The Transmission of US Monetary Policy Normalization to Emerging Markets

Kólver Hernández

Abstract

In this chapter, I analyze the potential macroeconomic effects of the normalization of US monetary policy for emerging market economies (EMEs), in particular for Mexico. I build on the work of Hernandez and Leblebicioğlu (2016) by adding monetary elements to their two-country DSGE model that endogenizes multiple transmission channels for the transmission of international shocks. Among those channels are the exchange rate, international bank lending, international trade and monetary policy rates. Based on a Bayesian estimation of the deep parameters of the model, I simulate scenarios that yield an equilibrium in which US monetary policy rate would increase in the last two quarters of 2015. The underlying conditions that promote the normalization of monetary policy in USA imply favorable growth of around 2.4% in GDP and an average increase of 25 basis points in US policy rate. For Mexico, those conditions carry positive international spillovers that result in an average GDP growth of 2.8%. The increase in US rate calls for a response in Mexico’s policy rate in more than one to one, i.e., it calls for an aggressive response. Mexico’s policy rate hike contains the depreciation of the exchange rate and stabilizes inflation.

Keywords: emerging market business cycles; transmission of foreign shocks; estimated two-country model; international transmission of monetary policy. JEL classification: E32, F41.

K. Hernández <kherandez@cemla.org>, researcher at Research Department, Banco de México. The author would like to thank comments from participants of the XI Meeting of Monetary Policy Responsibilities at Banco do Brasil and participants of the seminar Elaboración de Proyecciones en Ambientes de Alta Incertidumbre: Experiencias desde la Región, at CEPAL, Chile. The views expressed in this document correspond to the author and do not necessarily reflect the views of Banco de México. Additionally, this chapter was elaborated while the author was member of CEMLA.
1. INTRODUCTION

Through the lenses of a two-country dynamic stochastic general equilibrium (DSGE) model, this chapter analyzes multiple underlying conditions that yield an equilibrium in which USA normalizes its monetary policy by increasing the Federal Reserve funds rate. The question that I address is: What those conditions imply for emerging markets and in particular for Mexico? I build on the real business cycle model developed by Hernandez and Leblebicioğlu (2016) to add monetary features. The model features several channels for the international transmission of shocks, among them: the exchange rate channel, international bank lending, capital flows, USA and EME policy rates, and international trade. As shown first in Hernandez and Leblebicioğlu (2016), those channels are crucial to capturing the international transmission of shocks. In sharp contrast, Justiniano and Preston (2010) show that an estimated standard small open economy model fails to capture the international transmission of shocks from USA to a small open economy – Canada in that case.

In order to discipline the multiple channels modeled I use 20 time series from 2001Q1 to 2015Q2 for USA and Mexico to estimate the model. The model in-sample predictions are in line with the data. In particular, the model addresses very successfully the Justiniano and Preston (2010) criticism of estimated DSGE models in that this model predicts cross-country correlations consistent with the data.

With the purpose of produce policy normalization scenarios, I use the estimated model to simulate millions of paths for the full economy for the last two quarters of 2015 – which are out of sample. Then from the simulated paths I only consider those in which USA interest rate increases in one or both quarters. In the average policy normalization scenario, the model predicts conditions in USA that lead to a policy rate increase of 25 basis points jointly with an average growth of 2.4% in 2015. For Mexico those conditions imply a growth of 2.8%. The increase in US rate calls for an increase in Mexico’s policy rate. Mexico’s policy rate hike contains the ongoing depreciation of the real exchange rate and stabilizes inflation.

The rest of the chapter is organized as follows: Section 2 presents the two-country monetary DSGE model, Section 3 shows the scenario analysis and Section 4 concludes.
2. THE MODEL

In this section, I show the main ingredients of the two-country DSGE monetary model. The economy features domestic (EME) and foreign (US) households, two sectors of final goods producers (tradable and nontradable) in each economy. Following Christiano et al. (2014) it also features a capital owner, entrepreneurs and a financial intermediary, additionally it has a fiscal and a monetary authorities.

