Systemic banks and the lender of last resort*

Jorge Ponce† Marc Rennert‡

July 2, 2012
First version: October 2011
Preliminary and incomplete

Abstract

This paper proposes a model where systemic and non-systemic banks are exposed to liquidity shocks and an intervention of a lender of last resort is required because external sources of funding dried up. Troubles in a systemic bank may hurt non-systemic banks but not vice versa. We analyze the decision of the central banker and an unconditional bail-out rule to provide emergency liquidity assistance to illiquid banks whose solvency conditions are only observed through supervision. It is optimal to share the responsibilities between the central banker and the unconditional bailout rule. We find that the existence of systemic banks provides a rationale for the central banker to act as lender of last resort for the non-systemic bank in a larger range of liquidity shortfalls. The impact of the systemic risk on the allocation of responsibilities for systemic banks is ambiguous because the first-best as well as the central banker’s lending decision are less restrictive.

1 Introduction

Two lessons can be learned from the recent financial crisis. First, interbank markets can collapse so that even solvent banks are unable to borrow required funds to finance their

---

*We thank Bruno Biais, Fany Declerck, Dominik Grafenhofer, David Heilmann, Sebastien Pouget, Markus Reisinger and all seminar participants at the Toulouse School of Economics for their comments. All errors remain our own. The views expressed herein are those of the authors and do not necessarily represent the views of the institutions to which they are affiliated.

†Banco Central del Uruguay

‡Deutsche Bundesbank and IAE Toulouse
operations. Second, large, highly interconnected financial institutions are a source for the fragility of the financial system. The academic literature addressed both issues before the crisis. Concerning the first lesson Bagehot (1873) points out that in these circumstances a lender of last resort should provide emergency liquidity assistance to solvent banks. Rochet and Tirole (1996) stressed the risk arising from interconnected banks when they show that interconnectedness can lead to financial contagion so that other banks and the financial system can be affected by the failure of a highly interconnected financial institution.

The policy response to the Subprime crisis consisted of emergency interventions and structural reforms of the regulatory framework. During and after the crisis governments and central banks stepped in and supported banks with significant amount of liquidity independent of their solvency condition to stabilize the financial system and to prevent further contagion effects. In Europe e.g. governments mobilized liquidity assistance adding up to around 30% of its GDP. The structural reforms of the regulatory framework focused on two distinct objectives: the enhanced resilience of systemically important banks to reduce the likelihood of a failure and second the resolution of distressed systemically important financial institutions in an orderly manner. But the implications of systemically important banks for the design of the lender of last resort policy to provide funding to banks in case external sources of liquidity dried up has not received much attention. Although it is a very important issue for policy makers because despite implemented regulations after the Subprime crisis like the Dodd-Frank Wall Street Reform and Consumer Protection Act in the US till this day systemically important banks remained systemically important or

---

1Among others Brunnermeier (2008) and Mishkin (2010) describe the evolution of the financial crisis and its main events. Gorton and Metrick (2011) show that during the crisis a run on repo market occurred. Increasing haircuts of bilateral repo transactions combined with declining asset values increased reduced the funding capacity of the banking sector. Copeland, Martin, and Walker (2011) argue that also tri-party repo markets suffer a run, because the amount of funding decreased sharply. Acharya and Merrouche (2010) provide evidence for the liquidity hording and the effect on overnight interbank rates during the sub-prime crisis of 2007-08.

2Several papers like Acharya, Brownlees, Engle, Farazmand, and Richardson (2010) measured the systemic risk of individual financial institution during and after the Subprime crisis. They show that financial institution like Lehman Brothers, Merrill Lynch, Bear Stearns or AIG imposes a large systemic risk for the financial system.

3http://europa.eu/legislation_summaries/internal_market/single_market_services/financial_services_banking/mi0062_en.htm

4The Basel Committee on Banking Supervision and the Financial Stability Board published on the 4th of November 2011 press releases presenting specific requirements for globally systemically important banks (http://www.bis.org/press/p111104.htm) and http://www.financialstabilityboard.org/press/pr111104cc.pdf). They include higher capital requirements for systemically important institutions, elaboration of and coordination on recovery and resolution plans and more efficient supervision.
even grew larger.5

We offer a model to analyze the implications of the existence of systemically important banks for the design of the lender of last resort (LLR) policy. In our analyses we take for granted that other sources of external funding are not available. The Subprime crisis supports our assumption, but also several theoretical papers (Allen, Carletti, and Gale (2009), Flannery (1996), Freixas and Jorge (2008), Rochet and Vives (2004)) show that due to market imperfections like asymmetric information interbank markets may achieve only a second-best allocations and public interventions of a lender of last resort improve the allocation.

In our approach a systemic and a non-systemic bank coexist. Both banks engage in maturity transformation. They invest their demand deposits into risky illiquid long term assets. The banks are interconnected because a failure of the systemic bank may hurt the return of the non-systemic bank but not vice versa.6 A bank failure might occur because banks are exposed to liquidity shocks by the random withdrawal of demand deposits.7 In this situation only an emergency liquidity loan from the lender of last resort can ensure the bank’s continuation because other sources for external liquidity like interbank markets stop functioning. It is social optimal to provide emergency liquidity assistance if the illiquid bank has high quality asset. But the asset quality is a non-verifiable information of the lender of last resort based on qualitative assessment of supervisory information.

The policy maker must announce ex-ante the allocation of lender of last resort responsibilities with the objective to maximize social welfare. Assuming that the liquidity shortfall is verifiable the policy maker will allocate the responsibilities conditional on the size of the liquidity shock. Either the central banker can perform this task or an unconditional bailout rule can be apply which instructs the central banker to provide the emergency liquidity assistance independent of the bank’s solvency condition. The central banker’ objective does not coincide with the objective of the policy maker. She is concerned about her expected utility from the lender of last resort activities because she incurs monetary losses and political costs when a bank under her mandate fails. For this reason she will use the information about the asset quality to maximize her expected utility. With increas-


6Our assumption can be interpreted as banks’ exposure to counterparty risk in interbank markets or payment systems.

7The reason for the withdrawal of deposits is not modeled in detailed in this paper because the incentive of depositors are out of the scope. The liquidity shock can be interpreted in the spirit of Diamond and Dybvig (1983) with uncertainty about the consumption preferences of consumers.
ing liquidity shocks the central banker requires a higher asset quality because the central banker’s exposure increases with the size of the emergency liquidity assistance.

We show that in this framework it is optimal to share the responsibilities between the central banker and the unconditional bailout rule. The second-best optimal allocation of the lender of last resort responsibilities consists of two intervals. For small liquidity shocks the central banker should be assigned with the lender of last resort responsibility. For larger liquidity shortfalls the unconditional bailout rule should be applied. But the existence of systemic banks provides a rational for the central banker to act as the lender of last resort for the non-systemic bank with extended mandate. While the implication of the systemic risk for the allocation concerning the systemic bank is ambiguous.

The intuition for these results is the following. Given the expected losses of the central banker increase with the size of the emergency loan the central banker is too soft for small and too restrictive for large liquidity shocks. The unconditional bailout rule is always too soft because it does not require any minimum asset quality. There exists a liquidity shock where both achieve the same level of social welfare. Above this threshold the unconditional bailout rule dominates the central banker’s lending decision in terms of social welfare because the central banker is too tough. Below this threshold the central banker dominates since her lending decision is closer to the first-best solution.

The range of action for the central banker and the unconditional bailout rule differs between the non-systemic and the systemic bank. For the non-systemic bank the segmentation of the responsibilities depend on the state of the systemic bank. A failure of the systemic bank fails decreases the expected return of the non-systemic bank. This implies that from a first-best point of view the minimum requirement for asset quality is higher so that the central banker’s lending decision is closer to the first-best provision of emergency liquidity over a larger set of liquidity. The central banker obtains more responsibilities.

The division of responsibilities for the systemic bank is ambiguous because there are two counteracting effects. First, due to the systemic risk it is socially preferred to be more forbearing with the systemic bank. Ceteris paribus this implies more responsibilities for the center banker. But the central banker herself will be less strict to avoid the extended mandate for the non-systemic bank. Everything else constant this implies that the central banker should receive less responsibilities.

a unified regulatory architectures where the lender of last resort is combined with the de-
posit insurance in a single regulator dominates an architecture with separate agencies in a
framework with systemic and non-systemic banks. They find that a unified regulatory is
on the one hand more forbearance towards the systemic institution but on the other hand
can reduce systemic risk.

Our model differs in several points from Espinosa-Vega, Kahn, Matta, and Sole (2011).
First, we are interested in the optimal institutional allocation of lender of last resort re-
sponsibilities between the central banker and the unconditional bailout rule. Second, we
model the impact of a systemic bank in a different way. Espinosa-Vega, Kahn, Matta, and
Sole (2011) assume that a failure of the systemic bank reduces the probability of success
for the non-systemic bank. In our approach the failure of the systemic bank reduces the
return of the non-systemic bank. Both approaches lead to lower expected returns given
the closure of the systemic bank. Even if not modeled in detail we propose the following
interpretation based on banks’ exposures in interbank markets or payment systems. With
a closure of the systemic bank she defaults on its interbank or payment system claims. As
a consequence the non-systemic bank’s asset e.g. a portfolio consisting of several assets
classes thereunder interbank or payment system claims against the systemic bank yields a
lower return. Third, Espinosa-Vega, Kahn, Matta, and Sole (2011) assume like Kahn and
Santos (2005) that the regulator’s political cost of a bankruptcy exceeds the social cost of
a failure. This lead by construction to more forbearance by the regulators compared to the
first-best level. In our model the regulator’s policy cost of a bankruptcy are inferior to the
social cost like in Repullo (2000) and Ponce (2010). We argue that only a fraction of the
social cost arising from a bank failure can be attributed to the lender of last resort. For
this reason we observe that regulator’s level of forbearance can exceed or fall short of the
optimal level conditional on the regulators’ incentive structure and the bank’s solvency.
The last difference is that in Espinosa-Vega, Kahn, Matta, and Sole (2011) the maturity
of the risky asset differs between the systemic and the non-systemic bank. In our model
both banks invest into identical assets.

The rest of the paper is organized as followed. Section 2 provides a literature review.
In section 3 we present the model before the benchmark case without systemic risk is
analyzed in section 4. We introduce systemic risk into the model in section 5. In section 6
we extend the set of available agencies and consider the deposit insurer for the allocation
of responsibilities. Section 7 concludes.
2 Literature

The concept of a lender of last resort goes back to Bagehot (1873), who states that the central bank as the lender of last resort should lend to solvent banks at a penalty rate given adequate collateral. Despite criticism\(^8\) this doctrine is widely accepted and the theoretical literature on lender of last resort interventions grew significantly within recent years. A fraction of this literature\(^9\) focuses on coordination failures in interbank markets or payment systems and provide rationale for lender of last resort interventions. In these papers the existence of a lender of last resort can assure market participants and prevent inefficient closure of solvent banks or as in Acharya and Yorulmazer (2008) provide the surviving banks with necessary liquidity to acquire the illiquid banks’ assets and avoid efficiency losses due to misallocation of assets.

The optimal institutional allocation of lender of last resort responsibilities was initially studied by Repullo (2000). In his model the lender of last resort decides about the provision of emergency liquidity assistance to banks hit by a liquidity shock. The banks’ solvency is private information so that only the lender of last resort is given the authority to evaluate banks and receive a perfect but nonverifiable signal about their solvency. There are two agencies available to act as a lender of last resort: the central bank and the deposit insurance cooperation. Both agencies have the objective to maximize their expected final wealth. But they differ in their mandates so that their individual lending decisions as a lender of last resort do not coincide. The deposit insurance cooperation has the obligation to compensate depositors in case of a bank’s failure. When refusing the emergency loan she can liquidate banks in trouble, realize the liquidation value and limit her losses from the lender of last resort activities. For this reason the deposit insurance cooperation is biased towards liquidation. The central bank’s engagement is restricted to the emergency loan. She grants the emergency loan conditional on the bank’s solvency signal. Repullo shows that the deposit insurance cooperation does not provide socially optimal emergency liquidity assistance. The central bank on the contrary is too soft for small liquidity shocks, but too restrictive for large liquidity shortfalls. The second-best allocation involves both

\(^8\)Goodfriend and King (1988) argue that the existence of interbank market makes the liquidity provision to individual banks unnecessary. Opposing Goodfriend and King’s (1988) view Rochet (2004) provide a rational for a lender of last resort in a framework with sophisticated interbank markets. Goodhart (1999) points out that first it is difficult for the central bank to distinguish between solvent and insolvent banks and second that the lender of last resort might not be better informed than the market. Therefore the lender of last resort allocation should be inferior to the market allocation. Castiglionesi and Wagner (2012) show that under some conditions penalty rates increase bank moral hazard.

agencies. The central bank should be in charge of the lender of last resort responsibilities for small liquidity shortfalls while the deposit insurance should decide about the liquidity assistance for larger liquidity shocks.

Kahn and Santos (2005) and Kahn and Santos (2006) use Repullo’s (2000) framework to study the merits of centralization of lender of last resort responsibilities and the deposit insurance function. They introduce next to the illiquid asset a liquid asset in order to analyze the impact of the lender of last resort policy on the bank’s investment choice. They find that centralization induces more forbearance for large liquidity shocks and leads to inefficient investment into the risky asset. Keeping the functions separated causes softer lending decisions for small liquidity shortfalls. With information frictions about the bank’s solvency and liquidity shock they show that the central bank does not have an incentive to share its private information.

