Estimating Potential Output and Output Gap for Brazil*

Carlos Hamilton Vasconcelos Araujo†
Marta Baltar Moreira Areosa
Osmani Teixera de Carvalho Guillén
Banco Central do Brasil
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Abstract

This paper aims to estimate output gap for Brazilian economy through different methodologies. We use traditional univariate techniques and propose a new semi-structural methodology that combines HP filtering and the production function approach. In order to compare the different potential output estimates, we use a Phillips curve to predict free price inflation and a rolling forecast experiment as a test of forecast accuracy. Our results shows that the forecasts produced by the Local level and Watson models are even more inaccurate than those generated by the simplest univariate models. The main evidence is that the Beveridge-Nelson methodology outperforms all the models at all forecast horizons.

Resumo

Este artigo tem como objetivo estimar o produto potencial e o hiato do produto para a economia brasileira. Usamos técnicas univariadas tradicionais e propomos uma metodologia semi-estrutural que combina a filtragem de Hodrick-Prescott e a abordagem de função de produção. Para avaliar os resultados estimados do produto potencial, calculamos a inflação de preços livres prevista pela curva de Phillips e o erro quadrático médio em relação ao valor observado. Os resultados indicam que as previsões produzidas por modelos de componentes não observados são mais imprecisas que aquelas pelos modelos univariados mais simples. A metodologia de Beveridge-Nelson é mais eficiente em todos os horizontes de previsão.

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†Corresponding author, carlos.araujo@bcb.gov.br
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Palavras chave: hiato, modelos de espaço estado, funçaõ de produção, filtragem, curva de Phillips.

ANPEC classification: Área 2.

JEL classification: C32, E31, E52.
1 Introduction

Along the last fifteen years we have seen a worldwide adoption of inflation targeting regime as reported at Mishkin and Schmidt-Hebbel (2001). Those countries explicitly adopt a Taylor Rule reaction function. Although Blinder (1997) claims that Federal Reserve Bank (FED)’s strategy is different from the “deliberate desinflation” followed by inflation targeters, Bernanke (2003) asserts there is little difference between inflation targeting and FED’s strategy, known as “constrained discretion”.

In fact, FED has a legal mandate to pursue “maximum employment” and “stable prices”. In line with this framework, European Central Bank (ECB) has a statutory objective of maintaining price stability, but without prejudicing this target, it shall support economic policies with a view to contributing to the achievement of high level of employment and sustainable and non-inflationary growth.

Those facts give evidence that both FED and ECB implicitly adopt a Taylor Rule type reaction function as described in Taylor (1993). Thereafter, it seems that all modern central bankers operate monetary policy with a view at the short-run trade-off between inflation and unemployment. Blinder reports that the Federal Open Market Committee (FOMC)’s process of decision involves a consensus reached meeting-by-meeting, based on analysis of the current macroeconomic situation and the near-term outlooks. Under these circumstances, in order to change the interest rate (the policy instrument), policymakers demand both estimations of expected inflation and potential output.

Potential output, the associated output gap and the natural rate of unemployment are all concepts whose have received increase attention over the past few years in central banks, international organizations and among academics researchers. Measures of potential output and output gap are useful to identify the scope for sustainable noninflationary growth and to allow an assessment of the stance of macroeconomic policies.

