Long Memory and Nonlinearities in Peruvian Inflation Rate*

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Abstract

We analyze the dynamic properties of the Peruvian inflation at the aggregate and disaggregated levels using the approach based on the autocorrelation function (ACF). We concentrate in two properties: long memory and nonlinearities. These properties are introduced as a consequence of aggregation of heterogeneous and correlated unities. Because the total CPI is a weighted average of individual prices of goods, long memory and nonlinearities are found. We use two competing models to estimate the sample ACF: classical linear approach and the non-linear approach proposed by Abadir and Talmain (2002) and Abadir, Caggiano and Talmain (2005). The results show a strong persistence in aggregate unities. Selection of lag length is very heterogeneous. Estimations based on the non-linear approach are much better than estimations based on the traditional linear approach.

Keywords: Inflation, Autocorrelation Function, Long Memory, Nonlinearities

JEL: C22, E52, E58

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1 Introduction

In an AR(1) process, under the assumption that the autoregressive coefficient is less than unity, the autocorrelation function is strictly convex and decay towards zero at an exponential rate. However, financial and macroeconomic time series may have a complicated dependence structure which can not be adequately modeled using a lineal process.

Let $y$ be an aggregate time series obtained as the weighted sum of individual unities ($y_i$):

$$y = \sum_{i=1}^{N} h_i y_i,$$

where $h_i$ are the respective weights. If individual unities (sectors) are not correlated but they are persistent and heterogenous, Granger (1980) has shown that if $h_i = 1$ for every (sector) $i$, therefore, the aggregate series $y$ has long memory with autocorrelation function strictly convex decaying to zero at an hyperbolic rate. The result is generalized to the case where $\sum h_i = 1$ by Chambers (1998) and to the case of ARMA individual processes by Zaffaroni (2004). In another important contribution, Granger and Ding (1996) argue that aggregation of (sectorial) unities which are heterogenous and correlated introduce long memory and nonlinearities.

There are many different sources or explanations for the presence of heterogeneity. One source is the differences in the way as prices at the sectorial level are fixed; see Gadea and Mayoral (2005), Carvalho (2006). Another potential source of heterogeneity is the way as expectations are formed. In most cases, expectations adapt slowly at the individual level (Gagnon, 1996). However, even when expectations may adapt fastly, they may be very heterogenous among economic agents as Mankiw, Reis and Wolfers (2004) argue.

In the macroeconomic context, there are two additional aspects to be considered. First aspect is the existence of cross correlations between unities (sectors). Second aspect is the fact macroeconomic variables are constructed adding levels and not the logs of the unities (sectors). In other words:

$$\bar{y} = \ln[\sum_{i=1}^{N} h_i \exp(y_i)],$$

which is different to the process of (1). The process given by (2) is a non lineal function of the individual series (sectors) and its dynamic properties are very different of those given by (1).
Attanasio and Weber (1993), Abadir and Talmain (2002) summarizes above discussion indicating that the only fact to use a geometric average is responsible of the hyperbolic decaying behavior of the ACF. Aggregating using (2) produces changes in the concavity of the ACF. It is different of what suggests an ARFIMA model which is based on an aggregation of the type (1).

One illustrative example of the above described issues is the model with rigid prices of Rothemberg (1982). In this model each firm faces quadratic costs in the adjustment of prices. The dynamic of the sectorial prices is

\[ p_{it} = \phi_i p_{it-1} + \frac{1}{1 + \phi_i} p_{it}, \quad (3) \]

\[ p_{it}^* = \mu + \frac{1}{1 + \phi_i} u_t, \quad (4) \]

where \( p_{it} (\ln P_{it}) \) is the level of current prices of the firm \( i \) at the period \( t \), \( p_{it}^* (\ln P_{it}) \) is the level of optimal prices of the firm \( i \) at the period \( t \), and \( u_t \) is an inflationary shock which is distributed Normally.