Ponce (2010) extent Repullo’s (2000) framework by introducing an unconditional bailout rule meaning that an emergency loan will be provided to the bank in trouble regardless of the bank’s solvency. He shows that the second-best allocation changes given the larger set of policy tools. It consists of the application of the unconditional bailout rule for large liquidity shocks and the allocation of the lender of last resort responsibility to the central banker for small liquidity shocks. If bankers are able to manipulate the size of the liquidity shortfall he shows that applying the unconditional bailout rule should be combined with a punishment for the banker in order to incentivize him not to manipulate. Moreover he shows that first-best allocation can be achieved with an appropriate compensation scheme for the central banker.

Espinosa-Vega, Kahn, Matta, and Sole (2011) are the first analyzing the optimal allocation of lender of last resort responsibilities in a framework with systemic risk. Building on Repullo’s (2000) they introduce a systemic bank into the model which failure decreases the success probability of the non-systemic bank’s asset. This is in contrast to our paper where we assume that the systemic impact hurts the return of the non-systemic bank. Their objective is as in Kahn and Santos (2005) and Kahn and Santos (2006) to study the effect of centralization of regulatory arrangements on forbearance and information sharing. In this paper we focus on the optimal structure of lender of last resort policy with respect to the allocation of responsibilities between the central banker and the unconditional bailout rule. Next to the two points mention above our paper differs in two additional aspects. First, the regulator’s political cost of a failure do not exceed the social cost of failure and second, both banks invest into identical assets with the same maturity. In their model they find as we do that regulators are more forbearing towards the systemic institution.
Furthermore, they show that regulators have little incentives to share private information about the systemic relevance of an institution. They conclude that a centralized regulatory structure reduces forbearance and can avoid inefficient information sharing.

3 The model

We propose a model inspired by Espinosa-Vega, Kahn, Matta, and Sole (2011) and Repullo (2000) where banks are funded entirely by demand deposit contracts. Banks raise one unit of deposits at the beginning of their operations. We assume that deposits are fully insured by the deposit insurance and that they can be withdrawn either after the first or the second period of operation.

The banks invest their deposits into an illiquid risky asset which yields for each unit invested a random return \( \tilde{R} \) after two periods. The asset can either succeed, \( \tilde{R} = R \), or fail, \( \tilde{R} = 0 \). The asset is ex ante profitable: \( E(\tilde{R}) > 1 \), but it can not be sold at an intermediate date. However, the entire bank can be liquidated at this date. The liquidation value for both banks is equal to \( L \in (0, 1) \).

As Espinosa-Vega, Kahn, Matta, and Sole (2011) we consider two type of banks: a systemic bank (S) and a non-systemic one (N). A bank is considered as systemic if its failure has a contagion effect on the non-systemic bank. We assume that the systemic impact reduces the return of the non-systemic bank’s asset in the successful state to \( \tilde{R} = R - \gamma \). We differ in the modeling of the contagion effect from Espinosa-Vega, Kahn, Matta, and Sole (2011), but our approach follows Rochet and Tirole (1996) where ”systemic risk refers to the propagation of an agent’s economic distress to other agents linked to that agent through financial transactions”. From this point of view the systemic impact can be interpreted as losses from interbank or payment system claims against the systemic bank and is therefore related to the counterparty risk within a financial system. E.g. in interbank markets banks are connected through interbank lending in order to manager liquidity preferences. As a consequence of the systemic bank’s collapse the non-systemic bank’s asset e.g. a portfolio consisting of several asset thereunder claims against the systemic bank yields a lower return. In such a framework Freixas, Parigi, and Rochet (2000) show that the failure of a systemic bank spills over to other financial institutions and can trigger liquidations of non-systemic banks.

A bank failure can occur because after the first period of operation a fraction \( v_i \in (0, 1) \) of banks’ deposits are withdrawn. Since bank do not hold any liquid reserves and assets
are completely illiquid banks faces bankruptcy if \( v_i > 0 \) unless the lender of last resort provides emergency liquidity assistance. A closure of a bank causes social costs of \( c \). The social costs include, for example, bankruptcy costs and costs related to negative effects on the economy beyond the banking sector.

The sudden withdrawal of deposits can be interpreted as depositors’ consumption preferences like in Diamond and Dybvig (1983). However, we do not model depositors’ behavior in detail, because the focus of this paper is on the optimal allocation of lender of last resort responsibilities and therefore the depositors’ behavior is beyond the scope of this paper. The liquidity shock \( v_i \) is publicly verifiable because we assume that the withdrawal behavior of depositors like queuing in front of banks during a bank run is publicly observable. The liquidity shock \( v_i \) corresponds to the realization of a random variable \( \tilde{v}_i \) with a cumulative distribution \( G \). It has a support in \([0,1]\). Further we assume that the liquidity shocks of both banks are independent. This implies that we focus on individual liquidity situation and do not consider contagion effects of system liquidity crisis.

Additionally, there exists uncertainty about the success probability of the bank’s asset in the model. Simultaneously with the liquidity shock \( v_i \) a perfect but non-verifiable signal \( u_i \) with \( i \in S, N \) about the success probability of the bank’s asset at maturity is realized. The signal is privately observed only by the agency assigned with the LLR responsibilities, because it has the authority to collect all necessary information and the ability to assess the quality of banks’ assets by supervising them in order to fulfill this task. The signal is non-verifiable because it may be based on soft information obtained during asset quality assessment process. This assumption is decisive for the lender of last resort policy because ex ante allocation of responsibilities has to be conditional on the liquidity shortfall \( v_i \).

The policy maker can allocate the lender of last resort responsibilities between the central banker and the unconditional bailout rule in order to maximize social welfare. In the public sector many agencies have multidimensional mandates including the achievement of the agencies’ aims at reasonable cost. According to Tirole (1994) this does not prevent the policy maker to design a mechanism to motivate agencies if two concerns are considered. First, the quantification of some dimensions might be difficult. While the failure of a bank is publicly observable the decision making of the regulator to ensure the stability of the financial system might be private information. For this reason the central banker has to bear political cost in case of a failure under his mandate. Second, due to the existence of multiplicity of dimensions the allocation of weights to the different dimensions is of concern. We incorporate Tirole’s (1994) basic ideas and follow Ponce (2010) by setting up the objective function for the central banker so that she cares about their financial wealth
net of incurred political cost from a bank’s failure:

\[ U = I - \varphi \mathbb{1}_{\{\text{failure}\}} c, \tag{1} \]

where \( I \) corresponds to the agency’s net income. \( \mathbb{1}_{\{\text{failure}\}} \) is equal to one if the bank fails and zero otherwise. \( \varphi \in \{\alpha\} \) is the weight given to the political cost in case of a bank’s failure. The political cost for the central banker is \( \alpha c \) with \( \alpha < 1 \). Like Repullo (2000) and Ponce (2010) we assume that the political cost of a bank failure for the central banker do not exceed the social cost. We argue that the central banker can only be blamed for a fraction of the social cost caused by a bank failure because the society will hold the central banker responsible for the realized social cost at most and nothing beyond.

The central banker’s net income from the lender of last resort responsibilities is determined by its mandate. Her exposure corresponds to the amount of the emergency loan when she is engaged in liquidity provision. In case the troubled bank fails after being supported the central banker loses its emergency loan.

As in Ponce (2010) apart from allocating the responsibility to the central banker the policy maker can implement an unconditional bailout. In this case the central banker is instructed to provide liquidity to the troubled bank without any negative effect on her utility in case of default. Thus the central banker does not incur any political cost from a failure when the unconditional bailout rule is applied.

[Figure 1 about here.]

The timing of the model is summarized in figure 1 which will be explained in the following. For simplification but without loss of generality the systemic bank \( S \) starts to operate at date 0 while the starting date of operation for the non-systemic bank \( N \) is delayed to date 2. This sequential structure avoids the simultaneity of events and facilitates the analysis of the lender of last resort policies for both banks.

At date 0 the policy maker announces the lender of last resort policy for the systemic bank \( S \) and the non-systemic bank \( N \). Bank \( S \) raises one unit of deposits and invests it into a risky asset.

At date 1 bank \( S \)’s liquidity shortfall \( v_S \) is publicly observed. The lender of last resort observes in addition privately the solvency signal \( u_S \) of bank \( S \) and decides about the provision of the emergency liquidity loan. Either bank \( S \) receives an emergency loan and continues to operate or bank \( S \) is closed. Simultaneously bank \( N \) raises one unit of deposits and invests it into a risky asset.
At date 2 bank N’s public liquidity shock $v_N$ is realized. Bank N’s solvency signal $u_N$ is privately observed by the lender of last resort. The regulatory agency in charge applies the lender of last resort policy. Bank N is either closed or it remains open if the lender of last resort provides an emergency loan. In case bank S was not liquidated before bank S’s risky asset matures simultaneously and its return is realized.

In case bank N is still operating at date 3 the return of bank N’s risky asset is realized.

4 Benchmark case

In our benchmark case we analyze the first- and second-best lending decision within a framework consisting of only one single bank. In this section there is no contagion effect on other financial institutions. As described in section 3 the bank collects one unit of deposits and invests them into a illiquid risky asset with a random return after two periods. After one period of operation the bank faces a random but publicly observable liquidity shock $v$ and can only survive if the lender of last resort provides an emergency loan. The agency in charge of the lender of last resort responsibility uses a perfect but non-verifiable signal about the asset quality to decide whether or not to support the bank. Our benchmark is similar to the model studied in Ponce (2010). The main difference we do not consider the deposit insurance cooperation in our analysis.

4.1 First-best lender of last resort policy

In order to determine the first-best lending decision we assume that the liquidity shock $v$ as well as the solvency signal $u$ are both verifiable.

The expected social welfare from the bank is:

$$W_N = E[1_{LLR}(uR - (1 - u)c) + (1 - 1_{LLR})(L - c)] = E[1_{LLR}(u(R + c) - L) + (L - c)],$$

where $1_{LLR}$ is equal to 1 if the bank is supported and 0 otherwise. The expected continuation value of the bank including the social cost of a failure after two periods of operation is $(uR - (1 - u)c)$. In case the bank is not supported and liquidated after one period of operation the bank’s value net the social cost of the liquidation is $(L - c)$.

Since the bank’s liquidation value is constant it is social optimal to support the bank.
if the bank’s solvency signal is above the threshold $u^*$:

$$uR - (1-u)c \geq L - c,$$

$$u \geq u^* \equiv \frac{L}{R+c}. \quad (3)$$

If the solvency signal falls short of the threshold $u^*$ the bank should not receive emergency liquidity assistance.

### 4.2 Second-best lender of last resort policy

We analyze the second-best lender of last resort policy for the benchmark bank starting with the lending decision of the central banker followed by the provision of liquidity according to the unconditional bailout rule.

#### 4.2.1 Central banker as the LLR

Assume that the central banker is the lender of last resort. She will provide the emergency loan to the bank in trouble if the expected utility from supporting the bank exceeds the utility from closing the bank. If the emergency liquidity assistance with an amount of $v$ is provided the emergency loan will be repaid in case the supported bank is successful. Otherwise the amount $v$ of the emergency loan is lost. In addition the central banker has to bear the political cost of the bank’s failure. It follows that the central banker’s expected utility from providing the emergency liquidity assistance is equal to $-(1-u)(v+ac)$. If the central banker does not provides the emergency loan the bank is closed and the central banker incurs the political cost $ac$. Consequently the central banker will support the bank in trouble if the solvency signal is above the threshold $u^{CB}$:

$$(1-u)(v+ac) \leq ac,$$

$$u \geq u^{CB} \equiv \frac{v}{v+ac}. \quad (4)$$

Otherwise the central banker refuses the emergency loan and the bank is liquidated.
4.2.2 Unconditional bailout rule

The lending decision given the unconditional bailout rule is applied can be expressed in the following way:

\[ u \geq 0 \equiv u^{UBR}. \] (5)

According to the unconditional bailout rule the central banker is instructed to support banks in trouble with an emergency loan independently of the solvency signal \( u \).

Figure 2 plots the different lending decisions derived above in a \((u, v)\) plane. The first-best emergency liquidity provision requires minimum asset quality \( u^* \) independent of the size of the liquidity shock. It is therefore a horizontal line. The central banker’s threshold of the solvency signal depends on the size of the liquidity shock. With increasing liquidity shortfalls the central banker becomes tougher so that the central banker’s lending decision is a concave function passing through the origin. The unconditional bailout rule requires as the first-best liquidity provision a constant level of solvency independent of the size of the liquidity shortfall. But the minimum asset quality requirement is equal to zero. For this reason the unconditional bailout rule lending decision coincide with the abscissa in the \((u, v)\) plane.

The central banker’s lending decision is compare to the first-best provision of liquidity too soft for small liquidity shortfall and provides socially non-desirable emergency loans. For larger liquidity shocks the central banker is too tough and refuses to provide the socially desirable emergency liquidity assistance. The intuition of this observation is that for very small liquidity shocks close to zero the central banker has an incentive to lend to the bank in trouble. If the central bankers does so the expected cost from providing the emergency loan is \((1 - u)\alpha c\). If the central banker refuses the emergency loan the bank will be liquidated and the central banker will incur the political cost \(\alpha c\) with probability 1 which is larger than \((1 - u)\alpha c\). For larger liquidity shock the exposure of the central banker is more severe so that liquidity is only provided if the solvency signal is sufficiently large.

