

Testing Real Business Cycle Models in an Emerging Economy

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1. Introduction

One of the most dynamic areas of macroeconomic research in the last decades is that of Real Business Cycle (RBC) models. Since the seminal work by Kydland and Prescott (1982), a number of papers have tested the abilities of neoclassical general-equilibrium models to account for economic fluctuations. The original framework of Kydland and Prescott has been extended to include labor market rigidities (Hansen, 1985), taxes and government expenditures (McGrattan, 1994a), money and inflation (Cooley and Hansen, 1995), open economies (Backus et al, 1995), and increasing returns to scale in production (Weber, 2000). Each of these extensions has been successful in solving the limitations of calibrated models to replicate particularities of the data and can provide richer explanations of business cycles, although at the cost of increasing complexity.

Although RBC models have been successful when applied to developed economies, their abilities in replicating the data of developing countries remain largely untested. In the case of Chile, there are only a few noteworthy exceptions.¹ This paper provides the first systematic exploration of RBC models to the Chilean data, starting with the original Kydland and Prescott framework and introducing increasing degrees of complexity in the analysis. The purpose of this exercise is to test the capacities of RBC models to (1) replicate the salient characteristics of the observed aggregate fluctuations of the economy in the 1986-1998 period, and (2) provide insights regarding the contribution of fiscal and monetary policies to the cycle.

The challenge to RBC models posed by the Chilean experience is formidable. First, in the 1986-1998 period the economy experienced a rapid but unstable pace of growth. Although GDP grew at an average annual rate of 7.5%, it also experienced significant year to year fluctuations, from a high 10.1% growth in 1989 to only 2.1% in 1990. In contrast, in the same period GDP growth in the US was 2.6% and fluctuated within a narrower range of -1% to 4%. Second, in the 1986-1998 period Chile experienced a remarkable reduction in inflation, from a high annual rate of 27% in 1989 to only 4.6% in 1998, which suggests that the contribution of both nominal and real fluctuations might have played an important role during the period. Third, the economic structure of a developing country such as Chile differs markedly from that of industrial economies precisely in those key underlying parameters that govern the mechanics of RBC models. Particularly in terms of factor shares in GDP, the stock of

¹ Quiroz (1991) and Quiroz et al. (1991).

capital, the size and composition of government expenditures, the composition of consumption and investment, and the size of technological shocks.

The structure of the paper is as follows. Section 2 provides a snapshot of the salient features of economic cycles in Chile.² We use simple statistics to discuss the relative importance of the shocks to GDP and its components and to assess their temporal structure. Section 3 provides a brief description of the different general equilibrium models we use, stressing the role of technology shocks, the effect of real and monetary frictions (such as labor rigidities and cash-in-advance constraints), the impact of fiscal and monetary policy shocks, and the derived decision rules of optimizing agents. In this section we also discuss the solution methods employed in the numerical optimization. Section 4 of the paper describes the data –some of which has been collected especially for this study– and presents the parameterization of the different calibrated models. We also discuss the main difference between Chile’s key (deep) parameters and those of industrial economies, in particular the US. Section 5 presents the main empirical results, including the simulation of the models, the analysis of impulse-response functions, and a decomposition of relative contributions of the different sources of shocks to economic cycles. In section 6 we follow Canova (1994) by viewing our artificial economies as restricted versions of more general VAR models and testing these restrictions imposed by the structure of the model and the linearization process. Finally, section 7 collects the main conclusions and suggests future extensions of this work.

2. Characterizing the economic fluctuations of the Chilean economy

In this section we report the empirical regularities or stylized facts that characterize business cycles in Chile. Stylized facts were obtained from the longest available database with consistent information on a quarterly basis, which covers the 1986-1998 period. As expected, the Chilean economic fluctuations present important similarities when compared with the features of business cycles in industrialized countries (Backus et al. 1995), but they also present interesting peculiarities that our business cycle models should be able to address.

² For a complete description see Bergoing and Suarez (1999).

We follow Lucas (1977) in defining business cycles as those apparent deviations from a trend in which variables move in the long run. The definition and computation of this trend, nevertheless, are controversial. During the last years there has been a rich debate with respect to the abilities of different statistical methods to decompose time series into long and short term fluctuations (see Baxter and King, 1995; Guay and St-Amant, 1996). The relative advantages of techniques such as those of Beveridge and Nelson (1981), Watson (1986), Hodrick and Prescott (1997), and Baxter and King (1995) are, nevertheless, not established. Mechanical filters have been criticized, among others, by Harvey and Jaeger (1993) which show that the Hodrick-Prescott (HP) filter can induce spurious cyclical when applied to integrated data. Guay and St-Amant (1996) found that the HP and Baxter-King (BK) filters perform poorly in identifying the cyclical component of time series that have a spectrum with the shape characteristic of most macroeconomic time series. Baxter and King (1995) note that two-sided filters such as the HP and BK filters become ill-defined at the beginning and the end of samples.

Notwithstanding this debate, we follow the standard practice of the business cycle literature of reporting all stylized facts using the deviations of the variables of interest from their long-run trend obtained with the Hodrick and Prescott filter (HP). Since the purpose of our paper is to assess the capacities of this type of models in describing the regularities of Chile's economic cycles, this choice allows us to compare our results to the evidence gathered for other countries. Canova (1999) illustrates his support of the use of the Hodrick and Prescott's filter arguing that when comparing results among models we ought to be "looking through the same window".

We report several statistics for the detrended data.³ In particular, we consider: (1) the amplitude of fluctuations (volatility, in the business cycle jargon), represented by the standard deviation of the cyclical component of the series, (2) the ratio of the standard deviations of the series to the one of the GDP (labeled, relative volatility); (3) the contemporaneous correlation of the cyclical components of a variable and that of GDP;⁴ and (4) the phase shift, represented by the correlation coefficients between leads and lags of GDP and those of the variable. If, for example, a series is procyclical but peaks *i*

³All series are seasonally adjusted using the ARIMA X-11 procedure and expressed in natural logarithms before being filtered, with the exception of the percent variables, such as inflation and interest rates, which are in levels.

⁴ As customary, if the contemporaneous correlation is close to one we label the variable as procyclical; if it is close to minus one, we called it countercyclical, and if it is close to zero, we say is acyclical.

quarters before real GDP, we say the series leads GDP by these many quarters. Since all variables are in logarithms, the change in the trend component, $\hat{q} - \hat{q}_{-1}$, represents the growth rate.

Figure 1 shows the evolution of the cyclical GDP in the period under analysis. Clearly, at least four cycles have occurred in the sample, though they differ in magnitude and length, being the first two shorter than the subsequent two. This evidence is, in general terms, similar to the findings in Soto (1999) obtained using a switching-regime model based on Markov chains, although in the latter the size of the 1995 downturn is markedly smaller. The size and volatility of GDP cycles are rather large; considering that the quarterly trend is 1.8% in the sample, the peak of the cycle would be equivalent to observing an annualized growth rate of 15%, while at the trough it would amount to a zero growth rate.

Additional information is presented in Table 1 which reports numerical indicators of the amplitude and phase of the fluctuations of GDP and other key macroeconomic variables.⁵ In general terms this information points to several similarities in the Chilean business cycle with regards to that of industrialized countries, but it also highlights interesting differences.

In the first place, volatility of GDP in Chile –which reaches 2.00– is much higher than in most industrialized economies. For example, in the 1970-mid 1990s period volatility in Europe was only 1.01 on average.⁶ Nevertheless, the US exhibits a volatility that is comparable (1.92). In part, this higher volatility is a reflection of structural characteristics of the Chilean economy (relative absence of automatic stabilizers, shallow financial markets, less diversified production structure, etc.), but is also consistent with the pace of high growth sustained by Chile in the sample period. Between 1986 and 1998, GDP grew at a per-capita annual rate of 5.5%.

We also found that private consumption in Chile is as procyclical to GDP as it is in Canada, Italy, Japan, or the US. As displayed in Figure 2, GDP and consumption move in synchronicity, with a high correlation coefficient of 0.82. Moreover, consumption displays high volatility as it fluctuates by almost the same magnitude of output. This is consistent with the evidence found for France and Japan, but it is much higher than what was found in the US, Switzerland, and Italy where fluctuations are between 70% and 80% of the volatility of GDP.

⁵ A detailed analysis of these fluctuations is provided by Bergoing and Suárez (1998).

⁶We use the results of Backus et al (1995) for comparisons.

