A nonlinear specification of demand for narrow money in

Colombia

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Abstract

A nonlinear smooth transition regression (STR) model of the demand for narrow money in Colombia is specified using monthly data for cash, prices, the scale variable (industrial GDP), the interest rate and the rate of depreciation, within the single equation framework allowed by the data. In comparison with linear error correction model, the nonlinear specification is highly superior according to the statistics. The dynamics described by this model matches both the magnitudes and the behaviour of the aggregate demand for narrow money in Colombia during the sample period (1980.5-1998.11).

JEL classification: E41, C22, C52.

Key words: error correction; demand for money; target-bounds; buffer stocks; nonlinearity; smooth transition regression.

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1. Introduction

A significant part of the empirical research agenda in economics on the last years has been devoted to the search for robust econometric specifications of empirical models of the demand for money with the desired property of theoretical coherence. This tendency was provoked and supported by some complementary techniques coincident in a period spanning about the last twenty years. These techniques are the error correction model and the equilibrium long-run relationships of the variables involved in a model which have been added to some others used to investigate the issues termed weak, strong and super exogeneity, each with different implications on inference, forecasting, and policy, and related to the Lucas critique, parameter constancy, invertibility of the money demand function and to measurement problems of the monetary aggregate used in the specification. On the theoretical side, precautionary demand, risk aversion, asset demands, adjustment costs, target-bounds, buffer stocks, expectations, learning and financial innovation, have played a central role.

The works of Hendry and Ericson (1991 a, b) for the US and the UK demand for money have done an enormous contributions to the discussion on this area as well as the works done to the German case before and after the unification (e.g. Wolters et al., 1998; Beyer, 1998). Some works on money demand dealing with the above points have been done in Colombia using quarterly data, monetary base, M1 or M3 as monetary aggregates (an exception is Steiner who also dealt with cash), the GDP as the scale variable and linear specifications (see Carrasquilla and Renteria, 1991; Oliveros and Misas 1997; and Gómez, 1998, among others). However, the evidence of having an equation of money demand is rather mixed.

In this work we propose a different approach to arrive at the demand for money in Colombia. First, instead of using monetary base or some other broader aggregate, our definition of narrow money is cash. We assume that this variable reflects to a greater extent the preferences of agents to hold real balances while some other definitions could be affected by decisions of the Central Bank (e.g. changes in cash reserve requirements that would affect the measurement of monetary base), which could introduce an identification problem. Second, monthly data (1980:5-1998:11) are used to capture the short run dynamics of the demand for money on the belief that this is the most appropriate frequency to study the demand for real balances. Third, industrial GDP, computed on the basis of the Industrial Production Index, is used as a scale variable instead of (quarterly) GDP a variable not entirely accepted in Colombia because some supposed measurement problems. Finally, we specify a nonlinear dynamics of the demand for money characterised as a smooth transition regression (STR) (see Granger and Teräsvirta, 1993; Teräsvirta, 1998) in which the scale variable, the short term interest rate and the rate of depreciation appear as the explanatory variables in the empirical econometric model. In this paper, we can show that the demand for narrow money in Colombia can be represented by a, noninvertible, nonlinear logistic specification of the STR-type. Moreover, our nonlinear error correction representation is consistent with the theory.

Findings of nonlinearities in money demand functions are gaining in evidence overtime. They have been addressed, for example, by Hendry and Ericsson (1991b), Muscatelli and Spinelli (1996), Teräsvirta and Eliasson (1998), Ericsson et al. (1998), and Sarno (1999). The
nonlinear approach has been used to estimate models of constant parameters in samples of long span and low frequency (annual data for about one century or so). However, we also consider this approach more appropriate to approximate the DGP of a higher frequency series.

