Colombian economic growth under Markov switching regimes with endogenous transition probabilities

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ABSTRACT

In this paper, we modelled the Colombian long-run per capita economic growth (1925-2005) using a Markov switching regime model with both fixed (FTP) and time-varying transition probabilities (TVTP) to explain regime changes in the economic growth. We found evidence of nonlinearity in the per capita economic growth, and identified two different levels in the data associated with depression and sustainable growth regimes. In addition, the hypothesis of fixed probabilities is rejected in favour of the time-varying transitional probabilities, which mean that the correct model is the one with endogenous probabilities, when the probability of remaining in the sustainable growth regime increases with a rise in terms of trade and government expenditures; decreases with capital outflows. Increases in government expenditures and terms of trade decrease the probability of being in the depression state while an increase in capital outflows raises the probability. Finally, we found that the TVTP model gives more information than the FTP model because the probabilities have changed significantly during the period under analysis and the explanatory variables are very informative in dating the evolution of the state of the economy, especially those associated with external shocks.

JEL classification: O40, C22, E32, N16

Keywords: Markov endogenous switching regime model, time-varying transition probabilities, economic growth, Colombia

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1. Introduction

Following Hamilton (1989), the Markov switching regime model (MSRM) has become extensively used to study nonlinearities in economic growth. Nonlinearities are important since real output responds to a shock in a very different way depending on which state the economy is in; i.e. an expansion or a contraction. As Hamilton and Raj (2002) point out, the purpose of the regime switching model is to capture the asymmetry presented in the business cycle. The transition from one state of the cycle to another is modelled as a regime switch, and the probability of changing regime is inferred from the data. In the Hamilton’s original model, the transition probabilities were constants. However, constant or fixed transition probabilities are too restrictive to explain the behaviour of economic growth since economic variables are not allowed to affect transitional probabilities.

An extension of Hamilton (1989) allows time-varying transition probabilities. As explained by Filardo (1994) and Diebold et al. (1999), the Markov switching model with time-varying transition probability (TVTP) has the advantage over the fixed transition probabilities (FTP) in terms of flexibility. It can recognize systematic changes in the transition probabilities before and after turning points, capture more complex temporal persistence and allow expected duration to vary across time. In this context, economic fundamentals and policy shocks can influence the regime transition probabilities.

In this paper we modelled Colombian long-run per capita economic growth using a Markov switching regime model with time-varying transition probabilities. This is an extension of Misas and Ramírez (forthcoming 2007), which used a first-order MSRM with fixed transition probabilities to study Colombian’s long run economic growth. We found evidence of nonlinearity in the annual rate of growth. Two different states, depression and sustainable growth were identified, and transitions between them were
sudden and sporadic. The results show that the economy remained in the sustainable regime for most of the period and the turning points from the Markov switching model adequately capture the behaviour of Colombian real output through time.

Given these results, the purpose of the present paper is to determine the effect of some economic variables over the transition probabilities to explain regime changes in Colombian economic growth. To this end, we allow probabilities to be affected by policy variables, and analyse the asymmetric influence of these variables on the different growth regimes. Transition probabilities can be affected by exogenous shocks and economic policies.

Our main results can be summarized as follows: the hypothesis of fixed probabilities is rejected in favour of the time-varying transitional probabilities, which means that the correct model is the one with endogenous transition probabilities. The TVTP model is superior to the FTP model since the probabilities have changed significantly during the period under analysis and the explanatory variables are very informative in dating the evolution of the state of the economy. The probability of remaining in the sustainable growth regime increases with a rise in terms of trade and government expenditures, decreases with capital outflows. Increases in government expenditures and terms of trade also decrease the probability of being in the depression state, while an increase in capital outflows raises such probability.