In order to obtain sectorial inflation, we differentiate, we apply first differences to (3):

\[ \pi_{it} = \Delta p_{it} = \phi_i \Delta p_{it-1} + (1 - \phi_i) \Delta p_{it}^*. \quad (5) \]

In order to simplify some aspects, let assume an uniform distribution for the weights \( h_i = 1/N \) for each \( i \). Therefore, the CPI is

\[ P_t = N^{-1} \sum_{i=1}^{N} P_{it} \]

\[ = N^{-1} \sum_{i=1}^{N} \exp(p_{it}), \quad (6) \]

and therefore, the aggregate inflation is

\[ \pi_t = \ln[N^{-1} \sum_{i=1}^{N} \exp(p_{it})] - \ln[N^{-1} \sum_{i=1}^{N} \exp(p_{it-1})], \]

\[ = N^{-1} \sum_{i=1}^{N} \pi_{it} + R_t \neq N^{-1} \sum_{i=1}^{N} \pi_{it}, \quad (7) \]

where \( R_t \) is a residual term without exact analytical form. The fact that \( \pi_t \neq N^{-1} \sum_{i=1}^{N} \pi_{it} \) implies that inflation is potentially different of an ARFIMA(p,d,q) model.
This paper has two goals. First goal is to show empirically that aggregation introduces long memory and nonlinearities. We analyze it using Peruvian inflation rates at different degrees of aggregation. Another goal is the estimation of the ACF using the linear AR approach and the nonlinear ACT approach. Comparison of these estimations allow to identify the better performance of the ACT in capturing long memory and nonlinearities of the time series.

The paper is organized as follows. Section 2 describes briefly the two approaches used to estimate the ACF of the inflation rate at the different levels of disaggregation. Section 3 presents the data and the principal results obtained using the two approaches indicated in the methodology. Section 4 concludes.

2 Methodology

Two alternative approaches to estimate the ACF are briefly described.

The first approach is to use an AR(p) model which is widely used to measure persistence in financial and macroeconomic time series. Assume that we follow an AR(p) for the time series: $\pi_t = \phi_0 + \phi_1 \pi_{t-1} + \ldots + \phi_p \pi_{t-p} + \epsilon_t$, \hspace{1cm} (8)

where $\{\epsilon_t\} \sim N(0, \sigma^2_\epsilon)$. The ACF of (8) is denoted by $\rho^A_{:\pi}$ and its first $p$ values are given by the equations of Yule-Walker:

$$
\begin{bmatrix}
1 & \rho^A_1 & \rho^A_2 & \ldots & \rho^A_{p-1} \\
\rho^A_1 & 1 & \rho^A_1 & \ldots & \rho^A_{p-2} \\
\rho^A_2 & \rho^A_1 & 1 & \ldots & \rho^A_{p-3} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
\rho^A_{p-1} & \rho^A_{p-2} & \rho^A_{p-3} & \ldots & 1
\end{bmatrix}
\begin{bmatrix}
\phi_1 \\
\phi_2 \\
\phi_3 \\
\vdots \\
\phi_p
\end{bmatrix}
= 
\begin{bmatrix}
\rho^A_1 \\
\rho^A_2 \\
\rho^A_3 \\
\vdots \\
\rho^A_p
\end{bmatrix}.
\hspace{1cm} (9)
$$

Expression (9) is a system of $p$ equations in the $p$ values of $\{\rho^A_1, \rho^A_2, \ldots, \rho^A_p\}$ and therefore the values may be determined in an unique way. The other $\rho^A_\tau$ values are obtained using the following recursive expression:

$$
\rho^A_\tau = \phi_1 \rho^A_{\tau-1} + \ldots + \phi_p \rho^A_{\tau-p},
\hspace{1cm} (10)
$$

for all $\tau > p$. In terms of notation, this linear approach will be denoted by AR.

The second approach consists in a direct specification of the ACF of a time series potentially nonstationary. This approach is due to Abadir.
and Talmain (2002), Abadir, Caggiano and Talmain (2005). In terms of notation, this approach is denoted by ACT. According to this approach:

$$\rho_{\tau}^{ACT} = \frac{1 - a[1 - \cos(\omega \tau)]}{1 + b^{\tau c}},$$

(11)

where the numerator controls oscillations of the ACF and the denominator regulates the behavior of the memory. More exactly, parameter $c$ measures the rate of decay, parameter $b$ regulates the impact or slope of the ACF, parameter $a$ controls impact of the oscillations, and parameter $\omega$ regulates the frequency of the oscillations.

