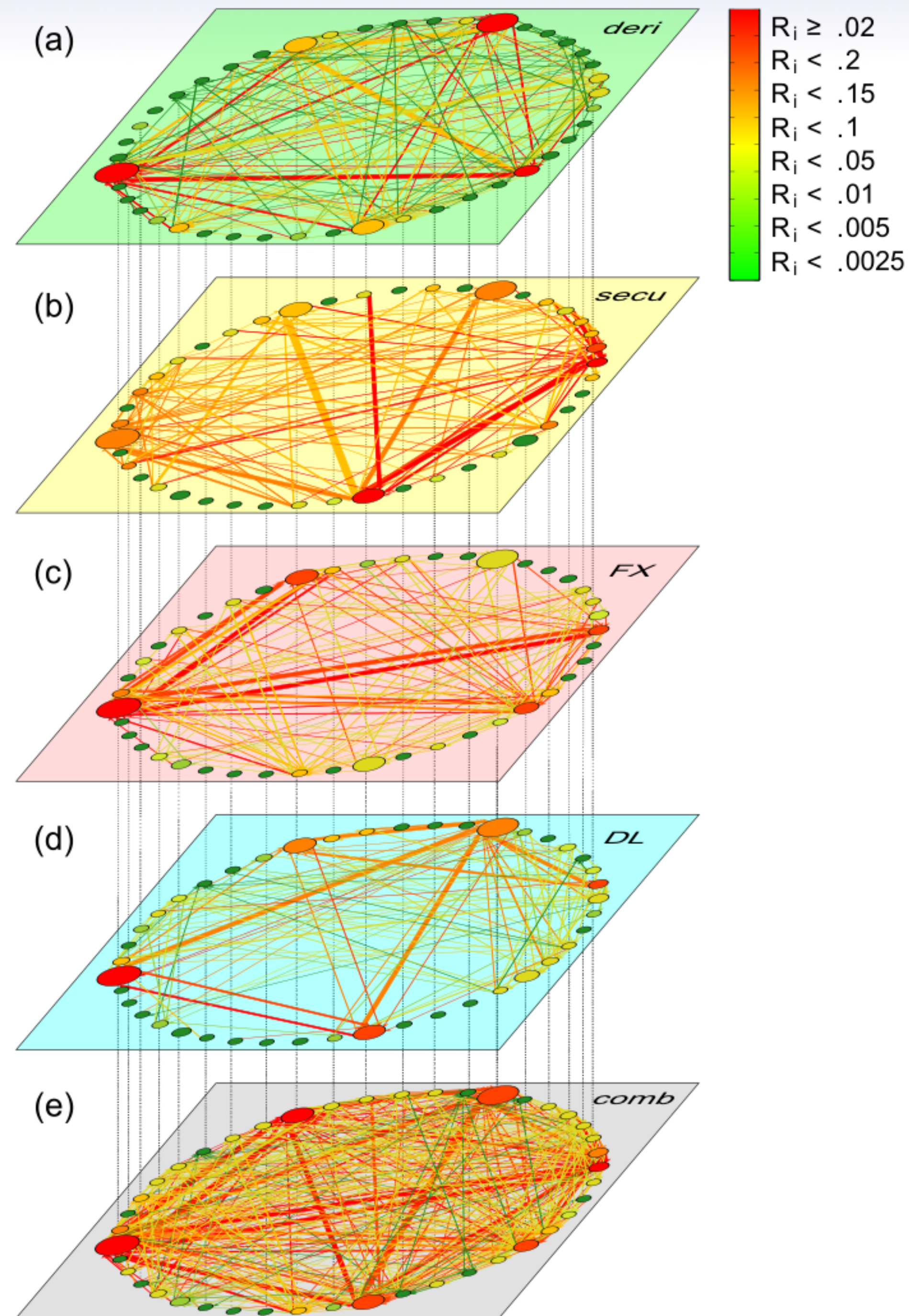
- Using only the layer of interbank loans underestimates systemic risk by 90%
- Systemic risk of the combined exposure network is higher than the sum of the 4 layers: *non-linear effect of combining layers*
- Financial markets underestimates current systemic risk
- The contribution of a credit transaction to expected systemic loss is up to a hundred times higher than the corresponding credit risk

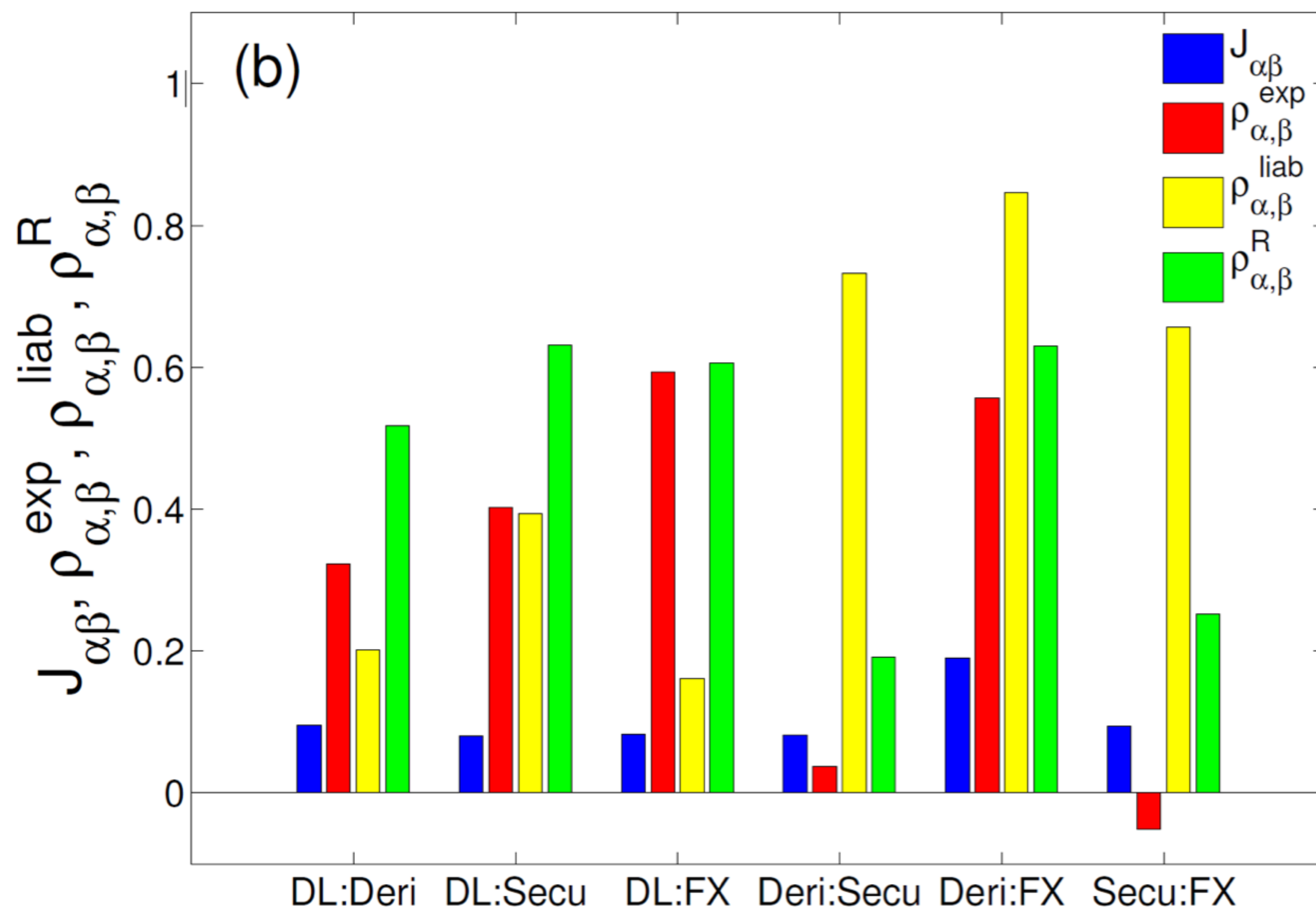
Data

Daily bilateral exposures on 43 banks in Mexico (from January 2, 2007 to May 30, 2013) arising from

1. Derivatives: valuation of derivatives transactions (swaps, forwards and options), repo transactions and securities trading.
2. Securities: securities issued by one bank that are held by another bank
3. Foreign Exchange: exposures from FX transactions
4. Deposits & Loans: Interbank deposits and loans in local and foreign currency, credit lines extended for settlement purposes.

The *combined* network of exposure is obtained by *aggregating the individual layer exposures*.

