Foreign exchange intervention and monetary policy design: a market microstructure analysis

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Abstract

In this paper we extend a new Keynesian open economy model to include FX dealers, information heterogeneity in the FX market and FX intervention by the monetary authority, in line with Bacchetta and van Wincoop (2006) and Vitale (2011). These ingredients generate deviations from the uncovered interest parity (UIP) condition. More precisely, in this setup portfolio decisions of the dealers add endogenously a time variant risk-premium element to the traditional UIP that depends on FX intervention by the central bank and FX orders by foreign investors. We analyse the effectiveness of different strategies of FX intervention (eg: unanticipated operations or via a preannounced rule) to affect the volatility of the exchange rate and the transmission mechanism of the interest rate. Also, to solve the model, we extend the methodology proposed by Townsend (1983) to solve dynamic stochastic general equilibrium (DSGE) models with heterogeneous expectations.

We find that in general equilibrium FX intervention can reduce the power of monetary policy, because it mutes the monetary transmission through exchange rates. Also, FX intervention under discretion can have larger effects on the exchange rate than under rules because of a surprise effect. However, rules can have stronger stabilisation power in response to shocks because they exploit the expectations channel.

Key words: Foreign Exchange Microstructure, Exchange rate dynamics, Exchange Rate Intervention, Monetary policy.

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1 Introduction

Interventions by central banks in FX markets have been common in many countries, and they have become even more frequent in both emerging market economies and some advanced economies in the most recent years. These interventions have been particularly large during periods of capital inflows, where central banks buy foreign currency to prevent an appreciation of the domestic currency. Also, they have been recurrent during periods of financial stress and capital outflows, where central banks used their reserves to prevent sharp depreciations of their currencies. For instance, in figure 1 we can see that during 2009 and March 2012 the amount of FX interventions as a percentage of FX reserves minus gold have been between 30% and 100% in some Latin American countries, and more than 100% in Switzerland. Also, these FX interventions have been sterilised in most cases, that is changes in central bank net foreign reserves have been accommodated by changes in net domestic, consistent in most cases with the use of an interest rate target as a policy instrument. Therefore, central banks have been able to intervene in FX markets without losing control of policy rates.

Given the considerable size of interventions in FX markets by some central banks, it is very important for those economies to include this factor in the policy analysis framework of central banks. There are different questions that need to be addressed, such as: how sterilised intervention affects the transmission mechanism of monetary policy?, which channels are at work?, are there benefits for intervention rules?, what should be the optimal monetary policy design in the context of FX intervention? To analyse these questions we need to have an adequate framework of exchange rate determination in macroeconomic models.

There is substantial empirical evidence that traditional approaches of exchange rate determination (eg assets markets) fail to explain exchange rate movements in the short-run, see for example Meese and Rogoff (1983) and Frankel and Rose (1995). This empirical evidence shows that most exchange fluctuations at short to medium term horizons are related to order flows - the flow of transactions between market participants -, as in the microstructure approach presented by Lyons (2001), and not to macroeconomic variables. However, in most of the models used for monetary policy analysis, the exchange rate is closely linked to macroeconomic fundamentals, as in the uncovered interest rate parity (UIP) condition. Such inconsistency between the model and the real exchange determination in practice could lead in some cases to incorrect policy prescriptions. For example, the overestimation of the impact of fundamentals corresponding underestimation of the impact of news and beliefs in the exchange rate.
Bacchetta and van Wincoop (2006) provide an alternative framework to analyse exchange rate determination. They introduce symmetric information dispersion about future macroeconomic fundamentals in a dynamic rational expectations model to explain some stylised facts of exchange rates. When introducing information heterogeneity their model can account for the short run disconnection between exchange rate fluctuations and observed fundamental, while both variables become closely related over longer horizons. In their model, exchange rates are closely related to order flow, defined as the private information component of FX orders. In a related work, Vitale (2010) extends Bacchetta and van Wincoop (2006) model to analyse the impact of FX intervention on FX markets. This model is useful to analyse how FX intervention influences exchange rates via both a portfolio-balance and a signalling channel.

In order to provide a framework for analysis of FX intervention together with monetary policy, we extend a standard new Keynesian small open economy model including market microstructure of exchange rate determination in the spirit of Bacchetta and van Wincoop (2006) and Vitale (2010). Different from them, we present a fully dynamic stochastic general equilibrium model with nominal rigidities, were we can analyse the interaction with interest rate policy and FX
intervention. In this setup, we introduce FX dealers and information heterogeneity in the FX market which generate deviations from the uncovered interest parity (UIP) condition. More precisely, in this alternative setup, the portfolio decision of dealers adds a time variant risk-premium element to the traditional UIP that depends on both FX intervention by the central bank and FX orders from foreign investors.

In our model central bank FX intervention affects exchange rate determination through two channels: the portfolio balance effect and the expectations/signalling effect. In the former, sterilized intervention alters the value of the currency because it modifies the ratio between domestic and foreign assets held by the private sector; and according to the latter, operations in foreign exchange markets by the monetary authority may signal changes in future monetary policy, affecting market expectations and hence the exchange rate. Our preliminary findings show that in general equilibrium FX intervention can reduce the power of monetary policy, because it mutes the monetary transmission mechanism through exchange rates. Also, FX intervention under discretion can have larger effect on the exchange rate than under rules because of a surprise effects. However, rules can have stronger stabilisation power in response to shocks because they exploit the expectations channel.

On the technical side, as the rational expectations equilibrium depends on portfolio decisions of FX dealers with heterogeneous expectations, which in turn depend on the conditional variance of the depreciation rate, the solution strategy follows an approach in line with Townsend (1983) and Bacchetta and van Wincoop (2006). That is, we solve a signal extraction problem of the investors to calculate the average expected depreciation rate in the modified uncovered interest parity condition with an endogenous risk premium, which feeds from the rational expectations solution of the model.

In the next section we introduce the model, with a special focus to the FX market. Section 3 presents the computational strategy and section 4 shows results from the simulation of the model. The last section concludes.

2 The Model

The model departs from a small open economy with nominal rigidities, in line with the contributions from Obstfeld and Rogoff (1995), Chari et al (2002), Gali and Monacelli (2005), Christiano et al (2005) and Devereaux et al (2006), among others. To maintain the concept of general equilibrium, we depart from a two-country model taking the size of one of these economies close to
zero, such that the small (domestic) economy does not affect the large (foreign) economy.

In this setup, dealers in the small domestic economy receive savings in domestic currency from households and customer sale orders in foreign currency from foreign investors and the central bank. Dealers invest each period in bonds in both currencies maximising their portfolio returns. We assume the frequency of decisions is the same between dealers and other economic agents. Households consume final goods, supply labour to intermediate goods producers and save in domestic bonds. Firms produce intermediate and final goods. Additionally, we include monopolistic competition and nominal rigidities in the retail sector, price discrimination and price to market in the export sector; and incomplete pass-through from exchange rate to imported good prices. Characteristics that are important to analyse the transmission mechanism of monetary policy in a small open economy. The monetary authority sets the nominal interest rate and intervenes directly in the spot FX market. The central bank can control the interest rate regardless of the FX intervention, that is we assume the central bank can always perform fully sterilised interventions\(^1\). We also consider as exogenous processes the foreign variables, such as output, inflation, interest rate and capital flows.

2.1 Dealers

In the domestic economy there is a continuum of dealers \(d\) in the interval \(d \in [0, 1]\). Each dealer \(d\) receive \(\omega_t^d\) in domestic currency from consumers, and \(\omega_t^{ds}\) and \(\omega_t^{dscb}\) in foreign currency from foreign investors and the central bank, respectively. These holdings are invested by dealers in domestic and foreign bonds, that is \(\omega_t^d + S_t (\omega_t^{ds} + \omega_t^{dscb}) = B_t^d + S_t B_t^{ds}\). As explained below, each dealer receive the same amount of savings from households and the same amount of foreign currency from the central bank, while the amount received from foreign investors differs between them. The exchange rate \(S_t\) is defined as the price of foreign currency in terms of domestic currency, that is a decrease (increase) of \(S_t\) corresponds to an appreciation (depreciation) of the domestic currency. Dealers are short-sighted, they invest taking into account only the return for one period. Dealer \(d\) problem is

\[
\max_{\Omega_{t+1}} -E_t^d e^{-\gamma \Omega_{t+1}}
\]

\(^1\)However, in practice sterilised interventions have limits. For example, the sale of foreign currency by the central bank is limited by the level of foreign reserves. On the other hand, the sterilised purchase of foreign currency is limited by the availability of instruments to sterilise those purchases (eg given by the stock of treasury bills in hands of the central bank or by the demand for central bank paper).
where $E_{d}^{t}$ is the expectation operator for dealer $d$ based on the information available at time $t$, $\gamma$ is the coefficient of absolute risk aversion and $\Omega_{d+1}^{t}$ is total investment after returns, given by:

$$
\Omega_{d+1}^{t} = (1 + i_{t}) B_{d}^{t} + (1 + i_{t}^{*}) S_{t+1} B_{d}^{d*}
$$

$$
\approx (1 + i_{t}) \left[ \omega_{d}^{t} + S_{t} (\omega_{d}^{d*} + \omega_{d}^{d*,cb}) \right] + (i_{t}^{*} - i_{t} + s_{t+1} - s_{t}) B_{d}^{d*}
$$

where we made use of the resource constraint of dealers, we have log-linearised the excess of return on investing in foreign bonds and $s_{t} = \ln S_{t}$. Since the only non-predetermined variable is $s_{t+1}$, assuming it is normal distributed with time-invariant variance $\sigma^{2}$, the first order condition for the dealers is:

$$
0 = -\gamma \left( i_{t}^{*} - i_{t} + E_{d}^{t} s_{t+1} - s_{t} \right) + \gamma^{2} B_{d}^{d*} \sigma^{2}
$$

where $\sigma^{2} = var_{t} (\Delta s_{t+1})$ is the conditional variance of the depreciation rate. Then, the demand for foreign bonds by dealer $d$ is given by the following portfolio condition:

$$
B_{d}^{d*} = \frac{i_{t}^{*} - i_{t} + E_{d}^{t} s_{t+1} - s_{t}}{\gamma \sigma^{2}}
$$

(2.1)

According to this expression, the demand for foreign bonds will be larger the higher its return, the lower the risk aversion or the lower the volatility of the exchange rate.

2.1.1 FX market equilibrium

Foreign bonds equilibrium in the domestic market should sum FX market orders from foreign investors (capital inflows) and central bank FX intervention, that is:

$$
\int_{0}^{1} B_{d}^{d*} dd = \int_{0}^{1} \left( \omega_{d}^{d*} + \omega_{d}^{d*,cb} \right) dd = \omega_{d}^{*} + \omega_{d}^{*,cb}.
$$

Replacing the FX market equilibrium condition in the aggregate demand for foreign bonds yields the following arbitrage condition:

$$
\bar{E}_{t} s_{t+1} - s_{t} = i_{t} - i_{t}^{*} + \gamma \sigma^{2} (\omega_{d}^{*} + \omega_{d}^{*,cb})
$$

(2.2)

where $\bar{E}_{t} s_{t+1}$ is the average rational expectation of the exchange rate next period across all dealers. Given that dealers have access to different sets of information, expected exchange rate depreciation would differ among them as well. This condition determines the exchange rate, and differs from the traditional uncovered interest parity condition on both the expectation term and

\textsuperscript{2}Conditions that are verified later that are satisfied.
the endogenous risk premium component. According to (2.2), an increase (decrease) in capital inflows or sales (purchases) of foreign bonds by the central bank\textsuperscript{3} appreciates (depreciates) the exchange rate $s_t$ (ceteris paribus the rest of variables). This effect is larger, the more risk averse dealers are (larger $\gamma$) or the more volatile the expected depreciation rate is (larger $\sigma^2$).

