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Inflation Expectations, Their Measurement and the Estimate of Their Degree of Anchoring

Editors:
Alexander Guarín, Luis Fernando Melo and Eliana González

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PREFACE

In 2005, CEMLA's Board of Governors agreed to bolster economic research and collaboration among its membership through the establishment of research activities on topics of common interest. After a careful analysis of the best way to implement such a program, the heads of economic studies of the central banks on the Steering Committee of CEMLA's Central Bank Research Network identified topics of interest and agreed that papers on these topics should be presented at the Network's Annual Meetings and subsequently published. The terms of reference for the first joint research project were established in 2006, and the first Joint Research Program book was published in 2008, entitled *Estimating and Using Unobservable Variables in the Region*.

Since then, research topics have been selected annually by the heads of economic studies at central banks within the Research Network Steering Committee, while representatives from the participating central banks have acted voluntarily as coordinators for each of these projects. Additional volumes have been published on topics such as inflationary dynamics, persistence, and prices and wages formation; domestic assets prices, global fundamentals, and financial stability; monetary policy and financial stability in Latin America and the Caribbean; international spillovers of monetary policy, and financial decisions of households and financial inclusion in Latin America and the Caribbean, among others.

All of the aforementioned subjects are of particular importance for the design and conduct of monetary policy and the preservation of financial stability. In its 2017 Meeting, the Research Network focused on a topic of particular interest for central banking, and whose importance has increased over recent years: that of the measurement of inflation expectations and their anchoring to an inflation target. One particular motive for this renewed interest was the shocks that affected inflation trends in the global economy in recent years, such as commodities price fluctuations and those associated with climate change phenomena, among others.

As argued in the literature (an overview of it is offered in the Introduction to the present volume), inflation expectations and, in particular, their degree of anchoring, are fundamental for determining price evolution and volatility developments. Therefore, an accurate measurement of inflation expectations and a better understanding of their determinants are fundamental for the design of an effective monetary policy. Nevertheless, such a measurement is a challenging task, which has been approached through survey-based or model-based methods, including their inference from market prices of financial instruments. Moreover, there has been a lively debate among authorities and researchers about the potential links between policy decisions and agents' expectations, and whether long-term expectations may be well-anchored.

The papers included in the present volume address these and other closely related topics (e.g. forecasts of inflation using novel techniques). They represent an effort by researchers of the central banks of Argentina, Bolivia, Brazil, Colombia, Costa Rica, Guatemala, Mexico, Paraguay, Peru, Spain, as well as researchers from CEMLA and the Bank for International Settlements (BIS), all of them coordinated by the *Banco de la República* (Colombia) with support provided by the Financial Stability Group of the Inter-American Development Bank.

We at CEMLA would like to thank the collaborators in this project, and hope that these documents serve as a showcase of the analysis carried out in the region and contribute towards the improvement of policy design related to the core activities of central banking in Latin America and the Caribbean.

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Introduction

Alexander Guarín
Luis Fernando Melo
Eliana González

The control of inflation and its volatility are fundamental issues for any country. Economies with a high level of inflation or uncertainty on its future value can lead, for example, to high costs for economic agents, distortions on future investment plans and welfare implications for society. On the contrary, economies with low levels of inflation and volatility, for instance, can enhance their population living conditions, access to credit sources, and confidence indicators for international investors (e.g., Madeira and Zafar, 2015; and Strohsal and Winkelmann, 2015).

Accordingly, keeping inflation under control becomes a crucial task for the monetary authority. In this regard, a strand of the economic literature has established an explicit relation between inflation, its long term expectations and their anchoring to a target level. In particular, the literature has underlined the relation of this anchoring to the ability of central banks to control inflation, set up an effective monetary policy strategy, and improve the transmission mechanisms (e.g., Haubrich *et al.*, 2012; Autrup and Grothe, 2014; and Strohsal *et al.*, 2016).

In this context, the appropriate measurement of inflation expectations and their degree of anchoring are essential elements for making monetary policy decisions by central banks. Nevertheless, these variables are unobservable and, hence, their monitoring and assessment are not straightforward.

In practice, inflation expectations are measured through surveys of specific population groups (e.g., financial market agents, firms, and consumers), or inferred from financial instruments' market prices (e.g., break-even inflation rates, inflation-linked bonds, swaps, and options). However, the analyses of such expectations from these two sources of information do not necessarily lead to the same conclusions (e.g., Pierdzioch and Rülke, 2013; and Nautz and Strohsal, 2015).

These measures have different features associated with their empirical counterparts. Survey-based expectations are a direct estimate of the probability distribution of inflation rates from different economic sectors. Nonetheless, these expectations are usually only available at low-frequencies (e.g., monthly or quarterly) and for a limited number of short-term horizons (typically, one or two years) (e.g., Autrup and Grothe, 2014; and Pierdzioch and Rülke, 2013).

By contrast, financial market-based expectations can be accessible in real time, at a higher-frequency (e.g., daily), and with multiple time horizons, including the long-term ones (e.g., five or ten-year). Nonetheless, these data are indirect measures of inflation expectations, whose measurement can be contaminated by several factors. For instance, break-even inflation rate[1] is considered a measure of inflation compensation that, in addition to inflation expectations, includes the inflation risk and liquidity premiums. The latter is associated with market conditions and the availability of liquid nominal and inflation-linked bonds (e.g., Antunes, 2015; and Strohsal and Winkelmann, 2015).

For authorities, another fundamental aspect is the formation process of inflation expectations. This process is essential to understanding how monetary policy decisions are transmitted to expectations (e.g., economic channels and their speed) and, in turn, to inflation. This enables central banks to design an effective policy strategy (e.g., Evans and Honkapohja, 2001; and Maertens and Rodríguez, 2013).

[1] These rates are derived from the spread between nominal and inflation-linked government bond yields.

The academic literature has directed its attention to two main schemes of expectations, namely, adaptive and rational. The former considers that inflation dynamics are based only on their own past values, and hence agents form their expectations using the observed price information (that is, a backward-looking rule). Under the latter scheme, each time expectations are formed, individuals consider all available information including, for example, the learning from previous prediction errors, the probable future actions of the central bank as well as the agents' beliefs (that is, a forward-looking rule) (e.g., Taylor, 1985; Kiley, 2007; and Golden and Monks, 2009). There have been other expectations formation mechanisms proposed in literature. For example, Gerberding (2001) has studied a combination of both adaptive and rational schemes, while Ekeblom (2012) has proposed some degree of learning in the formation of expectations. Other examples within this literature are Carlson and Valev (2002), Heinemann and Ullrich (2006), and Oral *et al.* (2011).

As mentioned, the anchoring of inflation expectations is fundamental for monetary policy. In fact, the literature points out that well-anchored expectations reduce the inflation risk premium, improve investment decisions, enhance the valuation of long-term assets, lower the volatility on long-term interest rates and make them less sensitive to shocks (e.g., Gürkaynak *et al.*, 2010; Mehrotra and Yetman, 2014; and Berument and Froyen, 2015).

Inflation expectations are *well-anchored* (that is, central bank's credibility is strong) if shocks affecting current inflation and its short-term expectation do not lead to long-run deviations from the target level. If they are well-anchored, then long-term expectations should be insensitive to macroeconomic shocks or other surprises, so that once shocks have dissipated, inflation should return to its long-run target. On the contrary, if central bank's credibility is weak, economic shocks could deviate long-term inflation expectations away from its inflation target (e.g., Demertzis *et al.*, 2009; Galati *et al.*, 2011; and Pagenhardt *et al.*, 2015).

Recent literature has carried out the measurement of the degree of anchoring through several methodologies, which capture theoretical aspects from two main lines of research. The first one evaluates if long-term inflation expectations are moving close to a target level, so the degree of anchoring depends on the deviation of these expectations regarding a specific inflation target (e.g., Mehrotra and Yetman, 2014; and Strohsal and Winkelmann, 2015).

The second line studies the dependence relation between short- and long-term inflation expectations. This literature assesses if shocks that affect short-term inflation expectations have effects on those of the long-term, so that the degree of anchoring depends on how statistically significant is the joint movement between short- and long-term inflation expectations in response to shocks (e.g., Gürkaynak *et al.*, 2010; and Antunes, 2015).

Figure 1 illustrates recent research works on the anchoring of inflation expectations. Each of these studies is characterized according to both the methodology considered and the source of data used in its empirical exercises.

A broad segment of this literature has investigated mainly two issues. The first one is the assessment and characterization of differences in the degree of anchoring between countries with and without an inflation-targeting regime. For instance, Gürkaynak *et al.* (2005), Gürkaynak *et al.* (2007), Demertzis *et al.* (2009), Gürkaynak *et al.* (2010), and Beechey *et al.* (2011) examined this matter for the United States (US) and the euro area, for sample periods between the 1990s and the end of 2000s. These studies find that a credible inflation-targeting strategy improves the anchoring of long-term inflation expectations, reduces their volatility and makes them less sensitive to inflation shocks.

The second issue is the evolution of the degree of anchoring over time. For example, the dynamics of anchoring in the pre- and post-Global Financial Crisis periods in the US between 2004 and 2014 is studied by Galati *et al.* (2011), Autrup and Grothe (2014), Nautz and Strohsal (2015), and Strohsal *et al.* (2016). The first three works state that inflation expectations have been deanchored since the Global Financial Crisis, while the latter work points out that the deanchoring lasted a short period in 2008, after which expectations were anchored again .

The works by Lemke and Strohsal (2013), Antunes (2015), Pagenhardt *et al.* (2015), and Scharnagl and Stapf (2015) carry out similar research for the euro area for sample periods between 2000 and 2015. Lemke and Strohsal (2013) and Scharnagl and Stapf (2015) stated that although the European Sovereign Debt Crisis increased the volatility of inflation expectations in 2011, these were not deanchored. On the other hand, Antunes (2015) and Pagenhardt *et al.* (2015) found that the same crisis' events increased the joint movement of short- and long-term inflation expectations, and since then the latter have been responding to economic shocks.

The variation in the degree of anchoring has also been studied in other countries for diverse samples between 1996 and 2013. De Pooter *et al.* (2014) find that inflation expectations in Brazil, Chile, and Mexico are anchored, and that these react to US news' surprises. Kabundi and Schaling (2013), and Çiçek and Akar (2014) provide evidence on the unsuccessful anchoring of inflation expectations in South Africa and Turkey. These are due to low credibility in each country. Mehrotra and Yetman (2014), and Berument and Froyen (2015) show that inflation expectations are more firmly anchored after the adoption of credible inflation-targeting regimes. Other recent examples are the studies about the degree of anchoring in Singapore by Ee and Supaat (2008); the US, European Monetary Union, United Kingdom and Sweden by Strohsal and Winkelmann (2015), and Colombia by Gamba *et al.* (2016).

Another topic associated directly with inflation expectations is the continuous monitoring and forecasting of inflation. This is highly relevant for central banks and their monetary policy strategies, particularly in economies with inflation-targeting regimes. Inflation forecasts are computed using various types of macroeconometric and time series methodologies. Recently, forecasting models based on large data sets, high numbers of predictors and direct combination of different forecast models have attracted the attention of modelers and practitioners. These techniques are useful considering that central banks have inflation forecasts coming from different models.

Recent examples of these forecasting tools are the Bayesian model averaging (BMA) (e.g., Wright, 2009), factor-augmented vector autoregressive (FAVAR) models (e.g., Bernanke *et al.*, 2005) and schemes for combining forecasts, proposed by Reid (1968), and Bates and Granger (1969). Hall and Mitchell (2007), and Geweke and Amisano (2011) consider combinations of forecasting densities instead of punctual predictions. Tian and Anderson (2014) proposed new schemes for combining forecasts with possible structural changes, and Kapetanios *et al.* (2015) extended the previous literature with weighting schemes.

A fundamental topic in forecasting is the performance evaluation of prediction models and their comparison with respect to a benchmark or other forecasts. The works by Giacomini and White (2006), and Giacomini and Rossi (2010) are recent examples of static

Figure 1

RECENT LITERATURE ON THE ANCHORING OF INFLATION EXPECTATIONS

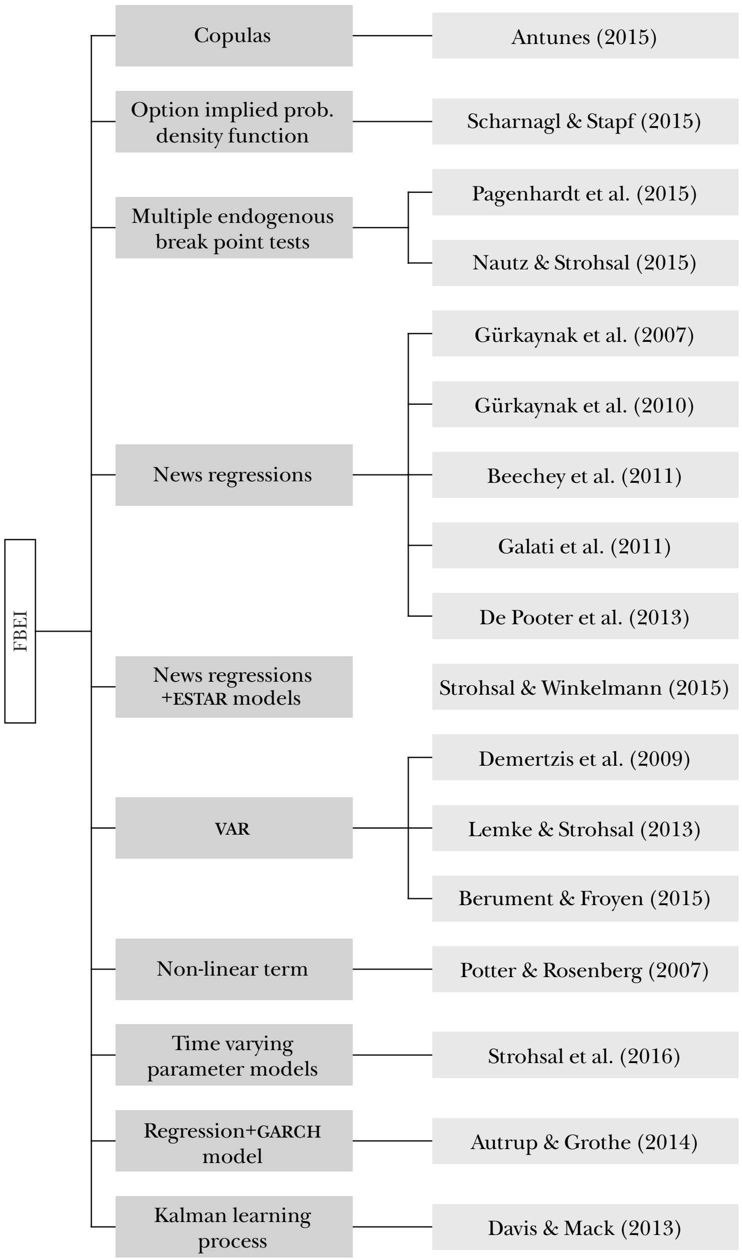
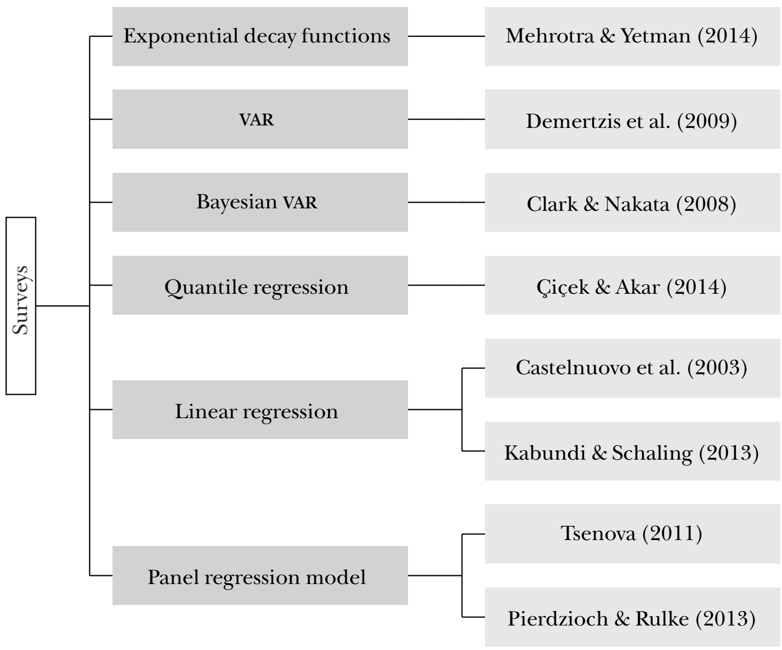


Figure 1 (cont.)

RECENT LITERATURE ON THE ANCHORING OF INFLATION EXPECTATIONS



and dynamic predictive ability tests, while Rossi and Sekhposyan (2010) is an application of these tests in inflation forecasting.

Currently, the literature on inflation expectations has gotten the attention of academics and policy makers. Their renewed interest in these issues is the result of recent shocks that have affected inflation. In particular, between the end of 2014 and the beginning of 2017, the global economy suffered a sudden and abrupt fall in oil prices with diverse effects on other prices and macroeconomic variables. Likewise, between 2015 and the first-half of 2016, some economies were affected by a climate phenomenon known as *El Niño*[2] with direct effects on the food supply and its prices, as well as indirect effects on core inflation through indexation mechanisms. The impact of these shocks on current inflation has underlined the relevance of bringing up old and new questions about the formation and measurement of inflation expectations, the estimation of their degree of anchoring, as well as the development of more accurate forecasts of future inflation and their relation with expectations.

This is inconsistent. Sometimes they use, for example, the empirical identification of inflation expectation formation processes (e.g., adaptive, rational, hybrid, or adaptive learning), their changes over time, the statistical validation of these schemes and the characterization of their main determinants. Likewise, these queries relate to the measurement of an informative signal of expectations, the choice of a suitable source of data and time horizons as well as the theoretical and empirical implications of such an election for monetary policy decision making.

Other questions are addressed, for instance, the measurement of the degree of inflation expectations anchoring over time and under different policy regimes, the implementation of existing methodologies, the design of new methods and the comparison of their results. A recent challenge is the prediction of variations in the degree of anchoring in response to diverse shocks (e.g., climate related shocks and commodity price's shocks). Other discussions arise on the evaluation of the measures of expectations as forecasts of future inflation, structural changes in these predictions and how to model them.

With the aim of providing empirical and theoretical support to the economic research and the policy decisions of central banks, the Center for Latin American Monetary Studies (CEMLA) in coordination with the Banco de la República (that is, the Central Bank of Colombia) organized the 2017 Joint Research Annual Program to study inflation expectations and other relevant topics associated with them. In the development of this program, the Financial Stability Group of the Inter-American Development Bank and the CEMLA provided academic support to the research groups through

[2] This is a season of high temperatures, shortage of rains and droughts.

academic feedback given by professors Olivier Coibion,[3] Massimiliano Marcellino,[4] and Andrea Tambalotti.[5]

This joint program was an opportunity to deal with some of the previous questions, learn about the current research on inflation expectations in central banks and contribute to the burgeoning economic literature on these issues. The results of this research agenda are compiled in this book, which includes 13 chapters. The first one is this Introduction. The remaining 12 chapters correspond to works from 10 central banks (Argentina, Bolivia, Brazil, Colombia, Costa Rica, Guatemala, Mexico, Paraguay, Peru, and Spain) and two international institutions (Bank for International Settlements – BIS, and CEMLA). These works address topics on the formation of inflation expectations, their measurement through surveys and financial market data, the estimation of the degree of anchoring adopting several methodological approaches, and the forecasts of inflation using novel techniques. The works were divided into four main sections, as follows.

1. THE FORMATION AND MEASUREMENT OF INFLATION EXPECTATIONS

In chapter 2, Alberto Fuertes, Ricardo Gimeno and José Manuel Marqués of the Banco de España use the affine model proposed by Gimeno and Marqués (2009) to decompose the nominal interest rates from Chile, Mexico, Colombia, and Brazil into real risk-free rates, inflation expectations and risk premium. For each country, the empirical exercises consider different sample periods between 2001 and 2016, depending on the availability of data on nominal government bonds. Results suggest that expectations in Mexico and Chile were anchored during the periods of study. On the contrary, in Colombia and Brazil, during the sample period analyzed, the inflation expectations deanchored and fluctuated over time.

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Chapter 3 presents the work by Alonso Alfaro and Aarón Mora from the Banco Central de Costa Rica. The authors use the model by Mankiw and Reis (2002) to examine information rigidities in inflation expectations of agents from several economic sectors between 2006 and 2017. Although previous studies suggest the existence of these rigidities in the expectations formation process in Costa Rica, the results of this research do not support these claims. Estimates show that the magnitude of the rigidities captured from data is not large enough to validate that statement.

The work by Pablo Alonso of the Banco Central del Paraguay is presented in chapter 4. Alonso estimates a model of determinants of the formation of inflation expectations in Paraguay since the adoption of the inflation-targeting scheme in 2011. His results show that expectations are a function of past inflation and the credibility in the central bank. Other variables such as the foreign exchange rate depreciation and the changes in oil prices do not seem to play a key role in their determination.

2. THE DEGREE OF ANCHORING OF INFLATION EXPECTATIONS

In chapter 5, Rocío Gondo and James Yetman of the Banco Central de Reserva del Perú and the BIS, respectively, use the work by Mehrotra and Yetman (2014) to infer from inflation expectations, for several Latin American countries between 1993 and 2016, an implicit anchor in the data. They also assess how it has evolved over time and compare it with the central bank's target level. Results show that most countries have an anchor whose importance has increased over time as a result of improvements in the credibility after the adoption of an inflation-targeting regime.

The research work by Mauricio Mora, Juan Carlos Heredia and David Zeballos of the Banco Central de Bolivia (BCB) is presented in chapter 6. Authors assess whether inflation expectations in Bolivia between 2005 and 2017 were anchored, in the sense that they were coherent with the inflation future path and the target level announced by the central bank.[6] Results indicate that long-term

[6] Bolivia is under a monetary-targeting scheme, such that the main reference for future inflation are the central bank's projections.

expectations were strongly anchored since 2014 due to a greater credibility of the BCB linked with a larger intervention in the money market, a more active communication strategy and a stable macroeconomic environment.

Chapter 7 presents the research by Fernando Nascimento de Oliveira and Wagner Gaglianone of Banco Central do Brasil (BCBr). They build several time-varying expectation anchoring indexes of the BCBr from 2002 to 2017, which are based on the monetary authority's capability to anchor long-term inflation expectations. Those indexes consider variables of fiscal and monetary policy in their estimation. Authors state that estimated indexes are consistent with the central bank's credibility perceived by economic agents in Brazil over the sample period.

3. INFLATION FORECASTING AND ITS PERFORMANCE EVALUATION

Chapters 8 and 9 present the research works developed by Lorena Garegnani and Maximiliano Gómez, and Luis Libonatti of the Banco Central de la República Argentina, respectively. Garegnani and Gómez estimate Bayesian VAR models with Argentinian data from 2004 to 2017, and forecast the headline inflation for several time horizons under a rolling window scheme. In the same line of research, Libonatti uses a mixed data sampling regression model to forecast the monthly core inflation of Argentina between 2015 and 2017 using a daily online price index captured by web scraping. In both works, authors compare their results to forecasts from traditional benchmark models and show, in general, a good performance of their predictions.

In chapter 10, the Economic Research Department of the Banco de Guatemala (Banguat) presents its work. This research assesses the performance of both unconditional and conditional inflation forecasts for several time horizons between 2011 and 2017. These predictions are built using time series tools and structural macroeconomic models used by the Banguat. In line with the traditional literature, their results show that forecasts computed with time series tools provide more accuracy in the shortest terms while structural macroeconometric models provide better predictions for medium- and long-term horizons.

In chapter 11, Héctor Zárate and Daniel Zapata from Banco de la República (Colombia) use artificial neural networks to forecast inflation expectations in a set of 16 countries with inflation-targeting regimes and a sample period between 1991 and 2016. Their predictions consider different expectations patterns depending on perceptions about the oil shock in 2014. Authors show that their exercises provide more accurate forecasts than the benchmark model and, anticipate turning points of inflation in most of the cases.

4. INFLATION EXPECTATIONS AND ITS RELATION WITH ECONOMIC POLICY

In chapter 12, Sebastián Cadavid and Alberto Ortiz from CEMLA examine empirically if the economic reforms implemented in Brazil, Chile, Colombia, Mexico, and Peru between 1999 and 2002 –particularly the adoption of an inflation-targeting regime and a flexible exchange rate– led to the observed reduction of inflation in these countries. Their empirical exercises consider counterfactual scenarios in an open economy with monetary factors. The authors show that if these reforms had not been adopted in these Latin American countries, they would have experienced higher inflation rates, variations in gross domestic product with small gains in economic growth and a large volatility in nominal variables.

Finally, chapter 13 presents the work by Bernabé López-Martin, Alberto Ramírez de Aguilar and Daniel Sámano from Banco de Mexico. They analyze the interaction between inflation, its expectations and fiscal deficits in Mexico between 1969 and 2016. The authors extend the model developed by Sargent et al. (2009) to study how fiscal policy can affect inflation expectations in a context of central bank independence. Their results suggest that the fiscal deficits financed through money creation are central to explain the behavior of Mexican inflation and its expectations during the sample period.

The editors trust that this brief introduction will motivate readers to carry on with each of the research works in this book. This publication is an opportunity to learn about relevant issues on inflation expectations for central banks in the region, as well as the current state of their research. We also think that this book will encourage relevant policy discussions on inflation, its expectations and other related issues, contributing to literature on monetary policy.

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Formation and Measurement of Inflation Expectations

Extraction of Inflation Expectations from Financial Instruments

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Abstract

In this paper, we estimate inflation expectations for several Latin American countries using an affine model that takes as factors the observed inflation and the parameters generated from zero-coupon yield curves of nominal bonds. By implementing this approach, we avoid the use of inflation-linked securities, which are scarce in many of these markets, and obtain market measures of inflation expectations free of any risk premium, eliminating potential biases included in other measures such as breakeven rates. Our method provides several advantages, as we can compute inflation expectations at any horizon and forward rates such as the expected inflation over the five-year period that begins five years from today. We find that inflation expectations in the long-run are fairly anchored in Chile and Mexico, while those in Brazil and Colombia are more volatile and less anchored. We also find that expected inflation increases at longer horizons in Brazil and Chile, while it is decreasing in Colombia and Mexico.

Keywords: inflation expectations, affine model, real interest rate, risk premium.

JEL classification: G12, E43, E44, C54.

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1. INTRODUCTION

Agents' inflation expectations are decisive when studying changes in many of the variables shaping households' and firms' decision making. One approach to obtain inflation expectations is based on the consensus view of specialist economic forecasters, such as the surveys of professional forecasters by the European Central Bank and the Federal Reserve Bank of Philadelphia, both of which are released quarterly. Other surveys also exist, such as the monthly University of Michigan Survey of Consumers in the United States, which elicits information from consumers rather than professional economic forecasters. In Latin America, several central banks also publish surveys about inflation expectations.[1] A drawback of these surveys is that they are released relatively infrequently and, thus, the information received has a time lag. Moreover, they only cover a small range of time horizons and, as identified in the literature (Ang et al., 2007; Chan et al., 2013), there is some bias and inertia in their responses.

An alternative way of obtaining agents' inflation expectations is to use prices of market-traded financial instruments employed to hedge against inflation such us inflation-linked bonds, inflation swaps, and inflation options. One may argue that, given that investors risk their funds when taking investment decisions based on expected future inflation and professional forecasters do not have any vested interest, they could provide a better forecast since they have more skin in the game. Another advantage to this approach is that it is possible to derive the whole probability function (Gimeno and Ibañez, 2017). This makes it possible to estimate, for example, the probability of the occurrence of certain extreme events or the uncertainty of future inflation. Another additional advantage in comparison with surveys is that changes in expectations can be observed almost in real time. This makes it easier to identify the effect of specific events or decisions on inflation expectations. Unfortunately, there are not many markets of inflation-linked securities available for most countries. For example, in Latin American only a few have inflation-linked bonds, and there are no markets for inflation

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[1] For example, the central banks of Chile, Colombia and Mexico publish a monthly survey about inflation expectations; the Bank of Brazil publishes a daily survey.

options at all. Another problem of obtaining inflation expectations using this approach is the presence of various risk premia, which are included in the prices of the underlying financial assets and which may also vary over time. The presence of these premia may distort the information content of these indicators, which may affect measures of agents' inflation expectations.

Due to the lack of inflation-linked securities in Latin American markets, we use an alternative approach developed by Gimeno and Marques (2012) to obtain inflation expectations: An affine model that takes as factors the observed inflation and the parameters generated in the zero-coupon yield curve estimation of nominal bonds. Also, by implementing this approach, we obtain a measure of inflation expectations free of any risk premia, since the model breaks down nominal interest rates as the sum of real risk-free interest rates, expected inflation, and the risk premium.

To the best of our knowledge, this is the first attempt to obtain pure inflation expectations using nominal government bonds for Latin American countries. We obtain government bond data for Brazil, Chile, Colombia, and Mexico, being able to estimate the zero-coupon yield curve and decompose that curve into the real risk-free rate, the risk premia, and inflation expectations. We can obtain inflation expectations for all of the horizons computed in the zero-coupon yield curve as well as forward rates such as the expected inflation over the five-year period that begins five years from today (the 5Y5Y forward rate). We find that inflation expectations in the long-term (5Y5Y) seem to be anchored in Chile and Mexico, although the level of expected inflation is above the central bank target rate of 3%. On the other hand, long-term inflation expectations in Brazil and Colombia are more volatile and have been fluctuating over time, experiencing a large decrease during 2017. These results may also point out that government bond markets in Brazil and Colombia do not provide as much information about future inflation as the other +markets.

We also find the expected inflation is currently increasing with the horizon in Brazil and Chile, while it is decreasing in Colombia and Mexico. For Mexico, there has been an important shock on expected inflation after the last US presidential elections, experiencing a large increase. None of the other countries analyzed have shown this pattern, limiting the spillovers effects of the results of the US presidential elections to inflation expectations in Mexico. Finally, we compare the forecasting power over one year of inflation expectations obtained using our approach with expected inflation obtained from surveys. Our approach performs better predicting inflation for Chile, while surveys do better for Brazil, Colombia, and Mexico.

Further analysis shows that inflation expectations from our model complement those from surveys and provide additional information. A simple average of the expected inflation obtained using our approach and expected inflation from surveys provides a better fit than using only expectations from surveys for all countries but Brazil. Overall there is a trade-off between the two ways of obtaining expected inflations, as surveys are less responsive to inflation shocks and our approach produces expected inflation levels that are more correlated with current inflation.

The paper proceeds as follows: Section 2 describes the financial instruments from which information about inflation expectations can be derived, analyzing their availability for Latin American markets. Section 3 summarizes the main features of the affine model we implement to obtain inflation expectations, and Section 4 shows the results. Section 5 concludes.

2. FINANCIAL INSTRUMENTS WITH INFORMATION ABOUT INFLATION EXPECTATIONS

2.1. Inflation-linked Bonds

One of the most popular metrics of inflation expectations based on financial asset prices is the one obtained from inflation-linked bonds (break-even inflation rates). This is calculated by comparing the yield of a conventional bond (whose associated coupon and principal payments are fixed in nominal terms), with that of an inflation-linked bond (indexed to a price index) of the same maturity from the same issuer.

The inflation-linked bond market is particularly active in the United States, where these assets (known as Treasury inflation-protected securities or TIPS) are issued in sufficient quantity to create a liquid market in which price formation is fluid. However, the situation in Europe is fragmentized due to the existence of multiple issuers (namely the traditional issuer of treasuries for France, Italy, and Germany, and the less frequent issuer Greece, later joined by

Spain in 2014) and the use of different consumer price indices (national and European) as a reference. These factors reduce liquidity and are an obstacle to obtaining a clear signal on the compensation demanded by investors for the expected increases in the cost of living. In Latin America, there are several markets of inflation-linked bonds in countries such as Brazil, Chile, and Mexico.

Besides the lack of market depth and liquidity, an additional problem with this indicator is that it includes other components as well as investors' expectations about future price developments. Firstly, given that investors are averse to inflation risk, they will demand a premium on conventional bonds that compensates them for the risk incurred, but not on inflation-linked bonds, as they are protected against this risk. For this reason, the indicator does not strictly measure the level of expectations, but rather the compensation for inflation that investors demand. Secondly, the different level of liquidity of the two instruments used to obtain the indicator (generally higher for conventional bonds than inflation-linked ones) means the yield spread between them is also influenced by their different liquidity premiums. As well as the aforementioned inflation-related factors, conventional bonds include a component reflecting the expected future course of the real interest rate, together with its associated risk premium. Finally, it should be borne in mind that the size of the premia present in the break-even rate (inflation risk and relative liquidity) may change over time, depending on changes in investors' risk appetite, the level of inflation risk, or market liquidity conditions.

The inflation compensation metric derived from inflation-linked bonds may also be temporarily affected by other factors in addition to those mentioned. Thus, for instance, changes in the supply and demand for conventional bonds relative to inflation-linked bonds, such as those associated with quantitative easing programs,[2] for

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[2] Only conventional government bonds were purchased in the Federal Reserve Board's first quantitative easing program. During the Federal Reserve Board's second quantitative easing program (QE II), a total of USD 600 billion-worth of government securities was purchased, of which 26 billion was in the form of inflation-linked bonds. The fact that more conventional bonds are being bought than inflation-linked bonds could push down their relative yield, and therefore depress the inflation expectations indicator in a way that is due to a mismatch in the supply and demand for bonds used to calculate the indicator rather than to agents' forecasts of future consumer price trends.

example, may cause distortions in these indicators. Given all these drawbacks, economists have developed extensive academic literature seeking to isolate different components of the inflation expectation indicators obtained from inflation-linked bonds.[3]

2.2. Inflation-linked Swaps

Along with inflation-linked bonds, inflation-linked swaps (ILS) are another type of financial asset containing information about agents' inflation expectations. In this derivative instrument, one of the contracting parties agrees to pay the counterparty a fixed sum on a future date in exchange for a payment linked to the future level of a price index. For example, in the case of a one-year ILS, the fixed-rate party could agree to pay 2% of €1 million in consideration for receiving a fraction of this nominal €1 million equivalent to the increase in the CPI over this 12-month period. Contrary to the case of inflation-linked bonds, the ILS market is more liquid in Europe than in United States (Gimeno and Ibáñez, 2017) and there are not ILS markets in Latin America, except in Brazil.

ILSs are bilaterally negotiated private contracts with no intermediary clearinghouse. This creates the risk that the other party will fail to meet its commitment at the end of the period, so the negotiated price incorporates the corresponding premium. Nevertheless, the absence of cash transfers before the expiry date reduces the size of this premium, as well as the liquidity premium, as there is no opportunity cost relative to alternative investments (Fleming and Sporn, 2013).

Like inflation-linked bonds, inflation swaps contain an inflation risk premium. Therefore, they measure compensation for inflation as well as inflation expectations. One of the main advantages of the ILS-based indicator relative to the one obtained from inflation-linked bonds is that, since it is not necessary to compare two different bonds, the distortions caused by ad hoc factors that affect the markets asymmetrically are eliminated. Particularly, these indicators would not have been directly affected by distortions linked to the implementation of central banks' asset purchase programs.

[3] See, for example, D'Amico et al. (2014) and Chernov and Mueller (2012).

2.3. Inflation-linked Options

Inflation options are contracts in which one of the parties agrees to pay the other an amount depending on whether a price index exceeds (cap) or falls below (floor) a given threshold (the strike rate) within a given period. If the condition is met, the payment would be the difference, in absolute terms, between the index and the threshold. Unlike both inflation-linked bonds and ILSs, which give estimates of the averages only at specific points in time, options can be used together with ILSs to obtain additional information such as the full probability distribution of the future course of inflation or implied volatility of inflation. This gives information about risk and uncertainty around the expected average value. In particular, an increase in the implied volatility suggests that agents are more concerned and there is more uncertainty over the future course of price indices.

As in the case of ILSs, options are negotiated bilaterally without the intervention of a clearinghouse, so prices may include a counterparty risk premium. Most of these derivatives are negotiated using the harmonized euro area CPI, the UK RPI (Retail Price Index), or the US CPI (Consumer Price Index), with maturities ranging from 1 to 30 years. The most liquid market is linked to the euro area index, followed by that of the UK (see Smith, 2012). It should also be noted that, as in the case above of the other financial instruments, option prices also contain premiums for inflation risk, and potentially, for liquidity risk. Currently, there are no markets for inflation options in Latin America.

The inflation risk premium is present in all three indicators, and the amount is the same. For its part, the liquidity risk premium is negative in the case of the bond-based metric, as conventional bonds are more liquid than interest-linked bonds, whereas, in the ILS, the sign of this premium is positive. The counterparty risk premium is only present in the case of ILSs and inflation options. Finally, the estimation error may be more significant for an indicator based on inflation-linked bonds.[4]

[4] Unlike ILSs, where the compensation for inflation is directly observable from the price, the bond-based indicator requires a comparison of the yields on inflation-linked bonds and conventional bonds. The differences in the features of both types of bonds, beyond the fact that in the case of inflation-linked bonds payments are linked to inflation (such as, for example, their expiry), may distort the inflation expecta-

2.4. Inflation Expectations from Financial Instruments in Latin America

Given the scarcity of financial instruments linked to price indexes in Latin American, obtaining indicators of inflation expectations from these securities is difficult and limited to a few countries. Also, the only indicator we can obtain is the break-even rate for those markets where inflation-linked bonds and conventional bonds exist and are liquid. This break-even rate is used as a proxy for expected inflation but, as we mentioned earlier, also includes several premia such as the risk and liquidity premia. We do not know the size of these premia, and thus we must keep in mind that this indicator provides only information about inflation compensation rather than pure inflation expectations.

Unfortunately, obtaining data on break-even rates for other countries is difficult because of the lack of inflation-linked securities. Table 1 shows the availability of each type of securities for Latin American countries. Even though there are several markets for inflation-linked bonds, it may be the case that, for some countries, it is difficult to obtain accurate prices, as there is either a small variety of bond maturities or bond markets are relatively illiquid. In the next section, we describe a different approach to obtain indicators about inflation expectations without the need for data on inflation-linked securities. This approach will provide two main advantages: First, it uses data only on conventional nominal bonds and realized inflation; second, it makes it possible to identify the risk premia component, obtaining a more accurate portrait of pure inflation expectations.

Table 1

INFLATION LINKED SECURITIES

Inflation linked bonds	Brazil, Chile, Mexico, Peru, Argentina, Colombia, Bolivia, Costa Rica, Uruguay
Inflation swaps	Brazil
Inflation options	–

tions indicator. The indicator is also seasonal, in a way that is linked to the behavior of inflation. To correct for these distortions, models or adjustments are often used that are subject to potential estimation errors.

3. MODELING INTEREST RATES FROM PUBLIC DEBT MARKETS

The methodology we implement decomposes nominal interest rates into three components from an affine model of the nominal term structure. This methodology is related to the macro-finance literature in which authors such as Diebold et al. (2006), Diebold et al. (2005), Carriero et al. (2006), and Ang et al. (2008) (ABW) incorporate macro-determinants into a multi-factor yield curve model with non-arbitrage opportunities. Our decomposition departs from previous approaches by extracting the risk premia from the difference between the nominal term structure and a notional term structure where the price of risk is set equal to zero.

We also propose an affine model where interest rates are affine relative to a vector of factors that includes inflation rates and exogenously determined factors based on the Nelson-Siegel exponential components of the yield curve (Nelson and Siegel, 1987), in a similar vein to Carriero et al. (2006) and Diebold and Li (2006). Moreover, in our case, we include the condition of non-arbitrage opportunities along the yield curve and take into account risk-aversion. Taking these two conditions together allows us to decompose nominal interest rates as the sum of real risk-free interest rates, expected inflation, and risk premium.

3.1. The Model

Affine term structure models allow the risk premium to be separated from expectations about future interest rates. An affine model assumes that interest rates can be explained as a linear function of certain factors,

$$y_{t,t+k} = \frac{-1}{k}\left(A_k + B_k' X_t\right) + u_{t,t+k} \quad u_t \quad N(0,\sigma^2 1),$$

where $y_{t,t+k}$ is the nominal interest rate in period t with term k, X_t is a vector of factors, A_k and $B_k^{'}$ are coefficients, and $u_{t,t+k}$ represents the measurement error. We also assume that X_t factors follow a VAR structure (in the same vein as Diebold et al., 2006):

$$X_t = \mu + \Phi X_{t-1} + \Sigma\varepsilon_t \quad \varepsilon_t \quad N(0,I),$$

where μ is a vector of the constant drifts in the affine variables X_t, Σ is the variance-covariance matrix of the noise term and Φ is a matrix of the autoregressive coefficients. To avoid arbitrage opportunities, the values of parameters A_k and $B_k^{'}$ should be restricted according to the following equation:

$$e^{A_{k+1}+B'_{k+1}X_t} = E_t[e^{A_1+B'_1 X_t} e^{A_k+B'_k X_{t+1}}]$$

The consideration of risk-aversion in this framework implies some compensation for the uncertainty of longer maturities, in which the random shocks ε_t accumulate. Coefficients that translate matrix Σ into the risk premium are called prices of risk (λ_t) and, following the literature, these coefficients are affine to the same factors X_t ,

$$\lambda_t = \lambda_0 + \lambda_1 X_t,$$

where λ_0 is a vector, and λ_1 a matrix of coefficients. If λ_1 is set to be equal to zero, then the risk premium will be constant, whereas if it is left unrestricted, we will obtain a time-varying risk premium.

We must consider the variables that could determine the term structure of interest rates in order to select the factors in the model. There is ample evidence in the literature that the information content of the whole term structure could be shortened to a small number of factors. The proposal of Diebold and Li (2006) is used, with the level (L_t), slope (S_t) and curvature (C_t) parameters from the Nelson and Siegel (1987) term structure specification as factors of an affine model. These factors can be found in most central bank estimations of the zero-coupon yield curve. This estimation implies that nominal interest rates can be modeled in the following equation,

$$y_{t,t+k} = L_t + S_t \frac{1-e^{-k/t}}{k/\tau} + C_t\left(\frac{1-e^{-k/\tau}}{k/\tau} - e^{-k/\tau}\right) + u_{t,t+k},$$

where τ , L_t , S_t , and C_t are the parameters that give us the interest rate at time t with maturity in k periods.

Although including a fourth factor in the model may not be necessary to obtain a good fitting of the interest rate term structure, if Nelson and Siegel's model is considered, adding the inflation rates allows us to take into account the yield curve information that could be useful in forecasting inflation.

$$X_t = \begin{bmatrix} L_t \\ S_t \\ C_t \\ \pi_t \end{bmatrix}$$

Once the affine model, represented by the previous equations, has been estimated, it is possible to decompose k-period nominal interest rates $(y_{t,t+k})$ into real risk-free rates $(Er_{t,t+k})$, inflation expectations $(E_t\left[\pi_{t,t+k}\right])$ and risk premia (denoted by $\gamma_{t,t+k}$), according to the following equation:

$$y_{t,t+k} = Er_{t,t+k} + E_t[\pi_{t,t+k}] + \gamma_{t,t+k}.$$

Therefore, real risk-free rates $(Er_{t,t+k})$ could be obtained by subtracting inflation expectations and risk premia from estimated nominal interest rates.

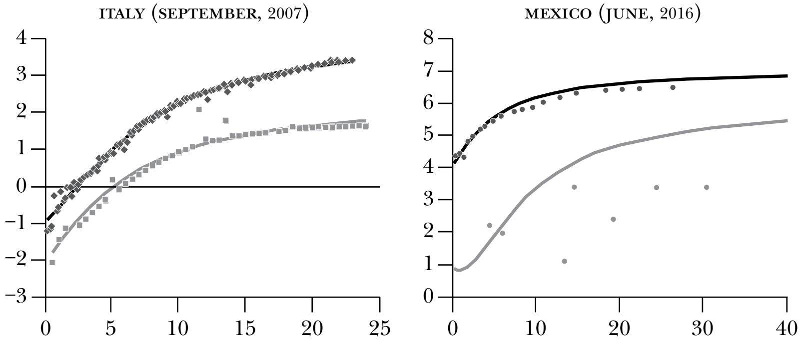
4. RESULTS OF INFLATION EXPECTATIONS FROM PUBLIC DEBT MARKETS

4.1 Yield Curve Estimation

To estimate the affine model proposed, we use monthly spot nominal interest rates for the Brazilian, Colombian, Chilean and Mexican government yield curve. These data have been obtained from a yield curve estimation that follows Diebold and Li (2006). We first analyze the yield curve estimates using both nominal interest rates, and inflation-indexed rates when available, to check the goodness of fit. For the sake of comparison, Figure 1 shows the yield curve estimates both for Mexican and Italian government bonds. The black (gray) line represents yield curve estimates for nominal government bonds (inflation-indexed government bonds). The dots represent the yield and maturity of traded bonds. Nominal yield curve estimates provide accurate estimates for both countries while inflation-indexed yield curve estimates only provide a good fit for Italy. Lack of inflation-indexed bonds for different maturities, low liquidity and low market depth make these yield curve estimates for Mexico unreliable. We find similar problems using inflation-linked bonds for Brazil, Chile, and Colombia. On the contrary, nominal yield curve estimates provide a reasonable fit for all these markets, and they will be the input to solve the affine model and obtain inflation expectations for the countries we analyze. We do also estimate the yield curve for the inflation-linked bonds in Chile. The Chilean market is one of the most active in Latin America, and we can compute the break-even rate as the difference between the estimated yield curves from nominal bonds and inflation-linked bonds. Figure 2 shows the one-year break-even rate for Chile obtained from the estimated yield curves. The break-even rate seems to be affected by the liquidity premia in the inflation-linked bond market as the rate decreases during the period when inflation rises.[5]

Figure 1

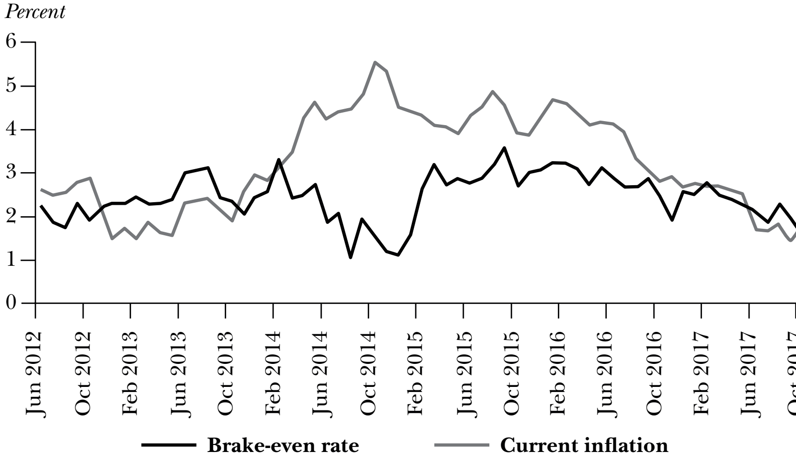
YIELD CURVE ESTIMATES
NOMINAL (BLACK) VS. INFLATION LINKED BONDS (GRAY)



[5] The break-even rate includes the spread between the liquidity premium of the nominal and the inflation-linked bond markets. Because of that, it decreases if the liquidity premium in the inflation-linked bond market rises more than the premium of the nominal bond market.

Figure 2

ONE YEAR BREAK EVEN RATE FROM YIELD CURVE ESTIMATES VS. CURRENT INFLACION FOR CHILE



The availability of nominal government bonds for the estimation of the zero-coupon yield curve is different for each country, both regarding the number of nominal bonds used and the length of the sample. Table 2 summarizes this information for each market.

Table 2

NOMINAL BONDS AVAILABILITY

	Number of bonds	*Period*	*Original bond maturity*
Brazil	104	Since Feb 2007	3 months – 11 years
Chile	15	Since July 2012	4 years – 30 years
Colombia	70	Since Feb 2005	1 year – 20 years
Mexico	47	Since May 2001	3 years – 30 years

4.2. Empirical Results

We mainly focus on the results related to inflation expectations, leaving aside a deeper interpretation of the term premia and the real yield curve. We obtain inflation expectations from the VAR equation. Since vector X_t includes current inflation (π_t), expectations on this variable can be computed from projections of the dynamics of the affine factors in the VAR equation.

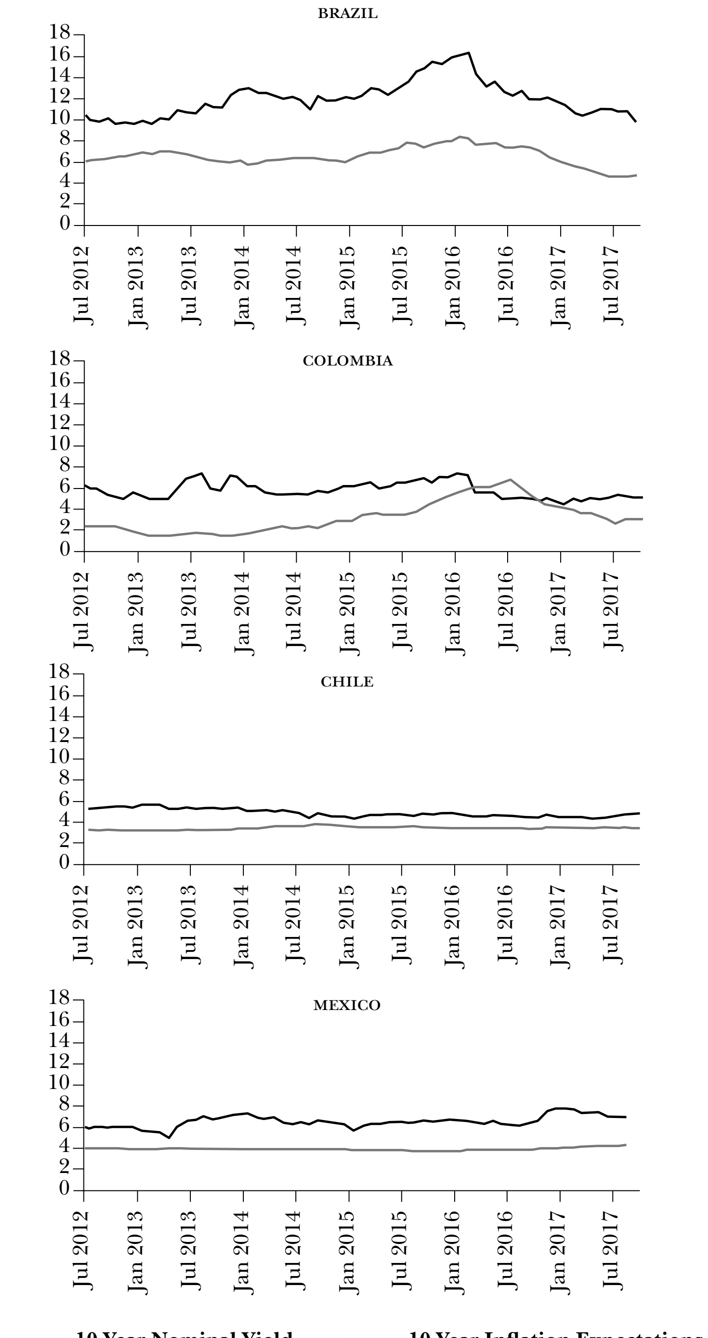
$$E_t[X_{t+h}] = (1 + \Phi + \Phi^2 + \ldots + \Phi^{h-1})\mu + \Phi^h X_t.$$

There are several advantages in using this method to obtain inflation expectations. First, there is a large degree of flexibility, as we can estimate expectations at different horizons. Moreover, we can also compute forward rates, allowing us to estimate, for example, the expected inflation over the five-year period that begins five years from today. This is a measure commonly used by central banks to analyze the anchoring of inflation expectations in the long-run. It is difficult to obtain these estimates in markets without inflation-linked securities and, to the best of our knowledge, this is the first time that these kinds of estimates are computed for Brazilian, Colombian, Chilean and Mexican markets. Also, as we pointed out in the introduction, using existing surveys on inflation expectations provides a limited picture, as the horizons are usually short and the frequency of publication is only monthly at best. Later we describe the characteristics of the surveys published by the central banks of the countries we analyze and compare the expectations obtained from these surveys with those we obtain.

Figure 3 shows the estimates of the nominal yield and inflation expectations over the ten-year horizon obtained from our proposed model. The difference between the two curves represents the real risk-free rate and the risk premium. For the sake of comparison, we restrict the sample period to be the same for the four countries. The results show two main features. First, inflation expectations seem to be more anchored both in Chile and Mexico, showing less volatility. Second, the level of inflation expectations is higher in Brazil, with the other three countries showing expected rates close to or below 4 percent.

Figure 3

10 YEAR NOMINAL BOND YIELD AND INFLATION EXPECTATIONS



As we previously mentioned, the model we propose allows us to compute inflation expectations at different horizons. Figure 4 shows inflation expectations for the one-year, five-year and ten-year horizons, as well as the inflation targeting level established by the central bank in each country. We can see again the different degree of anchoring by comparing the evolution of expectations for the one-year horizon with those for the five-years and ten-year horizons. Inflation expectations in Brazil and Colombia show a similar pattern for all horizons while expectations in Chile and Mexico are more volatile over the one year horizon, showing little changes over longer horizons.

Regarding the inflation targeting levels established by the central banks, most countries currently show inflation expectations at long horizons within the window limits,[6] although Brazil and Colombia have experienced recent periods where inflation expectations were well above these limits. Both countries showed inflation expectations above 6% before the large decreased experienced since the beginning of 2016. On the other hand, Mexico shows long-term inflation expectations slightly above the upper band of 4%, mainly due to the recent increase in expectations after the last US presidential elections. This effect is more apparent for the evolution of the one-year horizon, fading out at longer terms. Interestingly, it seems that the results of these elections have barely affected inflation expectations in other countries. For Brazil, the deep recession of 2015-2016 has affected expectations, with a large decrease experienced since the beginning of 2016. The path of inflation expectations changed again for Brazil at the end of 2016, with expectations turning higher at longer horizons, which signals a possible recovery. In the case of Colombia, the monetary policy implemented by the central bank during 2016, with increases in the policy rate from 4.5% in September 2015 to 7.75% in August 2016, have contained inflation expectations, being now closer to the inflation target. Longer-term inflation expectations continue to show lower levels than short-term ones for this country. Finally, Chile has experienced a decreasing trend in short-term expectations since mid-2014 which has been associated, first to the fall in oil prices, and since 2016 to the appreciation of the Chilean peso. Although short-term inflation expectations remain below the inflation target, expected inflation at long-term horizons is higher and have experienced little change.

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[6] The Bank of Brazil sets the inflation target at 4.5% with a window limit of ±1.5%. The central banks of Chile, Colombia and Mexico set the inflation target at 3% with a window limit of ±1 percent.

Figure 4

INFLATION EXPECTATIONS AT DIFFERENT HORIZONS

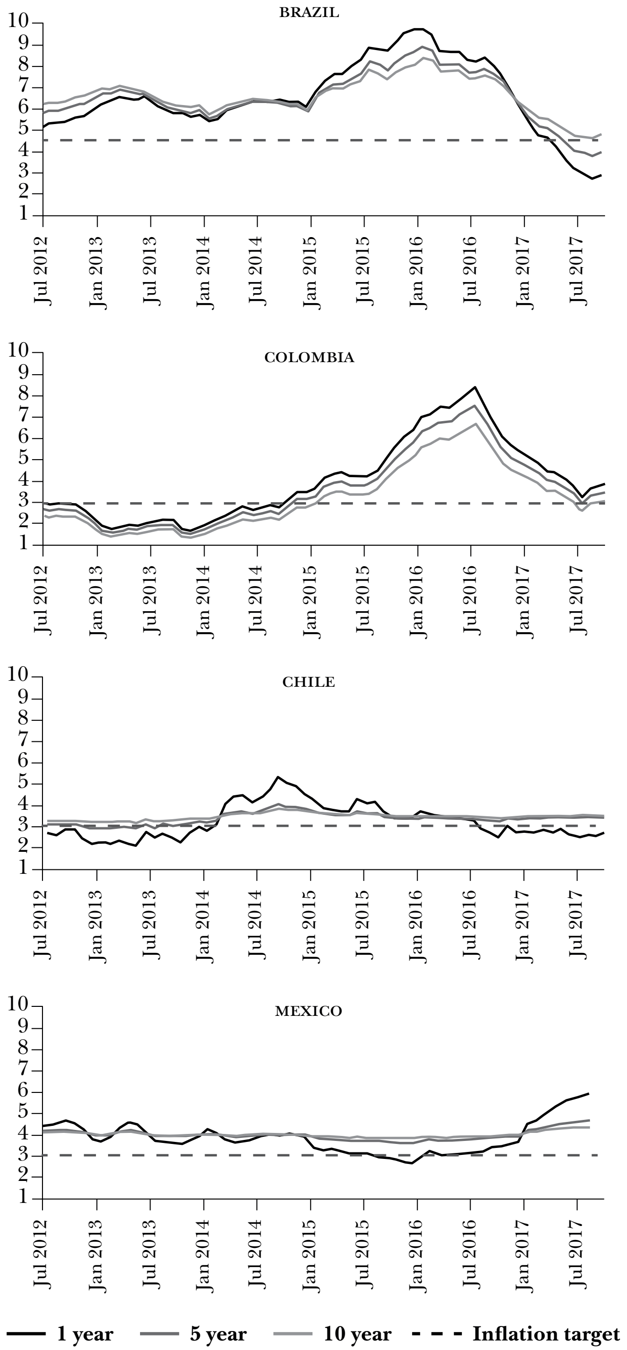


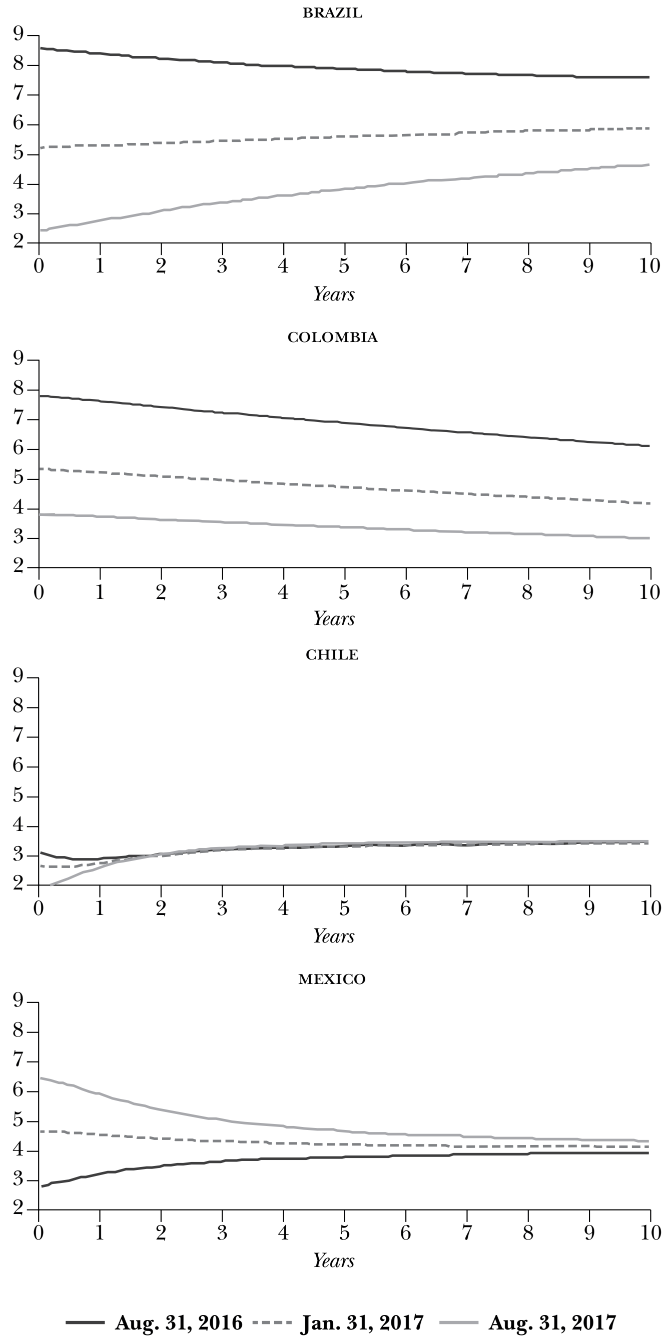
Figure 4 also provides information about the term structure of inflation expectations. Expected inflation in Colombia and Mexico is decreasing with the horizon, while in Brazil and Chile inflation is expected to increase in the future. Figure 5 shows the term structure of inflation expectations at three different dates for all the horizons we compute, giving an idea about how inflation expectations should evolve and how the term structure has changed since August 2016. The evolution of the term structure differs among the four countries. For Chile, expectations from the two-year horizon have barely changed at the three dates, experiencing a decrease over time for short-term expectations. For Brazil, there is an overall decrease at all horizons since August 2016, although the shape of the term structure has changed. At the end of August 2016, the term structure showed a decreasing trend that has currently change into an increasing one. For Mexico, the situation is the opposite, with inflation expectations increasing at all horizons since August 2016, and turning from an increasing trend to a decreasing one. The developments in the US have influenced these changes in Mexican inflation expectations after the last presidential elections. Finally, Colombia shows a decrease in the level of inflation expectations at all horizons, with a decreasing trend over time at the three dates.

Being able to decompose the yield curve and extracting inflation expectations at different horizons let us compute forward rates as well. This is especially useful in order to analyze the anchoring of inflation expectations over the medium and long-term. Forward rates such as the 5Y5Y (expected inflation over the five-year period that begins five years from today) are used by central banks to assess the level of long-term inflation anchoring. Figure 6 shows the 2Y2Y and 5Y5Y forward rates of inflation expectations together with the inflation target established by each central bank. Similarly, to the behavior of the ten-year horizon inflation expectations, the forward rates for Chile and Mexico are more stable and hardly move over time. The levels are above the inflation target but within the window of ±1% for Chile and almost within that window for Mexico. These results show that investors have almost kept unchanged the level of long-term expected inflation for these two countries.

On the contrary, inflation anchoring for Brazil and Colombia seems to be lower, with forward rates showing more volatility. In Brazil, long-term inflation expectations are above the target level but below the upper limit of ±1.5%, due to the large decrease experienced

Figure 5

TERM STRUCTURE OF INFLATION EXPECTATIONS



since the beginning of 2016. For Colombia, there is a similar pattern, with long-term inflation expectations currently below the target level of 3% after the decrease in the 5Y5Y forward rate experienced since mid-2016. The behavior of forward rates for Brazil and Colombia show that investors seem to face more uncertainty about the expected inflation in the long-term for these two countries. It could be also the case the government bond markets provide less information about future inflation for these two countries.

These results may question the effectiveness of monetary policy to anchor expected inflation. The results shown in Figure 5 indicate that the central banks of Chile and Mexico have been able to anchor long-term inflation expectations, although at levels above target, while central bank in Brazil and Colombia face more challenges to do so. Dincer and Eichengreen (2014) compute measures of central bank transparency and independence for a large set of countries. Regarding central bank transparency, among the four countries we analyze, the central banks of Brazil and Chile were the most transparent in 2010, the central bank of Colombia was less transparent and the central bank of Mexico was the least transparent.

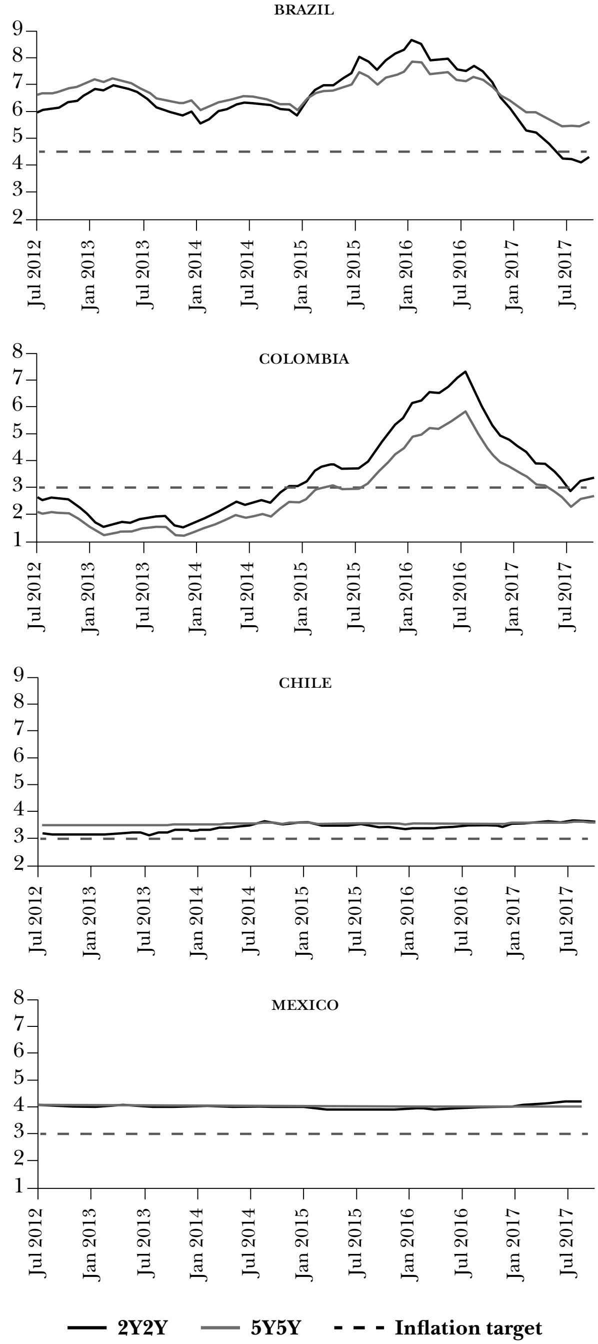
Their measure of central bank transparency does not seem to be related to the level of expected inflation anchoring we observe from our results. On the contrary, central bank independence may play a role. According to their measure of central bank independence, Chile and Mexico's central banks are more independent than the central bank of Colombia (unfortunately, they do not provide a measure of central bank independence for Brazil). In line with this result, Gutiérrez (2003) and Jácome and Vázquez (2008) find a relationship between central bank independence and inflation performance for Latin American countries.[7]

The purpose of our analysis is to identify the inflation expectations implicit on financial markets, something that would not necessarily be the best forecast for future inflation. However, we analyze the forecast capacity of this methodology in order to compare it with other alternatives frequently used by professional forecaster of inflation trends. In this vein, we compare the information about expected

7 Gutiérrez (2003) provides the values of the central bank independence indexes for the four countries in our study. Although we should be careful as the indexes were calculated long time ago, Mexico and Chile show the largest values of central bank independence.

Figure 6

INFLATION EXPECTATIONS OF FORWARD RATES



inflation obtained from our model with that provided by surveys. First, as we obtain expectations from nominal government bonds, expected inflation is derived from investor's perceptions, complementing the information from surveys which is usually obtained from the views of economists and forecasters. Second, we can obtain inflation expectations at different horizons and forward rates. Surveys usually provide few horizons, with limited information about long-term inflation expectations. Table 3 summarizes the information provided by the surveys published by the central banks in the four countries analyzed. Even though there is information about expected inflation at different horizons in the surveys, we cannot get all the different horizons we can compute using our proposed methodology. The surveys do not provide forward rates either. We next compare the forecasting accuracy of the inflation expectations obtained from our model with those provided by surveys and a simple autoregressive process AR(1). Figure 7 shows expected inflation obtained from surveys and our methodology as well as ex-post realized inflation for the 12-months horizon.[8] Inflation expectations obtained from surveys tend to be broadly stable over time and show little changes and reaction.

On the other hand, inflation expectations obtained from our model seem to be too reactive and more dependent on current inflation. Expected inflation from surveys fail to react to inflation shocks while our measures produce expectations that respond too late to inflation shocks. The AR(1) process provides similar inflation expectations to those obtained from our model although these expected values seem smoother. The difference between the inflation expectations obtained from the model and the AR(1) represents the additional information about future inflation once that we consider the inflation expectations embedded on bond prices. In order to analyze the forecast accuracy of the measures, we compute the mean square error (MSE) concerning ex-post realized inflation.

[8] In the case of Chile, it is 11-months horizon inflation expectations (annual change).

Table 3

SURVEYS ON INFLATION EXPECTATIONS–CENTRAL BANKS

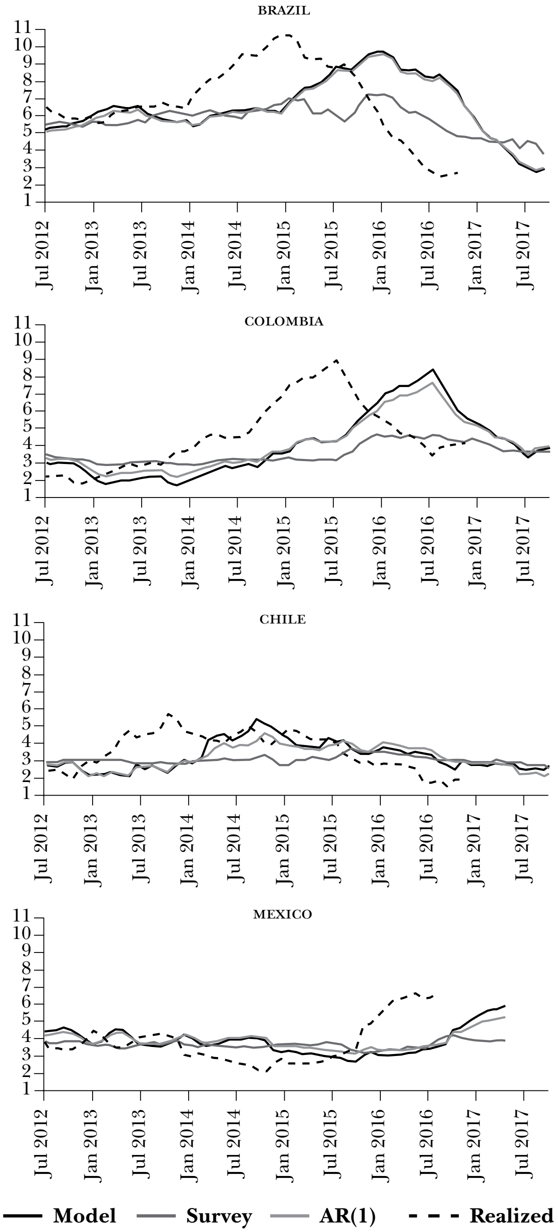
	Frequency	*Horizons*
Brazil	Daily	Next 12 months; current year (t) and t+1, t+2, t+3, t+4.
Chile	Monthly	Next 11 months; next 23 months; current year (t) and t+1, t+2.
Colombia	Monthly	Next 12 months; next 24 months; current year (t) and t+1.
Mexico	Monthly	Next 12 months; next 1-4 years; next 5-8 years.

Table 4 shows the ratio of the MSE obtained using expectations from surveys, as well as from our model and the AR(1) process, to the MSE computed using current inflation as the predicted future value (like in a unit root process). If the ratio is lower than one, it means that the expected values provide a better prediction of future inflation than assuming inflation will remain the same as today. The three measures, inflation expectations from surveys, from the AR(1) and our model show lower MSE than the unit root prediction. Comparing the three measures, expected inflation from surveys shows lower MSE for Brazil and Colombia. The model is the best predictor for Chile and the AR(1) process provides the lowest MSE for Mexico.

Inflation expectations from our model provide lower MSE for Chile and Mexico than for Brazil and Colombia. It seems that our measures of expected inflation are more accurate for countries where expectations are fairly anchored in the long-run. Our measures do complement those from surveys in terms of predictability, providing additional forecasting power and a much richer set of expected inflation horizons, and frequency.

Figure 7

12-MONTHS INFLATION EXPECTATIONS FROM SURVEY AND PROPOSED MODEL VS. REALIZED INFLATION



Source: DataStream, Banco Central de Chile, Banco de la República - Colombia, Banco Central do Brasil, Banco de México. Inflation expectations in 12 months for Brazil, Colombia and Mexico. Inflation expectations in 11 months for Chile.

Table 4

EXPECTED INFLATION FORECAST ERRORS

	Sample	*Survey*[1]	*Model*[1]	*AR(1)*[1]
Brazil	Feb 2007-Oct 2016	**0.5833**	0.8812	0.8415
Chile	Jul 2012-Dec 2016	0.7813	**0.6946**	0.7148
Colombia	Feb 2005-Nov 2016	**0.7956**	0.9354	0.8015
Mexico	May 2001-Nov 2016	0.6350	0.7078	**0.6324**

[1] Ratio of mean square error of expected inflation from surveys, an AR(1) process and our model with respect to a naïve prediction of expected inflation equal to current inflation. Expected inflation in 12 months for Brazil, Colombia and Mexico; 11 months for Chile.

5. CONCLUSIONS

Agents' inflation expectations are decisive when studying changes in many of the variables shaping households' and firms' decision making. We use a methodology to obtain inflation expectations from nominal government bonds and realized inflation, overcoming the problems of obtaining expected inflation using inflation-linked securities. This is especially useful for markets where inflation-linked securities are scarce and illiquid as it is the case of Latin America. In this article, we estimate inflation expectations for Brazil, Chile, Colombia, and Mexico. We find that inflation expectations seem to be anchored in Chile and Mexico in the long-term (5Y5Y forward rate), although the level of expected inflation is above the central bank target rate of 3 percent.

On the other hand, long-term inflation expectations in Brazil and Colombia are more volatile and have been fluctuating over time, experiencing a large decrease during 2017. These results advise further efforts from the Brazilian and Colombia central banks to anchor inflation expectations to make credible their inflation targets. Mexican and Chilean central banks should be more concerned in reducing the level of expected inflation as long-term expectations seem to be fairly anchored and show low levels of volatility.

We also find the expected inflation is currently increasing with the horizon in Brazil and Chile, while it is decreasing in Colombia and Mexico. For Mexico, there has been an important shock on expected inflation after the last US presidential elections, experiencing a large increase. None of the other countries analyzed have shown this pattern, limiting the spillovers effects of the results of the US presidential elections to inflation expectations in Mexico.

Finally, we compare the forecasting power over one year inflation expectations obtained using our approach with expected inflation obtained from surveys. Our approach performs better predicting inflation for Chile, while surveys do better for Brazil, Chile, and Colombia. There is a trade-off in terms of predictability as expected inflations from surveys is less responsive to inflation shocks, and our approach produces inflation expectations that are more correlated with current inflation.

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The Information Rigidities and Rationality of Costa Rican Inflation Expectations

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Aarón Mora Meléndez

Abstract

Costa Rican inflation expectations cannot be characterized as rational under any existing definition of the term. They cannot be categorized as adaptive either, since in addition to historical data on inflation, other macroeconomic variables are important in explaining inflation expectations. Instead, the sticky information model is considered a more sophisticated framework to assess inflation expectations of Costa Rican agents. Results are based on the Monthly Survey of Inflation and Exchange Rate Expectations elaborated and published by the Banco Central de Costa Rica. This chapter collects evidence to assess whether the expectations from this survey are subject to information rigidities. Additionally, this chapter shows how a simulated survey, based on a sticky information model, is capable of replicating features from the observed survey.

Keywords: inflation expectations, sticky information, adaptive learning.
JEL classification: C53, D84, E31, E58.

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1. INTRODUCTION

Conventional economic theory highlights the crucial influence of expectations on changes in macroeconomic variables. Changes in a variable affect the expectations related to its future movement and these expectations also influence the variable's underlying path. This bilateral relation puts the problem of how agents form their expectations into the front line of macroeconomic modeling.

Most central banks acknowledge the crucial role of expectations, and argue that managing inflation expectations is paramount for attaining price stability and conducting monetary policy. The Banco Central de Costa Rica (BCCR) operates under an inflation targeting regime, in order to accomplish its goal of a low and stable inflation level. It relies heavily on the inflation expectations of Costa Rican agents aligning closely with monetary policy. It is necessary to understand how inflation expectations are formed to anchor expectations to the ones targeted by the BCCR.

Until recently the research agenda on expectation formation was eclipsed by the rational expectations (RE) hypothesis started by Muth (1961). This hypothesis revolutionized macroeconomic thinking during the seventies by incorporating the effect of expectations into most economic models. As Thomas Sargent points out[1], the RE hypothesis allowed for the disappearance of any free parameters associated with expectations, so people's beliefs became outputs of the model in question. As a result, macroeconomists widely adopted the assumption of RE to arrive at tractable equilibrium solutions.

Nevertheless, a common critique for the RE hypothesis is that it assumes that people have much more information about the economy than they really do, since it implies that agents construct expectations and make decisions by gathering and conveying all available public information. This assumption is unrealistic and empirical studies often reject the RE hypothesis. There are three popular alternatives to the re hypothesis: *1)* agents use heterogeneous mechanisms to form their expectations, as in Branch (2004) and Honkapohja and Mitra (2006); *2)* agents use different information sets, as in Angeletos and Lian (2016); and *3)* agents have different abilities to process information, see for example Woodford (2001). A good survey of alternative approaches to the specification of expectations is presented in Woodford (2013)

[1] See Evans and Honkapohja (2005).

where the author presents how macroeconomic analysis under a new Keynesian framework could be performed without relying on the RE hypothesis. Regardless, there are well developed theoretical alternatives to RE, though many features observed in expectations survey are not entirely taken into account by these alternatives. Authors like Manski (2004) have pushed for more empirical studies that deepen our knowledge of how people elicit and revise their expectations.

One approach to analyzing expectations formation has focused on the role of information rigidities and has been supported by empirical evidence, see Mankiw and Reis (2002), Woodford (2001), and Sims (2003). In particular, Mankiw et al. (2003) depart from traditional empirical approaches to expectations measurement, which have traditionally relied on measures of central tendency, such as the mean or median; instead, they study the heterogeneity of inflation expectations using statistics of dispersion. The idea is that the disagreement among agents over inflation expectations can be explained by information stickiness. They use the sticky information model developed in Mankiw and Reis (2002) to explain the mean and dispersion of the United States' inflation expectations. Under this framework, just a fraction of the agents updates their expectations with the most recent information available. This fraction is derived from the bounded rationality associated with the cost of updating expectations. Pfajfar and Santoro (2010) build on this line of work and instead of using measures of central tendency, they perform percentile analysis to study the heterogeneity, learning, and information stickiness of inflation expectations.

Alfaro and Monge (2013) also document that Costa Rican inflation expectations can neither be characterized as rational nor adaptive. If expectations were rational, the realized bias between expected and realized inflation level could not be predicted: Costa Rican data fails this test even with relaxed assumptions of rationality. On the other hand, inflation expectations cannot be categorized as adaptive neither, since in addition to historical data on inflation, other macroeconomic variables hold significant explanatory power for inflation expectations.

Alfaro and Monge (2013) note the need to evaluate more sophisticated tools to model Costa Rican inflation expectations. This chapter will evaluate the sticky information model to determine whether this need is substantial. The main source of data for this research comes from the Monthly Survey of Inflation and Exchange Rate

Expectations conducted and published by the BCCR. For this chapter, we used 135 months of survey observations from January 2006 to March 2017. We identify individual participants and place them into four separate groups based on their profession. In the survey, respondents report their 12-month expected inflation as well as expected percentage variations (to different time horizons) of the exchange rate between the Costa Rican colon and United States dollar.

The remainder of the chapter is organized as follows: Section 2 describes the Monthly Survey of Inflation and Exchange Rate Expectations, presents its main features, and analyses the disagreement and the realized bias or forecast error presented in the survey. Section 3 presents the sticky information model of Mankiw et al. (2003), gathers evidence for information rigidities in the expectations of Costa Rican agents captured in the survey as a whole and within professional groups, and simulates a sticky information model that is based on a vector autoregressive model using Costa Rican macroeconomic data. Finally, Section 4 discusses the findings of the paper, which show nonconformity of the sticky information approach for the Costa Rican data, as well as the work ahead for modeling Costa Rican inflation expectations.

2. INFLATION EXPECTATIONS SURVEY

The BCCR has conducted the Monthly Survey of Inflation and Exchange Rate Expectations since 2006. This survey gathers data on expected inflation for the next 12 months and the expected percentage variation in the exchange rate between the Costa Rican colon (CRC) and the United States dollar (USD) for the next 3, 6, 12, 24, and 36 months[2]. The questionnaire of the survey can be found in Annex A. Responses to questions on inflation and exchange rate expectations are point expectations that ask for a numerical expectation along with the main factors that were considered to form these expectations.

The observation period starts on January 2006 and goes until March 2017, a total of 135 months. The individuals consulted in the survey are categorized into four different groups depending on their professional expertise: *1)* consulting, *2)* stock market analyst, *3)* academic,

[2] Consultancy of the 24- and 36-month variation in the CRC/USD exchange rate started on December 2016.

and *4)* business sector. The number of respondents to the survey and its composition have changed during the observation period; there were 27 respondents in January 2006, most of whom were stock market analysts and by March 2017, there were 61 respondents predominantly from the business sector. Figure 1 presents the composition of the sample group during the observation period.

Two features of the survey responses stand out: first, the total number of responses has increased more than twofold since the survey was first implemented, with a peak of 87 responses in June 2013[3]. Second, the composition of responses has drastically changed in the last years of the survey–the majority of responses have recently come from individuals working in the business sector–. This compositional shift has resulted from a change in the survey design from June 2012 to the present[4].

The BCCR computes the 12-month expected inflation by averaging the responses received during a particular month, expectations coming from the business sector are dominant in the expectations published, representing up to 80% of the responses since 2015. This dominance of the business sector in the average expected inflation can be observed in Figure 2 where the mean expectation is plotted for the whole sample and by group.

The average expectation has clearly declined, staying in the single digits since April 2009, and below 5% since April 2015. The behavior exhibited by the inflation expectations has been in accordance with the inflation target range of the BCCR (3%-5%) since April 2015. In January 2016, even though the inflation target range was downgraded to 2%-4%, expectations have continued to remain within the range up until the last month in our sample, March 2017.

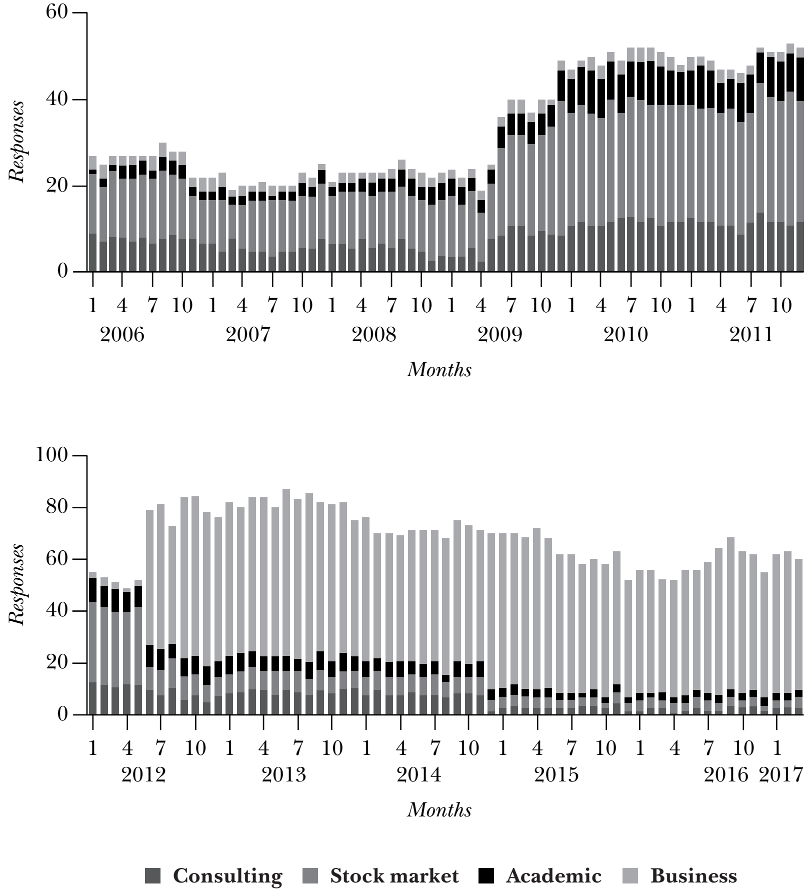
The alignment between the expected inflation rate and the target inflation range in recent years highlights the built-up credibility of BCCR towards society. For the thirty-year period preceding 2009, Costa Rica experienced double-digit inflation rates, but the BCCR has seemingly regained credibility. Agents trust the BCCR to steer the inflation rate, which thereby anchors inflation expectations. Despite this tendency for inflation expectations to lie within the target

3 With 64 of them from the business sector.

4 The two samples were active for several months, but the aggregate results did not differ.

Figure 1

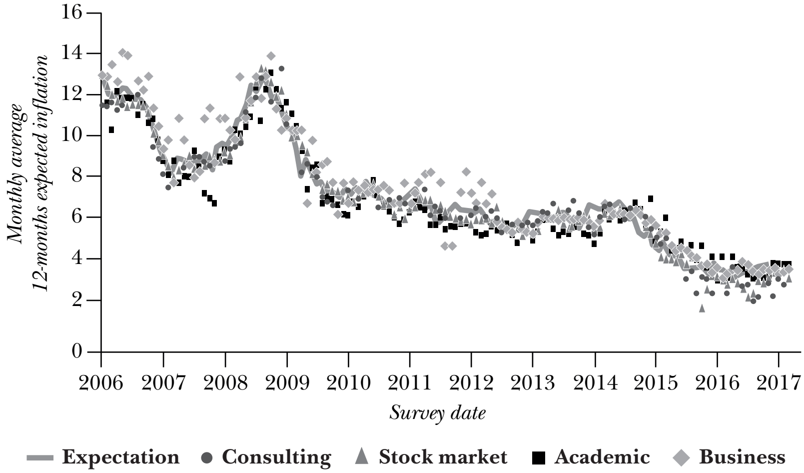
INFLATION EXPECTATIONS SURVEY: RESPONSES



Source: Own elaboration.

Figure 2

EXPECTED INFLATION MONTHLY AVERAGE



Source: Own elaboration.

range, disagreement about inflation expectations is present in the survey, not only between groups but also within groups[5].

2.1 Disagreement Among Expectations

Each individual in the survey sample has an identifier code and every month that an individual responds, the observations collected are registered with the relevant identifier (ID). This way the survey data can track respondent observations throughout the entire survey period, allowing for comparisons in the responses over time among individuals of the same group and within the full sample. In the survey there are 409 identifiers that correspond to at most 409 individuals[6] that respond the survey at some point during the observation period.

[5] Figure 9 in the Annex, shows the increase of outliers on the expectations from the business sector in recent years.

[6] Since the change in the design of the survey sample involved different nomenclature for the identifiers, the same individual can have two identifiers, one under the former sample and another one with the current sample.

Table 1

DISTRIBUTION ON THE NUMBER OF RESPONSES

	Identifiers (IDs) with equal or more responses	
Responses (⩾)	*Number of IDs*	*Percentage of IDs*
1	409	100.00
10	206	50.37
20	136	33.25
30	41	10.02
40	33	8.07
50	28	6.85
60	17	4.16
70	10	2.44
80	9	2.20
90	3	0.73

Source: Own elaboration.

The number of responses from a particular identifier range from 1 to 98, with an average of 16.46 during the 135-month observation period. The observed distribution on the number of responses by ID is shown on Table 1. Decomposing this distribution into the four aforementioned professional groups, we observe that the academic and consulting groups have the highest response rates. Even though the firm group dominates the survey responses, most of the firms' identifiers have less than 48 responses.

Given the number of individuals participating in the survey, their professional expertise, and background, disagreement among the inflation expectations can be observed on the survey. Mankiw et al. (2003) are primarily concerned with this disagreement, which is typical in most expectations surveys and they posit that this heterogeneity can be explained by bounded rationality, meaning that only a fraction of the agents adjusts their expectations as new information becomes available due to the cost associated with the adjustment.

In this context, dispersion statistics like the interquartile range can be used to discriminate between different models of expectations formation by pinning down their faculty to replicate features observed on the data. Figure 3 presents the interquartile range observed every month by group, along with the realized inflation rate for the month that these expectations were registered. This is done to assess whether the dispersion tends to increase when inflation is high, as has been suggested by Ballantyne et al. (2016) and Johannsen (2014), among others.

For the stock market analyst and academic groups, the interquartile range and inflation rate attain their maximum in the last months of 2008. For these two groups, it may seem to be a positive correlation between the level of inflation and interquartile range during years near the 2008-2009 financial crisis. Nonetheless, there are periods in which the inflation rate decreases but the dispersion of the sample expectations does not follow the same trend; the clearest example is the dispersion within business sector responses since 2015 the interquartile range has moved around 2% despite the sharp decline in inflation. This suggests that for the Costa Rican case there is no clear direct relation between the dispersion in inflation expectations and the level of inflation.

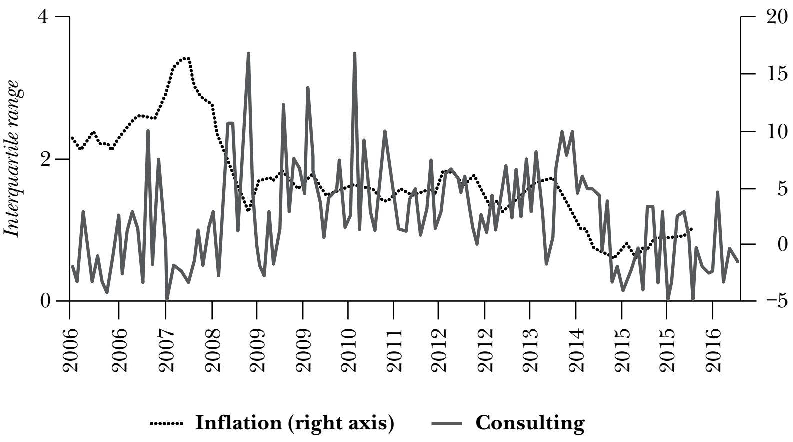
A basic regression exercise between dispersion as measured by the interquartile range and the inflation level is shown in Table 2. Regressing the interquartile range by the inflation rate does not illustrate a significant relation between the two groups: the associated coefficients are not significant when taking into account the whole survey or individual groups.

Elliott et al. (2008) and Engelberg et al. (2009) note that disagreement among inflation expectations does not necessarily indicate that agents face different degrees of uncertainty when forming their expectations. This is because the survey collects point predictions from which individual distributions or probabilistic beliefs of possible outcomes for future inflation cannot be inferred. It is possible that two forecasters who hold identical probabilistic beliefs provide different point predictions and it is also possible that two forecasters with different probabilistic beliefs provide the same point forecast. When using point forecasts, we can only interpret the phrase *disagreement among expectations* as an acknowledgment of distinct point forecasts; we cannot conclude anything about the uncertainty that forecasters face.

Figure 3

INTERQUARTILE RANGE BY GROUP

CONSULTING



ACADEMIC

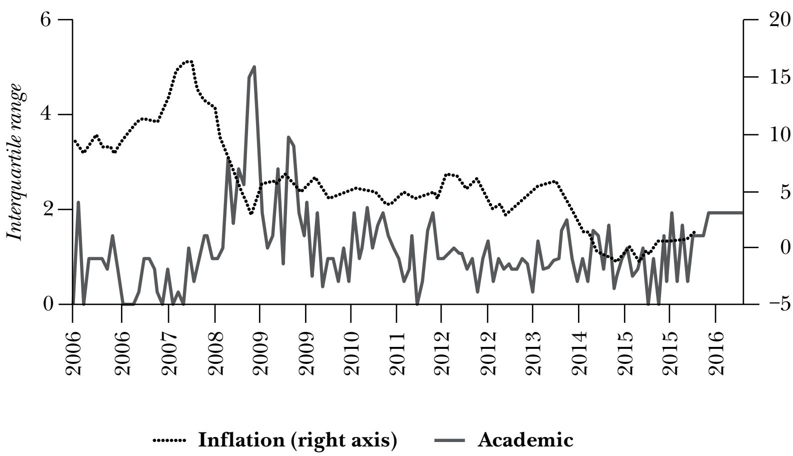
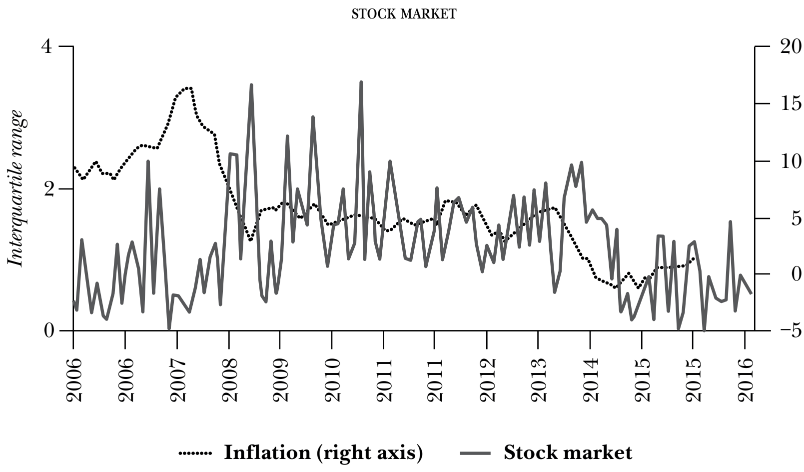
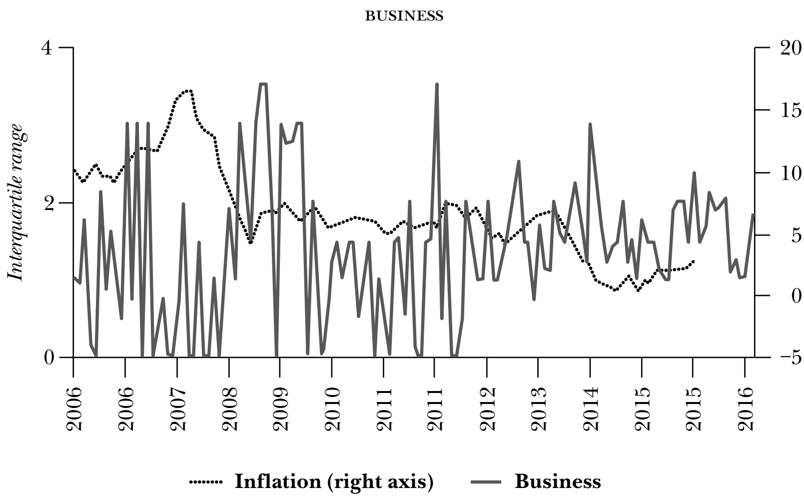


Figure 3 (cont.)

INTERQUARTILE RANGE BY GROUP





Source: Own elaboration.

Table 2

REGRESSION: INTERQUARTILE RANGE AND INFLATION

Coefficient	*Whole survey*	*Consulting*	*Stock market*	*Academic*	*Business*
Constant	1.591[c]	1.209[c]	1.135[c]	1.254[c]	1.446[c]
	(0.090)	(0.110)	(0.090)	(0.138)	(0.134)
Inflation	−0.013	−0.007	0.003	−0.002	−0.011
	(0.012)	(0.015)	(0.012)	(0.019)	(0.018)
N	135	135	135	135	135
R^2	0.0087	0.0014	0.0006	0.0001	0.0029

Note: [a] significance level 0.1, [b] 0.05, [c] 0.01.
Source: Own elaboration.

2.2 Realized Bias

We can also perform a second descriptive analysis of the survey inflation expectations focused on how well agents forecast the inflation level. If agents can successfully predict the path of future inflation, then the realized bias, that is the difference between the (forecasted) expected inflation level for time t and the realized inflation at time t, should be close to zero.

As a result of the survey design, when 12-month expected inflation is recorded at time t, its predictive power should be compared with the realized inflation level of time $t+11$, that is eleven months later from when the observation was collected. This is because even though agents form their expectations for each annual period, they are consulted during the first month of the forecast period. This does not present an issue since agents do not know the realized inflation of the month that is consulted[7]. For instance, the expected inflation of January 2006 should be compared with the inflation rate of December 2006 to compute the realized bias of December 2006.

With this adjustment only 124 months from January 2006 to April 2016 are used to analyze realized bias rather than all 135 months

[7] The Instituto Nacional de Estadísticas y Censos (INEC) of Costa Rica publishes the inflation rate of month t until the first days of month $t+1$.

of the survey. The last eleven months do not yet have a realized inflation level to compare to, since the last observed inflation in this paper is March 2017. Panel A of Figure 4 compares the expected and realized inflation rates, while panel B. shows the average realized bias.

Our measure of realized bias has exhibited cyclical behavior, reaching its minimum at the end of 2008 and its maximum at the end of 2009. While there are months where the realized bias has been practically zero, suggesting good predictive power, it has been positive since 2005, meaning that on average, inflation expectations have been greater than realized inflation.

The average realized bias seems to have a general upward trend across the entire observation period, standing above 5% during most of 2015 and part of 2016, but decreasing since the second semester of 2016. The average realized bias does not differ substantially by group–Figure 5 shows the average realized bias for each group and also for the entire survey sample–.

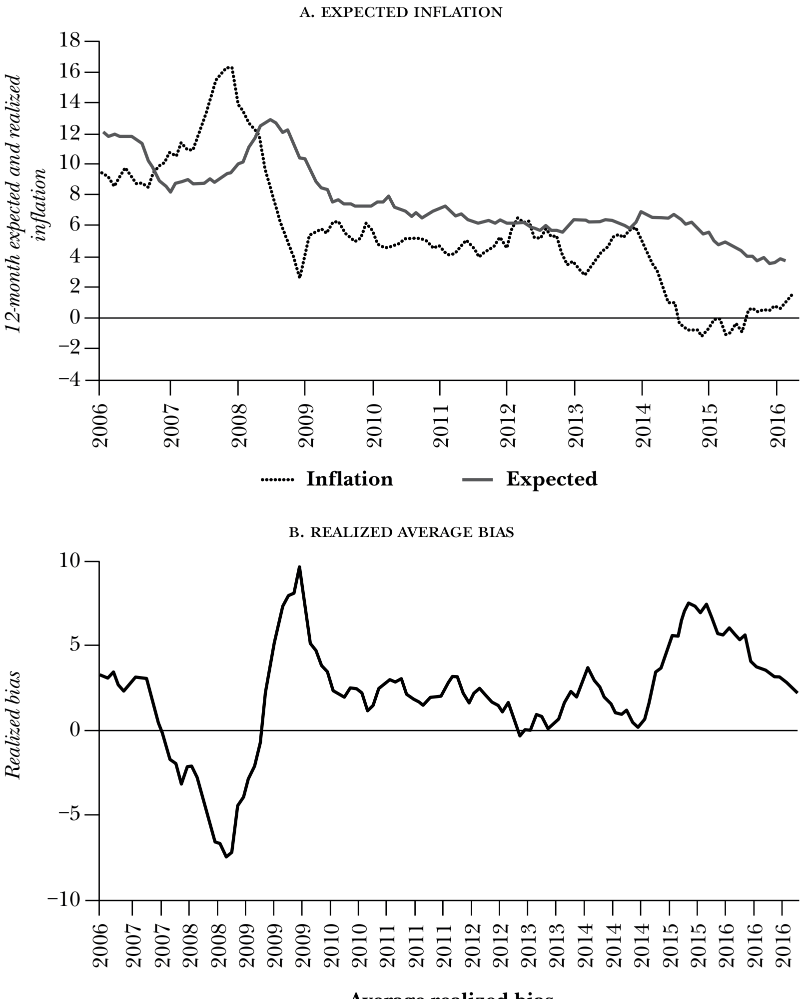
As expected, the business sector has dominated recent survey results–the average bias of the business sector has largely aligned with the average of the entire survey sample–. In addition, the average bias has increased over the years for all four groups. Figure 5 suggests that the differences among groups are not significant, but this can be explained as a result of using measures of central tendency such as the average. On the other hand, valuable information can be extracted by studying disagreement among inflation expectations via statistics of dispersion. The next section explores the role of information rigidities in explaining the heterogeneity in inflation expectations.

3. STICKY INFORMATION MODEL

Mankiw and Reis (2002) propose a model where information rigidities play a central role in the price and inflation dynamics. In their model, only a fraction λ of agents gather, process, and optimize their expectations with the most recent economic information available. The parameter λ, which is exogenous to the model, can be interpreted as the result of the bounded rationality associated with the cost of adjusting to new information. This model is conceived as an alternative to the new Keynesian Phillips curve since it highlights the role of information rigidities.

Figure 4

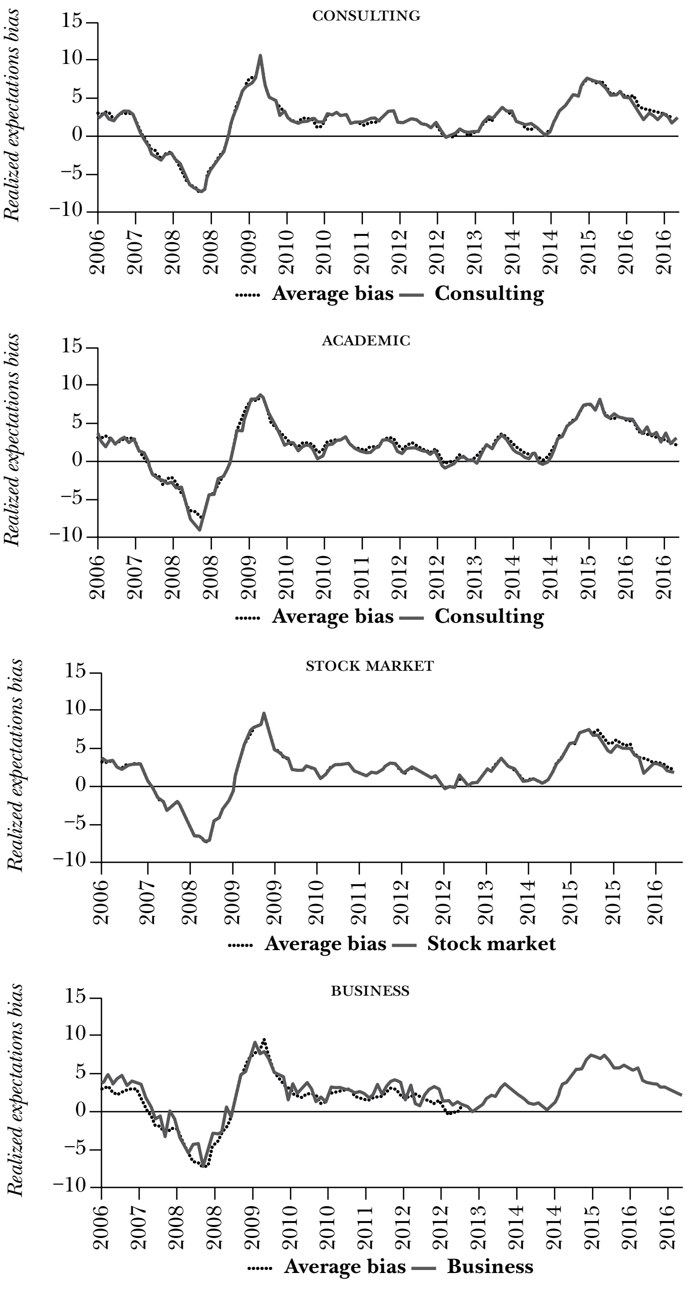
EXPECTED AND REALIZED INFLATION



Source: Own elaboration.

Figure 5

AVERAGE REALIZED BIAS BY GROUP



Source: Own elaboration.

The *sticky information Phillips* curve derived in Mankiw and Reis (2002) concludes that the relevant expectations of the agents are those made in the past about current conditions. Mankiw et al. (2003) follow this idea and study the disagreement about inflation expectations by assuming there is information stickiness, meaning that only a fraction of the agents generates their expectations of future inflation using all available economic information. With this specification, we can generate cross sectional samples of simulated expectations for each period, allowing us to study the features of a simulated survey beyond measures of central tendency.

In this section, we gather evidence of information rigidities present in the Monthly Survey on Inflation and Exchange Rate Expectations at the survey and group level. Moreover, a sticky information model is simulated, assuming that the process used to generate expectations is an econometric model and the way that rational agents form their expectations is through forecasts from this model. In particular, we use a vector autoregressive model with Costa Rican macroeconomic data to generate 12-month inflation forecasts.

3.1 Evidence for Information Rigidities

Following Coibion and Gorodnichenko (2015), we can exploit the conclusion from Mankiw and Reis (2002) that states that for an economic variable x under a sticky information model, the average forecast across agents at time t for time $t+h$, $F_t x_{t+h}$, is a weighted average of the current and past rational expectation forecast such that[8]:

$$F_t x_{t+h} = (1-\lambda)\sum_{j=0}^{\infty} \lambda^j E_{t-j} x_{t+h}. \quad (1)$$

Representing rational expectations as $E_t x_{t+h} = x_{t+h} - v_{t+h,t}$, where $v_{t+h,t}$ is the rational expectation error, which is uncorrelated with information dated t or earlier, we can find a predicted relation between the ex post mean forecast error and the ex ante mean forecast revision (see Coibion and Gorodnichenko, 2015, for its derivation):

[8] In this equation the probabilities of an update are reparametrized so only $(1-\lambda)$ percent of the agents update their information sets and acquire no new information with probability λ.

2 $$x_{t+h} - F_t x_{t+h} = \frac{\lambda}{1-\lambda}\left(F_t x_{t+h} - F_{t-1} x_{t+h}\right) + v_{t+h,t}.$$

The relation in 2 can be applied to the data. Since it requires the construction of a forecast revision, we will use data on the expected exchange rate variation instead of inflation expectations; only a 12-month expected inflation is available. Under a sticky information framework relation, 2 should be satisfied for the mean of any macroeconomic variable regardless of the frequency of t and the horizon h, so gathering evidence of information rigidities using the expected exchange rate variation should be comprehensive for all expectations in the survey. Specifically, quarterly data for the expected exchange rate e variation for three and six months is used to perform the following regression based on 2:

3 $$e_{t+1} - F_t e_{t+1} = \beta\left(F_t e_{t+1} - F_{t-1} e_{t+1}\right) + \varepsilon_t.$$

Estimates for Equation 3 at the survey and the group level are shown in Table 3. These regressions can be used to assemble evidence for information rigidities present on the survey. Under a sticky information model, the β coefficient in Equation 3 should be significant, which is the case at the survey level. An advantage of the relation between the ex post forecast error and the ex ante forecast revision on Equation 3 is that it enables us to map the estimated coefficient $\hat{\beta}$ to an estimate of the information rigidity parameter λ. In our case, this gives an estimate of $\hat{\lambda} = \hat{\beta}\big/\left(1+\hat{\beta}\right) \approx 0.1797/1.1797 \approx 0.15237$, which suggests that 84.76% of the agents update their information sets at a particular period and that on average an agent updates his or her information every 1.2 months.

At the group level, the estimates of Equation 3 suggest that the evidence for information rigidities is stronger among some groups compared to others. The β coefficient for Equation 3 is significant to various degrees among the groups, with the exception of the academic. For consultants and stock market analysts, the coefficient is significant at a 1% level and only at a 10% level for the businesspeople. The results imply different estimates for the rate of information acquisition λ among groups: 82.44% of the consultants, 83.61%, of the stock market analysts, 91.91% of academics, and 91.07% of the

Table 3

REGRESSION: EX POST MEAN FORECAST ON EX ANTE MEAN REVISION

	Dependent variable $e_{t+h}-F_te_{t+h}$				
	Survey	*Consulting*	*Stock market*	*Academic*	*Business*
$F_te_{t+1}-F_{t-1}e_{t+1}$	0.1797[c]	0.213[c]	0.196[c]	0.088	0.098[a]
	(0.056)	(0.058)	(0.056)	(0.056)	(0.056)
Observations	240	240	240	240	240
R^2	0.041	0.054	0.049	0.010	0.013
Adjusted R^2	0.037	0.050	0.045	0.006	0.009
Residual standard error (df = 239)	20.290	20.940	19.937	20.367	21.007
F statistic (df = 1; 239)	10.192[c]	13.643[c]	12.409[c]	2.492	3.129[a]

Note: [a] $p < 0.1$; [b] $p < 0.05$; [c] $p < 0.01$.
Source: Own elaboration.

businesspeople update their expectations with the most recent information available every period[9]. These results, however, show a relatively low degree of information rigidity. The evidence indicates that the sticky information assumption may not be particularly well suited to account for how the inflation expectations in the Costa Rican economy are formed. Nevertheless, we will stick to this assumption to evaluate how closely a model with sticky information can simulate the data.

3.2 Simulating a Sticky Information model

In this section, we generate a simulated survey using the following algorithm proposed in Mankiw et al. (2003). In this context, an agent's rationality is pin-downed so that we can use a vector autoregressive

9 One should keep in mind that the estimate for the academic group is not significant and for the business group is only significant at the 10% level. The coefficients are essentially unchanged if the model is estimated using a constant.

(VAR) model to generate rational forecasts[10]. The VAR model uses Costa Rican monthly data from January 1996 to March 2017 for inflation (π_t), interest rate (i_t), output gap (y_t), an inflation index of trade partners $\left(\pi_t^{C}\right)$, oil prices $\left(p_t^{oil}\right)$, and annual exchange variations (e_t). The design of the VAR model with two lags[11] is presented in 4.

$$z_t = A_1 z_{t-1} + A_2 z_{t-2} + u_t, \tag{4}$$

with

$$z_t := \begin{bmatrix} \pi_t \\ i_t \\ y_t \\ \pi_t^{C} \\ p_t^{oil} \\ e_t \end{bmatrix}.$$

As usual A_1 and A_2 are 6×6 matrices of coefficients and u_t stands for a process with a null expectation and a time invariant positive definite covariance matrix. Data used comes from different sources: *1)* monthly annual inflation (π_t) is measured using the CPI; *2)* the interest rate (i_t) is the basic passive interest rate (*tasa básica pasiva*, TBP); *3)* the output gap (y_t) is estimated following Hamilton (2017) using a series of the monthly index of economic activity (*índice mensual de actividad económica*, IMAE)[12]; *4)* the inflation index of trade partners $\left(\pi_t^{C}\right)$ is an index of the inflation of countries considered to be trade partners with Costa Rica (*indicador de inflación de socios comerciales*)[13]; *5)* oil prices $\left(p_t^{oil}\right)$ come from the monthly average of West Texas

[10] We attempted unsuccessfully to estimate the degree of information rigidity directly for inflation forecasts, using instrumental variables similarly to Coibion and Gorodnichenko (2015).

[11] Number of lags suggested by the Hannan-Quinn information criterion.

[12] We regress IMAE series at date t+24 (to include a two-year period) on the four most recent values as of date t. The residuals from this regression are set to be the cyclical component of the series.

[13] Mainly composed by the inflation of the United States, the euro zone, China and Central American countries.

Intermediate (WTI) crude prices; and *6)* the annual exchange variations (e_t) are relative annual variations on the BCCR's reference bid exchange rate between the US dollar and the Costa Rican colon by the end of the month. The TBP, IMAE, inflation index of trade partners, and reference bid exchange rate are computed and published by the Banco Central de Costa Rica.

The estimation of the VAR is done on a sample updating basis, meaning that at time t we estimate the VAR solely with information available up to time $t-1$, denoted by $I_{t-1} = \{z_{t-1}, z_{t-2}, \ldots\}$, and done for each month from January 2006 to March 2017. For example, for January 2006 Equation 4 is estimated using information on z_t from January 1996 up to December 2005, meaning that the initial sample size covers ten years; each subsequent month adds one observation to the sample size and the VAR model is reestimated with this updated sample. Using the estimates at time t, we forecast the 12-month forward inflation rate $\pi^e_{t+12|t}$ using the forecast for the next twelve months form the VAR updated up to time $t-1$:

5 $$\pi^e_{t+12|t} := \hat{\pi}_{t-1+12}.$$

The updating procedure of the parameters of the VAR is modeled as if the agents are econometricians who form their expectations about the future by incorporating new information on the sample when estimating the VAR.

With the VAR predicted values, especially for inflation $\{\hat{\pi}_t\}$, we generate cross sectional samples of expected inflation to obtain a simulated survey as follows:

1) Given that the Monthly Survey of Inflation and Exchange Rate Expectations includes data for 135 months, there will be 135 cross sectional samples, one for each t=1, ..., 135.

2) The cross-sectional sample size n is to be of 100 individuals for all periods, n=100.

3) In the first period each individual enters the simulated survey with the mean expectation observed from the survey in the first month.

4) For every $t=2, \ldots, 135$, and for each individual $i=1, \ldots, n$, a Bernoulli experiment with probability of success λ will be conducted.

   a) If the experiment is a success, individual i at time t will report his or her expected 12-month forward inflation rate $\pi^{e}_{t+12|t}$ using the 12-month forecast from the VAR model estimated with information up to time $t-1$:

6

$$\pi^{e}_{t+12|t} := \hat{\pi}_{t+11}.$$

   b) If the experiment is a failure, $\pi^{e}_{t+12|t}$ is set to the previous known expected value for individual i.

5) The previous steps give for each period t a series $\left\{\pi^{e}_{i,t+12|t}\right\}$ for $i=1, \ldots, n$. For each series the mean and the interquartile range (IQR) are recorded.

6) The value of λ is selected to minimize the difference between[14] the simulated mean expectation and the observed mean expectation from the survey.

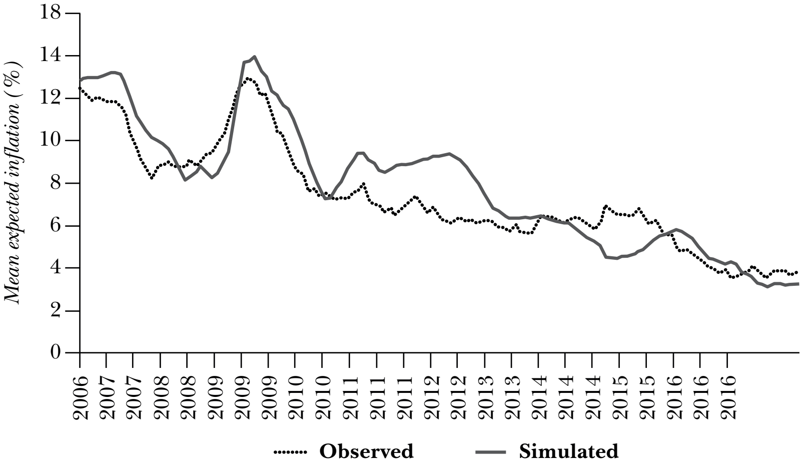
Running the previous algorithm gives the results presented in Figure 6: panel A, shows the generated average expectation, the observed average from the survey, and the realized inflation level at the survey date. We found the value of λ to be 0.17, meaning that only 17% of the agents in the simulated sample adjust their expectation with the most recent information, suggesting that an agent updates his or her information set every 5.9 months on average. The simulated mean expectations fit relatively well with the observed mean expectation from the survey, especially at the beginning and the end of the sample. The correlation between these two series is 91.15%. In the three months of 2017 included in the survey the observed mean expectations were 3.60% for January, 3.78% for February and 3.86% for March; while the simulated mean values are 3.23%, 3.25% and 3.23% respectively, illustrating the simulation's ability to replicate the real survey.

[14] We compute the mean of square differences between the simulated and observed series.

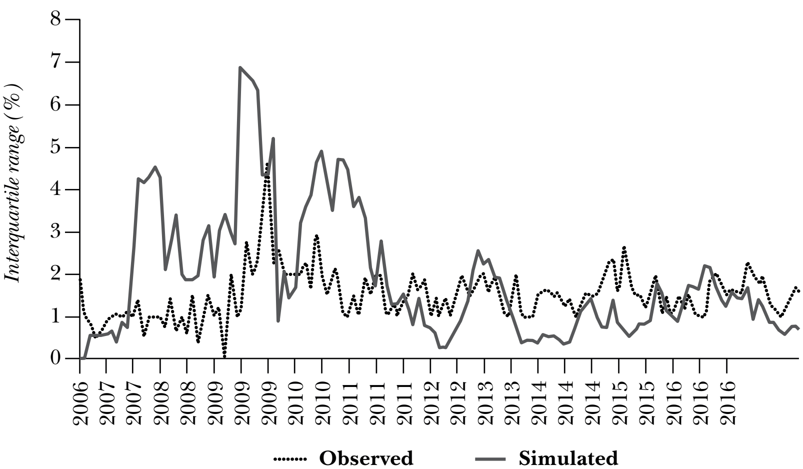
Figure 6

STICKY INFORMATION MODEL SIMULATION

A. EXPECTED INFLATION



B. REALIZED AVERAGE BIAS



Source: Own elaboration.

On the other hand, the simulated series for the interquartile range has a correlation of only 22.55% with the series from the survey. From panel B of Figure 6 we observe that simulated IQRs are close to the real IQRs only in the second half of the survey. This is due to a departure from the original algorithm in Mankiw et al. (2003) where λ is selected to maximize the correlation between the simulated series of IQRs and the survey series. Since we are interested in the mean expectation, our simulation was modified to put more emphasis on replicating the mean expectation.

The evidence of this simulated model also suggests that the sticky information assumption may not be appropriate. The value of the parameter λ required to match the dynamic of the mean forecast implies dynamics of disagreement that vary significantly from those found in the data.

4. CONCLUSIONS

This chapter builds on existing characterizations of Costa Rican inflation expectations by considering information rigidities in the expectation formation process. Our results are based on the Monthly Survey of Inflation and Exchange Rate Expectations. We analyze its panel structure to identify individual respondents and their groups of professional expertise (*consulting*, *stock market*, *academic*, and *business*). We found a set of stylized facts that describe the survey: *1)* responses are dominated by business sector respondents, implying that the mean expectations from the survey primarily reflect the mean expectation of the business sector; *2)* since April 2015 the mean expected inflation rate is within the inflation target range of the BCCR, (currently 2%-4%), suggesting that inflation expectations have been anchored by the BCCR's credibility and monetary policy; *3)* different groups have differing expectations and feature a positive interquartile range over time; *4)* there is no clear relation between the dispersion of inflation expectations and the inflation level, neither at the survey nor group level; *5)* on average agents, from the survey have positive forecast errors or realized bias, meaning that agents tend to expect greater inflation than in reality.

Because of these stylized facts, and the existing literature on Costa Rican inflation expectations, we proposed to test for information rigidities on the expectation formation process. We found some evidence suggesting that agents in the survey are subject to information stickiness and that only a fraction of agents form their expectations with the most recent information available. At the group level, we found that information rigidities are most prominent in the consulting and stock market analyst groups and less prominent in the academic and the business groups. However, the magnitude of the rigidity is not large enough to support the claim that the sticky information model is well suited to account for what we observe in the data.

Additionally, a simulated inflation expectations survey was generated using a sticky information algorithm and a vector autoregressive model to pin down the rationality of agents. This survey captured information on the inflation level, interest rates, output gap, inflation levels of trade partners, oil prices and annual exchange rate variations. The simulated survey replicated the mean expected inflation from the survey fairly well. Nevertheless, the level of stickiness required to match the data is low, and implies dynamics of disagreement that vary significantly from those found in the data.

Our findings show nonconformity of the sticky information approach for survey data along several dimensions, such as the Costa Rican data. We show that there is no correlation found between the level of inflation and the amount of disagreement among agents, the information rigidities for forecasts of exchange rates are much lower than what is needed to account for forecasts of inflation and finally, the value of needed to match dynamics of mean forecasts of inflation does not yield predictions for dynamics of disagreement that conform to those of the data.

Further work to deepen our knowledge about the expectation formation process of Costa Rican agents may consider the literature on the effects of learning on expectation formation. Moreover, we could redefine some questions in the survey to assess the probability beliefs of the respondents instead of point expectations. This would elicit information about the uncertainty agents' face when forming their expectations.

ANNEX

Annex A. Monthly Inflation and Exchange Rates Expectations Survey

Banco Central de Costa Rica, Economic Division
Monthly Survey on Inflation and Exchange Rate Expectations
July 2017
We appreciate your responses between July 10 and July 24

Respondent code:____________

1. What is your expected inflation rate, measure by the consumer price index, for the period between July 1, 2017 and June 30, 2018 (12 months)?
Answer:__________________(%)

2. Mention, in order of importance, the variables you take into consideration to form your expected inflation for the 12-month period:
*i)*____
*ii)*____
*iii)*____
*vi)*____
*v)*____

3. The reference bid rate calculated by the Banco Central de Costa Rica for June 30, 2017 was of 567.09 colones for US dollar. What is you your expected level for the reference bid exchange rate on the following dates?
3.1 On September 30, 2017 (3 months):____________________
3.2 On December 31, 2017 (6 months):____________________
3.3 On June 30, 2018 (12 months):________________________
3.4 On June 30, 2019 (24 months):________________________
3.5 On June 30, 2020 (36 months):________________________

4. Please detail the elements considered to form your exchange rate expectations in the short and long run:
Short run (3, 6 and 12 months):

i) ____

ii) ____

iii) ____

Long run (24 and 36 months)

i) ____

ii) ____

iii) ____

5. How do you consider that the general economic conditions for private production activities will evolve in the next six months in contrast with the past six months? (Please check one box)

Will improve	[ ]
Will be the same	[ ]
Will deteriorate	[ ]

Explain why: ____

6. How do you label the current conditions for firms to invest in the country? (Please check one box)

Good conditions	[ ]
Bad conditions	[ ]
Not sure	[ ]

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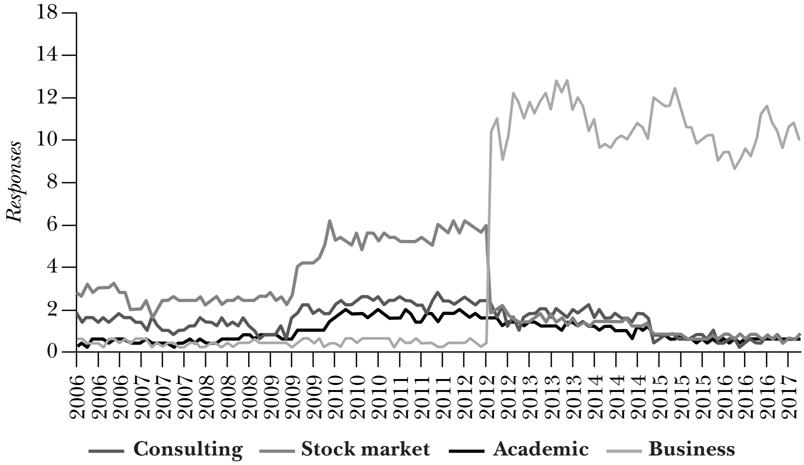
Annex B. Expected Inflation, Responses and Dispersion by Group

Figure 7

INFLATION EXPECTATIONS SURVEY, RESPONSES BY GROUP



Source: Own elaboration.