The paper is organized as follows. Section 2 describes the econometric model, section 3 presents the data and some stylized facts of Colombian long-run per capita economic growth. Section 4 reports and discusses the estimation results, first those from the model with fixed transition probabilities and then those from the model with time-varying transition probabilities. Section 5 concludes.
2. The Markov switching regime model with time-varying transition probabilities

We begin by applying the basic Hamilton (1989) model to the Colombian per capita economic growth. Let $y_t$ be the real per capita GDP annual rate of growth and $s_t$ an unobserved discrete variable that represents the state or regime the economy is in; we assume two states: 0 = depression and 1 = sustainable growth, such that:

$$s_t \in \{0,1\}$$

$$\varepsilon_t \sim N\left(0, \sigma^2_{s_t}\right)$$

$$\sigma^2_{s_t} = \sigma^2_0 + \sigma^2_1 s_t, \quad \sigma^2_0 > 0$$

$$\phi_{s_t} = \phi_0 + \phi_1 s_t$$

$$P[s_t = 0|s_{t-1} = 0] = p \quad P[s_t = 1|s_{t-1} = 1] = q$$

$$P[s_t = 1|s_{t-1} = 0] = 1 - p \quad P[s_t = 0|s_{t-1} = 1] = 1 - q$$

From equation (1), the growth rate ($y_t$) depends on $X_t$, which includes lags of $y_t$ and an iid random variable ($\varepsilon_t$), which follows a normal distribution with zero mean and $\sigma^2_{s_t}$ state-dependent variance (equations 2 and 3). Equation 4 describes the behaviour of the parameter $\phi_{s_t}$, which is also state-dependent. In equation 5 we specify that the switching of regimes follows a first-order Markov chain. Probabilities are noted by $p$ and $q$, where $p$ is the probability of remaining in state 0 at $t$, given that the economy is in regime 0 at $t-1$, and $q$ is the probability of staying in regime 1 at $t$, given that the economy is in state 1 at $t-1$; $1-p$ and $1-q$ are the transition probabilities for switching from one regime to the other.
In the basic Hamilton (1989) model, transition probabilities are assumed to be fixed; in that case, they are very restrictive in explaining changes in regimes. For this reason, we allow transition probabilities to depend on some macroeconomic variables in order to explain the probability of switching from one regime to another. We then analyse the results to determine whether changes in the economic variables can cause changes of regimes.

Following Diebold et al. (1999), we endogenized probabilities of changes of regime by incorporating economic variables as their determinants. Then, equation (5) becomes:

\[
P_{t}^{00} = \frac{\exp(Z'_{t-1} \beta_0)}{1 + \exp(Z'_{t-1} \beta_0)}
\]

\[
P_{t}^{01} = 1 - P_{t}^{00}
\]

\[
P_{t}^{10} = \frac{\exp(Z'_{t-1} \beta_1)}{1 + \exp(Z'_{t-1} \beta_1)}
\]

\[
P_{t}^{11} = \frac{1}{1 + \exp(Z'_{t-1} \beta_1)}
\]

where \(Z_{t-1}\) is a set of information variables. The transition probabilities are modelled as a logistic functional form such as (7):

Note: \(Z_{t-1} = \{1, Z_{1t-1}, Z_{2t-1}, \ldots, Z_{(k-1)t-1}\}\) y \(\beta = (\beta_{0}, \beta_{1}, \ldots, \beta_{(k-1)})\)
To estimate this regime switching model, we must specify the complete data likelihood function. Following Diebold et al. (1999), let $y_t$ be the sample path of a time series conditional upon $s_t$ as follows:

$$
(y_t|s_t = i; \alpha_i) \sim N(\mu_i, \sigma_i^2) \quad \text{where} \quad \alpha_i = (\mu_i, \sigma_i^2) \quad i = 0, 1 \quad (8)
$$

Thus, the conditional density of $y_t$ is:

$$
f(y_t|s_t = i; \alpha_i) = \frac{1}{\sqrt{2\pi \sigma_i}} \exp\left(-\frac{(y_t - \mu_i)^2}{2\sigma_i^2}\right) \quad i = 0, 1 \quad (9)
$$

In the likelihood function, a quantity of particular interest is $P(s_1)$, which denotes $P(S_1 = s_1)$ or the unknown initial state of the system. Regarding $z_t$, there are two cases to consider, stationary and nonstationary. When conditioning variables are stationary, $P(s_1)$ is simply the long-run probability of $S_1 = s_1$, which in turn is determined by $\beta$.

However, when the conditioning variables are nonstationary, the long-run probability does not exist and $P(S_1 = s_1)$ must be treated as an additional parameter that needs to be estimated. Diebold et al. (1999) show that $P(S_1 = 1)$, which denotes $(\rho)$ is all that is needed to construct the first likelihood term.

