Technology, Convergence and Business Cycles

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Abstract

In this paper we integrate Schumpeterian endogenous growth into a general equilibrium framework. By explicitly modelling the innovation and technology adoption process we are able to match some stylized economic facts such as entry rates and survival times of firms in the U.S. economy or the maximum convergence rates across countries. Additionally, it allows us to propose a new definition of what a technology shock is and to compare it with the standard definition. Results show how this framework provides a plausible description of how economies grow and respond to the arrival of new technologies.

Keywords: Medium-term business cycles, Schumpeterian growth, technology adoption.

JEL E3, O3, O4

1 Introduction

There seems to be widespread belief among many macroeconomists that technology is manna from heaven, since the seminal work by Kydland and Prescott (1982), who considered that business cycles were an optimal response by rational agents to exogenous changes in productivity. The real business cycle literature evolved by enriching the transmission mechanisms with more realistic features, but it kept the core assumption of exogenous “technology shocks” as a primary source of business cycle fluctuations.¹ The rise of the new Keynesian literature (e.g. Woodford; 2003) emphasized the importance of nominal variables in economic fluctuations, but retained the assumption of the existence of technology shocks, although reducing their contribution to output variance (Ireland, 2004; Smets and Wouters; 2007). Chari, Kehore and McGrattan (2009), who disagree with many of the

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¹See for example, Cooley and Prescott (1995).
shocks introduced in new Keynesian models, consider technology shocks to be structural, that is, interpretable and invariant to policy interventions.

The exogeneity of technology shocks is at odds with two decades of literature in endogenous growth theory, which has emphasized how technology development is the result of the actions of different types of agents (entrepreneurs, researchers, workers) that operate under specific sets of constraints in the general context of an economy. In these models, a policy intervention may have temporary or permanent effects on the long-run growth rate of the economy. For example, Blackburn and Pelloni (2005) show how in an economy with nominal rigidities where technology is endogenously generated by a process of learning-by-doing, a policy that aims at reducing output volatility may not be optimal in the terms of growth. More recently, Comin and Gertler (2006) have shown how introducing endogenous growth à la Romer (1990) into a model with imperfect competition is able to generate persistent fluctuations in the total factor productivity due to temporary shocks in non-technological variables such as wage mark-ups.

The aim of this paper is to integrate a model of endogenous growth into a simple DSGE model. The endogenous growth model is based on vertical innovations or “quality ladders” literature introduced by Grossman and Helpman (1991) and Aghion and Howitt (1992). In this literature, growth is endogenously driven by entrepreneurs’ attempts to innovate in order to climb up the quality ladder to capture a stream of monopoly profits. These models are typically defined as “Schumpeterian” as they naturally incorporate the concept of “creative destruction” by which new firms replace the less efficient old ones. The advantage of these models is that they are rigorously based on microeconomic theory and are suitable to answer a broad range of different questions, from entry-exit firm dynamics, to cross-country growth convergence and income differences, as discussed in Aghion and Howitt (1998).

By integrating growth into a DSGE framework we are able to provide quantitative responses in two different dimensions. In the first place, we are able to match the growth frontier observed for a wide set of countries since the 1960s and to provide an explanation to the growth and per capita income observed in most countries, which suggests that most “follower countries” have experienced higher barriers to entrepreneurship than the US. In the second place we are able to precisely define what a technology shock is in the context of the model. We consider that technology shocks are exogenous changes to the slope of the production function of entrepreneurs, which affect the expected costs and profits of potential innovators thus influencing the growth rate of productivity. We compare this

\[^2\] Introductory texts are, for example, Barro and Sala-i-Martin (2004), Grossman and Helpman (1991) and Aghion and Howitt (1998).

\[^3\] An early example of the integration between growth and cycles is Fatas (1998) in the context of a AK model.
definition of technology shocks with the standard ones in the context of real business cycle (RBC) models. Results show how our definition is able to produce similar output dynamics than traditional technology shocks, without the necessity of introducing exogenous persistence mechanisms.\footnote{It is in line with the results of Phillips and Wrase (2006).}

The structure of the paper is as follows. In section 2 we introduce the theoretical model. In section 3 we discuss a plausible parametrization and its implications to describe some economic features related to entrepreneurship. In section 4 we simulate a long-run perfect foresight simplified version of the model to see if it is able to describe the empirical growth pattern of countries in the growth frontier for the period 1960-2004. In section 5 we analyze the dynamics implication of the model, in terms of what a technology shock can be, and how it is related to the conventional definition. Finally in section 6 we conclude.