Figure 8

DISPERSION OF EXPECTED INFLATION BY GROUP

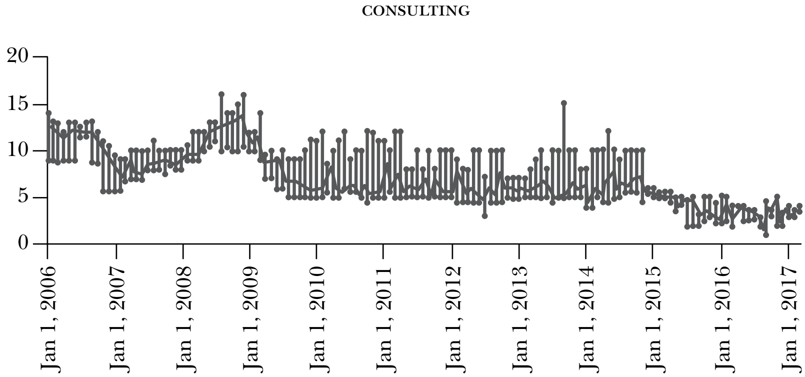
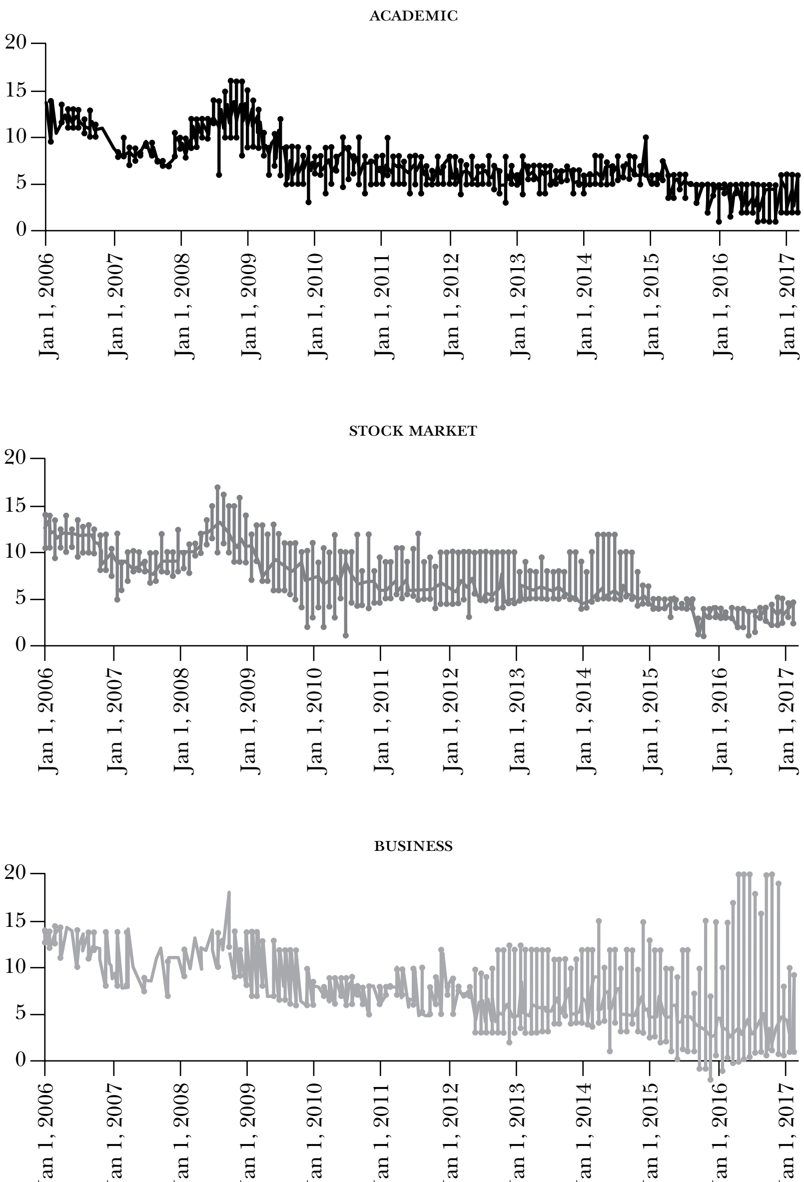


Figure 8 (cont.)

DISPERSION OF EXPECTED INFLATION BY GROUP



Source: Own elaboration.

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Formation and Evolution of Inflation Expectations in Paraguay

Pablo Agustín Alonso Méndez

Abstract

The establishment of the inflation targeting regime in Paraguay is relatively recent, however, the results have been satisfactory. This is because based on the observed data, from the implementation of this framework, it has been possible not only to reduce inflation levels, but also align inflation expectations along the medium-term inflation target. This chapter seeks to identify the main determinants of the formation of inflation expectations in Paraguay since the adoption of the inflation targeting regime. This work bases the analysis on the results obtained from the expectations surveys conducted by the country's monetary authority. The evolution of inflation should be an important factor to consider. Moreover, for the correct functioning of the expectations channel, it is essential that the monetary authority has sufficient credibility. A credibility index has been constructed to capture the effect of the credibility that the Banco Central del Paraguay has acquired during the inflation targeting regime. To guarantee the robustness of our results, the model we use has been estimated by three econometric methods: ordinary least squares (OLS), fully modified OLS (FMOLS),

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and the generalized method of moments (GMM). According to the outcomes of all these methods, inflation expectation formation in Paraguay is determined mainly by the inflation expectation of the previous month. In addition, the annual inflation information of the previous month is significant at the time of forming expectations.
Furthermore, the credibility index presents an expected negative sign, as inflation expectations have effectively aligned around the medium-term inflation target since the implementation of the inflation targeting regime. The exchange rate was not significant in the regressions. This could partly be due to a relatively low pass-through of the exchange rate to total inflation, especially in the last few years.

Keywords: inflation targeting, inflation expectations, monetary policy.
JEL classification: E31, E52, E58.

1. INTRODUCTION

The Banco Central del Paraguay (central bank of Paraguay, BCP) officially adopted the inflation targeting regime to regulate its monetary policy in May 2011. Prior to this policy framework, Paraguay exhibited marked levels of volatility even though there were no historical records of high inflation periods. Under the inflation targeting regime, volatility and inflationary levels have been reduced. These inflationary levels fostered uncertainty in economic agents when forming their inflation expectations. All this was reflected in the fact that these expectations showed considerable variability, in accordance with the results obtained in the expectations surveys of economic variables carried out by the central bank on a monthly basis.

The main purpose of this chapter is to try to identify some of the determinants that Paraguay's economic agents consider when forming their inflationary expectations. In view of the results of the survey, a series of factors that may influence the expectations formation of those who answered the survey have been considered. To do this, simple econometric regressions are carried out, and the results of these can be considered a first attempt to find the determinants of inflation expectations in Paraguay. In addition, the regressions highlight the importance of the establishment of the inflation targeting framework, not only in reducing inflation levels and their volatility, but also lowering inflation expectations. Furthermore, it can be affirmed that the BCP has managed to gain significant credibility with respect to the handling of the monetary policy in its attempt to maintain a low and stable inflation. This is reflected in the credibility index, which shows the alignment of expectations around the inflation target since the establishment of the inflation targeting regime.

Inflation expectations play a critical role in the process of price formation in the market. In addition, the decisions of households and firms depend heavily on the real return that could be expected on the savings and investments they make. Therefore, central banks closely monitor the development of inflation expectations in order to implement their monetary policy in a successful manner.

The results of the empirical model of this chapter show that the establishment of the inflation targeting scheme has helped to anchor expectations around the target, and that the dispersion of these expectations has been adjusted within the inflation range. Furthermore, this dispersion has been reduced with the decrease of the range during the consolidation process of the inflation targeting regime.

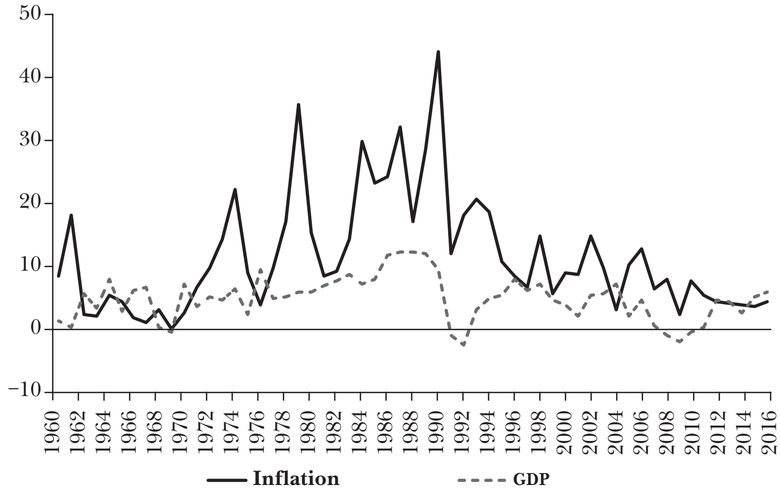
The first part of this chapter contains a brief narrative of monetary policy in Paraguay, highlighting their main characteristics, and delineates the most important results obtained from it, especially since the implementation of the inflation targeting framework. Next, the importance of inflation expectations in monetary policy, in general and specifically in Paraguay, is highlighted. Subsequently, after a description of the characteristics of the data according to the results of the economic variables survey, an estimation model of inflation expectations determinants in Paraguay is shown. The main outcomes of the model show the robustness of the results through different methodologies of estimating. Finally, in the last section some conclusions and final comments are presented.

2. MONETARY POLICY IN PARAGUAY

Throughout its history, the Paraguayan economy has not displayed significant macroeconomic imbalances, such as severe fiscal deficits or hyperinflationary episodes. The average growth of the gross domestic product (GDP) has been placed at relatively acceptable levels, although it has presented periods of high volatility. In regard to prices, inflation in Paraguay has been characterized by moderate levels,

Figure 1

INFLATION AND REAL GDP GROWTH
Percentage



Source: Banco Central del Paraguay

unlike most countries of the region (Figure 1). Likewise, the main problem regarding inflation has been its volatility. The macroeconomic performance of Paraguay can be attributed in part to the sound management of monetary policy. This is reflected partly in the fact that the *guarani*, the local currency of Paraguay, has not been modified since its inception, thus making it one of the oldest currencies in the region. The relatively prudent management of fiscal policy has contributed, to certain extent, to keeping inflation at a low level.

As pointed out in the document *Política monetaria en Paraguay: Metas de inflación, un nuevo esquema* (BCP, 2013), the design of monetary policy in Paraguay has considered the existence of a relation between the growth of money supply and inflation. Historically, this design has adopted a monetary policy scheme of intermediate objectives, in this case, setting targets for the growth of a specific monetary aggregate. Thus, the Central Bank used its instruments to control the money supply's growth to a level compatible with the inflation objective, which was based on the achievement of *low* *inflation*, using the quantitative theory of money as a conceptual framework reference.

Regarding economic activity, in general, the average growth of the Paraguayan economy has been acceptable, even though it has been characterized by its volatility. While the expansion of the economy was quite significant in the 1970s, mainly due to the construction of the Itaipu hydroelectric dam, there was a period of slowdown in the 1980s and 1990s. In this weakened situation and as a consequence of a weak financial system, and the fragility of the regulatory and supervisory frameworks, between 1995 and 1998, there were episodes of large financial crises. In this period, economic authorities needed a comprehensive reorganization of monetary and financial policy, which was attained through the enactment of important laws that allowed a much more stringent regulatory framework for financial institutions.[1]

In 2002, the Argentine economy fell into a deep crisis, causing the abandonment of the convertibility regime to which that country's exchange rate policy was subordinated. This episode also affected the Paraguayan economy. Despite the BCP's effort to curb capital outflows and exchange rate depreciation through sharp increases in the interest rates of monetary regulation instruments, the second financial crisis occurred towards the end of 2002, although of smaller magnitude than the first one.

Despite these episodes of crisis, the enactment of the aforementioned regulatory laws for the financial system allowed the BCP to focus more on the achievement and maintenance of low and stable inflation, driving its monetary policy of intermediate objectives, under a monetary aggregates framework.

As of 2004, the BCP began to lay the foundations for the establishment of an inflation targeting framework, albeit in an *experimental* way. Thus, the central bank modernized its monetary policy operational instruments with the establishment of a medium-term inflation target with a tolerance range. Under this scheme, it was possible to reduce the average inflation rate in the period from 2000 to 2010 to a single digit level.[2]

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1 The Law No. 489 of the BCP and the Law No. 861 "General of Banks, Finance, and other Credit Institutions."

2 In that period average inflation was 8.1%, while in the 1990-2000 period it was 15.1 percent.

With a more consolidated and orderly monetary policy framework, the BCP formally adopted the inflation targeting regime in May 2011, establishing a target of five percent annually with a tolerance range of +/−2.5 percentage points (pp). After the establishment of the inflation targeting regime, lower levels of inflation and volatility were recorded. For this reason, monetary authorities decided to reduce the tolerance range to +/−2 pp at the beggining of 2014, and at the end of that year, they also announced the reduction of the inflation target to 4.5% annually, which would apply in 2015 and 2016. In order to achieve its objective of maintaining low and stable inflation, at the beginning of 2017, the Central Bank announced a new reduction of the medium-term target to a rate of four percent annually, maintaining the tolerance range of +/−2 pp.

From the establishment of the inflation targeting regime, in the 2011-2016 period, average inflation was recorded at 3.9%. With these results and with the efforts of the monetary authorities to not only maintain low levels of inflation, but also reach a significant degree of credibility, inflationary expectations were aligned to values around the inflation target with less variability over the years.

3. INFLUENCE OF EXPECTATIONS ON INFLATION

Economics is a social science that somehow attempts to explain human behavior, so the perceptions of economic agents on the future evolution of a wide range of economic indicators are important. Therefore, an interesting challenge for monetary authorities is to try to interpret these perceptions in order to implement coherent policies that help guide them towards clear and precise objectives. Thus, it is in the macroeconomic field and particularly the theory of monetary policy, where expectations have become a powerful analytical tool.

Under the inflation targeting framework, the transmission mechanism of inflationary expectations is crucial for the achievement of a medium-term inflation target. The effectiveness of the expectations channel depends on the credibility of the central bank. Therefore, establishing a systematic and transparent decision-making process in monetary policy is key in facilitating the process of price formation and private expectations.

The achievement of the objectives proposed by the central bank, its transparency and communication increase its credibility, which contributes to that the expectations remain anchored to the target in the policy horizon. When a central bank has built a credible and transparent reputation, a monetary policy decision aimed at controlling inflation keeps inflation expectations anchored to the target. Therefore, in the face of an expectation of controlled inflation, decisions to adjust prices and wages will be made in line with the inflation target announced by the central bank.

Taking into account that the objective of clear and transparent communication is to give signals about the implications of monetary policy decisions, in general terms, the expectations channel may have a more rapid impact on the achievement of the inflation target compared to others transmission mechanisms that act with a greater lag. This makes the expectations channel an important and timely channel for the effectiveness of monetary policy.

Since the implementation of the inflation targeting regime, the Banco Central del Paraguay has made a great effort to improve its credibility. As mentioned above, Paraguay's main problem has not been high levels of inflation, but rather high volatility. Since the formal establishment of the inflation targeting scheme by the BCP, not only have inflation levels been reduced, but, above all, their volatility has been reduced (Figure 2). Likewise, it has been verified in the expectations data that there has been a decrease both in their levels and their volatility given the decrease in observed inflation rates. This suggests that the BCP has managed to increase its credibility in recent years.

As mentioned above, an interesting fact that has been observed with the implementation of the inflation targeting regime is the reduction of inflation expectations (average or median) to levels closer to the target (Figure 3 and 4). Additionally, the dispersion has been reduced, mainly because of the reduction of the tolerance range in 2014.

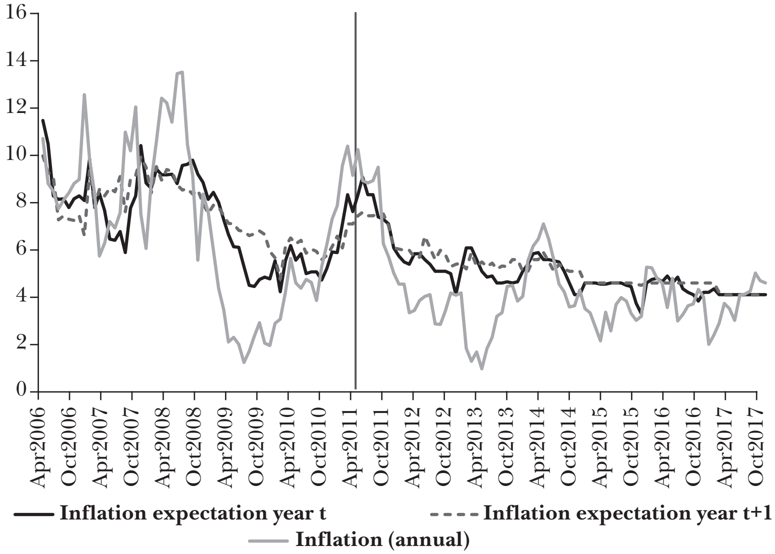
The reduction of the tolerance range can be proven through traditional statistics of variability, such as the standard deviation and the coefficient of variation (Figure 5), which effectively show a reduction (on average) in recent years, coinciding with the reduction of tolerance bands.

Finally, it was run, as an additional test, a simple model of the volatility statistics with respect to a dummy variable that takes the value of 1 if there is a reduction in the band. The variable is significant with an expected negative sign. In summation, these results suggest that

Figure 2

ANNUAL INFLATION AND INFLATION EXPECTATIONS FOR YEAR T AND T+1

Percentage

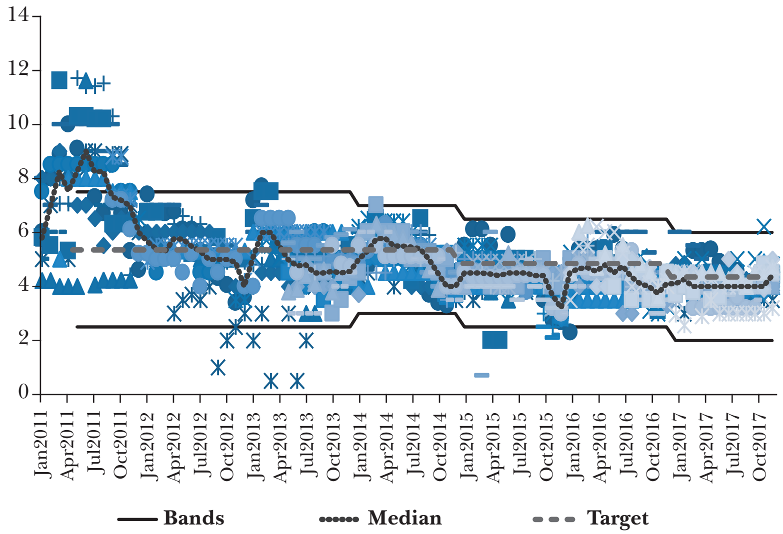


Source: Banco Central del Paraguay

Figure 3

DISPERSION OF INFLATION EXPECTATIONS FOR YEAR T[1]

Percentage

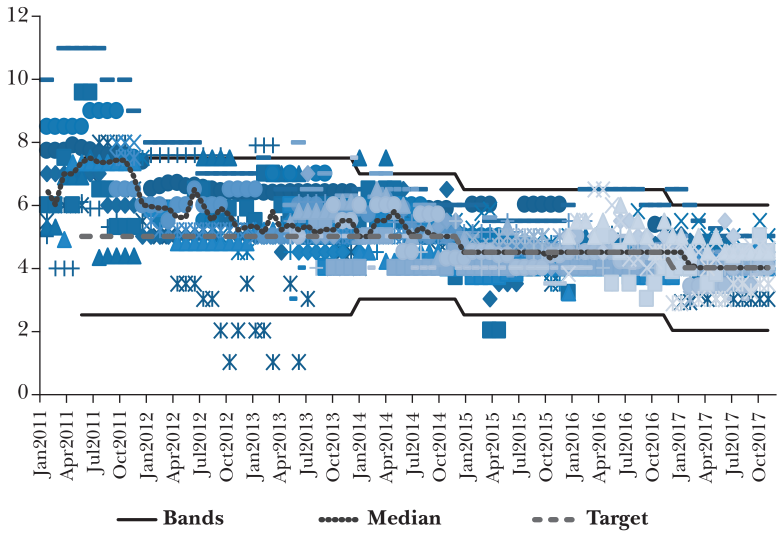


[1] The different dots correspond to the respondents in each period, which for ethical reasons cannot be identified individually.

Source: Banco Central del Paraguay

Figure 4

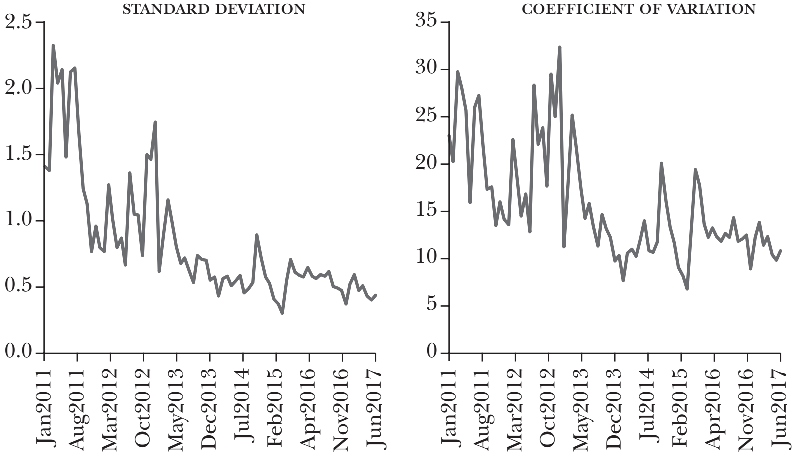
DISPERSION OF INFLATION EXPECTATIONS FOR YEAR T+1
Percentage



Source: Banco Central del Paraguay

Figure 5

STANDARD DEVIATION AND COEFFICIENT OF VARIATION
Percentage



Source: author’s calculations.

the reduction of the band contributed to decreasing the dispersion of the expectations of the economic agents.

4. EMPIRICAL MODEL FOR PARAGUAY

In the BCP, the expectations of the main macroeconomic variables are obtained with monthly frequency–as of April 2006, from the Economic Variables Survey (EVE). In its beginning, the EVE was mainly focused on representatives of some of the country's banks. Currently, this survey is aimed at agents representing different economic sectors that include banks and financial companies, risk rating agencies, brokerage firms, consulting firms, independent analysts, economic organizations, and universities. The number of respondents amounts to 34, of which, taking into account banks and financial companies, they comprise 22 representatives of financial institutions.

The EVE is divided into four blocks that include questions related to the expectations of economic agents with respect to total inflation, measured by the variation of the consumer price index, the evolution of the nominal exchange rate (guarani versus the United States dollar), GDP growth, and the trajectory of the monetary policy rate.

The set of questions corresponds to the expectations of the variables mentioned at different periods: for the end of the current month and the following, the current year, the next 12 months, the following year, and for the monetary policy horizon (which comprises between 18 and 24 months).

Considering that inflation expectations constitute an important tool for the BCP in the management of monetary policy under the inflation targeting scheme, this chapter aims to identify the main variables that affect the formation of inflation expectations.

4.1 Data Features

Taking into account the structure of the EVE surveys in relation to the expectations of the economic variables studied, the survey is designed to obtain information on the perspectives of the economic agents for the current year and for the following year. Thus, the survey data provide information for *fixed event forecasts*, which, to a certain extent, are limitations when estimating an econometric model.

In order to identify the main determinants of the process of forming expectations, it is necessary to have a series of *fixed horizon* inflation expectations. To carry out an approximation of fixed horizon forecasts from the fixed event forecasts of the EVE, we follow the work of Dovern et al. (2012), in which this approximation is made as a weighted average of *fixed-event forecasts* as follows:

1 $$F^{fh}_{y0,m,12}(x) = \frac{12-(m-1)}{12} F^{fe}_{y0,m,y0}(x) + \frac{m-1}{12} F^{fe}_{y0,m,y0+1}(x)$$

where $F^{fe}_{y0,m,y0}(x)$ is the fixed-event forecast of the variable x for the current year ($y0$) made in the month m of the year y0; $F^{fe}_{y0,m,y0+1}(x)$ is the fixed-event forecast of the variable x for the following year (y_0 + 1) made in the month m of the year y0; and $F^{fh}_{y0,m,12}(x)$ is the fixed horizon twelve-month-ahead forecast made in the month m of the year y0.[3]

For example, the inflation expectation made in October 2014 for the time period between October 2014 and October 2015 is approximated by the sum of $F^{fe}_{2014,10,2014}(\pi)$ and $F^{fe}_{2014,10,2015}(\pi)$, and weighted by 3/12 and 9/12, respectively.

In this section, we identify some variables that determine inflation expectations in Paraguay, according to empirical literature related to the subject, and as consider some characteristics of the Paraguayan economy.

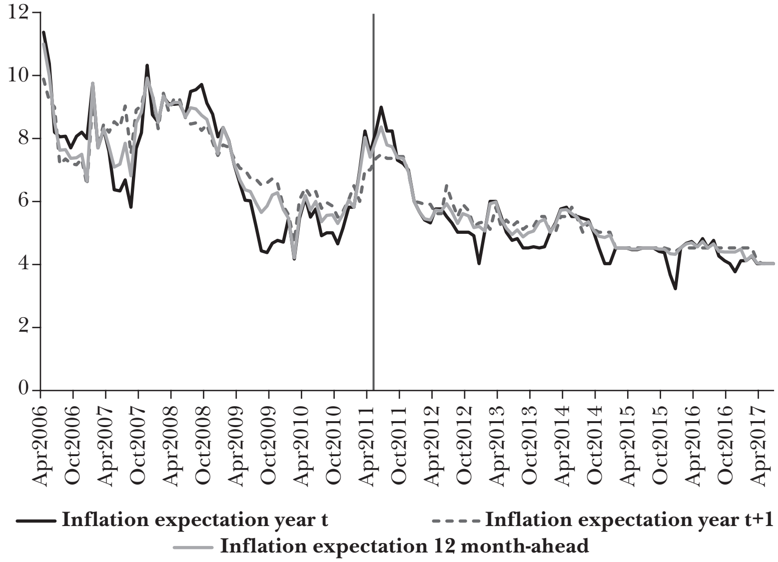
Taking into account that price formation has certain persistence in its adjustment process, for a certain period, the expectations of the recent past period should also be considered, since, in these expectations, agents are acquiring more information about events that may affect those expectations. In addition, the evolution of inflation should be an important factor to consider, since this evolution provides significant information when determining the future evolution of prices.

On the other hand, the establishment of the inflation targeting regime in Paraguay has been an important factor in the formation of inflation expectations, since it has led to a significant structural change in Paraguayan monetary policy, thus constituting an anchor that serves as a guide for the formation of these expectations (Figure 6). According to the observed inflation data, which were reduced both in levels and in variability, and the inflation targeting

Figure 6

INFLATION EXPECTATIONS FOR YEAR T AND T+1, AND 12 MONTHS FORWARD

Percentage



Source: Banco Central del Paraguay

framework, the monetary policy in Paraguay has achieved important credibility with economic agents. In part, this is reflected in the fact that when effective inflation data were adjusted around the target after the implementation of the inflation targeting scheme, expectations were also adjusted to the inflation target determined by the Banco Central del Paraguay.

For the correct functioning of the expectations channel, it is essential that the monetary authority has sufficient credibility. Economic agents must trust that the central bank will do everything necessary to achieve price stability and its inflationary objective in the medium term. Credibility would be able to neutralize, in part, the effects of economic shocks on prices that are transmitted through the channel of expectations.

In this sense, to try to capture the effect of the credibility that the Banco Central del Paraguay has acquired during the inflation targeting regime, a credibility index has been constructed following the work of Mendonça (2007), in which it is assumed that the central bank is able to guide inflation expectations towards the target and reaffirm its commitment to the inflation ranges. Thus, when expectations are equal to the inflation target the credibility index is equal to one, and decreases when expectations move away from the target. In cases where inflation expectations are located outside the inflation target bands, the index is equal to zero (see Annex).

Finally, it may be thought that a priori changes in the nominal exchange rate (guarani-dollar) should influence the formation of inflation expectations of economic agents on the cost side of imported goods (and inputs), especially when considering that Paraguay is a relatively open economy.[4] A similar analysis could be made when considering variations in oil price, since this product directly affects the price of fuels, an important input for any production process.

4.2 Estimation of the empirical model

To guarantee the robustness of our results, the model we use has been estimated by three econometric methods: ordinary least squares (OLS), fully modified OLS (FMOLS), and the generalized method of moments (GMM).[5] The FMOLS method assumes the existence of a cointegration relation between the variables, while the GMM method is created to avoid potential endogeneity problems with some regressors using OLS. The model has been estimated in monthly frequency. In accordance with the aforementioned information and taking into account some characteristics of the Paraguayan monetary policy, the estimated model is as follows:

2

$$\pi_t^e = \alpha_0 + \alpha_1 \pi_{t-1}^e + \alpha_2 \pi_{t-1} + \alpha_3 \Delta ner_{t-1} + \alpha_4 \Delta oil_{t-1} + \\ + \alpha_5 cred_{t-1} + \alpha_6 cred_{t-1} * \pi_{t-1}^e + a_7 dummy_t^{IT} + \varepsilon_t,$$

where π_t^e is the inflation expectation for twelve months ahead; π_{t-1} is the annual inflation of period $t-1$; Δner_{t-1} is the annual variation of the nominal exchange rate (guarani-dollar); Δoil_{t-1} is the annual variation in the price of oil; *cred* is a variable that measures the credibility of the central bank,[6] and $dummy_t^{IT}$ represents the period since the implementation of the inflation targeting regime.

According to our regressions' outcomes, inflation expectation formation in Paraguay (twelve-month-ahead) is determined mainly by the inflation expectation of the previous month (Table 1). In addition, the annual inflation information of the previous month is significant at the time of forming expectations.

On the other hand, the credibility index presents an expected negative sign, as inflation expectations have effectively aligned around the medium-term inflation target since the implementation of the inflation targeting scheme.

Changes in the exchange rate and the price of oil were not significant in the inflation expectation formation process. This could partly be due, to a relatively low pass-through of the exchange rate to total inflation, especially in the last few years.[7] Likewise, the oil price reduction in international markets has influenced the decrease of fuel prices in the local market.

Since the establishment of the inflation targeting scheme, both the level of inflation and its volatility have decreased. This behavior is also reflected in the results of the surveys, in which it is observed that inflation and its expectations present an important variability. The credibility achieved by the monetary authority has been essential in ensuring that expectations are adjusted to the inflationary objective of the medium term.

On the other hand, as of May 2011, the estimate of a dummy variable reflects the change in the monetary policy regime. In addition, it is proven that under the inflation targeting regime inflation expectations have been adjusted downward, as observed inflation data were aligned around the inflation target.

As previously indicated, since January 2014, the fluctuation bands have been reduced from +/− 2.5 pp to +/− 2 pp with respect to the inflation target. To test if the lower band has had a greater effect on inflation expectations, in the base equation, a dummy variable equal to 1 has been introduced since the period in which the decrease

[6] This index was constructed according to the work of Mendonça (2007), whose criterion is described in the Annex.

[7] See Banco Central del Paraguay (2015, recuadro 1).

Table 1

ESTIMATED EQUATIONS FOR INFLATION EXPECTATIONS

Dependent variable: inflation expectations (12 month-ahead)

	Models		
	OLS	*FMOLS*	*GMM*
Sample	2006M05-2017M12	2006M05-2017M12	2006M05-2017M12
Constant	2.41 (0.0000)	1.84 (0.0005)	2.57 (0.0071)
π^e_{t-1}	0.53 (0.0000)	0.62 (0.0000)	0.49 (0.0002)
π_{t-1}	0.16 (0.0000)	0.13 (0.0000)	0.17 (0.0000)
Δner_{t-1}	−0.0003 (0.9570)	−0.0006 −0.0008	−0.0003 −0.0015
Δoil_{t-1}	−0.0009 (0.5948)	−0.0008 (0.5962)	0.0015 (0.3086)
$cred_{t-1}$	−1.791 (0.0028)	−1.904 (0.0011)	−2.0967 (0.0061)
$cred_{t-1} * \pi^e_{t-1}$	0.27 (0.0047)	0.31 (0.0006)	0.32 (0.0022)
$dummy^{IT}$	−0.31 (0.0049)	−0.23 (0.0276)	−0.30 (0.0224)
Adjusted R^2	0.92	0.92	0.92

Note: *p*-values are in parenthesis.
Source: author's calculations.

Table 2

ESTIMATED EQUATIONS FOR INFLATION EXPECTATIONS FOR BANDS REDUCTION

Dependent variable: inflation expectations (12 month-ahead)

	Models		
	OLS	*FMOLS*	*GMM*
Sample	2006M05-2017M12	2006M05-2017M12	2006M05-2017M12
Constant	2.67	2.19	2.92
	(0.0000)	(0.0005)	(0.0000)
π^e_{t-1}	0.47	0.55	0.43
	(0.0000)	(0.0000)	(0.0000)
π_{t-1}	0.19	0.18	0.20
	(0.0000)	(0.0000)	(0.0000)
Δner_{t-1}	0.0055	0.0047	0.0020
	(0.3606)	(0.3744)	(0.7441)
Δoil_{t-1}	−0.0014	−0.0019	0.0029
	(0.3760)	(0.1929)	(0.0639)
$cred_{t-1}$	−0.502	−0.640	−0.7481
	(0.4371)	(0.2710)	(0.1869)
$cred_{t-1} * \pi^e_{t-1}$	0.03	0.07	0.07
	(0.7699)	(0.4350)	(0.4295)
$dummy^{IT}$	−0.20	−0.18	−0.20
	(0.0598)	(0.0564)	(0.0552)
$dummy^{bands}$	−0.57	−0.489	−0.570
	(0.0001)	(0.0001)	(0.0000)
Adjusted R^2	0.93	0.93	0.93

Note: *p*-values in parenthesis.
Sources: author's calculations.

in the range occurred. In this regard, interesting results are observed in all the estimation methodologies, as they show that the lower inflationary range has had an impact on getting inflation expectations adjusted to this new range (Table 2). This also shows that the BCP has had a significant influence on the credibility of economic agents in achieving the inflation goal under the inflation targeting regime.[8]

On the other hand, an exercise was carried out that reflects the behavior of the inflation expectations of the group of respondents categorized as financial entities (banks and financial companies). The results show that the expectations of the financial agents follow a similar pattern to the base equation (Table 3).

5. CONCLUSION

The implementation of an inflation targeting regime is relatively recent and because of this, economic agents have a learning curve with respect to the functioning of monetary policy transmission mechanisms and with respect to other macroeconomic variables that are relevant to explaining inflation. In the case of the Paraguayan economy, finding an econometric model that helps determine the main factors of inflation expectations is not a trivial task.

The establishment of the inflation targeting framework has led to an important structural change in the conduct of monetary policy in Paraguay. On top of helping reduce inflation levels and their volatility, this framework has also helped guide the inflation expectations of the economic agents through the nominal anchor of the medium-term inflation target.

Considering that the formation of prices is characterized by a change in persistence, it is reasonable to think that both the data of the observed inflation rate and that of their expectations in a previous period are important determinants at the time that economic agents define their expectations of inflation in the current period.

The observed trajectory of the inflation data shows that the implementation of the inflation targeting scheme has been satisfactory.

[8] The introduction of the band dummy variable diminishes the significance from the credibility index. This could be due to the fact that both variables reflect greater credibility in the inflation targeting scheme, so that the two variables cannot be together in the same base equation.

Table 3

ESTIMATED EQUATIONS FOR INFLATION EXPECTATIONS OF FINANCIAL ENTITIES

Dependent variable: inflation expectation (12 month-ahead)

	OLS
Sample	*2011M02-2017M12*
Constant	1.82
	(0.0045)
π^e_{t-1}	0.55
	(0.0000)
π_{t-1}	0.14
	(0.0001)
Δner_{t-1}	–0.0046
	(0.4850)
Δoil_{t-1}	0.0010
	(0.6300)
$cred_{t-1}$	–2.333
	(0.0019)
$cred_{t-1} * \pi^e_{t-1}$	0.46
	(0.0006)
Adjusted R^2	0.89

Note: *p*-values in parenthesis.
Sources: author's calculations.

This proves that the BCP has achieved significant credibility in its purpose of keeping inflation low and stable around the inflation target. Therefore, the alignment of inflation expectations around the target can be attributed to an increase in credibility.

It should be noted that the reduction in inflationary bands also reflects an adjustment of inflation expectations around the target, attesting likewise to greater credibility of economic agents in the management of monetary policy under the inflation targeting scheme. In addition, when the respondents are grouped in the category of financial entities, it is observed that the expectations of these agents follow a pattern similar to that observed in the base equation.

ANNEX

Annex A. Credibility Index

$$\left\{\begin{array}{ll}
1 & \text{if} \quad \pi_t^e - \pi^* \\
1 - \dfrac{1}{\pi^{lower} - \pi^*}\left[\pi_t^e - \pi^*\right] & \text{if} \quad \pi^{lower} < \pi_t^e \\
1 - \dfrac{1}{\pi^{upper} - \pi^*}\left[\pi_t^e - \pi^*\right] & \text{if} \quad \pi^{upper} > \pi_t^e \\
0 & \text{if} \quad \pi_t^e \geq \pi^{upper} \text{ or } \pi_t^e \leq \pi^{lower}
\end{array}\right\}.$$

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The Degree of Inflation Expectation Anchoring

Anchoring of Inflation Expectations in Latin America

Rocío Gondo
James Yetman

Abstract

We use inflation survey data from Consensus Economics to assess how firmly inflation expectations are anchored in Latin America. Following the methodology proposed by Mehrotra and Yetman (2018), we model inflation forecasts using a decay function, where forecasts monotonically diverge from an estimated anchor towards recent actual inflation as the forecast horizon shortens. Our results suggest that most countries do have an inflation anchor, with the estimated weight of the anchor increasing through time, indicating more strongly anchored expectations. This is consistent with the improving credibility of central banks' monetary policy management over our sample period (1993-2016). For countries with formal inflation targets, our results indicate that inflation targeting regimes are generally credible, with estimated anchors lying within the inflation target range for all countries in the most recent sample that we consider.

Keywords: inflation expectations, inflation anchoring, decay function.
JEL classification: E31, E58.

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1. INTRODUCTION

Monetary policy effectiveness, and especially the achievement of price stability, can be greatly assisted when inflation expectations are well anchored. In many models of inflation, for example, volatile inflation expectations directly increase the volatility of inflation outcomes. In Latin America, with a history of repeated episodes of high inflation, many countries have adopted inflation targeting (IT) as a framework to support a move to low and stable inflation and provide for better anchoring of inflation expectations. Some of these countries have adopted a schedule of decreasing targets over time with a view to gradually reducing inflation.

Challenges of inflation control for central banks in the region remain. In 2015-2016, some countries experienced inflation rates above the top of their target ranges, mainly commodity exporters who experienced large currency depreciations. In the cases of Colombia and Peru inflation expectations appear to have become de-anchored to some extent, with high inflation persisting (see Figure 1). Monetary policy tightening actions were taken in response to these developments, with their central banks raising policy rates by 3.25% and 1%, respectively.

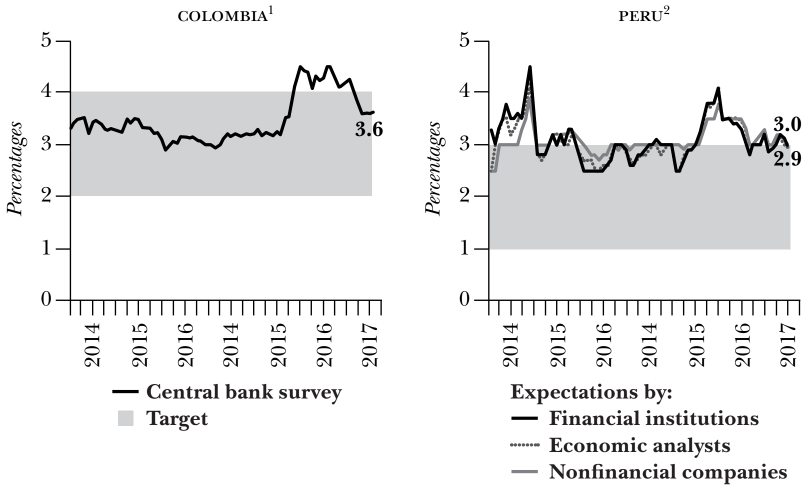
The goal of this paper is to assess whether or not countries have an inflation expectations anchor and, if they do, how strongly inflation expectations are anchored. For economies with formal IT frameworks, we also examine whether the anchor is consistent with the central bank's target. We define an inflation anchor as the expected level of inflation in the absence of any shocks to the economy. It should be noted that the inflation anchor is not necessarily equal to the inflation target for countries with an IT framework.

For each country, first, we evaluate whether there is an anchor for inflation expectations and, if so, how the anchor has evolved over time. Second, we analyze how well identified the inflation anchor is, using the standard deviation of the estimated anchor as an indicator of the degree of anchoring. Third, we compare the anchoring of inflation expectations between countries in the region that have inflation targets with such anchoring in those that do not.

We model inflation forecasts using a decay function, where forecasts monotonically diverge from the estimated anchor towards recent actual inflation as the forecast horizon shortens. We estimate

Figure 1

INFLATION EXPECTATIONS PUBLISHED BY CENTRAL BANKS



[1] Expectations for 12-month inflation. [2] Expectations of current year (December) 12-month inflation.
Sources: National data.

this relationship for each country over eight-year rolling samples using maximum likelihood, obtaining parameter estimates that define the decay function and the anchor.

Our results suggest that most countries do have an inflation anchor, although in some countries (including Argentina and Venezuela), the degree of anchoring declined in recent periods. For most countries, we observe a pattern of increasing anchoring of inflation expectations, consistent with the improved credibility of central banks' monetary policy management. This result stands in contrast with the results of Davis and Mack (2013), who found a low degree of anchoring of inflation expectations for Latin America compared with other regions, using a Phillips curve regression on core inflation.

In IT countries, inflation expectations appear to be well anchored. In addition, we find that the estimated anchors are generally consistent with their inflation targets; in the most recent sample that we examine, our estimated inflation anchors lie within the inflation target range for all countries with formal inflation targets. This result is consistent with the results in De Carvalho et al. (2006), where they find that the inflation anchor does not differ statistically from the inflation target for Brazil, Chile, and Mexico. For countries that adopted IT after 2009, the estimated anchor is slightly higher than the target, but this might be due to the rolling sample containing some years before the adoption of the regime.

We then consider some second-stage regressions based on these estimates, focusing on the estimated weight on the anchor at a two-year horizon, to explore what is driving our results. We show that IT and low levels of inflation persistence help explain strongly anchored inflation expectations.

Moreover, we find that inflation-targeting countries generally have more precisely estimated inflation expectations anchors. Capistran and Ramos-Francia (2010) report similar results: Countries with IT show a lower dispersion of long-run inflation expectations, especially in the case of emerging market countries. Similarly, for a sample of 15 advanced countries, Cecchetti and Hakkio (2009) find that the adoption of IT reduces the dispersion of inflation expectations.

In addition to the papers already cited, our work is related to models of inflation expectations extracted from financial data. For instance, Gurkaynak et al. (2007) find that ITers such as Canada and Chile have better anchored long-run inflation expectations than the United States (US), using break-even inflation rates from nominal and inflation-indexed bonds. For Latin America, De Pooter et al. (2014), using both survey-based and financial market-based data, find that inflation expectations have become better anchored over the past decade in Brazil, Chile, and Mexico. Focusing on Colombia, Espinosa-Torres et al. (2017) find that inflation expectations, obtained through break-even inflation measures, have remained anchored to values inside the inflation target range in the period following the Great Financial Crisis. Finally, for Brazil, Vicente and Guillen (2013) find that break-even inflation is an unbiased predictor of future inflation at short horizons, but is actually negatively correlated with inflation outcomes at 24- and 40-month horizons.

The paper is organized as follows. Section 2 provides a short description of the estimation methodology. Section 3 describes the data. Section 4 discusses the results. Section 5 then concludes.

2. METHODOLOGY

Following the methodology proposed by Mehrotra and Yetman (2018), we model inflation forecasts using a decay function, where forecasts diverge monotonically from an estimated anchor towards recent actual inflation as the forecast horizon shortens. This framework makes full use of the multiple-horizon dimension of the data to provide a measure of the level of the inflation anchor.

The functional form used to model inflation expectations is based on the cumulative density function of the Weibull distribution. This functional form assumes that, as the forecast horizon shortens, inflation expectations become increasingly sensitive to newly arriving information about inflation outcomes.

Given the observed behavior of inflation forecasts from the mean and median data from Latin American Consensus Forecasts, we model the expectations process for each country as follows:[1]

$$f(t,t-h)=\alpha(h)\pi^{*}+(1-\alpha(h))\pi(t-h)+\varepsilon(t,t-h)\,, \tag{1}$$

where $f(t,t-h)$ is the forecast of inflation for year t at horizon h; h is the number of months before the end of year being forecasted; $\alpha(h)$ is the weight on the anchor (which follows a decay function); π^{*} is the inflation anchor; $\pi(t-h)$ is the observed inflation at the time that the forecast is made; and $\varepsilon(t,t-h)$ is a residual term.

We assume that the decay function $\alpha(h)$ follows a Weibull cumulative density function:[2]

1 We parametrize the model to separately identify the anchor and the coefficients indicating the weight on the anchor. If there is a link between the two (for example, adopting an inflation target leads to a change in both the anchor and how strongly inflation is anchored), our estimation allows for this possibility but does not impose it. As such, it may be possible to improve the efficiency of the estimation approach taken here.

2 Our results are conditional on the decay function. Mehrotra and Yetman (2018) demonstrate that, provided inflation follows an autoregressive process, a monotonically decreasing decay function should fit inflation expectations.

2 $$\alpha(h) = 1 - \exp\left(-\left(\frac{h}{b}\right)^{c}\right).$$

The two parameters to estimate from the decay function are b and c. Higher values of b result in a smaller weight on the inflation anchor at short horizons, whereas higher values of c provide more curvature, and a more rapid decline the weight on the inflation anchor as the horizon shortens.

The variance of the residual is $\varepsilon(t,t-h)$ modeled as a function of the forecast horizon h:

3 $$V\left(\varepsilon(t,t-h)\right) = \exp\left(\delta_0 + \delta_1 h + \delta_2 h^2\right).$$

The use of the exponential function here ensures that the fitted values of the variance are positive for any values of the parameters defining the variance (δ_0, δ_1 and δ_2). Note that, aside from this restriction, our modeling assumptions for the variance are very flexible: It can be increasing or decreasing in the forecast horizon, or even follow a u-shaped (or inverse u-shaped) pattern across horizons.

Forecasts made at different horizons for the inflation outcome in a given year t are likely to be correlated, and more strongly so the closer the two horizons are. Therefore, the correlation between the residual at two different horizons h and k is modeled as:

4 $$corr\left(\varepsilon(t,t-h),\varepsilon(t,t-k)\right) = \phi_0 + \phi_1 |h-k|.$$

We estimate the set of parameters $\{\pi^*, b, c, \delta_0, \delta_1, \delta_2, \phi_0, \phi_1\}$ by maximum likelihood, economy by economy, based on eight-year rolling samples. Given the high degree of non-linearity of the model, we use 100 different sets of starting values in each case to ensure convergence to a global maximum. We then choose the estimates with the highest log-likelihood function value for which the parameters of the decay function are identified.

3. DATA

We use data on mean or median inflation forecasts from Latin American Consensus Forecasts. Our preference is median forecasts, constructed based on the full panel of inflation forecasts available from Consensus Economics at a monthly frequency. Medians are less affected by outlier forecasts than means, and may, therefore, be less vulnerable to data errors, for example. However, for some countries, forecaster-level data only becomes available partway through our sample. For other countries, only average forecasts are available for the full sample. Where we cannot construct median forecasts, we use mean forecasts instead.

Our sample covers 18 countries in the region, as listed in Table 1. The economies in our sample account for more than 95% of GDP for Latin America and the Caribbean in 2015 at market exchange rates. This sample includes countries with and without IT regimes, those that achieved low and stable inflation rates, and others where inflation has stayed relatively high and volatile.

Table 1

LIST OF COUNTRIES AND SAMPLE

	Data available from	*Inflation target adopted*		*Data available from*	*Inflation target adopted*
Argentina	1993		Guatemala	2009	2005
Bolivia	1993		Honduras	2009	
Brazil	1990	1999	Mexico	1990	2001
Chile	1993	1999	Nicaragua	2009	
Colombia	1993	1999	Panama	1993	
Costa Rica	1993	2005[a]	Paraguay	1993	2011
Dominican Republic	1993	2012	Peru	1993	2002
Ecuador	1993		Uruguay	1993	2007
El Salvador	2009		Venezuela	1993	

[a] Transition to an explicit IT regime started in 2005 with the announcement of an annual inflation target.

Arguably, there may be better inflation forecast datasets that could be used to answer this question, at least for some of the economies in our sample. For example, Consensus Economics' inflation forecasts are typically based on the annual average inflation rate, whereas most inflation targets are defined in terms of year-on-year inflation. Hence, central bankers are likely to care more strongly about anchoring in terms of year-on-year inflation, rather than annual average inflation. Offsetting this, we expect that measures of anchoring are likely to be highly correlated across the two measures. Further, using Consensus data, we are able to focus on a larger cross-section of countries, covering a longer period for many economies than would be possible with forecasts from other sources. The forecast surveys are also constructed using consistent methodology (in terms of variable definition and the timing of the forecasts, for example), so the results are likely to be comparable across countries.

Table 1 shows the availability of data for each country, including the starting date and the year of adoption of an IT regime, where applicable. Note that data availability is limited to bi-monthly for some economies in the early part of the sample, with monthly forecasts only published beginning in 2002. In these cases, we ensure that the contribution of the missing observations to the likelihood function is set to zero.

Figure A.1 in the Annex shows the evolution of inflation forecasts for each country in the sample. For countries that have had IT regimes for an extended period (displayed in Figure A.1, Section A: Brazil, Chile, Colombia, Mexico, and Peru), longer-horizon forecasts are more strongly anchored than for other countries in the sample. In particular, two-year-ahead inflation forecasts are close to the inflation target and the dispersion between the inflation forecasts for different years is quite small. In this set of countries, inflation forecasts only start to deviate from the target around 12 months ahead of the date being forecast, when observed inflation outcomes become more informative about the path of inflation.

The second group of countries (displayed in Figure A.1, Section B: Costa Rica, the Dominican Republic, Guatemala, Paraguay, and Uruguay) adopted IT more recently. For longer-horizon forecasts, e.g., 24 months ahead, we observe a wide dispersion in inflation forecasts across time, but a declining trend in the initial forecast point after the adoption of IT.

The last subset of countries is those without an explicit inflation target throughout our sample (see Figure A.1, Section C). These countries tend to show the largest dispersion between inflation forecasts at both short and long horizons.

4. RESULTS

We estimate our non-linear model by maximum likelihood using eight-year rolling samples. For each sample, we consider a large set of different starting values to ensure convergence to the global maximum. We consider that an inflation anchor exists if the estimated weight on the anchor at 24 months is higher than 0.10. Below this threshold, the estimated anchor tends to be very volatile and highly dependent on starting values, which we interpret as indicating that there is no inflation anchor.

4.1 Decay Function

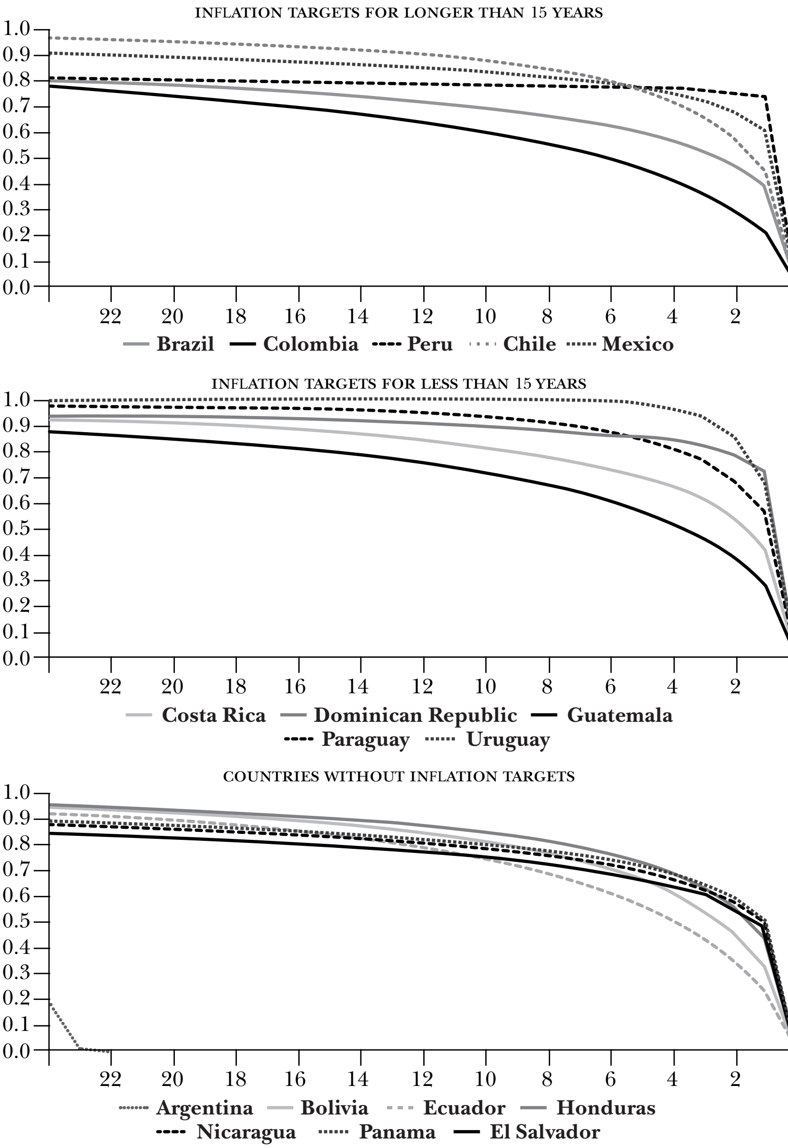
Figure 2 shows the estimated decay functions for all the countries in the sample, using the most recent rolling sample of 2009-2016. The figures show that the weight on the anchor is high–generally above 0.7–for all horizons longer than 12 months for all countries in our sample, with the exception of Argentina (which is barely visible in the bottom left corner of the right-hand panel). We generally observe a sharp decline in the weight assigned to the inflation anchor in horizons shorter than six months, when forecasters have more information about realized inflationary shocks that are likely to continue to influence inflation through to the inflation outcome being forecast. Qualitatively, there does not seem to be a large difference between countries with IT in our sample and other Latin American countries in terms of the estimated decay functions.

With respect to the evolution through time, Figure 3 shows the estimated weight on the anchor at a horizon of two years (i.e., $\alpha(24)$), the longest horizon for which we use the Consensus Forecast data.[3] We include all countries for which there are multiple rolling samples (i.e., forecasts are available before 2009). These results suggest

[3] Consensus Forecasts also publishes average forecasts at longer horizons, of up to ten years, for some economies in our sample, but these are only available twice per year.

Figure 2

DECAY FUNCTIONS 2009-2016

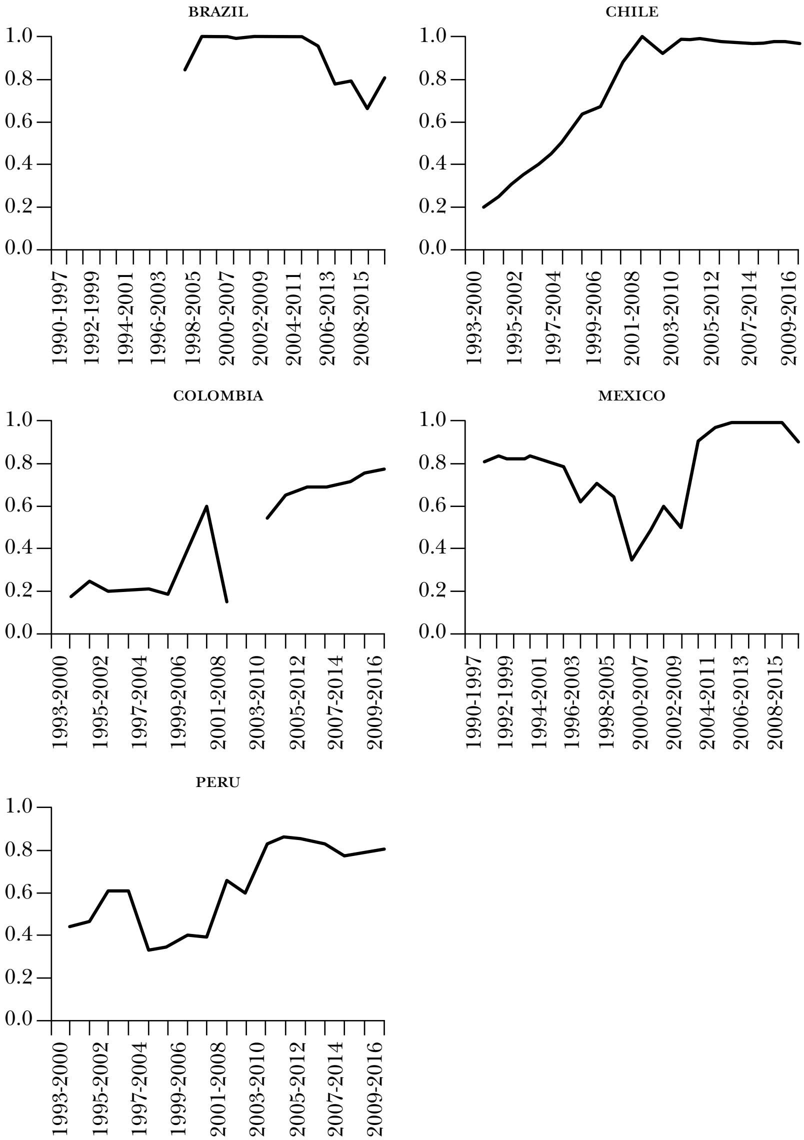


Notes: The horizontal axis represents the forecast horizon, defined as the number of months before the end of the calendar year being forecasted.
The graph does not include the decay function for Venezuela because the last available rolling sample is 2008–2015.
Sources: Authors' calculations.

Figure 3

ESTIMATED WEIGHT ON INFLATION ANCHOR (*h* = 24)

A. COUNTRIES WITH INFLATION TARGETS FOR LONGER THAN 15 YEARS

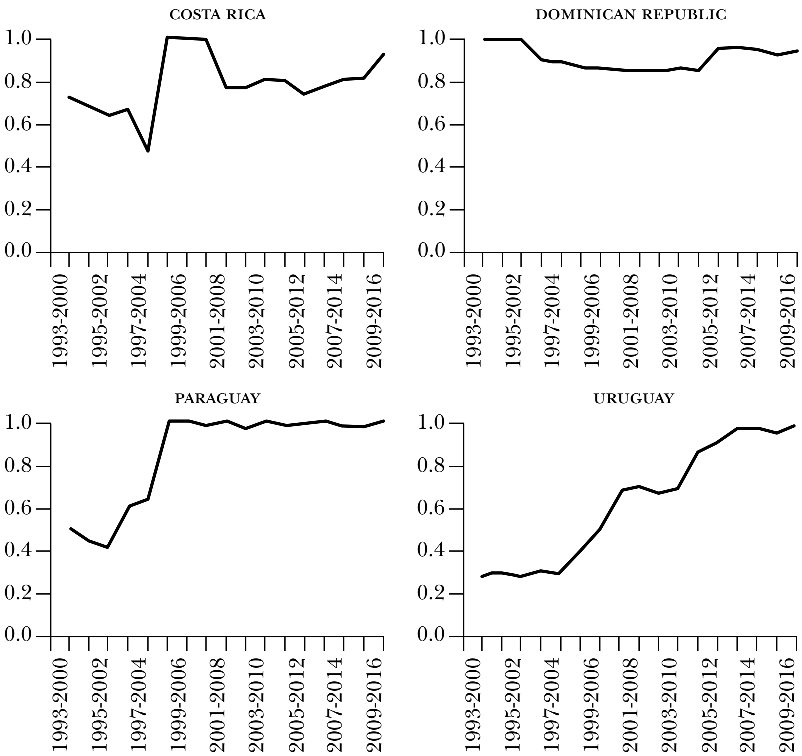


Notes: Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

Figure 3 (cont.)

ESTIMATED WEIGHT ON INFLATION ANCHOR (*h* = 24)

B. COUNTRIES WITH INFLATION TARGETS FOR LESS THAN 15 YEARS

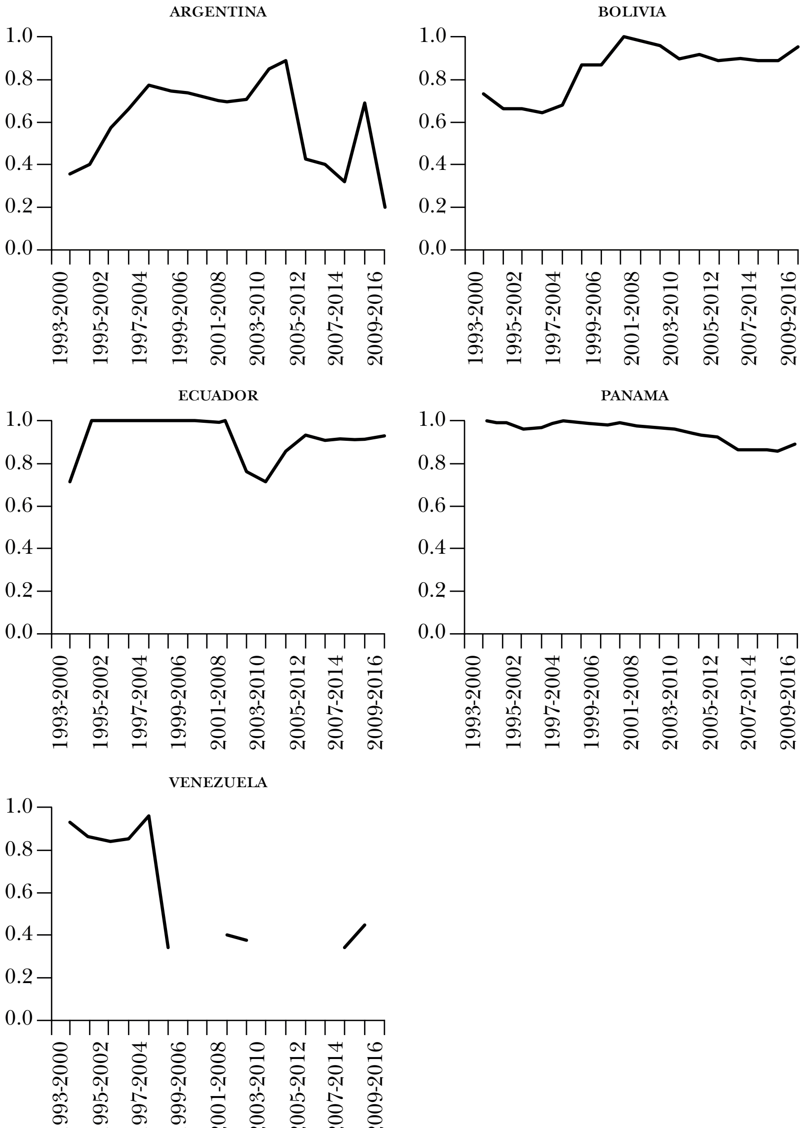


Notes: Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

Figure 3 (cont.)

ESTIMATED WEIGHT ON INFLATION ANCHOR (*h* = 24)

C. COUNTRIES WITHOUT INFLATION TARGETS



Notes: Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

that the degree of anchoring of long-run inflation expectations has generally increased over the sample, most notably for some of the economies with inflation targets (Chile, Colombia, Peru in Panel A, and Paraguay and Uruguay in Panel B).[4] In the most recent rolling sample, the weight on the anchor exceeds 0.7 for all economies except Argentina and Venezuela. Similar results are observed at other horizons too (see Figure A.2 in the Annex for anchoring at a 12-month horizon, for example).

Table 2 displays the key estimated parameters for the most recent rolling sample, 2009-2016. We report an estimated inflation anchor for all economies, including those for which this is poorly identified in the data. There is a wide variety of parameter estimates across countries. We note that Venezuela has a much higher estimated anchor than any of the other economies (at over 28%), and Argentina and Venezuela have much less precisely estimated anchors than the other countries in the sample, consistent with relatively weakly anchored inflation expectations for these countries.

Regarding the parameters that govern the shape of the decay function, most countries show a very low degree of curvature (i.e., low estimates of c), which means that the weight on the anchor remains high even as the forecast horizon shortens, as shown in Figure 2.

4.2 Estimated Inflation Anchors

Figure 4 shows the evolution of the estimated inflation expectations anchors, for the same set of countries displayed in Figure 3. Solid lines correspond to the point estimate of the anchor, while dashed lines represent the 95% confidence interval. Gray regions illustrate inflation target ranges where applicable.

Section A of the figure presents the results for countries that have had IT for more than 15 years. Since the adoption of IT, all these countries show a reduction in their anchor towards the inflation target.

[4] In our modeling of inflation expectations, we are implicitly assuming that changes in inflation persistence reflect changes in the anchoring of inflation expectations. To the extent that declining inflation persistence reflects changed price-setting mechanisms that results from greater anchoring of inflation expectations, this assumption is warranted (see Section 4.3). But there may be other, more mechanical sources of changes in inflation persistence–such as changes in the sectoral composition of the economy–that could bias our results.

Table 2

ESTIMATION RESULTS, 2009-2016

	b	c	π^*	$s.e.(\pi^*)$
Argentina	24.60	59.56	5.39	0.411
Bolivia	4.20	0.62	6.00	0.027
Brazil	6.37	0.38	4.88	0.028
Chile	2.58	0.55	2.98	0.004
Colombia	11.84	0.58	3.45	0.012
Costa Rica	3.39	0.49	6.09	0.043
Dominican Republic	0.35	0.25	5.84	0.044
Ecuador	6.25	0.72	4.17	0.015
El Salvador	3.47	0.33	3.06	0.014
Guatemala	6.62	0.59	7.83	0.032
Honduras	2.85	0.53	6.97	0.034
Mexico	1.29	0.29	3.54	0.006
Nicaragua	2.90	0.36	7.21	0.025
Panama	2.53	0.36	3.81	0.022
Paraguay	0.89	0.86	5.10	0.027
Peru	0.02	0.06	2.55	0.016
Uruguay	1.45	0.52	6.67	0.026
Venezuela[1]	29.64	2.39	28.35	0.328

[1] For Venezuela, results are for 2008-2015, since data are not available for 2016.

Moreover, for all countries except Brazil, estimates of the anchor are quite stable from one rolling sample to the next towards the latter end of the rolling samples.

The confidence bands (constructed from the standard deviation of the estimated anchor) indicate that the estimated anchors are generally tightly estimated.[5] Chile displays the most tightly estimated anchor across the rolling samples, whereas Colombia shows an increasing degree of tightness after the adoption of the inflation target, consistent with improving credibility.

Figure 4, Section B, shows the results for the more recent ITers. These countries, except for Uruguay, show a decreasing trend in their anchors. In the case of Costa Rica, this is consistent with their decreasing inflation target. In the case of Uruguay, the inflation target has remained at 5% since its adoption, but estimated inflation appears to be diverging from it towards the upper bound of the target range of 7%, at the same time as actual inflation has been close to 7%. This group of countries also shows a tightly estimated anchor for most countries and rolling samples; for Uruguay, the confidence band visibly narrows as time goes by.

For countries that are not ITers, displayed in Figure 4, Section C, there is generally more dispersion in both the estimated anchors and their trends. Ecuador has a stable estimated anchor of 4%, whereas Venezuela has many rolling samples without an identifiable anchor. The degree of tightness of the inflation anchor is, in general, lower for this group of countries too.

The degree of tightness of the inflation anchor exploits information from dispersion across the time series and horizons. We could also complement the estimation by further exploiting information on the standard deviation across forecasters for each country, although the availability of data would reduce the sample of countries. Thus, we leave this to future work.

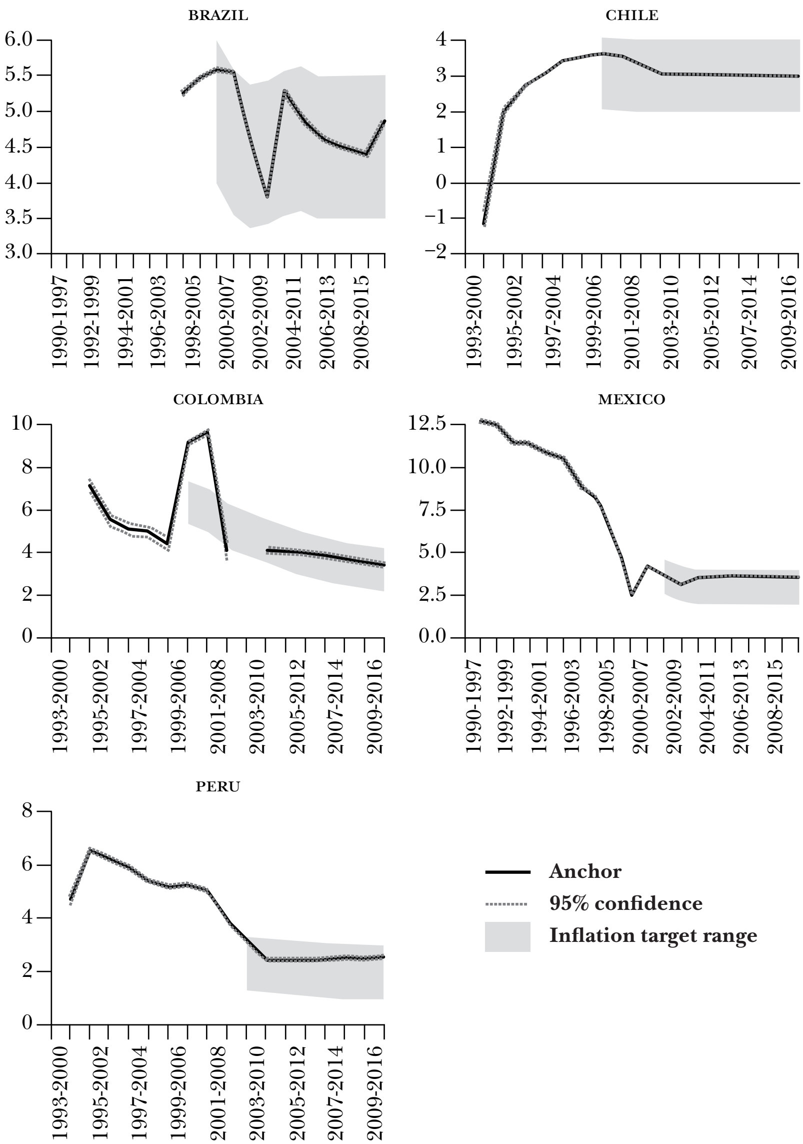
One caveat with the data used in the analysis is that inflation forecasts have a maximum horizon of two years, which might not be

5 The estimated confidence intervals for the inflation anchor depend on the functional form of the decay function. However, for a sample of advanced and emerging countries, Mehrotra and Yetman (2018) find that the Weibull-based decay function fits the data better than more restrictive forms, and more general forms do not increase explanatory power markedly.

Figure 4

EVOLUTION OF ESTIMATED INFLATION ANCHOR[1]

A. COUNTRIES WITH INFLATION TARGETS FOR MORE THAN 15 YEARS

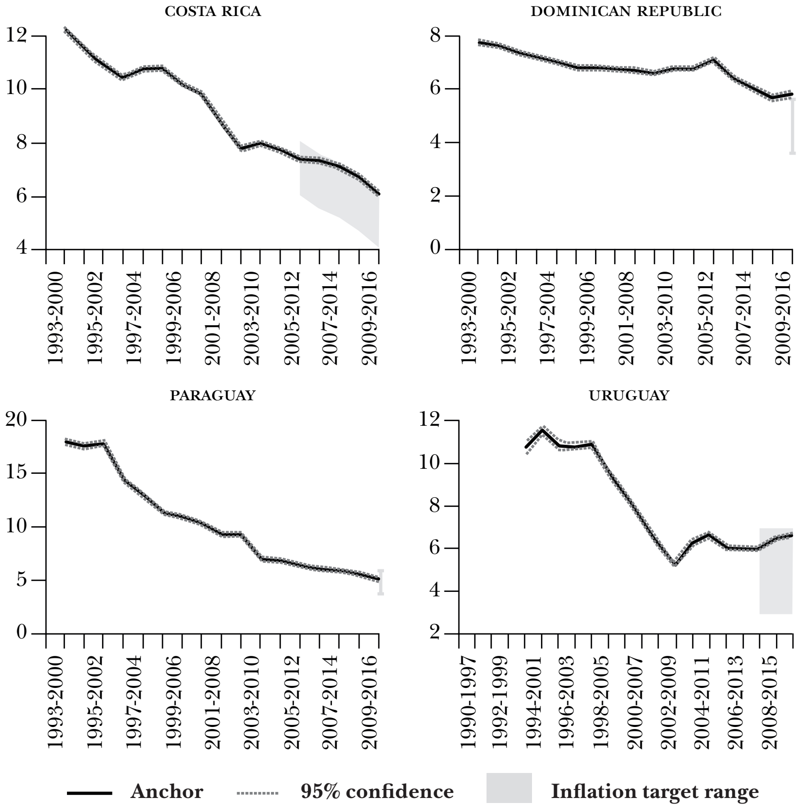


[1] Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

Figure 4 (cont.)

EVOLUTION OF ESTIMATED INFLATION ANCHOR[1]

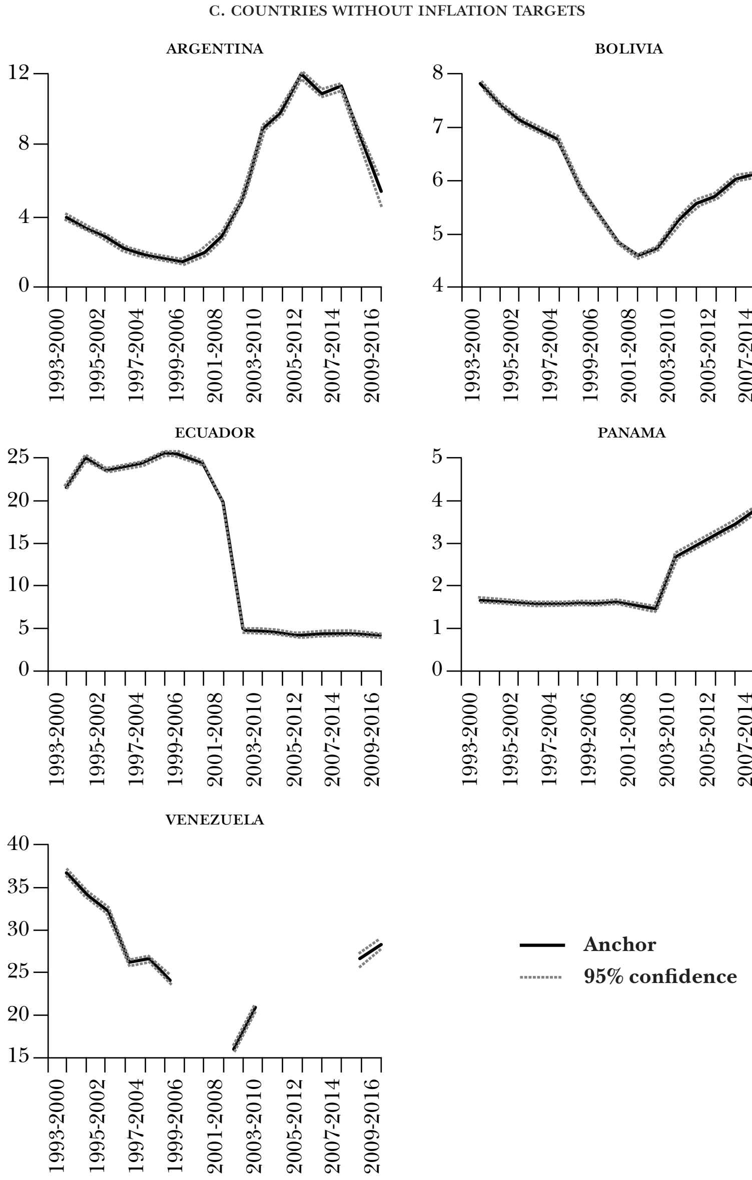
B. COUNTRIES WITH INFLATION TARGETS FOR LESS THAN 15 YEARS



[1] Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

Figure 4 (cont.)

EVOLUTION OF ESTIMATED INFLATION ANCHOR[1]



[1] Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

long enough to capture long-run inflation expectations.[6] We test this by plotting the longer-term Consensus inflation forecasts for six-to-ten years ahead, for the countries for which these are available, against the estimated anchors. Figure 5 shows that six-to-ten year ahead forecasts are highly correlated with the estimated anchor, with Venezuela being the main outlier, regardless of whether we take a particular sample period or the average. This is consistent with the results displayed in Mehrotra and Yetman (2018) for a larger sample of countries.

4.3 Effect of IT

Next, we focus on the sample of countries with IT and analyze whether or not the estimated anchor is consistent with the inflation target. By doing so, we are assessing whether our results are consistent with these countries building credibility for their IT monetary policy frameworks.[7] We focus on the average across all rolling samples where a country has an IT framework. Table 3 shows that the estimated

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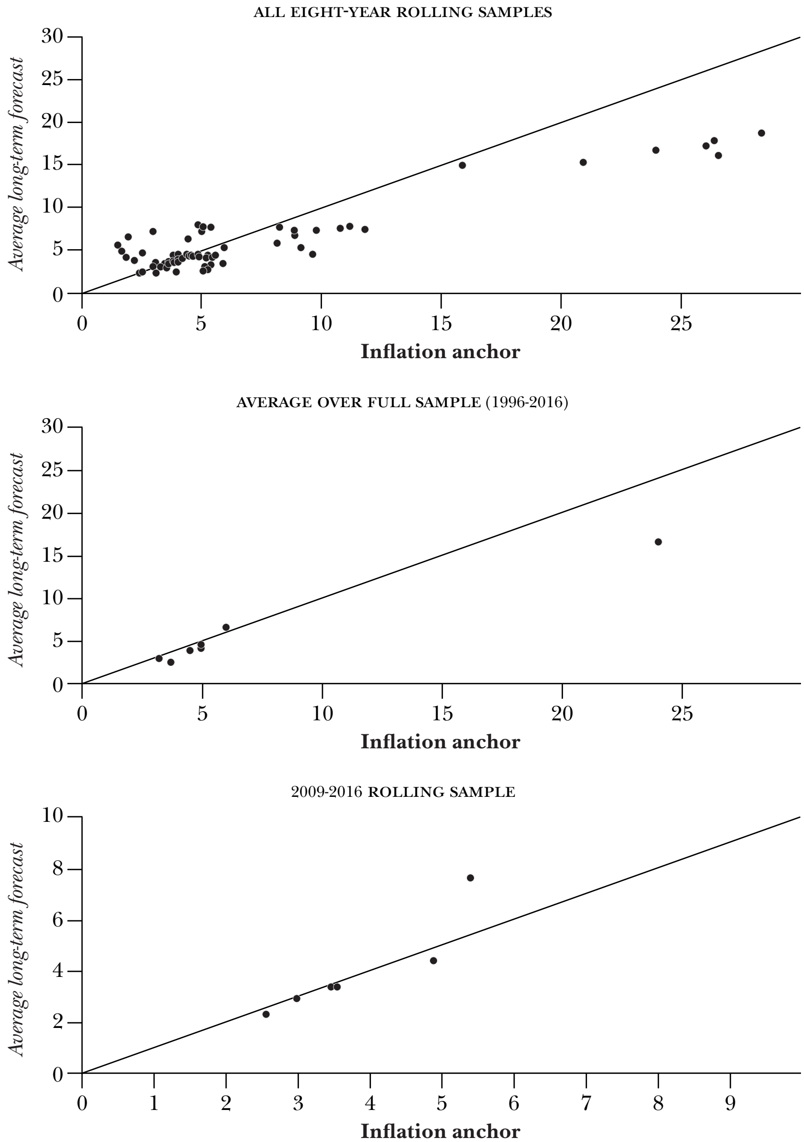
6 On the other hand, long-horizon forecasts (e.g., six-to-ten years ahead) might relate to outcomes too far into the future to be useful for monetary policy purposes. For monetary policy setting, the most relevant horizon is related to the frequency with which most prices and wages are adjusted, and hence has the greatest impact on inflation dynamics. Thus, one could imagine wage and price-setting decisions being influenced by inflation expectations that are anchored by a level of expected inflation that differs from expectations of long-run inflation (if, for example, forecasters anticipated that the monetary policy framework might be adjusted in a few years). In that case, six-to-ten year ahead inflation expectations might not be relevant for explaining inflation dynamics, but they could still be important for other economic decisions such as deciding to invest in fixed assets or determining long-term savings goals.

7 The anchor of inflation expectations could become more consistent with the inflation target, even if the central bank is not building credibility, e.g., if inflation moves towards the target for reasons unrelated to monetary policy or the inflation target is adjusted endogenously to track inflation. In the former case, these effects are likely to be transitory (so are mitigated against in part by our use of rolling samples). With respect to the latter case, we see limited evidence of inflation targets being adjusted strategically in response to deviations of inflation from target in the inflation targeters that we examine: Inflation targets are either constant over most of the 2009-2016 period (Brazil, Chile, Colombia, Guatemala, Mexico, Peru, and Uruguay) or follow a consistent declining path as inflation targets become more established over time (Costa Rica, the Dominican Republic, and Paraguay).

Figure 5

RELATION BETWEEN ESTIMATED INFLATION ANCHOR AND LONG-TERM FORECAST FROM CONSENSUS ECONOMICS



Note: Sample of countries with long-term forecasts from Consensus Economics includes Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela.
Sources: Consensus Economics©; authors' calculations.

Table 3

ESTIMATED ANCHOR AND INFLATION TARGET, 2009-2016

	Estimated anchor	*Inflation target*[1]		*Estimated anchor*	*Inflation target*[1]
Brazil	4.88	4.5	Guatemala	7.83	4.5
Chile	2.98	3.0	Mexico	3.54	3.0
Colombia	3.45	3.3	Paraguay[2]	5.10	4.8
Costa Rica	6.09	5.1	Peru	2.55	2.0
Dominican Republic[2]	5.84	4.6	Uruguay	6.67	5.0

[1] The inflation target is the simple average of the annual inflation target for each country in the given sample. [2] For countries that adopted IT later than 2009 such as the Dominican Republic and Paraguay, the sample starts in 2012 and 2011, respectively.

anchor is quite close to the average midpoint value of the inflation target in each country, and inside the range of +/– 1 percentage point for most countries. The gap between the two is wider in the case of the most recent ITers (such as Guatemala and the Dominican Republic) but, in those cases, the rolling sample includes years before the adoption of IT, so a wider deviation does not necessarily indicate a lack of central bank credibility.

We also estimate a modified version of our model only for countries with IT. Instead of estimating the anchor, we consider the midpoint value of the inflation target $\pi^T(t)$ and add a parameter d to capture deviations from the target.

$$f(t,t-h)=\alpha(h)(\pi^T(t)+d)+(1-\alpha(h))\pi(t-h)+\varepsilon(t,t-h).$$

A simple test with a null hypothesis of $d=0$ is then a test of whether the inflation target was credible or not. Note that, in cases where central banks have time-varying inflation targeting, we capture this with our $\pi^T(t)$, as we then use different values of the target for different years.

Table 4

ESTIMATION RESULTS WITH INFLATION TARGET, 2009-2016

	b	*c*	*d*	*s.e.(d)*
Brazil	6.37	0.38	0.38	0.028
Chile	2.58	0.56	−0.02	0.004
Colombia	11.27	0.74	0.35	0.014
Costa Rica	3.31	0.56	0.97	0.030
Dominican Republic	5.86	0.46	0.12	0.014
Guatemala	9.01	0.45	1.46	0.027
Mexico	1.29	0.29	0.54	0.005
Paraguay	1.34	1.17	0.10	0.017
Peru	0.32	0.12	0.55	0.016
Uruguay	1.45	0.52	1.67	0.025

Note: Uruguay has a target range of +/−2 percentage points; all other countries have a target range of +/−1 percentage point.

Table 4 shows the results of these estimations, for the most recent eight-year rolling sample. These confirm that the anchors of inflation expectations are in line with the inflation target range in all countries: within a +/−1 percentage point range in all cases except for Guatemala and Uruguay, the latter of which has an inflation target range of +/−2 percentage points. That is, we cannot reject the hypothesis that inflation expectations are anchored by the inflation targets for most countries.

In order to complement the comparison between countries with and without inflation targets, we further examine whether IT improves the anchoring of expectations. To do this, we perform a second step panel estimation. We regress the weight of the anchor $(\alpha(h))$ for each country for each eight-year rolling sample on a set of country characteristics. The set of regressors includes: *1)* a dummy variable that takes the value of 1 for countries with IT for the full rolling sample during the rolling sample; *2)* the number of years since the adoption of the IT regime; *3)* mean inflation; *4)* inflation variability, measured by the standard deviation of inflation; *5)* inflation persistence, based on an estimated AR(1) coefficient in a regression on annual inflation that includes a constant; and *6)* real GDP per capita.

The results, shown in Table 5, indicate that, aside from an intercept, only the coefficients for inflation persistence and the IT dummy are statistically significant. IT is associated with an increase in the degree of anchoring of inflation expectations by 0.25, whereas countries with less inflation persistence are associated with an increase in the degree of anchoring (the coefficient of −0.768 indicates that a decrease in inflation persistence from 0.9 to 0.8 corresponds to an increase in anchoring of 0.08). We obtain similar results when we repeat the regression with weights at shorter horizons, such as one year.[8] One way to interpret these results is that, even when we control for inflation persistence, which is negatively correlated with the IT dummy and anchoring, we still find that IT is associated with a significant increase in the anchoring of inflation expectations.

Table 6 displays second step estimation results where the dependent variable is the estimated standard error of the anchor. Here, the number of years since the adoption of IT and the persistence of inflation are marginally statistically significant, but the IT dummy is insignificant.

5. CONCLUSIONS

In this paper, we modeled inflation expectations from Consensus Forecasts to assess inflation expectations anchoring in Latin America. Our results suggest that most countries do have an inflation anchor, and that expectations have become more tightly anchored through time, consistent with the improving credibility of central banks' monetary policy management.

For countries with IT, we find that inflation targets are generally credible, in the sense that the estimated anchors lie within the inflation target range for all countries in the most recent sample that we estimate. Also, the adoption of IT is generally associated with an improvement in the degree of anchoring of expectations, both

8 At a forecast horizon of 12 months, being under an it regime is associated with an increase in the degree of anchoring of inflation expectations by 0.25, and a 0.1 drop in inflation persistence is associated with an increase in the degree of anchoring by 0.09.

Table 5

SECOND STEP ESTIMATION RESULTS
Dependent variable: inflation anchor weight ($h = 24$)

	Coefficient	*Standard error*
IT dummy	0.245[c]	0.0617
Years under IT	0.00682	0.01385
Inflation mean	4.39e-04	6.41e-04
Inflation standard deviation	4.55e-03	4.34e-03
Inflation AR(1) coefficient	–0.768[b]	0.343
GDP per capita	4.18e-06	7.30e-06
Constant	1.37[c]	0.322
R squared within	0.280	
Between	0.002	
Overall	0.107	
F-statistic	4.20	

Note: [a], [b], [c] indicates statistical significance at the 10%, 5%, and 1% levels, respectively.

Table 6

SECOND STEP ESTIMATION RESULTS
Dependent variable: standard error of the inflation anchor

	Coefficient	*Standard error*
IT dummy	–1.67e-03	13.2e-03
Years under IT	–6.03e-03[a]	2.92e-03
Inflation mean	2.97e-04	1.95e-04
Inflation standard deviation	5.42e-04	6.25e-04
Inflation AR(1) coefficient	0.0971[a]	0.0522
GDP per capita	3.68e-06	2.22e-06
Constant	–0.0733	0.0551
R squared within	0.176	
between	0.0007	
overall	0.004	
F-statistic	2.58	

Note: [a], [b], [c] indicates statistical significance at the 10%, 5%, and 1% levels, respectively.

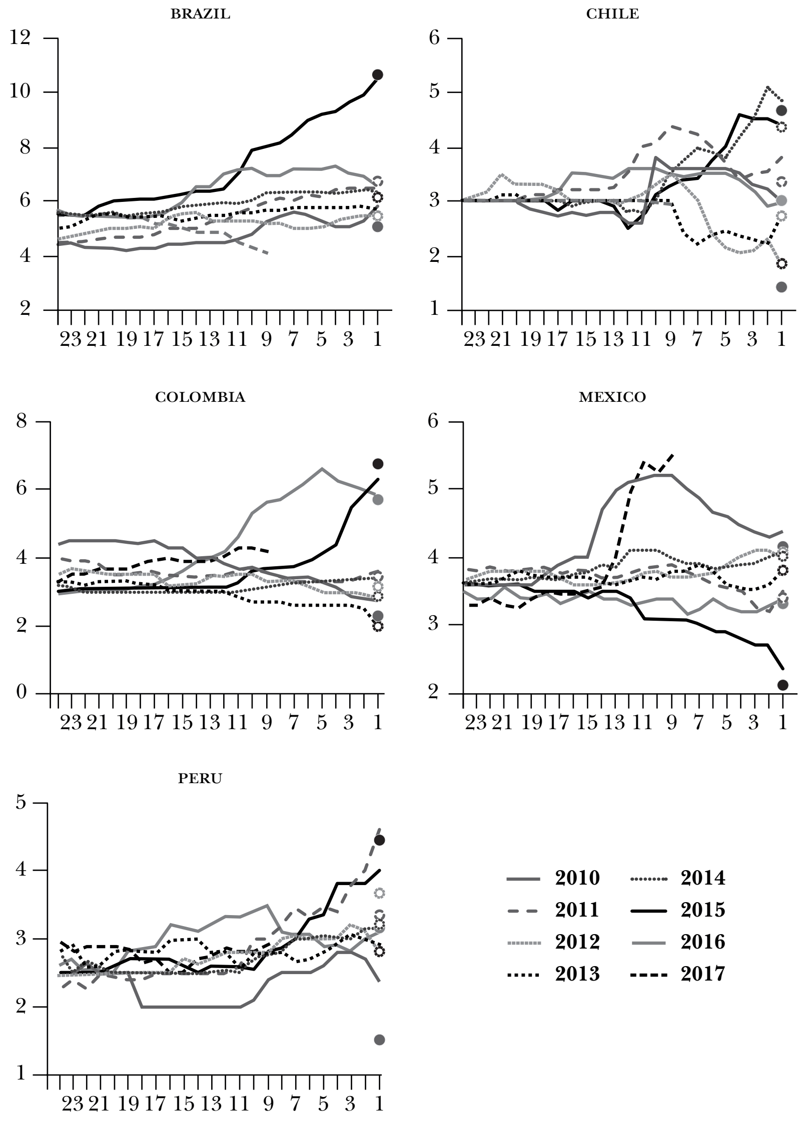
in terms of the weight on the anchor increasing and the anchor being more precisely identified by the data.

In future work, it would be possible to investigate inflation expectations anchoring further by focusing on the cross-sectional dispersion of forecasts. For example, Yetman (2017) focuses on forecaster-level data for Canada and the USA, while Hattori and Yetman (2017) conduct a similar exercise for Japan. However, for Latin America, similar data are only available from Consensus Economics for a limited subset (seven) of the countries that we study, and the number of forecasters for most of those countries is limited relative to those other studies.

Figure A.1

INFLATION FORECASTS AT DIFFERENT HORIZONS

A. COUNTRIES WITH INFLATION TARGETS FOR MORE THAN 15 YEARS

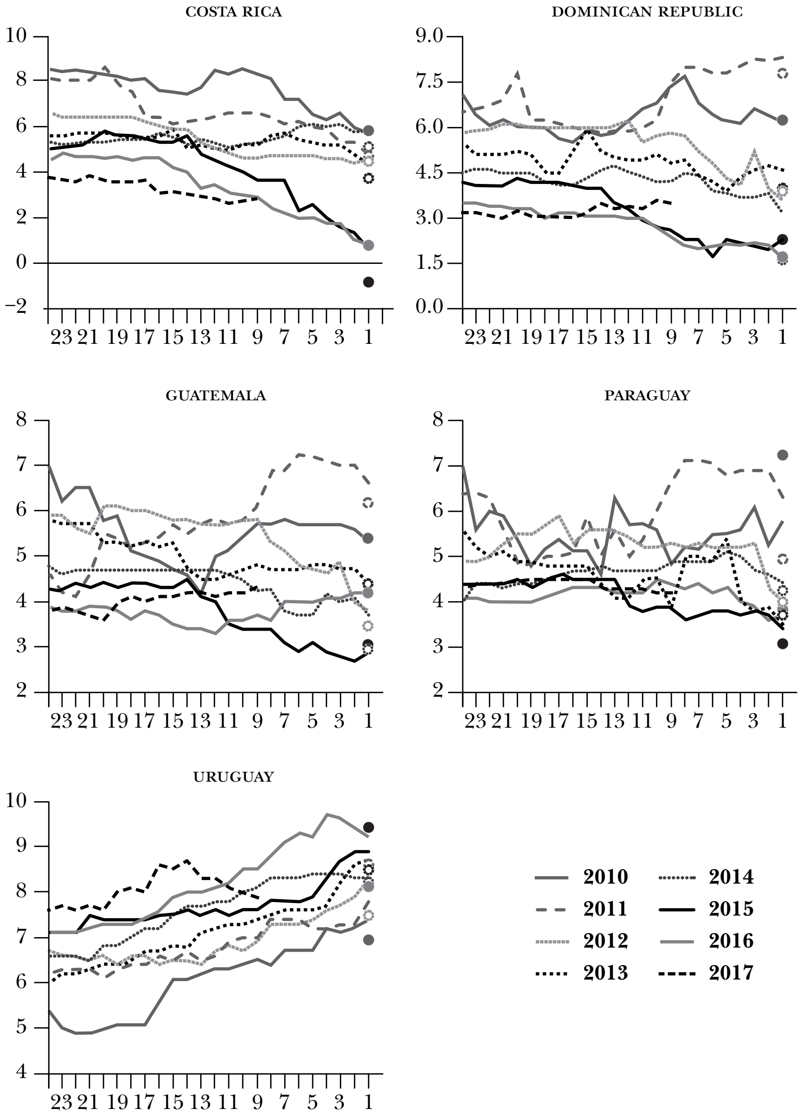


Notes: Horizontal axis represents the forecast horizon, defined as the number of months before the end of the calendar year being forecast. Dots represent the realized inflation at the end of year t.
Source: Consensus Economics ©; national data.

Figure A.1 (cont.)

INFLATION FORECASTS AT DIFFERENT HORIZONS

B. COUNTRIES WITH INFLATION TARGETS FOR LESS THAN 15 YEARS

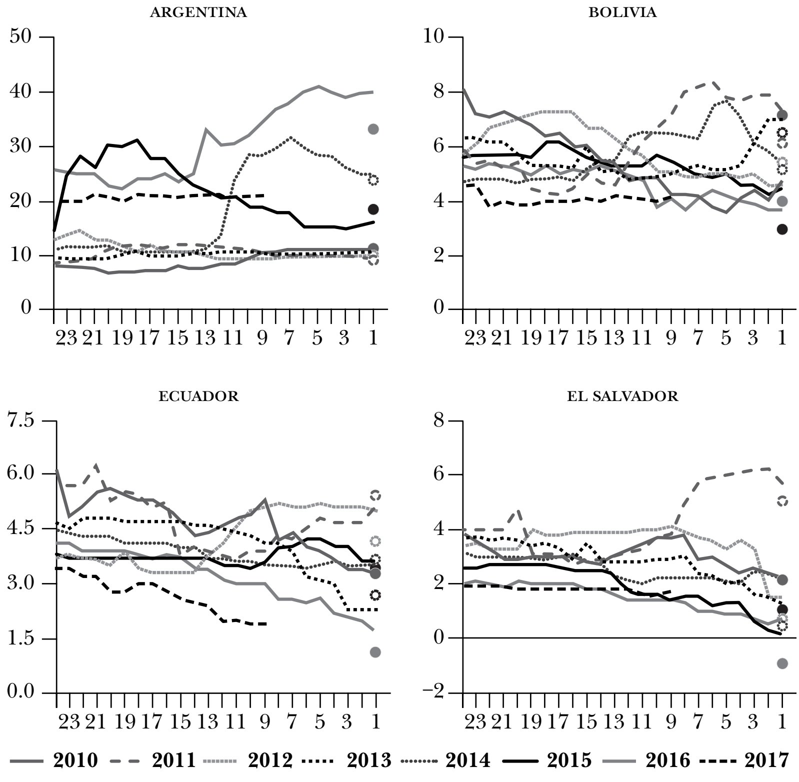


Notes: Horizontal axis represents the forecast horizon, defined as the number of months before the end of the calendar year being forecast. Dots represent the realized inflation at the end of year t.
Source: Consensus Economics ©; national data.

Figure A.1 (cont.)

INFLATION FORECASTS AT DIFFERENT HORIZONS

C. COUNTRIES WITHOUT INFLATION TARGETS

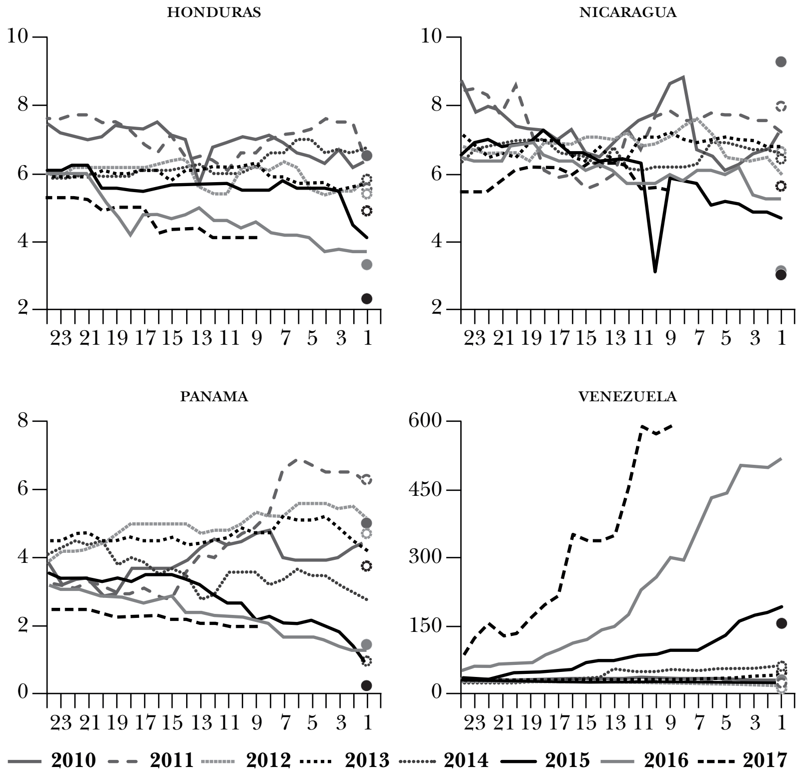


Notes: Horizontal axis represents the forecast horizon, defined as the number of months before the end of the calendar year being forecast. Dots represent the realized inflation at the end of year t.
Source: Consensus Economics ©; national data.

Figure A.1 (cont.)

INFLATION FORECASTS AT DIFFERENT HORIZONS

C. COUNTRIES WITHOUT INFLATION TARGETS (CONT.)

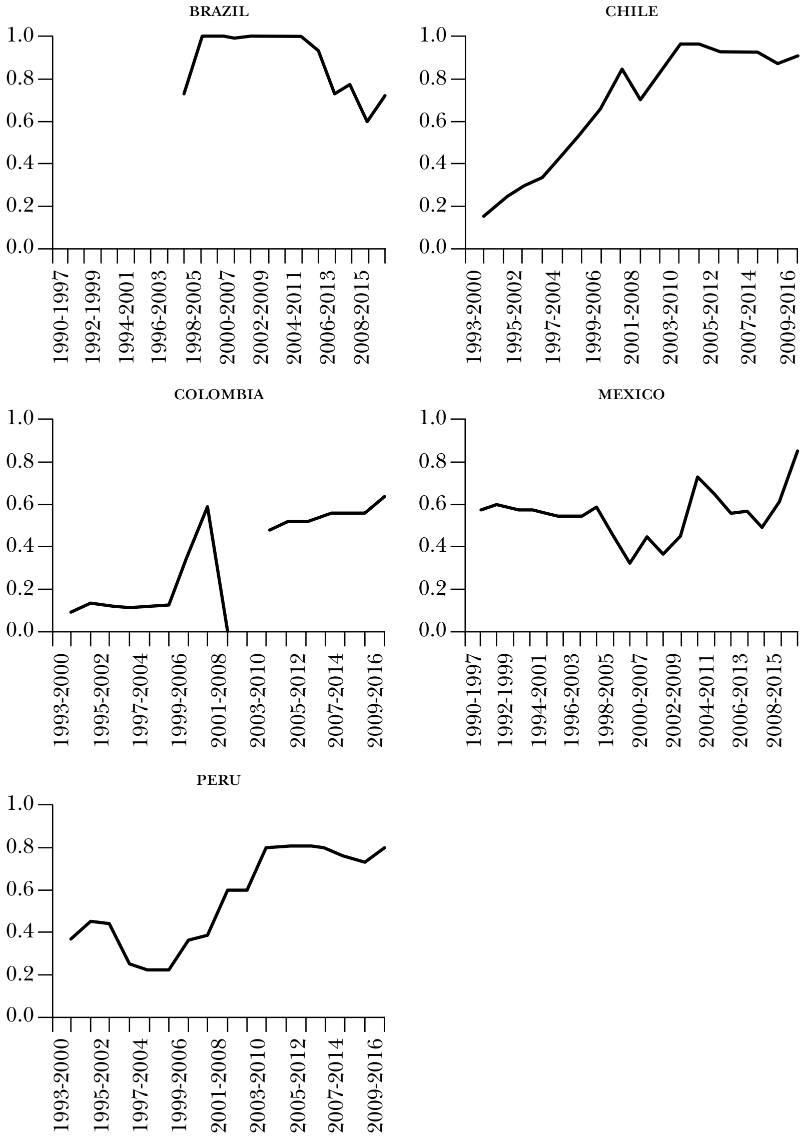


Notes: Horizontal axis represents the forecast horizon, defined as the number of months before the end of the calendar year being forecast. Dots represent the realized inflation at the end of year t.
Source: Consensus Economics ©; national data.

Figure A.2

ESTIMATED WEIGHT ON INFLATION ANCHOR (h = 12)

A. COUNTRIES WITH INFLATION TARGETS FOR MORE THAN 15 YEARS

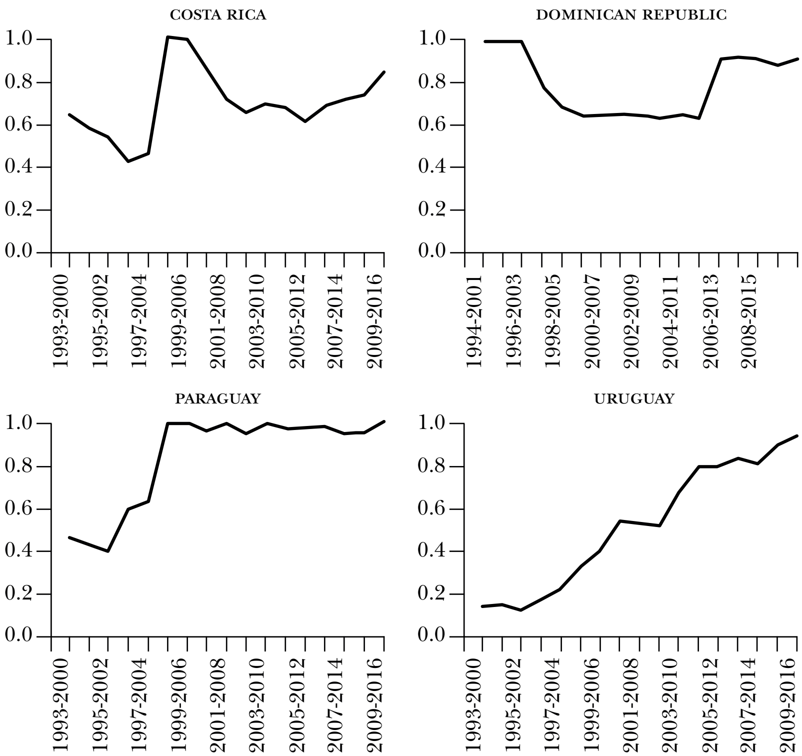


Notes: Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

Figure A.2 (cont.)

ESTIMATED WEIGHT ON INFLATION ANCHOR (h = 12)

B. COUNTRIES WITH INFLATION TARGETS FOR LESS THAN 15 YEARS

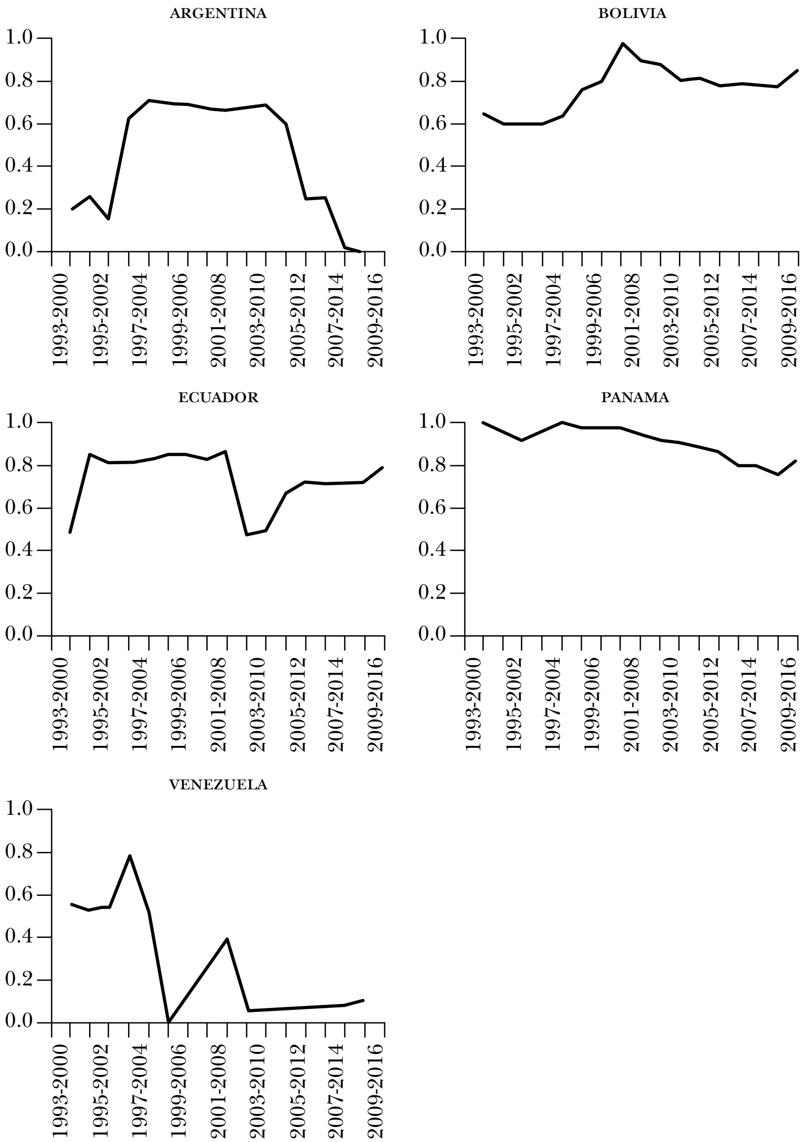


Notes: Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

Figure A.2 (cont.)

ESTIMATED WEIGHT ON INFLATION ANCHOR (*h* = 12)

C. COUNTRIES WITHOUT INFLATION TARGETS



Notes: Horizontal axis displays the eight-year rolling sample. Periods where no line is displayed correspond to rolling samples for which no anchor can be identified.
Source: Authors' calculations.

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The Time-Varying Degree of Inflation Expectation Anchoring in Bolivia

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David Esteban Zeballos Coria

Abstract

This chapter analyzes the time-varying degree of inflation expectations anchoring in Bolivia and, more precisely, whether inflation expectations have been in line with the inflation objectives announced by the Banco Central de Bolivia (central bank of Bolivia, BCB) and if they have become better anchored over time. Two considerations are particularly relevant in this regard. First, the main sources of information are the BCB survey and Focus Economics survey, which only have data for short- and medium-term inflation expectations. Second, monetary policy in Bolivia is under a monetary-targeting regime, so BCB projections represent the main references. The anchoring degree analysis of short-term inflation expectations was performed considering BCB projections, while the medium-term analysis used an implicit inflation target. In both cases, the results indicate there is a high degree of anchoring

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of inflation expectations in Bolivia, especially during the last four years. This study considers information from July 2005 to June 2017, with monthly frequency.

Keywords: inflation expectations, anchoring degree, monetary-targeting regime, BCB projections, time-varying parameters model.

JEL classification: E31, E52, E58, C32.

1. INTRODUCTION

The analysis of the behavior of the expectations of inflation of economic agents has been heavily studied in the past, especially with regards to the degree of anchoring of expectations, understood as the ability of monetary policymakers to manage inflation expectations (King, 2005). Theoretical literature and monetary policymakers agree that the anchoring of inflation expectations is of high importance in maintaining price stability, and expectations by private agents play an important role in macroeconomics since they can be a determinant of macroeconomic performance. Inflation expectations not only reflect private agents' perceptions about future inflation, but also directly impact current and future inflation.

Relatedly, a central bank should focus on the management of private expectations through communication for two reasons (Hubert, 2015). First, the expectations channel is one of the subtlest channels of monetary policy, because it depends on private agents' interpretation. As King (2005) notes, "because inflation expectations matter to the behavior of the households and firms, the critical aspect of monetary policy is how decisions of the central bank affect those expectations." Second, given the delay between policy actions and their real effects on macroeconomic variables, central bank communication provides policymakers with a way to promptly affect private expectations to shorten the transmission lag of monetary policy.

According to Blinder et al. (2008), central bank communication can take different forms: statements, minutes, interviews, speeches, or internal macroeconomic forecasts. We will focus on the latter instrument of communication because monetary policy in Bolivia is under a monetary-targeting regime. However, although the Banco Central de Bolivia (BCB, for its acronym in Spanish) does not have an explicit inflation target, its active communication policy and projections, announced twice per year in its Monetary Policy Report, become important reference points for agents at the time of forming their expectations.

Since the inflation expectations of private agents are not generally known, they can by approximated by: i) surveys of inflation expectations of professional forecasters or households and ii) market-based measures of inflation expectations. In the present document, we use information from the survey conducted by the BCB for the period between July 2005 and June 2017. This is a monthly survey of expectations for the rates of inflation (among other variables) for several short-term horizons. Additionally, we use information from the Latin Focus Consensus Forecast report of Focus Economics to gather data regarding medium-term inflation expectations in Bolivia.

There are not many studies that analyze the degree of anchoring of expectations in Bolivia. We can mention the work of Cerezo and Heredia (2013), who found that there was a greater degree of anchoring of inflation expectations in recent years than between 2008 and 2010. Nevertheless, they also found that expectations were not rational, suggesting that expectations reflect backward-looking behavior.

The main objective of this paper is to analyze the time-varying degree of inflation expectations anchoring in Bolivia. More precisely, we aim to assess whether inflation expectations have been in line with the inflation objectives announced by the BCB, and if they have become better anchored. The anchoring degree analysis of short-term inflation expectations was performed considering the BCB projections, while the medium-term analysis used an implicit inflation target. In both cases, the results indicate there is a high degree of anchoring of inflation expectations in Bolivia, especially during the last four years.

In the next section, there is a brief analysis about the behavior of inflation expectations in Bolivia and their stability. Subsequently, we show the results of the estimated models, analyzing the behavior of short-term inflation expectations with respect to the BCB projections, past inflation and other variables that could affect the formation of expectations. Then, the results of the analysis of medium-term expectations are presented. Finally, we present our conclusions.

2. INFLATION EXPECTATIONS IN BOLIVIA

In order to evaluate the evolution of the degree of anchoring of inflation expectations in Bolivia, we consider data from the survey conducted by the BCB for the period between July 2005 and June 2017.[1] This monthly survey contains information of the expectations of economic analysts, academics, members from financial sector and private business in Bolivia about the future behavior of economic variables of interest for BCB authorities such as inflation, exchange rate, GDP growth, trade balance, and fiscal balance, among others. In the case of inflation expectations, the survey focuses on: i) monthly inflation expected by the end of current month, ii) year-on-year inflation expected by the end of current year, iii) year-on-year inflation expected by the end of next calendar year and, iv) one year-ahead inflation expectations.

It is important to mention that, unlike surveys available in other countries, the BCB survey does not take into account long-term inflation expectations (e.g., five years-ahead expectations). Certainly, this issue restricts, to a certain extent, the variety of econometric analyses that can be implemented. Moreover, in Bolivian financial markets, no inflation-indexed bonds are traded, a feature that makes it impossible to estimate break-even inflation rates for this economy, which are a measure of inflation expectations widely used in topical literature.

Our analysis will be focused on approximately the last 12 years. During this period, important shocks (mainly foreign and supply-side shocks) hit the Bolivian economy and affected domestic inflation behavior. These shocks, along with some developments observed in monetary markets and the macroeconomic framework and changes in the dynamics of the local economy, may have affected the degree of anchoring of inflation expectations.

Between 2007 and 2008, the Bolivian economy went through an inflationary process triggered especially by a shock in international food and energy prices, reaching double-digit inflation rates not observed since the beginning of the previous decade. In this period, expectations of agents were significantly exacerbated, with median inflation expectations placing themselves above observed inflation rates. Subsequently, a process of disinflation took place associated

[1] Information for previous periods is not available.

with the global financial crisis in 2009, an episode characterized by a high degree of uncertainty about the performance of the world economy, with effects on Bolivian economic activity. Within this setting, inflation expectations followed a downward trend as well, although their decline was more moderate (Figure 1a).

In the period 2010-2011, new inflationary upsurges were noticed, although of smaller scale and persistence with respect to previous years. In this period, the main explanatory factors were a new rebound in the international prices of commodities and an increase in domestic prices caused by speculative activities after the Government temporarily readjusted fuel prices.[2] Beginning in 2012, the behavior of inflation was characterized by moderate fluctuations, exhibiting a downward trend during the last two years. In recent years, temporary hikes can be observed in the behavior of inflation, which are explained by increases of the prices of some foods, whose supply was affected by adverse weather events (like frosts, floods and droughts, among others). The trajectory of inflation expectations reflected a path similar to that of inflation between 2005 and 2011, although from 2012 onward it displayed stable behavior, with a median generally above observed inflation (Figure 1b).

The stability of inflation expectations is an important issue to consider, since it represents an initial approximation to its anchorage. A useful way to measure stability is through its degree of dispersion[3] (disagreement or uncertainty). Less dispersion can be interpreted as a signal of a better anchoring of inflation expectations.[4] For this purpose, we chose the cross-sectional standard deviation of inflation expectations (Figure 2). A higher degree of dispersion can be observed between mid-2007 and early 2011.[5] Afterwards, the degree

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2 It is important to note that fuels are subsidized in Bolivia. In December 2010, the government decided to withdraw the subsidy which generated an environment of uncertainty, causing expectations of inflation to increase. Although the measure was eliminated shortly, important second-round effects were generated during the following months.

3 Although, the dispersion of expectations in a survey is a measure of heterogeneity of beliefs rather than a measure of uncertainty (IMF, 2016), both tend to move together (Gürkaynak and Wolfers, 2007).

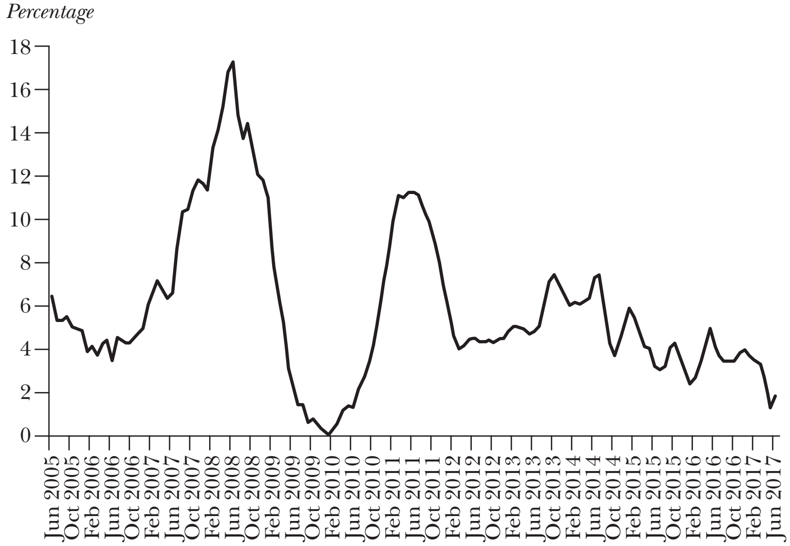
4 Dovern, Fritsche and Slacalek (2009), Capistran and Ramos-Francia (2010), Siklos (2013), and Ehrmann (2015).

5 During this period, Bolivian economy went through different circumstances that caused strong inflationary pressures: increased international

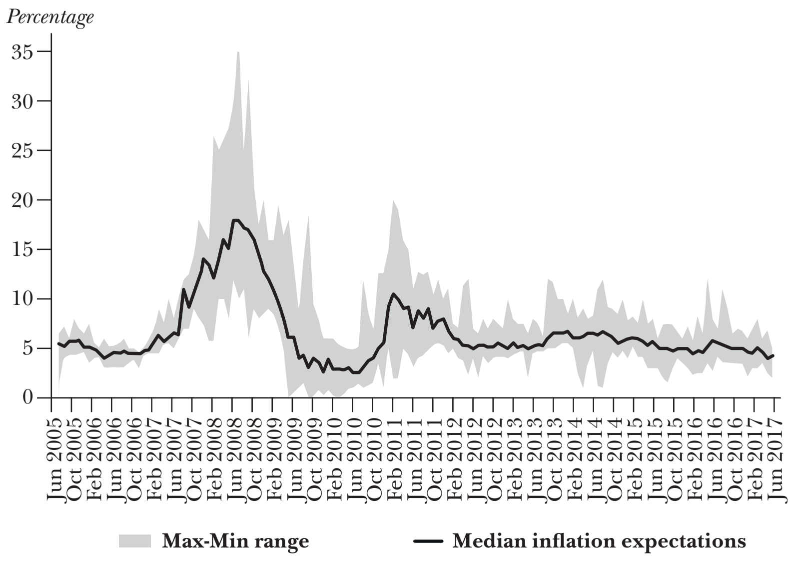
Figure 1

EVOLUTION OF HEADLINE INFLATION AND INFLATION EXPECTATIONS

A. HEADLINE INFLATION, YEAR TO YEAR



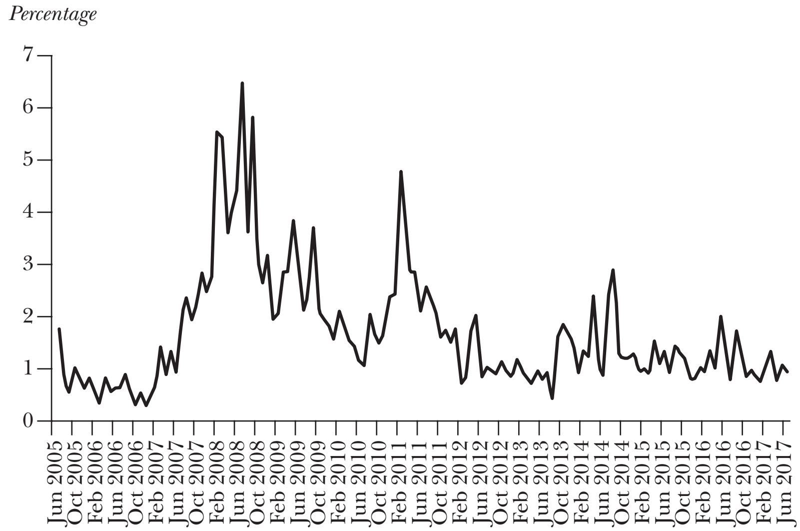
B. ONE YEAR-AHEAD INFLATION EXPECTATIONS



Note: National Statistics Institute and Central Bank of Bolivia.