Let $\theta = (\alpha', \beta', \rho')$ be the vector of all model parameters. The complete data likelihood function for a sample of size $T$ can be expressed as:

$$
f(y_T, s_T; z_T; \theta) = f(y_1, s_1; z_T; \theta) \prod_{t=2}^{T} f(y_t, s_t, y_{t-1}, s_{t-1}, z_T; \theta) \prod_{t=1}^{T} f(y_t, s_t, y_{t-1}, s_{t-1}, z_T; \theta) P(s_1) \prod_{t=1}^{T} f(y_t, s_t, y_{t-1}, s_{t-1}, z_T; \theta) X P(s_T|y_{T-1}, s_{T-1}, z_T; \theta)
$$
where \( f \) denotes any density and \((\leftarrow)\) denotes past history of the variable from \( t=1 \) to the variable subscript. Taking logs, the expression given above can be written in terms of indicator functions for the states, \( I(s_t) \). Thus, the complete data log likelihood function becomes:

\[(11)\]

\[
\log f\left(y_t, s_t \left| z_T; \theta \right.\right) = I(s_t = 1)\left[\log f\left(y_t \left| s_t = 1; \alpha_t \right.\right) + \log \rho\right] \\
+ I(s_t = 0)\left[\log f\left(y_t \left| s_t = 0; \alpha_0 \right.\right) + \log(1 - \rho)\right] \\
+ \sum_{s_{t-1}} I(s_t = 1, s_{t-1} = 1)\log\left(p_{i1}^{01}\right) \\
+ I(s_t = 0, s_{t-1} = 1)\log(1 - p_{i1}^{01}) \\
+ I(s_t = 1, s_{t-1} = 0)\log(1 - p_{i0}^{01}) \\
+ I(s_t = 0, s_{t-1} = 0)\log\left(p_{i0}^{00}\right)
\]

The complete data log likelihood cannot be constructed in practice because it is not observed. However, the incomplete data log likelihood function can be obtained by summing over all possible state sequences, as in equation (12), and then maximizing with respect to the vector of parameters \( \theta \).

\[
\log f\left(y_t; z_T; \theta \right) = \log\left(\sum_{s_t = 0}^{1} \sum_{s_t = 0}^{1} \sum_{s_t = 0}^{1} f\left(y_t, s_t \left| z_T; \theta \right.\right)\right) (12)
\]

As mentioned by Filardo (1994) and Diebold et al. (1999), in the TVTP model the vector of parameters \( \theta \), which includes the mean and variances of each state \( (\mu_i, \sigma_i^2) \forall i = 0, 1 \), the transition probabilities \( (p_{i0}^{00}, p_{i1}^{11}) \), their determinants \( (\beta) \) and the initial conditions \( (\rho) \) are jointly estimated with Maximum Likelihood (ML) methods.

In this particular case, we used the Expectation Maximization (EM) algorithm, which is a very general iterative procedure for maximising the incomplete data log likelihood when
some of the random variables involved are not observed. The EM algorithm proceeds, from an analytical point of view, in two stages. In the first, expectations of the smoothed transition probabilities are computed, conditional upon the parameters. In the second, the parameters are updated, conditional upon those smoothed transition probabilities. The algorithm iterates until the maximisation is reached.

3. Data

The above models are estimated with annual data of the real per capita GDP growth for the period 1925-2005 as a proxy of $y_t$ (graph 1). For information variables in $z_t$, we chose those that we consider to have been the main causes of variation in Colombian per capita economic growth during the twentieth century. Some variables reflect external shocks, such as terms of trade, international coffee prices, capital inflows and the USA real per capita GDP growth, and some comprise fiscal and monetary shocks. Graph 2 presents the evolution of these variables.