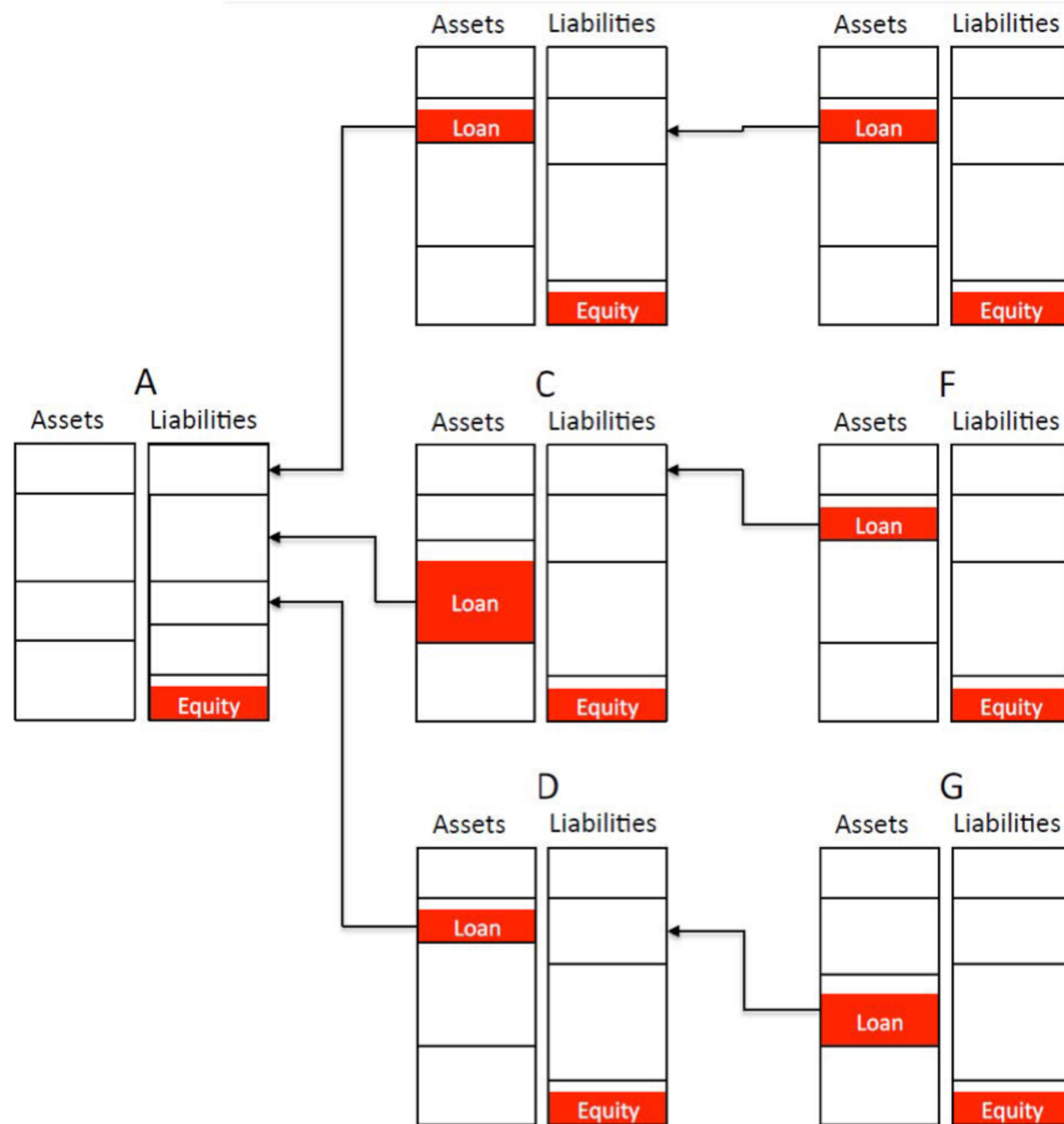


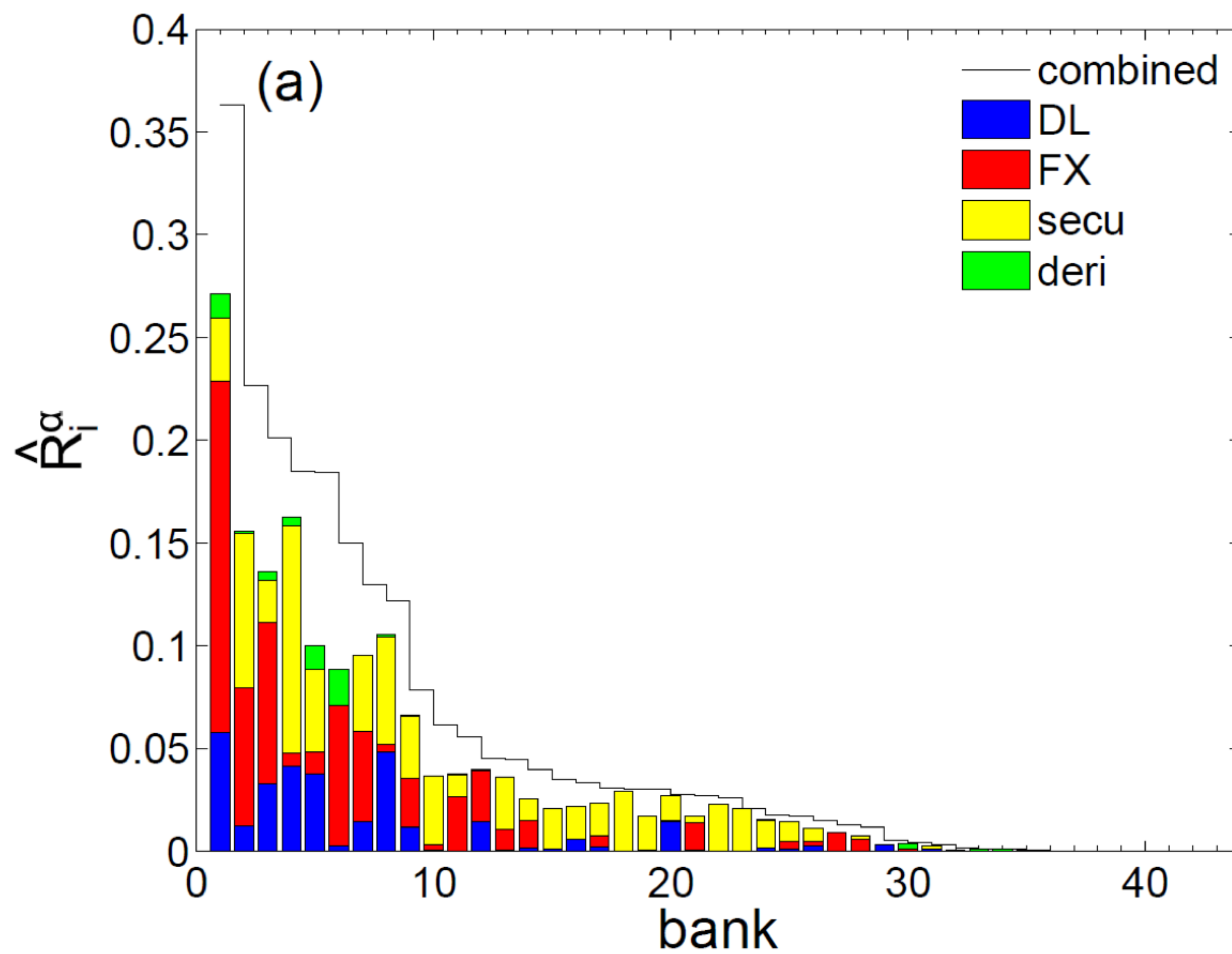


DebtRank

- Recursive method suggested by Battiston et al. (2012) to quantify the systemic importance of nodes in terms of losses
- Measures the fraction of the **total economic value** in the financial system that is potentially **lost by the default of a single bank**.
- \hat{R}_i^α is the DebtRank of bank i in layer α .

