Optimal Fiscal Policy in a Small Open Economy with Incomplete Markets and Interest Rate Shocks

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Abstract

This paper studies optimal fiscal policy in a small open economy model under incomplete financial markets, where interest rates, government spending and productivity are stochastic and taxes are distortionary. The contributions of the paper are twofold. First, I solve the Ramsey problem and characterize the properties of the optimal fiscal policy. Second, I show that the optimal fiscal policy smooths distortions over time. The tax rate and specially the public debt are very persistent irrespective of the degree of autocorrelation of the assumed processes for the shocks generating aggregate fluctuations. The government finances an increase in government spending or a decrease in the tax base partly by increasing debt and partly by increasing the tax rate. This reflects the government’s desire to smooth tax distortions over time.

Keywords: optimal fiscal policy, taxation, incomplete markets, debt policy.

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1 Introduction

In this paper, I analyze the properties of optimal fiscal policy in a small open economy where interest rates, government spending and productivity are stochastic, taxes are distortionary and markets are incomplete. This paper provides a new framework to analyze optimal fiscal policy in small open emerging economies. The present paper also contributes to the literature on optimal fiscal policy by studying the case of incomplete markets in a small open economy under uncertainty and distortionary taxation. Small open emerging economies typically face frequent and large fluctuations in interest rates, productivity and government spending. These economies differ from small open developed economies in several aspects: For example, output and consumption volatilities are higher in emerging economies than in developed economies. Also, as pointed out by Neumeyer and Perri (2005), interest rates are countercyclical in these economies, while in developed economies interest rates are acyclical.

In particular, it is important to answer the following questions: What are the properties of optimal fiscal policy in a small open emerging economy under incomplete markets and distortionary taxation? How should the tax rate and the level of public debt adjust to an innovation in government spending or productivity in a small open economy? How should the tax rate and the level of public debt adjust to an innovation in the interest rate in a small open economy? In this paper, I analyze these questions using a stochastic dynamic general equilibrium model of a small open economy with incomplete markets. I consider it is important to answer these questions, because, in my view, a model with incomplete markets captures better the financial environment faced by emerging economies than the
complete markets models, because even though emerging economies may have perfect access to international financial markets, they can not borrow contingent on the state of nature as the complete markets models assume. Developed countries have access to a richer menu of financial assets than developing countries, and as shown by Angeletos (2002) and Buera and Nicolini (2002) governments can use the maturity structure of non-contingent public debt to replicate the complete markets optimal allocation as long as they have access to a sufficiently rich maturity structure.

Therefore, in this paper I solve the optimal fiscal policy problem for a small open economy under incomplete markets and distortionary taxation. In the case of complete markets, any tax schedule can be implemented as long as it satisfies the economy’s resource constraints and as long as it can be sustained by a competitive equilibrium. When markets are incomplete, only a subset of the complete markets policies is available to the government. This introduces additional constraints on the set of competitive equilibrium allocations that the government can choose from, which makes the problem computationally more difficult.

In the model economy there are three agents, households, firms and a government. Households value leisure and consumption. Firms produce final goods using labor as the only input in production. In addition, firms have to pay for part of the wage bill before production takes place creating a need for working capital as in Neumeyer and Perri (2005). The government finances an exogenous and stochastic sequence of unproductive public consumption by issuing debt and by levying income taxes. The only taxes available to the government are proportional income taxes, which distort the consumption-leisure margin. All the agents
in the economy have access to international financial markets, where they can borrow or lend to foreigners. Financial markets are incomplete because agents can only buy and sell one-period non-contingent real bonds. Moreover, I assume that agents can commit to repay their debt. The model economy is subject to three types of shocks: productivity shocks, interest rate shocks and government spending shocks.

I follow the Ramsey approach in characterizing the optimal fiscal policy. In this approach the Ramsey planner chooses an allocation that maximizes the household’s utility subject to the condition that this allocation be implementable as a competitive equilibrium. In addition, I assume that the Ramsey planner commits to the announced policies.

The contributions of the paper are twofold. First, I solve the Ramsey problem for a small open economy with incomplete markets and stochastic interest rates, government spending and productivity. When markets are incomplete, as I have already mentioned, only a subset of the complete markets policies is available to the planner. This introduces into the Ramsey problem additional implementability constraints that arise from the requirement that the debt be risk-free. Since conditional expectations of future variables appear in these implementability constraints, the Ramsey problem is not recursive. Nevertheless, I show it is possible to recover a recursive formulation using the recursive contracts approach of Marcet and Marimon (1998). After setting up the Ramsey problem recursively, I compute the optimal policy by solving a log-linear approximation to the Ramsey planner’s optimality conditions.

Second, I find that the optimal fiscal policy smooths distortions over time.
The income tax rate, and specially the public debt are very persistent irrespective of the degree of autocorrelation of the assumed processes for the shocks generating aggregate fluctuations. This reflects the planner’s desire to smooth the cost of raising taxes over time.

The Ramsey planner finances an increase in government spending or a decrease in the tax base partly by increasing debt and partly by increasing the tax rate. In order to avoid a large distortion at the time of the shock, the planner smooths the tax increase over time. As a consequence, the stock of public debt displays a persistent increase. Debt plays in this model an important role as a shock absorber. After a positive innovation in the interest rate or in government spending, or after negative innovation in productivity, the level of public debt and the primary deficit increase. Government debt responds on impact less than the other variables, but it accumulates over time, and displays the most persistent impulse response function. The responses of debt and the primary deficit have the same sign in the first periods. However, the response of the primary deficit changes sign after a few periods, because a higher debt interest will have to be serviced in the future in response to an increase in debt today.

The paper proceeds as follows. Section 2 briefly discusses the literature on optimal fiscal policy. Section 3 presents the economic environment and the theoretical model. Section 4 presents the Ramsey problem, and develops a recursive representation. Section 5 describes the calibration of the model and analyzes the quantitative results to illustrate the dynamic properties of the optimal fiscal policy. Section 6 concludes.
Related Literature

This paper is related to several studies about optimal fiscal policy. An extensive literature on optimal fiscal policy has emerged since the seminal work of Lucas and Stokey (1983). Most of the existing work, however, has limited attention to closed economy environments. This paper instead studies optimal fiscal policy in a small open economy with incomplete markets. The two key elements that distinguish my analysis from the pertinent literature are the following: First, I consider a small open economy where the agents can only buy or sell risk-free debt. Second, I introduce stochastic interest rates to analyze how interest rates shocks affect the properties of the optimal fiscal policy.

In a closed economy environment with complete markets, Lucas and Stokey (1983) used the Ramsey approach of optimal taxation to study the properties of optimal fiscal policy. They found that it is optimal to respond to fiscal shocks by appropriately altering the state-contingent return on government debt and keeping the tax rate roughly constant, so state-contingent debt serves as an instrument to smooth tax distortions over time and states of nature. They also show that tax rates and debt inherit the serial correlation structure of the underlying shocks. Chari, Christiano and Kehoe (1994) analyzed the quantitative features of optimal fiscal policy in a standard real business cycle model with complete markets as in Lucas and Stokey (1983). They showed that another way to keep tax rates stable over the business cycle is to have non-state contingent debt with taxes on interest income that vary with the shocks, in this case state-contingent taxes on interest income should be used to provide insurance against adverse shocks. They found that in calibrated models to the
U.S., the standard deviation of optimal income taxes is close to zero while taxes on interest income are highly volatile and serially uncorrelated.

Aiyagari et al (2002) restricted the government to issue only one-period non-contingent debt. They showed that optimal fiscal policy under this environment imposes a near random walk behavior on taxes and debt irrespective of the degree of autocorrelation of the underlying shocks. They also found that the level of debt permanently increases after a fiscal shock, and that the response of the tax rate is a weighted average of a random walk and a serially uncorrelated process. Their results affirm partially the random walk hypothesis of Barro (1979).