**Graph 1**

Colombian real per capita GDP growth (%): 1925-2005

![Graph showing Colombian real per capita GDP growth](image)

Source: GRECO and DANE

Colombian real per capita GDP grew, on average 1.9 percent per year between 1925 and 2005. In general, the Colombian economy has stayed on a path of sustainable growth.
particular, positive per capita growth has characterized the Colombian economy for long periods, the longest being between 1959 and 1974. At that time the economy grew, on average, 2.6 percent per annum. However, there have been fluctuations throughout the period. In fact, between 1925 and 2005 the economy has suffered six main slowdowns. The first one, between 1930 and 1931, reflects the worldwide Great Depression. The second, occurred between 1940 and 1943 as a consequence of World War II. The economy again registered negative per capita rates of growth in 1950 and 1951 and between 1957 and 1958. With the decline of international coffee prices at the end of the seventies and the Latin American Debt Crisis, beginning in 1982, the Colombian economy fell by 1.4 per cent in 1982 and 0.8 percent in 1983. Nevertheless, the worst contraction of the twentieth century occurred between 1998 and 1999, as a result of the international financial crisis and the macroeconomic imbalances caused by excessive aggregate demand in previous years. In fact, in 1999, the economy declined in per capita terms by more than 6 percent.

As Misas and Ramírez (forthcoming 2007) shows the causes of fluctuations are principally based on external shocks that influenced the evolution of the terms of trade, capital inflows and international coffee prices, among other factors. As mentioned before, the purpose of the present paper is to determine precisely the effect of these and other economic variables over the transition probabilities in order to explain regime changes in Colombian economic growth.
4. Results

4.1 First step: fixed transition probabilities (FTP)

We first estimate the model using fixed transition probabilities in order to identify regime changes in the Colombian long-run per capita economic growth, along with their duration.
and the probabilities that the economy will remain in each state. We then compare these results with those from the time-varying transition probabilities estimation.

We included in \( X_t \) an intercept and the first four lags of the dependent variable \( y_t \), and a random variable \( \epsilon_t \), with a state-dependent variance. The results show that the lags of the dependent variable are not statistically significant. We then estimated the equation including only the intercept in \( X_t \). The results of this specification indicate that the random variable \( \epsilon_t \) is a state-dependent variance. Therefore, the model for the Colombian per capita economic growth is the following:

\[
\begin{align*}
y_t &= \mu_{sx} + \epsilon_t \quad s_t = 0, 1 \\
\mu_{sx} &= \mu_0(1 - s_t) + \mu_1 s_t \\
\epsilon_t &\sim N(0, \sigma^2_{sx}) \\
\sigma^2_{sx} &= \sigma_0^2(1 - s_t) + \sigma_1^2 s_t \\
P[s_t = 0 | s_{t-1} = 0] &= p \quad P[s_t = 1 | s_{t-1} = 1] = q \\
P[s_t = 1 | s_{t-1} = 0] = 1 - p \quad P[s_t = 0 | s_{t-1} = 1] = 1 - q
\end{align*}
\]

where the per capita GDP growth rate \( (y_t) \) depends on its average level \( \mu_{sx} \), which follows a Markov process, and on \( \epsilon_t \), a state-dependent variance. Switching of regimes follows a first-order Markov chain (equation 17), and \( s_t = 0, 1 \), where \( s_t = 0 \) corresponds to depression and \( s_t = 1 \) to sustainable growth, and \( p \) and \( q \) are the transition probabilities.

Therefore, using annual data of the first difference of the logarithm of real per capita GDP for the period 1925-2005, a first-order two-state Markov switching model was estimated for the Colombian per capita economic growth. In table 1 the maximum likelihood estimates of parameters are presented. As can be observed, all the parameters
are significant at 5 per cent. The results indicate that the two different levels presented in the data, \( \mu_0 \) and \( \mu_1 \), are statistically different and the \( \varepsilon_i \) process is a state-dependent variance.

**Table 1**

Maximum likelihood estimation of the parameters and asymptotic standard errors

<table>
<thead>
<tr>
<th>Parameters ( { \mu_0, \mu_1, \sigma_0^2, \sigma_1^2, p, q } )</th>
<th>Estimation</th>
<th>Standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_0 )</td>
<td>-1.0855</td>
<td>0.7065</td>
</tr>
<tr>
<td>( \mu_1 )</td>
<td>2.5178</td>
<td>0.3356</td>
</tr>
<tr>
<td>( \sigma_0^2 )</td>
<td>4.1955</td>
<td>2.1292</td>
</tr>
<tr>
<td>( \sigma_1^2 )</td>
<td>3.6945</td>
<td>0.7652</td>
</tr>
<tr>
<td>( p = P_{11} )</td>
<td>0.604</td>
<td>0.1723</td>
</tr>
<tr>
<td>( q = P_{22} )</td>
<td>0.9155</td>
<td>0.0575</td>
</tr>
</tbody>
</table>

\[ P(S_i = 1|y_1, \cdots, y_T; \theta) = 0.9906 \]