Nonlinear models of demand for money could be rationalised as either target-threshold or buffer stock models. The former type of models put forth by Miller-Orr (1966) and developed by Akerlof (1973, 1979), Milbourne (1983), Milbourne et al. (1983), and Smith (1986) provide microfoundations for the presence of a close to unity coefficient of the lagged dependent variable in the equation of money demand. Under these target-bound models, the agents define a target zone (bounded above and below) for their real-money balances. The upper and lower long-run thresholds are defined on the basis of expenditure plans and precautionary anticipations. Consequently, nominal balances are forced by the agents to stay near to the mean of the target zone (translated into nominal terms) when facing short-run deviations from the band. One type of the second class of models (see Laidler, 1984, 1988; Cuthbertson and Taylor, 1987, 1990) give a buffer stock role to money in the sense that it acts as an asset that absorbs temporary shocks for which agents cannot postpone adjustments which are assumed very costly. As we shall see below other different types of buffer stock models have been developed to account for the fact that, given the existence of adjustment costs, relatively small deviations from the long-run real-money holdings are allowed to persist in the short-run while relatively large are not.

The outline of the paper is as follows. The second section describes the data, shows some preliminary results and discusses the equilibrium long-run relationship and the exogeneity of the system. The third section deals with the linear error correction model of demand for money. The fourth section, extends the target-bound and buffer stock alternatives to rationalise nonlinearities of demand for money. The fifth section introduces the STR models, discusses some estimation and testing issues and shows the results. The sixth section makes some final remarks.

2. Data, cointegration system and exogeneity analysis

From the theoretical point of view, agents may hold money as an inventory to reduce differences between the streams of income and expenditure. However, agents may also hold money as an asset in a multi-asset portfolio. Consequently, we could have a customary long-run specification for nominal money demand \((M^d)\) such as:

\[
M^d = f (P, Y, I)
\]  

(1)

where \(P\) is the price level, \(Y\) is the scale variable and \(I\) is a set of rates of returns on assets. The empirical model considered in this work imposes long-run price homogeneity, takes the industrial GDP as the scale variable, and regards the interest rate from period \(t\) to \(t+1\) and the depreciation rate from \(t-1\) to \(t\) as the opportunity cost of holding money. The behaviour of these variables during the sample period is shown in Figure 1. The long-run dynamic model is expressed as:
\[ m - p = \tilde{\epsilon}_0 + \tilde{\epsilon}_1 y + \tilde{\epsilon}_2 i + \tilde{\epsilon}_3 e \]  

(2)

where \( m \) is the log of narrow money (cash), \( p \) is the log of CPI, \( y \) is the log of industrial GDP, \( i \) is the interest rate and \( e \) is the annual rate of depreciation\(^3\).

Following Wolters et al. (1998) we use seasonally unadjusted variables on the assumption that seasonal fluctuations are an important source of variation in economics time series for which it seems sensible to model them instead of dropping such fluctuations. The set of variables used in this work is monthly dated\(^4\) (1980:5-1998:11). The objective of using this frequency is to trace the behaviour of cash demand under the assumption that it is more accurate than annual or even quarterly data as a reference to compare with current events\(^5\).

Given that \((m-p), y, i, \) and \( e \) are \( I(1) \) under conventional tests, we use Johansen techniques to investigate the cointegrating properties of the stochastic VAR system. The number of cointegrating vectors was tested by using a lag length of twelve\(^6\), an intercept in the cointegration space without allowing for linear trends in the data\(^7\), and including centred seasonal dummies out of the long-run relationship\(^8\). The results of the cointegration analysis (Table 1) show that according to the trace statistic there is only one cointegrating vector among the stochastic variables of the system; the equation of the demand for narrow money has the correct signs after normalising by the coefficient of \((m-p)\); on the assumption that there is only a unique long-run relationship between the variables of the VAR system, we found evidence of joint weak exogeneity of \( y, i, \) and \( e \), which means that the stochastic processes \((y, i, \) and \( e \)) do not contain relevant information for the estimation of the parameters of interest and, consequently, the system of the demand for narrow money can be reduced to a single equation\(^9\). Furthermore, according to the results of Table 1 there is evidence of strong exogeneity, given the fact that the null hypothesis of block non Granger-causality cannot be rejected at conventional significance levels. This outcome suggests that no feedback from \((m-p)\) to the stochastic subsystem composed by \( y, i, \) and \( e \) could be found.