The high volatility of consumption is one of the challenges that business cycle models have to face. Since RBC models are essentially neoclassical, consumption is usually modeled under the permanent income hypothesis. In this setting, consumption volatility should be smaller than that of output since agents that optimize intertemporally tend to smooth out consumption. The apparent excess volatility of consumption is, in part, the result of using total consumption data. In fact, when consumption is separated into purchases of durable and nondurable goods, we find that their volatility is markedly different (see Figure 3). Volatility of durable goods is 7.5 times higher than that of nondurable goods. In what follows, we restrict consumption to nondurable goods and include purchases of durable consumption goods in investment.

Although the volatility of the purchases of non-durable consumption goods is smaller than that of total consumption or GDP, it remains rather high (1.60). In part, this may be the result of the existence of liquidity constraints (credit restrictions), a characteristic that our business cycle models should also address.

A second challenge posed by the Chilean data to business cycle models is the nature of shocks in labor markets. It can be seen in figure 4 that employment fluctuates far more than average hours worked. Theoretically, hours worked should fluctuate far more than employment if there are no rigidities or adjustment costs in labor markets. In a frictionless, neoclassical labor market employment should display very low volatility because most of the adjustments should fall on hours worked and the entry and exit of workers from the labor force should be minimal. In Chile, on the contrary, volatility is almost as high as that of GDP. This certainly reflects the fact that in our sample unemployment reduced from a high 15.4% in 1986 to 5.6% in 1998. But it could also point to the existence of labor market rigidities since average weekly hours worked fluctuate much less than total hours and than employment.

The evidence, then, suggests that most fluctuations in total hours represent movements in and out of the labor force rather than adjustments in average hours of work. As expected, the relative volatility of employment is markedly smaller than that in Europe (0.85), but much higher than that in Japan (0.34) or Australia (0.34). Naturally, we grant that part of this heterogeneity in the performance of the labor market reflects differences in institutional arrangements.

An additional puzzle posed by the behavior of agents in labor markets are the fluctuations in productivity levels and their correlation to hours worked. In the Chilean case, the volatility of average productivity is 1.94, almost as high as that of GDP, and substantially higher than that of developed economies. Nevertheless, its negative correlation with hours worked (-0.24) is equivalent to that in the US. This is a worrisome feature for our business cycle models because one of the weaknesses of the original Kydland and Prescott specification is its inability to replicate the low correlation between hours worked and productivity levels.

A third interesting feature of the business cycle in Chile is the presence of large fluctuations in investment. In the sample period, the volatility of investment reached 7.37, more than 3 times higher than that of GDP. This level is almost 50% more than what is observed in Europe (2.09), but not significantly larger than that of the US (3.27).

Another area in which the Chilean case seems to differ markedly from developed economies is the behavior of the government. The size of the government measured by public consumption (as percent of GDP) is much smaller in Chile (10.6%) than it is in the US (17.9%) or in Europe (18.3%) and comparable only to Japan (9.4%). Not only the size of the government is dissimilar but also their dynamic behavior is markedly different. In Chile, government expenditures are mildly correlated to GDP fluctuations, as depicted in figure 5. In developed economies such correlation is non-existent. In fact, the estimated correlation in Chile is 0.34, three times higher than in Europe. Moreover, public consumption is pro-cyclical and lead output by two quarters. Its volatility (1.12), on the other hand, is also very high for international standards (0.47 on average for Europe and 0.75 for the US).

A second important aspect of public consumption is that, for many groups of the society, these expenditures represent substantial transfers of goods (health and education, among others). Nevertheless, these groups also pay taxes, so that the net effect may be ambiguous. Business cycles models developed below explicitly address this issue.

There are also significant differences with regards to monetary issues. The volatility of nominal variables is higher in Chile than in industrialized countries. Money, as measured by per capita M1, presents a volatility of 5.84, almost four times higher than in the US, and it is also more strongly correlated with GDP. Correlation in Chile is 0.70, twice as high as in the US (0.33). In fact, money cycles display a striking synchronicity with GDP fluctuations, as depicted in figure 6. This may be the result

of the preference of the Central Bank towards using interest rate-based instruments –as opposed to targeting monetary aggregates– during the last ten years. Moreover, in Chile money leads the cycle by a quarter while M2A lags the cycle by a quarter.

Considering the evidence on monetary aggregates it is, then, not surprising to find that the volatilities of the price level and inflation are also markedly higher than those in developed economies. The relatively high volatility of inflation (1.13) is certainly related to the stabilization and anti inflationary policies implemented in the period. As a result, annual inflation decreased consistently from more than 25% in the late 1980s to a 2.5% in 1999. But anti-inflationary policies have also induced marked volatility in quarter-to-quarter inflation. The price level, not surprisingly, shows high persistency with an autocorrelation coefficient of 0.88. Persistency in inflation has also been compounded by the high degrees of indexation in the Chilean economy.

Finally, in small open economies, such as Chile, one should expect an important effect of international business cycles on the domestic economy. In particular, when foreign trade is largely dependent on commodity prices. In our case, this dependence arises from the large share of copper on exports, but also because Chile is dependent on oil imports. Commodity prices in both cases are very volatile. Hence, it is not surprising to find that the volatility of the terms of trade is 6 times higher than that of GDP. This is a much larger value than that of developed economies, which on average present a volatility of only 3.7. Nevertheless, it is not markedly different than most Latin American and African economies which in the 1950-1990 period hover around 12.⁷ The correlation between terms of trade shocks and output fluctuations, however, is very small (0.15), probably reflecting both the presence of commodity stabilization funds for copper and oil and the fact that the economy anticipates the effect of terms of trade shocks. Evidence for the latter effect is reflected in that terms of trade shocks lead GDP fluctuations by 3 quarters.

⁷ See Mendoza (1995).

Table 1
Main Indicators of the Business Cycle in Chile
1986.1-1998.4

Variable	Volatility (Std. Dev. x100)	Volatility Relative to that of GDP	Correlation with GDP	Persistence (autocorrelation)
Output	2.00	1.00	1.00	0.55
Consumption	1.98	0.99	0.82	0.60
• Non Durables	1.60	0.80	0.63	0.50
• Durables*	12.06	6.03	0.72	0.62
Investment	7.37	3.68	0.79	0.67
Capital	0.50	0.25	-0.17	0.81
Hours Worked	0.53	0.25	0.28	0.24
Employment	1.21	0.61	0.32	0.79
Labor Productivity	1.94	0.97	0.81	0.56
Government Cons.	1.12	0.56	0.39	0.60
Money	5.84	2.76	0.70	0.70
Price Level	2.39	1.17	-0.32	0.88
Inflation	1.13	0.54	-0.09	0.22
Terms of Trade	12.21	6.13	0.15	0.83

Note: (*) corresponds to the purchases of durable consumption goods.