Equation (2.2) is useful to understand both mechanisms through which FX intervention can affect the exchange rate. The last term on the right hand side captures the portfolio-balance channel. Given that dealers are risk-averse and hold domestic and foreign assets to diversify risk, FX intervention changes the composition of domestic and foreign asset held by the dealers. This will be possible only if there is a change in the expected relative rate of returns of these assets, which compensates for the change in the risk they bear. In other words, according to the portfolio-balance channel, a sale (purchase) of foreign bonds by the central bank augments (reduces) the ratio between foreign and domestic assets hold by dealers, inducing an appreciation (depreciation) of the domestic currency because dealers require a greater (smaller) risk premium to hold a larger (smaller) quantity of this currency.

The second mechanism at work is the expectations channel, also known as the signalling channel. When central banks intervene in the FX markets they also signal future changes in policy, which affect expectations as well. Therefore, changes in $E_t s_{t+1}$ in the left hand side caused by FX intervention will also have an effect in the spot exchange rate.

\subsection*{2.1.2 Information structure}

We include two sources of information heterogeneity among dealers: we assume they face idiosyncratic shocks in the amount of customer orders from foreign investors and also receive noisy signals about some future shocks.

We assume the foreign investor exposure for each dealer is equal to the average plus an idiosyncratic term:

$$d_t = \bar{d}_t + \epsilon^d_t$$

where $\epsilon^d_t$ has variance that approaches to infinity, so that knowing one's own foreign investor exposure provides no information about the average exposure as in Bacchetta and van Wincoop (2006). $\bar{d}_t$ is unobservable and follows an AR(1) process:

$$\bar{d}_t = \rho_{\bar{d}} \bar{d}_{t-1} + \epsilon^\omega_t$$

\textsuperscript{3}In this case, a sale (purchase) of foreign currency by the central bank implies purchases (sales) of domestic bonds by the monetary authority.\hfill 7
where $\varepsilon_t^{w_t} \sim N(0, \sigma_w^2)$. The assumed autoregressive process is known by all agents.

We assume that dealers observe past and current fundamental shocks, while they also receive private signals about some future shocks. More precisely, we assume dealers receive one signal each period about the foreign interest rate one period ahead\(^4\). That is, at time $t$ dealer $d$ receives a signal

$$v_t^d = i_t^* + \varepsilon_t^{vd}, \quad \varepsilon_t^{vd} \sim N(0, \sigma_{vd}^2)$$

where $\varepsilon_t^{vd}$ is independent from $i_{t+1}^*$ and other agent’s signals. This idiosyncratic signal can be reconciled with the fact that dealers have different models to forecast future fundamentals, so each can imperfectly observe future variables with an idiosyncratic noise. We also assume that the average signal received by investors is $i_{t+1}^*$, that is

$$\frac{1}{d} \sum_{d=1}^{D} v_t^d = i_t^*.$$ 

The foreign interest rate follows an AR(1) process known by dealers:

$$i_t^* = \rho_i i_{t-1}^* + \varepsilon_t^*$$

where $\varepsilon_t^* \sim N(0, \sigma_i^2)$. Dealers solve a signal extraction problem for the unknown innovations $(\varepsilon_t^{w_t}, i_{t+1}^*)$, given the observed depreciation rate and signal $(\Delta s_t, v_t^d)$\(^5\).

### 2.2 Households

#### 2.2.1 Preferences

The world economy is populated by a continuum of households of mass 1, where a fraction $n$ of them is allocated in the home economy, whereas the remaining $1 - n$ is in the foreign economy. Each household $j$ in the home economy enjoys utility from the consumption of a basket of final goods, $C_j^t$, and receives disutility from working, $L_j^t$. The households preferences are represented by the following utility function:

$$U_t = E_t \left[ \sum_{s=0}^{\infty} \beta^{t+s} U \left( C_{t+s}^j, L_{t+s}^j \right) \right],$$

where $E_t$ is the conditional expectation on the information set at period $t$ and $\beta$ is the intertemporal discount factor, with $0 < \beta < 1$.

The consumption basket of final goods is a composite of domestic and foreign goods, aggre-

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\(^4\)This assumption can be extended to the case where dealers receive each period a vector of signal of a set of fundamental variables.

\(^5\)As shown in the appendix, knowledge of the depreciation rate at times $t-1$ and earlier and of the interest rate shocks at time $t$ and earlier, reveal the shocks $\varepsilon^{w_t}$ at times $t-1$ and earlier. That is, $\varepsilon_{t-1}^{w_t}$ become observable at time $t$ for $s \geq 1$. 
gated using the following consumption index:

\[ C_t \equiv \left( (\gamma_H)^{\frac{1}{\varepsilon_H}} \left( C_t^H \right)^{\frac{\varepsilon_H-1}{\varepsilon_H}} + (1 - \gamma_H)^{\frac{1}{\varepsilon_H}} \left( C_t^M \right)^{\frac{\varepsilon_H-1}{\varepsilon_H}} \right)^{-\frac{1}{\varepsilon_H}}, \] (2.8)

where \( \varepsilon_H \) is the elasticity of substitution between domestic (\( C_t^H \)) and foreign goods (\( C_t^M \)), and \( \gamma_H \) is the share of domestically produced goods in the consumption basket of the domestic economy.

In turn, \( C_t^H \) and \( C_t^M \) are indexes of consumption across the continuum of differentiated goods produced in the home country and those imported from abroad, respectively. These consumption indices are defined as follows:

\[ C_t^H \equiv \left( \frac{1}{n} \int_0^n C_t^H(z)^{\frac{\varepsilon_H-1}{\varepsilon_H}} dz \right)^{\frac{1}{\varepsilon_H}}, C_t^M \equiv \left( \frac{1}{1-n} \int_n^1 C_t^M(z)^{\frac{\varepsilon_H-1}{\varepsilon_H}} dz \right)^{\frac{1}{\varepsilon_H}} \] (2.9)

where \( \varepsilon > 1 \) is the elasticity of substitution across goods produced within the home economy, denoted by \( C_t^H(z) \), and within the foreign economy, \( C_t^M(z) \). The household optimal demands for home and foreign consumption are given by:

\[ C_t^H(z) = \frac{1}{n} \gamma_H \left( \frac{P_t^H(z)}{P_t^H} \right)^{-\varepsilon} \left( \frac{P_t^H}{P_t^M} \right)^{-\varepsilon_H} C_t, \] (2.10)

\[ C_t^M(z) = \frac{1}{1-n} (1 - \gamma_H) \left( \frac{P_t^M(z)}{P_t^H} \right)^{-\varepsilon} \left( \frac{P_t^M}{P_t^M} \right)^{-\varepsilon_H} C_t \] (2.11)

This set of demand functions is obtained by minimizing the total expenditure in consumption \( P_t C_t \), where \( P_t \) is the consumer price index. Notice that the consumption of each type of goods is increasing in the consumption level, and decreasing in their corresponding relative prices. Also, it is easy to show that the consumer price index, under these preference assumptions, is determined by the following condition:

\[ P_t \equiv \left[ \gamma_H \left( P_t^H \right)^{1-\varepsilon_H} + (1 - \gamma_H) \left( P_t^M \right)^{1-\varepsilon_H} \right]^{\frac{1}{1-\varepsilon_H}} \] (2.12)

where \( P_t^H \) and \( P_t^M \) denote the price level of the home produced and imported goods, respectively. Each of these price indexes is defined as follows:

\[ P_t^H \equiv \left[ \frac{1}{n} \int_0^n P_t^H(z)^{1-\varepsilon} dz \right]^{\frac{1}{\varepsilon}}, P_t^M \equiv \left[ \frac{1}{1-n} \int_n^1 P_t^M(z)^{1-\varepsilon} dz \right]^{\frac{1}{\varepsilon}} \] (2.13)

where \( P_t^H(z) \) and \( P_t^M(z) \) represent the prices expressed in domestic currency of the variety \( z \) of home and imported goods, respectively.
2.2.2 Household’s budget constraint

For simplicity, we assume domestic households save only in domestic currency through the dealers. The budget constraint of the domestic household \((j)\) in units of home currency is given by:

\[
\varpi_t^j = (1 + i_{t-1}) \varpi_{t-1}^j - \frac{\psi}{2} \left( \varpi_t^j - \bar{\varpi} \right)^2 + W_t L_t^j - P_t C_t^j + P_t \Gamma_t^j
\]  

(2.14)

where \(\varpi_t^j\) is wealth in domestic assets, \(W_t\) is the nominal wage, \(i_t\) the domestic nominal interest rate, and \(\Gamma_t^j\) are nominal profits distributed from firms in the home economy to the household \(j\). An assumption is made that each household holds a fraction \(\frac{1}{n}\) of all firms in the economy and that there is no trade in firms shares. Households also face portfolio adjustment costs, for adjusting wealth from its long-run level. Households maximize (2.7) subject to (2.14).

2.2.3 Consumption decisions and the supply of labour

The conditions characterizing the optimal allocation of domestic consumption are given by the following equation:

\[
U_{C,t} = \beta E_t \left\{ U_{C,t+1} \left[ \frac{1 + i_t}{1 + \psi (\varpi_t^j - \bar{\varpi})} \right] \frac{P_t}{P_{t+1}} \right\}
\]  

(2.15)

We eliminate the index \(j\) for the assumption of representative agent. \(U_{C,t}\) denotes the marginal utility for consumption. Equation (2.15) corresponds to the Euler equation that determines the optimal path of consumption for households in the home economy, by equalizing the marginal benefits of savings to its corresponding marginal costs. The first-order conditions that determine the supply of labor are characterized by the following equation:

\[
\frac{U_{L,t}}{U_{C,t}} = \frac{W_t}{P_t}
\]  

(2.16)

where \(\frac{W_t}{P_t}\) denotes real wages. In a competitive labour market, the marginal rate of substitution equals the real wage, as in equation (2.16).

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6 This way the only portfolio decision is made by dealers, what simplifies the analysis.

7 This assumption allows us to work with the aggregate real economy as a representative agent model economy. Otherwise we would have to keep track of the wealth position of each household in the economy.