The unconditional bailout rule is always too soft in comparison with the first-best lending decision because the required asset quality is zero. Only in the origin coincide The unconditional bailout rule with the central banker’s lending decision because for positive
liquidity shortfalls the central banker is always tougher and requires a positive solvency signal.

4.2.3 Optimal allocation of LLR responsibilities

Following Ponce (2010) the expected social welfare function \( (2) \) given the first-best threshold \( u^* \equiv \frac{L}{R+c} \) for the provision of an emergency loan can be expressed as:

\[
W = E[\mathbb{1}_{LLR}(u - u^*)](R + c) + (L - c). \tag{6}
\]

To maximize (6) it is sufficient to maximize the normalized expected social welfare:

\[
w = E[\mathbb{1}_{LLR}(u - u^*)]. \tag{7}
\]

From 7 we can derive the normalized expected social welfare given either the central banker acts as the lender of last resort or the unconditional bailout rule is applied:

\[
w_{CB}(v) = \int_{u^B(v)}^{1} (u - u^*) \, dF(u), \tag{8}
\]

\[
w_{UBR} = \int_{0}^{1} (u - u^*) \, dF(u). \tag{9}
\]

Following Ponce (2010) we can shows that these functions have the following properties summarized in lemma 1.

**Lemma 1.** Assume \( E(\tilde{u} \mid u \leq u^{CB}(1)) > u^* \). Then, \( (1) \) \( w^{CB}(v) \) is increasing in \( v \) if \( v < v^{A} \equiv \frac{\alpha c L}{R-L-c} \), decreasing if \( v > v^{A} \), and has a global maximum at \( v = v^{A} \); \( 2 \) \( w^{CB}(0) = w^{UBR} \); and, \( 3 \) \( w^{CB}(0) > w^{CB}(1) > 0 \).

**Proof.** See Appendix A.1.

[Figure 3 about here.]

Figure 3 visualizes the properties of function (8) and (9) stated in lemma 1. They are presented as a function of the liquidity shortfall. The normalized expected social welfare function given the central banker is the lender of last resort is increasing for \( v < v^{A} \) and decreasing otherwise. At \( v^{A} \) the solvency requirement of the first-best and the central banker coincide so that the emergency liquidity assistance of the central banker corresponds to the first-best provision. For this reason the normalized expected social welfare function
has a maximum for \( v = v^A \). To the left and the right of \( v^A \) the solvency requirement of the central banker differs from the first-best requirement. On the left the central banker is too soft while on the right the central banker is too tough. Therefore \( w^{CB}(v) \) for \( v \neq v^A \) is lower than \( w^{CB}(v^A) \). The solvency requirement of the unconditional bailout rule has over the whole support of liquidity shocks constant to zero. For this reason the normalized expected social welfare function is a horizontal line.

Since only the liquidity shock \( v \) is public information the policy maker will allocate the lender of last resort responsibilities conditional on the size of the liquidity shock to maximize the expected social welfare. As in Ponce (2010) lemma 1 implies the following second-best optimal allocation:

**Proposition 1.** Assume that \( E(\tilde{u} | u \leq u^{CB}(1)) > u^\ast \). It is optimal to allocate the lender of last resort responsibilities to the central banker for liquidity shortfalls below the threshold \( v^\ast \in (v^A, 1) \). Otherwise, it is socially optimal to apply the unconditional bailout rule.

The condition \( E(\tilde{u} | u \leq u^{CB}(1)) > u^\ast \) implies that the asset quality of a random bank is more likely to be of average quality (i.e. \( u \in [u^\ast, u^{CB}(1)] \)) than of low quality (i.e. \( u \in [0, u^\ast] \)). In the interval \([0, u^{CB}(1)]\) the central banker might not provide socially desirable emergency loan depending on the size of the liquidity shortfall. But the average bank has a sufficient quality according to the first-best lending decision. For this reason, it is welfare improving to apply the unconditional bailout rule for large liquidity shocks, because for these shocks it is more likely that the central banker will be too restrictive and not provide socially desirable emergency loans. For small liquidity shocks the central banker’s lending decision is the closest to the first-best solution so that the allocation of lender of last resort responsibilities to the central banker for small liquidity shocks is welfare enhancing.

### 5 Financial system with a systemic bank

In this section we study the optimal lender of last resort policy for a financial system with a systemic and a non-systemic bank as described in section 3. In order to determine the optimal allocation of responsibilities for the systemic bank we solve the model backwards starting with the non-systemic bank followed by the systemic bank.

We define the following indicator variables with a value equal to one in case the below-mentioned conditions hold:
- $1_{SS}^S = 1$ if systemic bank S succeeds at date 2.
- $1_{SF}^S = 1$ if systemic bank S fails at date 2 or was closed at date 1.
- $1_S = 1$ if LLR loan is provides to systemic bank S.
- $1_{SS}^N = 1$ if LLR loan is provided to non-systemic bank N given systemic bank S succeeded.
- $1_{SF}^N = 1$ if LLR loan is provided to non-systemic bank N given systemic bank S failed at date 2 or was closed at date 1.

5.1 Non-systemic bank

5.1.1 First-best

For the determination of the socially optimal allocation of the LLR responsibilities we assume that the liquidity shock $v_S$ and the solvency signal $u_S$ are both public information and verifiable. The expected social welfare from bank N is:

$$W_N = E\{1_{SS}^S[(1_{SS}^N(u_NR - (1 - u_N)c) + (1 - 1_{SS}^N)(L - c)) + 1_{SF}^S[(1_{SF}^N(u_N(R - \gamma) - (1 - u_N)c) + (1 - 1_{SF}^N)(L - c))], \quad (10)$$

where the first term of this expression is the expected social welfare given a successful systemic bank (case SS). If the non-systemic bank is supported with an emergency loan the bank succeeds with probability $u_N$ and yields a return $R$. A failure occurs with a probability $(1 - u_N)$ and causes social cost $c$. If bank N is not supported the bank will be liquidated and the liquidation value $L$ will be realized. The closure causes social cost of $c$.

The second term of (10) is the expected social welfare in case bank S was liquidated at date 1 or its risky asset failed at date 2 (case SF). If bank N receives an emergency loan its risky asset succeeds with probability $u_N$ but yield only a return $R - \gamma$. The asset fails with probability $(1 - u_N)$ which causes social costs of $c$. If the emergency loan is refused bank N is liquidated and a liquidation value of $L$ is realized. A liquidation causes social cost of $c$.

For the determination of the first-best lending decision the thresholds on the solvency signal $u_N$ are derived separately for both states of the systemic bank S (case SS and case SF). First, the case of a successful bank S is analyzed. It is optimal to provide an emergency loan to bank N if the expected social welfare from bank N’s continuation exceeds the social
welfare of bank N’s liquidation. The social optimal lending decision to bank N in case SS is:

\[ u_N R - (1 - u_N) c \geq L - c, \]

\[ u_N \geq u_N^{SS} \equiv \frac{L}{R + c}, \]  \hspace{1cm} (11)

which is equivalent with the first-best lending decision in our benchmark case. If the solvency signal \( u_N \) is below \( u_N^{SS} \) it is not social optimal to provide the emergency liquidity assistance.

In case the systemic bank S failed it is optimal to provide emergency liquidity assistance to bank N if:

\[ u_N (R - \gamma) - (1 - u_N) c \geq L - c, \]

\[ u_N \geq u_N^{SF} \equiv \frac{L}{R + c - \gamma}. \]  \hspace{1cm} (12)

In equation (12) we observe the negative impact of the systemic bank’s failure on the non-systemic bank’s asset return in threshold \( u_N^{SF} \). Due to the lower asset return the first-best lending decision in case SF is tougher compared to the threshold in equation (11) for the case SS when the systemic bank is successful.

5.1.2 Central banker as the LLR

The central banker will only support the non-systemic bank N if the expected cost from providing the emergency loan is lower than the cost of closing bank N immediately. The central banker’s expected cost of an emergency loan to bank N is equal with the amount of the liquidity injection \( v_N \) and the political cost \( \alpha c \) due to a failure of bank N. If the central banker does not support bank N the bank will be closed and the central banker will incur the political cost \( \alpha c \) for the bank failure. The state of bank S has not impact on the central banker’s expected cost, because \( R - \gamma > 1 \). Thus the central banker’s expected utility from the lender of last resort activities is:

\[ B_N = 1^S_S \left[ (1 - u_N)(\alpha c + v_N) + (1 - 1^S_S)(-\alpha c) \right] \]

\[ + 1^S_F \left[ (1 - u_N)(\alpha c + v_N) + (1 - 1^S_F)(-\alpha c) \right]. \]  \hspace{1cm} (13)
From (13) it is obvious that the central banker will provide the emergency liquidity if:

\[ u_N(\alpha c + v_N) \geq v_N, \]
\[ u_N \geq u_N^{CB}(v_N) \equiv \frac{v_N}{v_N + \alpha c}, \]  

(14)

which is equivalent to the central banker’s lending decision in the benchmark case. If the solvency signal \( u_N \) is below \( u_N^{CB} \) the central banker will refuse the emergency loan and the non-systemic bank will be closed.

### 5.1.3 The unconditional bailout rule

The lending decision given the unconditional bailout rule is applied can be expressed in the following way:

\[ u_N \geq 0 \equiv u_N^{UBR}. \]  

(15)

It implies that banks with a positive liquidity shock \( v_N \) will always be supported independent of the solvency signal \( u_N \).

[Figure 4 about here.]

Figure 4 shows the liquidity provision thresholds for the non-systemic bank \( N \) defined above. The first-best lending decision depends on the state of bank \( S \) but is independent of the size of the liquidity shortfall \( v_N \). If bank \( S \) fails or was liquidated the first-best lending decision is more restrictive for the non-systemic bank and requires a higher solvency signal \( u_N^{SF} \). For this reason the first-best lending decision in case \( SF \) is above the one in case \( SS \). The central banker’s lending decision is independent of bank \( S \)’s state. It does only coincide with the socially optimal lending decision for a liquidity shock of size \( v_N^{A} (v_N^{C}) \) in case \( SS \) (SF). Since the central banker’s expected utility is decreasing with the size of the required emergency loan the central banker’s lending decision gets more restrictive with increasing liquidity shocks. The unconditional bailout rule provides always the emergency loan so that the lending decision in the \((v,u)\) plane coincide with the abscissa.

The comparison of the policies for the non-systemic bank \( N \) with the benchmark case yields the following proposition:

**Proposition 2.** The first-best lender of last resort policy for the non-systemic bank is more restrictive in the framework with a systemic bank compared to the benchmark case.
if the systemic bank was liquidated or failed. Otherwise the first-best lending decision for non-systemic bank corresponds to the benchmark case. The lending decisions of the central banker and the unconditional bailout rule to the non-systemic bank are equivalent with the benchmark case.

Proof. See Appendix A.2. \qed

5.1.4 Optimal allocation

Since the first-best lending decision for the non-systemic bank differs between case $SS$ and $SF$ we will study the optimal allocation of LLR responsibilities for both cases separately and therefore define $\nu \in \{SS, SF\}$. On the basis of case $SS$ we illustrate our approach to define the optimal second-best allocation of LLR responsibilities. As in the benchmark case the expected social welfare in (10) given the socially optimal threshold to provide emergency liquidity $u_{SSN} \equiv \frac{L}{R+c}$ when bank $S$ was successful can be expressed as:

$$W_{SSN} = E[\mathbb{1}_{SSN}(u_N - u_{SSN})](R + c) + (L - c).$$  \hspace{1cm} (16)

It is sufficient to maximize the normalized social welfare:

$$w_{SSN} = E[\mathbb{1}_{SSN}(u_N - u_{SSN})]$$  \hspace{1cm} (17)

in order to obtain the maximum of the social welfare in equation (16).

As the approach for $\nu = SF$ is analogous it follows for $\nu \in \{SF, SS\}$ that the normalized expected social welfare functions given the central banker is the lender of last resort or the unconditional bailout rule is applied are:

$$w_{CB,\nu}^{UBR} = \int_0^1 (u_N - u_{\nu N}) \, dF(u).$$  \hspace{1cm} (19)

Lemma 2 follows Ponce’s (2010) results adapted to the model studied here and proves some properties of the normalized expected social welfare functions (18) and (19).

Lemma 2. Assume that $E(\tilde{u}_N \mid u_N \leq u_{CB,\nu}^{UBR}(1)) > u_{SFN}$. Then, (i) if the systemic bank succeeded, $\nu = SS$ (respectively failed, $\nu = SF$), then (i) $w_{CB,SS}^{CB,SS}(v_N)$ is increasing in $v_N$ if $v_N < v_N^A \equiv \frac{\alpha cL}{R - L + c}$ (respectively $v_N < v_N^C \equiv \frac{\alpha cL}{R - L + c + \gamma}$), (ii) decreasing if $v_N > v_N^A$.
(respectively $v_N > v_N^C$), and (iii) has a global maximum at $v_N = v_N^A$ (respectively at $v_N = v_N^C$); (2) $w_{CB,\nu}^1(0) = w_{UBR,\nu}^1$; (3) $w_{CB,\nu}^1(0) > w_{CB,\nu}^1(1) > 0 \forall \nu \in \{SS, SF\}$.

Proof. See Appendix A.3.