DebtRank





Layers matter and may increase systemic risk

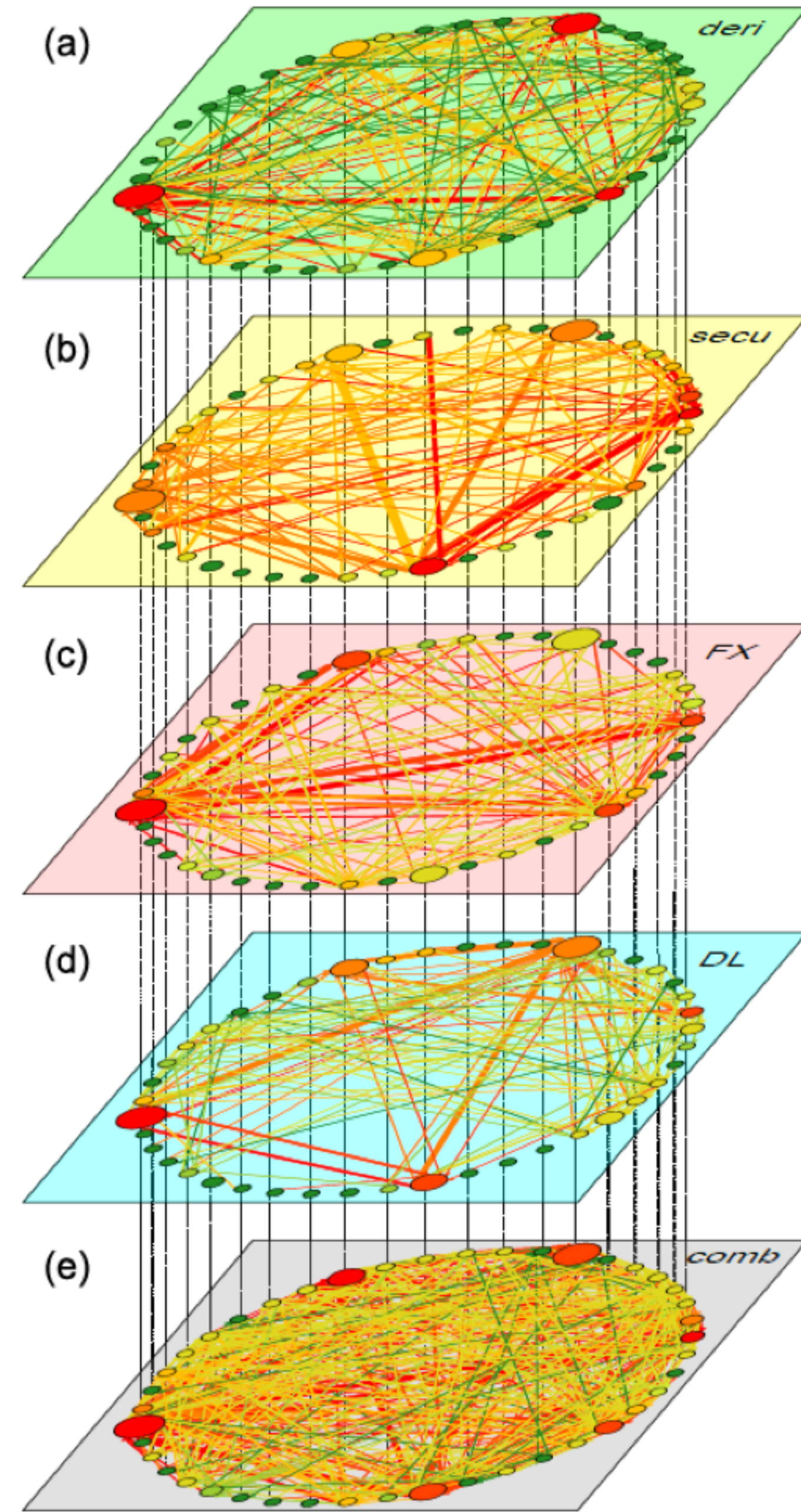
A

B

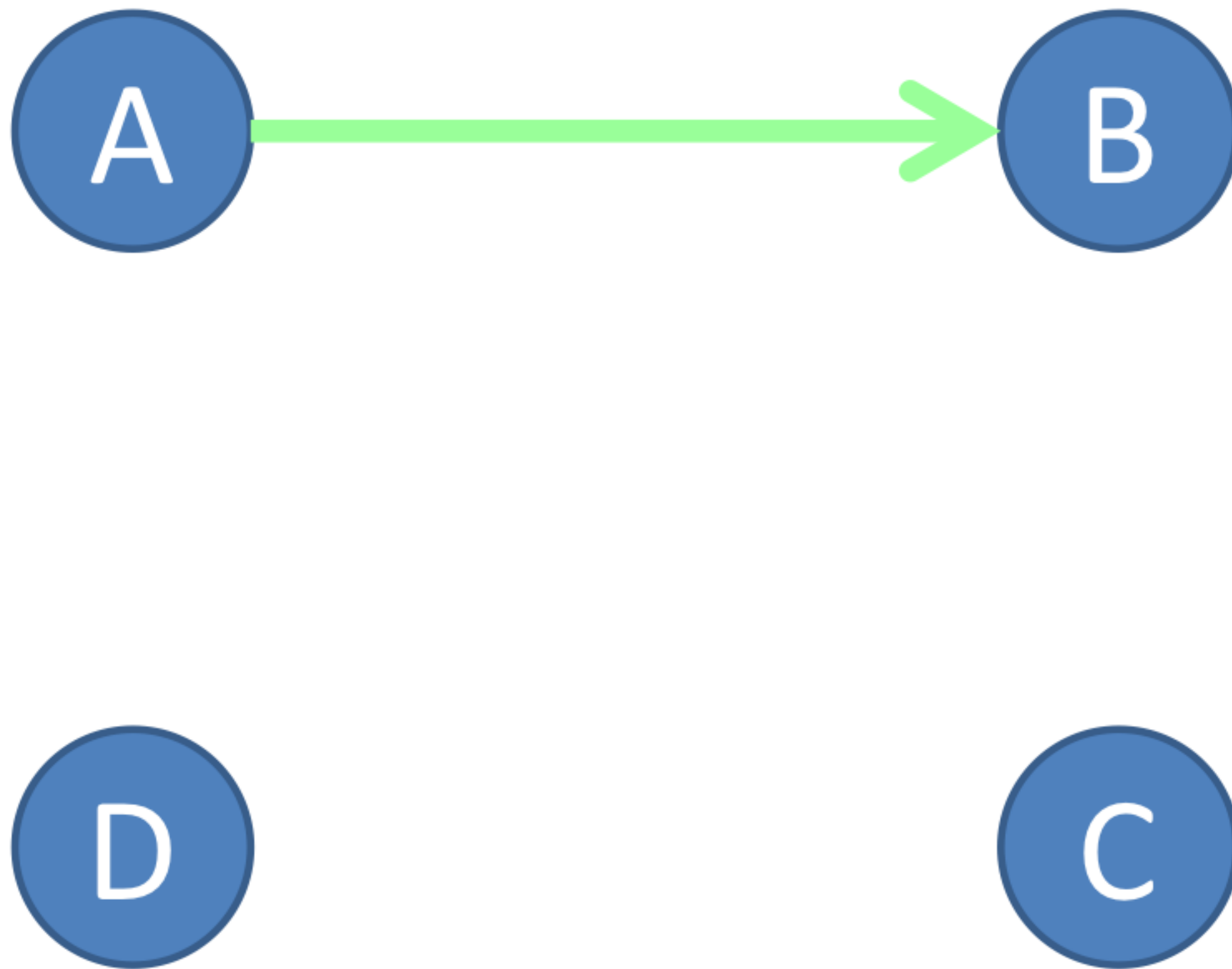
D

C

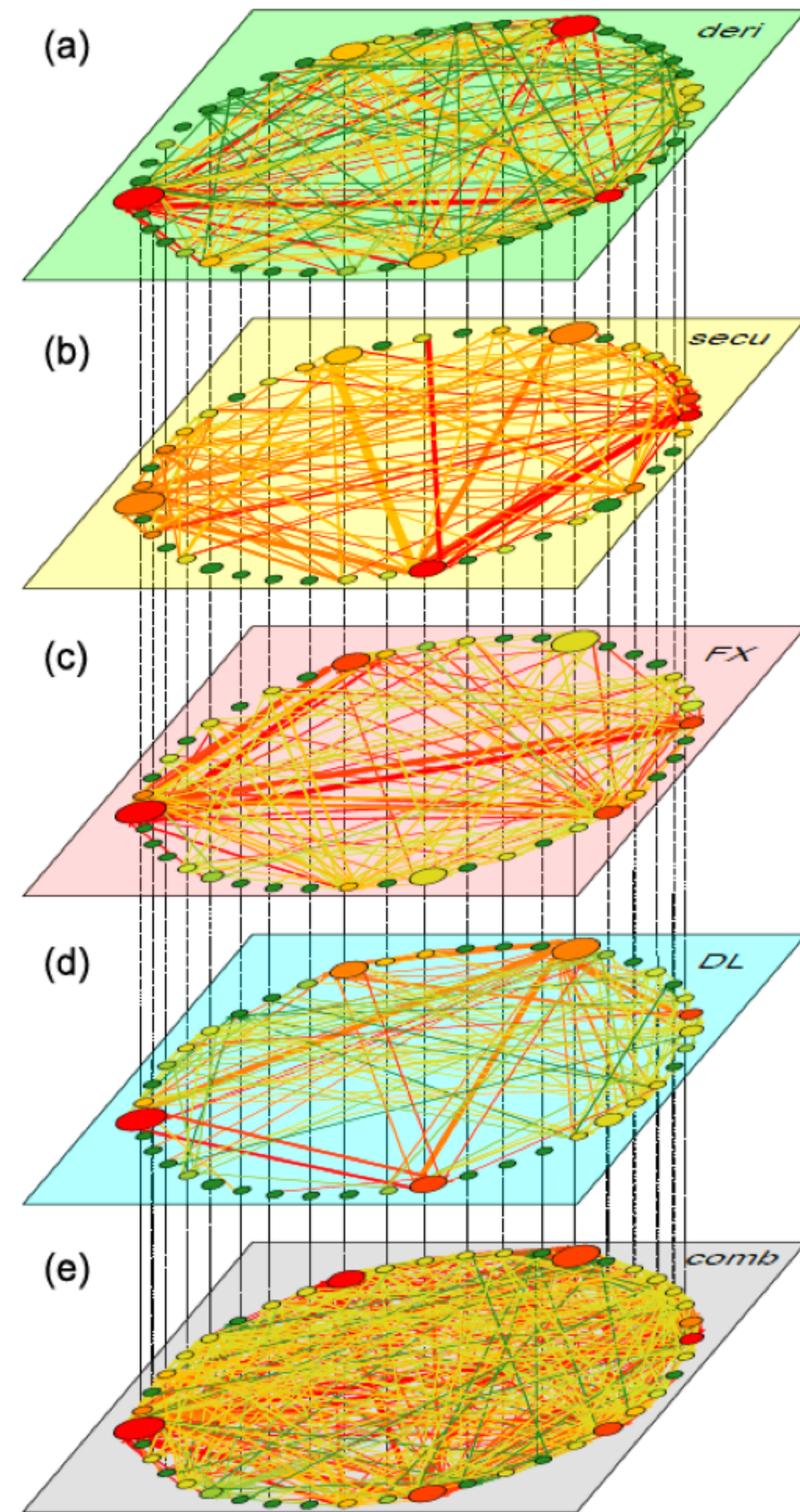