8 This assumption is necessary to provide stationarity in the asset position held by the households. See Schmitt-Grohe and Uribe (2004).
2.3 Foreign economy

The consumption basket of the foreign economy is similar to that of the foreign economy, and is given by:

\[
C_t^F = \left( \gamma^F \right)^{1/\varepsilon_F} \left( C_t^X \right)^{\varepsilon_F - 1} + \left( 1 - \gamma^F \right)^{1/\varepsilon_F} \left( C_t^F \right)^{\varepsilon_F - 1} \varepsilon_F^{-1} \varepsilon_F - 1 \tag{2.17}
\]

where \( \varepsilon_F \) is the elasticity of substitution between domestic (\( C_t^X \)) and foreign goods (\( C_t^F \)), respectively, and \( \gamma^F \) is the share of domestically produced goods in the consumption basket of the foreign economy. Also, \( C_t^X \) and \( C_t^F \) are indexes of consumption across the continuum of differentiated goods produced similar to \( C_t^H \) and \( C_t^M \) defined in equations (2.9). The demands for each type of good are given by:

\[
C_t^X(z) = \frac{1}{1-n} \gamma^F \left( \frac{P_t^X(z)}{P_t^H} \right)^{-\varepsilon_H} C_t^F \tag{2.18}
\]

\[
C_t^F(z) = \frac{1}{1-n} \gamma^F \left( \frac{P_t^X(z)}{P_t^H} \right)^{-\varepsilon_H} C_t^F \tag{2.19}
\]

where \( P_t^X \) and \( P_t^F \) correspond to the price indexes of exports and the goods produced abroad, respectively. \( P_t^* \) is the consumer price index of the foreign economy:

\[
P_t^* = \left[ \gamma^F \left( P_t^X \right)^{1-\varepsilon_F} + \left( 1 - \gamma^F \right) \left( P_t^F \right)^{1-\varepsilon_F} \right]^{\varepsilon_F - 1} \varepsilon_F - 1 \tag{2.20}
\]

2.3.1 The small open economy assumption

Following Sutherland (2005), we parameterize the participation of foreign goods in the consumption basket of home households, \( (1 - \gamma^H) \), as follows: \( (1 - \gamma^H) = (1 - n) (1 - \gamma) \). Where \( n \) represents the size of the home economy and \( (1 - \gamma) \) the degree of openness. In the same way, we assume the participation of home goods in the consumption basket of foreign households, as a function of the relative size of the home economy and the degree openness of the world economy, that is \( \gamma^F = n \left( 1 - \gamma^* \right) \).

This particular parameterization implies that as the economy becomes more open, the fraction of imported goods in the consumption basket of domestic households increases, whereas as the economy becomes larger, this fraction falls. This parameterization allows us to obtain the small open economy as the limiting case of a two-country economy model when the size of the domestic economy approaches towards zero, that is \( n \to 0 \). In this case, we have that \( \gamma^H \to \gamma \) and \( \gamma^F \to 0 \). Therefore, in the limiting case, the use in the foreign economy of any home-produced intermediate
goods is negligible, and the demand condition for domestic goods can be re-written as follows:

\[ Y_t^H = \gamma \left( \frac{P_t^H}{P_t} \right)^{-\varepsilon_H} C_t \]  
\[ M_t = (1 - \gamma) \left( \frac{P_t^M}{P_t} \right)^{-\varepsilon_H} C_t \]  
\[ X_t = (1 - \gamma^*) \left( \frac{P_t^X}{P_t} \right)^{-\varepsilon_F} C_t^* \]  

Thus, given the small open economy assumption, the consumer price index for the home and foreign economy can be expressed in the following way:

\[ P_t = P_t^F \]  
\[ P_t^* = \left[ \gamma \left( P_t^H \right)^{1-\varepsilon_H} + (1 - \gamma) \left( P_t^M \right)^{1-\varepsilon_H} \right]^{\frac{1}{1-\varepsilon_H}} \]  
\[ P_t^* = P_t^F \]  

Given the small open economy assumption, the foreign economy variables that affect the dynamics of the domestic economy are foreign output, \( Y_t^* \), the foreign interest rate, \( i^* \), the external inflation, \( \Pi^* \), and the capital inflows, \( \varpi_t^* \). To simplify the analysis, we assume these four variables follow an autoregressive process in logs.

### 2.4 Firms
#### 2.4.1 Intermediate goods producers

A continuum of mass \( n \) of \( z \) intermediate firms exists. These firms operate in a perfectly competitive market and use the following linear technology:

\[ Y_t^{int}(z) = A_t L_t(z) \]  

\( L_t(z) \) is the amount of labour demand from households, \( A_t \) is the level of technology.

These firms take as given the real wage, \( W_t/P_t \), paid to households and choose their labour demands by minimizing costs given the technology. The corresponding first order condition of this problem is:

\[ L_t(z) = \frac{MC_t(z)}{W_t/P_t} Y_t^{int}(z) \]  

where \( MC_t(z) \) represents the real marginal costs in terms of home prices. After replacing the labour demand in the production function, we can solve for the real marginal cost:

\[ MC_t(z) = \frac{W_t/P_t}{A_t} \]
Given that all intermediate firms face the same constant returns to scale technology, the real marginal costs for each intermediate firm \( z \) is the same, that is \( MC_t(z) = MC_t \). Also, given these firms operate in perfect competition, the price of each intermediate good is equal to the marginal cost. Therefore, the relative price \( P_t(z) / P_t \) is equal to the real marginal costs in terms of consumption unit \( (MC_t) \).

### 2.4.2 Final goods producers

**Goods sold domestically** Final goods producers purchase intermediate goods and transform them into differentiated final consumption goods. Therefore, the marginal costs of these firms equal the price of intermediate goods. These firms operate in a monopolistic competitive market, where each firm faces a downward sloping demand function, given below. Furthermore, we assume that each period \( t \) final goods producers face an exogenous probability of changing prices given by \( (1 - \theta^H) \). Following Calvo (1983), we assume that this probability is independent of the last time the firm set prices and the previous price level. Thus, given a price fixed from period \( t \), the present discounted value of the profits of firm \( z \) is given by:

\[
E_t \left\{ \sum_{k=0}^{\infty} (\theta^H)^k \Lambda_{t+k} \left[ \frac{P_{t+k}^{H,\omega}(z)}{P_{t+k}^H} - MC_{t+k}^H \right] Y_{t+k}^H(z) \right\} = 0
\]

where \( \Lambda_{t+k} = \beta^k \frac{UC_{t+k}}{UC_t} \) is the stochastic discount factor, \( MC_{t+k}^H = MC_{t+k} \frac{P_{t+k}}{P_{t+k}^H} \) is the real marginal cost expressed in units of goods produced domestically, and \( Y_{t+k}^H(z) \) is the demand for good \( z \) in \( t+k \) conditioned to a fixed price from period \( t \), given by

\[
Y_{t+k}^H(z) = \left[ \frac{P_{t+k}^{H,\omega}(z)}{P_{t+k}^H} \right]^{-\varepsilon} Y_{t+k}^H
\]

Each firm \( z \) chooses \( P_{t+k}^{H,\omega}(z) \) to maximize (2.28). The first order condition of this problem is:

\[
E_t \left\{ \sum_{k=0}^{\infty} (\theta^H)^k \Lambda_{t+k} \left[ \frac{P_{t+k}^{H,\omega}(z)}{P_{t+k}^H} F_{t+k}^H - \mu MC_{t+k}^H \right] (F_{t+k}^H)^{-\varepsilon} Y_{t+k}^H \right\} = 0
\]

where \( \mu \equiv \frac{\varepsilon}{\varepsilon-1} \) and \( F_{t+k}^H \equiv \frac{P_{t+k}^H}{P_{t+k}^H} \).

Following Benigno and Woodford (2005), the previous first order condition can be written recursively using two auxiliary variables, \( V_{t}^D \) and \( V_{t}^N \), defined as follows:

\[
\frac{P_{t+k}^{H,\omega}(z)}{P_{t+k}^H} = V_{t+k}^N \frac{V_{t+k}^D}{V_{t+k}^N}
\]
where

\[ V_t^N = \mu U_{C,t} Y_t^H MC_t^H + \theta^H \beta E_t \left[ V_{t+1}^N (\Pi_{t+1}^H) \right] \]  
(2.29)

\[ V_t^D = U_{C,t} Y_t^H + \theta^H \beta E_t \left[ V_{t+1}^D (\Pi_{t+1}^H) \right] \]  
(2.30)

Also, since in each period \( t \) only a fraction \( (1 - \theta^H) \) of these firms change prices, the gross rate of domestic inflation is determined by the following condition:

\[ \theta^H (\Pi_{t+1}^H)^{\epsilon^{-1}} = 1 - (1 - \theta^H) \left( \frac{V_t^N}{V_t^D} \right)^{1-\epsilon} \]  
(2.31)

The equations (2.29), (2.30) and (2.31) determine the supply (Phillips) curve of domestic production.

**Exported goods**  We assume that firms producing final goods can discriminate prices between domestic and external markets. Therefore, they can set the price of its exports in foreign currency. Also, when selling abroad they also face an environment of monopolistic competition with nominal rigidities, with a probability \( 1 - \theta^X \) of changing prices.

The problem of retailers selling abroad is very similar to that of firms that sell in the domestic market, which is summarized in the following three equations that determine the supply curve of exporters in foreign currency prices:

\[ V_t^{N,X} = \mu (Y_t^X U_{C,t}) MC_t^X + \theta^X \beta E_t \left[ V_{t+1}^{N,X} (\Pi_{t+1}^X) \right] \]  
(2.32)

\[ V_t^{D,X} = (Y_t^X U_{C,t}) + \theta^X \beta E_t \left[ V_{t+1}^{D,X} (\Pi_{t+1}^X) \right] \]  
(2.33)

\[ \theta^X (\Pi_{t+1}^X)^{\epsilon^{-1}} = 1 - (1 - \theta^X) \left( \frac{V_t^{N,X}}{V_t^{D,X}} \right)^{1-\epsilon} \]  
(2.34)

where the real marginal costs of the goods produced for export are given by:

\[ MC_t^X = \frac{P_t MC_t}{SP_t^X} = \frac{MC_t}{RER_t \left( \frac{P_t^X}{P_t} \right)} \]  
(2.35)

which depend inversely in the real exchange rate \( RER_t = \frac{S_t P_t}{P_t} \) and the relative price of exports to external prices \( \left( \frac{P_t^X}{P_t} \right) \).
2.4.3 Retailers of imported goods

Those firms that sell imported goods buy a homogeneous good in the world market and differentiate it into a final imported good \( Y_t^M(z) \). These firms also operate in an environment of monopolistic competition with nominal rigidities, with a probability \( 1 - \theta^M \) of changing prices.

The problem for retailers is also very similar to that of the producers of final goods. The Phillips curve for the importers is given by:

\[
V_t^{N,M} = \mu (Y_t^M U_{C,t}) MC_t^M + \theta^M \beta E_t \left[ V_{t+1}^{N,M} (\Pi_{t+1}^M)^{\varepsilon} \right] \tag{2.36}
\]

\[
V_t^{D,M} = (Y_t^M U_{C,t}) + \theta^M \beta E_t \left[ V_{t+1}^{D,M} (\Pi_{t+1}^M)^{\varepsilon-1} \right] \tag{2.37}
\]

\[
\theta^M (\Pi_t^M)^{\varepsilon-1} = 1 - (1 - \theta^M) \left( \frac{V_t^{N,M}}{V_t^{D,M}} \right)^{1-\varepsilon} \tag{2.38}
\]

where the real marginal cost for the importers is given by the cost of purchasing the goods abroad \((S_t P_t^*)\) to the price of imports \((P_t^M)\):

\[
MC_t^M = \frac{S_t P_t^*}{P_t^M} \tag{2.39}
\]

where \( MC_t^M \) also measures the deviations to the law of one price\(^9\).