Objective Function: -200.41

From the estimation, the average annual per capita economic growth is -1.09 percent in regime 0 (depression) and 2.52 percent in regime 1 (sustainable growth). Furthermore, the probability \( (p=P_{11}) \) of staying in a depression at time \( t \), given that the economy is in the same state at time \( (t-1) \), is 0.60. The probability \( (q=P_{22}) \) of being in sustainable growth in time \( t \), given that the economy was in the sustainable growth path at time \( (t-1) \) is large, 0.92 greater than \( (p=P_{11}) \). These high probabilities indicate that if the economy is in either sustainable growth or depression, it is likely to remain in such regime.

In addition, table 2 shows that the probability of switching from a depression state to sustainable growth \( (1-p=P_{12}) \) is almost 0.4, while the probability of changing from
sustainable growth to depression \((1-q=P_{21})\) is close to 0.09, which indicates change from depression to sustainable growth more likely than change from sustainable growth to depression.

### Table 2

Transition Matrix

\[
\begin{bmatrix}
P_{11} & P_{12} \\ P_{21} & P_{22}
\end{bmatrix} = \begin{bmatrix} 0.6040 & 0.3960 \\ 0.0845 & 0.9155 \end{bmatrix}
\]

With these results, the average length of each state can be inferred from equation (18), which represents the expected duration of state 1, and equation (19), which calculates the expected duration of state 0.\(^{12}\)

\[
\sum_{i=1}^{\infty} i \times q^{(i-1)} (1-q) \quad (18)
\]

\[
\sum_{i=1}^{\infty} i \times p^{(i-1)} (1-p) \quad (19)
\]

We found that the average length of sustainable growth is twelve years, whereas the expected duration of a depression regime is approximately three years. Similar results were found in Misas and Ramírez (forthcoming 2007) using the real GDP annual rate of growth. The larger duration of the sustainable growth regime can be explained by the fact that we defined sustainable growth as times when the economy experienced persistent growth, including periods of booms and very small slowdowns.

Graph 3 plots the probability of being in sustainable growth at each date in the sample; i.e., it depicts the evolution of the smoothed probabilities of state 1. The inference is based on the full sample and the estimated maximum likelihood parameters (table 1). The years in which the economy switched from each regime, based on \(P\left( s_i = 1 | y_1, \cdots, y_T ; \Theta \right) \leq 0.5 \), are shown in the graph.
The switching between regimes is sudden, deep and sporadic. The economy stays in sustainable growth most of the time. As we can see, the graph indicates four major changes from sustainable growth to depression in the sample. The first occurred between 1930 and 1931, the period of the Great Depression and the second, between 1940 and 1943 during World War II. After that, the economy experienced a long sustained per capita economic growth until 1981, when a major depression occurred. The last switch from sustainable growth to recession occurred in 1998, when the largest depression of the twentieth century started.

Finally, table 3 presents some specification tests proposed by Hamilton (1996) in order to verify the performance of the model. First, the White autocorrelation test suggests no evidence of autocorrelation. Second, the White specification test indicates that the Markov model not can be rejected against the alternative that there are no changes in regime. Therefore, evidence of nonlinearity in the Colombian economic growth is found. LM tests confirm the results of no autocorrelation. Similar results are also obtained when we examine each regime separately, and the LM test on ARCH effects shows that there is
no indication of the presence of such effects. Summing up, the tests suggest that there is no evidence of model misspecification.