Angeletos (2002), and Buera and Nicolini (2002) considered governments restricted to trading non-contingent real debt of different maturities. They showed that governments could use the maturity structure of non-contingent public debt to replicate the complete markets optimal allocation. However, Buera and Nicolini showed that the government might need to take extremely large long and short positions in debt of different maturities. Since all of the above papers study the properties of optimal fiscal policy in closed economies, they do not consider interest rate shocks.

In an open economy setting, Schmitt-Grohe and Uribe (2003b) study the properties of optimal policy in a monetary small open economy in which agents have access to complete international asset markets. They show that in a small open economy under complete markets optimal tax rates do not change in response to government spending shocks while in a closed economy they do. Riascos and Vegh (2004) consider an environment in which gov-
ernment spending is determined endogenously. They show that when markets are complete, the correlation between government consumption and output is zero. However, if markets are incomplete the correlation between government consumption and output is large and positive.

In terms of the existing literature, this paper is closest to Riascos and Vegh (2004). Like them, I study optimal fiscal policy in a small open economy. However, this paper differs in three key respects from their paper. First, the goal of the present paper is to characterize the behavior of optimal tax rates and government debt under incomplete markets in a small open economy, while the goal of Riascos and Vegh (2004) is to analyze the procyclicality of fiscal policy in developing countries, so they do not analyze the optimal behavior of public debt under incomplete markets. Second, these authors consider an endowment economy, while I consider a production economy with an elastic labor supply, so movements in the tax rate affect the labor supply and output. Third, they develop a model in which the interest rate is constant, while I build a model in which the interest rate is stochastic to study how interest rate shocks affect the properties of the optimal fiscal policy.

Optimal fiscal policy models for small open economies have not incorporated interest rate uncertainty, distortionary taxation and market incompleteness into a single framework. Previous papers either assume that agents have access to complete international asset markets or that interest rates are constant. In this paper, I solve the Ramsey problem for a small open economy under interest rate uncertainty, distortionary taxation and market incompleteness. Finally, by combining these elements in a general equilibrium model, I am
able to characterize the properties of optimal fiscal policy in this environment, on the basis
of Ramsey’s principle for optimal taxation.

3 The Model

Consider a small open economy populated by a large number of identical, infinitely lived
agents. Agents in the domestic economy are households, firms and a government. In each
period \( t = 0, 1, \ldots \) the economy experiences one of many events \( s_t \). We denote by \( s^t = (s_0, \ldots, s^t) \) the history of events up to and including period \( t \), and the probability as of period
0, of any particular history \( s^t \) by \( \mu (s^t) \). The initial realization \( s_0 \) is given. Asset markets
are incomplete since agents can only lend and borrow issuing and buying one-period non-
contingent bonds. Households derive utility from consumption and leisure. Firms produce
the final good \( y(s^t) \) using labor \( h^d(s^t) \) as the only input. In addition, firms have to pay a
fraction of the wage bill before production takes place, creating a need for working capital
as in Neumeyer and Perri (2005). The government finances an exogenous and stochastic
sequence of unproductive public consumption \( g(s^t) \) by issuing non-contingent debt and by
levying income taxes at the rate \( \tau (s^t) \).

The timing of events is as follows. In each period there are two sub-periods: in the
first sub-period, the asset and factor markets open, and in the second sub-period, the goods
market opens after the other two markets close. Let \( w(s^t) \) be the wage rate at date \( t \),
conditional on history \( s^t \), \( \theta \) the fraction of the wage bill that firms have to pay in advance in
the factor market, \( d(s^t) \) the quantity of bonds issued by the household at date \( t \), conditional
on history $s^t$ that pay one unit of consumption in every state in period $t + 1$, $b(s^t)$ the quantity of bonds issued by the government at date $t$, conditional on history $s^t$ that pay one unit of consumption in every state in period $t + 1$, and $q(s^t)$ the price of a bond at date $t$, conditional on history $s^t$ that pays one unit of consumption in every state in period $t + 1$. Therefore, $q(s^t)$ is inverse of the risk-free gross real interest rate $R(s^t) = \frac{1}{q(s^t)}$ at which agents borrow from foreigners in the asset market.

In the asset market, firms borrow $\theta w(s^t) h^d(s^t)$ units of the final good, households pay $d(s^{t-1})$ units of the consumption good issued in $s^{t-1}$, and issue $q(s^t)d(s^t)$ units of debt maturing in $t + 1$, and the government pays $b(s^{t-1})$ units of the consumption good issued in $s^{t-1}$, and issues $q(s^t)b(s^t)$ units of debt maturing in $t + 1$. In the factor market firms hire $h^d(s^t)$ units of labor, and pay $\theta w(s^t) h^d(s^t)$ units of the final good to households before production takes place. The goods market opens after the other two markets close. In this market, firms sell the final good $y(s^t)$, pay $(1 - \theta) w(s^t) h^d(s^t)$ to households, and keep $\theta w(s^t) h^d(s^t) R(s^t)$ to pay for the working capital they borrowed. In the asset market next period firms pay the working capital loan $\theta w(s^t) h^d(s^t) R(s^t)$. Households own the firms, so they use their income from the asset market, and their after tax wage and profit income to buy consumption goods $c(s^t)$. The government uses its income from the asset market and its tax revenues to buy public consumption $g(s^t)$. 
3.1 Households

Each household has preferences defined over consumption $c_t$ and labor $h_t$. The representative agent’s lifetime utility is given by:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) U(c(s^t), h(s^t))$$

(1)

where $0 < \beta < 1$ denotes the subjective discount factor, $c(s^t)$ and $h(s^t)$ denote consumption and labor conditional on the history of events $s^t$, and the single-period utility function $U$ is strictly increasing in consumption, decreasing in labor, strictly concave, and satisfies the Inada conditions.

Each period $t$, households supply labor in a competitive labor market, receive wage and profit income from the ownership of firms, and issue one-period non-contingent bonds $d(s^t)$, which pay one unit of consumption in every state in period $t+1$. Households spend their income on consumption, debt repayment, taxes on wage and profit and income, and on debt adjustment costs.

The period-by-period budget constraint is given by:

$$c(s^t) + d(s^{t-1}) + \psi(d(s^t)) \leq q(s^t) d(s^t) + (1 - \tau(s^t)) w(s^t) h(s^t) + (1 - \tau(s^t)) \Pi(s^t)$$

(2)

where $\tau$ denotes the income tax rate imposed by the government, $\Pi(s^t)$ profits, and the function $\psi(.)$ captures a convex cost of adjusting the household’s debt portfolio.

In addition to the budget constraint, the household is subject to the following borrowing
constraint that prevents it from engaging in Ponzi schemes:

\[
\lim_{j \to \infty} \prod_{i=0}^{j} q(s^{t+i}) d(s^{t+j}) \leq 0 \text{ for all } t, s^t
\] (3)

The assumptions on the utility function imply that households will always choose allocations such that constraints (2) and (3) hold with equality. The household’s problem is then to choose state-contingent plans \( \{c(s^t), h(s^t), d(s^t)\}_{t=0}^{\infty} \) to maximize (1) subject to (2) and (3) given the stochastic processes \( \{w(s^t), \tau(s^t), q(s^t), \Pi(s^t)\}_{t=0}^{\infty} \) and the initial condition \( d_{-1} \).

The first-order conditions associated with the household’s maximization problem are (2), and (3) holding with equality for all \( t, s^t \) and:

\[
- \frac{u_h(s^t)}{u_c(s^t)} = (1 - \tau(s^t)) w(s^t)
\] (4)

\[
u_c(s^t) [q(s^t) - \psi'(d(s^t))] = \beta \sum_{s^{t+1}} u_c(s^{t+1}) \mu(s^{t+1} | s^t)
\] (5)

where \( u_c \) denotes the derivative of the utility function with respect to consumption, and \( u_h \) the derivative of the utility function with respect to labor.