## 2 The Model

We develop here a model that integrates endogenous growth in an otherwise conventional real business cycle model with variable capital utilization. Endogenous growth is based on vertical innovations as in Aghion and Howitt (1998). The precise formulation of endogenous growth is a generalization of Howitt (2000). Final goods producers use labor and a continuum of differentiated intermediate goods as inputs. These intermediate goods differ in their relative productivity and each of them is produced by a monopolistic firm using capital. The amount of capital necessary to produce each intermediate good is proportional to its productivity, thus reflecting that more advanced products require increasingly capital-intensive techniques. Each period, there is a probability that the productivity of an intermediate good jumps to the technology frontier due to the innovation activities of entrepreneurs in each of the sectors. Entrepreneurs borrow resources and invest them in an attempt of increasing their probability of making a discovery. If a discovery happens, the successful entrepreneur introduces a new enhanced intermediate product in her sector and becomes the new incumbent. The entrepreneur will be the new monopolist until the moment another entrepreneur makes a discovery and produces a more advanced intermediate good in her sector. This mechanism of “creative destruction” by which new intermediate goods replace the previous ones is a key difference to endogenous growth models based on horizontal innovations, such as Comin and Gertler (2006).

In this model there are two important spillovers that affect the long-run growth rate. On the one hand, there is a positive intersectoral “technology spillover” since discoveries in one sector provide valuable knowledge tools to innovators in other sectors. On the other hand, there is a negative spillover in the form of a “business-stealing effect” as successful entrepreneurs destroy the surplus
attributable to the previous generation of intermediate goods by making them obsolete.

We first describe final good and intermediate good firms. We next characterize the innovation process by entrepreneurs and productivity dynamics. Then we turn to households, and finally characterize the complete equilibrium.

2.1 Final Goods Output

In the model, a country economy produces a final good under perfect competition by using labor and a continuum of intermediate products, according to the production function

\[ Y_t = L_t^{1-\alpha} \left( \int_0^1 A_{j,t} \varphi_{j,t}^\alpha d\varphi_j \right), \tag{1} \]

where \( \varphi_{j,t} \) is the flow output of intermediate product \( j \in [0, 1] \), and \( A_{j,t} \) is a productivity parameter attached to the latest version of intermediate product \( j \). The model displays decreasing returns to scale in each of the intermediate products. Solving the profit-maximization problem for the final-good firms the price of intermediate goods results in

\[ P_{j,t} = \alpha A_{j,t} L_t^{1-\alpha} \varphi_{j,t}^{\alpha-1}, \tag{2} \]

and the wages

\[ W_t = (1 - \alpha) \frac{Y_t}{L_t}. \tag{3} \]

2.2 Intermediate Goods Firms

Final output can be used interchangeably as a consumption or capital good, or as an input to innovation. Each intermediate product is produced by an incumbent monopolist using capital, according to the production function:

\[ \varphi_{j,t} = \bar{K}_{j,t}/A_{j,t}, \tag{4} \]

where \( \bar{K}_{j,t} \) is the effective capital in sector \( j \). Division by \( A_{j,t} \) indicates that successive vintages of the intermediate product are produces by increasingly capital-intensive techniques. The incumbent monopolist of each sector operates with a price schedule given by (2) and a cost function equal to \( Q_t \bar{K}_{j,t} \), where \( Q_t \) is the rental cost of capital.

All intermediate producers face a marginal cost \( A_{j,t} Q_t \) and marginal revenues \( A_{j,t} \alpha^2 (\varphi_{j,t})^{\alpha-1} L_t^{\alpha-1} \) proportional to \( A_{j,t} \), and therefore they all choose to supply the same amount of intermediate product \( \varphi_t = \varphi_{j,t} = \left( \frac{Q_t}{\alpha^2 L_t^{\alpha-1}} \right)^{1/(\alpha-1)}, \forall j \). The aggregate effective capital in the economy is
\[ K_t = \int_0^1 K_{j,t} dj = \int_0^1 A_{j,t} \varphi_{j,t} dj = \varphi_t \int_0^1 A_{j,t} dj = \varphi_t A_t, \]
where \( A_t = \int_0^1 A_{j,t} dj \) is the average productivity across all sectors. As a result, the aggregate production function of the economy (1) can be reduced to the standard constant returns to scale one \( Y_t = \bar{K}_t^\alpha (A_t L_t)^{1-\alpha} \).

The cost of capital can be expressed as a function of the aggregate level of capital

\[ Q_t = \alpha^2 \bar{K}_t^{\alpha-1} (A_t L_t)^{1-\alpha}, \]

and the flow of profits that each incumbent earns is

\[ \Pi_{j,t} = \alpha (1 - \alpha) Y_t \frac{A_{j,t}}{A_t}. \]

### 2.3 Entrepreneurs

Innovations result from entrepreneurship that uses technological knowledge. At any date there is a "leading-edge technology"

\[ A_t^{\max} \equiv \max \{ A_{j,t} | j \in [0, 1] \}. \]

This technology frontier just represents the most advanced technology across all the sectors.