Figure 1

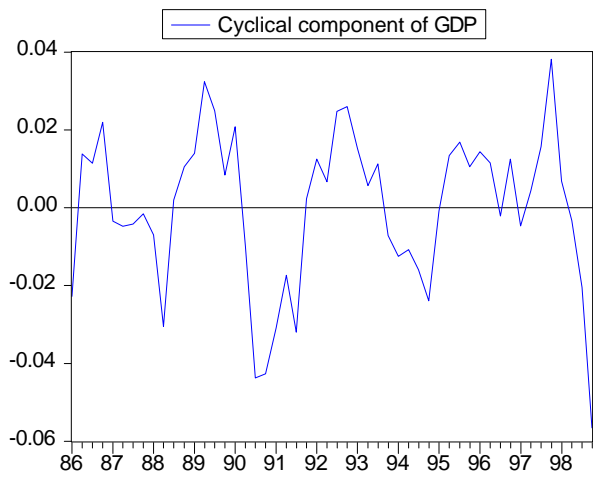


Figure 2

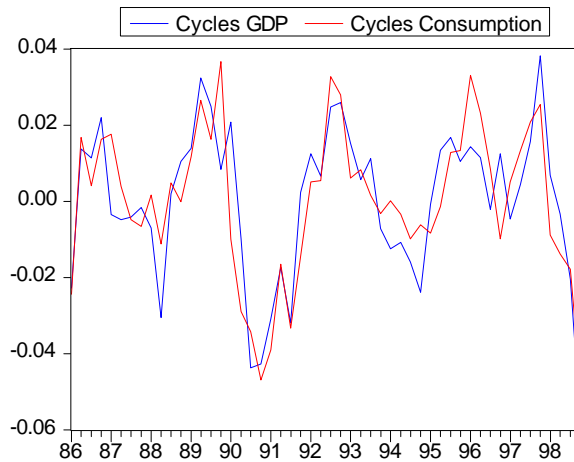


Figure 3

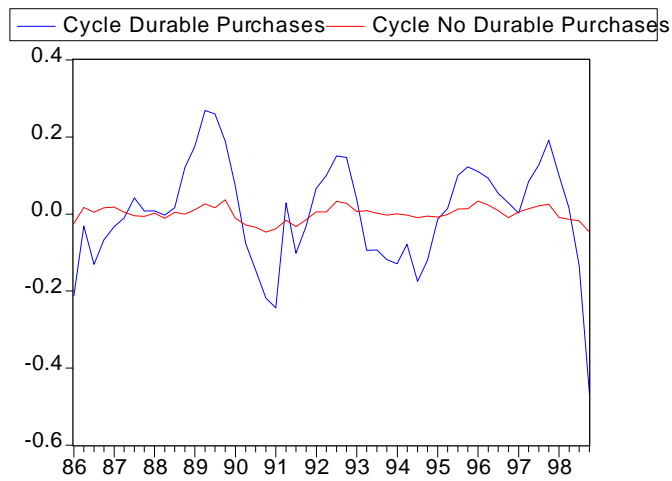


Figure 4

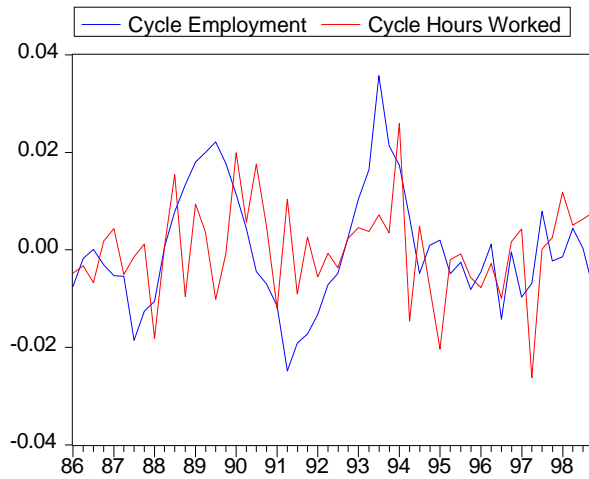


Figure 5

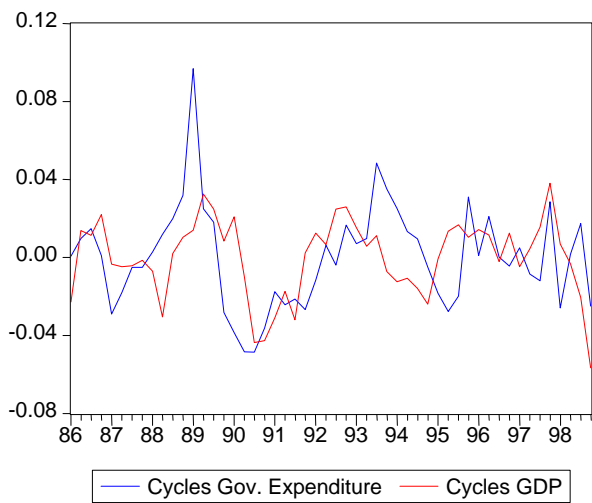


Figure 6

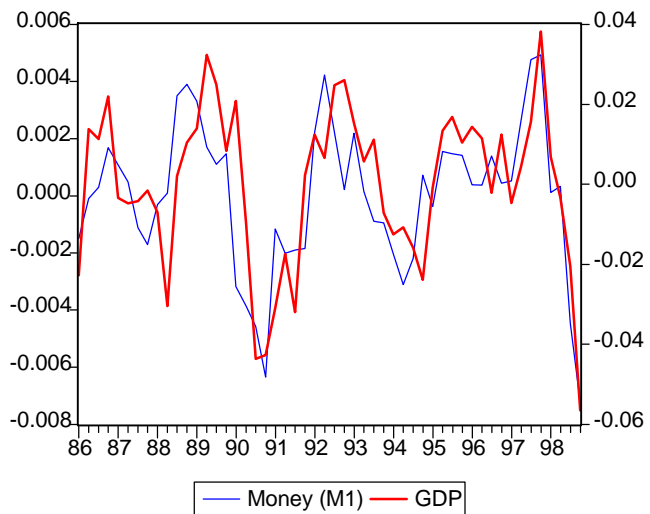


Figure 7

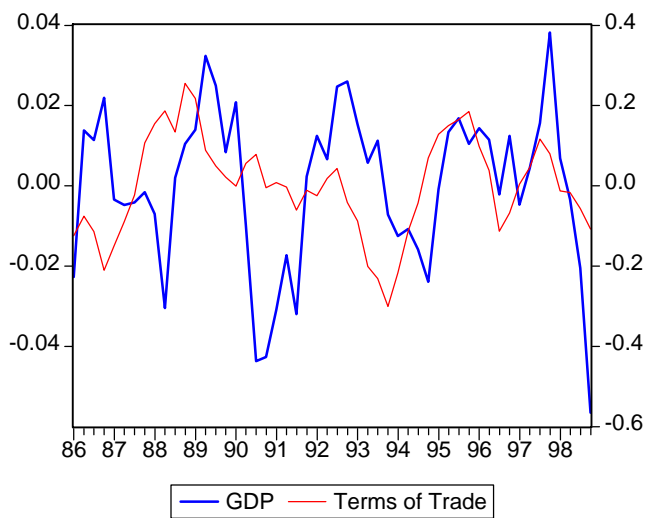
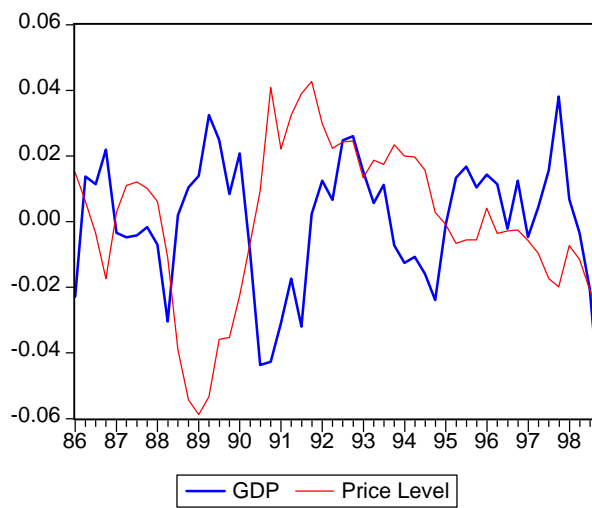


Figure 8



3. Business Cycles models

The original model by Kydland and Prescott has been extended to include, among other issues, household production (Benhabib, Rogerson and Wright, 1991); labor hoarding (Burnside, Eichenbaum, 1994); open economies (Backus, Kehoe and Kydland, 1995); money and inflation (Cooley and Hansen, 1995); incomplete markets and heterogenous agents (Rios-Rull, 1991); and increasing returns to scale (Devereux, Head, and Lapham, 1996).

In this section we present a stylized business cycle model for the Chilean economy and discuss the rationale for the main extensions we later test. Based on the description of the salient characteristics of economic cycles in Chile presented in section 2 and with the purpose of evaluating the relative contribution of macroeconomic policies, we develop a model that focuses on government expenditures and monetary shocks and includes real-side shocks as captured by technological shocks. The main characteristic of our model is to encompass within the framework of a general equilibrium setup an important number of features of the economy, including productivity growth, fiscal expenditures and monetary policy, and labor market rigidities. The main drawback of the present version of the model is that it neglects some the real and financial aspects of international business cycles and their effect on the private sector.

In addition, in this section we present the algorithms to obtain analytical and numerical solutions to the general-equilibrium optimization problem. For the latter, our discussion only sketches the main issues and we refer the reader to Cooley (1995) for detailed discussions on the different techniques.

3.1 The Basic Business Cycle Model

The model considers the government, a large number of identical firms, and a large number of infinitely-lived, identical consumers. In all models calibrated below, the production function is taken to be the same while the different specifications which we test are obtained by suitable changes in the utility function and the nature of government policies.