One main goal of the paper is the formal comparison of the two competing models (8) and (11). Both models are estimated by non linear squares. To select the order of the autoregressive process, we employ the Schwarz criterion (SIC). Given its consistency, the same criterion is used to compare the two competing models. The use of an information criterion allows us to compare two models not necessarily having the same degrees of freedom. The alternatives are Akaike and Hannan-Quinn criteria. The former has been shown to be inconsistent. The later is designed to pin down orders $p$ and $q$ of an ARMA($p,q$) process. Given that the ACT function does not belong to the ARMA class, we employ the SIC. We also follow the suggestion of Ng and Perron (2005) and hold the effective sample size fixed across models to be compared.

Both approaches above mentioned are used in the following steps. The first step is to estimate the ACF. It is performed using a definition that takes into account potential nonstationarity in the mean of the series under analysis. Second step identifies the lag value where the ACF is not statistically different of zero. Because there is a high degree of uncertainty in the estimation of the ACF of time series with potential flat tales, we use the bootstrap method recommended by Politis and Romano (2004). The third step estimates ACF using the (standard) AR approach. Final step estimates ACF using the non-linear approach ACT.

3 Results

Data under analysis is obtained from the Central Reserve Bank of Peru and it is monthly information covering period 1994:12-2008:08. Different levels of disaggregation are considered. A first level is the analysis of the total CPI and the indexes of total groups. In this case we have: food and beverages, clothes, housing, electricity, water, transport, etc. A second level
of disaggregation is the analysis of some particular division of the above groups. At this level we analyses milk products, clothes for women, clothes for men, electricity, rent, etc. Last level of analysis is inflation rates for specific products, for example: potatoes, rice, milk, rent, electricity, specific clothes, etc.

Inflation rate is measured as $\pi_{jt} = 100 \times [\ln(P_{jt}) - \ln(P_{jt-12})]$ where $j$ denotes the unity or sector under analysis.

Figure 1 plots the sample ACFs and the 90% confidence bands for the CPI and the eight groups of goods included in the sample (G-1, G-2, ..., G-8). Figure 2 shows similar information for selected goods of the sample. Most of the correlograms display a slow rate of decay, with frequent and long-lasting oscillations around a time-varying mean. They cross the zero-line, displaying statistically significant positive and negative values, but show the tendency to revert back to it. In Figure 1, most of sample ACFs are very similar except those of Housing and Electricity, Health, and Transport and Communications. In the case of Figure 2, most of sample ACFs of the individual goods are very different to the sample ACF of the total CPI. For instance the sample ACF of the chicken meal is clearly different to aggregate sample ACFs.

Table 1 shows some estimates obtained using the approaches AR and ACT. We may observe the estimate of $\tau^*$ and the level of persistence calculated by the AR approach. Furthermore we present the value of the SIC calculated for the AR and ACT approaches. Some conclusions emerge from these estimations. First, the value of the $\tau^*$ is high and very heterogeneous. The smallest value is obtained using the group of Housing and Electricity. Second, the selected $p$ value is very similar for all groups considered in Table 1. Third, the level of persistence is very high and close to unity. Four, the SIC is always minimized using the approach ACT. The only cases where SIC of the AR and ACT are very close are the group of Food and Beverages (G-1) and Other Goods and Services (G-8). Furthermore, there are big differences between the SIC obtained by AR and ACT for the groups Foods and Beverages (G-1), Housing and Electricity (G-3) and Transport and Communications (G-6).

Table 2 shows similar estimates as those of Table 1 but applied to selected goods. As before, estimated value of $\tau^*$ is very heterogeneous. Smallest values of $\tau^*$ are estimated for rice, chicken, electricity, potatoes. The level of persistence is smaller than values showed in Table 1 which indicates that level of persistence is higher for higher levels of aggregation. The estimates of the SIC are clearly favorable to the ACT approach. The differences are more clear than those observed in Table 1.
Figure 3 shows the sample and fitted ACFs for the CPI and the eight groups of consumption (G-1, G-2, ..., G-3). Figure 4 shows similar information but for a selected set of goods. The results allow to obtain the following comments. First, in all cases the ACT function fits the empirical ACF much better than the competing AR model. Second, in several cases the AR model delivers wrong sign of the concavity of the ACF in order to be close as possible to the sample ACF in the middle of the sample. It implies an overestimation of the ACF at low lags, and consequently an overestimation of the rate of decay of the shocks. It is worth to say that the sum of the autoregressive coefficients is very close to one. It implies that if the true data generating process is a linear autoregressive process, aggregate inflation has a unit root, or it is a near unit root process. Third, in some cases the AR estimations deliver implausible high-frequency oscillations.