2.1 Households

Both the domestic and foreign households supply labor to the tradable and non-tradable sectors and trade bonds with the rest of the world. The preferences are of the GHH—Greenwood et al. (1988) type:

\[
U = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ \xi_{C,t} - \varphi C_{t-1} - \frac{\vartheta}{1+\eta} L_t^{1+\eta} \right]^{1-\zeta},
\]

where \( C_t \) is consumption, \( L_t \) is labor, \( \xi_{C,t} \) is a preference shock, \( \varphi \in (0, 1) \) is a habit parameter, \( \eta \) determines the Frisch elasticity, and \( \vartheta \) is a preference parameter. The composite labor \( L_t \) is a CES basket with labor in the tradable sector \( L_{T,t} \) and labor in the nontradable sector \( L_{NT,t} \) with the elasticity of substitution \( \chi \). The consumption basket, \( C_t \), is defined by a CES aggregator for the tradable consumption basket \( C_{T,t} \) and the nontradable consumption basket \( C_{NT,t} \) with the elasticity of substitution \( \theta \). In turn, the CES tradable consumption basket is formed by consumption of the foreign good \( C_{F,t} \), the domestic good \( C_{H,t} \) and a consumption good that comes from the rest of the world \( C_{O,t} \). The elasticity of substitution across tradable goods is \( \nu \).

Households trade risk-free bonds with the rest of the world \( B^o_t \). The budget constraint is

\[
C_t + B^o_t + \frac{\sigma}{2} (B^o_t - \bar{B}^o)^2 = w^{*}_{T,t} L_{T,t} + w^{*}_{NT,t} L_{NT,t} + \frac{R_{t-1} B_{t-1}^o}{\pi_t} + \Omega_t - T_t,
\]

where \( w^{*}_{T,t} \) and \( w^{*}_{NT,t} \) are the wage rates, \( T_t \) denotes lump-sum taxes, \( \Omega_t \) is lump-sum payments to the households. Bond holdings are subject to quadratic costs of adjustment \( \frac{\sigma}{2} (B^o_t - \bar{B}^o)^2 \). The household chooses
\{C_t, L_t, L_{T,t}, L_{NT,t}, B_{t}^{\infty}\}_{t=0}^\infty \text{ to maximize Equation 1 subject to the budget constraint, Equation 2, the labor and consumption composites, and a no-Ponzi-game condition.}

2.2 Firms

There is a continuum of firms with mass one in each sector. They can be indexed by \(z \in [0, 1]\). Firms are monopolistic competitive and set prices subject to a Calvo pricing scheme, i.e., firms can change prices only when they receive a random signal that arrives with probability \((1-C)\) in every period. In the periods when the producer does not receive the random signal, it adjusts the nominal price according to the indexation rule:

\[ P_{j,t}(z) = (\pi_{t-1})^{\gamma} P_{j,t-1}(z), \quad j \in \{T, NT\} \]

where \(P_{j,t}(z)\) is the nominal price of the variety \(z\) in sector \(j\), \(\pi_t\) denotes aggregate inflation and \(\gamma \in [0, 1]\) is the indexation parameter. The firm \(z\) faces a demand of the form

\[ Y_t^j(z) = \left( \frac{P_{j,t}(z)}{P_{j,t}} \right)^{\lambda_t} Y_t^j \]

where \(\lambda_t\) follows an AR(1) process specified below, \(P_{j,t}\) is the aggregate price index in sector \(j\) and \(Y_t^j\) denotes total demand.

2.2.1 Technology

Firms in the tradable sector have the technology

\[ Y_{T,t} = \xi_{AT,t} (u_t K_{t-1})^{\alpha} L_{HT,t}^{1-\alpha}, \]

where \(u_t\) is the capital utilization rate, \(\alpha \in (0, 1)\), and \(\xi_{AT,t}\) denotes the productivity shock. In the non-tradable sector firms face the technology

\[ Y_{NT,t} = \xi_{AN,t} L_{NT,t}, \]

where \(\xi_{AN,t}\) denotes the productivity process. I allow for the sectoral technology shocks to be correlated.
corr\left(\xi_{AN,t}, \xi_{AT,t}\right) > 0.

Note that the correlation is across sectors within each country but there are not cross-country correlations among shocks.

Firms face a working capital constraint as in Neumeyer and Perri (2005) and Uribe and Yue (2006). They need to borrow a fraction \(\kappa_j\) of the payroll costs with an intra-period loan.