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\(^3\) The data corresponding to \( m, i \) and \( e \) are the month average instead of end of the month.
\(^4\) During the sample period there have been at least three remarkable events. First, the financial crisis occurred in the first part of the eighties. Second, the openness of the economy undertaken between the final of the last decade and the beginning of the nineties. Among the measures adopted within this framework, the change of the exchange rate determination regime from a crawling peg to a target zone is of particular interest for this study. Third, the new order brought to the monetary policy setting given the institutional changes, which allowed a more independent Central Bank.
\(^5\) To our knowledge, no investigation on demand for money using high frequency data has been done yet to the Colombian case.
\(^6\) The lag was chosen on the basis of the Likelihood ratio tests (see Lütkepohl, 1991).
\(^7\) A trend in the cointegration space happened to be statistically insignificant.
\(^8\) Because the system contains an unrestricted constant, the asymptotic values do not need any correction regardless of the seasonal dummies included there.
\(^9\) If we have a model such as \( x_t = f(z_t), z_t \) will be weakly exogenous if the joint distribution of \( w_t=(x_t, z_t) \) conditional on the past, can be factorised as the conditional distribution of \( x_t \) given \( z_t \) times the marginal distribution of \( z_t \). As a result, the parameters of the conditional and marginal distributions are not subject to cross-restrictions and the parameters of interest can be uniquely determined from the parameters of the conditional model.
Table 1. Cointegration system analysis
Trace test for the cointegration rank

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>Null hypothesis</th>
<th>Alternative hypothesis</th>
<th>Trace</th>
<th>T. Critical value (90%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1093</td>
<td>( r = 0 )</td>
<td>( r \geq 0 )</td>
<td>52.75</td>
<td>49.92</td>
</tr>
<tr>
<td>0.0835</td>
<td>( r \leq 1 )</td>
<td>( r \geq 2 )</td>
<td>28.32</td>
<td>31.88</td>
</tr>
<tr>
<td>0.0309</td>
<td>( r \leq 2 )</td>
<td>( r \geq 3 )</td>
<td>9.93</td>
<td>17.79</td>
</tr>
<tr>
<td>0.0156</td>
<td>( r \leq 3 )</td>
<td>( r \geq 3 )</td>
<td>3.31</td>
<td>7.50</td>
</tr>
</tbody>
</table>

Standardized long-run coefficients

<table>
<thead>
<tr>
<th>( (m-p) )</th>
<th>( Y )</th>
<th>( R )</th>
<th>( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>0.2929</td>
<td>-3.1336</td>
<td>-0.2899</td>
</tr>
</tbody>
</table>

P-values of testing

<table>
<thead>
<tr>
<th>Joint weak exogeneity</th>
<th>Block non causality</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3079</td>
<td>0.1336</td>
</tr>
</tbody>
</table>

With the aim of testing super exogeneity, we conduct a test of stability of the cointegrating rank by doing a recursive estimation (Hansen and Johansen, 1993). In Figure 2, the continuos (uppermost) line represents the null hypothesis \( H_0 : r = 0 \), while the next line down represents the null \( H_0 : r \leq 1 \), and so on. For conducting the test, we left out about 35 observations from the first sub-sample, and start to add an observation at time. In this way, we check whether the trace statistic would select a different rank from that selected for the full sample. A value of the ratio tests larger than one means that the hypothesis is rejected while a value smaller than one means that the hypothesis may not be rejected. The cointegrating rank \( [R] \) is stable if rank \( [R-1] \) is rejected for any sample, while cointegrating rank \( [R] \) is not rejected for any sample size. Accordingly, at 90% significance level, the test selects \( R = 1 \), a choice which corresponds to the rank we selected above (Table 1).

In summary we have one equilibrium long-run relation among the variables of the demand for money, in which the variables appear rightly signed. The scale variable, the interest rate and the rate of depreciation are joint weak exogenous; there is no feedback from \( (m-p) \) to the block composed by \( y \), \( i \), and \( e \) and there is stability of the cointegrating rank, so that both strong and super exogeneity obtain.

