Asymmetries of the Exchange Rate Pass Through to Domestic Prices: The Case of Costa Rica

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Abstract

By using a logistic smooth transition vector autoregressive model this paper examines whether the exchange rate had asymmetric effects on inflation in Costa Rica during the period 1991-2009. Three basic questions are tried to be answered: Is there any variable that significantly induces asymmetries on the exchange rate pass through? Do positive and negative shocks on exchange rate have symmetric effects on the price level? Is there any evidence that the size of the shocks determines the relative size of the impact on inflation of a given shock on the exchange rate? Among many variables, it is found that lagged change in oil prices is the most appropriate transition variable, which means that it induces the greatest asymmetric effects on the pass-through. On average, when this transition variable is above certain threshold level, the pass-through is two times higher. There is no strong evidence of sign or magnitude asymmetries, which means that the pass-through is expected to be essentially the same when there is a positive or a negative shock on the exchange rate and when such shocks are big or small.

Keywords: Exchange rate pass through, inflation, depreciation, asymmetries, smooth transition models

JEL classification: C32, C51, E31, E37

Resumen

Haciendo uso de un modelo autoregresivo de transición suave logística este trabajo estudia si las variaciones del tipo de cambio tienen efectos asimétricos sobre la inflación en Costa Rica durante el periodo 1991-2009. Se trata de responder a tres preguntas básicas: ¿Existe alguna variable que induzca asimetrías estadísticamente significativas en el efecto traspaso del tipo de cambio? ¿Acaso es diferente el impacto sobre los precios internos de choques positivos y negativos sobre el tipo de cambio? ¿Hay alguna diferencia significativa entre el efecto traspaso de choques grandes y pequeños sobre el tipo de cambio? Entre una extensa cantidad de variables sometidas a prueba, se encuentra que rezagos de la variación anual de los precios del petróleo producen la mayor evidencia de asimetría en el efecto traspaso del tipo de cambio. En promedio cuando esta variable de transición se ubica por sobre cierto umbral, el efecto traspaso es casi la dos veces mayor. No se halla fuerte evidencia de asimetrías de magnitud ni de signo; es decir, se espera que el efecto traspaso sea esencialmente igual en respuesta ya sea a choques grandes o pequeños o bien a choques positivos o negativos sobre el tipo de cambio.

Palabras claves: Efecto traspaso del tipo de cambio, inflación, depreciación, asimetrías, modelos de transición suave

Clasificación JEL: C32, C51, E31, E37

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1. Introduction

Under any monetary scheme, one of the elements that determine a successful application of monetary policy is a good understanding of the process that generates inflation. When the authorities explicitly adopt an inflation targeting regime, such element acquires even more importance because most of the attention is centered on the ability of the Central Bank to achieve his inflation target.

It is widely accepted that in the long run inflation is almost fully explained by monetary imbalances. However the process of inflation in the short run is more complex. Part of this complexity comes from the way variations of the exchange rate are passed to variations in the general level of prices. Such phenomenon has been called pass through (PT) of the exchange rate and it has been the subject of many theoretical and empirical research.

The Central Bank of Costa Rica has devoted efforts to understand how this effect takes place in the Costa Rican economy and to quantify it. Among the studies on the subject are León, Morera and Ramos (2001), León, Laverde and Durán (2002) and Castrillo and Laverde (2008). Those studies share a common issue in its methodological approach; all of them assume that the PT is a lineal phenomenon.

There have been an increasing number of empirical papers supporting the idea that the PT might show asymmetries. For instance it can be the case that the magnitude of the effect depends on the state of the economy. Highly inflationary environments, decreasing international reserves, production booms, all can be thought of as environments in which it is easier to pass a bigger portion of a given shock on the exchange rate to domestic prices of goods and services. If this is the case, state asymmetries would be present. It can also be the case that depreciations of the local currency have different impact on prices than those due to an appreciation; in this case there would be sign asymmetries. Finally, the size of the shock may matter; if big shocks have different relative impact on prices than small shocks, that will be evidence of size asymmetries.

If there are asymmetries on the mechanism that passes changes in the exchange rate to domestic prices, then conclusions about the magnitude of the PT measured by means of linear models can be misleading and so policy recommendations may lead to suboptimal results. This study extends the investigation of the PT effect in Costa Rica by analyzing the three kind of asymmetries mentioned above.

In order to investigate such subject, a logistic smooth transition vector autoregressive (LSTVAR) model is employed. Such approach has also been applied to Perú (Winkelried, 2003), Venezuela (Mendoza, 2004), Guatemala (Carpio and Mendoza, 2006) just to mention a few in Latin America.

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The paper is organized as follows: Section 2 offers a brief historical summary of the exchange rate regimes that the Costa Rican Economy has undergone. After that, section 3 exposes a theoretical background focused on the advantages of using a non linear approach for modeling the PT of the exchange rate. Section 4 is devoted to methodological aspects; it explains with some detail all relevant aspects in estimating a LSTVAR and the generalized impulse response function needed to analyze the asymmetries of the PT effect. Section 5 offers a description of the main results of the document. Section 6 concludes while section 7 and 8 shows all relevant references and appendices respectively.

2. Summary of Exchange Rate Regimes in Costa Rica

Costa Rican Economy has experienced several exchange rate regimes across history. Since the 1950’s decade and up to the beginning of the 1980’s a fixed Bretton Woods style scheme prevailed. Under such system, sporadic devaluations and revaluations occurred but were not quite common. Government avoided implementing such measures because they were associated with failures of the current economic policies.

By the beginning of the 1980’s, the fixed exchange rate regime became unbearable when the country faced the consequences of the debt crisis. The Central Bank lost most of its foreign reserves and the country as a whole became unable to honor the external debt. During the period of the crises multiple exchange rate regimes functioned and coexisted with an informal market that sometimes showed a very large margin spread when compared with the official exchange rate.

The program implemented in 1984 to stabilize the economy brought about the adoption of an exchange rate regime more flexible. The nominal exchange rate started to be adjusted periodically and the sizes of the changes were very small. This was the beginning of the system so called minidevaluaciones that was the official one for more than 20 years. Such scheme is more formally known as a crawling peg. The Central Bank constantly intervened in order to keep the exchange rate close to a previously determined variation path. The goal was to keep the real exchange rate as constant as possible based on the purchase parity power criteria. This system was consistent with the process of structural adjustment by which authorities tried to enhance competitiveness of exports, to promote stable capital inflows, reach balance of payments equilibrium, to encourage a rational usage of international reserves, to inset the national economy in the world market and, at the same time, to adjust the exchange rate system to the prevailing conditions in the international monetary system.