Figure 5 shows the properties proven in the lemma 2. The normalized expected social welfare given the central banker is the lender of last resort is increasing for liquidity shortfall smaller than $v_N^A$ ($v_N^C$) in case SS (SF) because the central banker’s lending decision converges to the first-best provision of liquidity. For liquidity shocks above these thresholds the normalized expected social welfare is decreasing because the central banker gets more restrictive and diverges from the first-best provision of liquidity. The normalized expected social welfare function given the unconditional bailout rule is applied is horizontal because like the first-best lending decision the unconditional bailout rule provides an emergency loan independent of the size of the liquidity shock. In case SS (SF) the normalized expected social welfare function given the central banker is the lender of last resort intersects with the normalized expected social welfare function if the unconditional bailout rule is applied for two liquidity shocks: 0 and $v_{NS}^S$ (0 and $v_{NS}^F$).

Since only the liquidity shock $v_N$ is verifiable the policy maker will allocate the lender of last resort responsibilities according to the size of the liquidity shortfall in order to maximize the expected social welfare. Lemma 2 implies the following second-best optimal allocation:

**Proposition 3.** Assume that $E\left(\tilde{u}_N \mid u_N \leq u_{CB,\nu}^1(1)\right) > u_{SF}$. If the systemic bank succeeded, $\nu = SS$ (respectively failed, $\nu = SF$), there exists a threshold for the liquidity shortfall of the non-systemic bank $v_{NS}^S \in (v_N^A, 1)$ (respectively $v_{NS}^F \in (v_N^C, 1)$) so that it is optimal to allocate the lender of last resort responsibilities for the non-systemic bank to the central banker for liquidity shortfalls below the threshold and to apply the unconditional bailout rule for liquidity shortfalls above it.

The intuition of proposition 3 can be explained as followed. For large liquidity shock the central banker’s lending decision is too restrictive. Given condition $E\left(\tilde{u}_N \mid u_N \leq u_{CB,\nu}^1(1)\right) > u_{SF}^1$ it is more likely that a random non-systemic bank’s asset is of average quality (i.e. $u \in [u_{SF}^1, u_{CB}^1(1)])$ than of low quality (i.e. $u \in [0, u_{SF}^1]$). With increasing liquidity shocks it is more likely that a non-systemic bank for which liquidity support is social optimal does not receive an emergency loan from the central banker than a non-systemic bank with
low quality assets is bailed out unconditionally. Therefore, the policy maker chooses to apply the unconditional bail out rule for large liquidity shortfalls. For small liquidity shock the central banker’s threshold is closer to the social optimal one than the unconditional bailout rule. Therefore, the LLR responsibilities for small liquidity shock are allocated to the central banker.

But we can show that the existence of the systemic bank provides a rational for the central banker to act as a lender of last resort with an extended mandate. Proposition 4 summarizes this finding:

**Proposition 4.** The central banker should act as a lender of last resort in a larger range of liquidity shortfalls of the non-systemic bank when the systemic bank failed than when it succeeded (i.e. \(v_{SN}^{SS} < v_{SN}^{SF}\)).

**Proof.** See Appendix A.4. \(\square\)

The intuition for this result is the following. With a failure of the systemic bank the expected return of the non-systemic bank falls so that for all liquidity shocks the social optimal threshold for the provision of the emergency loan increases. Since the central banker become less forbearing with increasing liquidity shocks the social optimal lending decision is closer to the central banker’s one for a larger interval of liquidity shocks. However, for very large liquidity shocks the central banker is still too tough, so that unconditional bailout rule still maximizes the expected social welfare in this interval.

### 5.2 Systemic bank

#### 5.2.1 First-best

As for the non-systemic bank we determine the first-best provision of emergency liquidity assistance by the comparison of the expected social welfare from supporting and not supporting the bank given that the liquidity shock \(v_S\) as well as the solvency shock \(u_S\) are both verifiable. The expected social welfare is:

\[
W_S = E\{1_S[u_S(R - (1 - u_S)c + W_{SN}^{SC})] + (1 - 1_S)[L - c + W_{SN}^{SL}]\},
\]

\[
W_S = E\{1_S[u_S(R + c) - L + W_{SN}^{SC} - W_{SN}^{SL}] + L - c + W_{SN}^{SL}\},
\]

(20)

where the first term is the social welfare given the systemic bank receives the emergency liquidity assistance. In this case the social welfare consists of the systemic bank’s expected
continuation value at date 2 net of social cost \((u_SR - (1 - u_S)c)\) and the expected social welfare of the non-systemic bank \(W_{SC}^{U}\) given the systemic bank continues to operate. The latter has to be considered because as we showed in section 5.1 the state of the systemic bank has an impact on the emergency liquidity provision for the non-systemic bank. The expected social welfare from the non-systemic bank given the systemic bank continues to operate is:

\[
W_{SC}^{U} = u_SE\{1_{SS}^{U}(u_NR - (1 - u_N)c) + (1 - 1_{SS}^{U})(L - c)\}
\]

which consists of the expected social welfare from the non-systemic bank given the systemic bank is successful or fails at date 2. If the systemic bank is successful the non-systemic bank’s expected continuation value net of the social cost of failure is \(u_NR - (1 - u_N)c\). If the emergency loan is refused the non-systemic bank will be liquidated so that the social welfare is equal to the liquidation value net of the social cost due to bank failure \(L - c\). In case the systemic bank fails at date 2 the non-systemic bank’s expected continuation value net of the expected social cost of a bank failure is \(u_N(R - \gamma) - (1 - u_N)c\). The liquidation value of the non-systemic bank net of the social cost of a bank failure is \(L - c\).

The second term of equation (20) is the social welfare in case the systemic bank is closed. The liquidation value net of social cost of a failure at date 1 is \(L - c\). As before the expected social welfare from the non-systemic bank given the systemic bank was closed has to be considered. This is:

\[
W_{SL}^{U} = E\{1_{SF}^{U}(u_N(R - \gamma) - (1 - u_N)c) + (1 - 1_{SF}^{U})(L - c)\},
\]

where the first term corresponds to the situation when the non-systemic bank receives emergency liquidity assistance while the second term refers to the situation when the emergency loan is refused. If the non-systemic bank is supported its expected continuation value net of the social cost of a bank failure is \(u_N(R - \gamma) - (1 - u_N)c\). If the non-systemic bank is not supported the liquidation value net of the social cost of a bank failure at date 2 is \(L - c\).

We define:

\[
W_{N}^{A} = E\{(1_{SS}^{U} - 1_{SF}^{U})(u_NR(c) - L) + 1_{SF}^{U} u_N\gamma\} \geq 0,
\]
so that

\[ W^{SC}_N - W^{SL}_N = u_SW^\Delta_N. \]  \(24\)

Given \((20)\) and \((24)\) it is social optimal to provide the emergency loan to the systemic bank if:

\[ u_S(R + c + W^\Delta_N) \geq L, \]
\[ u_S \geq u^*_S \equiv \frac{L}{R + c + W^\Delta_N}. \]  \(25\)

Is the solvency signal below \(u^*_S\) the systemic bank should be closed because it is socially not optimal to provide the emergency loan.

\subsection*{5.2.2 Central banker as the LLR}

Suppose the central banker is the lender of last resort for the systemic bank. If the central banker engages in the emergency liquidity assistance but the systemic bank fails the central banker loses the liquidity injection \(v_S\) and incurs the political costs \(\alpha c\). In addition the utility from the non-systemic bank \(N\) given the systemic bank continues to operate \(B^{SC}_N\) has to be considered because the state of the systemic bank influence the expected profitability of the bank \(N\) and the central banker’s responsibilities as a lender of last resort for the non-systemic bank. If the central banker refuses to support the systemic bank the central banker will not provide any liquidity and bank \(S\) will be closed. In this situation the central banker’s cost consists only of the political cost \(\alpha c\) and the central banker’s utility from the non-systemic bank \(N\) given the closure of the systemic bank \(B^{SL}_N\). Thus the central banker’s expected utility from her lender of last resort responsibilities for the systemic bank \(S\) is:

\[ B_S = E\{1_S (1 - u_S)(\alpha c + v_S) + B^{SC}_N) + (1 - 1_S)(-\alpha c + B^{SL}_N)\}, \]
\[ B_S = E\{1_S (u_S(\alpha c + v_S) - v_S + B^{SC}_N - B^{SL}_N) - \alpha c + B^{SL}_N\}. \]  \(26\)

The central banker’s utility from the non-systemic bank \(N\) given the systemic bank
continues to operate is:

$$B^S_N = u_S \int_{v^S_N}^0 \left[ \int_0^{u^C_B(v_N)} -(\alpha c)dF(u) + \int_{u^C_B(v_N)}^1 -(1 - u_N)(\alpha c + v_N)dF(u) \right] dG(v_N)$$

$$+ (1 - u_S) \int_{v^S_N}^{v^S_C(v_N)} \left[ \int_0^{u^C_B(v_N)} -(\alpha c)dF(u) + \int_{u^C_B(v_N)}^1 -(1 - u_N)(\alpha c + v_N)dF(u) \right] dG(v_N),$$

(27)

where it is considered that the systemic bank can be successful or fail. $v^S_N$ and $v^S_C(v_N)$ are the optimal second-best liquidity shock below which the central banker is the lender of last resort for the non-systemic bank as defined in proposition 3. When the central banker is responsible for the provision of the emergency loan the central banker will only support the bank in trouble if the solvency signal is above the threshold $u^C_B(v_N)$. In case the non-systemic bank fails while being supported the central banker will lose the emergency loan $v_N$ and incur the political cost of bank failure $\alpha c$. Below the solvency threshold the central banker will never support bank $N$ and incur the political cost $\alpha c$.

The utility from the non-systemic bank $N$ given a closure of the systemic bank $S$ is:

$$B^L_N = \int_{v^S_N}^{v^S_C(v_N)} \left[ \int_0^{u^C_B(v_N)} -(\alpha c)dF(u) + \int_{u^C_B(v_N)}^1 -(1 - u_N)(\alpha c + v_N)dF(u) \right] dG(v_N),$$

(28)

where $v^S_N$ is the non-systemic bank’s liquidity shock below which the central banker is the lender of last resort. The central banker will refuse the emergency loan if the non-systemic bank’s solvency signal is below $u^C_B(v_N)$. In this situation the non-systemic bank will be closed and the central banker will incur the political cost $\alpha c$. The central banker supports bank $N$ given the solvency signal is above the threshold $u^C_B(v_N)$. If the non-systemic bank fails the emergency loan $v_N$ will not be repaid and the central banker will incur additionally the political cost $\alpha c$.

We define:

$$B^\Delta_N = \int_{v^S_N}^{v^S_C(v_N)} \left[ \int_0^{u^C_B(v_N)} (\alpha c)dF(u) + \int_{u^C_B(v_N)}^1 (1 - u_N)(\alpha c + v_N)dF(u) \right] dG(v_N) \geq 0,$$

24
so that

\[ B_{N}^{SC} - B_{N}^{SL} = u_{S}B_{N}^{\Delta}. \]  

(29)

Given (26) and (29) the central banker lends to bank S if:

\[ u_{S}(v_{S} + \alpha c + B_{N}^{\Delta}) \geq v_{S}, \]

\[ u_{S} \geq u_{S}^{CB}(v_{S}) \equiv \frac{v_{S}}{v_{S} + \alpha c + B_{N}^{\Delta}}. \]  

(30)

The central banker will refuse the emergency loan if the solvency signal is below \( u_{S}^{CB}(v_{S}) \).

The solvency threshold of the central banker is softer compare to the threshold in the benchmark case in equation (4). The central banker considers for the systemic bank the effect of his lending decision on her responsibilities for the non-systemic bank. Since the central banker has more responsibilities for the non-systemic bank if the systemic bank failed she will be less strict with the latter one in order to avoid the extended mandate.

5.2.3 Unconditional bailout rule

The lending decision given the unconditional bailout rule is applied can be expressed in the following way:

\[ u_{S} \geq 0 \equiv u_{S}^{UBR}. \]  

(31)

Banks with a positive liquidity shock \( v_{S} \) will always be supported independent of the solvency signal \( u_{S} \).

Proposition 5 summaries the effect of the systemic bank on the lending decision compared to the benchmark case.

**Proposition 5.** The fist-best lender of last resort policy for the systemic bank is softer compare to the benchmark case, i.e. some banks that do not receive support in the benchmark should receive support if they are systemic. The central banker’s lending decision for the systemic bank is as well less strict compared to the benchmark case while the unconditional bailout rule remains unchanged.

*Proof.* See Appendix A.5.

The intuition of the softer first-best lender of last resort policy for systemic banks can be explained by the negative impact of a systemic bank’s failure on the expected return
of the non-systemic bank. The first-best lender of last resort policy is not only driven by the comparison between the expected continuation value and the liquidation value of the systemic bank but does also considers the continuation value of the non-systemic bank in both states of the systemic bank. If the systemic bank is not supported the expected profitability of the non-systemic bank is reduced and social welfare is harmed. For this reason, the first-best lender of last resort policy is more forbearing for the systemic bank compare to the benchmark case.

The central banker is as well softer compared to the benchmark case. This is due to the second-best optimal allocation of the responsibilities for the non-systemic bank. A closure of the systemic bank implies more responsibilities for the central banker as a lender of last resort for the non-systemic bank. The central banker is hence exposed to a larger expected loss from the lender of last resort activities because his mandate is extended. As a consequence the expected utility decreases. In order to avoid this negative impact on her utility the central banker is biased to forbearance for the systemic bank.