### 2.2.2 Pricing

Given the technology with constant returns to scale, real profits (in terms of the aggregate consumption basket) are given by

\[
\Pi_t^j(z) = p_{j,t}(z)Y_t^j(z) - mc_tY_t^j(z) \quad j \in \{NT,T\}
\]

where \(mc_t\) is the marginal cost and \(p_{j,t}(z) = \frac{P_{j,t}(z)}{P_t}\), where \(P_t\) is the aggregate price index. Firms receiving the Calvo signal to optimally change prices choose \(p_{j,t}(z)\) to maximize

\[
\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t C \cdot \frac{\Lambda_{t+1}}{\Lambda_t} \Pi_t^j(z)
\]

where \(\frac{\beta^t \Lambda_{t+1}}{\Lambda_t}\) is the household’s stochastic discount factor, subject to the demand, Equation 4, and the indexation rule, Equation 3.

The Appendix A shows that the pricing scheme yields the Phillips curves:

\[
\pi_{j,t} = \beta \mathbb{E}_t \pi_{j,t+1} + \nu_{t-1} - C \beta \nu_t + \frac{(1-C)(1-C \beta)}{C} \left\{mc_{j,t} - \frac{1}{\lambda-1} \lambda_{j,t} - C \beta \sigma_{ind,t}\right\}
\]

with \(j \in \{NT, T\}\), where

\[
\sigma_{ind,t} = \sigma_{ind,t-1} + \nu_t,
\]

and

\[
\pi_t = (1-a)\pi_{NT,t} + a\pi_{T,t}.
\]
2.3 Capital Producer, Entrepreneurs, and the Financial Intermediary

Following Christiano et al. (2014), the capitalist builds new raw capital with the technology

\[ K_t = (1 - \delta) K_{t-1} + \xi_{t,I_t} I_t \left[ 1 - \frac{\phi I_t}{2 \left( I_{t-1} - 1 \right)^2} \right], \]

and sells it to the entrepreneurs, where \( I_t \) is investment, \( \xi_{t,I_t} \) is an investment shock and \( \phi \) determines the convex adjustment cost of investment. The new capital is sold to the entrepreneur at the price \( Q_t^k \).

The entrepreneur receives a productivity shock \( \omega \), with \( \ln(\omega) \sim N(1, \sigma_\omega) \), that transforms the raw capital in effective capital \( \omega K \). The effective capital is rented to the final good producer and after it is used in production is sold back to the capitalist. The return on capital is \( \omega R_t^k \), where \( R_t^k = \frac{u_t^k - a(u_t^k) + Q_t^k (1 - \delta)}{Q_{t-1}} \), and

\[ a(u) := r^k \left[ \exp(\sigma_a (u - 1)) - 1 \right] \frac{1}{\sigma_a} \]

\( a(u) \) gives the utilization adjustment cost \( (\sigma_a > 0) \), and \( \delta \) is the depreciation rate.

The optimal contract maximizes the expected value of the entrepreneur subject to a zero profit condition for the intermediary. The optimality conditions imply:

\[ \mathbb{E}_t \Gamma_{t+1} = \mathbb{E}_t \left[ \frac{\Gamma_{t+1}^\prime - \mu G_{t+1}^\prime}{R_t - \Gamma_{t+1} - \mu G_{t+1}} R_{t+1}^k \right] \]

\[ R_{t+1} \left( L_{t-1}^N - 1 \right) = L_{t-1}^N \left( \Gamma_t - \mu G_t \right) R_t^k \]

where \( \omega_t \) is a threshold in the productivity shock that separates those that can repay the loan and those that default. \( F(\omega_t) \equiv \int_0^{\omega_t} dF(\omega) \) and \( G(\omega_t) \equiv \int_0^{\omega_t} \omega dF(\omega) \). \( \Gamma(\omega_t) = \left[ 1 - F(\omega_t) \right] \omega_t + G(\omega_t) \) and \( \Gamma^\prime \) and \( G^\prime \) are the corresponding derivatives with respect to \( \omega \).

2.4 Fiscal and Monetary Policies

The government purchases goods only from the domestic traded and nontraded sectors, which are combined in a composite good similar to the consumer’s consumption basket. The government spending follows the rule
\[ \text{Gov}_t = \left( \text{Gov} \right) \left( \frac{Y_t}{Y_{t-1}} \right)^{\psi_{G,Y}} \xi_{G,t}, \]

where \( \xi_{G,t} \) is an exogenous shock and \( \psi_{G,Y} \) is a reaction coefficient.