The minidevaluaciones regime led to several positive and negative situations. Among the positive ones stand a reduction in the exchange rate uncertainty, a gradual approximation to an equilibrium exchange rate level and to boost competitiveness by avoiding real appreciation of the national currency. On the other hand, among the negative features the system contributed to generate inflationary inertia. The rate at which the currency would be devaluated was previously known and so easily passed to price adjustments. There were also incentives for

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4 There was a short period during 1992 in which a trial of flotation failed.
short run capitals to enter the economy due to the low exchange rate uncertainty. Additionally, the constant intervention of the Central Bank generated high financial costs and reduced the effectiveness of the monetary policy. Finally, the regime also stimulated the dollarization of assets and liabilities, which increased vulnerability of the financial system.

The minidevaluaciones stop being the official system of exchange rate on October 17th 2006. Since then, the system is one of crawling bands whose initial width was moderate but it was meant to be increased. The regime was conceived as a transition stage since the goal is to implement managed floatation as part of an inflation targeting monetary policy regime. The lower and upper bounds of the band are determined by the Central Bank Board and they are allowed to change them according with the macroeconomic and financial conditions.

As Figure 1 shows, due to the excess supply of foreign currency at the beginning of the Crawling band regime, the exchange rate tended to be near the lower band. During this period the Central Bank bought foreign currency at the preset band’s price. During the second half of 2007 such tendency reverted and by July 2008 the exchange rate touched the upper bound of the band and the Central Bank had to sell foreign currency. With some short exception periods, such behavior keeps going on until August 2009 when the Central Bank made their final intervention in the wholesale market. Since then the exchange rate has been determined solely by market forces.

![Figure 1. Reference Exchange Rates. January 2005 - May 2010](image-url)
3. Theoretical background

3.1. Modelling asymmetries in autoregressive systems of equations

Numerous studies have found evidence that the PT of the exchange rate to domestic prices is an asymmetric phenomenon. This has been reported across a wide range of countries. Pollard and Coughlin (2004) report asymmetries in the response of US firms to variations in the exchange rate; Przystupa and Wróbel (2009) show evidence of the Poland case; Kumar (2004) reports similar results for the Indian economy. In Latin America for instance, Winkelried (2003) exposes the case of Peru; Mendoza (2004) the case of Venezuela; Carpio and Mendoza (2006) report the Guatemalan case and Silva and Minella (2006) expose the Brazilian evidence. The studies mentioned above, especially those applied to Latin American economies, share a common methodological line. Due to the frequent occurrence of structural breaks in the economic time series, and in order to capture the expected PT asymmetries caused by such breaks, they apply some form of regressive state dependent models.

The interest in applications that allow for nonlinear relations among economic time series has grown in the last two decades. A widely accepted line of work deals with regressive state dependant models. Those models describe the properties of the time series across different states or regimes and specify a rule that indicates when a regime switch occurs. Then the properties (conditional expectation and variance for instance) of the stochastic time series involved vary across regimes.

A wide range of regime switching regressive models have been applied in order to model such asymmetries. Most of the approaches can be classified in two groups depending on whether the regimes are deterministic or purely stochastic. When the regime at any time is known with 100% certainty, it is said that the regimes are deterministic. This is the case for instance when the state depends on past values of an observable variable. On the other hand, when agents have some degree of uncertainty, in other words they know only a probability that certain regime have occurred, then the system will be one of stochastic regimes. In the last case the typical methodological approach have been to use markovian models, while in the former, smooth transition autoregressive (STAR) and threshold models are typically employed.

Our interest in this section is to give a basic idea about the theory developed behind models of deterministic regimes with special interest in STAR models. Those are the kind of models that were applied in our empirical quantification of the PT in Costa Rica.

A good understanding of threshold models can be helpful in comprehending how STAR models work. If it is assumed that the regimes are deterministic, there will be some function mapping a known set of information (a variable $\tau_t$ for example) to the regimes. Threshold models specifically assume that the difference between $\tau_t$ and certain threshold parameter $\kappa$ defines the regime.

The simplest threshold model is the Self Exiting Threshold Autoregressive (SETAR$^5$), which assumes that $\tau_t$ is a certain lag ($l$) of the time series that is being modeled ($y_t$), this is $\tau_t = y_{t-l}$. A SETAR model then can be written down as:

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$^5$ See Tom and Lim (1980).
\[ y_t = (\phi_0 + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p})(1 - I(y_{t-1} > \kappa)) + (\Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p})I(y_{t-1} > \kappa) + \mu_t \]  

(2.1)

Where,

\[ E(\mu_t | \Omega_{t-1}) = 0 \]

\[ E(\mu_t^2 | \Omega_{t-1}) = \sigma^2 \]

\[ \Omega_{t-1} = [y_{t-1}, y_{t-2}, \ldots, y_{t-p},] \]

I(\Delta) is an indicative function such that I(\Delta) = 1 if \Delta is true and I(\Delta) = 0 otherwise.

Notice that in this way the stochastic system is decomposed in two (or more)\(^6\) lineal models, each one describing a different relation between the dependant and the respective explicative variables. When \( y_{t-1} > \kappa \) the system is governed by the \( \Phi_0, \Phi_1, \ldots, \Phi_p \) parameters and when \( y_{t-1} < \kappa \) the system follows the dynamics of the \( \phi_0, \phi_1, \ldots, \phi_p \) parameters.

The intuition for modelling asymmetries by using thresholds models is quite appealing. Agents may well react differently to shocks depending on the state of the economy. In the case of SETAR models, such state is represented by lags of the same dependent variable and the switch from one regime to the other is discrete due to the fact that the function \( I(\cdot) \) takes only two values, 0 or 1.

Although SETAR models allow for more flexibility than single-stage lineal models, they might be rigid for describing some potential asymmetric relationships among economic variables. If the indicative function \( I(\cdot) \) can take a continuum of values into the interval \([0,1]\), then the switch between regimes is gradual, or say smooth. This is the case of the STAR models where agents are assumed to smoothly change their reaction to shocks depending on the values taken by the variable signaling the state of the economy. Hereafter such variable will be named transition variable.

Teräsvirta (1994) introduces the most popular STAR model which takes the following form:

\[ y_t = (\phi_0 + \phi_1 y_{t-1} + \ldots + \phi_p y_{t-p}) + (\Phi_0 + \Phi_1 y_{t-1} + \ldots + \Phi_p y_{t-p})F(\tau_{t-1}, \gamma, \kappa) + \mu_t \]  

(2.2)

Where \( \mu_t \) and \( \Omega_{t-1} \) are defined as in (2.1). \( F(\cdot) \) is continuous, takes values in \([0,1]\) and is called transition function. Most applications assume that \( F(\cdot) \) is a logistic or exponential function. In this work the transition function will take the logistic form:

\[ F(\tau_{t-1}, \gamma, \kappa) = \frac{1}{1 + \exp[-\gamma(\tau_{t-1} - \kappa)]} \quad \text{con} \quad \gamma > 0 \]  

(2.3)

Notice that equation (2.2) has parameters that constantly change with \( \tau_{t-1} \). Then such expression allows for asymmetries in the relation of \( y_t \) with its past values as long as it keeps

\(^6\) There can be more that two regimes if more than one threshold is defined. If there are \( H \) thresholds, then there will be \( H+1 \) regimes.
away from the extreme cases where \( \lim_{\tau_{t-l} \to \infty} F(\tau_{t-l}, \gamma, \kappa) = 1 \) or \( \lim_{\tau_{t-l} \to -\infty} F(\tau_{t-l}, \gamma, \kappa) = 0 \), in which case (2.2) collapses to a lineal expression with fixed parameters. The \( \gamma \) factor signals how fast the structural parameters adjust when \( \tau_{t-l} \) changes. \( \kappa \) indicates the direction of such adjustments. It is said that threshold models are special cases of STAR models because they are generated from (2.2) when \( \gamma \to \infty \). In this case a very small change in \( \tau_{t-l} \) discretely changes the relation between \( y_t \) and its lags.