5.2.4 Optimal allocation

As above the social welfare function for the systemic bank $S$ is derived from (20) given

\[ u^*_{S} = \frac{L}{R + c + W^R_{S}}. \]

\[ W_{S} = E[\mathbb{1}_{S}(u_{N} - u^*_{S})](R + c) + (L - c + W^{SL}_{N}). \]

It is sufficient to maximize the normalized expected social welfare

\[ w_{S} = E[\mathbb{1}_{S}(u_{N} - u^*_{S})] \]

in order to maximize equation (32). The normalized expected social welfare function given the central banker is the lender of last resort or the unconditional bailout rule is applied are stated below:

\[ w^{CB}_{S}(v_{N}) = \int_{u^*_{S}}^{1} (u_{S} - u^*_{S}) dF(u), \]

\[ w^{UBR}_{S} = \int_{0}^{1} (u_{S} - u^*_{S}) dF(u). \]

We can show that these functions have the following properties summarized in lemma 3.
Lemma 3. Assume that $E(\widetilde{u}_S | u_S \leq u^{CB}_S (1)) > u^*_S$. Then, (1)(i) $w^{CB}_S (v_S)$ is increasing in $v_S$ if $v_S < v^A_S \equiv \frac{L(\alpha + B\Delta N)}{R - L + c + W \Delta N}$, (ii) decreasing if $v_S > v^A_S$, and (iii) has a global maximum at $v_S = v^*_S$; (2) $w^{CB}_S (0) = w^{UBR}_S$; (3) $w^{CB}_S (0) > w^{CB}_S (1) > 0$.

Proof. See Appendix A.6.

The policy maker will allocate the lender of last resort responsibilities among the central banker and the unconditional bailout rule conditional on the liquidity shortfall in order to maximize the normalized expected social welfare because only the liquidity shortfall is publicly available and verifiable. Lemma 3 implies the following proposition:

Proposition 6. Assume $E(\widetilde{u}_S | u_S \leq u^{CB}_S (1)) > u^*_S$. There exist an liquidity shortfall $v^*_S \in \{v^A_S, 1\}$ so that it is optimal to allocate the lender of last resort responsibilities to the central banker for all liquidity shock smaller than $v^*_S$. Above $v^*_S$ the it is optimal to apply the unconditional bail out rule.

The intuition of proposition 6 is as follows. Condition $E(\widetilde{u}_S | u_S \leq u^{CB}_S (1)) > u^*_S$ implies that the asset of a random systemic bank is more likely to be of an average quality (i.e. $u \in [u^*_S, u^{CB}_S (1)]$) than of low quality (i.e. $u \in [0, u^*_S]$). It is more likely that the central banker does not provide the social optimal emergency loan to a systemic bank if the liquidity shortfall is larger because the central banker’s lending decision is too restrictive. For this reason the policy maker chooses to apply the unconditional bail out rule for large liquidity shocks. But as above it is not always optimal to support illiquid banks. For small liquidity shock the central banker is still more restrictive than the unconditional bailout rule but the central banker’s threshold is closer to the social optimal one. Therefore, the LLR responsibilities for small liquidity shock are allocated to the central banker.

If the condition $E(\widetilde{u}_S | u_S \leq u^{CB}_S (1)) > u^*_S$ is not satisfied the systemic bank’s solvency is on average insufficient to receive an emergency loan from a first-best point of view. Instead of being supported the systemic bank should be closed and liquidated. In this situation the policy maker prefers not to apply the unconditional bailout rule because too many low quality systemic banks would receive social non-optimal emergency liquidity loans. The central banker will be responsible for the entire set of liquidity shocks because welfare losses from closing systemic banks with a sufficient first-best solvency are overcompensated by the restrictive lending decision of the central banker for small liquidity shocks.

Proposition 6 defines the threshold on the liquidity shock for the systemic bank where the responsibility is transferred from the central banker to the unconditional bailout rule.
For the determination of the range of action for the central banker and the unconditional bailout there exist two counteracting effects. First, the central banker’s lending decision is less strict for the systemic bank compared to the benchmark case because the central banker incorporates the consequences of the systemic bank’s collapse on her responsibilities for the non-systemic bank. Keeping the first-best lending decision constant this leads to more responsibilities for the central banker because for a larger interval of liquidity shocks the central banker’s behavior is closer to the first-best provision of liquidity. Second, the first-best lending decision itself is more forbearing with the systemic bank because the negative effect of the systemic bank’s collapse on the non-systemic bank’s profitability is considered. Other things being equal the lower solvency requirement of the first-best solution leads to less responsibilities for the central banker because the interval of liquidity shortfalls where the central banker is too tough increases. The parameter constellation of the model defines which of the two effects prevails. But the implication of these parameters for the effect of the systemic risk on both lending decisions are ambiguous so that the overall effect of the systemic risk on the optimal second-best allocation for the systemic bank is undetermined.
6 Extension

Until now the policy maker could allocate the lender of last resort responsibilities only among the central banker and the unconditional bailout rule. In his section we introduce the deposit insurer into the model so that the set of available policy instruments for the policy maker is enlarged. We do this to verify that the optimal allocation of responsibilities for the systemic and non-systemic bank derived above was not determined by the truncated set of policy instruments considered above.

The deposit insurer has to carry out the deposit insurance function. This obliges her to compensate depositors if a bank fails. She has two options to raise funds for the compensation payments. First, she has access to the failed bank’s asset and can realize the liquidation value $L$. Second, she is funded by banks through deposit insurance premiums. For simplicity, we assume that the deposit insurance premium is normalized to zero.

When appointed as the lender of last resort the deposit insurer observe the solvency signal $u$. In case the bank in trouble fails or is liquidated during her mandate she incurs political cost $\beta c$. As for the central banker we assume that the deposit insurer cares about the expected value of its final wealth net of political cost incurred in case of a bank failure.

In the following we will analyze the lending decision of the deposit insurer in the benchmark case as well as for the non-systemic and systemic bank and present the effect for the optimal second-best allocation.

6.1 Benchmark case

Suppose now that the deposit insurer has to decide in the benchmark case about the provision of an emergency loan to a bank hit by a liquidity shock $v$. The deposit insurer will support the bank if the expected utility from supporting exceeds the utility from liquidating the bank. The deposit insurer’s utility from lending to the bank the amount $v$ depends on whether the supported bank is successful or not. If the bank is successful the emergency loan $v$ is repaid, whereas when the bank fails the deposit insurer loses the emergency loan $v$. In addition she has to compensate the remaining depositors $(1 - v)$ and incurs the political cost of the bank’s failure $\beta c$. The expected utility from supporting the bank is therefore equal to $-(1 - u)(1 + \beta c)$. If the deposit insurer does not support the bank in trouble she will incur the political cost of the bank’s failure $\beta c$ and she has to compensate all depositors but can realize the liquidation value $L$. So the utility of the deposit insurer will be $L - 1 - \beta c$. The deposit insurer will lend the amount $v$ to the bank.
if:

$$-(1-u)(1+\beta c) \leq L - 1 - \beta c$$

$$u \geq u^{DI} \equiv \frac{L}{1+\beta c}. \quad (36)$$

The deposit insurer’s lending decision does not depend on the size of the liquidity shortfall because the liability of the deposit insurer is bounded above by the amount of deposits. The exposure is only reduced by the liquidation value in case of a closure. It is not affected by the substitution of deposits by an emergency loan. Comparing (3) with (36) it is obvious that the deposit insurer requires a higher solvency signal than the first-best lending decision in the benchmark case. Hence the deposit insurer is more restrictive and does not provide socially optimal emergency loans.

[Figure 6 about here.]

Figure 6 shows the lending decision of the deposit insurer in comparison with the one of the first-best solution, the central banker and the unconditional bailout rule. On the horizontal axis we find the liquidity shock while the solvency signal is plotted on the ordinate. The agencies’ lending decisions and the unconditional bailout rule do not coincide with each other. Both are constant over the whole range of liquidity shocks. They also do not match with the first-best solution. The deposit insurer is always too tough compared to the first-best liquidity provision while the unconditional bailout rule is always too soft. As mentioned above the central banker is too soft for small liquidity shocks and too tough for larger liquidity shortfalls compared to the first-best provision. As Ponce (2010) points it out the main reasons for the divergence between the agencies’ lending decisions is due to the differing impacts of an emergency loan on the agencies’ utility. While the deposit insurer has to compensate all depositors of a collapsed bank, but can realize the liquidation value the central banker’s exposure is restricted to the amount of the emergency loan. Additionally, the different weights of the political cost of a failure ($\alpha$, $\beta$) drive the lending decisions apart.

In order to determine the second-best optimal allocation when the deposit insurer is consider we derive the normalized expected social welfare function given the deposit insurer acts as the lender of last resort from the expected social welfare function in (2) and the
first-best threshold \( u^* = \frac{L}{R+c} \):

\[
w^{DI} = \int_{u^{DI}}^{1} (u - u^*) \, dF(u).
\]

(37)

Lemma 4 proves some properties of (37) and relates it to the normalized expected social welfare functions of the central banker and the unconditional bailout rule in equation (8) and (9).

**Lemma 4.** Assume \( E(\bar{u} | u \leq u^{DI}) > u^* \). Then, (1) \( w^{CB}(v) \) is increasing in \( v \) if \( v < v^A \equiv \frac{\alpha c L}{R-L+c} \), decreasing if \( v > v^A \), and has a global maximum at \( v = v^A \); (2) \( w^{CB}(0) = w^{UBR} > w^{DI} \); (3) If \( v < v^B \equiv \frac{\alpha c L}{1-L+\beta c} \), then \( w^{DI} < w^{CB}(v) \), otherwise \( w^{DI} \geq w^{CB}(v) \); and, (4) \( w^{DI} > w^{CB}(1) > 0 \).

Lemma 4 shows that for liquidity shocks below \( v^B \) the liquidity provision of the central banker is dominating the one of the deposit insurer in term of social welfare. Otherwise it is in the inverse. Furthermore, we observe that the deposit insurer as lender of last resort is always dominated by the unconditional bailout rule.

**Proof.** See Appendix A.7. \( \square \)

Since only the liquidity shock \( v \) is public information the policy maker will allocate the lender of last resort responsibilities conditional on the size of the liquidity shock to maximize the expected social welfare. Lemma 4 implies the following second-best optimal allocation:

**Proposition 7.** Assume that \( E(\bar{u} | u \leq u^{DI}) > u^* \). It is optimal to allocate the lender of last resort responsibilities to the central banker for liquidity shortfalls below the threshold \( v^* \in (v^A, v^B) \). Otherwise, it is socially optimal to apply the unconditional bailout rule.

Proposition 7 shows that the introduction of the deposit insurer does not affect the allocation of responsibilities in the benchmark case. However, the existence of the deposit insurer sets an upper bound \( v^B \) for the threshold \( v^* \) where the responsibilities are transferred from the central banker to the unconditional bailout rule. Above \( v^B \) the central banker is more strict and less in line with the first-best liquidity provision than the deposit insurer. So the central banker’s mandate should be restricted to \( v^B \).

Furthermore, the deposit insurer sets the condition which insure the existence of the allocation of responsibilities defined in proposition 7. Ponce (2010) points out that the condition \( E(\bar{u} | u \leq u^{DI}) > u^* \) implies that the asset quality of a random bank is more
likely to be of average quality (i.e. \( u \in [u^*, u^{DI}] \)) than of low quality (i.e. \( u \in [0, u^*] \)). Since in the interval \([u^*, u^{DI}]\) the deposit insurer will not provide socially desirable emergency loan, it is more likely that a socially desirable emergency loan is not provided compared to a socially non-desirable liquidity supports granted through the unconditional bailout rule for liquidity shortfalls in \( u \in [0, u^*] \). For this reason, it is welfare improving to apply the unconditional bailout rule instead of allocating lender of last resort responsibilities to the deposit insurer. For small liquidity shocks the central banker’s lending decision is the closest to the first-best solution. But when liquidity shock increase the central banker gets too tough, so that to allocate the lender of last resort responsibilities to the central banker for small liquidity shocks is welfare enhancing.

### 6.2 Non-systemic bank

Suppose that the deposit insurer is the lender of last resort for the non-systemic bank. The deposit insurer will only support the non-systemic bank if the expected utility from providing liquidity is superior to the utility of closing the non-systemic bank. Due to the assumption \( R - \gamma > 1 \) the non-systemic bank’s return is high enough to repaid all deposits in case systemic bank fails. This implies that the deposit insurer’s liquidity provision is independent of the systemic bank’s state. If the deposit insurer provides the emergency loan \( v_N \) the non-systemic bank will fail with probability of \((1 - u_N)\). It follows that the bank will be unable to repay \( v_N \). The deposit insurer has to compensate the remaining depositors \( 1 - v_N \) and incurs the political cost of a bank failure \( \beta c \). The expected utility of providing the liquidity is \((- (1 - u_N)(1 + \beta c))\). If the deposit insurer refuses the emergency loan the non-systemic bank will be closed and the deposit insurer will incur the political cost \( \beta c \). In addition the deposit insurer has to compensate all depositors, but she can realize the liquidation value \( L \). So the utility from closing the non-systemic bank is \( L - 1 - \beta c \). The deposit insurer will lend the amount \( v_N \) if:

\[
-(1 - u_N)(1 + \beta c) \geq L - 1 - \beta c,
\]

\[
u_N \geq u_N^{DI} \equiv \frac{L}{1 + \beta c}.
\]

If the solvency signal is below \( u_N^{DI} \) the deposit insurer will refuse the emergency loan. As in the benchmark case the threshold for the liquidity provision is independent of the liquidity shock. Comparing (38) with the threshold in the benchmark case in equation (36) shows that both lending decisions are equivalent.
Having defined the threshold on the solvency signal for the liquidity provision of the deposit insurer we drive from the social welfare function in (10) and the fist-best solvency signal thresholds in (11) and (12) the normalized expected social welfare function for the non-systemic bank given the deposit insurer is the lender of last resort:

$$w_{\text{DI}, \nu}^N = \int_{u_{\text{DI}}^N}^{1} (u_N - u_N') dF(u),$$

(39)

where $\nu \in \{SS, SF\}$ indicates the state of the systemic bank. Lemma 5 presents some properties of (39), the normalized expected social welfare functions of the central banker in equation (18) and the one of the unconditional bailout rule in equation (19).