Comments by Andre Lucas



Layers matter and may increase systemic risk



Comments by Andre Lucas



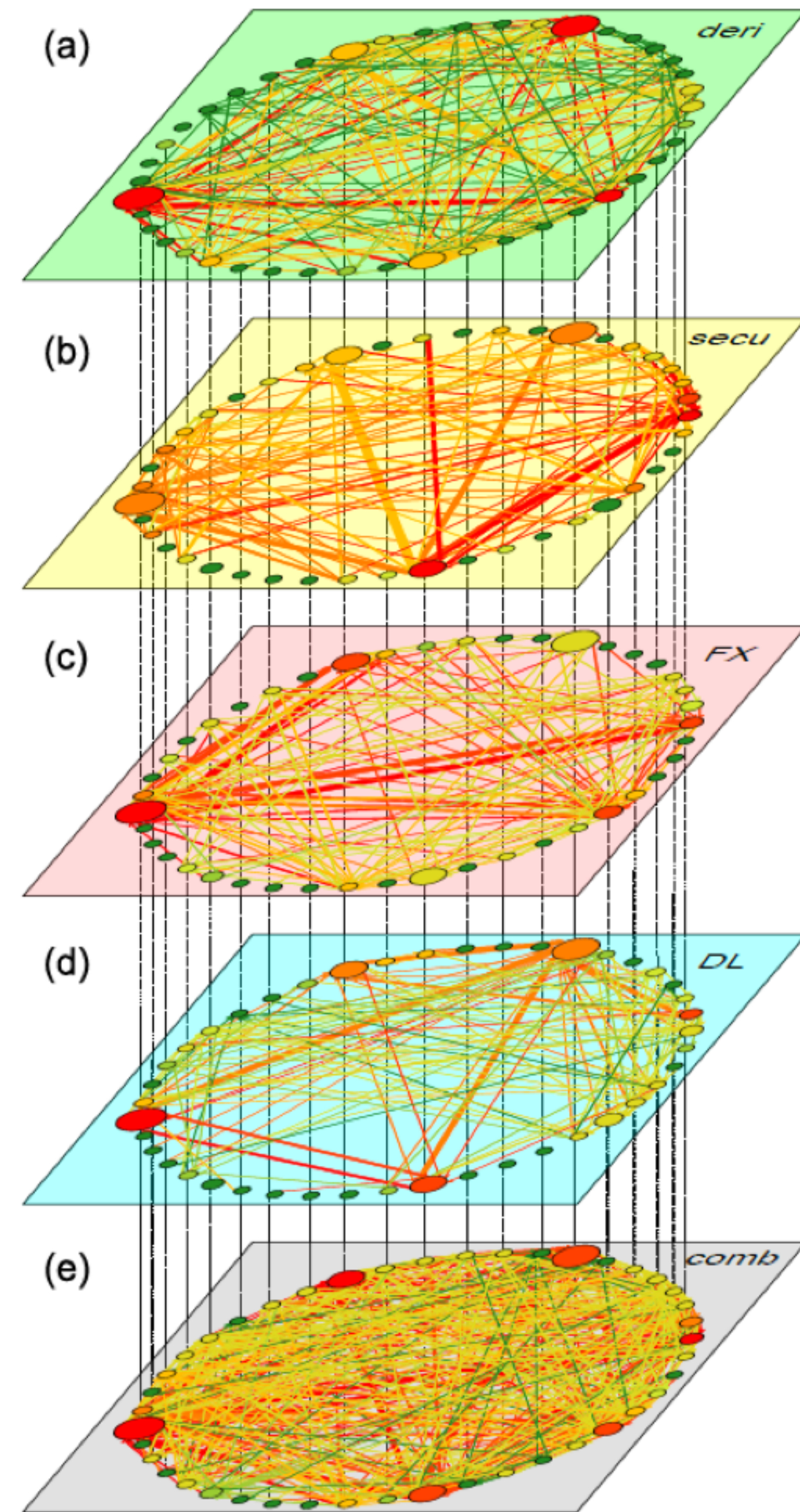
Layers matter and may increase systemic risk