3. The error correction representation of the demand for money

The results gathered in section 2 permit us to conduct the analysis by using a single equation framework. Consequently, the variations of \( (m-p) \) can be expressed in terms of an error correction mechanism, lags of the dependent variable, lags of the changes of \( y \), \( i \), and \( e \) and, possibly, deterministic components. OLS estimation can be used under the assumptions that the coefficients of money demand \( (m-p) \) are zero on the equations of \( y \), \( i \), and \( e \) and the structural shocks of \( (m-p) \) and \( y \), \( i \), and \( e \) are orthogonal.
Starting with twelve lags and seasonal centred dummies for each month, the error correction model we find, after excluding the variables found not statistically significant, is:

$$\Delta(m - p)_t = 0.2126 - 0.0445ecm_{t-1} - 0.0996d_{1,t} - 0.0230d_{2,t} + 0.0361d_{1,t-1} + 0.2412d_{12,t}$$

$$- 0.1533\Delta(m - p)_{t-2} - 0.0938\Delta(m - p)_{t-3} - 0.1154\Delta(m - p)_{t-4} - 0.1496\Delta(m - p)_{t-5}$$

$$+ 0.0598\Delta(m - p)_{t-7} + 0.1385\Delta y_{t-1} + 0.0821\Delta y_{t-2} + 0.1033\Delta y_{t-3} + 0.2937\Delta i_{t-7} + 0.2348\Delta i_{t-11}$$

where, $ecm_t = (m - p)_t - 0.293y_t + 3.134i_t + 0.290e_t$. \hspace{1cm} (3)

$T = 1981.5-1998.1 = 223; R^2 = 0.948; SEL = 0.020; P-value LB(36) = 0.305; ARCH(1) = 1.334(0.249); ARCH(4) = 0.6918(0.5984); JB = 11.5220(0.0031); RESET = 0.5332(0.000).$

The linear error correction model in (3) exhibits correct signs for the error correction term and the seasonal dummies corresponding to January, August, November and December. Also the coefficients have low standard errors (in parenthesis). The goodness of fit as measured by $R^2$ is very high and the standard error (SEL) seems of adequate magnitude. There is evidence of neither serial autocorrelation at one lag measured by the Durbin-Watson (DW) and 36 lags according to the $P$-value of the Ljung-Box coefficient nor ARCH-type nonlinearity at one and four lags. However, notice that, according to the Jarque-Bera statistic, the null of normality of the residuals can be rejected. This result together with the rejection of the null of no misspecification of the RESET test suggests that an alternative dynamic model should be considered. This suggestion is in line with that of Hendry and Ericsson (1991), Ericsson, Hendry and Prestwich (1998), Teräsvirta and Eliasson (1998), Lutkepohl et al. (1998), and Sarno (1999). The only alternative we consider here is that of having an error correction STR model of the demand for money.

4. Rationalising non linealirities within the money demand framework

As we mentioned in the introduction, nonlinear models of demand for money can be interpreted at least from two points of view: as the result of target-bounds and as a buffer stock, both micro-founded. With the target-bound models, first proposed by Miller-Orr (1966) and developed by Akerlof (1973, 1979), Milbourne (1983), Milbourne et al. (1983), and Smith (1986), the agents, based on expenditure plans and precautionary anticipations define a band, for their holdings of real-money.

The fact that any target band introduces nonlinearities to the behaviour of the targeted process, has been well documented in economics (e.g. Blatt, 1983, chapter 10, for the case of investment). It is also the case within the money demand framework (see Akerlof and Milbourne, 1980), since nominal balances are forced by the agents to keep near to the mean of
the target-bound when facing short-run deviations from it or even when the nominal balances are close to the upper and lower bounds\textsuperscript{10}.