The monetary authority follows the Taylor rule:

\[ R_t^p = \rho, R_{t-1}^p + \rho, \pi_t + \rho, Y_t + \epsilon_{mp,t} \]

where \( \rho, \) is the smoothing coefficient and \( \epsilon_{mp,t} \) is i.i.d. monetary-policy shock.

### 3. Estimation and Monetary Policy Scenario Design

As a general rule, I estimate all the parameters that govern shocks and frictions in the model. I use the Random Walk Metropolis-Hasting (RWMH) algorithm, as described in An and Schorfheide (2007), in particular, to solve the model I use the algorithm discussed in Hernandez (2013) jointly with the solution method of Klein (2000). I use quarterly data for Mexico and USA from 2001Q1 to 2015Q2. The time series used are: JP Morgan EMBI+ Spread Mexico, spread between BAA and 10-year Treasury for USA, shadow federal funds rate for USA, the 90-day CETES rate for Mexico, GDP-deflator inflation for Mexico and the USA, GDP growth for Mexico and the USA, consumption growth for Mexico and USA, investment growth for Mexico and USA, bilateral imports growth for Mexico, bilateral exports growth for Mexico, GDP-deflator-based bilateral real exchange rate depreciation, government spending growth for Mexico and USA, non-bilateral trade over GDP for Mexico and USA, and growth in per capita work hours for USA.

#### 3.1 The Transmission Mechanism of US Shocks

Figure 1 shows the impulse responses of key Mexico’s variables to US shocks. That is, it shows the transmission mechanisms of USA shocks into the Mexican economy. First, an expansionary US preference shock increases Mexico’s GDP, inflation, interest rates and depreciates the peso. The preference shock in USA acts as a USA demand
shock that increases GDP in USA, generates inflation in USA and as a result the US monetary policy has to increase the policy rate. Given the US rate hike, the peso depreciates, which together with the larger US demand for Mexican goods stimulates net exports in Mexico and thus GDP in Mexico gets stimulated. That is, the trade channel is of key importance for the international transmission of these types of shocks. In turn, the depreciation pass-through to domestic prices and is inflationary for Mexico; with higher GDP, a more depreciated peso and higher inflation, the monetary policy response in Mexico is to increase policy rates to restore the long-term equilibrium.

Second, a US technology shock increases US GDP, lowers US inflation and drops the real US interest rate—as in any standard DSGE model. In turn, the financial channel in Mexico takes more relevance for the international transmission of these type of shocks, because lower international rates make the US technology shock to act as a Mexico technology shock. That is, it lowers the marginal cost of production in Mexico as production financing costs are lower. In turn, lower marginal costs in Mexico lower inflation and stimulate GDP with higher net exports and, as a result, the peso gets appreciated to help restore the long-term equilibrium.

Finally, a monetary policy shock in USA is contractionary for USA and lowers US inflation. An interest rate hike in USA depreciates the peso, which is passed-through to domestic prices in Mexico and inflation hikes; as a result, the monetary policy increases the policy rate. The lower US demand for Mexican goods—despite the depreciated peso—drops domestic GDP.

Of course, these impulse responses are ceteris paribus exercises aimed to understand the transmission mechanisms of the model. The actual conditions under which one should expect a hike of US interest rates must be the end result of realizations of various shocks that determine a state of the US economy that calls for a less accommodative monetary policy. The next subsection addresses that issue.

3.2 Scenario Analysis

The scenario analysis is conducted as follows. First consider the model’s solution and the observables:

\[ S_t = TS_{t-1} + R_{t} \]

model’s law of motion

\[ D_t = ZS_{t-1} \]

observables
Figure 1
MEXICO: IMPULSE RESPONSES TO US SHOCKS IN THE ESTIMATED MODEL

US PREFERENCE SHOCK

US TECHNOLOGY SHOCK

US MONETARY POLICY SHOCK

The Transmission of US Monetary Policy Normalization
Figure 1 (cont.)