Economic variables are often included in VAR models in order to allow for endogenous dynamics among them. The presence of asymmetries in such multivariate systems can be modeled as an extension of expression (2.2) as follows:\(^7\)

\[
X_t = \Pi_1 + \sum_{m=1}^{q} \phi_m X_{t-m} + \left[ \Pi_2 + \sum_{m=1}^{q} \Phi_m X_{t-m} \right] F(\tau_{t-l}, \gamma, \kappa) + \varepsilon_t \quad (2.4)
\]

Where \( X_t \) is an \((n x 1)\) vector of endogenous variables; \( \Pi_1 \) and \( \Pi_2 \) are \( n x 1 \) vectors of regression constants; \( \phi_m \) and \( \Phi_m \) are, respectively, \((n x n)\) matrices of coefficients of lineal and no lineal part of the model; finally, \( F(\tau_{t-l}, \gamma, \kappa) \), \( \tau_{t-l} \), \( \gamma \) and \( \kappa \) are all defined as in (2.3).

A model like (2.4) is called Logistic Smooth Transition Vector Autoregressive (LSTVAR) and the estimation of the exchange rate PT that was carried out in this study is based on and approach alike.

According with Mendoza op.cit., an LSTVAR is quite appealing for investigating the PT effect of the exchange rate since it endogenously generates the switch from one regime, associated with particular economic conditions or the level of certain variable, to another. This kind of non linear relationship is known as state asymmetry. Additionally, by using LSTVAR it is also possible to study sign and magnitude asymmetries. The former refers to the case in which the PT is different for positive and negative variations of the exchange rate. The last one indicates that the PT coefficient is sensible to the magnitude of the shocks affecting the exchange rate series.

### 4. Methodology

According to Granger and Teräsvirta (1993), the process of estimating a LSTVAR model can be divided in three stages. Firstly a lineal models must be fit, in this case we estimate a VAR with 4 variables. Secondly it has to be tested whether there is evidence of non linearity in that VAR. This is done by following Luukkonen, Saikkonen and Teräsvirta (1988). Lastly, if there is evidence rejecting linearity, then a sequence of tests hypothesis should lead to choose between a specific form of transition, a logistic or exponential. Here the line of Mendoza op. cit. and Mendoza and Pedagua op. cit. was followed, they omit the last stage arguing that for capturing asymmetries of the PT effect it is theoretically more attractive to use a logistic form for the transition.

\(^7\) Extensive analysis of multivariate smooth transition models are offered in Granger and Teräsvirta (1993) and in Teräsvirta (1994). Mendoza (2006) offers a good survey on the subject.
4.1. Baseline Linear VAR

For studying the potential asymmetries of the PT effect in Costa Rica, this study is based on a VAR(4) model which constitutes the baseline model for carrying out an LSTVAR of the form (2.4). The four variables included in this VAR are: Percentage monthly change of the consumer price index \( \pi_{CT} \), percentage monthly change of nominal exchange rate \( \delta_{CT} \), output GAP \( gap_{CT} \) and real exchange rate misalignment \( tcr_{CT} \). Those variables were selected based on past studies of Costa Rican exchange rate PT and adequateness of VAR adjustment. The sample consists of monthly data from January 1991 up to June 2009.

All variables of the initial VAR were stationary according to standard augmented Dickey Fuller (ADF) and Phillip Perron (PP) tests (see appendix A). By applying a sequence of Wald exclusion tests, lags 1 through 6, 8 and 12 were selected at 10% level of significance. As expected from the stationary properties of the endogenous, all inverse roots of the characteristic polynomial lie inside the unit circle, which indicates that the VAR is stationary (see appendix A).

The second step is to test if there is evidence of non linearities. Such tests can consume a lot of degrees of freedom, which in turn might generate non efficient estimations. In order to increase the degrees of freedom available we follow Mendoza op.cit and Mendoza and Pedagua op.cit. who work with a coefficient restricted VAR (subset VAR). This was done by means of applying a sequential LR test for taking out of the system one lag of variable at each sequence. The first candidate to be eliminated in each sequence was the less significative variable lag. Notice that at this level we are no longer working with VAR tools, we use SUR methods in order to compute the solution of the system.

4.2. Linearity tests

It will make no sense to estimate a non linear model in the case that there exists no evidence signaling that such approach is more appropriate for describing the relationship among the variables involved. From equation (2.4) it is clear that non linearity is determined by a specific transition variable \( \tau_{CT} \). The transition variable may be endogenous or exogenous to the system. Economic theory should suggest a collection of candidates. The linearity test and a grid search procedure (explained in the next section) must provide a way to discriminate among them.

The list of candidates for transition variable that were tested in this study include all endogenous variables of the system and its respective (1 through 12) lags; additionally the next list of variables were also tested: the change of international oil prices, M1, international reserves of the Central Bank, real exchange rate index, producer price index, a dollarization proxy and the degree of commercial openness of the economy. For those variables we took the change in the last quarter, the last semester and the last year. Additionally the respective 1 through 12 lags of those were also considered. Hence a total of 396 different transition variables were tested.

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8 The output GAP is constructed as the log difference between SA monthly index of economic activity and its Hodrick-Prescott trend computed with a smooth factor of 14200, which is the adequate factor for Costa Rican monthly time series according with Esquivel and Rojas (2007).

9 Real exchange rate misalignment is computed as the log difference between SA multilateral real exchange rate index and its Hodrick-Prescott trend.

10 See León, Morera and Ramos (2001); León, Laverde and Durán (2002); Castrillo and Laverde (2008).
For each possible candidate for transition variable (TV) a linearity test was applied. In this test the linear model is taken as the null hypothesis and a non linear model is the alternative. The intuition behind such contrast is to find out which variables seem to improve the estimation due to non linearity.

Strictly, the test should be applied on equation (2.4) and the hypotheses are:

\[ H_0: \Phi_m = 0, \quad m = 1,2, \ldots q \]
\[ H_a: \Phi_m \neq 0 \quad \text{for at least one} \quad m = 1,2, \ldots q \]

Notice though that there is a problem on nuisance parameters. Under the null, both \( \gamma \) and \( \kappa \) parameters are not identified. This would lead to a non standard distribution of the test statistic. In order to solve this, Luukkonen, Saikkonen and Teräsvirta \textit{op.cit.} suggest considering a Taylor linearization of the function \( F(\tau_{t-l}, \gamma, \kappa) \).