Each period, the number of successful innovations in a sector \( j \) follows a Bernoulli distribution

\[ P(1 \text{ innovation at time } t \text{ in sector } j) = N_{j,t}. \]

This is the discrete-time version of a Poisson arrival rate of innovations, under the assumption that the probability of two or more successful innovations occurring in a single time period is negligible. The probability \( N_{j,t} \) is a function of the quantity of final output devoted to entrepreneurship in this sector \( X_{j,t} \):

\[ N_{j,t} = F \left( \frac{X_{j,t}}{\lambda_t A_t^{\max}} \right); \quad F(0) = 0, \quad F'(\cdot) \geq 0, \quad F''(\cdot) \leq 0. \]

Equation (9) displays decreasing returns to scale in innovation\(^5\). The parameter \( \lambda_t \) accounts for the productivity of resources devoted to innovation. The amount of resources is adjusted by the technology frontier variable \( A_t^{\max} \) to represent the increasing complexity of progress: as technology advances, the resource cost of further advances increases proportionally. For tractability we assume \( F(x) = \sqrt{2x} \).

Once an innovation happens, it creates an improved version of the existing product by raising its productivity \( A_{j,t} \) to the technology frontier \( A_t^{\max} \). The innovator then enters into Bertrand com-

\(^5\)Previous studies have found decreasing returns in R&D expenditure, such as Kortum (1993).
petition with the previous incumbent in that sector, who by definition produces a good of inferior quality. Rather than facing a price war with a superior rival, the incumbent exits. Having exited, the former incumbent cannot threaten to reenter. Therefore, in $t + 1$ the former entrepreneur has become the new incumbent.

The value of becoming the incumbent in period $t$, $V_{j,t}$, is the discounted flow of profits that it may obtain, taking into account the probability of obsolescence due to the arrival of a new innovation in this sector. We may define $v_{j,t}(A_{t-1}^{\text{max}}) \equiv \frac{V_{j,t}}{A_{t-1}^{\text{max}}}$ so

$$v_{j,t}(A_{t-1}^{\text{max}}) = \alpha (1 - \alpha) \frac{Y_t}{A_t} + \frac{(1 - N_{j,t})}{R_t} E_t \left[ v_{j,t+1}(A_{t-1}^{\text{max}}) \right],$$

(10)

where $R_t$ is the risk-free interest rate. The first term reflects the flow of profits of the monopolist whereas the second term is the discounted value of still being the incumbent at $t + 1$. Since the same amount of input (adjusted by $A_{t}^{\text{max}}$) will be invested in innovation in each intermediate sector because the prospective payoff is the same in each sector, we have $X_{j,t} = X_t$ and $N_{j,t} = N_t$.

We consider that each period there is a single innovator in each sector. She tries to maximize her discounted expected profits $N_tE_t[A_{t+1}^{\text{max}}(A_t^{\text{max}})]$ by investing $X_t$ units of final good subject to the innovation production function (9). The optimal condition governing the level of innovation is that the marginal costs of an extra unit of goods allocated to research equal the discounted marginal expected benefit. Hence we have the research arbitrage equation:

$$1 = \frac{E_t[v_{t+1}(A_{t+1}^{\text{max}})]}{N_t \lambda_t R_t},$$

(11)

and combining (10) and (11) we obtain the innovation equation:

$$N_t \lambda_t R_t = E_t \left[ \alpha (1 - \alpha) \frac{Y_{t+1}}{A_{t+1}} + G_{t+1}(1 - N_{t+1})N_{t+1} \lambda_{t+1} \right],$$

(12)

where $G_t \equiv \frac{A_{t+1}^{\text{max}}}{A_t^{\text{max}}}$ is the growth rate of the leading-edge technology. One possible interpretation of this equation is that the percentage of sectors where a successful innovation appears is inversely proportional to the real interest rates (as they rise the opportunity costs of innovation) and directly proportional to the expected output divided by productivity $\frac{Y_{t+1}}{A_{t+1}}$, as it reflects the adjusted expected profits of becoming the next monopolist. The effect of the efficiency parameter $\lambda_t$ is ambiguous: an increase of $\lambda_t$ tends to depress $N_t$ as it increases the amount of resources necessary to achieve an innovation; however an increase in $\lambda_{t+1}$ raises $N_t$ by making more difficult to entrepreneurs to enter this sector in the future, thus increasing the length of the monopoly period. This is a key feature of
this model in comparison to horizontal innovations models à-la-Romer (1990).

