

A Simple Model for Inflation Targeting in Brazil

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Based on a 6 equation model by Haldane and Battini (1999), we estimated a Phillips and an IS equations for Brazil after the Real Plan, in order to study the transmission mechanism of the monetary policy. The results show that interest rate affects output gap with a lag of one quarter and output is positively related with inflation with one lag only. The devaluation of the nominal exchange rate has also a contemporaneous effect in inflation. We also made stochastic simulations in order to depict the inflation and output gap volatility loci under alternative Taylor-type rules and under an optimal rule, which minimizes a loss function that depends on a weighted average of inflation and output gap variances. The stochastic simulation showed that output gap variance is more sensitive to the weights given in the loss function, compared to the variance in inflation. It also showed that optimization procedures involving more than 6 periods are inefficient and the most efficient frontier involves horizons between 2 and 4 periods. Finally, sub-optimal but simple rules, like Taylor type rules can perform as well as the optimal ones, depending on the parameters chosen and on the preferences of the Central Bank.

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1- Introduction

Brazil adopted a formal inflation-targeting regime in June 1999, six months after the switch in the exchange rate regime to a floating system². The main feature of the new monetary policy is to look at (expected) future inflation to decide the current interest rate. According to the most usual transmission mechanism, the forward-looking behavior is due to the lag that interest rate affects inflation through aggregate demand³.

Many Central Banks that adopted this new framework in the nineties were able to control inflation with more transparency and credibility than before. As the name suggests, in an inflation-targeting regime, the Central Bank main objective is to keep inflation within a predefined band. This contrasts with other nominal anchors, like exchange rate and monetary targets. Although there is a theoretical support for a relationship between monetary aggregates and inflation, empirical evidence shows that such relationships are not stable.

Among many papers, which present inflation targeting models and alternative interest rate rules we focus on three, one from Haldane and Battini (1999) Levin, Wieland and Williams (1999) and the other from Taylor (1999). Haldane and Battini (1999) emphasize the importance of a forward-looking interest rate rule, which performs almost as good as an optimal rule, even if the output variable is not directly included in the loss function to be minimized. The authors also summarize in a 6 equation model the most important features of the inflation target framework.

Levin, Wieland and Williams (1999) exhaust even more the subject, when they took four very extensive models and compare a wide set of policy rules and they conclude that:

“even in large models with hundreds of state variables, three variables (the current output gap, a moving average of current and lagged inflation rates, and lagged ‘interest rates’) summarize nearly all the information relevant to setting the ‘interest rate’ efficiently.” (page 31).

² England and Sweden are examples of countries that also introduced inflation targeting systems after a change in the exchange rate regime. Inflation targeting is also used in New Zealand, Australia, Chile, Israel and Spain.

³ References about the transmission mechanism include King (1997), Ball (1999) and Svenson (1998) .

Taylor (1999) summarizes the common features of the models that deal with inflation targeting. All of them are stochastic, dynamic and economy-wide. He also pointed out that :

“even the larger models can be described conceptually as “three relationship” system.(one relationship being the policy rule). Equation relating consumption, investment and net exports to interest rate and the exchange rate combine to form an IS block of equation; wage and price setting with exchange rate pass through combine to form a price adjustment block of equation.”

Taylor (1999) concludes that using more complicated ways to encompass the monetary transmission mechanism have a limited effect on the results of a simple policy rule. Another important conclusion is that more complex policy rule that incorporate inertial factors are more dependent on rational expectation assumptions.

In the present paper, the main objective is to estimate an IS and a Phillips equations for Brazil in order to simulate the effects of different interest rate rules on the variance of inflation and output gap. The benchmark is the optimal interest rate rule, obtained by minimizing a loss function, which is a weighted average of the variance of inflation and output gap.