According to a third degree Taylor expansion \( F(\tau_{t-l}, \gamma, \kappa) \approx \gamma (\tau_{t-l} - \kappa) - \frac{1}{48} \gamma^3 (\tau_{t-l} - \kappa)^3 \). In this case we can consider \( \kappa \) and \( \gamma \) as constants. This implies that the test we want to make has a third degree polynomial on \( \tau_{t-l} \). The form of the new test would then be:

\[ H_0: X_t = \Pi_1 + \sum_{m=1}^q \Phi_m \cdot y_{t-m} + \epsilon_t. \]
\[ H_a: X_t = \sum_{j=0}^3 (\Pi_1 + \sum_{m=1}^q \Phi_m \cdot y_{t-m}) \tau_{t-l}^j + \epsilon_t. \]

This test is much easier to compute and to interpret. It is as simple as to apply a LR test on the whole system by using SUR. The test statistic is:

\[ LR = T (\ln |\Omega_0| - \ln |\Omega_1|) \quad (3.1) \]

Where \( \Omega_0 \) and \( \Omega_1 \) are the variance-covariance matrices of the linear (restricted) and non-linear (non restricted) systems respectively. This LR value has a \( \chi^2 \) distribution, with degrees of freedom equal to the number of restricted coefficients.

According with the literature, it is very common to reject linearity when applying the test to the whole system\textsuperscript{11}. Then the tests were also applied in an equation by equation basis. In this case an F statistic was used:

\[ LM = \frac{SSR_0 - SSR_1}{\frac{\lambda_0 T}{\tau - \lambda_1}} \quad (3.2) \]

Here \( SSR_0 \) and \( SSR_1 \) are the sum of square residuals of both, linear and non-linear systems respectively. \( T \) is the sample size, \( \lambda_0 \) is the number of coefficients under the null and \( \lambda_1 \) is the number of coefficients of the non restricted system. \( LM \) has an \( F_{\lambda_0, T - \lambda_1} \) distribution.

Table 1 shows a sample of the main results of the linearity tests. The first column lists the transition variable candidates and the second the respective lag. Odd columns from 3 through 9 are the corresponding \( LM \) statistic computed for each equation in the system according to (3.2). Even columns in the same range are the matching \( P-values \). Finally the last two columns are the \( LR \) statistic computed from (3.1) and their \( P-values \). In order to discriminate among candidates

\textsuperscript{11} See Weise (1999), Winkelried \textit{op. cit.}, Mendoza \textit{op. cit.} and Mendoza and Carpio \textit{op. cit.}
for transition variable, the table is ordered according with two criteria. The candidates for which linearity was rejected in all four equations of the system are placed first. Then they were ordered, from high to low, in accordance with the respective \( LR \) value. Such ordering will place the best candidates at the top of the table.

From the table it is possible to note that among the strongest candidates for transition variable there are different lags of the change in the last \( t \) months of: Devaluation (DE\(_t\)), log difference of monetary aggregates (DLM\(_{1t}\)), log difference of Central Bank holdings of international reserves (DLRIN\(_{t}\)) and log difference of oil price (DLPPET\(_{t}\)). From a theoretical point of view, it seems reasonable to think about such variables as causing non linearity of the PT. When the economy faces shocks like a period of relatively high increase of oil prices, this causes an inflationary environment that probably will make agents pass more easily changes in nominal exchange rate to domestic prices. The same apply to positive shocks on monetary aggregates variables, when they are growing faster, eventually domestic prices will grow and it will turn easy to pass changes in nominal exchange rates. Finally, when the amount of international reserves is decreasing at a fast rate, the risk of devaluation goes up and so do devaluation expectations, that might facilitate to pass changes in devaluation more directly to domestic prices.

In order to decide which of those candidates to use for the LSTVAR free estimation, it would be better to pick the one that causes less problems of estimation. Such election was made, as in other studies on the field\(^{12}\), by means of a grid search routine.

\(^{12}\) Idem.
Table 1
Sample of the main results of the linearity tests

<table>
<thead>
<tr>
<th>Transition Variable</th>
<th>lag</th>
<th>F-Value in Pic</th>
<th>Prob</th>
<th>F-Value in de</th>
<th>Prob</th>
<th>F-Value on Gap</th>
<th>Prob</th>
<th>F-Value on Tcr</th>
<th>Prob</th>
<th>X²-Value</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>DE12</td>
<td>5</td>
<td>1.49</td>
<td>0.045</td>
<td>33.39</td>
<td>0.000</td>
<td>1.54</td>
<td>0.044</td>
<td>10.87</td>
<td>0.000</td>
<td>762.95</td>
<td>0.000</td>
</tr>
<tr>
<td>DLM112</td>
<td>10</td>
<td>1.08</td>
<td>0.003</td>
<td>30.22</td>
<td>0.000</td>
<td>1.92</td>
<td>0.005</td>
<td>8.42</td>
<td>0.000</td>
<td>684.54</td>
<td>0.000</td>
</tr>
<tr>
<td>DE8</td>
<td>11</td>
<td>1.63</td>
<td>0.018</td>
<td>25.01</td>
<td>0.000</td>
<td>1.59</td>
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<td>661.47</td>
<td>0.000</td>
</tr>
<tr>
<td>DE6</td>
<td>12</td>
<td>1.90</td>
<td>0.003</td>
<td>22.25</td>
<td>0.000</td>
<td>1.82</td>
<td>0.009</td>
<td>6.67</td>
<td>0.000</td>
<td>659.27</td>
<td>0.000</td>
</tr>
<tr>
<td>DE6</td>
<td>10</td>
<td>1.76</td>
<td>0.007</td>
<td>23.76</td>
<td>0.000</td>
<td>1.47</td>
<td>0.065</td>
<td>5.23</td>
<td>0.000</td>
<td>654.82</td>
<td>0.000</td>
</tr>
<tr>
<td>DLM112</td>
<td>12</td>
<td>1.99</td>
<td>0.001</td>
<td>22.49</td>
<td>0.000</td>
<td>2.02</td>
<td>0.003</td>
<td>8.92</td>
<td>0.000</td>
<td>652.93</td>
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</tr>
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4.3. The two-dimensional grid search

Due to the problem of nuisance parameters, the linearity test was applied to a Taylor approximation of the function of interest. It must be clear that up to this point the non linear system has not been estimated. In order to do so, the numerical algorithm for solving non linear SUR requires initial values for all parameters that are going to be estimated, including $\gamma$ and $\kappa$. Then there is still the question of finding out those adequate initial values. Mendoza op.cit. recommends an exhaustive search over possible initial values of $\gamma$ and $\kappa$. This so called grid search or two dimensional search process also serves as a final method for discriminating among strong candidates for transition variables.

