Estimation of a Time Varying Natural Interest Rate for Peru*

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Abstract

Following the approach of Mésonnier and Renne (2007), we estimate a Natural Interest Rate (NIR) using quarterly Peruvian data for the period 1996:3-2007:2. The model has six equations and it is estimated using the Kalman filter where the output gap and the NRI are treated as unobservable variables. The results indicate a relatively more stable NRI for the period 2001:3-2007:2 in comparison with the period 1996:3-2001:2. In comparison with the observed real interest rate, the NRI is more stable. The gap of the real interest rate, which measure the stance of the monetary policy, indicates a restrictive monetary policy for the periods 1996-2001 and 2003. The results suggest an expansive monetary policy for the periods 2002 and 2004-2007. Finally, we estimate the NRI using simple approaches as Hodrick and Prescott (1997), Baxter and King (1999), Christiano and Fitzgerald (2003). The results indicate strong differences between these estimates and our results.

Keywords: Interest Rate, Natural Interest Rate, Kalman Filter, Output Gap, Unobservable Components.

JEL: C32, E32, E43, E52.

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1 Introduction

The natural real rate of interest (hereafter NRI) is defined as the real short-term rate of interest of equilibrium or the neutral rate of interest. In more formal terms, the NRI is the real short-term interest rate consistent with the output at its potential level and with a stable inflation rate. Historically, the concept of a natural real rate of interest and its use for monetary policy is associated with Wicksell (1898, 1907). In recent years, the concept has revived due to the neo-wicksellian framework for monetary policy analysis advocated by Woodford (2003).

An important measure derived from the NRI is the real interest rate gap (IRG). It is calculated as the difference between the real short-term interest rate and the NRI. This indicator is a relevant candidate for assessing the current monetary policy stance. It is also a powerful reason why the central banks and central banks economists have dedicated a lot of attention to the theoretical developments and empirical strategies of estimation related to the NRI and IRG. Examples at this respect are Archibald and Hunter (2001), Christensen (2002), Williams (2003), Neiss and Nelson (2003), ECB (2004), Crespo-Cuaresma et al. (2004).

There is an enormous literature concerning the modelling and estimation of the NRI; see Giammarioli and Valla (2004) for an excellent survey. Two characteristics may be used to distinguish between the extended literature. The first concerns whether the focus of the model is on the short-term or medium to long-run implications of a nonzero gap. The second characteristic is related to the degree of structure put into the models to obtain the estimates of the NRI.

The first block of the literature follows the lines of Woodford (2003) and Neiss and Nelson (2003). The estimate of the NRI is obtained within the framework of a microfounded new Keynesian model, the so called dynamic stochastic general equilibrium (DSGE) models. In this context, the NRI equals the equilibrium real rate of return in an economy with full flexible prices. In other words, the NRI is the real short-term interest rate that equates aggregate demand and potential output for all periods. Interesting empirical applications for the Euro area are Giammarioli and Valla (2003),
Smets and Wouters (2003).

The second block of the literature follows Laubach and Williams (2003). In this kind of approach the procedure is to mix simple macroeconomic models belonging to the monetary policy literature with the use of the Kalman filter in order to estimate the NRI, the natural rate of unemployment and/or the potential level of the output as unobserved variables. This kind models are named semi-structural models. In this context, the NRI is the real short-term rate of interest consistent with output at its potential level and inflation stable in the medium run. It means that the effects of demand shocks on the output gap and supply shocks on inflation have completely vanished. Examples in this approach are Orphanides and Williams (2002), Crespo-Cuaresma, Gnan, and Ritzberger-Gründewald (2004), Basdevant, Björksten, and Karagedikli (2004), Larsen and McKeown (2004) and Garnier and Wilhelmsen (2005).

There are, of course, other simple ways to estimate the NRI as for example the application of statistical filters. Some of the most used are Hodrick and Prescott (1997), Baxter and King (1995) and Christiano and Fitzgerald (2003). However, these approaches are criticized because there are no economic support to the estimations. As Larsen and McKeown (2004) and Mésonnier and Renne (2007) suggest, the approaches belonging to the second brand of the literature represent a convenient compromise between the costly DSGE approach and the approaches purely statistic.