**Lemma 5.** Assume that $E(\tilde{u}_N \mid u_N \leq u_{\text{DI}}^N) > u_{\text{SF}}^N$. Then, (1) if the systemic bank succeeded, $\nu = SS$ (respectively failed, $\nu = SF$), then (i) $w_{\text{CB}, SS}^N(v_N)$ is increasing in $v_N$ if $v_N < v_A^N \equiv \frac{\alpha c L_{1+c}}{R - L + c}$ (respectively $v_N < v_C^N \equiv \frac{\alpha c L_{1+c}}{R - L + c - \gamma}$), (ii) decreasing if $v_N > v_A^N$ (respectively $v_N > v_C^N$); (2) $w_{\text{CB}, \nu}^N(0) = w_{\text{UBR}, \nu}^N > w_{\text{DI}, \nu}^N$; (3) If $v_N < v_B^N \equiv \frac{\alpha c L_{1+c}}{1-L_{1+c}}$, then $w_{\text{DI}, \nu}^N < w_{\text{CB}, \nu}^N(v_N)$, otherwise $w_{\text{DI}, \nu}^N \geq w_{\text{CB}, \nu}^N(v_N)$; (4) $w_{\text{DI}, \nu}^N > w_{\text{CB}, \nu}^N(1) > 0 \forall \nu \in \{SS, SF\}$.

**Proof.** See Appendix A.8. \qed

As for the benchmark case lemma 5 shows that applying the unconditional bailout rule dominates always the deposit insurer as a lender of last resort. For liquidity shocks below $v_B^N$ the central banker as the lender of last resort yields a higher normalized social welfare than appointing the deposit insurer. If the liquidity shocks are above $v_B^N$ the order inverses and the normalized social welfare given the deposit insurer acts as the lender of last resort is higher.

The policy maker will allocate the lender of last resort responsibilities according to the size of the verifiable liquidity shock $v_N$ to maximize the expected social welfare. Lemma 5 implies the following second-best optimal allocation:

**Proposition 8.** Assume that $E(\tilde{u}_N \mid u_N \leq u_{\text{DI}}^N) > u_{\text{SF}}^N$. If the systemic bank succeeded, $\nu = SS$ (respectively failed, $\nu = SF$), there exists a threshold for the liquidity shortfall of the non-systemic bank $v_{\text{DI}}^N \in (v_A^N, v_B^N)$ (respectively $v_{\text{DI}}^N \in (v_C^N, v_B^N)$) so that it is optimal to allocate the lender of last resort responsibilities for the non-systemic bank to the central banker for liquidity shortfalls below the threshold and to apply the unconditional bailout rule for liquidity shortfalls above it.

33
The introduction of the deposit insurer into the model implements a upper bound $v_{N}^{B}$ for the threshold where the mandate for the lender of last resort responsibilities is handed over from the central banker to the unconditional bailout rule. This threshold is equivalent with the one in the benchmark case when the deposit insurer is considered. Furthermore, the existence of the deposit insurer affects the condition which ensure proposition 8. Given condition $E(\bar{u}_{N} | u_{N} \leq u_{N}^{DI}) > u_{N}^{SF}$ it is more likely that a random non-systemic bank’s asset is of average quality (i.e. $u \in [u_{N}^{SF}, u_{N}^{DI}]$) than of low quality (i.e. $u \in [0, u_{N}^{SF}]$). It is more likely that a non-systemic bank for which liquidity support is social optimal does not receive an emergency loan from the deposit insurer than a non-systemic bank with low quality assets is bailed out unconditionally. Therefore, the policy maker chooses to apply the unconditional bail out rule instead of assigning the deposit insurer with the LLR responsibility.

It is not always optimal to support illiquid banks. For small liquidity shock the central banker’s threshold is closer to the social optimal one than the unconditional bailout rule. Therefore, the LLR responsibilities for small liquidity shock are allocated to the central banker. If the liquidity shock is large the central banker’s lending decision is too restrictive. The unconditional bailout rule maximizes the expected social welfare.

As stated in proposition 8 the existence of the deposit insurer does not change the allocation of the lender of last resort responsibilities. We can show that the extended set of policy instruments does not affect the responsibilities of the central banker given the state of the systemic bank. Proposition 9 summaries the central banker’s mandate in both cases.

**Proposition 9.** The central banker should act as a lender of last resort in a larger range of liquidity shortfalls of the non-systemic bank when the systemic bank failed than when it succeeded (i.e. $v_{N}^{SS} < v_{N}^{SF}$).

**Proof.** See Appendix A.9. □

The intuition for this result is equivalent to the explanation when only the central banker and the unconditional bailout rule are considered. With a failure of the systemic bank the expected return of the non-systemic bank falls so that for all liquidity shocks the social optimal threshold for the provision of the emergency loan increases. Since the central banker becomes less forbearing with increasing liquidity shocks the social optimal lending decision is closer to the central banker’s one for a larger interval of liquidity shocks. However, for very large liquidity shocks the central banker is still too tough so that unconditional bail out rule maximizes in this interval the expected social welfare.
6.3 Systemic bank

Suppose now that the deposit insurer is the lender of last resort for the systemic bank. The deposit insurer’s expected utility from the lender of last resort activities for the systemic bank is given by:

\[
D_S = E\{1_S[-(1 - u_S)(1 + \beta c) + D^{SC}_N] + (1 - 1_S)[L - 1 - \beta c + D^{SL}_N]\},
\]

\[
D_S = E\{1_S[u_S(1 + \beta c) - L + (D^{SC}_N - D^{SL}_N)] + L - 1 - \beta c + D^{SL}_N\}. \tag{40}
\]

If the deposit insurer provides the emergency loan the systemic bank will be successful with a probability \(u_S\) and repays the liquidity assistance so that the deposit insurer does not suffer any losses. With a probability of \((1 - u_S)\) the systemic bank fails. The deposit insurer loses the provided emergency loan \(v_S\) and has to compensate the remaining depositors \((1 - v_S)\). Additionally, the deposit insurer incurs the political cost \(\beta c\). The decision to support the systemic bank influences the profitability of the non-systemic bank. Consequently the utility for the deposit insurer from the non-systemic bank given the systemic bank continues to operate \(D^{SC}_N\) enters into the expected utility of the deposit insurer. In case the systemic bank is not supported the expected costs are \(L - 1 - \beta c + D^{SL}_N\) where \(L\) is the liquidation value of the systemic bank and \(D^{SL}_N\) is the deposit insurer’s utility from the non-systemic bank in case the systemic bank is closed.

According to proposition 8 the deposit insurer is not responsible for the provision of emergency loans to the non-systemic bank. For this reason it does not bear any political cost in case the non-systemic bank fails or is closed. But the deposit insurer still has to compensate the non-systemic bank’s depositors in case of distress. When the central banker is the lender of last resort and the non-systemic bank is not supported the deposit insurer has to compensate all depositors net of the liquidation value \((1 - L)\). If the central banker provides the emergency loan but the non-systemic bank fails the deposit insurer only has to compensate the remaining depositors because the emergency loan from the central banker is not insured by the deposit insurance. Thus, the expected cost for the deposit insurer is \((1 - u_N)(1 - v_N)\). If the unconditional bailout rule is applied the deposit insurer has to compensate all depositors in case of a non-systemic bank’s failure. Therefore, the expected cost is \((1 - u_N)\). This said the utility for the deposit insurer from the non-systemic bank
if the systemic bank S is not liquidated corresponds to:

\[
D_{SC}^N = u_S \left[ \int_{v_{SS}^N}^{v_{SF}^N} \left[ \int_0^{u_{CB}^N(v_N)} -(1 - L)dF(u) + \int_{u_{CB}^N(v_N)}^1 -(1 - u_N)(1 - v_N)dF(u) \right] dG(v_N) \\
+ \int_{v_{SS}^N}^{v_{SF}^N} \int_0^1 -(1 - u_N)(1 - v_N)dF(u)dG(v_N) \right] + (1 - u_S) \\
\left[ \int_{v_{SF}^N}^{v_{SS}^N} \left[ \int_0^{u_{CB}^N(v_N)} -(1 - L)dF(u) + \int_{u_{CB}^N(v_N)}^1 -(1 - u_N)(1 - v_N)(1 - v_N)dF(u) \right] dG(v_N) \\
+ \int_{v_{SF}^N}^{v_{SS}^N} \int_0^1 -(1 - u_N)dF(u)dG(v_N) \right],
\]

(41)

where \(v_{SS}^N\) and \(v_{SS}^{SSN}\) are the second-best threshold as defined in proposition 8.

The deposit insurer’s utility from the non-systemic bank if the systemic bank is liquidated is equal to:

\[
D_{SL}^N = \int_{v_{SF}^N}^{v_{SS}^N} \left[ \int_0^{u_{CB}^N(v_N)} -(1 - L)dF(u) + \int_{u_{CB}^N(v_N)}^1 -(1 - u_N)(1 - v_N)dF(u) \right] dG(v_N) \\
+ \int_{v_{SS}^N}^{v_{SF}^N} \int_0^1 -(1 - u_N)(1 - v_N)dF(u)dG(v_N),
\]

(42)

which follows the same reasoning as above. In case the central banker does not support bank N the deposit insurer liquidates the non-systemic bank and compensates all depositors. In this situation the expected cost for deposit insurer is equal to \(- (1 - L)\). If the central banker provided the emergency loan but bank N fails the deposit insurer has to compensate only the remaining depositors so that the expected costs are \(- (1 - u_N)(1 - v_N)\). If the unconditional bailout rule is applied and bank N fails the deposit insurer compensates all depositors which leads to expected costs equal to \(- (1 - u_N)(1 - v_N)\).

We define:

\[
D_N^\Delta = \int_{v_{SS}^N}^{v_{SF}^N} \left[ \int_0^{u_{CB}^N(v_N)} -(1 - L) -(1 - u_N)(1 - v_N)dF(u) \right] dG(v_N),
\]
so that

\[ D^{SC}_N - D^{SL}_N = u_SD^\Delta_N. \] (43)

Given (40) and (43) the deposit insurer lends to bank S if:

\[ u_S(1 + \beta c + D^\Delta_N) \geq L, \]
\[ u_S \geq u^{DI}_S \equiv \frac{L}{1 + \beta c + D^\Delta_N}, \] (44)

where \( D^\Delta_N \) represents the impact of the deposit insurer’s behavior towards systemic bank on the expected cost related to the non-systemic bank. If the solvency signal is below \( u^{DI}_S \) the deposit insurer will not provide the emergency loan and the systemic bank will be closed.

Using the solvency threshold of the deposit insurer defined in (44), the social welfare function from equation (20) and \( u^*_S = \frac{L}{R + c + W_N} \) we can derive the normalized expected social welfare function given the deposit insurer is the lender of last resort for the systemic bank:

\[ w^{DI}_S = \int_{u^{DI}_S}^{1} (u_S - u^*_S) dF(u). \] (45)

Lemma 6 proves some properties of (45) and relates it with the normalized expected social welfare function for the systemic bank given the central banker is the lender of last resort or the unconditional bailout rule is applied:

**Lemma 6.** Assume that \( E(\tilde{u}_S | u_S \leq u^{DI}_S) > u^*_S \). Then, (1)(i) \( w^{CB}_S (v_S) \) is increasing in \( v_S \) if \( v_S < v^A_S = \frac{L(\alpha c + B^\Delta_N)}{R - L + c + W_N} \); (ii) decreasing if \( v_S > v^A_S \), and (iii) has a global maximum at \( v_S = v^A_S \); (2) \( w^{CB}_S (0) = w^{UBR}_S > w^{DI}_S \); (3) If \( v_S < v^B_S = \frac{L(\alpha c + B^\Delta_N)}{1 - L + \beta c + D^\Delta_N} \), then \( w^{DI}_S < w^{CB}_S (v_S) \), otherwise \( w^{DI}_S \geq w^{CB}_S (v_S) \); (4) \( w^{DI}_S > w^{CB}_S (1) > 0 \).

**Proof.** See Appendix A.10.

Lemma 6 shows that the normalized social welfare from the allocation of the lender of last resort responsibilities to the deposit insurer is dominated by the unconditional bailout rule and for liquidity shocks below \( v^B_S \) also by the central banker. If the liquidity shocks are above \( v^B_S \) the normalized social welfare if the deposit insurer is the lender of last resort
exceeds the normalized social welfare when the central banker acts as the lender of last resort.