St-Aman and van Norden (1997) discussed some methodologies for estimating potential output and the output gap for the Canadian economy. Claus (1999) used a structural vector autoregression methodology to obtain a measure of potential output for the New Zealand economy, finding evidences that consumption increases in anticipation of higher future earnings. Kichian (1999) measured potential output using state-space models for the Canadian economy, finding that the obtained output gap can be quite useful in the formulation of monetary policy. Orphanides and van Nordem (1999) examined the reliability of alternative output detrending methods, with special attention to the accuracy of real-time estimates, they showed that ex post revisions of the output gap are of the same order of magnitude as the output gap itself, that these ex post revisions are highly persistent and that real-time estimates tend to be severely biased around business cycles turning points. Cerra and Saxena (2000) reviewed different methodologies that can be used to estimate potential output for Sweden, finding evidences of a large negative output gap. Proietti, Musso and Westermann
(2002) evaluated unobserved components models based production function approach for estimating the output gap and potential output for the Euro Area, they concluded that this models can be valuable for growth accounting and for reducing the uncertainty surrounding the output gap estimates. Rennison (2003) used a Monte Carlo experiment to evaluate the ability of a variety of output gap estimators to accurately measure the output gap in a model economy, his evidences indicate that an estimator that combines Hodrick-Prescott filter with a Blanchard-Quah structural vector autoregression yields an accurate estimate. Doménech and Gómez (2003) proposed a new method to obtain estimates of the potential output, core inflation and the NAIRU as latent variables, using a standard Okum’s law, a forward-looking Phillips curve and an investment equation. Ehrmann and Smets (2003) used a small forward-looking model of the euro-area economy to investigate the implications of incomplete information about potential output for the conduct of monetary policy; under optimal monetary policy, they found that output gap uncertainty leads to persistent deviations between the actual and the perceived output gap in response to supply and cost-push shocks. Arnold (2004) examined methods rely on purely statistical techniques (filtering, simultaneous econometrics models, multivariate time-series models) and others rely on statistical procedures grounded in economic theory (Solow growth model), highlighting the pros and cons of various approaches.

In July 1999, the Brazilian government officially adhered to an inflation targeting regime as described in Bogdanski, Tombini and Werlang (2000). This institutional change led the Central Bank of Brazil’s researches to invest in developing tools that appropriately delivery estimations of potential output and of inflation to members of Brazilian monetary committee. This effort is highlighted in Alves (2001), Bryan and Cecchetti (2001), Alves and Muínhos (2003) and Silva Filho (2002). This paper goes in the same line, so we try to contribute to the debate addressing these two issues.

First, we estimate potential output paths for the Brazilian economy through several different techniques, including time series filtering - such as Kalman filter (KF) and Beveridge-Nelson decomposition (BN) - and semi-structural approaches – a Hodrick-Prescott (HP) filter constrained by some structure. Time series filtering techniques, that basically extract a trend from Gross Domestic Product (GDP), have been largely used as reported in Orphanides and Norden (1999) and Kichian (1999). Even though semi-structural approaches have been equally popular1, as far as we know, our approach is the only that allows simultaneously setting paths for two latent variables - non-accelerating inflation rates of unemployment (NAIRU) and non-accelerating capacity utilization (NAICU) - to obtain output gap as by-product.

In a second stage, we will care about inflation. Through Ordinary Least square techniques we will identify the best Phillips Curve type equation for each output gap path generated before.

The rest of the paper is organized as follows. Section 2 presents the econometric

1St-Amant and van Norden (1997) and Alves and Muínhos (2003) uses this approach.
techniques considered to estimate potential output and output gap. Section 3 describes data used to estimate each model. Section 4 compares the inflation forecasts generated by a Phillips curve that is estimated using each output gap series. Section 5 concludes.

2 Econometric Approaches

Potential output is usually identified as the output trend. Many kinds of filtering have been proposed in order to decompose output into its high and low frequency components which were respectively assigned to output gap and potential output. The detrending methods considered in this paper are:

1. Deterministic Trends.
2. Moving Average.
3. The Hodrick-Prescott Filter.
5. Unobserved Component Models.
6. Hodrick-Prescott constrained to a Production Function

Next we briefly discuss each of these six groups and the variants of these methods which we apply.

2.1 Deterministic Trend

The first method we consider assume that the trend in (the logarithm of) output is well approximated as a simple deterministic function of time. The linear trend is the simplest of these models, it assumes that output may be decomposed into a cyclical component and a linear function of time:

\[ y_t = \alpha + \beta \cdot t + c_t \]  

where \( c_t \) is the output gap and \( y_t \) is our chosen measure of output (in logarithms).