The buffer stock models (see Laidler, 1984, 1988; Cuthbertson and Taylor, 1987; 1990; Milbourne, 1988) account for the fact that, given the existence of adjustment costs, relatively small deviations from the long-run real-money holdings are allowed to persist in the short-run while relatively large are not. Different approaches can be distinguish within the buffer stock models. At first place we find the disequilibrium models which can be of two different types: \textit{single equation disequilibrium estimates} of money demand that have an autoregressive component which has been associated to slow adjustment of short-run to long-run desired money holdings; and, \textit{complete disequilibrium models} in which an ample set of real and nominal variables can be affected\textsuperscript{11}. The latter require that, if the parameters of interest are the coefficients of the long-run money demand, then their estimates are conditional on the full specification of the entire model. At the second place, the stock absorber approach directly estimates the demand for money function although money supply is assumed to be held in transactions balances. Rational expectations have been assumed and unexpected money supply shocks are voluntarily held in money balances. Fully anticipated money supply changes are reflected in prices expectations and if prices are perfectly flexible, then real money balances remain unchanged (Carr et al., 1985). Carr and Darby (1981) and Cuthbertson and Taylor (1988) found evidence in favour of the shock absorber approach whereas Cuthbertson (1986) rejected it by using a different estimation techniques. The forward-looking is the third approach of the buffer stock models. Agents may temporarily hold unanticipated increases in money caused by income shocks rather than by money supply ones (see Muscatelli, 1988).

5. A smooth transition regressive model for money demand

We consider the smooth transition regressive models (STR) (see Granger and Teräsvirta, 1993; Teräsvirta, 1998) to represent the error correction model of the demand for money as the alternative to the linear error correction mechanism estimated in (3); see also Eliasson and Teräsvirta (1998), Lütkepohl et al. (1998), Sarno (1999), for applications of this approach.

The model can be written as:

\[
x_t = \mathbf{\beta} w_t + (\mathbf{\pi}_1 + \mathbf{\pi}_2 F(s_t, \mathbf{\alpha})) z_t + u_t
\]

(4)

where \(x_t\) is the dependent variable, \(w_t = (w_{1t}, \ldots, w_{Mt})\) is a vector of \(K\) regressors which enter linearly with constant parameter vector \(\mathbf{\beta}\), \(z_t = (z_{1t}, \ldots, z_{Mt})\) is a vector of \(M\) regressors, \(s_t = (s_{1t}, \ldots, s_{Lt})\) is a \((L \times 1)\) vector of regressors whose elements may include those of \(w_t\) and \(z_t\), and \(u_t\) is an \textit{iid} error process, \(E(u_t) = 0, \ Var(u_t) = \sigma^2\). \(F\) is a transition function bounded by 0

\textsuperscript{10} Akerlof and Milbourne´s model captures the stylised fact that the short run income elasticity is small and could even be negative.

\textsuperscript{11} These models use to consider an equation for a set of real and nominal variables (output, prices, interest rates, exchange rate, etc.) explained by predetermined variables and a lag polynomial of the differences between money supply and money demand; and other equations for the long-run money demand. (see for example, Laidler and Bentley, 1983; Davidson, 1984; Cuthbertson and Barlow, 1990).
and 1 whose parameters are denoted by \( \alpha \). It is assumed that \( E(w_t u_t) = 0 \), \( E(x_t u_t) = 0 \), \( E(s_t u_t) = 0 \). The \( x_t \), \( w_t \), and \( z_t \) processes are assumed to be stationary for the theory of linearity to work\(^\text{12}\). Some lagged elements of \( x_t \) may be included in \( w_t \) and \( z_t \) although weak exogeneity of the remaining elements of them, with respect to the parameters of interest in (4) is required.

Notice that when the transition function \( F = 0 \) the STR model (4) will be linear but, in general, the vector of regressor coefficients, \( \pi_1 + \pi_2 F(s_t; \alpha) \), will depend on the values of the transition variable \( s_t \). The transition function can be parameterised either as a logistic function, in whose case we have a logistic STR (LSTR) model:

\[
F(s_t; \alpha) = (1 + \exp\{-\gamma(s_t - c)\})^{-1}, \quad \gamma > 0
\]  

or as an exponential function, in whose case we have an exponential STR (ESTR) model:

\[
F(s_t; \alpha) = 1 - \exp\{-\gamma(s_t - c)^2\}, \quad \gamma > 0
\]  