Technology and Firm Behavior

Firms are assumed to operate with a constant-returns-to-scale technology in the two inputs, labor (N_t) and capital (K_t), and to maximize profits (π) in a competitive way, taking as given the price level (P_t), nominal wages (W_t) and the rental cost of capital (R_t):

$$\begin{aligned} \max \quad & \pi_t = P_t y_t - W_t N_t - R_t K_t \\ \text{s.t.} \quad & y_t = \lambda_t N_t^{(1-\theta)} K_t^\theta \end{aligned} \tag{1}$$

where λ_t is a stochastic technology shock and θ is a parameter. Since output is produced with constant returns to scale, it is appropriate to consider the existence of only one firm which obtains zero profits.

The first order conditions of the problem yield the following reduced-form functions for real wages and the rental price of capital:

$$\begin{aligned} W(\lambda_t, K_t, N_t) &= (1-\theta) \lambda_t \left(\frac{K_t}{N_t} \right)^\theta P_t \\ R(\lambda_t, K_t, N_t) &= \theta \lambda_t \left(\frac{N_t}{K_t} \right)^{1-\theta} P_t \end{aligned} \tag{2}$$

A second restriction is imposed to represent the process of capital accumulation. Following Prescott (1986), we assume that there are no gestation lags in investment (zero time to build) and that the depreciation rate is δ . Hence:

$$K_t = (1 - \delta) K_{t-1} + I_{t-1} \tag{3}$$

The Government

We use two different specifications of the government to test for the role of fiscal and monetary policies. In the first specification, we assume that government policies can be represented by a stochastic sequences of real current expenditures $\{g_t\}$ and ad-valorem taxes levied on consumption, $\{\hat{q}_t\}$. Note that we impose stochastic ad-valorem taxes instead of lump-sum taxes. These taxes are imposed on private consumption, c_t . The rationale for this specification is to recognize that compliance with tax payments is less than perfect and that tax distortions behave stochastically (e.g., commodity “stabilization” funds, specific- good taxes, import overcharges).⁸ We also define a transfer to consumers, T_t , that ensures that a balanced budget is achieved in every period.

$$P_t g_t + T_t = \tau_t P_t c_t \quad (4)$$

When considering monetary policies, we follow Cooley and Hansen (1995) and assume that the per-capita money supply grows at a stochastic rate of μ_t . Hence, the budget constraint of the government is:

$$\begin{aligned} P_t g_t + T_t &= M_{t+1} - M_t + B_{t+1} - B_t(1 + R_{t-1}) + \tau_t c_t P_t \\ M_{t+1} &= e^{\mu_t} M_t \end{aligned} \quad (5)$$

where M_t is the nominal per capita money supply, B_t are government bonds, and R_t is the nominal interest rate.

Since the presence of T_t ensures a balanced budget, Ricardian equivalence holds. Once the initial government debt is determined (B_0), for a given stochastic realization of money and government expenditures, the paths of $\{B_t\}$ and $\{T_t\}$ do not affect the optimal or equilibrium resource allocation (government bonds are not net wealth). For simplicity, then, it is convenient to assume $B_t = 0 \forall t$.

⁸In the empirical section we calibrate the tax process with a small variance to ensure that realized shocks are positive and close to the observed values of trade taxes.

Consumers

Consumers maximize the following expected intertemporal utility function:

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i u(c_t + \pi g_t, l_t) \quad (6)$$

where β is the discount factor, u is a strictly concave function in both arguments, c_t is private consumption, g_t is public consumption, δ is a constant, and l_t is leisure time. This specification, originally from McGrattan (1994a), assumes that whenever $\delta > 0$, public expenditures increase the utility of consumers. Alternatively, when $\delta = 0$ public consumption does not affect the utility of consumers.

In addition to consumption, consumers derive utility from leisure. We normalize time availability to 1 so that $1 - l_t$ is the fraction of time allocated to work. Two specifications of the labor market are tested below. In the first case, we allow consumers to freely determine the amount of work (number of hours) they offer in the labor market. In such case, we specify the utility function as:

$$u(c_t + \pi g_t, l_t) = \log(c_t + \pi g_t) + \gamma \log(l_t) \quad (7)$$

where γ is a curvature (elasticity) parameter. Since in this framework an optimizing agent can choose l_t to be any fraction of time, this is called the “divisible labor” model.

Assuming that labor is divisible, however, is at odds with the empirical evidence in the Chilean case which suggests that, in most cases, labor is contracted in fixed amounts (e.g., full-time positions, 8-hour shifts) while only a fraction of the labor force works variable time schedules (in 1998 only 20% of workers were classified as “self-employed”, a group that includes part-time and informal workers). Hansen (1985) suggests to model institutional rigidities in the labor markets using a “lottery” framework in which agents must decide between working a fixed amount of hours or not at all. The lottery is the probability of being hired to work N hours. If the utility function for individual i is $u(c_i, l_i) = \log(c_i) + \tilde{\alpha} \log(l_i)$, then the expected utility is $\log(c_i) + \tilde{\alpha} \log(l_i) \phi$, where ϕ is the probability of being employed.

In the aggregate, θ of the labor force work N hours and $(1-\theta)$ does not work. Then, per-capita hours worked are $n = N\theta$. The optimization problem can be written as:

$$u(c_t + \pi g_t, l_t) = \log(c_t + \pi g_t) + \gamma l_t \quad (8)$$

where γ is now proportional to $\log(1-N)/N$. The main effect of this “labor-rigidities” specification is to allow for more substitution between hours of work and leisure at different times.

Since the representative consumer own the capital stock, the budget constraint of his optimization problem includes the incomes derived from working activities ($W_t n_t$) and from renting the capital stock ($R_t k_t$). In addition, the consumer receives the lump-sum transfer (T_t) from the government (which can be positive or negative). Real income can be allocated to consumption, investment, and paying taxes (small case letters represent real values).

$$(1 + \tau_t) c_t + i_t \leq r_t k_t + w_t n_t + \frac{T_t}{P_t} \quad (9)$$

In order to introduce monetary issues, a cash-in-advance restriction is imposed. Cash-in-advance restrictions are equivalent to including money in the utility function (under general conditions) but are easier to tract when solving the numerical model. To allow for more realism in the modeling, we allow only a fraction of consumption (c_t^1) to be liquidity constrained. Our model is a variation of the “cash-good” specification of Cooley and Hansen (1995). Hence, the utility function and budget constraint of the consumer become:

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i u(c_t^1, c_t^2, g_t, l_t) \quad (10)$$

$$W_t n_t + R_t k_t + M_t + T_t \geq (1 + \tau_t) P_t (c_t^1 + c_t^2) + P_t i_t + M_{t+1}$$

$$T_t + M_t \geq (1 + \tau_t) P_t c_t^1$$

The utility function can be modified to reflect the “divisible” and “rigid” labor market in a consistent manner. The utility function becomes, respectively:

$$\begin{aligned} u(c_t^1, c_t^2, g_t, l_t) &= \alpha \log(c_t^1 + \pi g_t) + (1 - \alpha) \log c_t^2 + \gamma \log l_t \\ u(c_t^1, c_t^2, g_t, l_t) &= \alpha \log(c_t^1 + \pi g_t) + (1 - \alpha) \log c_t^2 + \gamma l_t \end{aligned} \quad (11)$$

where α is the proportion of goods that are bought with cash. Note that we have assigned government expenditures to the “cash good” portion of consumption.

Stochastic Processes

The models consider as much as four stochastic shocks: the technology parameter (\tilde{e}_t), the growth rate of money supply (μ_t), government expenditures (g) and consumption taxes (\hat{q}). For each of these shocks we fit an autoregressive model of the form:

$$z_t = \rho z_{t-1} + \epsilon_t \quad \epsilon_t \rightarrow N[(1 - \rho)\bar{z}, \Sigma] \quad (12)$$

where $|\tilde{\rho}| < 1$, $\tilde{\epsilon}_t$ is an i.i.d. innovation, and \bar{z} is a vector with the average growth rates of these variables. Parameters $\tilde{\rho}$ and \bar{z} are estimated using least squares techniques. In addition, we compute the cross-moments matrix of the sample and use it as the covariance matrix of innovations, $\hat{\Sigma}$.