Figure 2

CROSS-SECTIONAL STANDARD DEVIATION OF ONE YEAR-AHEAD INFLATION EXPECTATIONS



Source: Authors' calculations based on BCB data.

of dispersion tended to moderate, with a slight rebound between 2013 and 2014.[6] Except for those years, a lower degree of uncertainty about rates of inflation expected by economic agents can be observed beginning in 2012. Hence, the trajectory of expectations observed in recent years suggests a strengthening of their degree of anchoring over time.

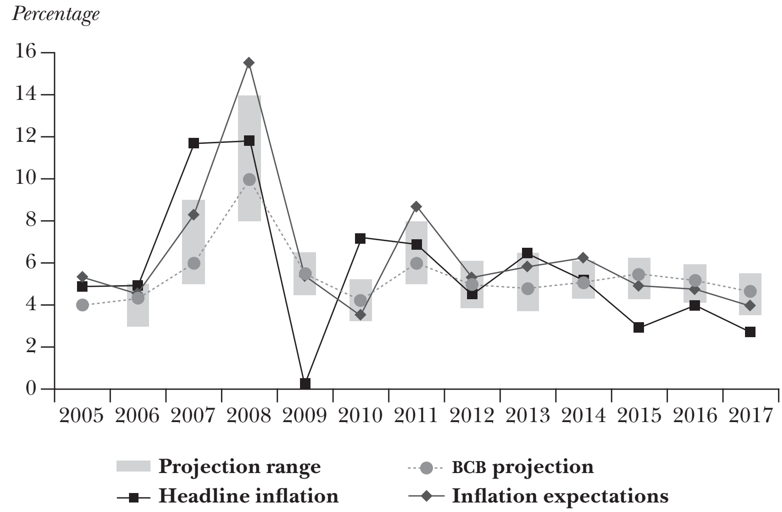
Inflation expectations in Bolivia seem to be more homogeneous in recent years. This homogeneity may reflect the existence of a common reference point that is taken into account by economic agents while forming their inflation expectations. One of these possible

commodity prices, economic acceleration, regulated price adjustments and others. All these factors created an environment of uncertainty regarding the future level of prices.

6 In 2013 and 2014 inflationary pressures were observed due to the rise in prices of some foods because adverse weather events reduced agricultural supply in local markets.

Figure 3

INFLATION EXPECTATIONS, HEADLINE INFLATION AND BCB PROJECTION



Note: Inflation expectations are computed as the mean of inflation expectations for a given year. BCB projections and the projection range are computed as the average of the inflation projections announced at the beginning and middle of the year.
Source: National Statistics Institute and Central Bank of Bolivia.

reference points is the inflation projection of the Central Bank announced in its Monetary Policy Report twice per year. Between 2005 and 2011, headline inflation and inflation expectations ended the year above the BCB projection, except for in 2009, and, in some cases, even above the projected range (Figure 3). The shocks noted above generated an environment of uncertainty, making it difficult for the BCB and private agents to project inflation. It seems that during this time economic agents mainly considered past headline inflation or possibly other variables to formulate their expectations. In 2012, this situation changed, a result of the expectations of the agents landing closer to the BCB projection, especially between 2015 and 2017. This could indicate that there is a significant degree of anchoring of expectations in recent years. This item will be studied empirically in the next section of the paper.

3. EMPIRICAL ANALYSIS OF THE SHORT TERM

While this study focuses mainly on assessing the anchoring of short-term inflation expectations over time, it should nonetheless be noted that the behavior of short-term expectations is also relevant to policymakers. According to Łyziak and Paloviita (2016), the credibility of a central bank should not only be measured in terms of its ability to anchor long-term expectations, but also in terms of its ability to affect short and medium-term expectations, since these have an important role in wage adjustments and price-setting by firms.

In addition, another point that must be emphasized is that in Bolivia, monetary policy is not based on an inflation-targeting regime. On the contrary, the monetary regime of Bolivia is one of monetary-targeting. However, although the BCB does not have an explicit inflation target, its active communication policy and projections announced twice per year in its Monetary Policy Report become important reference points for agents at the time of forming their expectations.

In a similar vein, the work of Anderson and Maule (2014) assesses the anchoring of short-term inflation expectations in the United Kingdom considering the Bank of England's inflation projections as one of its determinants. Likewise, Hubert (2015) showed that the projections of the European Central Bank play an important role in the formulation of short and medium-term expectations in the Eurozone.

In this context, an econometric model is estimated to analyze the evolution of the degree of anchoring of inflation expectations. Before we start, two aspects must be considered. First, most of the surveys contain "fixed-event" (FE) information (i.e., information always points to a single moment, like the end of the current or next calendar year) on the expectations of different variables, so they constitute an abundant source of information. Notwithstanding their availability, this paper requires the use of "fixed-horizons" (FH) variables (i.e., those that keep an *n* horizon, such as 12 months ahead) with the purpose of working with econometric models because forecasting horizons of FE forecasts (or expectations) vary from month to month (the horizon shrinks as time passes).

We, therefore, employ a technique that allows us to use the FE information. Following Dovern, Fritsche and Slacalek (2009), we create a FH variable as a weighted average of FE forecasts; the weights are determined by the number of months forecasted in both the current and subsequent years. Denote $F^{fe}_{y0,m,yo}(x)$ as the FE forecast of variable *x* for year *Y0* made in month *m* of year *Y0* and $F^{fe}_{y0,m,y1}(x)$ the FE forecast of variable *x* for year *Y1* made in month *m* of year *Y0*. Then $F^{fh}_{y0,m,12}(x)$ represent the FH forecast 12 months ahead made in month *m* of year *Y0*. We approximate the FH forecast for the next 12 months as an average of the forecast for the current and next calendar year weighted by their share in forecasting horizon:

1 $$F^{fh}_{y0,m,12}(x) = \frac{12-m+1}{12} * F^{fe}_{y0,m,yo}(x) + \frac{m-1}{12} * F^{fe}_{y0,m,y1}(x)$$

According to Winkelried (2017), a survey that registers FE expectations for horizons *Y0* and *Y1* does contain information for expectations at any intermediate horizon; for instance, expectations for 12 months ahead are implicitly contained in current and next year forecast. Therefore, the inflation expectation obtained with this technique (Figure 4a) is equal to the inflation expectation one year ahead shown in Figure 1b. This technique was also used with the information from the BCB projection for the current and next calendar year (Figure 4b).

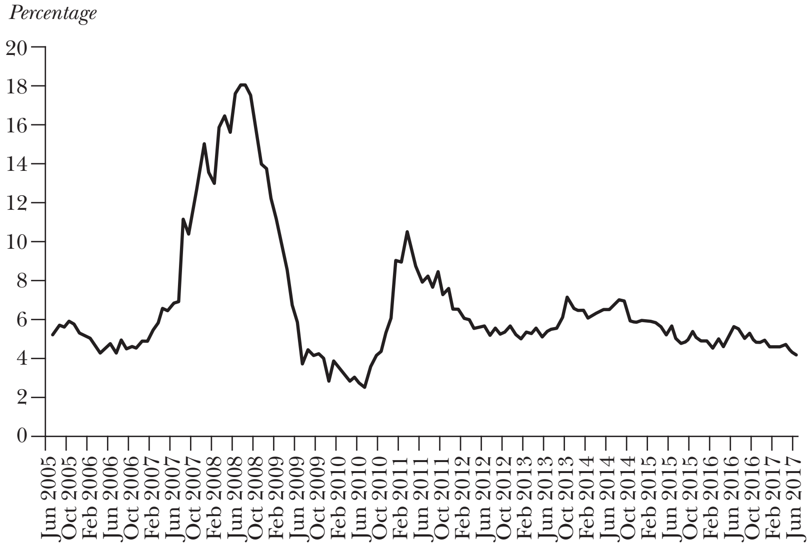
A second point we should consider is the effect of new inflation information on the formulation of economic agents' expectations. According to Hubert (2015), the effects of central bank inflation projections on private agents are stronger at the beginning of each year than at the end, when much more information is available on the actual behavior of inflation. Consequently, this document mainly considers the projections announced by the BCB at the beginning of each year. However, a second variable was created to reflect the BCB projection, which also includes updates of the projection announced after the first semester of every year, mainly with the purpose of performing robustness analysis.[7]

[7] Annex 1 presents the evolution of the BCB projection for the current and next calendar year separated, and the BCB inflation projections constructed using the technique of equation (1) that includes the updates at middle of each year.

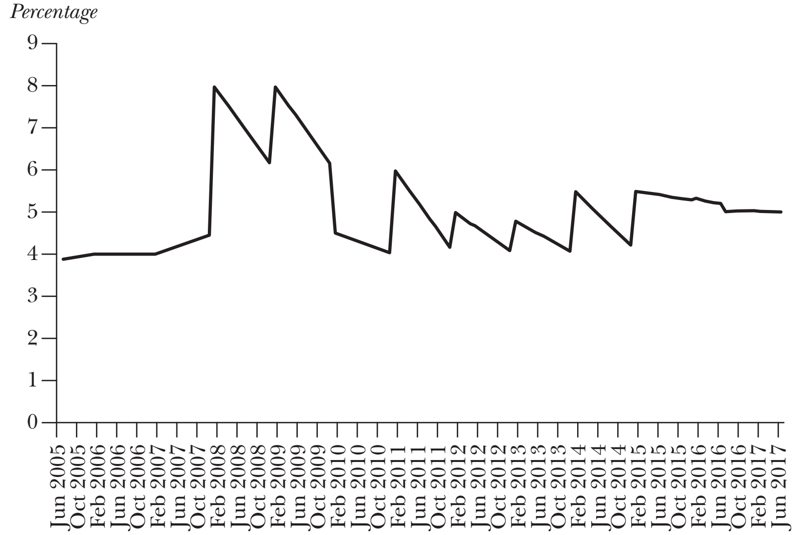
Figure 4

FIXED HORIZONS VARIABLES FOR SHORT TERM

A. NEW ONE YEAR-AHEAD INFLATION EXPECTATIONS



B. NEW BCB PROJECTION



Source: Authors' calculations based on BCB data.

3.1 BCB Projection against Headline Inflation

In this section, the specification of the model is based on the methodology applied by Łyziak and Paloviita (2016), who estimate different models to measure the degree of anchoring of inflation expectations for the Euro Zone. The specified equation is as follows:

2
$$\pi^e_{t|t+n} = \gamma^{proj} \pi^{proj}_{t+n} + \gamma^{\pi} \pi_{t-1} + \mu_t$$

where:

$$\gamma^{proj} + \gamma^{\pi} = 1$$

where $\pi^e_{t|t+n}$ represents the inflation expectations in period t for the horizon; $t+n; \pi^{proj}_{t+n}$ is the inflation projection for the horizon; $t+n; \pi_{t-1}$ represents observed inflation lagged one period and n is equal to 12 months. Additionally, an error term (μ_t) is included in the equation. Note that, by construction, the sum of the coefficients of the model must be equal to one. If the coefficient γ^{proj} reaches a value as close as possible to one, it would reflect a significant degree of anchoring of expectations.

According to Strohsal, Melnick and Nautz (2015), the central bank's credibility can be gained, but it can also be lost. As a consequence, the degree of inflation expectations anchoring might not be constant over time. Meanwhile, Orphanides (2015) once pointed out that inflation expectations are well anchored until they are not. This means that the degree of anchoring can change over time, so using a model with constant parameters may not be the best option. In that sense, in the present document a time-varying parameter model is estimated, in line with other works such as Demertzis, Marcellino and Viegi (2012) and Strohsal, Melnick and Nautz (2015).

In the name of simplification, we assume that the state parameters follow a random walk process. We use the Kalman filter (Kalman, 1960) to compute the one-step ahead estimates of the means and variances[8] of the states by maximum likelihood.

8 During the estimation, the variances parameters are expressed in exponential form to ensure that the variances themselves are non-negative.

The results for this first estimation showed that the coefficient γ^{proj} attained a value close to 0.80, which implies that there is a significant degree of anchoring of short-term expectations in Bolivia (Table 1). On the other hand, the coefficient γ^{π} for lagged inflation is significant at 10 percent.

Table 1

RESULTS FROM TIME-VARYING PARAMETER MODEL 1

	Projections	*Past Inflation*
Coefficient	0.80	0.20
RMSE	0.13	0.12
z-Statistic	6.28	1.66
p-value	(0.00)	(0.09)

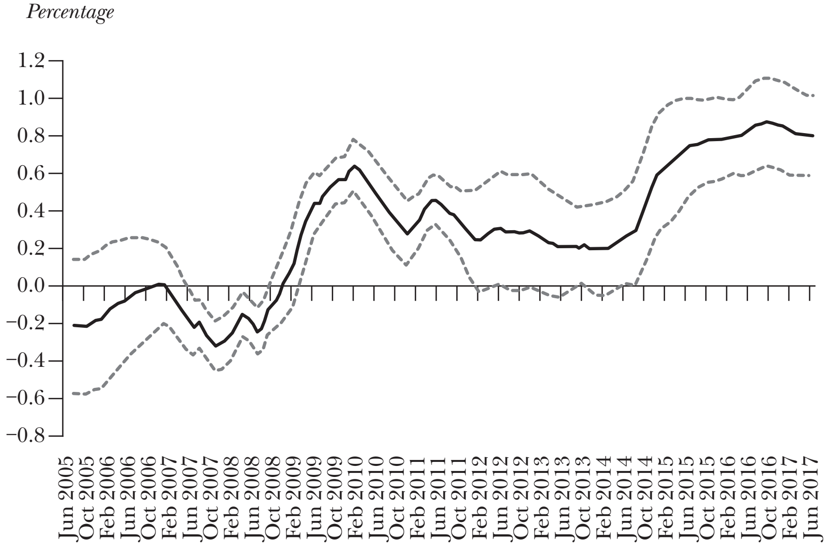
A strength of state-space models is that they permit observe the evolution of the different coefficients over time. It can be seen that the value of the coefficient γ^{proj} was negative between mid-2005 and late 2008 (Figure 5a), in line with the overshooting of expectations that took place then. In this period the anchoring degree of expectations was null. Later, an improvement in the degree of anchoring of expectations can be observed as of 2009,[9] reaching values near 0.6 until mid-2010, when it fell again because of a new inflationary rebound. The BCB projections coefficient reflected stable behavior around 0.25 from 2012 until mid-2014. In July 2014 this coefficient begins important growth, reaching 0.80 in the last two years under consideration.

[9] It is also interesting to note that the degree of anchoring of expectations did not decline in time of the international financial crisis, something that was analyzed in different documents such as Galati, Poelhekke and Zhou (2011), Autrup and Grothe (2014), and Nautz and Strohsal (2015). However, this does not imply that in that period there was a greater degree of central bank credibility.

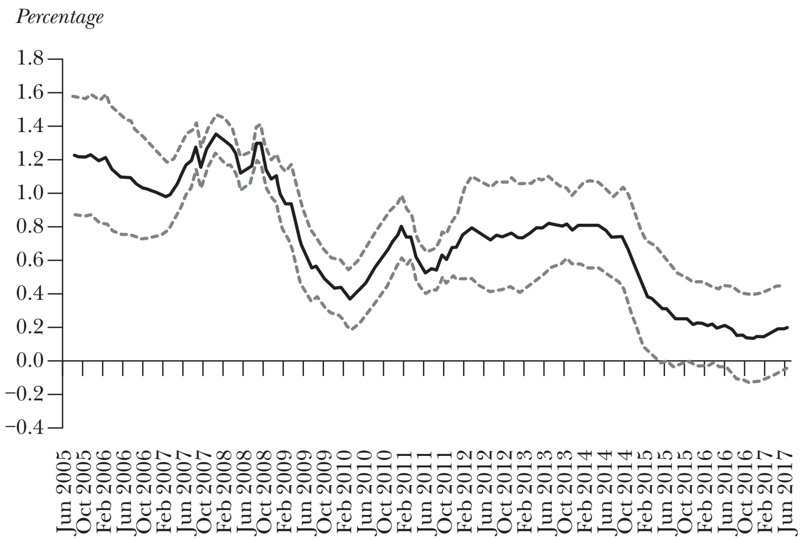
Figure 5

EVOLUTION OF COEFFICIENTS IN MODEL 1

A. BCB PROJECTION COEFFICIENT (γ^{proj})



B. HEADLINE LAGGED INFLATION COEFFICIENT (γ^{π})



Note: Smoothed coefficient ± 2RMSE.

In the case of headline lagged inflation (Figure 5b), the highest values were observed between 2007 and 2008 when it reached values higher than one, which shows the exacerbation of expectations during this time. Later, values tended to decrease and seemingly lose importance in the formulation of agents' expectations.

Annex 2 contains the results using the updated BCB projection under this specification. The results obtained are similar to those found with Model 1; there also exists a significant degree of anchoring of short-term expectations with respect to updated BCB projections. These first results showed that short-term inflation expectations are anchoring,[10] since the BCB projection had a bigger impact on economic agents than headline inflation. However, information from other variables may affect the formulation of expectations.

3.2. BCB Projection against Other Variables

Economic agents are exposed to a great diffusion of local and international information, especially in light of advances in communication. This means that the behavior of other variables may affect the formulation of private agents' expectations. Relatedly, there exists a strand of literature that investigates how inflation expectations respond to macroeconomic news (Beechey and Wright, 2009, and Beechey, Johannsen and Levin, 2011), though with a long-term focus. Since short-term inflation expectations respond to observed inflation, they should be more sensitive to changes in other variables. With the objective of analyzing the effects of information from other variables on the behavior of inflation expectations, in this section we make estimates with different models, including a broad set of external variables in addition to BCB projections and observed inflation.

$$\pi^{e}_{t|t+n} = \beta_1 \pi^{proj}_{t+n} + \beta_2 \pi_{t-1} + \beta_m X_t(L) + \mu_t \tag{3}$$

Once again $\pi^{e}_{t|t+n}$ represents one year-ahead inflation expectation; π^{proj}_{t+n} is the BCB inflation projection for the horizon t + n; where n is equal to 12 months and π_{t-1} represents observed inflation lagged one period. We include X_t, which represents the battery of different

[10] This result does not imply that inflations expectations are rational; that issue is not analyzed in this study.

external variables used to estimate the models; some of them will be introduced with lags. Additionally, an error term $\left(\mu_t\right)$ is included in the equation.

In order to guide our selection of external variables, we follow the works of Celasun, Gelos and Prati (2004), Cerisola and Gelos (2005), Bevilaqua, Mesquita and Minella (2007), and Carrasco and Ferreiro (2013). The variables chosen were output gap,[11] one year-ahead expectations of nominal depreciation,[12] and expectations of fiscal balance in percent of GDP.[13]

We also incorporate other variables that may be related to the characteristics of the Bolivian economy, such as shocks from climatic events[14] (as food represents an important part of the CPI in Bolivia, nearly 28 percent) and external shocks[15] (as previously noted, the Bolivian economy was exposed to major external shocks during the last decade)[16]. In the case of inflation expectations and BCB projections, we use the variables created in the previous section.

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[11] The information was obtained from the Global Index of Economic Activity (IGAE, for its acronym in Spanish) which represents a proxy variable of economic activity in monthly frequency, minus its trend value (where the trend is approximated through a Hodrick-Prescott filter).

[12] Most of the documents use movements in the nominal exchange rate. However, in Bolivia, the exchange rate has been fixed since 2011, and it is an important variable since it works as a nominal anchor. For this reason, we use economic agents' expectations of future depreciation.

[13] We use expectations of fiscal balance as a proxy of the primary fiscal balance in order to have a variable with monthly data. For this case and the expectations of nominal depreciation we use the information from the BCB survey employing the technique of equation (1).

[14] We employ the Multivariate ENSO (El Niño/Southern Oscillation) Index (MEI) of the United States National Oceanic and Atmospheric Administration (NOAA) as a proxy variable to reflect the changes in the weather condition.

[15] The Food Price Index of the International Monetary Fund (IMF) was considered. International food price shocks have a significant impact on inflation in Bolivia because of the high share of food in the country's CPI.

[16] We also use other variables like IGAE growth YoY, economic agents' expectations of economic growth and the imf international energy price index; none of these, however, showed satisfactory results.

As in the previous section, for the estimation we use time-varying parameter models with different specifications, and we suppose that the state parameters of all the variables follow a random walk process. The results of the different models' specifications can be observed in Table 2.

Table 2

DETERMINANTS OF SHORT-TERM INFLATION EXPECTATIONS

	Model 2	*Model 3*	*Model 4*	*Model 5*
BCB projection	0.74	0.75	0.74	0.73
	(0.00)	(0.00)	(0.00)	(0.00)
Inflation (*t*–1)	0.29	0.29	0.28	0.28
	(0.06)	(0.07)	(0.08)	(0.09)
Nominal depreciation expectations	0.20	0.21	0.21	0.20
	(0.76)	(0.75)	(0.75)	(0.77)
International food price index (*t*–1)	0.01	0.01	0.01	0.004
	(0.77)	(0.75)	(0.75)	(0.81)
Output gap (*t*–2)		0.29		0.30
		(0.36)		(0.35)
Climatic events			−0.03	−0.06
			(0.91)	(0.82)
Fiscal Balance/GDP Expectations				−0.03
				(0.43)

Note: The values in parentheses represent the *p*-values.

We created four different models, and in each one the BCB projection remained the most important explanatory variable with coefficients around to 0.74, close to those obtained in Section 3.1. Also, lagged inflation was significant (at 10 percent) in all models, with a coefficient near 0.28. The remaining variables were not statistically significant. The least relevant were the international price food index,[17] expectations of the fiscal balance in percent of GDP, and the climatic event variable.[18] The lagged output gap displayed a high coefficient, but it was not significant.[19]

The evolution of the coefficients of the BCB projection and headline inflation is similar to that found in Section 3.1 (Figure 6). It can be observed that headline inflation had a greater impact on inflation expectations between 2005 and 2010, while BCB projections had a greater effect in recent years. The effect of BCB projections at the beginning of the sample, however, are around 0.45 (in the model used in Section 3.1, the coefficient was close to 0 during this period). Meanwhile, the coefficient of observed inflation was near 0.65 (in the results of previous model, it was near 1).

It seems that the inclusion of other variables simply tended to reduce the explanatory value of observed inflation over inflation expectations. Most of the additional variables also work as determinants

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[17] During 2007-2008 and 2010-2011, international food prices rose exponentially, so national producers decided to sell most of their production to foreign markets, generating a shortage in local markets. This caused an increase in the prices of some foods (like sugar) or inputs (such as soybeans that are important for poultry farms), which translated into an inflationary process. However, in recent years international food prices have fallen and shown less dynamism; in addition, limits were applied to exports in order to ensure supply to local markets. These factors may have diminished the index's relationship with local food prices, so this variable turned out to be not significant in the formulation of expectations.

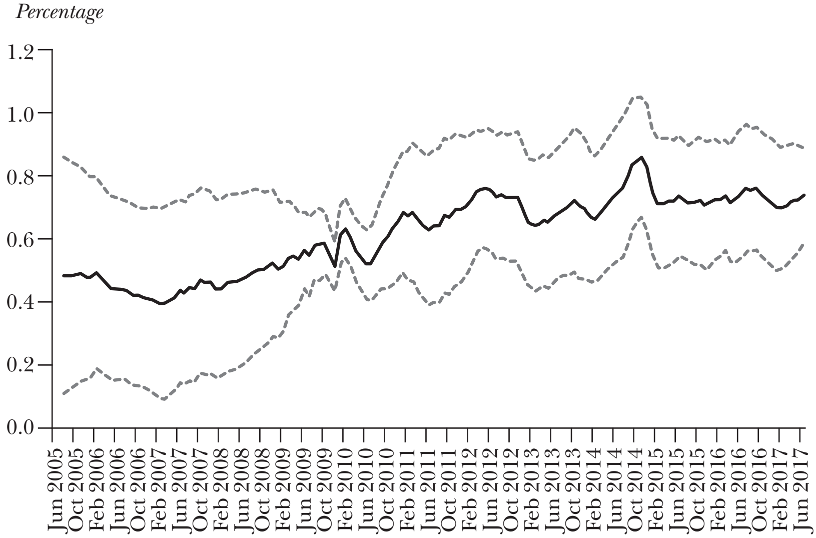
[18] The sign of the coefficient of climatic events was negative in the models. Since the MEI was used as a proxy variable, when it presents negative values it denotes the presence of the La Niña phenomenon. This phenomenon can generate heavy rains, floods and landslides, especially in the eastern part of Bolivia, where most of the agricultural production is located. Therefore, it can be inferred that when the La Niña phenomenon occurs, the inflation expectations of economic agents would increase, although not significantly. This variable's lack of significance is possibly explained by the fact that the effects of climatic events generally affect food prices for no longer than three months; prices subsequently decrease as supply normalizes in local markets. Economic agents thus do not expect there to be a constant rise in prices in following months.

[19] It is worth mentioning that, unlike the rest of the variables, the IGAE information is available to the general public with a greater lag time. In that sense, the output gap entered the model with a lag of two periods.

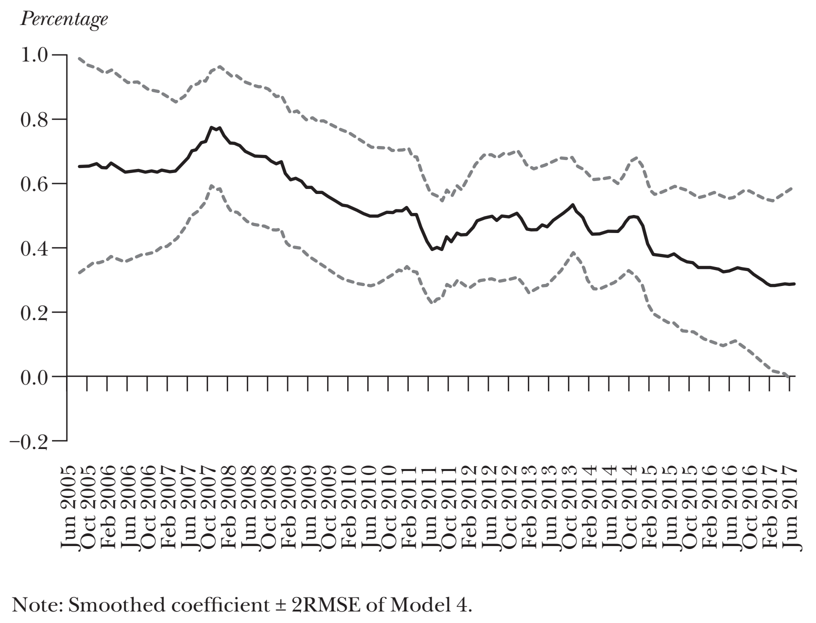
Figure 6

EVOLUTION OF COEFFICIENTS IN MODEL 2

A. BCB PROJECTION COEFFICIENT



B. HEADLINE LAGGED INFLATION COEFFICIENT



Note: Smoothed coefficient ± 2RMSE of Model 4.

of headline inflation; this could be the reason why none of them are significant, since their impacts are already contained in the path of the inflation. The evolution of this last variable reflects the impacts of imported inflation, demand pressures or climatic events. Therefore, the agents maybe only need to see the path of inflation, which already includes a lot of additional underlying information.

A special analysis deserves depreciation expectations, although these were found to be non-significant, there was a time when they had a more relevant role. The exchange rate in Bolivia has been under a crawling-peg regime since the late 1980s, and during the 1990s the local currency was continually depreciated in order to maintain the country's external competitiveness. This caused a significant process of dollarization (Berg and Borensztein, 2000), and a high pass-through effect (Laguna, 2010). In addition, in such a situation the population becomes accustomed to seeing depreciation as a normal process of the economic system (Humérez and De la Barra, 2007). However, this pattern changed radically after 2006. In 2007 and 2008 the local currency appreciated in order to mitigate the effects of the external environment on internal prices (Figure 7b). This measure had the effect of reducing expectations of inflation (Figure 7a), illustrating the important role of exchange policy in maintaining price stability.

Since 2011 the exchange rate has remained stable in order to anchor expectations and contain external inflationary pressures. This may have caused agents to stop considering the exchange rate as a relevant variable for the formation of their expectations in recent years.

The inclusion of other variables did not affect the previous results from Section 3.1, and it supports the possibility that short-term inflation expectations are anchoring in Bolivia. However, it would be good to analyze whether BCB announcements have effects on the inflation expectations of a longer horizon, such as the medium term.

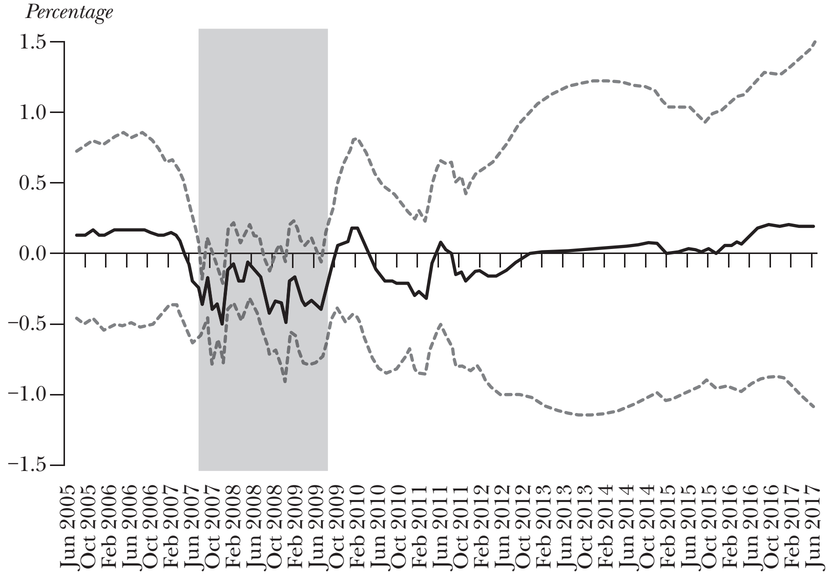
4. EMPIRICAL ANALYSIS IN THE MEDIUM TERM

Although, our main analysis has been done with the BCB survey and, therefore, with short-term information; there are other sources where anyone can find information on the expectations of economic agents. Most of the research papers on this topic consider data from international private companies that conduct surveys on different

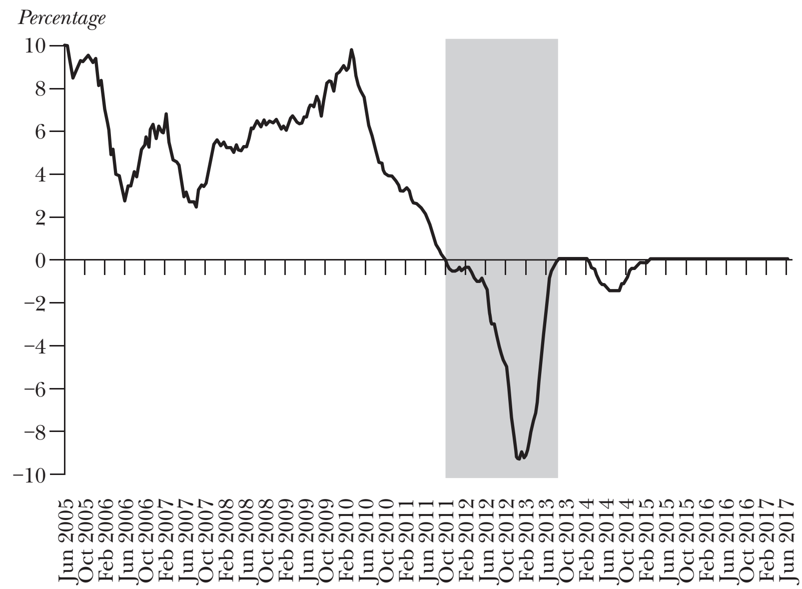
Figure 7

EXCHANGE RATE IN BOLIVIA

A. COEFFICIENT OF DEPRECIATION EXPECTATIONS MODEL 2



B. EXCHANGE RATE VARIATION YEAR TO YEAR IN PERCENTAGE



Note: Smoothed coefficient ± 2RMSE of Model 2.

variables in a large number of countries. In this case we choose to use the information provided by the Latin Focus Consensus Forecast[20] report from Focus Economics.[21] While the large sample size allows us to study the expectations of private agents, we chose this database mainly because it offers information not only on forecasts for the current and next calendar year, but also for years further ahead.[22]

In order to compare the information offered by the Focus Economics survey with the BCB survey, we use the technique from equation (1) in Section 3 to transform the data of inflation expectations for the current and next calendar year. The series obtained reflect similar behavior in general terms (Figure 8). Between 2007-2008 and 2010-2011 both series show an increase, although one of less magnitude in the case of Focus Economics expectations. Since 2012, both series have stabilized, except for a slight increase in BCB expectations between 2013 and 2014, and from 2015 on they present similar values. By performing a cross correlation analysis considering the whole sample (July 2005 - June 2017), a high level of correlation (0.92) was obtained. Therefore, the Focus Economics information on inflation expectations can be considered a complement to BCB survey data.

The forecast information of interest in the Focus Economics surveys, conducted with a monthly frequency, is that from April 2010.[23] We gathered information for the current year, the next calendar year, and the third, fourth, and fifth years ahead, so we have data on inflation expectations up to five years ahead. Although the information

[20] The Latin Focus Consensus Forecast report is a monthly publication, which contains macroeconomic projections from nearly 200 different sources. It covers approximately 30 macroeconomic indicators per country for a five-year forecast horizon including economic activity (GDP), industrial production, business confidence, consumer confidence, inflation, monetary policy decisions and exchange rate movement.

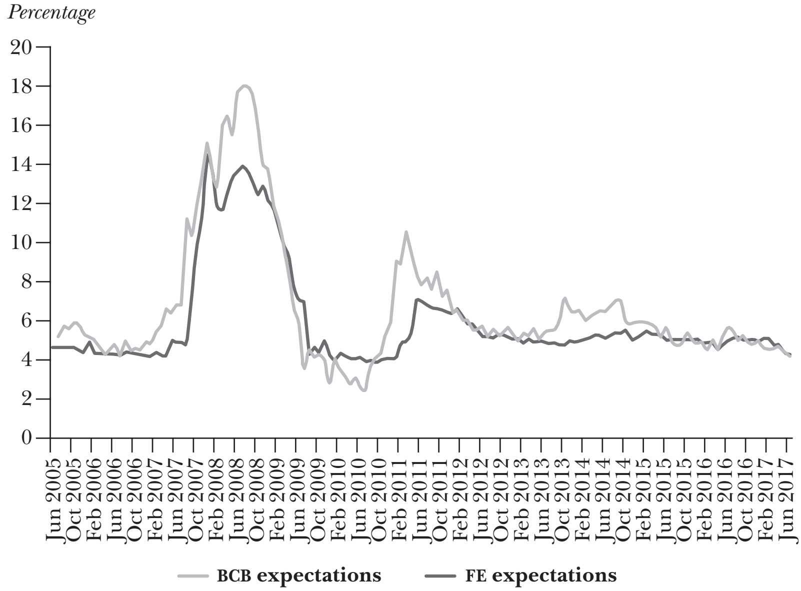
[21] Focus Economics is a company that has information on economic forecasts for many key indicators in 127 countries. Its reports draw on many economic and commodities price forecasts and on economic analysts around the world.

[22] There exist other institutions that provide information about economic forecast; one of the most famous is Consensus Economics. Nevertheless, in the case of Bolivia its report has only forecast information for the current and next calendar year of the variables of interest for the present document.

[23] There exists forecast information for the current and next calendar year for a longer period, but, not for the rest of the years.

Figure 8

EVOLUTION OF INFLATION EXPECTATIONS FROM BCB SURVEY AND FOCUS ECONOMICS SURVEY



Source: Authors' calculations based on BCB and Focus Economics data.

is on fixed-event variables, in order to work with these data we also convert them into fixed-horizon variables using the technique from equation (1) in Section 3. We end with information on inflation expectations for the current year (first), the next calendar year (second), and the third and fourth years (Figure 9a). The last years would be used to study the degree of anchoring in the medium term.[24]

The four variables show high values between 2011 and the beginning of 2012, and later they reflect more moderate behavior, similar to that observed with expectations from the BCB survey. A rebound

[24] Although most of the literature defines the medium term as beginning with the fifth year ahead (see, Carrasco and Ferreiro, 2013; imf, 2016), this document defines the medium term as beginning with the second year ahead, like Łyziak and Paloviita, 2016.

can be observed by the end of 2015 for all cases, except the first year. In the last six months, the inflation expectations at the second, third, and fourth years stabilize around 4.78 percent, while the expectations for the present year (first year) fall to 4.31 percent.

In the case of BCB projections, we have the projections for the current and next calendar year from the Monetary Policy Reports. The BCB does not undertake projections for longer periods in their reports, which poses a challenge for analyzing the degree of anchoring in the medium term. To deal with this issue, we use an implicit inflation target as a reference for inflation expectations in the medium term.[25] We considered the level of inflation that is normally used in the medium-term projections for internal analysis in the BCB. In this case, it would be precisely 5 percent,[26] which is in line with the projections made for the Economic and Social Development Plan 2016–2020 for Bolivia. As in the previous case, we take fixed-event variables and use equation (1) to change them to fixed-horizon variables (Figure 9b).

With the variables prepared, the first step was to analyze the behavior of short-term inflation expectations (current year) in order to compare the results with those obtained with the expectations from the BCB survey in Section 3.1[27] with equation (2). The results show an important role of headline inflation, especially in 2007, 2008, and 2011 (Figure A5b). Nevertheless, since 2012 the coefficient of BCB projections (degree of anchoring) has reflected an upward trend with slight fluctuations, reaching a value of 0.83 at the end of the sample (Figure A5a). The results have the same observed pattern as those obtained in Section 3.1, showing a greater degree of anchorage in recent years. This shows the importance the BCB's projections acquired in the last few years, not only for local economic agents but also for foreign forecasters.

In order to compare the results from the degree of anchoring of inflation expectations in the short term and medium term, we use the same time-varying parameter model from equation (2) with the same assumptions from the previous section. We introduce

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25 There exist research papers that have used implicit inflation targets such as Mumtaz and Theodoridis (2017).

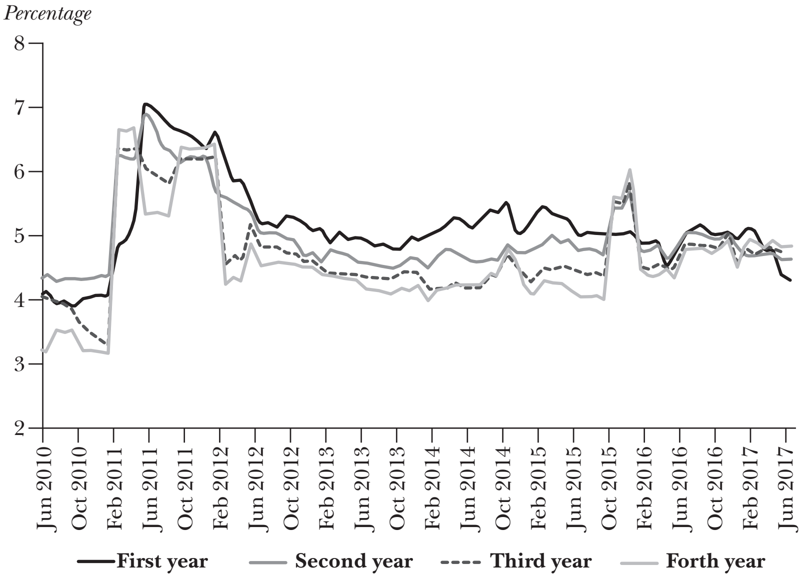
26 Also, this level has been used as reference for the next calendar year's projections in the BCB Monetary Policy Report since 2015.

27 The results of Model 6 can be found in Annex 3.

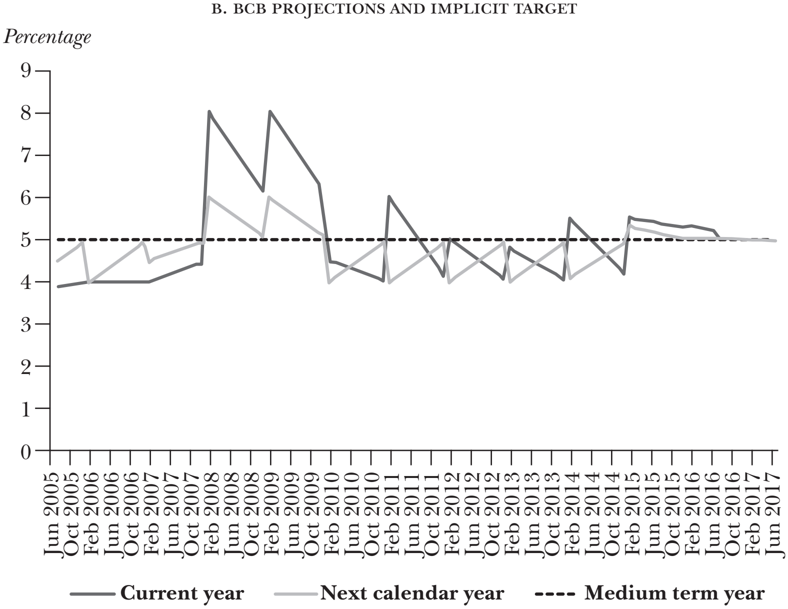
Figure 9

FIXED HORIZONS VARIABLES FOR MEDIUM TERM

A. INFLATION EXPECTATIONS FROM FOCUS ECONOMICS



B. BCB PROJECTIONS AND IMPLICIT TARGET



Note: Authors' calculations based on Focus Economics and BCB data.

the inflation expectations by year horizon with the respective BCB projection; for example, the BCB projection for the first and second year will be included in the models with the inflation expectations for the current and next calendar year, respectively. Meanwhile, the implicit inflation target will be introduced into the models with inflation expectations for the third and fourth years. Thus, we have four models, whose results are in Table 3.

Table 3

DEGREE OF ANCHORING OF INFLATION EXPECTATIONS IN THE SHORT TERM AND MEDIUM TERM

	Model 6	*Model 7*	*Model 8*	*Model 9*
	First year (current year)	*Second year (next year)*	*Third year*	*Fourth year*
BCB projection (implicit target)	0.83	0.90	0.91	0.93
	(0.00)	(0.00)	(0.00)	(0.00)
Past inflation	0.17	0.10	0.09	0.07
	(0.09)	(0.19)	(0.45)	(0.65)

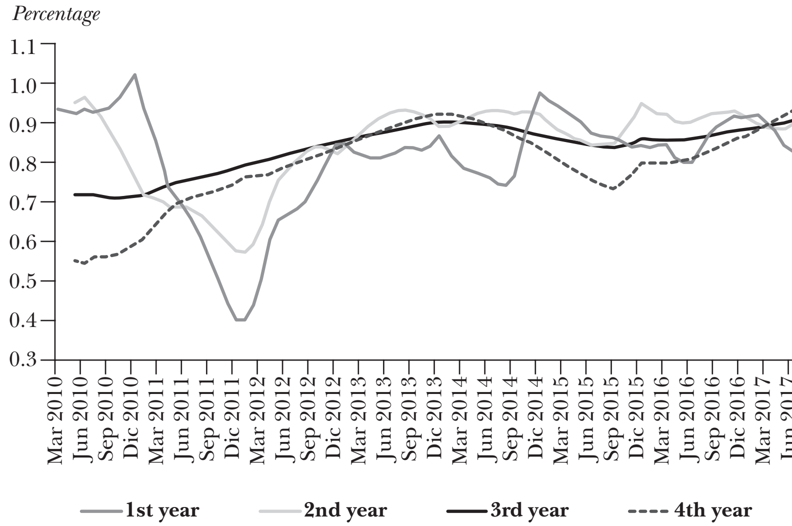
Note: The values in parentheses represent the *p*-values.

The results show a greater degree of anchoring in the medium term than in the short term, in line with the results of Carrasco and Ferreiro (2013), Strohsal, Melnick and Nautz (2015) or IMF (2016). The coefficient of past inflation becomes smaller and not significant in the second, third, and fourth years. Meanwhile the degree of anchoring (coefficient of BCB forecast) is stronger in recent years; it is a difference of almost 10 percentage points between the coefficients in the first and fourth years. The coefficients for the first and second years reflect more volatile behavior over the time (Figure 10). In all of these cases, an improvement in the degree of anchoring can be seen since 2012, with higher or lower fluctuations. The degree of anchoring of inflation expectations is generally greater in the medium term than in the short term.

The BCB does not publish an inflation target for medium-term. Nevertheless, as Strohsal, Melnick and Nautz (2015) mentioned, inflation targets do not have to be officially announced to be effective. Many central

Figure 10

COEFFICIENTS OF BCB PROJECTIONS FOR DIFFERENT YEAR HORIZONS



Source: Authors' compilation based on BCB data.

banks, including the European Central Bank or the U.S. Federal Reserve, do not publish official inflation targets but are able to communicate the level of their inflation objective to the markets.

Although inflation expectations appear to be well anchored in the medium term with respect to past inflation, there is a strand of literature that postulates that long-term (medium-term) expectations should not respond to changes in short-term inflation expectations either (Jochmann, Koop and Potter, 2010; Łyziak and Paloviita, 2016). In that sense, we additionally create a model to study if there is a relationship between medium-term and short-term inflation expectations using the information from Focus Economics.

If medium-term inflation expectations are well anchored, they should not respond to changes from short-term inflation expectations. In this case, following the work of Strohsal, Melnick and Nautz

(2015), medium-term inflation expectations[28] ($\pi^e_{m,t}$) are a function of observed inflation (π_{t-1}), short-term expectations[29] ($\pi^e_{s,t}$) and the implicit inflation target (π^*):

$$\pi^e_{m,t} = \alpha_1 \pi_{t-1} + \alpha_2 \pi^e_{s,t-1} + \alpha_3 \pi^* + \epsilon_t \tag{4}$$

where:

$$\alpha_1 + \alpha_2 + \alpha_3 = 1$$

If $\alpha_1 > 0$ it means that medium-term inflation expectations follow past inflation. If $\alpha_2 > 0$, the information from short-term inflation expectations is relevant for the medium term. With these considerations, medium-term inflation expectations will show a greater degree of anchorage as long as the value of α_3 is close to 1. For inflation expectations to be perfectly anchored it is necessary that $\alpha_1 = \alpha_2 = 0$.

As in the previous cases, a time-varying parameter model is used with monthly data from April 2010 to June 2017. The state parameters follow a random walk process for simplification and variances parameters are expressed in exponential form. The Kalman filter is used to compute the one-step ahead estimates of the means and variances of the states by maximum likelihood. The results of the estimation are shown in Table 4.

Table 4

DEGREE OF ANCHORING OF INFLATION EXPECTATIONS IN THE MEDIUM TERM, MODEL 10

Past inflationn	*Short-term expectations*	*BCB implicit target*
alpha 1	*alpha 2*	*alpha 3*
0.03	0.25	0.71
(0.82)	(0.17)	(0.00)

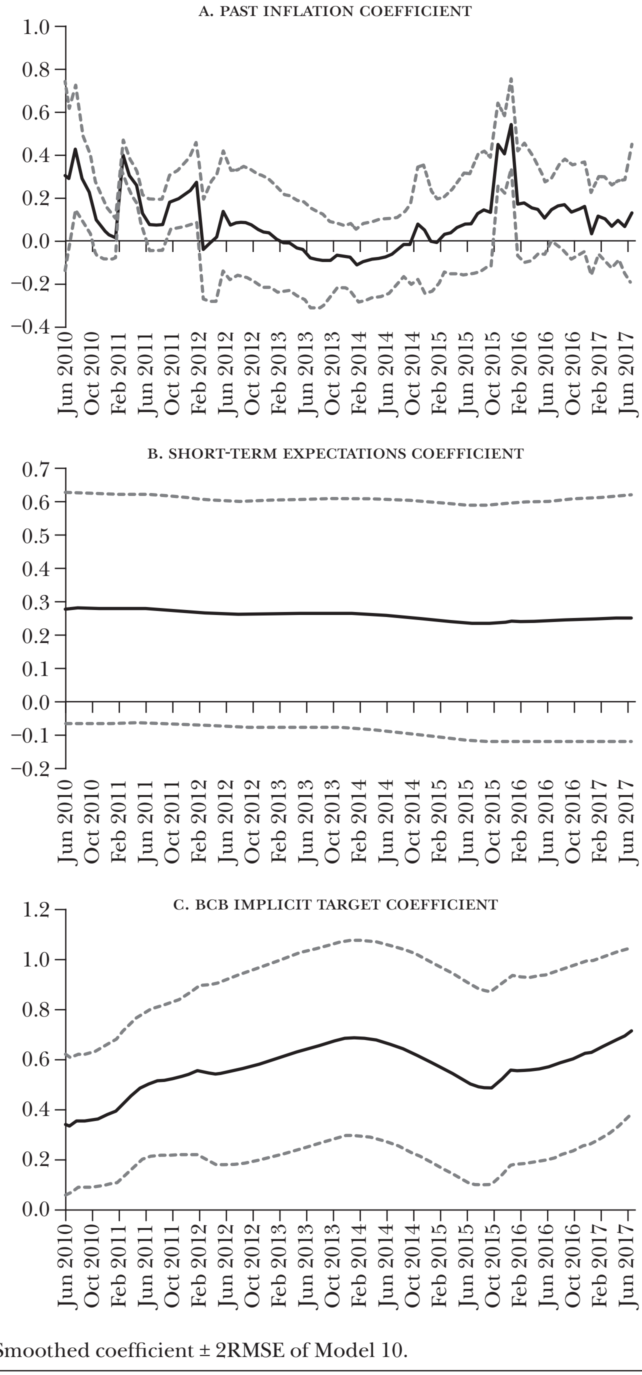
Note: The values in parentheses represent the *p*-values.

[28] As a reference of medium-term we choose the inflation expectations for the fourth year of Focus Economics.

[29] As a reference of short-term we choose the inflation expectations for the first year of Focus Economics, in order to work with the same survey sample.

Figure 11

EVOLUTION OF COEFFICIENTS IN MODEL 10



Note: Smoothed coefficient ± 2RMSE of Model 10.

The past inflation coefficient (Figure 11a) shows erratic behavior over time, reaching its highest values during 2010 and the end of 2015, in the last months its value decreased to 0.03, a low and insignificant value. The short-term expectations coefficient (Figure 11b) displays a value of about 0.25 for the whole sample, being almost constant. However, it is not significant; the effect that this variable could have on medium-term expectations seems to be already rescued with the information of past inflation so it does not present any significant changes to its behavior.

Finally, the BCB implicit target coefficient (Figure 11c) exhibits an upward trend, similar to those observed in other models, with a temporary fall between the second quarter of 2014 and the third quarter of 2015. This coefficient rose from 0.34 in mid-2010 to 0.71 in mid-2017. Under this specification, medium-term inflation expectations reflect a high degree of anchoring since past inflation ceased to be significant and short-term inflation expectations did not have a significant effect throughout the analysis period.

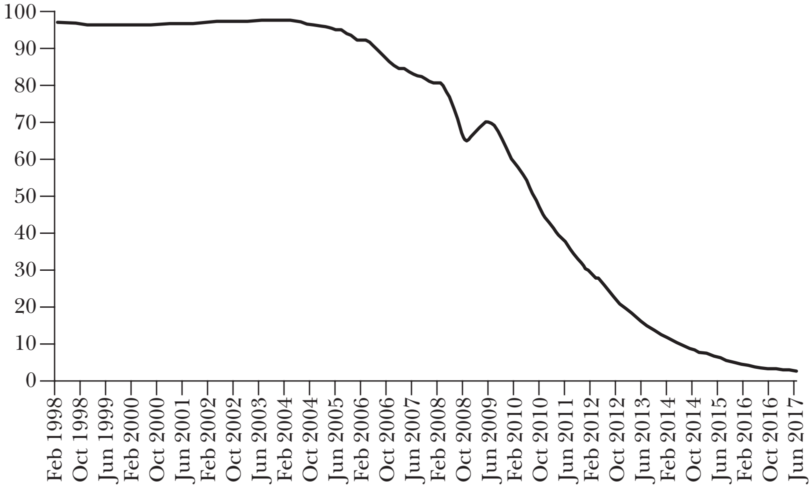
5. SOME CONSIDERATIONS REGARDING THE RESULTS

The results obtained show that there could be a significant degree of anchoring of inflation expectations in Bolivia, both in the short and medium-term, mainly since 2014. In the case of short-term expectations, it is quite noticeable that BCB's projections have greater effect than observed inflation and other variables, unlike other studies that indicate that past inflation has a high relevance in this time horizon (Łyziak and Paloviita, 2016). However, in the medium term (fourth year), as expected, there is a greater degree of anchoring than in the short term (first year). It is also remarkable considering this result was obtained with two different samples (BCB survey and Focus Economics survey).

This behavior indicates a significant improvement in the degree of credibility of the BCB, and it could be associated with several factors. These include the adoption of a more active role by the monetary authority (with a higher degree of intervention in the money market and a more active communication policy), a stable macroeconomic environment, and the progress made in the process of financial de-dollarization.

Figure 12

PERCENTAGE OF DOLLARIZATION OF FINANCIAL LOANS



Source: Central Bank of Bolivia.

During the 1990s and the first five years of the 2000s, almost all of the loans and deposits in the financial system were denominated in U.S. dollars because people in Bolivia had greater confidence in the dollar to carry out their daily transactions. This situation can be attributed to the constant depreciations during this period, which led to a loss of the value of the local currency. In 2006, when the Bolivian appreciated, the degree of financial dollarization in Bolivia began to decrease. This aspect, with other measures applied by the local authorities, allowed the de-dollarization process to accelerate. This in turn created a more favorable environment for monetary policy and a greater role for the BCB in local economic activity. While 97 percent of loans were made in dollars at the beginning of 1998, by mid-2017 this figure had fallen to 2.7 percent (Figure 12). In the same period, deposits in dollars declined from 92.7 percent to 15.6 percent. These developments apparently helped to create a more predictable environment for economic agents.

6. CONCLUSIONS

This study with different specifications of time-varying parameters models shows that a high degree of anchoring of inflation expectations in Bolivia could exist. Our main analysis was performed considering information from the BCB survey, which was complemented with data from Focus Economics survey. Considering the limitations of these data sources, our study focuses mainly on the analysis of the short and medium-term expectations, obtaining good results in both cases.

The results show that the BCB's projections, presented in its Monetary Policy Report have a significant effect on short-term inflation expectations, unlike other studies that indicate that past inflation has a high relevance in this time horizon (Łyziak and Paloviita, 2016). The anchoring of short-term inflation expectations for central banks is not of less importance since these have a relevant role in wage adjustments and price setting by firms. It is remarkable that we found a high level of anchoring degree with two different samples (BCB survey and Focus Economics survey).

In the case of medium-term inflation expectations, we use an implicit inflation target of five percent for time horizons longer than two years. Also, we use information from Focus Economics, which has data on inflation expectations up to five years ahead. Following the work of Łyziak and Paloviita (2016) and Strohsal, Melnick and Nautz (2015), we found that past inflation and short-term expectations do not have a significant impact. Meanwhile, the implicit target would be the main reference for the formulation of medium-term inflation expectations.

This research paper represents a first step in understanding the behavior of inflation expectations in Bolivia. There are not many studies that have analyzed their conduct or how they react to the announcements made by the BCB about the future trajectory of inflation. Since 2006, the BCB has actively participated in press conferences, seminars and presentations in order to forge a closer relationship with the population in general (academics, experts, students, reporters, and others). The results of this paper show that the BCB's projections may have excerted a greater influence on agents' inflation expectations in recent years. However, more studies should be carried out to understand and evaluate better the capacity of the BCB to anchor the inflation expectations of the Bolivian population.

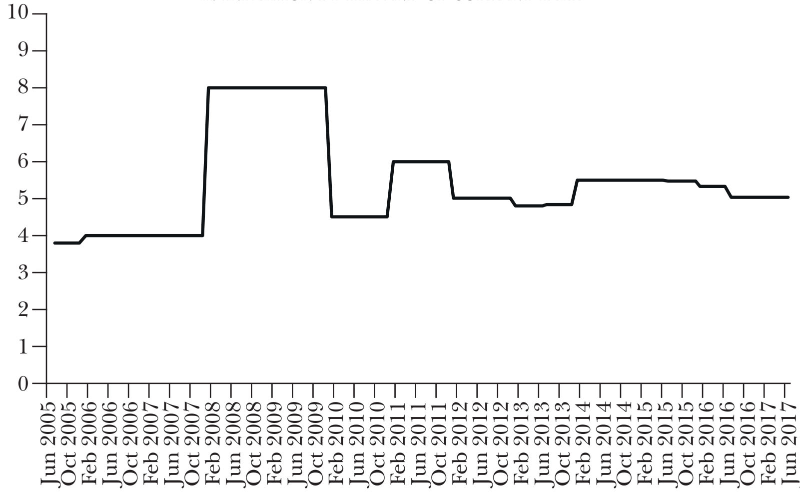
ANNEXES

Annex 1. BCB Projections

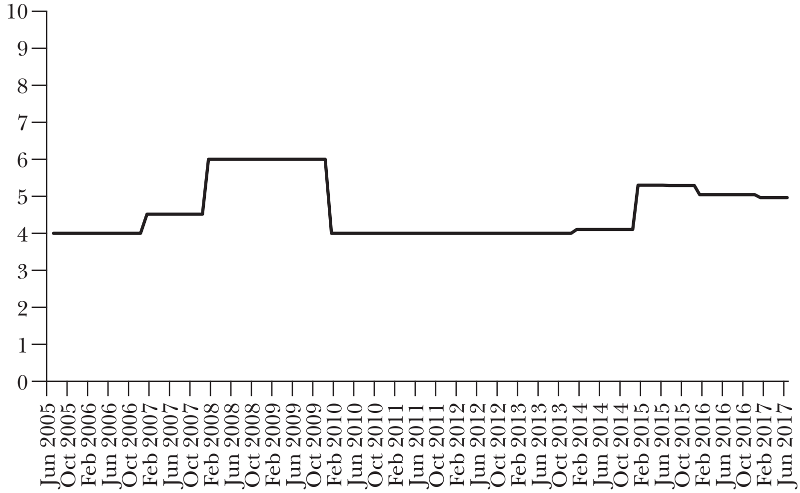
Figure A.1

ORIGINAL BCB PROJECTIONS

A. INFLATION BY THE END OF CURRENT YEAR



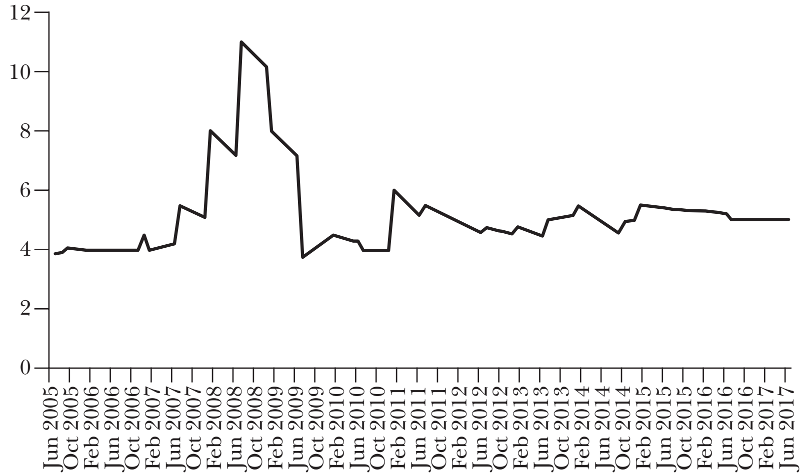
B. INFLATION BY THE END OF NEXT CALENDAR YEAR



Note: Central Bank of Bolivia.

Figure A.2

UPDATED BCB PROJECTION (INFLATION BY THE END OF CURRENT YEAR)



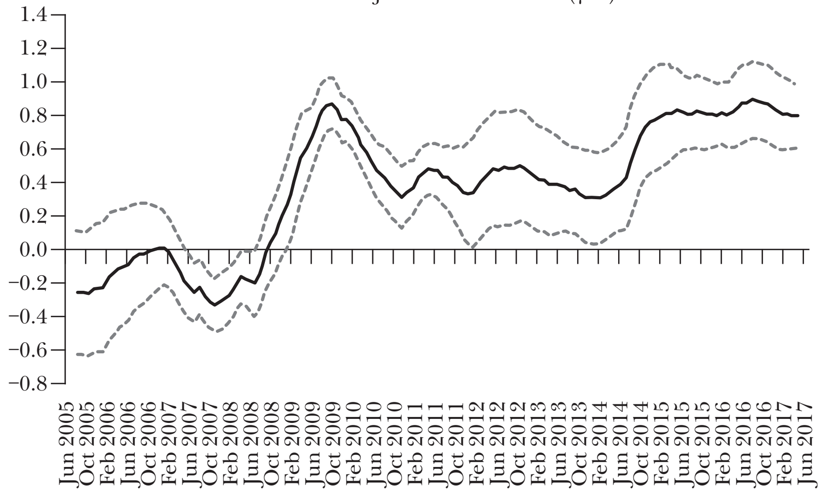
Source: Central Bank of Bolivia.

Annex 2

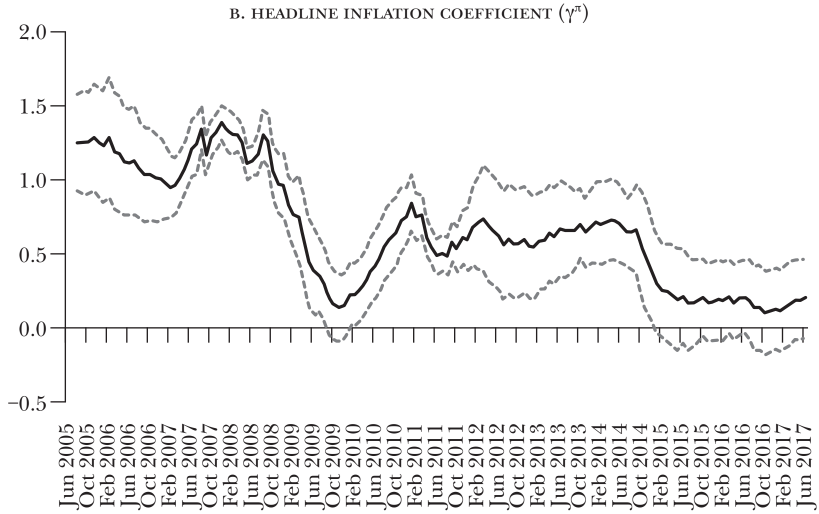
Figure A.3

EVOLUTION OF COEFFICIENTS IN ALTERNATIVE MODEL 1

A. UPDATED BCB PROJECTION COEFFICIENT (γ^{proj})



B. HEADLINE INFLATION COEFFICIENT (γ^{π})



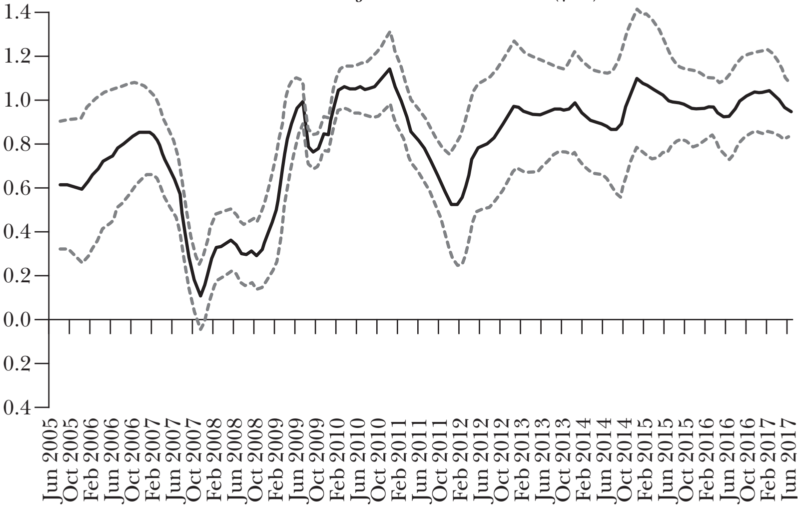
Note: Smoothed coefficient ± 2RMSE.

Annex 3

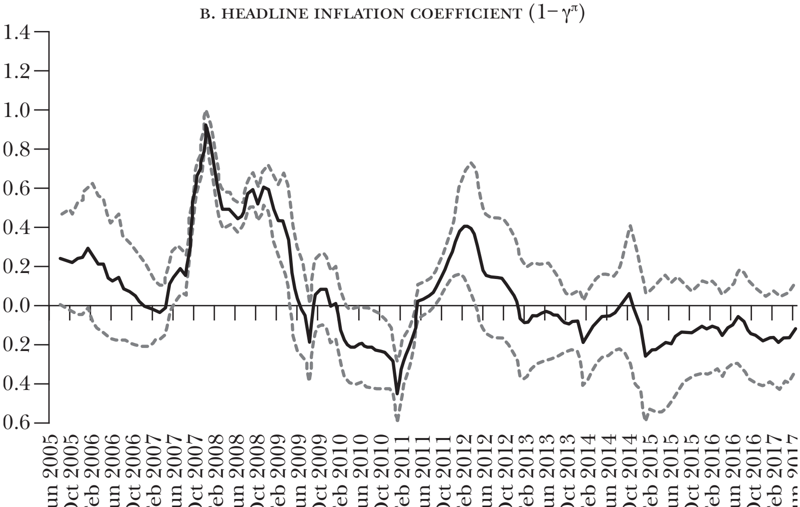
Figure A.4

EVOLUTION OF COEFFICIENTS IN MODEL 6

A. UPDATED BCB PROJECTION COEFFICIENT (γ^{proj})



B. HEADLINE INFLATION COEFFICIENT ($1-\gamma^{\pi}$)



Note: Smoothed coefficient ± 2RMSE.

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Expectations Anchoring Indexes for Brazil Using Kalman Filter: Exploring Signals of Inflation Anchoring in the Long Term

Fernando Nascimento de Oliveira
Wagner Piazza Gaglianone

Abstract

Our objective in this paper is to build expectations anchoring indexes for inflation in Brazil that are fundamentally driven by the monetary authority's capacity to anchor long-term inflation expectations vis-à-vis short-run inflation expectations. The expectations anchoring indexes are generated from a Kalman filter, based on a state-space model that also takes into account fiscal policy dynamics. The model's signals are constructed using inflation expectations from the Focus survey of professional forecasters, conducted by the Banco Central do Brasil, and from the swap and federal government bond markets, which convey daily information of long-term inflation expectations. Although varying across specifications, the expectations anchoring indexes that we propose tend to display a downward trajectory, more clearly in 2009, and show a recovery starting in 2016 until the end of the sample (mid-2017).

Keywords: credibility index, inflation expectation, inflation anchoring, Kalman filter, Banco Central do Brasil.

JEL classification: E50, E52, E58.

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1. INTRODUCTION

Well-anchored inflation expectations are fundamental for the conduct of monetary policy. Properly anchoring inflation expectations requires the central bank to be regarded as credible, that is, economic agents should be confident that the central bank will react to the various shocks that affect the economy to maintain price stability.

Cukierman and Meltzer (1986) stressed that the future objectives of central banks depend on inflation expectations. In this sense, a credible commitment to an explicit inflation objective helps to anchor inflation expectations to the desired level. This anchoring contributes to delivering price stability, which is the main objective of central banks.

In turn, Blinder (2000) sent questionnaires to 127 heads of central banks around the world asking their opinion on the importance of central bank credibility. The answers showed clearly that credibility matters in practice. A credible central bank is one that can make a believable commitment to low inflation policy and has complete dedication to price stability. This will make disinflation less costly and decrease the sacrifice ratio.

Nonetheless, building credibility is costly and takes repeated successes to establish. Moreover, credibility evolves in asymmetric fashion and can be lost rapidly, depending on the perception by economic agents that the central bank is able (or not) to achieve its objectives. As famously put by Benjamin Franklin: "It takes many good deeds to build a good reputation, and only one bad one to lose it."[1]

Central banks have imperfect control over inflation in the short run. As Gomme (2006) remarked, current inflation provides a noisy signal of a central bank's long-term intentions, and therefore of its type. According to the author, a central bank is credible when the public assigns a high probability of low inflation-type to the central bank. In this context, a central bank will lose credibility when this probability decreases. The credibility of central banks is very much concerned with people's beliefs about what the central bank will do in the future.

[1] See Isaacson (2004).

On the other hand, central bank credibility is a latent variable[2] and, consequently, it is not easy to measure in practice. One possibility is to look for measures that reflect the capacity central banks have to anchor inflation expectations. In the literature, this is done mostly by looking at how closely short-run expectations match the central bank's explicit or implicit inflation target (see Bordo and Siklos, 2015). The problem with these measures, in our view, is that other signals can exist in the economy that may also help to give an idea of how well inflation expectations are anchored.

Figure 1 compares the consensus inflation forecast in Brazil (horizon of one year) with the inflation target and respective tolerance bands. Based on these series, Figure 2 shows the evolution of some credibility indexes (hereafter CIS) for the Banco Central do Brasil (BCB) from January 2002 to June 2017. The measures are, respectively, CI-CK (Cecchetti and Krause, 2002), CI-M (Mendonça, 2004) and CI-MS (Mendonça and Souza, 2009).

These indexes measure deviations of short-run inflation expectations from BCB's inflation target.[3] For instance, note that at the end of 2002, before the presidential election, these indexes had a substantial decline in credibility. This fact can be related to an exogenous shock to BCB: the uncertainty about the policy regime with a likely victory of the presidential candidate Lula, which triggered the country sovereign risk premium (EMBI+BR) to sharply rise during this period. This was a situation completely out of BCB's control.[4]

Also, note that Figure 2 shows a very volatile CI-M, considering the whole sample, indicating a fast loss and recovery of credibility. The other indexes show different behavior of credibility: CI-CK varies very little, while CI-MS looks constant almost all the time. In fact,

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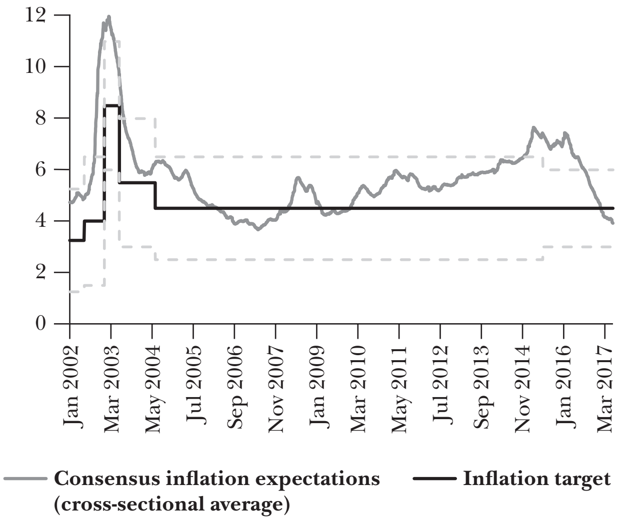
[2] The international literature on credibility indexes of central banks is vast. They are many theoretical as well as empirical papers on the subject. See, for example, Gomme (2006), Svenson (1993), Clarida and Waldman (2007), Ceccheti and Krause (2002), Kaseeream (2012) and Bordo and Siklos (2015).

[3] Other papers also build credibility indexes for the Banco Central do Brasil focusing on deviations of short-term inflation expectations from inflation target, such as Teles and Nemoto (2005), Sicsú (2002), Nahon and Meurer (2005), and Lowenkron and Garcia (2007).

[4] Note that CI-M decreases substantially during the subprime crisis, which like Lula's election is also exogenous to BCB. At the end of the period, CI-M shows a steep credibility recovery that also seems counterfactual.

Figure 1

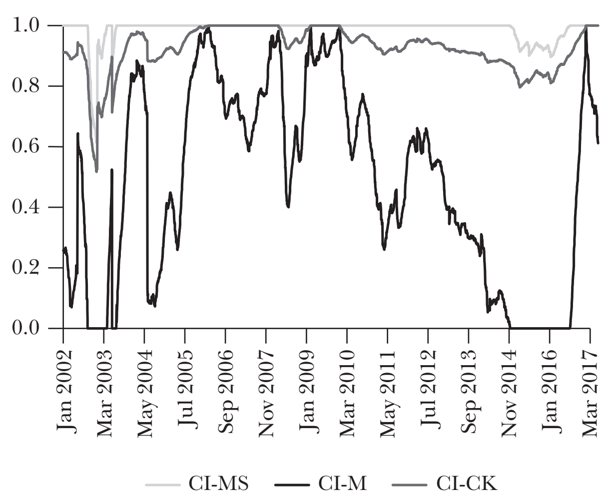
SURVEY-BASED INFLATION EXPECTATIONS, INFLATION TARGET AND TOLERANCE BANDS



Note: Average inflation expectations (Focus survey) with forecast horizon of one year. Inflation targets and tolerance bands from <http://www.bcb.gov.br/pec/metas/InflationTargetingTable.pdf>.

Figure 2

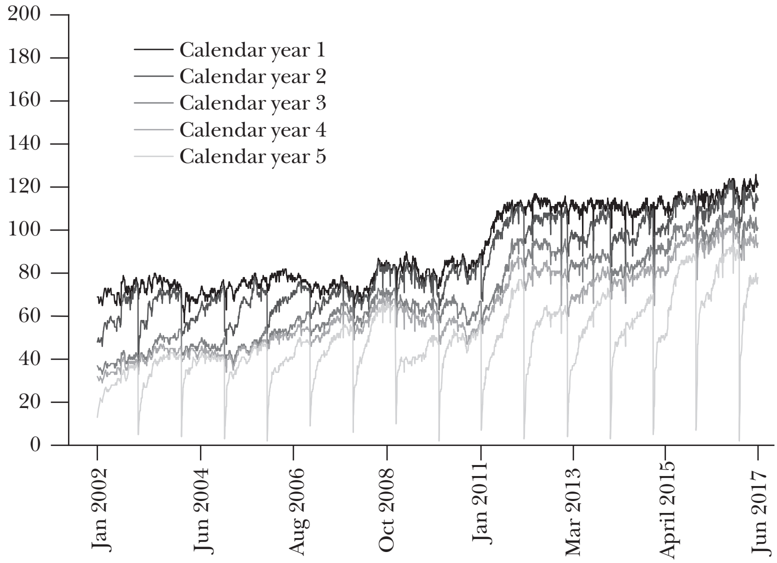
CREDIBILITY INDEXES FROM THE LITERATURE



Note: CI-CK means Cecchetti and Krause (2002), CI-M denotes Medoça (2004) and CI-MS means Mendoça and Souza (2009). Inflation expectations are the survey-based cross-sectional average expectations with fixed horizon of one year.

Figure 3

DAILY NUMBER OF SURVEY PARTICIPANTS THAT REPORT INFLATION FORECAST FOR THE CURRENT AND THE FOLLOWING CALENDAR YEARS (END-OF-YEAR FIXED-EVENT FORECAST)



Source: Banco Central do Brasil and authors' calculations.

the credibility dynamics implied by these indexes seem not to appropriately represent the dynamics of mean and standard deviation inflation expectations measured in fixed horizons and taken from BCB's daily survey of expectations (*Focus*), presented in Figure 5. The first graph shows that the cross-sectional mean of inflation expectations with a forecast horizon of four years–a measure of long-term expectations–has much less volatility than the one-year (short-term) inflation expectations. Not only that but in the run-up to Lula's election and the subprime crises, the four-year expectations varied much less than the one-year counterpart. The second graph of Figure 5 shows a similar dynamic pattern for the short-run (one year) and long-run (four years) standard deviation of inflation expectations.[5]

[5] There are other papers in the literature that build credibility indexes for the BCB taking different approaches from those that look at short-term deviations of inflation expectations from the target. This is the

In practice, one should examine a variety of signals to construct a measure that really reflects the ability of central banks to anchor inflation expectations (see Demertzis *et al.*, 2012). We think that the problem with most traditional CIS available in the literature is that they focus on the short-run deviations of inflation expectations from the inflation target. In contrast, we construct in this paper expectations anchoring indexes (hereafter, EAIs) that are specifically designed to measure the degree of anchoring of long-term inflation expectations vis-à-vis the short-run.

The bottom-line of our argument is that a central bank is credible if it has the capability to properly anchor long-run inflation expectations. The extent of long-term inflation anchoring will serve as a proxy for anchoring. If the central bank is credible and anchors long-term inflation expectations, then the long-run expectations will become less responsive to short-run economic news.[6] This means that in the presence of a negative or positive short-term shock to inflation, economic agents believe the central bank will take appropriate countervailing actions to keep inflation on target in the long run.

Our view is in line with Demertzis *et al.* (2012) and Buono and Formai (2016). Demertzis *et al.* point out that the credibility of the central bank decouples long-run inflation expectations from short-run expectations. Buono and Formai notice that inflation expectations are anchored when movements in short-run expectations do not affect movements in the long term.[7]

To build expectations anchoring indexes for inflation in Brazil that decouple long-term from short-term inflation expectations, we also need to incorporate explicitly in our approach some measure

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case of Garcia and Guillén (2011), Leal et al. (2012), Issler and Santos (2017), and Val et al. (2017).

6 Bernanke (2007) describes inflation anchoring in the following manner: "…"anchored" to mean relatively insensitive to incoming data. So, for example, if the public experiences a spell of inflation higher than their long-run expectation, but their long-run expectation of inflation changes little as a result, then inflation expectations are well anchored. If, on the other hand, the public reacts to a short period of higher-than-expected inflation by marking up their long-run expectation considerably, then expectations are poorly anchored".

7 For other empirical papers with definitions of credibility, see Davis (2012), Levieuge *et al.* (2015) and Dimitris *et al.* (2016). For theoretical papers with definitions of central bank credibility, see Barro and Gordon (1983), Walsh (1995) and Blackburn and Christensen (1989).

of fiscal policy. The reason is that, in some periods in Brazil, perceptions about fiscal policy and fiscal sustainability seemed to have played an important role in explaining inflation expectations. If we do not control for that, processes of deanchoring of expectation may be attributed to the BCB's policies and not to broader economic policies. In emerging countries where the public debt is high (in terms of GDP) and with short average maturity, periods of fiscal dominance may occur.

As Sargent and Wallace (1981) argue, under fiscal dominance, the monetary authority faces the constraints imposed by the demand for government bonds. If the fiscal authority cannot finance its deficits solely by new bond sales, then the monetary authority is forced to create money and tolerate additional inflation. Although such a monetary authority might still be able to control inflation over the long run, it is less capable than a monetary authority under a no fiscal dominance situation. Blanchard (2004) argues that fiscal dominance describes the situation of the Brazilian economy in 2002 and 2003.

In periods of fiscal dominance, there may be a reversal of the traditional roles of monetary and fiscal policies: central banks are inclined to reduce interest rates when inflation rises, the opposite of their standard response, in order to guarantee the stability and solvency of debts and deficits. Therefore, in such periods even a credible central bank may find difficulty in keeping long-term inflation expectations unaffected by short-term shocks on inflation or short-term inflation expectations.

Our objective in this paper is to build EAIs for BCB that are fundamentally driven by the capacity the BCB has to anchor long-term inflation expectations vis-à-vis short-run expectations. The EAIs will be constructed from a Kalman filter, based on a linear state-space model that also takes into account fiscal policy dynamics. The signals of the state-space model will give information on the anchoring of long-term inflation expectations.

There are many possible signals of long-term inflation anchoring in the literature,[8] based on nonparametric or parametric approaches. We use as many signals as possible from all sources that are available. In this sense, we have disaggregated daily data (from January 2002 to June 2017) of inflation expectations from the *Focus*

[8] See Natoli and Sigalotti (2017).

survey of professional forecasters conducted by the BCB. From this survey, we extracted 17 signals. We also have market data of nominal federal government bonds (*Letras do Tesouro Nacional,* hereafter LTN) and inflation-indexed bonds (*Notas do Tesouro Nacional*, hereafter NTN-B) from April 2005 to June 2017. Finally, we have information on swaps of fixed interest rate instruments against inflation from January 2005 to June 2017. From the bond and swap markets, we extracted 14 signals.

We contribute to the literature in several manners. Firstly, as far as we know, this is the first paper to use a large number of signals of long-term inflation expectation anchoring, coming from both surveys and market data. Secondly, we focus on long-term inflation expectations, unlike the great majority of empirical papers on the subject in Brazil.[9] We can update our EAIs on a daily basis with disaggregated and aggregated data obtained through surveys or through market information. By construction, our EAIs give a prompt idea of how well the long-term inflation expectations are anchored, which is very important in the implementation of monetary policy, especially in an inflation targeting regime.

In the third place, we take into account both fiscal policy and monetary policy when estimating the state-space model using our survey and market data for long-term inflation expectation anchoring compared to short-run inflation expectations. Finally, the disaggregated confidential survey data of the BCB–an essential part of our database–is unique and enables us to have a much better grasp of inflation expectations of economic agents in Brazil, and hence of BCB's ability to anchor them.

The rest of the paper is organized as follows: Section 2 describes the data; Section 3 presents the empirical analyses, and Section 4 concludes.

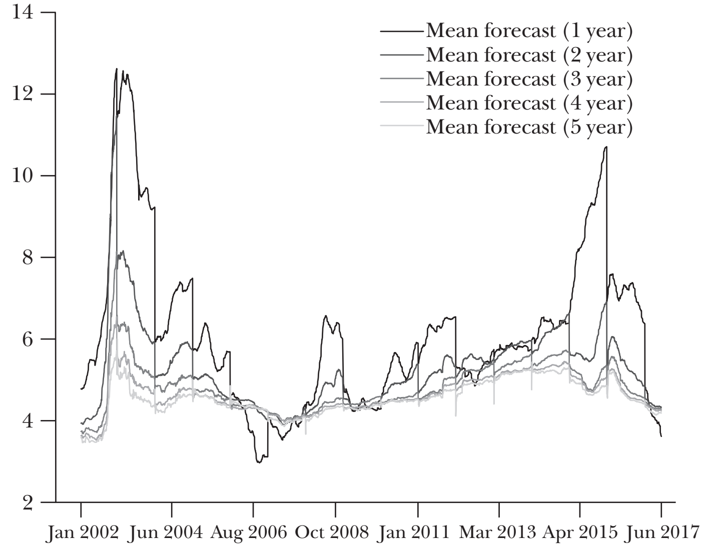
2. DATA

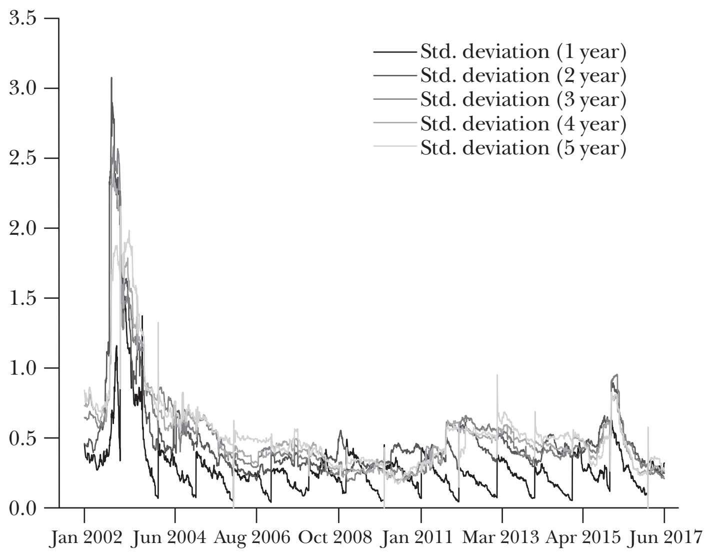
We have survey and market data. In the former case, we have data from January 2002 to June 2017. In the latter case, we have data from April 2005 to June 2017.

[9] See Gaglianone (2017) for a recent survey of applied research on inflation expectations in Brazil.

Figure 4

SURVEY DATA: CROSS-SECTIONAL MEAN, MEDIAN, STANDARD DEVIATION AND INTER-QUARTILE RANGE OF INDIVIDUAL SURVEY-BASED INFLATION FORECASTS (FIXED EVENTS)
Raw data from the focus survey (calendar-year forecasts)

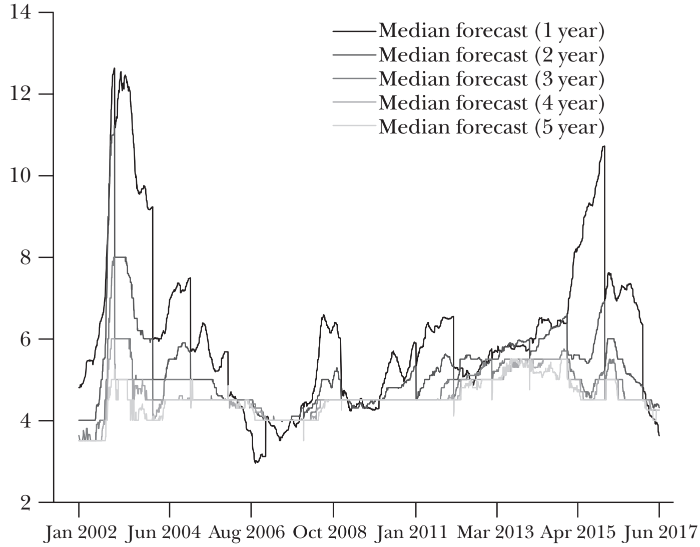


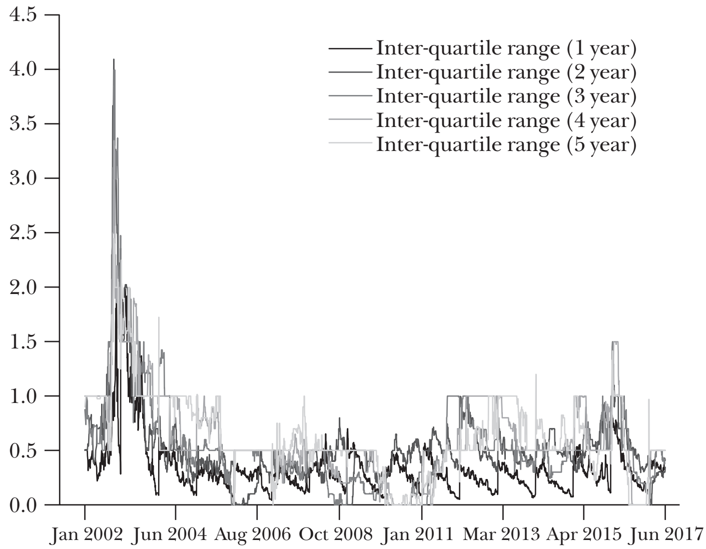


Source: Banco Central do Brasil and authors' calculations.

Figure 4 (cont.)

SURVEY DATA: CROSS-SECTIONAL MEAN, MEDIAN, STANDARD DEVIATION AND INTER-QUARTILE RANGE OF INDIVIDUAL SURVEY-BASED INFLATION FORECASTS (FIXED EVENTS)
Raw data from the focus survey (calendar-year forecasts)





Source: Banco Central do Brasil and authors' calculations.

Our survey data are proprietary, with confidential information at the individual level and publicly available data at the aggregate level. The data were obtained from the *Focus* survey organized by the BCB, collected every workday by the BCB.[10] We have the distribution of inflation expectations for every workday.

We have unbalanced panel data of survey inflation expectations. The number of registered institutions that take part in the survey is 277 in our sample. The number of workdays in our sample is 3,781. The average number of institutions that report inflation forecasts is 83 for the forecast horizon of one year and 48 for the four-year horizon.

Figure 3 presents the number of institutions that forecast inflation every workday for one year up to five years. As can be seen, there are some workdays on which very few institutions reported. This is particularly relevant in the case of forecasts for four or five years. In addition, for each end-of-year inflation, the number of institutions reporting forecasts increases as long as the forecast horizon diminishes. To avoid problems in our estimations, we consider that when there were fewer than 10 institutions reporting on a certain workday, we repeat the forecasts of the previous workday in which there were more than 10 institutions reporting for the same period.

Raw information on inflation expectations pertains to fixed events (e.g., end-of-year inflation forecasts for the current and following years); see Figure 4. We transform them to fixed-horizon inflation expectations by linear interpolation using the daily (decreasing) forecast horizon of the fixed-event inflation forecasts; see Figure 5. Since the longest horizon of inflation forecasts available in the *Focus* survey involves the five-year-ahead forecast (calendar year), we employ the inflation expectations for the following four and five calendar years to build the interpolated forecast with a maximum fixed horizon of four years.

On the other hand, there is no inflation target set for such long horizons. Since the beginning of the inflation targeting regime in 1999 and up to the inflation target announced for 2019, the inflation target

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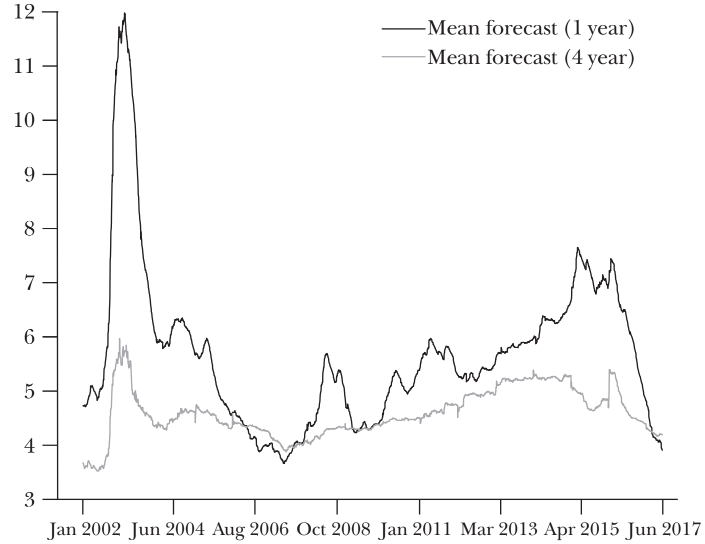
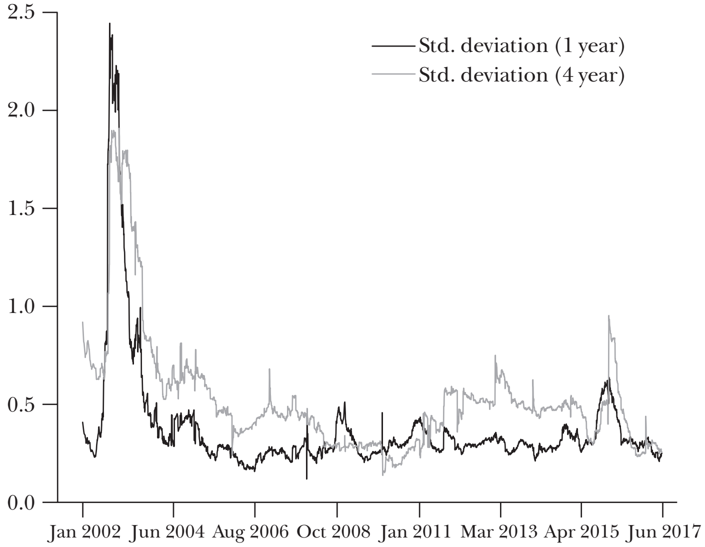
[10] Nowadays, the BCB releases on the internet the micro data of the Focus survey of expectations, in a panel data with fake IDs (i.e., the identity of the survey participants is preserved and the disclosed database only contains anonymous participants). For more details, see the website: http://dadosabertos.bcb.gov.br/dataset/expectativas-mercado/resource/23f6c983-f9bd-48f8-a889-72def3ae17c8

Figure 5

SURVEY DATA: CROSS-SECTIONAL MEAN, MEDIAN, STANDARD DEVIATION AND INTER-QUARTILE RANGE OF INDIVIDUAL SURVEY-BASED INFLATION FORECASTS (FIXED HORIZONS)
Transformed data (fixed-horizons forecasts)

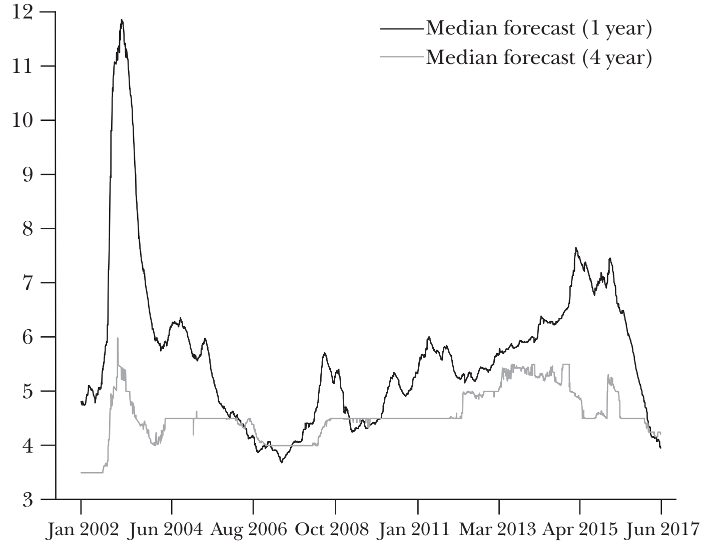


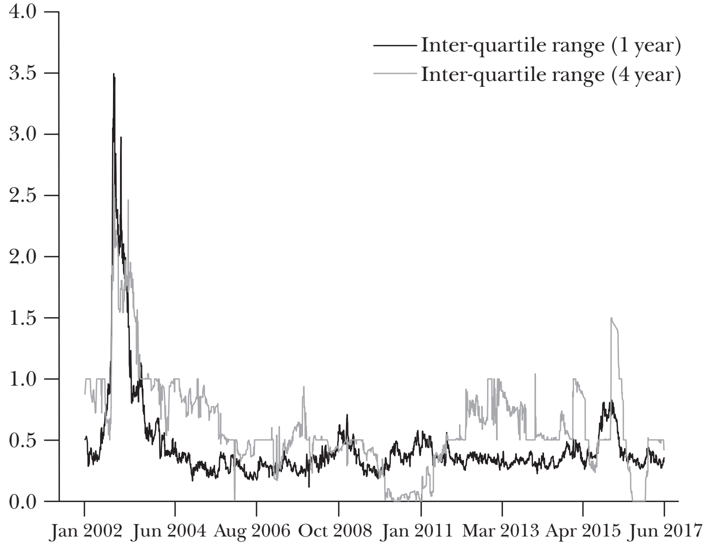


Source: Banco Central do Brasil and authors' calculations.

Figure 5 (cont.)

SURVEY DATA: CROSS-SECTIONAL MEAN, MEDIAN, STANDARD DEVIATION AND INTER-QUARTILE RANGE OF INDIVIDUAL SURVEY-BASED INFLATION FORECASTS (FIXED HORIZONS)
Transformed data (fixed-horizons forecasts)

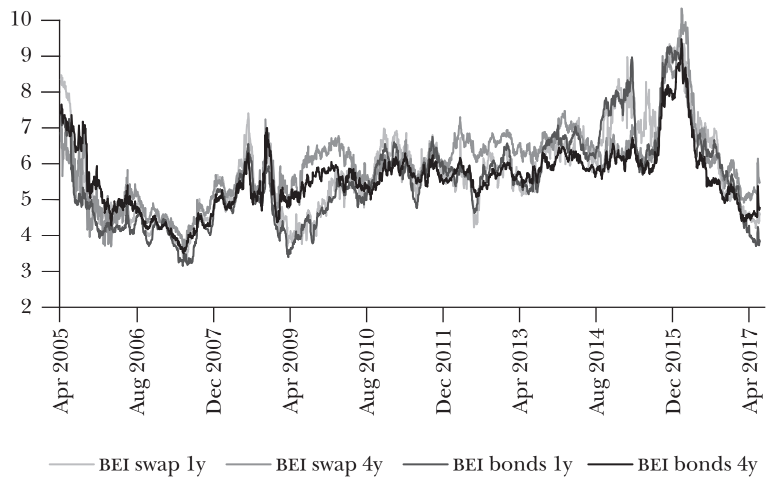




Source: Banco Central do Brasil and authors' calculations.

Figure 6

MARKET DATA: BREAKEVEN INFLATION
BEI, percentage 12 months



Source: Anbima, B3 and authors' calculations.

and tolerance bands had been set up to June of year *t* for the calendar year *t+2*. Nowadays, the new target is announced up to June of year *t* for the calendar year *t+3*.[11] Since many signals depend on the inflation target, and since our longest forecast horizon is four years, we assume that the inflation target four years ahead is equal to the target set for the calendar year *t+2* (or *t+3*, whenever available).

In the case of market data, we have publicly available information on federal government bonds and swaps of fixed interest rate against inflation and a coupon from April 2005 to June 2017. The former are obtained from Anbima (Brazilian Financial and Capital Market Association) and the latter are registered by B3 (a Brazilian company that operates securities, commodities and futures exchange, among others, previously known as BM&FBOVESPA). Federal government bonds are nominal bonds (LTNs) and inflation-indexed bonds

[11] See <https://www.bcb.gov.br/pec/metas/InflationTargetingTable.pdf>.

(NTN-Bs). The yields of these bonds for different maturities are calculated by fitting LTN and NTN-B with the Nelson-Siegel-Svensson functional form.

The difference between yields of the same maturity of LTNs and NTN-Bs is known as breakeven inflation (hereafter BEI). According to Shen (2006): "An increase in the breakeven rate is sometimes viewed as a sign that market inflation expectations may be on the rise. For example, the FOMC frequently refers to the yield spread as a measure of 'inflation compensation' and considers the yield spread an indicator of inflation expectations in policy deliberations."[12] In this paper, we use BEI series as proxies of market inflation expectations. It is important to note that these measures are embedded with a liquidity premium as well as an inflation risk premium that might distort it from pure measures of inflation expectations.

Swaps of inflation plus a coupon against fixed interest rates are registered by B3. The BCB collects workday information in this respect. The difference between fixed rate and coupon gives BEIs of swaps. One advantage of BEIs coming from swaps–compared to BEIs from federal government bonds–is that they have very low liquidity premiums.[13] Figure 6 shows the dynamics of BEI from swaps and federal government bonds with maturities of one and four years.

In both Figures 5 and 6, it is easy to observe that four-year survey inflation expectations and four-year BEIs have lower variance and are more persistent than one-year inflation expectations and one-year BEIs, respectively.

As for an indicator of high frequency fiscal policy, we use workday expectations of primary balance as a percentage of GDP. These data are also collected from the *Focus* survey. We use in our empirical analyses the one-year ahead expectations. The raw data on the expectations are for fixed events and we transform them for a fixed horizon by linear interpolation in exactly the same way as we do for inflation expectations.

Figure 7 shows the dynamics of this series. As can be seen, there is a clear turning point in fiscal expectations in our sample. Until

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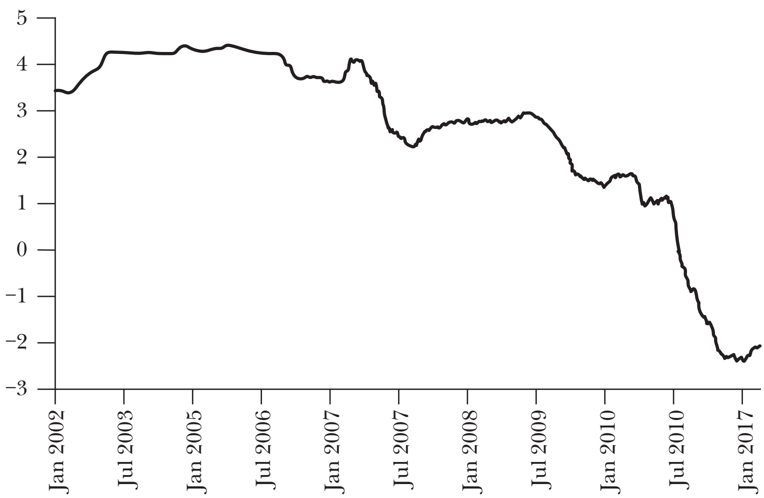
12 FOMC means the Federal Open Market Committee of the U.S. Federal Reserve.

13 We have yields for fixed-interest bonds with maturities of one, three and ten years. We interpolate linearly the three- and ten-year yields to get the four-year yields that we used to construct BEIs for the swap market.

Figure 7

CONSENSUS SURVEY-BASED EXPECTATIONS OF PRIMARY FISCAL BALANCE (z_t)
Percentage of GDP, forecast horizon of 12 months



Source: Banco Central do Brasil, *Focus* survey, cross-section average expectations.

2009, the expectations were relatively stable around a primary surplus of 4% of GDP. From mid-2009 until mid-2012, expectations fluctuated near a primary surplus of 3% of GDP. However, from mid-2012 on there was clear deterioration of these expectations, reaching a primary balance of -2% of GDP in the beginning of 2017.

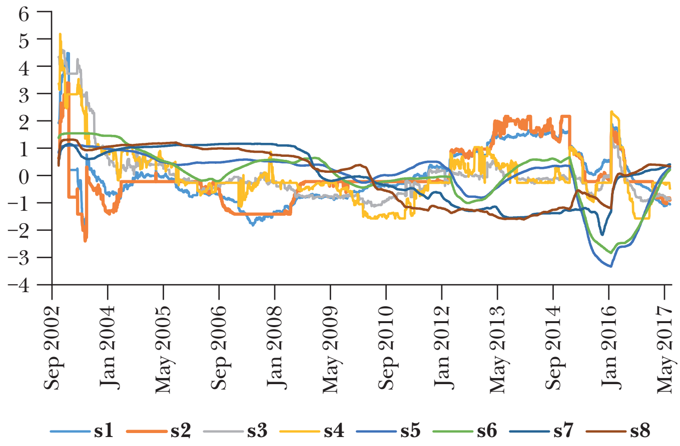
3. EMPIRICAL ANALYSIS

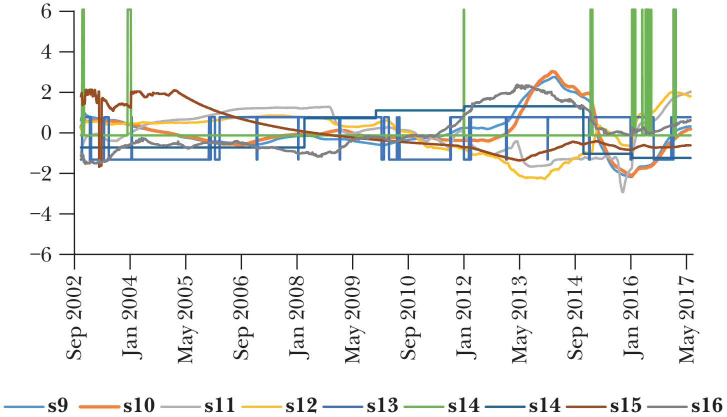
Our method to construct the expectations anchoring indexes can be summarized as follows:

1) we build a set of normalized (i.e., zero mean and unit variance) signals from both survey and market data; 2) we employ factor analysis to summarize the panel data information of signals into a single "common factor" series that contains the core dynamics of long-term inflation expectation anchoring with respect to the short-run

Figure 8

SURVEY SIGNALS
Exponential smoothing, half-life of one year





inflation expectations; 3) we estimate a state-space model using a Kalman filter to build two separate states for monetary policy credibility and fiscal stance; and 4) we employ a logit transformation to set the scale of states into the [0;1] interval.

We next describe the signals of long-term inflation anchoring that we used in the paper.

3.1 Signals of Long-term Inflation Anchoring

Some of our signals are based on recursive correlations or recursive regressions. In these cases, we used a training sample of six months (126 workdays) in order to generate the first signal observation. Moreover, we treated the observations of our recursive analyses in three different ways: each observation was weighted by exponentially smoothed weights with a half-life of one or two years,[14] or by using a rolling window of three years. Moreover, all the signals that we used to build our EAIs were normalized z-scores (i.e., with zero mean and standard deviation equal to 1).

3.1.1 Signals from Survey Data

Table 1 lists the signals that we extracted from the BCB survey. We built signals based on recursive Pearson correlation and recursive ordinary least squares (OLS) of mean and median four-year inflation expectations against one-year inflation expectations. We also built signals based on recursive correlations and recursive OLS between the standard deviation and inter-quartile range of four- and one-year inflation expectations. In the case of regressions, our signals are the slope coefficients of the regressors related to one-year inflation expectations.

We built a signal based on the estimation of time-varying VAR as in Demertzis *et al.* (2012). The estimation is based on Stock and Watson (1996). The coefficients vary through time like random walks. The coefficient of interest is the one that measures the elasticity of four-year inflation expectations in relation to one-year inflation expectations.

[14] In other words, for a given sample, a weight equal to 1 is attached to the most recent observation. After a half-life period (e.g., 1 year =252 workdays), the weight exponentially decays to 0.5.

We built two signals based on the evolution of the distribution of the four-year inflation expectations. One signal is equal to 0 if the median of the distribution is equal to the inflation target and 1 otherwise. The other signal is equal to 0 on workday *t* if the distribution on this day is equal to the distribution on workday *t-21* (previous month) and 1 otherwise, based on the Kolmogorov-Smirnov test.[15]

We built another signal based on Nautz and Strohsal (2015). The authors estimate by OLS a multiple regression between long-term inflation expectations and lag of long-term inflation expectations and surprises in macroeconomic variables. We tested for the possibility of structural breaks between the dependent variable and the regressors that measure macroeconomic surprises according to Andrews (1993) and Quandt (1960)[16]. We used as macroeconomic variables levels of the nominal foreign exchange rate (R$/US$), EMBI+BR and the yield of the 360 days interest rate swap. We considered a surprise in these macroeconomic variables when the value of the series is higher (or lower) than the mean of the series plus (minus) one standard deviation. Our coefficient of interest is the one related to the nominal foreign exchange rate.

We built a signal based on recursive logistic regressions, with equal weights for the time series observations, such as in Natoli and Sigalotti (2017). The model estimates the probability that four-year inflation expectations will be higher or lower than the 75% percentile of the workday distribution of this series (the dependent variable is 1 if it is higher and 0 if it is lower). This probability is estimated given that the one-year inflation expectations were higher or lower than the 75% percentile of the distribution of the same workday of this series (the regressor is 1 if it is higher and 0 if it is lower). Our coefficient of interest is the one related to the one-year inflation expectations.

Figures 8, 9 and 10 show the evolution of the signals above–normalized z-scores with zero mean and standard deviation equal to 1– of recursive regressions estimated with exponentially smoothed

[15] See Massey (2012).

[16] In this paper, we employ the idea behind the Quandt-Andrews test, in which a single Chow (1960) breakpoint test is performed for every observation between two dates. The test statistics from those Chow tests are used to build dummy variables representing the different regimes between breakpoints.

weights with a half-life of one or two years or using weights from a rolling window of three years.

3.1.2 Signals from Market Data

In the case of market data, we built signals based on BEIs of one year and four years obtained in the swap and bond markets. Several of the signals were obtained in exact ways described in the previous section. We included two different signals from the survey signals: one is the difference between BEI and the inflation target and the other one is the square of this difference. Table 2 lists the market signals and Figures 11, 12 and 13 show the evolution of the market signals.

3.1.3 Selection of Signals Based on Correlation Analysis

We have a total of 31 signals: 17 are selected from survey data and 14 are selected from market data. To obtain our benchmark EAIs that we present in Section 3.4, we select from these 31 signals the ones whose correlations are less than 0.7. Table 3 shows the correlation matrix of the selected signals. As a result, the following 14 signals were selected: S3, S9, S12, S13, S14, S15, S17, SM3, SM4, SM7, SM8, SM9, SM12, and SM14.

3.2 Factor Analysis

Next, we employ factor analysis (FA) to extract common factors from the set of signals chosen. There are many ways suggested in the literature to combine the set of signals into a single indicator (e.g., equal weights or PCA–principal component analysis). We adopt the factor analysis setup,[17] since our goal here is to build a single time series that reflects long-term anchoring of inflation expectations (in respect to short-run inflation expectations) by extracting common movements from the set of selected signals.

To do so, we use the *principal factors* as the factor extraction method and the *ordinary correlation* for covariance analysis. The idea is to

[17] Factor analysis (FA) and principal component analysis (PCA) are similar statistical techniques in the sense that both generate linear combinations of the original series. However, pca is used to retain the maximum amount of information from data in terms of total variation, whereas fa accounts for common variance. Thus, fa is often employed to build factors (latent variables), while pca is often used in data reduction frameworks. See Johnson and Wichern (1992) for further details.

Table 1

SIGNALS CONSTRUCTED FROM SURVEY-BASED INFLATION EXPECTATIONS

Group	*Signals*	*Description*
1	S1	cross-section mean forecast long run - inflation target
1	S2	cross-section median forecast long run - inflation target
1	S3	cross-section standard deviation (forecast long run - inflation target)
1	S4	cross-section inter-quartile range (forecast long run - inflation target)
2	S5	recursive Pearson correlation between (cross-section mean) short and long run inflation expectations
2	S6	recursive Pearson correlation between (cross-section median) short and long run inflation expectations
2	S7	recursive Pearson correlation between (cross-section std. dev.) short and long run inflation expectations
2	S8	recursive Pearson correlation between (cross-section inter-quartile range) short and long run expectations
3	S9	recursive OLS regression with (cross-section mean) short and long run inflation expectations
3	S10	recursive OLS regression with (cross-section median) short and long run inflation expectations
3	S11	recursive OLS regression with (cross-section std. dev.) short and long run inflation expectations
3	S12	recursive OLS regression with (cross-section inter-quartile range) short and long run inflation expectations
4	S13	binary variable from the hypothesis test (Ho: median expectation = inflation target) for the long run expectations
4	S14	binary variable from the hypothesis test Ho: distr(t) = distr(t–21) for the long-run cross-section distribution
5	S15	Nautz and Strohsal (2015), FX-rate slope from OLS (median expectation, macro shocks)
6	S16	Natoli and Sigalotti (2017), slope from logit regression, median inflation expectations (short, long)
7	S17	Demertzis *et al.* (2012), time-varying VAR, median inflation expectations (short, long)

obtain a vector of loadings that maximizes the cumulative communality using a number of *n* factors. This way, each considered signal (s_{it}) can be decomposed into a common component and an idiosyncratic component:

1 $$s_{it} = \Lambda_i F_t + \varepsilon_{it}$$

The common component captures the bulk of the covariation between s_{it} and the other signals, whereas the idiosyncratic term affects only s_{it} by assumption. Thus, it is simply a scaled common factor (Ft), which is estimated using the entire set of signals. The long-term inflation-anchoring indicator is defined to be this common factor.

We adopt here a parsimonious model with two factors (*n = 2*), since alternative models with more factors, in general, deliver estimations with higher uniqueness and lower communality (in the additional variables and/or factors) in relation to a model with fewer factors.[18]

As a result, the first factor accounts for 37% of the total variance of the set of 14 selected signals, whereas the first and second factors together represent 55% of the fraction of total variance.[19] Next, we use those figures to build a combined single factor, as a linear combination of the two original factors, as follows: $F_t = F_{1,t} * 0.37/0.55 + (1-0.37/0.55) * F_{2,t}$. Table 4 summarizes the factor loadings and Figure 14 shows the factors in the baseline case.

3.3 State-space Model

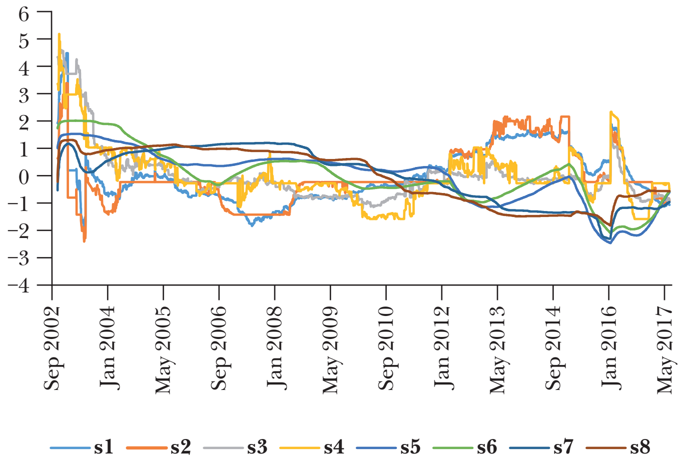
We build our expectations anchoring indexes based on the maximum likelihood estimation of a linear state-space model as described in the system of Equations 2-3, presented next. The idea is to disentangle the fiscal policy effect from the common factor F_t, constructed

[18] We use the parsimonious number of two factors since they account for more than half of the fraction of total variance of the set of signals. Nonetheless, there are many alternative factor selection tools available in the literature, such as the ones proposed by Bai and Ng (2002) or Alessi, Barigozzi and Capasso (2010).

[19] These figures are computed using the eigenvalues obtained in the solution of each factor's linear combination, as explained in Jolliffe (2002).

Figure 9

SURVEY SIGNALS
Exponential smoothing, half-life of two years



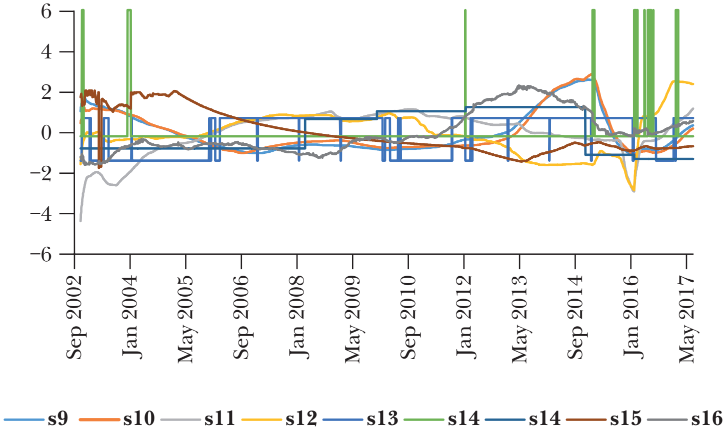


Figure 10

SURVEY SIGNALS

Rolling window weights, window of three years

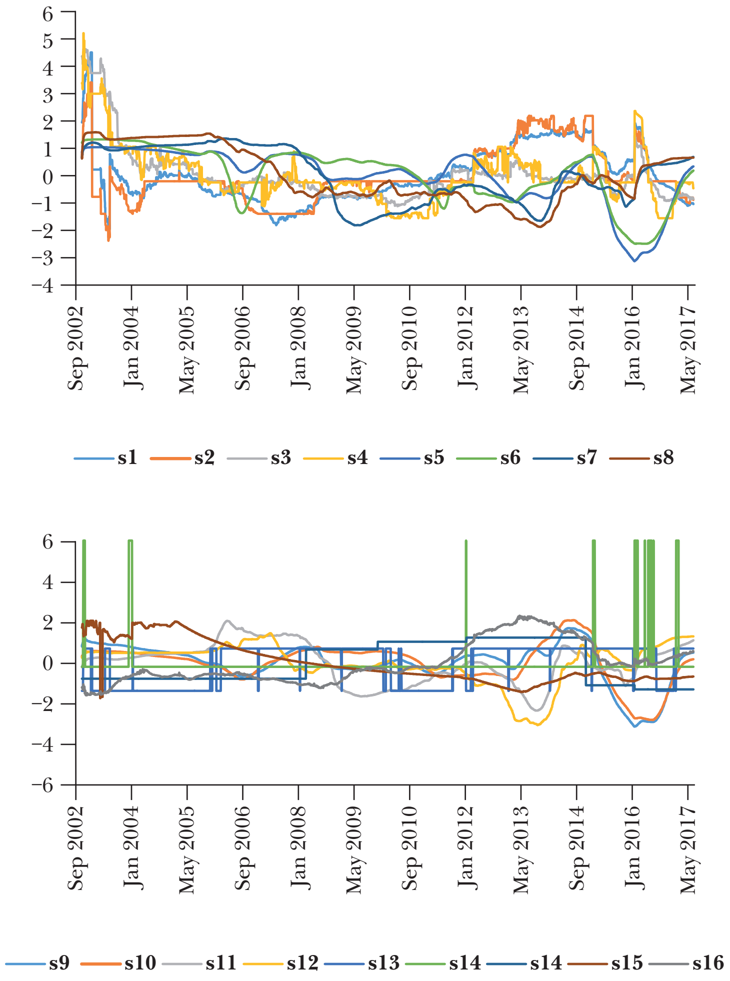
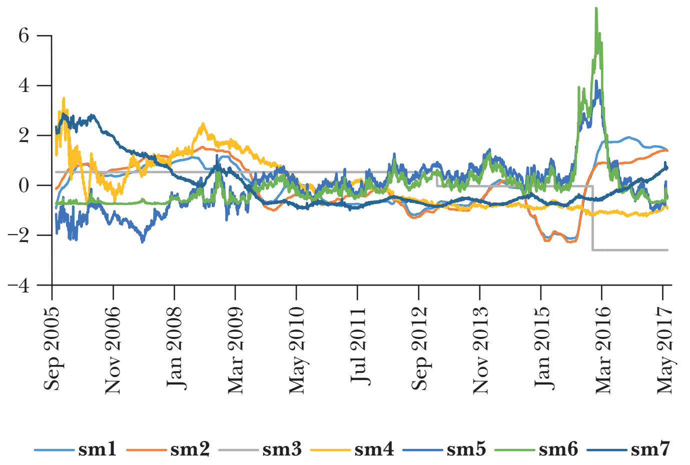


Figure 11

MARKET SIGNALS
Exponential smoothing, half-life of one year



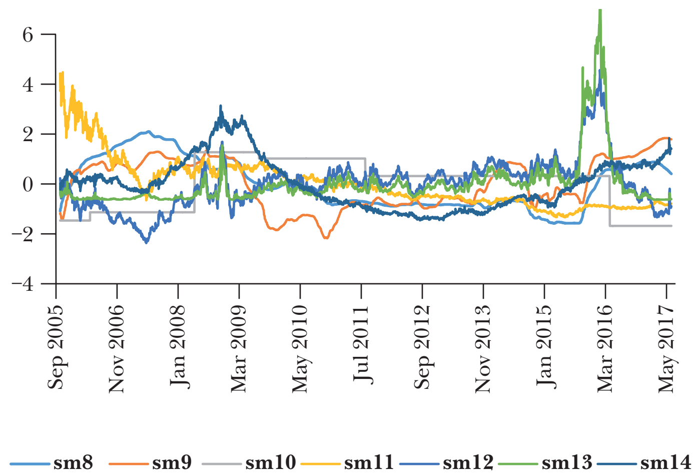


Figure 12

MARKET SIGNALS
Exponential smoothing, half-life of two years

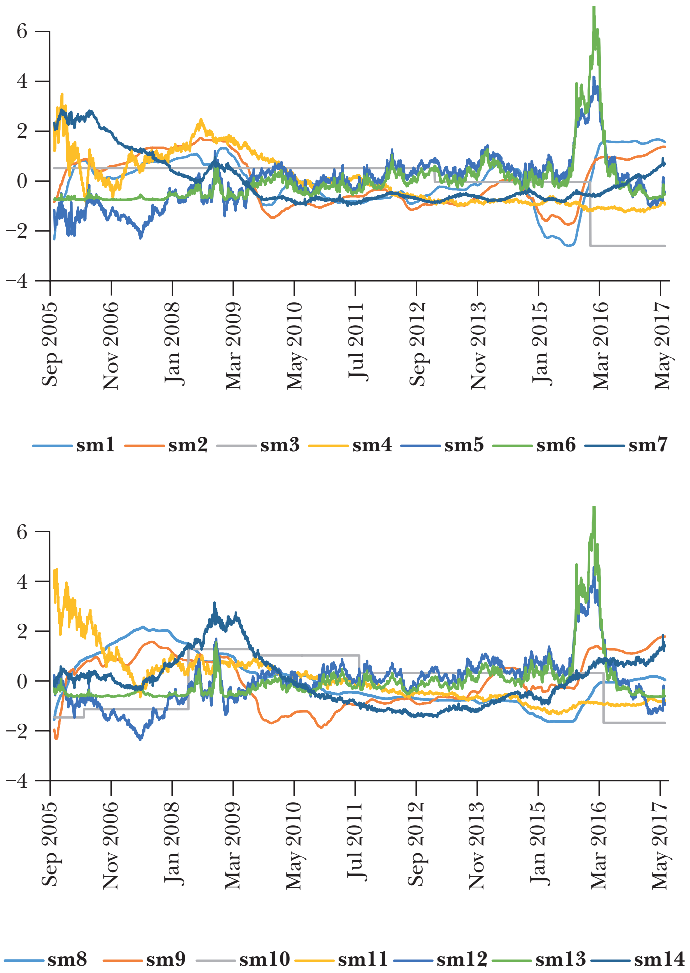
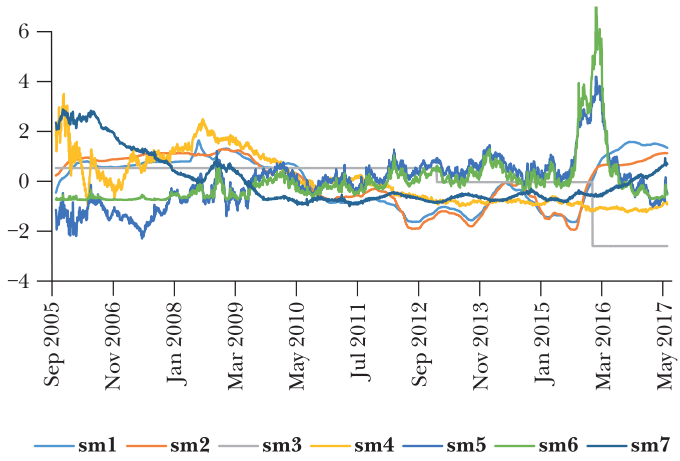


Figure 13

MARKET SIGNALS
Rolling window weights, window of three years



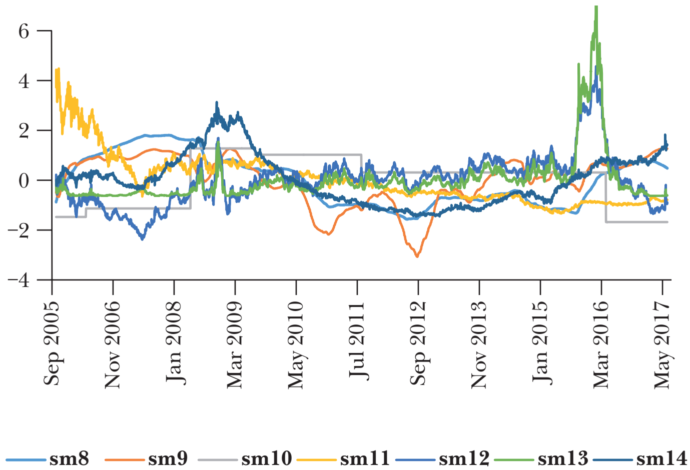


Figure 14

FACTORS FROM LONG-TERM INFLATION EXPECTATION ANCHORING
Baseline ES2y

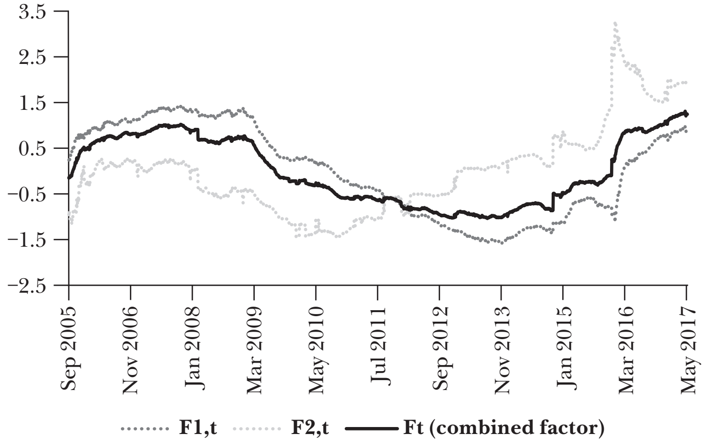


Table 2

SIGNALS CONSTRUCTED FROM BREAKEVEN INFLATION (BEI) MARKET DATA

Signals	*Description*
sm1	slope from recursive OLS regression, BEI four years against BEI one year (swaps)
sm2	recursive correlation between BEI four years and one year (swaps)
sm3	Nautz and Strohsal (2015), FX-rate slope from OLS (BEI 4y swaps, macro shocks)
sm4	Natoli and Sigalotti (2017), slope from logit regression, Δ BEI swaps (1y, 4y)
sm5	(BEI 4y swaps-inflation target)
sm6	$(\text{BEI 4y swaps-inflation target})^2$
sm7	Demertzis *et al.* (2012), time-varying VAR, BEI swaps (1y, 4y)
sm8	slope from recursive OLS regression, BEI four years against BEI one year (bonds)
sm9	recursive correlation between BEI four years and one year (bonds)
sm10	Nautz and Strohsal (2015), FX-rate slope from OLS (BEI 4y bonds, macro shocks)
sm11	Natoli and Sigalotti (2017), slope from logit regression, Δ BEI bonds (1y, 4y)(BEI 4y bonds-inflation target)
sm12	(BEI 4y bonds-inflation target)
sm13	$(\text{BEI 4y bonds-inflation target})^2$
sm14	Demertzis *et al.* (2012), time-varying VAR BEI bonds (1y, 4y)

in the previous section, and build a filtered anchoring indicator from the state-space model:

2 $$x_t = Ax_{t-1} + B\varepsilon_t,$$

3 $$y_t = Cx_t + Dv_t,$$

where $x_t = [c_t; f_t; o_t]'$ is a vector of states and $y_t = [z_t; F_t; 1_t]'$ is a vector of observable variables, and ε_t and v_t are uncorrelated Gaussian residuals. First, c_t is the monetary policy (espectations anchoring) state of interest, f_t is a state designed to capture the fiscal stance dynamics, and o_t is an auxiliary state to include the intercepts in the equations.