The null hypothesis is that of linearity (\( H_0 : \gamma = 0 \)). However, (4) either with (5) or (6) is only identified under the alternative (\( H_1 : \gamma > 0 \)) which invalidates the asymptotic distribution theory. It has been shown that the problem can be solved by using the auxiliary regression (Granger and Teräsvirta, 1993; Teräsvirta, 1994; Teräsvirta, 1998):

\[
x_t = \beta' w_t + \lambda_0 z_t + \lambda_1 z_t s_t + \lambda_2 z_t s_t^2 + \lambda_3 z_t s_t^3 + v_t
\]  

where the null of nonlinearity becomes \( H_0 : \lambda_0 = \lambda_2 = \lambda_3 = 0 \), with power against both the LSTR and ESTR. The choice between LSTR and ESTR can also be done using equation (7), following the sequence proposed by Teräsvirta (1998). When \( t \) takes the place of the transition variable \( (s_t \equiv t) \), the transition function is either:

\[
F(t; \alpha) = (1 + \exp\{-\gamma(t - c_1)(t - c_2)(t - c_3)\})^{-1};
\]

\[
\gamma > 0, \quad c_1 \leq c_2 \leq c_3
\]  

or

\[
F(t; \alpha) = (1 + \exp\{-\gamma(t - c_1)(t - c_2)\})^{-1};
\]

\[
\gamma > 0, \quad c_1 \leq c_2
\]  

or

\[
F(t; \alpha) = (1 + \exp\{-\gamma(t - c_1)\})^{-1};
\]

\[
\gamma > 0
\]  

Equation (7) can be used as a test of parameter constancy in the linear case (Lin and Teräsvirta, 1994; Jansen and Teräsvirta, 1996). For testing parameter constancy in the

\(^{12}\) However, the theory still applies when \( s_t \) is non stationary dominated by a polynomial in \( t \) (Lin and Teräsvirta, 1994).
nonlinear case we use equation (4.6) of Eitrheim and Teräsvirta, (1996). If the test is carried out with either (8), (9), or (10), they will be termed LM3, LM2, or LM1, respectively. The LM1 is a test of \( H_0: \lambda_1 = 0|\lambda_2 = \lambda_3 = 0 \), which has good power against a smooth change in the parameters and, depending on the value of gamma, also against a single structural brake. The LM2 is a test of \( H_0: \lambda_4 = \lambda_5 = 0|\lambda_6 = 0 \), and the transition function is a LSTR2 (a logistic transition function with a rest path in the middle regime).

The results of the linearity test are reported in Table 2. All variables of the linear error correction model were used as possible transition variables. Furthermore, we have assumed that the seasonal dummies enter the money demand function only linearly. The results do not indicate any nonlinearity at 5% of significance when the possible transition variable is one of the variables used in the linear model (3). Nonlinearity can not be rejected when the transition variable is \( \Delta(m-p)_{t-11} \) given that the \( P \)-value is the lowest (0.0008). Some other good candidates were \( \Delta(m-p)_{t-10} \) (0.01) and \( \Delta y_{t-10} \) (0.009). The error correction term did not work properly as a transition variable.

With this result the selection procedure described in detail in Teräsvirta (1998) allowed us to choose a logistic STR model whose final specification is:

\[
\begin{align*}
\Delta(m-p)_{t} &= 0.8632 - 0.1763ecm_{t-1} - 0.1113d_{1,t} - 0.0245d_{S,t} + 0.0366d_{11,t} + 0.1956d_{12,t} \\
&- 0.1012\Delta(m-p)_{t-3} - 0.1257\Delta(m-p)_{t-4} - 1.7563\Delta(m-p)_{t-5} - 0.0604\Delta(m-p)_{t-7} + 0.0943\Delta y_{t-3} \\
&+ 2.6616\Delta r_{t-7} + \{-0.6865 + 0.1385ecm_{t-1} + 0.1397\Delta y_{t-1} + 0.0764\Delta y_{t-2} - 0.1574\Delta(m-p)_{t-2} \\
&+ 1.6092\Delta(m-p)_{t-5} - 2.4409\Delta i_{t-7}\} \ast \{1 + \exp[-9.4941(\Delta(m-p)_{t-11} + 0.1051)/\sigma_{\Delta(m-p)}]\}^{-1} \\
&= 0.391(0.283) - 0.799(0.290) - 0.242(0.058) - 0.300(0.033) - 0.242(0.027) + 1.6092(0.391) - 2.4409(0.810) - 2.4409(6.628) - 2.4409(0.283)
\end{align*}
\]

where, \( ecm_t = (m-p)_{t} - 0.293y_t + 3.134i_t + 0.290e_t \).