### 2.4 Productivity

The evolution of the average productivity of the economy is given by the number of sectors that experience an innovation:

\[
A_t = \int_0^1 \left[ N_{j,t-1} A_{t-1}^{\max} + (1 - N_{j,t-1}) A_{j,t-1} \right] \, dj = N_{t-1} \left( A_{t-1}^{\max} - A_{t-1} \right) + A_{t-1}, \tag{13}
\]

which describes how the productivity increases due to the distance to the technology frontier \(A_{t-1}^{\max} - A_{t-1}\) multiplied by the entry rate of new firms \(N_{t-1}\) (the percentage of sector where a new incumbent appears).

Growth in the leading-edge parameter \(A_{t}^{\max}\) occurs as a result of the knowledge spillovers produced by innovations. At any moment in time, the technology frontier is available to any successful innovator, and this publicly available knowledge grows at a rate proportional to the aggregate rate of innovations. Therefore we have

\[
G_t = 1 + \sigma N_{t-1}, \tag{14}
\]

where \(\sigma\) is the spillover coefficient.

In section 4 we will test the theory in a multicountry framework. In order to do so, we should specify whether different countries share the same technology frontier or each of them generates its own. Following Howitt (2000) and the literature in technology adoption, such as Parente and Prescott (1993) or Lucas (2009), we consider that there is a world technology frontier, resulting from innovation spillovers in the technology leader. Entrepreneurs in different countries may access to this frontier if they happen to be successful in their (costly) adaptation attempts. Therefore, the model is the same for all countries other than the technology leader, with the particularity that \(G_t\) is exogenous to their economies.

### 2.5 Households

Our formulation of the household sector is reasonably standard. Let \(C_t\) be consumption. Then the household maximizes the present discounted utility as given by:

\[
E_t \sum_{i=0}^{\infty} \beta^i \left[ \log (C_{t+i}) - \mu \frac{L_{t+i}}{1 + s} \right], \tag{15}
\]
with \(0 < \beta < 1\), subject to the budget constraint

\[
C_t + I_t + \frac{B_t}{R_t} = W_tL_t + Q_t\hat{K}_t + (\Pi_t - X_t) + B_{t-1},
\]

(16)

and to the capital accumulation equation

\[
K_t = I_t + (1 - \delta(U_t))K_{t-1},
\]

(17)

where \(I_t\) is investment, \(B_t\) is the amount of (possibly state-contingent) bonds, \(\Pi_t = \int_0^1 \Pi_{j,t} d\gamma\) are the total profits that households receive from the ownership of the monopolist firms and \(K_t\) reflects the total aggregate capital in the economy. Installed and effective capitals are related by the capital utilization rate \(U_t\) so that the effective capital in period \(t\) is the product of the utilization rate and the installed capital at the end of the previous period, \(\hat{K}_t = U_tK_{t-1}\). The household’s decision problem is to choose the consumption, investment, labor supply and capacity utilization to maximize (15) subject to (16) and (17).

The solution of the households’ problem yields the standard Euler equations for the risk-free interest rate and the cost of capital:

\[
1 = E_t \left[ \left( \frac{\beta C_t}{C_{t+1}} \right) R_t \right],
\]

(18)

\[
1 = E_t \left[ \left( \frac{\beta C_t}{C_{t+1}} \right) (Q_{t+1}U_{t+1} + (1 - \delta(U_{t+1})) \right],
\]

(19)

the relationship between the marginal costs of the utilization rate and the cost of capital

\[
Q_t = \delta'(U_t),
\]

(20)

and the relationship of wages with the marginal rate of substitution between consumption and labor

\[
W_t = L_t C_t.
\]

(21)

2.6 Equilibrium

The economy has a symmetric sequence of markets equilibrium. The endogenous state variables are the aggregate capital stock \(K_t\), the productivity level \(A_t\) and the entry rate of new firms in the economy \(N_t\). The final goods market is in equilibrium if production equals demand for consumption, capital accumulation and entrepreneurship \(Y_t = C_t + I_t + X_t\). The capital rental market is in equilib-
rium when the demand for capital by intermediate good producers equals the supply by households. The labor market is in equilibrium if firms’ demand for labor equals labor supply by households.

The model has a deterministic steady state that displays a balanced-growth path, where variables $Y_t$, $C_t$, $I_t$, $X_t$, $A_t$, $A_t^{\text{max}}$, $W_t$, $\Pi_t$, $K_t$ and $\bar{K}_t$ grow at a rate $G = 1 + \sigma N$ (where $N$ is the steady-state value of $N_t$), whereas $U_t$, $N_t$, $Q_t$, $L_t$, and $G_t$ are stationary. To make all the variables stationary we divide $Y_t$, $C_t$, $I_t$, $X_t$, $A_t$, $W_t$, $\Pi_t$, $K_t$ and $\bar{K}_t$ by $A_t^{\text{max}}$.