In turn, z_t is the consensus expectation (*Focus* survey) of the primary fiscal balance as a percentage of GDP, one-year ahead, F_t is the long-term anchoring factor and 1_t is a constant series with unit values to play the role of the intercept. The matrices A, B, C, and D are *3 x 3* null matrices, except for eight parameters estimated by maximum likelihood (ML) within a standard Kalman filter.

4 $$A = \begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & \theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}; B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}; C = \begin{bmatrix} 0 & \theta_3 & \theta_4 \\ \theta_5 & \theta_6 & \theta_7 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } D = \begin{bmatrix} \theta_8 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Note that the state $o_t = 1_t$ plays the role of the intercept and states $c_t = \theta_1 c_{t-1} + \varepsilon_{1,t}$ and $f_t = \theta_2 f_{t-1} + \varepsilon_{2,t}$ are AR(1) processes with zero mean. On the other hand, the observable fiscal expectation (z_t) is driven by the fiscal state (f_t) plus an intercept and the idiosyncratic shock $v_{1,t}$. The long-term anchoring factor F_t is decomposed into two states, c_t and f_t, which are designed to capture, respectively, the dynamics of monetary and fiscal policies.

5 $$z_t = \theta_3 f_t + \theta_4 + \theta_8 v_{1,t},$$

6 $$F_t = \theta_5 c_t + \theta_6 f_t + \theta_7$$

Table 3

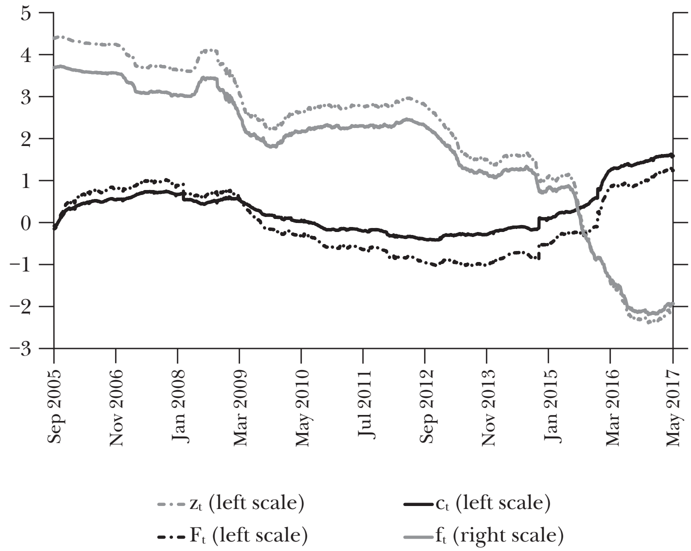
CORRELATION MATRIX OF SELECTED SIGNALS (SURVEY AND MARKET)

	S3	*S9*	*S12*	*S13*	*S14*	*S15*	*S17*	*SM3*	*SM4*	*SM7*	*SM8*	*SM9*	*SM12*	*SM14*
S3	1.00													
S9	0.30	1.00												
S12	–0.56	–0.62	1.00											
S13	0.35	0.26	–0.22	1.00										
S14	0.16	0.00	0.01	0.08	1.00									
S15	0.00	0.26	–0.31	–0.11	–0.21	1.00								
S17	0.48	0.69	–0.61	0.15	0.00	0.46	1.00							
SM3	–0.07	–0.04	–0.22	0.03	–0.33	0.51	–0.15	1.00						
SM4	–0.34	–0.43	0.39	0.00	–0.16	0.07	–0.68	0.54	1.00					
SM7	–0.08	–0.34	0.38	0.12	–0.05	–0.52	–0.51	0.10	0.47	1.00				
SM8	–0.25	–0.57	0.66	0.11	–0.05	–0.26	–0.69	0.16	0.60	0.61	1.00			
SM9	0.08	–0.08	0.29	0.38	0.15	–0.65	–0.31	–0.48	0.01	0.47	0.52	1.00		
SM12	0.30	0.25	–0.63	–0.01	0.16	0.13	0.34	–0.21	–0.41	–0.57	–0.66	–0.23	1.00	
SM14	–0.49	–0.40	0.56	0.14	0.07	–0.38	–0.68	–0.22	0.54	0.36	0.53	0.52	–0.15	1.00

Note: Only signals with pairwise absolute correlation below 0.7 are selected for the ES2y baseline case.

Figure 15

MONETARY POLICY CREDIBILITY STATE (c_t), FISCAL POLICY STATE (f_t), EXPECTATION OF PRIMARY FISCAL BALANCE (z_t) AND LONG-TERM AND ANCHORING FACTOR (F_t)



The following restrictions are employed in the ML estimation: $0 < \theta_1 < 1; 0 < \theta_2 < 1; \theta_3 > 0; \; \theta_5 > 0; \theta_6 > 0; \theta_8 > 0,$ such that increases in the states c_t and f_t represent a better anchored expectations state and a better fiscal stance, respectively. Also note, from (5), that the fiscal expectations series z_t is not linked to the monetary policy credibility state–which is a restriction adopted to properly identify the model parameters–and that there is no residual in (6) to guarantee that all the dynamics observed in the common factor F_t are either driven by the monetary policy state or by the fiscal policy state.[20]

[20] This assumption, in principle, could be relaxed by including an error term with zero mean and low variance (set as initial condition in the Kalman filter estimation).

Table 4

FACTOR MODEL LOADINGS (BASELINE *ES2Y*)

Signal	*Loadings F1*	*Loadings F2*	*Communality*	*Uniqueness*
S3	−0.47	0.32	0.33	0.67
S9	−0.65	0.15	0.45	0.55
S12	0.80	−0.01	0.65	0.35
S13	−0.02	0.31	0.10	0.90
S14	−0.03	0.34	0.11	0.89
S15	−0.47	−0.68	0.69	0.31
S17	−0.87	0.11	0.77	0.23
SM3	0.02	−0.83	0.69	0.31
SM4	0.67	−0.50	0.70	0.30
SM7	0.67	0.11	0.47	0.53
SM8	0.86	−0.06	0.74	0.26
SM9	0.50	0.71	0.75	0.25
SM12	−0.62	0.20	0.42	0.58
SM14	0.74	0.19	0.59	0.41

Notes: Sample from September 28, 2005, to June 2, 2017 (2,916 workdays). Unrotated loadings and prior communalities via squared multiple correlation. The variation explained by the first factor is 37%, whereas the first and second factors explain 55% of total variance.

As is well known, the model described in the system of equations 2-3 has only one global maximum, so initial conditions of the state variable do not have any influence on its estimation by maximum likelihood, except maybe on the number of interactions until convergence is reached.[21] Finally, the EAI is defined as the logit-transformed[22] smoothed Kalman filtered state c_t. Table 5 presents the Kalman filter parameter estimates and Figure 15 exhibits the states and observable variables in the baseline case.

We should stress that the results obtained from the reduced-form model represented by equations (1) to (6) hinge on the assessment

[21] We limit to 1,000 the number of interactions of the maximum likelihood estimations. In all estimations presented in this paper, maximum likelihood converged before reaching the limit of interactions. For the Kalman filter, we considered the expectation of initial state vector equal to zero.

[22] To guarantee the EAI to be inside the [0;1] interval.

that the expectations anchoring indexes concerning monetary policy have been disentangled from fiscal policy. Our strategy to implement such separation of policies is based on a standard state-space model using survey and market data. We acknowledge that the simplified setup, due to several modelling assumptions, might not entirely purge the fiscal policy outlook from the proposed expectations anchoring index.[23] The empirical results next presented should be interpreted with this caveat in mind.

3.4 Baseline EAIs

Our baseline EAIs are the ones in which we used both signals from survey and market data (total of 14 signals), selected with correlation analysis (see Section 3.1.3). We create three versions of these indexes depending on whether the signals are constructed from recursive correlations (or regressions) weighting the observations with exponentially smoothed weights with a half-life of one or two years or using a rolling window of three years (see Figure 16).

Because we have market data only starting from 2005, the baseline-EAIs start then. Overall, they indicate that in the beginning of the sample (2005-2008), the degree of expectations anchoring showed a reasonably high and stable pattern. In other words, market inflation expectations reflected the commitment of the BCB to keep inflation at the center of the inflation target.

When the subprime crisis hit Brazil's economy, the expectations anchoring indexes dropped and only started to improve again in the second quarter of 2013, when a contractionist monetary cycle (increases in the Selic interest rate) took place. By the end of the sample (mid-2017), the EAIs reached similar levels to those observed in the beginning of the sample, reflecting the BCB clear objective to curb

[23] For instance, the single fiscal expectations series, coupled with an autoregressive structure assumed for the fiscal state f_t, might not properly capture the core standpoint of fiscal policy. Alternative approaches to tackle this issue could consider, for instance, a state-space model containing an entire block of equations (instead of a single one) to model the fiscal policy in a disaggregate way. On the other hand, the set of observable variables could include data from credit default swaps and/or real interest rates (e.g., long-maturity forwards) or even risk premium estimates using satellite term-structure models.

inflation with the help of fiscal measures that intended to signal better public debt dynamics.

3.5 Robustness Analyses

We conduct a robustness analysis in three main dimensions. First, we create two other groups of EAIs based only on survey data or on market data. Each one is divided into three other groups, again depending on whether the signals are created from recursive correlations (or regressions) in which observations are weighted by exponential smoothing with a half-life of one or two years or a rolling window of three years. Figures 17 and 18 show the evolution of these EAIs.

The dynamics of survey-EAIs are similar to the baseline ones, with one important difference. Survey EAIs obtained with rolling windows are more volatile (in particular, after 2006) when compared to the other survey EAIs. We do not have a precise explanation for this. However, we suspect that this may have to do with the fact that we use binary survey signals, which may have had a greater impact on this EAI due to the rolling windows.

As a second robustness exercise, we estimate and remove from the breakeven inflation (BEI) series the risk premium, which is expected to be nontrivial, particularly in the short run. To do so, we regress each BEI series against an intercept and the cross-section interquartile range constructed from the survey-based inflation expectations data (using the same forecast horizon). For instance, in the case of the BEI from swaps with one-year maturity, we use the following regression:

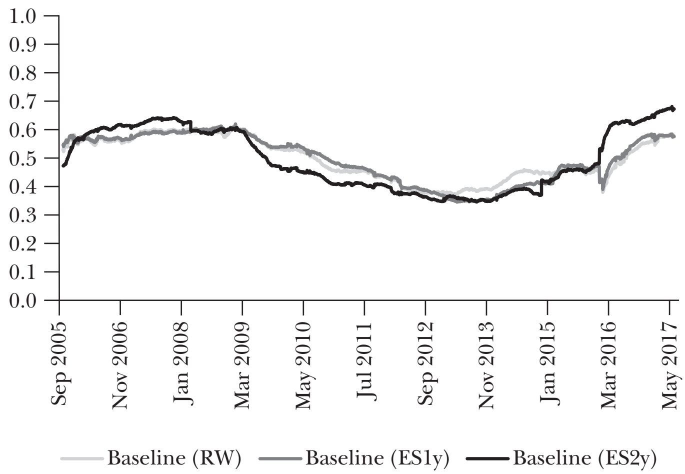
7 $$BEI\ swap1y(t) = a + b * IQR1y(t) + e(t).$$

The risk premium series is proxied by $\hat{b} * IQR1y(t)$, whereas the BEI series without risk premium is given by $\hat{a}$+e(t).[24] In the case of BEI from bonds, we include an additional regressor to account for liquidity premium (given by the ratio between the market value of NTN-Bs and LTNs outstanding). Figure 19 shows the original BEI series and those

[24] The advantage of our approach is that the estimated risk premium is "model-free" in the sense that it is not grounded on a specific theoretical model, but instead is solely based on survey data at the micro level.

Figure 16

EXPECTATIONS ANCHORING INDEX
Baseline



Notes: ES1y and ES2y denote the exponenially smoothed weights with half-life of one year and 2 years, respectively, and rw means rolling window weights (window of three years). Only signals with pairwise absolute correlation below 0.7 are selected for the baseline case. The following signals are selected: S3, S9, S12, S13, S14, S15, S17, SM3, SM4, SM7, SM8, SM9, SM12 and SM14.

Table 5

KALMAN FILTER ESTIMATION OF THE EXPECTATIONS ANCHORING INDEX (BASELINE *ES2Y*)

Parameter	*Estimate*	*S.E.*	
θ_1	0.9897	0.0004	a
θ_2	0.9900	0.0004	a
θ_3	5.7601	0.0682	a
θ_4	5.8999	0.0669	a
θ_5	1.5670	0.0105	a
θ_6	1.0880	0.0552	a
θ_7	0.2627	0.0016	a
θ_8	0.0004	0.0546	

Note: Sample from September 28, 2005, to June 2, 2017 (2,916 observations). “a” indicates statistical significance at 1% level. Only signals with pairwise absolute correlation below 0.7 are selected for the ES2y baseline case.
The following signals are selected: S3, S9, S12, S13, S14, S15, S17, SM3, SM4, SM7, SM8, SM9, SM12, and SM14.

without the risk premium. Figure 20 presents the effect of the risk premium extraction in the expectations anchoring index constructed with market data. They show similar dynamics to our baseline EAIs.

The third robustness check consists of using a different method in the factor analysis. Instead of extracting two factors, we employ here the minimum average partial (MAP) criterion for selecting the number of factors. In the baseline case, the method suggests a single factor, which is used as Ft in model (2)-(3). Figure 21 presents the expectations anchoring index obtained from the single factor using MAP; with a very similar trajectory compared to the baseline EAI.

4. CONCLUSION

According to Blinder (1998): "In the real world, credibility is not created by incentive compatible compensation schemes or by rigid precommitment. Rather, it is painstakingly built up by a history of matching deeds to words."

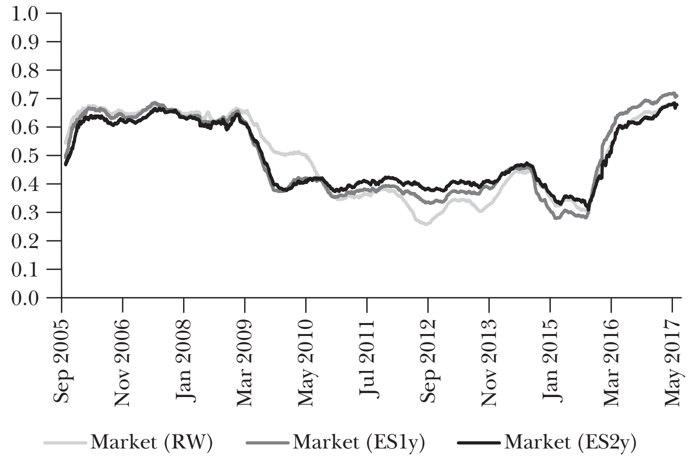
Our objective in this paper is to build expectations anchoring indexes for inflation in Brazil that are essentially driven from the BCB's ability to anchor long-term inflation expectations. The EAIs are smoothed Kalman filtered maximum likelihood estimates from a linear statespace model, which also includes expected fiscal dynamics from survey data. The model signals give information on the degree of long-term inflation expectation anchoring.

We derive our EAIs from surveys of inflation expectations and from market data. Although varying across specifications, the expectations anchoring indexes that we propose tend to display a downward trajectory, more clearly in 2009, and show a recovery starting in 2016 until the end of the sample (mid-2017).

Future extensions of the paper could include other signals of long-term inflation anchoring. We also think that our method can be extended to the creation of EAIs for other central banks around the world, despite different data on long-term inflation expectations from those we have in Brazil and used in this paper.

Figure 17

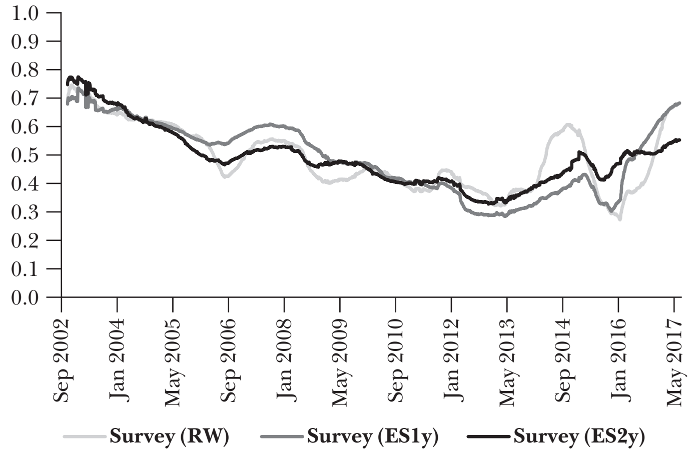
EXPECTATIONS ANCHORING INDEX
Market signals



Notes: ES1y and ES2y denote the exponenially smoothed weights with half-life of one year and two years, respectively, and RW means rolling window weights (window of three years).

Figure 18

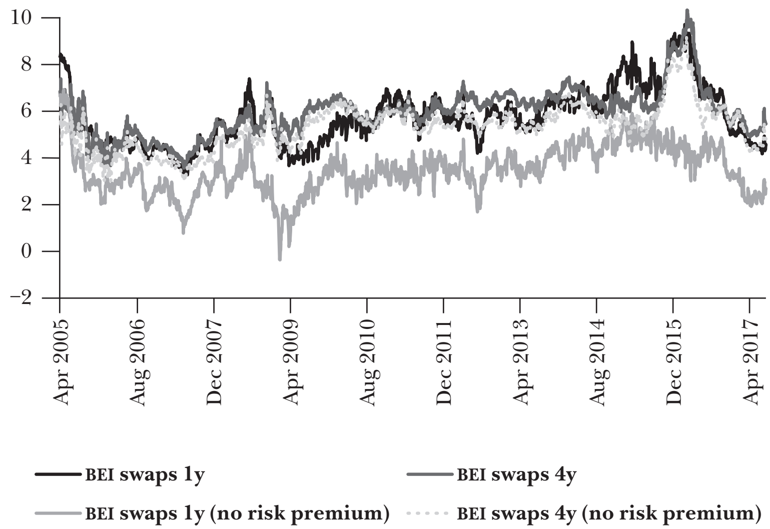
CREDIBILITY INDEX
Survey signals



Notes: ES1y and ES2y denote the exponentially smoothed weights with half-life of one year and two years, respectively, and RW means rolling window weights (window of three years).

Figure 19

MARKET DATA: BREAKEVEN INFLATION AND RISK PREMIUM EXTRACTION
BEI, percentage 12 months



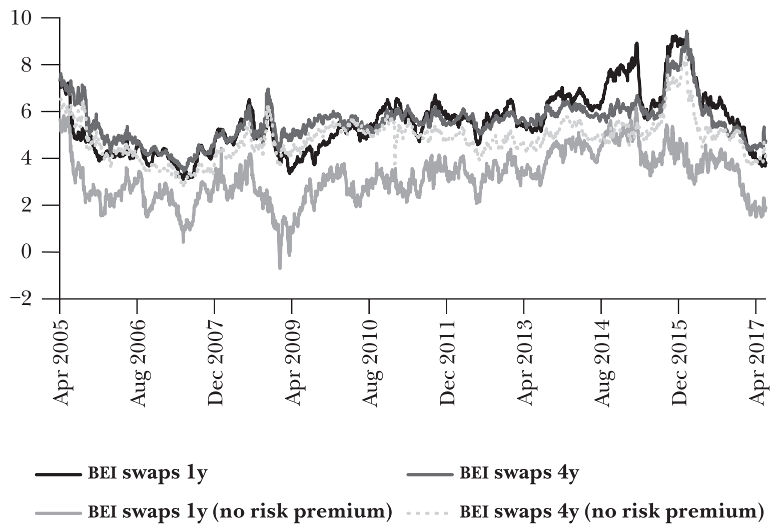


Figure 20

EXPECTATIONS ANCHORING INDEX AND THE EFFECT OF RISK PREMIUM EXTRACTION FROM MARKET DATA

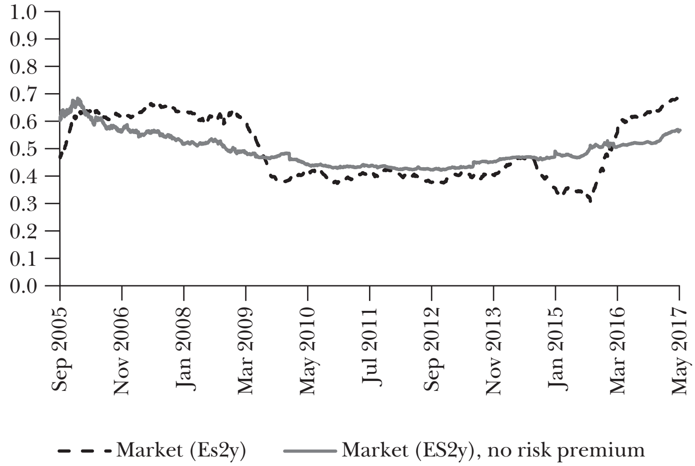
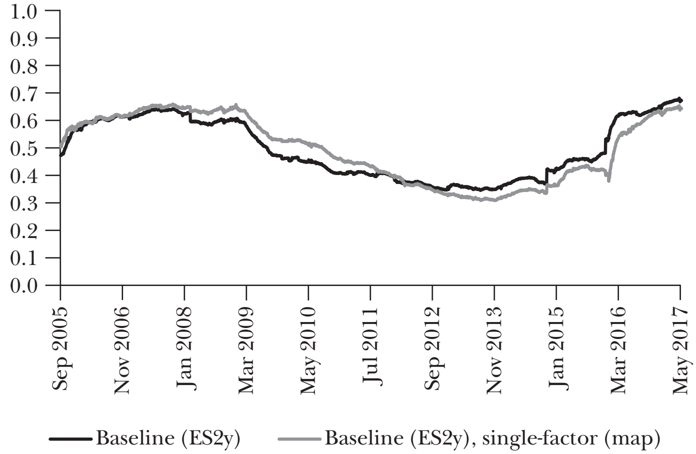


Figure 21

EXPECTATIONS ANCHORING USING A DIFFERENT METHOD TO CONSTRUCT THE COMMON FACTOR Ft



Note: The single-factor comes from the “minimum average partial” criterion for selecting the number of factors.

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Inflation Forecasting and Its Performance Evaluation

Forecasting Inflation in Argentina

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Abstract

During the year 2016, the Banco Central de la República Argentina has begun to announce inflation targets. In this context, providing the authorities of good estimates of relevant macroeconomic variables turns out to be crucial to make the pertinent corrections in order to reach the desired policy goals. This paper develops a group of models to forecast inflation for Argentina, which includes autoregressive models, and different scale Bayesian VARs (BVAR), and compares their relative accuracy. The results show that the BVAR model can improve the forecast ability of the univariate autoregressive benchmark's model of inflation. The Giacomini-White test indicates that a BVAR performs better than the benchmark in all forecast horizons. Statistical differences between the two BVAR model specifications (small and large-scale) are not found. However, looking at the RMSEs, one can see that the larger model seems to perform better for larger forecast horizons.

Keywords: Bayesian vector autoregressive, forecasting, prior specification, marginal likelihood, small-scale and large-scale models.

JEL classification: C11, C13, C33, C53.

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1. INTRODUCTION

Several long-term nominal commitments such as labor contracts, mortgages and other debt are widespread features of modern economies. Forecasting how the general price level will evolve over the life of a commitment is an essential part of private sector decision-making.

The existence of long-term nominal obligations is also among the primary reasons economists generally believe that monetary policy is not neutral, at least over moderate horizons.

Central banks aim is to keep inflation stable, and perhaps also to keep output near an efficient level. With these objectives, the New Keynesian model makes explicit that optimal policy will depend on optimal forecasts (e.g., Svensson, 2005), and further, that policy will be most effective when it is well understood by the public.

Under *inflation targeting* the central banks generally released forecasts in quarterly *Inflation Reports* in a way to be more transparent in their actions. The costs and benefits of transparency are widely debated, but the need for a central bank to be concerned with inflation forecasting is broadly agreed. In short, inflation forecasting is of foremost importance to households, businesses, and policymakers.

During the year 2016, the Banco Central de la República Argentina (BCRA) has begun to announce inflation targets. In this context, providing the authorities of good estimates of relevant macroeconomic variables turns out to be crucial to make the pertinent corrections in order to reach the desired policy goals.

A standard tool in macroeconomics that is widely employed in forecasting is vector autoregressive (VAR) analysis. VARs are flexible time series models that can capture complex dynamic relationships among macroeconomic aggregates. However, their dense parameterization often leads to unstable inference and inaccurate out-of-sample forecasts, particularly for models with many variables, due to the estimation uncertainty of the parameters.

Litterman (1980) and Doan, Litterman, and Sims (1984) have proposed to combine the likelihood function (the data) with some informative prior distributions (the researcher's belief about the values of coefficients) to improve the forecasting performance of VAR models, introducing a Bayesian approach into VAR modeling.

In any Bayesian inference, a fundamental yet challenging step is prior specification, which influences posterior distributions of the unknown parameters and, consequently, the forecasts (Geweke, 2005). Fortunately, the literature has proposed some methodologies to set how informative the prior distributions should be.

Regarding prior selection, Litterman (1980) and Doan, Litterman, and Sims (1984) set the tightness of the prior by maximizing the out-of-sample forecasting performance of a small-scale model. Many authors follow this strategy, such as Robertson and Tallman (1999) and Wright (2009), and Giannone et al. (2014), who minimize the root mean square error (RMSE) of the forecasts.

On the other hand, Banbura et al. (2008) propose to control the overfitting caused by the considerable number of variables in the model, by selecting the shrinkage of the coefficients in such a way as to give an adequate fitting *in-sample*. Within this second selection strategy, we can find authors such as Giannone et al. (2012), Bloor and Mathenson (2009), Carriero et al. (2015) and Koop (2011).

Banbura, Giannone, and Reichlin (2008) showed that, by applying Bayesian VAR methodology, they were able to handle large unrestricted VARs models and therefore they demonstrated that VAR framework can be applied to empirical problems that require the analysis of more than a few sets of time series. The authors showed that a Bayesian VAR is a viable alternative to factor models or panel VARs for analysis of large dynamic systems.

This paper develops a group of models to forecast inflation for Argentina, which includes autoregressive models, and different scale Bayesian VARs (BVAR), and compares their relative accuracy.

The paper is organized as follows: Section 2 presents the methodological aspects related to the application of Bayesian analysis in a VAR framework, Section 3 presents a brief description of the data, Section 4 goes through the empirical results, and finally, Section 5 concludes.

2. BAYESIAN VAR METHODOLOGY

A VAR model has the following structure

$$y_t = c + B_1 y_{t-1} + \quad + B_p y_{t-p} + \varepsilon_t, \tag{1}$$

where $\boldsymbol{y}_t$ is a $n\times 1$ vector of endogenous variables, $\varepsilon_t \quad N(0,\Sigma)$ is a $n\times 1$ vector of exogenous shocks, $\boldsymbol{c}$ is a $n\times 1$ vector of constants, $\boldsymbol{B}_1$ to $\boldsymbol{B}_p$ are $n\times n$ matrices, and $\boldsymbol{\Sigma}$ is $n\times n$ covariance matrix.

The BVAR coefficients are a weighted average of the prior mean (researcher's belief) and the maximum likelihood (ML) estimators (inferred from the data), with the inverse covariance of the prior and the ML estimators as weights.

Consider the following posterior distribution for the VAR coefficients

2 $$\beta \big| \Omega \sim N(\beta_0, \Omega^{-1}\xi)$$

where the vector β_0 is the prior mean (whose elements will represent the coefficient in Equation 1, the matrix $\boldsymbol{\Omega}$ is the known variance of the prior, and ξ is a scalar parameter controlling the tightness of the prior information. Even though $\boldsymbol{\Omega}$ could have many shapes, gamma and Wishart distributions are frequently used in the literature, since they ensure a normally distributed posterior.[1]

The conditional posterior of $\boldsymbol{\beta}$ can be obtained by multiplying the prior by the likelihood function. The posterior takes the form

3 $$\beta \big| \Omega, \boldsymbol{y} \sim N\left(\hat{\beta}(\xi), \hat{\boldsymbol{V}}(\xi)\right),$$

where

4 $$\hat{\beta}(\xi) \equiv vec\left(\hat{\beta}(\xi)\right),$$

and

5 $$\hat{\boldsymbol{B}}(\xi) \equiv (\boldsymbol{x}'\boldsymbol{x}\,\Sigma^{-1} + (\Omega\xi)^{-1})^{-1}(\boldsymbol{x}'\boldsymbol{y}\,\Sigma^{-1} + (\Omega\xi)^{-1}\beta_0),$$

1 If the posterior distributions are in the same family as the prior probability distribution, the prior and posterior are then called conjugate distributions.

6 $$\hat{V}(\xi) \equiv (x'x\Sigma^{-1} + (\Omega\xi)^{-1})^{-1}.$$

Vectors y and x represent observed data while β_0 is a matrix where each column corresponds to the prior mean of each equation.

It is important to note that if we choose a large value for ξ, the prior will have little weight into the posterior. This translates to large volatility of the prior and not enough information coming from the prior. On the other hand, if the ξ is set to a small value (i.e., close to zero), the prior becomes more informative and the posterior mean moves towards the prior mean. To see this point, we can express 5 as follows:

7 $$\hat{B}(\xi) \equiv \hat{\Omega}[\Omega_0^{-1}\beta_0 + (\Sigma^{-1} \otimes x)y]$$

and

8 $$\hat{\Omega} = [\Omega_0^{-1} + \Sigma^{-1} \otimes x'x]^{-1}.$$

If the second element between brackets in Equation 7 is multiplied by $(x'x)^{-1}(x'x)$, we obtain the following equations:

9 $$\hat{B}(\xi) \equiv \hat{\Omega}[\Omega_0^{-1}\beta_0] + \hat{\Omega}[\Sigma^{-1} \otimes x'x(x'x)^{-1}x'y]$$

10 $$\hat{B}(\xi) \equiv \hat{\Omega}[\Omega_0^{-1}\beta_0] + \hat{\Omega}[\Sigma^{-1} \otimes x'x(x'x)\beta_{ols}]$$

As can be seen, the posterior is a weighted average between the prior and the ordinary least square (OLS) estimators,[2] where the weights are the reciprocal of the prior covariance matrix and the reciprocal of the OLS covariance matrix respectively. As a result, if the information contained in the data is good enough to describe the process

[2] The ols estimators of a var coincide exactly with the ml estimators conditional on the initial values.

behind it, the posterior will move towards the OLS estimators. However, it is important to underscore that, even if the available series are adequate to describe the data generating process, the researcher could still formulate a hypothesis about the distribution of the parameters based on his own beliefs. That would imply ignoring the information contained in the data, and usually that kind of decisions are based on strong beliefs.

The issue mentioned in the last paragraph demonstrates the need to be cautious about choosing the prior mean and the hyperpriors. In the following subsections, these aspects are discussed in more detail.

2.1 Level or Growth Rate

It is unclear a priori whether transforming variables into their growth rates can enhance the forecast performance of a BVAR model. On one hand, the level specification can better accommodate the existence of long-run (cointegrating) relationships across the variables, which would be omitted in a VAR in differences. On the other hand, Clements and Hendry (1996) have shown that in a classical framework, differencing can improve forecasting performance in the presence of instability.

There has been little effort in the BVAR literature to compare specifications in levels versus differences. Carriero et al. (2015) work with this specific topic and found that models in growth rates generally yield more accurate forecasts than those obtained from the models in levels. However, we can find both approaches in the literature. Following the Litterman (1986) tradition, some authors considered BVARs with variables in levels (e.g., Banbura et al., 2008; Giannone et al., 2014, and Giannone et al., 2012). Other authors used BVARs with variables in differences or growth rates (e.g., Clark and McCracken, 2007, and Del Negro et al., 2004).

As mentioned above, there is no apparent reason to opt for series in levels or in differences to work with; nevertheless, choosing a representation ex-ante, gives us information about the characteristics of the prior distribution (values of the mean prior). For example, working with variables in differences implies that the persistence of those variables should be low, and that one should impose a number close to zero as a prior mean of the first lag, denoting low persistence in the series.

Since it is a good practice to start with some idea about the value that the prior could take and encouraged by the evidence found by Carriero et al. (2015), we have opted to work with variables in differences.

In the next subsection, we will treat the variance of the prior as another aspect of prior distribution.

2.2 Choice of Hyperparameters and Lag Length Strategy

To select the hyperparameters and the lag length we will follow the strategy suggested by Banbura et al. (2008), Carriero, et al. (2015) and Giannone et al. (2012). Suppose, that a model is described by a likelihood function $p(\mathbf{y}|\theta)$ and a prior distribution $p_\gamma(\theta)$, where θ is a vector parameter of the model and γ is a vector of hyperparameters affecting the distribution of all the priors of the model. It is natural to choose these hyperparameters by interpreting the model as a hierarchical one, i.e. replacing $p_\gamma(\theta)$ with $p(\theta|\gamma)$ and evaluating their posterior (Berger, 1985; Koop, 2003). In this way, the posterior can be obtained by applying Bayes' law

11 $$\mathrm{p}(\gamma|y) \approx p(y|\gamma)p(\gamma),$$

where $p(\gamma)$ is the density of the hyperparameters and $p(y|\gamma)$ is the marginal likelihood. In turn, the marginal likelihood is the density that comes from the data when the hyperparameters change–in other words, the marginal likelihood can be obtained after integrating out the uncertainty about the parameters in the model,

12 $$p(\gamma|y) = \int p(y|\theta,\gamma)p(\theta|\gamma)d\theta.$$

For every conjugate prior, the density $p(\gamma|y)$ can be computed in closed form. To obtain the Bayesian hierarchical structure, it is necessary to obtain the distribution of $p(\theta)$ by integrating out the hyperparameters

13 $$p(\theta) = \int p(\theta,\gamma)\pi(\gamma)d\gamma.$$

More precisely, we can find different values of the prior distribution from different hyperparameter values and, in this way, we can represent the posterior as:

14 $$p(\theta,\gamma \mid y) = p(y \mid \theta,\gamma)\, p(\theta,\gamma)\, p(\gamma).$$

The marginal likelihood should be sufficient to discriminate among models; in this sense, we can choose models with different hyperparameters and different likelihood specification (more precisely, lags length structure). To make this point operational, we estimate different models, following Giannone et al. (2012), who introduce a procedure allowing to optimize the values of the hyperparameters that maximize the value of the marginal likelihood of the model. This implies that the hyperparameter values are not set a priori but are estimated.

Then the marginal likelihood can be estimated for every combination of hyperparameter values within specified ranges and for different lag length structures, and the optimal combination is retained as the one that maximizes that value.

2.3 Comparison Strategy

In this subsection, we present some details about our strategy for model comparison. We will mention the steps that we will follow to do it and then give more details about the predictive ability tests used for comparison:

a) Estimate a univariate AR model.

b) Compute the relative RMSE to the AR from (a).

c) Compute the relative RMSE to the BVAR.[3]

d) Run the test of Giacomini and White (2006) to compare both models.

Our benchmark is a univariate model. This means that we have at hand different statistical measures that cover both the frequentist

[3] The mean of the predictive density is considered.

and the Bayesian approaches. While frequentist literature tends to compare the forecasts with actual values, Bayesian literature compares the realized values with the whole posterior predictive density.

The testing methodology of Giacomini and White (2006) consists on evaluating relative forecast accuracy with a Diebold-Mariano (1995) like test, but with one central difference: The size of the in-sample estimation window is kept fixed, instead of expanding. Using the sample observations available at time t, forecasts of $y_{t+\tau}$ are produced for different t for given τ periods into the future, with rolling windows of estimation with the two models that are being compared. The sequences of forecasts are then evaluated according to some loss function and then the difference of forecast losses is computed. This way, a time series of differences in forecast losses $\Delta L_{t+\tau}(\hat{\theta})$ that depends on the estimated parameters is constructed. The test then consists on a Wald test on the coefficients of the regression of that series against a constant, the unconditional version of the test in Equation 15, or against other explanatory variables, the conditional version in Equation 16:

15 $$\Delta L_{t+\tau}(\hat{\theta}) = \mu + \varepsilon_t,$$

16 $$\Delta L_{t+\tau}(\hat{\theta}) = \beta' X_t + \varepsilon_t.$$

Standard errors may be calculated using the Newey-West covariances estimator, controlling for heteroskedasticity and autocorrelation. In this paper, the unconditional version is used.

The Giacomini-White test[4] has many advantages: It captures the effect of estimation uncertainty on relative forecast performance, it allows for comparison between either nested or non-nested models, and, finally, it is quite easy to compute.

[4] See chapter 17 of the book by Hashimzade and Thornton (2013) for a detailed discussion about this test.

2.4 Model Specification

We follow Banbura et al. (2008) and analyze two VAR models that incorporate variables of special interest, including indicators of real economic activity, consumer prices, and monetary variables. We consider the following two alternative models:

Small-scale model. This is a small monetary VAR including three key variables:

a) Prices: We used the consumer price index constructed by the *Instituto Nacional de Estadística y Censos* de la República Argentina (INDEC). After December 2006 until July 2012, the previous series is linked with the evolution of the consumer price index provided by the Instituto Provincial de Estadísticas y Censos de San Luis and, after July 2012, series is again linked with the evolution of the consumer price index of the city of Buenos Aires.[5]

b) Economic activity: We used a monthly economic activity indicator known as EMAE (Estimador Mensual de Actividad Económica) published by the INDEC. The EMAE is based on the value added for each activity at a base price plus net taxes (without subsidies), and it uses weights provided by Argentina's National Accounts (2004). It tries to replicate quarterly GDP at a monthly frequency.

c) Interest rate: We used data from the BCRA on 30 to 59-day fixed term deposit rates.

Large-scale Model. In addition to the variables included in the small-scale model, this version also includes the rest of the variables in the data set. These are detailed in the next section.

In September 2016, Argentina transitioned to an inflation targeting regime. This could generate a structural break in the mean and variance. To account for this possible change in the mean of the

[5] From December 2006 to October 2015, the index by the INDEC presented severe discrepancies with provincial and private price index, and hence was discarded for that period.

process, we incorporate a dummy variable in both specifications (Marcelino and Mizon, 2000).[6]

As we compare models of different sizes, we need a strategy on how to choose the shrinkage hyperparameter as models become larger. As the dimension increases, we want more shrinkage, as suggested by the analysis in De Mol et al. (2008) to control for overfitting. We set the tightness of the prior for the model to have better in-sample fit; in this way, we are shrinking more in a larger dimension model.

3. DATA

Our data set is composed of a group of 16 monthly macroeconomic variables of Argentina available on a monthly frequency. Sources of the series, the transformations did on them and their stationarity characteristics are described in the Annex.

4. RESULTS

4.1 Estimation of the BVAR Model

4.1.1 The Optimal Hyperparameters

We work with a Normal-Wishart BVAR specification. In this type of specification, there are two hyperparameters and two parameters. We estimate the overall tightness λ_1, lag decay λ_3, and the lag length as we have described in Section 2.2, and then we impose the value of the prior mean (the autoregressive coefficient) equal to zero as discussed earlier.

The hyperparameter of the overall tightness λ_1 is the standard deviation of the prior of all the coefficients in the system other than the constant. In other words, it determines how all the coefficients are concentrated around their prior means.

The term λ_3 is a decay factor and $1/(L^{\lambda_3})$ controls the tightness on lag L relative to the first lag. Since the coefficients of higher order

[6] In the Annex, we show the posterior estimation of the whole sample to see the effect of this. We controlled the change in the mean due the transition to an inflation targeting regime and indeed obtained a significant coefficient in both models.

lags are more likely to be close to zero than those of lower order lags, the prior for the standard deviations of the coefficients decrease as the lag length increases. The values usually used in the literature are 1 or 2, so we settle for $\lambda_3 = 2$.

The prior variance of the parameters of $\hat{\beta}(\xi)$ is set according to:

17 $$\sigma_{ij}^2 = \left(\frac{1}{\sigma_j^2}\right)\left(\frac{\lambda_1}{L^{\lambda_3}}\right)^2$$

where σ_j^2 denotes the OLS residual variance of the autoregressive coefficient for variable j, λ_1 is an overall tightness parameter, L is the current lag, and λ_3 is a scaling coefficient controlling the speed at which coefficients for lags greater than 1 converge to 0.

For exogenous variables, we define the variances as:

18 $$\sigma^2 = \left(\lambda_1\lambda_4\right)^2$$

The results for the hyperparameters and prior means of the small and the big scale model are shown in Table 1. All the hyperparameters are equal for both type of models except for the hyperparameter λ_1.

The characteristics of our hyperparameters after the optimization procedure is as follow:

Table 1

LIST OF HYPERPARAMETER VALUES

Hyperparameters values	*Large-scale model*	*Small-scale model*
Autoregressive coefficient:	0	0
Overall tightness (λ_1).	0.05	0.23
Lag decay (λ_3):	2	2
Exogenous variable tightness	1	1
Lag length	1	1

The hyperparameter λ_1 is equal to 0.05 for the large-scale model while the hyperparameter λ_1 for the small-scale is 0.23. From a practical point of view, this means that the *true* value of the coefficients estimated (posterior) is probably to be farther from the prior mean in the small-scale model than in the large-scale one.

Another aspect to consider about λ_1 is the fact that this hyperparameter impacts on the distribution of the parameters of lagged endogenous and exogenous variables of each equation in the system. In this sense, with more shrinkage, for example, it is less probable that the posterior coefficients of the lagged endogenous and exogenous variables depart from the prior.

As can see in Table 1, the posterior coefficients of the variables in the large-scale model are less probable to depart from the prior than the small-scale ones. Models with lots of variables will tend to have a better in-sample fit even when λ_1 is set to *loose* value.

The posteriors obtained for the small- and the large-scale model of the inflation equation in each type of model are shown in the Annex.

4.1.2 Forecasting Exercise

Our forecasting exercise is conducted in the following way. We estimate the hyperparameters considering the whole sample, through the maximization of the marginal likelihood; and then, we compute the forecasts.

As we mentioned before, the data set goes from January 2004 to July 2017. We compute one-, three- and six-step-ahead forecasts with rolling windows. The size of the estimation sample is the same for each forecast horizon. Out-of-sample forecast accuracy is measured in terms of RMSE of the forecasts. Therefore, we obtained three RMSEs for each model.

Relative forecast accuracy is analyzed in Table 2, by computed the different combinations of RMSE ratios. On average, the BVAR presents better accuracy than the benchmark independently of the forecast horizon. For immediate horizons, the small-scale model slightly outperforms the larger one, but the large-scale model outperforms the small one for further forecast horizons.

Table 2								
RELATIVE FORECAST ACCURACY								
One-step-ahead			*Three-steps-ahead*			*Six-steps-ahead*		
Ratio small model-benchmark	*Ratio large model-benchmark*	*Ratio large model-small model*	*Ratio small model-benchmark*	*Ratio large model-benchmark*	*Ratio large model-small model*	*Ratio small model-benchmark*	*Ratio large model-benchmark*	*Ratio large model-small model*
0.77	0.90	1.69	0.78	0.77	1.02	0.87	0.82	0.94

In the next subsection, we analyze these results with a Giacomini-White test.

4.2 Forecast Evaluation

To evaluate the predictive performance of the different models, we used the tests described earlier. Each column of Table 3 contains the probability value of Giacomini-White test statistic for the different models.

Table 3			
GIACOMINI-WHITE TEST			
Forecast horizon	*Large BVAR vs. benchmark*	*Small BVAR vs. benchmark*	*Difference between BVAR models*
One-step-ahead	0.03	0.01	0.29
Three-steps-ahead	0.00	0.00	0.49
Six-steps-ahead	0.09	0.05	0.41

The result of the Giacomini-White test shows that, at a 5% of significance level, the large BVAR model outperforms the benchmark for one step and three steps ahead forecast horizon, while the small BVAR outperforms the benchmark at a 5% significance level for all forecast horizons. The last column of the table shows the Giacomini-White test applied to the differences in predictive ability between the small- and large-scale BVAR models, but in this case, the differences are not significant for all forecast horizons.

5. CONCLUSIONS

This paper assesses the performance of Bayesian VAR to forecast inflation in Argentina. We considered a Normal-Wishart BVAR specification for a small- and a large-scale model of differentiated variables setting the prior mean according to standard recommendations in previous studies. The overall tightness hyperprior and the lag length of the different models were set by optimization of the marginal likelihood. We found that large-scale models have narrower priors, giving more weight to the priors mean than small-scale models.

Overall, the results show that the BVAR model can improve the forecast ability of the univariate autoregressive benchmark's model of inflation. The Giacomini-White test indicates that a BVAR performs better than the benchmark in all forecast horizons. Statistical differences between the two BVAR model specifications (small and large-scale) are not found. However, looking at the RMSEs, one can see that the larger model seems to perform better for larger forecast horizons.

Annex A. Data Characteristics

Table A.1

LIST OF ENDOGENOUS VARIABLES

	Source	*Description*	*Transf.*	*Characteristics*	
1	INDEC	EMAE	log	SA	Unit-root
2	INDEC	CPI inflation	–	–	Trend
3	INDEC	Core CPI inflation (ex. seasonal and regulated)	–	–	Trend
4	INDEC	Industrial employment	log	SA	Unit-root
5	INDEC	Construction employment	log	SA	Unit-root
6	INDEC	Retail trade employment	log	SA	Stationary
7	BCRA	M2 monetary aggregate	log	SA	Unit-root
8	BCRA	Multilateral nominal exchange rate	log	–	Unit-root
9	BCRA	30 to 59-day deposit rate	–	–	Unit-root
10	INDEC	Imports of intermediate goods	log	SA	Unit-root
11	INDEC	Total exports	log	SA	Unit-root
12	UTDT	Consumer confidence index	–	–	Unit-root
13	INDEC	Monthly supermarket sales	log	SA	Unit-root
14	AFCP	Cement sales	log	SA	Unit-root
15	MINEM	Asphalt sales	log	–	Stationary
16	MERVAL	Stock market index	log	–	Unit-root

Annex B. Results characteristics

Table B.1

SMALL BVAR CHARACTERISTICS

Endogenous variables:	Inflation, interest rate, real activity
Exogenous variables:	Constant, dummy 2016-11
Estimation sample:	July 2004 to July 2017
Sample size (omitting initial conditions):	156
Number of lags included in regression:	1
Prior:	Normal-Wishart
Autoregressive coefficient:	0
Overall tightness:	0.23
Lag decay:	2
Exogenous variable tightness:	1

Table B.2

SMALL BVAR INFLATION EQUATION COEFFICIENT VALUES

	Median	*SD*	*lb*	*ub*
INF(−1)	0.468	0.066	0.338	0.598
I(−1)	0.901	0.640	−0.356	2.157
Y(−1)	2.631	3.500	−4.237	9.499
Constant	0.280	0.071	0.140	0.420
d112016	−0.197	0.144	−0.479	0.086

Sum of squared residuals: 91.05
R-squared: 0.291
Adj. R-squared: 0.272

Table B.3

LARGE BVAR CHARACTERISTICS

Endogenous variables	Inflation, interest rate, real activity, multilateral exchange rate, industrial employment, cement sales, asphalts sales, imports of intermediate goods, total exports, M2, core inflation, construction employment, consumer confidence index, supermarket sales, stock market index
Exogenous variables	Constant, dummy 2016-11
Estimation sample	July 2004 to July 2017
Sample size	156
Number of lags	1
Prior	Normal-Wishart
Autoregressive coefficient	0
Overall tightness	0.05
Lag decay	2
Exogenous variable tightness	1

Table B.4

LARGE BVAR INFLATION EQUATION COEFFICIENT VALUES

	Median	*SD*	*LB*	*UB*
INF(−1)	0.145	0.045	0.057	0.234
I(−1)	0.436	0.407	−0.362	1.235
Y(−1)	1.177	2.131	−3.005	5.359
E(−1)	7.261	3.431	0.528	13.994
EMPI(−1)	16.644	11.611	−6.143	39.431
CEM(−1)	−0.680	0.556	−1.771	0.410
ASPH(−1)	0.083	0.411	−0.723	0.888
IMP(−1)	0.125	0.477	−0.810	1.061
EXP(−1)	0.091	0.491	−0.873	1.055
M2(−1)	4.093	2.410	−0.637	8.823
INFC(−1)	0.183	0.047	0.091	0.275
EMPC(−1)	−1.452	2.933	−7.207	4.303
ICC(−1)	−0.011	0.013	−0.036	0.013
SUP(−1)	2.243	1.322	−0.351	4.837
STK(−1)	0.133	1.110	−2.045	2.310
Constant	0.056	0.039	−0.021	0.132
d112016	−0.014	0.042	−0.096	0.067

Sum of squared residuals: 89.33
R-squared: 0.304
Adj. R-squared: 0.224

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MIDAS Modeling for Core Inflation Forecasting

Luis Libonatti

Abstract

This paper presents a forecasting exercise that assesses the predictive potential of a daily price index based on online prices. Prices are compiled using web scraping services provided by the private company PriceStats in cooperation with a finance research corporation, State Street Global Markets. This online price index is tested as a predictor of the monthly core inflation rate in Argentina, known as "resto IPCBA" and published by the Statistics Office of the City of Buenos Aires. Mixed frequency regression models offer a convenient arrangement to accommodate variables sampled at different frequencies and hence many specifications are evaluated. Different classes of these models are found to produce a slight boost in out-of-sample predictive performance at immediate horizons when compared to benchmark naïve models and estimators. Additionally, an analysis of intra-period forecasts, reveals a slight trend towards increased forecast accuracy as the daily variable approaches one full month for certain horizons.

Keywords: MIDAS, distributed lags, core inflation, forecasting.

JEL Classification: C22, C53, E37.

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1. INTRODUCTION

Forecasting inflation has become increasingly important in Argentina as it is essential for economic agents to adjust wages and prices—particularly in recent years—in a context of high and volatile inflation. Having timely updates about the future trajectory of the inflation rate is essential for conducting monetary policy, specially, since the Central Bank is transitioning towards an inflation targeting regime. Recent developments in the use of "big data" have greatly facilitated tracking macroeconomic variables in real-time. A remarkable example is the construction of online price indexes that are sampled daily, rather than monthly, as it is standard for traditional price indexes from statistical offices. The question naturally arises of whether this information can help predict the future trajectory of traditional consumer price indexes. Ghysels *et al.* (2004) introduced a regression framework that allows for the exploitation of time series sampled at different frequencies, known in the literature as Mixed Data Sampling (MIDAS) regression models. The methodology reduces to fitting a regression model to a low-frequency variable using high-frequency data as regressors. As it will be shown later, this technique closely resembles distributed lag models. This paper employs this methodology to assess whether the combination of price series sampled at different frequencies is an effective tool for improving forecast accuracy compared to naïve models, using the online price index constructed by PriceStats in cooperation with State Street Global Markets.

The rest of the paper is organized as follows. In the next section, a brief introduction to MIDAS models is presented. In the third section, existing theoretical research on MIDAS regressions as well as some applications in forecasting inflation are briefly reviewed. In the fourth section, the forecasting exercise is described, and the results are discussed. And finally, the fifth section concludes.

2. MIDAS REGRESSION MODELS

MIDAS regression models propose a data-driven method to aggregate high frequency variables into lower-frequency predictors. They provide an alternative to the well-known "bridge" approach (Schumacher, 2016) in which high frequency variables are aggregated with equal weights (flat aggregation).[1] Ghysels *et al.* (2004) suggested combining y_t, a low frequency process, and x_τ, a high frequency process that is observed a discrete and fixed number of times m each time a new value of y_t is observed, in a plain regression equation,

2.1
$$y_t = \sum_{(j=0)}^{(m-1)} \theta_j x_{t-j/m} + u_t,$$

or more compactly,

2.2
$$y_t = \left(\theta' x_t'\right)' + u_t$$

where $\boldsymbol{x}_t \equiv \left[x_t \cdots x_{t-(m-1)/m} \right]$ is a $1\times m$ row vector that collects all the x_τ corresponding to period t and $\boldsymbol{\theta} \equiv \left[\theta_0 \ldots \theta_{m-1}\right]'$ is the $m\times 1$ vector of weight coefficients.[2] Each j high frequency observation $x_{t-j/m}$ within the low frequency period t enters the model linearly as a variable accompanied by its specific weight, θ_j, totaling m explanatory variables and m weights, plus an error term. The high frequency subindex τ needs to be represented in terms of the low frequency index t by noting that $\tau = t-1+j/m$ for $j=1,\ldots,m$ since m is fixed, where $x_{t-0/m}$ would be the most recent observation. This structure actually conceals a high frequency lag polynomial $\theta\left(L^{1/m}\right) \equiv \sum_{j=0}^{m-1} \theta_j L^{j/m} x_t$ so that $L^{j/m} x_t = x_{t-j/m}$ is similar in fashion to a distributed lags model.

To provide a clearer perspective, it is perhaps easier to introduce matrix notation. Defining $X \equiv [x_1' \ldots x_T']$ as the $T\times m$ matrix that groups all the $\boldsymbol{x}_t$ vectors together; $\boldsymbol{y} \equiv \left[y_1 \cdots y_T \right]'$, the collection of the low frequency observations of size $T\times 1$; and $u \equiv [u_1 \ldots u_T]$ the residuals of the same length as $\boldsymbol{y}$, it is possible to unveil a simple multiple regression equation,

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1 In fact, this can be considered a special case of a MIDAS regression.

2 This equation may also include constants, trends, seasonal terms or other low frequency explanatory variables.

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{T-1} \\ y_T \end{bmatrix} = \begin{bmatrix} x_1 & \cdots & x_{1-(m-1)/m} \\ x_2 & \cdots & x_{2-(m-1)/m} \\ \vdots & \ddots & \vdots \\ x_{T-1} & \cdots & x_{T-1-(m-1)/m} \\ x_T & \cdots & x_{T-(m-1)/m} \end{bmatrix} \begin{bmatrix} \theta_0 \\ \vdots \\ \theta_{m-1} \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_{T-1} \\ u_T \end{bmatrix}. \tag{2.3}$$

Indeed, this problem can be solved by ordinary least squares (OLS) and this method will produce consistent coefficient estimates. Equation (2.1) is usually referred to as the unrestricted MIDAS regression model (U-MIDAS). [3] However, an inconvenience arises when m, the length of the vector θ, is large relative to the sample size T, as is usually the case in MIDAS regressions. When this occurs, the models suffer from parameter proliferation and OLS induces poor estimates and consequently, poor forecasts. A straightforward way to overcome this deficiency is to impose restrictions on the coefficients of the high frequency lag polynomial and restate each θ_j as a function of some q hyperparameters and its subindex j (its position within the low frequency lag polynomial) in such a way that $q \gg m$. Each θ_j is redefined as $\theta_j \equiv w_j(\gamma; j)$ where the vector γ is the collection of q hyperparameters that characterize the weight function $w_j(\bullet)$. Equation (2.1) is transformed to,

$$y_t = \lambda \sum_{j=0}^{m-1} \left(\frac{w_j(\boldsymbol{\gamma}; j)}{\sum_{m-1}^{j=0} w_j(\boldsymbol{\gamma}; j)} \right) x_{t-j/m} + u_t. \tag{2.4}$$

where λ is an impact parameter and the weights are normalized so that they sum up to unity. Ghysels *et al.* (2004) initially recommended what is known as the exponential Almon polynomial as a candidate for weight function as it allows for many different shapes and depends only on a few parameters. This is an exponentiated version of an Almon lag polynomial, which is well known in the distributed lags literature,[4]

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[3] Foroni et al. (2015) present a detailed assessment of this strategy.

[4] See for example the book by Judge *et al.* (1985).

2.5 $$w_j\left(\gamma_1,\ldots,\gamma_q;j\right)=e^{\Sigma_{s=1}^{q}\gamma_s j^s}.$$

Another conventional candidate is the beta probability density,

2.6 $$w_j\left(\gamma_1,\gamma_2;j\right)=z_j^{\gamma_1-1}\left(1-z_j\right)^{\gamma_2-1},$$

with $z_j \equiv j/\left(m-1\right), \gamma_1 > 0$ and $\gamma_2 > 0$.

Parameterization as in equation (2.5) has proved to be quite popular and has become the standard among researchers, particularly when q=*2*.

The introduction of constrained coefficients has many far-reaching implications. The model turns nonlinear and lacks a closed form solution. It is necessary to resort to nonlinear least squares and approximate the solution by numerical optimization routines. Additionally, the constraints are highly likely to introduce a bias in each θ_j. However, based on Monte Carlo simulations, when the sample size is small relative to the number of parameters, Ghysels *et al.* (2016) argue that both, parameter estimation precision and out-of-sample forecast accuracy, gained by the increase in degrees of freedom, far offset the effects of the bias generated by misspecified constraints.

MIDAS models are generally intended as a direct forecasting tool since this could prove to be more robust against misspecification (Marcellino *et al.*, 2006). This implies that estimation additionally depends on the time displacement of the variables, $d \in \mathbb{Q}$, and the forecast horizon,[5] $h \in \mathbb{N}$. The direct strategy requires estimation of as many models as per pair (d, h) is required. If T_Y is the time index of latest y_t available for estimation, and T_X is the time index of the latest x_τ available for both estimation and forecasting, then d can be defined as $d \equiv T_Y - T_X$. Setting $W\left(L^{1/m};\boldsymbol{\gamma}\right) \equiv \sum_{j=0}^{m-1} w_j\left(\boldsymbol{\gamma};j\right)L^{1/m}$, a forecast can be computed with,

2.7 $$\hat{y}_{T+h}=\hat{\lambda}_{d,h}W\left(L^{1/m};\hat{\boldsymbol{\gamma}}_{d,h}\right)x_{T-d}.$$

[5] How many periods into the future it is necessary to forecast.

The "nowcast" can be retrieved when d=−1 and h=1. Note also that, the fact that d is a rational number implies that it is possible to generate intra-period forecasts.

To arrive at equation (2.7), it is first necessary to estimate,

2.8

$$y_t = \lambda W\left(L^{1/m};\hat{\boldsymbol{\gamma}}\right)x_{t-h-d} + u_t,$$

and then compute $\hat{y}_{T+h}$ with the estimated parameters, $\hat{\lambda}_{d,h}$ and $\hat{\boldsymbol{\gamma}}_{d,h}$, and the vector x_{T-d}.

It is possible to extend the MIDAS model by allowing for more than m high frequency regressors. For example, by including p_X lags of the vector x_t totaling $m \times L_X$ high frequency variates where $L_X = p_x + 1$, the MIDAS-DL model is formed,

2.9

$$y_t = \sum_{r=0}^{p_X}\left(\boldsymbol{\theta}'_r \boldsymbol{x}'_{t-r}\right)' + u_t,$$

or equivalently,

2.10

$$y_t = \sum_{r=0}^{p_X}\sum_{j=0}^{m-1}\boldsymbol{\theta}_{r,j}x_{t-r-j/m} + u_t.$$

In matrix notation, this can be represented by,

2.11

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{T-1} \\ y_T \end{bmatrix} = \begin{bmatrix} x_1 & \cdots & x_{1-(m-1)/m} & \cdots & x_{1-p_X} & \cdots & x_{1-p_X-(m-1)/m} \\ x_2 & \cdots & x_{2-(m-1)/m} & \cdots & x_{2-p_X} & \cdots & x_{2-p_X-(m-1)/m} \\ \vdots & \ddots & \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{T-1} & \cdots & x_{T-1-(m-1)/m} & \cdots & x_{T-p_X-1} & \cdots & x_{T-p_X-1-(m-1)/m} \\ x_T & \cdots & x_{T-(m-1)/m} & \cdots & x_{T-p_X} & \cdots & x_{T-p_X-(m-1)/m} \end{bmatrix} \begin{bmatrix} \theta_{0,0} \\ \vdots \\ \theta_{0,m-1} \\ \vdots \\ \theta_{p_X,0} \\ \vdots \\ \theta_{p_X,m-1} \end{bmatrix} + \begin{bmatrix} u_1 \\ u_1 \\ \vdots \\ u_{T-1} \\ u_T \end{bmatrix}.$$

If different weight functions for each θ_r in equation (2.9), then the multiplicative or aggregates-based MIDAS model is obtained (Ghysels *et al.*, 2016). On the contrary, employing a single weight function for all $m \times L_X$ coefficients vectors θ_r is also possible. The first method allows for greater flexibility but at the cost of more parameters to estimate, so this possibility will not be considered, as this may not be convenient for a very short sample size.

Other possible extensions include constructing high frequency factors (Marcellino and Schumacher, 2010), incorporating cointegration relations (Miller, 2013), integrating Markov switching (Guérin and Marcellino, 2013), estimating multivariate models (Ghysels *et al.*, 2007), using infinite polynomials (Ghysels *et al.*, 2007) or adding low frequency autoregressive augmentations (Ghysels *et al.*, 2007; Clements and Galvão, 2008; Duarte, 2014), for example. Foroni and Marcellino (2013) provide a comprehensive survey of possible extensions in a recent survey about mixed frequency models.

3. LITERATURE REVIEW

Clements and Galvão (2008) were among the first to study applications of MIDAS regressions to macroeconomic variables. In their paper, they forecast U.S. real quarterly output growth in combination with three different monthly variables: *i)* industrial production, *ii)* employment growth, and *iii)* capacity utilization. They find a slight increase in out-of-sample forecast accuracy with both vintage and revised data compared to two benchmarks models, an autoregression and an ADL model in particular, for short-term horizons. They also derive and assess a model with autoregressive dynamics introduced as a common factor shared by the low and the high-frequency lag polynomials. Based on comments by Ghysels *et al.* (2007), they argue that including an autoregressive term in a standard MIDAS model, as in the next equation,

3.1 $$y_t = \phi y_{t-1} + \lambda W\left(L^{1/m};\gamma\right)x_t + u_t,$$

induces a seasonal response from y_t to x_t irrespective of whether x_t exhibits a seasonal pattern. They suggest further restricting the model by adding a common lag polynomial shared between y_t and x_t,

3.2
$$(1-\phi L)y_t = \lambda(1-\phi L)W(L^{(1/m)};\gamma)x_t + u_t,$$

so that when writing the model in distributed lag representation, the polynomial in L cancels out, eliminating the spurious seasonal response. A multi-step generalization of (3.2) for h-step-ahead forecasts would be,

3.3
$$\left(1-\phi L^h\right)y_t = \lambda\left(1-\phi L^h\right)W\left(L^{1/m};\gamma\right)x_t + u_t.$$

Armesto *et al.* (2010) analyze the performance of MIDAS models for the US economy for four different variable combinations: *i)* quarterly GDP growth and monthly employment growth; *ii)* monthly CPI inflation and daily Fed funds rate; *iii)* monthly industrial production growth and a measure of term spread, and *iv)* employment growth and again a measure of term spread. They contrast the results of flat aggregation, the exponential Almon polynomial and a step weight function, but are unable to find a dominant model specification. They provide detailed results for one-step-ahead intra-period forecasting performance of the models, computed by accumulating leads[6] as the high frequency variable approaches a full low frequency period. They find an erratic pattern for the root mean square forecast error (RMSFE) of the models as a function of the leads included in the regression. Thus, in a real-time setting, which intra-period forecasts could be the most accurate would not be trivial.

Monteforte and Moretti (2013) develop MIDAS models to forecast the euro area harmonized price index inflation. They put forward a two-step approach involving low and high frequency variables. In the first place, they estimate a generalized dynamic factor model (Forni *et al.*, 2000) for the inflation rate based on a set of variables,

[6] In this instance "lead" refers to an observation of the high-frequency predictor that corresponds to the same temporal period of the low frequency variable.

and then they extract a common component and separate that into a long-run and a cyclical, or short-run, component. The second step consists in fitting the model of Clements and Galvão (2008) to capture short-term dynamics and use financial time series as high frequency regressors, in addition to the long-run component previously estimated as well as other low frequency variables. They design three MIDAS models, M1, M2 and M3, each with different high frequency regressors: *i)* M1 includes the short-term interest rate, changes in interest rate spread and oil future prices; *ii)* M2 uses changes in the wheat price, oil future quotes and the exchange rate; and finally, *iii)* M3 consists of long-term rates, changes in the interest rate spreads, and changes in the short-term rate. They contrast the out-of-sample performance in terms of RSMFE of these models against the equations for the inflation rate of two different low frequency vector autoregressions, and univariate random walks, autoregressions and autoregressive-moving average models. They compute all the intra-period forecasts for the MIDAS models and the monthly average of these daily forecasts, and compare this average to all the low frequency models. All the analysis is conducted for one-month-ahead and two-month-ahead forecasts. They find on average a 20% reduction in forecast error dispersion. The authors also provide a final empirical exercise by using forecast combinations with the MIDAS models and the inflation rate implied by financial derivatives, but this approach does not produce any significant gains.

Duarte (2014) discusses in detail the implications of autoregressive augmentations in MIDAS regression models and diverse ways to incorporate them. She explores the out-of-sample performance of MIDAS models with autoregressive augmentations with no restrictions, with an autoregressive augmentation with a common factor restriction, and models with autoregressive augmentations with no restrictions and a multiplicative scheme to aggregation. She then compares these models to the same models but without the autoregressive component, and to two low frequency benchmark models, a low frequency autoregression and multiple regression model. She computes forecasts for quarterly euro area GDP growth based on three different series: *i)* industrial production, *ii)* an economic sentiment indicator and *iii)* the Dow Jones Euro Stoxx index. She disregards the seasonal spikes impulse responses as the relevant impulse responses, as she argues that it is not possible to single out a particularly relevant impulse response for a mixed-frequency process since responses vary depending on when the shocks occur within the low-frequency process. Although there is no superior model among all tested, Duarte finds once again that there are sizable gains compared to the benchmarks at all horizons.

Breitung and Roling (2015) propose a "nonparametric" MIDAS model to forecast monthly inflation rates using a daily predictor. Instead of imposing any particular polynomial parameterization, the nonparametric approach consists on enforcing some degree of smoothness to the lag distribution by minimizing a penalized least squares cost function,

3.4 $$S(\theta)=(y-\boldsymbol{X}\theta)'(y-\boldsymbol{X}\theta)+\eta\theta'\boldsymbol{D}'D\theta$$

where D is a $(m-1)\times(m+1)$ matrix such that

3.5 $$D=\begin{bmatrix} 1 & -2 & 1 & 0 & \cdots & 0 \\ 0 & 1 & -2 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 1 & -2 & 1 \end{bmatrix},$$

and η is a pre-specified smoothing parameter. They refer to this estimator the Smoothed Least Squares estimator, and its structure closely resembles the well-known Hodrick-Prescott filter. If η is not known, they suggest solving for the η that minimizes the Akaike Information Criterion. Their target variable is the harmonized index of consumer prices for the euro area and they use a commodity price index as a high frequency regressor. They compare their model against the unconditional mean and the parametric MIDAS model (exponential Almon weights) for two different forecast horizons. They conclude that the commodity index paired with the nonparametric MIDAS results in a reasonably good one-month-ahead forecasts. Additionally, the authors conduct a Monte Carlo experiment and compare their model to four parametric MIDAS alternatives: *i)* the exponential Almon polynomial, *ii)* a hump shaped function, *iii)* a declining linear function, and *iv)* a sinusoidal function. They find that the nonparametric method performs on par with the parametric competitors.

4. DATA, EXERCISE, AND RESULTS

The out-of-sample predictive performance of an online price index will be analyzed to forecast the core inflation rate in real-time. To be more specific, this will be assessed using many different MIDAS specifications discussed in the previous sections and these estimations will be compared with benchmark single frequency naïve models and estimators. MIDAS turn out to be intuitive for this purpose since the monthly inflation rate can be approximately decomposed as the aggregation of daily inflation rates of the corresponding month, when evaluated in logarithmic differences, $\pi_t^m \approx \sum_{\tau \in t} \left(\log p_\tau^d - \log p_{\tau-1}^d \right)$

Atkeson and Ohanian (2001), Stock and Watson (2007) and Faust and Wright (2009) have shown that simple benchmarks are not easily beaten by more sophisticated models (at least in the case of the US economy), and so these could serve as a good starting point to gauge the predictive power of the daily series.

4.1 Data

The online price index is compiled by the company PriceStats in cooperation with State Street Global Markets, a leading financial research corporation. PriceStates is a spin-off company that emerged from the Billion Prices Project at MIT, founded by professors Alberto Cavallo and Roberto Rigobón. It is the first company, institution, or organization to apply a big data approach to produce real-time (daily) price indexes to track general price inflation and other related metrics. Essentially, they collect daily data of prices from online retailers by "web scraping" (i.e. recording price information contained inside specific HyperText Markup Language tags in the retailers' websites) and aggregate the data by replicating the methodology of a traditional consumer price index, as is done by National Statistics Offices with offline prices. Cavallo (2013) goes through the methodology and provides comparisons between online and offline price indexes for Argentina, Brazil, Chile, Colombia, and Venezuela. He concludes that online price indexes can track the dynamic behavior of inflation rates over time fairly well with the exception of Argentina. In fact, the construction of online price indexes was initially motivated by the desire to provide the public with an alternate measure of the inflation rate in Argentina because from the years

2007 to 2015 there were large discrepancies between the official price indexes compiled by the National Institute of Statistics and Census (INDEC) and price indexes compiled by provincial statistics offices or those compiled by private consultants. Throughout the rest of the paper, this price index will be referred to as the State Street PriceStats Index (SSPS). Data for Argentina is available since November 1, 2007 with a three-day publication lag.

A provincial price index that raised itself to prominence in recent years is the consumer price index compiled by the General Department of Statistics and Censuses of the Government of the Autonomous City of Buenos Aires, known as IPCBA. Although this index only takes into account the territory of the City of Buenos Aires (with a population close to 3 million), it should be reasonable to expect that price dynamics in the Buenos Aires Metropolitan Area (which encompasses a much larger population, close to 14 million or 1 / 3 of the total population of Argentina) share most of its features with the pricing structure of the City of Buenos Aires, resulting from arbitrage by reason of geographical proximity, as this should prevent large distortions, at least in nonregulated markets. A more restricted version of the index is also published, called "resto IPCBA" (rIPCBA) witch serves as a measure of core inflation. Compared to the headline version, it excludes products with strong seasonal patterns and regulated prices (e.g. public utility services) and represents 78.15% of the headline index. rIPCBA is available from July 2012 onward and is released monthly, with approximately a two-week publication lag.

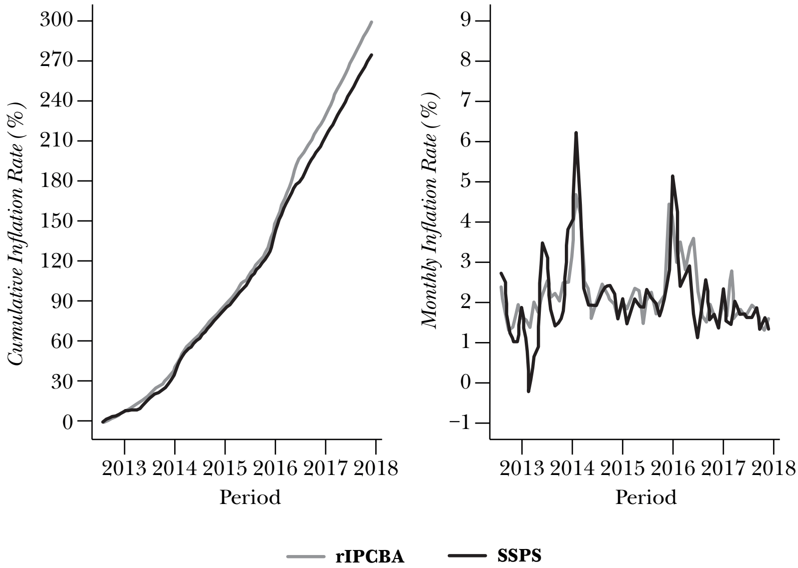
These two indexes, as well as other provincial private and public price indexes, are closely monitored by the monetary authorities as well as the general public. This is particularly true for INDEC's recently introduced National Consumer Price Index. As the name implies, this is the only index with full national coverage. However, this index so far consists of less than two years of data points and this limits the possibility of drawing any relevant inferences.

Inflation in Argentina in recent years has been high, unstable and volatile, particularly from 2012 to most of 2016 when Argentina experienced high monetization of fiscal deficits, strict capital controls and two major devaluations of the currency.[7] The average monthly

[7] The last one coinciding with the lifting of the majority of the capital controls in December 2015 and a subsequent transition to a flexible exchange rate regime and inflation targeting.

Figure 1

COMPARISON BETWEEN rIPCBA INFLATION AND SPSS INFLATION AGGREGATED TO MONTHLY FREQUENCY



inflation rate has been fluctuating around 2.2% for rIPCBA and 2.1% for the monthly aggregated SSPS series, with coefficients of variation at 35% and 49% respectively. This should pose a significant challenge for economists' ability to formulate accurate forecasts. Figure 1 illustrates the comparison between these two indexes and provides a quick glimpse at the potential predictive power of the high-frequency index. Overall and for the scope of this work, rIPCBA is available from July 2012 to December 2017 (66 data points) while SSPS ranges from November 1, 2007 to December 31, 2017 (3,714 data points).

4.2 Forecasting Exercise

The MIDAS specifications tested were the MIDAS-DL, the unrestricted autoregressive MIDAS-DL (MIDAS-ADL), and the autoregressive MIDAS-DL with the common factor restriction (MIDAS-ADL-CF). All MIDAS specifications were evaluated with several high frequency regressors equal to $m \times L_X$,[8] with $L_X \in \{1,2,3\}$, and forecasts were computed for horizons $h \in \{1,2,3\}$ over a 36-observation evaluation sample, spanning from January 2015 to December 2017, and an 18 observation subsample from July 2016 to December 2017 (a period with a more stable inflation rate), using recursive (expanding) windows. MIDAS-ADL-CF models included quadratic and cubic variations of the standard Almon polynomial and the exponential version, as well as the Beta probability density function. MIDAS-ADL models further added flat aggregation (equal weights); and finally, MIDAS-DL models added the nonparametric (NP) model described in Section 3. Forecast combinations of the various MIDAS models with equal weights were also considered. In addition, all these models were compared to two benchmarks: *i)* the low-frequency unconditional mean and *ii)* a low-frequency first order autoregression.[9]

In a first stage, the models were estimated with a balanced dataset. In other words, there is exact frequency matching: m daily observations from the same month or L_X groups of m daily observations from the same months correspond to a specific low-frequency monthly observation of the dependent variable. In total, two sets of RMSFE were computed, one corresponding to the large sample and the other to a reduced subsample. For all forecast horizons, d was set to $d = -1$.

A second stage involved estimating intra-period forecasts for the best selected L_X for each forecast horizon based on the results from the large sample of the first stage and briefly analyzing the stability of the forecasts as more recent information is incorporated in the models. When intra-period forecasts were computed, d is a fraction in the interval $[-1, 0)$. More specifically, $d = -1 + i/m$ for i in $1,\ldots,m$ where m is the frequency. Forecasts from the autoregression and the unconddditional mean remained the same throughout the month.

To account for the fact that SSPS is an irregularly spaced series, the frequency was assumed fixed at $m = 28$, and so days 29, 30 and 31 of each month are discarded. Daily inflation rates were first computed with the full dataset and then the observations beyond day 28 of each month were discarded.

8 First order MIDAS-ADL-CF models include $m \times \left[L_X + \min\left(L_X, h\right)\right]$. high-frequency regressors since the common factor restriction increases the number of variates depending on the forecast horizon and the number of high frequency lags.

9 A detailed list of the models can be found in Appendix A.

Estimation was conducted in R with the midasr package developed by Ghysels *et al.* (2016) while optimization was performed with three routines included in optimx[10] for nonlinear models or with the lm function from the stats package for linear ones. Models that require optimx were solved simultaneously with three optimization routines (ucminf, nlminb and Nelder-Mead) for each model, forecast horizon h, number of high frequency regressors L_X, and out-of-sample period. Only the best solution was kept. The algorithm was initialized taking the hypothesis of equal weights and a null impact parameter as starting conditions. This strategy delivered reasonable results empirically and serves as a check on whether the high-frequency regressors are actually relevant.

4.3 Empirical Results

Tables 1 and 2 summarize the main results of the first stage. In general, for $h=1$ (nowcasts), larger values of L_X produce better results while this tends to reverse when forecasting further into the future, i.e. $h=3$. For $h=2$, the results are ambiguous and indicate that $L_X=2$ or $L_X=3$ perform best. All three classes of MIDAS models exhibit similar performance irrespective of the inclusion of the autoregressive term or how it is incorporated. For all h, most MIDAS models for at least some L_X are able to produce a small gain at around 10% when compared to the autoregression and a larger 25% against the unconditional mean.[11] The smaller sample greatly amplifies these results. Note that for each h, there is a flat aggregation model that performed very well and, at times, even better than standard MIDAS models, but overall, there is not a single MIDAS model that systematically outperforms the rest. The forecast combination tested does not seem to improve over any particular MIDAS model.

Figures (2)-(4) condense the main findings of the second stage. Forecasts for $h=1$ display a clear trend towards better accuracy as the high frequency variable reaches a full low frequency period. In day 1 to day 28 point to point comparison, the RMSFE is reduced by approximately 20% and particularly, in the second half of the month, the models start to surpass the accuracy of the autoregression

[10] A comprehensive description about this package can be found in Nash and Varadhan (2011).

[11] Tables with RMSFE ratios are presented in Appendix B.

by up to 15% at most for some days. The improved performance, when evaluated in the subsample, suggests that it is even possible to obtain even better results as the inflation rate stabilizes. Similar behavior, although less evident, is observed for forecasts for period $h=3$ in the case of MIDAS-DL models. Forecasts for horizon $h=2$ display a rather erratic pattern excepting the flat aggregation MIDAS-DL and MIDAS-ADL models.

Figure 5 zooms in on the evolution of all intra-period forecasts for selected models, either $h=1$, $h=2$ or $h=3$. Despite the intra-period forecasts evidencing some volatility within the month, this does not seem to be a major concern as inflation stabilizes at the end of the sample. Additionally, note that forecasting further into the future yields a dynamic closer to the unconditional mean of the whole process. In the future, these results could be used as a training sample from which to compute inverse mean square error weights and perform forecast combinations, which could prove to be effective in mitigating intra-period forecast volatility.

Although the results look promising, they should be interpreted with caution. The predictive ability of the models was tested with the methodology by Giacomini and White (2006)[12] and both the unconditional and the conditional versions of the test were examined. The MIDAS models were evaluated against the two naïve benchmarks, modeling the difference in forecast accuracy as a constant (unconditional) and also as a first order autoregression (conditional). The results do not indicate that the difference in forecast accuracy is significant (at 0.05) for most MIDAS models. However, since the "large" out-of-sample evaluation set actually constitutes a small sample by literature standards, the result of the tests cannot be taken as final. As more observations become available, the tests could be updated with a larger sample to arrive at a more robust conclusion.

12 This is similar to the standard test by Diebold and Mariano (1995). The key difference lies in that the estimation sample size is kept fixed instead of ever expanding, as this allows to better incorporate estimation uncertainty and to compare nested models.

Table 1

OUT-OF-SAMPLE PREDICTIVE PERFORMANCE, RMSFE

	$h=1$			$h=2$			$h=3$		
	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$
Almon ($q=2$)									
MIDAS-DL	0.630	0.627	**0.564**	0.790	*0.717*	***0.712***	*0.740*	***0.721***	*0.751*
MIDAS-ADL	*0.578*	*0.589*	**0.564**	*0.775*	0.729	**0.724**	0.760	**0.741**	0.765
MIDAS-ADL-CF	0.592	0.625	***0.561***	0.785	0.722	**0.719**	**0.748**	0.733	0.768
Almon ($q=3$)									
MIDAS-DL	0.660	0.623	***0.571***	0.819	*0.731*	***0.720***	*0.755*	***0.724***	*0.757*
MIDAS-ADL	*0.609*	*0.609*	**0.574**	*0.805*	0.745	**0.731**	0.770	**0.739**	0.770
MIDAS-ADL-CF	0.617	0.636	**0.576**	0.827	0.741	***0.719***	0.762	**0.734**	0.777
Exp. Almon ($q=2$)									
MIDAS-DL	0.705	0.646	***0.566***	0.816	0.755	***0.749***	***0.803***	0.863	0.837
MIDAS-ADL	***0.627***	0.633	0.632	0.775	0.768	**0.766**	0.839	0.840	***0.835***
MIDAS-ADL-CF	0.639	*0.629*	***0.557***	*0.765*	***0.745***	0.885	0.875	***0.831***	0.860
Exp. Almon (q = 3)									
MIDAS-DL	0.731	**0.648**	0.661	0.834	*0.826*	***0.742***	*0.809*	*0.785*	***0.778***
MIDAS-ADL	***0.628***	*0.633*	0.645	0.822	0.834	**0.814**	0.827	**0.792**	0.807
MIDAS-ADL-CF	0.663	0.661	***0.563***	***0.812***	0.863	0.866	0.824	**0.800**	0.824

Beta									
MIDAS-DL	0.668	0.624	**0.574**	0.768	***0.694***	*0.701*	0.747	***0.730***	0.746
MIDAS-ADL	*0.571*	*0.622*	**0.568**	*0.728*	**0.707**	0.716	***0.732***	0.750	0.748
MIDAS-ADL-CF	0.614	*0.619*	***0.558***	0.739	**0.697**	0.704	0.740	**0.736**	*0.737*
Flat									
MIDAS-DL	1.158	0.617	***0.568***	0.745	***0.673***	*0.690*	***0.713***	*0.736*	*0.745*
MIDAS-ADL	*0.944*	*0.592*	***0.568***	*0.733*	**0.694**	0.746	**0.729**	0.766	0.777
Nonparametric									
MIDAS-DL	0.623	0.629	**0.567**	0.782	**0.717**	**0.717**	**0.718**	0.721	0.755
EW Forecast Combination									
MIDAS-DL	0.657	0.621	**0.569**	0.780	*0.719*	***0.710***	***0.738***	*0.741*	*0.750*
MIDAS-ADL	*0.585*	*0.595*	**0.570**	*0.765*	0.729	**0.728**	0.763	**0.760**	0.764
MIDAS-ADL-CF	0.609	0.627	***0.563***	0.768	**0.734**	0.754	0.770	**0.756**	0.770
Autoregression									
$p = 1$	0.619	0.619	0.619	0.757	0.757	0.757	0.757	0.757	0.757
Unconditional Mean									
$\overline{y}$	0.790	0.790	0.790	0.800	0.800	0.800	0.806	0.806	0.806

Notes: The evaluation sample comprises 36 data points, from January 2015 to December 2017. Characters in **bold** indicate the best number of variables, L_X, for each model and forecast horizon, h. Characters in *italics* indicate the best model for each number of variables, L_X, and forecast horizon, h.

Table 2

OUT-OF-SAMPLE PREDICTIVE PERFORMANCE, RMSFE

	$h=1$			$h=2$			$h=3$		
	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$
Almon ($q=2$)									
MIDAS-DL	0.555	0.521	**0.427**	0.646	*0.541*	***0.538***	*0.568*	*0.548*	***0.547***
MIDAS-ADL	*0.460*	*0.482*	**0.430**	*0.608*	0.547	**0.544**	0.579	**0.562**	0.569
MIDAS-ADL-CF	0.508	0.528	***0.422***	0.625	0.548	**0.543**	0.586	0.574	**0.573**
Almon ($q=3$)									
MIDAS-DL	0.564	0.560	***0.430***	0.683	*0.572*	***0.546***	*0.580*	***0.547***	*0.559*
MIDAS-ADL	*0.465*	*0.523*	**0.435**	*0.642*	0.581	**0.551**	0.578	**0.561**	0.584
MIDAS-ADL-CF	0.533	0.569	**0.433**	0.663	0.581	**0.549**	0.596	**0.581**	0.601
Exp. Almon ($q=2$)									
MIDAS-DL	0.568	0.523	**0.432**	0.656	0.540	**0.539**	0.574	***0.561***	***0.561***
MIDAS-ADL	*0.470*	***0.453***	**0.453**	*0.600*	0.538	***0.535***	***0.568***	0.572	0.578
MIDAS-ADL-CF	0.581	0.531	***0.425***	0.620	***0.533***	0.821	0.669	0.585	**0.575**
Exp. Almon (q = 3)									
MIDAS-DL	0.620	0.549	**0.430**	*0.566*	*0.551*	***0.533***	***0.555***	*0.559*	*0.559*
MIDAS-ADL	*0.469*	***0.460***	0.456	0.640	0.651	**0.540**	0.565	**0.561**	0.565
MIDAS-ADL-CF	0.613	0.545	***0.424***	0.720	0.657	**0.634**	**0.573**	0.579	0.579

Beta									
MIDAS-DL	0.598	0.491	**0.444**	0.592	***0.506***	*0.523*	0.587	***0.570***	0.583
MIDAS-ADL	*0.489*	***0.437***	*0.441*	*0.550*	**0.513**	0.529	*0.578*	0.585	***0.569***
MIDAS-ADL-CF	0.548	0.492	***0.432***	0.568	**0.511**	0.525	0.580	***0.570***	0.571
Flat									
MIDAS-DL	0.542	0.515	**0.417**	0.610	***0.501***	*0.526*	***0.539***	*0.577*	*0.594*
MIDAS-ADL	*0.540*	*0.479*	***0.405***	*0.577*	**0.515**	0.547	**0.544**	0.580	0.604
Nonparametric									
MIDAS-DL	0.551	0.539	**0.432**	0.633	0.543	**0.540**	0.554	**0.548**	0.557
EW Forecast Combination									
MIDAS-DL	0.488	0.522	**0.429**	0.621	***0.533***	***0.533***	*0.562*	*0.555*	***0.561***
MIDAS-ADL	***0.417***	*0.462*	*0.426*	*0.597*	0.544	**0.534**	**0.565**	0.566	0.573
MIDAS-ADL-CF	0.541	0.527	***0.426***	0.624	**0.551**	0.592	0.595	**0.573**	0.575
Autoregression									
$p = 1$	0.552	0.552	0.552	0.675	0.675	0.675	0.692	0.692	0.692
Unconditional Mean									
$\overline{y}$	0.697	0.697	0.697	0.691	0.691	0.691	0.685	0.685	0.685

Notes: The evaluation sample comprises 18 data points, from July 2016 to December 2017. Characters in **bold** indicate the best number of variables, L_X, for each model and forecast horizon, h. Characters in *italics* indicate the best model for each number of variables, L_X, and forecast horizon, h.

Figure 2

EVOLUTION OF THE RMSFE FOR HORIZON $h = 1$ WITHIN A MONTH FOR SELECTED MODELS WITH $L_X = 3$

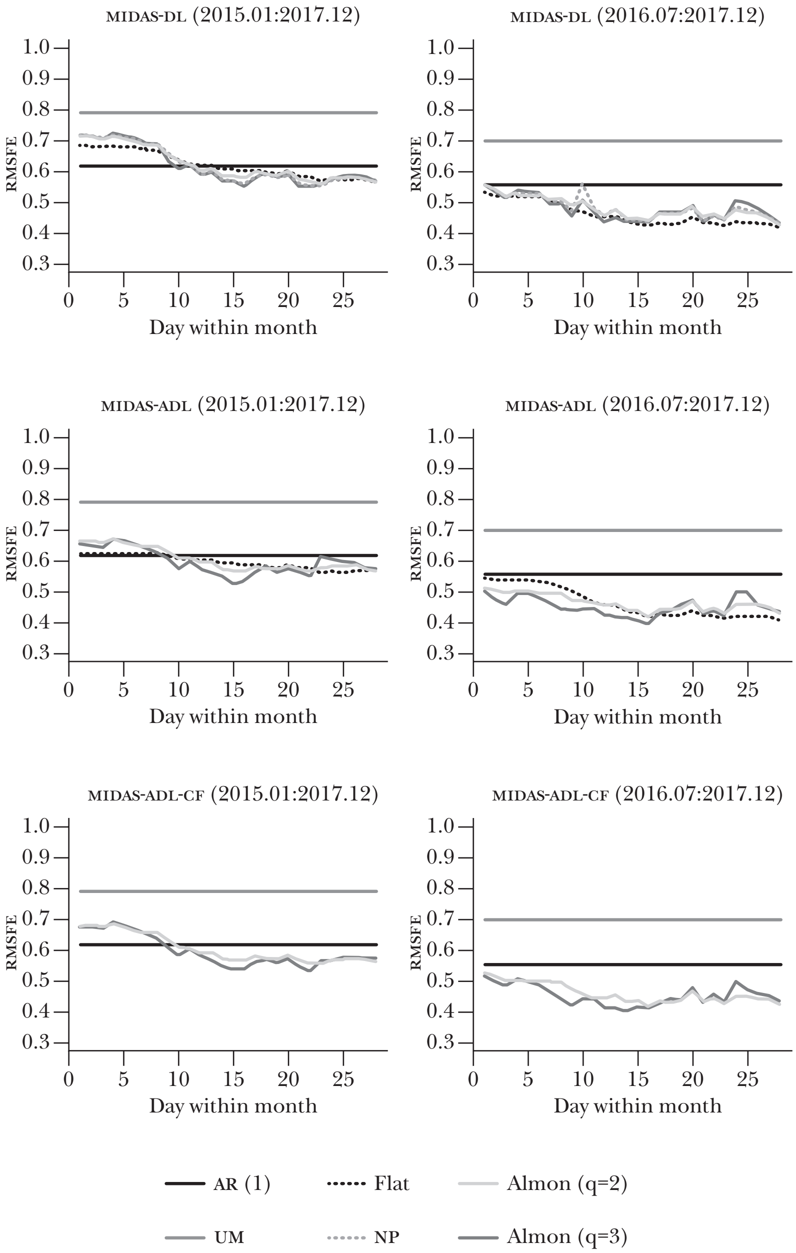


Figure 3

EVOLUTION OF THE RMSFE FOR HORIZON $h = 2$ WITHIN A MONTH FOR SELECTED MODELS WITH $L_X = 3$

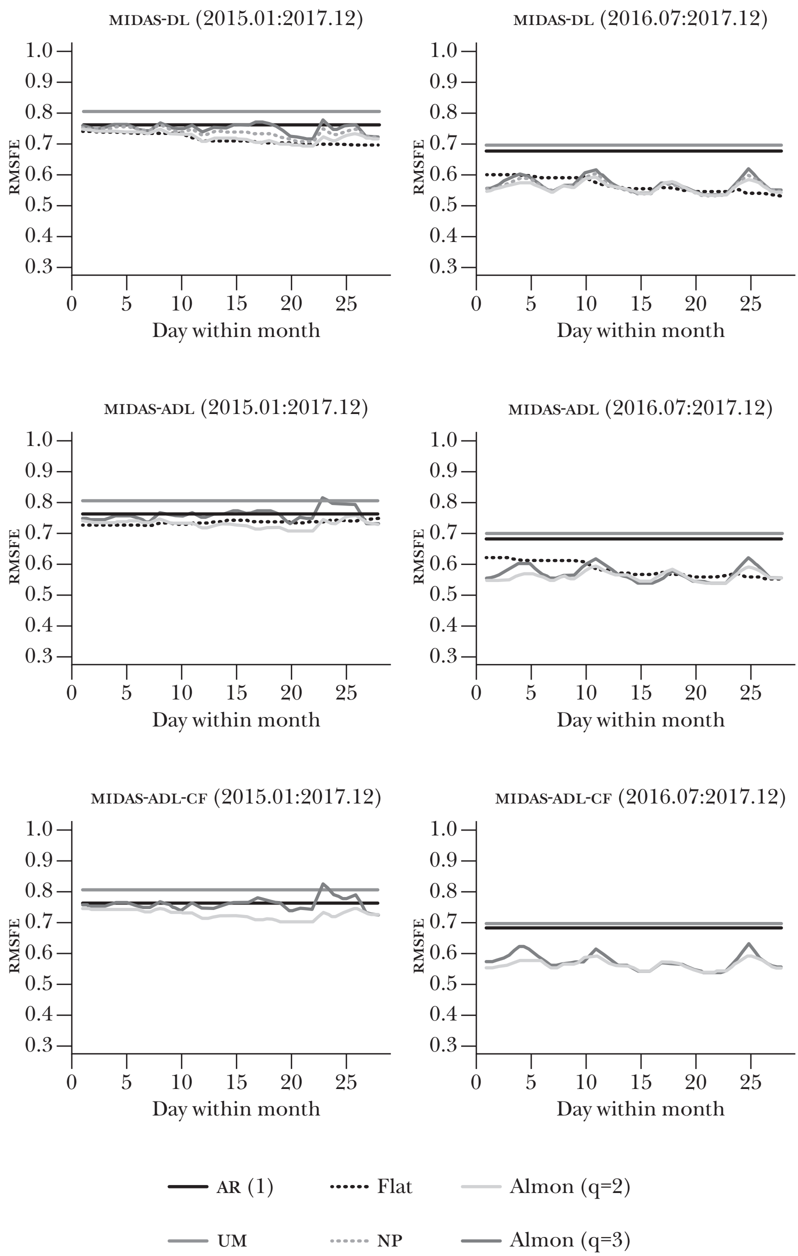


Figure 4

EVOLUTION OF THE RMSFE FOR HORIZON h =3 WITHIN A MONTH FOR SELECTED MODELS WITH $L_X = 2$

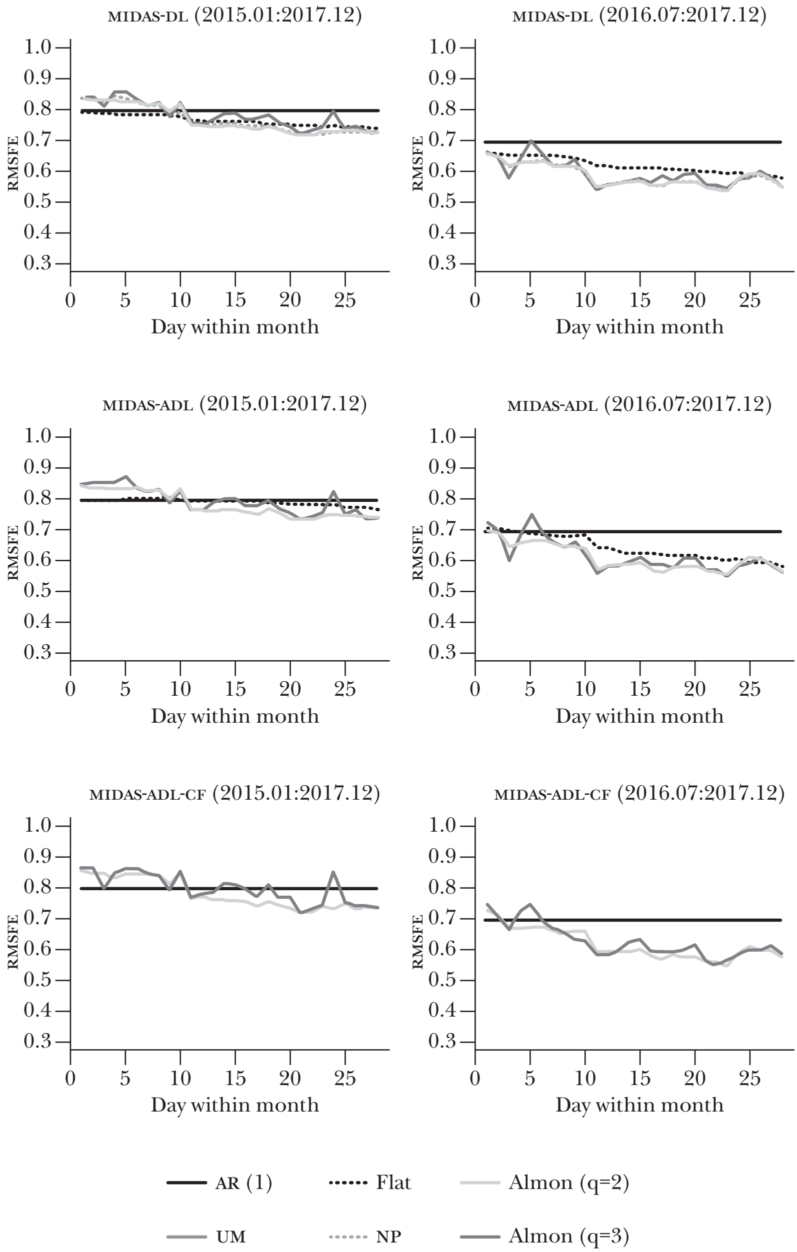
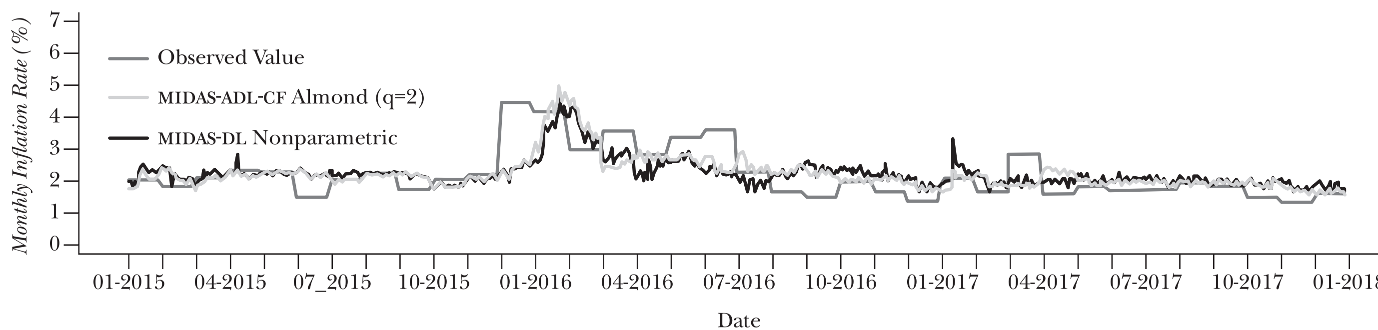


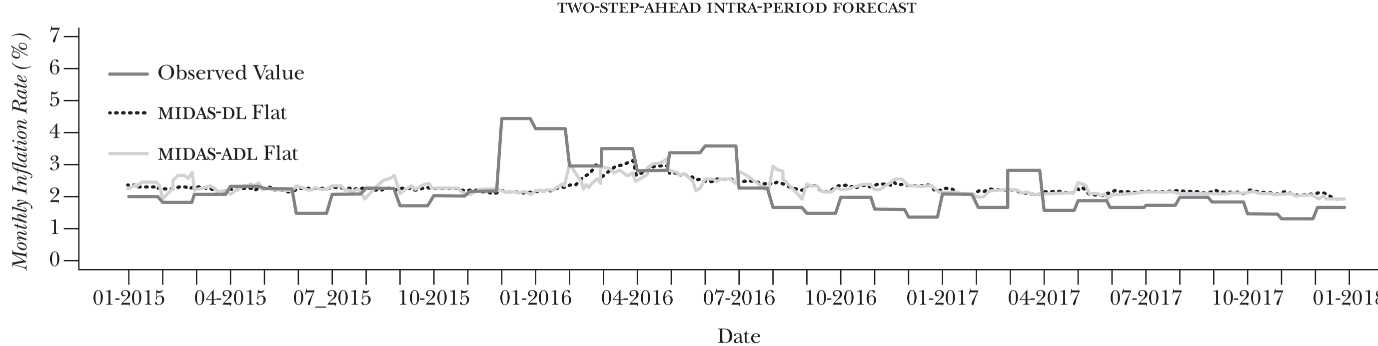
Figure 5

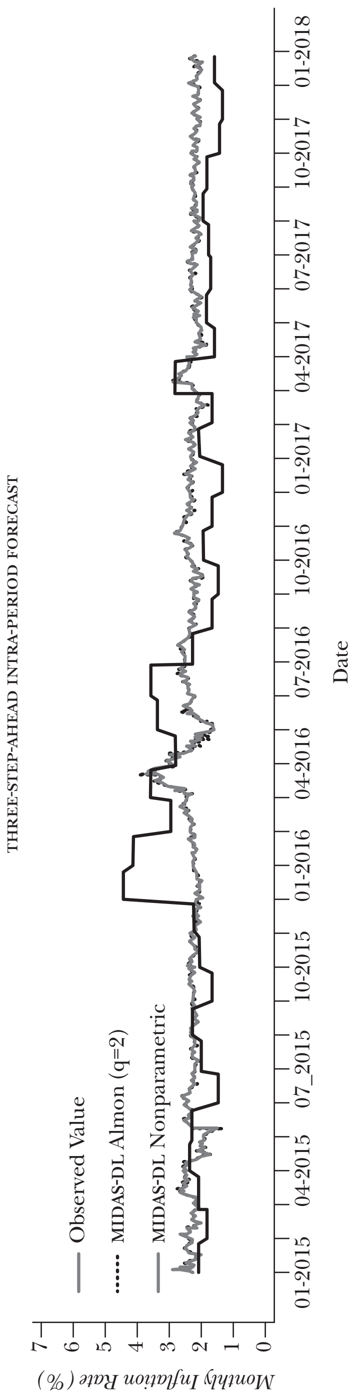
EVOLUTION OF INTRA-PERIOD FORECASTS FOR SELECTED MODELS AND FORECAST HORIZONS

ONE-STEP-AHEAD INTRA-PERIOD FORECAST



TWO-STEP-AHEAD INTRA-PERIOD FORECAST




THREE-STEP-AHEAD INTRA-PERIOD FORECAST
Observed Value
MIDAS-DL Almon (q=2)
MIDAS-DL Nonparametric
Monthly Inflation Rate (%)
0
1
2
3
4
5
6
7
01-2015
04-2015
07_2015
10-2015
01-2016
04-2016
07-2016
10-2016
01-2017
04-2017
07-2017
10-2017
01-2018
Date

5. CONCLUSION

For some particular MIDAS specifications, there is a slight improvement compared to the low-frequency benchmark autoregression and the unconditional mean. In principle, this would imply that high-frequency online price indices have a good potential to forecast future behavior of consumer inflation for immediate horizons in Argentina, but these results are still not robust. This could serve as a useful complementary tool to assess the out-of-sample performance of perhaps more sophisticated models. Future research could focus on building an alternative variable such as a daily financial factor as suggested by Monteforte and Moretti (2013) or comparing with measures of market expectations in order to further validate the findings of this paper.

ANNEX

Appendix A: MIDAS Specifications

The full set of specifications of the models is detailed below. All models were estimated with $L_X \in \{1,2,3\}$, $h \in \{1,2,3\}$ and d as explained in subsection 4.2. The subscript (d,h) on parameter estimates denoting dependence on d and h has been suppressed for simplicity.

MIDAS-DL:

A.1
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\lambda} \sum_{j=0}^{m \times L_X - 1} \left(\sum_{s=1}^{2} \gamma_s j^s \right) \pi_{T-d-j/m}^{SSPS}$$

A.2
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\lambda} \sum_{j=0}^{m \times L_X - 1} \left(\sum_{s=1}^{3} \hat{\gamma}_s j^s \right) \pi_{T-d-j/m}^{SSPS}$$

A.3
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\lambda} \sum_{j=0}^{m \times L_X - 1} \left(\frac{e^{\sum_{s=1}^{2} \gamma_s j^s}}{\sum_{j=0}^{m \times L_X - 1} e^{\sum_{s=1}^{2} \gamma_s j^s}} \right) \pi_{T-d-j/m}^{SSPS}$$

A.4
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\lambda}\sum_{j=0}^{m\times L_X-1}\left(\frac{e^{\sum_{s=1}^{3}\gamma_s j^s}}{\sum_{j=0}^{m\times L_X-1} e^{\sum_{s=1}^{3}\gamma_s j^s}}\right)\pi_{T-d-j/m}^{SSPS}$$

A.5
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\lambda}\sum_{j=0}^{m\times L_x-1}\left(\frac{z_j^{\gamma_1-1}\left(1-z_j\right)^{\gamma_2-1}}{\sum_{j=0}^{m\times L_x-1} z_j^{\gamma_1-1}\left(1-z_j\right)^{\gamma_2-1}}\right)\pi_{T-d-j/m}^{SSPS}$$

A.6
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\lambda}\sum_{j=0}^{m\times L_x-1}\pi_{T-d-j/m}^{SSPS}$$

A.7
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \sum_{j=0}^{m\times L_x-1}\hat{\theta}_j^{NP}\pi_{T-d-j/m}^{SSPS}$$

MIDAS-ADL:

A.8
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \phi\hat{\pi}_T^{rIPCBA} + \hat{\lambda}\sum_{j=0}^{m\times L_X-1}\left(\sum_{s=1}^{2}\hat{\gamma}_s j^s\right)\pi_{T-d-j/m}^{SSPS}$$

A.9
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\phi}\pi_T^{rIPCBA} + \hat{\lambda}\sum_{j=0}^{m\times L_X-1}\left(\sum_{s=1}^{3}\hat{\gamma}_s j^s\right)\pi_{T-d-j/m}^{SSPS}$$

A.10
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\phi}\pi_T^{rIPCBA} + \hat{\lambda}\sum_{j=0}^{m\times L_x-1}\left(\frac{e^{\sum_{s=1}^{2}\hat{\gamma}_s j^s}}{\sum_{j=0}^{m\times L_x-1} e^{\sum_{s=1}^{2}\hat{\gamma}_s j^s}}\right)\pi_{T-d-j/m}^{SSPS}$$

A.11
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\phi}\pi_T^{rIPCBA} + \hat{\lambda}\sum_{j=0}^{m\times L_x-1}\left(\frac{e^{\sum_{s=1}^{3}\hat{\gamma}_s j^s}}{\sum_{j=0}^{m\times L_x-1} e^{\sum_{s=1}^{3}\hat{\gamma}_s j^s}}\right)\pi_{T-d-j/m}^{SSPS}$$

A.12
$$\hat{\pi}_{T+h}^{rIPCBA} = \hat{\alpha} + \hat{\phi}\pi_T^{rIPCBA} + \hat{\lambda}\sum_{j=0}^{m\times L_x-1}\left(\frac{z_j^{\hat{\gamma}_1-1}\left(1-z_j\right)^{\hat{\gamma}_2-1}}{\sum_{j=0}^{m\times L_x-1} z_j^{\hat{\gamma}_1-1}\left(1-z_j\right)^{\hat{\gamma}_2-1}}\right)\pi_{T-d-j/m}^{SSPS}$$

A.13

$$\hat{\pi}^{rIPCBA}_{T+h} = \hat{\alpha} + \hat{\phi}\pi^{rIPCBA}_{T} + \hat{\lambda}\sum_{j=0}^{m\times L_x -1} \pi^{SSPS}_{T-d-j/m}$$

MIDAS-ADL-CF:

A.14

$$\hat{\pi}^{rIPCBA}_{T+h} = \hat{\alpha} + \hat{\phi}\pi^{rIPCBA}_{T} + \hat{\lambda}\left(1+\hat{\phi}L^{h}\right)\sum_{j=0}^{m\times L_X -1}\left(\sum_{s=1}^{2}\hat{\gamma}_s j^s\right)\pi^{SSPS}_{T-d-j/m}$$

A.15

$$\hat{\pi}^{rIPCBA}_{T+h} = \hat{\alpha} + \hat{\phi}\pi^{rIPCBA}_{T} + \hat{\lambda}\left(1+\hat{\phi}L^{h}\right)\sum_{j=0}^{m\times L_X -1}\left(\sum_{s=1}^{3}\hat{\gamma}_s j^s\right)\pi^{SSPS}_{T-d-j/m}$$

A.16

$$\hat{\pi}^{rIPCBA}_{T+h} = \hat{\alpha} + \hat{\phi}\pi^{rIPCBA}_{T} + \hat{\lambda}\left(1+\hat{\phi}L^{h}\right)\sum_{j=0}^{m\times L_x -1}\left(\frac{e^{\sum_{s=1}^{2}\hat{\gamma}_s j^s}}{\sum_{j=0}^{m\times L_x -1} e^{\sum_{s=1}^{2}\hat{\gamma}_s j^s}}\right)\pi^{SSPS}_{T-d-j/m}$$

A.17

$$\hat{\pi}^{rIPCBA}_{T+h} = \hat{\alpha} + \hat{\phi}\pi^{rIPCBA}_{T} + \hat{\lambda}\left(1+\hat{\phi}L^{h}\right)\sum_{j=0}^{m\times L_x -1}\left(\frac{e^{\sum_{s=1}^{3}\hat{\gamma}_s j^s}}{\sum_{j=0}^{m\times L_x -1} e^{\sum_{s=1}^{3}\hat{\gamma}_s j^s}}\right)\pi^{SSPS}_{T-d-j/m}$$

A.18

$$\hat{\pi}^{rIPCBA}_{T+h} = \hat{\alpha} + \hat{\phi}\pi^{rIPCBA}_{T} + \hat{\lambda}\left(1+\hat{\phi}L^{h}\right)\sum_{j=0}^{m\times L_x -1}\left(\frac{z_j^{\hat{\gamma}_1 -1}\left(1-z_j\right)^{\hat{\gamma}_2 -1}}{\sum_{j=0}^{m\times L_x -1} z_j^{\hat{\gamma}_1 -1}\left(1-z_j\right)^{\hat{\gamma}_2 -1}}\right)\pi^{SSPS}_{T-d-j/m}$$

Other:

A.19

$$\hat{\pi}^{rIPCBA}_{T+h} = \hat{\alpha} + \hat{\phi}\pi^{rIPCBA}_{T}$$

A.20

$$\hat{\pi}^{rIPCBA}_{T+h} = \left(1/T\right)\sum_{t=1}^{T}\pi^{rIPCBA}_{t}$$

Appendix B: Additional Tables

Table B.1

OUT-OF-SAMPLE PREDICTIVE PERFORMANCE, RATIO TO RMSFE OF AUTOREGRESSION ×100

	$h=1$			$h=2$			$h=3$		
	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$
Almon ($q=2$)									
MIDAS-DL	101.8	101.4	**91.2**	104.2	*94.7*	***93.9***	*93.1*	***90.8***	*94.5*
MIDAS-ADL	*93.4*	*95.1*	**91.1**	*102.3*	96.2	**95.6**	95.6	**93.2**	96.2
MIDAS-ADL-CF	95.7	101.0	***90.6***	103.6	95.3	**94.9**	94.1	**92.2**	96.6
Almon ($q=3$)									
MIDAS-DL	106.6	100.7	***92.2***	108.1	*96.5*	**95.0**	*95.0*	***91.2***	*95.2*
MIDAS-ADL	*98.3*	*98.4*	**92.7**	*106.3*	98.4	**96.5**	96.9	**93.0**	96.9
MIDAS-ADL-CF	99.8	102.7	**93.0**	109.2	97.8	***94.9***	95.8	**92.3**	97.7
Exp. Almon ($q=2$)									
MIDAS-DL	113.8	104.4	**91.5**	107.7	99.7	***98.8***	***101.0***	108.5	105.3
MIDAS-ADL	***101.3***	102.2	102.1	102.4	101.4	**101.1**	105.5	105.7	***105.1***
MIDAS-ADL-CF	103.2	*101.6*	***90.0***	*100.9*	***98.4***	116.8	110.1	***104.6***	108.2
Exp. Almon (q $=3$)									
MIDAS-DL	118.2	**104.7**	106.8	110.1	*109.1*	***97.9***	*101.8*	*98.8*	***97.9***

MIDAS-ADL	101.4	*102.3*	104.2	108.5	110.1	**107.4**	104.0	99.7	**101.5**
MIDAS-ADL-CF	107.1	106.8	***90.9***	*107.3*	114.0	**114.4**	103.7	**100.6**	103.7
Beta									
MIDAS-DL	108.0	100.8	**92.7**	101.4	***91.6***	*92.6*	*94.0*	***91.8***	*93.8*
MIDAS-ADL	*92.3*	100.5	**91.7**	*96.1*	**93.4**	94.5	**92.1**	94.4	94.2
MIDAS-ADL-CF	99.2	*100.1*	***90.2***	97.6	**92.0**	92.9	*93.1*	**92.6**	92.7
Flat									
MIDAS-DL	187.1	99.8	***91.8***	98.3	***88.8***	*91.1*	***89.7***	*92.6*	*93.8*
MIDAS-ADL	*152.6*	*95.6*	***91.8***	*96.7*	**91.6**	98.5	**91.7**	96.4	97.8
Nonparametric									
MIDAS-DL	100.6	101.6	**91.7**	103.3	**94.6**	**94.6**	**90.4**	90.7	95.0
EW Forecast Combination									
MIDAS-DL	106.2	100.4	**92.0**	102.9	*95.0*	***93.7***	***92.9***	*93.3*	*94.3*
MIDAS-ADL	*94.5*	*96.1*	**92.2**	*100.9*	96.3	**96.1**	96.0	**95.6**	96.2
MIDAS-ADL-CF	98.4	101.4	***90.9***	101.4	**96.9**	99.5	96.9	**95.1**	96.9
Unconditional Mean									
$\overline{y}$	127.7	127.7	127.7	105.6	105.6	105.6	101.4	101.4	101.4

Notes: The evaluation sample comprises 36 data points, from January 2015 to December 2017. Characters in **bold** indicate the best number of variables, L_X, for each model and forecast horizon, h. Characters in *italics* indicate the best model for each number of variables, L_X, and forecast horizon, h.

Table B.2

OUT-OF-SAMPLE PREDICTIVE PERFORMANCE, RATIO TO RMSFE OF AUTOREGRESSION×100

	$h=1$			$h=2$			$h=3$		
	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$
Almon ($q=2$)									
MIDAS-DL	100.6	94.4	***77.4***	95.7	*80.1*	***79.6***	*82.2*	*79.2*	***79.0***
MIDAS-ADL	*83.4*	*87.4*	**77.9**	*90.1*	81.0	**80.6**	83.8	**81.2**	82.4
MIDAS-ADL-CF	92.0	95.7	**76.5**	92.6	81.2	**80.5**	84.8	82.9	**82.8**
Almon ($q=3$)									
MIDAS-DL	102.3	101.5	***78.0***	101.2	*84.8*	***80.8***	83.9	***79.2***	*80.8*
MIDAS-ADL	84.2	*94.7*	**78.9**	*95.1*	86.0	**81.6**	*83.6*	**81.1**	84.4
MIDAS-ADL-CF	96.6	103.2	**78.5**	98.3	86.0	**81.3**	86.1	**84.1**	86.9
Exp. Almon ($q=2$)									
MIDAS-DL	103.0	94.8	***78.3***	97.1	80.0	**79.9**	83.1	***81.2***	***81.2***
MIDAS-ADL	*85.3*	***82.1***	**82.1**	*88.9*	79.7	***79.2***	***82.1***	82.8	83.5
MIDAS-ADL-CF	105.4	96.2	**77.1**	91.9	***79.0***	121.6	96.8	84.6	**83.2**
Exp. Almon (q $=3$)									
MIDAS-DL	112.4	99.5	**77.9**	*83.8*	*81.6*	***79.0***	***80.3***	80.8	80.8

MIDAS-ADL	*85.0*	*83.4*	**82.7**	94.8	96.4	**80.0**	81.7	**81.2**	81.7
MIDAS-ADL-CF	111.1	98.8	***77.0***	106.6	97.3	**94.0**	**82.9**	83.7	83.7
Beta									
MIDAS-DL	108.5	89.1	**80.5**	87.7	***74.9***	*77.5*	84.9	***82.4***	84.3
MIDAS-ADL	*88.6*	***79.2***	80.0	*81.5*	**75.9**	78.4	*83.5*	84.6	***82.3***
MIDAS-ADL-CF	99.4	89.3	***78.3***	84.1	**75.7**	77.7	83.8	***82.4***	82.5
Flat									
MIDAS-DL	98.3	93.4	**75.6**	90.3	***74.2***	*77.9*	***77.9***	*83.4*	*85.9*
MIDAS-ADL	*98.0*	*86.8*	***73.4***	*85.5*	**76.3**	81.0	**78.7**	83.8	87.3
Nonparametric									
MIDAS-DL	100.0	97.8	**78.3**	93.8	**80.4**	80.0	80.1	**79.2**	80.6
EW Forecast Combination									
MIDAS-DL	88.4	94.7	**77.7**	91.9	***78.9***	***78.9***	*81.2*	***80.2***	*81.2*
MIDAS-ADL	***75.5***	*83.9*	*77.3*	*88.4*	80.6	**79.1**	**81.8**	81.9	82.8
MIDAS-ADL-CF	98.1	95.5	***77.3***	92.4	**81.6**	87.7	86.0	**82.9**	83.1
Unconditional Mean									
$\overline{y}$	126.3	126.3	126.3	102.4	102.4	102.4	99.0	99.0	99.0

Notes: The evaluation sample comprises 18 data points, from July 2016 to December 2017. Characters in **bold** indicate the best number of variables, L_X , for each model and forecast horizon, h . Characters in *italics* indicate the best model for each number of variables, L_X , and forecast horizon, h .