2.2 Moving average

The second method assume that the logarithm of output may be decomposed into a trend component, \( \tau_t \), and a cyclical component \( c_t \),

\[ y_t = \tau_t + c_t \]  

where \( \tau_t \) is the moving average of the output \( (\tau_t = (y_t + y_{t-1} + y_{t-2} + y_{t-3}) / 4) \).
2.3 The Hodrick-Prescott Filter

The Hodrick-Prescott Filter (1997), commonly called HP filter, is a simple smoothing procedure that has become increasingly popular because its flexibility in tracking the characteristics of the fluctuations in trend output. Output trend, $\overline{y}_t$, derived using the HP filter is obtained by minimizing a combination of the gap between actual output, $y_t$, trend output and the rate of change in trend output for the whole sample of observations, $N$. Formally, The HP-filtered trend is given by

$$\{\overline{y}_t\}_{t=1}^N = \arg\min\sum_{t=1}^{N} (y_t - y^g_t)^2 + \lambda \sum_{t=2}^{T-1} [(\overline{y}_{t+1} - \overline{y}_t) - (\overline{y}_t - \overline{y}_{t-1})]^2$$

and $y^c_t$ is the resulting measure of the output gap,

$$\{y^c_t\}_{t=1}^N = \{y_t - \overline{y}_t\}_{t=1}^N$$

$\lambda$ is called the “smoothing parameter” and penalizes the variability in the growth component. The larger the value of $\lambda$, the smoother the growth component and the greater the variability of the output gap. As $\lambda$ approaches infinity, the growth component correspond to a linear time trend. For quarterly data, Hodrick and Prescott propose setting $\lambda$ equal to 1600.

2.4 The Beveridge-Nelson Decomposition.

Beveridge-Nelson decomposition is a detrending method using unobserved components. Output is assumed to contain unobserved permanent and temporary component consisting of a random walk with drift and a stationary autoregressive process.

Suppose that $y_t$ follows an ARIMA(p,1,q) process. In this case, $\Delta y_t$ evolves according an ARMA(p, q) model that may be represented in its state-space form as:

$$\xi_t = F \cdot \xi_{t-1} + v_t$$

$$\Delta y_t - \mu = G \cdot \xi_t$$

where $\xi_t$ is the state-space vector, $v_t \sim N(0, \Omega)$ and $\mu$ is the unconditional mean of $\Delta y_t$. The Kalman filter may be used to estimate this model and $\xi_t$.

Beveridge and Nelson (1981) consider that $y_t$, could be decomposed into a cyclical component, $c_t$, and a trend, $\tau_t$, defined as the minimum MSE forecast of the long-run level of $y_t$ minus its deterministic drift. That is

$$y_t = \tau_t + c_t$$

where
\[ \tau_t \equiv \lim_{J \to \infty} E_t [y_{t+j} - J\mu] \]

As suggested in Morley (2002), the state-space representation expressed in equations 4 and 5 can be used to calculate the exact trend and cycle, given by

\[ \tau_t = y_t + \lim_{J \to \infty} \sum_{j=1}^{J} E_t [\Delta y_{t+j} - \mu] = y_t + G \lim_{J \to \infty} \sum_{j=1}^{J} F^j E_t [\xi_t] = y_t + GF(I - F)^{-1} E_t [\xi_t] \]

and

\[ \epsilon_t = y_t - \tau_t = -GF(I - F)^{-1} E_t [\xi_t] \]

It is important to highlight that, since all eigenvalues of F are supposed to lie inside the unit root circle, \((I - F)\) is always an invertible matrix.

Additionally, using equations 4 and 5, it is easy to see that

\[ \tau_t - \tau_{t-1} = \mu + G(I - F)^{-1} \nu_t \]  

proving that the trend is a random walk with a drift.

### 2.5 Unobserved Component Models

Unobserved component models attempt to specify the time series properties of output and and use the resulting model to identify cyclic and trend component. The simplest unobserved component is the Local Level Model,

\[ y_t = \mu_t + \epsilon_t \]
\[ \mu_t = \mu_{t-1} + \eta_t, \]
\[ \epsilon_t \sim iid \ N(0, \sigma^2_\epsilon) \]
\[ \eta_t \sim iid \ N(0, \sigma^2_\eta) \]
\[ E[\epsilon_t, \eta_s] = 0, \forall t, \forall s \]