First-order condition (4) shows that the tax rate introduces a wedge between the consumption-leisure marginal rate of substitution and the wage rate. First-order condition (5) is the stochastic Euler equation. This equation show that at the optimum, the marginal benefit of issuing an additional unit of debt must equal its marginal cost.
3.2 Firms

Firms are identical and perfectly competitive. They transform labor into a final good using the following technology:

\[ y(s_t) = z(s_t) f(h^d(s_t)) \]  

(6)

where \( y(s_t) \) denotes output in state \( s_t \), \( z(s_t) \) an exogenous and stochastic productivity shock, and \( h^d(s_t) \) labor demand in state \( s_t \). The function \( f \) is strictly increasing, strictly concave, and homogeneous of degree \( \eta \leq 1 \).

The profits of the firm in state \( s_t \) are:

\[ \Pi(s_t) = y(s_t) - w(s_t) h^d(s_t) - \theta w(s_t) h^d(s_t) (R(s_t) - 1) \]  

(7)

where \( \theta w(s_t) h^d(s_t) (R(s_t) - 1) \) represents the interests that firms have to pay for the working capital they borrowed. The firm’s problem is to choose labor \( h^d(s_t) \) to maximize (7) subject to (6) given \( w(s_t), z(s_t), \) and \( R(s_t) \). The first-order condition for all \( t, s_t \) is:

\[ w(s_t) = \frac{z(s_t) f'(h^d(s_t))}{1 + \theta (R(s_t) - 1)} \]  

(8)

First-order condition (8) shows that the need for working capital to finance the wage bill makes labor’s demand sensitive to the interest rate. Since firms have to borrow to pay for a fraction of the wage bill, an increase in the interest rate makes the effective labor cost higher, so the firm’s labor demand falls for any level of wages.
3.3 The Government

In each period $t$, the government collects taxes on wage and profit income, and issues one-period non-contingent bonds $b(s^t)$, which pay one unit of consumption in every state in period $t+1$. The government uses its income to buy public consumption $g(s^t)$, which is exogenous, stochastic and unproductive, to repay debt, and to pay for adjustment costs on its debt portfolio.

The government’s period-by-period budget constraint is given by

$$g(s^t) + b(s^{t-1}) + \psi (b(s^t)) \leq q(s^t) b(s^t) + \tau (s^t) w(s^t) h(s^t) + \tau (s^t) \Pi (s^t)$$

(9)

where $\psi(.)$ captures a convex cost of adjusting the government’s debt portfolio.

In addition to this budget constraint, the government is subject to the following borrowing constraint that prevents it from engaging in Ponzi schemes:

$$\lim_{j \to \infty} \prod_{i=0}^{j} q(s^{t+i}) b(s^{t+j}) \leq 0 \text{ for all } t, s^t$$

(10)

Constraint (10) is a requirement for the existence of a well defined Ramsey equilibrium. The no-Ponzi game constraint cannot be ignored because without it the first best allocation is feasible. A benevolent government seeking to maximize the welfare of private agents will always choose state-contingent allocations such that (9) and (10) hold with equality. The fiscal policy consists in the announcement of state-contingent plans for $\{\tau(s^t), b(s^t)\}_{t=0}^{\infty}$. 
3.4 Competitive Equilibrium with Income Taxes

Given the initial conditions \(d_{-1}, b_{-1}\), and the stochastic processes \(\{g(s^t), z(s^t), q(s^t)\}_{t=0}^{\infty}\), a competitive equilibrium is a set of state-contingent sequences \(\{c(s^t), h(s^t), h^d(s^t), d(s^t)\}_{t=0}^{\infty}\), a state-contingent sequence of prices \(\{w(s^t)\}_{t=0}^{\infty}\), and a fiscal policy \(\{\tau(s^t), b(s^t)\}_{t=0}^{\infty}\) satisfying the following conditions for all \(t, s^t\)

\[
c(s^t) + d(s^{t-1}) + \psi(d(s^t)) = q(s^t) d(s^t) + (1 - \tau(s^t)) w(s^t) h(s^t) + (1 - \tau(s^t)) \Pi(s^t)
\]

\[
\lim_{j \to \infty} \prod_{i=0}^{j} q(s^{t+i}) d(s^{t+i}) = 0
\]

\[
-\frac{u_h(s^t)}{u_c(s^t)} = (1 - \tau(s^t)) w(s^t)
\]

\[
u_c(s^t) [q(s^t) - \psi'(d(s^t))] = \beta \sum_{s^{t+1}} u_c(s^{t+1}) \mu(s^{t+1} | s^t)
\]

\[
w(s^t) = \frac{z(s^t) f'(h^d(s^t))}{1 + \theta(R(s^t) - 1)}
\]

\[
\Pi(s^t) = (1 - \eta) z(s^t) f(h^d(s^t))
\]

\[
g(s^t) + b(s^{t-1}) + \psi(b(s^t)) = q(s^t) b(s^t) + \tau(s^t) w(s^t) h(s^t) + \tau(s^t) \Pi(s^t)
\]

\[
\lim_{j \to \infty} \prod_{i=0}^{j} q(s^{t+i}) b(s^{t+i}) = 0
\]

\[
h(s^t) = h^d(s^t)
\]

Since the domestic economy is small, \(q(s^t)\) is exogenous. The tax rate, and the fraction of the wage bill that firms pay before production takes place create a wedge between the consumption-leisure marginal rate of substitution and the marginal rate of transformation. Combining the firm’s and the household’s first-order conditions for labor we get
\[
- \frac{u_h(s^t)}{u_c(s^t)} = z(s^t) f'(h(s^t)) \frac{(1 - \tau(s^t))}{1 + \theta(R(s^t) - 1)}
\]

where \(\frac{(1 - \tau(s^t))}{1 + \theta(R(s^t) - 1)}\) is the wedge between the consumption-leisure marginal rate of substitution and the marginal rate of transformation. Combining the household’s and the government’s budget constraints, and substituting the wage rate and profits and from equations (8) and (11) we obtain the economy’s resource constraint

\[
q(s^t) \left( d(s^t) + b(s^t) \right) + z(s^t) f(h(s^t)) \left[ \frac{\eta}{1 + \theta(R(s^t) - 1)} + 1 - \eta \right] = d(s^{t-1}) + b(s^{t-1}) + c(s^t) + g(s^t) + \psi(d(s^t)) + \psi(b(s^t))
\]

where \(\frac{\eta}{1 + \theta(R(s^t) - 1)} = \frac{w(s^t)h(s^t)}{y(s^t)}\) is the labor’s share in output, and \(1 - \eta = \frac{n(s^t)}{y(s^t)}\) is the profit’s share in output.

Let \(TB(s^t)\) be the trade balance of the economy at date \(t\) conditional on history \(s^t\)

\[
TB(s^t) = z(s^t) f(h(s^t)) - c(s^t) - g(s^t) - \psi(d(s^t)) - \psi(b(s^t)) \quad (13)
\]

The optimal fiscal policy is a set of state-contingent sequences \(\{\tau(s^t), b(s^t)\}_{t=0}^{\infty}\) associated with the competitive equilibrium that yield the highest level of utility to the representative household, that is, that maximizes (1). To find the optimal policy, it is convenient to use a simpler representation of the competitive equilibrium known as the primal form. Finding the primal form involves the elimination of all prices and tax rates from the equilibrium conditions, so that the resulting reduced form involves only quantities.
3.4.1 The Primal Form

**Proposition 1** Given the initial conditions $d_{-1}$, $b_{-1}$, and the exogenous stochastic processes \( \{ g(s^t), z(s^t), q(s^t) \}_{t=0}^{\infty} \), state-contingent plans \( \{ c(s^t), h(s^t), d(s^t), b(s^t) \}_{t=0}^{\infty} \) satisfy (5)

\[
c(s^t) + d(s^{t-1}) + \psi(d(s^t)) = q(s^t) d(s^t) - \frac{u_h(s^t)}{u_c(s^t)} h(s^t) \left[ 1 + (1 + \theta (R(s^t) - 1)) \frac{1 - \eta}{\eta} \right]
\]

\[
q(s^t) (d(s^t) + b(s^t)) + z(s^t) f(h(s^t)) \left[ \frac{\eta}{1 + \theta (R(s^t) - 1)} + 1 - \eta \right] = \quad \text{(14)}
\]

\[
d(s^{t-1}) + b(s^{t-1}) + c(s^t) + g(s^t) + \psi(d(s^t)) + \psi(b(s^t))
\]

if and only if they satisfy (2), (4), (5), (8), (9) and (11)