It is important to let the system endogenously generate initial values for $\gamma$ and $\kappa$. It will be seen later that the free estimation of $\gamma$ is particularly problematic. On the other hand, an ad hoc imposition of the initial value of $\kappa$ may seem appealing. For instance, taking $\kappa$ as the long run equilibrium level of the transition variable might seem reasonable. Notice though that such equilibrium level is not necessarily the level for which the system will change from a low to a high regime. It is better to let the system identify this critical level.

Once the list of candidates for transition variable has been reduced by means of the linearity tests, it is possible to have a good idea about an interval that contains an adequate initial value for $\kappa$ for each candidate. On the other hand, the parameter $\gamma$ in principle does not depend on which transition variable is chosen.

For a particular candidate for transition variable, the grid search consists of a systematic evaluation of different LSTVAR models with different pairs of $\gamma$ and $\kappa$. Then the pair that yields a higher level of the likelihood function will be chosen as initial values for a later free estimation of the system. For each candidate the problem is then to find $(\hat{\kappa}, \hat{\gamma})$ such that:

$$(\hat{\kappa}, \hat{\gamma}) = \arg\max_{(\kappa, \gamma) \in G \times K} \ell(\Omega(\kappa, \gamma), \Pi(\kappa, \gamma)) \quad (3.3)$$

Here $G$ and $K$ are the sets that contains all possible values for $\gamma$ and $\kappa$ respectively. If the likelihood function is normal, this would mean to find:

$$(\hat{\kappa}, \hat{\gamma}) = \arg\max_{(\kappa, \gamma) \in G \times K} \left[ -\frac{Tn \cdot \ln 2\pi}{2} + \frac{T \cdot \ln |\Omega^{-1}(\kappa, \gamma)|}{2} - \frac{Tn}{2} \right] \quad (3.4)$$

Since only the second term of the right hand side of (3.4) is non constant across $G \times K$, the problem is equivalent to find:

$$(\hat{\kappa}, \hat{\gamma}) = \arg\max_{(\kappa, \gamma) \in G \times K} \frac{T \cdot \ln |\Omega^{-1}(\kappa, \gamma)|}{2} \quad (3.5)$$

The determinant function is multiplicative and $\Omega(\kappa, \gamma)$ is positive definite (thus invertible), then the problem is reduced to find:

$$(\hat{\kappa}, \hat{\gamma}) = \arg\min_{(\kappa, \gamma) \in G \times K} \left[ \ln |\Omega(\kappa, \gamma)| \right] \quad (3.6)$$

There is still the question about how to define $K$ and $G$. It is important to set $K$ such that each possible $\kappa$ has enough observations below and above it. Those will be later the high and low regime observations in the LSTVAR. According with such criteria, a lower limit ($ll$) and a high limit ($hl$) were set. Then $K$ is defined as:
\[ K := \left\{ \kappa \mid \kappa = ll + n \cdot \left( \frac{lh-ll}{20} \right), n \in \{0,1,\ldots,20\} \right\} \]

Notice that in this way the interval \([ll, hl]\) is split into twenty parts and \(\kappa\) is allowed to take 21 possible values. When choosing among candidates for transition variables, those which generate values of \(\kappa\) far from the limits \([ll, hl]\) were preferred\(^\text{13}\).

\(G\) consists of positive numbers that belong to the interval \([0.5, 80]\). As mentioned before, the higher the value of \(\gamma\), the faster the change from low regime to high regime. Several authors report a difficult estimation of \(\gamma\) and problems for estimating \(\kappa\) when \(\gamma\) is high\(^\text{14}\). In order to mitigate this problem, Granger and Teräsvirta \textit{op.cit.} suggest specifying the value of \(\gamma\) relative to the magnitude of the standard deviation of the transition variable \(\sigma_{tv}\)\(^\text{15}\). In this study \(G\) is defined as:

\[ G := \left\{ \gamma \mid \gamma = \frac{n}{2}, n \in \{1,2,3,\ldots,160\} \right\} \]

Notice though that since its values do not depend on the model, how to choose \(G\) is actually up to the researcher. Additionally, as mentioned above, higher values of \(\gamma\) will make difficult the estimation of \(\kappa\), then we prefer transition variables that generate relatively low values of \(\gamma\) in the grid search.

The best behavior of the likelihood function in the grid search was found when the transition variable was the 10th lag of the annual change of oil prices \((DLPPET_{12}^{10})\), then this was the finally chosen transition variable. Figure 2 shows the result of the grid search for \(DLPPET_{12}^{10}\). The horizontal (X-Y) axes of the figure correspond to the range of values of \(\gamma\) and \(\kappa\) over which the search was applied, the vertical (Z) axis represents the logarithm of the determinant of the variance-covariance matrix of the LSTVAR estimation for the corresponding \((\kappa, \gamma)\) pair.

When \((\kappa, \gamma) = (-0.055, 18)\) there is an absolute min of \(\ln|\Omega(\kappa, \gamma)|\), this is why this transition variable was chosen over other strong candidates. The value of \(\kappa = -0.055\) is relatively far from the limits of the set \(K\)\(^\text{16}\) and \(\gamma = 18\) is a relatively small parameter, which favors a smooth transition between regimes. It is worth to mention that when other strong candidates for transition variables were analyzed, the respective grid search yielded values of \(\kappa\) close to the limits of \(K\), which let few observation in one of the regimes, or very high values of \(\gamma\), which is not appealing for an LSTVAR. Appendix B shows the results of the grid search for other strong candidates for transition variable.

\(^{13}\) This criteria is used in order to maximize the number of available observations in the high and low regime.

\(^{14}\) See Mendoza \textit{op.cit.} and Carpio and Mendoza \textit{op.cit.} for instance.

\(^{15}\) In this case the transition function will take the form: \(F(t_{i-t}, \gamma, k) = \frac{1}{1+\exp[-\gamma(t_{i-t}-k)/\sigma_{tv}]}\).

\(^{16}\) When \(DLPPET_{12}^{10}\) is the transition variable, the limits of the set \(C\) are \(ll = -0.89\) and \(lh = 0.89\).
4.4. Free estimation of the system

Once a particular transition variable has been chosen, the result of the grid search provides initial values for all the coefficients of the system. Then it is possible to apply non linear SUR estimation in order to get a free estimation of all coefficients. This is when all the above process pays off. If the search for initial values was accurate, then it is not likely that the free estimation largely modifies the value of the coefficients.

There were some problems of overflow when trying to estimate freely the system as a whole. According with Mendoza op.cit. this is a common issue. Eviews algorithm for solving non linear SUR models finds difficulties especially when the \( \gamma \) coefficient is relatively high. In this case, the same author recommends fixing \( \gamma \) at the level found in the grid search process. This will not significantly affect the estimation of the remaining parameters because the transition function responds quite weakly to high values of \( \gamma \).