### Table 3
Specification tests

<table>
<thead>
<tr>
<th>Test</th>
<th>Value</th>
<th>P-Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>White autocorrelation test $\chi^2(4)$</td>
<td>5.452</td>
<td>0.243</td>
</tr>
<tr>
<td>White Markov specification test $\chi^2(4)$</td>
<td>4.748</td>
<td>0.314</td>
</tr>
<tr>
<td>LM test on autocorrelation in state 0, $\chi^2(1)$</td>
<td>3.959</td>
<td>0.046</td>
</tr>
<tr>
<td>LM test on autocorrelation in state 1, $\chi^2(1)$</td>
<td>0.481</td>
<td>0.488</td>
</tr>
<tr>
<td>LM test on autocorrelation across states, $\chi^2(1)$</td>
<td>0.663</td>
<td>0.415</td>
</tr>
<tr>
<td>LM test on ARCH effects, $\chi^2(1)$</td>
<td>1.503</td>
<td>0.220</td>
</tr>
</tbody>
</table>

#### 4.2 Second step: time-varying transition probabilities (TVTP)

In a second step, we estimate the model under transitional endogenous probabilities, allowing a set of economic variables to explain the evolution of such probabilities. As mentioned above, from an initial set of possible explanatory variables, we include the real international price of coffee, terms of trade, capital inflows, USA per capita real GDP growth, government expenditures as a percentage of GDP, and the real interest rate. With this information, we established different models to select the one that presents the smooth transition probabilities consistent with the economic history of the country.\textsuperscript{15}

Table 4 presents the results of the selected model, which includes terms of trade, per capita capital inflows, and government expenditure as percentage of GDP. As expected, the external shocks have significantly affected the evolution of GDP growth. The model
was selected based on the gradients and on a likelihood test that compares the Hamilton model, with fixed transition probabilities, with the model with time-varying transition probabilities.16

The initial values associated with $\mu_0, \mu_1, \sigma_0^2, \sigma_1^2$ and $\rho$ are obtained through the Hamilton model. It is worth mentioning, that the algorithm is robust to different initial values of these parameters. In addition, the initial values of the coefficients in the vector $\beta$, equation 7, are taken from the OLS regressions. In those regressions the dependent variable is the smooth probability vector obtained from the Hamilton model, as a proxy for the endogenous transition probabilities and explanatory of the selected variables.

The TVTP estimations from table 4 also indicate that two different states of the economy with magnitudes that differ considerably can be identified: a depression regime with a negative $\mu_0$, and a sustainable growth regime with a positive $\mu_1$. Allowing TVTP the average annual per capita economic growth rates in regime 0 and 1 are similar to those from the FTP estimations.

We observe that the explanatory variables of the transition probabilities present, in almost all cases, the sign suggested by economic intuition. In fact, the probability of remaining in the sustainable growth period increases with a rise in terms of trade and government expenditure, and decreases with capital outflows. If the economy is in depression, an increase in government expenditures and terms of trade decreases the probability of remaining in this state, while an increase in capital outflows increases this probability.17
Following Diebold et al. (1999), we conducted a likelihood test that compares the model of time-varying transition probabilities with the model of fixed probabilities. The first model is the unrestricted one and the second is the restricted one. We obtain a p-value of 1.7E-06, which allows us to reject the null hypothesis of constant probabilities in favor of

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_0$</td>
<td>-0.879</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>2.536</td>
</tr>
<tr>
<td>$\sigma^2_0$</td>
<td>4.338</td>
</tr>
<tr>
<td>$\sigma^2_i$</td>
<td>3.749</td>
</tr>
</tbody>
</table>

**Regime 0 (depression)**

| $\beta_{00}$ (intercept) | 0.175               |
| $\beta_{01}$ (terms of trade) | 3.032              |
| $\beta_{02}$ (capital flows)  | -0.174             |
| $\beta_{03}$ (government expenditures) | 4.676E1         |

**Regime 1 (sustainable growth)**

| $\beta_{10}$ (intercept) | -1.947             |
| $\beta_{11}$ (terms of trade) | 1.337              |
| $\beta_{12}$ (capital flows)  | 0.663              |
| $\beta_{13}$ (government expenditures) | -1.056E2    |
| $\rho$                     | 0.98               |

Convergence: 8.16E9  
Objective Function: -185.56
the TVTP model. Therefore, the right model is the one with endogenous transition probabilities.