3.2 Model Solution

The six models specified above (the economy with and without labor market rigidities, with and without government, and with and without money) are solved in a similar manner but the decision rules in each case differ markedly reflecting the nature of the specifications. In the most comprehensive economy, the problem that the representative household must solve is to maximize expected utility subject to the budget constraint and cash-in-advance restrictions (equation 10), subject to stochastic shocks to state variables (equation 12), the law of motion for capital (equation 3), the demand functions for labor and capital (derived from the first order conditions of the firm’s problem in equation 2), the government’s budget constraint (equation 5), the monetary policy, and the law of motion of the stock

of capital (equation 3). For the comprehensive version of the model with labor rigidities, the Bellman's equation representing the household's problem is given by:

$$\begin{aligned}
v(z_t, K_t, k_t, \hat{m}_t) &= \max \left[\alpha \log(c_t^1 + \pi g_t) + (1 - \alpha) \log c_t^2 - \gamma n_t + \beta E_t v(z_{t+1}, K_{t+1}, k_{t+1}, \hat{m}_{t+1}) \right] \\
s.t. \\
w_t(\lambda_t, K_t, N_t) &= (1 - \theta) \lambda_t K_t^\theta N_t^{-\theta} \\
r_t(\lambda_t, K_t, N_t) &= \theta \lambda_t N_t^{1-\theta} K_t^{\theta-1} \\
z_t &= \rho z_{t-1} + \epsilon_t \\
c_t^1 &= \frac{\hat{m}_t + e^{\mu_t} - 1}{e^{\mu_t} \hat{p}_t} + c_t^2 \tau_t - g_t \\
(1 + \tau_t) c_t^2 + k_{t+1} + \frac{\hat{m}_{t+1}}{\hat{p}_t} &= [r_t(\lambda_t, K_t, N_t) + 1 - \delta] k_t + w_t(\lambda_t, K_t, N_t) n_t \\
K_{t+1} &= K(z_t, K_t) \\
H_{t+1} &= H(z_t, K_t) \\
\hat{p}_{t+1} &= p(z_t, K_t)
\end{aligned} \tag{13}$$

where z is vector comprising the stochastic processes of the technology parameter (ϵ), money supply (μ), government expenditures (g), and consumption taxes (δ).

The problem is solved using Hansen and Prescott's (1995) method, which requires to derive the linear per-capita decision rules from a recursive linear-quadratic approximation of the economy. Each model is solved to obtain the decision rules of the agents with which they determine the allocation of time to work $\{n_t\}$, investment levels $\{i_t\}$, and the amount of money to be held to satisfy the cash-in-advance restriction $\{m_t\}$. These decision rules are a function of the structural form of the models, the deep parameters that describe technologies, preferences, and restrictions, and the stochastic processes that govern disturbances and shocks. A useful transformation allows for an easier solution. Defining $\hat{m}_t = m_t / M_t$ and $\hat{p}_t = p_t / M_{t+1}$ it is possible to eliminate m_t and p_t from equation (13) and solve the problem directly.

A recursive competitive equilibrium in this economy is defined as a set of decision rules for the household, $c^1(s)$, $c^2(s)$, $k'(s)$, $\hat{m}'(s)$, and $n(s)$, where $s = (z, K, k, \hat{m})$; a set of per-capita decision rules, $K'(z, K, N)$ and $N(z, K, N)$; pricing functions $p(z, K)$, $w(\tilde{e}, K, N)$, and $r(\tilde{e}, K, N)$; and a value function, $v(s)$, such that:

1. Households optimize: given the pricing functions and the per capita decision rules, $v(s)$ solves the functional equation (13), and c^1 , c^2 , k' , \hat{m}' , and n are the associated decisions rules;
2. The firm optimizes: the functions w and r are given by equation (2)
3. Individual decisions are consistent with aggregate outcomes:

$$\begin{aligned} k_{t+1}(z_t, K_t, K_t, 1) &= K_{t+1}(z_t, K_t, K_t, 1) \\ n_t(z_t, K_t, K_t, 1) &= N_t(z_t, K_t, K_t, 1) \\ \hat{m}_{t+1} &= (z_t, K_t, K_t, 1) = 1 \quad \forall (z, K) \end{aligned}$$

4. Parameterization and Data

The parameterization of the models was undertaken using quarterly data for the 1986-98 period. The sources and detailed definitions of the data are described in the appendix. National account figures for the main macroeconomic variables such as GDP and consumption were obtained from Banco Central. The available data presents some limitations that needed to be fixed. First, we use Gallego and Soto (2000) methodology to obtain the breakdown between durable and non-durable goods, since only total private consumption is available in national accounts. Government consumption, on the other hand, was obtained from national accounts and excludes public investment.

The aggregate capital stock series was obtained from Coeymans (1999), which uses sectoral accounts and endogenous depreciation rates in its computation. Coeymans figures, however, are very similar to what could be obtained using total fixed capital formation, the perpetual inventory method, and a standard depreciation rate of 3.5%. For the 1986-98 period a capital to quarterly output ratio of 11.2 is found. This figure is smaller than that for the US employed by Cooley and Hansen (1995) and McGrattan (1994b) which is around 13.5. Investment figures (including public investment) were obtained from national accounts but adjusted to include consumption in durable goods and changes in

stocks. Depreciation rates (δ) were obtained by subtracting the change in the capital stock (net investment) from gross investment [$i_t + k_{t+1} - k_t$] and expressing it as a proportion of k_t . An average value of 3.22% was found for the 1986-98 period, slightly higher than that for the US which is usually between 2.0% and 2.5%. An alternative procedure suggested by McGrattan (1994b) –regressing the computed depreciation on k_t – yields a similar estimate. See Table 2 for the complete parameterization.

The breakdown of time between work and leisure was estimated as follows. Total available hours per week were computed by multiplying the labor force by 112 (16 hours, seven days a week). Total worked hours per week were computed as average hours worked times employment (all figures were obtained from the National Bureau of Statistics, INE). We obtained an estimated share of leisure of 64%, slightly below the standard 70% used among others by Kydland and Prescott (1982), Hansen (1985) and McGrattan (1994b) for the US. Casual evidence suggests our estimate is likely to be accurate since part-time work is very uncommon in the formal labor market in Chile and occasional surveys tend to support the notion that work schedules are markedly longer than in developed economies.

Some of the parameters were obtained from the Euler conditions of the general equilibrium models described before. For example, the discount factor $\hat{\alpha}$ was obtained from the Euler condition for consumption which implies that $\beta = (1 + r)^{-1}$. We used the 1986-98 average between the deposit annual rate (6.2%) and lending rate (9.1%) and obtained an estimated $\hat{\alpha}$ of 0.9817.

The share of capital in output, θ , was obtained from the first-order conditions of the problem but it depends on whether the model includes or excludes government expenditures and taxes. For the no government case, the capital-output ratio is 12.3 for the sample period, so that the estimated θ is 0.6668. For the models that include the government the calibrated value of θ is 0.6096. This value is quite close to the factor share of capital obtained from national accounts (0.5885).

$$\theta = \frac{[1 - \beta(1 - \delta)]}{\beta} \frac{1}{(1 + \tau)} \frac{k_t}{y_t} \quad (14)$$

The parameter of leisure in the utility function, on the other hand, depends on the specification of the labor market and the presence of the government. For the frictionless models, parameter $\tilde{\alpha}$ was calibrated as:

$$\gamma = \frac{(1 - \theta)}{(1 + \tau)n \left(1 - \delta \frac{k}{y} \right)} \quad (15)$$

while in the case of labor rigidities the parameter corresponds to:

$$\gamma = \frac{[1 - \theta]l}{(1 + \tau)n \left[1 - \frac{\delta k - g(1 - \pi)}{y} \right]} \quad (16)$$

The calibrated parameters are displayed in Table 2. In general, the obtained curvatures are much smaller than that of the US reflecting the smaller amount of leisure time allocated by Chilean workers, as well as the larger share of capital in factor incomes.

In the absence of microeconomic studies of the Chilean case, the proportion of government expenditures that is valued by consumers, δ , was estimated using the following Euler equation:

$$\frac{U'(C_t)}{\beta U'(C_{t+1})} = 1 + r_t - \delta = \frac{C_{t+1} + \pi g_{t+1}}{\beta [C_t + \pi g_t]} \quad (17)$$

From this first order condition we run the following regression:

$$C_t = \frac{1}{\beta(1+r_t-\delta)} C_{t-1} + \pi \left(g_t - \frac{g_{t-1}}{\beta(1+r_t-\delta)} \right) + \epsilon_t \quad (18)$$

the estimated parameter is $\delta=0.40$, implying that less than half of government expenditures is valued by consumers as substitute of private consumption. This estimate is much higher the value of zero used by McGrattan (1994b) for her study of the US economy. We deem our value reasonable for the Chilean case since around 37% of government current expenditures in the 1994-1998 period were direct transfers to the population in terms of health, education, and housing subsidies (Banco Central de Chile, 1999).