Another important ingredient is the stability of the NRI. The available empirical evidence points to the plausibility of significant time variation in the NRI in many economies. For example Rapach and Wohar (2005) find evidence of multiple structural breaks in the mean of real interest rates over the last four decades in 13 industrialized countries. It recommends the use of estimation methods that allow for large persistent fluctuations in the NRI.

We thus follow the approach suggested by Mésonnier and Renne (2007) which is derived from Laubach and Williams (2003). The approach of Mésonnier and Renne (2007) has two advantages respect to the way suggested by Laubach and Williams (2003). First, unlike Laubach and Williams (2007), we allow for stationarity (but highly persistency) of the unobservable component that drives the low-frequency common fluctuations of the
NRI and the potential output growth. It is advantageous because assuming nonstationarity in the output growth and the NRI, as in Laubach and Williams (2003), enters in contradiction with economic theory and intuition. Second, the real interest rate is calculated as a model-consistent ex ante real rate of interest using inflation expectations provided by the model. Other studies, including Laubach and Williams (2003) use inflation expectations using univariate autoregressive models.

We apply the recommended approach of Mésonnier and Renne (2007) to quarterly Peruvian data for the period 1996:3-2007:2. The results are relatively sensible to the calibration of two parameters. However, in most cases the estimates of the NRI are very stable. The gap on the real interest rate indicates a restrictive monetary policy for the periods 1996-2002 and 2003. The monetary policy appears to be relatively expansive for the periods 2004-2007. The behavior of the gap is very stable in the second period.

This paper is organized as follows. In Section 2, the model is described. Section 3 briefly describes the data and deals with the econometric results. Section 4 concludes.

2 The Model


The rationale for the dynamic of the NRI follows from the basic optimal growth model. In this model, the intertemporal utility maximization yields a log-linear relationship between the real interest rate \( r^* \) and the rate of labor-
augmenting technological change $a$ which is also the rate of growth of per capita output along a balanced-growth path. This relationship is expressed as $r^* = \theta a + \rho$, where $\theta$ is the constant relative risk aversion (the inverse of the intertemporal elasticity of substitution) and $\rho$ is the rate of time preference of the household. It is possible to assume that the trend growth rate $A_t$ is subject to low-frequency fluctuations. If it is true, it is possible to find a link between long run fluctuations in the growth rate of potential output and the NRI. In this sense, this approach is located between those of Laubach and Williams (2003) and Orphanides and Williams (2002). In the former, the NRI is the sum of the trend growth rate and a second nonstationary component. The first component drives the low-frequency fluctuations of potential output growth rate. In the latter study, the NRI and potential output growth are completely unrelated which is difficult to conciliate with theoretical intuition$^1$.

The model consists of the following six equations:

\begin{align}
\pi_t &= \alpha(L)\pi_t + \beta(L)z_t + \epsilon^\pi_t, \\
z_t &= \phi(L)z_t + \lambda(L)(i_t - \pi_{t+1} - r^*_t) + \epsilon^z_t, \\
r^*_t &= \mu_r + \theta a_t, \\
\Delta y^*_t &= \mu_y + a_t + \epsilon^y_t, \\
a_t &= \psi a_{t-1} + \epsilon^a_t, \\
y_t &= y^*_t + z_t,
\end{align}

where the four shocks are independently and normally distributed with variances $\sigma^2_\pi$, $\sigma^2_z$, $\sigma^2_y$ and $\sigma^2_a$.

The first equation may be interpreted as an aggregate supply equation or “Phillips curve”. It specifies that consumer price inflation is related to its own lags and lags of the output gap. The second equation is reduced form of an aggregate demand equation, or “IS curve”, relating the output gap to its own lags and lags of the IRG. Stable inflation is consistent with a zero output gap and zero interest rate gap. In this sense, the NRI may be named a nonaccelerating-inflation rate of interest. In this model, the monetary policy affects inflation rate towards the output gap. Furthermore, the nominal

$^1$It may result in a nonoptimal exploitiation of the data.
short-term interest rate is assumed to be exogenous which implies that the reaction function is implicit.

In the literature a common specification for the NRI is a random walk\(^2\). In the present approach, the NRI is assumed to follow a highly autoregressive process as specified by (4) and (6)\(^3\). Even when the random walk assumption may be advantageous from some perspective\(^4\), it hinders the economic interpretation of the model. It is true if in particular we assume that potential growth (\(\Delta y_t^*\)) shares common fluctuations with \(r_t^*\). Estimates (see next section) show that this process is highly persistent which is consistent with the purpose of capturing large and low-frequency fluctuations in the level of the equilibrium real rate.