MEXICO: IMPULSE RESPONSES TO US SHOCKS IN THE ESTIMATED MODEL

<table>
<thead>
<tr>
<th>Shock Type</th>
<th>Interest Rate</th>
<th>Depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>US Preference Shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>0.03</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>-0.06</td>
<td>-0.08</td>
</tr>
<tr>
<td></td>
<td>-0.08</td>
<td>-0.10</td>
</tr>
<tr>
<td></td>
<td>-0.1</td>
<td>-0.12</td>
</tr>
<tr>
<td>US Technology Shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.02</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>-0.04</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>-0.06</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td>-0.08</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td>-0.1</td>
<td>-0.3</td>
</tr>
<tr>
<td></td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>US Monetary Policy Shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.14</td>
<td>0.7</td>
</tr>
<tr>
<td></td>
<td>0.12</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>0.08</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.06</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>0.04</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>0.02</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>-0.1</td>
</tr>
</tbody>
</table>
where $T$, $R$, and $Z$ are matrices formed by functions of the deep parameters of the model.

- Use the Kalman filter to obtain an estimate of $S_t$ and $D_t$ for $t = 1 \ldots n$.

- Draw $f$ draws of $t$ and obtain $S_{n+f}$ and $D_{n+f}$. Repeat many times to obtain many possible histories.

- Form a loss function to weight all draws of $S_{n+f}$ and $D_{n+f}$. The weighted average is the forecast.

- The loss function can be very sophisticated for central banks.

- Here, I only impose more weight to those draws consistent with an increase of the US interest rate consistent with the FOMC announcement.

Figure 2 shows the model predictions for the effects of the normalization of US monetary policy. The model predicts conditions in USA that lead to a policy rate increase of 25 basis points and average growth of 2.5% in 2015. For Mexico those conditions imply a growth of 2.4%. The increase in US rates calls for an aggressive response of Mexico’s policy rate. Mexico’s policy rate hike will contain the ongoing depreciation of the real exchange rate and stabilize inflation.

\begin{figure}[h]
\centering
\begin{minipage}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{gdpavg245.png}
\caption{GDP\textsuperscript{avg}: 2.45 (IMF 2.5)}
\end{minipage} \hfill
\begin{minipage}{0.45\textwidth}
\centering
\includegraphics[width=\textwidth]{gdpavg282.png}
\caption{GDP\textsuperscript{avg}: 2.82 (IMF 2.4)}
\end{minipage}
\end{figure}
4. CONCLUSIONS

This chapter presents a DSGE model for the Mexican economy that contains important channels for the international transmission of US shocks to Mexico. Among the transmission channels are: the exchange rate channel, international bank lending, capital flows, monetary policy rates and international bilateral trade. Based on a Bayesian estimation of the deep parameters of the model, I simulate millions of scenarios under which the US monetary policy rate would increase in the last two (out of sample) quarters of 2015. Those scenarios are built by drawing stochastic macroeconomic shocks for the whole economy, that is, USA, Mexico and other international shocks are simultaneously considered. Out of those stochastic draws, I only consider those that yield an equilibrium in which the US monetary policy rate increases as a result. In average, those equilibria are characterized by favorable GDP growth in both countries, a modest increase in the Federal Reserve funds rate and a more than one-to-one response in Mexico’s policy rate. The general conclusion is that those conditions that are needed for the normalization of US monetary policy are good conditions for both, USA and Mexico.

APPENDIX: PHILLIPS CURVE

In this Appendix I show the details to obtain the Phillips curve of the model. First I show how to write the optimal price chosen by a firm in a recursive fashion then I combine that optimal price with the aggregate price index to obtain the Phillips curve of the model.

A.1 Optimal Price Recursion

Consider a firm that can re-optimize its price in period \( t \), the firm chooses \( P_{j,t}(z) \) to maximize—we only show the relevant part of profits, that is, the case when the firm has to keep the non-optimal price \( P_{z,i}(z) \) \( \forall i = 1, ..., \), which happens with probability \( C \) in each future period:

\[
\mathbb{E}_t \sum_{t=0}^{\infty} C^t \beta^t \lambda_{t+1} \Pi_{t+1} = \mathbb{E}_t \sum_{t=0}^{\infty} C^t \beta^t \lambda_{t+1} \left[ P_{N,t+1}(z) Y^j_{t+1}(z) - MC_{t+1} Y^j_{t+1}(z) \right]
\]
Using the indexation rule (3) profits can be written as

\[
\mathbb{E}_t \sum_{i=0}^{\infty} C^i \beta^i \lambda_{t+i} \left[ \text{ind}_{t+i} P_{j,t}(z) \left( \frac{\text{ind}_{t+i} P_{j,t}(z)}{P_{j,t+i}} \right)^{-\lambda_{j+i}} (Y^i_{t+i}) \right] - MC_{t+i} \left( \frac{\text{ind}_{t+i} P_{j,t}(z)}{P_{j,t+i}} \right)^{-\lambda_{j+i}} (Y^j_{t+i}) \]