In order to obtain an estimate of the proportion of transactions made by consumers using only cash, we use the Euler equations for consumption which implies:

$$\frac{C_t}{C_t^1} = \frac{1}{\alpha} + \frac{1-\alpha}{\alpha} R_t \quad (19)$$

where C/C_1 is the inverse proportion of cash goods in total consumption. Note that, since cash-in-advance restrictions hold, $C/C_1 = C/M_1$. Following Cooley and Hansen (1995), we regress the ratio non-durable consumption to M1 on the interest rate. The model was estimated using non-linear least squares and obtained an point-estimate value of $\hat{\alpha}$ is 0.725. Arguably, the estimation is not necessarily an accurate measure of cash goods since M1 includes money held by firms –a very small proportion in the Chilean case– and the estimation is not robust.

Technology shocks were obtained directly from the data using the calibrated $\hat{\alpha}$ as:

$$\lambda_t = \frac{y_t}{k_t^\theta n_t^{1-\theta}} \quad (20)$$

To characterize the stochastic processes of shocks we proceeded as follows. First, the values for $(\bar{\mu}, \bar{g}, \bar{\tau})$ were obtained directly from the data as the average sample values. The growth in the per-capita money supply is 2.38%, while government consumption amounts to 9.1% of GDP and taxes 8.3%. Second, $\bar{\lambda}$ was set at 1, since it is only a scale parameter. Third, an AR(1) process was fitted to the detrended log of $(\lambda_t, g_t, \tau_t, MI_t)$ and we obtained the respective autoregressive coefficients: (0.9835,

0.9531, 0.9317, 0.4000). Fourth, we computed the covariance matrix of the shocks, $\hat{\Omega}$, as shown in table 2.

Table 2
Parameterization of the Business Cycle Models for Chile

<i>Model</i>	<i>CALIBRATED PARAMETERS</i>									
	$\bar{\epsilon}$	\bar{a}	\hat{a}	\bar{a}	$\bar{\lambda}$	\bar{g}	$\bar{\tau}$	$\bar{\delta}$	μ	\hat{a}
<i>No government Labor divisible</i>	0.669	0.03	0.978	0.975	1	0	0	0	0	0
<i>Government Labor divisible</i>	0.608	0.03	0.978	1.08	1	0.09	0.08	0.4	0	0
<i>No government Labor indivisible</i>	0.669	0.03	0.978	1.523	1	0	0	0	0	0
<i>Government Labor indivisible</i>	0.608	0.03	0.978	1.687	1	0.09	0.08	0.4	0	0
<i>Money Labor indivisible</i>	0.669	0.03	0.978	1.523	1	0	0	0	0.02	0.725
<i>Money, Government, Labor indivisible</i>	0.608	0.03	0.978	1.687	1	0.09	0.08	0.4	0.02	0.725

	<i>AUTOCORRELATION COEFFICIENT</i>
$\bar{\epsilon}$	0.9835
μ	0.4000
$\hat{\delta}$	0.9317
g	0.9531

	<i>COVARIANCE MATRIX OF INNOVATIONS</i>			
	$\bar{\epsilon}$	μ	$\hat{\delta}$	g
$\bar{\epsilon}$	0.000241			
μ	0.000217	0.000756		
$\hat{\delta}$	0.000355	0.000107	0.000258	
g	-0.002937	0.000121	0.000021	0.001246

5. **Main Results**

Before presenting the results it is interesting to evaluate how different are the parameters of the Chilean economy when compared to those used in studies of the developed economies. Table 3 presents a summary of the key parameters. It can be easily seen how significant are the differences in the nature of the developing economies when compared to more advanced countries.

The share of capital in GDP is substantially higher in Chile, a reflection of the relative scarcity of physical capital. Depreciation rates are much higher and, consequently, the discount factor is smaller. It is important to note that real interest rates are substantially higher in Chile than in all studies of developed economies: McGrattan (1994b) and Cooley and Hansen (1995) use annual rates well below 4% while in Chile average interest rates reach 7.7%.

The labor market is also markedly different. Although leisure time is substantially lower in Chile, the most significant difference is in the “curvature” of labor in the utility function. In the Chilean case, substitution is only one half of that in the US and increases only marginally when labor market rigidities are allowed. It should be noted that increasing this parameter from 2.33 to 3.22 allowed Hansen (1985) to improve substantially the abilities of real business cycles models in replicating the data in output and labor markets.

The two other important differences are the valuation of public consumption by consumers and the proportion of cash goods in the economy. The latter is substantially lower than in the US, as estimated by Cooley and Hansen (1985).

It is in this table, perhaps, where it is clearer the extent to which we are testing the real business cycle model. We will require the model in section 3 to be able to replicate the similarities and differences in economic fluctuations between Chile and developed countries, but using a very different set of parameters within a framework which is undoubtedly not very sophisticated.

Table 3
Key Parameters Comparison

	Chile (1986-1998) this study	US (1947-1987) Mc Grattan (1994)	US (1954-1991) Cooley - Hansen (1995)
Share of Capital in GDP	0.608	0.360	0.400
Depreciation Rates	0.032	0.025	0.019
Discount Rate	0.978	0.985	0.989
Labor Curvature			
• Divisible	1.523	2.330	
• Rigidities	1.636	3.220	2.530
Leisure time	0.640	0.733	0.690
Share of Govt. Expend. in Utility function	0.600	0.000	0.000
Proportion of Cash Goods	0.725	0.000	0.840

Simulation Results

The first column in table 4 reproduces the principal indicators of the Chilean business cycle we would like to replicate using the different models. The simplest model is contained in the column labeled 1 and corresponds to the case in which we exclude the government, allow for divisible labor, and introduce only one source of stochasticity in the form of technological shocks. It can be seen that the model is successful in replicating an important number of the features of the data we attempt to model.⁹ In fact, it reproduces in general terms the volatility of output and capital, but falls short of matching that of consumption, labor productivity, and investment, while it overstates the fluctuations in hours worked. In addition, it produces a positive and significant correlation between hours worked and productivity, which is at odds with the data, being the latter negative. This simple model also replicates some of the correlation between the variables, but in general terms is unsatisfactory. For some variables it generates excessive contemporaneous correlation (e.g., consumption, investment, and labor

⁹It should be acknowledged that the business cycle literature lacks a metric to determine how close are simulated variable to the actual data. In the analysis we consider that the volatility of a simulated variable is “close” to the actual volatility when it is within a 10% range.

productivity), while in others it fails to capture the true relationship, in particular in the case of capital and hours worked. By construction the model does not replicate any nominal variable.

The second column considers the introduction of the fiscal side of government activities. As displayed in column 2, the introduction of government expenditures improves significantly the abilities of the business cycle model to replicate the volatility of consumption, investment, and labor productivity. It is interesting to note that despite the substantial improvement in replicating the volatility of average labor productivity, the ability of the model to reproduce the functioning of the labor market is still disappointing, as apparent in the insufficient volatility of hours worked and the positive correlation between hours worked and productivity. In addition, the correlation between consumption and output is too small. Nevertheless, this model matches the correlation of hours worked and GDP.

The third model we tested allows for labor market rigidities. The purpose of introducing indivisible labor is to improve the limitations of the above models in describing the trade off between hours worked and employment. It can be clearly seen that the specification is unsatisfactory.

However, when this model with friction in the labor market is coupled with model 2 (allowing for government expenditures), the quality of the calibrated model improves substantially. In fact, the model is able to replicate most of the real side indicators we consider, with the only exception of consumption which fluctuates very little and is not correlated to output as in the data. In the results it is noteworthy that the model replicates the volatility of output, hours worked, productivity and investment within a close range, and overestimates to some extent that of capital. Moreover, it produces a positive (but insignificant) correlation between hours worked and labor productivity. Nevertheless, the simulated contemporaneous correlations are in general overestimated.

In summary, these results suggest that (1) business cycles models are able to replicate most of the observed fluctuations of the real side of the economy, (2) in comparative terms, the introduction of government expenditures is able to explain substantially more of fluctuations than labor market rigidities, and (3) replicating the fluctuations in consumption requires to place additional constraints to the optimizing behavior of agents contained in our model.