**Proof.** See the Appendix. \(\blacksquare\)

4 Ramsey Problem

It follows from the previous Proposition that the Ramsey problem can be stated as choosing state-contingent plans \( \{ c(s^t), h(s^t), d(s^t), b(s^t) \}_{t=0}^{\infty} \) to maximize

\[
\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) U (c(s^t), h(s^t))
\]

subject to

\[
u_c(s^t) [q(s^t) - \psi(d(s^t))] = \beta \sum_{s^{t+1}} u_c(s^{t+1}) \mu(s^{t+1} | s^t)
\]

\[
c(s^t) + d(s^{t-1}) + \psi(d(s^t)) = q(s^t) d(s^t) - \frac{u_h(s^t)}{u_c(s^t)} h(s^t) \left[ 1 + (1 + \theta (R(s^t) - 1)) \frac{1 - \eta}{\eta} \right]
\]

(14)
\[
q (s^t) (d(s^t) + b(s^t)) + z(s^t) f(h(s^t)) \left[ \frac{\eta}{1 + \theta (R(s^t) - 1)} + 1 - \eta \right]
\]

\[
= d(s^{t-1}) + b(s^{t-1}) + c(s^t) + g(s^t) + \psi(d(s^t)) + \psi(b(s^t))
\]

Given the initial conditions \(d_{-1}, b_{-1}\)

This problem is not recursive because constraint (5) involves a conditional expectation of future control variables. Therefore, the usual Bellman equation is not satisfied, so the optimal choice at \(t, s^t\) is not a time invariant function of the state variables \(\{g_t, z_t, q_t, d_{t-1}, b_{t-1}\}\) as in standard dynamic programming, so the whole history of shocks can matter for today’s optimal decision. Nevertheless, Marcet and Marimon (1998) show that when the original maximization problem is not recursive because implementability constraints depend on plans for future variables, an equivalent saddle point problem can be constructed leading to a recursive formulation. The resulting saddle point problem expands the state space by including new state variables that summarize the evolution of the lagrange multipliers of the original problem. To solve the Ramsey problem, we need to write the problem in a recursive framework. The first step is to transform the original problem into a recursive saddle point problem.

The corresponding Lagrangian is

\[
\Gamma = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu(s^t) \begin{cases} 
U(c(s^t), h(s^t)) + \gamma(s^t) \\
[ u_c(s^t) [q(s^t) - \psi'(d(s^t))] - \beta \sum_{s^{t+1}} u_c(s^{t+1}) \mu(s^{t+1} | s^t) ] 
\end{cases}
\]

subject to (14) and (15), where \(\beta^t \mu(s^t) \gamma(s^t)\) is the Lagrange multiplier associated to constraint (5). Using the law of iterated expectations and reordering terms, one can show
that the function $H$ defined as

$$
H = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu (s^t) \{ U (c (s^t), h (s^t)) + \gamma (s^t) u_c (s^t) \left[ q (s^t) - \psi' (d (s^t)) \right] - \zeta (s^t) u_c (s^t) \}
$$

(16)

$$
\zeta (s^{t+1}) = \gamma (s^t) \text{ for all } t \geq 0
$$

(17)

$$\zeta_0 = 0$$

is such that, for all feasible sequences $\Gamma = H$

Therefore, any solution to the original Ramsey problem must also be a solution to the problem of maximizing (16) subject to (17), (14) and (15).

Here $\zeta (s^t)$ acts as a co-state variable. Notice that this saddle point problem does not have any future variables in the constraints, and that all the functions in the constraints are known. If we include $\zeta (s^t)$ in the set of state variables, the problem becomes recursive.

This saddle point problem has a recursive formulation, in the sense that there exists a unique value function $W (d, b, \zeta, s)$ satisfying

$$
W (d, b, \zeta, s) = \min_{\gamma} \max_{c, h, d', b'} \left\{ u (c, h) + \gamma u_c (c, h) \left[ q - \psi' (d') \right] - \zeta u_c (c, h) + \beta E \left[ W (d, b, \zeta, s) \mid s \right] \right\}
$$

subject to:

$$
c + d + \psi (d') = q d' - \frac{u_h (c, h)}{u_c (c, h)} h \left[ 1 + (1 + \theta (R-1)) \frac{1-\eta}{\eta} \right]
$$

$$
q (d' + b') + z f (h) \left[ \frac{\eta}{1 + \theta (R-1)} + 1 - \eta \right]
$$

$$
= d + b + c + g + \psi (d') + \psi (b')
$$
\[ \zeta' = \gamma \]

where \( s = (z, q, g) \)

The solution to this functional equation yields a stationary policy function \( \chi \), so that the optimal solution to the Ramsey problem satisfies

\[
(c(s_t), h(s_t), d(s_t), b(s_t), \gamma(s_t)) = \chi(d(s_{t-1}), b(s_{t-1}), \zeta(s_t), s_t)
\]

for all \( t \) and \( \zeta_0 = 0 \).

The solution is recursive since only the values  \((d(s_{t-1}), b(s_{t-1}), \zeta(s_t), s_t)\) are relevant from history, and the policy function \( \chi \) is time invariant. Dependence of the optimal solution on \( \zeta(s_t) \) is the reason that the model is not recursive in the standard sense of having a time invariant policy function of \((d(s_{t-1}), b(s_{t-1}), s_t)\).

The Lagrangian for this problem, after substituting \( \zeta(s^t) = \gamma(s^{t-1}) \) in the objective function, is given by:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l}
U(c_t, h_t) + \gamma_t u_c(t) [q_t - \psi'(d_t)] - \gamma_{t-1} u_c(t) + \lambda_t \\
q_t d_t - \frac{w_t(t)}{w_c(t)} h_t \left[ 1 + (1 + \theta(R_t - 1)) \frac{1-q}{n} \right] - d_{t-1} - c_t - \psi(d_t) + \phi_t \\
q_t (d_t + b_t) + z_t f(h_t) \left[ \frac{n}{1+n(R_t - 1)} + 1 - \eta \right] \\
-d_{t-1} - b_{t-1} - c_t - g_t - \psi(d_t) - \psi(b_t) \end{array} \right\}
\]

where \( \beta^t \lambda_t \) and \( \beta^t \phi_t \) are the Lagrange multipliers on constraints (14) and (15) respectively.

The first-order conditions for all \( t \) are given by:
\[ u_c(t) + \gamma_t u_{cc}(t) [q_t - \psi'(d_t)] - \gamma_{t-1} u_{cc}(t) - \phi_t \]
\[ = \lambda_t \left[ \left( 1 + \frac{1 - \eta}{\eta} [1 + \theta (R_t - 1)] \right) \left( \frac{u_{ch}(t)}{u_c(t)} h_t - \frac{u_h(t) u_{cc}(t)}{(u_c(t))^2} h_t \right) + 1 \right] \]

\[ u_h(t) + \gamma_t u_{ch}(t) [q_t - \psi'(d_t)] - \gamma_{t-1} u_{ch}(t) = \lambda_t \left( 1 + \frac{1 - \eta}{\eta} [1 + \theta (R_t - 1)] \right) \]
\[ \left( \frac{u_h(t)}{u_c(t)} + \frac{u_{hh}(t)}{u_c(t)} h_t - \frac{u_h(t) u_{ch}(t)}{(u_c(t))^2} h_t \right) - \phi_t z_t f(t) h_t \left( \frac{\eta}{1 + \theta (R_t - 1)} + 1 - \eta \right) \]

\[ q_t d_t - \frac{u_h(t)}{u_c(t)} h_t \left( 1 + \frac{1 - \eta}{\eta} [1 + \theta (R_t - 1)] \right) - d_{t-1} - c_t - \psi(d_t) = 0 \]

\[ q_t (d_t + b_t) + z_t f(h_t) \left( \frac{\eta}{1 + \theta (R_t - 1)} + 1 - \eta \right) = d_{t-1} + b_{t-1} + c_t + g_t + \psi(d_t) + \psi(b_t) \]

\[ u_c(t) [q_t - \psi'(d_t)] - \beta E_t [u_c(t + 1)] = 0 \]

\[ -\gamma_t u_c(t) \psi''(d_t) + (\lambda_t + \phi_t) [q_t - \psi'(d_t)] - \beta E_t [\lambda_{t+1} + \phi_{t+1}] = 0 \]

\[ \phi_t [q_t - \psi'(d_t)] - \beta E_t [\phi_{t+1}] = 0 \]

\[ \lim_{t \to \infty} \prod_{i=0}^{t} q_i d_t = 0 \]
$$\lim_{t \to \infty} \prod_{i=0}^{t} q_i b_t = 0$$

$$\lim_{t \to \infty} \prod_{i=0}^{t} q_i \gamma_t = 0$$

$d_{-1}, b_{-1}$ given, and $\gamma_{-1} = 0$.