Appendix B shows a comparison between the initial and free estimated values of the coefficients. As an example, the initial value of \( \kappa \) was set, according with the grid search, at a level of -0.055 while the free estimation of the LSTVAR sets it at -0.049, which is a relatively small change as expected when the two dimensional search is exhaustive. This final level and the one used for \( \gamma \) (which is 18) are the ones used for computing the impulse response functions and the corresponding PT estimations. Appendix C illustrates the historic level of the transition function and its shape when plotted against a sorted series of the transition variable.
4.5. Measure of the Pass-Through based on Generalized Impulse Response Functions

Due to the fact that impulse response functions of non linear models have no analytic form, in order to compute measures of the PT effect we rely on the proposal of Koop, Pesaran and Potter (1996) for dealing with multivariate nonlinear models. This is to consider the impulse response function a random variable and, without imposing Gaussian assumptions, to generate the so-called Generalized Impulse Response Function (GIRF).

Consider the following LSTVAR which is a specific version of the general model (2.4).

\[
\begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{pmatrix}_t = \Pi_1 + \sum_{m=1}^{q} \Phi_m \cdot \begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{pmatrix}_{t-m} + \Pi_2 + \sum_{m=1}^{q} \Phi_m \cdot \begin{pmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{pmatrix}_{t-m} \cdot F(\tau_t - l, \gamma, \kappa) + \begin{pmatrix}
\epsilon_1 \\
\epsilon_2 \\
\epsilon_3 \\
\epsilon_4 \\
\end{pmatrix}_t
\]

(3.7)

Where, \( F(\tau_t - l, \gamma, \kappa) = \frac{1}{1 + e^{-\gamma(\tau_t - l)}} \)

In order to compute the GIRF, a definition of conditional expectation is needed. The interest is in knowing the value expectation of the \( j \)-th variable after \( h \) periods conditioned on the history until time \( t-1 \) (that will be called \( \Omega_{t-1} \)) and a given shock (called \( S_{y_j} \)) on the \( i \)-th variable at time \( t \). Then the GIRF is derived by taking the difference between such conditional expectation and the same expectation, but conditioned only on a particular history realization \( \omega_{t-1} \in \Omega_{t-1} \).

This is:

\[
GIRF_{y_j}(y_j, h) = E\left( y_{j_{t+h}} \mid e_{i_{t}} = S_{y_j}, \omega_{t-1} \right) - E\left( y_{j_{t+h}} \mid \omega_{t-1} \right)
\]

(3.8)

A particular history is, for instance, that the economy is experiencing high inflation, or that oil prices are growing at a high rate, etc. Notice that in addition to the shock which is given in the initial period, for computing the GIRF it is necessary to have a sequence of innovations which are obtained by means of simulating the model with the available information. By definition such a sequence has zero mean. The underlying assumption for this method is that there are no sample variability, or that the non linear model is known.

The accumulated impulse response up to time \( H \) of \( y_j \) to shocks on \( y_1 \) would then be:

\[
\sum_{h=1}^{H} GIRF_{y_j}(y_j, h)
\]

(3.9)

Since an expectation function and particularly a conditional expectation function is linear, computing the accumulated impulse response is the same as separately computing all expectation values and then calculate its difference, or to calculate all expectation differences period to period and then add them all.

\[
\sum_{h=1}^{H} GIRF_{y_j}(y_j, h) = E\left( \sum_{h=1}^{H} y_{j_{t+h}} \mid e_{i_{t}} = S_{y_j}, \omega_{t-1} \right) - E\left( \sum_{h=1}^{H} y_{j_{t+h}} \mid \omega_{t-1} \right)
\]

(3.10)

\(17\) Notice that Gaussian assumptions will not be adequate when there exist contemporaneous dependence across shocks in the equations of the system.
Based on the GIRF definition, the PT of a shock on $y_t$ over $y_j$ after $H$ periods is then computed as:

$$PT_H = \frac{\sum_{h=1}^{H} GIRF_{y_t}(y_j, h)}{\sum_{h=1}^{H} GIRF_{y_t}(y_j, h)}$$  \hspace{1cm} (3.11)$$

Notice that since $GIRF_{y_t}(y_j, h)$ is exactly $\partial y_{j,t+h}/\partial e_t$, then expression (3.11) can be rewritten as:

$$PT_H = \frac{\sum_{h=1}^{H} \partial y_{j,t+h}/\partial e_t}{\sum_{h=1}^{H} \partial y_{j,t+h}/\partial e_t}$$ \hspace{1cm} (3.12)$$

Which is the same notation used in Goldfang and Werland (2000) and in most of the resent studies that apply LSTVAR to measure PT effects.

If in (3.11) $y_t$ is devaluation and $y_j$ is inflation, then by measuring the relative change in inflation up to period $H$ as a consequence of a shock on depreciation on time 0 with respect to the accumulated response, up to period $H$, of the depreciation as a consequence of a shock on itself in period 0, expression (3.11) prevents a potential overestimation of the PT effect\textsuperscript{18}

According to expression (3.11) computing the PT requires simulating the model $H+1$ periods ahead considering a history of the vector innovations \{$e_t, e_{t+1}, e_{t+2}, \ldots, e_{t+H}$\}, where $e_t = (e_1, e_2, e_3, e_4)'$. In order to simulate whether it is a low or high regime shock, the particular history is constructed by means of bootstrapping from the corresponding collection of vector errors of the free estimated system. Appendix D offers a detailed explanation about how the PT is computed.

Asymmetries are tested by comparing the PT effect that the LSTVAR generates when different definitions of shocks are provided. As explained in appendix D, we rely on the Cholesky decomposition of the residual variance-covariance matrix such that the series of vector innovations can be expressed as $e_t = A\nu_t$, where matrix $A$ is the upper Cholesky factor. Then if we want to give variable two a shock of a size equivalent to one standard deviation, we must set $\nu_t = (0, 1, 0, 0)'$; a negative shock of three standard deviation will require to set $\nu_t = (0, -3, 0, 0)'$ and so on.

5. Results

With the lagged variation of oil prices as the transition variable, the estimated LSTVAR shows evidence of important state asymmetries of the PT effect and almost no proof of either sign or size asymmetries for the sample of Costa Rican data. Those results are summarized in Table 2, which includes the most important outcomes of this study.

Regarding the organization of Table 2, notice that all relevant PT estimations are located into the six columns under the title Pass-Through Effect. The three main rows of the body of the chart refer to the high and low regimes and the linear case. The high and low regime determines whether the shock on devaluation is given when the 10th lag of annual oil prices change is higher or lower than the critical level of -4.9%. By comparing those two rows it is possible to know about potential state asymmetries.

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\textsuperscript{18} See Goldfang and Werland \textit{op.cit.}
Secondary rows correspond to different size of the shocks. We considered only two cases: a small shock (one standard deviation) and a big shock (three standard deviations) in devaluation. Note that in this case the small and big shocks are equivalent to 0.8% and 2.4% monthly devaluation respectively. By comparing those secondary rows it is possible to reach conclusions about possible magnitude asymmetries.