It assumes that the observed output series \( y_t \) may be decomposed in a random walk component \( \mu_t \) and a white noise \( \epsilon_t \). \( \epsilon_t \) and the increments of the random walk are assumed to be mutually uncorrelated and follow independent Gaussian distributions. This implies that \( y_t \) follows an IMA(1,1), with the size of the MA term determined by the relative variances of \( \epsilon \) and \( \mu \).
The second unobserved component model we consider is

\[ y_t = \mu_t + c_t \]
\[ \mu_t = \delta + \mu_{t-1} + \eta_t \]
\[ c_t = \alpha_1 c_{t-1} + \alpha_2 c_{t-2} + \varepsilon_t \]
\[ \varepsilon_t \sim iid \ N(0, \sigma^2_{\varepsilon}) \]
\[ \eta_t \sim iid \ N(0, \sigma^2_{\eta}) \]
\[ E[\varepsilon_t, \eta_s] = 0, \forall t, \forall s \]

(9)

due to Watson (1986), where the output time series \( y_t \) is decomposed into a trend component, \( \mu_t \), and a cyclical component, \( c_t \). The trend component, \( \mu_t \), is assumed to follow a random walk with drift and the cyclical component, \( c_t \), is assumed to follow an AR(2) process, to allow for more persistence.

### 2.6 Hodrick-Prescott constrained to a Production Function

Areosa (2004) suggests the following methodology to estimate potential output and output gap:

In line with the production function approach presented in Alves and Muinhos (2003), we also assume that a Cobb-Douglas production function with constant return to scale is used to assess output and potential output. That is:

\[ y_t = A_t(K_t)^{\alpha_t}(L_t(1 - u_t))^{(1-\alpha_t)} \]  
\[ \overline{y}_t = A_t(K_t)^{\alpha_t}(L_t(1 - \text{nairu}_t))^{(1-\alpha_t)} \]  

(10)  
(11)

where \( y_t \) is the output, \( \overline{y}_t \) is the potential output, \( A_t \) is the productivity factor, \( K_t \) is the capital stock, \( L_t \) is the labor force, \( \alpha_t \) is the income capital share, \( c_t \) is the capacity utilization, \( u_t \) is the rate of unemployment, \( \text{nairu}_t \) is the non-accelerating inflation capacity utilization and \( \text{nairu}_t \) is the non-accelerating inflation rate of unemployment.

Defined output gap as

\[ h_t = \ln \left( \frac{y_t}{\overline{y}_t} \right) \]  

(12)

we can use equations 10 and 11 to assess the output gap as:

\[ h_t = \alpha_t [\ln(c_t) - \ln(\text{nairu}_t)] + (1 - \alpha_t) [\ln(1 - u_t) - \ln(1 - \text{nairu}_t)] \]
\[ \overline{y}_t = \frac{y_t}{\exp(h_t)} = y_t \left( \frac{\text{nairu}_t}{c_t} \right)^{\alpha_t} \left( \frac{1 - \text{nairu}_t}{1 - u_t} \right)^{1-\alpha_t} \]

(13)  
(14)

It is important to emphasize that \( \alpha \) is not supposed to be constant. Data from national accounts gives evidence that \( \alpha \) has grown significantly during the last decade.
Since there is no reliable quarterly series for $\alpha$, we choose to use the modified optimization problem proposed in Areosa (2004).

As an attempt to combine HP filtering and the production function approach just presented, potential output and its unobserved components - nairu and naicu - are estimated by solving the following problem:

\begin{equation}
\begin{aligned}
\min_{\{\text{nairu}_t\}_{t=1}^N, \{\text{naicu}_t\}_{t=1}^N, \{\alpha_t\}_{t=1}^N} & \ 
\left\{ \begin{array}{l}
\beta_1 \sum_{t=1}^N (\text{nairu}_t - \text{ru}_t)^2 + \lambda \sum_{t=2}^{N-1} (\Delta^2\text{nairu}_t)^2 \\
\beta_2 \sum_{t=1}^N (\text{naicu}_t - \text{cu}_t)^2 + \lambda \sum_{t=2}^{N-1} (\Delta^2\text{naicu}_t)^2 \\
\beta_3 \sum_{t=1}^N (\bar{y}_t - y_t)^2 + \lambda \sum_{t=2}^{N-1} (\Delta^2\bar{y}_t)^2 \\
\beta_4 \left[ \lambda \sum_{t=2}^{N-1} (\Delta^2\alpha_t)^2 \right]
\end{array} \right.
\end{aligned}
\end{equation}

s.t.