3 Model Parametrization

In this section we explore the plausibility of different parameter values in order to describe diverse features of the economy. This parametrization is meant as a benchmark, as we have found our results to be robust to reasonable variations around this benchmark. In order to do so, we first log-linearize the model around its deterministic steady state and we present the results in the Appendix A. Then we parameterize the model with data from the United States. To the possible extent, we use the restrictions of balanced growth to pin down parameter values. Otherwise, we look for evidence in the literature and comment the implications of different assumptions.

3.1 General Economy

We first determine the values of the set of parameters $\alpha$, $\beta$, $\delta$, $\zeta$ and $\xi$ by using aggregate magnitudes of the United States economy and typical parameters in the literature. As we have not included any population growth in the model, we assume that all the magnitudes are expressed in terms of the working-age population. Variables $Y_t$ and $I_t$ are assumed to describe the real gross domestic product and the real private fixed investment divided by the civilian noninstituional population. $L_t$ is the product of the average weekly number of hours per employee, expressed in percentage of a 40-hour week, multiplied by the civilian employment and divided by the civilian noninstitutional population. By considering this definition of labour, coupled with equation (1), we avoid the sort of scale effects commented in Jones (1995). All the variables are annual as opposed to quarterly as the focus of the model is in fluctuations over a longer horizon than is typically studied in business cycle research.

In steady state, the growth rate of the economy $G$ is set to 1.02, which is the average growth of $Y_t$. Therefore we set $\beta = \frac{G}{R} = 0.99$ so the steady state real interest rate is 3%, as in Christiano, Eichenbaum and Evans (2005) ($R = 1.03$). The steady-state depreciation rate $\delta \equiv \delta(U)$ is set to 0.1, as is standard in the literature. By setting $\alpha = 0.36$, we obtain a steady state investment share

6If not indicated otherwise, all data come from the Bureau of Economic Analysis. Population is the civilian noninstitutional population older than 16.
I \equiv \frac{\alpha^2 (G - (1-\delta))}{(R - (1-\delta))} \text{ of 12\% and a labor share } WL \text{ of 64\%, which are consistent with their historical averages in the post-war period. Finally we set the inverse of the Frisch elasticity of labor } \varsigma \text{ at unity and the elasticity of the change in the depreciation rate with respect to the utilization rate } \xi \equiv \frac{U''(U)}{\delta'(U)} \text{ at 0.33, as intermediate values for the range of estimates across the micro and macro literature and the same values as in other studies, such as in Comin and Gertler (2006).}

3.2 Entrepreneurship

To pin down } N, \text{ or equivalently } N, \text{ we need to include information related to entrepreneurship in the United States. As variable } N_t \text{ is the entry rate of new firms in the economy, corrected by the growth in the working-age population, we may employ aggregate data on firm turnover. Using data from the U.S. Small Business Administration, the value of } N \text{ is around 9\% for the period 1990-2003. This value is higher than the one considered in Howitt (2000), who sets } N \text{ as 3.6\%, based on empirical evidence from Caballero and Jaffe (1993) who estimated that the average U.S. company that does not innovate loses value at a 3.6\%. To obtain a value of } N = 0.09 \text{ we set } \lambda = 13 \text{ and then we determine the value of the spillover coefficient } \sigma \text{ that is consistent with long-run growth } G, \sigma = 0.22.

Figure 1 shows the survivor function implied by this value of } N, \text{ i.e. the percentage of firms of a given cohort that are still alive after a number of years. Here we compare the results of the model for } N \text{ equal to 9\% and 3.6\% with empirical data for the 1963 and 1976 cohorts of U.S. manufacturing firms obtained from Dunne, Roberts and Samuelson (1988) and Audretsch (1991), respectively. Results show that the value of 9\% provides a better approximation to the U.S. data than 3.6\%.

4 Implications for Economic Growth

In the previous section we have shown that the model seems to be able to describe some of the long run features of the US economy, such as the investment share, the entry rate of new firms and the survival rates. In this section we test whether its long-run growth path is consistent with the empirical evidence across countries. To do so, we work with a simplified version, stripped off of some business cycles features. We simulate the model under the assumptions of perfect foresight and no stochastic shocks for different initial values out of the steady state, and compare it with the historical data for a broad set of countries.

