Expectations, Learning and Monetary Policy

George W. Evans, University of Oregon and University of Saint Andrews
and
Seppo Honkapohja, Bank of Finland

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Introduction

Occasionally economic circumstances can change quickly. Then information and understanding become very imperfect.
=> Agents then try to improve their knowledge.
=> Learning behavior becomes a major driver of economic dynamics.

- Imperfect knowledge and learning can be contrasted with rational expectations (RE), the standard paradigm in macroeconomics.
  - Agents are dynamic optimizers and their beliefs are the same as the (model) reality, except for unforecastable random shocks.
  - Learning is not inconsistent with RE: RE may emerge if the agents' environment remains stationary for sufficiently long period.

- Dynamics under learning can be different from RE dynamics.
  - From policy point of view it is important to take into account the learning process.

- This webinar provides and overview to various issues in learning and monetary policy.

- Section 1 gives an introduction to adaptive learning.

- Section 2 discusses basic issues in learning and monetary policy:
  - learning and Taylor rules,
  - learning and optimal monetary policy,
  - non-fundamental equilibria, long-horizon forecasts,
  - internal CB forecasting, speed of convergence.

- Section 3 continues with further issues:
  - perpetual learning,
  - liquidity traps and other topics.

Learning and bounded rationality

- Many macroeconomic models are summarized in the reduced form

$$y_t = F(y_{t-1}, \{y^e_{t+j}\}^{\infty}_{j=0}, w_t, \eta_t), \tag{1}$$

  where y_t is a vector of endogenous aggregate variables, and w_t is a vector of stochastic exogenous variables, often taken to be a VAR.

- Comments:
  - (1) assumes a representative agents setting. This can be easily relaxed.
  - Agents are assumed to know their own characteristics, but otherwise structural knowledge is imperfect.

- Comments (continued):

  - The precise information set of agents.

  - Must specify the degree of structural information: do agents only use a reduced form; if so, is it correctly specified?

  - The horizon for decisions and expectations must be specified.

- **Cognitive consistency principle**: economic agents are about as smart as good economists (theorists or econometricians).

  => we mostly focus on adaptive or econometric learning.

  => economic agents use time-series econometric techniques, possibly with some structural information.

- **Simple general model**

$$y_t = \mu + \alpha y_t^e + \delta' w_{t-1} + \eta_t. \tag{2}$$

y_t is a scalar endogenous variables, w_{t-1} is a vector of exogenous observables, and η_t is an unobservable random shock. $y_t^e = E_{t-1}^* y_t$ is is the expectation of y_t based on observables dated $t-1$ or earlier.

- The unique REE is

$$y_t = \bar{a} + \bar{b}' w_{t-1} + \eta_t,\ \bar{a} = (1-\alpha)^{-1}\mu, \bar{b} = (1-\alpha)^{-1}\delta. \tag{3}$$

- **Example**: (Muth 1961 market model). A competitive market with one-period production lag. Supply decisions based on $p_t^e = E_{t-1}^* p_t$.
  - Demand and supply are

$$d_t = m_I - m_p p_t + v_{1t}, \text{ and } s_t = r_I + r_p p_t^e + r_w' w_{t-1} + v_{2t},$$

  where v_{1t}, v_{2t} are white noise.
  (Lucas 1973 model is another example.)
  Learning: Agents estimate the parameters a, b of their forecasting model or **perceived law of motion (PLM)**

$$y_t = a + b' w_{t-1} + \eta_t, \tag{4}$$

  and to use the estimates a_{t-1}, b_{t-1} to make forecasts

$$y_t^e = a_{t-1} + b_{t-1}' w_{t-1}.$$

- **Temporary equilibrium at time** t: w_{t-1} and η_t and y_t^e determine actual y_t according to (2).
  Agents update the parameters a, b in t to (a_t, b_t) using least squares (LS). This defines a sequence of temporary equilibria for $t, t+1, \ldots$RE are attained asymptotically if $a_t, b_t \rightarrow \bar{a}, \bar{b}$ as $t \rightarrow \infty$.

- **Result:** (Bray & Savin 1986, Marcet & Sargent 1989) There is asymptotic convergence to RE with prob. one if $\alpha < 1$. If $\alpha > 1$, the system diverges.

- **E-stability** (Evans and Honkapohja 2001, 2009 are recent expositions) is a general technique to derive the convergence condition. In the current example, inserting (4) into (2) yields the temporary equilibrium

$$y_t = \mu + \alpha a + (\delta + \alpha b)' w_{t-1} + \eta_t$$

  and a corresponding mapping $T(a, b) = (\mu + \alpha a, \delta + \alpha b)$ in the parameter space. E-stability is stability of $(\bar{a}, \bar{b})$ under $d(a, b)/d\tau = T(a, b) - (a, b)$, where τ is virtual time. One immediately sees the condition $\alpha < 1$.