Table B.3

OUT-OF-SAMPLE PREDICTIVE PERFORMANCE, RATIO TO RMSFE OF UNCONDITIONAL MEAN×100

	$h=1$			$h=2$			$h=3$		
	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$
Almon ($q=2$)									
MIDAS-DL	79.7	79.4	**71.4**	98.7	*89.6*	**88.9**	*91.8*	***89.5***	*93.2*
MIDAS-ADL	*73.1*	*74.5*	**71.4**	*96.9*	91.1	**90.5**	94.2	**91.9**	94.9
MIDAS-ADL-CF	75.0	79.1	***71.0***	98.1	90.3	***89.8***	92.8	**90.9**	95.3
Almon ($q=3$)									
MIDAS-DL	83.5	78.9	***72.2***	102.4	*91.3*	**89.9**	*93.7*	***89.9***	*93.9*
MIDAS-ADL	*77.0*	*77.1*	**72.6**	*100.6*	93.2	**91.3**	95.5	**91.7**	95.6
MIDAS-ADL-CF	78.1	80.4	**72.9**	103.4	92.6	***89.8***	94.5	**91.0**	96.3
Exp. Almon ($q=2$)									
MIDAS-DL	89.1	81.7	**71.7**	101.9	94.4	***93.6***	***99.6***	107.0	103.8
MIDAS-ADL	***79.4***	80.1	79.9	96.9	96.0	**95.7**	104.0	104.2	***103.6***
MIDAS-ADL-CF	80.8	*79.5*	***70.5***	*95.6*	***93.1***	110.6	108.5	***103.1***	106.7
Exp. Almon (q = 3)									
MIDAS-DL	92.5	**82.0**	83.6	104.3	*103.3*	***92.7***	*100.3*	*97.4*	***96.6***
MIDAS-ADL	***79.4***	*80.1*	81.6	102.7	**104.2**	101.7	102.6	**98.3**	100.1

MIDAS-ADL-CF	83.8	83.7	***71.2***	***101.6***	107.9	108.3	102.2	**99.2**	102.2
Beta									
MIDAS-DL	84.5	79.0	**72.6**	96.0	***86.7***	*87.6*	92.6	***90.5***	92.5
MIDAS-ADL	*72.3*	78.7	**71.8**	*91.0*	**88.4**	89.4	***90.8***	93.0	92.8
MIDAS-ADL-CF	77.6	*78.4*	***70.6***	92.4	**87.1**	88.0	91.8	**91.3**	***91.4***
Flat									
MIDAS-DL	146.5	78.1	**71.9**	93.1	***84.1***	*86.2*	***88.4***	*91.3*	*92.5*
MIDAS-ADL	*119.5*	*74.9*	***71.9***	*91.6*	**86.7**	93.3	**90.5**	95.0	96.4
Nonparametric									
MIDAS-DL	78.8	79.6	**71.8**	97.8	**89.6**	**89.6**	**89.1**	89.4	93.7
EW Forecast Combination									
MIDAS-DL	83.2	78.6	**72.1**	97.5	*89.9*	***88.7***	***91.6***	*92.0*	*93.0*
MIDAS-ADL	*74.0*	*75.3*	**72.2**	*95.6*	91.2	**90.9**	94.6	**94.3**	94.8
MIDAS-ADL-CF	77.1	79.4	***71.2***	96.0	**91.7**	94.2	95.5	**93.7**	95.5
Autoregression									
$p = 1$	78.3	78.3	78.3	94.7	94.7	94.7	98.6	98.6	98.6

Notes: The evaluation sample comprises 36 data points, from January 2015 to December 2017. Characters in **bold** indicate the best number of variables, L_X , for each model and forecast horizon, h . Characters in *italics* indicate the best model for each number of variables, L_X , and forecast horizon, h .

Table B.4

OUT-OF-SAMPLE PREDICTIVE PERFORMANCE, RATIO TO RMSFE OF UNCONDITIONAL MEAN×100

	$h=1$			$h=2$			$h=3$		
	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$	$L_X=1$	$L_X=2$	$L_X=3$
Almon ($q=2$)									
MIDAS-DL	79.6	74.8	**61.3**	93.5	*78.2*	***77.7***	*83.0*	*80.0*	***79.8***
MIDAS-ADL	*66.1*	*69.2*	**61.7**	*88.0*	79.1	**78.7**	84.6	**82.0**	83.2
MIDAS-ADL-CF	72.9	75.8	***60.6***	90.4	79.2	**78.5**	85.6	83.8	**83.7**
Almon ($q=3$)									
MIDAS-DL	81.0	80.3	***61.8***	98.8	*82.8*	***78.9***	84.7	***79.9***	*81.6*
MIDAS-ADL	*66.7*	*75.0*	**62.4**	*92.9*	*92.9*	84.0	**79.7**	*84.5*	**81.9**
MIDAS-ADL-CF	76.5	81.7	**62.2**	95.9	84.0	**79.4**	87.0	**84.9**	87.7
Exp. Almon ($q=2$)									
MIDAS-DL	81.5	75.0	**62.0**	94.8	78.1	**78.0**	83.9	***82.0***	***82.0***
MIDAS-ADL	67.5	***65.0***	**65.0**	*86.8*	77.8	***77.3***	***82.9***	83.6	84.4
MIDAS-ADL-CF	83.5	76.2	***61.0***	89.7	***77.1***	118.7	**97.7**	85.4	84.0
Exp. Almon (q $=3$)									
MIDAS-DL	89.0	78.8	***61.7***	*81.8*	*79.6*	***77.2***	***81.1***	*81.6*	*81.6*
MIDAS-ADL	*67.3*	*66.0*	**65.5**	92.6	94.1	**78.1**	82.5	**82.0**	82.5

MIDAS-ADL-CF	87.9	78.2	**60.9**	104.1	**95.0**	91.8	**83.7**	84.5	84.5
Beta									
MIDAS-DL	85.9	70.5	**63.7**	85.6	***73.2***	*75.7*	85.7	***83.2***	*85.2*
MIDAS-ADL	*70.2*	***62.7***	63.3	*79.6*	**74.1**	76.5	*84.3*	85.5	**83.1**
MIDAS-ADL-CF	78.7	70.7	***62.0***	82.1	**73.9**	75.9	84.6	***83.2***	83.3
Flat									
MIDAS-DL	77.8	73.9	**59.8**	88.2	***72.4***	*76.1*	***78.7***	*84.2*	*86.8*
MIDAS-ADL	*77.6*	*68.7*	***58.1***	*83.5*	**74.5**	79.1	**79.4**	84.6	88.2
Nonparametric									
MIDAS-DL	79.1	77.4	**62.0**	91.5	78.5	**78.1**	**80.9**	80.0	81.4
EW Forecast Combination									
MIDAS-DL	70.0	75.0	**61.5**	89.7	***77.0***	***77.0***	*82.0*	***81.0***	*82.0*
MIDAS-ADL	***59.8***	*66.4*	*61.2*	*86.3*	78.7	**77.2**	**82.6**	82.7	83.6
MIDAS-ADL-CF	77.6	75.6	***61.2***	90.2	**79.7**	85.6	86.8	**83.7**	84.0
Autoregression									
$p=1$	79.2	79.2	79.2	97.6	97.6	97.6	101.0	101.0	101.0

Notes: The evaluation sample comprises 18 data points, from July 2016 to December 2017. Characters in **bold** indicate the best number of variables, L_X, for each model and forecast horizon, h. Characters in *italics* indicate the best model for each number of variables, L_X, and forecast horizon, h

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Evaluation of Inflation Forecasting Models in Guatemala

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Abstract

Inflation forecasts are a relevant input for monetary policy decision in central banks, particularly for those operating under inflation targeting. Therefore, central banks must continuously evaluate the forecasting accuracy of the models used to produce inflation forecasts. In this study, we present the results of an exhaustive evaluation of historical forecast performance for the most important models used to forecast inflation at Banco de Guatemala. We find evidence supporting the claim that time series models perform better for short time horizons while conditional DSGE models (both structural and semi-structural) do better in medium and long time horizons.

Keywords: economic forecasting, forecasting accuracy, forecasting efficiency.

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1. INTRODUCTION

Banco de Guatemala adopted a monetary policy framework based on inflation targeting (IT) in 2005. Because of the forward-looking nature of that regime, central bank authorities should base their policy decisions on reliable inflation forecasts. In fact, Banco de Guatemala employs an array of models to forecast inflation, which include OLS, ARIMA, structural and semi-structural DSGE type of models, as well as forecast combinations of all, or some of these approaches. Since each of these models provides different information about the future path of inflation, a rigorous evaluation of their performance is required in order to determine their reliability, so that the central bank staff could give more weight to more reliable models, and improve the less reliable ones or get rid of them.

This document presents the results of a thorough evaluation of the most frequently used models by Banco de Guatemala to forecast inflation. Our evaluation is divided according to the type of model employed to produce a forecast. First, we evaluate models that produce unconditional forecasts, based on four different approaches: *1)* forecasting accuracy and bias; *2)* ability to predict a change of trend; *3)* prediction similarity; and *4)* forecast efficiency. Second, we assess the performance of models that produce conditional forecasts, by generating in-sample projections for different scenarios of exogenous and endogenous variables. Our main findings indicate that time series models perform better for short time horizons, while the DSGE models are more efficient forecasting longer time horizons.

The remaining of this document is organized as follows. Section 2 presents a description of all unconditional and conditional models employed by Banco de Guatemala to generate inflation forecasts. Section 3 describes the data and methodology employed for evaluation purposes. Section 4 shows the results obtained. Finally, Section 5 concludes.

2. FORECASTING EVALUATION AT THE CENTRAL BANK OF GUATEMALA

The prediction of the inflation rate is very important in the case of an inflation targeting regime, because it allows the central bank to take the monetary policy actions to keep inflation on target and keep the credibility of the regime. Therefore, Banco de Guatemala uses an array of models to forecast the inflation rate. The main forecast models are divided between those that produce unconditional forecasts and those producing conditional forecasts.

2.1 Unconditional-forecasts Models

In this section, we describe the main models used in this paper to evaluate unconditional inflation forecasts. We start by explaining the three main models used to explain the inflation rate. The first one is the indicator variable (IV), which is the inflation forecast employed at Banco de Guatemala as the main short-term forecast in the conduction of its monetary policy, and it is estimated by the Department of Macroeconomic Analysis and Forecasts. The forecast is based on a set of time series models plus the expert knowledge that the economic analysts have about the inflation series. In particular, they complement the inflation forecasts generated by the models with considerations about trend, seasonality, and temporary shocks, in addition to the overall domestic and foreign economic conditions. The second one is the forecast combination through individual time-varying efficient weights (EFP). This model is based on assessing past forecast performance efficiency at each of eight quarters ahead, according to an algorithm called the efficient forecast path (EFP), described in Castillo y Ortiz (2017). The model is explained in detail in Annex 3, which is delivered upon request. The third one is the average macroeconomic models (AMM), used by the Economic Research Department (DIE[1]). The DIE uses two macroeconomic models to make forecasts: the semi-structural macroeconomic model 4.0.1 (MMS) and the macroeconomic structural model (MME).

Furthermore, we evaluate inflation expectations with two measures available at Banco de Guatemala. Both are measured monthly. The first one is from an Economic Expert Panel (EEP). Banco de Guatemala surveys an independent panel of experts from the private sector every month on economics, finance, and business in Guatemala. The objective of the survey is to assess their perception of the future trend of inflation, economic activity, and public confidence in the economy. The second one is from the DIE, which also carries out an inflation expectations survey among its staff.

2.2. Conditional-forecasts Models

In this section, we evaluate the performance of three conditional models to predict the inflation rate. The first model is the MMS 4.0.1 which is a reduced form model, characterized by a difference-equations system, representing the transmission mechanisms of monetary policy for quarterly data. The current version (MMS 4.0.1) is part of the set of non-micro funded general equilibrium macroeconomic models used at Banco de Guatemala that have evolved from the first version launched in 2006. It was built on the basis proposed by Berg, et al. (2006a and 2006b), who provided a practical guide to non-micro funded DSGE models and their implementations for central banks. In this regard, the MMS 4.0.1 is a semi-structural model (non-micro funded) for a small, open economy, where monetary authorities operate policy within an inflation-targeting framework and implement monetary policy through a Taylor-type rule. All variables in the model are specified in annual growth rates. The MMS 4.0.1 has 40 equations (and 40 variables), of which 28 (70%) are endogenous and 12 (30%) are exogenous variables. The model delivers forecasts for both core inflation and headline inflation, and it is currently used for producing inflation and monetary policy interest rate forecasts that are inputs for Banco de Guatemala's monetary policymaking process. Those variables that display high volatility are transformed through a moving sum (or average) scheme in order to reduce that volatility and avoid possible outliers. At that respect, we get smoothed series.

The second model is a macroeconomic model of inflation forecast for Guatemala (PIGU). It is also a semi-structural macroeconomic model, very similar to the MMS 4.0.1. Variables in PIGU are also expressed as annual rates of change. There are three main differences between PIGU and MMS 4.0.1: the set of exogenous variables, the exogenous variables' volatility, and the type of inflation. First, the set of exogenous variables: Even though some exogenous variables are common to both models, others are not. For example, foreign inflation in MMS 4.0.1 is the US core-PCE inflation, while in PIGU is US Headline CPI inflation. Second, the exogenous variables' volatility: many MMS 4.0.1's exogenous variables are smoothed (four-quarter averages), while PIGU uses quarterly variables. Finally, the type of inflation: MMS 4.0.1 forecasts both core and headline inflation, while PIGU forecasts headline inflation only. The model is currently available to all the central bank's staff, through a custom-made interface.

The third model is the macroeconomic structural model (MME), which is a medium scale DSGE model, built within the new-Keynesian framework. It features a financial accelerator à la Bernanke, Gertler and Gilchrist (1999) and other frictions relevant for emerging or developing economies, such as deviations from the law of one price and the UIP. It is a model of heterogeneous agents; households supply labor services to entrepreneurs. They consume domestic and foreign goods, constitute deposits in domestic currency, take foreign debt and collect remittances from abroad. Firms, operating in a perfectly competitive market, assemble differentiated varieties to produce the home (or domestic) homogeneous final good. There are other firms producing the intermediate good, operating in a monopolistic competitive market; they buy a homogeneous wholesale good from entrepreneurs to differentiate it and produce a particular variety. When these firms decide to change their prices, they face adjustment costs, à la Rotenberg (1982), introducing nominal price rigidities into the model. Entrepreneurs use three inputs to produce the wholesale good: capital, labor, and imported raw materials. They buy capital from capital producing firms using their own wealth and loans granted by banks since they are not able to self-finance their entire capital purchases. The financial sector is comprised of private banks divided into two activities: narrow banks that carry out passive operations gathering deposits from households and retail banks using those deposits to grant loans to entrepreneurs. There is also a central bank setting the short-term interest rate–the policy rate–according to a Taylor-type rule and a central government carrying out unproductive spending.

3. DATA AND FORECAST EVALUATION METHODOLOGY

In this section, we describe the data and explain the methodology chosen in order to examine the forecasting accuracy of both the unconditional and conditional models. In the case of the forecast evaluation of unconditional models, the statistical tests are not included in this paper; however, they can deliver upon request (see Annex 3).

3.1 Data

First, we begin describing the dataset used for the unconditional models. First, we use quarterly data to evaluate the forecasting accuracy of the unconditional models. Each quarter, the IV and the AMM model forecast inflation for the next eight quarters, starting at 2011Q1 and finishing at 2017Q2. The EFP model starts forecasting inflation every quarter for the next eight quarters only from 2014Q2 to 2017Q2. Then, we classify the forecasts of each quantitative model into different time-horizons (one, two, three, four, and eight quarter) to evaluate the forecasting performance of each time horizon, in order to find which model is best to forecast the inflation patterns in every one of them. The evaluation sample is rather short, especially in the case of the EFP's forecasts, for which there are only 13 quarters. Also, we evaluate how well the quantitative models predict the inflation rate in December the current and the next year. Second, we use the monthly data on inflation expectations from both an economic experts' panel (EEP) and the DIE to examine the accuracy of the inflation expectations in prediction the inflation of December over a one and two-year horizon. The sample of forecasting errors is from 2015M07 to 2017M06 in the case of the one-year horizon and from 2016M07 to 2017M06 in the case of two-year horizon predictions.

Second, we describe the data used in the case of the conditional models. For each of the three evaluated models, we generate quarterly headline inflation forecasts with a sample from 2011Q1 to 2017Q2.[2] In addition, we consider five forecasting horizons: One quarter, two quarters, four quarters, six quarters, and eight quarters.

3.2. Forecast Evaluation Methodology

First, we explain the methodology to evaluate the forecasting accuracy of the unconditional models. We evaluate the key properties of the forecasting errors; i.e., we perform precision, accuracy, directional

[2] A first evaluation was conducted considering a wider sample (2006Q1-2017Q2), but results from this exercise were not as expected, in particular for headline inflation forecasts. This could be due to some periods of high volatility in headline inflation. For example, inflation went from 14.16% in the third quarter of 2008 towards a negative value (–0.73%) one year later (in August 2009). Therefore, in order to get robust results, we began our evaluation from 2011Q1.

change, and efficiency tests to evaluate which model is best to predict the future path of inflation. We start examining the residuals distribution of the forecast, checking for normality and skewness. Then, we compare the root mean square error (RMSE) values to find which model predicts the inflation rate best. After that, we use the Diebold-Mariano (DM) test to examine if the difference between the MSE of the two competing models is statistically significant at least at the 10% level. Also, we use the Giacomini-Rossi fluctuation (GR) test to examine the forecasting accuracy between the two competing models over forecasting horizons with rolling windows of four. With this test, we examine if the forecasts of one model are better than another in every rolling window or if there is a change (fluctuation) in the accuracy. In addition, we use the Pesaran-Timmerman (PT) test to determine if the forecasts of the models can correctly predict the directional change of inflation. Finally, we test the efficiency of the forecasts by calculating the weak and strong efficiency tests.

Second, we explain the methodology to evaluate the performance of the conditional models to predict the inflation rate. The quality of any variable's conditional forecasts depends on two elements: The performance of the forecasting model (as such) and the quality of the forecasting model's inputs on which the forecasts are conditioned (e.g., the quality of the exogenous variables' forecasts). We evaluate the forecasting model's performance by generating in-sample forecasts in hindsight for different scenarios for the exogenous variables and for some endogenous variables as well. Some of these scenarios involve historically observed values for the exogenous and some endogenous variables, to evaluate forecasts as if we had the best possible forecast for these variables and thus, eliminate one source of error. In the case of the semi-structural models (MMS and PIGU), we plug, for each forecasted period, the historically observed values of exogenous and some endogenous variables. In the case of the structural model (MME), exogenous variables are represented by stochastic processes, typically of autoregressive nature. Therefore, alternative scenarios are only conditioned by historically observed values of two endogenous variables: inflation and output.

First, the MMS 4.0.1 considers the scenarios: free, anchor 1, anchor 2, and anchor 3. In the free scenario, the exogenous variables' forecasts are generated by the model's laws of motion and all endogenous' forecasts are generated by the model. In the anchor 1 scenario, the exogenous variables' forecasts are generated by the corresponding historically observed data, and some endogenous variables' forecasts generated by the corresponding historically observed data: monetary aggregates and economic output. The anchor 2 scenario considers that the inflation forecast for the first quarter in the forecasting horizon is anchored by the corresponding historically observed data, besides the characteristics of the anchor 1 scenario. The last scenario (anchor 3) considers that the monetary policy interest rate is anchored by the corresponding historically observed data, as well as the characteristics of the anchor 2 scenario.

Second, PIGU considers the scenarios: free, anchor 1, anchor 2, and anchor 3. The free scenario contains the same characteristics than in the case of the MMS 4.0.1. In the anchor 1 scenario, the exogenous variables' forecasts are generated by the corresponding historically observed data, and all endogenous variables' forecasts are generated by the model. In the anchor 2 scenario, the exogenous variables' forecasts are generated by the corresponding historically observed data, the inflation forecasts for the first two quarters in the forecasting horizon are anchored by the corresponding historically observed data, while all other endogenous forecasts are generated by the model. In the anchor 3 scenario, the exogenous variables' forecasts are generated by the corresponding historically observed data, the inflation forecasts for the first two quarters in the forecasting horizon are anchored by the corresponding historically observed data, and all other endogenous variables' forecasts are anchored by the corresponding historically observed data.

Third, the MME considers two scenarios: free and anchor 1. In the free scenario, the exogenous variables forecasts are generated by the model's law of motion. In the anchor 1 scenario, the exogenous variables are generated by the model's laws of motion; and the inflation and output forecasts for the first quarter in the forecasting horizon are anchored by the corresponding historically observed data.[3]

For each model's horizon-scenario combination, we compute the mean error and the root mean squared error. The quantitative results allow us to compare the models' forecasting performances (provided that they are fed with the best possible inputs; i.e., they

[3] Anchored values of inflation are slightly different from the corresponding observed values because the inflation series generated by the model has a quarterly frequency; hence, its annualized inflation rate is the sum of four quarterly values rather than a 12-month variation rate.

are fed with historically observed data for the relevant variables) and to assess the informative contribution of exogenous and endogenous variables for forecasting headline inflation.

4. RESULTS

In this section, we present the main results of the forecasting accuracy of both the unconditional and the conditional models. Most of the tables and figures are presented in Annex 5, which do not appear in this paper. However, they are delivered upon request.

4.1. Unconditional Forecast Evaluation

We compare the forecasting performance to predict the inflation patterns between the AMM, the IV, and the EFP model. Also, we evaluate the forecasting performance of the inflation expectations generated by both the EEP and the DIE. First, we compare the performance of the forecasts of the models to predict inflation one, two, three, four, and eight quarters ahead. Second, we analyze the accuracy of the forecasts to predict the inflation rate in December in either the current or the following year. The December inflation forecast is a monetary policy indicator variable at Banco de Guatemala; hence, its evaluation is very important.

4.1.1. Skewness and Normality

We start by evaluating the key properties of the forecasting error distribution: normality and bias. To examine normality, we use the JB test developed by Jarque and Bera (1980). The tables are included in Annex 5, which is delivered upon request. First, we evaluate the properties of the forecasts through different forecasting horizons. The forecast errors of the three models follow a normal distribution according to the Jarque-Bera test, at the conventional levels of significance. Also, the IV's forecast shows a negative skewness while the AMM's and EFP's forecasts show a positive skewness. However, the skewness is low in all cases. Also, the forecast errors of the inflation expectation predictions (both the EEP and the DIE) also follow a normal distribution. There is a positive bias in the inflation expectations predictions in the case of the DIE in both one- and the two-year horizons.

Second, we evaluate the properties of the forecasts in the case of December evaluation. The forecast errors of the three models also follow a normal distribution in all forecasting horizons. In addition, there is a positive bias in the EFP's forecast in the first three quarters while there is no skewness in the remaining ones. IV's and AMM's forecasts both tend to have a negative bias.

4.1.2. RMSE and MPE

We compute the RMSE and MPE to determine which forecasting model performs best, in the case of both the quantitative and the inflation expectations. The tables are included in Annex 5, which is delivered upon request. In the case of the quantitative models, the forecasts of the IV model are better in the short run–one and two quarters–based on the RMSE. In the middle run, the forecasts of the AMM model are more accurate. However, in the long run–eight quarters–, the forecasts of the EFP model outperform the others. Also, we also analyze the inflation expectations predictions. Based on the RMSE, the EEP's inflation expectations are more accurate than those of the DIE's in both the one- and two-years horizons

Second, we proceed to analyze the forecasting accuracy of the quantitative models in their ability to predict the inflation rate in December for the current and the following year, based on the RMSE. We observe that the forecasts of the AMM model are better than the others in the first five forecasting horizons, while the IV's forecasts are best for the last three horizons.

4.1.3. Diebold-Mariano Test

First, we use the DM test developed by Diebold and Mariano (1999) to compare the predictive accuracy between two competing models, of both the quantitative and the inflation expectations predictions. The null hypothesis is that the two models have equal accuracy. The results of the DM test in the case of the quantitative models are presented in Table 1 (the *p*-values of the test are shown in parenthesis). In Column 2, it is shown the test between the AMM and the IV model. Only in the case of four- and eight-quarter forward forecasting horizons, the DM-statistic is negative and statistically significant at 5% level; therefore, we reject the null hypothesis, and conclude that the forecasting accuracy of the AMM model is best for both the intermediate and long time horizons. Then, the DM-statist between the EFP and the IV model is presented in Column 3. The statistic is positive and statistically significant at 5% level in all forecasting horizons, which means that all MSE of the IV model are lower than those of the EFP model; therefore, the forecasting accuracy of the IV model is best to predict inflation.

After that, the DM-statistic between the EFP and the AMM model is presented in Column 4. The statistic is only positive and statistically significant for the one-, two-, and three-quarter forward forecasting horizons. This means that for those horizons, the MSE of the AMM model are lower than those of the EFP model; therefore, the AMM's forecasts are best to predict inflation in the short run. Also, we evaluate the predictive performance of the inflation expectations of both the EEP and the DIE. The DM-statistic is only statistically significant for the two-year horizon with a sample of 12 months. This means that the MSE of the EEP is lower than the MSE of the DIE. Thus, we conclude that the inflation expectation predictions of the EEP are more accurate than those of the DIE, only on this horizon.

Second, we compare the forecasting accuracy of the quantitative models to predict the December inflation rate, from different horizons. The results of the DM test are presented in Table 2. In Column 2, it is shown the test between the AMM and the IV model. The DM-statistics are negative and statistically significant starting from three-quarter forward forecasting horizon, so the MSE of the AMM model are lower than those of the IV model. Therefore, the forecasts of the AMM model are best to predict inflation.

Table 1

DM TEST, QUANTITATIVE MODELS

Forecasting horizons in quarters	*dm statistic (AMM-IV)*	*dm Statistic (EFP-IV)*	*dm statistic (EFP-AMM)*
1	1.44 (0.15)	1.71 (0.087)	1.65 (0.09)
2	1.30 (0.19)	1.97 (0.049)	2.03 (0.04)
3	0.21 (0.84)	1.79 (0.074)	1.70 (0.09)
4	–2.95 (0.00)	1.76 (0.079)	1.61 (0.11)
8	–3.35 (0.02)	2.91 (0.004)	–0.87 (0.38)

Sources: author's elaboration, central bank's forecasts.

Table 2

DM TEST, QUANTITATIVE MODELS

Forecasting horizons in quarters	*dm statistic (AMM-IV)*	*dm statistic (EFP-IV)*	*dm statistic (EFP-AMM)*
1	1.44 (0.15)	2.10 (0.036)	1.65 (0.10)
2	−0.95 (0.34)	2.70 (0.007)	2.55 (0.01)
3	−4.60 (0.00)	2.62 (0.009)	2.58 (0.00)
4	−2.33 (0.01)	4.75 (0.000)	7.32 (0.00)
5	−3.20 (0.00)	2.09 (0.036)	3.16 (0.00)
6	−2.93 (0.00)	−5.61 (0.000)	−22.50 (0.00)
7	−2.98 (0.00)	−62.39 (0.000)	2.58 (0.01)
8	−1.95 (0.05)	–	–

Sources: author´s elaboration, central bank's forecasts.

Then, the DM-statistic between the EFP and the IV model is presented in Column 3. The statistic is statistically significant in all forecasting horizons, which means that the MSEs of the IV model are lower than those of the EFP model. Hence, we reject the null hypothesis of equal accuracy. Also, the statistic is positive for the one- to five-quarter horizons, which means that the MSEs of the IV model are lower than those of the EFP. Hence, the IV model is more accurate in its prediction of December inflation rate in the short and intermediate time horizons. On the other hand, the statistic is negative from six to seven quarters ahead; therefore, the EFP model is best in the long run to predict the inflation rate. After that, the DM-statistic between the EFP and the AMM model is presented in Column 4. This is statistically significant in all forecasting horizons, which means that we reject the null hypothesis of equal accuracy. Also, in almost all forecasting horizons, the MSEs of the AMM model are lower than those of the EFP model. Therefore, the AMM model is best to predict inflation rate in December.

4.1.4. Pesaran-Timmerman Test

We use the PT test developed by Pesaran and Timmerman (1992) to evaluate the directional forecasting of both the quantitative models and the inflation expectations predictions. The critical values to reject the null hypothesis of independence are ± 1.645 for 10% level of significance. First, we examine the directional forecasting accuracy in the case of the IV model (see Annex 1, Table A.1.1). The S_n statistic is only higher than its critical value in the case of one-, two- and three-quarter horizons, so we can reject the null hypothesis of independence and conclude that the forecasts of the IV model can predict successfully the direction of inflation in the short run. Now, we evaluate the directional accuracy in the case of the AMM model (see Column 3). We observe that the S_n statistic is higher than its critical value only in the case of one- and two-quarter horizons, so we can reject the null hypothesis of independence only for those two horizons and conclude that the model can successfully predict the direction of the inflation in the short run. We proceed to analyze the directional accuracy of the forecast in the case of the EFP mode (see Column 4). The S_n statistic is higher than the critical value only in the case of one-quarter horizon; therefore, we can only reject the null hypothesis of independence for this horizon and conclude that the forecast of the EFP model can predict successfully the direction of the inflation in the case of that particular horizon. Also, we analyze the directional forecasting accuracy of the inflation expectations predictions of both the EEP and the DIE. We reject the null hypothesis of independence only in the case of the EEP's forecasts in the case of a two-year horizon. Hence, we can conclude that the panel can predict successfully the direction of inflation.

Second, we examine the directional forecasting accuracy of the inflation rate for December (see Table Annex 2, A.2, which is delivered upon request) only for the case of the IV and AMM models, since we do not have enough data for the case of the EFP model. We start with the IV model (see the first column). We can reject the null hypothesis of independence in the case of one-, three-, four-, five-, and six-quarter horizons, so the model can predict successfully the directional change of inflation in the short and middle run. Then, evaluate the performance of the AMM model (see the second column). We can reject the null hypothesis of independence in the case of one-, two-, three-, six-, seven-, and eight-quarter horizons, which implies that the model can predict successfully the directional change of inflation in both the short and the long run.

4.1.5. Giacomini-Rossi Fluctuation Test

We use the Giacomini and Rossi fluctuation test developed by Giacomini and Rossi (2010) to examine the performance of two competing models in the presence of possible instabilities. We use the IV model as the benchmark model in the case of the quantitative model, and the inflation expectations' predictions of the EEP in the case of expectations' forecasts. The test is only used in some of the forecasting horizons due to data availability. We set the rolling windows equal to four quarters to make the forecasting analysis. Also, we use graphical analysis to examine the performance of the forecasts of the two competing models in the different rolling windows to see whether there is a fluctuation in the forecasting accuracy. This is available in Annex 4, which is delivered upon request.

First, we start with the forecasting accuracy evaluation of the quantitative models (see Annex 1, Table A1.3). We define the loss function between the AMM and the IV model in Equation 1. If the loss function turns out to be negative, we conclude that the forecasts of the AMM model are more accurate than those of the IV model. On the other hand, if the loss function turns out to be positive, the forecasts of the IV model are better at predicting inflation than those of the AMM model. We observe that we reject the null hypothesis of equal forecasting accuracy over every forecasting horizon since the GR-statistic is higher than its critical value (see Table A1.3, Column 2). This means that one model displays better predictive ability to forecast inflation in at least one period of time. Also, the graphical analysis reveals that the forecasts of the IV model are more accurate than those of the AMM one step ahead. However, it seems that the forecasts of the AMM model predict better the inflation patterns in four- and eight-quarter horizons.

$$L_t\left(\hat{\theta}_{j-h,R},\hat{\gamma}_{j-h,R}\right)=MSE_{AMM,t}-MSE_{IV,t} \tag{1}$$

Then, we compare the forecasting accuracy between the EFP and the IV model with the use of the GR test (see Column 3). The loss function between the two models is defined by Equation 2. In this case, the null hypothesis of equal accuracy is rejected in every forecasting horizon since the GR-statistic is higher than the critical value. This means that, at least in one period, one model generates more accurate forecasts of inflation. The graphical analysis shows that the forecasts of the IV model are more accurate in almost all the evaluation sample in each forecasting horizon. Therefore, the forecasts of the IV model seem to be more accurate than the EFP model in all forecasting horizons (see Annex 5, delivered upon request).

2 $$L_t\left(\hat{\theta}_{j-h,R},\hat{\gamma}_{j-h,R}\right)=MSE_{EFP,t}-MSE_{IV,t}$$

Second, we use the GR test to examine the performance of the inflation expectations predictions from the DIE and the EEP. We consider the EEP data as a benchmark model. The loss function is set up in Equation 3. The graphical analysis shows that there is a fluctuation of the forecasting accuracy of the inflation expectations between the two models in the case of one-year horizon. However, the inflation expectations of the EEE predict better the inflation patterns in the case of the two-year horizon (see Annex 4, delivered upon request).

3 $$L_t\left(\hat{\theta}_{j-h,R},\hat{\gamma}_{j-h,R}\right)=MSE_{EEP,t}-MSE_{DIE,t}$$

4.1.6. Weak Efficiency Test

We examine the efficiency of the unconditional forecasts of both the quantitative and the qualitative models with a variant of the weak efficiency test developed by Mincer and Zarnowitz (1969). First, we start with the quantitative models (see Annex 1, Table A1.4). From the second column, we observe that AMM's forecasts satisfy the weak efficiency hypothesis only in the case of one quarter ahead. From the third column, we analyze the weak efficiency of the IV forecasts (see the third column) We observe that forecasts of the model satisfy the weak efficiency only in the case of one and two forecasting horizons. From the fourth column, we evaluate the weak efficiency of the EFP forecasts (see the fourth column). We observe that the forecasts of the model satisfy the weak efficiency in almost all forecasting horizons with the exception of four quarters ahead. In sum, the forecast of the EFP is more efficient than those of the other models based on the results of the weak efficiency test. Also, the forecast of the AMM and the IV are weakly efficient in the short run. In addition, the inflation expectations predictions of both the EEP and the DIE model do not satisfy de weak efficiency test at 5% level in all forecasting horizons.

Second, we test for the weak efficiency only in the case of the AMM and the IV models, in the prediction of the inflation rate of December, because of data availability (see Annex 5, which is delivered upon request). In the case of the AMM's forecasts, we cannot reject the null hypothesis of weak efficiency only in the case of two and three quarters ahead. Also, the forecasts of the IV model satisfy the weak efficiency tests in five out of eight forecasting horizons. In sum, the forecasts of the IV model are more efficient than those of the AMM model in evaluating the December predictability of inflation.

4.1.7. Strong Efficiency Test

We perform the strong efficiency test for the two econometric models: IV and EFP. The null hypothesis establishes that a new variable (which is not included in the econometric models) does not explain the forecasting error. Therefore, the rejection of the null hypothesis means that the errors are strongly efficient. Otherwise, if the null hypothesis is not rejected, then the inclusion of a new variable can add information to improve the forecasts. We consider five variables in logs of the structural model of the Banco de Guatemala to make the test: consumption, index of raw materials, investment, government spending, and credit.

First, we start with the IV model; the tests are shown in Annex 5, Table A5.7, which is delivered upon request. In the second column, we list the coefficient of consumption. We cannot reject the null hypothesis at the 5% level of significance in the case of one and two quarters ahead. Therefore, the forecasts are strongly efficient for those horizons. However, for three to eight quarters ahead, consumption does explain the forecasting error, which means that they are not strongly efficient for these horizons. Similarly, in the third column, the null hypothesis is not rejected at the 5% significance level. Therefore, the forecasts are strongly efficient in those horizons. However, from three to eight quarters ahead, the inclusion of the raw material index can improve the forecasts, which mean that they are not strongly efficient. Then, in the fourth column, we observe that the null hypothesis is not rejected in one, two and three quarters ahead, which means that the forecasts are strongly efficient in those horizons. However, from four to eight quarters ahead, investment explains the forecasting errors, therefore; the forecasts are not strongly efficient. After that, in the fifth column, we observe that the null hypothesis is not rejected in all forecasting horizons, which means that the forecasts are strongly efficient, and the inclusion of government spending will not improve them. Finally, in the sixth column, we observe that the forecasts are strongly efficient from one to three quarters ahead. However, from four to eight quarters ahead, the inclusion of credit can improve the forecasts, which implies that they are not strongly efficient in those horizons.

We continue with the EFP model; the tests are shown in Annex 5, Table A5.8, which is delivered upon request. We observe that we reject the null hypothesis for one-quarter predictions for the five variables, which means that the forecasts of the IV model are not strongly efficient and the inclusion of the consumption, raw material index, investment, government spending, and credit can improve the forecasts for this forecasting horizon. However, the forecasts are strongly efficient in the case of the remaining forecasting horizons for the five variables, because we cannot reject the null hypothesis.

Second, we perform the strong efficiency tests in the case of the evaluation of December, only for the IV model due to data availability (see Annex 5, Table A5.9). We observe that we cannot reject the null hypothesis for all forecasting horizons in the case of the raw material index, investment, government spending, and credit, at the 5% level of significance, which means that the forecast are strongly efficient. However, in the case of consumption, we cannot reject the null hypothesis in all forecasting horizons except for the three quarters ahead, which means that the forecast is strongly efficient for most horizons.

4.2 Conditional Forecast Evaluation

We make a headline inflation forecasting exercise in hindsight for the three models. Also, we consider four scenarios for both the MMS 4.01.1 and PIGU and two scenarios for MME. The forecasting horizon begins on 2011Q1. First, we show the inflation patterns and the forecasts of each model (see Annex 5, Figures A5.1, A5.2, and A5.3, which are delivered upon request). Second, we calculate the ME and the RMSE (see Annex 1, Tables A1.6, A1.7 y A1.8).

In the case of the MMS 4.0.1, the model generates core inflation forecasts, and therefore headline inflation is constructed based on those projections. This explains that, in the case of anchor 2 and anchor 3, we have values different from zero in 1 and 2 quarters ahead for the ME and RMSE (see Annex 1, Table A1.6). PIGU model minimizes the RMSE in the fourth scenario (anchoring exogenous variables, all other endogenous variables and two quarters of inflation) for all forecasting horizons (see Annex 1, Table A1.7). In this case, the model's forecasts are negatively biased for all relevant horizons (the first two horizons are trivially unbiased since the historically observed inflation values are imposed as the model's forecasts). In order to compare the two models' forecasting performances, we pick the best scenario for each model. In particular, we compare the MMS 4.0.1's performance in the third scenario with the PIGU's performance in the fourth scenario. We focus on the last three forecasting horizons since PIGU's RMSE for the first two horizons is trivially equal to zero. The results show that PIGU's RMSE for the three relevant horizons are less than the corresponding values for MMS 4.0.1 and, hence, PIGU is preferred in this evaluation exercise, even though its forecasts tend to underestimate inflation (i.e., its forecasts are negatively biased). See Table 3.

For the MME, the ME suggests that there is a positive inflation bias (see Annex 1, Table A1.8). Results also suggest that forecasts generated by the model can benefit from anchoring inflation and output one quarter ahead since doing so reduces the RMSE (or its mean across different forecasting horizons). This improvement will require that better short-term projections (from outside the model) are available.

Table 3

COMPARISON OF THE BEST SCENARIOS BETWEEN MMS 4.0.1 AND PIGU

Forecasting horizons in years	*mms 4.0.1, anchor 2*	*PIGU, anchoring exogenous and endogenous variables, plus two periods of inflation*
4	1.37	0.61
6	1.36	0.62
8	1.57	0.65

Source: author's elaboration, central bank's forecasts.

5. CONCLUSIONS

In this paper, we evaluated Banco de Guatemala's most important models used to forecast inflation. Forecast accuracy for unconditional models (i.e., IV, AMM, and forecast combinations of OLS and time series models) was evaluated for end of the year forecasts, and for a two-year forecast horizon, using a variety of measurements and tests (i.e., normality, RMSE, DM, PT, GR, and weak and strong efficiency tests). In the case of a conditional forecast, we evaluated the forecasting accuracy of three models: MMS 4.0.1, PIGU, and MME.

We found empirical evidence supporting a higher degree of accuracy for time series models for the short forecast-horizons, and better performance for models generating conditional-forecasts in longer forecast-horizons. The main purpose of this study was to assess the accuracy and precision of the main inflation forecasts generated at Banco de Guatemala. The next step is to take advantage of the obtained results in order to improve the quality of the inflation forecasting models in use at the central bank. In particular, we should continuously reevaluate model specifications, the quality of the data sets, and the variable-transformation procedures. In addition, we should perform a complete evaluation of the inflation forecasts at least once a year, as some central banks already do.

ANNEX

Annex 1. Tables of the Unconditional Forecast Evaluation

Table A1.1

PT TEST, QUANTITATIVE MODELS

Forecasting horizons in quarters	*S_n statistic (IV)*	*S_n Statistic (AMM)*	*S_n statistic (EFP)*
1	4.28	3.98	2.41
2	3.77	2.93	1.62
3	2.57	1.54	0.73
4	0.00	0.88	−1.01
8	0.00	−1.49	−1.53

Sources: author's elaboration, central bank's forecasts.

Table A1.2

PT TEST, QUANTITATIVE MODELS, DECEMBER EVALUATION

Forecasting horizons in quarters	*S_n statistic (IV)*	*S_n statistic (AMM)*
1	1.67	1.67
2	1.02	1.67
3	−1.67	1.67
4	1.67	−1.46
5	−2.31	−1.33
6	−2.31	−2.31
7	–	−2.31
8	–	−2.31

Sources: author's elaboration, central bank's forecasts.

Table A1.3

GR TEST, QUANTITATIVE MODELS

Forecasting horizons in quarters	*GR statistic (AMM-IV)*	*GR statistic (EFP-IV)*
1	4.77	5.68
2	15.28	5.93
3	9.93	11.29
4	9.07	7.39
8	11.28	–

Sources: author's elaboration, central bank's forecasts.

Table A1.4

WEAK EFFICIENCY TEST, QUANTITATIVE MODELS

Forecasting horizons in quarters	*Weak efficiency test (AMM)*	*Weak efficiency test (IV)*	*Weak efficiency test (EFP)*
1	0.11 (0.89)	6.33 (0.24)	3.58 (0.08)
2	4.41 (0.02)	3.29 (0.12)	0.22 (0.89)
3	6.18 (0.00)	11.57 (0.01)	12.08 (0.97)
4	5.39 (0.01)	21.81 (0.00)	–
8	104.62 (0.00)	62.16 (0.00)	0.20 (0.83)

Sources: author's elaboration, central bank's forecasts.

Table A1.5

WEAK EFFICIENCY TEST, QUANTITATIVE MODELS, DECEMBER EVALUATION

Forecasting horizons in quarters	*Weak efficiency test (AMM)*	*Weak efficiency test (IV)*
1	83.48 (0.00)	1.17E+12 (0.00)
2	1.36 (0.35)	1.5268 (0.32)
3	1.45 (0.34)	1.2242 (0.38)
4	8.87 (0.034)	9.5156 (0.03)
5	1.71E+11 (0.00)	14.1267 (0.03)
6	1.85E+11 (0.00)	1.6197 (0.33)
7	2.03E+10 (0.00)	0.9950 (0.47)
8	1.66E+10 (0.00)	1.8451 (0.30)

Sources: author's elaboration, central bank's forecasts.

Tables of the Conditional Forecast Evaluation

Table A1.6

ME AND RMSE, MMS 4.01, 2011Q1-2017Q2

Forecasting horizons in quarters	*Free model*		*Anchor 1*		*Anchor 2*		*Anchor 3*	
	ME	*RMSE*	*ME*	*RMSE*	*ME*	*RMSE*	*ME*	*RMSE*
1	–0.11	0.73	–0.03	0.71	0.01	0.33	0.01	0.33
2	–0.13	1.21	–0.02	1.27	0.01	0.87	0.01	0.87
4	0.22	1.43	0.29	1.58	0.29	1.37	0.29	1.37
6	0.55	1.47	0.54	1.4	0.52	1.36	0.52	1.36
8	0.27	1.72	0.55	1.63	0.54	1.57	0.54	1.57
Mean	0.16	1.31	0.26	1.32	0.27	1.1	0.27	1.1

Sources: author's elaboration, central bank's forecasts.

Table A1.7

ME AND RMSE, PIGU, 2011Q1-2017Q2

Forecasting horizons in quarters	*Free model*		*Anchoring exogenous variables*		*Anchoring exogenous variables and two periods of inflation*		*Anchoring exogenous and endogenous variables, plus two periods of inflation*	
	ME	*RMSE*	*ME*	*RMSE*	*ME*	*RMSE*	*ME*	*RMSE*
1	−0.22	0.83	−0.25	0.72	0	0	0	0
2	−0.3	1.26	−0.38	0.9	0	0	0	0
4	0	1.44	−0.47	0.88	−0.39	0.82	−0.27	0.61
6	0.34	1.11	−0.58	1.12	−0.56	1.13	−0.32	0.62
8	0.41	0.89	−0.79	1.29	−0.79	1.29	−0.38	0.65
Mean	0.05	1.11	−0.49	0.98	−0.35	0.65	−0.19	0.38

Sources: author's elaboration, central bank's forecasts.

Table A1.8

ME AND RMSE, MME, 2011Q1-2017Q2

Forecasting horizons in quarters	*Free model*		*Anchor 1*	
	ME	*RMSE*	*ME*	*RMSE*
1	0.3	0.62	−0.09	0.1
2	0.89	1.28	0.36	0.61
4	2.37	2.72	1.81	2.09
6	2.82	2.98	2.87	3.04
8	2.82	2.93	2.86	2.96
Mean	1.84	2.11	1.56	1.76

Sources: author's elaboration, central bank's forecasts.

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Forecasting Inflation Expectations from the CESifo World Economic Survey: An Empirical Application in Inflation Targeting Countries

Héctor M. Zárate Solano
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Abstract

This paper has two purposes. First, it evaluates the responses to the questions on inflation expectations in the World Economic Survey (WES) for sixteen inflation targeting countries. Second, it compares inflation expectation forecasts across countries by using a two-step approach that selects the most accurate linear or non-linear forecasting method for each country. Then, Self-Organizing Maps are used to cluster inflation expectations, setting as a benchmark June 2014, when there was a sharp decline in oil prices. Analyzing inflation expectations in the context of this price change makes it possible to distinguish between countries that anticipated the oil shock smoothly and those that had to adjust their expectations significantly. The main findings from the WES in-sample comparison suggest that expert forecasts of inflation expectations are systematically distorted in 83 percent of the countries in the sample. On the other hand, the out of sample forecast analysis indicates that Non-linear Artificial Neural Networks combined with Bayesian regularization outperform ARIMA linear models for longer forecasting horizons. This holds true for countries with both soft and brisk changes of expectations. However, when forecasting one step ahead, the performance between the two methods is similar.

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JEL classifications: C02, C222, C45, C63, E27.
Keywords: Inflation expectations, Machine learning, Self-organizing maps, Nonlinear auto-regressive neural network, Expectation surveys, Time series models.

1. INTRODUCTION

Cross-country data from economic expectations surveys have recently highlighted the importance of analyzing and forecasting public expectations to gain insight into crucial empirical issues in macroeconomics. Expectations can influence the future path of real economic variables and help guide policy decision-makers, and inflation expectations are particularly important for countries that utilize inflation targeting as their primary monetary policy framework. The usefulness of inflation expectations is manifested in various realms of economic analysis. They are critical for *i)* testing theories of informational inflation rigidity (Coibion *et al.*, 2012); *ii)* estimating key structural parameters, such as the intertemporal substitution elasticity (Crump *et al.*, 2015); *iii)* testing public understanding of monetary policy, such as the Taylor rule (Carvalho and Nechio, 2014); and *iv)* assessing how well inflation expectations may be anchored among economic agents, which is key in assessing the effectiveness of central bank communication. Lastly, New Keynesian macroeconomic models have successfully used inflation expectations to predict real inflation (Henzel and Wollmershäuserab, 2008).

Expectation surveys have featured a wide range of respondents, including economic experts, central bankers, financial agents, consumers, and firms. Those surveyed often have to make important decisions that take into account inflation and survey data, and their responses provide information on the effectiveness of economic policies and institutional confidence. The World Economic Survey (WES) collects data on inflation expectations across countries and surveys more than 1,000 economic experts in approximately 120 countries. The respondents evaluate present economic conditions and predict the economic outlook of the country in which they reside, giving special attention to price trends in their answers to both qualitative and quantitative questions.

Thus, we must assess the suitability of WES data surveys and select the appropriate methods to accurately forecast inflation expectations.

In regard to suitability, we can use simple exploratory data analysis based on time plots and correlations, and we can calculate the in-sample forecast errors within a sample of 16 inflation-targeting countries. To find the appropriate forecasting method, we use a two-step approach centered on both clustering and forecasting techniques. Specifically, we analyze the June 2014 oil price shock and its effect on inflation expectations and other macroeconomic indicators. We consider this oil shock relevant because the decline in oil prices was significantly larger than in any previous episode during the past 30 years. The decline weakened fiscal policy and reduced the economic activity of oil exporters, but for oil importers, inflationary and fiscal pressures were alleviated. The oil price shock is also significant because it affected growth and inflation through two channels: input costs and real income shifts. Changes through either of these channels then led to changes in inflation expectations. Thus, we evaluate different forecasting methods in the period after the oil shock from Q3 2014 to Q2 2016. To obtain optimal forecasts, a combination of clustering and forecasting analysis can be used. Data visualization techniques are useful for discovering important characteristics and potential clusters of economic agents. In addition, we use machine learning and statistical methodologies to improve inflation expectation forecasts based on qualitative and quantitative questions from the WES.

This paper examines the data on inflation expectations from the WES for 16 inflation-targeting countries. Then, by making use of Self-Organizing Maps (SOM) we cluster agents' expectations for these countries to classify them either as "soft" or "brisk" based on the speed of their expectations change after the oil shock of 2014 (Claveria, Monte and Torra, 2016). After that, we combine the SOM representations with different forecasting methods to select models for inflation expectation forecasting. The ARIMA model reflects the linear class of models and the Non-linear Auto-regressive Neural network (NAR-NN) reflects the non-linear class of models.

Our main findings are the following. First, we present evidence of heterogeneity in the correlation patterns between inflation expectations and observed inflation. There are increasing, descending, and inverted U-shaped correlations over time. Regarding frequency domain analysis, the highest coherence values were often found in periods of higher frequencies in most countries, implying that there is a strong relationship between cycles of short periods.

According to the WES forecast error analysis, we observe that even though the forecasts meet at least the minimum standard when compared to a random walk, economic experts have made systematic errors in their predictions. That is, inflation was under- predicted while increasing and over-predicted while declining in most of the countries. Moreover, the mean squared error decomposition illustrated that there were systematic distortions in the inflation forecasts in around 83 percent of the countries. The evidence suggests that although the accuracy of the forecasts increases as the forecasting horizon decreases, this relationship is not monotonic. This finding does not support the hypothesis that forecasts have improved over time, which may signal that there is a non-linear data- generating process.

Second, turning to a much more complex analysis, the SOM representation allows us to cluster countries based on the evolution of inflation expectations before the oil price shock. It is important to note that the low inflation expectations cluster is relatively small compared to the high and neutral clusters for inflation-targeting countries. We find that in the one step- forward forecasts, the neural network only slightly improves on forecasts of the ARIMA, but that it outperforms the ARIMA model in the two step-forward forecasts for Canada, Colombia, Chile, Poland, Hungary, and Sweden. Therefore, using a non-linear neural network along with Bayesian regularization leads to an improvement in expectations forecasts.

This paper contains five sections apart from this introduction and proceeds as follows. In Section 2 we describe the WES data and evaluate the responses to both qualitative and quantitative inflation questions. In Section 3, we provide the methodologies for clustering and forecasting, emphasizing the merits of the artificial neural network approach. In Section 4, we summarize the main results, including the cluster analysis and forecasting accuracy. Finally, in Section 5 we present our conclusions and propose future lines of research.

2. WORLD ECONOMIC SURVEY DATA AND THEIR SUITABILITY FOR FORECASTING INFLATION

Surveying economic experts across different countries, the CESifo World Economic Survey (WES) carried out by the IFO Institute for Economic Research collects data on how experts view their country's economic outlook. In this paper, we use the term economic experts to include representatives of multinational enterprises, banks, chambers of commerce, academic institutions, and individual economists.

The questionnaire is distributed every quarter (January, April, July, and October) with qualitative and quantitative questions related to the general economic situation and expectations regarding key macroeconomic indicators: economic growth, interest rates, consumption, capital, exchange rates, and inflation, among others.[1] The questions on the expected inflation rate, which are the main focus of this paper, reveal qualitative and quantitative information on the economic experts of each country. Thus, the participants are asked to give their expectations of what the inflation rate will be by the end of the next six months. They indicate "HIGHER" for an expected rise in the inflation rate, "ABOUT THE SAME" for no change in the inflation rate, and "LOWER" for an expected fall in the expected inflation rate by the end of the next six months. We transformed these responses into a cardinal time series of expected inflation by applying the following standard approach: where the response is considered high, a numerical value of 9 is coded; where the response is considered neutral, a value of 5 is coded; and where the response is considered low, a value of 1 is recorded. Next, we calculate the average rating for each question for each country. Traditionally, analysts have categorized these country ratings by terming an average greater than 5 a positive zone and an average below 5 a negative zone. The neutral zone depends simply on the analyst's subjective decision. One of the results of this paper is to establish the limitations that come with this three-zone categorization and instead, we let the data speak for itself.

In the quantitative question the experts of each country are asked to predict the future inflation rate: "the rate of inflation on average this year will be: % p.a." We analyze the responses to this question through an in-sample statistical analysis of forecasting error. Further information on the WES can be found in Stangl (2007a and 2007b).

We analyze expectations for 16 inflation-targeting countries from Q3 1991 to Q2 2016. The countries included in our analysis are Brazil, Canada, Switzerland, Chile, Colombia, Czech Republic, United Kingdom, Hungary, Korea Republic, Mexico, Norway, Philippines,

[1] A survey form of the World Economic Survey, the WES questionnaire, is included in Appendix A, see Figure 14.

Poland, Sweden, Thailand, and South Africa.[2] The relationship between the indicator of WES inflation expectations and the observed annual inflation rate is illustrated through a simple exploratory analysis that uses time plots and correlation statistics.[3] The observed inflation rate and the corresponding inflation expectations are depicted in Figure 1 for some selected countries. For each country, inflation was measured by annual changes in the Consumer Price Index. According to Figure 1, WES expectations move in tandem with actual inflation for most of the period under study except during idiosyncratic and global shocks that affected specific national economies.[4]

Figure 2 displays the correlation coefficient over time and the coherence as a function of the frequency between the WES inflation expectations and real annual inflation. The plot of the correlation coefficient shows the existence of different patterns of linear association. For example, while the correlation in Mexico has increased over time, it has decreased in Canada. On the other hand, Colombia has experienced an inverted u-shaped correlation pattern that peaks in the middle of 2002. According to frequency domain analysis, higher coherence was found in higher frequencies of the spectral distribution in most of the countries, which suggests that the relationship between inflation expectations and observed inflation is strong predominantly during short cycles. It is important to note that Asian countries have higher coherence in lower frequencies, which points to a different trend between expectation and observed inflation.[5]

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[2] Figure 11 in Appendix A contains the full-time series length.

[3] To see the other countries' inflation expectations, see Figure 15 in the Appendix.

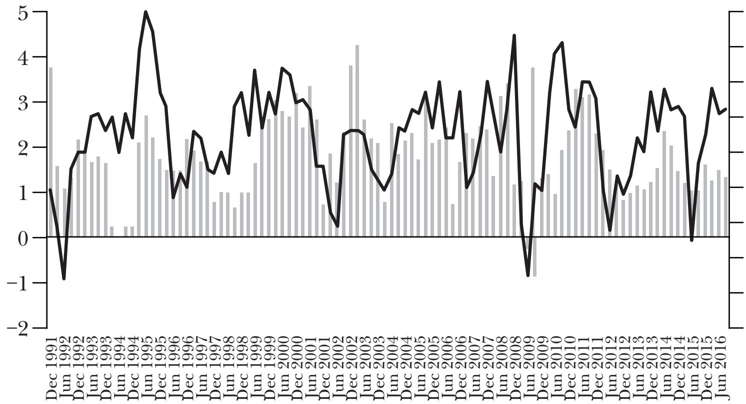
[4] In addition, we include a summary of the data, their histograms and correlations which are relevant to the SOM analysis: Figure 12 in the Appendix reveals the heterogeneity of the variables, and Figure 13 displays the correlation between them. Table 9 in the Appendix shows a brief summary of the WES expectations data.

[5] To see the spectral decomposition of the other countries, see Figure 16 in the Appendix.

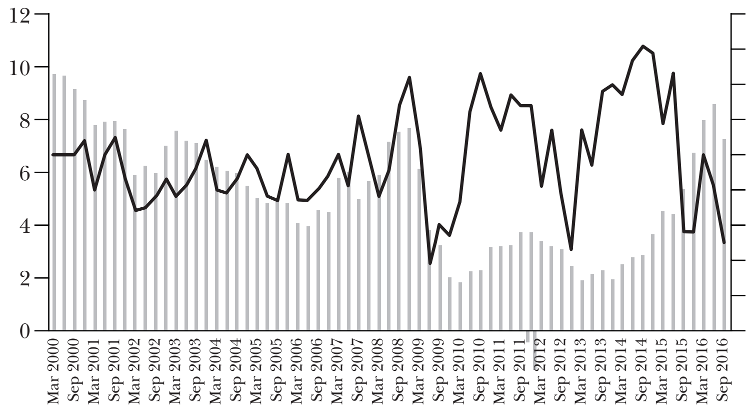
Figure 1

COMPARISON OF THE INFLATION EXPECTATIONS WITH THE OBSERVED ANNUAL INFLATION

(a) canada



(b) colombia



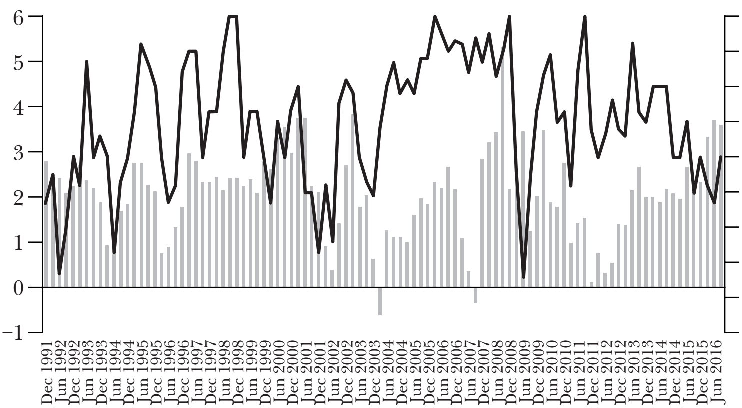
Annual Inflation — Inflation rate next 6 months

Source: WES survey and OECD statistics. Some selected countries.

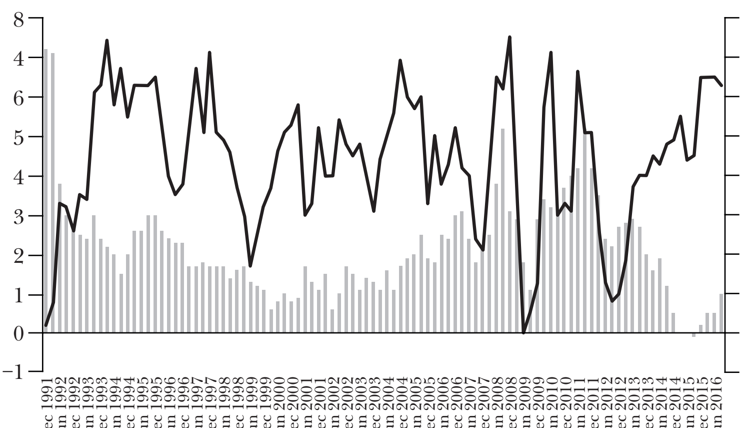
Figure 1 (cont.)

COMPARISON OF THE INFLATION EXPECTATIONS WITH THE OBSERVED ANNUAL INFLATION

(c) Norway



(d) United Kingdom

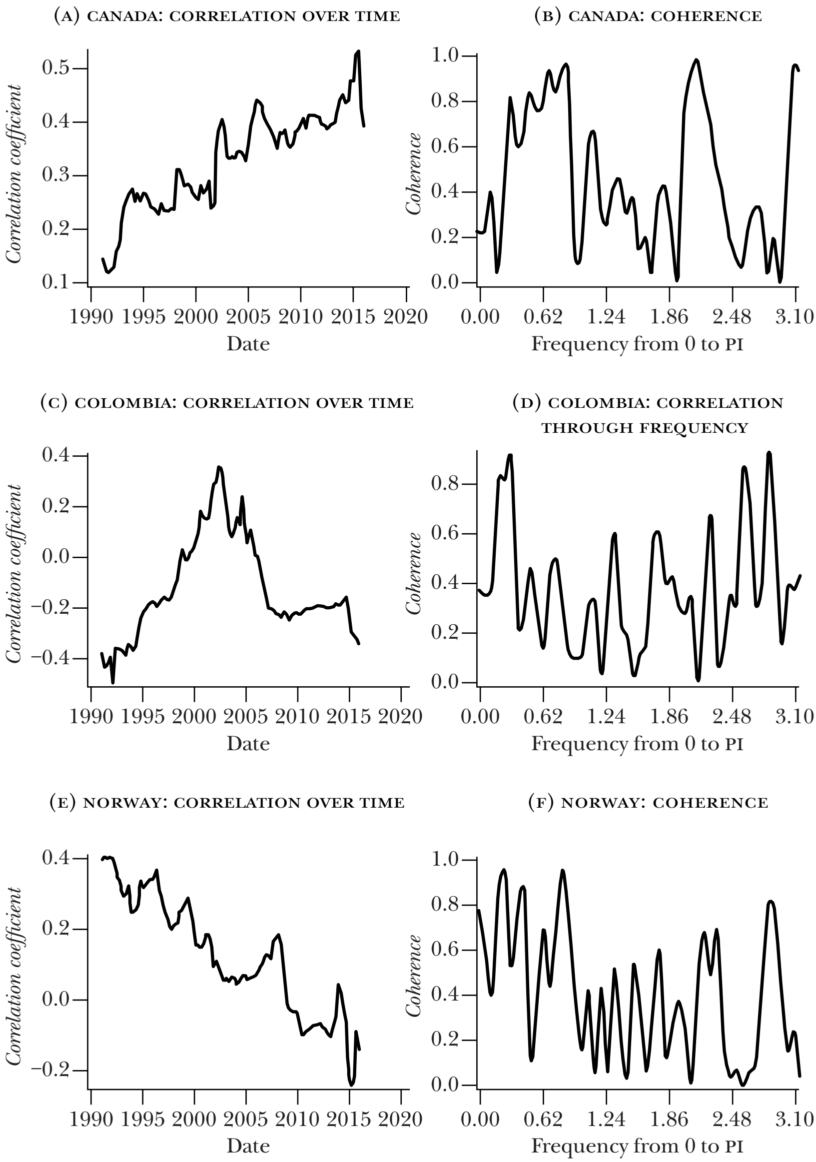


Annual Inflation — Inflation rate next 6 months

Source: WES survey and OECD statistics. Some selected countries.

Figure 2

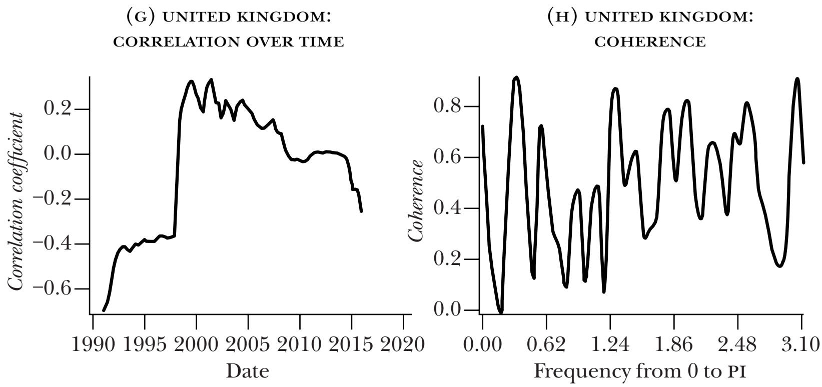
CORRELATION AND COHERENCE COEFFICIENTS OF QUALITATIVE WES INFLATION EXPECTATION WITH OBSERVED ANNUAL INFLATION



Source: WES survey and OECD statistics and IMF data.

Figure 2 (cont.)

CORRELATION AND COHERENCE COEFFICIENTS OF QUALITATIVE WES INFLATION EXPECTATION WITH OBSERVED ANNUAL INFLATION



Source: WES survey and OECD statistics and IMF data.

2.1 Quantitative Forecasting Inflation Expectations

In this section, we perform an in-sample forecasting analysis based on the forecasting error. We compute the forecasting error as the difference between annual average inflation based on the CPI and the corresponding quantitative WES inflation assessment from the survey question "the rate of inflation on average this year will be: % p.a.". We follow previous work by Fildes and Stekler (2002) and Hammella and Haupt (2007) to quantify and examine the accuracy of WES forecasts at different horizons. It is important to note that the experts receive more information from quarter to quarter during the year as data on the observed inflation rate is released.

2.1.1 Statistical Analysis of the Forecasting Error

The forecasting error is calculated in the following way:

$$e\big(L,Q(h),t\big)=\overline{p}(L,t)-q\big(L,Q(h),t\big) \tag{1}$$

where L = countries, h = I, II, III, IV, and t = 1991,. . . , 2016. First, we compute some standard error statistics for each quarter including the RMSFE (root mean squared forecast error), MAE (mean absolute error), and Theil U-statistic. See Hamella and Haupt (2007).[6]

Second, we used the additive mean squared error decomposition proposed by Theil in 1966 (see Theil *et al.*, 1975) to obtain insight into the structure of the forecast error. The decomposition is meant to illustrate how the error changes conditional on the different forecasting horizons through three components: the bias share Vh, the spread share Sh, and the covariance share Kh. The Vh bias component measures systematic distortions in the forecast, where bias should decrease through forecast horizons only if the expectations are anchored. Sh measures the dispersion between observed inflation and the WES forecast. Finally, Kh assesses the linear association between average inflation and the WES forecast; if the correlation is perfect then K=0. Notice that the components should sum up to one.

2.1.2 Quantitative Inflation Expectation Results

Tables 1 and 2 summarize the RMSFE and its decomposition for the sample of countries at different time horizons. The results illustrate that the RMSFE decreases throughout the year for countries such as Switzerland, Colombia, Korea, and Norway. Nevertheless, there are some countries which exhibit a different pattern in which the last forecast is more uncertain. The countries in this group include Brazil, Canada, Chile, Czech Republic, and United Kingdom. The heterogeneity among RMSFE values across countries can be explained by the fact that the RMSFE relies on the restricted assumption that survey forecasters have a symmetric loss function. The RMSFE also depends on the unit of measurement and the inflation rate in each country. These diagnoses remain by observing the MAE and U- statistics. Figure 3 compares the respective observed annual inflation (bar line) and the WES expectation for each quarter for some selected countries.[7,8]

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6 The respective statistics equations are presented in Appendix A.3, and MAE and U-statistic results are in Tables 10 and 11, respectively. See Appendix.

7 To see the other countries quantitative inflation expectation, see Figure 17 in Appendix A.3.

8 The quarter-specific forecasting error by country is plotted in Figure 18, Appendix A.3.

The evidence for Colombia suggests that actual annual inflation was overestimated during the period from 2000 to 2003, and from 2003 to 2007 the expectations were close to the observed inflation rate. The 2008 financial crises led expectations to undershoot observed inflation for a short period of time, but soon after, expectations began to overshoot observed inflation until 2014. Eventually, the 2014 oil shock induced a period of undershooting. There are different patterns across the countries. For example, in Mexico expectations were close to actual inflation until the oil shock, but after the shock, they overestimated observed inflation rates. In Tables 3 and 4 we count the number of years in which inflation was overestimated and underestimated respectively by respondents, to the quarterly WES survey. For instance, the results indicate that annual inflation in Colombia was overestimated, on average, in 14 of 25 years and for Mexico in 17 of 26 years. There is evidence that systematic overestimation was greater than underestimation. The exception occurs in the case of Brazil in which, on average, in 15 of 26 years inflation was underestimated by economic experts.

Finally, a cross-country comparison using the U-statistic confirms that the WES-forecasts in every country at least meet the minimum standard when compared with the random walk alternative.

Table 1

ROOT MEAN SQUARED FORECAST ERRORS OF WES SURVEY QUANTITATIVE INFLATION QUESTION Q1 1991 TO Q3 2016

Countries	*4-step forecast (QI)*	*3-step forecast (QII)*	*2-step forecast (QIII)*	*1-step forecast (QIV)*
Brazil	182.71	321.48	354.44	431.01
Canada	0.70	0.57	0.42	0.58
Switzerland	0.75	0.50	0.41	0.38
Chile	1.23	1.46	1.36	1.66
Colombia	1.80	1.67	1.43	1.00
Czech Republic	4.97	4.81	6.87	3.08
United Kingdom	0.89	0.88	0.90	0.99
Korea	1.61	1.41	1.16	1.09
Mexico	3.37	2.03	4.48	3.62
Norway	0.78	0.65	0.52	0.39
Hungary	2.12	1.32	1.12	1.54
Philippines	2.29	1.77	1.29	1.22
Poland	5.48	2.07	10.48	11.47
Sweden	1.05	0.80	0.99	1.19
Thailand	2.05	1.56	1.51	1.04
South Africa	1.77	1.57	1.49	1.27

Table 2

THEIL ERROR DECOMPOSITION OF THE WES FORECAST ERRORS Q1 1991 TO Q2 2016

Countries	*Error decomposition*	*4-step forecast (QI)*	*3-step forecast (QII)*	*2-step forecast (QIII)*	*1-step forecast (QIV)*
Brazil	V	0.13	0.06	0.07	0.01
	S	0.84	0.81	0.53	0.10
	K	0.06	0.14	0.45	0.92
Canada	V	0.16	0.20	0.31	0.16
	S	0.05	0.14	0.26	0.26
	K	0.83	0.70	0.46	0.61
Switzerland	V	0.22	0.32	0.30	0.19
	S	0.22	0.28	0.37	0.54
	K	0.60	0.55	0.35	0.31
Chile	V	0.00	0.02	0.05	0.02
	S	0.02	0.20	0.74	0.75
	K	1.02	0.84	0.25	0.27
Colombia	V	0.003	0.06	0.04	0.01
	S	0.08	0.02	0.05	0.33
	K	0.96	0.99	0.95	0.71
Czech R.	V	0.10	0.14	0.06	0.02
	S	0.17	0.21	0.33	0.002
	K	0.77	0.77	0.65	1.02
United K.	V	0.23	0.26	0.18	0.14
	S	0.16	0.28	0.43	0.30
	K	0.64	0.56	0.43	0.60
Korea	V	0.37	0.44	0.52	0.39
	S	0.03	0.002	0.0003	0.02
	K	0.62	0.45	0.50	0.62

Countries	*Error decomposition*	*4-step forecast (QI)*	*3-step forecast (QII)*	*2-step forecast (QIII)*	*1-step forecast (QIV)*
Mexico	V	0.03	0.13	0.01	0.01
	S	0.43	0.002	0.11	0.03
	K	0.57	1.04	0.92	1.01
Norway	V	0.06	0.02	0.08	0.02
	S	0.19	0.14	0.13	0.18
	K	0.79	0.79	0.83	0.84
Hungary	V	0.04	0.15	0.01	0.01
	S	0.00	0.08	0.11	0.27
	K	1.00	0.96	0.92	0.76
Philippines	V	0.20	0.15	0.18	0.35
	S	0.01	0.15	0.27	0.07
	K	0.81	0.76	0.59	0.61
Poland	V	0.06	1.20	0.07	0.05
	S	0.44	0.05	0.58	0.36
	K	0.54	0.96	0.39	0.62
Sweden	V	0.35	0.28	0.16	0.03
	S	0.07	0.39	0.49	0.74
	K	0.61	0.39	0.39	0.27
Thailand	V	0.18	0.47	0.40	0.56
	S	0.005	0.001	0.001	0.002
	K	0.84	0.77	0.62	0.45
South A.	V	0.14	0.25	0.27	0.32
	S	0.17	0.21	0.25	0.18
	K	0.72	0.65	0.51	0.53

Table 3

OVERESTIMATION OF WES FORECASTS QI 1991 TO Q2 2016

Countries	*4-step forecast (QI)*	*3-step forecast (QII)*	*2-step forecast (QIII)*	*1-step forecast (QIV)*
Brazil	10 casesof (26)	10 cases of (26)	10 cases of (25)	13 cases of (25)
Mean	−0.95	−5.09	−4.54	−70.26
Std. Deviation	1	11.83	11.75	201.59
Canada	16 cases of (26)	16 cases of (26)	17 cases of (25)	20 cases of (25)
Mean	−0.64	−0.55	−0.42	−0.4
Std. Deviation	0.58	0.42	0.27	0.42
Switzerland	20 cases of (26)	18 cases of (26)	19 cases of (25)	19 cases of (25)
Mean	−0.61	−0.45	−0.36	−0.3
Std. Deviation	0.44	0.27	0.27	0.24
Chile	15 cases of (26)	15 cases of (26)	12 cases of (25)	15 cases of (25)
Mean	−0.9	−0.91	−0.61	−0.56
Std. Deviation	0.72	0.7	0.54	0.45
Colombia	13 cases of (26)	15 cases of (26)	13 cases of (25)	13 cases of (25)
Mean	−1.23	−1.23	−1.15	−0.54
Std. Deviation	1.41	1.53	1.32	0.36
Czech Republic	21 cases of (26)	18 cases of (26)	19 cases of (25)	20 cases of (25)
Mean	−2.36	−2.05	−2.48	−1.15
Std. Deviation	4.94	5.48	7.6	2.53
United Kingdom	20 cases of (26)	19 cases of (26)	17 cases of (25)	20 cases of (25)
Mean	−0.78	−0.76	−0.78	−0.71
Std. Deviation	0.5	0.49	0.52	0.47
Korea	20 cases of (26)	24 cases of (26)	22 cases of (25)	20 cases of (25)
Mean	−1.46	−1.18	−0.99	−0.91
Std. Deviation	1.06	0.89	0.74	0.82

Countries	*4−step forecast (QI)*	*3−step forecast (QII)*	*2−step forecast (QIII)*	*1−step forecast (QIV)*
Mexico	17 cases of (26)	18 cases of (26)	15 cases of (25)	17 cases of (25)
Mean	−0.8	−0.96	−2.15	−1.12
Std. Deviation	0.71	1.73	4.94	3.28
Norway	16 cases of (26)	18 cases of (26)	15 cases of (25)	14 cases of (25)
Mean	−0.64	−0.52	−0.48	−0.33
Std. Deviation	0.51	0.43	0.32	0.27
Hungary	17 cases of (26)	13 cases of (26)	15 cases of (25)	13 cases of (25)
Mean	−1.5	−1.02	−0.77	−0.76
Std. Deviation	1.3	0.67	0.74	0.77
Philippines	21 cases of (26)	19 cases of (26)	19 cases of (25)	20 cases of (25)
Mean	−1.77	−1.41	−1	−1.1
Std. Deviation	1.42	0.9	0.73	0.64
Poland	17 cases of (26)	19 cases of (26)	14 cases of (25)	13 cases of (25)
Mean	−3.19	−1.21	−0.65	−0.45
Std. Deviation	5.62	1.28	0.39	0.28
Sweden	21 cases of (26)	21 cases of (26)	21 cases of (25)	20 cases of (25)
Mean	−0.89	−0.66	−0.66	−0.59
Std. Deviation	0.71	0.41	0.44	0.3
Thailand	17 cases of (26)	20 cases of (26)	21 cases of (25)	22 cases of (25)
Mean	−1.69	−1.27	−1.26	−0.91
Std. Deviation	1.83	1.2	1.04	0.65
South Africa	17 cases of (26)	18 cases of (26)	20 cases of (25)	21 cases of (25)
Mean	−1.51	−1.27	−1.12	−0.93
Std. Deviation	1.15	0.41	0.62	0.23

Table 4

UNDERESTIMATION OF WES FORECASTS Q1 1991 TO Q2 2016

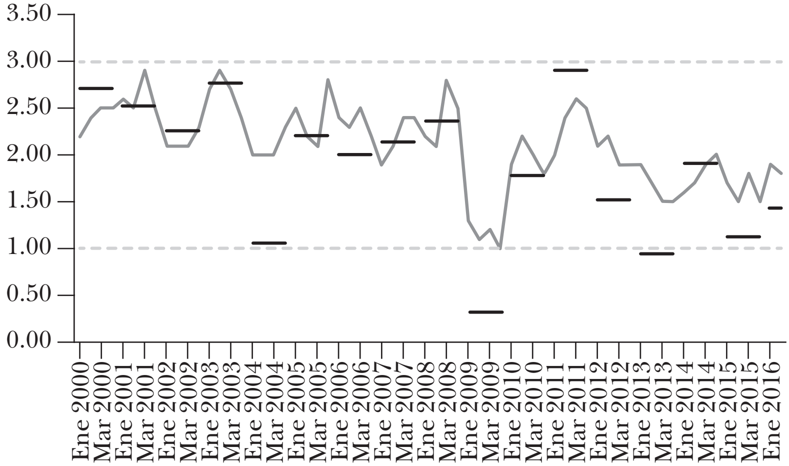
Countries	*4-step forecast (QI)*	*3-step forecast (QII)*	*2-step forecast (QIII)*	*1-step forecast (QIV)*
Brazil	16 cases of (26)	16 cases of (26)	15 cases of (25)	12 cases of (25)
Mean	109.13	157.78	153.64	178.45
Std. Deviation	212.5	390.49	446.02	580.77
Canada	10 cases of (26)	10 cases of (26)	8 cases of (25)	5 cases of (25)
Mean	0.29	0.23	0.16	0.45
Std. Deviation	0.25	0.19	0.1	0.46
Switzerland	6 cases of (26)	8 cases of (26)	6 cases of (25)	6 cases of (25)
Mean	0.53	0.29	0.2	0.27
Std. Deviation	0.6	0.37	0.21	0.27
Chile	11 cases of (26)	11 cases of (26)	13 cases of (25)	10 cases of (25)
Mean	1.07	1.22	1.15	1.48
Std. Deviation	0.87	1.42	1.33	2.09
Colombia	13 cases of (26)	11 cases of (26)	12 cases of (25)	12 cases of (25)
Mean	1.43	1.01	0.67	0.78
Std. Deviation	1.1	0.73	0.83	1.06
Czech Republic	5 cases of (26)	8 cases of (26)	6 cases of (25)	5 cases of (25)
Mean	1.76	0.79	1.03	2.33
Std. Deviation	2.32	1.15	1.74	3.93
United Kingdom	6 cases of (26)	7 cases of (26)	8 cases of (25)	5 cases of (25)
Mean	0.72	0.62	0.47	1
Std. Deviation	0.36	0.57	0.75	1.13
Korea	6 cases of (26)	2 cases of (26)	3 cases of (25)	5 cases of (25)
Mean	0.61	0.34	0.29	0.24
Std. Deviation	0.46	0.46	0.33	0.15

Countries	*4-step forecast (QI)*	*3-step forecast (QII)*	*2-step forecast (QIII)*	*1-step forecast (QIV)*
Mexico	9 cases of (26)	8 cases of (26)	10 cases of (25)	8 cases of (25)
Mean	3.2	1.8	2.12	3.22
Std. Deviation	4.8	1.38	2.25	2.69
Norway	10 cases of (26)	8 cases of (26)	10 cases of (25)	11 cases of (25)
Mean	0.55	0.51	0.35	0.29
Std. Deviation	0.5	0.39	0.26	0.23
Hungary	9 cases of (26)	13 cases of (26)	10 cases of (25)	12 cases of (25)
Mean	1.58	0.94	0.9	1.21
Std. Deviation	1.92	1.1	0.86	1.57
Philippines	5 cases of (26)	7 cases of (26)	6 cases of (25)	5 cases of (25)
Mean	2.04	1.56	0.87	0.76
Std. Deviation	1.52	1.41	1.32	0.82
Poland	9 cases of (26)	7 cases of (26)	11 cases of (25)	12 cases of (25)
Mean	2.04	2.04	7.17	6.03
Std. Deviation	2.87	2.03	14.73	16.1
Sweden	5 cases of (26)	5 cases of (26)	4 cases of (25)	5 cases of (25)
Mean	0.52	0.68	0.97	1.36
Std. Deviation	0.3	0.67	1.61	2.09
Thailand	9 cases of (26)	6 cases of (26)	4 cases of (25)	3 cases of (25)
Mean	0.66	0.76	0.63	0.14
Std. Deviation	0.56	0.34	0.39	0.07
South Africa	9 cases of (26)	8 cases of (26)	5 cases of (25)	4 cases of (25)
Mean	0.96	0.72	0.6	0.4
Std. Deviation	1.15	0.41	0.62	0.23

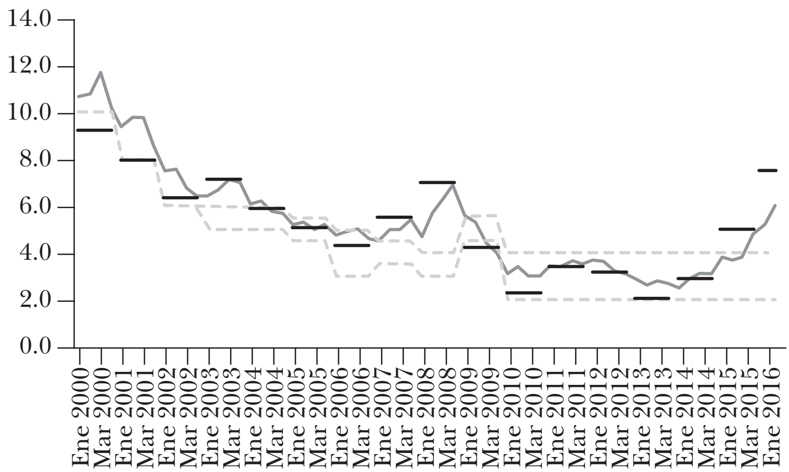
Figure 3

COUNTRIES WES QUANTITATIVE INFLATION EXPECTATIONS, ANNUAL INFLATION, AND INFLATION TARGETS

(A) CANADA



(B) COLOMBIA

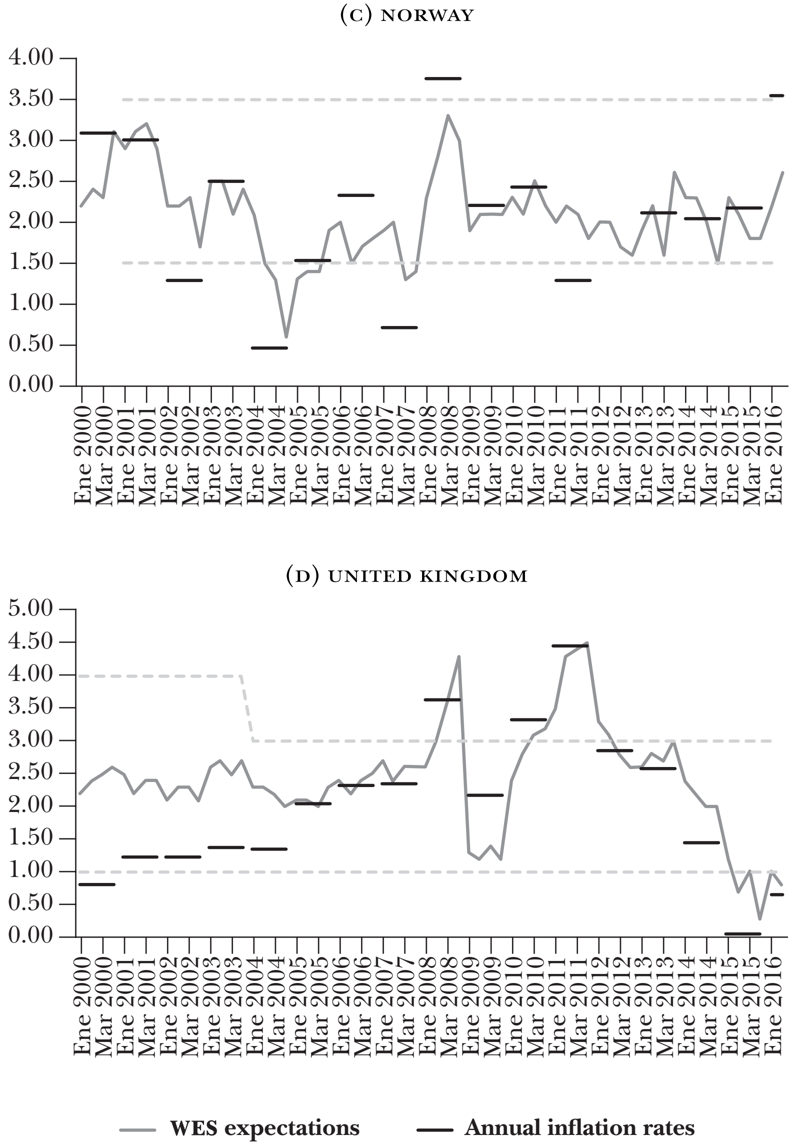


WES expectations **Annual inflation rates**

Source: WES survey and OECD statistics and IMF data.

Figure 3

COUNTRIES WES QUANTITATIVE INFLATION EXPECTATIONS, ANNUAL INFLATION, AND INFLATION TARGETS



Source: WES survey and OECD statistics and IMF data.

3. METHODOLOGY

In this section, we describe the Artificial Neural Networks (ANNs) models applied to cluster and forecast inflation expectations from the WES surveys. To cluster we relied on Kohonen self-organizing maps (SOMs), and to forecast we employed the multilayer perceptron from which the Non-linear autoregressive neuronal network, NAR-NN, is a subclass. The learning procedures to train ANNs is a statistical technique from which the weights are the relevant statistics that could be found through an optimal solution, White (1989). Previous work that employed ANNs to forecast inflation include Stock and Watson (1998) and Marcellino (2004) who conducted an extensive successful forecasting study on EMU macroeconomic variables. On the other hand, Kock and Teräsvirta (2016) considered macroeconomic forecasting with a flexible single-hidden layer fed-forward neural network.

3.1 Artificial Neural Networks

In order to explain the ANNs framework, we start looking at the key points of the simple neural network model that form the base of the SOM and NAR-NN models.

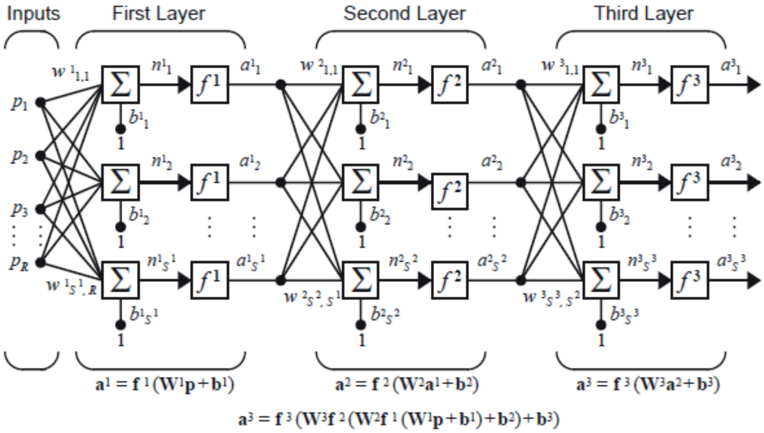
ANNs are a type of parallel computing system consisting of several simple interconnected processors called neurons or nodes, through which there is a learning process that adjusts the system parameters to approximate non-linear functions between a set of inputs (variables) and the output (results). For more information, see Jain, Mao and Mohiuddin (1996).

Following Hagan *et al.* (2014), the simplest neuron model is composed of a scalar input p, called a single variable, which is multiplied by a scalar weight w. Then, wp plus the bias b form the called net input n, which is sent to the activation function f, to produce the scalar neuron output a. However, the ANN's architecture may be more complex; they can have multiple inputs, layers, and neurons as shown in Figure 4.

The parameters are constrained by weights and biases and are adjusted with some learning rule (e.g., Kohonen's learning rule), while the activation function is chosen according to the task at hand. For example, in the SOM, the competitive function is applied. These networks are fed forward, which means that there are no loops between

Figure 4

A THREE-LAYER NEURAL NETWORK



Source: Hagan *et al.* (2014).

the outputs and inputs.[9] To see more details about ANNs see Hagan *et al.* (2014).

3.2 Self-Organizing Maps

In this paper, Self-Organized Maps, proposed by Kohonen in 1982 (see Kohonen, 2001), were used to cluster economic agents' expectations before the oil shock. Furthermore, mapping those expectations after the shock in the resulting cluster map, we divide the observations into two groups based on whether the expectations adjusted briskly or softly. It is important to note that SOMs are competitive feed-forward networks based on unsupervised training and have the topology preservation property. This means that nearby input

[9] In the NAR-NN Model, to perform multi-step forecasts, the network is transformed into a recurrent network after their parameters were trained as a feed-forward network.

patterns should be represented on the map by nearby output units; see Kohonen (2001).

The SOM architecture consists of a two-layer network: in the first layer the inputs are multiplied with weights that were initialized as small numbers. Then the results are evaluated by a competitive function that produces a wining neuron (Best Matching unit). The weights are updated according to the learning rule, equation (2), and the neuron's neighborhood is updated as well. See Figure 5 below.

$$\mathbf{w}_i(q) = (1-\alpha)\mathbf{w}_i(q-1) + \alpha(\mathbf{p}(q)) \tag{2}$$

The training stage for each iteration consists of weight adjustments for the winning neuron and its neighbors and these adjustments are undertaken using the learning rule. This process guarantees similarity between the inputs and the neurons represented on the feature map (the second layer of the map). At the end of the process, the resulting learned weights capture the data characteristics on the two-dimensional feature map (Hagan *et al.*, 2014).

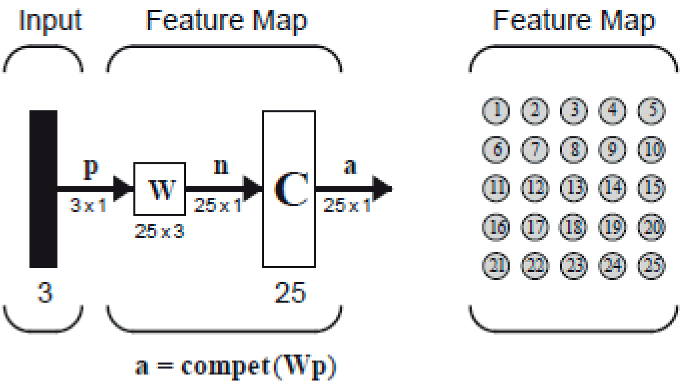
Kohonen suggested using rectangular and hexagonal neighborhoods. Furthermore, to improve the SOM's performance, we considered gradually decreasing the neighbor size during the training so that it only includes the winning neuron. Moreover, to consider the trade-off between fast learning and stability, the learning rate can be also decreased in this phase. This is because a high learning rate at the beginning of the training phase allows for quick but unstable learning. On the other hand, with a low rate, learning becomes slow but more stable.

3.3 Nonlinear Auto-Regressive Neural Network

In this subsection, we describe the main issues of the NAR-NN methodology, including the selection of the training algorithm. The model assumes the current observation is explained by the compromise of two components: signal and noise. The first is an unknown function that is approximated by the neural network to the inflation expectation time series with an autoregressive structure. The second component is noise, which is assumed to be independent with zero mean. The model equation is stated below:

Figure 5

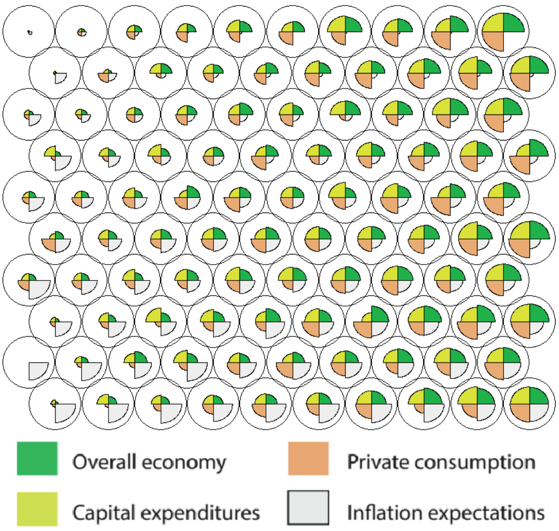
A SELF-ORGANIZING MAP OF 5X5 DIMENSION



Source: Hagan *et al.* (2014).

Figure 6

WEIGHT SOM VECTORS OF WES EXPECTATIONS FOR THE NEXT 6 MONTHS



3 $$Y_t = g\left(Y_{t-1} + Y_{t-2} + \ldots + Y_{t-p}\right) + e_t$$

4 $$Y_{t+1} = \mathbf{f}^2\left(\mathbf{W}^2\,\mathbf{f}^1\left(\mathbf{W}^1\left[Y_t, Y_{t-1}, \ldots, Y_{t-p}\right] + \mathbf{b}^1\right) + \mathbf{b}^2\right) + e_{t+1}$$

In order to obtain the best approximation for g, the neural network architecture should meet the following three standard conditions: it has to avoid overfitting,[10] the predicted error should be uncorrelated over time, and the cross-correlation function between the predicted errors and the observed time series should be close to zero. In this paper, we rely on the Bayesian regularization framework to approximate g in a parsimonious manner (Titterington, 2004). The objective function for the Bayesian regularization setup is given by:

5 $$F(x) = \beta \sum_{t=1}^{T}\left(Y_t - \widehat{Y}_t\right)^T \left(Y_t - \widehat{Y}_t\right) + \alpha \sum_{i=1}^{n} x_i^2$$

This is the weighted combination between the model fit and the smoothness. The parameter α penalizes model complexity and β reflects the goodness of fit. The term x_i^2 is the sum of the squared parameters values of the network, weights and biases.

Using the Bayes theorem sequentially, the joint posterior distribution of the parameters α and β, given the data D and the neural network model chosen M, is computed by multiplying the likelihood times the joint a priori distribution of α and β divided by the evidence:

6 $$P(\alpha, \beta \mid D, M) = \frac{P(D \mid \alpha, \beta, M)\, P(\alpha, \beta \mid M)}{P(D \mid M)}$$

The prior joint density for α and β is assumed from the uniform distribution. Consequently, the posterior can be obtained by computing the following probabilities:

[10] Overfitting is a characteristic that should be avoided and occurs when the neural network fit the data closely in the training set, but in the testing set and out of sample, the fitting is poor.

7 $$P(D|\alpha,\beta,M)=\frac{P(D|X,\beta,M)P(X|\alpha,M)}{P(X|D,\alpha,\beta,M)}$$

8 $$P(X|D,\alpha,\beta,M)=\frac{P(D|X,\beta,M)P(X|\alpha,M)}{P(D|\alpha,\beta,M)}$$

For more technical details and the full training algorithm see Hagan *et al.* (2014).

The adaptation of the algorithm requires a neural network architecture, *M*, which means we have to pick the number of neurons in the input layer, the number of hidden layers, the number of neurons per hidden layer, and the number of neurons in the output layer. For more details see Zhang, Patuwo and Hu (1998).

Bayesian regularization guarantees that the parameter sum is the optimal given data. In order to optimize the regularization parameters, the objective function $F(x)$ should be minimized following the Levenberg-Marquardt Back propagation algorithm.

The Bayesian regularization results exhibit flexibility to model the network architecture. Thus, for the hidden layer, we set a fixed number of nodes and we used just one hidden layer due to the length of the time series. However, we observed that an extra layer did not significantly change the results. With respect to the output layer, one node is used because the forecast is one-step-ahead. The selection of the adequate number of input nodes or lags will be explained in the NAR-NN results section. In order to improve the generalization of the network, the methodology usually requires one to divide the data into three sets: training, validation, and testing. However, Bayesian regularization avoids the validation stage because the solution is based on the optimization of equation (3).

Moreover, we employed the hyperbolic Tangent Sigmoid as an activation function for the nodes in the hidden layer as shown below. This function is frequently used in forecasting.

9 $$a=\frac{e^{n}-e^{-n}}{e^{n}+e^{-n}}$$

10 $$a = n$$

For the output layer the linear function is used.[11] The final architecture in matrix notations and scalar is:

11 $$Y_{t+1} = \mathbf{f}^2\left(\mathbf{W}^2\,\mathbf{f}^1\left(\mathbf{W}^1\left[Y_t, Y_{t-1}, \ldots, Y_{t-p}\right] + \mathbf{b}^1\right) + \mathbf{b}^2\right)$$

$$y_{t+1} = \sum_{j=1}^{10} w_j^2\,\mathbf{f}_j^1\left(\sum_{i=0}^{p} w_{i+p}^1 Y_{t+i-p}^1 + b^1\right) + b^2$$

$$p^n = \frac{2\left(p - p^{\min}\right)}{\left(p^{\max} - p^{\min}\right)} - 1$$

where w_{i+p}^1, i=1, ... , p, w_j^2, i=1, ... , p are the weights of the output layer, b^1 is the biases of the first layer, and b^2 the biases of the second layer.

Figure 7 displays the observed data (black line), the fit in the training set (blue line), the forecasts in horizons 1 and 2 (green and orange lines, respectively), and the out-of-sample forecasts eight steps ahead (yellow line). Also, the figure is divided into three blocks. The block on the left corresponds to the training set from Q31991 to Q2 2014; the center block corresponds to the testing set from Q3 2014 to Q2 2016, which occurs after the oil shock period, and the right block is the forecasting period.

3.4 ARIMA

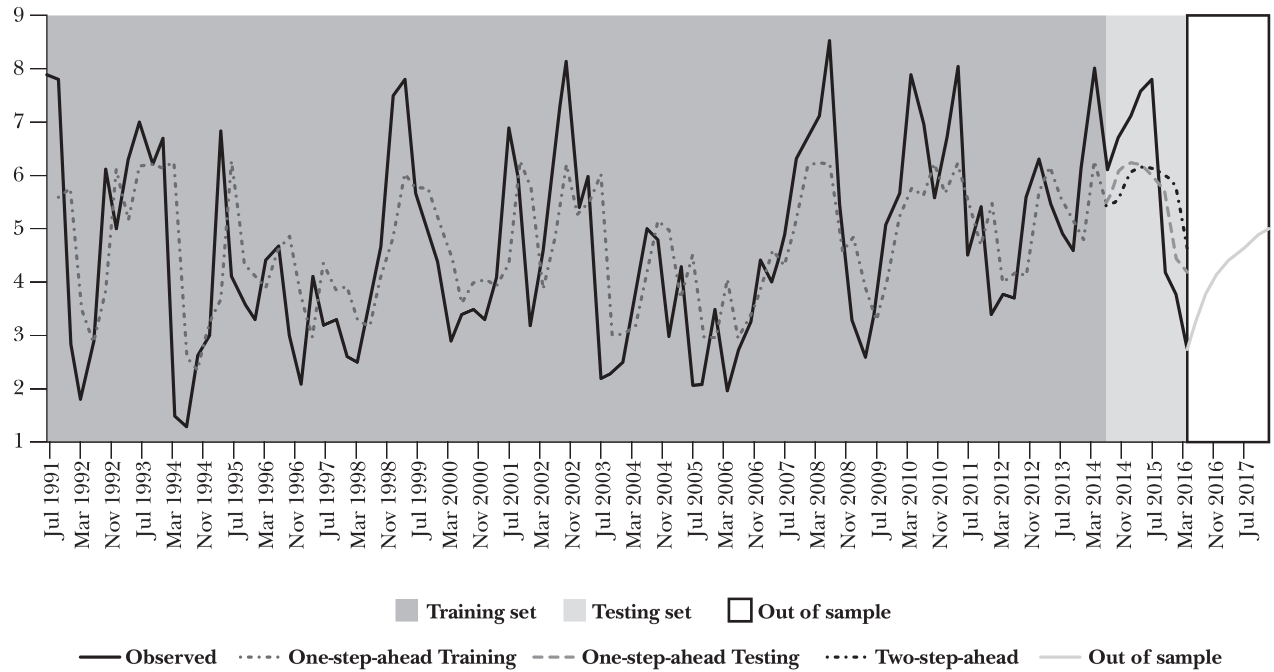
Box and Jenkins proposed the ARIMA model in 1970 (Box *et al.*, 2016). The general expression of an ARIMA model is the following:

12 $$Y_t = \frac{\Theta_s\left(L_s\right)\theta\left(L\right)}{\Phi_s\left(L^s\right)\varphi\left(L\right)\Delta_s^D \Delta^d}\varepsilon_t$$

[11] Notice that before training the network, data normalization, which transforms the data in the interval between [-1, 1], is required to make the training algorithm faster.

Figure 7

DATA BLOCK DIVISION AND OUT OF SAMPLE SETS



where $\Theta_s\left(L^s\right)=\left(1-\Theta_s L^s-\Theta_{2s}L^{2s}-\Theta_{3s}L^{3s}-\ldots-\Theta_{Qs}L^{Qs}\right)$ is a seasonal moving average polynomial, $\Phi_s\left(L^s\right)=\left(1-\Phi_s L^s-\Phi_{2s}L^{2s}-\ldots-\Phi_{3s}L^{3s}\right)$ is the seasonal auto-regressive polynomial, $\theta(L)=\left(1-\theta_1 L_1-\theta_2 L_2-\ldots-\theta_q L_q\right)$ is the regular moving average polynomial, and $\varphi(L)=\left(1-\varphi_1 L^1-\varphi_2 L^2-\ldots-\varphi_p L^p\right)$ is a regular auto-regressive polynomial, Δ_s^D is the seasonal difference operator, Δ^d is the difference operator, s is the periodicity of the considered series (s=4 for quarterly data), and ε_t is the innovation which is assumed to represent white noise.[12]

4. RESULTS

In this section, we present the main results of the clustering and forecasting for inflation expectations across countries. First, we present the SOM analysis that includes three sequential steps: the choice of the map topology based on data, the training and validation stages of the SOM neural network, and the elaboration of the clustering map of agent expectations (in Appendix B we include a detailed explanation of these steps). Then we overlap agents' inflation expectations on the resulting SOM map. Finally, the NAR-NN results are provided.

4.1 Self-Organizing Maps of Agents' Expectations

In this subsection, we briefly describe technical details on the implementation of the SOM analysis. We set a 10x10 hexagonal map with a learning rate varying from 0.05 to 0.0001, and we used 1,000 iterations. The computation was accomplished by the Kohonen package in R developed by Wehrens and Buydens (2007). The training step used observations before the oil shock identified on Q2 2014 and it covers a sample of 84 observations per country for the expected situation by the end of the next six months of the overall economy, capital expenditures, private consumption, and inflation.[13]

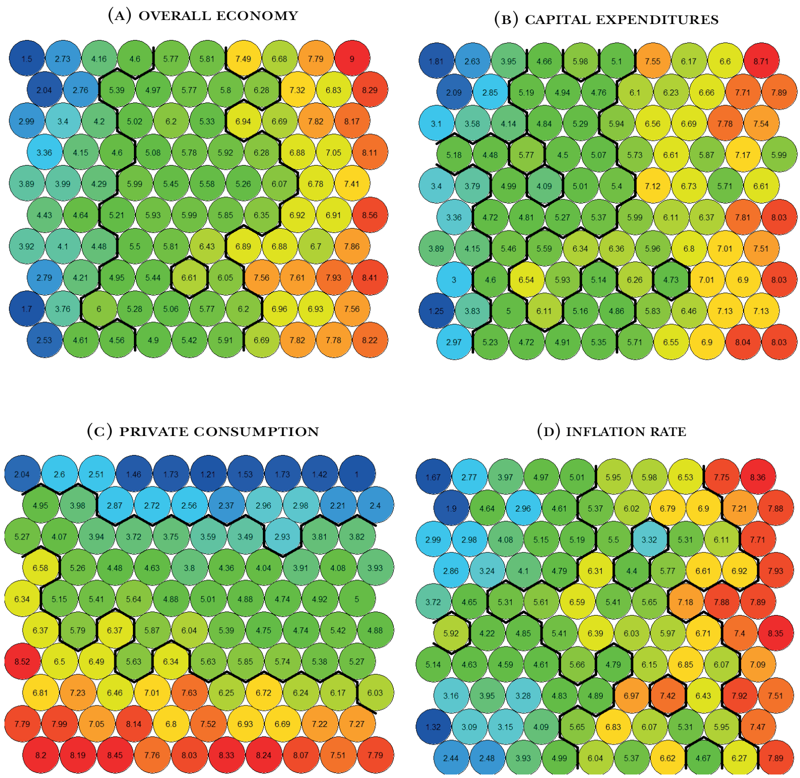
---

[12] The ARIMA models chosen are described in Appendix D.

[13] Appendix B explains the choice of topology as well as the post-training analysis of the results.

Figure 8

SOMs OF COUNTRIES' ECONOMY SITUATION EXPECTATIONS FOR THE NEXT 6 MONTS (Q3 1991 TO Q4 2014)



A key tool in this analysis is *the feature map or heat map* that is the representation of a single variable across the map (Figure 6). In this application, the colors identify the intensity of the indicator. For example: while the blue color is associated with low expectations, the red is associated with high expectations. Clustering can be performed by using hierarchical clustering on the weight learned vectors of the variable. This procedure requires one to set the number of clusters.

Thus, given the nature of the expectations, we choose three clusters to represent low, neutral, and high expectations.

4.2 Overlapping Agents' Inflation Expectations by Country

In order to categorize agents' inflation expectation patterns after the oil price shock that took place on June 2014, we overlap those expectations from the third quarter of 2014 with the second quarter of 2016 on the resulting heatmap. Next, we classified the expectations patterns by country into two categories: smooth and brisk expectation trajectories. For smooth transitions, we expected to find a path that moves through a single cluster. Otherwise, we identify a brisk trajectory by observing a changing path among several clusters. In Figure 9, the black arrow represents the trajectory of the inflation expectation with the initial node marked by a black start symbol.

For instance, in the case of Colombia, Figure 9(b), the observed inflation expectations for July 2014 are in the higher expectation cluster, then move through the heatmap ending in the lower expectation cluster. We classified this pattern as one of brisk expectations. Conversely, for the United Kingdom in Figure 9(d), inflation expectations vary only between two clusters. Thus, it can be categorized into the group with a smooth pattern. Table 5 summarizes the classification results for our sample of countries. From this table it is plausible that changes in expectations in countries heavily dependent on oil revenues were brisk, as exemplified by Colombia and Canada. However, in countries such as Mexico, the change in expectations is smooth because this economy is much more diversified. However, we should consider that each country faces global and idiosyncratic shocks that could have produced this heterogeneity as well.

Figure 9

COUNTRIES' INFLATION RATE NEXT SIX MONTHS (Q3 2014 TO Q2 2016) ON THE EXPECTED INFLATION RATE SOM MAP

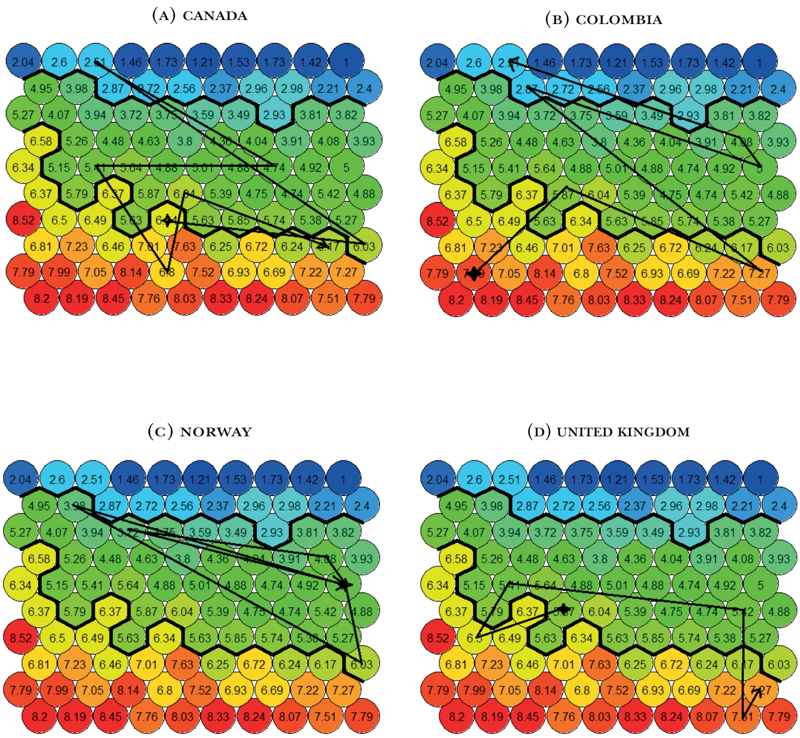


Table 5

CLASSIFICATION OF INFLATION EXPECTATIONS AND LAG SELECTED IN THE NAR-NN MODEL

Country	*Inflation expectation*	*Lag selected*
Brazil	Brisk	1
Canada	Brisk	8
Chile	Smooth	4
Colombia	Brisk	5
Czech R.	Smooth	6
Korea R.	Smooth	2
Mexico	Smooth	6
Norway	Smooth	1
Switzerland	Brisk	8
United K.	Smooth	6
Hungary	Smooth	10
Philippines	Brisk	1
Poland	Smooth	7
Sweden	Smooth	1
Thailand	Brisk	4
South A.	Brisk	1

4.3 Non-Linear Auto-Regressive Neural Network Results

We have to select a model M to apply the Bayesian regulation framework to the NAR-NN in order to improve its generalization ability. For each country, the sum of the parameters is conditional on the complexity of the data. In this context, we chose a flexible network where regularization guarantees the minimum sum of parameters. Thus, we set an architecture with one hidden layer of 10 neurons. Moreover, at the input layer we have to specify the number of neurons that correspond to the lag order used to forecast one step ahead. We used the Neural Network Toolbox (Hagan, Demuth and Beale, 2002).

The lag order selection was based on different criteria: the mean squared error resulting from the testing data, the error auto-correlation function, and the cross-correlation between the errors and the observed data. In this way, from lags 1 to 10 we generated 30 neural networks per lag and obtain the MSE for the training, testing, and the complete sample. Then, we select the lag that reports the smallest median from the testing data sample, considering the auto-correlation diagnostics.[14] The lags chosen for each country are presented in Table 5, and the overall results from lags 1 to 10 are shown in Table 6.[15] A similar procedure was developed by Ruiz *et al.* (2016). Next, we present the forecast results for some selected countries.[16,17,18]

[14] In most of the cases mean and median, of the lag chosen, are both the smallest. However, in Colombia, Czech Republic and Switzerland this is not the case, even though the lag's mean is closer to the smallest mean.

[15] These results for all datasets and training sets are presented in Tables 12 and 13, respectively, in Appendix 3.

[16] To see the other countries, see Figure 24 in Appendix C.

[17] A summary of results of the neural networks parameters is presented in Table 14 in Appendix 3.

[18] A simulation of 1000 networks was performed to ensure that the MSE presented belongs to the average neural network find after specifying the model previously described. See Table 15 and Appendix 3.

Table 6

LAG STATISTICS TEST DATA - ONE-STEP AHEAD FORECAST. SAMPLE = 30

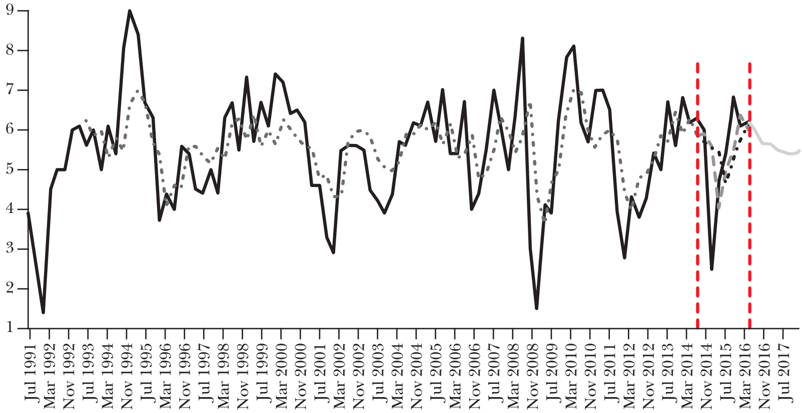
Countries	*Lags*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Brazil	mean	**1.65**	2.08	1.85	1.86	1.89	1.85	1.83	1.84	1.92	2.04
	median	**1.57**	2.07	1.85	1.86	1.83	1.85	1.83	1.84	1.92	2.04
Canada	mean	2.04	1.84	1.59	1.63	1.62	1.54	1.56	**1.52**	1.75	1.73
	median	2.04	1.74	1.59	1.63	1.62	1.54	1.56	**1.52**	1.75	1.73
Switzerland	mean	1.32	1.21	1.04	1.04	1.02	0.98	1.06	0.79	0.93	**0.78**
	median	1.30	1.22	1.04	1.04	1.02	0.98	1.06	**0.78**	0.94	0.83
Chile	mean	4.34	2.76	2.77	**2.70**	2.74	2.80	2.94	3.13	3.13	2.93
	median	4.38	2.76	2.79	**2.68**	2.76	2.81	3.00	3.06	3.13	2.91
Colombia	mean	4.82	2.88	2.91	2.90	**2.83**	2.88	2.86	3.27	3.21	3.27
	median	4.94	2.88	2.92	2.88	**2.78**	2.83	2.81	3.23	3.15	3.20
Czech R.	mean	0.72	0.74	0.73	0.70	0.93	**0.68**	1.20	1.24	2.18	1.46
	median	0.73	0.74	0.73	0.70	0.93	**0.67**	1.20	1.24	2.10	1.15
United K.	mean	0.87	0.87	0.95	0.95	0.88	**0.82**	0.85	1.14	0.87	0.86
	median	0.87	0.86	0.95	0.95	0.88	**0.82**	0.83	0.84	0.87	0.83
Korea R.	mean	2.24	**1.87**	1.99	2.02	2.03	2.21	2.40	2.06	2.11	2.09
	median	2.21	**1.86**	1.99	2.02	2.03	2.21	2.40	2.06	2.06	2.05

Countries	*Lags*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Mexico	mean	0.38	0.42	0.52	0.48	0.48	**0.31**	0.50	0.36	0.45	1.07
	median	0.38	0.42	0.52	0.49	0.48	**0.30**	0.57	0.37	0.31	0.53
Norway	mean	**1.41**	1.44	1.61	1.67	1.59	2.01	2.04	2.10	1.88	1.80
	median	**1.41**	1.44	1.61	1.67	1.59	2.01	2.04	2.10	1.88	1.80
Hungary	mean	3.49	2.92	2.94	3.47	3.75	3.54	3.73	3.46	2.87	**2.77**
	median	3.52	2.91	2.94	3.47	3.71	3.54	3.73	3.46	2.86	**2.75**
Philippines	mean	**3.41**	3.99	3.86	3.78	3.78	3.83	3.47	3.60	4.14	3.65
	median	**3.41**	3.99	3.86	3.78	3.78	3.83	3.47	3.60	4.05	3.45
Poland	mean	1.12	1.02	1.04	1.07	1.37	1.07	**0.72**	0.87	3.52	7.12
	median	1.12	1.04	1.01	1.06	1.37	1.07	**0.72**	0.86	2.99	7.12
Sweden	mean	**1.09**	1.59	1.52	1.58	1.68	1.74	1.72	1.73	1.74	1.67
	median	**1.10**	1.59	1.52	1.58	1.68	1.74	1.72	1.72	1.74	1.67
Thailand	mean	1.67	1.01	1.03	**0.90**	0.95	1.00	1.06	1.10	1.05	1.02
	median	1.68	1.01	1.03	**0.91**	0.95	1.00	1.06	1.10	1.05	1.01
South A.	mean	**2.63**	3.31	3.48	3.56	3.97	3.85	4.27	4.51	4.59	4.64
	median	**2.63**	3.31	3.48	3.56	3.97	3.85	4.11	4.51	4.59	4.97

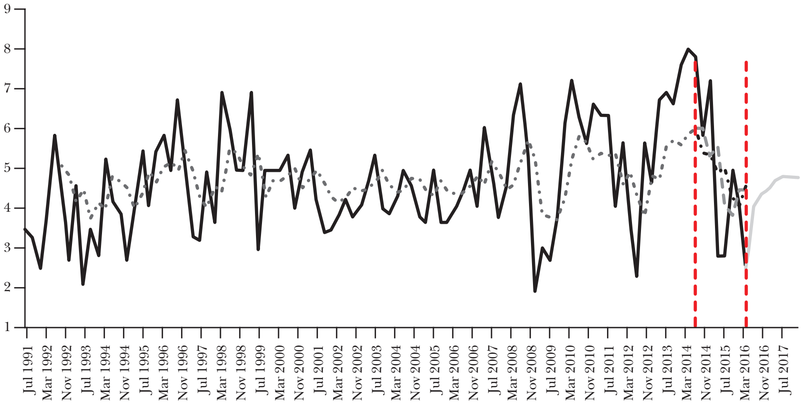
Figure 10

FORECASTS OF INFLATION EXPECTATIONS USING THE NAR-NN MODEL

(A) CANADA



(B) COLOMBIA



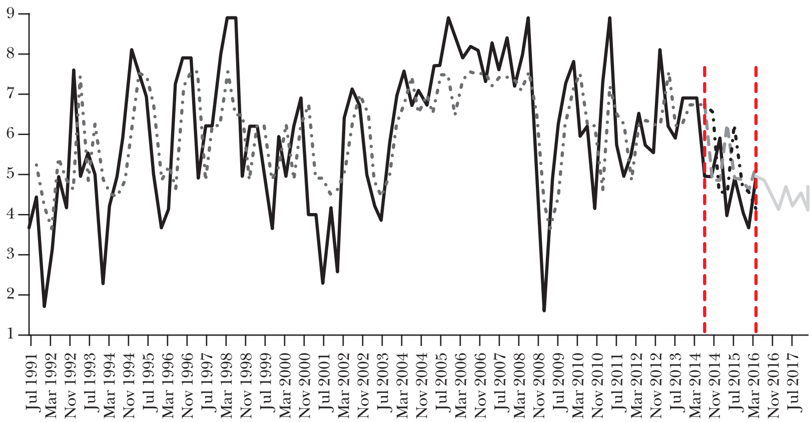
Observed **One-step-ahead Training**

One-step-ahead Testing **Two-step-ahead** **Out of sample**

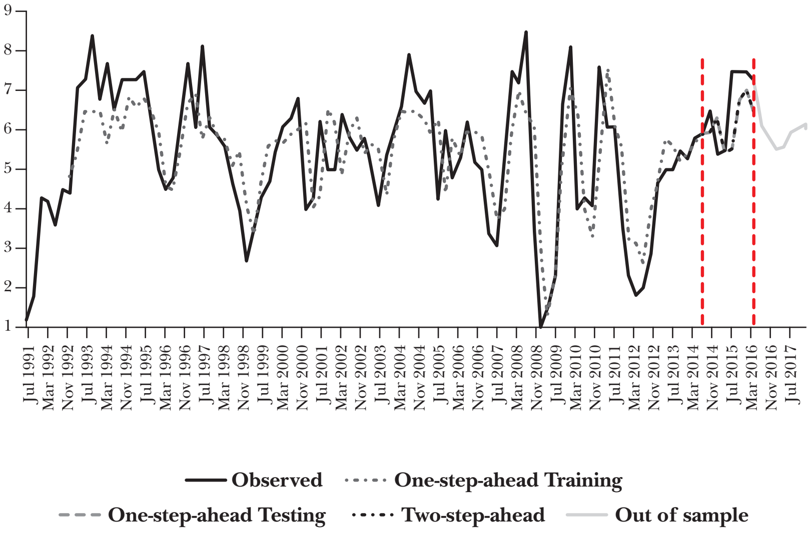
Figure 10 (cont.)

FORECASTS OF INFLATION EXPECTATIONS USING THE NAR-NN MODEL

(C) NORWAY



(D) UNITED KINGDOM



4.4 Forecast Accuracy

Table 7

MSE COMPARISON AT TESTING DATA SETS FOR COUNTRIES WITH BRISK INFLATION EXPECTATIONS

	Arima		*NAR*		*Diebold*	*Diebold*
	Testing set		*Testing set*		*Testing set*	*Testing set*
Countries	*One-step ahead*	*Two-step ahead*	*One-step ahead*	*Two-step ahead*	*One-step ahead*	*Two-step ahead*
Brazil	1.909	3.408	**1.470**	**2.616**	−0.988	−1.252
Canada	1.732	2.173	**1.519**	**1.834**	−1.402	**−2.097**
Colombia	2.913	2.926	**2.776**	**2.648**	−0.467	−1.763
Philippines	**3.052**	**3.223**	3.435	4.291	0.751	2.426
South A.	3.892	6.929	**2.580**	**6.045**	−1.571	−0.448
Switzerland	0.894	**1.136**	**0.781**	1.414	−0.343	1.041
Thailand	**0.797**	**0.885**	0.914	1.041	0.519	0.555
Brisk	**2.018**	**2.961**	**1.734**	**2.632**	**−0.693**	**−0.702**

Table 8

MSE COMPARISON AT TESTING DATA SETS FOR COUNTRIES WITH SOFT INFLATION EXPECTATIONS

	Arima		*NAR*		*Diebold*	*Diebold*
	Testing set		*Testing set*		*Testing set*	*Testing set*
Countries	*One-step ahead*	*Two-step ahead*	*One-step ahead*	*Two-step ahead*	*One-step ahead*	*Two-step ahead*
Chile	3.577	4.181	**2.680**	**2.429**	−1.349	**−2.539**
Czech R	0.918	2.230	**0.665**	**1.464**	−0.763	−1.080
Hungary	3.485	6.850	**2.746**	**4.734**	−1.380	−1.610
Korea	**1.764**	**2.812**	1.857	3.028	**2.870**	**8.936**
Mexico	**0.279**	0.474	0.299	**0.341**	0.215	−0.945
Norway	1.484	2.019	**1.419**	**1.221**	−0.248	−1.043
Poland	1.028	2.263	**0.716**	**0.925**	−1.296	**−3.950**
Sweden	1.822	2.467	**0.905**	**0.913**	**−2.087**	**−2.183**
United K.	0.947	2.101	**0.820**	**1.465**	−0.945	−1.510
Soft	**1.205**	**2.12**	**1.043**	**1.544**	**−0.033**	**0.33**

5. CONCLUSIONS

Evaluating and forecasting inflation expectations from international surveys of economics experts can be valuable for monetary macroeconomic modeling. In this research, we set two goals. First, we analyzed WES inflation expectations data for 16 countries that adopted inflation targeting regimes as the basis of their monetary policy. Given that the quarterly questions on the evolution of prices in these surveys consider both qualitative and quantitative scales, we used a descriptive analysis for the relationship between inflation expectations and observed inflation, and we study the structure of the in-sample forecasting errors.