In figure 2 we show the relationship between initial income and average growth rate for a set of countries for the period 1960-2004. The countries have been chosen to represent a considerable
Figure 1: Survival Rates

![Survival Rates Graph]

Figure 2: Income and Growth Rates, 31 Countries

![Income and Growth Rates Graph]
share of the World’s GDP and population, and they constitute a broad sample of developed and emerging economies. Our selection includes 31 countries, but the growth rate distribution is consistent with studies with a higher number of countries, such as Lucas (2009). The GDP is divided by the population aged 15-64, and the initial income is normalized so that the technology leader’s income in 1960 is 1.\footnote{Countries are: Algeria, Argentina, Australia, Austria, Bangladesh, Belgium, Brazil, Chile, China, Colombia, Egypt, France, Hong Kong, China, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Nigeria, Pakistan, Philippines, South Africa, Spain, Sweden, Thailand, United Kingdom, United States and Venezuela. GDP data is in PPP and comes from the World Bank and population figures from the United Nations.}

The triangular pattern in the data scatter plot is quite significative. The rich countries -mainly from Europe, North America and Japan- have had growth rates close to 2 percent. The poorest countries -mainly in Africa, Asia and Latin America- show extreme variety in growth rates, ranging from the miraculous growth of South Korea or Hong Kong to the stagnation and even negative growth of others such as Venezuela or South Africa. We define the growth frontier as the maximum average growth rate that can be achieved given the initial income.

Previous literature in convergence has analyzed why some countries successfully engage in the convergence process, whereas others do not. Mankiw, Romer and Weil (1992) have shown that the neoclassical model can explain the convergence process if it is augmented to include human capital. Howitt (2000) presents a Schumpeterian model that is able to offer an explanation for convergence rates and productivity differences. His model implies that all countries that successfully engage in technology adaptation efforts will converge in growth rates to the one of the technology leader. Convergence is restricted however to this select group of countries. Those in which there is not a strong enough incentive to entrepreneurship will not grow at all in the long run, a phenomenon defined by Quah (1996) as “club convergence”. Differences in the incentives to innovate may be due to the existence of “barriers”, as commented in Parente and Prescott (1993), such as regulatory and legal constraints, bribes that must be paid, violence, sabotage or worker strikes.

Additionally, empirical research by Hall and Jones (1999) and others shows that productivity differences are substantially even among advanced countries, a feature that has no answer in the neoclassical level. The Schumpeterian model may account for them: for a given country \(i\), the steady state average distance to the technology frontier is given by

\[
\frac{A^i}{A_{\text{max}}} = \frac{N^i}{N^i + G - 1},
\]

where \(N^i\) is the steady state probability of a successful innovation in country \(i\). Therefore, the lower the value \(N^i\), the lower the country’s productivity will be related to the leader, even if the country
has successfully converged to its steady state and it is growing at the leader’s growth rate.

4.1 Growth Frontier

Does the Schumpeterian model allow us to quantitatively replicate the pattern of the growth frontier found in the data? To test it, we should simulate the model for different initial incomes and see which are the associate average growth rates. To find a solution for the long-run evolution of the countries, we applied the adapted version of the log-linearize model in the Appendix A. We make some useful simplifications in a number of dimensions. Firstly, we consider that labor supply is inelastic, thus abstracting from any form of participation or unemployment. Secondly we assume that capital utilization is constant. Thirdly we consider the deterministic case when there are no exogenous stochastic shocks of any kind.

We solve the model by a standard algorithm, such as the Blanchard and Kahn (1980) method. In general, the solution of dynamic stochastic general equilibrium models can be expressed as $z_t = P(\theta)z_{t-1} + Q(\theta)\epsilon_t$ where $z_t$ is a vector of the endogenous variables of the model, $\epsilon_t$ is a vector of stochastic innovations and $P(\theta)$ and $Q(\theta)$ are transition matrices whose elements are (nonlinear) functions of the structural parameters $\theta$. In the deterministic case, we may approximate the solution as $z_t = P(\theta)z_{t-1}$. Therefore, once we set the vector of structural parameters $\theta = \{\alpha, \beta, \delta, \lambda, \sigma\}$, we should just provide initial values to the state variables $\{A_0, K_0, N_0\}$ to simulate the evolution of a country over its growth path. We simulate the model for different initial values for a period of 44 years that is assumed to represent the period 1960-2004, and compute the average growth rate of the GDP for each growth path.

In Figure 3 we show the results of simulating the model presented in this paper for the same parametrization discussed in section 3, that is, $G$ set to 1.02, $\beta$ to 0.99, $\delta$ to 0.1 and $\alpha = 0.36$. We set $\lambda$ to consider the values of $N$ of 9% and 3.6%. By proceeding like this, we are assuming that countries in the growth frontier share the same structural parameters as the leader, with the possible exception of the barriers to entrepreneurship, which seems a plausible assumption. The difference in the value of $\lambda$ may reflect different types of barriers between the leader and the followers. The results show how the model is able to correctly reproduce the post-war growth frontier in the case of $N = 3.6\%$. This may suggest that most of the follower countries have faced during this period higher marginal innovation costs than the U.S.