- Further directions:
  - When the model has multiple REE, E-stability is a **selection criterion** to find REE that are attainable under learning. Multiple REE arise when expectations about the future y^e_{t+1} influence y_t.
  - The learning approach generates **new dynamics** not found under RE hypothesis.

Methodological issues

- **Misspecification** of the forecasting model.

- **Structural change** of unknown form: agents use **constant-gain learning**.

- **Heterogeneous expectations, e.g. dynamic predictor selection**.

- **Bayesian learning and model averaging**.

- **Structural knowledge.**

Learning and monetary policy

- The basic New Keynesian model
  (i) households: supply labour, purchase goods, hold money and bonds,
  (ii) firms hire labour and produce differentiated goods under monopolistic competition,
  (iii) firms face a constraint on price setting and there is no capital,
  (iv) central bank sets monetary policy by controlling the interest rate.

- The linearized NK model: Aggregate demand and Phillips curves are

$$\begin{aligned} x_t &= -\varphi(i_t - E_t^* \pi_{t+1}) + E_t^* x_{t+1} + g_t, \\ \pi_t &= \lambda x_t + \beta E_t^* \pi_{t+1} + u_t, \end{aligned}$$

  where x_t and π_t denote the output gap and inflation, i_t is the nominal interest rate.

- $E_t^* x_{t+1}$ and $E_t^* \pi_{t+1}$ denote the private sector (not necessarily rational) expectations. (E_t without $*$ denotes rational expectations).
  - Parameters φ and λ are positive and the discount factor is $0 < \beta < 1$.
  - Shocks g_t and u_t are assumed to be observable and follow

$$\begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix}, \; F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix}$$

  where $0 < |\mu| < 1$, $0 < |\rho| < 1$ and $\tilde{g}_t \sim iid(0, \sigma_g^2)$, $\tilde{u}_t \sim iid(0, \sigma_u^2)$.

Interest rate rules

- **Instrument rules** for i_t are without consideration of policy optimization. A prominent example is the Taylor (1993) rule

$$i_t = \pi_t + 0.5(\pi_t - \bar{\pi}) + 0.5x_t,$$

where $\bar{\pi}$ is the target inflation level and the output gap target is zero.
- More generally Taylor-type rules are, wlog setting $\bar{\pi} = 0$,

$$i_t = \chi_\pi \pi_t + \chi_x x_t, \text{ where } \chi_\pi, \chi_x > 0.$$

- Variations of the Taylor rule replace π_t and x_t by lagged values

$$i_t = \chi_\pi \pi_{t-1} + \chi_x x_{t-1} \text{ where } \chi_\pi, \chi_x > 0.$$

or by forecasts of current or future values

$$i_t = \chi_\pi E_t \pi_{t+1} + \chi_x E_t x_{t+1} \text{ where } \chi_\pi, \chi_x > 0.$$

Determinacy

- How many nonexplosive RE solutions exist for a given model?

- **Results for Taylor rules**: Assume $\chi_\pi, \chi_x \geq 0$ and F has nonnegative diagonal elements.
  (i) Basic Taylor rule yields determinacy if

$$\lambda(\chi_\pi - 1) + (1 - \beta)\chi_x > 0.$$

(ii) Lagged data and forward-looking Taylor rules

$$\begin{aligned} i_t &= \chi_\pi \pi_{t-1} + \chi_x x_{t-1} \text{ and} \\ i_t &= \chi_\pi E_t \pi_{t+1} + \chi_x E_t x_{t+1} \end{aligned}$$

deliver determinacy if $\chi_\pi > 1$ and $\chi_x > 0$ sufficiently small.

Learning in the NK Model

- Assume non-rational expectations $E_t^*(.)$ and consider the general forward-looking model

$$y_t = ME_t^* y_{t+1} + Pv_t,\ v_t = Fv_{t-1} + \tilde{v}_t.$$

E-stability: we guess a PLM of the form

$$y_t = a + cv_t,$$

and the 2×1 vector a and the 2×2 matrix c are not known. Agents would form expectations as $E_t^* y_{t+1} = a + cFv_t$.

- The mapping from PLM to ALM is

$$T(a,c) = (Ma, P + McF),$$

and MSV REE $(0, \bar{c})$ is a fixed point. The E-stability ODE is

$$d(a,c)/d\tau = T(a,c) - (a,c),$$

which is a vector-valued ode for a and matrix-valued ode for c.
- System for a is two-dimensional and E-stability requires $tr(M - I) < 0$ and $\det(M - I) > 0$.
- The system for c: E-stability requires that the eigenvalues of $(F' \otimes M) - I$ have negative real parts.