2.5 Monetary authority

2.5.1 Monetary policy

The central bank implements monetary policy by setting the nominal interest rate according to a Taylor-type feedback rule that depends on CPI inflation. The generic form of the interest rate rule that the central bank uses is given by:

\[
\frac{(1 + \hat{i}_t)}{(1 + \hat{i})} = \left( \frac{\Pi_t}{\Pi} \right)^{\varphi_{\pi}} \exp (\varepsilon_t^{MON}) \tag{2.40}
\]

where \( \varphi_{\pi} > 1 \). \( \Pi \) and \( \hat{i} \) are the levels in steady state of inflation and the nominal interest rate. The term \( \varepsilon_t^{MON} \) is a random monetary policy shock distributed according to \( N \sim (0, \sigma_{MON}^2) \).

\(^9\)See Monacelli (2005) for a similar formulation.
2.5.2 FX intervention

We assume the central bank can always perform fully sterilised FX interventions, therefore they maintain the control of the interest rate regardless of the intervention. We assume FX intervention is operated directly with dealers in the (spot) FX market. The central bank negotiates every period directly with the dealers, such that every dealer receive the same amount of sales/purchases of foreign bonds from the central bank. Each period any dealer \( d \) receives a market order \( \omega_t^{d*,cb} \) from the central bank, where \( \omega_t^{d*,cb} > 0 (\omega_t^{d*,cb} < 0) \) when the central bank sells (purchases) foreign bonds. Total customer flow received by dealer \( d \) equals \( \omega_t^{d*} + \omega_t^{d*,cb} \). We also assume the central bank can have three different strategies of FX intervention.

First, the central bank can perform pure discretional intervention:

\[
    w_t^{cb} = \varepsilon_t^{cb1}
\]

where the central bank intervenes via unanticipated or secret interventions. We also assume the monetary authority can intervene by a preannounced rule. As a second case, the central bank can perform rule based intervention taking into account the changes in the exchange rate:

\[
    w_t^{cb} = \phi_{\Delta s} s_t + \varepsilon_t^{cb2}
\]

According to this rule, when there are pressures of depreciation (appreciation) of the domestic currency, the central bank sells (purchases) foreign bonds to prevent the exchange rate to fluctuate. \( \phi_{\Delta s} \) captures the intensity of the response of the FX intervention to pressures in the FX market.

Finally, the monetary authority can take into account misalignments of the real exchange rate as a benchmark for FX intervention:

\[
    w_t^{cb} = \phi_{rer} r_t + \varepsilon_t^{cb3}
\]

where \( r_t \) capture deviations of the real exchange rate with respect to its steady state.

2.6 Market clearing

Total domestic production is given by:

\[
    P_{t}^{def} Y_t = P_t^H Y_t^H + S_t P_t^X Y_t^X
\]

16
after using the equations (2.21) and (2.22) and the definition of the consumer price index (2.24),
the equation (2.44) can be decomposed in:

\[ P_{t}^{\text{def}} Y_t = P_t C_t + S_t P_t^X Y_t^X - P_t^M Y_t^M \]  \hspace{1cm} (2.45)

To identify the gross domestic product (GDP) of this economy, \( Y_t \), it is necessary to define the GDP deflator, \( P_t^{\text{def}} \), which is the weighted sum of the consumer, export and import price indexes:

\[ P_t^{\text{def}} = \phi_C P_t + \phi_X S_t P_t^X - \phi_M P_t^M \]  \hspace{1cm} (2.46)

where \( \phi_C, \phi_X \) and \( \phi_M \) are steady state values of the ratios of consumption, exports and imports to GDP, respectively. The demand for intermediate goods is obtained by aggregating the production for home consumption and exports:

\[ Y_t^{\text{int}} (z) = Y_t^H (z) + Y_t^X (z) \]

\[ = \left( \frac{P_t^H (z)}{P_t^H} \right)^{-\varepsilon} Y_t^H + \left( \frac{P_t^X (z)}{P_t^X} \right)^{-\varepsilon} Y_t^X \]  \hspace{1cm} (2.47)

Aggregating (2.47) with respect to \( z \), we obtain:

\[ Y_t^{\text{int}} = \frac{1}{n} \int_0^n Y_t^{\text{int}} (z) \, dz = \Delta_t^H Y_t^H + \Delta_t^X Y_t^X \]  \hspace{1cm} (2.48)

where \( \Delta_t^H = \frac{1}{n} \int_0^n \left( \frac{P_t^H (z)}{P_t^H} \right)^{-\varepsilon} \, dz \) and \( \Delta_t^X = \frac{1}{n} \int_0^n \left( \frac{P_t^X (z)}{P_t^X} \right)^{-\varepsilon} \, dz \) are measures of relative price dispersion, which have a null impact on the dynamic in a first order approximation of the model.

Similarly, the aggregate demand for labour is:

\[ L_t = \frac{MC_t}{W_t/P_t} \left( \Delta_t^H Y_t^H + \Delta_t^X Y_t^X \right) \]  \hspace{1cm} (2.49)

After aggregating the household’s budget constraints, the firm’s profits and including the equilibrium condition in the financial market that equates household wealth with the stock of total bonds (\( \bar{B}_t = B_t + S_t B_t^* \)), we obtain the aggregate resources constraint of the home economy:

\[ \frac{\bar{B}_t}{P_t} - \frac{\bar{B}_{t-1}}{P_{t-1}} = \frac{P_t^{\text{def}}}{P_t} Y_t - C_t \]

\[ + \left\{ \frac{(1 + i_{t-1})}{\Pi_t} - 1 \right\} \frac{\bar{B}_{t-1}}{P_{t-1}} + REST_t \]  \hspace{1cm} (2.50)
The equation (2.50) corresponds to the current account of the home economy. The left hand side is the change in net asset position in terms of consumption units. The right hand side is the trade balance, the difference between GDP and consumption which is equal to net exports, and the investment income. The last term, \( REST_t \), is negligible and takes into account the monopolistic profits of retail firms:

\[
REST_t = \frac{P_t^M}{P_t} Y_t^M \left( 1 - \Delta_t^M MC_t^M \right)
\]  

(2.51)

### 3 Computational strategy

The computational strategy consists in dividing the system of log-linearised equations in two blocks. In the first block, we take into account all the equations except the modified UIP condition. In this group, the depreciation rate only appears in the real exchange rate equation:

\[
rer_t = rer_{t-1} + \Delta s_t + \pi_t^* - \pi_t
\]  

(3.1)

This system of equations can be written as:

\[
A_0 \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_2 \Delta s_t + B_0 \epsilon_t
\]  

(3.2)

where \( X_t = [rer_t, i_t, \pi_t^*, w_t^*, i_t^*, ...]' \) is a size \( n_1 \) vector of backward looking variables, \( Y_t = [\pi_t, ...]' \) is a size \( n_2 \) vector of forward looking variables, such as \( n = n_1 + n_2 + 1 \) is the number of endogenous variables. \( \epsilon_t \) is the vector of observable shocks in the model. \( A_2 = [1, 0, 0]' \) is a \((n_1 + n_2) \times 1\) matrix.\(^{10}\)

The second block corresponds to the modified UIP condition:

\[
E_t \Delta s_{t+1} = i_t - i_t^* + \gamma \sigma^2 \left( \omega_t^* + \omega_t^{*, cb} \right)
\]  

(3.3)

In the first stage we find the rational expectations solution of the system in (3.2) using the perturbation method\(^{11}\), taking as exogenous \( \Delta s_t \). That is, we find the policy functions:

\(^{10}\)In the first block we assume that information heterogeneity only enters the model through the depreciation rate determination, allowing to exclude the unobservable shocks from the solution in this step.

\(^{11}\)We use Dynare to solve for the rational expectations of the first block. More information see: Villemont (2011) and Adjemian et al (2012).
\[ Y_t = M_1 X_{t-1} \]  \hspace{1cm} (3.4) \\
\[ X_t = M_2 X_{t-1} + M_3 \Delta s_t + M_4 \epsilon_t \]  \hspace{1cm} (3.5)

In the second stage we use the solution in the first part to find the policy function of \( \Delta s_t \) using Townsend (1983) method. More precisely, we conjecture a solution for \( \Delta s_t \) as a function of infinite lag polynomials of the shocks in the model.

\[ \Delta s_t = \mathcal{A}(L) \varepsilon_t^\tau + \mathcal{B}(L) \varepsilon_t^\omega + \mathcal{D}(L) \zeta_t \]

where \( \varepsilon_t^\tau \) represent the innovations to the future fundamentals over which agents receive a signal \((i_{t+1}^*)\), \( \varepsilon_t^\omega \) is the shock to the unobservable process, which can be inferred with a lag as the direct observation of the depreciation rate and the rest of the shocks reveals the value of the this shock. \( \mathcal{A}(L) \) and \( \mathcal{B}(L) \) are infinite lag polynomials, while \( \mathcal{D}(L) \) is a vector of infinite lag polynomials operating \( \zeta_t \), the vector of remaining shocks. \(^{12}\)

In the second stage we solve for the signal extraction problem of the dealers for the unobserved innovations \(( \varepsilon_t^\omega, \varepsilon_{t+1}^\omega)\), using both the depreciation rate and their signal \(( \Delta s_t, \nu_{i_t}^d)\). In this manner we can link the unobservables to the observables and then to the shocks. In this way the average expectation of the future depreciation rate and the conditional volatility is calculated and introduced in the solution obtained for (3.3). In turn, given the solution (3.4 and 3.5) found from (3.2), we can express the endogenous variables as a function of shocks as well.

What follows is equating the coefficients of both results, generating a system of nonlinear equations on the unknown components in \( A(L), B(L) \) and \( D(L) \). Following Bacchetta and van Wincoop (2006), we transform the infinite order set of equations by exploiting the recursive patterns in the coefficients and truncating the order of the polynomial after guaranteeing the stability of the equilibrium for higher orders. See appendix B for more details on the computational strategy.