The first order condition is

\[
\mathbb{E}_t \sum_{i=0}^{\infty} C^i \beta^i \lambda_{t+i} \left[ (\text{ind}_{t+i})^{-\lambda_{j+i}+1} (-\lambda_{j+i} + 1)(P_{j,t}(z))^{-\lambda_{j+i}} \left( \frac{1}{P_{j,t+i}} \right)^{-\lambda_{j+i}} (Y^j_{t+i}) \right] + MC_{j+i\lambda_{j+i}+t+i} \left( P_{j,t}(z) \right)^{-\lambda_{j+i}+1} \left( \frac{\text{ind}_{t+i}}{P_{j,t+i}} \right)^{-\lambda_{j+i}} (Y^j_{t+i}) = 0.
\]

Note that

\[
\frac{P_{j,t+i}}{P_t} = \frac{P_{j,t}}{P_t} \frac{P_{j,t+1}}{P_{j,t}} \ldots \frac{P_{j,t+i-1}}{P_{j,t+i-1}} = p_{j,t} \Pi_{s=0}^{t+i} := p_{j,t} \text{ind}_{j,t+i}
\]

where I use \( \Pi_{j=0}^{t} := 1 \). Divide the expression above by \( P_t \) and rewrite it as a note that I multiply by \(-1\) in the term \(-1(\lambda_{N,t+i} - 1) = (1-\lambda_{N,t+i}) -\):

\[
\mathbb{E}_t \sum_{i=0}^{\infty} C^i \beta^i \lambda_{t+i} \left[ \text{ind}_{t+i}^{-\lambda_{j+i}+1} (\lambda_{j+i} - 1)(P_{j,t}(z))^{-\lambda_{j+i}} \left( \frac{1}{p_{j,t} \text{ind}_{j,t+i}} \right)^{-\lambda_{j+i}} (Y^j_{t+i}) \right] - MC_{j+i\lambda_{j+i}+t+i} \left( P_{j,t}(z) \right)^{-\lambda_{j+i}+1} \left( \frac{\text{ind}_{t+i}}{p_{j,t} \text{ind}_{j,t+i}} \right)^{-\lambda_{j+i}} (Y^j_{t+i}) = 0.
\]

Linearizing the expression above and using the steady-state relation \( mc_j = \frac{\lambda_{j-1}}{\lambda_j} \) we get

\[
\mathbb{E}_t \left\{ \lambda (\lambda_j - 1) Y_j \left\{ P_{j,t}(z) - MC_{j,t} \right\} + \lambda Y_j \lambda_j \right. \\
+ C \beta \lambda (\lambda_j - 1) Y_j \left\{ + \text{ind}_{t+1} + P_{j,t}(z) - MC_{j,t+1} \right\} + C \beta \lambda Y_j \left\{ \lambda_{j,t+1} \right\} \\
+ C^2 \beta^2 \lambda (\lambda_j - 1) Y_j \left\{ + \text{ind}_{t+2} + P_{j,t}(z) - MC_{j,t+2} \right\} + C^2 \beta^2 \lambda Y_j \left\{ \lambda_{j,t+2} \right\} \\
+ \ldots \right\} = 0.
\]
simplifying and solving for \( P_{N,t}(z) \)

\[
P_{j,t}(z) \sum_{i=0}^{\infty} C^i \beta^i = \left\{ MC_{j,t} - \frac{1}{\lambda_j - 1} \lambda_{j,t} \right\} \\
+ C \beta \left\{ -ind_{t+1} + MC_{j,t+1} - \frac{1}{\lambda_j - 1} \lambda_{j,t+1} \right\} \\
+ C^2 \beta^2 \left\{ -ind_{t+2} + MC_{j,t+2} - \frac{1}{\lambda_j - 1} \lambda_{j,t+2} \right\} + \\
+ C^3 \beta^3 \left\{ -ind_{t+3} + MC_{j,t+3} - \frac{1}{\lambda_j - 1} \lambda_{j,t+3} \right\} + ...
\]