Second, we generated-out-of-sample forecasts for the inflation expectations of the countries by relying on a two-step approach to sequentially cluster and forecast inflation expectations. Thus, the clustering technique known as Self-Organizing Maps and a predictive model based on artificial neural networks allow us to visualize and predict different patterns of inflation expectations according to their perceptions before the oil shock that took place in the middle of 2014.

We cluster the countries according to the evolution of their inflation expectations during the transition period to the recent minimum oil price mark. Then, we obtain forecasts of survey expectations by using linear and non-linear NAR-NN methods. For the SOM analysis, we find that some countries exhibited brisk behavior that is associated with signs that inflation expectations were de-anchoring. At the same time, there were countries with a soft evolution of inflation expectations.

The correlation analysis from the time and frequency domain indicates the existence of different patterns of linear associations over time and frequency: increasing, descending, and inverted U-shaped. Moreover, the highest coherence between inflation and expectations was found mainly in higher frequencies, which suggests that the relationship between inflation expectations and observed inflation is present in short duration cycles.

Concerning the statistical evaluation based on the forecasting errors of the quantitative inflation expectation, we detected uncertainty in the predictions of average annual inflation across countries that could be classified into two groups. In the first group, the closer the expert is to the end of the year, the smaller the prediction bias. This group includes Colombia and Switzerland among others. The other group of countries exhibit increasing bias in the last quarter of the prediction period and include Brazil, Canada, and Chile.

Additionally, the quality of the quantitative question is judged by standard measures of forecast evaluation at different horizons: RMSE, MAE, and U-Theil. Thus, we concluded that the forecasts meet a minimum standard compared to the random walk reference and that economic experts have made systematic errors in their predictions. Inflation was under- predicted when it was rising and over-predicted when it was declining in most of the countries. The Theil decomposition of the MAE illustrated that 83 percent of the countries experienced systematic distortion in their forecasts, which means that the increase in accuracy with shorter forecast horizons is not monotonic. The evidence does not support the claim that forecasts have improved over time due to a non-linear generating data process. The evidence also suggests that turning points of observed average inflation were mostly anticipated in most cases. This issue may be an interesting area for further research.

On the other hand, a Self-Organizing Map analysis of surveys expectations before the impending oil shock allows us to classify inflation expectations as either brisk or soft based on the speed with which expectations shift. Using this classification, we can select the most appropriate forecasting method. We notice that the low-inflation expectations cluster is relatively small compared to high and neutral clusters for inflation targeting countries. The Nonlinear auto-regressive neural network and ARIMA methods were used as competing candidates to forecast inflation expectations. The results indicate that in the one step ahead forecasts the neural network is slightly better, but in two step-ahead forecasts, it outperforms the ARIMA model significantly. For Canada, Colombia, Chile, Poland, Hungary, and Sweden in particular, the neural network produces significant improvement in the two-step ahead forecasts.

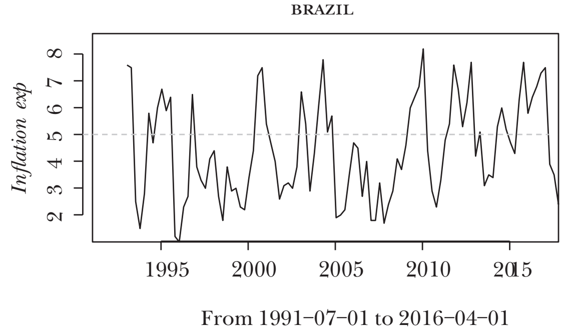
Further research is required to provide theoretical economic explanations for the results of each country. Moreover, this combination between machine learning and statistics can be implemented in a follow-up paper to forecast actual inflation.

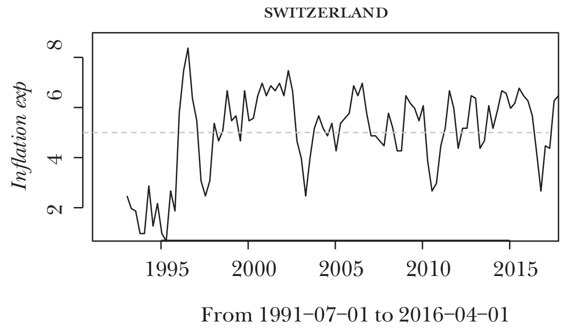
ANNEX A. DATA

A.1 Qualitative Series

Figure 11

EXPECTED INFLATION RATE FOR THE NEXT SIX MONTHS
Wes Qualitative Question





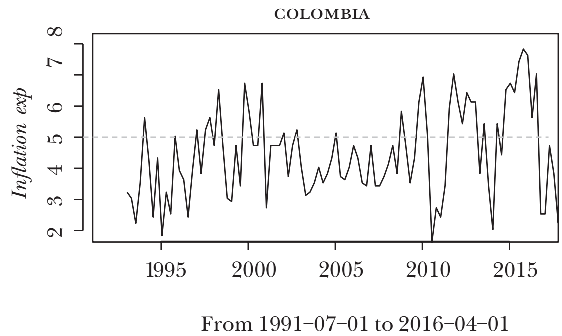
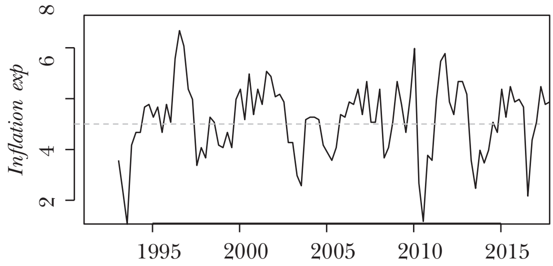


Figure 11 (cont.)

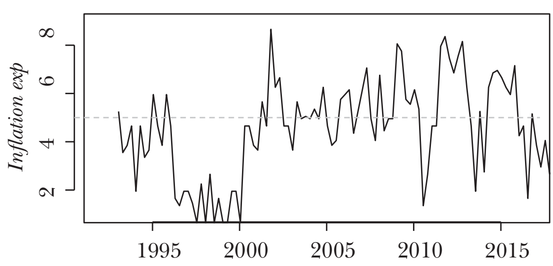
EXPECTED INFLATION RATE FOR THE NEXT SIX MONTHS
Wes Qualitative Question

CANADA



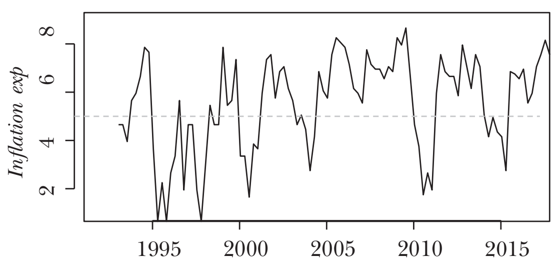
From 1991−07−01 to 2016−04−01

CHILE



From 1991−07−01 to 2016−04−01

CZECH REPUBLIC

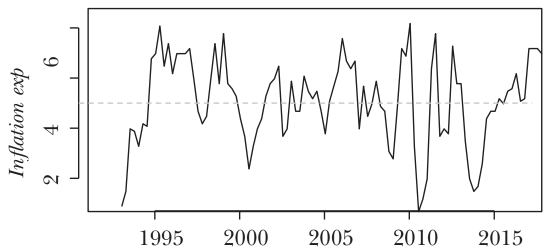


From 1991−07−01 to 2016−04−01

Figure 11 (cont.)

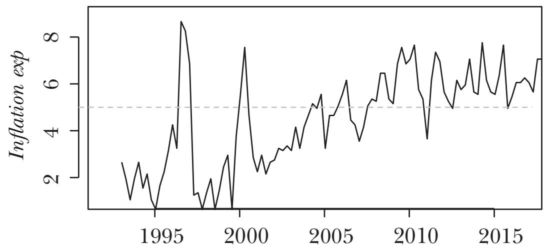
EXPECTED INFLATION RATE FOR THE NEXT SIX MONTHS
Wes Qualitative Question

UNITED KINGDOM



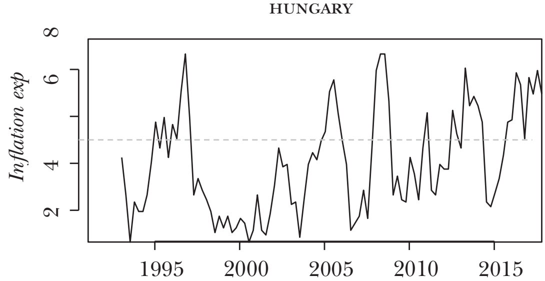
From 1991-07-01 to 2016-04-01

MEXICO



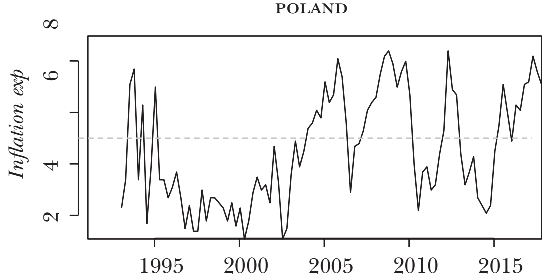
From 1991−07−01 to 2016−04−01

HUNGARY



From 1991−07−01 to 2**01**6−04−

POLAND



From 1991−07−01 to 2016−04−01

Figure 11 (cont.)

EXPECTED INFLATION RATE FOR THE NEXT SIX MONTHS
Wes Qualitative Question

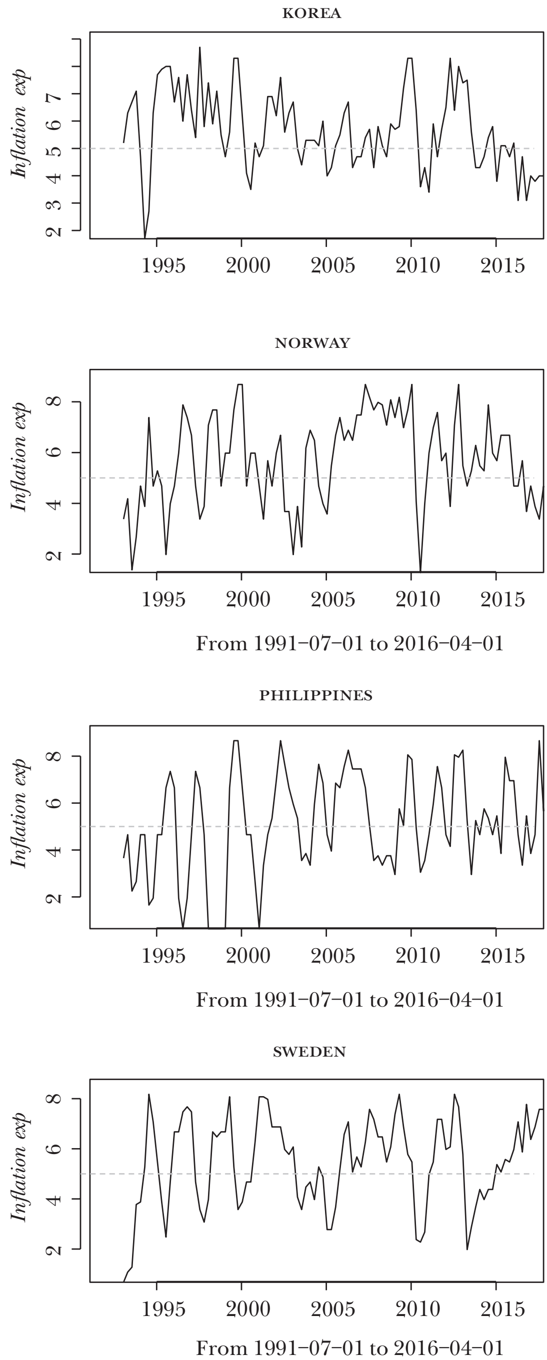


Figure 12

HISTOGRAMS OF AGENTS' EXPECTATIONS OF ECONOMIC SITUATION FOR NEXT SIX MONTHS IN MACROECONOMIC VARIABLES

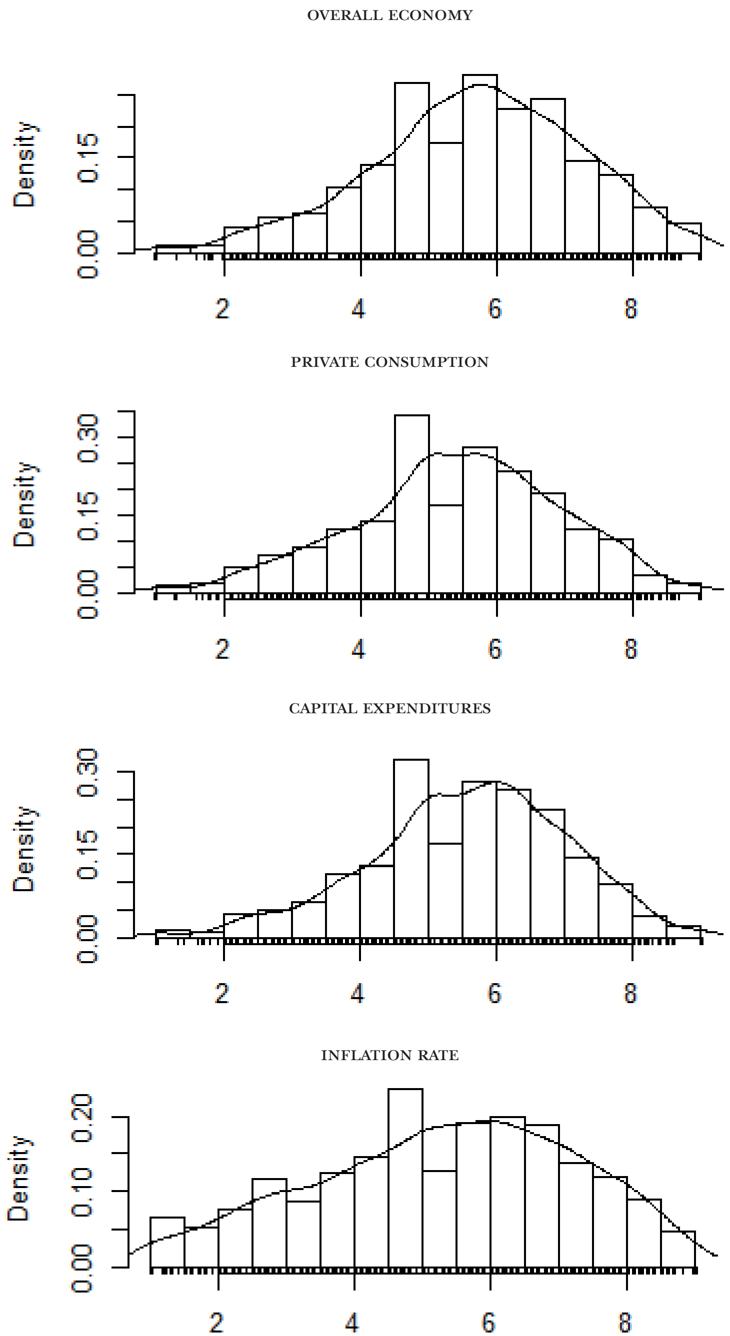


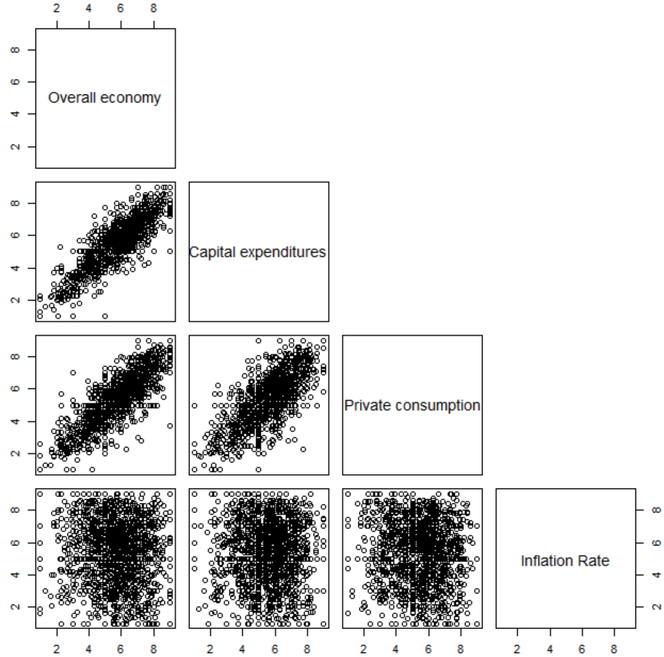
Table 9

DATA SUMMARY OF WES EXPECTATIONS FROM Q3 1991 TO Q2 2016
Selected countries

	Overall economy	*Capital expenditures*	*Private consumption*	*Inflation rate*
Min	1	1	1	1
1stQ	4.8	4.7	4.57	4
Median	5.8	5.7	5.5	5.5
Mean	5.79	5.59	5.44	5.32
3rdQ	6.8	6.6	6.5	6.8
Max	9	9	9	9

Figure 13

SCATTER PLOT OF AGENTS' EXPECTATIONS OF ECONOMIC SITUATION FOR NEXT SIX MONTHS



A.2 WES Survey Questionnaire

Figure 14

EXAMPLE OF WORLD ECONOMIC SURVEY (WES) QUESTIONNAIRE

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Telefax: + 49 (0)89 985369 or + 49 (0)89 9224-1463 or -2227

World Economic Survey WES

The individual survey results will be treated as absolutely confidential. Please mark the appropriate boxes. No mark means: "Not applicable" or "no judgement". The answer "no change" implies no remarkable change.

Data requested for **Code-Nr.:**

1. This country's **general situation** regarding

	present judgement			compared to the same time last year			from now on: expected situation by the end of the next 6 months		
	good	satis-factory	bad	better	about the same	worse	better	about the same	worse
- overall economy	☐	☐	☐	☐	☐	☐	☐	☐	☐
- capital expenditures	☐	☐	☐	☐	☐	☐	☐	☐	☐
- private consumption	☐	☐	☐	☐	☐	☐	☐	☐	☐

2. Expected **foreign trade volume** by the end of the next 6 months (in convertible currency)

	higher	about the same	lower
exports	☐	☐	☐
imports	☐	☐	☐

3. Expected **trade balance** within the next 6 months (in convertible currency)

improve-ment (a)	no change	deterio-ration (b)
☐	☐	☐

(a) increasing surplus or decreasing deficit
(b) decreasing surplus or increasing deficit

4. Expected **inflation rate** by the end of the next 6 months (change of consumer prices compared to the same month previous year)

higher	about the same	lower
☐	☐	☐

The rate of inflation on average of 2004 will be ________ % (p.a.)

5. Expected **interest rates** by the end of the next 6 months

	higher	about the same	lower
- short term rates (3-month money market rates)	☐	☐	☐
- long-term rates (government bonds with 10 and more years of maturity)	☐	☐	☐

6. At present, **in relation to this country's currency** the following currencies (US-$; Euro; UK £; Yen) are...

	US $	Euro	UK	Yen
overvalued	☐	☐	☐	☐
about at proper value	☐	☐	☐	☐
undervalued	☐	☐	☐	☐

7. **The value of the US $** in relation to this country's currency by the end of the next 6 months will be

higher	about the same	lower
☐	☐	☐

8. The level of **domestic share prices** by the end of the next 6 months will be

higher	about the same	lower
☐	☐	☐

9. Please try to asses the **importance** of the following **problems** the economy of your country is facing **at present**

	most important	important	not so important
- Lack of confidence in the government's economic policy	☐	☐	☐
- Insufficient demand	☐	☐	☐
- Unemployment	☐	☐	☐
- Inflation	☐	☐	☐
- Lack of international competitiveness	☐	☐	☐
- Trade barriers to exports	☐	☐	☐
- Lack of skilled labour	☐	☐	☐
- Public deficits	☐	☐	☐
- Foreign debts	☐	☐	☐
- Capital shortage	☐	☐	☐

- Others: ____________________

10. **ICC Special Question**
The current round of **multilateral trade** negotiations, launched in Doha in 2001, is stalled. Leaders of the world's major industrial democracies are meeting in June 2004 at the G8 Summit in the US to discuss world trade and protectionist pressures.

1. How important is it to your business / the economy of your country that the **Doha Round gets back on track**?

very important	important	not important	no answer
☐	☐	☐	☐

2. How satisfied are you with the **efforts of the world's political leaders to contain protectionist pressures** and to promote a better understanding among electorates of the benefits of an open international trade system?

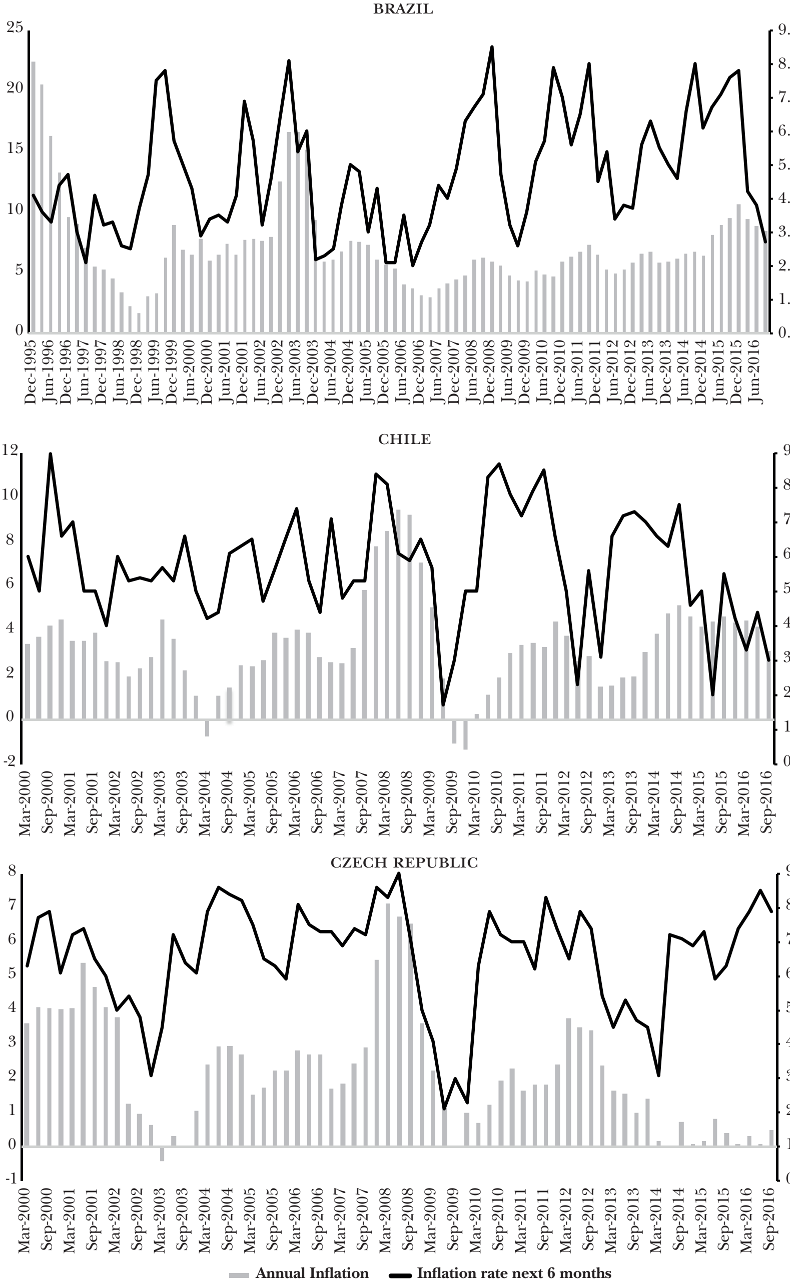
fully satisfied	satisfied	dissatisfied	no answer
☐	☐	☐	☐

11. Expected growth of **real Gross Domestic Product (GDP)** this year in % ________

Please return the questionnaire by April 14, 2004

Figure 15

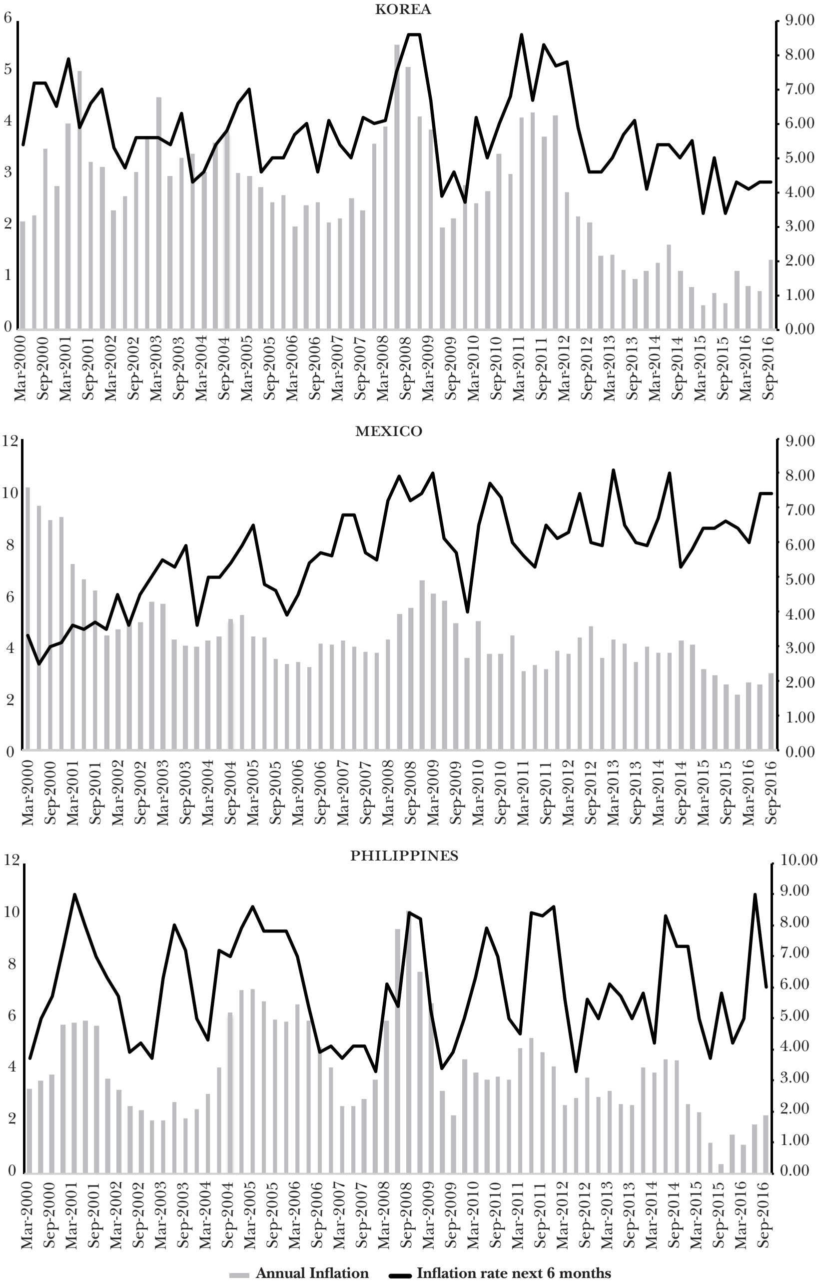
COUNTRIES' INFLATION EXPECTATIONS AND ANNUAL INFLATION



Source: WES and OECD statistics.

Figure 15 (cont.)

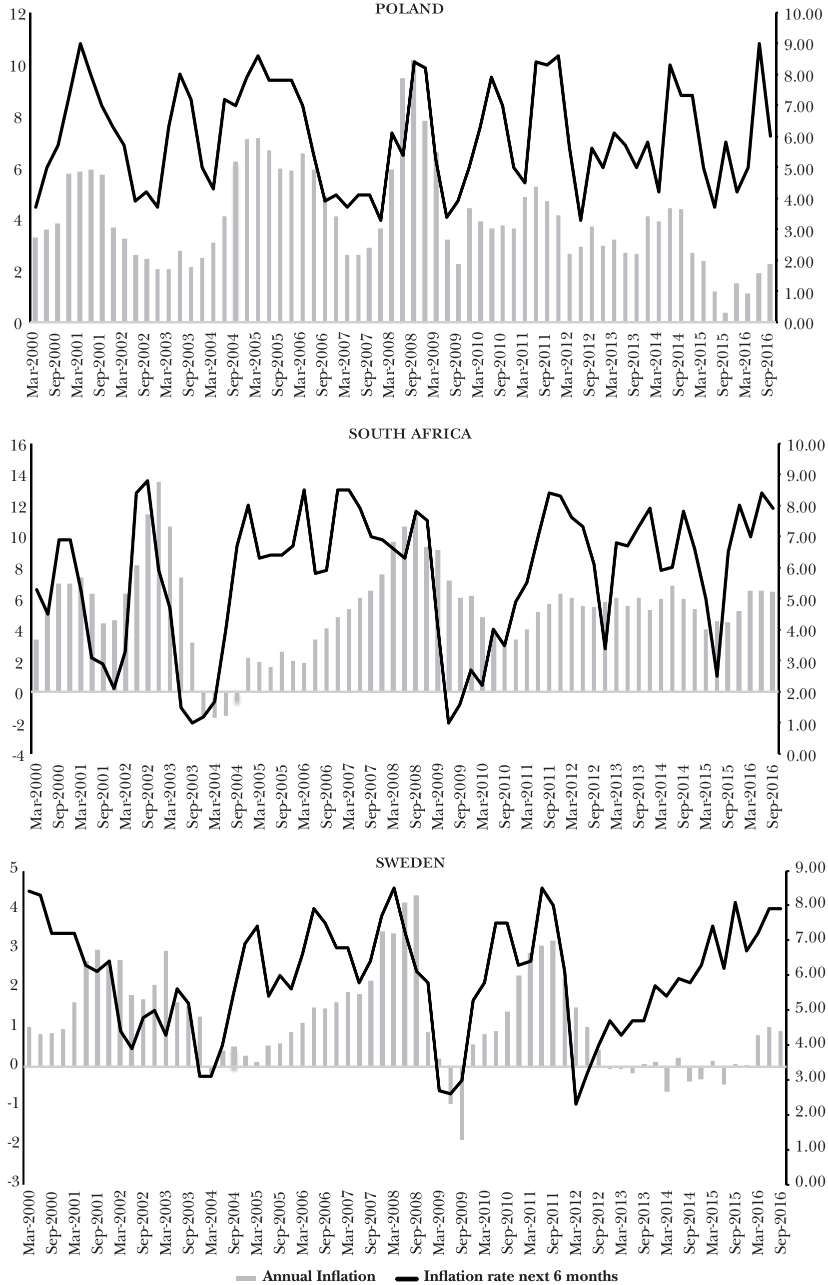
COUNTRIES' INFLATION EXPECTATIONS AND ANNUAL INFLATION



Source: WES and OECD statistics.

Figure 15 (cont.)

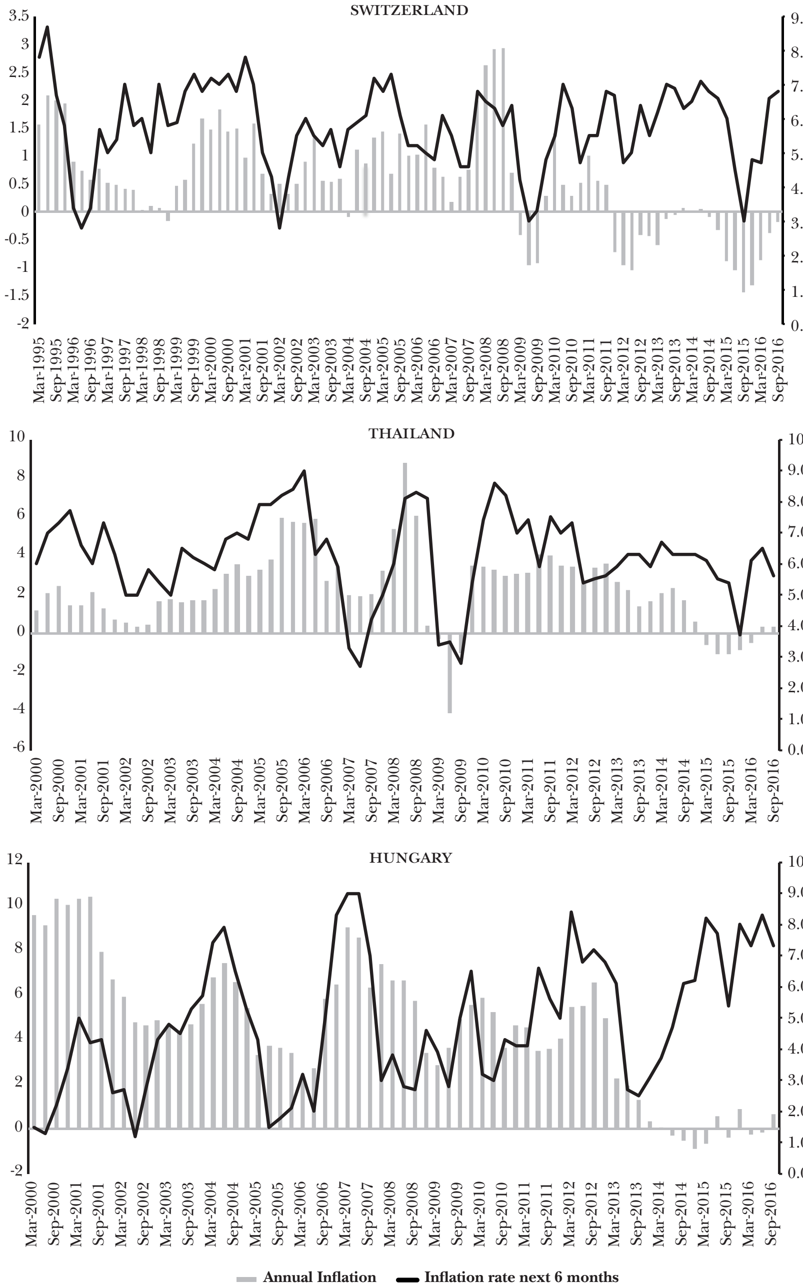
COUNTRIES' INFLATION EXPECTATIONS AND ANNUAL INFLATION



Source: WES and OECD statistics.

Figure 15 (cont.)

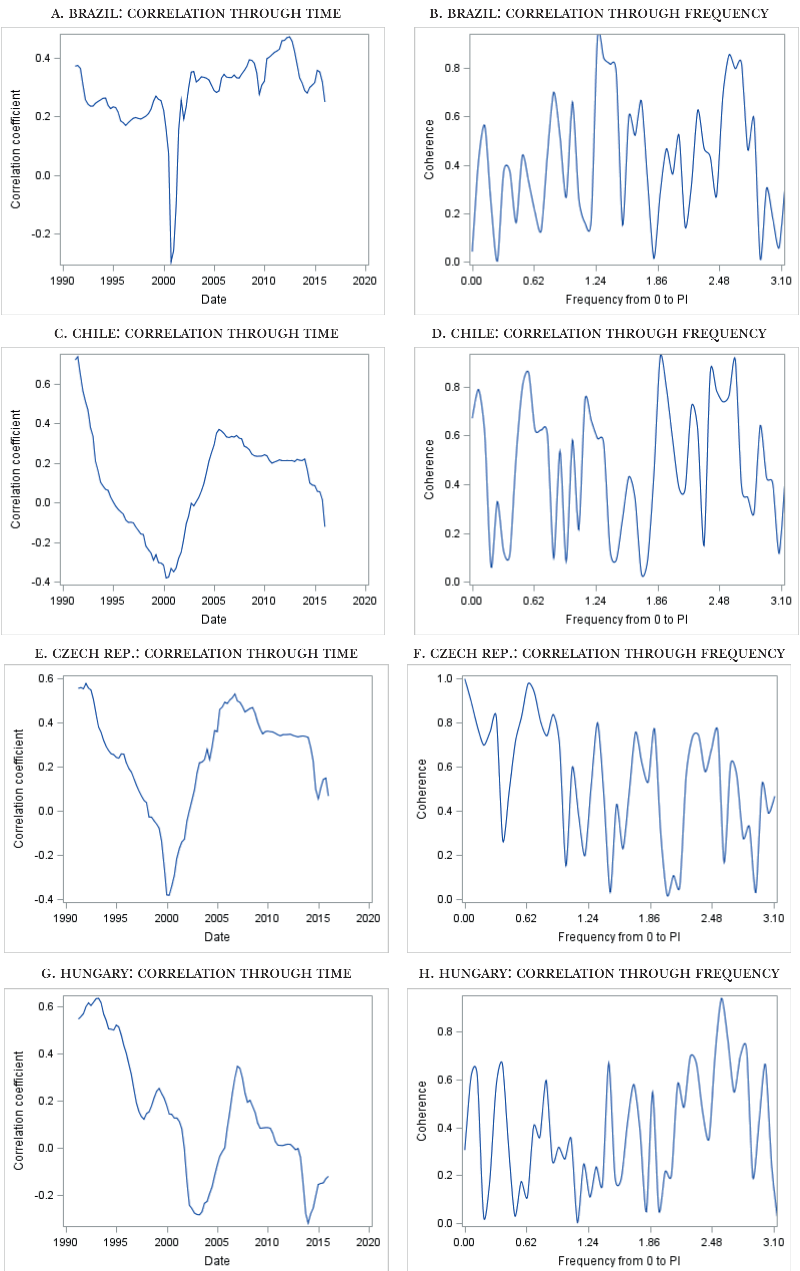
COUNTRIES' INFLATION EXPECTATIONS AND ANNUAL INFLATION



Source: WES and OECD statistics.

Figure 16

CORRELATION COEFFICIENTS BETWEEN WES QUALITATIVE INFLATION EXPECTATION AND ANNUAL INFLATION

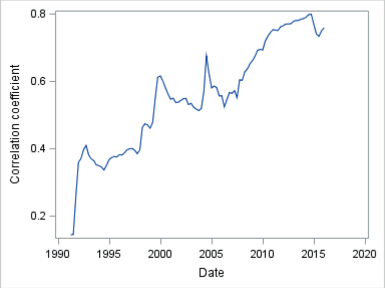


Source: WES survey, OECD statistics and IMF data.

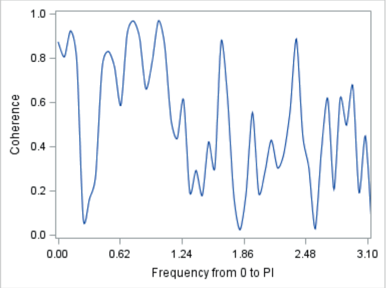
Figure 16 (cont.)

CORRELATION COEFFICIENTS BETWEEN WES QUALITATIVE INFLATION EXPECTATION AND ANNUAL INFLATION

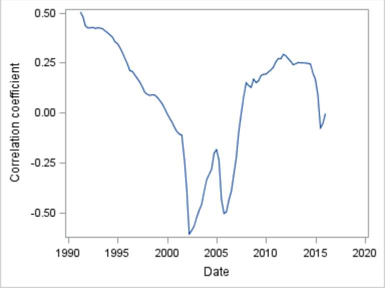
I. KOREA: CORRELATION THROUGH TIME



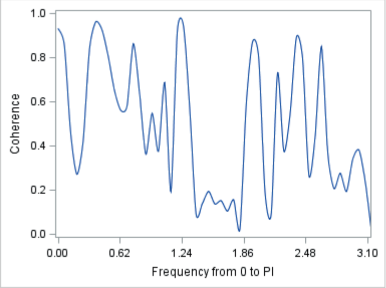
J. KOREA: CORRELATION THROUGH FREQUENCY



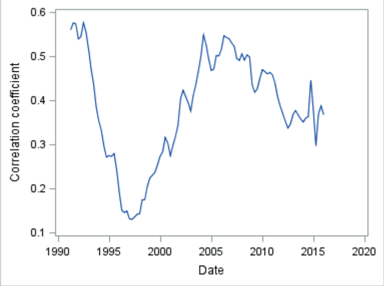
K. MEXICO: CORRELATION THROUGH TIME



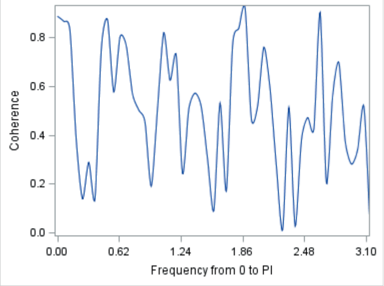
L. MEXICO: CORRELATION THROUGH FREQUENCY



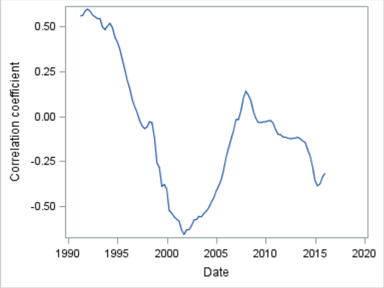
M. PHILIPPINES: CORRELATION THROUGH TIME



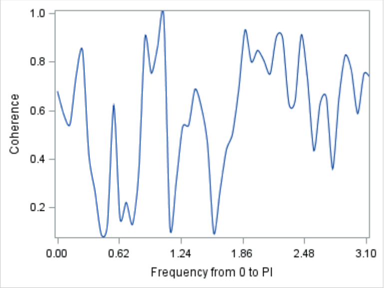
N. PHILIPPINES: CORRELATION THROUGH FREQUENCY



O. POLAND: CORRELATION THROUGH TIME



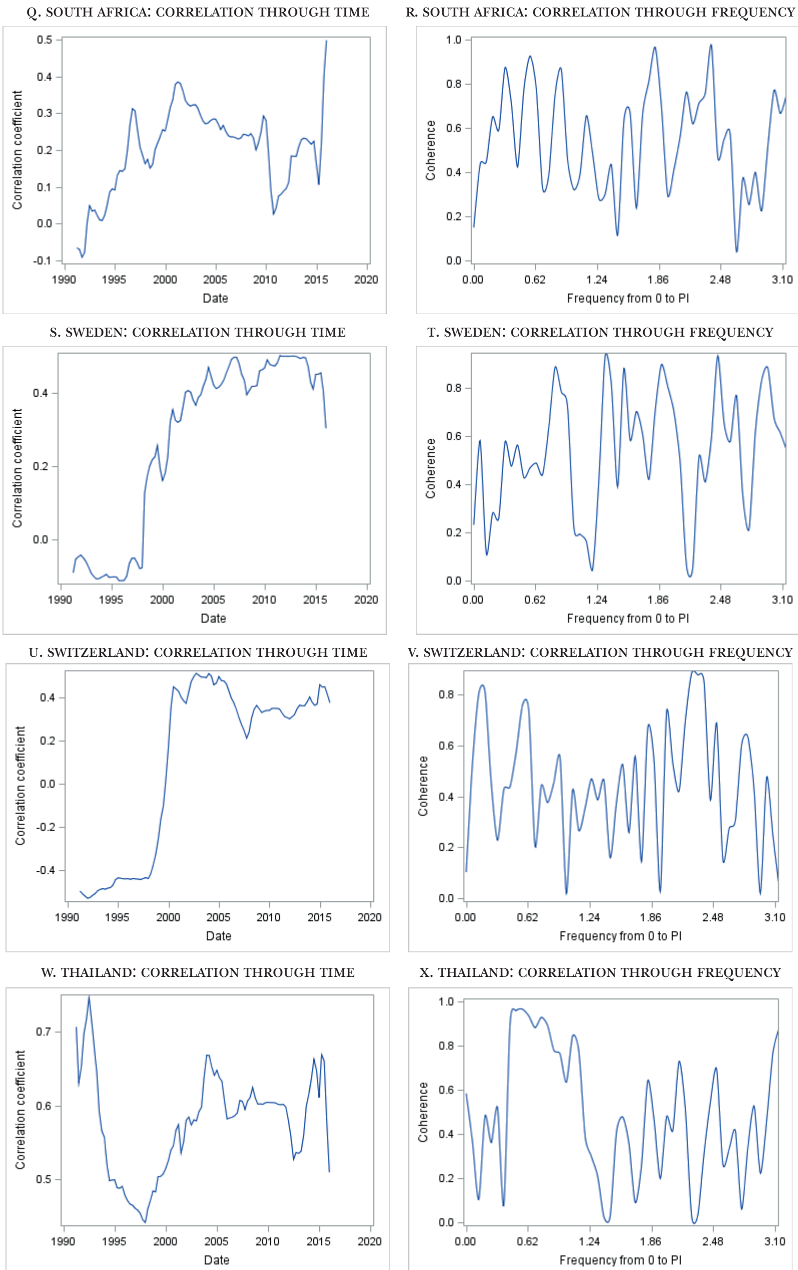
P. POLAND: CORRELATION THROUGH FREQUENCY



Source: WES survey, OECD statistics and IMF data.

Figure 16 (cont.)

CORRELATION COEFFICIENTS BETWEEN WES QUALITATIVE INFLATION EXPECTATION AND ANNUAL INFLATION



Source: WES survey, OECD statistics and IMF data.

A.3 Quantitative Forecasting Inflation Expectations

A.3.1 Equations of the Statistical Analysis Forecasting Error

Root mean squared forecast error (RMSFE):

13 $$\sqrt{\frac{1}{26}\sum_{1991}^{2016} e\left(L, Q(h), t\right)}$$

Mean absolute error (MAE):

14 $$\frac{1}{26}\sum_{1991}^{2016} e\left(L, Q(h), t\right)^2$$

Theil U.statistic:

15 $$\frac{\sqrt{\frac{1}{26}\sum_{1991}^{2016} e\left(L, Q(h), t\right)^2}}{\sqrt{\frac{1}{26}\sum_{1991}^{2016} q\left(L, Q(h), t\right)^2}\sqrt{\frac{1}{26}\sum_{1991}^{2016} \bar{p}\left(L, t\right)^2}}$$

Bias share:

16 $$V(h)=\frac{\left[\frac{1}{26}\sum_{1991}^{2016} q\left(L, Q(h), t\right)^2-\frac{1}{26}\sum_{1991}^{2016} \bar{p}\left(L, t\right)^2\right]}{\frac{1}{26}\sum_{1991}^{2016} e\left(L, Q(h), t\right)^2}$$

The spread share:

17 $$S(h)=\frac{\left[S_q(h)-S_{\bar{q}}(h)\right]^2}{\frac{1}{26}\sum_{1991}^{2016} e\left(L, Q(h), t\right)^2}$$

where $S_q(h)$ and $S_{\bar{q}}(h)$ are the standard deviations of the respective quarter. The covariance share:

18 $$K(h)=\frac{2\left(1-r_{q,\bar{p}}(h)\right)S_q(h)-S(h)}{\frac{1}{26}\sum_{1991}^{2016} e\left(L, Q(h), t\right)^2}$$

where $r_{q,\bar{p}}(h)$ is the correlation coefficient between q and p. Thus $V(h)+S(h)+K(h)=1$.

Table 10

MAE OF WES SURVEY QUANTITATIVE INFLATION QUESTION

	4-step forecast (QI)	*3-step forecast (QII)*	*2-step forecast (QIII)*	*1-step forecast (QIV)*
Brazil	67.52	99.05	94.00	122.19
Canada	0.51	0.43	0.34	0.41
Switzerland	0.59	0.41	0.32	0.30
Chile	0.97	1.04	0.89	0.93
Colombia	1.33	1.14	0.92	0.65
Czech Republic	2.25	1.66	2.14	1.39
United Kingdom	0.76	0.72	0.68	0.77
Korea	1.26	1.11	0.91	0.78
Mexico	1.63	1.22	2.14	1.79
Norway	0.61	0.51	0.43	0.31
Hungary	1.53	0.98	0.82	0.98
Philippines	1.82	1.45	0.97	1.03
Poland	2.79	1.44	3.52	3.13
Sweden	0.82	0.67	0.71	0.75
Thailand	1.34	1.15	1.16	0.81
South Africa	1.32	1.10	1.02	0.84

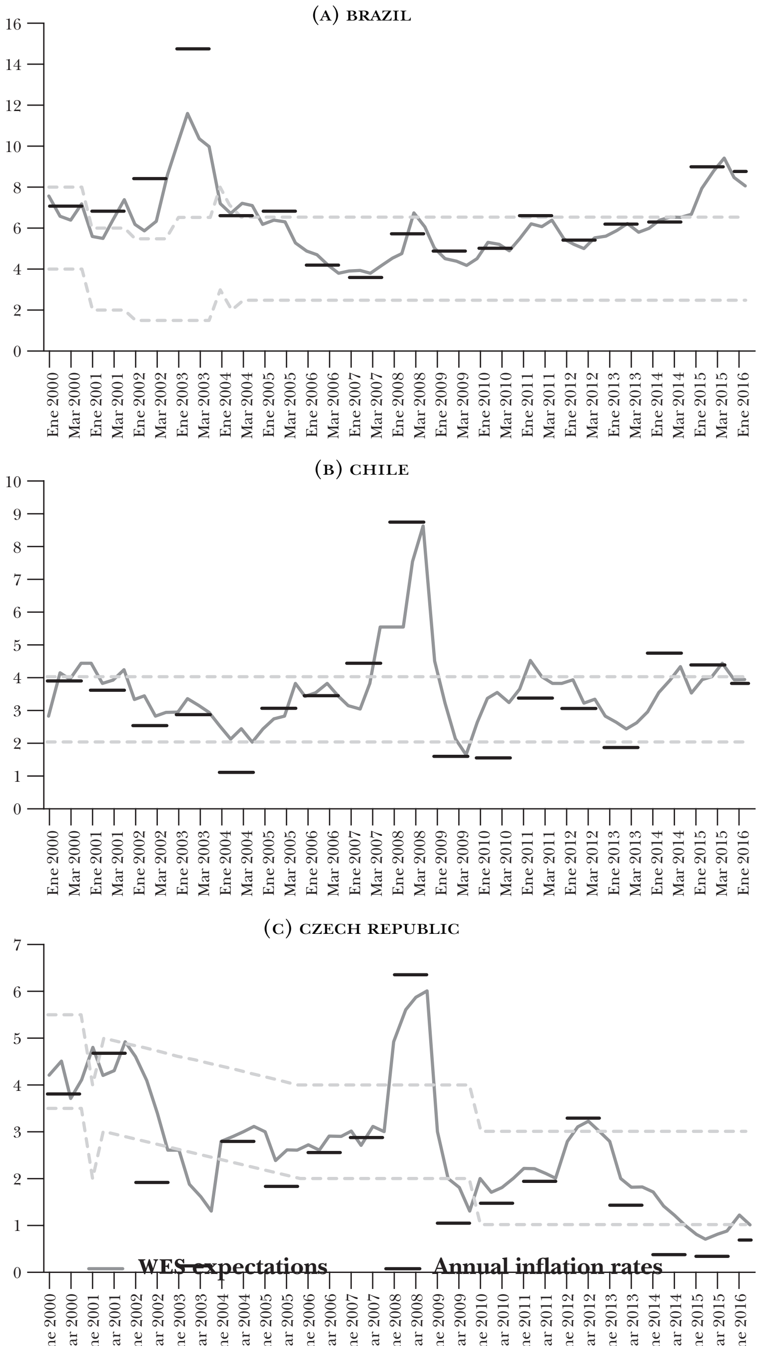
Table 11

U-STATISTIC OF WES SURVEY QUANTITATIVE INFLATION QUESTION

	4-step forecast (QI)	*3-step forecast (QII)*	*2-step forecast (QIII)*	*1-step forecast (QIV)*
Brazil	0.001	0.001	0.001	0.001
Canada	0.138	0.113	0.083	0.115
Switzerland	0.237	0.162	0.126	0.120
Chile	0.022	0.028	0.027	0.033
Colombia	0.010	0.009	0.007	0.005
Czech Republic	0.075	0.074	0.087	0.057
United Kingdom	0.110	0.111	0.112	0.122
Korea	0.075	0.067	0.055	0.054
Mexico	0.022	0.011	0.022	0.019
Norway	0.143	0.118	0.101	0.079
Hungary	0.011	0.007	0.006	0.008
Philippines	0.046	0.039	0.027	0.025
Poland	0.010	0.004	0.032	0.033
Sweden	0.141	0.121	0.151	0.210
Thailand	0.118	0.091	0.081	0.058
South Africa	0.030	0.026	0.024	0.021

Figure 17

COUNTRIES' QUANTITATIVE INFLATION EXPECTATIONS AND ANNUAL INFLATION

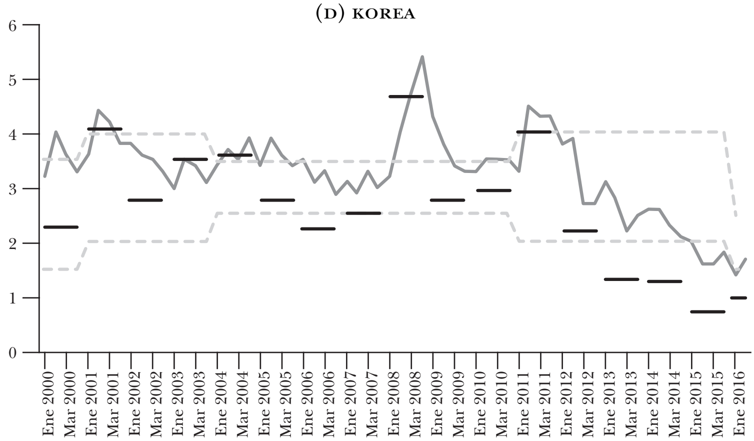


Source: WES survey and OECD statistics and IMF data.

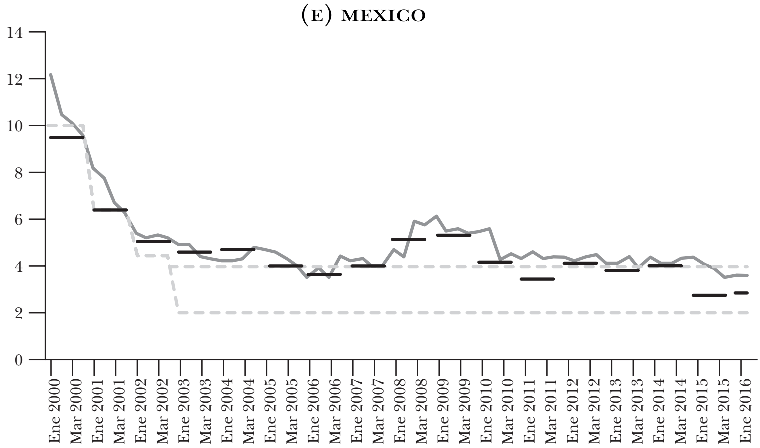
Figure 17 (cont.)

COUNTRIES' QUANTITATIVE INFLATION EXPECTATIONS AND ANNUAL INFLATION

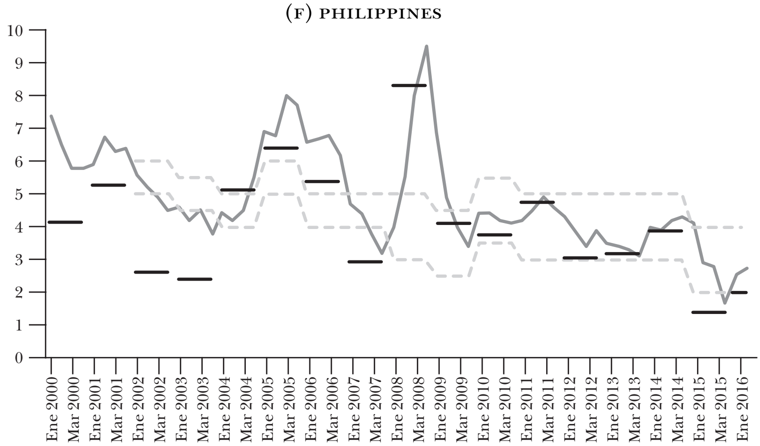
(D) KOREA



(E) MEXICO



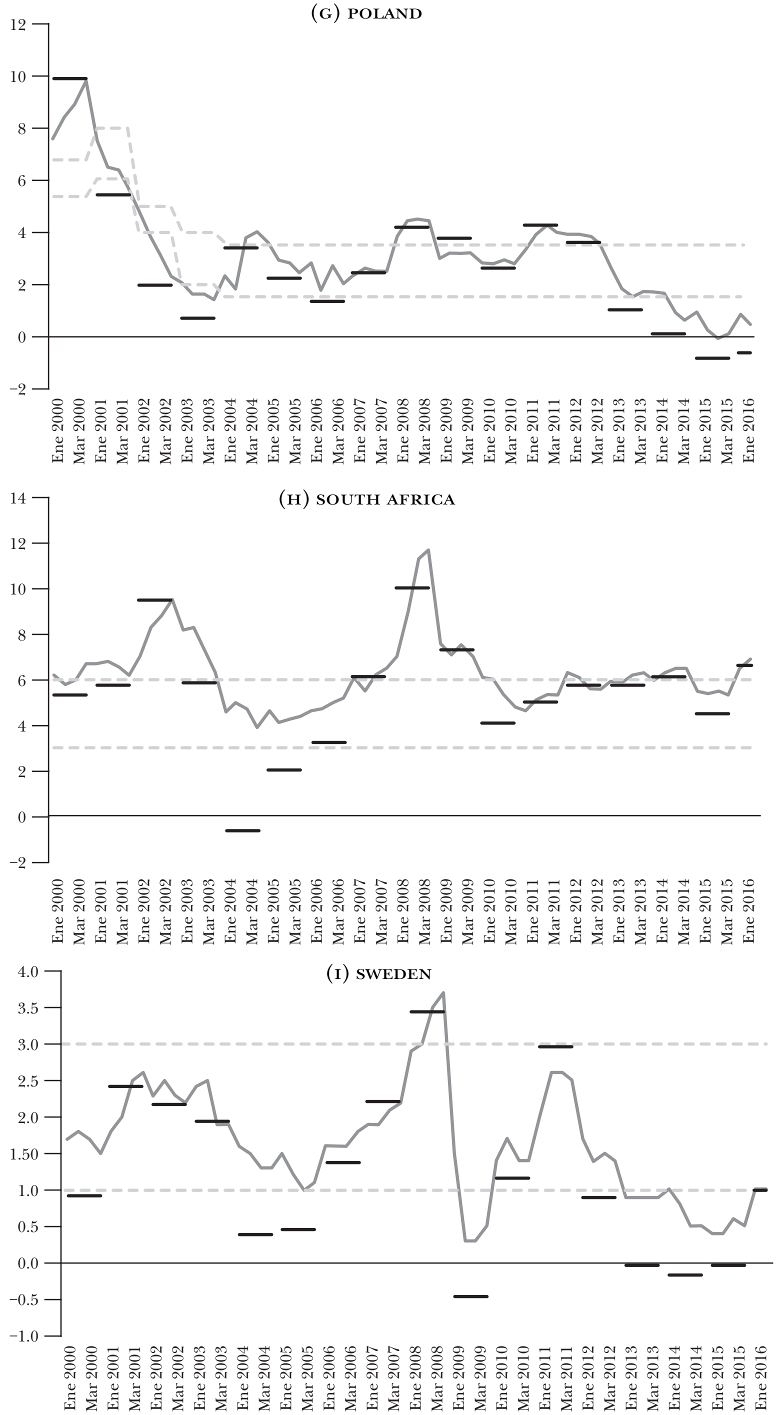
(F) PHILIPPINES



Source: WES survey and OECD statistics and IMF data.

Figure 17 (cont.)

COUNTRIES' QUANTITATIVE INFLATION EXPECTATIONS AND ANNUAL INFLATION

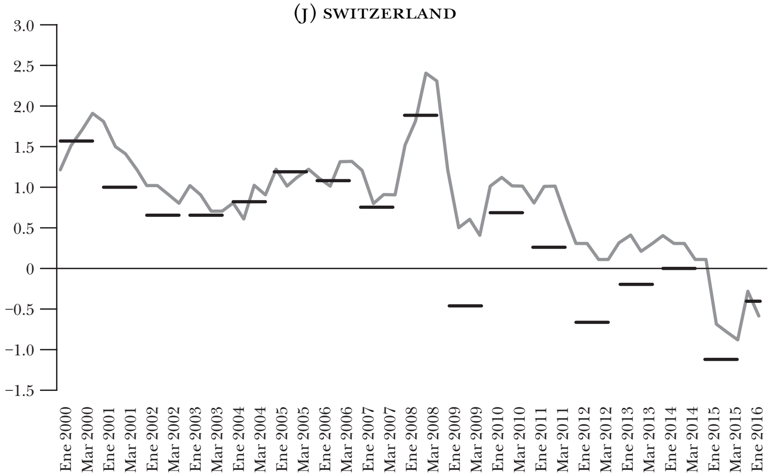


Source: WES survey and OECD statistics and IMF data.

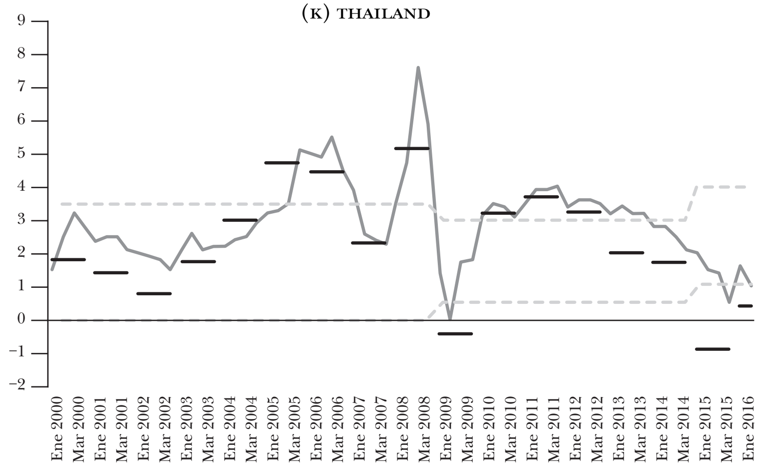
Figure 17 (cont.)

COUNTRIES' QUANTITATIVE INFLATION EXPECTATIONS AND ANNUAL INFLATION

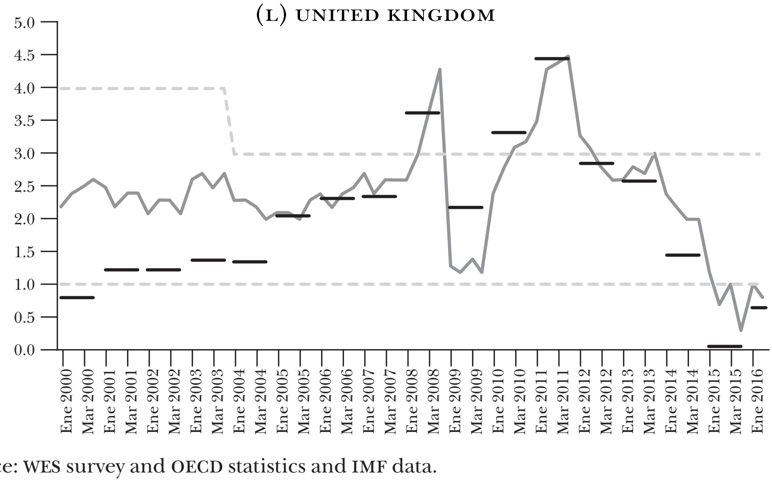
(J) SWITZERLAND



(K) THAILAND



(L) UNITED KINGDOM

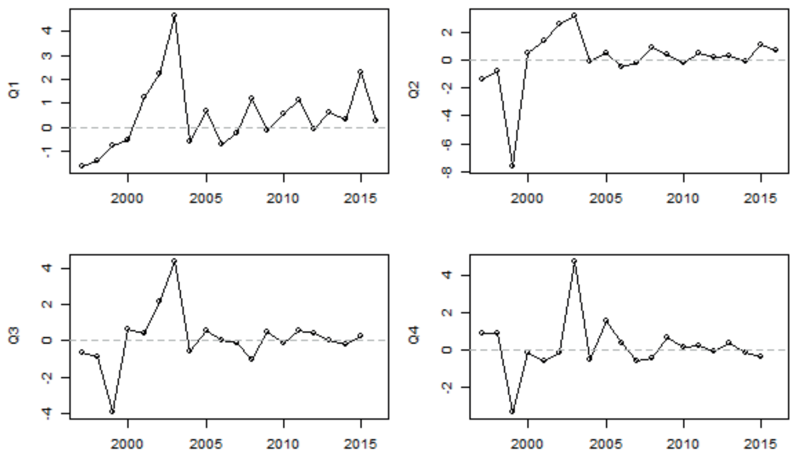


Source: WES survey and OECD statistics and IMF data.

Figure 18

QUARTER-SPECIFIC FORECASTING ERROR BY COUNTRY

A. QUARTER-SPECIFIC FORECASTING ERRORS BRAZIL



B. QUARTER-SPECIFIC FORECASTING ERRORS CANADA

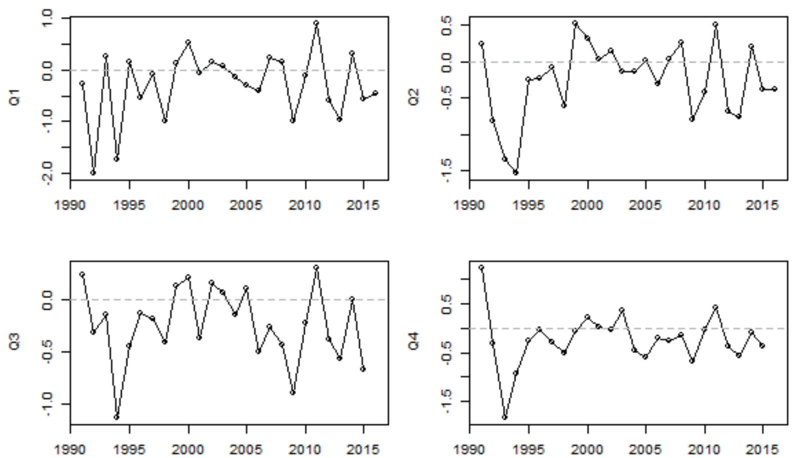
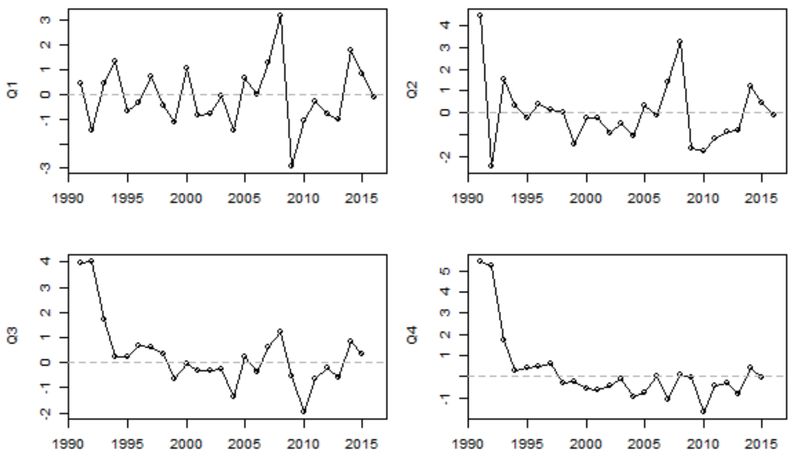


Figure 18 (cont.)

QUARTER-SPECIFIC FORECASTING ERROR BY COUNTRY

C. QUARTER-SPECIFIC FORECASTING ERRORS CHILE



D. QUARTER-SPECIFIC FORECASTING ERRORS COLOMBIA

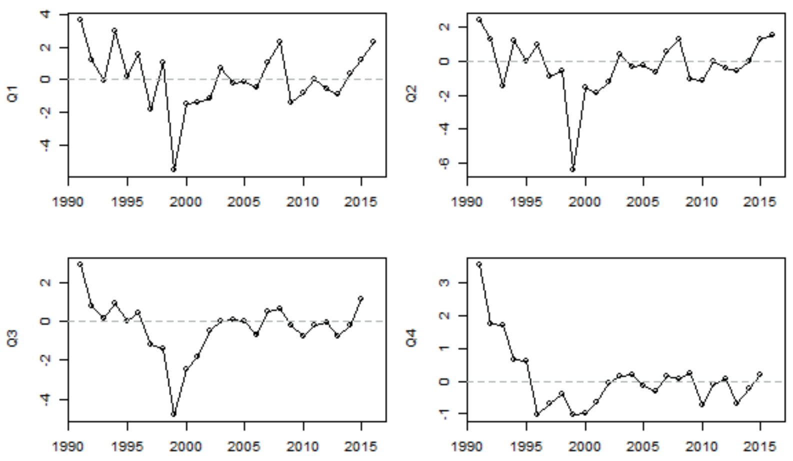
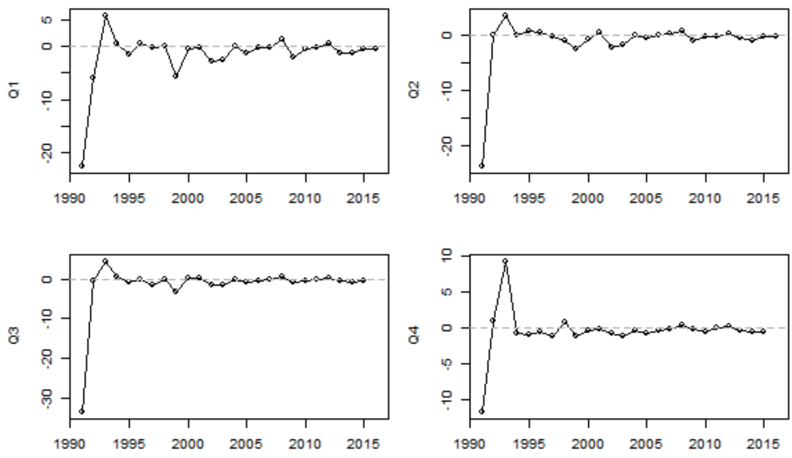


Figure 18 (cont.)

QUARTER-SPECIFIC FORECASTING ERROR BY COUNTRY

E. QUARTER-SPECIFIC FORECASTING ERRORS CZECH REPUBLIC



F. QUARTER-SPECIFIC FORECASTING ERRORS HUNGARY

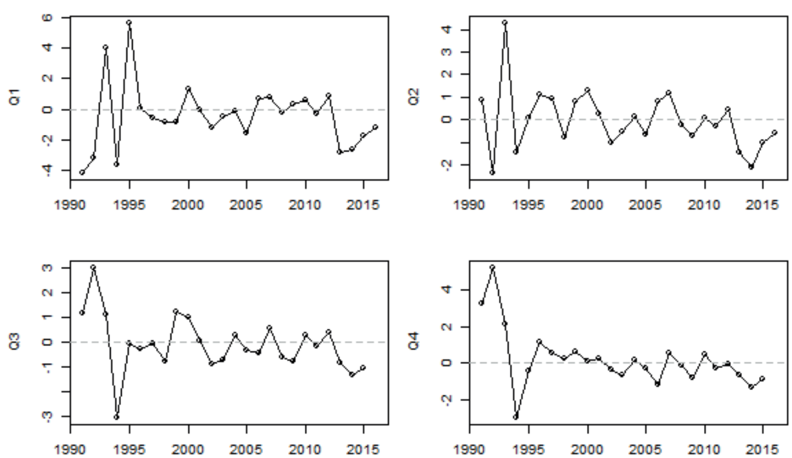
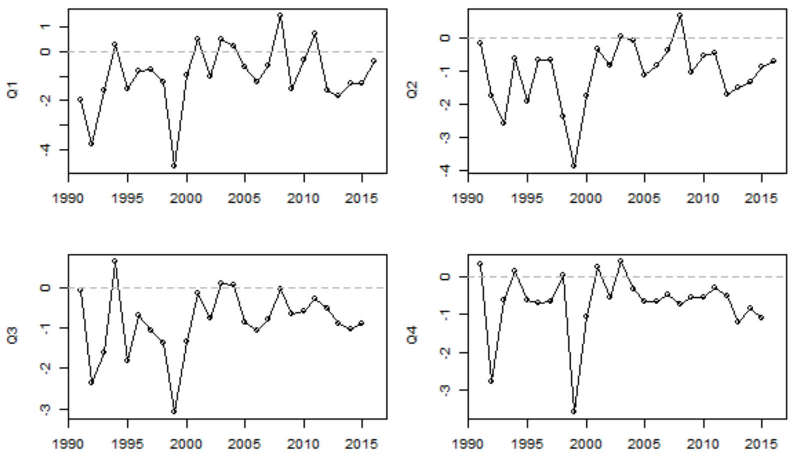


Figure 18 (cont.)

QUARTER-SPECIFIC FORECASTING ERROR BY COUNTRY

G. QUARTER-SPECIFIC FORECASTING ERRORS KOREA



H. QUARTER-SPECIFIC FORECASTING ERRORS MEXICO

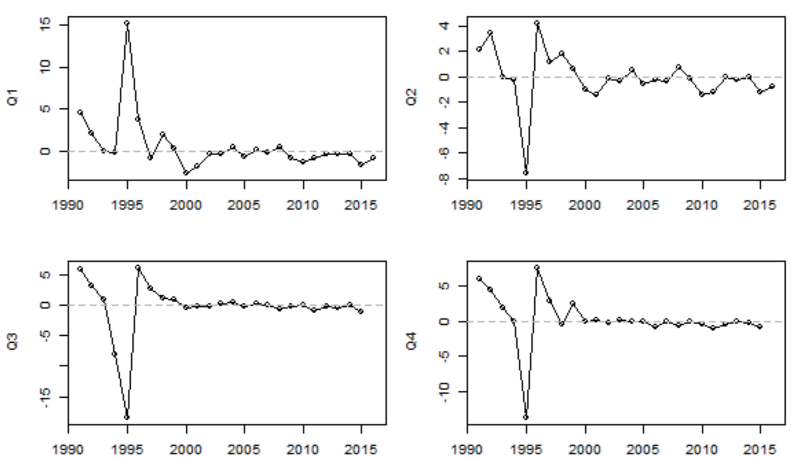
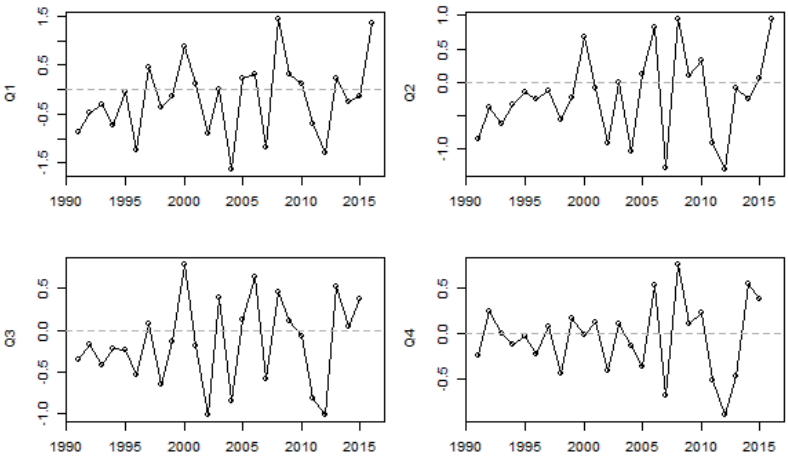


Figure 18 (cont.)

QUARTER-SPECIFIC FORECASTING ERROR BY COUNTRY

I. QUARTER-SPECIFIC FORECASTING ERRORS NORWAY



J. QUARTER-SPECIFIC FORECASTING ERRORS PHILIPPINES

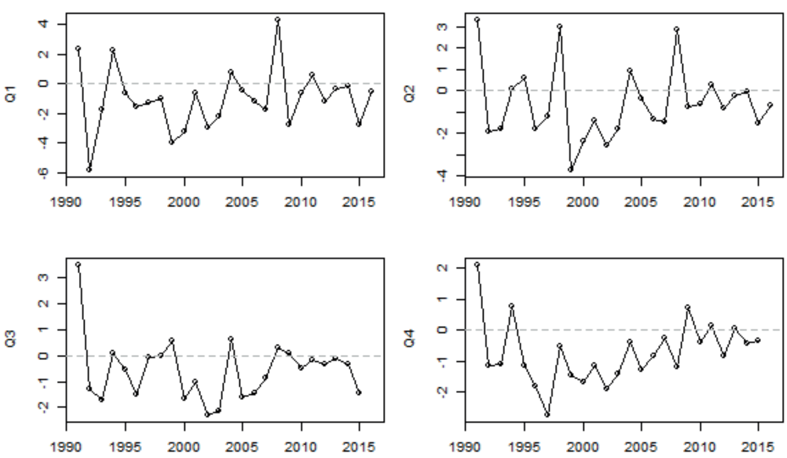
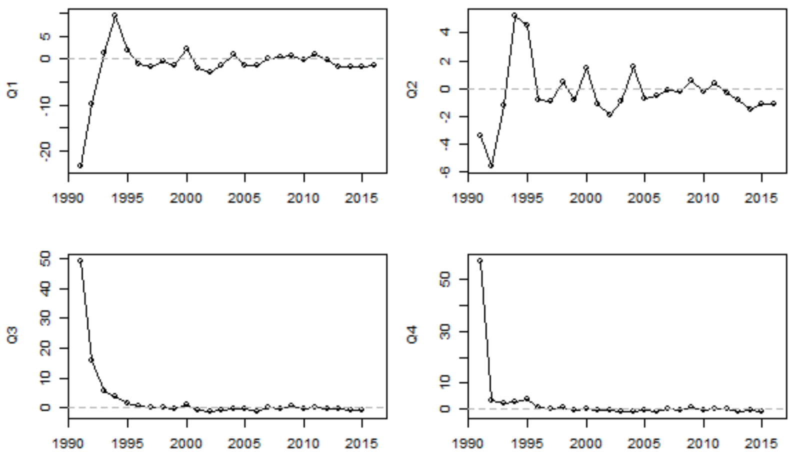


Figure 18 (cont.)

QUARTER-SPECIFIC FORECASTING ERROR BY COUNTRY

K. QUARTER-SPECIFIC FORECASTING ERRORS POLAND



L. QUARTER-SPECIFIC FORECASTING ERRORS SOUTH AFRICA

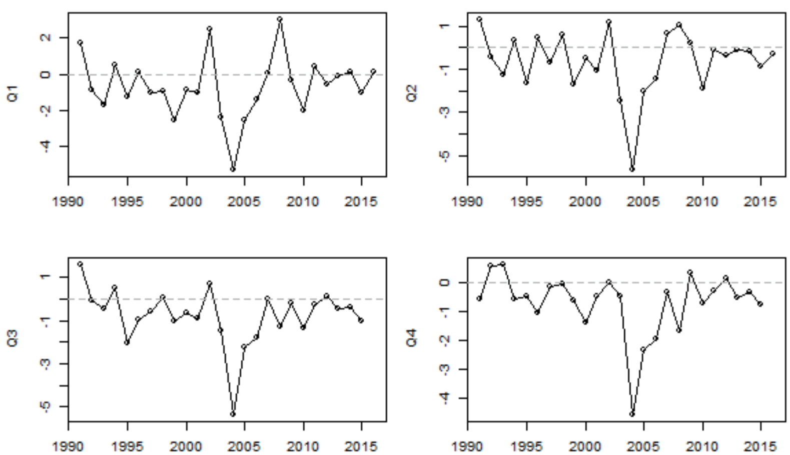
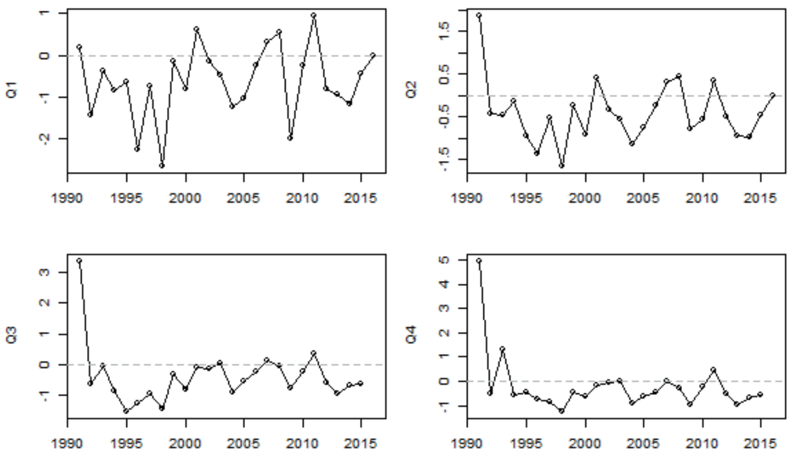


Figure 18 (cont.)

QUARTER-SPECIFIC FORECASTING ERROR BY COUNTRY

M. QUARTER-SPECIFIC FORECASTING ERRORS SWEDEN



N. QUARTER-SPECIFIC FORECASTING ERRORS SWITZERLAND

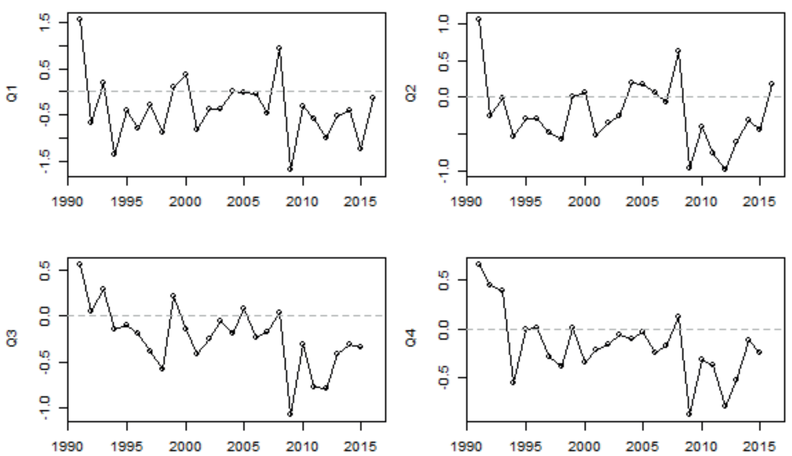
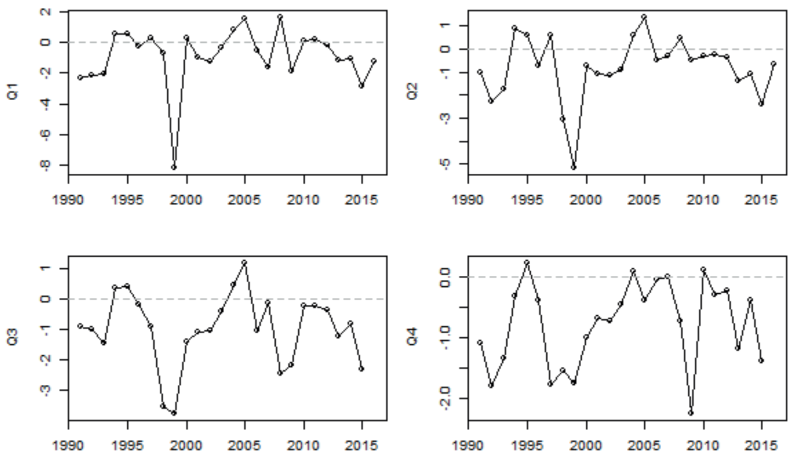


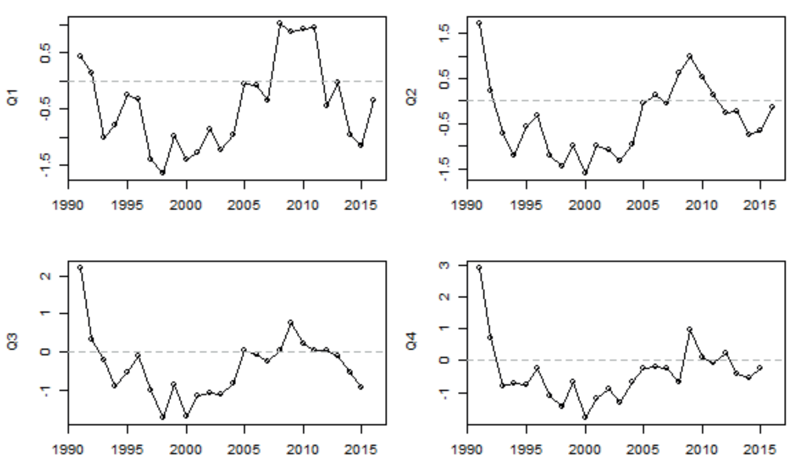
Figure 18 (cont.)

QUARTER-SPECIFIC FORECASTING ERROR BY COUNTRY

O. QUARTER-SPECIFIC FORECASTING ERRORS THAILAND



P. QUARTER-SPECIFIC FORECASTING ERRORS UNITED KINGDOM



ANNEX B. SELF-ORGANIZING MAP VALIDATION

B.1 Choice of Topology

In this section, we present the best topology according to available data. This includes presenting the dimensions of the map and the form of the neighborhood. In order to have more neighbors around the winning neuron, we choose the hexagonal topology that allocates six neurons around the center one. For the dimensions we found several empirical rules. The first rule is to have the number of neurons increase with the square root y of the number of data points. This give us a map of 40 neurons. The second rule is to have 10 samples per neuron, which gives a total of 192 neurons.

We tried different architectures to try to get enough granularity on the map with small topographic error. Unfortunately, there is not a set criterion by which to judge performance in SOM networks. Therefore, to complete our goal of finding the agent's clusters before the oil price shock, we divide our data into two sets, before and after the shock. Thus, the training data will be from the third quarter of 1991 to the second quarter of 2014.

Using the R software, we analyzed various architectures: the dimensions of the map (3x10 vs. 18x10), the storage of their topographic errors, and their granularity.[19] Figure 19 shows us the choice of hexagonal topology of 10x10.

B.2 Post-Training Analysis

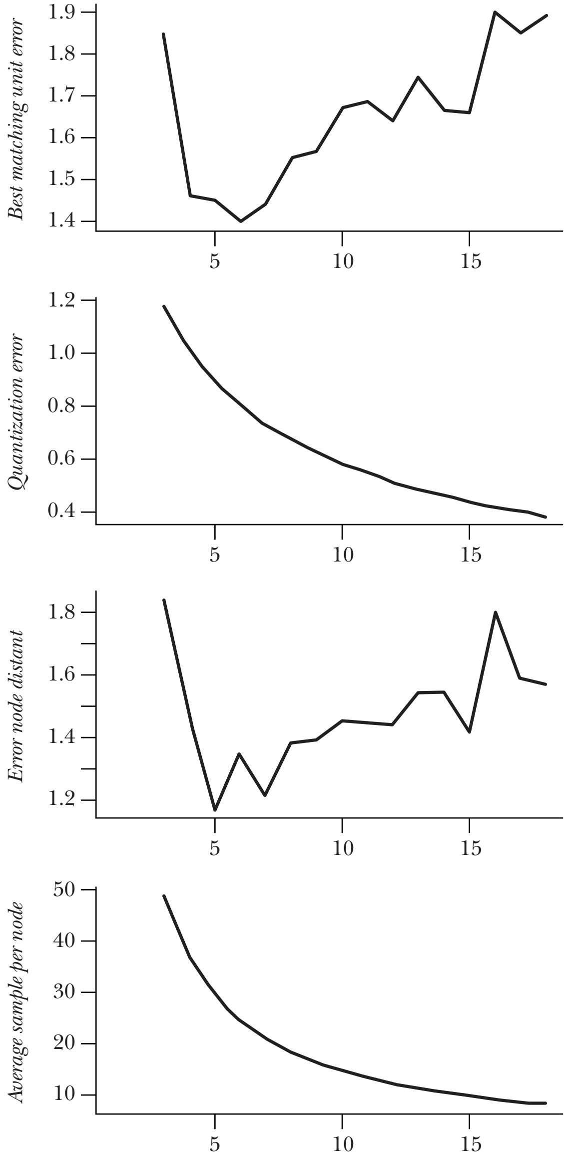
Following Wehrens (2007) and Lynn (2014) we analyze the results from the trained map to validate the previous results. The training progress shows the mean distance between neuron's weights to the samples represented through each iteration. When the training progress reaches a minimum, no more iterations are required. See Figure 20.

---

[19] The quantization error is not comparable between maps because it is susceptible to map size. To see more about topographic errors see the Post-training analysis section.

Figure 19

BEST MATCHING UNIT ERROR, ERROR NODE DISTANCE, QUANTIZATION ERROR, AND SAMPLE PER NEURON VS. MAP WIDTH NODE SIZE

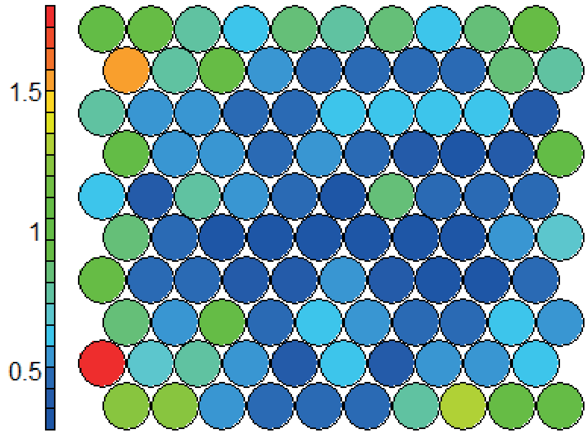


Source: WES survey and OECD statistics and IMF data.

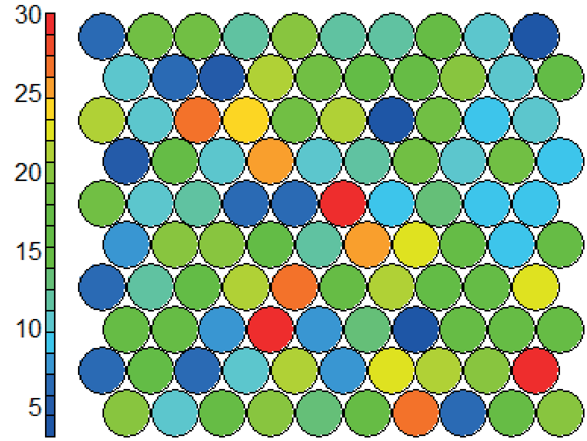
Figure 20

POST-TRAINING ANALYSIS

A. NODE QUALITY/DISTANCE



B. NODE COUNTS



In Figure 20(a), the node or quality distance map is shown. This map displays an approximation of the distance per node to the sample that they are representing; this is known as the quantization error. According to the quantization error, the smaller the distance, the better the map. When it is large, some input vectors are not adequately represented on the map. However, the error is also subject to map sizes: if the map is large, it could be close to zero. This would represent overfitting because the number of neurons on the map should be significantly smaller that the sample size. The mean quantization error found is 0.5888693.

In Figure 20(b), one can analyze how many samples are mapped to each node on the map. Ideally, we want the sample distributions to be relatively uniform. Our map is relatively uniform, including between 10 to 15 samples per neuron, and there are non-empty neurons.

Figure 21(b) shows a map that is also named the U-matrix and which shows the distance between each neuron and its immediate neighbors. Because we choose a hexagonal neighbor, each neuron has six neurons in it neighborhood. This map also assists in identifying similar neurons.

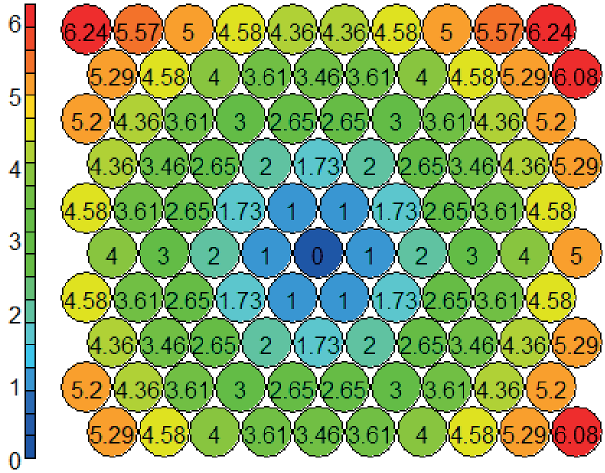
The weight vectors plot, Figure 22, shows the weights associated with each neuron. Each weight vector is similar to the variable that it represents due to Kohonen's learning rule. The weight distributions on the map represent: green for the overall economy, yellow for capital expenditures, orange for private consumption, and white for inflation expectations. This allows us to distinguish patterns of the variables.

Finally, we present three measures of topographic errors. We already looked at the first one, the quantization error, which is the average distance between each variable and the closest neuron. To reiterate our quantization error is 0.5888693. The best-matching error is the average distance between the best matching unit and the following, which is 1.568656. This error is in terms of coordinates in the map. Similarly, the node distance error is the average distance between all pairs of most similar codebook vectors, which is 1.387984.

Figure 21

THE U-MATRIX

A. DISTANT TO UNIT 45



B. NEIGHBOURHOOD DISTANCE

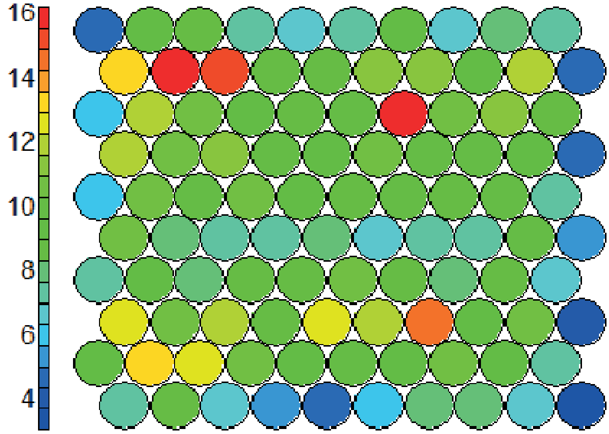
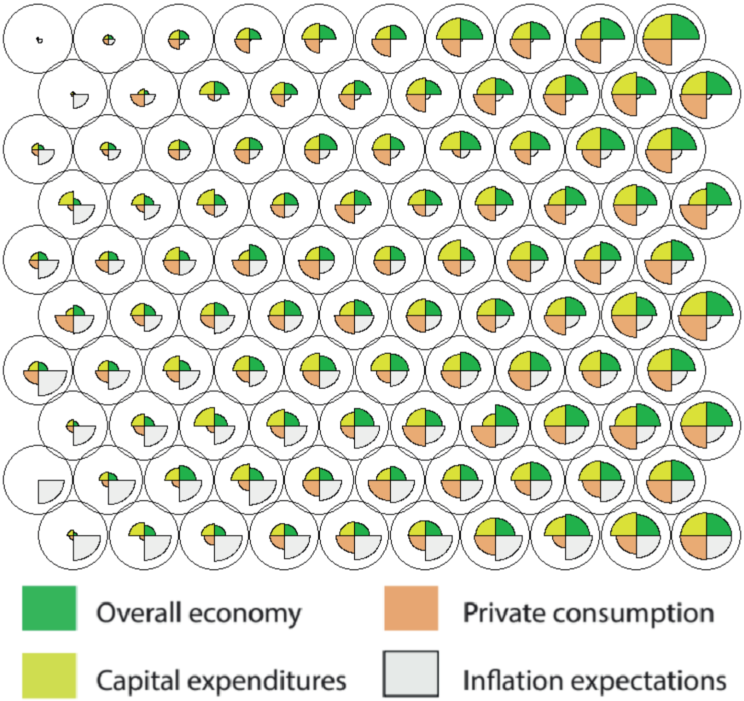


Figure 22

WEIGHT VECTORS



B.3 Non-Linear Auto-Regressive Neural Networks Validation and Other Results

B.3.1 Lag Selection

Table 12

LAG STATISTICS ON ALL DATA, ONE STEP-AHEAD FORECASTS, SAMPLE OF 30

Countries	*Lags*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Brazil	mean	2.01	1.99	1.88	1.90	1.93	**1.86**	1.87	1.88	1.89	1.88
	median	1.99	1.99	1.88	1.90	1.90	**1.86**	1.87	1.88	1.89	1.88
Canada	mean	**1.17**	1.38	1.30	1.29	1.30	1.30	1.31	1.32	1.29	1.30
	median	**1.16**	1.38	1.31	1.29	1.30	1.30	1.31	1.32	1.29	1.30
Switzerland	mean	1.03	1.08	1.04	1.04	1.03	1.03	0.83	**0.33**	0.50	0.65
	median	1.03	1.09	1.04	1.04	1.03	1.03	0.87	**0.31**	0.50	0.83
Chile	mean	2.22	2.09	2.08	2.19	2.10	**2.04**	2.07	2.10	2.04	2.09
	median	2.22	2.09	2.05	2.23	2.17	**1.95**	2.19	2.15	2.04	2.10
Colombia	mean	**1.56**	1.58	1.58	1.60	1.59	1.61	1.57	1.61	1.57	1.60
	median	1.56	1.58	1.58	1.59	1.57	1.59	1.55	1.58	**1.54**	1.57
Czech R.	mean	1.86	2.05	1.99	1.94	1.85	0.89	1.73	1.78	0.39	**0.37**
	median	1.87	2.05	1.99	1.94	1.85	0.89	1.73	1.78	0.32	**0.29**
United K.	mean	1.28	1.40	1.39	1.38	1.26	1.21	1.09	0.99	1.15	**0.94**
	median	1.27	1.41	1.39	1.38	1.26	1.21	1.10	1.00	1.16	**0.85**

Korea R.	mean	1.36	1.47	1.45	1.46	1.45	1.35	1.36	**1.30**	1.31	1.32
	median	1.36	1.47	1.45	1.46	1.45	1.35	1.36	1.30	**1.29**	1.31
Mexico	mean	1.27	1.37	1.33	1.14	1.12	0.91	1.24	1.31	1.02	**0.28**
	median	1.24	1.37	1.32	1.14	1.12	0.90	1.44	1.35	1.18	**0.25**
Norway	mean	1.86	2.08	1.97	1.98	1.92	1.91	1.83	1.81	1.76	**1.75**
	median	1.86	2.08	1.97	1.98	1.92	1.91	1.83	1.81	1.76	**1.75**
Hungary	mean	1.64	1.86	1.78	1.68	1.72	1.63	1.52	**1.51**	1.64	1.60
	median	1.64	1.86	1.78	1.68	1.73	1.63	1.52	**1.51**	1.64	1.59
Philippines	mean	2.66	2.50	2.39	2.41	2.43	2.45	2.26	2.17	**1.51**	1.71
	median	2.65	2.50	2.39	2.41	2.43	2.45	2.26	2.23	1.94	**1.70**
Poland	mean	1.74	1.82	1.25	0.95	1.35	1.30	0.89	0.87	**0.32**	0.63
	median	1.73	1.84	1.26	0.95	1.35	1.30	0.89	0.87	**0.27**	0.63
Sweden	mean	1.25	1.16	1.11	1.13	1.11	1.12	1.07	1.07	1.09	**1.06**
	median	1.24	1.16	1.11	1.13	1.11	1.12	1.07	1.07	1.09	**1.06**
Thailand	mean	1.67	1.87	1.46	1.68	1.35	1.35	**1.27**	1.29	1.32	1.35
	median	1.67	1.87	1.46	1.63	1.35	1.35	**1.27**	1.29	1.32	1.36
South A.	mean	2.25	2.41	2.25	2.25	2.18	2.19	2.19	2.16	**1.74**	1.92
	median	2.25	2.41	2.25	2.25	2.18	2.19	2.21	2.16	**1.69**	1.85

Table 13

LAG STATISTICS ON TRAIN DATA, ONE STEP-AHEAD FORECAST, SAMPLE OF 30

Countries	*Lags*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Brazil	mean	2.04	1.98	1.88	1.90	1.94	1.86	1.87	1.89	1.89	**1.86**
	median	2.03	1.98	1.88	1.90	1.90	1.86	1.87	1.89	1.89	**1.86**
Canada	mean	**1.10**	1.34	1.28	1.26	1.27	1.27	1.28	1.30	1.24	1.26
	median	**1.07**	1.34	1.28	1.26	1.27	1.27	1.28	1.30	1.24	1.26
Switzerland	mean	1.01	1.07	1.05	1.04	1.03	1.03	0.80	**0.29**	0.46	0.64
	median	1.01	1.07	1.05	1.04	1.03	1.03	0.85	**0.27**	0.46	0.83
Chile	mean	2.04	2.03	2.02	2.15	2.04	1.96	1.99	2.00	**1.94**	2.01
	median	2.03	2.03	1.98	2.18	2.11	**1.87**	2.11	2.07	1.94	2.02
Colombia	mean	**1.27**	1.46	1.46	1.48	1.48	1.49	1.45	1.45	1.41	1.44
	median	**1.26**	1.46	1.46	1.48	1.46	1.47	1.43	1.43	1.38	1.41
Czech R.	mean	1.96	2.17	2.11	2.05	1.94	0.91	1.78	1.83	**0.22**	0.26
	median	1.97	2.17	2.10	2.05	1.94	0.91	1.78	1.83	**0.15**	0.19
United K.	mean	1.31	1.45	1.43	1.42	1.29	1.25	1.12	0.98	1.18	**0.95**
	median	1.31	1.46	1.43	1.42	1.29	1.25	1.13	1.01	1.18	**0.85**
Korea R.	mean	1.28	1.44	1.40	1.41	1.39	1.27	1.26	**1.23**	1.23	1.24
	median	1.28	1.44	1.40	1.41	1.39	1.27	1.26	1.23	**1.22**	1.23

Mexico	mean	1.34	1.45	1.40	1.20	1.18	0.97	1.31	1.40	1.07	**0.20**
	median	1.32	1.45	1.39	1.20	1.18	0.95	1.53	1.44	1.27	**0.22**
Norway	mean	1.91	2.14	2.00	2.01	1.95	1.90	1.81	1.79	**1.75**	1.75
	median	1.90	2.14	2.00	2.01	1.95	1.90	1.81	1.79	**1.75**	1.75
Hungary	mean	1.48	1.76	1.67	1.52	1.53	1.45	**1.31**	1.33	1.52	1.48
	median	1.48	1.77	1.67	1.52	1.55	1.45	**1.31**	1.33	1.52	1.48
Philippines	mean	2.59	2.37	2.26	2.29	2.31	2.32	2.15	2.03	**1.26**	1.52
	median	2.58	2.37	2.26	2.29	2.31	2.32	2.15	2.10	1.74	**1.53**
Poland	mean	1.79	1.90	1.27	0.94	1.34	1.32	0.90	0.87	0.01	**0.00**
	median	1.78	1.91	1.29	0.94	1.34	1.32	0.90	0.87	0.01	**0.00**
Sweden	mean	1.27	1.12	1.08	1.09	1.05	1.06	1.00	1.01	1.03	**1.00**
	median	1.26	1.13	1.08	1.09	1.05	1.06	1.00	1.01	1.03	**1.00**
Thailand	mean	1.67	1.95	1.50	1.75	1.39	1.38	**1.29**	1.31	1.35	1.38
	median	1.67	1.95	1.50	1.69	1.39	1.38	**1.29**	1.31	1.35	1.40
South A.	mean	2.22	2.33	2.13	2.13	2.02	2.03	1.99	1.94	**1.46**	1.66
	median	2.21	2.33	2.13	2.13	2.02	2.03	2.03	1.94	**1.41**	1.54

B.3.2 Post-Training Analysis

Table 14

NEURAL NETWORKS RESULTS OF TRAINING PHASE

Countries	*Total number of*	*Effective number of parameters*	*Maximum sum squared of parameters*	*Sum squared of parameters*	*Total epoch*
Brazil	31	2.88	2760	1.74	355
Canada	101	7.66	53	1.01	622
Chile	61	4.68	91.3	1.11	228
Colombia	71	5.02	72	0.72	1000
Czech Republic	81	31.17	61.6	20.96	314
Korea	41	2.99	280	1.10	1000
Mexico	81	20.71	61.7	9.91	114
Norway	31	2.96	2760	1.49	70
Switzerland	101	38.81	53.4	22.16	330
United Kingdom	81	10.20	64.7	3.39	245
Hungary	121	14.04	46	2.9722	889
Philippines	31	2.04	2760	1.30	108
Poland	91	19.48	58.2	7.43	156
Sweden	31	2.75	2760	1.53	484
Thailand	61	9.16	91.3	4.09	298
South A.	31	2.64	2760	1.54	502

Countries	Best epoch	Error Autocorrelation	Input-error Correlation	*Correlation coefficient* Training R	Testing R	All R
Brazil	2	1	0	0.605	0.877	0.632
Canada	99	1	0	0.570	0.334	0.551
Chile	56	1	0	0.702	-0.049	0.678
Colombia	429	1	0	0.445	0.560	0.463
Czech Republic	253	1	0	0.885	0.607	0.884
Korea	1000	1	0	0.523	-0.464	0.554
Mexico	64	1	0	0.875	0.474	0.879
Norway	4	1	0	0.641	-0.041	0.640
Switzerland	240	1	0	0.935	0.759	0.921
United Kingdom	77	1	0	0.740	0.473	0.743
Hungary	103	1	0	0.820	-0.157	0.826
Philippines	12	0	0	0.678	0.077	0.652
Poland	129	1	0	0.887	0.605	0.895
Sweden	9	1	0	0.741	0.108	0.746
Thailand	151	1	0	0.674	0.181	0.664
South A.	8	0	0	0.744	0.545	0.739

B.3.3 MSE Evaluation

Table 15

NEURAL NETWORK SIMULATIONS STATISTICS BY DATASETS, SAMPLE OF 1,000

	Brazil			*Korea*		
	All data	*Training set*	*Testing set*	*All data*	*Training set*	*Testing set*
mean	2.01	2.05	1.65	1.47	1.44	1.86
median	2.00	2.04	1.61	1.47	1.44	1.86
std	0.04	0.03	0.18	0.01	0.01	0.03
maximum	2.09	2.09	2.11	1.52	1.44	2.51
minimum	1.92	1.97	1.24	1.36	1.34	1.62
	Canada			*Mexico*		
	All data	*Training set*	*Testing set*	*All data*	*Training set*	*Testing set*
mean	1.33	1.31	1.52	0.97	1.03	0.34
median	1.32	1.30	1.52	0.90	0.95	0.30
std	0.06	0.06	0.01	0.24	0.24	0.17
maximum	2.09	2.09	2.11	3.91	4.00	2.94
minimum	1.92	1.97	1.24	0.90	0.95	0.30
	Chile			*Norway*		
	All data	*Training set*	*Testing set*	*All data*	*Training set*	*Testing set*
mean	2.21	2.17	2.69	1.87	1.91	1.41
median	2.23	2.18	2.68	1.86	1.90	1.42
std	0.06	0.07	0.03	0.04	0.04	0.03
maximum	1.33	1.36	1.01	2.06	2.12	1.51
minimum	0.55	0.54	0.67	1.82	1.86	1.25
	Colombia			*Switzerland*		
	All data	*Training set*	*Testing set*	*All data*	*Training set*	*Testing set*
mean	1.59	1.48	2.83	0.31	0.27	0.78
median	1.57	1.46	2.78	0.31	0.27	0.78
std	0.08	0.07	0.21	0.00	0.00	0.00
maximum	1.93	1.76	3.69	0.31	0.27	0.78
minimum	1.57	1.46	2.77	0.31	0.27	0.78

	Czech Republic			United Kingdom		
	All data	Training set	Testing set	All data	Training set	Testing set
mean	0.90	0.92	0.69	1.21	1.25	0.82
median	0.89	0.91	0.67	1.21	1.25	0.82
std	0.08	0.08	0.08	0.00	0.00	0.00
maximun	1.21	1.25	0.82	1.21	1.25	0.82
minimum	1.21	1.25	0.82	1.21	1.25	0.82

	Hungary			Philippines		
	All data	Training set	Testing set	All data	Training set	Testing set
mean	1.59	1.47	2.85	2.66	2.60	3.42
median	1.59	1.48	2.75	2.66	2.59	3.43
std	0.11	0.17	0.54	0.04	0.05	0.07
maximun	1.73	1.58	7.60	2.81	2.74	3.88
minimum	0.68	0.00	2.74	2.59	2.51	2.98

	Poland			Sweden		
	All data	Training set	Testing set	All data	Training set	Testing set
mean	0.89	0.90	0.72	1.25	1.26	1.14
median	0.89	0.90	0.72	1.24	1.25	1.15
std	0.01	0.01	0.01	0.03	0.04	0.09
maximun	1.02	1.04	0.80	1.35	1.37	1.31
minimum	0.88	0.90	0.66	1.21	1.21	0.86

	Thailand			South Africa		
	All data	Training set	Testing set	All data	Training set	Testing set
mean	1.69	1.76	0.90	2.27	2.23	2.64
median	1.63	1.69	0.91	2.25	2.22	2.63
std	0.09	0.10	0.02	0.07	0.07	0.10
maximum	1.82	1.91	0.91	2.64	2.58	3.35
minimum	1.63	1.69	0.86	2.22	2.18	2.43

B.3.4 Results, Other Countries

Figure 23

COUNTRIES' INFLATION RATE FORECAST FOR THE NEXT SIX MONTHS BY NAR-NN

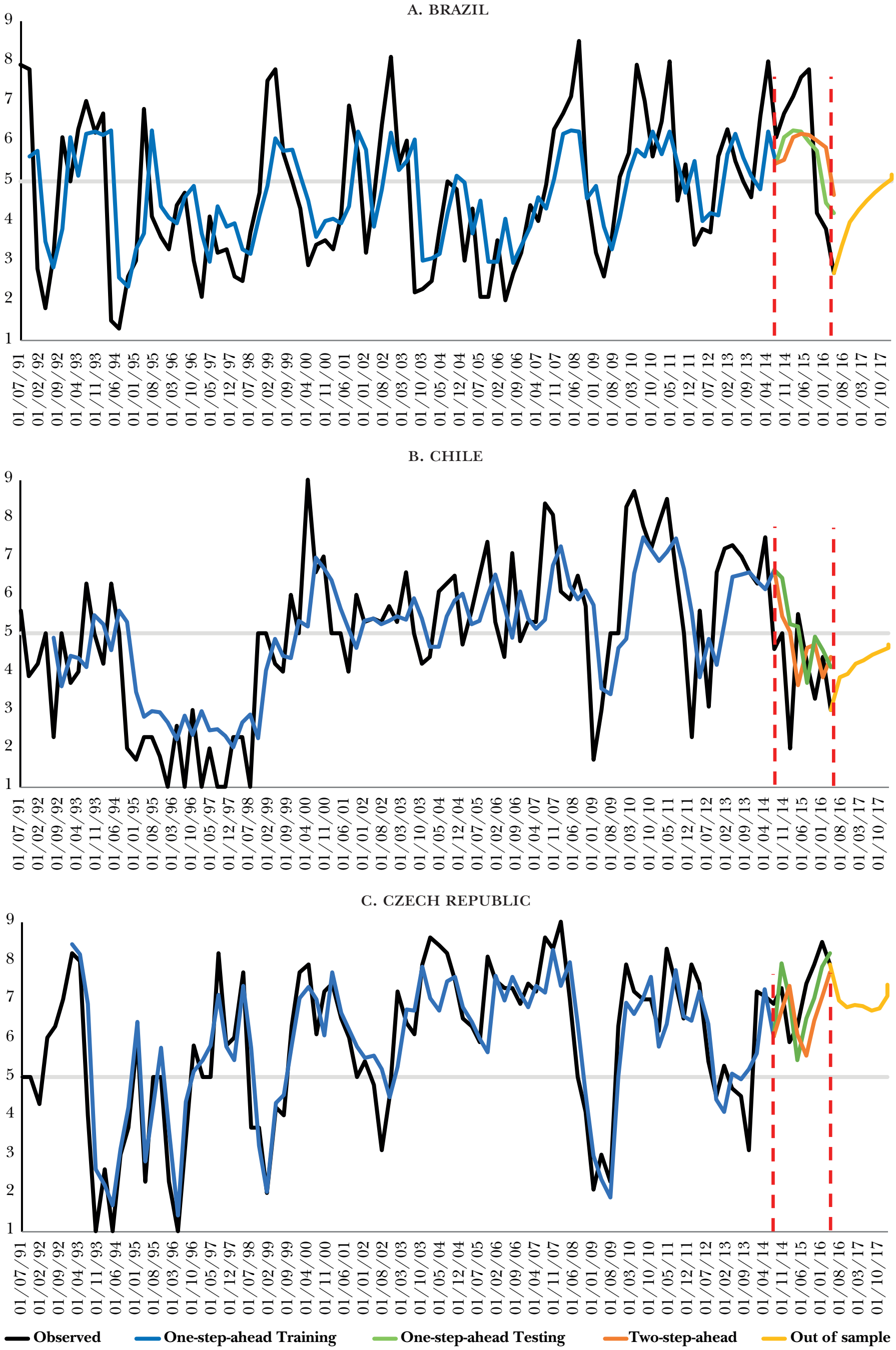


Figure 23 (cont.)

COUNTRIES' INFLATION RATE FORECAST FOR THE NEXT SIX MONTHS BY NAR-NN

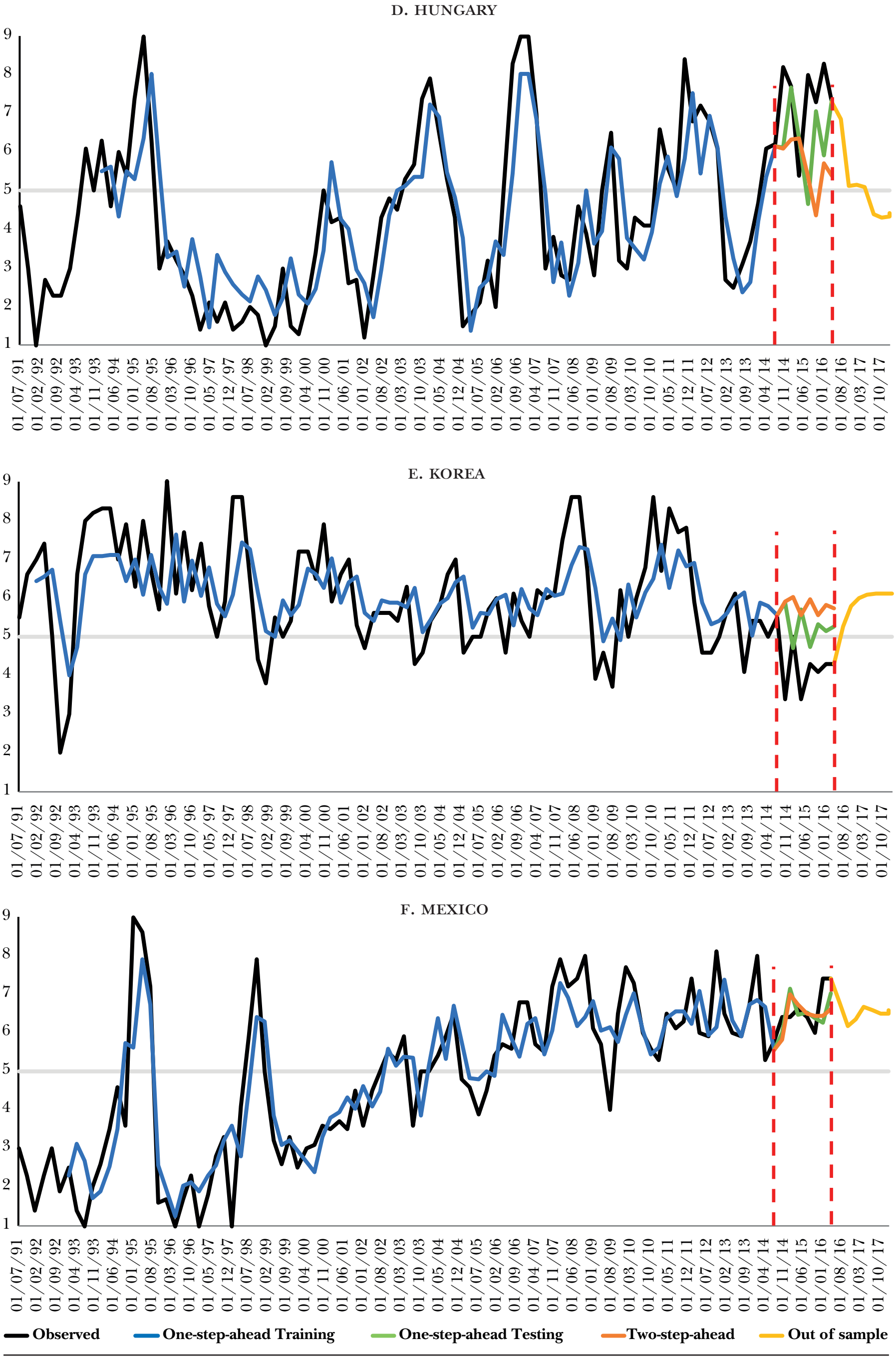
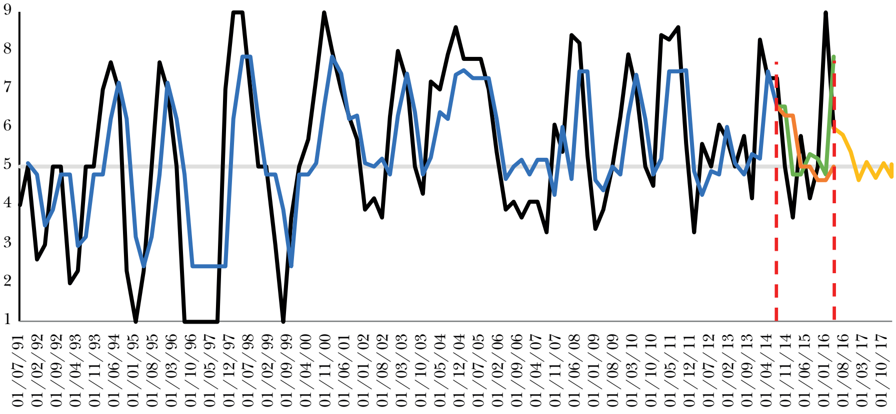


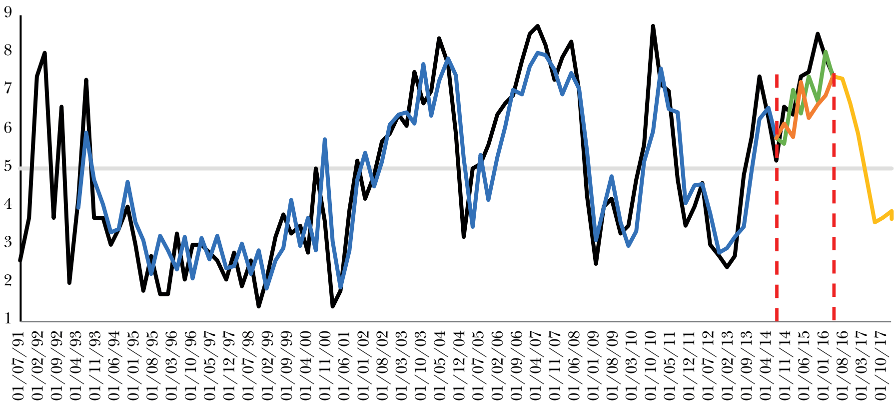
Figure 23 (cont.)