\(^{12}\)Notice that \( \epsilon_t \) and \( \zeta_t \) are not the same, since the latter only has the observable shocks, while the former also incorporates the unobservable shocks.
4 Results

4.1 Calibration

We calibrate most of the parameters according to the parameterisation of Castillo et al (2009), as shown in Table 1. The parameters used are standard in the new open economics literature. The standard deviation of all the exogenous process was set to 0.01 and the autocorrelation coefficient to 0.5. The coefficient of absolute risk aversion for dealers was set to 500 as in Bacchetta and van Wincoop (2006).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9975</td>
<td>consumers time-preference parameter.</td>
</tr>
<tr>
<td>$\chi$</td>
<td>2</td>
<td>labour supply elasticity.</td>
</tr>
<tr>
<td>$\gamma_c$</td>
<td>3</td>
<td>risk aversion parameter.</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>0.75</td>
<td>elasticity of substitution btw. home and foreign goods.</td>
</tr>
<tr>
<td>$\varepsilon_X$</td>
<td>0.75</td>
<td>elasticity of substitution btw. exports and foreign goods.</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.6</td>
<td>Share of domestic tradables in domestic consumption.</td>
</tr>
<tr>
<td>$\theta_H$</td>
<td>0.75</td>
<td>Domestic goods price rigidity.</td>
</tr>
<tr>
<td>$\theta_M$</td>
<td>0.95</td>
<td>Imported goods price rigidity.</td>
</tr>
<tr>
<td>$\theta_X$</td>
<td>0.1</td>
<td>Exported goods price rigidity.</td>
</tr>
<tr>
<td>$\psi_b$</td>
<td>0.1</td>
<td>portfolio adjustment costs.</td>
</tr>
<tr>
<td>$\varphi_\pi$</td>
<td>1.5</td>
<td>Taylor rule reaction to inflation deviations.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>500</td>
<td>Absolute risk aversion parameter (dealers)</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>0.5</td>
<td>Net asset position over GDP ratio.</td>
</tr>
<tr>
<td>$\phi_C$</td>
<td>0.68</td>
<td>Consumption over GDP ratio.</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>0.01</td>
<td>S.D. of all shocks x</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>0.5</td>
<td>AR(1) coefficient for all the exogenous processes</td>
</tr>
</tbody>
</table>

4.2 Model dynamics

4.2.1 Disconnection from fundamentals

As in the case of Bacchetta and van Wincoop (2006), the model also presents disconnection from fundamentals in the short run. In figure 2 we show impulses responses to a capital inflows shock. We compare two models: the full information rational expectations model (FI) and the heterogeneous information (HI) model. In the former, dealers have common knowledge about the shocks, while in the latter they solve the signal extraction problem to learn about the capital inflows shock and the next period interest rate shock. According to this graph, the exchange rate responds much less to a capital inflow shock when dealers do not observe the shock and have to
extract information to learn about it. Also, the response of all the macroeconomic variables is much smaller than in the full information case.

[Insert figure 2 here]

4.2.2 Anticipation effect

Similar to the exercise in figure 4, we compare in figure 4 the impulse responses to a interest rate shock next period \((t + 1)\) in the model under full information with respect to the model with heterogeneous information. As shown, the depreciation rate under HI responds one period earlier than in the FI case, however the response is slightly smaller. Also, all the other macroeconomic variables have a similar behaviour, with the exception of inflation and the interest rate, which have a larger response under HI. This exercise illustrates the effects of macroeconomic news in the FX market. When dealers anticipate there is going to be a change in a macroeconomic fundamental, the exchange rate responds to those beliefs, which also have effects in other macroeconomic variables.

[Insert figure 3 here]

4.2.3 Intervention at work

We present three exercises to show how FX intervention works in the model. In figure 4 we show the effects of an increase in 1% of sales of foreign currency by the central bank. In the semi-continuous blue line the monetary authority performs this policy under discretion, while in the continuous green line intervention is made by the rule that responds to the depreciation rate (rule 1). We also assume intervention under discretion has some persistence, to match the behaviour of the intervention under a rule. As shown, the initial appreciation of the exchange in response to the sale of foreign currency by the central bank is larger under discretion than under rule 1. This is because under discretion dealers do not expect fluctuations of the exchange rate to be smoothed in the future as in the case of rules, therefore the response of the exchange rate is larger. In other words, under discretion the central bank can surprise the market, generating a larger effect response of the exchange rate than under a rule.

In figure 5 we show the effects of an increase in the policy rate in 1% in the first period under both no intervention and rule 1 of FX intervention. In both cases the exchange rate appreciates, but this is larger in the case of no FX intervention. Consequently, inflation decreases by more because of the pass-through from exchange rate to inflation. Also, the decrease in output is larger.
on impact under no FX intervention because of a larger real exchange rate appreciation. This exercise sheds light on how the power of monetary policy is reduced when central banks intervene in the FX market, because it mutes the transmission of monetary policy through fluctuations of the exchange rate.

Finally, in figure 6 we compare the responses to a capital inflows shock under different intervention regimes. In the semi-continuous blue line the central bank do not intervene in the FX market, in the dotted black line the central bank intervenes by the rule that takes into account the depreciation rate (rule 1) and in the continuous green line intervention with persistence, such that it much the initial intervention under rule 1. As shown, there is little difference in the evolution of the exchange rate under discretion than under no intervention. In both cases, the initial appreciation is close in magnitude. However, FX intervention under rule 1 is more efficient in reducing the fluctuations of the exchange rate. The main channel at work here is expectations. While discretionale interventions fail to anchor expectations of future exchange rates, pre-announced rules can have an additional stabilisation effect through expectations. In other words, given that it is known the central bank will enter in the FX market to prevent large fluctuations in the exchange rate, the amount of intervention necessary to reduce fluctuations is smaller. The amount of FX purchases and sales from the central bank needs to stabilise the exchange rate will be much higher under discretion because it does not influence expectations as in the case of an intervention rule.

[Insert figure 4,5,6 here]

Among other exercises we would like to perform with this model, we would like to compare the performance of the FX intervention rule that responds to real exchange rate misalignments (rule 2). Since real exchange movements are more persistent and less volatile than the depreciation rate, we expect FX intervention under this rule will be also long-lived. We also would like to test the connection between the exchange and order flows, measured as in Bacchetta and van Wincoop (2006) as the private information component of total FX orders. We expect the connection of the exchange rate with this variable to be also high. Finally, we would also like to perform some welfare analysis, showing how welfare frontiers (measured by the volatility of macroeconomic variables) evolve under different parameterisation of the rules.
5 Conclusions

In this paper we present a model to analyse the interaction between monetary policy and FX intervention by central banks, which also have microstructure fundamentals in the determination of the exchange rate. We introduce a portfolio decision of short-sighted dealers which adds an endogenous risk premium to the traditional uncovered interest rate condition. Also, the heterogeneous information assumption provides the disconnection observed in the data between exchange rates and macroeconomic fundamentals. In this model, FX intervention affects the exchange rate through both a portfolio-balance and an expectations/signalling channel. On the technical side, we also propose an extension of the Townsend (1983) method to solve DSGE models with heterogeneous information.

Our preliminary results shed light that FX intervention reduces the power of monetary policy. Also, FX intervention under discretion can be more powerful than rules to affect the exchange because of a surprise effect. However, when we analyse the response to a capital inflows shock, we showed that FX intervention rules have some advantages as an stabilisation tool, because they anchor expectations about future exchange rates. Therefore, the amount of FX intervention needed to stabilise the exchange rate under rules is much smaller than under discretion.

This work can be extended in many ways, to analyse for example different puzzles of the new international economy literature in the context of 2-country models, such as the consumption-real exchange rate anomaly.
Figure 2: Impulse responses to a capital inflows shocks. Comparison between full information (FI) and heterogeneous information (HI) model.
Figure 3: Impulse responses to a shock in $i_{t+1}^*$. Comparison between full information (FI) and heterogeneous information (HI) model.

Figure 4: Impulse responses to a 1% FX intervention shocks. Comparison between model with FX intervention under discretion and under rule 1 ($\Delta s_t$) heterogeneous information (HI) model.
Figure 5: Impulse responses to a 1% increase in the monetary policy rate. Comparison between model with FX intervention under discretion and under rule 1 ($\Delta s_t$) heterogeneous information (HI) model.

Figure 6: Impulse responses to a 1% increase in capital inflows. Comparison among models under (1) no FX intervention, (2) FX intervention under discretion, and (3) FX intervention under rule 1 ($\Delta s_t$).
References


A  The log-linear version of the model

A.1  Aggregate Demand

Aggregate demand \((y_t)\)

\[
y_t = \phi_C(c_t) + \phi_X(x_t) - \phi_M(m_t) + g_t \tag{A.1}
\]

GDP deflator \((t_t^{\text{def}})\)

\[
t_t^{\text{def}} = \phi_X(rer_t + t_t^X) - \phi_M t_t^M \tag{A.2}
\]

Real exchange rate \((rer_t)\)

\[
rer_t = rer_{t-1} + \Delta s_t + \pi_t^* - \pi_t \tag{A.3}
\]

Euler equation \((\lambda_t)\)

\[
\lambda_t = \lambda_{t+1} + E_t(\lambda_{t+1} - \pi_t + \pi_t^*) - \psi b_t \tag{A.4}
\]

Marginal utility \((\lambda_t)\)

\[
\lambda_t = -\gamma_a c_t \tag{A.5}
\]

Exports \((x_t)\)

\[
x_t = -\varepsilon^X(t_t^X)^* + y_t^*; \tag{A.6}
\]

Relative price of exports \((t_t^X)\)

\[
t_t^X = t_{t-1}^X + \pi_t^X - \pi_t^*; \tag{A.7}
\]

Imports \((m_t)\)

\[
m_t = -\varepsilon(t_t^M) + c_t; \tag{A.8}
\]

Relative price of imports \((t_t^M)\)

\[
t_t^M = t_{t-1}^M + \pi_t^M - \pi_t; \tag{A.9}
\]

Home produced goods demand \((y_t^H)\)

\[
y_t^H = -\varepsilon(t_t^H) + c_t; \tag{A.10}
\]

Relative price of home produced goods \((t_t^H)\)

\[
t_t^H = -\left(\frac{1 - \psi}{\psi}\right) t_t^M \tag{A.11}
\]
\textbf{A.2 Aggregate Supply}

Total CPI ($\pi_t$):
\[
\pi_t = \psi \pi_t^H + (1 - \psi) \pi_t^M + \mu_t \tag{A.12}
\]

Phillips curve for home-produced goods ($\pi_t^H$):
\[
\pi_t^H = \kappa_H m_c + \beta E_t \pi_{t+1}^H \tag{A.13}
\]

Real marginal costs ($m_c$)
\[
m_c = w_p - a_t; \tag{A.14}
\]

Phillips curve for imported goods ($\pi_t^M$):
\[
\pi_t^M = \kappa_M m_c^M + \beta E_t \pi_{t+1}^M \tag{A.15}
\]

Marginal costs for imports ($m_c^M$)
\[
m_c^M = r_e - t^M \tag{A.16}
\]

Phillips curve for exports ($\pi_t^X$)
\[
\pi_t^X = \kappa_X m_c^X + \beta E_t \pi_{t+1}^X \tag{A.17}
\]

Marginal costs for imports ($m_c^X$)
\[
m_c^X = m_c - r_e - t^X \tag{A.18}
\]

\textbf{A.3 Labour Market}

Labour demand ($l_t$)
\[
l_t = y_t - a_t; \tag{A.19}
\]

Labour supply ($w_p$)
\[
w_p = \gamma_a c_t + \chi l_t \tag{A.20}
\]

\textbf{A.4 FX Markets and Current Account}

Modified UIP ($\Delta s_t$)
\[
E_t \Delta s_{t+1} = \delta_t \delta_t^* - \gamma \sigma^2 \left( \omega_t^* + \omega_t^{*,cb} \right) \tag{A.21}
\]
Current account \((b_t)\)

\[
\phi_w (b_t - \beta^{-1}b_{t-1}) = t^{def}_t + y_t - \phi_cc_t + \frac{\phi_w}{\beta} (a_{t-1} - \pi_t)
\] (A.22)