Columns under the titles: Positive Shocks(%) and Negative Shocks(%) are self explained. Notice though that we also organized in columns the horizons of the PT estimations. Then by comparing the corresponding 6, 12 or 24 months of positive and negative shocks it will be possible to know about any possible sign asymmetry.

In order to contrast results, we present a linear estimation of the PT effect. Such estimation comes from the impulse response functions applied to the initial VAR(4) which was the baseline model for the LSTVAR.

Let us focus firstly on state asymmetries. Notice that the PT in high regime is consistently higher than the low regime PT. This is the case for positive and negative shocks, for every horizon larger than 6 months and for the two sizes of shocks. Actually, on average the high regime PT is about twice the size of the low regime PT. In the long run, it is expected that when the transition variable is above its threshold level the shocks on the exchange rate will pass to prices in an amount that doubles the one expected when the transition variable is below the threshold. This seems to be strong evidence of high state asymmetries of the PT effect in Costa Rica. However Figure 3, which shows the PT of the high and low regimes for up to 24 months horizon together with their respective confidence intervals, reveals that those differences in magnitude are actually statistically no significative.19

Table 2 shows no strong evidence of sign asymmetries. On one side, in the high regime when there is a small or big shock, the PT tends to be slightly bigger when the shock is positive and the horizon larger. On the other side, in the low regime the PT is somewhat smaller when the shock is positive independently of the size of the shock or the horizon.

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19 Appendix C explains how those confidence intervals were built. Solid lines show sign and magnitude averages PT level for each horizon. Notice that the low regime confidence interval is much wider than the one for the high regime; this is because we have much less observations in the low regime so the estimation is less precise.

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Nor there is convincing evidence of magnitude asymmetries. In the high regime for either positive or negative shocks, the PT is almost the same for small and big shocks across all horizons. On the other hand, in low regime it seems that the size of the shocks matters a little bit more. For positive shocks it seems that the big ones induce slightly larger PT. On the opposite, for negative shocks the smaller ones produces somewhat larger PT. In either case the differences are very small, which lead us to the conclusion that the size of the shocks have no important effects on the magnitude of the PT.

It is important to mention how the estimated PT behaves at different horizons. From the chart it can be inferred that in a year period most of the impact on prices is given in the first 6 months. Notice that the PT is almost the same across all 6 and 12 months columns. Between 12 and 24 months there are big differences, on average the 24 months PT is twice the 12 month PT.

Contrasting the predicted PT of this LSTVAR with those estimated by means of the lineal version and the levels reported in past studies that uses Costa Rican data is also interesting. Notice that for all horizons (up to 28 months), signs and magnitudes, the linear PT overestimates the predicted level of the LSTVAR. Figure 4, which shows average PT effect of the LSTVAR and the linear PT, demonstrates that such difference is statistically significative for most horizons.

Long run PT of the linear version also tends to be higher than previous work for Costa Rican data. Leon, Morera and Ramos (2001) estimated a long run PT of about 0.55, which is in between the 12 and 24 months levels estimated by our lineal version, yet in our case the lineal PT reaches up to 0.7 after 30 months. The LSTVAR signals an average long run PT of nearly 0.6.
With a more recent data base, Castrillo and Laverde (2008) estimated a long run PT of 0.33, which is more or less consistent with the levels predicted by our LSTVAR between 12 and 24 months but only in the case of the high regime. Our study suggests that their estimations can be misleading in cases when the 10th lag of the annual change of oil prices is below -4.9%. In such cases our average measures of long run PT effect is about 0.24.

Since both studies, Leon, Morera and Ramos *op.cit.* and Castrillo and Laverde *op.cit.* identify only one lag as an estimator of the short run PT, it is hard to compare shorter horizons estimations. The former paper estimates a PT coefficient of 0.16 after 2 months, while the last one identifies a smaller PT of almost 0.06 after 4 months. Our estimations (not shown in Table 2) are of an average PT of 0.04 and 0.07 after 2 and 4 months respectively.

All information summarized in Table 2 is illustrated in the appendix E where graphics of the PT in high and low regime, for positive and negative shocks and for small and big shocks are illustrated.
6. Conclusions

This study carries out an LSTVAR model for analyzing non linearities in the PT of the exchange rate to domestic prices in the Costa Rican economy. It was found that the evolution of changes in oil prices plays an important role in generating state asymmetries because it explains the transition between regimes. It was found that such transition is relatively smooth and it happens when the 10\textsuperscript{th} lag of the annual change of oil prices is about -4.9\%. In other words, there is evidence of state asymmetries, the state is determined by the 10\textsuperscript{th} lag of the annual variation of oil prices and the threshold is -4.9\%.

The fact that the change in oil prices determines the states might have at least two explanations. High change of oil prices can generate inflationary expectations which might induce price setters to easily pass shocks on exchange rate to their domestic prices. On the other hand, a higher oil price means a bigger imports bill for the economy and so a higher trade deficit for every level of exports, then agents might interpret this as a situation in which the probability that a given shock on the exchange rate becomes permanent is elevated. Clearly that will make them prone to pass such changes to their domestic prices.

There was no robust evidence signaling neither sign nor magnitude asymmetries. This means that in the estimated model, given a state of the economy defined by the transition variable, positive and negative shocks on exchange rate are expected to impact equally domestic prices. The same would be true for big and small shocks; the impact on domestic prices of big and small shock was found to be proportional. It is worth to mention that this is in no way robust evidence of the absence of such asymmetries. Notice that the final transition variable chosen is exogenous to the VAR system, in such cases it is hard to generate sign or magnitude asymmetries.

In the high regime, that is when the 10\textsuperscript{th} lag of annual oil prices change is more than -4.9\%, it is expected that the long run PT is about 47\%, while in the low regime the portion of a given shock on the exchange rate that is passed to domestic prices is about 24.7\%.

The base line VAR tends to overestimate the PT effect. According with impulse response functions of the base line VAR, the long run PT reaches 68\%. The difference between the PT obtained by means of the lineal version and the average PT from the LSTVAR is statistically significative for all periods from 4 through 30 months after the shock. Such overestimation becomes greater if the shock happens in the low regime.

Most of the PT effect happens in the first six months after the shock. No matter the regime, on average half of the impact of a given exchange rate movement takes place during the first 6 months, the remaining effect impacts prices across the next 18 months.

For shorter horizons (less than 6 months after the shock) this study confirms what Castrillo and Laverde op.cit reported, that is a decrease in the PT effect with respect to what Leon, Morera and Ramos op.cit. found. The later study reports a PT of 16\% after 2 months, while the former found evidence of 6\% after 4 months. Our findings point to an average 4\% after 2 months and 7\% once 4 months has passed.
7. References


Winkelried, Diego (2003). ¿Es asimétrico el pass-through en el Perú? Un análisis agregado. VIII Reunión de Red de Investigadores de Banco Central del Continente Americano. CEMLA.
Due to the rejection of the null hypothesis of unit root, some other unit root test that allow for structural breaks on the series were not applied. It has been shown that under such rejections, ADF and PP test are much more robust in leading to a right identification of order of integration. See Castrillo and Rodriguez (2009).
Appendix B

Grid search results

Chart B1 shows the result of the grid search process for a few candidates for transition variable. As mentioned before DLPPET12(-10) is the transition variable chosen. The remaining variables where dismissed due to few observations in one of the regimes or because the γ parameter was too high. DE12(-5) is the fifth lag of the annual change of nominal exchange rate; DLM112(-10) is the annual variation of the monetary aggregate M1 lagged 10 periods; DLITCER12(-9) is the ninth lag of the annual change in real exchange index and DLRIN12(-8) is the annual variation of international reserves lagged 8 periods.