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8We have employed the Dynare package to numerically solve the model.

9An important limitation of this approach is the approximation error due to the linearization incurred when the initial state is very far from the steady state. However, this approach is equivalent to the growth regressions in Mankiw, Romer and Weil (1992) or Howitt (2000) so we consider it a valid first order approximation.

10If a country large enough would have had a set of structural parameters that produce a steady state productivity level higher than the U.S. one, it should have become the new technology leader, something that is refuted by the data.
Table 1. Parameter Values

<table>
<thead>
<tr>
<th>α</th>
<th>β</th>
<th>δ</th>
<th>G</th>
<th>N</th>
</tr>
</thead>
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<tr>
<td>0.36</td>
<td>0.99</td>
<td>0.10</td>
<td>1.02</td>
<td>0.09/0.036</td>
</tr>
</tbody>
</table>

5 Technology and Business Cycles

Assuming that the proposed model provides a superior description of long-term technology dynamics than the standard neoclassical one, which are its medium-term implications? In this section we define what is a technology shock in the proposed model and compare it with the standard definition in the context of the real business cycle (and new Keynesian) literature.

5.1 Technology Shocks

Since the seminal work of Kydland and Prescott (1982), the literature has defined aggregate technology shocks as exogenous changes in the productivity level of a country. In the context of a standard RBC model the standard assumption is to consider that final output is given by a production function \( Y_t = \bar{K}_t^\alpha (A_t e^{\mu_t} L_t)^{1-\alpha} \) where \( \mu_t \) is a transitory technology shock that follows an AR(1) process with innovations \( \varepsilon_t^\mu \). \( A_t \) is assumed to grow exogenously at a rate \( G \). The innovations in \( \mu_t \), \( \varepsilon_t^\mu \) are
assumed to be independent normally distributed i.i.d. process with mean zero and variance unity.\textsuperscript{11}

In the Schumpeterian model, a technology shock can be defined as any perturbation that changes the slope $\lambda_t$ of the entrepreneurs production function (9). They are modeled as $\dot{\lambda}_t = \rho^{\lambda}_{t-1} \dot{\lambda} + \sigma^\lambda \varepsilon^\lambda_t$, with $\dot{\lambda}_t = \log(\lambda_t/\lambda)$. When a positive technology shock arrives, entrepreneurs face lower cost to achieve the same probability of success. It reduces the cost for a given success probability, but at the same time it may reduce the expected profits, as more innovation can also be done in the future, thus shortening the length of the monopolistic period.

In figure 4 we compare the impulse responses of a real business cycle model with those from the Schumpeterian model. The period length is a year. The parametrization is similar as the one presented in section 3.1. The value of the persistence of the technology shock $\rho^\mu$ is taken to be 0.8, equivalent to a quarterly 0.95, a typical value in the literature. The persistence of the shock $\rho^\lambda$ is set to 0.1.

<table>
<thead>
<tr>
<th>Table 2. Parameter Values</th>
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<tbody>
<tr>
<td>$\alpha$</td>
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<tr>
<td>Schumpeterian</td>
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<tr>
<td>RBC</td>
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When a positive technology shock arrives in a RBC model, it temporary raises the aggregate productivity. This increase pushes up both wages and capital returns, and therefore households decide to increase their investment, capital utilization and labor supply. As the effects of the shock disappear, the endogenous variables return to their steady state values. Cogley and Nason (1995) showed how the persistence of the aggregate variables of figure 4 is just that of the technology shock, that is, the persistence mechanisms of the model are very weak.

In contrast, in the Schumpeterian case, when a positive temporary technology shocks arrives entrepreneurs face lower costs in the present; but they expect them to rise later thus reducing the probability of new successful innovations in the future. As a consequence, entrepreneurs decide to invest more resources in technology adoption, which temporary reduces investment and consumption and increases the average entry rate of new firms in the economy. The more active the innovation process is, the higher the average productivity increases in the medium term as a result of both the direct effect (old firms are replaced by more efficient new ones) and the indirect one (the technology frontier increases faster due to technology spillovers). This is not a temporary change in the TFP, but a permanent one, as the new firms will remain in the economy until they will be replaced by a (more productive) new vintage. This is the key difference with the traditional transitory shocks, and

the source of the strong persistence displayed in the simulation despite the much lower persistence of the shock. Due to the increase in resources devoted to technology adoption, capital investment is therefore reduced during the first years after the arrival of a new shock and capital utilization rises. After a few years, the amount of innovation resources returns to its steady state value and the positive effects of the rise in the TFP begin to steadily increase wages and output. Households then reduce their labor supply meanwhile consumption and investment reach new and higher levels.