In addition to replicating real side fluctuations, for policy purposes one would like business cycle models to replicate nominal variables such as inflation and monetary aggregates. Moreover, one should

expect a further gain on the real side if the inability of these four initial models to replicate the volatility of consumption is linked to the existence of liquidity constraints. As discussed above, we introduce cash in advance constraints in models 5 and 6.

Table 4
Simulated Business Cycle Models for the Chilean Economy

Variables	Actual Data	Divisible Labor		Labor Rigidities			
		Without Govment.	With Govment	Without Govment	With Govment	Money	Money and Govment
	1986-98	1	2	3	4	5	6
<u>Volatility</u>							
GDP	2.00	2.15	1.94	2.42	1.97	2.27	2.47
Consumption	1.60	0.74	0.97	0.83	0.98	1.51	1.52
Investment	7.37	4.52	7.15	5.25	7.18	4.36	5.63
Capital	0.50	0.48	0.76	0.58	0.76	0.48	0.60
Hours worked	0.53	0.95	0.23	1.69	0.40	1.35	1.64
Labor Product	1.94	1.23	1.89	0.87	1.87	1.03	0.98
Money	5.84	-	-	-	-	5.84	5.34
Prices	5.17	-	-	-	-	5.84	6.16
Inflation	1.13	-	-	-	-	1.12	2.09
<u>Contemporaneous correlation to GDP</u>							
GDP	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.63	0.89	0.13	0.85	0.13	0.71	0.68
Investment	0.79	0.99	0.95	0.99	0.95	0.94	0.96
Capital	-0.17	0.07	0.24	0.09	0.24	0.05	0.07
Hours worked	0.28	0.98	0.27	0.97	0.32	0.97	0.96
Labor Product	0.81	0.99	0.97	0.90	0.98	0.94	0.90
Money	0.70	-	-	-	-	0.63	0.88
Prices	-0.32	-	-	-	-	-0.02	-0.19
Inflation	-0.09	-	-	-	-	-0.03	-0.12
Corr. Hours Productivity	-0.23	0.93	0.15	0.78	0.12	0.83	0.77

Model 5 introduces cash-in-advance constraints but excludes the fiscal side of government activities. The model successfully replicates the volatility of money and inflation and overestimates slightly that of the price level. It can be seen that money constraints also allow us to replicate the volatility of consumption and its correlation with output. Nevertheless, the model continues to produce a labor market equilibrium solution which does not match the data and is unable to find a significant correlation between output and the price level and inflation. As in all previous models that exclude government expenditures, the correlation between hours worked and average productivity levels is disappointingly high.

Model 6 introduces cash in advance restrictions within the framework of labor rigidities and government consumption. Once again, a number of features of the nominal side of the data are adequately reproduced, including the volatilities of money and prices, although it overstates that of inflation. In addition, the correlation of nominal variables and GDP is better reproduced. Several characteristics of real side fluctuations are also reproduced by the simulations: the volatility of GDP, consumption, capital, and the contemporaneous correlation of most variables to GDP. Nevertheless, the model fails to improve the representation of labor market data when compared to model 5.

Sensibility analysis

The above parameterization imposes a number of assumptions on the structure of the economy that render the calibrated business cycle model a particular vision of the Chilean economy. Section 6 of the paper formally tests these restrictions. An alternative methodology would be to change the structure of parameters and the dynamic nature of forcing variables. For space reasons, we report only some of the indicators in Table 6.

The sensibility analysis is performed on the most ambitious specification (model 5) and focus basically on the two crucial policy parameters –the proportion of government expenditures valued by consumers, δ , and the proportion of cash goods, α . Likewise, we change the dynamic structure of technology shocks to allow for permanent shocks (unit roots).

Table 5
Sensibility Analysis of the Business Cycle Model of the Chilean Economy

Variables	Actual Data	Model	5	Increase δ from 0.4 to 0.6	Increase α from 0.725 to 0.850	Change ϵ from 0.9835 to 1	Increase μ from 4% to 6%
	1986-98	1	2	3	4	5	
<u>Volatility</u>							
GDP	2.00	2.47	2.51	2.49	2.29	2.48	
Consumption	1.60	1.52	1.54	1.75	1.67	1.59	
Investment	7.37	5.63	5.68	5.73	4.73	5.52	
Capital	0.50	0.60	0.61	0.61	0.53	0.60	
Hours worked	0.53	1.64	1.73	1.69	1.22	1.65	
Labor Product	1.94	0.98	0.93	0.96	1.17	0.97	
Money	5.84	5.34	5.84	5.84	5.84	4.47	
Prices	5.17	6.16	6.19	6.20	6.23	4.92	
Inflation	1.13	2.09	2.12	2.21	2.21	2.06	
<u>Contemporaneous correlation to GDP</u>							
GDP	1.00	1.00	1.00	1.00	1.00	1.00	
Consumption	0.63	0.68	0.68	0.61	0.77	0.68	
Investment	0.79	0.96	0.96	0.94	0.95	0.95	
Capital	-0.17	0.07	0.07	0.07	0.06	0.07	
Hours worked	0.28	0.96	0.97	0.97	0.96	0.97	
Labor Product	0.81	0.90	0.89	0.89	0.96	0.90	
Money	0.70	0.88	0.88	0.87	0.97	0.87	
Prices	-0.32	-0.19	-0.15	-0.15	-0.20	-0.19	
Inflation	-0.09	-0.12	-0.22	-0.21	-0.24	-0.23	
Corr. Hours Productivity	-0.23	0.77	0.75	0.75	0.84	0.77	

The results, presented in Table 5, suggest the following conclusions:

- The matching of variances and correlations is not affected in a significant manner when parameter δ –the valuation of public goods in the utility function– is increased from 0.4 to 0.6.
- When liquidity constraints are made more stringent –parameter α increases from 0.725 to 0.85– the general matching of variances and correlations becomes only marginally affected, improving only on the variances of nominal variables.
- Modeling productivity shocks as unit roots –i.e., allowing for permanent shocks– improves the matching of moments in several dimensions: the variances of GDP, consumption, capital, hours worked and labor productivity become closer to the actual data, but the matching worsens for nominal variables (prices and inflation). Likewise, in the model with permanent shocks the correlations between real variables and GDP are higher than in real data.
- Finally, increasing the exogenous growth rate of money from 4% to 6% does not induce any significant change in real variables –with the exception of a marginal increase in the variance of consumption– but it worsens the matching of nominal variables and the real quantity of money.

In summary, changing the main parameters of this real business cycle of model does not produce significant changes in the qualitative conclusions reached above, although in some cases it modifies the numerical outcomes of the model and their distance from the actual data. Although this is not a formal test, the results suggest that the parameterization does in fact reflect the underlying structure of the Chilean economy and that the selection of crucial parameters is not too arbitrary.

6. *Evaluating the simulated RBC models* (this section is to be finished)

Business cycle models can be viewed as restricted versions of more general VAR models. These restrictions imposed by the structure of the model and the linearization process can be tested using relatively simple statistical procedures (see Canova et al, 1994). The debate among econometricians about the empirical evaluation of these models remains, nevertheless, controversial (Kydland and Prescott, 1996; Hansen and Heckman, 1996; and Sims, 1996).

Following Canova et al. (1994), consider the following representation of model 5 (including government expenditures, taxes, labor rigidities, and cash in advance restrictions) employed in section 5:

$$\begin{aligned} y_t &= Az_t \\ z_t &= Fz_{t-1} + G\epsilon_t \end{aligned} \tag{21}$$

where y is the vector of variables of interest, z are the controlled and uncontrolled states (the latter labeled x), ϵ are the innovations, and A , F , and G are matrices of coefficients. These matrices are in general combinations of the “deep parameters” presented in Table 2; consequently model 5 imposes particular structures to matrices A and F which can be tested directly against the sample data.

The first type of test arises from the long-run restrictions contained in matrix A . When the forcing variables (or uncontrolled states, x) are integrated variables, matrix F takes the following particular form:

$$\begin{bmatrix} \gamma & \delta \\ 0 & I_p \end{bmatrix}$$

where p of the eigenvalues of F are unity while the rest are the eigenvalues of \tilde{a} . Since the latter are assumed to be less than one in business cycle models, there must be $(n-p)$ cointegrating vectors among the states. This is the first testable hypothesis that can be confronted to the data. In our particular case, the z vector includes \tilde{e} , g , μ , \hat{o} , and k .