4 Conclusions

Peruvian monthly inflation rates have been used in order to identify for the presence of long memory and nonlinearities. Aggregate data and different levels of disaggregation have been used. The linear AR and nonlinear ACT approaches have been used to estimate the sample ACF.

The empirical results confirm that the ACT functional form, which is derived from a long memory and nonlinear process, replicates the sample ACFs much better than a autoregressive model, which is based on the assumption of log-linearity. Therefore the results confirm results of Caballero and Engel (2003) in the sense that estimates of persistence based on partial-adjustment ARMA models are likely to be incorrect.

According to our estimates, the SIC is always minimized using the approach ACT. The only cases where SIC of the AR and ACT are very close are the group of Food and Beverages (G-1) and Other Goods and Services (G-8). Furthermore, there are big differences between the SIC obtained by AR and ACT for the groups Foods and Beverages (G-1), Housing and Electricity (G-3) and Transport and Communications (G-6). It is more clear when disaggregated data is used.

Furthermore the results indicate that the sample and fitted ACFs for the CPI and the eight groups of consumption (G-1, G-2, ..., G-3). In several cases the AR model delivers wrong sign of the concavity of the ACF in order to be close as possible to the sample ACF in the middle of the sample. It implies an overestimation of the ACF at low lags, and consequently an overestimation of the rate of decay of the shocks. It is worth to say that the
sum of the autoregressive coefficients is very close to one. It implies that is the true data generating process is a linear autoregressive process, aggregate inflation is a unit root, or a near unit root process.

References


<table>
<thead>
<tr>
<th>Grupos</th>
<th>$\tau^*$</th>
<th>$\phi$</th>
<th>$\sum_{i=1}^p \phi_i$</th>
<th>SIC</th>
<th>SIC</th>
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<tr>
<td>Total IPC</td>
<td>76</td>
<td>3</td>
<td>0.998</td>
<td>-4.271</td>
<td>-5.436</td>
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<tr>
<td>Alimentos y Bebidas (G-1)</td>
<td>84</td>
<td>2</td>
<td>0.996</td>
<td>-3.246</td>
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<tr>
<td>Vestido y Calzado (G-2)</td>
<td>100</td>
<td>2</td>
<td>0.999</td>
<td>-4.008</td>
<td>-3.992</td>
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<tr>
<td>Alq. Vivienda, Comb. y Electricidad (G-3)</td>
<td>56</td>
<td>2</td>
<td>0.979</td>
<td>-2.658</td>
<td>-4.568</td>
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<tr>
<td>Muebles, Enseres y Mant. Vivienda (G-4)</td>
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<td>0.999</td>
<td>-4.602</td>
<td>-4.704</td>
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<tr>
<td>Cuidado Salud y Serv. Médicos (G-5)</td>
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<td>-3.792</td>
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<tr>
<td>Transporte y Comunicaciones (G-6)</td>
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<td>-3.592</td>
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<tr>
<td>Espace., Serv. Cultural y Enseñanza (G-7)</td>
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<td>2</td>
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<td>Otros Bienes y Servicios (G-8)</td>
<td>78</td>
<td>2</td>
<td>0.998</td>
<td>-5.220</td>
<td>-5.241</td>
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Table 2. Estimates of AR and ACT (Selected Productos)

<table>
<thead>
<tr>
<th>Productos</th>
<th>τ*</th>
<th>AR</th>
<th>ACT</th>
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<tr>
<td></td>
<td>p</td>
<td>( \sum_{i=1}^{p} \phi_i )</td>
<td>SIC</td>
</tr>
<tr>
<td>IPC</td>
<td>76</td>
<td>3</td>
<td>0.998</td>
</tr>
<tr>
<td>Arroz</td>
<td>10</td>
<td>4</td>
<td>0.947</td>
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<td>Pan</td>
<td>37</td>
<td>2</td>
<td>0.984</td>
</tr>
<tr>
<td>Carne de Res</td>
<td>92</td>
<td>2</td>
<td>0.998</td>
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<tr>
<td>Carne de Pollo</td>
<td>19</td>
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<td>2</td>
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<td>Combustibles</td>
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<td>55</td>
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Figure 1. ACF and Confidence Intervals (CPI and Groups)
Figure 2. ACF of CPI and Selected Products
Figure 3. Estimations AR and ACT of CPI and Groups
Figure 4. Estimations AR and ACT of Selected Products