note

\[ ind_{t+1} = \tilde{\pi}_{t+1} \]

\[ ind_{t+2} = \tilde{\pi}_{t+1} + \tilde{\pi}_{t+2} \]

\[ ind_{t+3} = \tilde{\pi}_{t+1} + \tilde{\pi}_{t+2} + \tilde{\pi}_{t+3} \]

thus define

\[ \sigma_{ind,t} = \sigma_{ind,t-1} + \tilde{\pi}_{t+1} \]

with \( \sigma_{ind,0} = 0 \). Then rewrite the price as

\[
P_{j,t}(z) \sum_{i=0}^{\infty} C^i \beta^i = \left\{ MC_{j,t} - \frac{1}{\lambda_j - 1} \lambda_{j,t} \right\} \\
+ C \beta \mathbb{E}_t \left\{ -\sigma_{ind,t} + MC_{j,t+1} - \frac{1}{\lambda_j - 1} \lambda_{j,t+1} \right\} \\
+ C^2 \beta^2 \mathbb{E}_t \left\{ -\sigma_{ind,t+1} + MC_{j,t+2} - \frac{1}{\lambda_j - 1} \lambda_{j,t+2} \right\} + \\
+ C^3 \beta^3 \mathbb{E}_t \left\{ -\sigma_{ind,t+2} + MC_{j,t+3} - \frac{1}{\lambda_j - 1} \lambda_{j,t+3} \right\} + ...
\]
or recursively:

\[ P_{j,t}(z) = (1-C \beta) \left\{ MC_{j,t} - \frac{1}{\lambda_j - 1} \lambda_{j,t} - C \beta \sigma_{ind,t} \right\} + C \beta \mathbb{E}_t P_{j,t+1}(z) . \]

**A.2 Phillips Curve**

Dropping the index \( z \) because all firms choose the same price, from the price index:

\[ \left( P_{j,t} \right)^{1-\lambda_{j,t}} = C \left( \tilde{\pi}_t P_{j,t-1} \right)^{1-\lambda_{j,t}} + (1-C) \left( P_{j,t}^* \right)^{1-\lambda_{j,t}} . \]

In log-linear and solving for \( P_{j,t+1}^* \) from the price index

\[ P_{j,t+1}^* = \frac{1}{1-C} P_{j,t+1} - \frac{C}{1-C} \left\{ P_{j,t} + \tilde{\pi}_{t+1} \right\} = \frac{1}{1-C} \pi_{j,t+1} + P_{j,t} - \frac{C}{1-C} \tilde{\pi}_{t+1} \]

using this in the optimal price

\[ P_{j,t}^* = (1-C \beta) \left\{ MC_{j,t} - \frac{1}{\lambda_j - 1} \lambda_{j,t} - C \beta \sigma_{ind,t} \right\} + C \beta \mathbb{E}_t \left\{ \frac{1}{1-C} \pi_{j,t+1} + P_{j,t} - \frac{C}{1-C} \tilde{\pi}_{t+1} \right\} \]

and using this back in the price index

\[ P_{j,t} = C \left\{ P_{j,t-1} + \tilde{\pi}_t \right\} + (1-C) \left\{ (1-C \beta) \left\{ MC_{j,t} - \frac{1}{\lambda_j - 1} \lambda_{j,t} - C \beta \sigma_{ind,t} \right\} \right\} + C \beta \mathbb{E}_t \left\{ \frac{1}{1-C} \pi_{j,t+1} + P_{j,t} - \frac{C}{1-C} \tilde{\pi}_{t+1} \right\} \]

or subtracting \( P_{j,t-1} \) on both sides we obtain:

\[ \pi_{j,t} = \beta \mathbb{E}_t \pi_{j,t+1} + \tilde{\pi}_t - C \beta \mathbb{E}_t \tilde{\pi}_{t+1} + \frac{(1-C)(1-C \beta)}{C} \left\{ mc_{j,t} - \frac{1}{\lambda_j - 1} \lambda_{j,t} - C \beta \sigma_{ind,t} \right\} \]

where \( mc_{j,t} := MC_{j,t} / P_{j,t} \).
References