The next section presents the Haldane-Battini model and the structure of the IS and Phillips equations. Section 3 contains the estimations of the 2 equations, section 4 shows the results of the stochastic simulations for optimal and alternative Taylor-type rules. The last section presents the conclusive remarks. The stochastic simulation showed that output gap variance is more sensitive to the weights given in the loss function, compared to the variance in inflation. It also showed that optimization procedures involving more than 6 periods are inefficient and the most efficient frontier involves horizons between 2 and 4 periods. Finally, sub-optimal but simple rules, like Taylor type rules can perform as well as the optimal ones, depending on the parameters chosen and on the weights given in the loss function. This is a similar result to the one found in Levin, Wieland and Williams (1999) and is good news in the sense that the monetary authority may be able to adopt simpler rules with few consequences in terms of generating excessive output and inflation volatility.

2- A model for the transmission mechanism using output gap

2.1 Haldane and Battini Model

As a starting point we used a small, open-economy, log-linear rational expectations macro-model found in Haldane and Battini (1999), which has 6 equations:

$$y_t - y_t^* = \alpha_1 y_{t-1} + \alpha_2 y_{t+1} + \alpha_3 [i_t - E(\pi_{t+1})] + \alpha_4 (e_t + p_t^c - p_t^{cf}) + \varepsilon_{1t} \quad (1)$$

$$m_t - p_t^c = \beta_1 y_t + \beta_2 i_t + \varepsilon_{2t} \quad (2)$$

$$E_t(e_{t+1}) = e_t + i_t - i_t^f + \varepsilon_{3t} \quad (3)$$

$$p_t^d = 1/2[w_t + w_{t-1}] \quad (4)$$

$$w_t - p_t^c = \chi_0 [E_t(w_{t+1}) - E_t(p_{t+1}^c)] + (1 - \chi_0)[w_{t-1} - p_{t-1}^c] + \chi_1 (y_t - y_t^*) + \varepsilon_{4t} \quad (5)$$

$$p_t^c = \phi p_t^d + (1 - \phi)e_t \quad (6)$$

Equation (1) is a usual IS curve, where the output gap depends negatively on the real interest rate and positively on the real exchange rate. Equation (2) is a regular LM curve, in which money demand depends on nominal interest rate and output. Equation (3) is an uncovered interest parity, that does not include a risk premium variable.

Equation (4) and (5) are the supply side. Equation (4) is a mark-up over weighted average contract wages. Equation (5) is a wage contracting equation. The lag/lead weights sum up to one to generate a vertical long-run Phillips curve. The output gap also is included in this equation.

Equation (6) is consumption price index, depending on the domestic goods and imported goods.

After some manipulation in (4)–(6), the authors come with a reduced-form Phillips curve of the model:

$$\pi_t = \chi_0 E_t(\pi_{t+1}) + (1 + \chi_0)\pi_{t-1} + \chi_1 (y_t - y_{t-1}) + \mu[(1 - \chi_0)\Delta c_t - \chi_0 E_t \Delta c_{t+1}] + \varepsilon_5 \quad (7)$$

where ϵ is the real exchange rate. There is a term for the output gap (which is the most important to explain the interest rate transmission mechanism), and the weighted backward and forward-looking inflation terms represent the inflation persistence. The last two terms are the exchange rate pass-through.

2.2 Two Equation Model

This paper is going to work with an even simpler model, with three equations: an IS curve, a Phillips curve; and an equation for nominal exchange rate:

$$y_t - y_t^* = \beta_1 r_{t-1} + \beta_3 (y_{t-1} - y_{t-1}^*) + \beta_2 p_{t-1} + \beta_3 \Delta c_{t-1} + \epsilon_t \quad (8)$$

$$\pi_t = \alpha_1 \pi_{t-1} + \alpha_2 y_{t-1} + \alpha_3 (e_t - e_{t-1}) + \eta_t \quad (9)$$

$$e_t = e_{t-1} + \upsilon_t \quad (10)$$

Equation (8) is an open economy IS curve, where output gap ($y - y^*$) depends on its own lags, on the lagged real interest rate (r), on the lagged real exchange rate first difference (Δc), on the lagged fiscal deficit (p) and on a demand shock (ϵ).