The equation (6) is the autoregressive representation for \(a_t\). It captures low-frequency variations in potential output growth assuming that these variations are common with those of the NRI. Notice that equation (5) specifies the behavior of the potential output growth. It says that potential output growth has another stationary component which may account for other sources of discrepancies with the NRI (shocks to preferences, changes in fiscal policies, for example). A simple white noise insufficient to model this second stationary component.

All specifications are consistent with the hypothesis that potential output is an I(1) process. Application of simple unit root tests reject the null hypothesis of an I(2) log real output.

\(^2\)Nonstationarity is also specified for the growth rate of potential output. Some examples are Laubach and Williams (2003), Orphanides and Williams (2002), Larsen and McKeown (2004), and Fabiani and Mestre (2001).

\(^3\)Another exception in the stationary specification of the NRI is Gerlach and Smets (1999). Furthermore, they assume that potential output is I(1).

\(^4\)The advantage is to combine persistent changes in the unobservable component with smooth accommodation of plausible but unspecified structural breaks in the effective interest rate series over the period of estimation.

\(^5\)A nonstationary specification for the NRI and then potential output growth would indeed imply that potential output is integrated of order two. In terms of the standard optimal growth model, it would mean a nonstationary path of output to the stock of capital.
3 Results

Peruvian quarterly data for the period 1996:3-2007:2 is used in the estimations. Inflation rate is defined as the annualized quarterly growth rate of the CPI series. The ex ante real short-term rate of interest is obtained by deducting from the current level of the nominal rate of interest the one-quarter-ahead expectation of the (quarterly annualized) inflation rate. The data set is complete with the log of the real GDP. All variables have been seasonal adjusted using the procedure Tramo-Seats of Gómez and Maravall (1999).

The six-equations model is written in the state-space form, and the parameters are estimated by maximization of the likelihood function provided by the Kalman filter. The filter is a recursive algorithm for sequentially updating a linear projection for a dynamic system. Given a set of measurement and transition equations, the Kalman filter provides the best linear unbiased estimate of the state variables. A particular feature of this approach is its ability to quantify uncertainty around the estimated state variables. In this sense, a filtered estimate of the state variables uses information only up to time $t$, whereas a smoothed estimate uses information from the whole sample, that is, up to time $T$. The former is frequently named a one-sided estimate whereas the second is a two-sided estimated.

Direct estimation (without restrictions) by maximum likelihood arises two difficulties. The first difficulty is the estimation of the parameter $\theta$. Unconstrained estimation of this parameter appears to be very instable and not statistically significant\(^6\). This parameter links two unobservable variables which may do its estimation ambitious if we consider the sample size used in the estimations. The second difficulty found in the unconstrained estimation is the zero estimated of $\sigma_y$. In some cases a zero estimated is also found for the parameter $\sigma_z$. It implies that the idiosyncratic shocks to the output are not possible to be distingued from the transitory shocks to the

\(^6\)Similar difficulties have been found by Larsen and McKeown (2004) applying the methodology of Laubanch and Williams (2003) to UK data. Because they interpret the problem as a dimensionality issue, they decide to reduce the number of parameter using a calibration similar to Mésonnier and Renne (2007) which is also applied here. Further instability determine to calibrate another parameter.
output. It is not surprising if we think in the high persistence of the output gap.

In order to face these difficulties, two calibrations are used. The first one is the calibration of the ratio $\sigma_y/\sigma_z$. Basis to calibrate this ratio is difficult to find. Even for the cases of the US and EU economies, the evidence does not suggest a basis for a consensus calibration. Fabiani and Mestre (2004) find a ratio of 0.94 for their Euro area model. Peersman and Smets (1999) find a value of 0.42 for a model including five countries of the EU. For the US some estimates are due to Peersman and Smets (1999), Smets (2002) and Laubachs and Williams (2003). The range of values is 1.7-3.3.