A

B

D

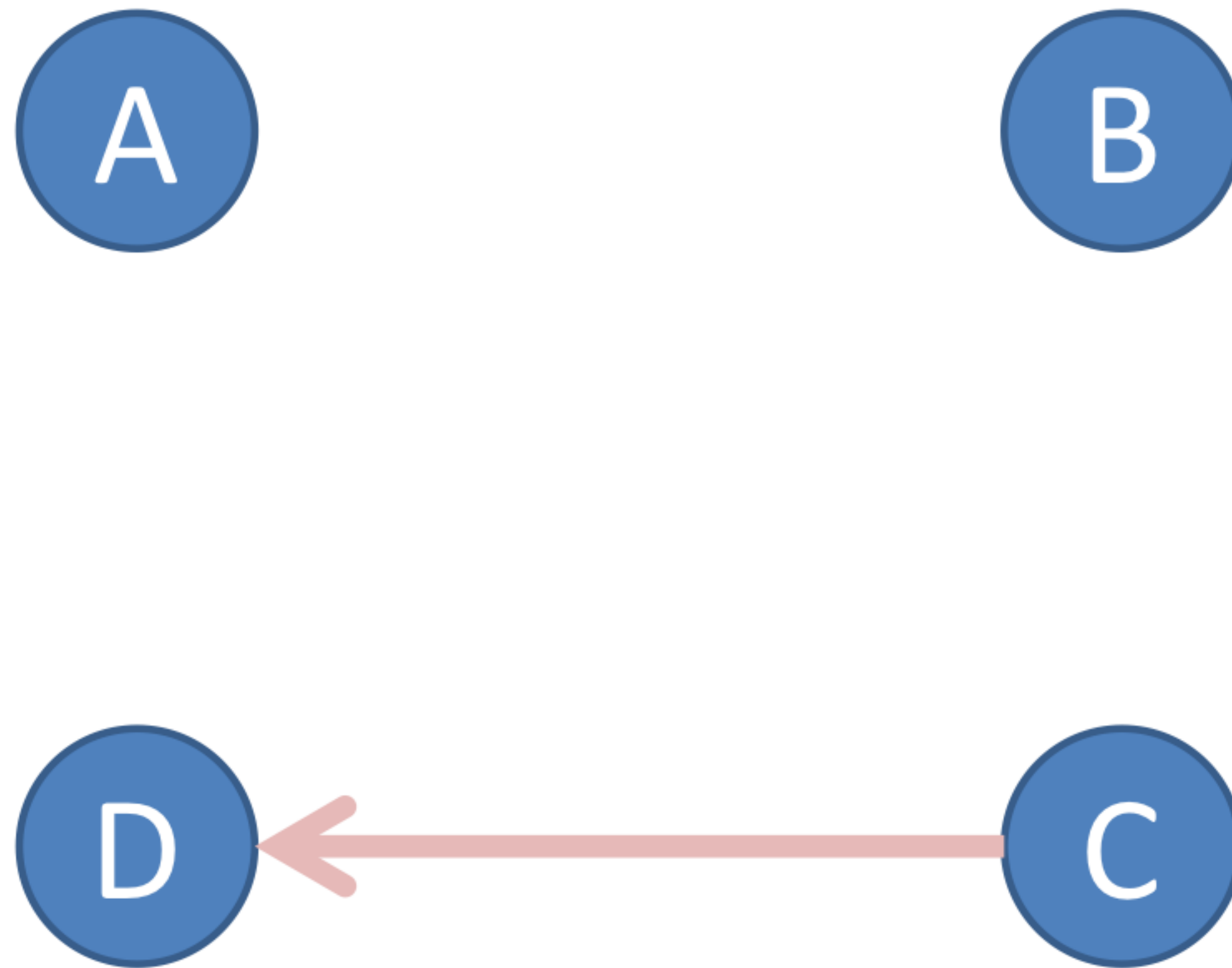
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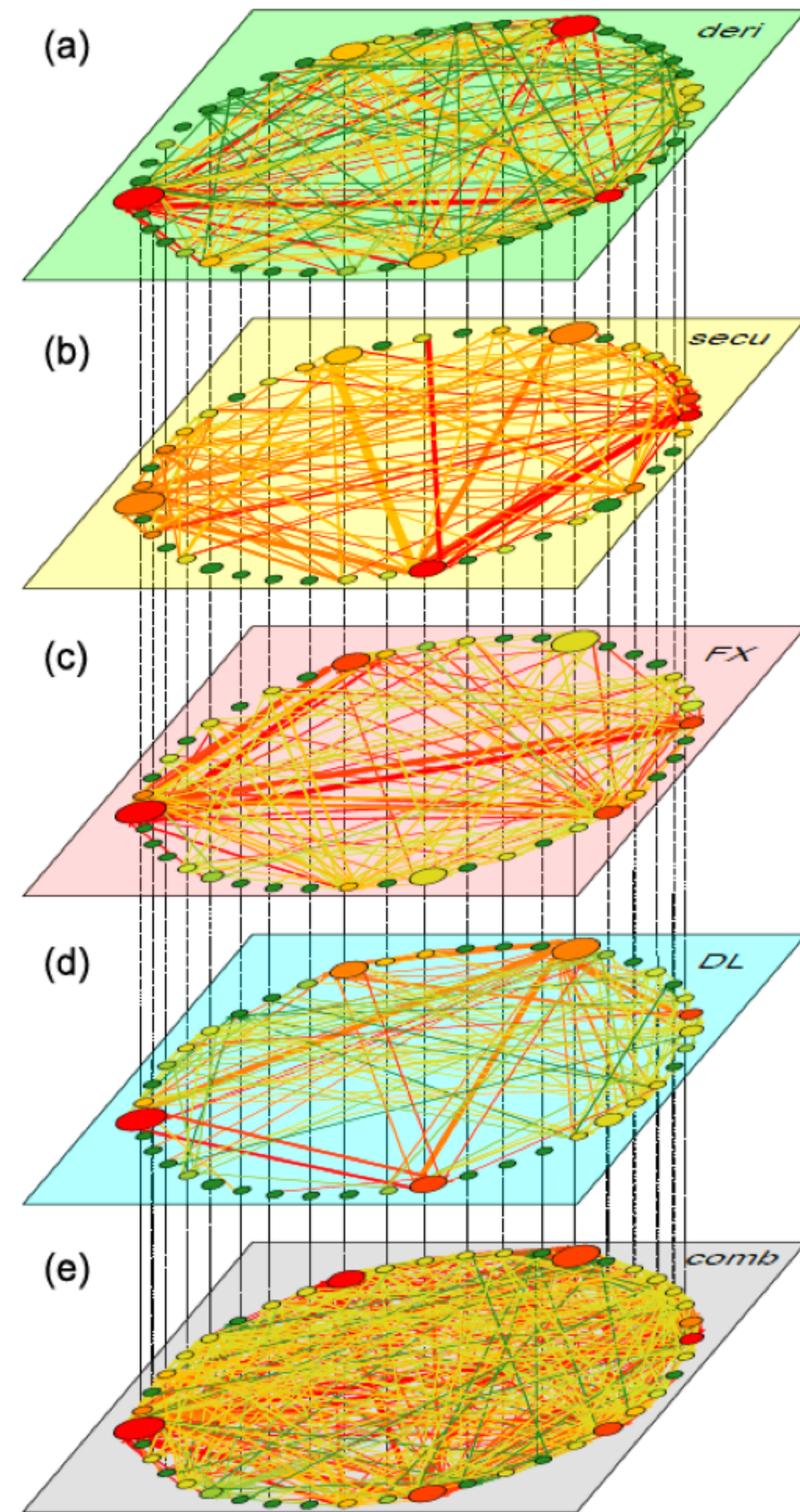
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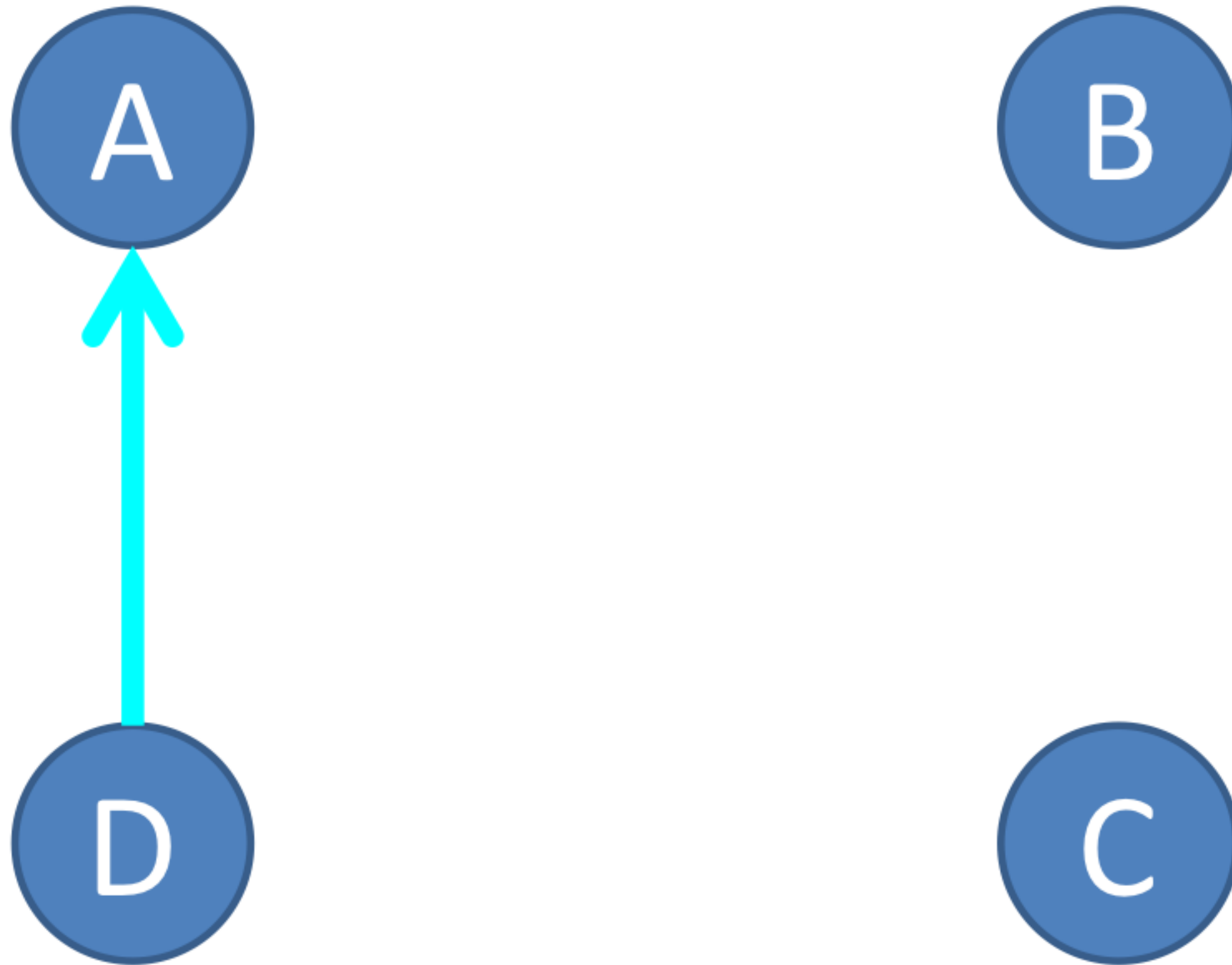
Layers matter and may increase systemic risk