Graph 4 presents the smoothed TVTP obtained from the model of endogenous probabilities. As we see, the length of both depression and sustainable growth states are longer than in the FTP (graph 3). In particular, the probabilities of being in sustainable growth decrease earlier in time in the TVTP model than in the FTP model. It also captures the date of the turning points better. Therefore, the variables included in the set of explanatory variables, \( z_t \), are informative in dating the state of the economy.

Graph 4

Probabilities of being in sustainable growth (state 1) TVTP

Finally, graph 5 shows the time-varying transition probabilities against the fixed probabilities from Hamilton’s model. As we can see, terms of trade, capital inflows and government expenditures are important determinants of transition probabilities, and of the switching between regimes. Clearly, the TVTP model gives more information than the FTP model since the probabilities have changed significantly during the period under analysis.
5. Conclusions

In this paper, we modelled the Colombian long-run per capita economic growth using a Markov switching regime model with both fixed transition probabilities and time-varying transition probabilities to explain regime changes in Colombian economic growth. In particular, in the model of endogenous probabilities, we allowed probabilities to be affected by policy variables to analyze the asymmetric influence of these variables on the different growth states.

Our findings can be summarized as follows. First, we find evidence of nonlinearity in the per capita economic growth. Second, we can identify two different levels in the data that can be associated with depression and sustainable growth regimes. Third, changes between regimes are sudden, deep and sporadic. Fourth, the hypothesis of fixed probabilities can be rejected in favor of the time-varying transitional probabilities, which means that the correct model is the one with endogenous transition probabilities. Fifth, external shocks have affected the evolution of the economy significantly. The probability of remaining in the sustainable growth regime increases with a rise in terms of trade and
government expenditures and decreases with capital outflows. Also, if the economy is in
the depression state, an increase in government expenditures and terms of trade decreases
the probability of remaining in this state, while an increase in capital outflows increases
this probability. Finally, we find that the TVTP model is superior to the FTP model since
the probabilities have changed considerably during the period under analysis and the
explanatory variables are very informative in dating the evolution of the state of the
economy.
REFERENCES


Bureau of Economic Analysis, [www.bea.gov](http://www.bea.gov)


Notes


2 See Hamilton (1989)


4 For details see Filardo (1994) and Diebold et al (1999).


6 Most of the studies of Colombian business cycle assumes that the growth rate follows a linear process, for instance see Posada (1999), Fernández and Gonzáles (2000) and Urrutia and Fernández, (2003). The exceptions are Arango and Melo (2006) and Misas and Ramírez (forthcoming 2007).

7 This notation follows closely Misas and Ramírez (forthcoming 2007)


9 The review of the Colombian business cycle literature was useful to identify the variables that were used in the estimation. See Ocampo (1987), Posada (1999), Fernández y Gonzáles (2000), and Misas and Ramírez (forthcoming 2007), among others.

10 See Misas and Ramírez (forthcoming 2007)

11 Similar estimation of the first step was presented in Misas and Ramírez (2006). However, they estimated the model using real GDP growth instead of real per capita GDP growth.

12 For practical reasons, equations (18) and (19) are truncated in 100.000.

13 White autocorrelation test verifies the score correlation at time $t$ with respect to $\mu_i$ and the score of time $(t-1)$ with respect to $\mu_j$ with $i, j = 1, 2$.

14 The Markov assumption that $P(s_t = i)$ depends only of the state in $(t-1)$ can be tested against two alternatives hypothesis: (i) $P(s_t = i)$ depends on several previous states or (ii) $P(s_t = i)$ depends on the realization of $y_{t-1}$. The test verifies if the score with respect to the transition probabilities can be forecasted by its lags or by the score with respect to the average.

15 The estimation was made by a code developed by the authors in SAS version 9, IML procedure. In addition, Gretchen C. Weinbach provided us with her original code in Matlab, which was used to cross check our code.

16 It is important to mention that the likelihood function (equation 11) in our paper presents some issues regarding the indeterminacy of the logarithmic function. This problem was overcome by replacing the indeterminacy by values close to zero or one. However, this fact did not allow us to construct the standard deviation associated with the estimated parameters. We are working in this direction to overcome this issue.

17 The magnitude of the capital flows coefficients is due to the levels that this variable presents. During almost all the period under analysis, this variable corresponds mainly to capital outflows. This supports the sign of the coefficients in the estimation procedure.