The policy maker will allocate the lender of last resort responsibilities among the central banker, the deposit insurer and the unconditional bailout rule conditional on the liquidity shortfall in order to maximize the normalized social welfare. The following proposition is derived from the properties proven in lemma 6:

**Proposition 10.** Assume $E(\tilde{u}_S | u_S \leq u^{DI}_S) > u^*_S$. There exist an liquidity shortfall $v^*_S \in \{v^A_S, v^B_S\}$ so that it is optimal to allocate the lender of last resort responsibilities to the central banker for all liquidity shock smaller than $v^*_S$. Above $v^*_S$ the it is optimal to apply the unconditional bailout rule.

The extended set of policy instruments does not affect the allocation of lender of last resort responsibilities for the systemic bank. It provides however an upper bound $v^B_S$ for the threshold where the mandate is handed over from the central banker to the unconditional bailout rule because from a social welfare point of view the central banker’s lending decision for $v_S > v^B_S$ is worse than the deposit insurer’s liquidity provision.

As for the benchmark case and the non-systemic bank the deposit insurer determine the condition ensuring proposition 10. Condition $E(\tilde{u}_S | u_S \leq u^{DI}_S) > u^*_S$ implies that the asset of a random systemic bank is more likely to be of an average quality (i.e. $u \in [u^*_S, u^{DI}_S]$) than of low quality (i.e. $u \in [0, u^*_S]$). For this reason it is more likely that the deposit insurer does not provide the social optimal emergency loan to a systemic bank than a systemic bank with low quality assets is bailed out unconditionally. Therefore, the policy maker chooses to apply the unconditional bail out rule instead of assigning the deposit insurer with the LLR responsibility.

As above it is not always optimal to support illiquid banks. For small liquidity shock the central banker’s threshold are closer to the social optimal one than the unconditional bailout rule. Therefore, the LLR responsibilities for small liquidity shock are allocated to the central banker. If the liquidity shock is large the central banker’s lending decision is too restrictive. The unconditional bailout rule maximizes the expected social welfare.

7 Conclusion

This paper analyses the optimal institutional allocation of lender of last resort responsibilities in a framework with a systemic and a non-systemic bank. The implication of this paper contributes to the ongoing revision of the regulatory structure after the Subprime crisis in
2007/2008. The recent reform proposals focused on capital and liquidity requirements as well as on recovery and resolution plans to increase the resilience of the financial sector. But the lender of last resort as the last line of defense when external source of finance run dry did not attract much attention.

In our approach a systemic and non-systemic bank coexist. The failure of the systemic bank hurts the return of the non-systemic bank but not vice-versa. Both banks are exposed to a liquidity shock. Taking for granted that other source of external funding are not available a public intervention by the lender of last resort is necessary to avoid socially inefficient bank failures.

We show that the lender of last resort responsibilities are shared between the central banker and the unconditional bailout rule. On the one hand the unconditional bailout rule provides too often socially undesirable emergency loans. For small liquidity shocks the central banker can improve the situation because her emergency liquidity assistance is conditional on the bank’s solvency. On the other hand the central banker gets more restrictive with increasing liquidity shortfalls and might refuse socially optimal emergency liquidity assistance. For this reason the unconditional bailout rule should be applied for large liquidity shocks.

The allocation of lender of last resort responsibilities for the non-systemic bank should be conditional on the state of the systemic bank. Given the negative impact of a systemic bank’s failure on the profitability of the non-systemic bank the central banker should obtain more responsibilities in case the systemic bank collapsed. An symmetric application of the unconditional bailout rule for the non-systemic banks is welfare decreasing because it does not considers the systemic risk within the banking sector.

For the systemic banks the determination of the range of action for the central banker and the unconditional bailout rule is ambiguous because there are counteracting effects. On the one hand from the social optimum point of view more forbearance for systemic bank is desirable. This implies less responsibilities for the central banker and extended application of the unconditional bailout rule. On the other hand the central banker itself will be less restrictive in order to limit the responsibilities for the non-systemic bank. This leads to more responsibilities for the central banker and a smaller range where the unconditional bailout rule is applied.
A Appendix

A.1 Proof of Lemma 1

(1) The first derivative of \( w^{CB} (v) \) is: \( w^{CB} (v) = -\hat{w}^{CB} (v) [u^{CB} (v) - u^*] f (u) \), where \( f \) is the density function of the random variable \( \tilde{u} \). Since \( \hat{w}^{CB} (v) \) and \( f (u) \) are positive for all \( v \) and \( u \), \( w^{CB} (v) \) is increasing in \( v \) if \( u^{CB} (v) < u^* \), decreasing if \( u^{CB} (v) > u^* \), and has a global maximum for \( u^{CB} (v) = u^* \). Since \( u^{CB} (v) > 0 \) and \( v^A \) is so that \( u^{CB} (v^A) = u^* \) (see Figure 3), the result follows.

(2) Since \( u^{CB} (0) = 0 = u^{UBR} \), then \( u^{CB} (0) = u^{UBR} \).

(3)(a) Assume \( w^{CB} (0) - u^{CB} (1) \leq 0 \). Then \( \int_{u^{CB} (0)}^{1} (u - u^*) dF (u) - \int_{u^{CB} (1)}^{1} (u - u^*) dF (u) \leq 0 \), \( \int_{u^{CB} (0)}^{1} (u - u^*) dF (u) \leq 0 \), \( \left[ E \left( \tilde{u} \mid u^{CB} (0) \leq u \leq u^{CB} (1) \right) - u^* \right] \) \( \leq 0 \), and \( E \left( \tilde{u} \mid u^{CB} (0) \leq u \leq u^{CB} (1) \right) \) \( \leq u^* \). A contradiction.

(b) \( w^{CB} (1) = \int_{u^{CB} (1)}^{1} (u - u^*) dF (u) = \left[ E \left( \tilde{u} \mid u > u^{CB} (1) \right) - u^* \right] [1 - F \left( u^{CB} (1) \right)] \). Since \( u^* = \frac{L}{\bar{R} + c} < \frac{1}{1 + ac} = u^{CB} (1) \), both factors are positive, then \( w^{CB} (1) > 0 \).

\( \square \)

A.2 Proof of Proposition 2

(1) \( u^* = u^{SS}_N < u^{SF}_N \) because \( \gamma > 0 \) (2) \( u^{CB} = u^{CB}_N \) (3) \( u^{UBR} = u^{UBR}_N \)

\( \square \)

A.3 Proof of Lemma 2

(1) The first derivative of \( u^{CB, \nu}_N (v_N) \) is: \( u^{CB, \nu}_N (v_N) = -\hat{u}^{CB, \nu}_N (v_N) [u^{CB, \nu}_N (v_N) - u^\nu_N] f (u) \), where \( f \) is the density function of the random variable \( \tilde{u}_N \). Since \( \hat{u}^{CB, \nu}_N (v_N) \) and \( f (u) \) are positive for all \( v_N \) and \( u_N \), \( u^{CB, \nu}_N (v_N) \) is increasing in \( v_N \) if \( u^{CB, \nu}_N (v_N) < u^\nu_N \), decreasing if \( u^{CB, \nu}_N (v_N) > u^\nu_N \), and has a global maximum for \( u^{CB, \nu}_N (v_N) = u^\nu_N \). Since \( u^{CB, \nu}_N (v_N) > 0 \) and \( v^A_N \) (respectively \( v^C_N \)) is so that \( u^{CB, \nu}_N (v^A_N) = u^{SS}_N \) (respectively \( u^{CB, \nu}_N (v^C_N) = u^{SF}_N \)) (see Figure 4), the result follows.

(2) Since \( u^{CB}_N (0) = 0 = u^{UBR}_N \), then \( u^{CB, \nu}_N (0) = w^{UBR, \nu}_N \).

(3)(a) Assume \( w^{CB, \nu}_N (0) - u^{CB, \nu}_N (1) \leq 0 \). Then \( \int_{u^{CB, \nu}_N (0)}^{1} (u - u^\nu_N) dF (u) - \int_{u^{CB, \nu}_N (1)}^{1} (u - u^\nu_N) dF (u) \leq 0 \), \( \left[ E \left( \tilde{u}_N \mid u^{CB, \nu}_N (0) \leq u \leq u^{CB, \nu}_N (1) \right) - u^\nu_N \right] \leq 0 \), and \( E \left( \tilde{u}_N \mid u^{CB, \nu}_N (0) \leq u \leq u^{CB, \nu}_N (1) \right) \leq u^\nu_N \). A contradiction.

(b) \( w^{CB}_N (1) = \int_{u^{CB, \nu}_N (1)}^{1} (u - u^\nu_N) dF (u) = \left[ E \left( \tilde{u}_N \mid u > u^{CB, \nu}_N (1) \right) - u^\nu_N \right] [1 - F \left( u^{CB, \nu}_N (1) \right)] \). Since \( u^\nu_N = \frac{1}{1 + ac} = u^{CB}_N (1) \), both factors are positive, then \( w^{CB}_N (1) > 0 \).

\( \square \)
A.4 Proof of Proposition 4

Given equation (18) and (19) (a) \( w_{N}^{UBR,SS} - w_{N}^{UBR, SF} = u_{N}^{SF} - u_{N}^{SS} \), (b) \( w_{N}^{CB,SS}(v_{N}) - w_{N}^{CB, SF}(v_{N}) = (u_{N}^{SF} - u_{N}^{SS})[1 - F(u_{N}^{CB}(v_{N}))] \), (c) \( w_{N}^{CB,SS}(v_{N}) - w_{N}^{CB, SF}(v_{N}) \) is non-increasing in \( v_{N} \), (d) To the right of \( v_{N}^{C} \) both \( w_{N}^{CB,SS}(v_{N}) \) and \( w_{N}^{CB, SF}(v_{N}) \) are decreasing. It follows that \( v_{N}^{SS} < v_{N}^{SF} \).\( \square \)

A.5 Proof of Proposition 5

The minimum solvency requirement in the first-best is \( u_{S}^{*} \equiv \frac{L}{R + c + W_{N}^{A}} \) where \( W_{N}^{A} = E\{1_{N}^{SS} - 1_{N}^{SF}(u_{N}(R + c) - L) + 1_{N}^{SF}u_{N}^{\gamma}\} \geq 0 \). It follows that \( u_{S}^{*} \leq u^{*} \).\( \square \)

A.6 Proof of Lemma 3

(1) The first derivative of \( w_{S}^{CB}(v_{S}) \) is: \( \dot{w}_{S}^{CB}(v_{S}) = -\dot{u}_{S}^{CB}(v_{S})\left[u_{S}^{CB}(v_{S}) - u_{S}^{*}\right]f(u) \), where \( f \) is the density function of the random variable \( \tilde{u}_{S} \). Since \( \dot{u}_{S}^{CB}(v_{S}) \) and \( f(u) \) are positive for all \( v_{S} \) and \( u_{S} \), \( \dot{w}_{S}^{CB}(v_{S}) \) is increasing in \( v_{S} \) if \( u_{S}^{CB}(v_{S}) < u_{S}^{*} \), decreasing if \( u_{S}^{CB}(v_{S}) > u_{S}^{*} \), and has a global maximum for \( u_{S}^{CB}(v_{S}) = u_{S}^{*} \). Since \( \dot{u}_{S}^{CB}(v_{S}) > 0 \) and \( v_{S}^{A} = \frac{1}{\gamma} \), the result follows.\( \square \)

(2) Since \( u_{S}^{CB}(0) = 0 = u_{S}^{UBR} \), then \( w_{S}^{CB}(0) = w_{S}^{UBR} \).

(3)(a) Assume \( w_{S}^{CB}(0) - w_{S}^{CB}(1) \leq 0 \). Then \( \int_{u_{S}^{CB}(0)}^{1} (u_{S} - u_{S}^{*})dF(u) - \int_{u_{S}^{CB}(1)}^{1} (u_{S} - u_{S}^{*})dF(u) \leq 0 \), \( \int_{u_{S}^{CB}(0)}^{u_{S}^{CB}(1)} (u_{S} - u_{S}^{*})dF(u) \leq 0 \), \( \left[E(\tilde{u}_{S} | u_{S}^{CB}(0) \leq u_{S} \leq u_{S}^{CB}(1)) - u_{S}^{*}\right] \left[F(u_{S}^{CB}(1)) - F(u_{S}^{CB}(0))\right] \leq 0 \), and \( E(\tilde{u}_{S} | u_{S} \leq u_{S}^{CB}(1)) \leq u_{S}^{*} \). A contradiction. Together with property (2) this implies \( w_{S}^{UBR} > w_{S}^{CB} \).

(b) \( w_{S}^{CB}(1) = \int_{u_{S}^{CB}(0)}^{1} (u_{S} - u_{S}^{*})dF(u) = \left[E(\tilde{u}_{S} | u_{S} > u_{S}^{CB}(1)) - u_{S}^{*}\right] \left[1 - F(u_{S}^{CB}(1))\right] \). Since \( u_{S}^{CB}(1) = \frac{1}{\gamma} < 1 \) and assumption \( E(\tilde{u}_{S} | u_{S} \leq u_{S}^{CB}(1)) > u_{S}^{*} \) implies \( E(\tilde{u}_{S} | u_{S} \geq u_{S}^{CB}(1)) > u_{S}^{*} \) both factors are positive, then \( w_{S}^{CB}(1) > 0 \).\( \square \)

A.7 Proof of Lemma 4

The proof of lemma 4 is taken from Ponce (2010).