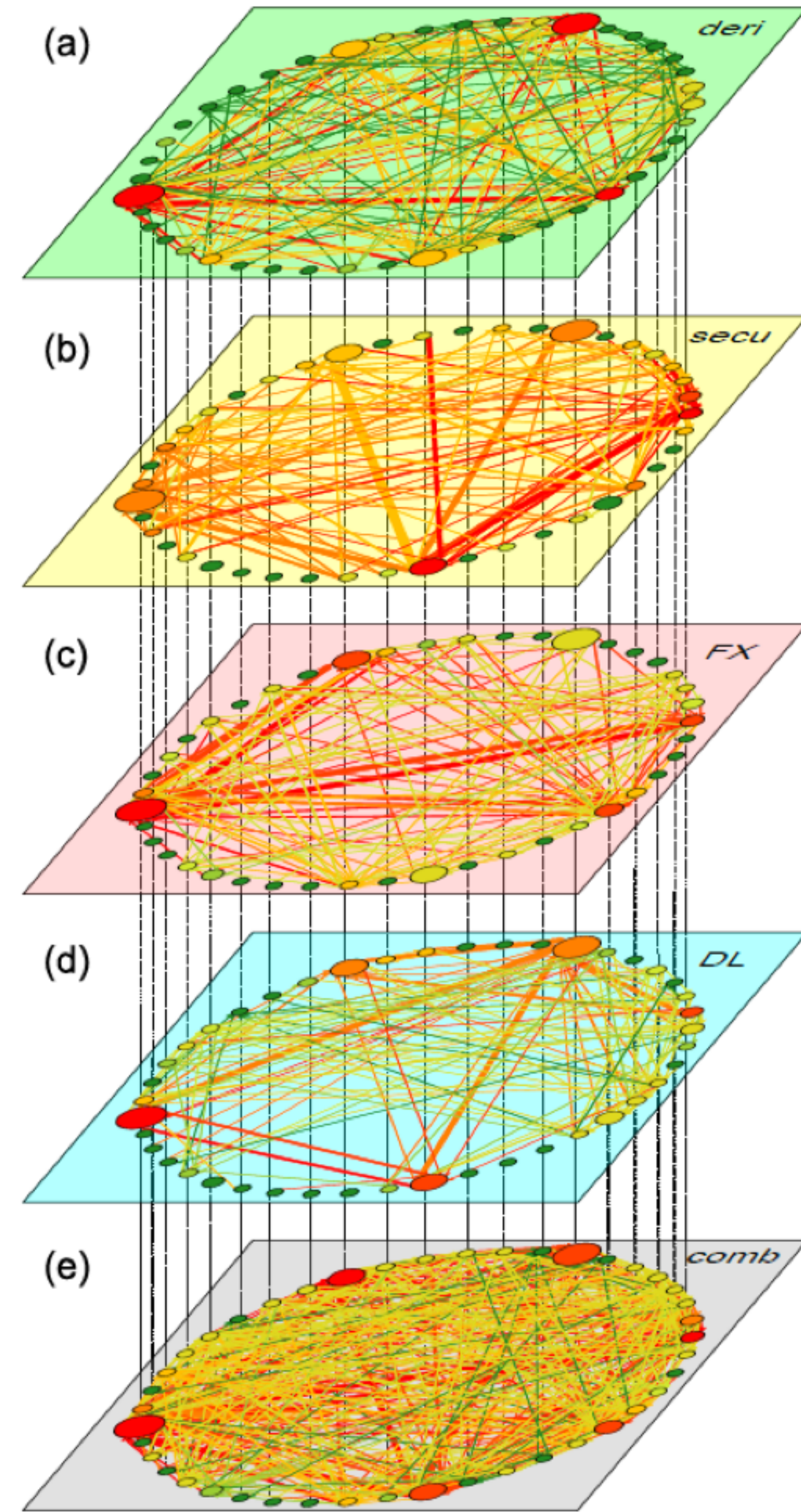
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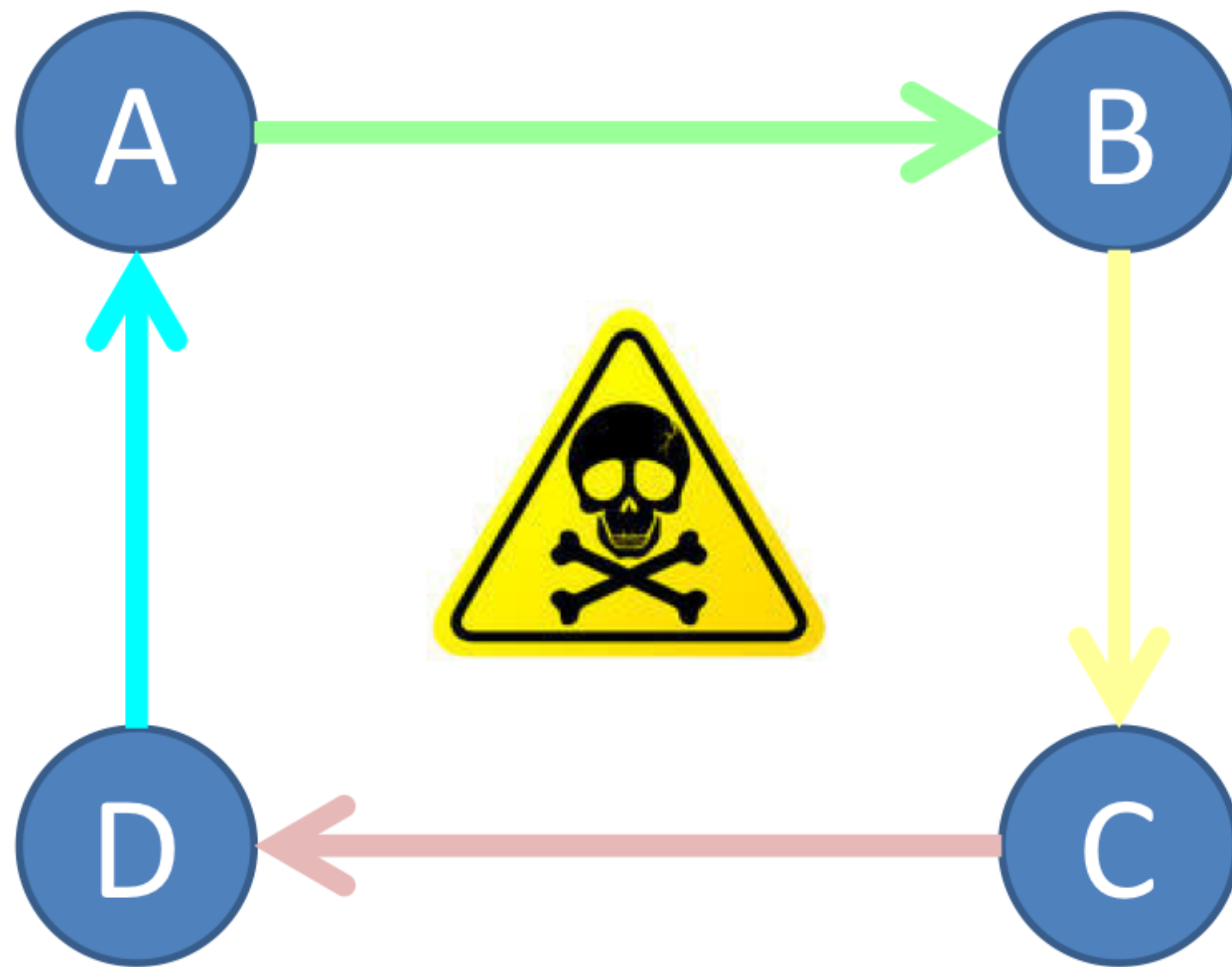
Layers matter and may increase systemic risk