\( T = 1981.5-1998.11= 211; R^2 = 0.956; Senl = 0.0187; Varl/Varl = 0.868; P-value \ LB(4) = 0.871; ARCH(1) = 2.5359(0.112); ARCH(4) = 1.2177(0.3045); JB = 3.7067(0.1567). \)

From the statistics of the error correction STR regression in (11), it can be seen that this model outperforms the linear error correction model of equation (3). Thus, the standard error (SEnl) is smaller to the extent that the ratio of the residual variances is 0.868, the residuals are white noise and fulfils normality requirements according to Jarque-Bera statistic. The nonlinear model seems more accurate in capturing the behaviour of the demand for cash in Colombia than the linear error correction model (Figures 3 and 4). It can also be observed by looking at the residuals of the respective models (Figures 5 and 6). In Figure 7 we can see the

\[11\] We thank Timo Teräsvirta for providing us with some of the Gauss code used in this application and also Munir Jalil for further developments of it.
difference between the residuals and the exact gains of using a nonlinear specification. On the other hand, Figures 8 and 9 show the linear and nonlinear error correction mechanisms.

In addition to these specification tests we also implement the LM-type tests of non remaining autocorrelation, non remaining nonlinearity and parameter constancy developed by Eitrheim and Teräsvirta (1996) in order to check the adequacy of the estimated model in the nonlinear framework. The test of non remaining autocorrelation has under the alternative hypothesis a nonlinear model with autocorrelation of order \( q \). Under the null the test is distributed as a \( \chi^2 \), but with the purpose of improving its power in small samples we use the \( F \)-version. The results (Table 2) indicate that the test fails to reject the null hypothesis of non error autocorrelation of order 1, 4, and 20 at any usual level of significance.

Table 2. P-values of the LM test of non error autocorrelation against an AR(\( q \)) and MA (\( q \)) error processes

<table>
<thead>
<tr>
<th>Maximum lag q</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>12</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>No error</td>
<td>0.358</td>
<td>0.634</td>
<td>0.891</td>
<td>0.700</td>
<td>0.601</td>
<td>0.736</td>
<td>0.805</td>
</tr>
</tbody>
</table>

Following Eitrheim and Teräsvirta (1996) we also check whether equation (11) is an adequate characterisation of the nonlinear features rendered by the data by looking at any non remaining nonlinearity. This test is a generalisation in the nonlinear framework of the classical RESET test. Under the alternative it tests an additive nonlinear model and has a \( \chi^2 \) distribution under the null. However, the model is not identified under the null but this problem has been solved in the same way in which the linearity test is solved by Granger and Teräsvirta (1993) and Teräsvirta (1994). In this case, all variables in the linear error correction model of equation (3) were tried as potential transition variables in the additive nonlinear model. Nonetheless the results do not give any evidence in favour of remaining nonlinerity in the model of equation (11).

Finally, we have applied the test of parameter constancy under the alternative hypothesis of a smooth change in the parameters of the model which also includes the test of an abrupt change. The test under the alternative is parameterised using three different functions in which the transition variable is the time (equations 8, 9, and 10 above). These possible parameterisations allow a wide range of non constancy possibilities. Here, the constancy test is carried out over three different sets of parameters (seasonal, linear and nonlinear) of equation (11). The results in Table 3 show that the non constancy test can be rejected at any level of significance for the parameters in the linear and the nonlinear part of the model. However, the null can be rejected for the parameters of the seasonal dummies. This result is generated by the non constancy of the dummy corresponding to August (\( H_0 \); (5) in Table 3), however we have no explanation to this phenomenon. The other parameters are constant over time and hence maintain our final specification of equation (11).
Table 3. P-values of the LM test of parameters constancy of the LSTR model (11) against STR-type non constancy