A.5 Monetary Policy

Interest rate \((i_t)\)

\[
i_t = \varphi(\pi_t) + \varepsilon^{int}_t
\] (A.23)

FX intervention \((\omega_i^{*,cb})\)

\[
\omega_i^{*,cb} = \varphi_{\Delta s} s_t + \varphi_{rer} r_{et} + \varepsilon^{cb}_t
\] (A.24)

A.6 Foreign Economy

Foreign output \((y^*_t)\):

\[
y^*_t = \rho_y y^*_t - 1 + \varepsilon^{y^*_t}_t
\] (A.25)

Foreign inflation \((\pi^*_t)\):

\[
\pi^*_t = \rho_{\pi^*} \pi^*_t - 1 + \varepsilon^{\pi^*_t}_t
\] (A.26)

Foreign interest rates \((i^*_t)\):

\[
i^*_t = \rho_{i^*} i^*_t - 1 + \varepsilon^{i^*_t}_t
\] (A.27)

Capital inflows-order flows \((\omega^{*}_i)\)

\[
\omega^{*}_t = \rho_{\omega^*} \omega^{*}_{t-1} + \varepsilon^{\omega^*_t}_t
\] (A.28)

A.7 Domestic Shocks

Productivity shocks \((a_t)\):

\[
a_t = \rho_a a_{t-1} + \varepsilon^a_t
\] (A.29)

Demand shocks \((g_t)\):

\[
g_t = \rho_g g_{t-1} + \varepsilon^g_t
\] (A.30)

Mark up shocks \((\mu_t)\):

\[
\mu_t = \rho_{\mu} \mu_{t-1} + \varepsilon^\mu_t
\] (A.31)

So we have in total 31 equations, 24 from the original model and 7 auxiliary equations. We have included two exogenous shocks process: demand \((g_t)\) and mark-up/inflation \((\mu_t)\) shocks to perform additional analysis. The variables in the model are: \(a_t, y_t, c_t, x_t, m_t, y^*_t, y^H_t, l_t, \lambda_t, \omega^*_t, \ldots\)
The minimum state variable (MSV) set is composed by 12 variables: \( m_i, mc_i^X, mc_i^M, t_i^\text{def}, t_i^X, t_i^M, t_i^H, \pi_t, \pi_t^H, \pi_t^X, \pi_t^M, \pi_t^*, rer_t, \Delta s_t, i_t, i_t^*, \omega_t^*, g_t, \mu_t \).

The log-linearised system of equations of the model can be written as:

\[
B \text{ Details of the computational strategy}
\]

\[
\begin{align*}
A_0 \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} &= A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_2 \Delta s_t + B_0 \epsilon_t \\
&= (B.1)
\end{align*}
\]

and

\[
E_t \Delta s_{t+1} = i_t - i_t^* + \gamma a^2 (\omega_t^* + \omega_t^{*, cb})
\]

where \( A_2 = [1, 0, \ldots, 0]^T \) is a \((n_1 + n_2) \times 1\) matrix and the definitions of the other matrices and vectors are in section 3. This is the state space form of the model.

B.1 Solving the first block

As an illustration, we will solve the system in (B.1) under some simplifying assumptions. For a more general solution, see Villemot (2011). The system in (B.1) can be written as:\(^{13}\)

\[
\begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A_0^{-1} A_1 \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + A_0^{-1} A_2 \Delta s_t + A_0^{-1} B_0 \epsilon_t
\]

or

\[
\begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = A \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} a_{11} \Delta s_t \\ 0_{(n_1 + n_2 - 1) \times 1} \end{bmatrix} + B \epsilon_t
\]

after making \( A = A_0^{-1} A_1 \), \( B = A_0^{-1} B_0 \) and \( a_{11} \) the \((1, 1)\) element of \( A_0^{-1} \). Using the Jordan decomposition of \( A = PAP^{-1} \), it becomes:

\[
P^{-1} \begin{bmatrix} X_t \\ E_t Y_{t+1} \end{bmatrix} = \Lambda P^{-1} A \begin{bmatrix} X_{t-1} \\ Y_t \end{bmatrix} + \begin{bmatrix} p_{11} a_{11} \Delta s_t \\ 0_{(n_1 + n_2 - 1) \times 1} \end{bmatrix} P^{-1} B \Delta s_t + P^{-1} C \epsilon_t
\]

Making \( R = P^{-1} B, \Lambda = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & \Lambda_2 \end{bmatrix}, P^{-1} = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \), \( R = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} \) and \( p_{11} \) the \((1, 1)\)

\(^{13}\)Assuming \( A_0 \) is invertible, otherwise we can generalise this for the case of non-invertible matrix.
element of $P^{-1}$. $\Lambda_1$ ($\Lambda_2$) is the diagonal matrix of stable (unstable) eigenvalues of size $n_1$ ($n_2$).

The system of equations become:

$$
\begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix}
\begin{bmatrix}
X_t \\
E_t Y_{t+1}
\end{bmatrix}
= \begin{bmatrix}
\Lambda_1 & 0 \\
0 & \Lambda_2
\end{bmatrix}
\begin{bmatrix}
P_{11} & P_{12} \\
P_{21} & P_{22}
\end{bmatrix}
\begin{bmatrix}
X_{t-1} \\
Y_t
\end{bmatrix}
+ \begin{bmatrix}
p_{11}a_{11}\Delta s_t \\
0_{(n_1+n_2-1)\times 1}
\end{bmatrix}
+ \begin{bmatrix}
R_1 \\
R_2
\end{bmatrix}
\epsilon_t.
$$

Making $\tilde{X}_{t-1} = P_{11}X_{t-1} + P_{12}Y_t$, $\tilde{Y}_t = P_{21}X_{t-1} + P_{22}Y_t$, the system becomes:

$$
\begin{bmatrix}
\tilde{X}_t \\
E_t \tilde{Y}_{t+1}
\end{bmatrix}
= \begin{bmatrix}
\Lambda_1 & 0 \\
0 & \Lambda_2
\end{bmatrix}
\begin{bmatrix}
\tilde{X}_{t-1} \\
\tilde{Y}_t
\end{bmatrix}
+ \begin{bmatrix}
p_{11}a_{11}\Delta s_t \\
0_{(n_1+n_2-1)\times 1}
\end{bmatrix}
+ \begin{bmatrix}
R_1 \\
R_2
\end{bmatrix}
\epsilon_t
$$

According to Blanchard & Kahn, given that $\Lambda_2$ is the diagonal of unstable eigenvalues, the only estable solution is given by: $\tilde{Y}_t = 0 = P_{21}X_{t-1} + P_{22}Y_t$.

Then, the solution for the forward looking variables is:

$$
Y_t = (P_{22})^{-1}P_{21}X_{t-1}. \quad (B.3)
$$

The solution for the system of stable (backward looking) equations is:

$$
\tilde{X}_t = \Lambda_1\tilde{X}_{t-1} + \begin{bmatrix}
p_{11}a_{11}\Delta s_t \\
0_{(n_1+n_2-1)\times 1}
\end{bmatrix}
+ R_1\epsilon_t \quad (B.4)
$$

### B.2 Solving the second block

#### B.2.1 The $MA(\infty)$ representation of the first block

Now we change the classification of endogenous variables in the block 1 to focus in the ones which are part of the minimum state variables (MSV) set. We call these variables $Z_t$, while the rest of endogenous variables is referred as $Z_t^-$. In our case the $Z_t$ is formed by 12 variables as defined in appendix A.

The transition and policy functions can be written as:

$$
\begin{bmatrix}
Z_t \\
Z_t^-
\end{bmatrix}
= \begin{bmatrix}
W \\
W^-
\end{bmatrix}
Z_{t-1} + \begin{bmatrix}
V \\
V^-
\end{bmatrix}
\epsilon_t^* \quad (B.5)
$$

where $\epsilon_t^* = [\epsilon_t', \Delta s_t]$ appends the depreciation rate in the vector of shocks. Evaluating the transition function in $t - 1$ and replacing it in (B.5), we have:
Repeating this process many times, we get:

\[
\begin{bmatrix}
Z_t \\
Z_t^-
\end{bmatrix} = \begin{bmatrix} W & V \\
W^- & V-
\end{bmatrix} (WZ_{t-2} + V\epsilon_{t-1}^*) + \begin{bmatrix} V \\
V-
\end{bmatrix} \epsilon_t^*
\]

Which allows us to write the solution as a MA(\infty):

\[
\begin{bmatrix}
Z_t \\
Z_t^-
\end{bmatrix} = \begin{bmatrix} W & V \\
W^- & V-
\end{bmatrix} \sum_{i=1}^{\infty} (W)^{i-1} V\epsilon_{t-i}^* + \begin{bmatrix} V \\
V-
\end{bmatrix} \epsilon_t^*
\]

Given the form of matrix \(W\), the impact of shocks diminish over time, allowing us to approximate the solution using a fixed number of lags. We focus in the solution for \(i_t\) in this step and replace it back into (B.2). In our setup \(i_t^*\) follows an exogenous process which is easy to express as a function of shocks. Finally, the last term, \(\gamma \sigma^2 (\overline{\omega}_t^* + \overline{\omega}_t^{*cb})\) is a combination of another first order autoregressive process \(\overline{\omega}_t^*\) and the conditional volatility term \(\sigma^2\).

### B.2.2 Conditional moments

In order to calculate the conditional volatility of the depreciation rate, we need to make use of the strategy proposed by Bacchetta and van Wincoop (2006), based on the Townsend’s method.

First we conjecture a solution for the depreciation of exchange rate of the form:

\[
\Delta s_t = A(L)\epsilon_{t+1}^* + B(L)\overline{\omega}_t^* + D(L)\zeta_t
\]

where \(A(L)\) and \(B(L)\) are infinite lag polynomials, while \(D(L)\) is a vector of infinite lag polynomials operating \(\zeta_t\), the vector of remaining shocks. Writing \(A(L) = a_1 + a_2 L + a_3 L^2 + \ldots\) (and a similar definition for \(B(L)\) and \(D(L)\)), we proceed to forward the conjecture to obtain the value in \(t + 1\).

\[
\Delta s_{t+1} = a_1 \epsilon_{t+2}^* + b_1 \overline{\omega}_{t+1}^* + d_1 \zeta_{t+1} + \vartheta' \zeta_t + A'(L)\epsilon_t^* + B'(L)\overline{\omega}_{t-1}^* + D'(L)\zeta_{t-1}
\]

where \(\zeta_t' = (\epsilon_{t+1}^*, \overline{\omega}_t^*)\) contains the unobservable innovations, \(\vartheta' = (a_2, b_2)\) stands for the parameters associated to these shocks, \(A'(L) = a_3 + a_4 L + \ldots\) (similar definition for \(B'(L)\)) and \(D'(L) = d_2' + d_3' L + \ldots\). \(A'(L)\epsilon_t^* + B'(L)\overline{\omega}_{t-1}^* + D'(L)\zeta_{t-1}\) represents the term of all observable and past known shocks. Taking expectations over the previous equations yields.
\[ E^d_t(\Delta s_{t+1}) = \vartheta E^d_t(\xi_t) + A^*(L)\varepsilon^*_t + B^*(L)\varepsilon^*_{t-1} + D^*(L)\zeta_t \]

while the conditional expectation as a function of unobservable innovations:

\[ \sigma^2_{\Delta s} = \text{var}_t(\Delta s_{t+1}) = a_1^2\text{var}_t(\varepsilon^*_{t+2}) + b_1^2\text{var}_t(\varepsilon^*_{t+1}) + (d_1')\text{var}_t(\zeta_{t+1})d_1 + \vartheta'\text{var}_t(\xi_t)\vartheta. \]

In order to obtain the conditional moments we need to obtain the conditional expectation and variance of the unobservable component \( \xi_t \).