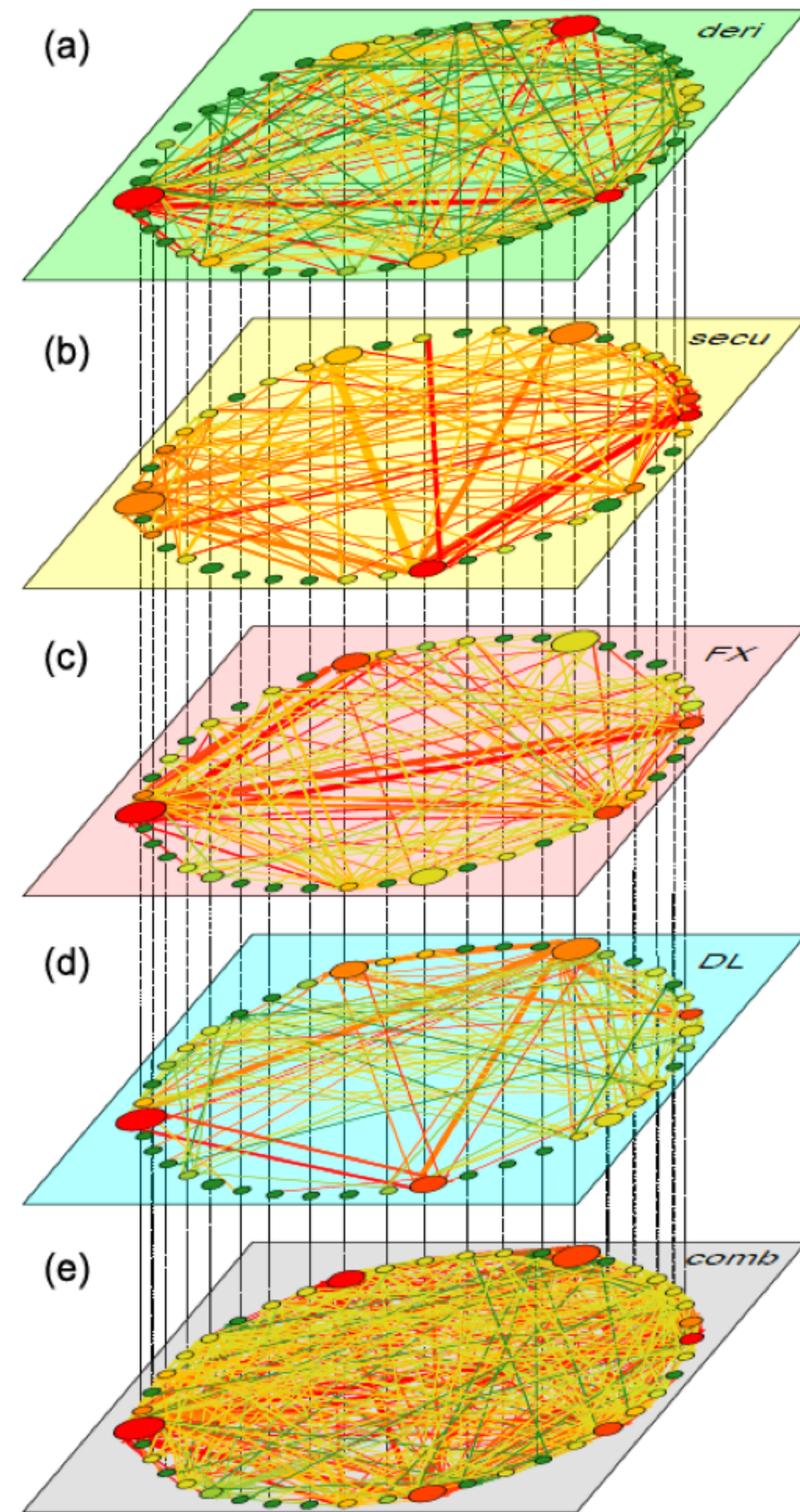
Comments by Andre Lucas



Layers matter and may increase systemic risk

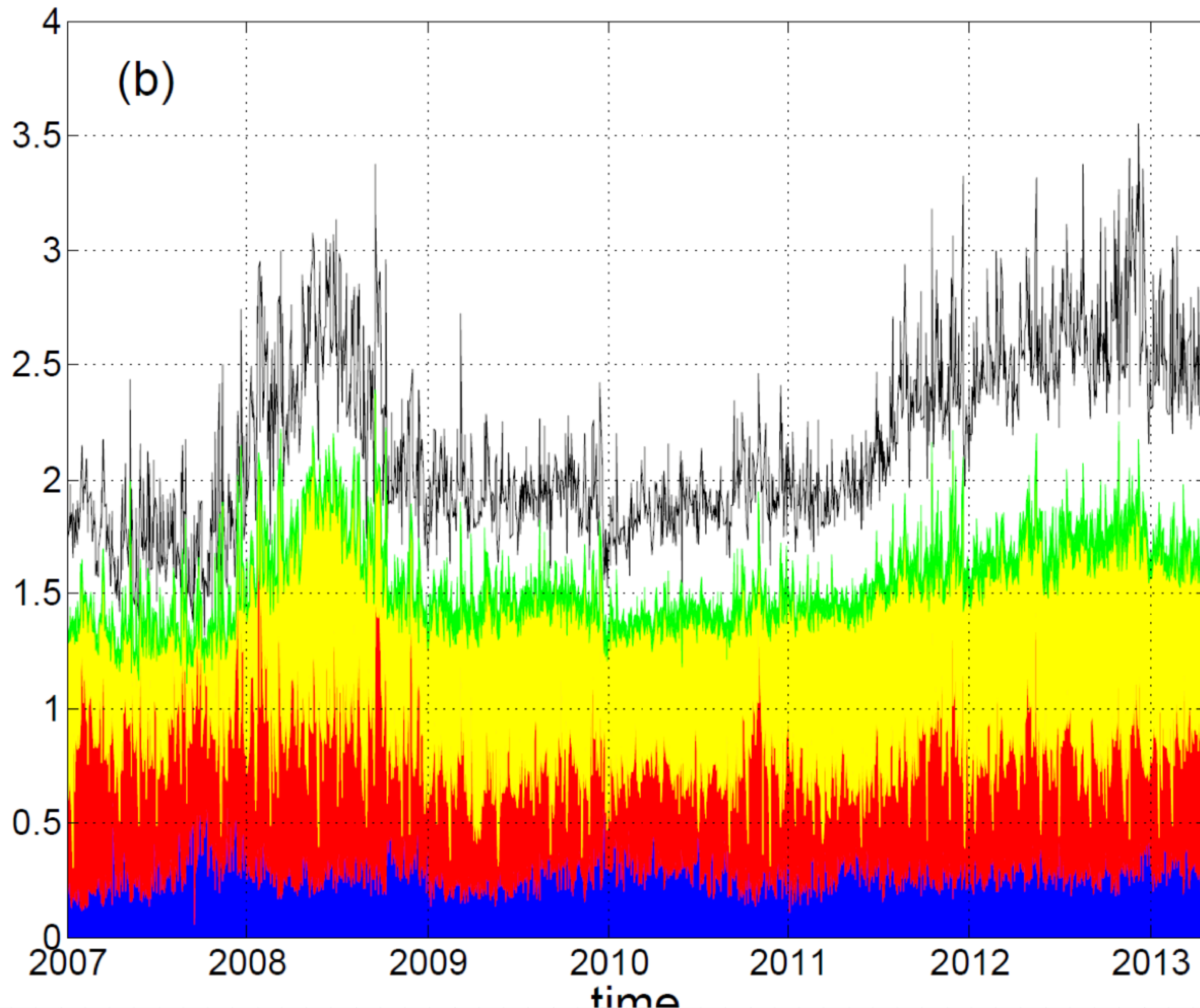


Comments by Andre Lucas



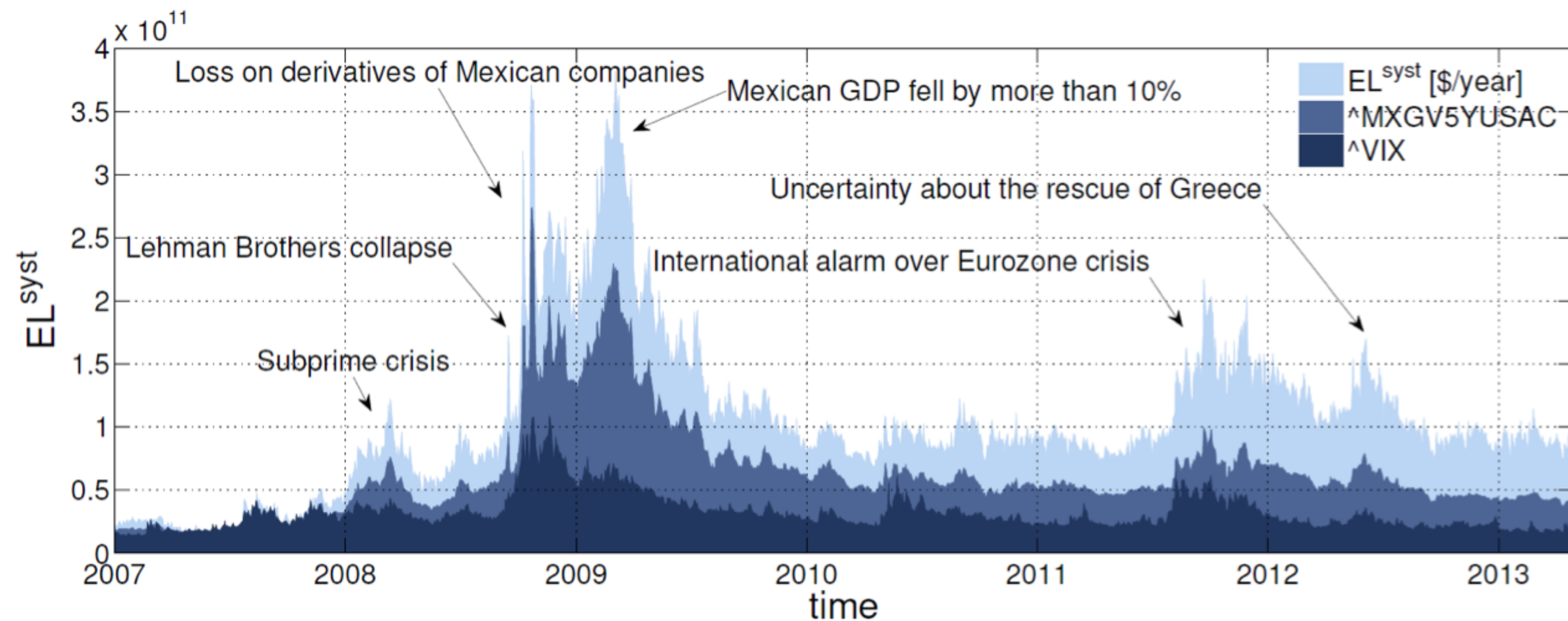
Systemic Risk for the Entire System

Define an SR index for the entire system: $SRI^\alpha(t) = \sum_{i=1}^B \hat{R}_i^\alpha(t)$.



Expected Systemic Losses

- Calculated as $EL^{syst} = V^{comb} \sum_{i=1}^B p_i^{def} R_i^{comb}$.
- Combines SR contributions from networks and default rates.
- Compare to measure of CDS spread on 5-year Mexican government bond, and VIX volatility index



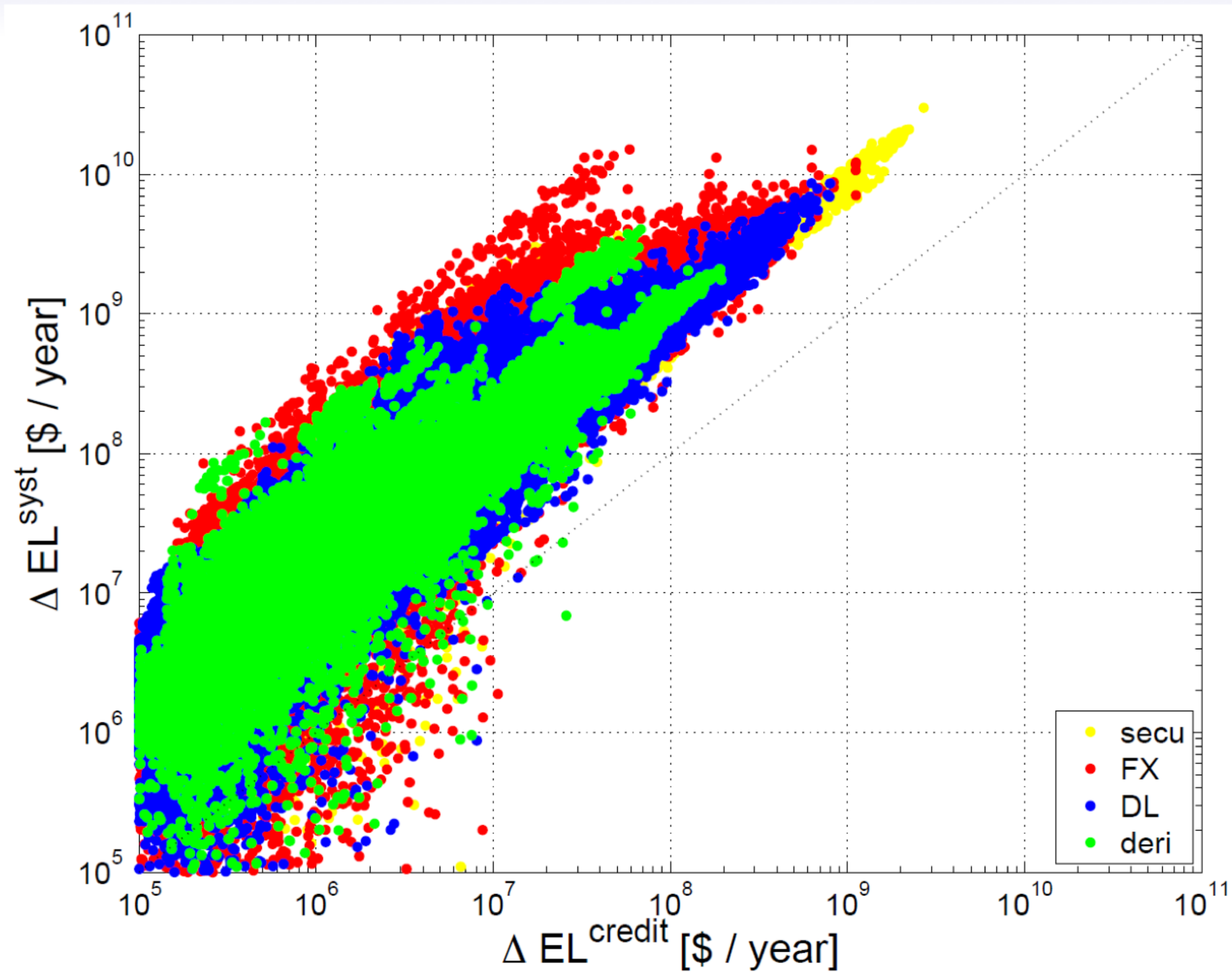
Marginal contribution of individual transactions

Consider an individual loan between bank i and j :

- Marginal contribution to systemic risk: ΔEL^{syst}

Compare to

- Marginal contribution to idiosyncratic credit risk: ΔEL^{credit}



Conclusion

In this paper we

- consider a data set of different exposures between banks in Mexico
- analyze individual layers and the combined multilayer network
- using systemic risk measures based on DebtRank

Conclusion

We find that

- Using only interbank loans underestimates systemic risk by 90%
- Systemic risk of the combined exposure network is higher than the sum of the 4 layers: non-linear effect of combining layers
- Financial markets underestimates current systemic risk
- The contribution of a credit transaction to expected systemic loss is up to a hundred times higher than the corresponding credit risk

Learning goals

Part 1

- Explain 3 main channels of financial contagion:
 - Default cascades,
 - Funding contagion / liquidity hoarding
 - Fire sales externality
- Compute by hand:
 - Fictitious default algorithm of Eisenberg & Noe (2001)
 - DebtRank algorithm of Bardoscia, Battiston, Caccioli et Cardarelli (2015)

Learning goals

Part 2

- Give typical characteristics of large networks
- Construct financial network data from balance sheet data and large exposures
- Compute measures of financial contagion:
 - System level: systemic risk, expected systemic loss
 - Bank level: systemic importance, vulnerability
- Explain what is a multilayer network and why it is important for assessing systemic risk
 - Poledna et al. (JFS, 2015)



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