\begin{align*}
\frac{1}{4} \sum_{j=1}^4 \alpha_4(T-1)+j = \bar{\alpha}_T, & \forall T \in \{1, ..., N/4\} \\
0 \leq \alpha_t \leq 1, & \forall t \in \{1, ..., N\} \\
\bar{y}_t = y_t \left( \frac{\text{naicu}_t}{\alpha_t} \right)^{\alpha_t} \left( \frac{\text{nairu}_t}{\alpha_t} \right)^{1-\alpha_t}
\end{align*}

Two sets were chosen to represent the obtained results. They emphasize the importance of calibrating this model, since the results generated are considerably different.

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\beta_3$</th>
<th>$\beta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Set 1</td>
<td>9.347</td>
<td>9.159</td>
<td>4</td>
<td>0.056</td>
</tr>
<tr>
<td>Set 2</td>
<td>9.347</td>
<td>9.159</td>
<td>12</td>
<td>0.056</td>
</tr>
</tbody>
</table>

If we had an quarterly series for $\bar{\alpha}$, we would not need to built the series $\{\alpha_t\}_{t=1}^N$. In order to isolate the effect of estimating $\{\alpha_t\}_{t=1}^N$, we have chosen to run 15 assuming that $\alpha_t$ remained constant during every given year $T$, that is

\begin{align*}
\alpha_4(T-1)+1 = \alpha_4(T-1)+2 = \alpha_4(T-1)+3 = \alpha_4(T-1)+4 = \bar{\alpha}_T, & \forall T \in \{1, ..., N/4\}
\end{align*}

\footnote{On the following problem $\Delta^2$ represents the second centred difference. For instance, $\Delta^2y_t = y_{t+1} - 2y_t + y_{t-1}$.}
3 Data

The following set of time-series (1995 to 2003) was used in our exercises, which from now on will be represented by the letter in brackets.

- Gross Domestic Product – GDP ($y_t$); Fundação Instituto Brasileiro de Geografia e Estatística (IBGE) - the official statistics bureau - is in charge of measuring this quarterly series.

- Unemployment Rate ($u_t$); IBGE produces the unemployment series that embraces the widest survey region. Nevertheless, IBGE has recently changed its measuring methodology, discontinuing its original series at December 2002 and starting another one at March 2002. In this paper, we extended this old series supposing that all variation occurred in the new series would also occur in the old one. We changed a monthly series into a quarterly one by considering the mean for each quarter.

- Installed Capacity Utilization ($c_t$); Conselho Nacional da Indústria (CNI) produces this monthly series. We changed this series into a quarterly one by considering the mean for each quarter (series 1341 available at www4.bcb.gov.br/pec/series/ingl/)

- Income Capital Share ($\alpha_t$); We estimated an yearly series based on the empirical labor share obtained from national accounts (available at tab04.xls, extracted from sinoticas.zip at ftp://ftp.ibge.gov.br/Contas_Nacionais/Sistema_de_Contas_Nacionais/2000_2002).

- Broad Consumer Price Index Inflation Rate ($\pi_t$); IBGE is in charge of measuring the IPCA - the official consumer price index used in the inflation targeting regime (series 433 available at www4.bcb.gov.br/pec/series/ingl/).

- Inflation of Market Prices ($\pi_t'$); This series is obtained excluding the managed price items from the IPCA headline index.

- Expected Headline Inflation($E_t[\pi_{t+1}]$); Since we did not have a long series for this variable, we chose to build according to two different approaches. One of them considered that agents could perfectly forecast inflation.$(E_t[\pi_{t+1}] = \pi_{t+1})$. On the other approach, an error was added to this series $(E_t[\pi_{t+1}] = \pi_{t+1} + \varepsilon_t)$. For simplicity, we assumed that this error was normally distributed and had zero mean.

Next we briefly discuss the Phillips Curve specification used to compare the output gaps generated by each detrending method.
4 Comparative Analysis

We used full sample to estimate the models presented in section 2 and obtain potential output. Figure 1, presented in appendix 1, shows these estimations which can be classified in two groups: low and high variance trend. The former encompasses Watson, Hodrick-Prescott and deterministic trends. As we can notice in Figure 1, when we used the observed values (HPCPF2a) instead of estimating a quarterly series for the income capital share (HPCPF2), the potential output becomes much more volatile.