The second testable implication of the RBC model as represented by equation (31) is that the residual of $y_t - Az_t$ ought to be stationary and the cointegrating vector must be A . Hence, a simple test of stationarity can be conducted to test this restriction.

Finally, the third testable implication of RBC models arises from the reparameterization of the system as a VAR of the following form:

$$\begin{aligned}\Delta y_t &= A\alpha\beta'z_{t-1} + AG\epsilon_t \\ &= A\alpha v_{t-1} + AG\epsilon_t\end{aligned}\tag{22}$$

where \hat{a} are the cointegrating vectors described above. The RBC imposes the following two testable restrictions: (a) only state variables appear in Δy_t , and (b) coefficients on v_{t-1} should be $A\hat{a}$.

When the forcing variables are not integrated, it becomes very difficult not to reject the restrictions embedded in the model $y_t = Az_t$. Nevertheless, the following VAR model obtained from equation (21):

$$y_t = AFz_{t-1} + AG\epsilon_t$$

encompasses directly the restrictions imposed by the RBC model. In this case, again (a) only state variables appear in Δy_t , and (b) coefficients on z_{t-1} should be AF .

Evaluation results

Table 6 presents unit-root tests for the deseasonalized data. It can be seen that unit root tests do not reject the null of non-stationarity in the state variables k and n nor in the main variables of interest (GDP, consumption and investment), but the null is rejected in all forcing variables except tax rates. For technology shocks the evidence is less robust. It is widely accepted that unit root tests can be very misleading due to low power, structural breaks, etc (Hamilton, 1994).

Table 6
Unit Root Tests: Phillips-Perron Methodology
 1986-1998

		Without trend	With trend
Money growth		-5.15	-5.48
Technology shocks		-1.1	-3.3
Government expenditures		-1.69	-7.62
Taxes		-1.68	-1.16
Capital Stock		12.74	-0.23
Output		-1.46	-2.53
Consumption		0.9	-3.03
Investment		-2.19	-1.69
Rejection Values	5%	-2.92	-3.50
	10%	-2.60	-3.18

All data seasonally adjusted, 3-lag truncation.

Integrated Forcing Variables

Treating forcing variables as integrated processes implies that, according to the business cycle model, there should be three cointegrating vectors ($n=5$, $p=2$). Table 6 presents the result of estimating cointegrating vectors within the sample data. The RBC restrictions are weakly supported by the data in the sense that one cannot reject the null hypothesis of three cointegrating vectors. A stronger test would be to determine whether the implied calibrated decision functions are actually stationary and match the estimated cointegrating vectors.

Table 7
Cointegration Tests: Johansen's methodology
 Sample: 1986:1 1997:4 , 4 lags

Series: Capital, Hours, Technology Shocks, Money growth, Government Expenditures

Eigenvalue	Likelihood Ratio	5 Percent Critical Value	1 Percent Critical Value	Hypothesized No. of CE(s)
0.749819	157.2708	87.31	96.58	None**
0.709188	99.07681	62.99	70.05	At most 1**
0.414048	47.20346	42.44	48.45	At most 2*
0.281425	24.75370	25.32	30.45	At most 3
0.228091	10.87332	12.25	16.26	At most 4

Test assumption: Linear deterministic trends in the data.

*(**) denotes rejection of the hypothesis at 5%(1%) significance level.

L.R. test indicates 3 cointegrating equation(s) at 5% significance level.

The second set of tests considers the implied reduced form of output, consumption, and investment, as described in equation (23), in terms of combination of the deep parameters. Since, all endogenous variables are I(1), ζ should be I(1). Cointegration tests are reported in Table 8.

$$\begin{aligned}
 y_t &= f_1(k_t, \tau_t, \lambda_t, g_t) + \eta_{y_t} \\
 c_t &= f_2(k_t, \tau_t, \lambda_t, g_t) + \eta_{c_t} \\
 i_t &= f_3(k_t, \tau_t, \lambda_t, g_t) + \eta_{i_t}
 \end{aligned}
 \tag{23}$$

Table 8
 Cointegration Tests of Equations 23

	ADF test on ζ s	ECM tests
Output		
Consumption		
Investment		

The final set of tests considers the implications for the dynamic structure imposed by the RBC model. We obtained the implied calibrated VAR representation of the RBC model along the lines above mentioned and compare it to an estimated VAR on the sample data. In equation (24) the numbers in brackets are estimated while plain numbers are combinations of the calibrated parameters and the decision functions.

$$\begin{aligned}
\Delta y_{t-1} &= x.xx k_{t-1} + x.xx n_{t-1} + x.xx \lambda_{t-1} + x.xx g_{t-1} + x.xx \mu_{t-1} + \eta_{y_t} \\
&\quad (-0.142) \quad (0.705) \quad (-0.065) \quad (0.280) \quad (0.237) \\
\Delta c_{t-1} &= x.xx k_{t-1} + x.xx n_{t-1} + x.xx \lambda_{t-1} + x.xx g_{t-1} + x.xx \mu_{t-1} + \eta_{c_t} \\
&\quad (-0.139) \quad (-0.298) \quad (0.105) \quad (0.135) \quad (0.066) \\
\Delta i_{t-1} &= x.xx k_{t-1} + x.xx n_{t-1} + x.xx \lambda_{t-1} + x.xx g_{t-1} + x.xx \mu_{t-1} + \eta_{i_t} \\
&\quad (-0.040) \quad (1.854) \quad (-0.316) \quad (-0.202) \quad (0.167)
\end{aligned} \tag{24}$$

Non integrated forcing variables

When assuming that forcing variables are not integrated, the following VAR in levels was fitted to the data (numbers in brackets are estimated parameters, plain numbers are those implied by the structure of the model and the calibration):

$$\begin{aligned}
y_{t-1} &= x.xx k_{t-1} + x.xx n_{t-1} + x.xx \lambda_{t-1} + x.xx g_{t-1} + x.xx \mu_{t-1} + \eta_{y_t} \\
&\quad (-0.142) \quad (0.705) \quad (-0.065) \quad (0.280) \quad (0.237) \\
c_{t-1} &= x.xx k_{t-1} + x.xx n_{t-1} + x.xx \lambda_{t-1} + x.xx g_{t-1} + x.xx \mu_{t-1} + \eta_{c_t} \\
&\quad (-0.139) \quad (-0.298) \quad (0.105) \quad (0.135) \quad (0.066) \\
i_{t-1} &= -0.004 k_{t-1} + x.xx n_{t-1} + 0.115 \lambda_{t-1} - 0.265 g_{t-1} + 0.503 \mu_{t-1} + \eta_{i_t} \\
&\quad (-0.040) \quad (1.854) \quad (-0.316) \quad (-0.202) \quad (0.167)
\end{aligned} \tag{25}$$

7. Conclusions

This paper tested the performance of six business cycles models when replicating the salient features of the Chilean data. The Chilean economic cycle provides an interesting case to study because, while it presents similar characteristics to cycles in developed economies, it also displays important idiosyncratic features. Common characteristics allow us to compare our results to those of developed economies, idiosyncracies put RBC models to test and make for interesting analysis.

The models have been evaluated, as customary in this literature, with regards to their capacities in reproducing deviations from their long-run trend, correlations with GDP, and phase shifts. When applied to Chilean data the simplest RBC model is capable of reproducing a number of real side characteristics of the Chilean economy, but fails to provide a satisfactory result in consumption and hours worked, where it displays too little and too much variation, respectively. This suggests that the model needs to be extended to include additional restrictions. We follow Hansen (1985) when including labor market rigidities and McGrattan (1994b) to include government expenditures.

In summary, these results suggest that (1) business cycles models are able to replicate much of the observed fluctuations of the real side of the economy, (2) in comparative terms, the introduction of government expenditures is able to explain substantially more of economic fluctuations than labor market rigidities, and (3) replicating the fluctuations in consumption requires to place additional constraints to the optimizing behavior of agents contained in our model.

To improve on our understanding of business fluctuations in Chile, we extend the previous model to include monetary features, which are modeled using cash in advance restrictions. The model successfully replicates most of the features of monetary aggregates, price levels, and inflation. As intended, money constraints also allow us to replicate the volatility of consumption and its correlation with output. Nevertheless, the model produce labor market equilibrium solution which does not match the data adequately.

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Appendix