6 Summary and Conclusions

By assuming that productivity changes happen exogenously, macroeconomists have ignored an important link between growth and business cycles. Key economic issues such as innovation or entrepreneurship play little roles in most of the models as they simple assume that productivity follows a random walk with drift, independently of the state of other economic variables. In contrast to this view, the work by Comin and Gertler (2006) has shown how endogenous technology adoption may generate persistent fluctuations in output and productivity due to non-technological shocks. Their results seem to be confirmed by the existence of “medium-term business cycles” or correlations between high and medium frequency components of the economic time-series.
In this paper we have shown how endogenous technology adoption provides a better description of the productivity growth process than the standard exogenous one. In the first place, we show how the model is able to replicate the shape of the growth frontier, a complementary result to those of Howitt (2000) who has already showed how the model was superior to the Solow one in explaining the convergence rates, per capita incomes and existence of convergence clubs. In the second place, we also show how the model is able to generate high persistence in the dynamics of macrovariable even in the presence of low-persistence technology shocks, a result in line in the ones of Phillips and Wrase (2006).

Finally, by explicitly modelling the productivity growth process, this model may be a first step to analyze other interesting issues such as the relation between output growth and volatility or how nominal frictions affect output.

References


A Appendix: The Complete Log-Linearized Model

A.1 General Case

Let lower case variables denote percent deviations from the steady state, and let ratios of capital letters without time subscript denotes the steady state value of the respective ratio. It is convenient to express the complete loglinearized model in terms of four blocks of equations: (1) aggregate demand; (2) aggregate supply; (3) evolution of the state variables; and (4) shock processes.

**Aggregate Demand**

\[ c_t = E_t [c_{t+1} + g_{t+1}] - r_t \]  \hspace{1cm} (23)

\[ r_t = (1 - (1 - \delta)/R) E_t [q_{t+1}] \]  \hspace{1cm} (24)

\[ q_t = \xi u_t \]  \hspace{1cm} (25)

\[ q_t = y_t + g_t - u_t - k_{t-1} \]  \hspace{1cm} (26)

\[ y_t = \frac{I}{Y} (i_t - c_t) + \frac{X}{Y} (2n_t - c_t + \lambda_t) \]  \hspace{1cm} (27)

**Aggregate Supply**

\[ y_t = (1 - \alpha) (l_t + a_t) + \alpha (u_t + k_{t-1} - g_t) \]  \hspace{1cm} (28)

\[ y_t = (1 + \zeta) l_t + c_t \]  \hspace{1cm} (29)

\[ g_t = \frac{G - 1}{G} n_{t-1} \]  \hspace{1cm} (30)

**Evolution of State Variables**

\[ a_t = (1 - N) a_{t-1} \]  \hspace{1cm} (31)

\[ k_t = \frac{\delta + G - 1}{G} u_t + \frac{1 - \delta}{G} (k_{t-1} + g_t) + \frac{R + \delta - 1}{G} u_t \]  \hspace{1cm} (32)

\[ n_t = -r_t - \dot{\lambda}_t + (1 - 2N) \frac{G}{R} E_t [n_{t+1}] + \frac{(1 - N)G}{R} E_t [\dot{\lambda}_{t+1} + g_{t+1}] + \frac{(R - 1 + N)}{R} E_t [y_{t+1} - a_{t+1}] \]  \hspace{1cm} (33)

**Shock Process**

\[ \dot{\lambda}_t = \rho \lambda \dot{\lambda}_{t-1} + \sigma \varepsilon_t^{\lambda} \]  \hspace{1cm} (34)
with $G = 1 + \sigma N$, $R = \frac{G}{\beta}$, $\delta = \delta(U)$, $\xi = \frac{U^2(U)}{\sigma(U)}$, $I = \frac{a^2(G-(1-\delta))}{(R-(1-\delta))}$, $X = \frac{a(1-\alpha)N(1+\sigma)}{2(R-G(1-N))}$. $\varepsilon_t^\lambda$ is a normally distributed i.i.d. process with mean zero and variance unity.

### A.2 Follower Countries

In the case of technology adoption by follower countries presented in section 3, we should set $u_t$, $l_t$ and $g_t$ as zero. Besides, neither equations (25),(30) nor (34) do apply and the evolution of the distance to the technology frontier (equation 31) is given by $a_t = \frac{(1-N)}{G}a_{t-1} + \frac{G-1}{G}a_{t-1}$. The rest of equations are the same.