Equation (9) is an open economy Phillips curve. Inflation (π) depends on a lag of itself, on a lag of output gap, on a change in the nominal exchange rate (Δe) and on a shock (η). The exchange rate affects inflation directly through the price of imports and indirectly through its effects on the output gap.

Equation (10) is the exchange rate determination, which is assumed to follow a random walk.

In this model, the transmission mechanism from interest rate to inflation occurs only through the aggregate demand channel, and it takes two periods between the interest rate decision by the monetary authority and its effect on inflation. Observe that modeling exchange rate through a random walk precludes the often mentioned exchange rate channel, where exchange rate appreciates following increases in interest rates via UIP. Despite UIP is a more attractive way to model exchange rate because of

its theoretical appealing, predictions of future exchange rate using a random walk specification usually outperforms the predictions based on UIP⁴.

Given equations (8)-(10), the Central Bank chooses interest rate at each period and such decision will affect inflation from two periods ahead on. It is therefore necessary to specify the Monetary Authority decision mechanism. Among the rules used to describe the interest rate decision, Taylor type rules and optimal rules are the most popular.

Taylor rules are simple ones, where the interest rate is set as a linear function of to the current behavior of two variables: the wedge between observed inflation and the target and the output gap. Such rules can also be extended including other variables, like exchange rate or past interest rate, as well as using expected variables instead of observed ones.

Optimal rules are ones where the interest rate is the solution of a minimization of a loss function subject to some constraints that should characterize the transmission mechanism. For example, the Central bank chooses the real interest rate that minimize the following loss function:

$$\text{Minimize}_{r_t} L_t = \sum_{i=0}^{\infty} \delta^i \{ \lambda E_t [(\pi_{t+i} - \pi_{t+i}^*)^2] + (1-\lambda) E_t [(y_{t+i} - y_{t+i}^*)^2] \} \quad (11)$$

s.t. equations (8) - (10).

This is a typical problem in the literature of inflation targeting⁵, where λ is the weight the Central Bank places on the variance of inflation, δ is the discount factor, y is output and y^* is potential output.

3. Estimations

The estimation used quarterly data and the variables are expressed in their logarithms. Inflation is measured by the general price index IGP-DI/FGV. The output proxy is GDP, and the gap is estimated by GDP minus potential output. The estimation

⁴ See, for example, Wadhvani (1999).

⁵ See, for instance, Svensson (1995) and Svensson (1997/98).

of the potential output is a difficult and controversial task and we used the Hodrick-Prescott filter. The nominal exchange rate is the period average of the ask prices, and the nominal interest rate is the Selic rate, the equivalent to US Fed funds rate. The fiscal variable chosen is the federal government primary deficit, measured as percentage of the GDP. The real interest rate was calculated by deflating the nominal interest rate by the IGP-DI. In order to obtain the real exchange rate, the nominal one was multiplied by the American producer price index and divided by the IGP-DI.

The sample period ranged from 1992:4 to 1999:1 to estimate the IS equation and ranged from 1995:1 to 1999:2 to estimate the Phillips curve. We decided to restrict the data range for the Phillips equation, restricting the sample after the Real Plan in order to avoid high inflation contamination in the coefficients.

The choice to use quarterly data for such a short sample period poses the problem of too few degrees of freedom. On the other hand, quarterly data are less noisy than monthly ones. Besides, the transmission mechanism should take at least 2 quarters to occur. Consequently it would not be feasible to work with monthly data to forecast inflation one or two years ahead, as it is needed in an inflation target framework, since the standard errors of the forecasts would become very large. Finally, despite the few degrees of freedom, most of the estimated coefficients turned out to be statistically significant.