The computation of the conditional moments is then obtained following Townsend (1983). We focus in the case of only one signal and one unobservable shock as in the base case of Bacchetta & van Wincoop (2006). In this case, FX traders will extract information from the observed depreciation exchange rate \( \Delta s_t \) and the signal \( v^d_t \). To focus on the informational content of observable variables, we subtract the known components from these observables and define these new variables as \( \Delta s^*_t \) and \( v^{d*}_t \). We follow the authors’ notation, hence in this case the observation equation of this part of the problem is given by:

\[ Y^d_t = H'\xi_t + w^d_t \]

where \( w^d_t = (0, \varepsilon^*_{t+1})' \), \( Y^d_t = (\Delta s^*_t, v^{d*}_t)' \) and

\[ H' = \begin{bmatrix} a_1 & b_1 \\ 1 & 0 \end{bmatrix} \]

The parameters of \( A(L), B(L) \) and \( D(L) \) need to be solved for by matching coefficients at both sides of the solution. The unconditional means of \( \xi_t \) and \( w^d_t \) are zero while we define their unconditional variances as \( \tilde{P} \) and \( R \) respectively. Hence we can write:

\[ E^d_t(\xi_t) = MY^d_t \]

where:

\[ M = \tilde{P}H[H'\tilde{P}H + R]^{-1}. \]

For the variance of the unobservable component we have, \( P \equiv \text{var}_t(\xi_t) \), where

\[ P = \tilde{P} - MH'\tilde{P}. \]

By substituting the solution back into the fundamental equation, we obtain a mapping which
is a function of the solution obtained in the first step. We obtain now from (B.2):
\[ E_t s_{t+1} = \Delta s_t + \phi s_t \]

where \( Fz(L) \epsilon_t^* \) stands for \( z_t = \{i_t, rer_t\} \) for \( i_t^* \), and \( J(L)\zeta_t \) for \( \omega_t^* \). This is the "fundamental equation" \( MA(\infty) \) representation. We must contrast this result with the solution from the Townsend method. Here we obtain:
\[ E_t s_{t+1} = \psi' MH' \zeta_t + A^*(L)\epsilon_t^* + B^*(L)\zeta_{t-1} + D^*(L)\zeta_t \] (B.9)

here we start the process of identifying the parameters of \( A(L), B(L) \) and \( D(L) \).

### B.2.3 Solution of parameters

Now we go through the algebra, define \( P_i \) as an auxiliary vector indicating the position of the interest rate \( (i_t) \) in the vector \( Z_t \).

\[ P_i' = \begin{bmatrix} 0 & 0 & \cdots & 1 & \cdots & 0 \end{bmatrix} \]

hence, we can express the interest rate as (given that is in the MSV set):
\[ i_t = P_i'Z_t = P_i' \sum_{j=0}^{\infty} (W)^j V^s_i - j \] (B.10)

where \( \epsilon_t^* = [\epsilon_t', \Delta s_t] \) and
\[ \Delta s_t = A(L)\delta_t^* + B(L)\zeta_{t-1} + D(L)\zeta_t \]

Now we start identifying the parameters multiplying shocks. For this purpose we define extra auxiliary vectors \( Q_k \) where the row in which the variable \( k \) appears in \( Z \) has the value of 1 and the rest of elements are zero.

With these vectors we can begin identifying the parameters. For this, we use the method of undetermined coefficients comparing equations (B.9) and (B.8) .
Solution without FX intervention  We solve first for the parameters assuming first there is no FX intervention, that is: \( \varphi_{\Delta s} = \varphi_{rer} = 0 \).

We start with \( \varepsilon_t^s, \varepsilon_{t-1}^s, \ldots, \varepsilon_{t-s+3}^s \):

\[
\begin{align*}
    a_3 &= \frac{d_i}{d\varepsilon_t^s} + \left( \frac{d_i}{d\Delta s_t} + \frac{d_i}{d\Delta s_{t-1}} \right) - \frac{d_i^*}{d\varepsilon_t^s} \\
    a_4 &= \frac{d_i}{d\varepsilon_{t-1}^s} + \left( \frac{d_i}{d\Delta s_{t-1}} + \frac{d_i}{d\Delta s_{t-2}} \right) - \frac{d_i^*}{d\varepsilon_{t-1}^s} \\
    \vdots \\
    a_s &= \frac{d_i}{d\varepsilon_{t-s+3}^s} + \sum_{j=1}^{s-1} \left( \frac{d_i}{d\Delta s_{t+1-j}} \right) - \frac{d_i^*}{d\varepsilon_{t-s+3}^s} \\
    a_s &= \mathcal{P}_i \left(W^{s-3}\right) V Q_{\omega} + \mathcal{P}_i \left( \sum_{j=1}^{s-1} (W^{s-1-j}V) Q_{\Delta s} a_j \right) - \rho_{\omega}^{s-3}
\end{align*}
\]

which is valid for \( s \geq 3 \). In this case the direct effect is zero, because \( i_t^* \) only appear in the modified UIP condition, that is \( \frac{\partial i_t}{\partial \varepsilon_{t-s+3}^s} = 0 \). Then the solution for the \( a \)'s is given by:

\[
    a_s = \sum_{j=1}^{s-1} \mathcal{P}_i' \left(W^{s-1-j}V\right) Q_{\Delta s} a_j - \rho_{\omega}^{s-3} \quad \text{for } s \geq 3
\]

For \( \varepsilon_{t-1}^\omega, \varepsilon_{t-2}^\omega, \ldots, \varepsilon_{t-s+2}^\omega \):

\[
\begin{align*}
    b_3 &= \frac{d_i}{d\varepsilon_{t-1}^\omega} + \left( \frac{d_i}{d\Delta s_{t-1}} + \frac{d_i}{d\Delta s_{t-2}} \right) + \gamma \sigma^2 \frac{d\omega_t}{d\varepsilon_{t-1}^\omega} \\
    \vdots \\
    b_s &= \frac{d_i}{d\varepsilon_{t-s+2}^\omega} + \sum_{j=1}^{s-1} \left( \frac{d_i}{d\Delta s_{t+1-j}} \right) + \gamma \sigma^2 \frac{d\omega_t}{d\varepsilon_{t-s+2}^\omega} \\
    b_s &= \mathcal{P}_i \left(W^{s-2}\right) V Q_{\omega} + \mathcal{P}_i \left( \sum_{j=1}^{s-1} (W^{s-1-j}V) Q_{\Delta s} b_j \right) + \gamma \sigma^2 \rho_{\omega}^{s-2}
\end{align*}
\]

which is valid for \( s \geq 3 \). Similarly to the previous case, the direct effect is zero here, that is \( \frac{\partial b_t}{\partial \varepsilon_{t-s+2}^\omega} = 0 \). Then the solution for the \( b \)'s is given by:

\[
    b_s = \sum_{j=1}^{s-1} \mathcal{P}_i' \left(W^{s-1-j}V\right) Q_{\Delta s} b_j + \gamma \sigma^2 \rho_{\omega}^{s-2} \quad \text{for } s \geq 3
\]

37
For the rest of shocks $\varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_{t-s}$ :

\[
\begin{align*}
    d_2 &= \frac{d_i}{d_{t-1}} + \left( \frac{d_i}{d_{s_{t-1}} d_{t-1}} + \frac{d_i}{d_{s_t} d_{t-1}} \right) + \gamma \sigma^2 (I_{\zeta=\varepsilon^{cb}}) \\
    d_3 &= \frac{d_i}{d_{t-2}} + \left( \frac{d_i}{d_{s_{t-2}} d_{t-2}} + \frac{d_i}{d_{s_{t-1}} d_{t-1}} \right) + \gamma \sigma^2 (\rho_{\zeta} I_{\zeta=\varepsilon^{cb}}) \\
    &\vdots \\
    d_s &= \frac{d_i}{d_{t-s+2}} + \sum_{j=1}^{s-1} \left( \frac{d_i}{d_{s_{t-1+j}} d_{t-s+2}} \right) + \gamma \sigma^2 (\rho_{\zeta}^{s-2} I_{\zeta=\varepsilon^{cb}}) \\
    d_s &= \mathcal{P}_i ((W)^{s-2} VQ_\zeta + \sum_{j=1}^{s-1} \mathcal{P}_i (W^{s-1-j} V) Q_{\Delta s} d_j + \gamma \sigma^2 (\rho_{\zeta}^{s-2} I_{\zeta=\varepsilon^{cb}}) \quad (B.15)
\end{align*}
\]

which is valid for $s \geq 2$. $I_{\zeta=\varepsilon^{cb}}$ is an indicator value of 1 when the shock $\zeta$ equals $\varepsilon^{cb}$. Note also that $\mathcal{P}_i W^{s-2} VQ_\zeta = 0$ when $\zeta = \varepsilon^{cb}$

This set of equations allows us to express the whole system as a function of parameters $a_1, a_2, b_1, b_2$ and the vector of parameters $d_1$.

For the two unobservable $\{\varepsilon_{t+1}^*, \varepsilon_t^*\}$ shocks we get:

\[
\begin{align*}
    (\theta' MH')_1 &= (\mathcal{P}_i VQ_{\Delta s}) a_1 \quad (B.16) \\
    (\theta' MH')_2 &= [(\mathcal{P}_i VQ_{\Delta s}) b_1 + \mathcal{P}_i VQ_{\omega^*}] + \gamma \sigma^2 \quad (B.17)
\end{align*}
\]

substituting back the values for the matrices, we obtain a non-linear system of equations on the unknowns:

\[
\begin{align*}
    [a_2 \ b_2] M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} &= (\mathcal{P}_i VQ_{\Delta s}) a_1 \quad (B.18) \\
    [a_2 \ b_2] M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} &= (\mathcal{P}_i VQ_{\Delta s}) b_1 + \gamma \sigma^2 \quad (B.19)
\end{align*}
\]

[Hence we have two equations and four unknowns, which impedes us to solve for the system. Bacchetta & van Wincoop (2006), solve the system of difference equations and impose a non-explosive solution, obtaining the two additional restrictions. In our case, the presence of auxiliary matrices in the recursive pattern followed by the coefficients difficults obtaining a solution on the initial values for the system of difference equations. Hence we decide to follow a numerical approach by limiting the number of lags and analyzing the stability of the solution. The numerical]
strategy relies then on the convergence of the values of $A(L)$, $B(L)$ and $D(L)$. We set up the nonlinear system of equations on the first elements of both infinite lag polynomials and search for a numerical solution using the trust-region-dogleg method implemented by MATLAB. The extra restrictions in our case are given by selecting a limit to the lags and setting the parameters associated with this lag in zero. Since these are functions of the first parameters (the unknowns), we can solve the system and obtain the solution. We change sequentially this limit and obtain solutions in each step. The algorithm stops when a fixed point is achieved, revealing that the inclusion of additional lags has a negligible effect on the result. We set the convergence criteria on $1e-003$. The result will be sensitive to the initial values, since non-linear problems will yield more than one solution. For this reason we start each step using the previous step result as initial point.] Marco: change this accordingly

The system of equations: We can represent the system of equations using some auxiliary matrices.