The second calibration is the parameter $\theta$. Reasonable values for this parameters should be consistent with the order of magnitude of empirical estimate of the inverse of the intertemporal elasticities of substitution found in the literature. Hall (1988) find a small and not statistically different from zero. It corresponds to an infinite risk aversion coefficient. Other estimates of the intertemporal elasticity of substitution (ranging from 0.27 to 0.77) are due to Ogaki and Reinhart (1998). For Peru, using a stochastic dynamic equilibrium model, Castillo, Montoro and Tuesta (2007) find a value of 4.00 for the risk aversion coefficient. Given the precedent discussion, we consider the range [0, 20] as a reasonable interval for plausible candidate values of the risk aversion parameter $\theta$.

The equations (1)-(2) need to select lag length. Based on the statistical significance, the equation (1) uses three lags of the inflation and one lag of the output gap. In the equation (2), one lag of the output gap and the second lag of the gap between the interest rate have been selected. Furthermore, the null hypothesis that the coefficients of the inflation sum unity is not rejected. Therefore, we impose this condition implying that an accelarionist form of the Phillips curve is adopted. In other words, inflation depends only of nominal factors in the long run.

Table 1 presents different scenarios of estimation according to different values of the calibrated parameters. Last column presents the unrestricted

\footnote{Castillo, Montoro and Tuesta (2007) consider habits in the utility function. Using a habit coefficient of 0.75, the intertemporal elasticity of substitution corresponds to 0.25. Therefore, it implies a risk aversion coefficient of 4 in quarter terms.}
estimates. The results related to the values $\theta = 1$ and $\sigma_y/\sigma_z = 0.5$ appears to be the preferred scenario.

Figure 1 shows the output gap (with 90% confidence intervals), the NRI, the NRI versus the observed real short-term interest rate and the gap of the NRI (with confidence bands). The picture of the output gap indicates excess of supply until 2000-2001. Unlike it, in recent years, the output gap appears close to zero with wide confidence regions. The estimation of the NRI shows narrow bands indicating precise estimation. In comparison with the observed real short-term interest rate, the NRI appears very stable. The gap of the interest rate indicates a restrictive monetary policy for the periods 1996-2001 and 2003. A slight expansive monetary policy is observed for the periods 2002 and 2004-2007.

Another interesting issue is the fact that the estimates of the NRI and the gap of the interest rate appears to be very stable and similar among the different scenarios of estimation.

Other methods to estimate the NRI are available. The simplest way to calculate the NRI is to use statistical procedures as the Hodrick and Prescott (1997) filter, the Baxter and King (1999) filter, and the Christiano and Fitzgerald (2003) filter. Other alternatives is to use the unobserved component model of Clark (1987) or the space-state representation suggested by Kim (2001). Estimates using these methods have been obtained and compared with the estimation obtained from our model. The striking result is the negative correlation between our estimates and the statistical estimates of the NRI. The only positive correlation (0.419) is obtained with the method of Kim (2001). This result is important because it indicate strong differences between simple methods and our estimates.

The correlations are positive if we compare our estimates of the gap of the interest rate obtained from our model versus those obtained from the statistical methods. Surprisingly the filter of Hodrick and Prescott (1997) presents the highest correlation (0.753). It is surprising because the correlation is negative (-0.537) between its estimate of NRI and our estimate of NRI. Therefore, considering only correlation of the gap of interest rate could be misleading if we do not also see the correlation of the NRI. The correlation of the gap of the interest rate obtained using Kim (2001) and our
estimate is 0.734. This method appears to be the only rival to our estimates.

4 Conclusions

This paper uses a semi-structural model to estimate the NRI using Peruvian data for the period 1996:3-2007:2. A model without restrictions has been estimated. Some scenarios with two calibrated parameters are also estimated. In comparison with the behavior of the real short-term interest rate, the NRI is very stable. It is true for all considered scenarios. The gap of the interest rate is also stable and it describes the periods of restrictive and expansive monetary policy. Other statistical procedures to estimate the NRI have been used to compare with those from our model. The results indicate strong differences between both set of estimates. It suggest that care should be taken when we use simple statistical procedures to estimate the NRI or the gap of the interest rate.

References


Table 1. Parameter Estimates

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<th>(\theta = 16)</th>
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Estimados usando $\sigma_y/\sigma_z = 0.5$ y $\theta = 1$
Estimados usando $\sigma_y/\sigma_z = 0.5$ y $\theta = 4$
Estimados usando $\sigma_y/\sigma_z = 1$ y $\theta = 1$
Estimados sin restricciones