<table>
<thead>
<tr>
<th>Null hypothesis</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LM1</td>
<td>0.401</td>
<td>0.435</td>
<td>0.003</td>
<td>0.437</td>
<td>0.000</td>
</tr>
<tr>
<td>LM2</td>
<td>0.514</td>
<td>0.472</td>
<td>0.0003</td>
<td>0.694</td>
<td>0.000</td>
</tr>
<tr>
<td>LM3</td>
<td>0.871</td>
<td>0.837</td>
<td>0.00003</td>
<td>0.889</td>
<td>0.000</td>
</tr>
</tbody>
</table>

(1) $H_0$: “All parameters of the linear part of the model except the constant are constant”
(2) $H_0$: “All parameters of the nonlinear part of the model except the nonlinear intercept are constant”.
(3) $H_0$: “All parameters of the seasonal part of the model are constant”
(4) $H_0$: “All parameters of the seasonal part of the model without the seasonal parameter of August are constant”
(5) $H_0$: “The seasonal parameter of August is constant”

Note: The parameters not under test are assumed constant also under the alternative

6. Some final remarks

The final specification we have obtained could be analysed as follows. There is an equilibrium long-run demand for narrow money in which price homogeneity has been imposed and the normalised coefficients are correctly signed. There is evidence of weak, strong and super exogeneity. According to the statistics our nonlinear model outperforms the linear error correction of equation (3).

The logistic STR model (11) contains a nonlinear error correction adjustment (necm) which we reproduce here as:

$$necm = 0.8632 - 0.1763ecm_{t-1}$$
$$+ \{-0.6865 + 0.1385ecm_{t-1}\}*\{1 + \exp\{-9.4941[\Delta(m-p)_{t-11} + 0.1051]/\sigma_{\Delta(m-p)}\}\}^{-1}$$

To analyse the local dynamics, notice that in the extreme regimes of the transition function, ($F = 0$ and $F = 1$ of Figures 10 and 11), the necm becomes:

$$necm_{F=0} = 0.8632 - 0.1763ecm_{t-1}$$
$$necm_{F=1} = (0.8632 - 0.6865) - (0.1763 - 0.1385)ecm_{t-1}$$

thus, when $\Delta(m-p)_{t-11}$ is close or less than the threshold value ($c = -10.5\%$), where $F = 0$, the error correction is 4.67 [=-0.1763/(0.1763-0.1385)] times the error correction when is $\Delta(m-p)_{t-11}$ greater than the threshold value, where $F = 1$. These different adjustment process towards the equilibrium is indicative of the asymmetric dynamics rendered by the logistic STR which could match the buffer stock models, in the sense that the agents adjust...
their holdings of real money to the desired level in different magnitudes and speeds depending on the value (and the sign) of $\Delta(m - p)_{t-11}$.

The nonlinear dynamics works sometimes by January and December depending on the value of $\Delta(m - p)_{t-11}$. This result is highly intuitive since during these months the agents change their portfolio composition: in December in favour of cash while reversing this decision by January.

The non constancy of the parameter corresponding to August could be explained by some institutional changes not captured by the model occurred during the sample period. Thus, a possible extension of this work could be to modelling this non constancy using the STR-type models with the aim of figuring it out.
References


Figure 1. Set of variables (monthly data, 1980:5 – 1998:11)

Sources: narrow money, interest rate and depreciation rate from the Banco de la República. 
CPI and Industrial Production Index from DANE
Figure 2. Parameter stability of the cointegrating rank

Figure 3. Money demand: observed vs. linear estimation

Figure 4. Money demand: observed vs. nonlinear estimation
Figure 5. Residual from the linear model

Figure 6. Residual from the nonlinear model

Figure 7. Residual from the linear (LRES) and nonlinear (NRES) models
Figure 8. Linear error correction

Figure 9. Nonlinear error correction

Figure 10. Transition function over time

Figure 11. Transition function