5 Dynamic Properties of the Optimal Fiscal Policy

In this section we carry out some simulations to study the dynamic properties of the model economy under the Ramsey policy with incomplete markets. We compute the equilibrium dynamics by solving a linear approximation to the Ramsey planner’s optimality conditions. First, we present the baseline calibration of the model. Second, we show and discuss the impulse response functions of the model. Third, we present and analyze the moments of the simulated time series. Finally, we conduct a sensitivity analysis to evaluate how the results change when we vary some of the parameter values.

5.1 Calibration

The benchmark model is calibrated so as to make it consistent with some of the empirical regularities that reflect the structure of a typical emerging economy. In particular, data from Mexico is used to calibrate the parameters of the benchmark model. The data considered corresponds to quarterly observations for the period 1980-2006. In general the results are
robust to changes in the parameters, therefore, I only report sensitivity analyses for those parameters that are crucial for determining the effects of the shocks in the model economy.

The time unit is one quarter, and the time endowment, which can be divided between labor and leisure is normalized to one. We assume that the period utility function follows the GHH specification.

\[ u(c, h) = \frac{(c - \varphi h^\nu)^{1-\sigma} - 1}{1 - \sigma} \]

The parameter \( \sigma \), the coefficient of relative risk aversion, is set equal to 2, which is a standard value. The parameter \( \nu \) is set to 1.6 following Neumeyer and Perri (2005). This parameter determines the labor supply elasticity, which is equal to \( \frac{1}{\nu - 1} \). We assume that households devote on average 20\% of their time to work. We set \( \beta \) to match an average real interest rate of 10.17\% per year. We determine the value of \( \beta \) from the first-order condition for bonds evaluated in steady-state.

\[ \beta = \frac{1}{\bar{R}} \]

We assume that all the wage bill is paid in advance, so we set \( \theta = 1 \).

The production function takes the following form

\[ zf(h) = zh^\eta \]

We assume that the labor’s share in output \( \frac{w^h}{y} = 2/3 \) in steady-state. This value and the firm’s first-order condition for labor evaluated in steady-state imply that:

\[ \eta = \frac{w^h}{y} (1 + \theta (\bar{R} - 1)) \]
The parameter \( \eta \) determines the wage elasticity of labor demand, which is equal to \( -\frac{1}{\eta - 1} \). The productivity shock is equal to 1 in steady-state, therefore output in steady-state is equal to:
\[
\bar{y} = \bar{h}^\eta \]
Profits in steady-state are equal to:
\[
\Pi = \bar{y} (1 - \eta) \]
The government’s budget constraint in steady-state after substituting the firm’s optimality conditions is equal to:
\[
\frac{\bar{y}}{\bar{y}} + (1 - \beta) \frac{\bar{b}}{\bar{y}} = \tau \left[ (1 - \eta) + \frac{\eta}{1 + \theta (R - 1)} \right] \]
The average public-debt to GDP ratio in Mexico for the period 1980-2006 is 0.375, and the average share of government spending in GDP is 13.5%. These values imply that the income tax rate in steady-state is equal to:
\[
\tau = \frac{\frac{\bar{y}}{\bar{y}} + (1 - \beta) \frac{\bar{b}}{\bar{y}}}{\left[ (1 - \eta) + \frac{\eta}{1 + \theta (R - 1)} \right]} \]
We calibrate \( \varphi \) from first-order condition (4) evaluated in steady-state
\[
\varphi = \frac{\bar{w} (1 - \bar{\tau})}{\nu \bar{h}^{\varphi - 1}} \]
The economy’s resource constraint in steady-state is equal to
\[
\bar{y} - \bar{c} - \bar{g} = (1 - \beta) (\bar{d} + \bar{b}) + (R - 1) \theta \frac{\eta \bar{y}}{1 + \theta (R - 1)} \]
where \((\bar{R} - 1) \theta \frac{\eta y}{1 + \theta (\bar{R} - 1)} = (\bar{R} - 1) \theta \bar{w} \bar{h}\) are the interests paid on the firm’s working capital.

The private-debt to GDP ratio in steady-state is equal to:

\[
\frac{d}{y} = D - \frac{\theta \bar{w} \bar{h}}{y} - \beta \frac{\bar{b}}{y}
\]

where \(D\) is the foreign-debt to GDP ratio in steady-state. The average foreign-debt to GDP ratio in Mexico for the period 1980-2006 is approximately equal to 0.4.

We can calculate the consumption’s share in GDP in steady-state from the economy’s resource constraint.

\[
\frac{\bar{c}}{\bar{y}} = 1 - \frac{\bar{g}}{\bar{y}} - (\bar{R} - 1) \frac{\theta \bar{w} \bar{h}}{\bar{y}} - (1 - \beta) \frac{(\bar{d} + \bar{b})}{\bar{y}}
\]

We assume that the adjustment costs functions for the household and the government are respectively.

\[
\psi (d) = \frac{\psi}{2} (d_t - \bar{d})^2
\]

\[
\psi (b) = \frac{\psi}{2} (b_t - \bar{b})^2
\]

where \(\psi\) is a constant determining the size of the debt holding costs and \(\bar{d}, \bar{b}\) are the steady-state values of the household’s debt and the government’s debt respectively. We set \(\psi\) to the minimum value that guarantees that the equilibrium solution is stationary. We also assume that the exogenous stochastic processes \(z_t, R_t, g_t\) follow independent AR (1) processes.

\[
\log z_t = \rho z^2 \log z_{t-1} + \varepsilon_t^z
\]

\[
\log \left( \frac{R_t}{\bar{R}} \right) = \rho^R \log \left( \frac{R_{t-1}}{\bar{R}} \right) + \varepsilon_t^R
\]

\[
\log \left( \frac{g_t}{\bar{g}} \right) = \rho^g \log \left( \frac{g_{t-1}}{\bar{g}} \right) + \varepsilon_t^g
\]
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<tr>
<th>Parameter</th>
<th>Value</th>
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</tr>
<tr>
<td>$\sigma^\theta_\epsilon$</td>
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</tbody>
</table>

Table 1: Parameters
5.2 Impulse Response Functions

In this section we present the impulse response functions of the model to illustrate the dynamic properties of the optimal fiscal policy. The interest rate is expressed in percentage points. Output, productivity, hours worked, the wage rate, the tax rate, tax revenues and government spending are expressed in percentage deviations from their steady state values. The deviations of government debt, the primary deficit, the total deficit, the trade balance, the current account and consumption are expressed as a percent of steady state output.

The fraction of the wage bill that firms have to pay in advance, and the tax rate create a wedge between the consumption-leisure marginal rate of substitution and the marginal rate of transformation. This wedge is equal to:

\[
\frac{(1 - \tau(s^t))}{1 + \theta(R(s^t) - 1)}
\]

If we log-linearize this wedge around the steady-state, we get the following expression:

\[
\text{wedge}_t = -\frac{\tau}{1 - \tau} \tilde{\tau}_t - \frac{\theta R}{1 + \theta (R - 1)} \tilde{R}_t
\]

The Ramsey planner smooths out this wedge over time. Combining the firm’s and households first order conditions we get that:

\[
-\frac{u_h(s^t)}{u_c(s^t) (1 - \tau(s^t))} \frac{1}{1 + \theta (R(s^t) - 1)} = z(s^t) f'(h(s^t)) \frac{1}{1 + \theta (R(s^t) - 1)}
\]

where the left hand side can be interpreted as the labor supply and the right hand side as the labor demand in state \(s^t\).