COUNTRIES' INFLATION RATE FORECAST FOR THE NEXT SIX MONTHS BY NAR-NN

G. PHILIPPINES



H. POLAND



I. SOUTH AFRICA

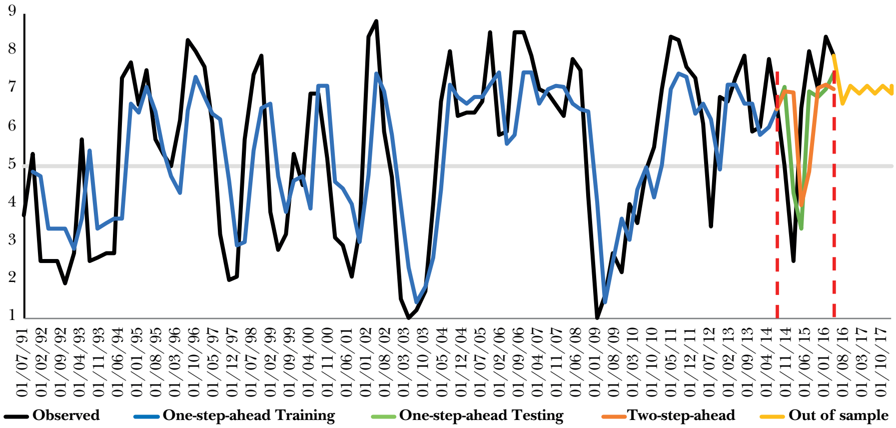
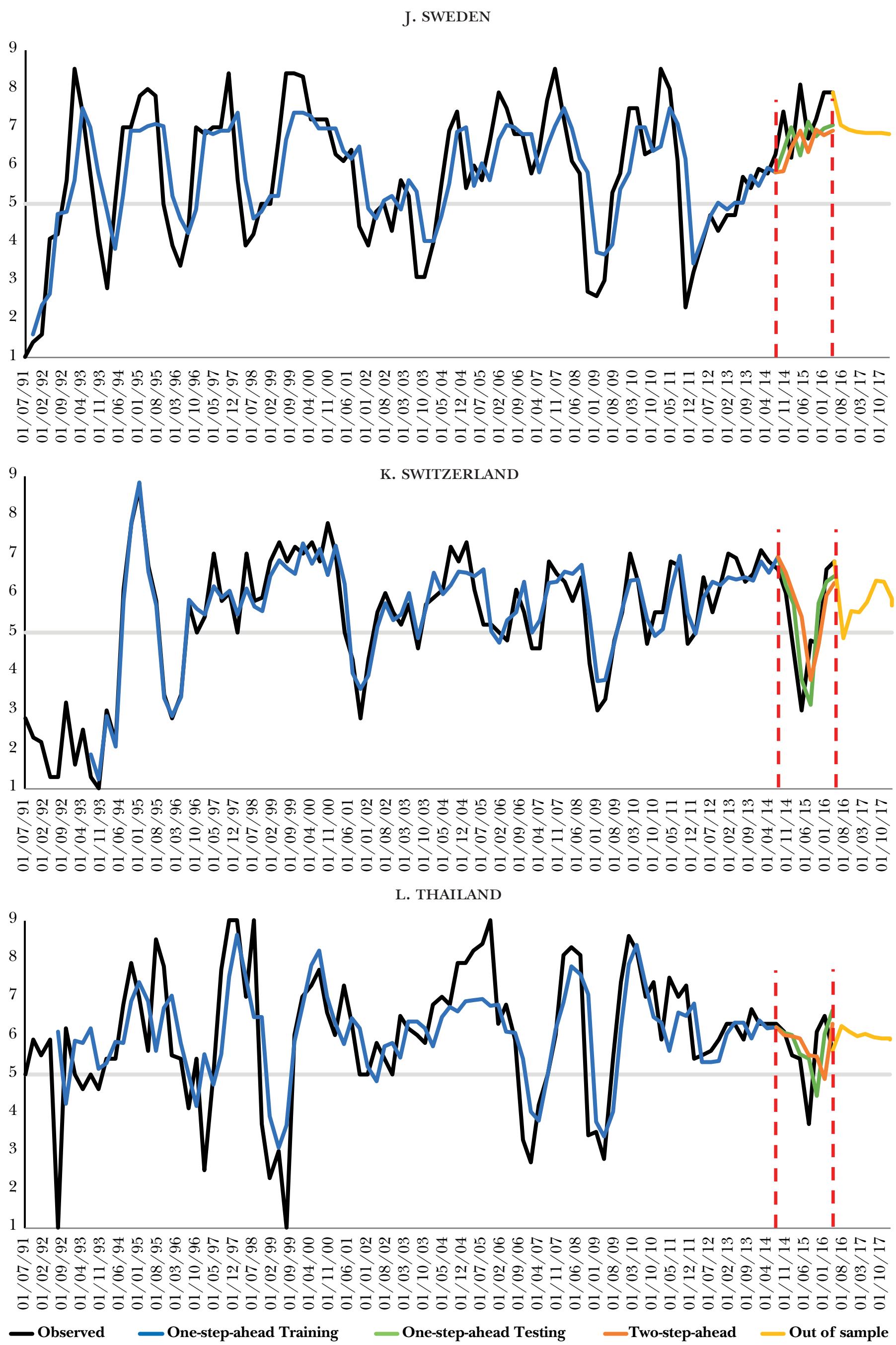


Figure 23 (cont.)

COUNTRIES' INFLATION RATE FORECAST FOR THE NEXT SIX MONTHS BY NAR-NN



B.4 ARIMA

In the ARIMA modeling, various tests were performed before modeling the series in order to understand the generating data process and find the best (p,d,q)(P,D,Q) order suit to the series. We began to perform the Augmented Dickey-Fuller (ADF) test (see Dickey and Fuller, 1981) and the Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test (see Kwiatkowski *et al.*, 1992 to find the differentiation order (Table 16). In the Dickey-Fuller test, we started including the trend and constant over the regression for which all the series rejected the null hypothesis of the unit root. For the KPSS test, like the ADF test, we included the trend and constant terms and almost all the series did not reject the null hypothesis of stationary except for Switzerland and Norway, where the Switzerland series became stationary after the first 8 observations were excluded from the tests. To find the seasonal difference order, the Canova-Hansen test (see Canova and Hansen, 1995) was implemented, which has a null hypothesis of no unit roots at seasonal frequencies. This test complements the HEGGY test of seasonal unit roots.

Once the difference orders were determined and the respective transformations were applied, such as applying logarithms if necessary, we proceed to explore the autocorrelation function, partial autocorrelation, extended autocorrelation function, and information criterion AIC and BIC. We used these factors to find the autoregressive and moving average coefficients. A group of possible models were tested on each country, for which the most suitable model had to accomplish five conditions:

- Low BIC, AICc, and RMSE
- coefficients statistically different to zero.
- the residuals should be uncorrelated through time.
- the cross-correlation function between the predicted errors and the observed time series should be close to zero.
- The high order closest model should fail in comparison.

Then, after we found the best ARIMA model possible, we forecast one step ahead and two step ahead on the testing set and calculate the respective MSE to compare with the NAR-NN Model.

Table 16

UNIT ROOT, STATIONARITY TESTS AND MODEL IDENTIFICATION

	ADF t-Stat	*KPSS Stat*	*(p,d,q)(P,D,Q) order*
Brazil	5.871	0.089	(1,0,0)
Canada	5.357	0.040	(1,0,0)
Switzerland	4.085	0.188	(2,0,1)
Chile	3.377	0.143	(1,1,1)
Colombia*	4.892	0.059	(1,0,0)
Czech Republic*	4.431	0.086	(1,1,1)
United Kingdom	5.294	0.069	(1,0,0)(1,0,0)
Korea	4.997	0.065	(1,0,1)
Mexico	5.179	0.056	(1,1,1)
Norway	4.846	0.150	(1,1,1)
Hungary*	4.022	0.089	(1,0,0)
Philippines*	6.370	0.077	(1,0,0)
Poland*	3.537	0.122	(0,1,2)
Sweden	5.545	0.065	(2,0,0)
Thailand*	4.928	0.045	(1,0,0)
South Africa	5.515	0.044	(1,0,0)(1,0,0)
Test critical values:			
1% level	-4.04	0.216	
5% level	-3.45	0.146	
10% level	-3.15	0.119	

*Log transformation

References

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Inflation Expectations and Its Relation with Economic Policy

Did the Introduction of Inflation Targeting Represent a Regime Switch of Monetary Policy in Latin America?

Sebastián Cadavid Sánchez
Alberto Ortiz Bolaños

Abstract

In the 1990s, after experiencing high levels of inflation, several countries in Latin America passed constitutional amendments providing greater autonomy to their central banks. A few years later, many central banks increased their exchange rate flexibility and later adopted inflation targeting frameworks. These institutional changes coincided with sharp reductions in inflation and its variability. In this paper, we ask if the observed reduction of inflation is possibly related to changes in monetary policy. To answer this question, we build and estimate a Markov-Switching DSGE model for an open economy with monetary factors for Brazil, Chile, Colombia, Mexico, and Peru, all of whom formally adopted inflation targeting regimes between 1999 and 2002. Regimes are classified according to their relative weights of inflation in an interest rate reaction function. Although ex-ante these regimes need not be associated with the introduction of the inflation targeting framework, the coincidence of a regime switch with a more responsive interest rate - inflation relationship is striking. Furthermore, the Markov-Switching DSGE model allows us to generate counterfactuals of what could have happened if the observed change towards a more aggressive fight against inflation had not taken place. In general, we observe that if monetary policy had remained dovish, these countries would have experienced higher and more variable levels of inflation and more pronounced variations in GDP with small gains in average economic growth. Therefore, we conclude that

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the introduction of inflation targeting represented a favorable regime switch in the implementation of monetary policy in Latin America.

Keywords: Monetary policy, inflation, Markov-switching DSGE, Bayesian Maximum Likelihood methods.

JEL: E31, E37, E52, E58, C11.

1. INTRODUCTION

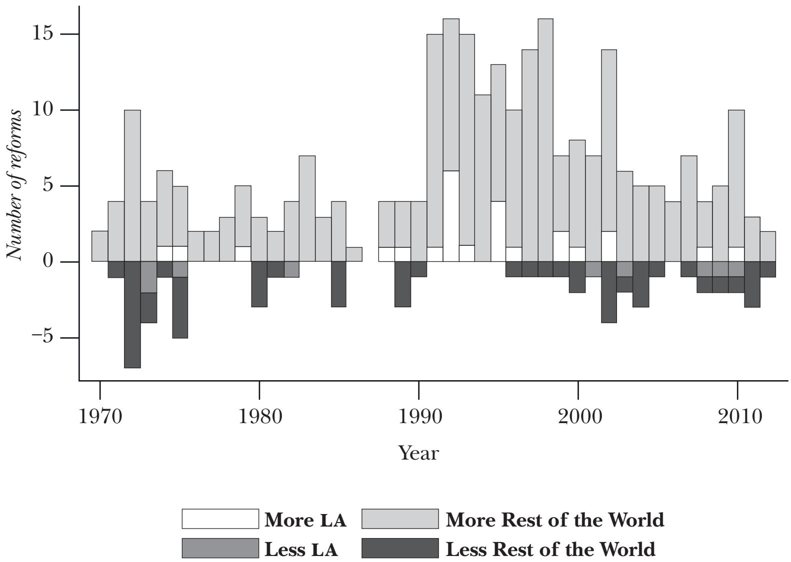
Beginning in the late 1980s, many countries around the world enacted new central banking legislation to grant more autonomy to their monetary authorities. For example, see Figure 1, which uses a sample of indexes of central bank independence from 182 countries since 1970, produced by Garriga (2016). Figure 1 shows a sharp increase in the number of reforms toward increased central bank independence in the 1990s. This shift came in response to the traumatic inflationary and hyper-inflationary episodes experienced in the previous decades, and it was reinforced by evidence showing that "central bank independence promotes price stability" without "measurable impact on real economic performance" (e.g., Alesina and Summers (1993)).

In Latin America, starting with Venezuela in 1974, several countries had reforms to strengthen the independence of their central banks[1]. In some countries, and for different reasons (from depletion of reserves to the desire to gain greater control of monetary policy), many central banks increased their exchange rate flexibility. The process continued with the adoption of inflation targeting frameworks to direct monetary policy. These institutional changes coincided with

[1] According to Garriga (2016), since 1970, countries that took positive reforms towards independence were the following: Venezuela in 1974; Chile in 1975; Haiti in 1979; Mexico in 1985; Brazil in 1988; Chile in 1989; El Salvador in 1991; Argentina, Colombia, Ecuador, Nicaragua, Peru, and Venezuela in 1992; Mexico in 1993; Bolivia, Costa Rica, Paraguay, and Uruguay in 1995; Honduras in 1996; Cuba in 1997; Nicaragua and Venezuela in 1999; El Salvador in 2000; Guatemala and the Dominican Republic in 2002; and Uruguay in 2008 and 2010. Meanwhile, negative reforms hindering Central Bank independence include the following: Argentina and El Salvador in 1973, Panama in 1975, El Salvador in 1982, Uruguay in 1997, Venezuela in 2001, Argentina in 2003, Ecuador in 2008, Venezuela in 2009, Nicaragua in 2010, and Argentina in 2012.

Figure 1

REFORMS TO THE INDEPENDENCE OF CENTRAL BANKS AROUND THE WORLD



sharp reductions of inflation and its variability. Table 1 summarizes the average inflation for each decade together with the years when positive reforms toward central bank independence were enacted, greater exchange rate flexibility was pursued, and inflation targeting was introduced. The selected countries for this analysis are Brazil, Chile, Colombia, Mexico, and Peru, which were early adopters of inflation targeting in Latin America between 1999 and 2002.

Although common sense provides a reason to believe that there could be a relation between institutional changes and inflation reduction, to the best of our knowledge, there is no quantitative evidence measuring if and how these changes determined inflation. In this paper, we provide this evidence by analyzing a Markov-Switching Dynamic Stochastic General Equilibrium (MS-DSGE) model for an open economy with monetary factors estimated for Brazil, Chile, Colombia, Mexico, and Peru. Regimes are classified according to their

Table 1

INFLATION AND CENTRAL BANKS CHANGES IN SELECTED COUNTRIES OF LATIN AMERICA

Average inflation	*1980-1989*	*1990-1999*	*2000-2009*	*2010-2015*	*Positive reforms towards independence*	*Exchange rate flexibility*	*Year of Inflation Targeting introduction*
Brazil	121.7	147.1	6.6	6.2	1988	1999	1999
Chile	19.9	11.8	3.5	3	1975 and 1989	1999	1999
Colombia	20.8	19.9	6.1	3.1	1992	1999	1999
Mexico	53.1	18.3	5.1	3.6	1985 and 1993	1995	2001
Peru	111	78.5	2.6	3	1992	2002	2002

relative weights of inflation in an interest rate reaction function. Although ex-ante these regimes need not be associated with the introduction of the inflation targeting framework, the coincidence of a more responsive monetary policy with inflation targeting is striking. Furthermore, the model allows us to generate counterfactuals of what could have happened if the observed change toward a more aggressive fight against inflation would not have taken place. In general, we observe that if monetary policy had remained dovish, these countries would have experienced higher and more variable levels of inflation and more pronounced variations in GDP with small gains in average economic growth. Therefore, we conclude that the introduction of inflation targeting represented a favorable regime switch in the regulation of monetary policy in Latin America.

The rest of the paper is organized as follows. Section 2 presents a Markov-Switching open-economy DSGE model with monetary factors that will serve as the theoretical basis used to perform our analysis. Section 3 describes the tools used to solve and estimate the Markov-switching DSGE model. Section 4 presents results for the five countries discussed. Specifically, (4.1) displays the probabilities of the high inflation responses and high volatility regimes;

(4.2) reports the parameter estimates; (4.3) shows the model's impulse response functions for the high and low inflation response regimes to analyze the mechanisms; and (4.4) counterfactual simulated variables under the high and low inflation response regimes to analyze what could have happened during the sample period if monetary policy had been conducted differently, together with tables summarizing the average standard deviation and coefficient of variation of the observed variables and the hypothetical series generated in the counterfactuals. Section 5 concludes.

2. MODEL

Our model is based on the monetary open economy model presented by Gali and Monacelli (2005) and later estimated for the Commonwealth countries by Lubik and Schorfheide (2007) and for a large set of emerging market countries by Ortiz and Sturzenegger (2007). In essence the economy is summarized by the following three equations: an open economy Investment-Savings (IS) curve, an open economy Phillips curve and an interest rate rule.

To capture potential regime changes, we specify a Markov-switching DSGE model where we allow for changes in the parameters associated with the monetary authority reaction function and the price formation process, and use a state variable ξ^{sp} to denote the structural parameters *sp* regime at time *t*. To allow for regime changes in the stochastic volatilities we model a second, independent, Markov-Switching process and use a state variable ξ^{vo} to distinguish the volatility *vo* regime at time *t*.

In log linearized form, the open economy IS-curve is:

$$y_t = E_t y_{t+1} - \left[\tau + \alpha\left(2-\alpha\right)\left(1-\tau\right)\right]\left(R_t - E_t \pi_{t+1} - \rho_a a_t + \alpha E_t \Delta q_{t+1}\right) + \alpha\left(2-\alpha\right)\frac{1-\tau}{\tau} E_t \Delta y^*_{t+1} \tag{2.1}$$

where y_t denotes aggregate output, R_t nominal interest rate, π_t CPI inflation, a_t is the growth rate of a non-stationary technology process A_t, q_t terms of trade, defined as the relative price of exports in terms of imports, and y^*_t world output. E_t denotes the conditional expectation operator. The parameter τ represents the elasticity of inter-temporal substitution and α is the import share.[2] Technology follows an exogenous process: $ln\left(A_t / A_{t-1}\right) = \bar{a} + a_t, a_t = \rho_a a_{t-1} + \sigma_{a,\xi^{vo}} \varepsilon_{a,t}$, where ρ_a is the autoregressive coefficient and $\sigma_{a,\xi^{vo}}$ is the standard deviation of the stochastic volatility of the technology innovations $\varepsilon_{a,t}$, whose ξ^{vo} subscript denotes that it is allowed to change across regimes at time t. The same convention in notation follows for the other exogenous processes as world output y_t^* that is treated as an unobservable and is assumed to follow the process $y_t^* = \rho_{y^*} y_{t-1}^* + \sigma_{y^*,\xi_t^{vo}} \varepsilon_{y^*,t}, \varepsilon_{y^*,t} \sim N\left(0,1\right)$. In order to guarantee stationarity of the model, all real variables are expressed in terms of percentage deviations from A_t.

The log-linear version of the open economy Phillips curve is:

2.2
$$\pi_t = \frac{\beta}{1+\beta\chi_{p,\xi_t^{sp}}} E_t \pi_{t+1} + \frac{\chi_{p,\xi_t^{sp}}}{1+\beta\chi_{p,\xi_t^{sp}}} \pi_{t-1} + \beta\alpha\Delta q_{t+1} - \alpha\Delta q_t + \frac{\kappa_{\xi_t^{sp}}}{\tau + \alpha\left(2-\alpha\right)\left(1-\tau\right)}\left(y_t - \bar{y}_t\right)$$

where $\bar{y}_t = -\alpha\left(2-\alpha\right)\frac{1-\tau}{\tau} y_t^*$ is potential output in the absence of nominal rigidities. β represents the discount factor, χ_p is the degree of lagged price inflation, κ is the structural parameter associated to the Phillips curve and the ξ^{sp} subscript indicates that these parameters are allowed to change across regimes at time t.

The log-linear version of the interest rate rule is given by:

2.3
$$R_t = \rho_{R,\xi_t^{sp}} R_{t-1} + \left(1-\rho_{R,\xi_t^{sp}}\right)\left[\psi_{\pi,\xi_t^{sp}} \pi_t + \psi_{y,\xi_t^{sp}} y_t + \psi_{\Delta e,\xi_t^{sp}} \Delta e_t\right] + \sigma_{R,\xi_t^{vo}} \varepsilon_{R,t}$$

where e_t is the nominal effective exchange rate, defined as the price of domestic currency in terms of foreign currency. The parameter ρ_R captures the degree of interest rate smoothing, while ψ_π, ψ_y and $\psi_{\Delta e}$ capture the sensitivity of the interest rate with respect to inflation, output deviation from its steady-state and nominal exchange rate

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2 The equation reduces to the closed economy variant when $\alpha = 0$.

depreciation, Δe_t, respectively. The ξ^{sp} subscript indicates that these parameters are allowed to change across regimes at time t. $\sigma_{R,\xi^{vo}}$ is the standard deviation of the stochastic volatility of the interest rate $\varepsilon_{R,t} \sim N(0,1)$, whose ξ^{vo} subscript denotes that it is allowed to change across regimes at time t.

The exchange rate is introduced via CPI inflation according to:

2.4
$$\pi_t = \Delta e_t + (1-\alpha)\Delta q_t + \pi_t^*$$

where π_t^* is a world inflation shock which is treated as an unobservable and is assumed to follow an exogenous process: $\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \sigma_{\pi^*,\xi_t^{vo}}\varepsilon_{\pi^*,t}, \varepsilon_{\pi^*,t} \sim N(0,1)$. Terms of trade, in turn, are assumed to follow a law of motion for their growth rate:

2.5
$$\Delta q_t = \rho_q \Delta q_{t-1} + \sigma_{q,\xi_t^{vo}}\varepsilon_{q,t}$$

with $\varepsilon_{q,t} \sim N(0,1)$. Equations (2.1) to (2.5), plus the exogenous processes for technology, world output and world inflation, constitute the whole model.

3. SOLUTION AND ESTIMATION OF THE MARKOV-SWITCHING DSGE MODEL

The DSGE system with constant parameters has the following matrix form:

3.1
$$\Gamma_o X_{t+1} = \Gamma_1 X_t + \Theta Z_t + \varphi\varepsilon_t$$

where Γ_o, Γ_1, Θ and φ matrices contain the model's parameters. x_t stands for the $(n\times 1)$ vector of endogenous variables,[3] Z_t is the $(k\times 1)$ vector of exogenous processes and η_t corresponds to the $(\ell\times 1)$

[3] with $X_t = \left[y_t \pi_t R_t \Delta q_t \Delta e_t \pi_t^* y_t^* a_t\right]'$.

disturbances vector. The conditions for existence and uniqueness of the solution (3.1) depend on the generalized eigenvalues of the system's matrices (Farmer *et al.*, 2008).

Using the solution algorithm proposed by Sims (2002) or Schmitt-Grohé and Uribe (2003) the unique solution for the system (3.2) is combined with an observation equation:

3.2
$$X_t = G(\Lambda)x_{t-1} + AZ_t$$

3.3
$$Y_t^{obs} = MX_t$$

where Λ stands for the parameters of the model, Y_t^{obs} are the observed variables,[4] and M provides the policy function for the observables. Following Bianchi and Ilut (2017), we introduce the possibility of regime change for the structural parameters and the volatilities through two Markov chains, ξ^{sp} and ξ^{vo}. The former denotes the unobserved regime associated with the monetary parameters subject to regime shifts and takes on discrete values $sp \in \{1,2\}$,[5] and the latter stands for the shock volatilities, assumes discrete values, $vo \in \{1,2\}$,[6] and evolves independently of *sp*.

Both state variables *sp* and *vo* are assumed to follow a first-order Markov chain with the following transition matrices, respectively:

3.4
$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \text{ and } \begin{pmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{pmatrix}$$

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4 GDP growth, inflation rate, interest rate, change in the terms of trade and nominal depreciation.

5 Where 1 and 2 are the high and low response to inflation regimes $\left(\text{i.e } \psi_{\pi,\xi_t^{sp=1}} > \psi_{\pi,\xi_t^{sp=2}}\right)$, respectively.

6 Where 1 and 2 are the low and high volatility regimes. In order to define the high volatility regime, we included into the model the following restriction: $\sigma_{a,\xi^{vol=1}} < \sigma_{a,\xi^{vol=2}}$.

where $H_{ij}=p\,(sp_t=j|sp_{t-1}=i)$, for $i, j=1, 2$, and $Q_{ij}=p(vo_t=j \mid vo_{t-1}=i)$ for $i, j=1, 2$. Then, H_{ij} stands for the probability of being in regime j at t given that one was in regime i. The analysis is symmetric for Q_{ij}.

The Markov switching system can be cast in a state-space form by collecting all the endogenous variables in a vector X_t and all the exogenous variables in a vector Z_t:

$$B_1\left(\xi_t^{sp}\right)X_t = E_t\left\{A_1\left(\xi_t^{sp},\xi_{t+1}^{sp}\right)X_{t+1}\right\} + B_2\left(\xi_t^{sp}\right)X_{t-1} + C_1\left(\xi_t^{sp}\right)Z_t \tag{3.5}$$

$$Z_t = S\left(\xi_t^{sp}\right)Z_{t-1} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N\left(0,\Sigma\left(\xi^{vo}\right)\right) \tag{3.6}$$

where the matrices $A_1\left(\xi_t^{sp}\right), B_1\left(\xi_t^{sp}\right), B_2\left(\xi_t^{sp}\right), C_1\left(\xi_t^{sp}\right)$ and $S\left(\xi_t^{sp}\right)$ are functions of the model parameters. $\Sigma\left(\xi^{vo}\right)$ is the covariance matrix of the shocks,[7] which depends on the unobserved state ξ^{vo}, controlled by the transition matrix Q. Therefore, note that, in contrast with (3.1), (3.5) has a presence of unobserved variables and unobserved Markov states of the Markov chains.

There are several studies in the MS-DSGE literature that analyze the technical aspects of solving this state-space system (Farmer *et al.* (2008, 2011); Foerster *et al.* (2014); Maih (2015)[8] and Cho (2016)), in the sense that solution algorithms developed for solving DSGE models with fixed parameters (e.g. Sims (2002) and Schmitt-Grohé and Uribe (2003)) are unsuitable. To solve the system we use the Newton methods developed in Maih (2015), which expand on the method proposed by Farmer *et al.* (2011) and concentrates on minimum state variable solutions (MSV) of the form:

7 Where: $\Sigma\left(\xi^{vo}\right) = diag\left(\sigma_{q,\xi_t^{vo}}, \sigma_{a,\xi_t^{vo}}, \sigma_{R,\xi_t^{vo}}, \sigma_{y^*,\xi_t^{vo}}, \sigma_{\pi^*,\xi_t^{vo}}\right)$.

8 The routines used for the computations were implemented using RISE, an object-oriented Matlab toolbox for solving and estimating Markov switching rational expectation models, developed by Junior Maih.

3.7
$$X_t = \Omega^*\left(\xi^{sp},\theta^{sp},H\right)X_{t-1} + \Gamma^*\left(\xi^{sp},\theta^{sp},H\right)Z_t\left(\xi^{vo},\theta^{vo}\right)$$

Where θ^{sp} and θ^{vo} are the switching parameters controlled by ξ_t^{sp} and ξ_t^{sp}, respectively.

The complete state form of the model combines (3.7) with the measurement equations (3.8):

3.8
$$Y_t^{obs} = L\left(\theta^{ss}\right) + MX_t$$

where:

$$Y_t^{obs} = \begin{bmatrix} \Delta GDP_t \\ Inflation \\ Interest\ rate_t \\ \Delta Terms\ of\ trade \\ \Delta Exchange\ rate \end{bmatrix}, L\left(\theta^{ss}\right) = \begin{bmatrix} 0 \\ 4\pi^{ss} \\ 4\left(\pi^{ss}+r^{ss}\right) \\ 0 \\ 0 \end{bmatrix}, M\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The presence of unobserved DSGE states X_t and unobserved parameters (controlled by the Markov chains), implies that the standard Kalman filter cannot be used to compute the likelihood. So, in correspondence with Bianchi and Ilut (2017) we use the Kim *et al.* (1999) filter.

We use the Bayesian approach to estimate the model:

- Using Kim *et al.* (1999) algorithm, we compute the likelihood introducing non-linearities and unobserved chains employing the filter with prior distribution of the parameters.
- We construct the posterior kernel with our results from the Bee_gate[9] optimizer routine.
- We use the posterior mode as the initial value for the Metropolis Hasting algorithm, with 100.000 iterations.
- We compute moments utilizing the mean and variance of the last 50.000 iterations.

9 RISE toolbox optimization routine.

3.1 Counterfactuals

To explore the characteristics of the MS-DSGE model with multiple regimes, we generate a counterfactual series based on conditional forecast simulations. Specifically, this analysis allows us to get an idea of what would have happened if the monetary policy had not changed, given the smoothed shocks estimated by the model. The model is resolved introducing a law of motion consistent with the fact that no other regime would have been observed. In this section the algorithm to generate the simulated series is briefly explained.

Once the model is estimated, we generate forecasts from the ms-dsge model conditional on the realized path of the five model shocks: terms of trade, technology, monetary, world output, and world inflation. Our conditional forecasts are generated over the full sample period for each of the five countries. The data from the first quarter in every sample are used as initial conditions. The parameters utilized are the estimated posterior distribution of the coefficients for each regime.

We trace out the counterfactuals' paths by generating a new data vector for Z_t in (3.7), which includes the smoothed shocks. As different paths for the endogenous variables (one for each regime) are obtained for this regime switching model, we utilize the "expected smoothed series of the shocks, correspond to the weighted average paths of the exogenous variables.

Once the system is integrated, as in the previous subsection, the data are filtered and the counterfactual paths for the unobserved and observable variables are generated.

4. RESULTS

4.1 Regime probabilities

Figures 2 to 6 show the smoothed probabilities for the two Markov-switching processes. The top panel of each figure shows the probability that monetary policy is conducted under a high interest rate response to inflation regime based on the structural parameters of the interest rate rule. The bottom panel presents the probability of being on a high volatility regime based on the relative volatility of the non-stationary technology process. The first thing one must notice is that high interest rate response regimes have been the most prevalent forms of regime during the sample periods. The percentage of periods where our estimation assigns a probability higher than 50% of Brazil, Chile, Colombia, Mexico, and Peru being in a high response regime are 77%, 90%, 77%, 65% and 69%, respectively. Regarding the transition matrix, the mean (and 10%-90% confidence interval in parenthesis) parameter estimates for the probability of going from a high response to a low response regime, $H_{1,2}^{coef=1}$, are 0.1603 (0.039, 0.4719), 0.0808 (0.0141, 0.21), 0.0863 (0.0239, 0.2236), 0.1161 (0.0707, 0.1842) and 0.0721 (0.0276, 0.1129), respectively, while the probability of moving from a low response to a high response regime $H_{2,1}^{coef=2}$, are 0.2257 (0.0997, 0.4375), 0.0521 (0.0225, 0.0942), 0.1566 (0.048, 0.3472), 0.2108 (0.097, 0.3049) and 0.0565 (0.0191, 0.101), respectively.

4.1.1 High interest rate response regimes

With the introduction of inflation targeting and greater exchange rate flexibility, after a 35% real depreciation in 1999, Brazil experienced a regime switch to high response in 1999Q3. Our analysis captures the 2002 depreciation and the Cardoso-da Silva government transition as a transitory change of the monetary policy regime from 2002Q4 to 2003Q4. From 2004Q1 onwards, the probability of being under a high response monetary policy is close to 1.

Chile fully adopted inflation targeting in 1999, but as stated in Corbo *et al.* (2002) the scheme began to be implemented in the 1990s. Our estimation captures a high response to inflation from the beginning of the sample in 1996 until 2007Q4. In 2008Q1 and until 2009Q4, there was a marked shift in policy with smaller weight on inflation and larger weight on output during a stagflationary period. From 2010Q1 onwards, the interest response of interest rates to inflation is estimated to be strong with high probability.

Colombia experienced a strong shift in monetary policy during 2000Q1 shortly after the introduction of inflation targeting and greater exchange rate flexibility.

Mexico has three periods during which our estimation assigns a high probability to a high response regime: from 1988Q2 to 1988Q3, from 1992Q1 to 1994Q4 and from 1997Q2 onwards. The first period coincides with Pacto de Solidaridad y Estabilidad Económica, signed in December 1987, which was a heterodox plan committing labor unions and public and private sectors to limit their price revisions

Figure 2

SMOOTHED PROBABILITIES FOR BRAZIL

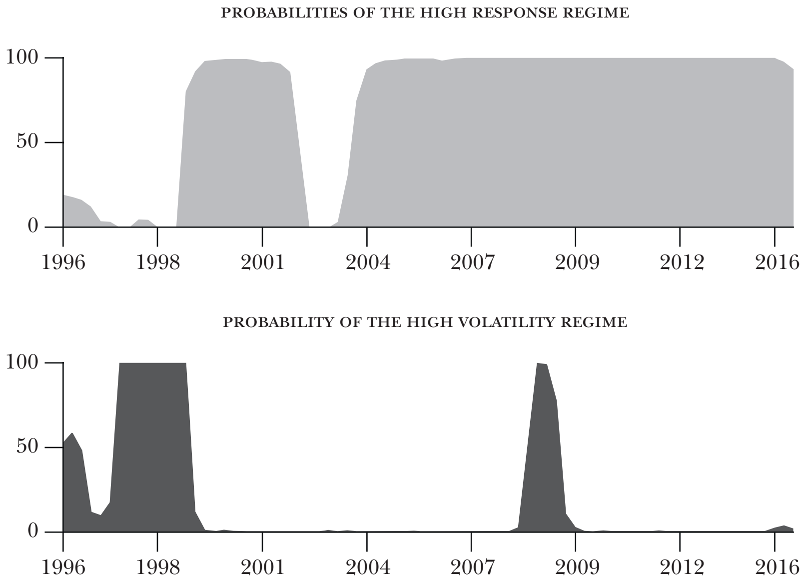


Figure 3

SMOOTHED PROBABILITIES FOR CHILE

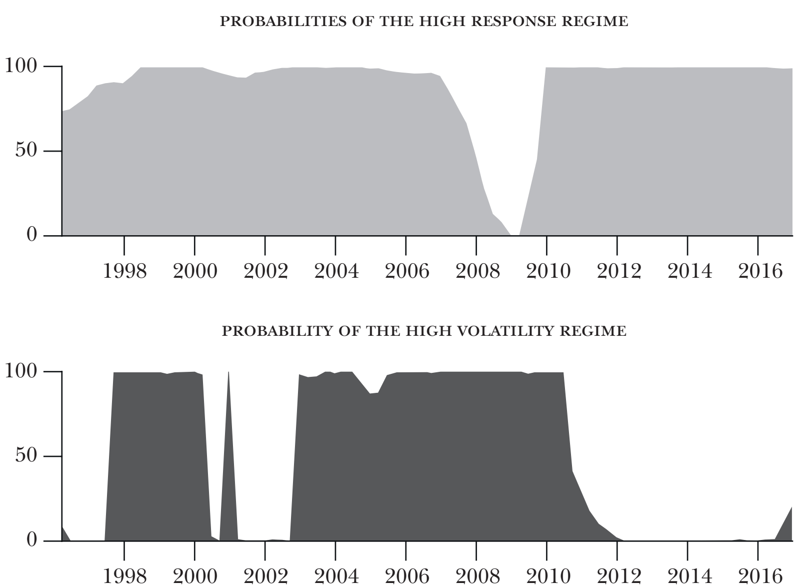


Figure 4

SMOOTHED PROBABILITIES FOR COLOMBIA

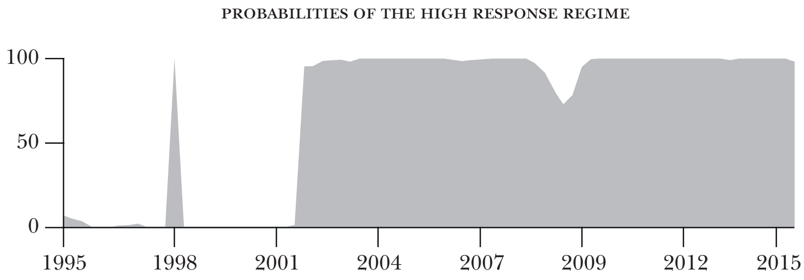


Figure 5

SMOOTHED PROBABILITIES FOR MEXICO

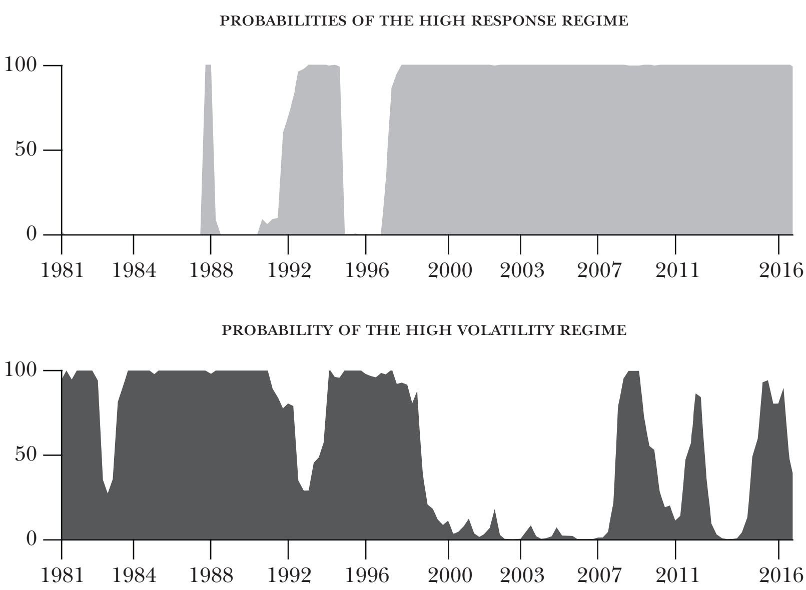
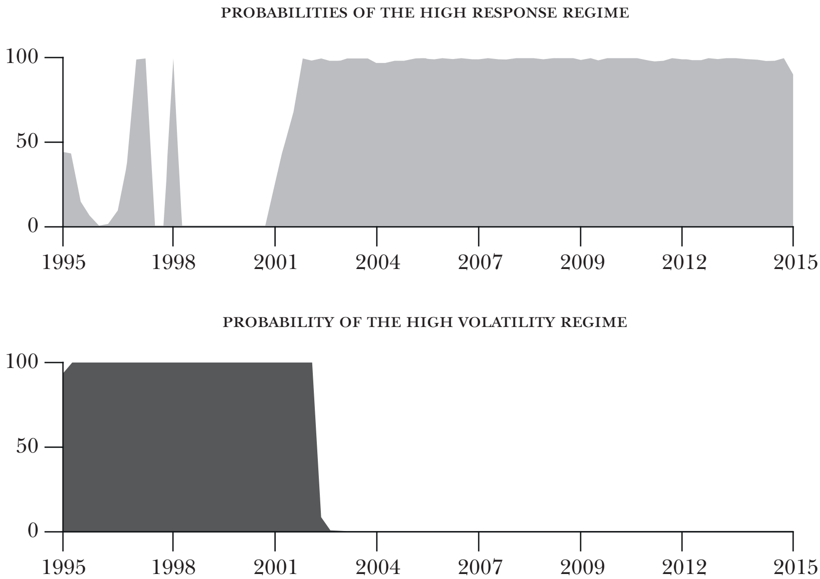


Figure 6

SMOOTHED PROBABILITIES FOR PERU



to anchor inflation expectations. The second period was shortly after the exchange rate policy changed from fixed exchange rate to a band system with a floor and a ceiling both adjustable over time. It includes the 1993 Constitutional reform granting legal autonomy to the Central Bank and the establishment of the price stability objective while it recognized that no government authority could force the Central Bank to grant financing. The December 1994 Tequila crisis forced the Central Bank to adopt a floating exchange rate regime. The crisis required balancing nominal pressures with an output contraction which required postponing the adoption of a high response regime until 1997Q2 consolidated in 2001 with the introduction of inflation targeting.

In addition, our analysis estimates Peru had three periods with a high probability of high response regime: from 1997Q4 to 1998Q1, in 1998Q4, and from 2002Q1 onwards. Therefore, after brief episodes of monetary tightening in 1997/1998, monetary policy switched towards greater responsiveness to inflation in 2002 which coincides with the adoption of the inflation targeting regime.

4.1.2 High volatility shock regimes

Cogley and Sargent (2005), Sims and Zha (2006) and Bianchi (2012) highlight the importance of accounting for stochastic volatility of exogenous shocks when a regime switch in monetary policy is analyzed. Additionally, Liu and Mumtaz (2011) and Goncalves *et al.* (2016) show that the fit of the model is improved when a Markov-Switching process for regime volatilities is introduced. In our estimation, we classify a regime as one of high volatility if the standard deviation of the stochastic volatility of the non-stationary technology shock is large. Given that in order to guarantee stationarity of the model, all real variables must be expressed in terms of percentage deviations from A_t, the growth rate of the non-stationary technology process enters the IS-curve. Organizing countries alphabetically, the percentage of periods where the estimation assigns a probability higher than 50% of being in a high volatility regime are 18%, 51%, 22%, 56% and 35%, respectively. Regarding the transition matrix, the mean (and 10%-90% confidence interval in parenthesis) parameter estimates for the probability of going from a low volatility to a high volatility regime, $H_{1,2}^{vol=1}$, are 0.3071 (0.1241, 0.5589), 0.0307 (0.0107, 0.0589), 0.0607 (0.0089, 0.2931), 0.1922 (0.0958, 0.339) and 0.0849 (0.0103, 0.4463), respectively, while the probability of moving from a low response to a high response regime $H_{2,1}^{coef=2}$, are 0.1458 (0.0278, 0.4982), 0.182 (0.1096, 0.2873), 0.1023 (0.0257, 0.2056), 0.109 (0.0577, 0.1836) and 0.1719 (0.0427, 0.4136), respectively. High volatility periods for Brazil are 1996Q2-1996Q3, 1997Q4-1999Q3, and 2008Q3-2009Q2; while for Chile they are 1997Q4-2000Q2, 2001Q1, and 2003Q1-2010Q3; for Colombia they are 1995Q4-1996Q3, 1998Q2-2000Q2, 2002Q3-2003Q1, and 2008Q4-2009Q1; for Mexico they are 1981Q1-1983Q1, 1984Q1-1992Q2, 1994Q1-1998Q3, 2008Q2-2010Q1, 2011Q4-2012Q2, and 2015Q1-2016Q3; and for Peru it is 1995Q4-2002Q3.

4.2 Estimation results

Table 2, below, reports the mean for the estimated parameters of the model for each country, while the appendix has individual tables for each country with the mean, mode, standard deviation

Table 2

MEAN FOR THE ESTIMATED PARAMETERS FOR BRAZIL, CHILE, COLOMBIA, MEXICO AND PERU

Parameter	*Distribution*	*Country*				
		Brazil	*Chile*	*Colombia*	*Mexico*	*Peru*
$\chi_{p,\xi^{coef=1}}$	Beta	0.1738	0.2053	0.7092	0.8564	0.1318
$\chi_{p,\xi^{coef=2}}$	Beta	0.4471	0.5124	0.313	0.6134	0.1471
$\kappa_{p,\xi^{coef=1}}$	Gamma	1.1362	0.0765	0.5845	2.1643	0.5011
$\kappa_{p,\xi^{coef=2}}$	Gamma	0.6296	0.0631	1.9982	2.3736	0.0565
$\rho_{R,\xi^{coef=1}}$	Beta	0.7629	0.9215	0.7298	0.458	0.697
$\rho_{R,\xi^{coef=2}}$	Beta	0.6113	0.4912	0.7065	0.6279	0.6254
$\psi_{\pi,\xi^{coef=1}}$	Gamma	3.4901	2.7337	3.2941	1.8458	1.9066
$\psi_{\pi,\xi^{coef=2}}$	Gamma	1.0417	0.8692	0.9746	0.6154	0.9226
$\psi_{y,\xi^{coef=1}}$	Gamma	0.3013	0.5594	0.3849	0.7265	0.4092
$\psi_{y,\xi^{coef=2}}$	Gamma	0.8799	0.434	0.7379	0.8310	0.5639
$\psi_{\Delta,\xi^{coef=1}}$	Gamma	0.0435	0.0816	0.137	0.1108	0.1725
$\psi_{\Delta,\xi^{coef=2}}$	Gamma	0.0422	0.0662	0.0463	0.3408	0.1506
α	Beta	0.076	0.0539	0.1132	0.2689	0.0393
r	Gamma	3.6731	2.2813	6.8509	2.1004	8.8041
τ	Beta	0.2792	0.16	0.2445	0.3256	0.1306
ρ_a	Beta	0.3014	0.1599	0.1291	0.2007	0.3924

Parameter	*Distribution*	*Country*				
		Brazil	*Chile*	*Colombia*	*Mexico*	*Peru*
ρ_q	Beta	0.424	0.1553	0.1628	0.4305	0.3605
ρ_{y*}	Beta	0.9818	0.9579	0.9659	0.9042	0.9682
$\rho_{\pi*}$	Beta	0.3715	0.3129	0.2303	0.7824	0.416
$H_{1,2}^{coef=1}$	Beta	0.1603	0.0808	0.0863	0.1161	0.0721
$H_{2,1}^{coef=2}$	Beta	0.2257	0.0521	0.1566	0.2108	0.0565
$\sigma_{R,\xi^{vol=1}}$	Inv.Gamma	5.3145	0.5788	0.8134	4.5438	2.4271
$\sigma_{R,\xi^{vol=2}}$	Inv.Gamma	3.3642	3.3239	6.8695	5.8216	7.6316
$\sigma_{q,\xi^{vol=1}}$	Inv.Gamma	5.791	6.4758	5.5065	3.121	4.1378
$\sigma_{q,\xi^{vol=2}}$	Inv.Gamma	4.2554	5.3403	7.2084	4.4066	5.1138
$\sigma_{a,\xi^{vol=1}}$	Inv.Gamma	4.6972	3.9563	5.0036	3.2222	2.7075
$\sigma_{a,\xi^{vol=2}}$	Inv.Gamma	4.7999	6.1979	6.0725	7.4444	6.0456
$\sigma_{y^*,\xi^{vol=1}}$	Inv.Gamma	3.5522	3.4781	1.6996	6.7571	2.1448
$\sigma_{y^*,\xi^{vol=2}}$	Inv.Gamma	6.9291	5.4652	3.0673	7.3328	3.5942
$\sigma_{\pi^*,\xi^{vol=1}}$	Inv.Gamma	4.8214	7.2118	5.0864	5.09	5.0435
$\sigma_{\pi^*,\xi^{vol=2}}$	Inv.Gamma	6.1201	4.6023	2.4292	9.5155	5.0472
$H_{1,2}^{vol=1}$	Beta	0.3071	0.0307	0.0607	0.1922	0.0849
$H_{2,1}^{vol=2}$	Beta	0.1458	0.182	0.1023	0.109	0.1719

and confidence intervals. When describing the parameter estimates, we follow the convention of reporting values of countries ordered as Brazil, Chile, Colombia, Mexico, and Peru. First, we describe the values for the high interest rates responses to inflation regimes and then for the low response regimes, followed by a comparison. We report the mean for the estimated parameters and, in parenthesis, the estimated values for the 10% and 90% confidence intervals. Here, we focus on talking about the parameters related to the inflation formation process of the Phillips curve and the interest rate reaction function.

The persistence of inflation is captured by the parameter χ_p in the Phillips Curve. The parameter estimates for the high interest rate response regime, $\chi_{p,\xi_{coef=1}}$, are 0.1738 (0.0319, 0.4303), 0.2053 (0.1027, 0.3366), 0.7092 (0.4474, 0.8981), 0.8564 (0.6316, 0.9739) and 0.1318 (0.0321, 0.2885), respectively, while for the low interest rate response regimes, $\chi_{p,\xi_{coef=2}}$, they are 0.4471 (0.1352, 0.8285), 0.5124 (0.1913, 0.8204), 0.313 (0.1498, 0.5307), 0.6134 (0.496, 0.7669), and 0.1471 (0.0352, 0.286), respectively. Therefore, average inflation persistence has been lower for the high interest rate response regimes in Brazil and Chile, while it has been higher in Colombia and Mexico, and has remained almost unchanged in Peru. The counterpart to this persistence of inflation is the relative weight that expectations have in the inflation formation process.

The sensitivity of inflation to the output gap is partially captured by the parameter κ in the Phillips Curve. The parameter estimates for the high interest rate response regime, $\kappa\xi_{coef=1}$, are 1.1362 (0.8484, 1.6328), 0.0765 (0.0368, 0.1346), 0.5845 (0.3863, 0.8068), 2.1643 (1.9357,2.3318) and 0.5011(0.3481,0.6833), respectively, while for the low interest rate response regimes, $\kappa\xi_{coef=2}$, they are 0.6296 (0.27, 1.2559), 0.0631 (0.0331, 0.1008), 1.9982 (1.6591,2.3484), 2.3736 (1.7729,3.3246) and 0.0565 (0.0294,0.0863), respectively. Therefore, average sensitivity of inflation to the output gap has been lower for the high interest rate response regime in Colombia, higher in Brazil and Peru, and it has remained almost unchanged at a fairly low value in Chile and a high value in Mexico.

Therefore, in the context of the inflation formation process, going from a low interest response to a high one, as happened chronologically in all countries except Chile, Brazil experienced a drop in inflation inertia and a more responsive trade-off between output gap and inflation, Colombia has higher inflation inertia and a less responsive trade-off, Mexico has higher inflation inertia and moderate decrease in the responsiveness of the trade-off, and Peru has the same level of inertia and a more responsive trade-off. Meanwhile, as stated before, Chile started the sample with a high interest rate response to inflation and loosened the policy from 2008Q1 to 2009Q4. Then, when moving from a high interest rate response to a low one, Chile had an increase in inflation inertia without changes in the slope of its Phillips curve.

Turning to the interest rate reaction function, the persistence of interest rates is captured by the parameter ρ_R. The parameter estimates for the high interest rate response regime, $\rho_{R,\xi_{coef}=1}$, are 0.7629 (0.6917,0.8144), 0.9215 (0.8525,0.9788), 0.7298 (0.6633, 0.8071), 0.458 (0.3897,0.5541) and 0.697 (0.6211,0.753), respectively, while for the low interest rate response regime, $\rho_{R,\xi_{coef}=2}$, they are 0.6113 (0.2252,0.813), 0.4912 (0.4328, 0.5514), 0.7065 (0.6491, 0.7621), 0.6279 (0.3992, 0.7734) and 0.6254 (0.5227, 0.7344), respectively.

Therefore, average persistence of interest rates has been higher for the high interest rate response regime in Brazil, Chile and Peru, it has decreased in Mexico and it has remained relatively unchanged in Colombia.

The sensitivity of interest rates to inflation is captured by the parameter ψ_π. The parameter estimates for the high interest rate response regime, $\psi_{\pi,\xi_{coef}=1}$, are 3.4901 (2.733, 3.8618), 2.7337 (1.079, 5.4875), 3.2941 (1.8292, 4.9853), 1.8458 (1.7431, 1.9526) and 1.9066 (1.3059,3.309), respectively, while for the low interest rate response regime, $\psi_{\pi,\xi_{coef}=2}$, are 1.0417 (0.6815,1.4375), 0.8692 (0.7058,1.0166), 0.9746 (0.7722, 1.1641), 0.6154 (0.4424, 0.823) and 0.9226 (0.444, 1.7992), respectively.

The sensitivity of interest rates to output deviations is captured by the parameter ψ_y. The parameter estimates for the high interest rate response regime, $\psi_{y,\xi_{coef}=1}$, are 0.3013 (0.075, 0.9818), 0.5594 (0.3015, 0.8963), 0.3849 (0.1969, 0.6058), 0.7265 (0.602, 0.8016) and 0.4092 (0.1659,0.859), respectively, while for the low interest rate response regime, $\psi_{y,\xi_{coef}=2}$, are 0.8799 (0.2204, 2.0191), 0.434 (0.2317, 0.7397), 0.7379(0.3355, 1.2305), 0.831 (0.8039, 0.8562) and 0.5639 (0.3263, 1.0481), respectively. Therefore, average sensitivity of interest rates to output deviations has been lower for the high interest rate response regime in Brazil, Colombia, Mexico and Peru, while it has been higher in Chile.

The sensitivity of interest rates to exchange rate depreciations is captured by the parameter $\psi_{\Delta e}$. The parameter estimates for the high interest rate response regime, $\psi_{\Delta e,\xi^{coef=1}}$, are 0.0435 (0.0156,0.098), 0.0816 (0.0229,0.2694), 0.137 (0.1068,0.1752), 0.1108 (0.0961,0.1254) and 0.1725 (0.1215,0.2283), respectively, while for the low interest rate response regimes, $\psi_{\Delta e,\xi^{coef=2}}$, are 0.0422 (0.0139,0.1547), 0.0662 (0.026,0.1325), 0.0463 (0.0148,0.0844), 0.3408 (0.0775,0.6386) and 0.1506 (0.1139,0.1925), respectively. Therefore, average sensitivity of interest rates to exchange rate depreciations has been higher for the high interest rate response regime in Colombia, it has decreased in Mexico and it has remained almost unchanged for Brazil, Chile and Peru.

Therefore, in terms of the interest rate reaction function, going from a low interest response to a high one as happened chronologically in all countries except Chile, Brazil exhibited a greater persistence of interest rates, less sensitivity to output deviations, and no change in the response to exchange rate fluctuations. Colombia exhibited similar persistence of interest rates, decreased sensitivity to output deviations and larger sensitivity to exchange rate fluctuations. Mexico exhibited less persistence of interest rates, and smaller sensitivity to output deviations and exchange rate fluctuations. Peru exhibited larger persistence of interest rates, diminished sensitivity to output deviations, and similar response to exchange rate fluctuations. Finally, for Chile, when moving from a high interest rate response to a low one, interest rates exhibited less persistence and the weight on output deviations was larger, as expected from the countercyclical stance of their monetary policy.

4.3 Impulse response functions

Figures 7 to 11 show the impulse response functions regarding monetary policy, non-stationary technology, terms of trade, world output, and world inflation shocks, respectively. Each graph compares the responses under the high and low interest rate response to inflation regimes. Inspecting the different mechanisms prevalent in each country under each policy stance will allow us to understand the counterfactuals that are presented later where we ask what may have happened if another regime had been in place for the entire sample.

An unexpected expansion of monetary policy appreciates the currency, while it lowers inflation and output. Under the high policy response regime, appreciations are larger in Chile and Peru, where real interest rates increase by more and inflation drops are larger. Only in the case of Chile has the observed output contraction been larger under the high policy response regime, which could be due to the fact that the low response regime was implemented for countercyclical motives only once the inflation targeting regime was consolidated.

Technology is assumed to be difference stationary, so innovations in productivity have permanent effects on output. On average, output increases, inflation is positive, currency depreciates, and real interest rates decrease. These movements are slightly smaller under the high policy response regime.

An unexpected improvement in terms of trade raises output, appreciates the currency, and lowers inflation (except for the high policy response regime in Peru, where prices increase). On average, these movements prompt the central banks to loosen policy (except for the high policy response of Chile). Appreciations are of similar magnitude under both policy response regimes. Under the high policy response regime, output expansions are larger in Colombia and Mexico, the reduction of inflation is smaller in Brazil, Chile and Mexico, and the real interest rate drops by more in all countries except Chile.

World demand shocks lower domestic output, increase inflation, and potentially cause an exchange rate depreciation. These results arise because, under the estimated elasticities of intertemporal substitution, world output shocks lower domestic potential output in all countries. Despite the fact that nominal interest rates increase, real interest rates decrease. Under high policy response regimes output contractions are larger, inflation increases less, nominal exchange rate depreciation is smaller, and the central banks cut real interest rates by less.

Shocks to import price inflation appreciate the currency, but raise inflation because, in addition to the inherent foreign price inflation, the central bank reacts to movements in the exchange rate, and lowers real interest rates. Under high policy response regimes output increases by less, except in the case of Colombia, inflation increases by less, except in the case of Peru and the nominal exchange rate depreciation is of similar magnitude, except for Mexico where it is larger under high response.

Figure 7

MONETARY POLICY SHOCK IRFs

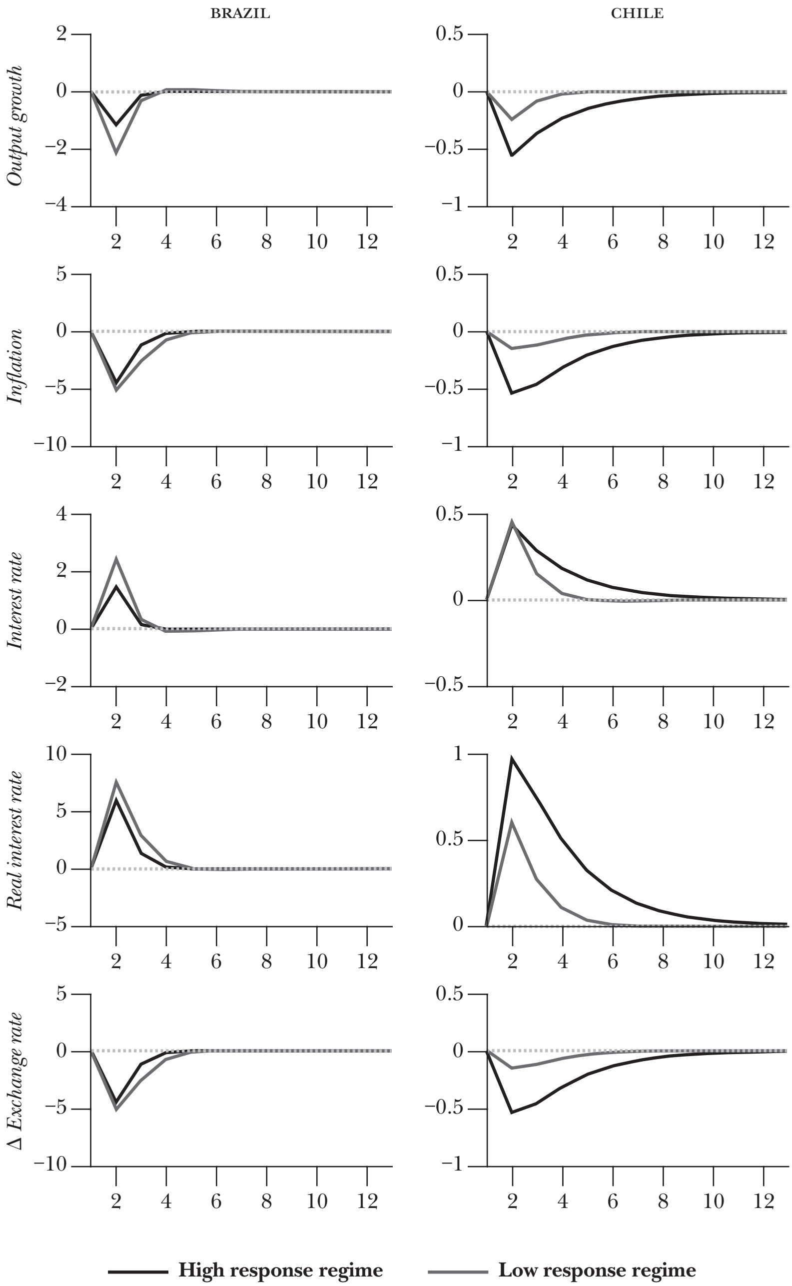


Figure 7 (cont.)

MONETARY POLICY SHOCK IRFs

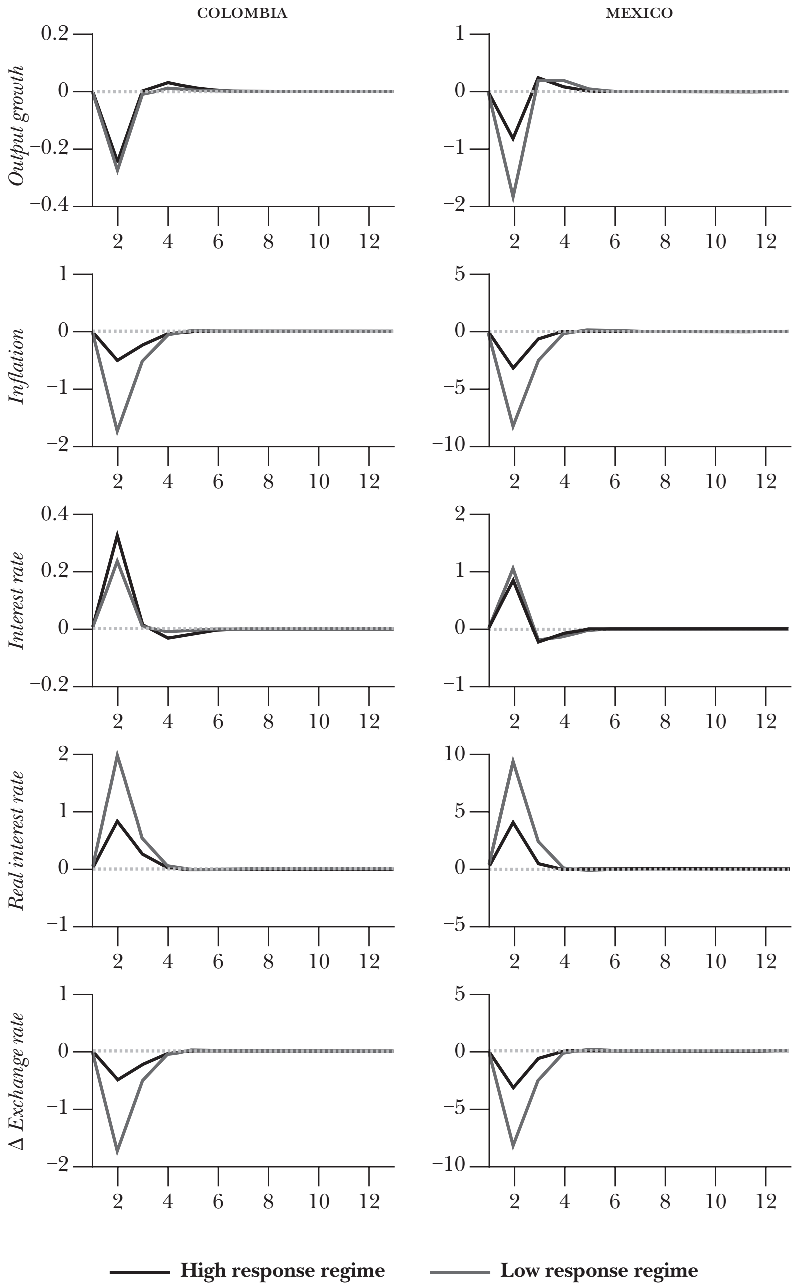


Figure 7 (cont.)

MONETARY POLICY SHOCK IRFs

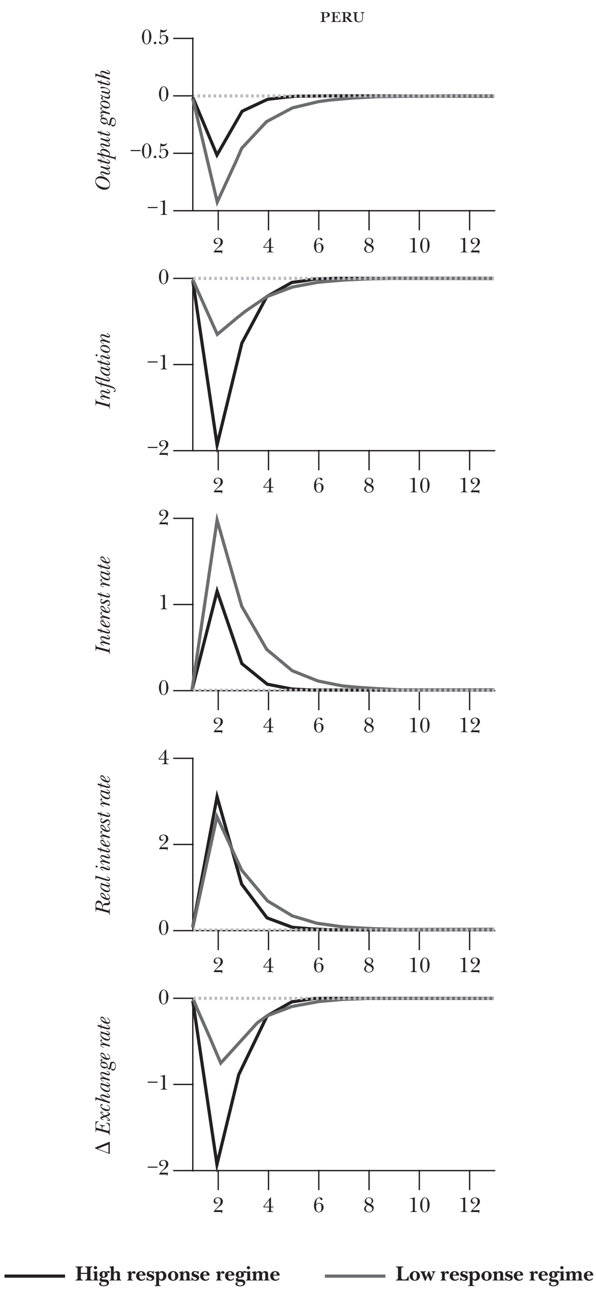


Figure 8

TECHNOLOGY SHOCK IRFs

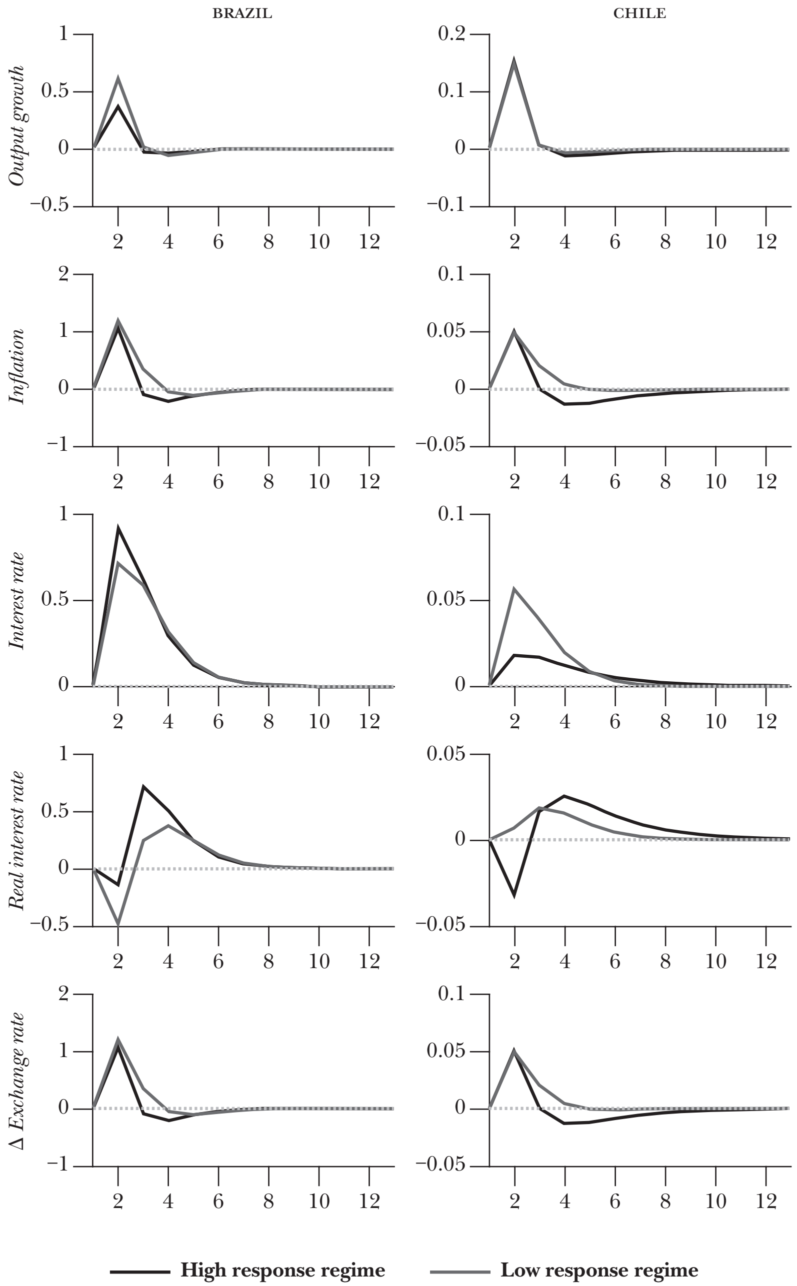


Figure 8 (cont.)

TECHNOLOGY SHOCK IRFs

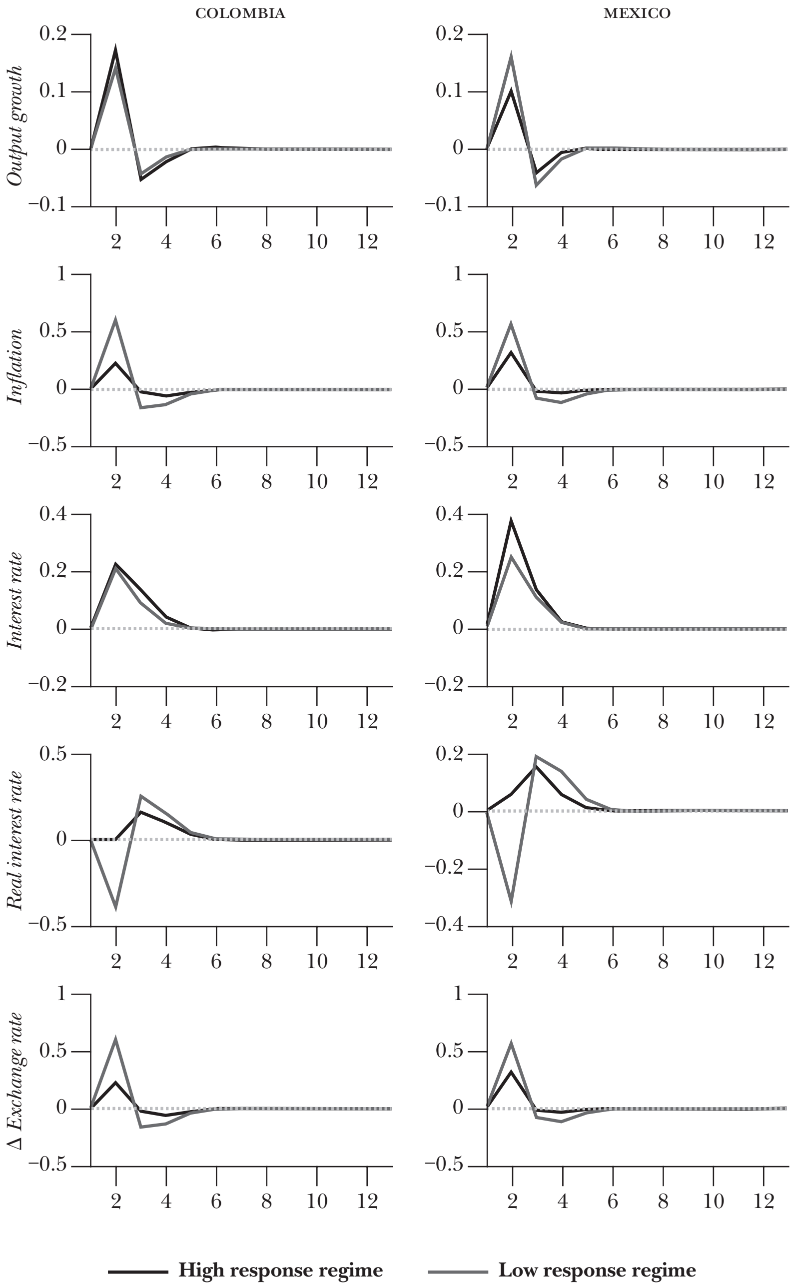


Figure 8 (cont.)

TECHNOLOGY SHOCK IRFs

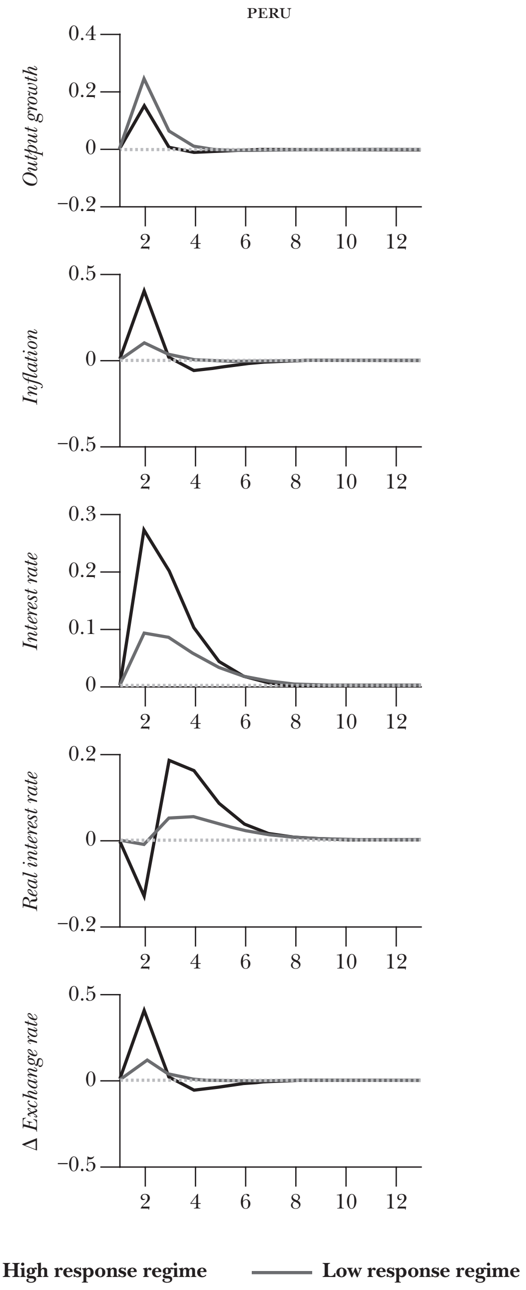


Figure 9

TERMS OF TRADE SHOCK IRFs

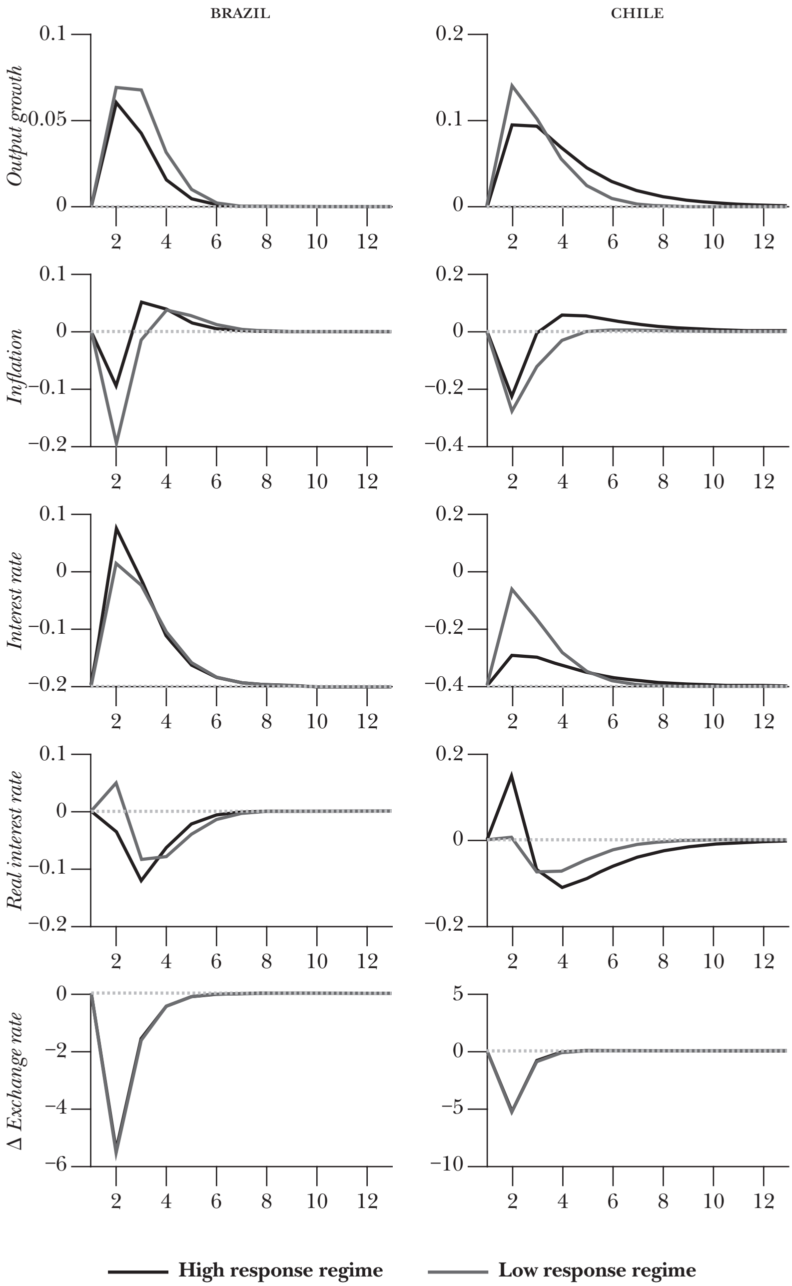


Figure 9 (cont.)

TERMS OF TRADE SHOCK IRFs

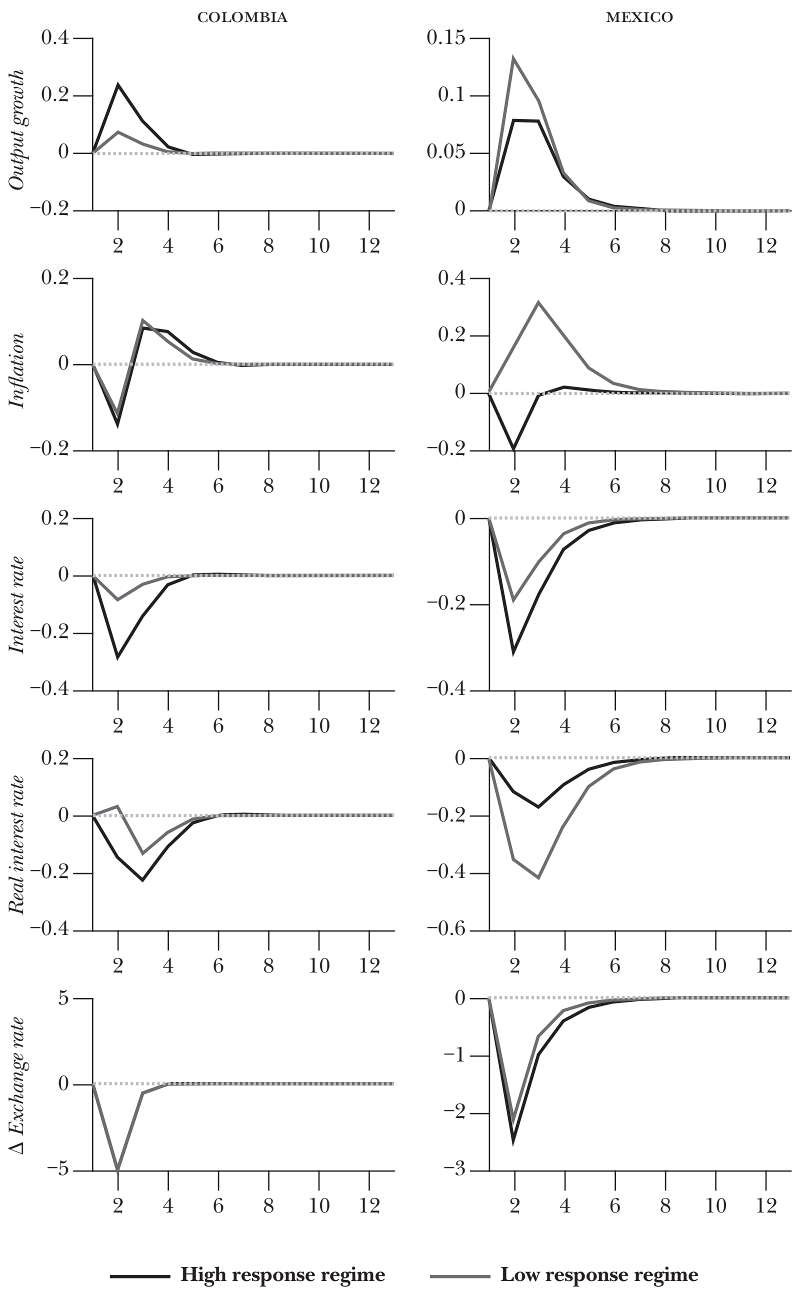


Figure 9 (cont.)

TERMS OF TRADE SHOCK IRFs

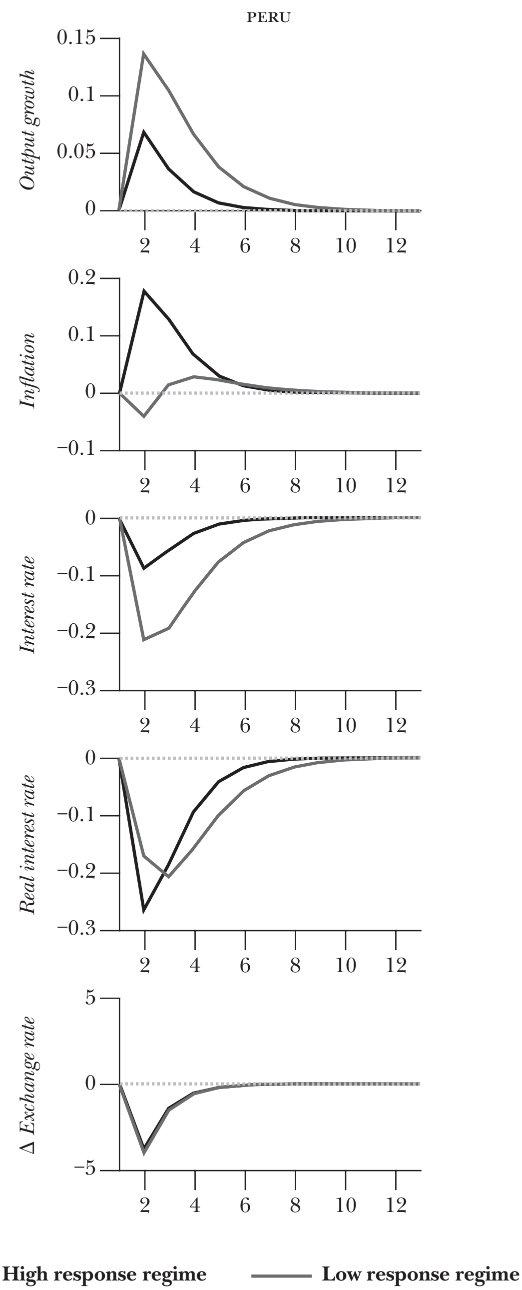


Figure 10

WORLD OUTPUT SHOCK IRFs

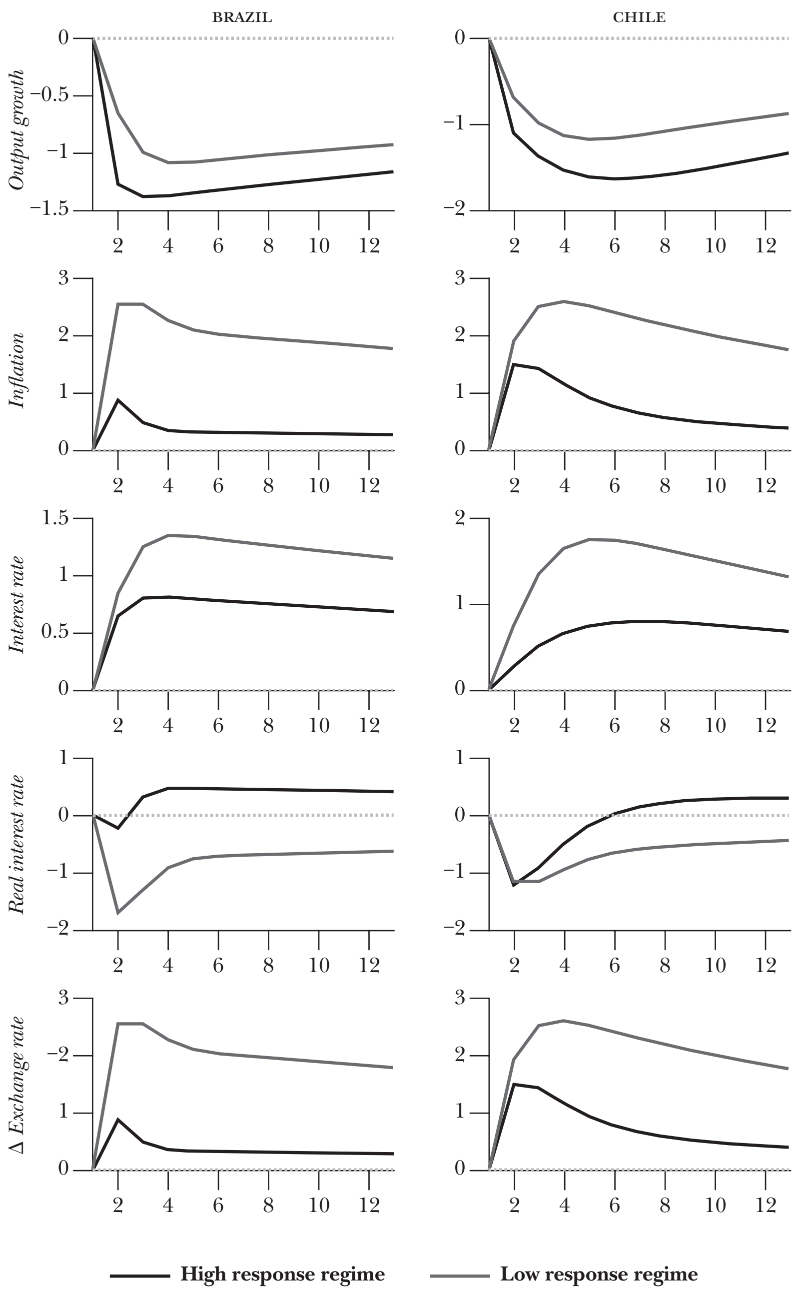


Figure 10 (cont.)

WORLD OUTPUT SHOCK IRFs

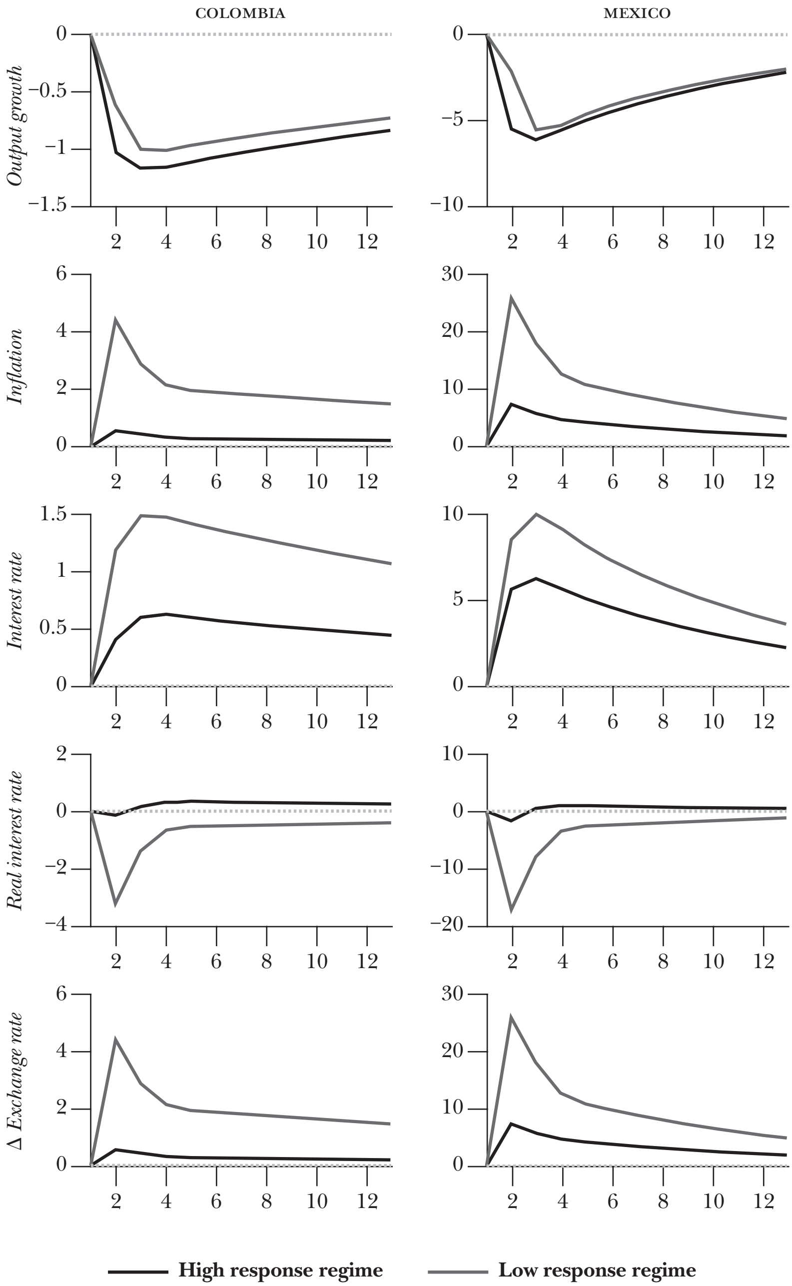


Figure 10 (cont.)

WORLD OUTPUT SHOCK IRFs

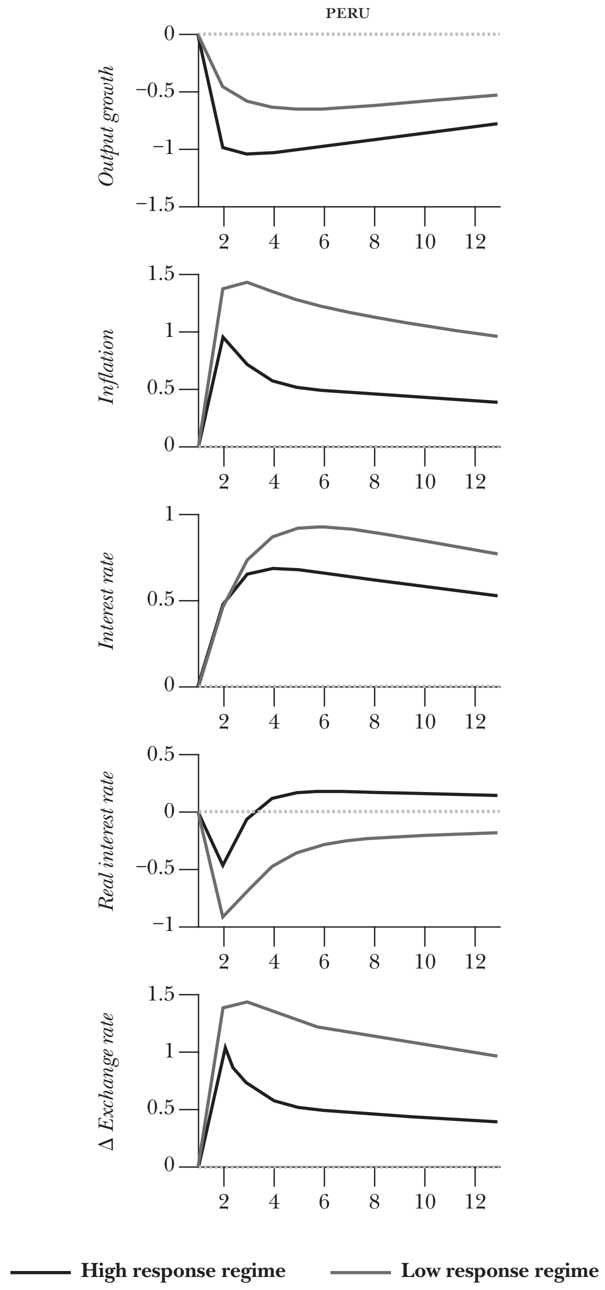


Figure 11

WORLD INFLATION SHOCK IRFs

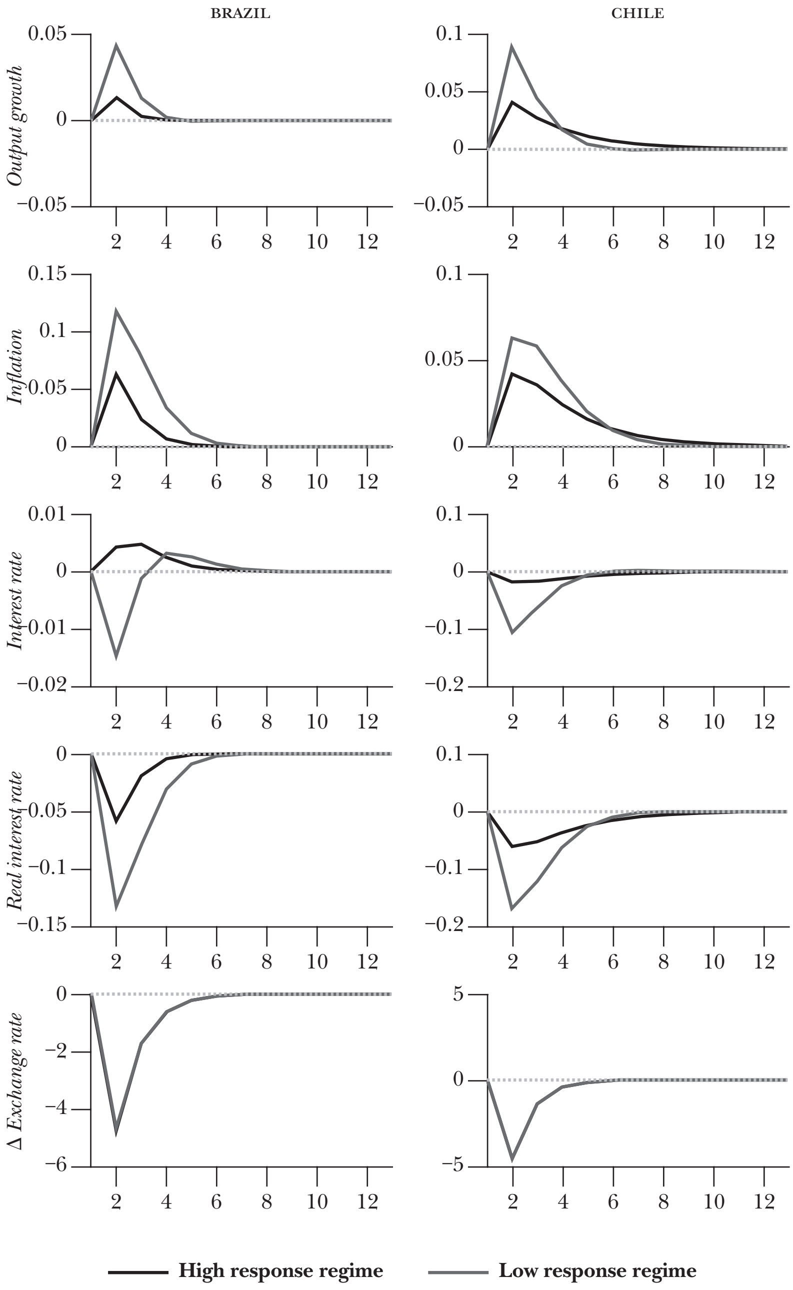


Figure 11 (cont.)

WORLD INFLATION SHOCK IRFs

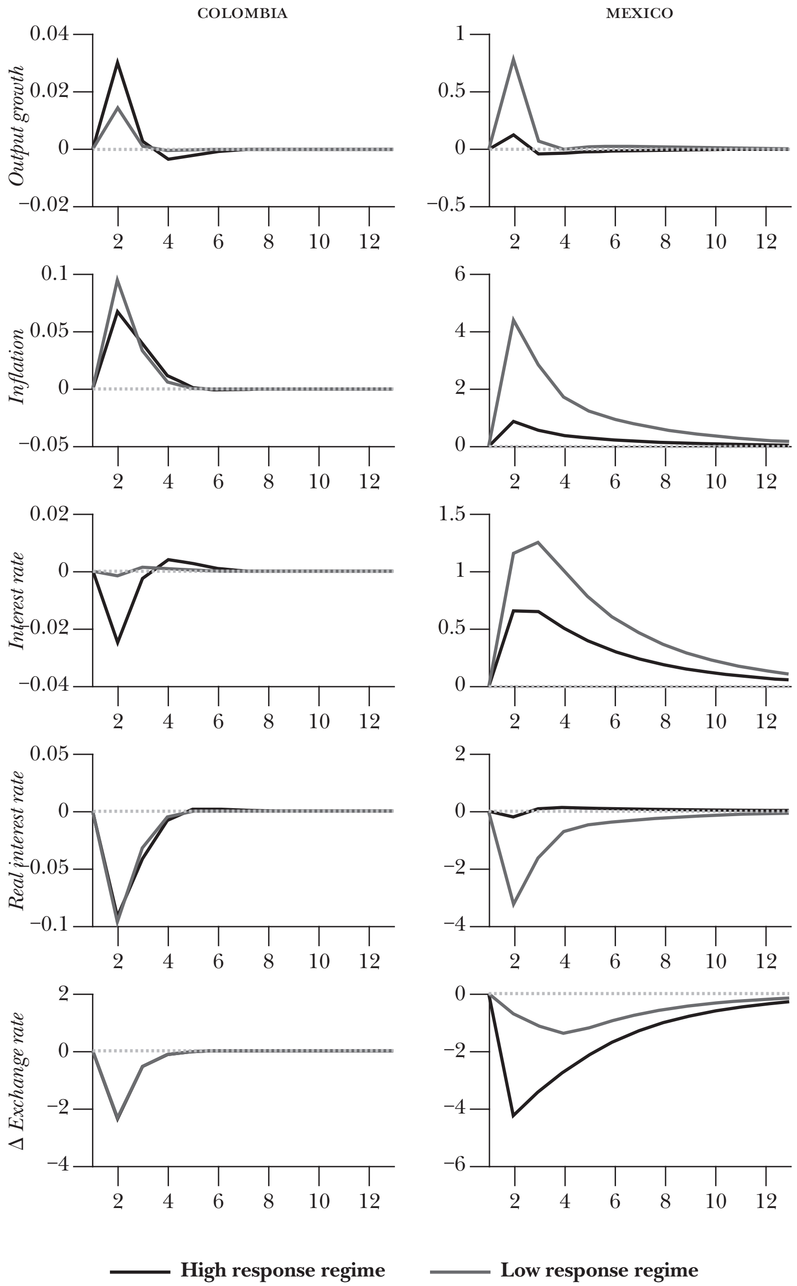
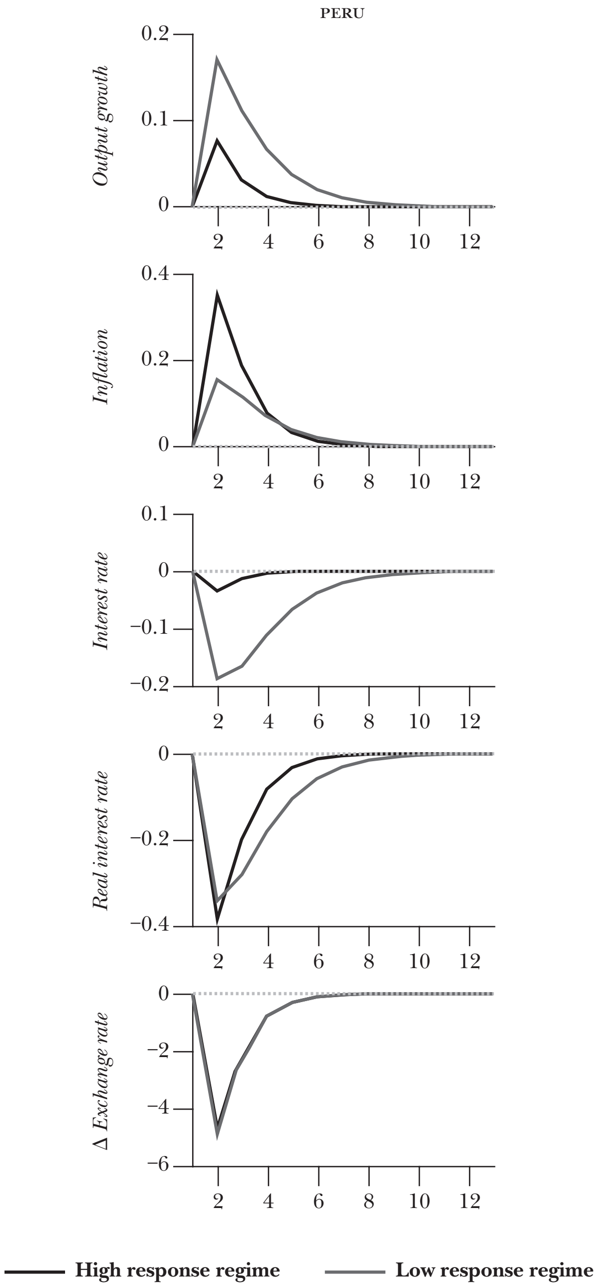


Figure 11 (cont.)

WORLD INFLATION SHOCK IRFs



4.4 Counterfactuals

As shown by the impulse response functions, there are differences in the magnitudes and even signs of the responses under the different regimes. Our estimated model allows one to perform counterfactual analysis of what could have happened if policies had been different. In Figures 12 to 16, we show the actual behavior of five observables: GDP growth, inflation, nominal interest rate, ex-post real interest rate, and nominal depreciation, and compare them with the hypothetical behavior that may have been observed under a constant high interest rate response regime and a constant low response regime. Table 3 reports the average, standard deviation and coefficient of variation of the actual observables and their simulated counterfactuals.

Looking at the figures one realizes that the regime switches that occurred throughout Latin America towards more responsive interest rate reaction functions helped to prevent many inflationary runs, several large nominal exchange rate depreciations, and large volatility of the nominal variables. Table 3 confirms that there would have been less average inflation under the high interest rate response regime than the observed average inflation, which is lower than the average inflation under the low interest rate response regime. Not only would average inflation have been lower, but the standard deviation of inflation would also have been lower under the counterfactual high response regime than in the observed one, which is lower than the counterfactual low response regime. The high response regime does not imply higher average nominal interest rates or higher average real interest rates, while their variability under that high response regime would have been less than the observed ones. Average nominal depreciation under the high response regime turned out to be smaller and less volatile. The reduction in the level and volatility of the nominal variables under the high response regime does not imply a sacrifice in terms of output growth, or on its volatility.

Table3

SUMMARY STATISTICS

Variable	*Series*	*Brazil*			*Chile*			*Colombia*			*Mexico*			*Peru*		
		Average	*SD*	*CV*	*Average*	*SD*	*CV*	*Average*	*SD*	*CV*	*Average*	*SD*	*CV*	*Average*	*SD*	*CV*
	Observed	0.64	1.26	1.97	3.85	4.21	1.09	3.44	4.27	1.11	2.26	5.73	2.53	4.65	3.31	0.71
Output growth	High response	0.99	3.28	3.30	3.75	2.78	0.74	3.42	4.11	1.22	1.77	4.84	2.73	4.97	2.74	0.55
	Low response	1.00	3.63	3.62	3.65	4.75	1.30	3.37	4.47	1.31	3.46	8.85	2.56	5.38	5.71	1.06
	Observed	6.31	3.72	0.59	3.06	2.51	0.82	9.84	7.43	1.12	20.15	24.78	1.23	3.62	3.18	0.88
Inflation	High response	3.89	2.73	0.70	2.93	1.90	0.65	7.13	4.93	1.04	11.08	7.65	0.69	3.34	2.12	0.64
	Low response	15.73	5.80	0.37	3.14	3.88	1.23	17.13	12.26	0.89	26.83	23.69	0.88	6.89	8.68	1.26
	Observed	16.49	7.00	0.42	4.59	2.04	0.44	12.56	10.50	2.33	25.38	26.36	1.04	6.91	6.03	0.87

Variable	Series	Brazil			Chile			Colombia			Mexico			Peru		
		Average	*SD*	*CV*	*Average*	*SD*	*CV*	*Average*	*SD*	*CV*	*Average*	*SD*	*CV*	*Average*	*SD*	*CV*
Interest rate	High response	10.69	3.84	0.36	4.76	1.44	0.30	9.01	6.41	1.61	18.90	13.06	0.69	5.43	2.61	0.48
	Low response	12.59	4.17	0.33	4.76	3.02	0.63	12.58	8.48	1.57	30.81	27.84	0.90	4.42	5.26	1.19
	Observed	10.18	7.58	0.74	1.53	2.15	1.40	2.72	5.13	2.43	5.23	9.08	1.74	3.29	5.73	1.74
Real interest rate	High response	6.80	3.49	0.51	1.83	2.04	1.11	1.89	2.80	3.64	7.82	7.94	1.01	2.09	2.61	1.25
	Low response	−3.14	7.36	2.34	1.63	1.48	0.91	−4.56	8.94	1.07	3.98	15.61	3.92	−2.47	13.33	5.40
	Observed	1.64	9.13	5.58	0.67	4.87	7.25	1.36	6.60	0.89	−0.59	4.81	8.10	0.45	2.70	6.02
Nominal depreciation	High response	−0.60	8.61	14.26	0.72	2.86	4.00	−3.15	8.44	1.34	−5.59	9.92	1.77	0.25	3.29	13.40
	Low response	11.24	9.23	0.82	1.28	7.50	5.88	6.33	9.15	0.77	5.74	14.88	2.59	3.77	8.15	2.16

Figure 12

COUNTERFACTUAL FOR HIGH AND LOW RESPONSE REGIMES FOR BRAZIL

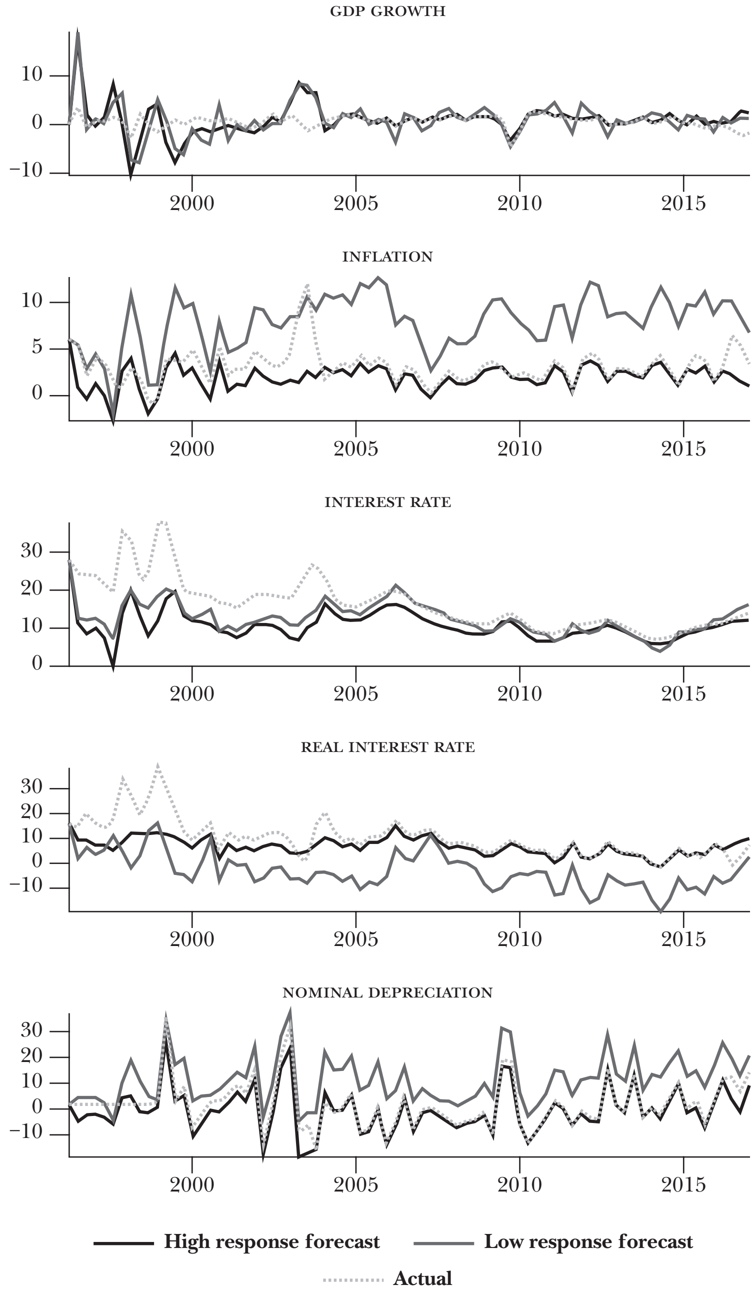


Figure 13

COUNTERFACTUAL FOR HIGH AND LOW RESPONSE REGIMES FOR CHILE

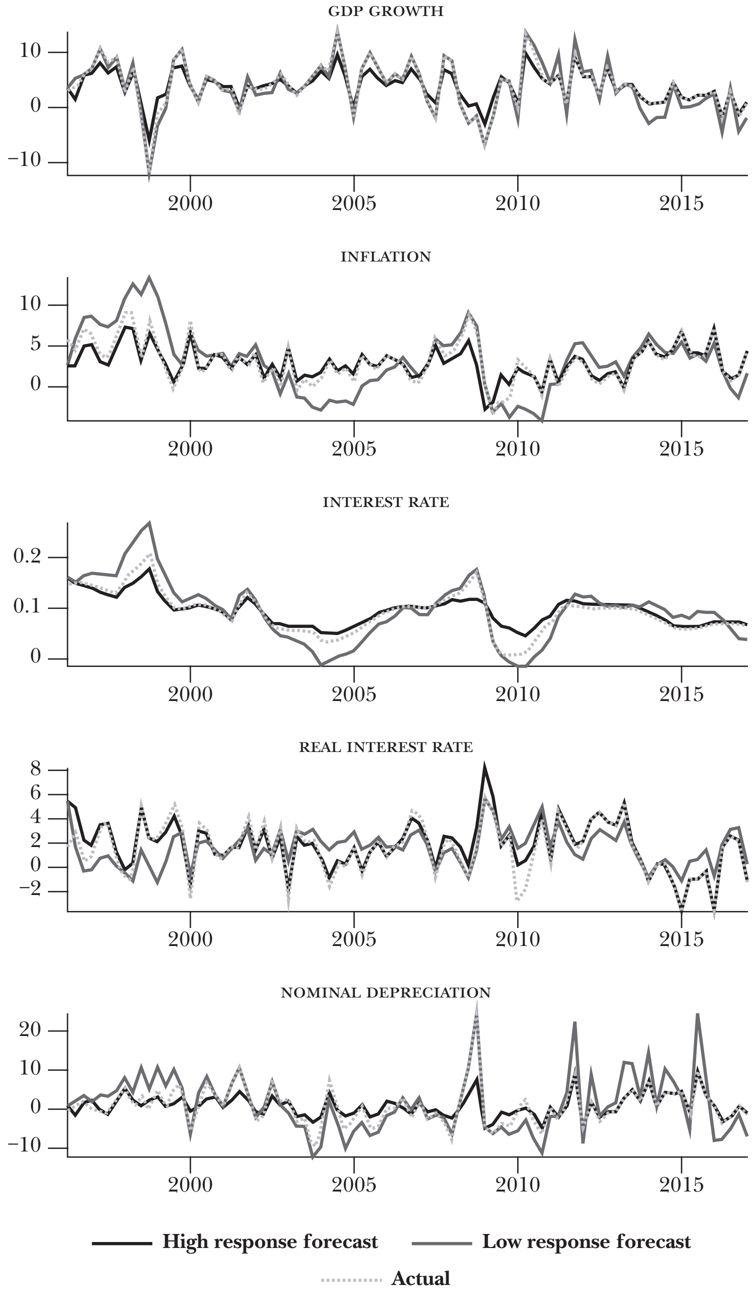


Figure 14

COUNTERFACTUAL FOR HIGH AND LOW RESPONSE REGIMES FOR COLOMBIA

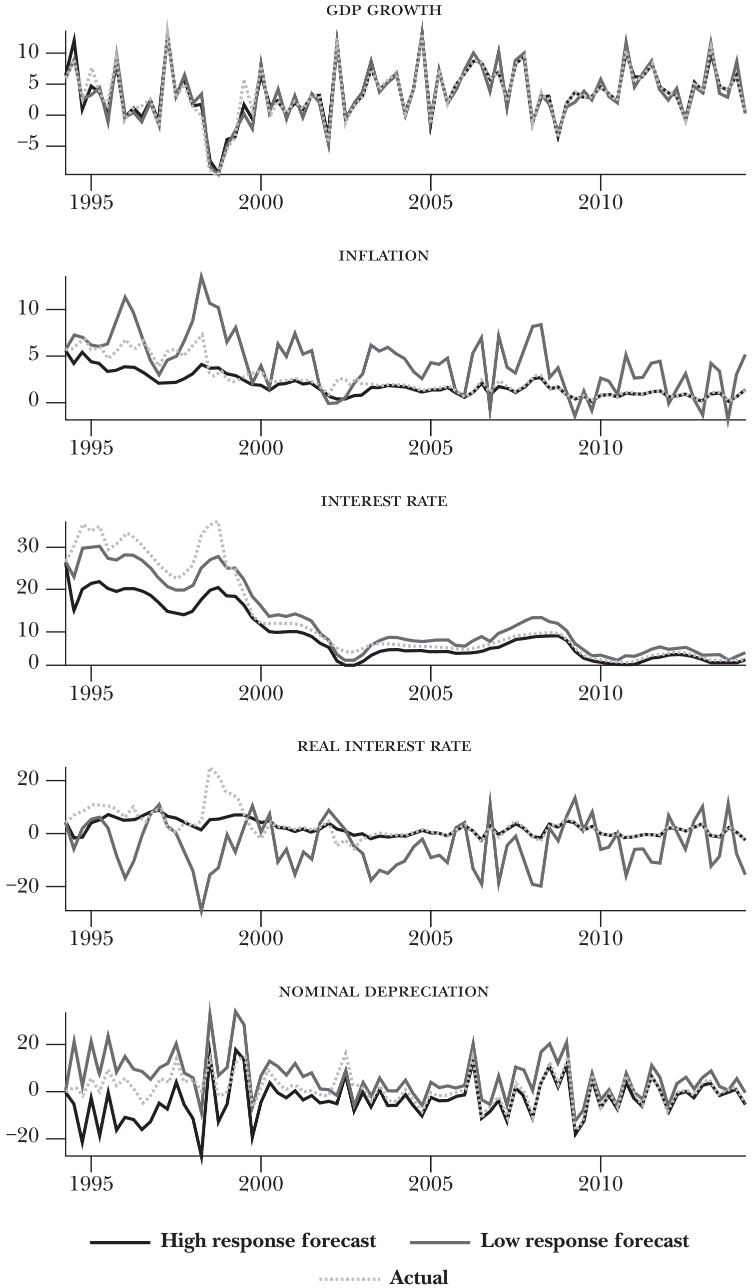


Figure 15

COUNTERFACTUAL FOR HIGH AND LOW RESPONSE REGIMES FOR MEXICO

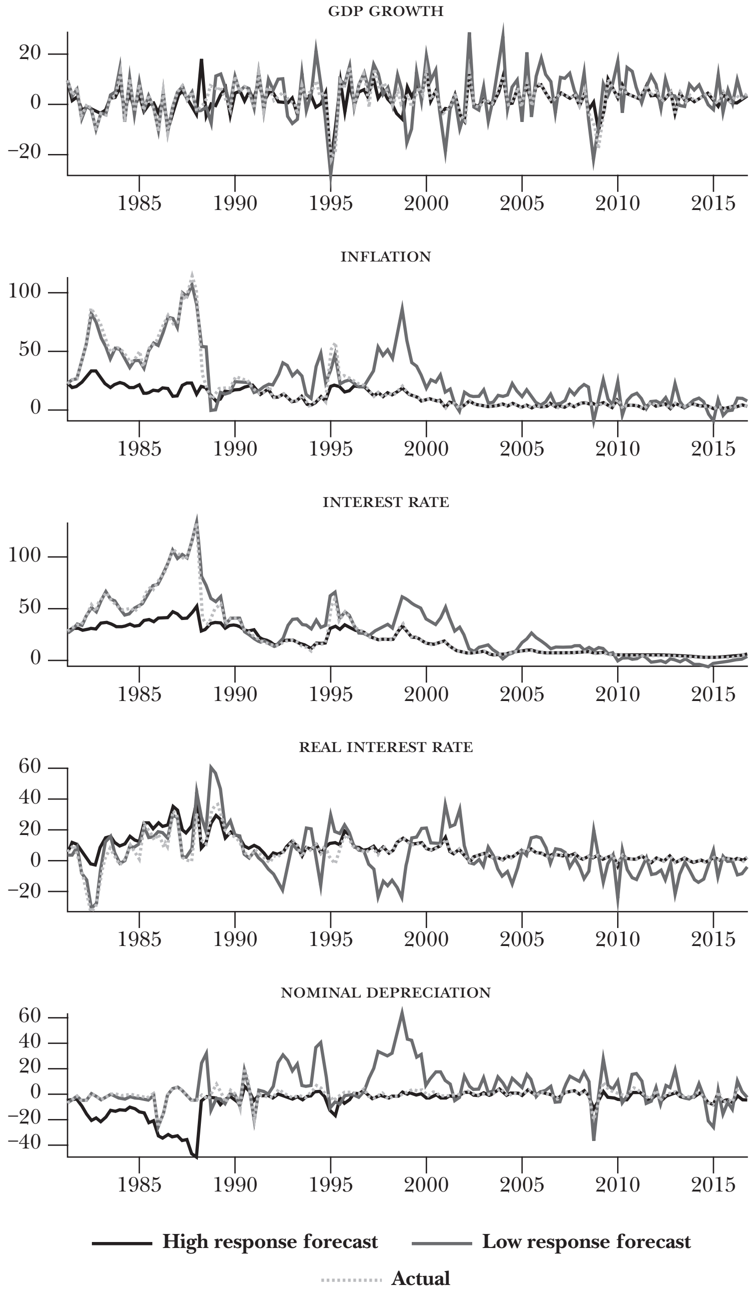
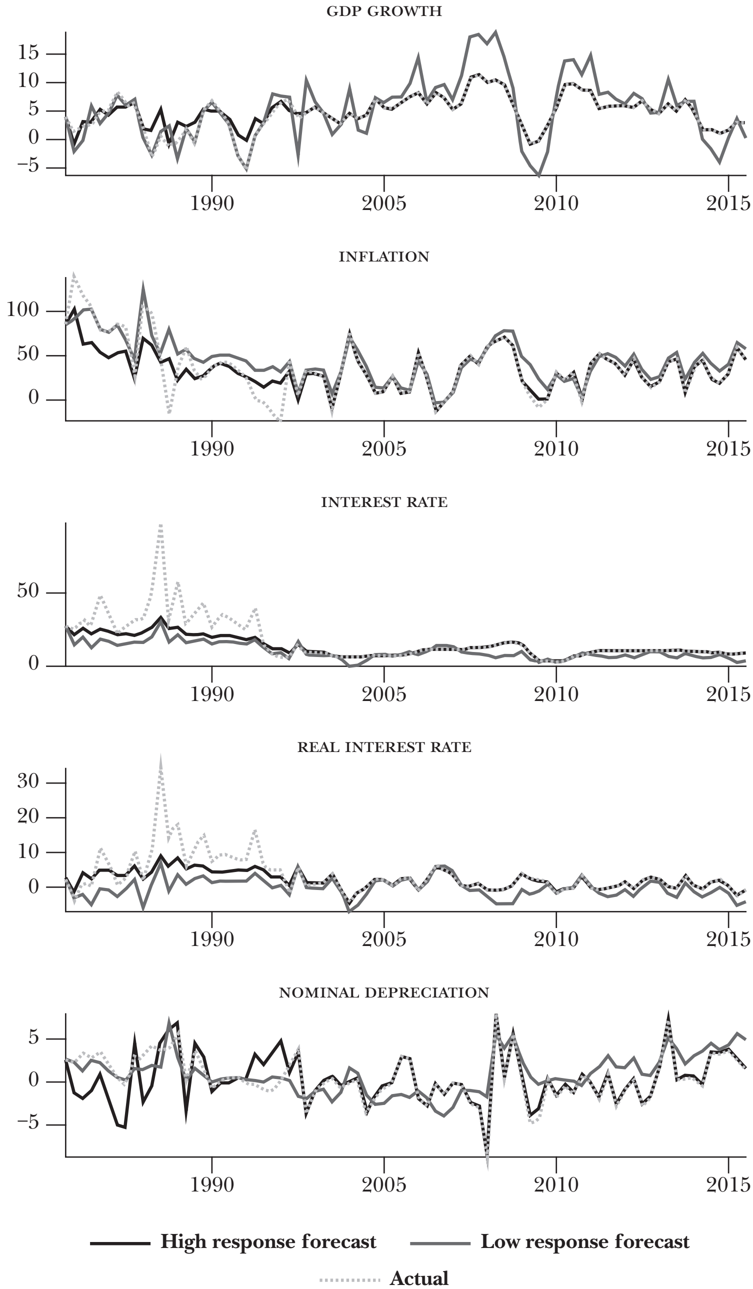


Figure 16

COUNTERFACTUAL FOR HIGH AND LOW RESPONSE REGIMES FOR PERU



5. CONCLUSIONS

In this paper we explore whether the central bank reforms implemented in the 1990s in Brazil, Chile, Colombia, Mexico and Peru, which lead to an inflation targeting framework, represented a regime switch in their monetary policies. The estimation of a Markov-switching DSGE open economy monetary model allows us to identify regime shifts of an interest rate reaction function together with the inflation determination process of a hybrid New Keynesian open economy Phillips curve. Our estimation identifies the following periods as having high interest rate responses to inflation: from 1999Q3 to 2002Q3 and from 2004Q1 onwards for Brazil; from the beginning of the sample in 1996Q2 to 2007Q4 and from 2010Q1 onwards for Chile; from 2000Q1 onwards for Colombia; from 1988Q2 to 1988Q3, from 1992Q1 to 1994Q4, and from 1997Q2 onwards for Mexico; 1997Q4 to 1998Q1, in 1998Q4, and from 2002Q1 onwards for Peru. The introduction of inflation targeting is associated with a marked regime switch towards a more reactive interest rate policy.

The estimation of the structural parameters associated with the hybrid New Keynesian open economy Phillips curve indicates that when changing from a low interest response to a high one as it happened chronologically in all countries (except Chile), Brazil experienced a drop in inflation inertia and a more responsive trade-off between output gap and inflation, Colombia experienced a higher inflation inertia and a reduction in the slope of the Phillips curve, Mexico also experienced higher inflation inertia and a slightly reduction in the large slope of the Phillips curve, and Peru experienced the same level of inertia and a more responsive trade-off. Meanwhile, as stated before, Chile began our sample with a high interest rate response to inflation and loosened the policy from 2008Q1 to 2009Q4. Then, when moving from a high interest rate response to a low one, Chile had an increase in inflation inertia without changes in the small slope of the Phillips curve.