After estimating potential output through different methodologies, we assessed the output gap using equation 12. In order to compare these series, a comparative measure should be established. Following Proietti, Musso and Westermann (2002), we used as comparative criterium their accuracy in forecasting. Since output gap is to represents a measure of inflation pressures, we expect it to increase accuracy in our inflation forecasts.

Analyzing the predictive power of each output gap series is a complex task. First of all, we should estimate a plain Phillips curve that captures the trade-off between inflation and unemployment. Based on Bodanski, Tombini and Werlang (2000), we have chosen the following specification:

$$\pi^f_t = \alpha_1 \pi_{t-1} + \alpha_2 E_t[\pi_{t+1}] + \alpha_3 h_{t-2} + \varepsilon_t,$$

(16)

where $\pi^f_t$ is the (log of) inflation of market prices, $\pi_t$ is the (log of) headline inflation, $E_t[\pi_{t+1}]$ is the expected inflation and $h_t$ is the output gap.

It is important to highlight that the dependent variable in this equation is the inflation of market prices. This specification is in line with Banco Central do Brasil Inflation Report (2002) and it can be understood considering that a relevant part of inflation is due to managed prices and consequently does not respond to monetary policy. It is also important to note that expected inflation takes an important part on this specification.

We used a rolling forecast experiment as a test of forecast accuracy as proposed in Proietti, Musso and Westermann (2002). For each output gap series, we estimated the Phillips curve changing the sample from 1995:1 - 2001:4 to 1995:1 - 2003:4 and, each time, forecasting up to 4 steps ahead. The mean square error of these forecasts was chosen as a measure of accuracy for each model.

Figure 2 shows the observed and forecasted free prices inflation presented in Table 2. Several features are readily apparent. First, the forecasts produced by the Local level and Watson models are even more inaccurate than those generated by the simplest univariate models. Second, the deterministic trend, moving average, Hodrick-Prescott, Beveridge-Nelson and production function models have strong short-term comovements, appearing to be moving upward and downward at roughly the same time, although the amount of these moves vary from one method to another. Third, despite having similar short-term movements, the different methods typically
give rise to a wide range of different estimates of free prices inflation. The differences between highest and lowest estimates is 1.09 % in 2002:4 and 0.53 in 2003:4.

### Table 1 - Mean Square Error Results

<table>
<thead>
<tr>
<th>Method</th>
<th>1 step</th>
<th>2 steps</th>
<th>3 steps</th>
<th>4 steps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic Trend</td>
<td>0.000048</td>
<td>0.000040</td>
<td>0.000037</td>
<td>0.000042</td>
</tr>
<tr>
<td>Moving Average</td>
<td>0.000047</td>
<td>0.000039</td>
<td>0.000039</td>
<td>0.000041</td>
</tr>
<tr>
<td>Hodrick-Prescott</td>
<td>0.000050</td>
<td>0.000048</td>
<td>0.000048</td>
<td>0.000049</td>
</tr>
<tr>
<td>Beveridge-Nelson</td>
<td>0.000032</td>
<td>0.000034</td>
<td>0.000034</td>
<td>0.000038</td>
</tr>
<tr>
<td>Local Level Model</td>
<td>0.000162</td>
<td>0.000165</td>
<td>0.000155</td>
<td>0.000187</td>
</tr>
<tr>
<td>Watson(1986)</td>
<td>0.000138</td>
<td>0.000156</td>
<td>0.000147</td>
<td>0.000149</td>
</tr>
<tr>
<td>HPPF1</td>
<td>0.000062</td>
<td>0.000056</td>
<td>0.000060</td>
<td>0.000075</td>
</tr>
<tr>
<td>HPCPF2</td>
<td>0.000053</td>
<td>0.000054</td>
<td>0.000057</td>
<td>0.000059</td>
</tr>
<tr>
<td>HPCPF2a</td>
<td>0.000066</td>
<td>0.000075</td>
<td>0.000092</td>
<td>0.000101</td>
</tr>
</tbody>
</table>

HPCPF stands for Hodrick-Prescott filter constrained to a Production Function. The number 1 and 2 refers to the parameter set used. In HPCPF2a we used the observed income capital share series, as described in equation 23.