Using the proposed utility and production functions, and taking logs, we can express the
labor demand and labor supply curves respectively as follows:

\[
\ln w_t = -(1 - \eta) \ln h_t - \ln [1 + \theta (R_t - 1)] + \ln z_t
\]

\[
\ln w_t = (\nu - 1) \ln h_t - \ln (1 - \tau_t)
\]

We can also write these equations in percentage deviations from the steady-state as follows:

\[
\hat{w}_t = -(1 - \eta) \hat{h}_t - \frac{\theta \hat{R}}{1 + \theta (\overline{R} - 1)} \hat{R}_t + \hat{z}_t
\]

\[
\hat{w}_t = (\nu - 1) \hat{h}_t + \frac{\tau}{1 - \tau} \hat{\tau}_t
\]

Combining the previous equations we obtain an expression for the percentage deviations of hours worked around the steady-state:

\[
\hat{h}_t = \frac{1}{\nu - \eta} \left[ \hat{z}_t - \frac{\theta \overline{R}}{1 + \theta (\overline{R} - 1)} \hat{R}_t - \frac{\tau}{1 - \tau} \hat{\tau}_t \right]
\]

The log-linearized Euler equation is equal to:

\[
E_t (\hat{c}_{t+1} - \hat{c}_t) = \frac{1}{\sigma} \left( 1 - \frac{\varphi \overline{h}^{\nu}}{\bar{e}} \right) \hat{R}_t + \frac{\nu \varphi \overline{h}^{\nu}}{\bar{e}} E_t (\hat{h}_{t+1} - \hat{h}_t)
\]

5.2.1 Interest Rate Shocks

The log-linearized equilibrium conditions after an interest rate shock under the benchmark parameterization include the following equations:

\[
\hat{\text{wedge}}_t = -\frac{\tau}{1 - \tau} \hat{\tau}_t - \hat{R}_t
\]

\[
\hat{h}_t = -\frac{1}{\nu - \eta} \left[ \hat{R}_t + \frac{\tau}{1 - \tau} \hat{\tau}_t \right]
\]
\[
E_t(\tilde{c}_{t+1} - \tilde{c}_t) = \frac{1}{\sigma} \left( 1 - \frac{\varphi \tilde{h}}{c} \right) \tilde{R}_t + \frac{\nu \varphi \tilde{h}}{c} E_t(\tilde{h}_{t+1} - \tilde{h}_t)
\]
\[
\tilde{w}_t = -(1 - \eta) \tilde{h}_t - \tilde{R}_t
\]
\[
\tilde{w}_t = (\nu - 1) \tilde{h}_t + \frac{\tau}{1 - \tau} \tilde{r}_t
\]

Figure 1 displays the impulse response functions under the optimal fiscal policy to a one-percentage point increase in the interest rate. The optimal fiscal policy smooths distortions over time. An increase in the interest rate increases the wedge between the consumption-leisure marginal rate of substitution and its marginal rate of transformation, so the government reduces the tax rate and increases its public debt to smooth the distortion over time. An increase in the interest rate shifts the labor demand curve to the left on impact, while the decrease in the tax rate shifts the labor supply curve to the right. Therefore, the wage rate falls when the shock is realized. Hours worked fall, not only because the percentage change in the interest rate is greater than the percentage in the tax rate, but also because an increase of 1% in the interest rate decreases hours worked in \((\frac{1}{\nu - 1})\) %, while a decrease of 1% in the tax rate increases hours by \((\frac{1}{\nu - 1}) \left( \frac{\tau}{\nu - 1} \right)\) %. Output falls because hours worked fall and productivity remains constant. The interest rate shock has two effects on consumption, the direct effect induces agents to substitute consumption intertemporally, and it depends on the intertemporal elasticity of substitution, while the indirect effect makes consumption move when hours worked move since the utility function is not separable, and agents smooth the marginal utility of consumption over time. The sum of the two effects makes consumption more sensitive to interest rate shocks than output. Tax revenues decrease, not only because the tax rate decreases, but also because the tax base falls. The primary deficit
increases because tax revenues decrease and government spending remains constant, while the total deficit increases even more since debt interests increase. Public debt responds less on impact to the shock than the other variables, but it accumulates over time. The impulse response functions of the tax rate, the tax revenues, and the primary deficit change sign after some periods, since a higher debt interest will have to be serviced in the future. The trade balance increases because consumption falls more than output and government spending remains constant. The current account becomes negative, since the net asset position of the economy deteriorates. Public debt and the tax rate display the most persistent impulse response functions.
Figure 1: Impulse Responses to an Interest Rate Shock.
Figure 1 (Cont): Impulse Responses to an Interest Rate Shock.
5.2.2 Productivity Shocks

The log-linearized equilibrium conditions after a productivity shock include the following equations:

\[ \hat{\text{wedge}}_t = -\frac{\tau}{1 - \tau} \hat{\zeta}_t \]

\[ \hat{h}_t = \frac{1}{\nu - \eta} \left[ \hat{\zeta}_t - \frac{\tau}{1 - \tau} \hat{\tau}_t \right] \]
\[ E_t (\bar{c}_{t+1} - \bar{c}_t) = \frac{\nu \phi \bar{h}^t}{\tau} E_t \left( \hat{h}_{t+1} - \hat{h}_t \right) \]

\[ \hat{w}_t = -(1 - \eta) \hat{h}_t + \hat{z}_t \]

\[ \hat{w}_t = (\nu - 1) \hat{h}_t + \frac{\tau}{1 - \tau} \hat{z}_t \]

Figure 2 displays the impulse response of the economy under the optimal fiscal policy to a one percent decrease in productivity. For a given tax rate, employment, output, consumption, and tax revenues decrease in response to a negative productive shock, while the primary deficit increases. Since productivity shocks do not introduce any distortions in the economy, and since the tax rate is distortionary, the government finances its deficit partly by an increase in public debt, and partly by a small and persistent increase in the tax rate that pays off the increase in public debt gradually over time. A decrease in productivity shifts the labor demand curve to the left on impact. The labor supply also shifts to the left on impact, because the tax rate increases, so hours worked decrease. Output decreases more than hours because the fall in productivity decreases output directly, but also indirectly through labor. Consumption decreases less than hours and output, because the interest rate remains constant, so productivity affects consumption only through the effect that it has on labor. The wage rate decreases, not only because the percentage decrease in productivity is greater than the percentage increase in the tax rate, but also because a 1% decrease in productivity decreases the wage rate in 1% for any level of hours worked, while a 1% increase in the tax rate increases the wage rate in \((\frac{\tau}{1 - \tau})\)% for any level of hours worked. The impulse response functions of the tax revenues and the primary deficit change sign after some periods to pay for the additional debt. The trade balance deteriorates because consumption falls.
less than output, and the current account becomes negative, since the net asset position of the economy deteriorates. Public debt and the tax rate display again the most persistent impulse response functions.