Chart B1

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<td>Upper</td>
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</table>

Chart B2 shows the coefficient estimation from the grid search process when $DLPPET_{12}^{10}$ is TV. Notice that both, γ and κ, are fixed. Parameters in this chart are those used as initial values when solving the non-linear SUR model. Chart B3 includes the parameters freely estimated, in this case only γ is fixed.

---

21 10th lag of annual change of oil prices.
### Initial values of VAR

#### Chart B2

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*Initial Values for the Free Estimation*
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Free Estimated Parameters

Chart B3
Appendix C

Lets define the high regime as the set of observations for which the transition function (TF) is higher than 0.9 and the low regime as the set for which TF is lower than 0.1. Then, according with expression (2.3), when $(y, \kappa) = (18, -0.049)$ the high regime occurs when annual variation of oil prices is above -1% and the low regime when it is below -8.8%. Such defined, the high regime groups 60% of the sample, while 27% belongs to the low regime. The remaining 13% are transition observations. Figure C1 shows both, TV and TF, chronologically sorted. Figure C2 plots both variables against each other when the series are sorted according with TV. Notice that TF reaches a value of exactly 0.5 when $TV = \kappa = -0.049$ which is the vertical line.

![Figure C1](image1.png)

**Figure C1**
Transition Function and Transition Variable

![Figure C2](image2.png)

**Figure C2**
Transition function vs. Transition variable.
**Appendix D:**

**Impulse response computation**

Simulating a shock on the variable $y_i$ and the impact that the LSTVAR (3.7) will yield on variable $y_j$ is the interest. If the shock is $S_{y_i}$, such impact will be given by the generalized impulse response function defined as in (3.8), that is:

$$GIRF_{y_i}(y_j, h) = E\left(y_{j_t+h}\bigg|\omega_{t-1} = S_{y_i}, \omega_{t-1}\right) - E\left(y_{j_t+h}\bigg|\omega_{t-1}\right) \tag{3.13}$$

Notice that $\omega_{t-1}$ is all history observed up to the time $t$. Then we follow the next sequence of steps for computing the PT:

1. After having estimated the complete LSTVAR, to establish the conditions of the shock. That means to set:
   i. The regime for which the shock will be simulated. This is to select randomly a point for which $\tau_{t-1} > \kappa$ (high regime) or $\tau_{t-1} < \kappa$ (low regime).
   ii. The sign of the shock. That is to define whether $S_{y_i}$ is positive or negative.
   iii. The magnitude of the shock. As in Mendoza (2004) we define a small shock as one standard deviation and a big shock as three standard deviations of the shocked variable.
2. To simulate the model $H$ periods ahead incorporating, from $t+1$ up to $t+H$, a particular history for the elements of the vector $e_t = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)'_t$. This history is constructed by means of bootstrapping from the observed history of errors a sample of size $H$. This is the way the component $E\left(y_{j_t+h}\bigg|\omega_{t-1}\right)$ is formed for $h=0,1,2,...,H$.
3. As specified in step 1, shock element $\varepsilon_i$ of vector $e_t$ in time $t$ and simulate the model $H$ periods ahead considering the same history for the elements of $e_t$ from step 2. This allows to compute $E\left(y_{j_t+h}\bigg|\varepsilon_{t} = S_{y_i}, \omega_{t-1}\right)$ for $h=0,1,2,...,H$.
4. Calculate $GIRF_{y_i}(y_j, h)$ according with (3.13).
5. Execute steps 2 to 4 $R$ times and compute the PT following equation (3.11), this will yield $R$ measures of PT that were averaged out. We use $R=500$.
6. Use nonparametric bootstrap percentile confidence intervals to infer the observed significance level of the effects. This is done by repeating steps 1 through 5 $N$ times. The 5th and 95th percentiles of the empirical distribution formed the limits for the 90% bootstrap percentile confidence interval. We set $N=500$.

In order to be able to compare the PT effect that non-linear models yield with those coming from the base lineal model, we use orthogonalized shocks constructed by means of Cholesky decomposition. This also permits us to relay on an identified interpretation of the structural errors of the reduced form of the system. Specifically we use the following structure for the vector of errors:

$$
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4
\end{pmatrix} =
\begin{pmatrix}
\mu_1 \cdot \alpha_{11} + \mu_2 \cdot \alpha_{12} + \mu_3 \cdot \alpha_{13} + \mu_4 \cdot \alpha_{14} \\
\mu_2 \cdot \alpha_{22} + \mu_3 \cdot \alpha_{23} + \mu_4 \cdot \alpha_{24} \\
\mu_3 \cdot \alpha_{33} + \mu_4 \cdot \alpha_{34} \\
\mu_4 \cdot \alpha_{44}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4
\end{pmatrix} =
\begin{pmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{13} & \alpha_{14} \\
0 & \alpha_{22} & \alpha_{23} & \alpha_{24} \\
0 & 0 & \alpha_{33} & \alpha_{34} \\
0 & 0 & 0 & \alpha_{44}
\end{pmatrix}
\begin{pmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4
\end{pmatrix}
$$
Or written in compact form: $e_t = Av_t$

In such a form we assume that $v_t$ is a vector written on an orthonormal basis of errors, which means that if we want to give variable two a shock of a size equivalent to one standard deviation, we must set $v_t = (0, 1, 0, 0)'$; a negative shock of three standard deviation will require to set $v_t = (0, -3, 0, 0)'$ and so on. Matrix $A$ is the upper Cholesky factor of the residual variance-covariance matrix. Extended details can be found in Hamilton (1994).
Appendix E

Pass-Through effects and confidence intervals.

Figure E1
Pass-Through in High and Low Regimes

Small and Positive Shocks.

Big and Positive Shocks.

Small and Negative Shocks.

Big and Negative Shocks.
Figure E2
Pass-Through for Positive and Negative Shocks

High Regime and Small Shocks.

- Positive Shock
- Negative Shock

High Regime and Big Shocks.

Low Regime and Small Shocks.

Low Regime and Big Shocks.
Figure E3

Pass-Through for Big and Small Shocks

High Regime and Positive Shocks.

Small Shock

Big Shock

High Regime and Negative Shocks.

Small Shock

Big Shock

Low Regime and Positive Shocks.

Small Shock

Big Shock

Low Regime and Negative Shocks.

Small Shock

Big Shock