- **E-stability with Taylor rules:** (Bullard and Mitra 2002) Basic Taylor rule yields E-stability if

$$\lambda(\chi_\pi - 1) + (1 - \beta)\chi_x > 0.$$

  Lagged data and forward-looking Taylor rules deliver E-stability for $\chi_\pi > 1$ and sufficiently small $\chi_x > 0$.

- **Note:** These conditions are the same as for determinacy.

- If the model includes interest rate inertia in the Taylor rule, then conditions determinacy and E-stability become less stringent (Bullard and Mitra 2007).

Optimal policy rules

- Assume RE. For optimal policies the quadratic loss function

$$E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \alpha(x_{t+i} - x^*)^2],$$

approximates the indirect utility of households. x^* is efficient steady state output, wlog set $x^* = 0$.

- This objective is called **flexible inflation targeting**. Strict inflation targeting would be $\alpha = 0$.

- **Optimal rules** minimize the quadratic loss function. **Optimal discretionary policy** is given by the FOC

$$\lambda\pi_t + \alpha x_t = 0.$$

- Under commitment the FOC is

$$\lambda\pi_t + \alpha(x_t - x_{t-1}) = 0$$

  in every period. This policy exhibits time inconsistency (following the FOC is "timeless perspective").

- These FOC's must be converted to **reaction functions** specifying the decision about the actual policy instrument. **Implementations** of optimal policy divide into "fundamentals-based" and "expectations-based" rules.

- The fundamentals-based rule is in the case of discretionary policy:

$$i_t = \psi_g g_t + \psi_u u_t.$$

- Under policy with commitment the fundamentals-based rule depends on x_{t-1}:

$$i_t = \psi_x x_{t-1} + \psi_g g_t + \psi_u u_t,$$

where the optimal coefficients depend on the structural parameters and the objective function. (Computing the ψ_i requires first calculating the optimal REE and then solving the IS curve to obtain the i_t rule.).

Determinacy and learning stability under fundamentals-based rules

- Consider optimal discretionary policy rule $i_t = \psi_g g_t + \psi_u u_t$. The system is

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + \begin{pmatrix} -\varphi\psi_u \\ 1 - \lambda\varphi\psi_u \end{pmatrix} u_t.$$

  Now $\det(M) = \beta$ and $tr(M) = 1 + \beta + \lambda\varphi > 1 + \det(M)$, so indeterminacy.

- Determinacy for optimal fundamentals-based rule under commitment depends on the values of structural parameters.

- **E-stability:** Under fundamentals.based optimal discretionary rule the system is

$$\begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = \begin{pmatrix} 1 & \varphi \\ \lambda & \beta + \lambda\varphi \end{pmatrix} \begin{pmatrix} E_t^* x_{t+1} \\ E_t^* \pi_{t+1} \end{pmatrix} + \begin{pmatrix} -\varphi\psi_u \\ 1 - \lambda\varphi\psi_u \end{pmatrix} u_t,$$

  Consider the intercept term:

$$da/d\tau = (M - I)a, \text{ where } M - I = \begin{pmatrix} 0 & \varphi \\ \lambda & \beta + \lambda\varphi - 1 \end{pmatrix}.$$

  We have $tr(M - I) \lesseqgtr 0$ but $\det(M - I) = -\lambda\varphi < 0$, so E-instability.

- **Fundamentals-based optimal rule under commitment** leads always to E-instability (proof omitted).

Expectations-Based Optimal Rules

- If private expectations are observable then incorporate them into **optimal expectations-based rules** (Evans and Honkapohja 2003a,b, 2006).

$$i_t = \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t,$$

under discretion or

$$i_t = \delta_L x_{t-1} + \delta_\pi E_t^* \pi_{t+1} + \delta_x E_t^* x_{t+1} + \delta_g g_t + \delta_u u_t$$

under commitment. The coefficients are

$$\begin{aligned} \delta_L &= \frac{-\alpha}{(\alpha+\lambda^2)},\ \delta_\pi = 1 + \frac{\lambda\beta}{\varphi(\alpha+\lambda^2)},\ \delta_x = \varphi^{-1}, \\ \delta_g &= \varphi^{-1},\ \delta_u = \frac{\lambda}{\varphi(\alpha+\lambda^2)}. \end{aligned}$$

Under optimal discretionary policy the coefficients are identical except that $\delta_L = 0$.