Figure 2. Impulse Responses to a Productivity Shock
Figure 2 (Cont): Impulse Responses to a Productivity Shock
5.2.3 Government Spending Shocks

The log-linearized equilibrium conditions after a government spending shock include the following equations:

\[
\text{wedge}_t = -\frac{\tau}{1 - \tau} \hat{r}_t
\]

\[
\hat{h}_t = -\frac{1}{\nu - \eta} \left[ \frac{\tau}{1 - \tau} \hat{r}_t \right]
\]
\[ E_t (\tilde{c}_{t+1} - \tilde{c}_t) = \frac{\nu^{\prime} \tilde{h}^\nu}{\tilde{c}} E_t (\tilde{h}_{t+1} - \tilde{h}_t) \]

\[ \hat{w}_t = - (1 - \eta) \tilde{h}_t \]

\[ \hat{w}_t = (\nu - 1) \tilde{h}_t + \frac{\tau}{1 - \tau} \tilde{\tau}_t \]

Figure 3 displays the impulse response of the economy under the optimal fiscal policy to a one percent increase in government spending. For a given tax rate, a government spending shock does not affect hours worked, and consequently, it does not affect output and consumption. Since the tax rate is distortionary, the government finances the increase in government spending by increasing its debt and by a small but persistent increase in the tax rate that will pay off the increase in debt gradually over time. Tax revenues increase less than government spending, so the primary deficit increases. The impulse response function of the primary deficit changes sign after some periods to prevent debt from exploding. Since the tax rate increases after a positive government spending shock, the labor supply curve shifts to the left, while the labor demand curve remains constant on impact. Therefore, the wage rate increases, and hours worked fall after a positive government spending shock. Since hours worked fall, output and consumption also fall. The trade balance deteriorates, but it changes sign after some periods, since the debt service increases in response to an increase in external debt. The current account also deteriorates since the net asset position of the economy deteriorates. Public debt displays the most persistent impulse response function.
Figure 3: Impulse Responses to a Government Spending Shock
Figure 3 (Cont): Impulse Responses to a Government Spending Shock
5.3 Second Moments and Simulations

The following table displays a number of sample moments of key macroeconomic variables under the optimal fiscal policy. Table 2 reports the volatility, correlations and autocorrelations of the variables. We use the baseline calibration to compute these moments.

Some interesting facts emerge from this table:
1. The tax rate, and specially the public debt are quite persistent. The reason is that the government finances an increase in government spending or a decrease in the tax base partly by increasing public debt, and partly by increasing the tax rate. In order to avoid a large distortion when a shock hits the economy, the government smooths distortions over time.

2. The government smooths the tax rate over the business cycle; the standard deviation of the tax rate is smaller than the standard deviations of the other variables in the economy. The government uses public debt as an instrument to smooth tax distortions over time.

3. Public debt is negatively correlated with productivity and output, and positively correlated with government spending and the interest rate.

4. The tax rate is negatively correlated with productivity, output and the interest rate, and positively correlated with government spending, while output, consumption and
hours are negatively correlated with government spending.

5. Hours, output and consumption are negatively correlated with the interest rate.

5.4 Sensitivity Analysis

In this section, I conduct a sensitivity analysis to evaluate how the results change when we vary some of the parameter values. First, I consider the elements of the model that are crucial for determining the effects of interest rate fluctuations on the Ramsey allocation. These elements are the wage elasticity of labor supply \( \frac{1}{\nu - 1} \), and the fraction of the wage bill that firms have to pay in advance \( \theta \). Second, I conduct a sensitivity analysis on the parameter values that determine the serial correlation properties of the shocks. In particular, I evaluate how the results change when we consider i.i.d shocks instead of the serially correlated shocks of the baseline calibration. In all the cases, I keep all the other parameters at their baseline values, and I only analyze the impulse response functions of the model to interest rate shocks.

5.4.1 Wage Elasticity of Labor Supply

In this section, I evaluate how the results change if we reduce \( \nu \) from its baseline value of 1.6 to 1.1. The log-linearized equilibrium conditions after an interest rate shock include the following equations:

\[
\text{wedge}_t = -\frac{\tau}{1 - \tau} \hat{\tau}_t - \hat{R}_t
\]

\[
\hat{h}_t = -\frac{1}{\nu - \eta} \left[ \hat{R}_t + \frac{\tau}{1 - \tau} \hat{\tau}_t \right]
\]
\[ E_t (\tilde{c}_{t+1} - \tilde{c}_t) = \frac{1}{\sigma} \left( 1 - \frac{\varphi \tilde{h}^\nu}{c} \right) \tilde{R}_t + \frac{\nu \varphi \tilde{h}^\nu}{c} E_t (\tilde{h}_{t+1} - \tilde{h}_t) \]

\[ \hat{w}_t = -(1 - \eta) \tilde{h}_t - \tilde{R}_t \]

\[ \hat{w}_t = (\nu - 1) \tilde{h}_t + \frac{\tau}{1 - \tau} \tilde{r}_t \]

Notice that the lower \( \nu \) is, the higher the wage elasticity of labor supply \( \frac{1}{\nu - 1} \) is. For a given tax rate, as the wage elasticity of labor supply increases, hours worked, and consequently output and consumption, decrease more in response to an interest rate shock. The government minimizes distortions over time, so it decreases the tax rate after a positive interest rate shock. Nevertheless, the tax rate decreases less than in the benchmark model because a higher deficit has to be financed. Public debt also grows more than in the baseline model. Hours worked fall more, not only because the wage elasticity of labor supply increases, but also because the tax rate falls less than in the baseline model. Consequently, output and consumption also fall more than in the benchmark model, while the wage rate falls less. The impulse response functions of the tax rate and the primary deficit change sign after some periods to finance the additional debt interests. The trade balance increases less than in the baseline model because output falls more, and the current account deteriorates more than in the benchmark case. Public debt displays again the most persistent impulse response function.
Figure 4: Impulse Responses to an Interest Rate Shock $\nu = 1.1$
Figure 4 (Cont): Impulse Responses to an Interest Rate Shock $\nu = 1.1$
The following table reports the volatility, correlations, and autocorrelations of key macroeconomic variables under the optimal fiscal policy for a value of $\nu$ of 1.1.

Table 3 reveals some interesting results:

1. Increasing the wage elasticity of labor supply, increases the volatilities of all the variables. The reason is that an increase in the interest rate paid by firms on working capital induces a fall in hours that depends mainly on the wage elasticity of the labor
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<th>Variable</th>
<th>Std. Dev.%</th>
<th>Autocorr.</th>
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<th>Corr($x, g$)</th>
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<td>Public Debt</td>
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<td>0.69</td>
<td>-0.55</td>
<td>0.72</td>
<td>-0.06</td>
<td>-0.63</td>
</tr>
<tr>
<td>Output</td>
<td>8.27</td>
<td>0.62</td>
<td>0.92</td>
<td>-0.16</td>
<td>-0.33</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>7.55</td>
<td>0.60</td>
<td>0.81</td>
<td>-0.24</td>
<td>-0.49</td>
<td>0.97</td>
</tr>
<tr>
<td>Hours</td>
<td>8.76</td>
<td>0.61</td>
<td>0.84</td>
<td>-0.21</td>
<td>-0.46</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Table 3: Moments $\nu=1.1$

demand $\frac{1}{\eta-1}$ and on the wage elasticity of the labor supply $\frac{1}{\nu-1}$. The lower is $\nu$ or the higher is $\eta$, the more hours fluctuate in response to interest rate shocks. Consequently, output and consumption also become more volatile. The volatilities of the tax rate and of the public debt also increase.

2. The correlations of the tax rate and of public debt with the interest rate increase when we increase the wage elasticity of labor supply, since the tax rate decreases less, and the public debt increases more in response to interest rate shocks than in the benchmark model.

5.4.2 Fraction of the wage bill paid in advance

In this section, I evaluate how the results change if we assume that firms only have to pay half of the wage bill in advance. The log-linearized equilibrium conditions after an interest
An interest rate shock affects less the wedge between the consumption-leisure marginal rate of substitution and its marginal rate of transformation when firms have to pay only half of the wage bill in advance, so the government decreases the tax rate less than in the baseline model. The labor demand and labor supply curves shift less than in the benchmark model, so the wage rate falls less after an interest rate shock. Hours worked fall, because the negative effect of an increase in the interest rate on hours is greater than the positive effect of a decrease in the tax rate. Nevertheless, hours worked decrease less than in the baseline model, since a 1 percentage point increase in the interest rate decreases hours in \( \left( \frac{1}{\nu - 1} \right) \%) \) in the benchmark model, while it decreases hours in \( \left( \frac{1}{\nu - 1} \right) \frac{\tau}{1 + R} \%) \) when \( \theta = 0.5 \). Output and consumption also decrease less than in the baseline model, because hours decrease less. Tax revenues also decrease less than in the benchmark model, so the primary deficit, and public debt also increases less. The impulse response functions of the tax rate, tax revenues and the primary deficit, change sign after some periods to pay for the additional debt interests.