The A system

The equations for $a_3...a_{n+2}$ can be written as:

$$
\begin{bmatrix}
    a_3 \\
    a_4 \\
    \vdots \\
    a_{n+1} \\
    a_{n+2}
\end{bmatrix} = 
\begin{bmatrix}
    \# & 0 & \ldots & 0 & 0 \\
    \#: & \# & \ldots & 0 & 0 \\
    \vdots \\
    \#: & \# & \ldots & \#: & \# \\
    \#: & \# & \ldots & \#: & 
\end{bmatrix} 
\begin{bmatrix}
    a_2 \\
    a_3 \\
    \vdots \\
    a_n \\
    a_{n+1}
\end{bmatrix} - 
\begin{bmatrix}
    1 \\
    \rho_i^* \\
    \vdots \\
    (\rho_i^*)^{n-2} \\
    (\rho_i^*)^{n-1}
\end{bmatrix}
$$

These equations can be written in the matrix form, after assuming that the value of $a_{n+2} \rightarrow 0$:

$$Z_1 A = Z_2 A - X_i^*$$

where $Z_1 = \begin{bmatrix} 0_{(n-1)\times 1} & I_{n-1} \\ 0 & 0_{1\times(n-1)} \end{bmatrix}$, $X_i^* = \left[1, \rho_i^*, ... , (\rho_i^*)^{n-1}\right]^\prime$. $A = [a_2, ..., a_{n+1}]'$ is a $n \times 1$ vector. $Z_2$ is a lower triangular matrix:

$$Z_2^\dagger = 
\begin{bmatrix}
    P_i^\prime VQ_{\Delta s} & 0 & \ldots & 0 & 0 \\
    P_i^\prime WQ_{\Delta s} & P_i^\prime VQ_{\Delta s} & \ldots & 0 & 0 \\
    \vdots \\
    P_i^\prime W^{n-2} VQ_{\Delta s} & P_i^\prime W^{n-3} VQ_{\Delta s} & \ldots & P_i^\prime VQ_{\Delta s} & 0 \\
    P_i^\prime W^{n-1} VQ_{\Delta s} & P_i^\prime W^{n-2} VQ_{\Delta s} & \ldots & P_i^\prime WVQ_{\Delta s} & P_i^\prime VQ_{\Delta s}
\end{bmatrix}$$

39
The B system:
Equation (B.19) can be written as:

\[ b_2 = (P_i V Q_{\Delta s}) b_1 + \gamma \sigma^2 + \left\{ b_2 - [a_2 \ b_2] M \begin{bmatrix} b_1 \\ 0 \end{bmatrix} \right\} \]  

(B.20)

Then, the system of equations for the b’s can be written as:

\[ Z_1 B = Z_2 B + \gamma \sigma^2 X_{\omega^*} + \hat{B} \]

where \( B = [b_1, b_2, ..., b_n]' \) and \( \hat{B} = [b_2 - [a_2 \ b_2] M \begin{bmatrix} b_1 \\ 0 \end{bmatrix}, 0, ..., 0]' \).

The D system
Similarly, the system for \( D^c = [d_1^c, d_2^c, ..., d_n^c]' \) is the following

\[ Z_1 D^c = Z_2^c D^c + Z_3 \]  
when \( \zeta \neq \varepsilon^{s,cb} \)

\[ Z_1 D^{\omega^*,cb} = Z_2^c D^{\omega^*,cb} + \gamma^2 X_{\omega^*,cb} \]  
otherwise

where \( Z_3 = \{ P_i'[V, WV, ..., W^{n-1}V] Q_\zeta \}' \).

The complete system of equations.
Then, after making use of \( Z = Z_1 - Z_2 \), the total system of non-linear equations become:

\[
\begin{bmatrix} a_2 \\ b_2 \end{bmatrix} M \begin{bmatrix} a_1 \\ 0 \end{bmatrix} = (P_i V Q_{\Delta s}) a_1 \\
A = -Z^{-1} X_{t^*} \\
B = Z^{-1} \left( \gamma \sigma^2 X_{\omega^*} + \hat{B} \right) \\
D^c = Z^{-1} Z_3 \\
D^{\omega^*,cb} = (\gamma \sigma^2) Z^{-1} X_{\omega^*,cb} \\
\sigma^2 = a_1^2 var_t(\varepsilon_{t+2}^r) + b_1^2 var_t(\varepsilon_{t+1}^{\omega^*}) + (d_1)' var_t(\zeta_{t+1}) d_1 + \vartheta' var_t(\xi_t) \vartheta \]  

(B.21)

Note the system has \( n \times \# \) of shocks+2 equations and unknowns, which only \( n \times 2 + 2 \) are non-linear equations.

Solving for \( M \)

\[
\hat{P} = \begin{bmatrix} \sigma_{t^*}^2 & 0 \\ 0 & \sigma_{\omega^*}^2 \end{bmatrix}, R = \begin{bmatrix} 0 & 0 \\ 0 & \sigma_\vartheta^2 \end{bmatrix}
\]
\[ M = \hat{P}H[H'\hat{P}H + R]^{-1} \]
\[ = \begin{bmatrix}
\sigma^2_{\tau} & 0 \\
0 & \sigma^2_{\omega^*}
\end{bmatrix}
\begin{bmatrix}
a_1 & b_1 \\
b_1 & 0
\end{bmatrix}
\left[ \begin{bmatrix}
\sigma^2_{\tau} & 0 \\
0 & \sigma^2_{\omega^*}
\end{bmatrix}
\begin{bmatrix}
a_1 & 1 \\
b_1 & 0
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 \\
0 & \sigma^2_v
\end{bmatrix}\right]^{-1} \]
\[ = \begin{bmatrix}
1 & 0 \\
a_1\sigma^2_{\tau} & \sigma^2_{\omega^*}
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{\tau} + \sigma^2_v \\
\sigma^2_v
\end{bmatrix}
- \begin{bmatrix}
1 & 0 \\
a_1\sigma^2_{\tau} & \sigma^2_{\omega^*}
\end{bmatrix}
\left[ \begin{bmatrix}
1 & 2 \\
2 & 1
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{\tau} + \sigma^2_v \\
\sigma^2_v
\end{bmatrix}
- \begin{bmatrix}
1 & 2 \\
2 & 1
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{\tau} + \sigma^2_v \\
\sigma^2_v
\end{bmatrix}
\right]
\]
\[ = \begin{bmatrix}
1 & 0 \\
1 & 2
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{\tau} + \sigma^2_v \\
\sigma^2_v
\end{bmatrix}
- \begin{bmatrix}
1 & 0 \\
1 & 2
\end{bmatrix}
\left[ \begin{bmatrix}
1 & 2 \\
2 & 1
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{\tau} + \sigma^2_v \\
\sigma^2_v
\end{bmatrix}
- \begin{bmatrix}
1 & 2 \\
2 & 1
\end{bmatrix}
\begin{bmatrix}
\sigma^2_{\tau} + \sigma^2_v \\
\sigma^2_v
\end{bmatrix}
\right]
\]

**Solution with FX intervention rules**

When we allow for FX intervention, the equations (B.11), (B.13) and (B.15) are replaced by:

\[ a_s = p_i \left( \sum_{j=1}^{s-1} W^{s-1-j}VQs a_j \right) - \rho u_s^{-3} + \gamma \sigma^2 \left[ \varphi_{\Delta s} a_{s-1} + \varphi_{rer} P_{rer} \left( \sum_{j=1}^{s-1} W^{s-1-j}V Q_{\Delta s} a_j \right) \right] \]  
(B.22a)

\[ b_s = p_i \left( \sum_{j=1}^{s-1} (W^{s-1-j}V)Qs b_j \right) + \gamma \sigma^2 \left[ \rho u_s^{-2} + \varphi_{\Delta s} b_{s-1} + \varphi_{rer} P_{rer} \left( \sum_{j=1}^{s-1} W^{s-1-j}V Q_{\Delta s} b_j \right) \right] \]  
(B.22b)

\[ d_s = p_i (W)u_{s-2}VQc + \sum_{j=1}^{s-1} p_i (W^{s-1-j}V)Qs d_j + \gamma \sigma^2 \left[ \varphi_{\Delta s} d_{s-1} + \varphi_{rer} P_{rer} \left( \sum_{j=1}^{s-1} W^{s-1-j}V Q_{\Delta s} d_j \right) \right] \]  
(B.22c)

\[ (\delta'MH')_1 = (P_i V Q_{\Delta s}) a_1 + \gamma \sigma^2 (\varphi_{\Delta s} a_1 + \varphi_{rer} P_{rer} V Q_{\Delta s} a_1) \]  
(B.22d)

\[ (\delta'MH')_2 = [(P_i V Q_{\Delta s}) b_1 + P_i V Q_{\omega^*}] + \gamma \sigma^2 (1 + \varphi_{\Delta s} b_1 + \varphi_{rer} P_{rer} V Q_{\Delta s} b_1) \]  
(B.22e)

We can also express this with linear algebra. For example, the A system can be written as:

\[ Z_1 A = Z_2 A + \gamma \sigma^2 (\varphi_{\Delta s} I_n + \varphi_{rer} Z_2^{rer}) A - X_i \]
Then, after making use of $Z^{FX} = Z_1 - Z_2^i - \gamma \sigma^2 (\varphi_{\Delta} I_n + \varphi_{rer} Z_{rer}^2)$, the total system of non-linear equations become:

\[
\begin{bmatrix} a_2 & b_2 \end{bmatrix} M \begin{bmatrix} a_1 \\ 1 \end{bmatrix} = (P_i V Q_{\Delta a}) a_1 + \gamma \sigma^2 (\varphi_{\Delta} a_1 + \varphi_{rer} P_{rer} V Q_{\Delta a} a_1)
\]

\[
A = - \left( Z^{FX} \right)^{-1} X_i^*
\]

\[
B = \left( Z^{FX} \right)^{-1} \left( \gamma \sigma^2 X_{\varpi^*} + \hat{B} \right)
\]

\[
D^c = \left( Z^{FX} \right)^{-1} Z_3
\]

\[
D^{\varpi^*,ch} = \left( \gamma \sigma^2 \right) \left( Z^{FX} \right)^{-1} X_{\varpi^*,ch}
\]

\[
\sigma^2 = a_1^2 \text{var}_t(z_{t+2}^*) + b_1^2 \text{var}_t(z_{t+1}^*) + (d_1)' \text{var}_t(\xi_{t+1}) d_1 + \vartheta' \text{var}_t(\xi_t) \vartheta \quad (B.23)
\]

where $\hat{B}$ is defined as before.