(1) The first derivative of \( w_{S}^{CB}(v) \) is: \( \dot{w}_{S}^{CB}(v) = -\dot{u}_{S}^{CB}(v)\left[u_{S}^{CB}(v) - u_{S}^{*}\right]f(u) \), where \( f \) is the density function of the random variable \( \tilde{u} \). Since \( \dot{u}_{S}^{CB}(v) \) and \( f(u) \) are positive for all \( v \) and \( u \), \( w_{S}^{CB}(v) \) is increasing in \( v \) if \( u_{S}^{CB}(v) < u_{S}^{*} \), decreasing if \( u_{S}^{CB}(v) > u_{S}^{*} \), and has a global maximum for \( u_{S}^{CB}(v) = u_{S}^{*} \). Since \( \dot{u}_{S}^{CB}(v) > 0 \) and \( v^{A} \) is such that \( u_{S}^{CB}(v^{A}) = u_{S}^{*} \), the result follows.
factors are positive, then $w^{A.9}$ Proof of Proposition 9

(2) (a) Since $u^{CB}(0) = 0 = u^{UBR}$, then $w^{CB}(0) = u^{UBR}$. (b) Assume $w^{UBR} - w^{DI} \leq 0$. Then $\int_0^1 (u - u^*) dF(u) - \int_{w^{DI}} (u - u^*) dF(u) \leq 0$, $\int_0^1 (u - u^*) dF(u) \leq 0$, $[E(\tilde{u} \mid u \leq w^{DI}) - u^*] F(w^{DI}) \leq 0$, and $E(\tilde{u} \mid u \leq w^{DI}) \leq u^*$. A contradiction.

(3) Since $v^B$ is so that $u^{CB}(v^B) = u^{DI}$, then $w^{CB}(v^B) = w^{DI}$. Properties 1 and 2 imply that $w^{DI} < w^{UBR} \leq w^{CB}(v)$ for $v < v^B$ and that $w^{DI} \geq w^{CB}(v)$ for $v \geq v^B$.

(4) Since $v^B < 1$, property 3 implies that $w^{DI} > w^{CB}(1)$. $w^{CB}(1) = \int_{u^{CB}(1)}^1 (u - u^*) dF(u) = [E(\tilde{u} \mid u > u^{CB}(1)) - u^*] [1 - F(u^{CB}(1))].$ Since $u^* = \frac{L}{R+c} < \frac{1}{1+\alpha c} = u^{CB}(1) < 1$ both factors are positive, then $w^{CB}(1) > 0$.

A.8 Proof of Lemma 5

(1) The first derivative of $w^{CB,\nu}_N(v_N)$ is: $w^{CB,\nu}_N(v_N) = -\tilde{u}^{CB}_N(v_N) [u^\nu_N(v_N) - u^\nu_N] f(u)$, where $f$ is the density function of the random variable $\tilde{u}_N$. Since $u^{CB}_N(v_N)$ and $f(u)$ are positive for all $v_N$ and $u_N$, $w^{CB,\nu}_N(v_N)$ is increasing in $v_N$ if $u^{CB}_N(v_N) < u^\nu_N$, decreasing if $u^{CB}_N(v_N) > u^\nu_N$, and has a global maximum for $u^{CB}_N(v_N) = u^\nu_N$. Since $u^{CB}_N(v_N) > 0$ and $\nu_N$ (respectively $v_N$) is so that $u^{CB}_N(v_N^A) = u^{SS}_N$ (respectively $u^{CB}_N(v_N^C) = u^{SF}_N$), the result follows.

(2) (a) Since $u^{CB}_N(0) = 0 = u^{UBR}$, then $w^{CB,\nu}_N(0) = w^{UBR,\nu}_N$. (b) Assume $w^{UBR,\nu}_N - w^{DI,\nu}_N \leq 0$. Then $\int_0^1 (u_N - u^\nu_N) dF(u) - \int_{w^{DI}_N} (u_N - u^\nu_N) dF(u) \leq 0$, $\int_{w^{DI}_N} (u_N - u^\nu_N) dF(u) \leq 0$, $[E(\tilde{u}_N \mid u_N \leq w^{DI}_N) - u^\nu_N] F(w^{DI}_N) \leq 0$, and $E(\tilde{u}_N \mid u_N \leq w^{DI}_N) \leq u^\nu_N$. A contradiction.

(3) Since $v^B_N$ is so that $u^{CB}_N(v^B_N) = u^{DI}_N$, then $w^{CB,\nu}_N(v^B_N) = w^{DI,\nu}_N$. Properties 1 and 2a imply that $w^{DI,\nu}_N < w^{UBR,\nu}_N \leq w^{CB,\nu}_N(v_N)$ for $v_N > v^B$ and that $w^{DI,\nu}_N \geq w^{CB,\nu}_N(v_N)$ for $v_N \geq v^B$.

(4) Since $v^B_N < 1$, property 3 implies that $w^{DI,\nu}_N > w^{CB,\nu}_N(1)$. $w^{CB}_N(1) = \int_{u^{CB,\nu}_N(1)}^1 (u_N - u^\nu_N) dF(u) = [E(\tilde{u}_N \mid u_N > u^{CB}_N(1)) - u^\nu_N] [1 - F(u^{CB}_N(1))].$ Since $u^\nu_N < \frac{1}{1+\alpha c} = u^{CB}_N(1) < 1$ both factors are positive, then $w^{CB}_N(1) > 0$.

A.9 Proof of Proposition 9

Given equation (18) and (19) (a) $w^{UBR,SS}_N - w^{UBR,SS}_N = u^{SF}_N - u^{SS}_N$, (b) $w^{CB,SS}_N(v_N) - w^{CB,SS}_N(v_N) = (u^{SF}_N - u^{SS}_N) [1 - F(u^{CB}_N(v_N))]$, (c) $w^{CB,SS}_N(v_N) - w^{CB,SS}_N(v_N)$ is non-increasing in $v_N$, (d) To the right of $v^C_N$ both $w^{CB,SS}_N(v_N)$ and $w^{CB,SS}_N(v_N)$ are decreasing. It follows that $v^{SS}_N < v^{SF}_N$. 

42
A.10 Proof of Lemma 6

(1) The first derivative of $w_{s}^{CB}(v_s)$ is: $\dot{w}_{s}^{CB}(v_s) = -u_{s}^{CB}(v_s) \left[ u_{s}^{CB}(v_s) - u_{s}^v \right] f(u)$, where $f$ is the density function of the random variable $\bar{u}_s$. Since $u_{s}^{CB}(v_s)$ and $f(u)$ are positive for all $v_s$ and $u_s$, $w_{s}^{CB}(v_s)$ is increasing in $v_s$ if $u_{s}^{CB}(v_s) < u_{s}^v$, decreasing if $u_{s}^{CB}(v_s) > u_{s}^v$, and has a global maximum for $u_{s}^{CB}(v_s) = u_{s}^v$. Since $u_{s}^{CB}(v_s) > 0$ and $v_s^A$ is so that $u_{s}^{CB}(v_s^A) = u_{s}^*$, the result follows.

(2) (a) Since $u_{s}^{CB}(0) = 0 = u_{s}^{UBR}$, then $w_{s}^{CB}(0) = w_{s}^{UBR}$. (b) Assume $w_{s}^{UBR} - w_{s}^{DI} \leq 0$. Then $\int_0^1 (u_s - u_s^v) dF(u) - \int_{u_s^v}^{u_s} (u_s - u_s^v) dF(u) \leq 0$, $\int_{u_s}^{u_s^v} (u_s - u_s^v) dF(u) \leq 0$, $\left[ E\left(\bar{u}_s | u_s \leq u_s^{DI}\right) - u_s^v\right] F\left(u_s^{DI}\right) \leq 0$, and $E\left(\bar{u}_s | u_s \leq u_s^{DI}\right) \leq u_s^v$. A contradiction.

(3) Since $v_s^B$ is so that $u_{s}^{CB}(v_s^B) = u_{s}^{DI}$, then $w_{s}^{CB}(v_s^B) = w_{s}^{DI}$. Properties 1 and 2a imply that $w_{s}^{DI} < w_{s}^{UBR} \leq w_{s}^{CB}(v_s)$ for $v_s < v_s^B$ and that $w_{s}^{DI} \geq w_{s}^{CB}(v_s)$ for $v_s \geq v_s^B$.

(4) Since $v_s^B < 1$, property 3 implies that $w_{s}^{DI} > w_{s}^{CB}(1)$. $w_{s}^{CB}(1) = \int_{u_s}^{u_s^v} (u_s - u_s^v) dF(u) = E\left(\bar{u}_s | u_s > u_s^{CB}(1)\right) - u_s^v [1 - F\left(u_s^{CB}(1)\right)]$. Since $u_s^v = \frac{L}{R + e^{-W_s^N}} < \frac{1}{1 + \alpha c + B_s^N} = u_s^{CB}(1) < 1$ both factors are positive, then $w_{s}^{CB}(1) > 0$. $\blacksquare$

References


Invest deposits into risky asset

Figure 1: Timing of the model.
Figure 2: Lending decisions in the benchmark case. It is socially optimal to lend to benchmark banks with solvency signals above $u^*$. In region $\circ$ the central banker (CB) provides socially non-desirable emergency loans; in region $\square$ she does not provide socially desirable emergency loans. In regions $\bullet$ and $\mathbb{D}$, socially non-desirable emergency loans are provided by following the unconditional bailout rule (UBR). Let $v^A \equiv \frac{\alpha c L}{\mathcal{R} - L + c}$ be the value for $v$ so that $u^{CB}(v) = u^*$. It is immediate that $0 < v^A < 1$. 
Figure 3: Normalized expected social welfare for the benchmark bank. The optimal allocation of the lender of last resort activity for the benchmark bank follows the upper envelope of solid functions: for $v < v^*$ the central banker’s (CB) decision maximizes $w$; for $v \geq v^*$ the unconditional bailout rule (UBR) maximizes $w$. 
Liquidity Shortfall ($v_N$)

Solvency Signal ($u_N$)

CB’s threshold, $u_{CB}^N(v_N)$.

UBR’s threshold, $u_{UBR}^N(v_N)$.

Socially optimal threshold in SF, $u_{SF}^N$.

Socially optimal threshold in SS, $u_{SS}^N$.

Figure 4: Lending decisions for the non-systemic bank. It is socially optimal to lend to non-systemic banks with solvency signals above $u_i^N$ for $i \in \{SS, SF\}$. In region @ the central banker (CB) provides socially non-desirable emergency loans; in region © she does not provide socially desirable emergency loans. In regions @ and ©, socially non-desirable emergency loans are provided by following the unconditional bailout rule (UBR).

In state SS the systemic bank’s asset was successful while in state SF the systemic bank either was liquidated or its asset failed.

Let $v_A^N = \frac{\alpha c L}{R - L + \epsilon}$ be the value for $v_N$ so that $u_{CB}^N(v_N) = u_{SS}^N$ and $v_C^N = \frac{\alpha c L}{R - L + \epsilon - \gamma}$ be the value for $v_N$ so that $u_{CB}^N(v_N) = u_{SF}^N$. It is immediate that $0 < v_A^N < v_C^N < 1$. 

49
Liquidity Shortfall ($v_N$) 

Normalized Expected Social Welfare ($w_N$)

The CB is the LLR in SS, $w_{CB,SS}^N(v_N)$. 

UBR is applied in SS, $w_{UBR,SS}^N(v_N)$. 

UBR is applied in SF, $w_{UBR,SF}^N(v_N)$. 

The CB is the LLR in SS, $w_{CB,SS}^N(v_N)$. 

The CB is the LLR in SF, $w_{CB,CS}^N(v_N)$. 

Figure 5: Normalized expected social welfare for the non-systemic bank. The optimal allocation of the lender of last resort activity for the non-systemic bank follows the upper envelope of solid functions in case the systemic bank survives and is successful. Otherwise it follows the upper envelope of the dashed functions: for $v_N < v^i_N$ the central banker’s (CB) decision maximizes $w_N^i(v_N)$; for $v_N \geq v^i_N$ the unconditional bailout rule (UBR) maximizes $w_N^i(v_N)$ for $i \in \{SS, SF\}$. 

In state SS the systemic bank’s asset was successful while in state SF the systemic bank either was liquidated or its asset failed.
Figure 6: Lending decisions in the benchmark case. It is socially optimal to lend to benchmark banks with solvency signals above $u^*$. In region $\text{(a)}$ the central banker (CB) provides socially non-desirable emergency loans; in regions $\text{(c)}$ and $\text{(d)}$ she does not provide socially desirable emergency loans. In regions $\text{(c)}$ and $\text{(e)}$ the deposit insurer (DI) does not provide socially desirable emergency loans. In regions $\text{(a)}$ and $\text{(b)}$, socially non-desirable emergency loans are provided by following the unconditional bailout rule (UBR).

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*$ and $v^B = \frac{\alpha c L}{1 - L + \beta c}$ the value for $v$ so that $u^{CB}(v) = u^{DI}$. It is immediate that $0 < v^A < v^B$. Moreover, $c < \frac{1 - L}{L}$ implies that $v^B < 1$. 

Let $v^A = \frac{\alpha c L}{K - L + \epsilon}$ be the value for $v$ so that $u^{CB}(v) = u^*