The trade balance increases more than under the baseline parameterization, because output
decreases less when $\theta = 0.5$, and the current account deteriorates less than in the benchmark model. Public debt and the tax rate display the most persistent impulse response functions.

Figure 5: Impulse Responses to an Interest Rate Shock $\theta = 0.5$
Figure 5 (Cont): Impulse Responses to an Interest Rate Shock $\theta = 0.5$
The following table reports the volatility, correlations and autocorrelations of key macroeconomic variables under the optimal fiscal policy for a value of $\theta = 0.5$.

Table 4 also reveals some interesting results:

1. The standard deviations of all the variables decrease. The reason is that when we reduce the fraction of the wage bill that firms have to pay in advance, the negative impact that interest rate shocks have on labor demand decreases. Thus, hours worked
<table>
<thead>
<tr>
<th>Variable</th>
<th>Std. Dev.%</th>
<th>Autocorr.</th>
<th>Corr(x, z)</th>
<th>Corr(x, g)</th>
<th>Corr(x, r)</th>
<th>Corr(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Public Debt</td>
<td>24.20</td>
<td>0.94</td>
<td>-0.06</td>
<td>0.16</td>
<td>0.07</td>
<td>-0.13</td>
</tr>
<tr>
<td>Tax Rate</td>
<td>2.90</td>
<td>0.70</td>
<td>-0.18</td>
<td>0.89</td>
<td>-0.08</td>
<td>-0.28</td>
</tr>
<tr>
<td>Output</td>
<td>4.64</td>
<td>0.65</td>
<td>0.98</td>
<td>-0.10</td>
<td>-0.13</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>3.65</td>
<td>0.60</td>
<td>0.78</td>
<td>-0.24</td>
<td>-0.52</td>
<td>0.88</td>
</tr>
<tr>
<td>Hours</td>
<td>3.19</td>
<td>0.63</td>
<td>0.92</td>
<td>-0.21</td>
<td>-0.28</td>
<td>0.97</td>
</tr>
</tbody>
</table>

Table 4: Moments theta=0.5

and output decrease less than in the benchmark model in response to an interest rate shock. Since the volatility of the tax base decreases, the volatilities of the tax rate and of the public debt decrease as well.

2. The absolute values of the correlations of hours worked, output and consumption with the interest rate decrease when we decrease \( \theta \), since the negative impact that interest rate shocks have on labor demand decreases. Since the tax base becomes less correlated with the interest rate, the absolute value of the correlations of public debt and of the tax rate with the interest rate also decrease.

5.4.3 Serial Correlation Properties of the Shocks

In this section, I evaluate how the results change when we consider serially uncorrelated shocks. Table 5 reports the volatility, correlations and autocorrelations of key macroeconomic variables under the Ramsey policy when interest rate shocks are uncorrelated, table 6 shows the case of uncorrelated productivity shocks and table 7 the case of uncorrelated government
spending shocks.

The moments in tables 5, 6 and 7 reveal the same result as before: the optimal fiscal policy smooths distortions over time. Public debt is very persistent irrespective of the degree of autocorrelation of the underlying shocks. The results also show that the tax rate is quite persistent independently of the assumed processes for the shocks generating aggregate fluctuations; this reflects the planner’s desire to smooth tax distortions over time. The planner finances any innovation to government purchases or to the tax base partly by
Table 7: Moments: Uncorrelated Government Spending Shocks

increasing public debt and partly by increasing the tax rate. Therefore, public debt plays an important role as a shock absorber in this model.

6 Conclusions

I have characterized the properties of optimal fiscal policy in a small open economy where interest rates, government spending and productivity are stochastic, taxes are distortionary and markets are incomplete. This paper provides a new framework to analyze optimal fiscal policy in small open emerging economies, and extends the existing literature on optimal fiscal policy by studying the case of incomplete markets in a small open economy under uncertainty and distortionary taxation.

The main contributions of the paper are the following. First, I solve the Ramsey problem for a small open economy with incomplete markets and stochastic interest rates, government spending and productivity. I show that if we restrict the agents in a small open economy to
buy and sell only one-period non-contingent real bonds, the Ramsey planner is confronted with stochastic sequences of implementability constraints that arise from the requirement that the debt be risk-free. Since conditional expectations of future variables appear in these implementability constraints, the Ramsey problem is not recursive. However, I show that it is possible to recover a recursive formulation using the recursive contracts approach of Marcet and Marimon (1998).

Second, I show that the optimal fiscal policy smooths distortions over time. When the government can only issue one-period non-contingent debt, the market value of outstanding debt is completely independent of the realization of the state of nature, so the government needs to adjust the tax rate and the public debt in response to shocks that affect government spending or the tax base. The tax rate, and specially the public debt are very persistent irrespective of the degree of autocorrelation of the shocks that generate aggregate fluctuations. This reflects the planner’s desire to smooth distortions over time. Debt plays an important role as a shock absorber. After a positive innovation in the interest rate or in government spending, or after negative innovation in productivity, the level of public debt and the primary deficit increase. Government debt responds on impact less than the other variables, but it accumulates over time, and displays the most persistent impulse response function. The responses of debt and the primary deficit have the same sign in the first periods. However, the response of the primary deficit changes sign after a few periods, because a higher debt interest will have to be serviced in the future in response to the increase in debt.
7 References


8 Appendix

**Proposition.** Given the initial conditions $d_{-1}, b_{-1}$, and the exogenous stochastic processes

$\{g(s^t), z(s^t), q(s^t)\}_{t=0}^\infty$, state-contingent plans $\{c(s^t), h(s^t), d(s^t), b(s^t)\}_{t=0}^\infty$ satisfy (5)

$$c(s^t) + d(s^{t-1}) + \psi(d(s^t)) = q(s^t) d(s^t) - \frac{u_h(s^t)}{u_c(s^t)} h(s^t) \left[ 1 + (1 + \theta(R(s^t) - 1)) \frac{1 - \eta}{\eta} \right]$$

(14)

$$q(s^t) (d(s^t) + b(s^t)) + z(s^t) f(h(s^t)) \left[ \frac{\eta}{1 + \theta(R(s^t) - 1)} + 1 - \eta \right] =$$

(15)

d(s^{t-1}) + b(s^{t-1}) + c(s^t) + g(s^t) + \psi(d(s^t)) + \psi(b(s^t))

if and only if they satisfy (2), (4), (5), (8), (9) and (11).

**Proof.** First, I show that if state contingent plans $\{c(s^t), h(s^t), d(s^t), b(s^t)\}_{t=0}^\infty$ satisfy (2), (4), (5), (8), (9) and (11), then they also satisfy (14), (15), and (5). To obtain (14), solve (8) for $w(s^t)$, (11) for $\Pi(s^t)$, and (4) for $\tau(s^t)$. Then, use the resulting expressions to eliminate $w(s^t)$, $\Pi(s^t)$, and $\tau(s^t)$ from (2). The resulting equation is (14). To obtain (15), substitute (2) into (9), then solve (8) for $w(s^t)$, and (11) for $\Pi(s^t)$, and use the resulting
expressions to eliminate \( w(s^t) \), and \( \Pi(s^t) \) from this equation. The resulting expression is (15). Now, it must be shown that if state-contingent plans \( \{c(s^t), h(s^t), d(s^t), b(s^t)\}_{t=0}^{\infty} \) satisfy (14), (15), and (5), then they also satisfy (2), (4), (5), (8), (9) and (11). Set \( w(s^t) \) such that (8) holds, \( \tau(s^t) \) such that (4) holds and \( \Pi(s^t) \) such that (11) holds. Therefore, (4), (8), and (11) are satisfied by construction. Then, substitute \( w(s^t), \tau(s^t), \) and \( \Pi(s^t) \) in (14) to obtain (2). Finally, substitute \( w(s^t), \tau(s^t), \) and \( \Pi(s^t) \) in (14) and (15), and combine the resulting expressions to obtain (9). □