The estimation of the structural parameters associated with the interest rate reaction function indicates that when going from a low interest response to a high one as it happened chronologically in all countries (except Chile), Brazil exhibited increased persistence of interest rates, decreased sensitivity to output deviations, and no change in response to exchange rate fluctuations. Colombia exhibited similar persistence of interest rates, less sensitivity to output deviations, and more sensitivity to exchange rate fluctuations. Mexico exhibited smaller persistence of interest rates and smaller sensitivity to output deviations and exchange rate fluctuations. Peru exhibited higher persistence of interest rates, lower sensitivity to output deviations and similar responses to exchange rate fluctuations. Finally, for Chile, when moving from a high interest rate response to a low one, interest rates exhibited less persistence and the weight on output deviations was larger, as expected from the countercyclical stance of their monetary policy.

When comparing the impulse response functions under the two regimes, we notice some differences in magnitude and sign. An unexpected increase in monetary policy, appreciates the currency, while it lowers inflation and output. Under high policy response regimes appreciations are larger in Chile and Peru, where real interest rates increase by more and inflation drops are larger. Only in the case of Chile has the observed output contraction been larger under the high policy response regime. This may be explained by the fact that the Chile's low response regime was implemented with countercyclical motives only once the inflation targeting regime was consolidated.

Our counterfactual analysis allows us to argue that the regime switches towards more responsive interest rate reaction functions helped to avoid many inflationary runs, several large nominal exchange rate depreciations and large volatility of the nominal variables. This reduction of nominal volatility did not come at the cost of smaller output growth or the need of larger output fluctuations. Therefore, we conclude that the introduction of inflation targeting represented a favorable regime switch in the conduct of monetary policy in Latin America.

ANNEX

A. Estimated Parameters

Table 4

ESTIMATED PARAMETERS OF BRAZIL

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
$\chi_{p,\xi^{coef=1}}$	Beta	0.1738	0.0482	0.1299	0.0319	0.4303
$\chi_{p,\xi^{coef=2}}$	Beta	0.4471	0.3213	0.2214	0.1352	0.8285
$\kappa_{p,\xi^{coef=1}}$	Gamma	1.1362	0.9582	0.2401	0.8484	1.6328
$\kappa_{p,\xi^{coef=2}}$	Gamma	0.6296	0.4708	0.3204	0.27	1.2559
$\rho_{R,\xi^{coef=1}}$	Beta	0.7629	0.7847	0.048	0.6917	0.8144
$\rho_{R,\xi^{coef=2}}$	Beta	0.6113	0.7513	0.1814	0.2252	0.813
$\psi_{\pi,\xi^{coef=1}}$	Gamma	3.4901	3.6914	0.3406	2.733	3.8618
$\psi_{\pi,\xi^{coef=2}}$	Gamma	1.0417	0.7656	0.296	0.6815	1.4375
$\psi_{y,\xi^{coef=1}}$	Gamma	0.3013	0.1377	0.3081	0.075	0.9818
$\psi_{y,\xi^{coef=2}}$	Gamma	0.8799	0.378	0.6633	0.2204	2.0191
$\psi_{\Delta,\xi^{coef=1}}$	Gamma	0.0435	0.0323	0.025	0.0156	0.098
$\psi_{\Delta,\xi^{coef=2}}$	Gamma	0.0422	0.0268	0.0391	0.0139	0.1547
α	Beta	0.076	0.0436	0.0454	0.0291	0.1778
r	Gamma	3.6731	3.0345	0.7512	2.6661	4.8276
τ	Beta	0.2792	0.2896	0.1012	0.147	0.4755
ρ_a	Beta	0.424	0.0606	0.2548	0.0349	0.7505

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
ρ_q	Beta	0.3014	0.1127	0.2563	0.0512	0.8563
ρ_{y*}	Beta	0.9818	0.999	0.0356	0.9387	0.9992
$\rho_{\pi*}$	Beta	0.3715	0.3471	0.0892	0.2455	0.535
$H_{1,2}^{coef=1}$	Beta	0.1603	0.0428	0.1461	0.039	0.4719
$H_{2,1}^{coef=2}$	Beta	0.2257	0.1262	0.1173	0.0997	0.4375
$\sigma_{R,\xi^{vol=1}}$	Inv.Gamma	5.3145	6.0774	0.704	4.0033	6.2124
$\sigma_{R,\xi^{vol=2}}$	Inv.Gamma	3.3642	2.3893	1.0409	2.262	5.2165
$\sigma_{q,\xi^{vol=1}}$	Inv.Gamma	5.791	6.202	0.342	5.2568	6.2795
$\sigma_{q,\xi^{vol=2}}$	Inv.Gamma	4.2554	3.7875	0.9045	3.1056	5.8516
$\sigma_{a,\xi^{vol=1}}$	Inv.Gamma	4.6972	4.5965	0.1259	4.5092	4.9424
$\sigma_{a,\xi^{vol=2}}$	Inv.Gamma	4.7999	4.6046	0.2162	4.5204	5.188
$\sigma_{y^*,\xi^{vol=1}}$	Inv.Gamma	3.5522	2.4173	0.9944	2.3191	5.2352
$\sigma_{y^*,\xi^{vol=2}}$	Inv.Gamma	6.9291	7.8618	1.0058	5.0694	8.0384
$\sigma_{\pi^*,\xi^{vol=1}}$	Inv.Gamma	4.8214	4.0789	0.6289	4.002	5.8516
$\sigma_{\pi^*,\xi^{vol=2}}$	Inv.Gamma	6.1201	6.72	0.7868	4.811	7.1394
$H_{1,2}^{vol=1}$	Beta	0.3071	0.2284	0.1399	0.1241	0.5589
$H_{2,1}^{vol=2}$	Beta	0.1458	0.0487	0.1472	0.0278	0.4982

Table 5

ESTIMATED PARAMETERS OF CHILE

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
$\chi_{p,\xi^{coef=1}}$	Beta	0.2053	0.1535	0.0715	0.1027	0.3366
$\chi_{p,\xi^{coef=2}}$	Beta	0.5124	0.7801	0.1992	0.1913	0.8204
$\kappa_{p,\xi^{coef=1}}$	Gamma	0.0765	0.0648	0.03	0.0368	0.1346
$\kappa_{p,\xi^{coef=2}}$	Gamma	0.0631	0.0363	0.0208	0.0331	0.1008
$\rho_{R,\xi^{coef=1}}$	Beta	0.9215	0.8787	0.0462	0.8525	0.9788
$\rho_{R,\xi^{coef=2}}$	Beta	0.4912	0.4859	0.0359	0.4328	0.5514
$\psi_{\pi,\xi^{coef=1}}$	Gamma	2.7337	1.2134	1.6982	1.079	5.4875
$\psi_{\pi,\xi^{coef=2}}$	Gamma	0.8692	0.8601	0.0915	0.7058	1.0166
$\psi_{y,\xi^{coef=1}}$	Gamma	0.5594	0.3792	0.2495	0.3015	0.8963
$\psi_{y,\xi^{coef=2}}$	Gamma	0.434	0.4119	0.1508	0.2317	0.7397
$\psi_{\Delta,\xi^{coef=1}}$	Gamma	0.0816	0.0441	0.0774	0.0229	0.2694
$\psi_{\Delta,\xi^{coef=2}}$	Gamma	0.0662	0.0561	0.0334	0.026	0.1325
α	Beta	0.0539	0.0526	0.0144	0.0331	0.0798
r	Gamma	2.2813	1.6134	0.9063	0.9927	3.9223
τ	Beta	0.16	0.1537	0.0389	0.1068	0.2349
ρ_a	Beta	0.1553	0.1155	0.0403	0.0925	0.224
ρ_q	Beta	0.1599	0.173	0.0632	0.0639	0.2696

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
ρ_{y^*}	Beta	0.9579	0.9601	0.0134	0.9344	0.9784
ρ_{π^*}	Beta	0.3129	0.2894	0.0601	0.2253	0.4259
$H_{1,2}^{coef=1}$	Beta	0.0808	0.0234	0.0633	0.0141	0.21
$H_{2,1}^{coef=2}$	Beta	0.0521	0.0288	0.0222	0.0225	0.0942
$\sigma_{R,\xi^{vol=1}}$	Inv.Gamma	0.5788	0.708	0.0994	0.3924	0.7324
$\sigma_{R,\xi^{vol=2}}$	Inv.Gamma	3.3239	0.5593	1.72	0.5233	5.5594
$\sigma_{q,\xi^{vol=1}}$	Inv.Gamma	6.4758	8.2896	0.9905	5.2872	8.4767
$\sigma_{q,\xi^{vol=2}}$	Inv.Gamma	5.3403	2.6141	1.7776	2.2494	7.698
$\sigma_{a,\xi^{vol=1}}$	Inv.Gamma	3.9563	3.8918	0.3844	3.4115	4.7018
$\sigma_{a,\xi^{vol=2}}$	Inv.Gamma	6.1979	3.9948	1.5389	3.8555	8.1539
$\sigma_{y^*,\xi^{vol=1}}$	Inv.Gamma	3.4781	3.5359	0.5735	2.7111	4.5585
$\sigma_{y^*,\xi^{vol=2}}$	Inv.Gamma	5.4652	3.8204	1.2679	3.5727	7.7182
$\sigma_{\pi^*,\xi^{vol=1}}$	Inv.Gamma	7.2118	8.1589	0.9146	5.7006	8.754
$\sigma_{\pi^*,\xi^{vol=2}}$	Inv.Gamma	4.6023	2.8905	1.2114	2.8584	6.702
$H_{1,2}^{vol=1}$	Beta	0.0307	0.0275	0.0149	0.0107	0.0589
$H_{2,1}^{vol=2}$	Beta	0.182	0.1298	0.0535	0.1096	0.2873

Table 6

ESTIMATED PARAMETERS OF COLOMBIA

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
$\chi_{p,\xi^{coef=1}}$	Beta	0.7092	0.3151	0.1375	0.4474	0.8981
$\chi_{p,\xi^{coef=2}}$	Beta	0.313	0.592	0.1163	0.1498	0.5307
$\kappa_{p,\xi^{coef=1}}$	Gamma	0.5845	0.6924	0.1267	0.3863	0.8068
$\kappa_{p,\xi^{coef=2}}$	Gamma	1.9982	1.6185	0.2196	1.6591	2.3484
$\rho_{R,\xi^{coef=1}}$	Beta	0.7298	0.8046	0.0436	0.6633	0.8071
$\rho_{R,\xi^{coef=2}}$	Beta	0.7065	0.6574	0.0349	0.6491	0.7621
$\psi_{\pi,\xi^{coef=1}}$	Gamma	3.2941	1.8019	1.1685	1.8292	4.9853
$\psi_{\pi,\xi^{coef=2}}$	Gamma	0.9746	0.8772	0.1204	0.7722	1.1641
$\psi_{y,\xi^{coef=1}}$	Gamma	0.3849	0.2018	0.1315	0.1969	0.6058
$\psi_{y,\xi^{coef=2}}$	Gamma	0.7379	0.5394	0.3263	0.3355	1.2305
$\psi_{\Delta,\xi^{coef=1}}$	Gamma	0.137	0.1147	0.0206	0.1068	0.1752
$\psi_{\Delta,\xi^{coef=2}}$	Gamma	0.0463	0.0296	0.042	0.0148	0.0844
α	Beta	0.1132	0.084	0.0292	0.0722	0.1641
r	Gamma	6.8509	5.4332	0.5786	5.8481	7.6151
τ	Beta	0.2445	0.1537	0.0852	0.1398	0.4253
ρ_a	Beta	0.1628	0.1377	0.0336	0.1122	0.2207
ρ_q	Beta	0.1291	0.1212	0.0685	0.0355	0.2556

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
ρ_{y^*}	Beta	0.9659	0.9619	0.015	0.9391	0.9864
ρ_{π^*}	Beta	0.2303	0.2119	0.0575	0.1442	0.3319
$H_{1,2}^{coef=1}$	Beta	0.0863	0.0163	0.0566	0.0239	0.2236
$H_{2,1}^{coef=2}$	Beta	0.1566	0.0367	0.0871	0.048	0.3472
$\sigma_{R,\xi^{vol=1}}$	Inv.Gamma	0.8134	0.3186	0.4212	0.3368	1.5494
$\sigma_{R,\xi^{vol=2}}$	Inv.Gamma	6.8695	5.962	0.4012	6.2046	7.4314
$\sigma_{q,\xi^{vol=1}}$	Inv.Gamma	5.5065	5.5265	0.7479	4.3785	6.5215
$\sigma_{q,\xi^{vol=2}}$	Inv.Gamma	7.2084	6.3208	0.5752	6.3062	8.2431
$\sigma_{a,\xi^{vol=1}}$	Inv.Gamma	5.0036	5.7915	0.6898	3.9735	5.8133
$\sigma_{a,\xi^{vol=2}}$	Inv.Gamma	6.0725	8.2629	1.1733	4.3541	7.8001
$\sigma_{y^*,\xi^{vol=1}}$	Inv.Gamma	1.6996	0.6976	0.8043	0.5943	2.9033
$\sigma_{y^*,\xi^{vol=2}}$	Inv.Gamma	3.0673	2.673	0.3084	2.6163	3.5536
$\sigma_{\pi^*,\xi^{vol=1}}$	Inv.Gamma	5.0864	5.0467	0.3212	4.3974	5.4164
$\sigma_{\pi^*,\xi^{vol=2}}$	Inv.Gamma	2.4292	2.907	0.4745	1.6841	3.073
$H_{1,2}^{vol=1}$	Beta	0.0607	0.0553	0.0809	0.0089	0.2931
$H_{2,1}^{vol=2}$	Beta	0.1023	0.136	0.0601	0.0257	0.2056

Table 7

ESTIMATED PARAMETERS OF MEXICO

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
$\chi_{p,\xi^{coef=1}}$	Beta	0.8564	0.9444	0.1206	0.6316	0.9739
$\chi_{p,\xi^{coef=2}}$	Beta	0.6134	0.7351	0.0816	0.496	0.7669
$\kappa_{p,\xi^{coef=1}}$	Gamma	2.1643	2.2281	0.1162	1.9357	2.3318
$\kappa_{p,\xi^{coef=2}}$	Gamma	2.3736	2.0484	0.4645	1.7729	3.3246
$\rho_{R,\xi^{coef=1}}$	Beta	0.458	0.4138	0.0551	0.3897	0.5541
$\rho_{R,\xi^{coef=2}}$	Beta	0.6279	0.735	0.142	0.3992	0.7734
$\psi_{\pi,\xi^{coef=1}}$	Gamma	1.8458	1.7333	0.0627	1.7431	1.9526
$\psi_{\pi,\xi^{coef=2}}$	Gamma	0.6154	0.8004	0.1313	0.4424	0.823
$\psi_{y,\xi^{coef=1}}$	Gamma	0.7265	0.7031	0.0629	0.602	0.8016
$\psi_{y,\xi^{coef=2}}$	Gamma	0.8310	0.8491	0.1824	0.8039	0.8562
$\psi_{\Delta,\xi^{coef=1}}$	Gamma	0.1108	0.1093	0.0335	0.0961	0.1254
$\psi_{\Delta,\xi^{coef=2}}$	Gamma	0.3408	0.0899	0.2613	0.0775	0.6386
α	Beta	0.2689	0.238	0.0362	0.2123	0.3289
r	Gamma	2.1004	1.7134	0.2971	1.6491	2.5185
τ	Beta	0.3256	0.3347	0.0216	0.2756	0.3478
ρ_a	Beta	0.2007	0.2273	0.0444	0.1302	0.2724
ρ_q	Beta	0.4305	0.3102	0.0879	0.2889	0.5608

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
ρ_{y^*}	Beta	0.9042	0.9236	0.0217	0.8646	0.9359
ρ_{π^*}	Beta	0.7824	0.8252	0.0428	0.7059	0.8408
$H_{1,2}^{coef=1}$	Beta	0.1161	0.1094	0.0361	0.0707	0.1842
$H_{2,1}^{coef=2}$	Beta	0.2108	0.2528	0.061	0.097	0.3049
$\sigma_{R,\xi^{vol=1}}$	Inv.Gamma	4.5438	4.5083	0.1139	4.3641	4.7386
$\sigma_{R,\xi^{vol=2}}$	Inv.Gamma	5.8216	5.8513	0.038	5.7552	5.8765
$\sigma_{q,\xi^{vol=1}}$	Inv.Gamma	3.121	3.0513	0.0634	3.012	3.2223
$\sigma_{q,\xi^{vol=2}}$	Inv.Gamma	4.4066	4.3941	0.0598	4.3069	4.5035
$\sigma_{a,\xi^{vol=1}}$	Inv.Gamma	3.2222	3.2116	0.0709	3.101	3.3199
$\sigma_{a,\xi^{vol=2}}$	Inv.Gamma	7.4444	7.3618	0.1203	7.2862	7.6952
$\sigma_{y^*,\xi^{vol=1}}$	Inv.Gamma	6.7571	6.7489	0.0777	6.6572	6.9247
$\sigma_{y^*,\xi^{vol=2}}$	Inv.Gamma	7.3328	7.3367	0.0746	7.2085	7.4538
$\sigma_{\pi^*,\xi^{vol=1}}$	Inv.Gamma	5.09	5.0717	0.0477	5.0186	5.1741
$\sigma_{\pi^*,\xi^{vol=2}}$	Inv.Gamma	9.5155	9.4522	0.0774	9.3967	9.6475
$H_{1,2}^{vol=1}$	Beta	0.1922	0.1021	0.0825	0.0958	0.339
$H_{2,1}^{vol=2}$	Beta	0.109	0.0925	0.0397	0.0577	0.1836

Table 8

ESTIMATED PARAMETERS OF PERU

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
$\chi_{p,\xi^{coef=1}}$	Beta	0.1318	0.0928	0.0787	0.0321	0.2885
$\chi_{p,\xi^{coef=2}}$	Beta	0.1471	0.1609	0.0779	0.0352	0.286
$\kappa_{p,\xi^{coef=1}}$	Gamma	0.5011	0.3816	0.1019	0.3481	0.6833
$\kappa_{p,\xi^{coef=2}}$	Gamma	0.0565	0.0672	0.0171	0.0294	0.0863
$\rho_{R,\xi^{coef=1}}$	Beta	0.697	0.7132	0.0412	0.6211	0.753
$\rho_{R,\xi^{coef=2}}$	Beta	0.6254	0.6094	0.0656	0.5227	0.7344
$\psi_{\pi,\xi^{coef=1}}$	Gamma	1.9066	1.4844	0.5911	1.3059	3.309
$\psi_{\pi,\xi^{coef=2}}$	Gamma	0.9226	0.5921	0.5032	0.444	1.7992
$\psi_{y,\xi^{coef=1}}$	Gamma	0.4092	0.2172	0.2179	0.1659	0.859
$\psi_{y,\xi^{coef=2}}$	Gamma	0.5639	0.4629	0.2286	0.3263	1.0481
$\psi_{\Delta,\xi^{coef=1}}$	Gamma	0.1725	0.1612	0.0326	0.1215	0.2283
$\psi_{\Delta,\xi^{coef=2}}$	Gamma	0.1506	0.1693	0.0247	0.1139	0.1925
α	Beta	0.0393	0.0389	0.0166	0.0201	0.0757
r	Gamma	8.8041	1.8227	4.3353	1.4432	13.308
τ	Beta	0.1306	0.0582	0.0516	0.0522	0.2153
ρ_a	Beta	0.3605	0.3714	0.0521	0.2759	0.4462
ρ_q	Beta	0.3924	0.3134	0.0721	0.2687	0.5028

Parameter	*Distribution*	*Mean*	*Mode*	*Standard dev.*	*10%*	*90%*
ρ_{y*}	Beta	0.9682	0.9756	0.0133	0.9445	0.9877
$\rho_{\pi*}$	Beta	0.416	0.3717	0.0559	0.3277	0.5085
$H_{1,2}^{coef=1}$	Beta	0.0721	0.0662	0.0257	0.0276	0.1129
$H_{2,1}^{coef=2}$	Beta	0.0565	0.0615	0.0265	0.0191	0.101
$\sigma_{R,\xi^{vol=1}}$	Inv.Gamma	2.4271	1.0314	1.8471	0.9785	6.2415
$\sigma_{R,\xi^{vol=2}}$	Inv.Gamma	7.6316	7.4037	1.2162	5.286	9.3854
$\sigma_{q,\xi^{vol=1}}$	Inv.Gamma	4.1378	3.6176	0.7587	3.5144	6.1017
$\sigma_{q,\xi^{vol=2}}$	Inv.Gamma	5.1138	3.2364	2.213	2.7457	8.9518
$\sigma_{a,\xi^{vol=1}}$	Inv.Gamma	2.7075	2.2299	0.8397	2.0503	4.9969
$\sigma_{a,\xi^{vol=2}}$	Inv.Gamma	6.0456	3.837	1.7465	3.4937	8.618
$\sigma_{y^*,\xi^{vol=1}}$	Inv.Gamma	2.1448	0.2633	1.2789	0.2842	4.4775
$\sigma_{y^*,\xi^{vol=2}}$	Inv.Gamma	3.5942	0.2823	2.6484	0.3459	8.0066
$\sigma_{\pi^*,\xi^{vol=1}}$	Inv.Gamma	5.0435	4.6914	0.5071	4.3391	6.001
$\sigma_{\pi^*,\xi^{vol=2}}$	Inv.Gamma	5.0472	4.5065	1.3062	3.1803	7.6507
$H_{1,2}^{vol=1}$	Beta	0.0849	0.0213	0.1287	0.0103	0.4463
$H_{2,1}^{vol=2}$	Beta	0.1719	0.0582	0.1171	0.0427	0.4136

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Fiscal Policy and Inflation: Understanding the Role of Expectations in Mexico

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Abstract

We estimate a hidden Markov model where inflation is determined by government deficits financed through money creation and/or by destabilizing expectations dynamics (expectations can potentially divorce inflation from fundamentals). The baseline model, proposed by Sargent et al. (2009), is used to analyze the interaction between fiscal deficits, inflation expectations, and inflation in Mexico. The model is able to distinguish between causes and remedies of hyperinflation, such as persistent or transitory shocks to seigniorage-financed fiscal deficits, de-anchoring of inflation expectations from fiscal fundamentals, and cosmetic (non-fundamental) monetary reforms. The behavior of monetized deficits provides an adequate account of high inflation episodes and stabilizations for the period 1969-1994. We then extend

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the model to analyze the possibility that fiscal policy can affect inflation expectations in a context of Central Bank independence, as is the case of Mexico after 1994. We find evidence that the exchange rate and sovereign interest rate spreads influence the evolution of aggregate prices.
Keywords: inflation, inflation expectations, fiscal policy.
JEL: E31, E42, E52, E63.

1.INTRODUCTION

As in other countries in Latin America during the second half of the twentieth century, Mexico suffered several episodes of annual inflation rates above one hundred percent. These high inflation episodes were typically accompanied by elevated levels of public deficit financed with monetary expansions.[1] Until 1994, a regime of fiscal dominance prevailed, where the Central Bank adjusted its monetary policy to the financial requirements of the fiscal authority. Thereafter, the autonomy of Banco de México was established and inflation started a process of moderation.

To analyze the interaction between inflation, inflation expectations, and fiscal deficits in Mexico, we utilize the model developed by Sargent *et al.* (2009). This model has been used to infer the determinants of hyperinflations and stabilizations in different countries in Latin America (Argentina, Bolivia, Brazil, Chile, and Peru). It gives a central role to government deficits financed through money creation, but also to destabilizing expectations that can, under certain conditions, divorce inflation from fundamentals. The baseline framework consists of a non-linear hidden Markov model with the following key components: (i) a standard demand function for real balances, an adaptive scheme for the expected rate of inflation,[2] (iii) a government budget constraint that relates fiscal deficits to monetary

[1] Fischer *et al.* (2002), Catao and Terrones (2005), and Lin and Chu (2013), among others, document international evidence regarding the relationship between ination rates, scal decits, and money supply. Rogers and Wang (1994) estimate that between 1977 and 1990, scal and monetary shocks accounted for 60 percent of the variance of ination in Mexico.

[2] Agents have adaptive expectations or backward-looking expectations when these are formed by extrapolating past values of the variable being predicted.

supply, and (iv) a stochastic fiscal deficit that follows a hidden Markov process. With these components, the model is able to distinguish between the causes and remedies of hyperinflations, such as persistent or transitory shocks to seigniorage-financed fiscal deficits, de-anchoring of inflation expectations from fiscal fundamentals, and cosmetic (non-fundamental) monetary reforms. Sargent *et al.* (2009) conclude that the behavior of monetized deficits determined most hyperinflations and stabilizations for the set of countries they studied.

We first use the baseline model to account for the evolution of inflation in Mexico between 1969 and 2016. The methodology uses a series for inflation, interpreting the density of the inflation series as a likelihood function in order to estimate the history of fiscal deficits and the process of the formation of inflation expectations that better account for the evolution of inflation. This approach is convenient given numerous methodological modifications in the construction of public accounts, the sometimes less-than-ideal transparency in historical series, and the fluctuations in the perception of economic agents of what constitutes fiscal responsibility for the government (e.g., bailouts of the financial system or sub-national governments). These problems plague historical accounts of events in developing economies. The estimated sequence of fiscal deficits is then compared to available data for government deficits and a historic narrative of the events associated with episodes of high inflation and stabilizations. In line with the results for other countries, the model suggests that the evolution of fiscal deficits is central in explaining the behavior of inflation in Mexico. Furthermore, it provides a description of the formation of inflation expectations. For example, the parameters of the model suggest that inflation must be high for several consecutive periods in order to de-anchor inflation expectations and generate an inflation spiral.

For the period of decreasing inflation that started in the second half of the 1990s, the baseline model suggests that the level of fiscal deficits financed through monetary expansion is modest. This interpretation, however, is not fully satisfactory as the Central Bank became independent in 1994. Thus, a theory that contemporaneously links inflation to fiscal deficits through the monetary channel seems lacking if we aim to understand inflation after 1994. This motivates the following question; can we find evidence that fiscal policy affects inflation and inflation expectations even in the context of Central Bank independence?

A strand of the macroeconomic literature proposes that fiscal policy is relevant to achieving price stability even in an environment where monetary policy is conducted by an independent Central Bank.[3] We extend the baseline model along several dimensions with the objective of documenting evidence, perhaps indirect, or rebutting the possibility that fiscal policy is relevant in determining inflation and inflation expectations in a context of Central Bank independence. A variable of interest we consider is the spread in the sovereign interest rate EMBI. This variable, which can be considered forward-looking, reflects the fiscal situation of the government. To the extent that economic agents perceive potential risks in terms of the ability of the government to make debt payments, it may also affect the credibility of the Central Bank. The perception of this type of risk is incorporated in the prices of sovereign debt. The state of public finances is often considered to affect the exchange rate; this is the second variable we assess in the model. The results indicate that both variables are relevant in determining inflation expectations and inflation.[4]

We proceed as follows: Section 2 presents the baseline model and describes the mechanisms that drive the behavior of the different variables. Section 3 presents the main results for the baseline model: (1) the parameter values of the model and their implications in terms

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[3] There exists a vast literature studying the relevance of fiscal policy and its interaction with monetary policy for the determination of inflation, a seminal paper is Sargent and Wallace (1981). Though we will not attempt to provide an exhaustive set of references, some additional examples are provided by Sims (2016), Leeper (1991), Davig *et al.* (2011), Sargent and Zeira (2011), Woodford (2001), and Bianchi and Ilut (2017). For an introductory treatment of the fiscal theory of the price level, see Christiano and Fitzgerald (2000). Central Banks frequently express concern related to how fiscal imbalances may affect the effectiveness of monetary policy (e.g., Carstens and Jácome (2005) and Ramos-Francia and Torres-Garcia (2005)).

[4] There are different mechanisms through which these variables could potentially be relevant; we explore the impact through expectations and the demand for real money balances. We discuss the evidence of the extent to which these variables are influenced by international and exogenous factors, with a focus on the case of Mexico, such as prices of commodities in global markets.

of the behavior of the main variables, (2) a comparison of the inflation series generated by the model and those observed in the data, with a historical account of the events associated with the different inflation and stabilization episodes, and (3) a comparison of the series for fiscal deficits generated by the model with the historical series. Section 4 presents the extensions of the model and the main results. Section 5 provides our concluding remarks.

2. THE BASELINE MODEL

The baseline model is the one featured in Sargent *et al.* (2009), constructed to study the relationship between inflation, fiscal deficits, and inflation expectations. An ad- vantage of this model is its simple structure, which allows for the estimation of its parameters using only the historic series of one of the main variables, in our case the monthly inflation series (the estimation algorithm is described briefly in the next section and in the Appendix). With these parameters, the model accounts for an observed sequence of inflation as a result of fiscal deficits and a particular process for the formation of inflation expectations. The framework consists of three main components: a money demand function, the budget constraint of the government, a process that models the formation of expectations, and the (exogenous and stochastic) evolution of deficits. We now describe each of these components.

2.1 The Money Demand and the Government Budget Constraint

A standard money demand equation (e.g. Cagan (1956)) establishes a relationship between the nominal balances as a percentage of output Mt at time t, the price level P_t at time t, and the expectations of agents of the price level P^e_{t+1} for period $t+1$:[5]

1
$$\frac{M_t}{P_t} = \frac{1}{\gamma} - \frac{\lambda}{\gamma}\frac{P^e_{t+1}}{P_t},$$

[5] In a seminal paper, Cagan (1956) specifies a demand for real balances and backward-looking expectations to explain several European hyperinflation episodes.

where $\lambda \in (0,1)$ represents the weight that the expected price level P^e_{t+1} has on the current price level P_t, and $\gamma > 0$ is the weight that the nominal balances relative to output have on the price level at time t.[6] Thus, if the public expects a higher price level in $t+1$, their real balances demand M_t/P_t will fall.

The next equation represents the budget constraint of the government, where d_t (a stochastic variable) is the part of the real deficit of the government that is monetized (net of debt emissions, so it must be covered by printing money). Thus, the growth of nominal balances per unit of output is determined according to the following equation:

$$M_t = \theta M_{t-1} + d_t P_t, \tag{2}$$

where parameter $\theta \in (0,1)$ adjusts for growth in real output and taxes on cash balances.[7] This equation implies that larger fiscal deficits are associated with increases in the level of nominal balances as a percentage of GDP.[8]

We let $\beta_t = P^e_{t+1}/P_t$ denote the gross expected inflation rate. Using (1) and (2) it can be shown that the gross inflation rate at time t is:

6 Equation (1) can be written as $P_t = \gamma M_t + \lambda P^e_{t+1}$. Hence, $\{\lambda, \gamma\}$ represent the weights that P^e_{t+1} and M_t have on P_t, respectively.

7 Parameter θ is related to output growth in the model. Let $M_t = \frac{\hat{M}_t}{Y_t}$ where $\hat{M}_t$ are the nominal balances at time t and Y_t is output. If D_t represents the level of real fiscal deficit at time t, then the government budget constraint is $\hat{M}_t = \hat{M}_{t-1} + P_t D_t$. Dividing this equation by Y_t then: $M_t = \frac{Y_{t-1}}{Y_t} M_{t-1} + P_t d_t$. Therefore, θ can be interpreted as the inverse of the output growth factor. Consequently, this model is assuming a constant output growth rate. Quantitatively, this parameter is not relevant for our results.

8 We are defining the fiscal deficit as $d_t = g_t - \tau_t + (1+r_t) b_t - b_{t+1}$, where g_t and τ_t represent government expenditures and revenues relative to output, b_t is the level of sovereign debt relative to output and r_t is the interest rate on sovereign debt.

3 $$\pi_t = \frac{P_t}{P_{t-1}} = \frac{\theta\left(1-\lambda\beta_{t-1}\right)}{1-\lambda\beta_t - \gamma d_t}.$$

This equation suggests that inflation is a function of two variables: the expected gross inflation rate and the real fiscal deficit. According to (3), if the expected gross inflation rate β_t or fiscal deficit d_t rise, current inflation π_t will also increase.[9] It is worth mentioning that, equation (3) does not depend on the particular process through which inflation expectations are formed, or the stochastic process assumed for fiscal deficits. Nevertheless, these assumptions are crucial to determine a sequence of inflation rates $\left\{\pi_t, \pi_{t+1}, \ldots\right\}$ according to the model. The next two sections will explain the specification for the evolution of expectations and the dynamics followed by the real fiscal deficit.

2.2 Inflation Expectations

The baseline specification follows, for example, Marcet and Nicolini (2003), assuming that the public updates their beliefs on future inflation β_t using adaptive expectations. According to Sargent and Wallace (1973), agents have adaptive expectations when they take into account past information to extrapolate it to form their expectations. Specifically in this model, the gross expected inflation rate is a weighted average between the gross inflation rate and the gross expected inflation lagged one period:

4 $$\beta_{t+1} = \left(1-v\right)\beta_t + v\pi_t,$$

where $0 < v < 1$ is the weight that expectations give to past observed inflation. In related literature, this particular type of adaptive expectations is known as constant-gain expectations, given the constant weight in the process that determines the formation of expectations.[10]

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[9] This is obtained with $\lambda \in (0,1)$, $\theta \in (0,1)$, and $\gamma > 0$.

[10] For example, Branch (2004) develops a micro-founded model where agents optimally choose not to update their beliefs according to a rational expectations algorithm because the information it requires is too costly (rational expectations algorithms usually require a lot of information). In the type of models we are considering, adaptive

Assuming constant-gain expectations (CGE) is key in determining the dynamics of the model. Panel (a) of Figure 1 shows the change in gross inflation $\pi_{t+1}-\pi_t$ as a function of expectations β_t, with a constant real fiscal deficit. As shown in the Figure, there are two values of β that imply a constant inflation equilibrium: β_1 and β_2. In the adaptive expectations literature, β_1 and β_2 are known as *self-confirming* equilibria. As implied by the Figure, β_1 is a locally stable equilibrium, thus, if the beliefs of the public regarding future inflation are not sufficiently high then $\pi_{t+1}-\pi_t$ will converge to zero and β_{t+1} to β_1. Additionally, equation (4) implies that π_t will also converge to β_1. However, if $\beta_t > \beta_2$, then $\pi_{t+1}-\pi_t$ will increase, with unbounded dynamics. Therefore, $\beta_t>\beta_2$ implies that the model will eventually generate a hyperinflation episode. This phenomenon is called escape dynamics by Sargent *et al.* (2009).[11]

Panel (b) of Figure 1 presents another result of CGE: assuming β_t induces escape dynamics, a hyperinflation episode can be prevented if the deficit is reduced. This Panel shows two dynamic paths for $\pi_{t+1}-\pi_t$ as a function of β_t. The only difference between these paths is the level of fiscal deficit. The dynamics shown in blue correspond to a high fiscal deficit, while the dynamics in green correspond to a low fiscal deficit. Assuming a high deficit and $\beta_t = \hat{\beta}$, if the deficit is not reduced then it will provoke an escape dynamics of inflation and expectations as shown with blue arrows in Panel (b) of Figure 1. However, if the government reduces its fiscal deficit to a sufficiently low level then, even when $\beta_t = \hat{\beta}$, it will be able to prevent an escape dynamics. Furthermore, $\pi_{t+1}-\pi_t$ will converge to a low and stable inflation equilibrium as shown by the green arrows in the Figure.

Finally, CGE implies a non-trivial computational advantage: given the complexity of the function that will be used to estimate all the parameters involved in the model, assuming this type of expectations

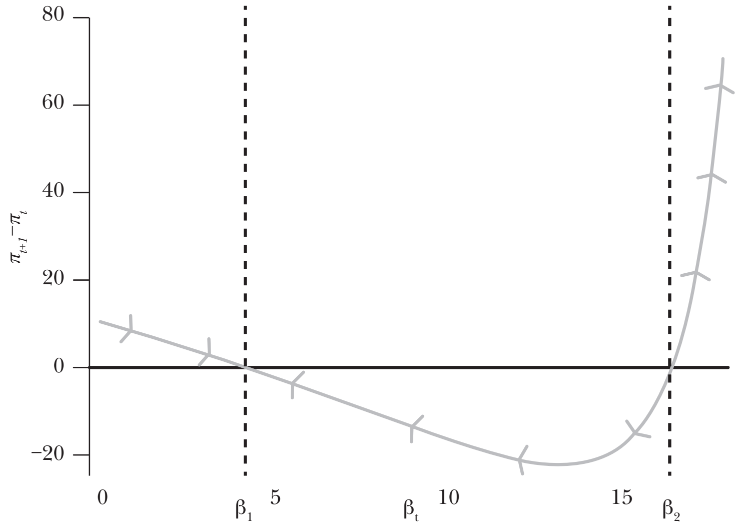
expectations or other deviations from rational expectations, can be necessary to generate hyperinflation episodes (e.g. Sallum *et al.* (2005)). See Sargent *et al.* (2009) for a list of references in a growing literature using calibration or econometric techniques to compare time-series data with models in which agents use this type of algorithm to form their beliefs.

[11] Williams (2016) characterizes how adaptive expectations can lead to escape dynamics and ex- plains how the likelihood, frequency and direction of the variables during an escape dynamics can be characterized by a deterministic control problem.

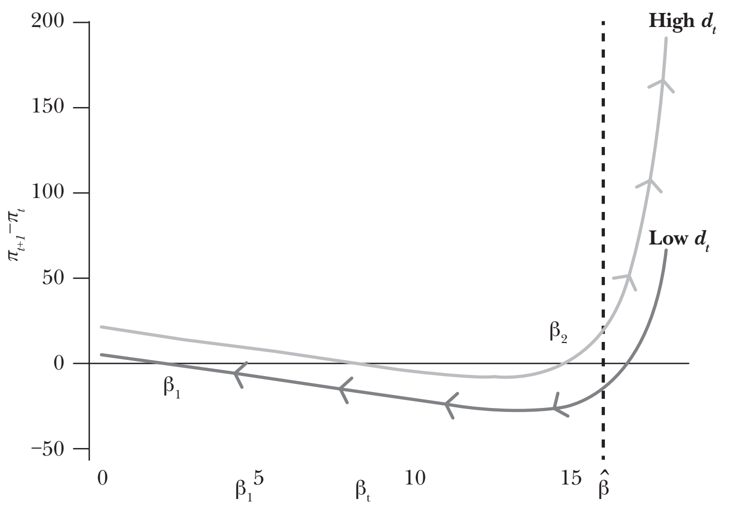
Figure 1

DYNAMICS INDUCED BY ADAPTIVE EXPECTATIONS

A. INFLATION AND EXPECTATIONS



B. FISCAL DEFICIT AND EXPECTATIONS



Note: these figures considers β_{t-1} =1.02 and the estimated parameters shown in Table 1.

allows us to reduce the computational burden.[12] We discuss the implications of using rational expectations in the Appendix.

2.3 The Process for Fiscal Deficits

The last key variable that determines inflation rates is the level of real fiscal deficit relative to output d_t. The fact that d_t is assumed to be a random variable is motivated by, among other factors according to our interpretation, exogenous conditions in global financial markets, the international price of commodities that are crucial in determining the fiscal situation of many governments in developing economies, and political processes. With these considerations, in an admittedly reduced form, it is assumed that d_t is a random variable with the following conditional distribution:

$$log\left(d_t \mid \bar{d}_t, \upsilon_t\right) \sim N\left(log\left(\bar{d}_t\right), \upsilon_t\right). \tag{5}$$

Thus, d_t is a random variable with a log-normal distribution that has a median of $\bar{d}_t$ and a variance parameter v_t. A restriction of assuming a log-normal distribution for fiscal deficits relative to output is that d_t cannot be negative (a fiscal surplus is not feasible). Sargent *et al.* (2009) explain that even when they allow the distribution of d_t to have negative values, there is not a significant improvement in the fit of the model. Furthermore, a log-normal distribution captures the skewness of inflation shown in the data. In the case of Mexico, we will see that three values for $\bar{d}_t$ are sufficient to adequately capture the evolution of deficits during the period we analyze.

Each period, $\bar{d}_t$ is determined by a discrete Markov process with D possible states.[13] In the same manner, v_t follows another Markov process with V states that is independent of the process that determines $\bar{d}_t$. In related literature, the stochastic process followed by d_t

[12] The next section explains some of the details involved in estimating the parameters of model.

[13] A stochastic process x_t is said to be a discrete Markov process if x_t takes values in a set I with $|I| \in \mathbb{N}$ and for all $t = 1, 2, \ldots$ the Markov property is satisfied: $P\left[x_{t+1} = i \mid x_0, x_1, \ldots, x_t\right] = P\left[x_{t+1} = i \mid x_t\right]$. This property states that past realizations of the process $\left\{x_0, x_1, \ldots, x_{t-1}\right\}$ do not affect future values, only the present state x_t affects x_{t+1}.

is called a Hidden Markov Process.[14] Each Markov process involved in the model is related to a matrix where the elements represent the transition probabilities from one state of the process to another. We let $Q_d \in \mathbb{R}^{D\times D}, Q_v \in \mathbb{R}^{V\times V}$ be the transition matrix associated to the $\left\{\bar{d}_t, v_t\right\}$ processes, respectively.[15]

Another important property of the model is that it generates a non-linear relation- ship between inflation, its expectations, and fiscal deficits. The impact that current inflationary expectations β_t have on inflation π_t and future expectations β_{t+1} is a function of the hidden Markov state that governs the median fiscal deficit $\bar{d}_t$. An example of the non-linearity generated by the hidden Markov process of the model can be seen in Panels (a) and (b) of Figure II. Panel (a) shows that, for the same level of β_t, the effect of the fiscal deficit on inflation is magnified as the median level of fiscal deficit $\bar{d}_t$ rises (this Figure considers $\bar{d}_1 > \bar{d}_2 > \bar{d}_3$). Panel (b) displays a similar effect of fiscal deficit on the evolution of inflation expectations. This non-linearity between the inflation rate, its expectations, and fiscal deficits in the model is consistent with empirical studies. For example, Catao and Terrones (2005) and Lin and Chu (2013) provide evidence, utilizing data for more than 100 countries, that fiscal deficits have a strong and weak impact on the inflation rate in high and low inflation episodes, respectively. Thus, the data and the model suggest that there is a non-linear impact of fiscal deficits on inflation and expectations of inflation.

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14 Formally, a hidden Markov process is a pair $\{x_t, y_t\}$ such that x_t is a (standard) Markov process and there exists a function f such that for all $t = 1, 2, \ldots, y_t = f(x_t)$ and:

$$P\left[y_{t+1} = y \mid x_0, x_1, \ldots, x_{t+1}, y_0, y_1, \ldots, y_t\right] = P\left[y_{t+1} = y \mid x_{t+1}\right].$$

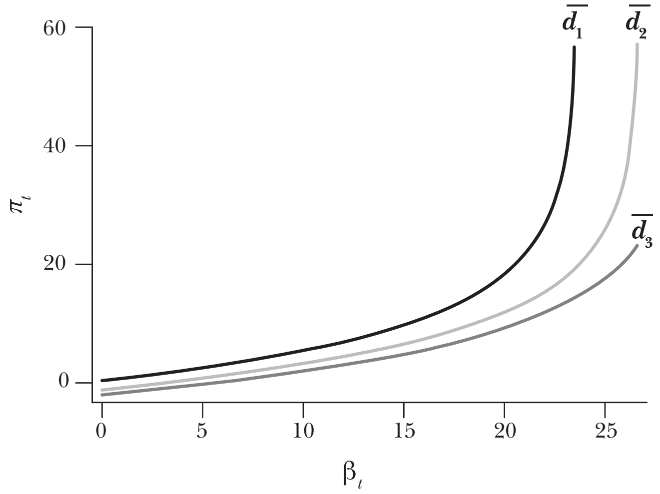
In these type of processes, y_t is known as the observable part of the process and x_t is the hidden component. In the model presented in this section, y_t is the real fiscal deficit relative to output while x_t is a vector that contains the median $\bar{d}_t$ and variance v_t of fiscal deficit at each t.

15 This means, in the case of $\bar{d}_t, Q_d$ in its (i, j) component contains the probability of being in a state j in t+1 conditional on $d_t = i : Q_d(i, j) = P\left[\bar{d}_{t+1} = j \mid \bar{d}_t = i\right]$.

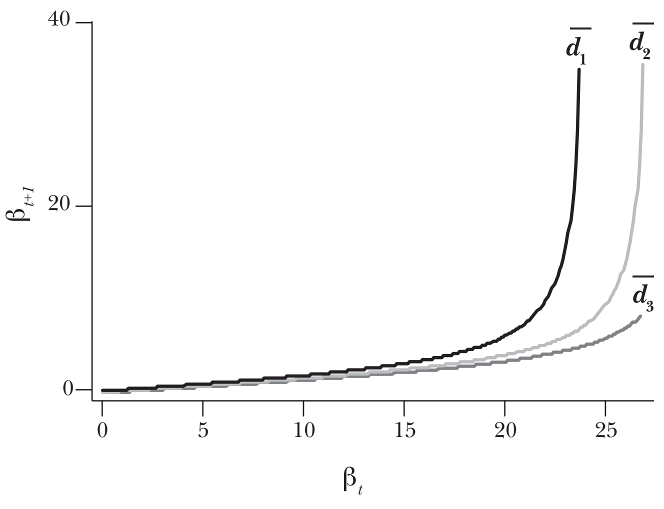
Figure 2

NON-LINEAR EFFECT OF FISCAL DEFICITS

A. EFFECT ON INFLATION



B. EFFECT ON EXPECTATIONS



Note: these figures consider β_{t-1}=1.02 and the estimated parameters as described in the next section.

2.4 Model Restrictions on Expectations

Equation (3) implies that inflation in the model is well defined only if at each t:$1-\lambda\beta_{t-1}>0$ and $1-\lambda\beta_t-\gamma d_t>0$ (otherwise the real balances demand could become negative). However, there is no restriction within the model preventing these constraints from being violated. Furthermore, (3) implies that the gross inflation rate is not bounded.[16] Given the numerical problems that this can generate when estimating the parameters, it is assumed that there exists a constant $\delta>0$ such that $\pi_t<\delta$ for every t.

The two restrictions that need to be considered such that π_t is well defined and bounded are:

6 $$1-\lambda\beta_{t-1}>0 \quad \text{and} \quad \delta\left(1-\lambda\beta_t-\gamma d_t\right)>\theta\left(1-\lambda\beta_{t-1}\right).$$

If any of these constraints is violated, then it is assumed that the gross inflation rate is not determined following (3). Instead, π_t will be determined randomly according to the following log-normal distribution:

7 $$log\left(\pi_t\right)\sim N\left(log\left(\bar{\pi}_t\left(d_t\right)\right),\upsilon_\pi\right),$$

where $\bar{\pi}_t\left(d_t\right)$ is the inflation equilibrium determined by (3) in the model without uncertainty and conditional to a certain fiscal deficit d_t,[17] whereas υ_π represents the variance of inflation when it is determined following (7). Additionally, if $\delta\left(1-\lambda\beta_t-\gamma d_t\right)\leq\theta\left(1-\lambda\beta_{t-1}\right)$, Sargent *et al.* (2009) suggest resetting expected inflation to $\beta_{t+1}=\pi_t$, otherwise the dynamics between β_{t+1} and inflation will provoke $\pi_{t+1}\geq\delta$ and eventually $\beta,\pi\rightarrow\infty$.

Whenever the current hidden Markov state $\left\{\bar{d}_t,\upsilon_t\right\}$ provokes dynamics that will eventually make $\left\{\pi,\beta\right\}$ violate (6) or that will

16 If $1-\lambda\beta_t-\gamma d_t\rightarrow 0$, then $\pi_t\rightarrow\infty$.

17 Certainty in the model implies $\pi_t=\beta_t$. In equilibrium, $\pi_t=\pi_{t-1}$. Using (3) it can be shown that: $\bar{\pi}_t\left(d_t\right)=\left(1+\theta\lambda-d_t-\sqrt{\left(1+\theta\lambda-d_t\right)^2-4\theta\lambda}\right)\Big/2\lambda.$

generate an escape dynamics, the government can implement a *reform* to prevent this from happening. Sargent *et al.* (2009) define two types of reforms: a reform is said to be cosmetic if the government is able to (temporarily) control inflation but the median level of fiscal deficit is not altered. Following Panel (a) of Figure 1, a *cosmetic* reform can fail if the expected inflation rate associated with inflation β_{t+1} is such that $\beta_{t+1} > \beta_2$. However, a cosmetic reform can be successful if $\beta_{t+1} \leq \beta_2$.[18] A *structural reform*, on the other hand, occurs when the government is able to control the inflation rate by reducing the median level of fiscal deficit, $\bar{d}_t$. Panel (b) of Figure 1 is an example of a structural reform where the government succeeded in controlling an escape dynamics.

An important contribution of the model is its ability to identify whether a reform is cosmetic or structural. Previous literature had only studied structural reforms, even though the notion of a cosmetic reform was part of academic and economic policy discussions. The inclusion of cosmetic reforms in the model represents a reduced form approach to consider different episodes in Latin America, when governments attempted to control inflation without tackling fiscal deficits. Discussions of eco- nomic events often point to the role of the exchange rate, which is not explicitly included in the baseline model, and we explore below through different extensions of the baseline model.

3. BASELINE MODEL RESULTS

In this section, we present the main results of the baseline model. We present the fit of the model for real fiscal deficits, inflation, and its expectations between 1969 and 2016. Then, as a validation procedure, we compare these model-fitted series with data available for different variables.

3.1 Baseline Model Estimation

Heuristically, the estimated parameters are obtained as the vector of values that maximize the likelihood function, which consists of the

[18] Sargent *et al.* (2009) argue that in Peru a cosmetic reform was enough to control the inflationary crisis this country experienced in 1985.

marginal density of the sequence of inflation.[19] The inflation data corresponds to the *Índice Nacional de Precios al Consumidor* (INPC) between 1969 and 2016, at a monthly frequency. The INPC is the Consumer Price Index (CPI) computed by the National Institute of Statistics and Geography, *Instituto Nacional de Estadística y Geografía* (INEGI) since 2011, and by Banco de México before that year.

We consider a monthly frequency for the model estimation, consistent with the data. Before estimating the parameters, one must choose the number of states of nature for $\left\{\bar{d}, v\right\}$: denoted D and V, respectively. As D or V become larger, the fit of the model in terms of approximating the data tends to improve at the expense of increasing the computational burden. Sargent *et al.* (2009) estimate two models for each country they study: a model with D=3, V=2 and a model with D=2, V=3. Then, using the Schwarz information criterion (SIC), we select the model that provides a better fit to the data.[20] Table 1 shows the estimation results for a model with three possible states for $\bar{d}\left(D=3\right)$ and two states for v (V=2). We choose this model because, after estimating the two models with data for Mexico, the SIC suggests that D= 3, V= 2 provides a better approximation to the data.

The estimated parameters suggest interesting facts about the price formation process in Mexico: λ=0.7556 implies that the price level reflects agents' expectations on the future price level. Hence, if inflation expectations are volatile, then the observed inflation will also have a high variance. This result implies that a necessary condition to have stable inflation is to anchor expectations. Mexico's λ is similar to the estimation by Sargent *et al.* (2009) for Argentina (λ=0.730) and Peru (λ=0.740).

The estimated value of ν=0.1147 for Mexico implies that to anchor expectations, observed inflation must remain stable for several months.[21] On the other hand, this also implies that the expected

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[19] In the Appendix we provide further details regarding the estimation of the model. Ramirez de Aguilar (2017) describes the computational procedure.

[20] The SIC is a Bayesian selection criterion between two models, A and B. Let L_x, P_x, n_x be the log-likelihood, the number of parameters, and the sample size in model $x \in \{A, B\}$, respectively. Then, the Schwarz criterion for model x is computed as $SIC_x = log(n_x)P_x - 2L_x$. If $SIC_A < SIC_B$, then model A is preferred.

[21] The estimation of ν=0.1147 implies that the weight agents give to their past expectations is 0.8853. Hence, if inflation is stable for only

Table 1

PARAMETER ESTIMATION

Parameter	*Estimation*	*Description*
λ	0.7556 (0.0022)	weight of expectations on the price level
v	0.1147 (0.0081)	weight of past inflation on expectations
$\bar{d}_1$	0.0075 (0.0001)	monthly high median level of fiscal deficits
$\bar{d}_2$	0.0039 (0.0004)	monthly moderate median level of fiscal deficits
$\bar{d}_3$	0.0023 (0.0002)	monthly low median level of fiscal deficits
v_1	0.0671 (0.0087)	high variance of monthly fiscal deficits
v_2	0.0295 (0.0012)	low variance of monthly fiscal deficits
v_π	0.0753 (0.0010)	variance of inflation when it is determined randomly
p_{11}^d	0.9731 (0.0361)	probability of $\bar{d}_{t+1} = \bar{d}_1$ conditional on $\bar{d}_t = \bar{d}_1$
p_{22}^d	0.9787 (0.0390)	probability of $\bar{d}_{t+1} = \bar{d}_2$ conditional on $\bar{d}_t = \bar{d}_2$
p_{33}^d	0.9924 (0.0056)	probability of $\bar{d}_{t+1} = \bar{d}_3$ conditional on $\bar{d}_t = \bar{d}_3$
p_{11}^v	0.7493 (0.1072)	probability of $v_{t+1}=v_1$ conditional on $v_t=v_1$
p_{22}^v	0.7789 (0.0879)	probability of $v_{t+1}=v_2$ conditional on $v_t=v_2$

Note: the numbers shown in parentheses represent the standard deviation of each parameter, computed using the Hessian matrix of the maximum likelihood problem (see MacDonald and Zuccini (2009)).

inflation rate de-anchors only if the observed inflation is high for an extended period. Sargent *et al.* (2009)'s estimations for Argentina (ν=0.023), Chile (ν=0.025), and Peru (ν=0.069) indicate that, in these countries, observed inflation has a relatively limited effect on inflation expectations, while the estimates for Bolivia (ν=0.232) and Brazil (ν=0.189), suggest that observed inflation has a stronger impact on expectations.

Regarding fiscal deficits, according to the estimation, when the government generates a high fiscal deficit for one year ($\bar{d} = \bar{d}_1$ for twelve consecutive months), fiscal deficit represents approximately 9.12% of GDP. If the government generates a moderate deficit for one year, this will amount to approximately 4.76% of GDP. Finally, if fiscal deficits are low for one year, then it represents 2.78% of the GDP. These levels of deficit are associated, in steady state, with average annual inflation rates of 69.41%, 17.53% and 3.54%, respectively. As it will be shown, these estimates are consistent with fiscal deficit data between 1977 and 2016.

3.2 Fiscal Deficits, Inflation, and Expectations

Once the parameters are estimated, fiscal deficits relative to output can be computed in each period exploiting the assumptions made for $\left\{d_t \mid \bar{d}_t, \upsilon_t\right\}$ and considering that $\left\{\bar{d}_t, \upsilon_t\right\}$ follow a discrete Markov process. We estimate the conditional density of fiscal deficits given the sequence of inflation observed in the data π^T and the parameter estimation, $p\left(d_t \mid \pi^T, \hat{\phi}\right)$. Then, we use the median of each density to construct a sequence $\left\{d_t\right\}_{t=1}^T$ that is used to compute $\left\{\pi_t, \beta_t\right\}_{t=1}^T$ according to the model. Finally, we compare the model implied sequence of inflation $\left\{\pi_t\right\}_{t=1}^T$ with the empirical series.

Figure 3 presents the model simulation for fiscal deficits, inflation expectations, observed inflation, and the probability of a regime change in $\bar{d}$.

- Between 1969 and 1972, marked as Region (1) in Figure 3, a low rate of inflation is associated with the lowest hidden state

one month, this will not be enough to reduce β because past beliefs have more weight on expectations. Only if the inflation rate is stable for several consecutive months will β also become stable.

of median deficit $\bar{d}_3$. This is consistent with the economic history of Mexico; during the decade of the 1960s, the inflation rate in Mexico achieved its lowest value during the second half of the twentieth century: an average of 2.8%, which is replicated by the model.[22]

- Between 1973 and 1982, marked as Region (2) of the Figure, the model suggests that fiscal deficits increased from a low to a moderate median level, accompanied by an increase of the inflation rate. Since this level of deficit remained constant for several years, inflation expectations de-anchored. Consequently, the observed inflation rate also presented an increase between 1973-1982. At the end of 1971, a global recession reduced international credit. Fearing a period of stagnation, the government responded by increasing public expenditures financed with monetary emission, foreign credit, and reserves of private financial institutions at the Central Bank. The fiscal deficit relative to output increased from 2.5% of GDP in 1971 to 4.9% in 1972, while the monetary base grew 14.8% during 1972, the rate of inflation registered an average of 14% during 1973-1976. Meanwhile, government expenditures increased from 30.9% relative to output to 40.6% in 1981; the fiscal deficit relative to output rose from 6.7% in 1977 to 14.1% in 1982.
- In 1981, the world economy was going through another recession that once again reduced international credit. In Mexico, there was not a significant reduction in expenditures and by 1982 the lack of foreign credit led the government to finance most of its expenditures with monetary emission: between 1981 and 1983, the monetary base was growing at an average rate of approximately 90% and the inflation rate was 63.1% on average. During 1983, the model generates an inflation rate above 80% as a result of an increase in fiscal deficits, which reached their highest median level. During 1983-1986, the government raised taxes and renegotiated its foreign debt.

[22] In this section we draw from Cardenas (2015), who provides an exhaustive narrative of the economic history of Mexico during the period of our analysis. Historical series for output and the inflation rate data presented in this section were obtained in the Historic Statistics of Mexico published by INEGI.

However, there was not a significant adjustment of expenditures; by 1986 the fiscal deficit reached the same level it registered in 1982, equal to 14.1% of GDP. In 1985 world oil prices fell and by 1986 the price of the Mexican oil mix suffered a drop of 65%, generating a loss equivalent to 6.5% of GDP and a reduction of 26% in federal income. By 1987, the annual inflation rate was 159%.[23]

- Region (3) of Figure 3 presents evidence of a cosmetic reform, to control inflation: during 1984 the government was able to reduce inflation from 85% to 56%, according to the model, due to a temporal reduction of its fiscal deficit. However, as shown by Panels (a) and (d) the median fiscal deficit between 1985-1987 remained at the highest possible (estimated) value. As a consequence, inflation began to grow once again in 1985.
- After the 1987 crisis, in 1988 the Mexican government reached an agreement with representatives of the private sector called the Economic Solidarity Plan (in Spanish: Pacto de Solidaridad Económica) in which the government com- mitted to reducing expenditures and inflation. The fiscal deficit came to historic lows and even achieved surpluses, and the government was able to restructure its debt. By 1989 the annual inflation rate was lowered to 20.3%. The model is consistent with this episode of economic history in Mexico; through the lens of the model, the government conducted a structural reform: between 1988 and 1993 (Region (4) of the Figure), fiscal deficits were reduced from the highest possible median $\bar{d}_1$ to a moderate level $\bar{d}_2$ in 1989 and then in 1993 to a lower median $\bar{d}_1$. This reduction of the fiscal deficit had an immediate impact on inflation and its expectations.
- Several factors induced another crisis at the end of 1994 and during 1995. The re-privatization of the banks was financed with foreign debt, which left the financial sector exposed to sudden exchange rate movements and increments in interest rates. Additionally, the government issued bonds that were paid in pesos but with dollar nominal values (the

[23] Cardenas (2015) argues that the crisis presented during 1987 is a direct consequence of the unwillingness of the government to reduce its deficit during 1982-1987.

Tesobonos), which required a stable exchange rate in order to keep this debt sustainable. However, political events led to a significant depreciation of the domestic currency in 1994 accompanied by capital outflows (Calvo and Mendoza (1996), Cole and Kehoe (1996) analyze these events). The government faced a debt crisis, the private financial sector found itself in bankruptcy, and the inflation rate reached 51% in 1995. The government negotiated loans with the International Monetary Fund (IMF) and with the United States in order to finance its debt.

- The model attributes, in Region (5), the escalation in inflation during 1995 to an increase in fiscal deficit between 1994 and 1995. However, this escalation was a consequence, to a significant extent, of the nominal exchange rate depre- ciation at the end of 1994 and the collapse of the financial sector in 1995. In this case, there is a discrepancy between the in-sample predictions of the model concerning fiscal deficit and what is observed in the data. This discrepancy between the model and the data motivates the introduction of the nominal exchange rate in the model. It will be shown that by introducing this variable we can better account for the behavior of inflation during 1995 and in general.
- After a constitutional reform in 1993, Banco de México became independent in 1994. The reform established as its primary mandate to preserve the purchasing power of the national currency.[24] The average annual inflation rate fell from 10.95% between 1996-2002 to 3.98% between 2003-2016, achieving historic minimums during 2015 and 2016.[25]

[24] Some of the policies adopted by the Central Bank after 1994 were: (i) restoration of the level of international reserves to gain credibility, (ii) the use of an objective of cumulative current account balances that private banks held at the Central Bank as the primary monetary policy instrument, (iii) adoption of an inflation-targeting policy, and (iv) to improve transparency, the Central Bank began to publish reports communicating monetary policy decisions as well as quarterly reports of the economy. For a more detailed description of these policies see Ramos-Francia and Torres- Garcia (2005).

[25] Furthermore, as documented by Chiquiar *et al.* (2010), the inflation rate after 2000-2001 became a stationary process and initiated its convergence towards the inflation target.

Meanwhile, fiscal deficits remained relatively low and stable during 1997-2016.[26]

- During the last sub-period (Region (6) of Figure 3), the model predicts that fiscal deficits were at the lowest median and variance hidden states. The model also shows that the expected inflation rate has fluctuated within the range of the target of Banco de México: an inflation rate of 3% that can vary between 2% and 4%. The model proposes that a necessary condition to anchor inflation and its expectations is a low monetization of fiscal deficit. The only year in which the fiscal deficit had a slight probability of being at a higher median state was in 2009, in the course of the global financial crisis. However, since the inflation rate remained low after 2009, the baseline model predicts that Mexico has remained in a low fiscal deficit regime.

Considering the inflation history previously described, we observe that the model predicts a deficit distribution with an elevated mean and variance during those years in which the inflation rate was elevated, as in 1987 (a year characterized by the highest inflation rate presented in Mexico during the second half of the twentieth century). In those years in which the inflation rate was moderately high, as in 1975, the model predicts a fiscal deficit with a moderate mean and lower variance than in 1987. Finally, in those years where the inflation rate is low, the fiscal deficit density is characterized by a low mean and variance.

3.3 Fiscal Deficits: Data and Model Simulation

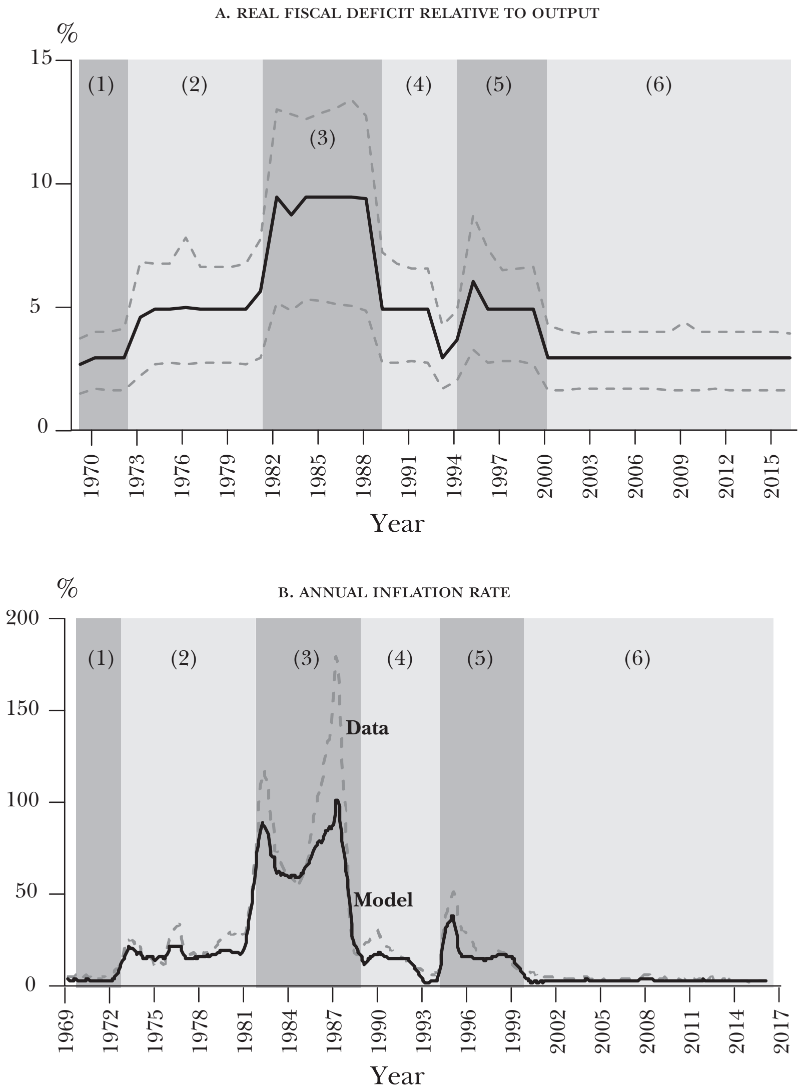
The Ministry of Public Finance of Mexico, Secretaría de Hacienda y Crédito Público (SHCP), computes a measure of the fiscal deficit called Balance Público Tradicional (BPT) since 1977. This measure represents the difference between current and capital expenditures and revenue of almost all of the public sector.[27] Since 1990, the SHCP

[26] In 2008 there was a methodological modification in BPT, it became a wider measure of fiscal deficits: after 2008 the BPT considers part of the investments made by two important state- owned firms (PEMEX and CFE) that before were considered as long-term debt (this type of investments are called PIDIREGAS).

[27] The BPT does not consider the revenue and expenditures of Banco de México or the public financial sector. The financial sector of the

Figure 3

DYNAMICS OF THE MODEL

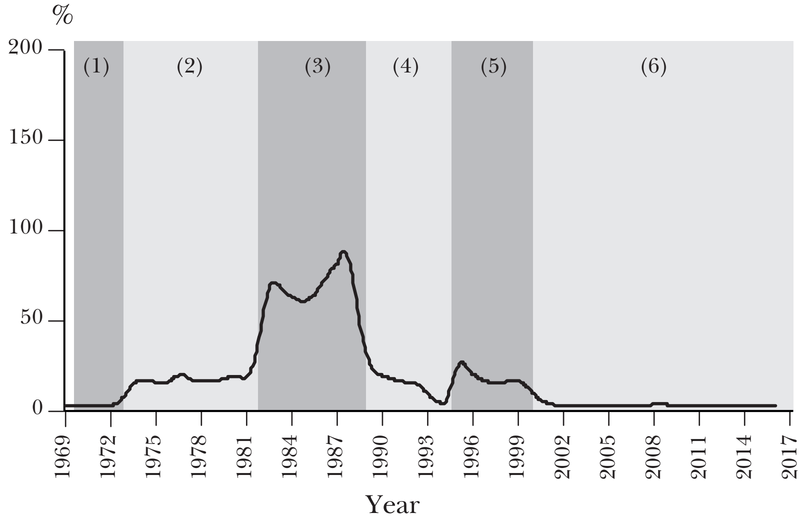


Notes: Panel (a) plots the median real scal deficit relative to output together with the 10th and 90th percentile of the annual deficit distribution. Panel (b) shows the annual inflation rate predicted by the model given the real scal deficit, and the data. Panel (c) shows the expected inflation rate according to the CGE algorithm (4). Panel (d) plots $P[\bar{d}_t = \bar{d}_2|\pi^t, \hat{\phi}] + P[\bar{d}_t = \bar{d}_3|\pi^t, \hat{\phi}]$ where $\bar{d}_2$ and $\bar{d}_3$ are the moderate and low levels of mean fiscal deficit.
Source: INEGI and model results.

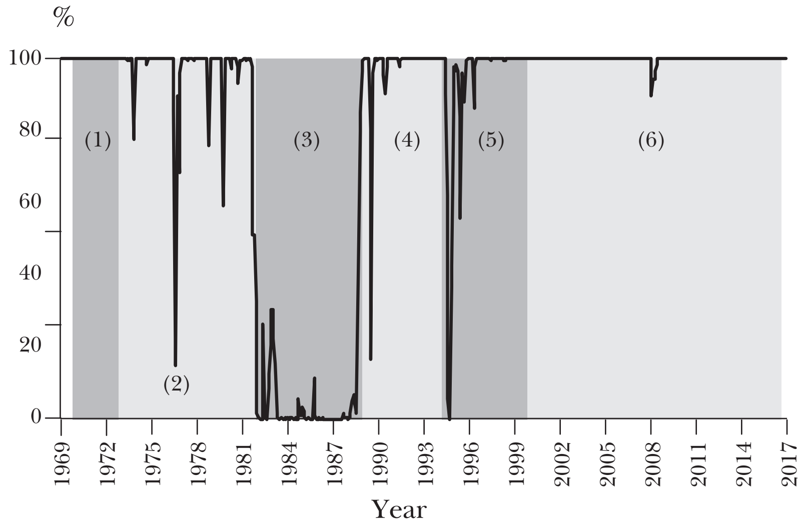
Figure 3 (cont.)

DYNAMICS OF THE MODEL

C. EXPECTED INFLATION RATE



D. PROBABILITY OF A LOW INFLATION REGIME



Notes: Panel (a) plots the median real scal deficit relative to output together with the 10th and 90th percentile of the annual deficit distribution. Panel (b) shows the annual inflation rate predicted by the model given the real scal deficit, and the data. Panel (c) shows the expected inflation rate according to the CGE algorithm (4). Panel (d) plots $P[\bar{d}_t = \bar{d}_2|\pi^t, \hat{\phi}] + P[\bar{d}_t = \bar{d}_3|\pi^t, \hat{\phi}]$ where $\bar{d}_2$ and $\bar{d}_3$ are the moderate and low levels of mean fiscal deficit.
Source: INEGI and model results.

computes an alternative fiscal deficit measure called Requerimientos Financieros del Sector Público (RFSP), which incorporates the financial requirements of the government at the federal level. This is a broader measure of fiscal deficit since it includes the BPT in addition to all revenues and expenditures of the public financial sector that provide funds for public policy.[28]

Panel (a) of Figure 4 displays the estimated sequence of fiscal deficits from the model, as well as the BPT and the RFSP relative to GDP between 1977 and 2016. As shown in the Figure, there is an adequate approximation of the model to the BPT data before 1991 and to the RFSP after 1993. During 1991 and 1992, both series show a fiscal surplus. The model cannot match this feature of the data given the assumption of a log-normal distribution, and deficits cannot be negative. Addition- ally, the model predicts a higher deficit during 1994-1996 relative to those observed in the data; in 1995 the model predicts a fiscal deficit relative to output of 6.1% of GDP, while the RFSP exhibits a fiscal deficit of 2.5% of GDP. The baseline model can only attribute the spike in inflation of that year to fiscal deficits. We will see that the extensions of this model can better account for the rates of inflation during this episode. During 1977-2016, the model's median deficit variance is 53.7% of the variance presented in the fiscal deficit data.[29]

Panel (b) of Figure4 displays the model's implied monetary base growth rate compared with Banco de México's data between 1969 and 1970.[30] The Figure shows that the model approximates the data's

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government includes, among others, trust funds and banks administered by the federal government.

[28] For example, during 1990-1998 the government managed a trust fund called FOBAPROA, its objective was to insure private banks against overdue accounts in case of a financial crisis. If the fund provided resources to a private bank to cover its overdue accounts, this would be considered in the RFSP but not in the BPT. The RFSP are a better approximation of the concept of deficits considered in the model. However, before 1990 the only official deficit measure available is the BPT. We are grateful to Nicolas Amoroso, Oscar Budar, and Juan Sherwell for their invaluable guidance in understanding historical accounts and providing these series.

[29] For these results, we considered the BPT before 1991 and the RFSP after this year.

[30] To compute the monetary base growth according to the model, we considered equation (1) to show that: $\frac{M_t}{M_{t-12}}=\frac{P_t}{P_{t-12}}\left(\frac{1-\lambda\beta_t}{1-\lambda\beta_{t-12}}\right)$. Ramirez de Aguilar (2017) presents further details.

sequence reasonably well, although there are differences in 1990-1992. The model's monetary base growth rate variance accounts for 82% of the variance presented in the data.

4. BEYOND THE BASELINE MODEL

Considering that, since 1994, Banco de México has been an independent Central Bank and no longer finances the federal government through money creation, in this section we present modifications to the baseline model.[31] Before we discuss these extensions, we should be explicit about the fact that the model by itself does not distinguish between periods of monetary or fiscal dominance. Formally, the estimation of the model will propose a series of deficits that are financed with monetary emission, while the classification of different periods in terms of the regime rests on the interpretation of the historical narrative we previously presented.[32] In a similar manner, Meza (2017) concludes that the change in legislation that granted independence to Banco de México in 1993 represented a credible change from fiscal to monetary dominance, and that the transition to an independent Central Bank has been successful. Furthermore, Central Bank independence does not imply $d \approx 0$ if the target for inflation is, for example, 3%. Through the lens of the model, the Central Bank would target a long-run level of money growth such that inflation fluctuates around the target of this institution.[33]

The extensions we present will allow us to illustrate some of the channels through which fiscal policy may potentially influence inflation even in a context of autonomy of the Central Bank. These

[31] As explained by Meza (2017), the Central Bank transfers resources to the Ministry of Finance (equivalent to the Treasury in the U.s.), after determining its earnings and following legally specified rules. This is called the Remanente de Operación de Banco de México. In the United States, the Federal Reserve transfers to the Treasury most of its interest earnings from government debt. As further discussed below, this can be perfectly consistent with a regime of monetary dominance.

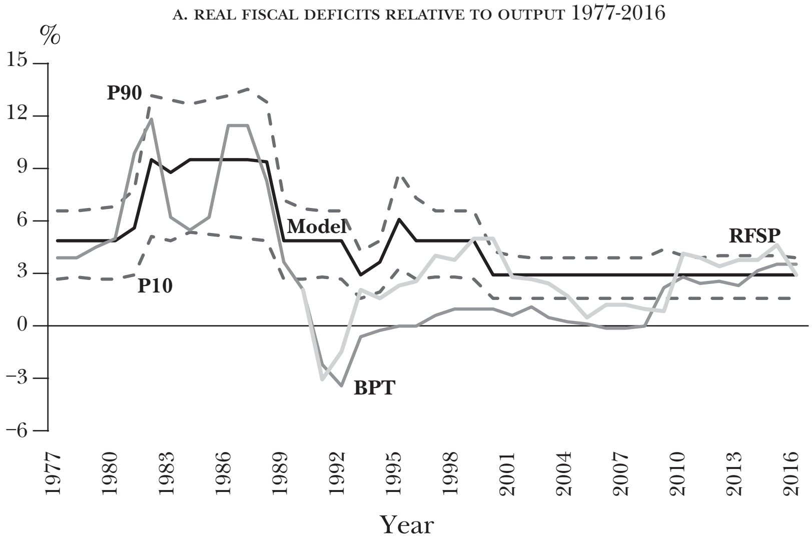
[32] In this sense, the approach is complementary to models that consider regime-switching environments, e.g. Chung *et al.* (2007), Cadavid-Sanchez *et al.* (2017), and Bianchi and Ilut (2017).

[33] For the period, Meza (2017) estimates seigniorage at an average of 0.66 p.p. of GDP for the period 1995-2016.

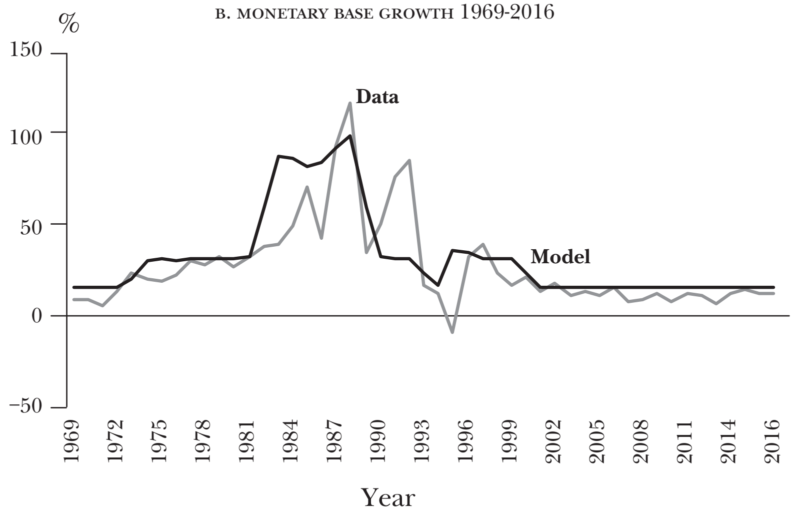
Figure 4

DATA AND MODEL COMPARISON

A. REAL FISCAL DEFICITS RELATIVE TO OUTPUT 1977-2016



B. MONETARY BASE GROWTH 1969-2016



Notes: the series presented are - Panel (a): in blue the estimated scal deficit with the 10th/ 90th percentiles of the estimated deficit distribution. In red/orange the BPT or the RFSP relative to GDP. Panel (b): In blue/red the model/data monetary base annual growth rate, respectively.
Source: Banco de México and SHCP.

modifications are inspired by the literature that studies the interactions between fiscal and monetary policy, which suggests that, even with an independent Central Bank, fiscal policy can still affect inflation. For example, if agents observe an increasing deficit that translates into higher debt, they may anticipate a regime change to make the fiscal path sustainable, hence, they may increase their current inflationary expectations and inflation itself.

First, we present an extension where we consider that the expected inflation rate may be influenced by fluctuations in the nominal exchange rate (NER) between the Mexican peso and the U.S. dollar. An important result of this model is that the effect that the NER has on inflation (known in the literature as Exchange Rate Pass-Through, ERPT) is a function of the fiscal deficit. According to our estimation, in a situation with elevated fiscal deficits that generate high inflation rates, the ERPT is considerable. After 1995, the year in which the NER changed from a fixed to a flexible regime and after Banco de México became an independent institution, the ERPT to inflation and its expectations has become rather limited.

The second extension considers the sovereign interest rate spread EMBI of J.P. Morgan as a variable that reflects the fiscal situation of governments. We estimate that the EMBI has a moderate impact on inflation and its expectations, although its effect is positive and statistically significant. An increase in the EMBI spread is associated with the perception that the government is not in a solid fiscal situation. Hence, following the example illustrated by Kocherlakota (2012), agents may incorporate in their inflation expectations the possibility that the Central Bank may lose independence to the fiscal authority, and consequently raise their inflation expectations. This, according to the model, generates an increase in observed inflation as well.

In the third extension we specify a real-balances demand function that incorporates the exchange rate, as an alternative channel through which this variable may influence inflation.[34]

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[34] We have explored additional extensions of the model. For example, incorporating the CETES interest rate, and another specification that includes the target for the inflation rate of Banco de México. However, the fit of these alternative specifications is less favorable (results available upon request). Further exploration of alternative specifications would certainly be an interesting topic for future research.

Empirical evidence shows that sovereign interest rate spreads are, to a large extent, driven by international factors such as risk appetite, market volatility, terms of trade, global liquidity, contagion from events such as the Russian crisis or the LTCM collapse in 1998, and even U.s. macroeconomic news.[35] In the same fashion, exchange rate fluctuations are linked to global financial factors (to give some recent examples, Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017)), and the Mexican peso is sometimes considered a commodity currency (see Kohlscheen (2010)). The state of public accounts can make the economy vulnerable to these external shocks.[36]

Our model allows us to explore empirically the possibility that fiscal policy can make the evolution of inflation sensitive to events in international financial markets. The results motivate the need for further theoretical developments in this area, in particular for developing economies, where sovereign interest rate spreads and exchange rates seem to be of primary relevance. The historical narrative of events in Mexico for the period 1969-1994 supports this interpretation; events such as significant drops in the price of oil or sudden-stops make the economy vulnerable when fiscal accounts are in a dire situation and the government may be forced to turn to the Central Bank to cover its financial needs. Even in a context of de jure monetary dominance, economic agents may consider that these risks are still present, and thus we aim to capture this possibility in the estimation of our model.[37]

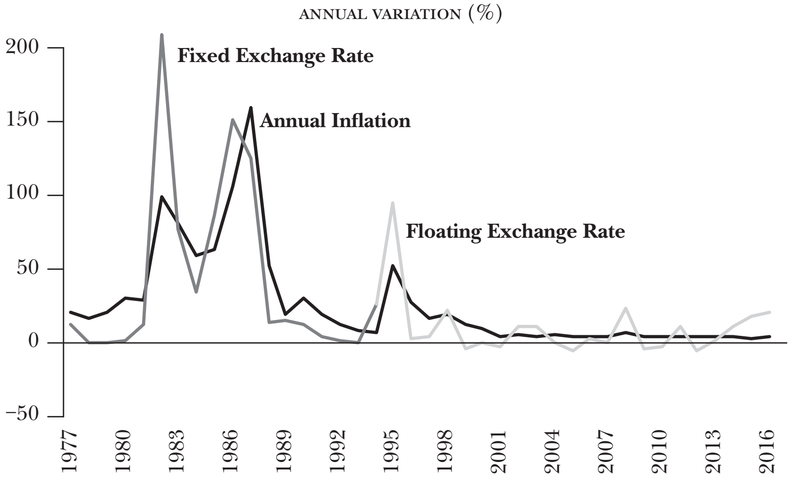
[35] There is an extensive literature that documents these facts, including Longstaff *et al.* (2011), González-Rozada and Levy-Yeyati (2008), Bunda *et al.* (2009), Ciarlone *et al.* (2009), Hilscher and Nosbusch (2010), and Ozatay *et al.* (2009).

[36] The issue of endogeneity is addressed by exploiting alternative methodologies in Cortés-Espada (2013) and Lopez-Villavicencio and Mignon (2016).

[37] These channels have been considered by Zoli (2005) in the case of Brazil, by assessing the impact of news concerning fiscal variables and fiscal policy on sovereign interest rate spreads and the exchange rate and discussing the potential implications for monetary policy. Cerisola and Gelos (2005) find that the stance of fiscal policy (proxied by the ratio of the consolidated primary surplus to GDP) is important to determine inflation expectations in the case of Brazil and argue that fiscal policy is instrumental in anchoring inflation expectations.

Figure 5

ANNUAL INFLATION AND VARIATION OF THE NER



Source: Banco de México and INEGI.

4.1 The Role of the Exchange Rate

As documented by Rogers and Wang (1994) and Carrasco and Ferreiro (2013), an important variable in determining inflation expectations is the nominal exchange rate (NER). Figure 5 presents, as a motivation for this extension, the annual inflation rate and the annual variation of the NER between 1977 and 2016. This Figure shows a significant correlation between these variables, particularly during episodes of high inflation. An important fact to consider is that before 1995 Mexico had a fixed exchange rate with bounded depreciations.[38] After 1994, the peso-dollar NER entered a floating regime.

In this extension, we consider that the exchange rate variation ΔNER is a variable that can affect inflation expectations. We assume

[38] In the Appendix, we describe the different exchange rate regimes in Mexico.

that this variation has a weight ξ on expectations. Hence, for each period t, the expected inflation rate is determined as follows:

8 $$\beta_t = \left(1 - \upsilon - \xi\right)\beta_{t-1} + \upsilon\pi_{t-1} + \xi \Delta NER_t.$$

Given that Mexico had a fixed NER during 1969-1994 and after 1995 the NER is in a floating regime, we estimate the model allowing ξ to change during 1969-1994 and 1995-2016. Hence, the model allows agents to give a weight ξ_1 to the NER variation during a fixed exchange rate regime and a weight ξ_2 when the NER is in a floating regime. To estimate this model, again we consider the monthly inflation sequence according to the INPC between January of 1969 and December of 2016, and the sequence of the monthly variation in the peso-dollar NER documented by Banco de México for that period. Table 2 presents the estimated parameters of this version compared with the baseline model estimation. Considering the exchange rate as a variable that can influence inflation expectations (and hence, inflation), the model can account for 75.8% of the variance observed in the inflation data, while the baseline model can explain 61.6% of this variance. Also, as suggested by the Diebold-Mariano test, during 2000-2016 the NER and baseline models produce different in-sample forecasts of observed inflation (at a 1% significance level) and the modified model has a higher correlation with the inflation data.[39] This result emphasizes the relevance of the exchange rate for the determination of the inflation rate in Mexico.[40]

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39 The hypothesis test proposed in Diebold and Mariano (1995) allows to assess if two forecasts $\left\{y_{it}, y_{jt}\right\}_{t=1}^{T}$ related to a series $\left\{y_t\right\}_{t=1}^{T}$ are statistically different. Defining $e_{kt}=y_{kt}-y_t$ for $k\in\{i,j\}$ and considering a loss-function $g(e)$, the null hypothesis in the Diebold-Mariano test is that $\mathbb{E}\left[g\left(e_{it}\right)-g\left(e_{jt}\right)\right]=0$. These authors construct a statistic function that involves the autocorrelations of the forecasts and show that, if the time series considered are covariance stationary and short memory, it has a t-Student distribution. Then, they construct a statistic that, under the same assumptions, is asymptotically $N(0, 1)$.

40 More formally, according to the SIC comparison, the ordering of the models is the following: the model with the EMBI spread and the NER in the for-

Table 2

EXTENDED MODEL PARAMETER ESTIMATION

parameter	*NER model*	*baseline model*	*description*
λ	0.7730 (0.0013)	0.7556 (0.0022)	weight of expectations on the price level
ν	0.1152 (0.0049)	0.1147 (0.0081)	weight of past inflation on expectations
ξ_1	0.0215 (0.0006)	-	weight of NER on expectations in a fixed regime
ξ_2	0.0047 (0.0001)	-	weight of NER on expectations in a floating regime
$\bar{d}_1$	0.0077 (0.0001)	0.0075 (0.0001)	monthly *high* median level of fiscal deficits
$\bar{d}_2$	0.0039 (0.0003)	0.0039 (0.0004)	monthly *moderate* median level of fiscal deficits
$\bar{d}_3$	0.0022 (0.0003)	0.0023 (0.0002)	monthly *low* median level of fiscal deficits

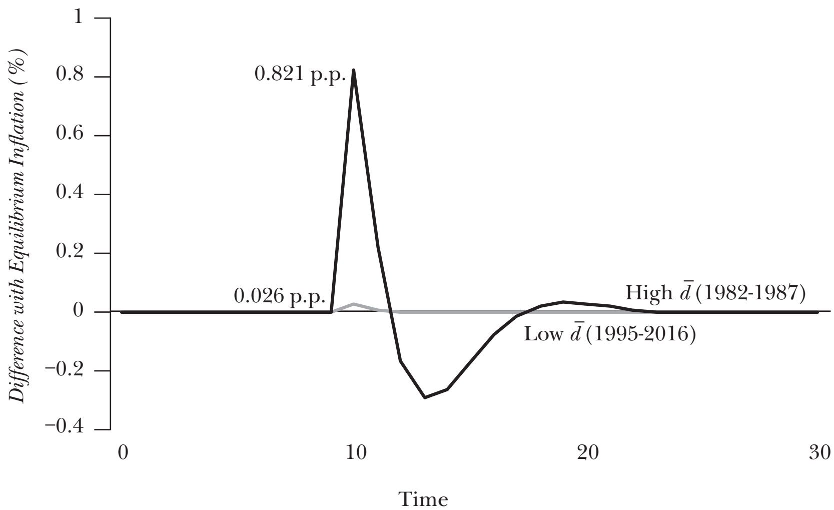
Notes: the numbers shown in parentheses represent the standard deviation of each parameter, computed using the Hessian matrix of the maximum likelihood problem (see MacDonald and Zuccini (2009)).

The parameters $\{\xi_1, \xi_2\}$ are statistically different, a result that can be interpreted as follows: between 1969 and 1994 the ERPT to expectations was 0.0215 p.p. given 1% depreciation of the NER. After 1995 the ERPT shows a considerable reduction: a 1% exchange rate depreciation translates to an increase in the expected inflation rate of 0.0047 p.p. To assess the ERPT into the observed inflation, we must consider not only the ERPT to expectations, but also the fiscal deficit level relative to GDP. This is because, within the model, both variables jointly determine the inflation rate. As we detailed in the previous

mation of expectations, the model with the NER in the real balances demand function (presented in the following section), the model with only the NER in the formation of expectations and, finally, the baseline model.

Figure 6

IMPULSE-RESPONSE FUNCTIONS OF INFLATION



Source: Banco de México and INEGI.

section, a higher fiscal deficit magnifies the effect that β_t has on inflation (in fact this effect is nonlinear). Hence, if fiscal deficit increases, the effect that the NER variation has on π_t will grow because this variation affects β_t. It can be shown that:

9
$$\frac{\partial \pi_t}{\partial \Delta NER} = \frac{\partial \pi_t}{\partial \beta_t} \frac{\partial \beta_t}{\partial \Delta NER} = \frac{\lambda \xi}{1 - \lambda \beta_t - d_t} \pi_t .$$

This equation highlights two important results: (i) the ERPT is increasing in d_t; (ii) a higher inflation rate implies a higher ERPT. Figure 6 shows the impulse-response function of inflation given a 1% depreciation in the NER. As this Figure suggests, when fiscal deficit is high (e.g., during 1982-1987) the ERPT to inflation is 0.821 p.p. However, if fiscal deficit is low the ERPT of a 1% depreciation is 0.026 p.p. Hence, a low fiscal deficit financed by the Central Bank not only translates into low inflation, but also into a limited ERPT. A low pass-through contributes to a steady and anchored expected and observed inflation rate.[41]

Figure 7 shows that the model that considers the NER as a variable that influences inflation expectations is able to provide a better account of the behavior of inflation dynamics in general, but especially during 1982 and 1994-1995, relative to the model that does not consider the NER, given the depreciation of the NER observed during those years.

4.2 The Role of the EMBI Spread

In this section we analyze an extension of the baseline model that considers the sovereign interest rate spread EMBI, a variable that captures the perception of the fiscal situation in Mexico and may influence inflation expectations. To the extent that this variable is relevant according to the estimation then this would suggest that, even though Mexico has an independent Central Bank, fiscal policy must be relevant for monetary policy through its influence on the inflation rate and its expectations.[42]

As a motivation for this extension, Figure 8 displays, in Panel (a), the interest rate spread EMBI and the NER between 1998 and 2016. This Figure shows that these variables are weakly correlated. Hence, if we consider the EMBI and the NER, we will be able to identify the effect that each variable has on inflation and its expectations. Panel (b)

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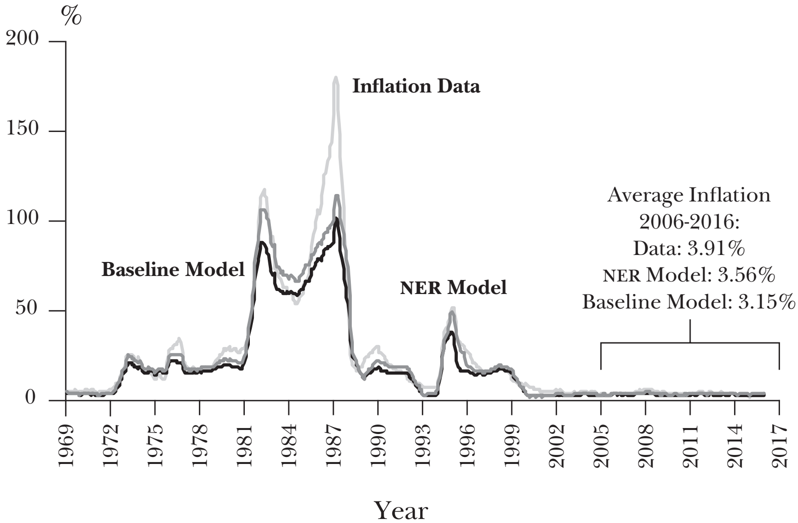
[41] The low level of pass-through is consistent with estimates in the literature for Mexico, see Albagli *et al.* (2015), Capistrán *et al.* (2011), Cortés-Espada (2013), and Kochen and Samano (2016). Furthermore, there is evidence of a declining ERPT in environments with more stable inflation and with the adoption of inflation targets (see Baqueiro *et al.* (2003), Choudhri and Hakura (2006) and Lopez-Villavicencio and Mignon (2016)). Capistrán *et al.* (2011) and Cortés-Espada (2013) document a lower ERPT for Mexico under the inflation targeting regime.

[42] The perception of economic agents of the fiscal responsibility of the government may depend on the particular historical context. For example, Sargent and Zeira (2011) describe how the anticipation of a future government bailout of banks caused a jump in inflation in Israel in 1983. They argue that the public anticipated that this bailout would eventually be financed by monetary expansion. Alternatively, Chung *et al.* (2007) explore an environment where monetary and fiscal regimes evolve according to a Markov process, this possibility can change the impact of policy shocks. These authors argue that, to the extent that there has been a history of changes in policy regimes, private agents can ascribe a probability distribution over the different regimes.

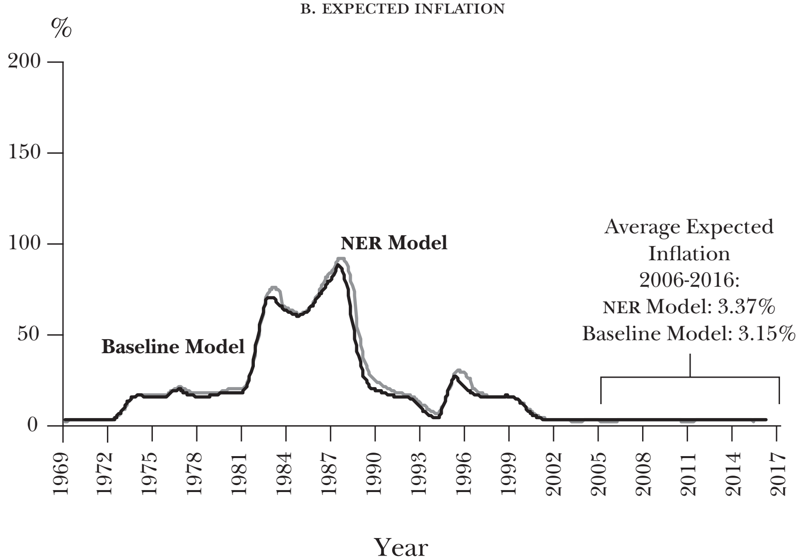
Figure 7

INFLATION AND EXPECTATIONS IN THE NER MODEL

A. ANNUAL INFLATION RATE



B. EXPECTED INFLATION



Source: INEGI.

of this Figure shows the relationship between the annual inflation rate and the variation (in basis points) of the EMBI spread.

In this extension, we consider two regimes: a fiscal dominance regime where the fiscal authority can use money creation to finance its deficit, and Central Bank autonomy, where it cannot. The interpretation we propose is that Mexico had a fiscal dominance regime between 1969 and 1994. Under fiscal dominance, Mexico had a fixed NER and under monetary dominance, the peso-dollar NER is under a floating regime (see the Appendix for a more detailed description of the exchange rate regimes). We assume that under fiscal dominance, agents determine their expectations according to:

$$\beta_t = \left(1 - \upsilon_1 - \xi_1\right)\beta_{t-1} + \upsilon_1 \pi_{t-1} + \xi_1 \Delta NER_t. \quad (10)$$

After 1994 we allow agents to give some weight σ to the current fiscal situation (which is reflected in the sovereign EMBI spread). Hence, agents determine their inflation expectations according to:

$$\beta_t = \left(1 - \upsilon_2 - \xi_2 - \sigma\right)\beta_{t-1} + \upsilon_2 \pi_{t-1} + \xi_2 \Delta NER + \sigma \Delta EMBI_t. \quad (11)$$

We allow the parameters $\{\nu, \xi\}$ to vary because the NER had a change in its regime.

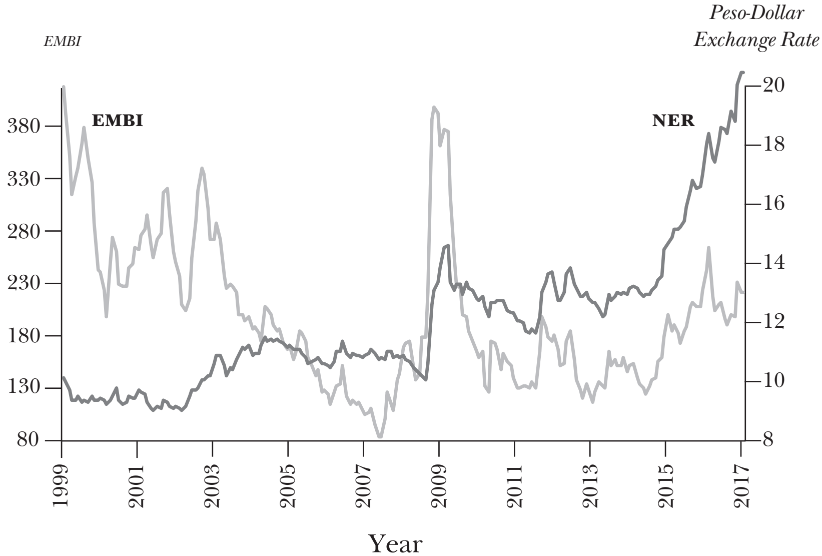
If parameters ξ and σ are positive and statistically significant, it would imply that the EMBI spread and the NER influence inflation. In fact, these variables can generate the escape dynamics that in the baseline model could only be ignited by the behavior of fiscal deficits.[43] Figure 9 exemplifies how an escape dynamics, that leads to high or hyperinflation, can occur in this scenario: suppose that initially $\beta_t = \beta^*$ and that ΔNER_t, $\Delta EMBI_t$ are limited. This implies that inflation and its expectations will converge to a low inflation equilibrium as the blue arrows show. However, if the fiscal authority starts to considerably increase its deficit (which is no longer financed with money creation and is therefore translated into debt) this would be reflected

[43] In the baseline model, an escape dynamics can only occur if fiscal deficit increases for a considerable period, because it is the only way to raise inflation expectations.

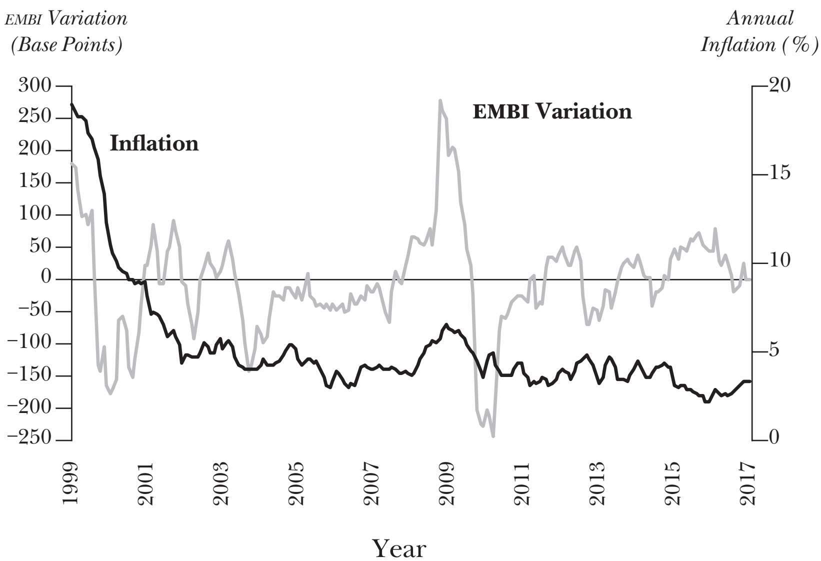
Figure 8

EMBI, NER AND INFLATION

A. EMBI AND NER



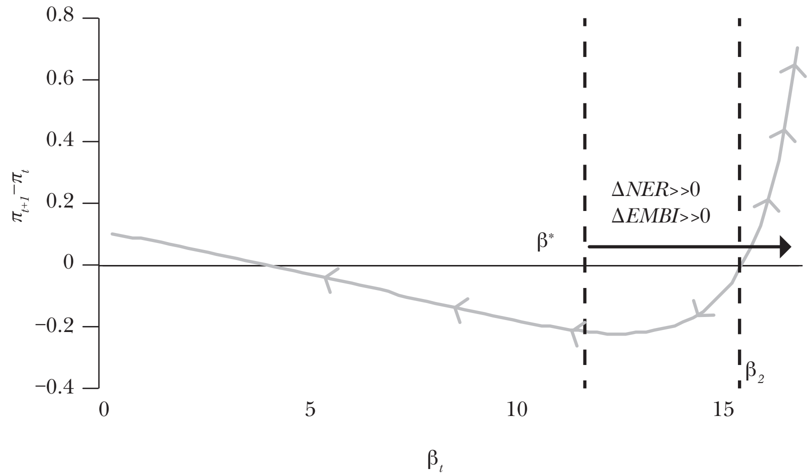
B. EMBI VARIATION AND INFLATION



Source: Banco de México, Bloomberg and INEGI.

Figure 9

ESCAPE DYNAMICS IN THE MODIFIED MODEL



Note: these figures considers β_{t-1}=1.02 and the estimated parameters of the EMBI extension.

in the EMBI spread and influence the NER. In our model, the increment in these variables will affect inflation expectations. Furthermore, if this effect is large enough, as shown with an orange arrow in the Figure, it will cause that $\beta_t>\beta_2$, which will lead to high inflation (as shown with red arrows). Consequently, if σ and ξ are significant and positive then, even in a context of monetary dominance, our model suggests the possibility of high inflation caused by the fiscal authority via expectations.

To estimate this model, once again we consider the inflation sequence according to the INPC during 1969-2016, the NER variation registered by Banco de México, and the EMBI spread reported by Bloomberg after 1994. The main results of this extension are:

- The estimation for σ suggests that, everything else constant, if the EMBI spread increases 100 basis points, the rate of inflation rises by 0.24 p.p.[44]
- On the other hand, the estimation of ξ_2 implies that under monetary dominance the inflation rate increases 0.011 p.p. given a 1% depreciation of the NER.
- Finally, with this specification for inflation expectations, the model estimates that $\bar{d}(3)$ is almost zero, which is the deficit regime for the period of independence of the Central Bank.

Figure 10 shows that, if we consider the interest rate spread EMBI and the NER, then the inflation generated by this model is closer to the inflation sequence presented in the data. Actually, the incorporation of these variables allows the model to explain 0.65 p.p. more of the inflation rate during 2006-2016 compared to the baseline model. The Diebold-Mariano test also suggests that the in-sample forecast for the inflation sequence between these years is statistically different (at a 1% confidence level) between the EMBI extension and the baseline model. Hence, these extensions suggest that the fiscal situation, to some extent, have caused the inflation rate to be above Banco de México's inflation target of 3%.

4.3 The Exchange Rate: An Alternative Channel

A variable such as the exchange rate may affect inflation through several channels and not only through inflation expectations. We now discuss an extension where the NER has an effect on inflation through its direct influence on the price level Pt. We assume that $P_t = \gamma M_t + \lambda P^e_{t+1} + \psi NER_t$.[45] Hence, the NER has a weight ψ on the price level, parameter that can be interpreted as the pass-through of the NER to the price level. This modification implies that the inflation rate is now given by the following expression:

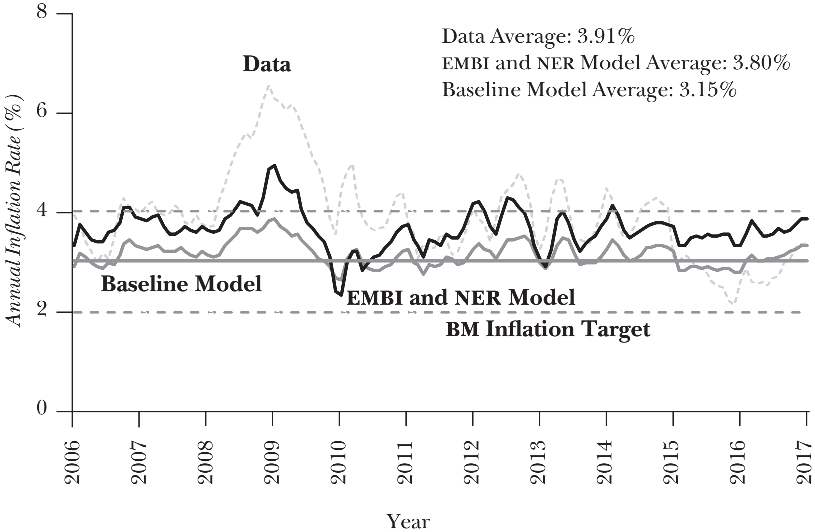
12 $$\pi_t = \frac{\theta\left(1-\lambda\beta_{t-1}-\psi NER_{t-1}\right)}{1-\lambda\beta_t-\psi NER_t-\gamma d_t}.$$

[44] To find the impact that the EMBI spread has on inflation, we again have to consider an impulse-response function as in Figure 6.

[45] Alternatively, this expression can be rewritten as a demand for real balances that depends on the exchange rate.

Figure 10

EVOLUTION OF INFLATION: MODELS AND DATA DURING 2006-2016



Source: Banco de México, Bloomberg and INEGI.

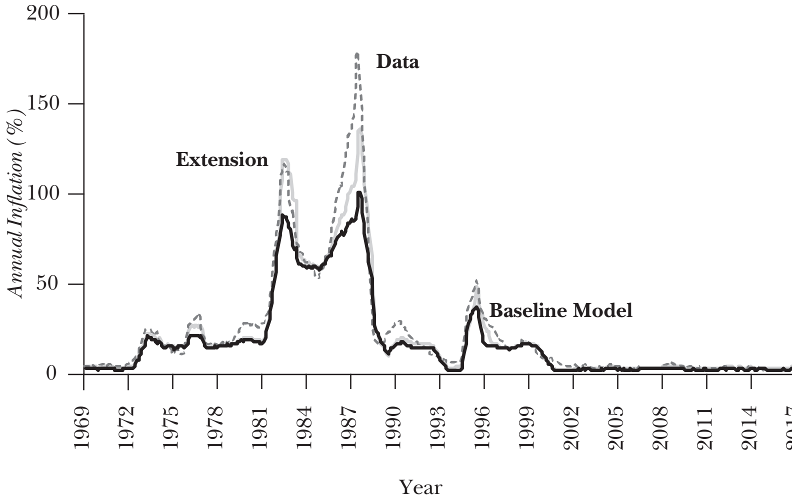
Expectations are given by the CGE algorithm $\beta_{t+1}=(1-\nu)\beta_t+\nu\pi_t$, when there is fiscal dominance (i.e. before 1994) and by $\beta_{t+1}=(1-\nu-\sigma)\beta_t+\nu\pi_t+\sigma\Delta EMBI_t$ under Central Bank independence. The main difference between assuming that the NER affects expectations or P_t is that, in this extension, inflation is a function of the NER dynamics in two consecutive periods: (NER_{t-1}, NER_t). Hence, if the NER depreciates considerably between $t-1$ and t, this will have a higher impact on inflation and on future inflationary expectations.

Figure 11 presents the main results of this extension. As this Figure shows, the extended model better accounts for the inflation rate during 1970-2016 than the baseline model. This model performs particularly better in those periods in which the NER registers a considerable depreciation. For example, during 1982, the peso-dollar NER suffered a depreciation of over 200% and the model predicts that inflation at the end of that year was 118.1%. Additionally, during 1995 the NER had a depreciation that surpassed 100%, which implied, according to the model, an inflation of 49.1% by the end of this year.

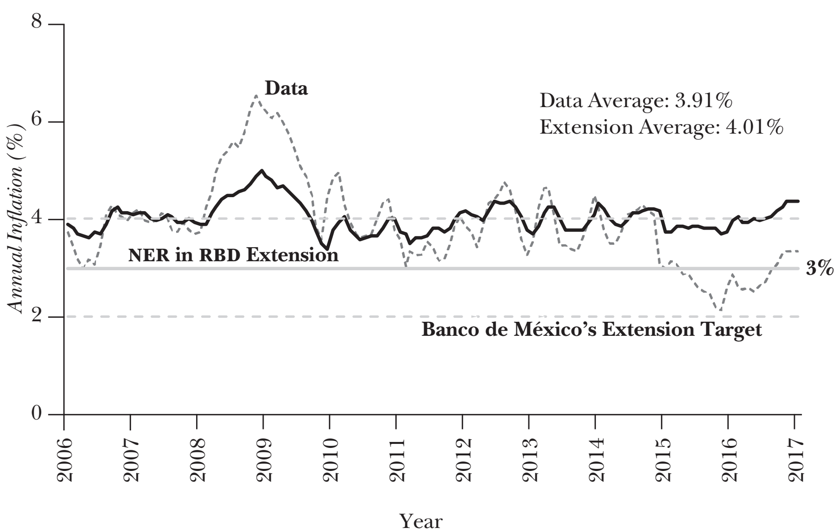
Figure 11

INFLATION IN THE EXTENDED MODEL

A. 1970-2016



B. 2006-2016



Source: Banco de México and INEGI.

5. CONCLUDING REMARKS

The baseline model and the extensions that we have presented allow us to assess the role of fiscal policy in the determination of inflation and its expectations. Even in a context of Central Bank independence, a large literature has explored the role of fiscal policy in determining inflation. We exploit a simple model and provide evidence of the relevance of fiscal policy in determining the behavior of aggregate prices in Mexico as well as the importance of expectations.

Admittedly, the theoretical framework we utilize is relatively simple and models with more structure, perhaps in the inter-temporal dimension, would increase our understanding of the relationship between fiscal policy and inflation in emerging economies. Furthermore, it is sometimes argued that Central Bank independence acts as a mechanism that increases fiscal responsibility of the government in developing countries (Bodea and Higashijima (2015), Minea and Tapsoba (2014)). We believe that further research is necessary to understand the institutional arrangements that govern the relationship between a central bank and the fiscal authority in the presence of competing objectives and constraints.

6. APPENDIX

6.1 Parameter Estimation

The following equations, together with transition matrices $\{Q_d, Q_v\}$ define inflation, expected inflation, and fiscal deficits at each t according to the baseline model:

$$\pi_t = \chi_t \frac{\theta\left(1-\lambda\beta_{t-1}\right)}{1-\lambda\beta_t-\gamma d_t} + \left(1-\chi_t\right)\pi_t^*\left(d_t\right),$$

$$\beta_{t+1} = \left(1-v\right)\beta_t + \pi_t, \quad log\left(d_t \mid \bar{d}_t, v_t\right) \sim N\left(log\left(\bar{d}_t\right), v_t\right),$$

where Xt is a constant equal to 1 if {πt, βt.dt} satisfy $1-\lambda\beta_{t-1}>0$ and $\delta(1-\lambda\beta_t-\gamma d_t)>\theta(1-\lambda\beta_{t-1})$.[46] Assuming $\beta_0=\pi_0$ and given a sequence of fiscal deficits $\left\{d_t\right\}_{t=0}^{T}$, the model can generate a sequence for the expected inflation rate $\left\{\beta_t\right\}_{t=0}^{T}$ and for the actual inflation rate $\left\{\pi_t\right\}_{t=0}^{T}$. However, the hidden Markov states $\left\{\bar{d},v\right\}$, among other parameters, must be estimated to generate a sequence of fiscal deficits. Table 3 shows the parameters that need to be estimated.

Table 3

MODEL PARAMETERS

parameter	*restrictions*	*description*
λ	$0<\lambda<1$	weight of expectations on the price level
ν	$0<\nu<1$	weight of past inflation on expectations
γ	$\gamma>0$	weight of monetary base on the price level
θ	$0<\theta<1$	persistence of the monetary base
δ	$\delta>0$	constant that bounds inflation
$\bar{d}_1,\bar{d}_2,\ldots,\bar{d}_D$	$\bar{d}_1>\bar{d}_2>\ldots>\bar{d}_D>0$	median values of fiscal deficits
$v_1,v_2,\ldots,v_V$	$v_1>v_2>\ldots>v_V>0$	variance values of fiscal deficits
v_π	$v_\pi>0$	inflation variance when determined randomly
p_{ij}^d	$0\le p_{i,j}^d\le 1,\sum_j p_{i,j}^d=1$	*i, j*-component of the transition matrix Q_d
p_{ij}^v	$0\le p_{i,j}^v\le 1,\sum_j p_{i,j}^v=1$	*i, j*-component of the transition matrix Q_v

[46] These constraints guarantee that the model's inflation rate is bounded and that the real balances demand is positive.

Let ϕ be the vector of all the parameters in the model. Given that d_t is a random variable and because $\{\pi_t, \beta_t\}$ are a function of fiscal deficits, we can construct a joint density function for a sequence of T periods of inflation, its expectations and fiscal deficit: $p\left(\pi^T, \beta^T, d^T \mid \phi\right)$. If there was available data on inflation, its expectations and fiscal deficit for a large T, the estimated parameters $\hat{\phi}$ can be obtained using the maximum-likelihood method applied to the joint density $p\left(\pi^T, \beta^T, d^T \mid \phi\right)$. However, data on inflation expectations and fiscal deficit is hard to find for a large T, or may not be reliable. Furthermore, we find that historical series often go through methodological modifications. This is particularly true in the case of Mexico, as we have already discussed.

INPC (consumer price index) data is available since January 1969 at a monthly frequency. Therefore, to estimate the parameters we use the marginal density of a sequence of inflation π^T between January of 1969 and December of 2016. This marginal density is denoted $p\left(\pi^T \mid \phi\right)$. The estimated parameters are obtained as the vector $\hat{\phi}$ that maximizes $p\left(\pi^T \mid \phi\right)$ given the gross inflation rate sequence π^T (subject to constraints):

$$\hat{\phi} = \arg\max_{\phi \in \Omega} \; p\left(\pi^T \mid \phi\right), \tag{13}$$

where Ω is the set of all the vectors φ that satisfy the constraints relevant for each parameter. Because there is no analytical solution to this maximization problem, $\hat{\phi}$ has to be approximated numerically. To do this, we used a constrained optimization algorithm based on the BFGS (Broyden-Fletcher-Goldfarb-Shanno) method of Nocedal and Wright (2006) and the block-wise method of Sims *et al.* (2006).

Given the computational burden of the maximum-likelihood optimization problem, Sargent *et al.* (2009) fix three parameters to reduce the complexity on the estimation. These parameters are: $\theta = 0.99$, $\delta = 100$, and $\gamma = 1$. The value assigned to θ is consistent with the behavior of nominal balances in the five countries these authors studied. Fixing $\delta = 100$ implies that, in every period, inflation cannot surpass 10,000%. Finally, γ was fixed because the maximum-likelihood algorithm cannot identify γ and d_t separately. Once d_t is estimated for each period, γ is re-normalized so that the mean of fiscal deficits estimated by the model matches the mean observed in the data (in our case, for Mexico for the period 1977-2016).

6.2 Adaptive vs. Rational Expectations

In this part of the Appendix we discuss some of the implications that rational expectations have in the baseline model presented in this paper. Additionally, we compare the main differences induced in the dynamics of the model between these types of expectations and CGE. One way of modeling that agents are rational when forming their beliefs on future inflation is to assume:

14 $$\beta_{t+1} = \mathbb{E}_t\left[\pi_{t+1} \mid \bar{d}_t, v_t\right].$$

Equation (14) points out one important difference between rational expectations and CGE in this model. If agents are rational, they condition their expectations on the median level $\bar{d}_t$ and the variance v_t of current fiscal deficit since the evolution of the median and variance of fiscal deficit is known to agents when they are rational. Assuming CGE does not require agents to condition their expectations on $\left\{\bar{d}_{t+1}, v_{t+1}\right\}$ because they update their beliefs according to (4).

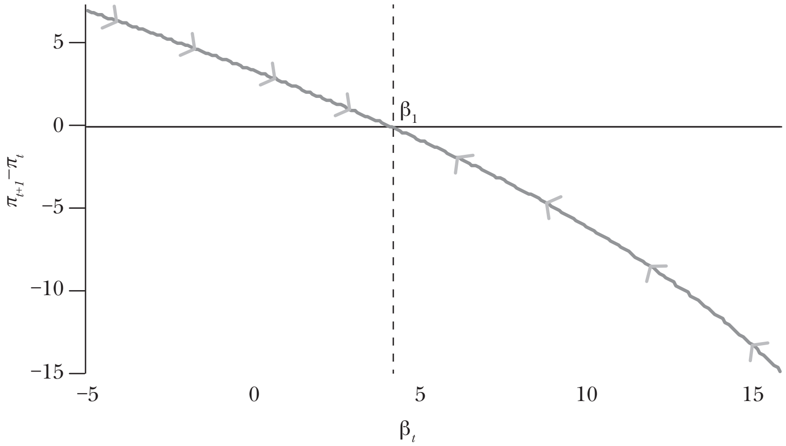
Assuming rational expectations also affects the dynamics between the gross inflation rate of two consecutive periods $\{\pi_t, \pi_{t+1}\}$ as a function of β_t. Panel (a) of Figure 12 plots $\pi_{t+1} - \pi_t$ as a function of β_t assuming β_{t+1} is determined according to (14) and using the same median and variance of fiscal deficit in t and $t+1$. As this Figure shows, there is only one value of β_t that induce a constant inflation (and expectations) over time (β_1). As the Figure suggests, β_1 is a stable equilibrium. Thus, if fiscal deficit remains with the same median and variance level, $\pi_{t+1} - \pi_t$ will converge to zero and β_t to β_1.

With rational expectations, contrary to CGE, if inflation is high ($\beta_t > \beta_1$), agents will not allow their expectations to provoke the escape dynamics. Their expectations will adjust and converge to β_1. However, the government could prevent expectations from converging to a high inflation equilibrium by reducing its fiscal deficits as shown in Panel (b) of Figure 12. This Figure plots $\pi_{t+1}\ \pi_t$ as a function of β_t for two different $\bar{d}$ values (low and high). Assuming $\beta = \hat{\beta}$ and that the median fiscal deficit level is high, if the government continues with this deficit level, inflation will con- verge to a high equilibrium

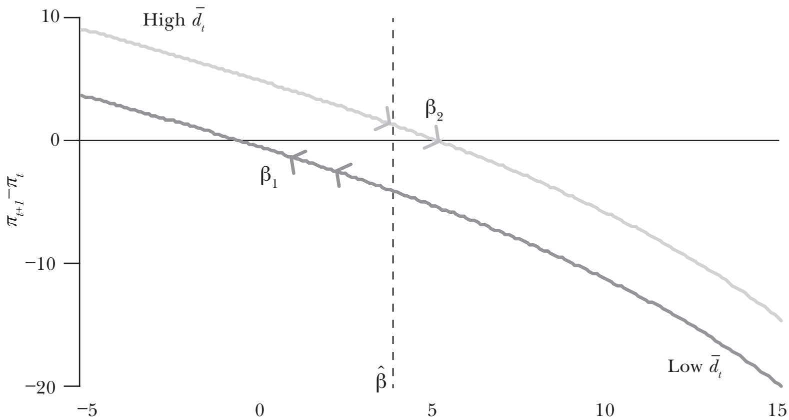
Figure 12

DYNAMICS INDUCED BY RATIONAL EXPECTATIONS

A. INFLATION AND EXPECTATIONS



B. FISCAL DEFICIT, INFLATION AND EXPECTATIONS



Note: These figures considers β_{t-1} =1.02 and the estimated parameters shown in Table 1.

and its expectations to β_2. However, if the government reduces its fiscal deficits, it will change the dynamics on inflation and its expectations inducing a convergence to β_1.

Figure 12 points out an important difference between rational expectations and CGE: when agents use the CGE algorithm, if the inflation rate induces a high β_t then this could provoke an escape dynamics and eventually a hyperinflation episode, where the dynamics between inflation and its expectations are unbounded. However, with rational expectations, even with an extremely high fiscal deficit, agents always adapt their expectations to prevent a hyperinflation spiral. If fiscal deficit is high, rational expectations imply a stable equilibrium with a high inflation rate and no escapes.

Even though CGE and rational expectations induce different dynamics on the variables involved in the model, the inflation equilibria they predict are similar. Sargent *et al.* (2009) argue that, in the context of hyperinflation models, "an adaptive expectations version of the model shares steady states with the rational expectations version, but has more plausible out-of-steady state dynamics." Besides, rational expectations may induce multiple equilibria that are hard to compute. Given the computational problem rational expectations may induce and the fact that some Latin American countries have experienced hyperinflation episodes with escape dynamics which a strictly rational expectations model cannot account for, CGE are necessary for the purposes of this study.

6.3 Exchange Rate Regimes

The table in this Annex presents the different regimes that the peso-dollar NER has had between 1954 and 2016. Before 1994, this NER had several regimes that can be considered slight variations of a fixed NER rule. For example: (i) controlled variation, in which the Banco de México established an interval in which the NER was allowed to vary; (ii) generalized controlled system, in which all credit institutions needed an authorization from the Central Bank to sell or buy currencies; and (iii) controlled flotation, in which Banco de México established an interval, changed daily, within which the NER was allowed to fluctuate.

Table 4

EXCHANGE RATE REGIMES IN MEXICO DURING 1954-2016

Date			*NER*	
Begining	*End*	*Regime*	*Begining*	*End*
April 1954	August 1976	Fixed	12.50	12.50
September 1976	August 1982	Controlled Variation	20.50	48.80
September 1982	December 1982	Generalized Controlled System	50.00	70.00
December 1982	August 1985	Controlled System	95.00	281.00
August 1985	November 1991	Controlled Flotation	282.30	3,073.00
November 1991	December 1994	Floating Intervals with Controlled Variation	3,074.10	*N* 3.99
December 1994	December 2016	Floating	*N* 4.88	*N* 20.51

Notes: *N* denotes New Mexican Pesos.
Source: Banco de México.

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