Table 1 reports the mean square forecast error relative to the free prices inflation, resulting from the rolling forecast of the equation (16), the target variable being the quarterly inflation rate of the free prices. The main evidence is that the Beveridge-Nelson methodology outperforms all the models at all forecast horizons.

### Table 2 - Observed and forecasted free price inflation

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Deterministic Trend</td>
<td>1.58</td>
<td>0.62</td>
<td>2.53</td>
<td>6.15</td>
<td>4.02</td>
<td>1.60</td>
<td>0.64</td>
<td>1.25</td>
<td>2.16</td>
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<tr>
<td>Moving Average</td>
<td>1.58</td>
<td>1.16</td>
<td>1.48</td>
<td>6.32</td>
<td>4.79</td>
<td>0.65</td>
<td>1.03</td>
<td>0.18</td>
<td>2.01</td>
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<tr>
<td>Hodrick-Prescott</td>
<td>1.61</td>
<td>1.19</td>
<td>1.35</td>
<td>5.78</td>
<td>4.66</td>
<td>0.77</td>
<td>1.15</td>
<td>0.19</td>
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<td>Beveridge-Nelson</td>
<td>1.58</td>
<td>1.19</td>
<td>1.80</td>
<td>6.19</td>
<td>4.45</td>
<td>0.93</td>
<td>0.95</td>
<td>0.15</td>
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<td>Local Level Model</td>
<td>1.20</td>
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<td>2.24</td>
<td>3.00</td>
<td>2.23</td>
<td>1.73</td>
<td>0.46</td>
<td>1.16</td>
<td>1.19</td>
</tr>
<tr>
<td>Watson(1986)</td>
<td>1.43</td>
<td>1.18</td>
<td>3.11</td>
<td>4.00</td>
<td>2.25</td>
<td>0.89</td>
<td>0.08</td>
<td>0.67</td>
<td>0.48</td>
</tr>
<tr>
<td>HPPF1</td>
<td>1.36</td>
<td>1.32</td>
<td>1.48</td>
<td>5.42</td>
<td>4.59</td>
<td>0.88</td>
<td>1.48</td>
<td>0.05</td>
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<tr>
<td>HPCPF2</td>
<td>1.55</td>
<td>1.17</td>
<td>1.35</td>
<td>5.57</td>
<td>4.61</td>
<td>0.79</td>
<td>1.22</td>
<td>0.16</td>
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</tr>
<tr>
<td>HPCPF2a</td>
<td>1.31</td>
<td>1.22</td>
<td>1.39</td>
<td>5.33</td>
<td>4.50</td>
<td>0.78</td>
<td>1.38</td>
<td>-0.03</td>
<td>1.47</td>
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</table>

HPCPF stands for Hodrick-Prescott filter constrained to a Production Function. The number 1 and 2 refers to the parameter set used. In HPCPF2a we used the observed income capital share series, as described in equation 23.
5 Conclusion

In this paper we have calculated different measures of potential output and output gap for the Brazilian economy, using different models largely used in the literature. In order to compare the output gaps generated from different methodologies, we used a forward-looking Phillips curve and a rolling forecast experiment as a test of forecast accuracy.

Our findings are the following. The potential output estimated by the different models can be classified in two groups: low and high variance trend. The forecasts produced by the unobserved components models are even more inaccurate than those generated by the simplest univariate models. The deterministic trend, moving average, Hodrick-Prescott, Beveridge-Nelson and production function models have strong short-term comovements, appearing to be moving upward and downward at roughly the same time. The main evidence is that the Beveridge-Nelson methodology outperforms all the models at all forecast horizons.
References


Appendix 1 - Figures

Figure 1 - Potential output estimated by different methodologies
Figure 2 - Free prices inflation forecast

- log(free prices)
- HPCPF2a
- HPCPF2
- HPCPF1
- Local Level
- Watson
- Beveridge-Nelson
- Hodrick-Prescott
- Deterministic Trend
- Moving Average