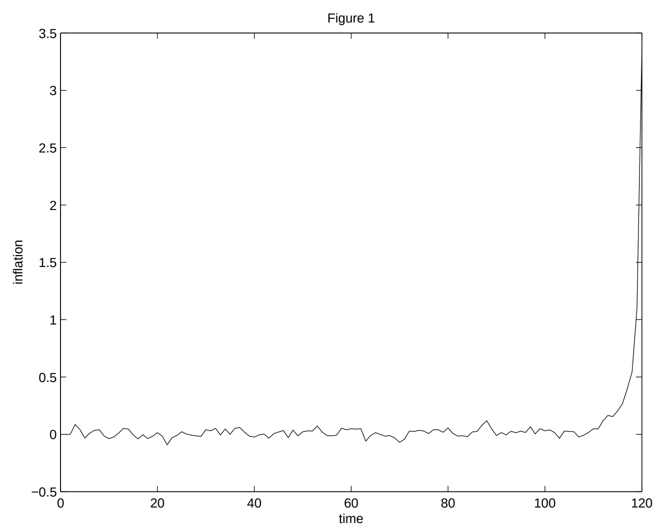
- **Result:** For all possible values of structural parameters the system is determinate and the optimal REE is stable under private agent learning.
  Proof: (only the discretionary case). The reduced form is

$$\begin{pmatrix} \pi_t \\ x_t \end{pmatrix} = \begin{pmatrix} \beta\alpha(\lambda^2+\alpha)^{-1} & 0 \\ -\beta\lambda(\lambda^2+\alpha)^{-1} & 0 \end{pmatrix} \begin{pmatrix} \hat{E}_t\pi_{t+1} \\ \hat{E}_t x_{t+1} \end{pmatrix} + \begin{pmatrix} 1-\varphi\delta_u\lambda \\ -\varphi\delta_u \end{pmatrix} u_t.$$

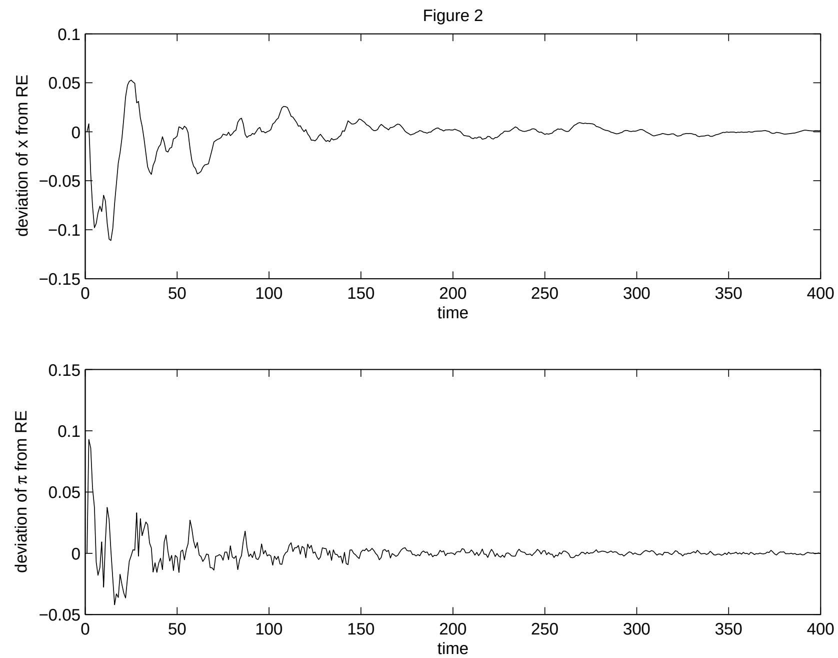
  Since the eigenvalues of the matrix are 0 and $0 < \beta\alpha(\lambda^2+\alpha)^{-1} < 1$, we have determinacy and also E-stability under this interest rate rule.

- A partial intuition: for example, an increase in $\hat{E}_t\pi_{t+1}$ leads to an increase in i_t which more than offsets the direct effect of $\hat{E}_t\pi_{t+1}$ on x_t since $\delta_\pi > 1$.

Here are some simulations (commitment case):



Instability under fundamentals-based rule



Stability under expectations-based rule

Final comments

- The presentation has focused on the basic setting for learning dynamics under Taylor rules and optimal policy rules. Many further questions have been studied in the literature (some will be covered in part II by George W. Evans).

- One topic is **operationality**:
  - Issue of current data for the Taylor rule: replace π_t and x_t by their expectations $E_t^*\pi_t$ and $E_t^*x_t$.
  - Observability of private expectations $E_t^*\pi_{t+1}$ and $E_t^*x_{t+1}$ in expectations-based rules: CB could run its own learning and forecasts.
  - Knowledge of structural parameters of the economy in expectations-based rules: CB can directly estimate IS and Phillips curve parameters and use estimated values in the rule.