The equations were estimated using OLS. The IS curve does not contains a real exchange rate term neither a fiscal term in the present specification. But all the others term are very significant and with the expected sign. The real interest rate was included with 1 lag. A lag of the output gap, a pulse dummy for the Real plan and a dummy for the third quarter of 1998 were also included as explanatory variables. Equation (12) shows the coefficients and their respective t-statistics.

$$h_{t+1} = 0.02 - 0.39r_t + 0.73h_t + 0.38DReal_t - 0.22D983_t + \eta_{t+1} \quad (12)$$

N=29

R-squared =0.79

F-statistic = 22.8

The Phillips equation was estimated with one lagged term for inflation. The output gap entered the equation with one lag, and the devaluation of the nominal exchange rate seems to be contemporaneous, with a pass-through of 20%.

Equation (13) shows the coefficients and their respective t-statistics.

$$\pi_{t+2} = -0.006 + 0.80\pi_{t+1} + 0.31h_{t+1} + 0.20\Delta e_{t+2} + \varepsilon_{t+2} \quad (13)$$

(-1.63) (3.2) (2.26)

N = 18 R-Squared = 0.72 F-statistic = 12,31

We are imposing a long-run vertically in the Phillips equation, restricting to 1 the sum of the coefficients of lagged inflation and of nominal exchange rate variation. What means that any devaluation in the exchange rate will be completely passed through prices in the long run. Although we did not impose the restriction of no constant when estimating the parameters, this term was dropped in the simulations presented below.

The effect of interest rate on inflation is indirect and takes two periods to occur. So, the control lag is two quarters. A one percentage point increase in the real interest rate will affect negatively the output gap in 0,39 percentage point. Given that a decrease of 1 percentage point in the output gap reduces inflation by 0,31 percentage point, the final effect of the increase of 1 percentage point in interest rate will be a reduction of 0,12 percentage point in inflation in the short run. In the long run, taking into consideration the auto-regressive coefficients, the final effect would be a reduction in inflation of 0.6 pp.

In both equations we test the residuals for auto correlation. The correlograms do not show evidence of this problem. The test of the cross-correlation of the two equations residuals was also done and the result shows no evidence of that.

4. The Optimal Rule

The Central Bank is assumed to choose an interest rate path that minimize the following loss function:

$$\min_{\{i_{t+j}\}} \ell = \frac{\lambda}{2} \sum_{j=1}^{\infty} \rho^j E_t (\pi_{t+j} - \pi_t)^2 + \frac{(1-\lambda)}{2} \sum_{j=1}^{\infty} \rho^j E_t (h_{t+j})^2 \quad 6$$

⁶ Stella and Michkin (data) shows that if model parameters are uncorrelated with interest rate, the optimization problem above is the same as:

subject to (8)–(10), ie, the IS, Phillips and exchange rate equations, and to:

$$\dot{i} = \dot{i}_{t+1} = \dot{i}_{t+2} = \dots = \dot{i}_{t+n-1} = \dot{i} \quad (i)$$

where λ is the weight the Central Bank gives to inflation variance compared to output gap variance. When the value of λ is 0, the optimal rule puts all weight on the output. Conversely, when $\lambda=1$ means the Central Bank cares only about inflation variance.

Restriction (i) is equivalent to assume that, when setting interest rates at t , the Central Bank commits to keep the interest rate unchanged between period t and $t+T-1$. Observe that this procedure may be considered a myopic optimization problem because it only takes into account expected inflation and output gap T periods ahead, ignoring the effects of interest rate on inflation and output gap from period $T+1$ on. The use of such myopic approach, though, simplifies considerably the solution. Besides, we calculated optimal interest rates with T ranging from 2 to 8 periods and it could be concluded that it was inefficient for the Central Bank to take into consideration more than 6 quarters in the loss function.

The optimal interest rate is given by⁷:

$$i_t^* = \frac{- \left\{ \lambda \sum_{j=1}^n \rho^j (E_t \tilde{\pi}_{t+j} - \pi_t^*) a_{i,j} + (1-\lambda) \sum_{j=1}^n \rho^j E_t h_{t+j} b_{1,j} \right\}}{\lambda \sum_{j=1}^n \rho^j a_{i,j} (a_{i,j} + a_2) + (1-\lambda) \sum_{j=1}^n \rho^j b_{1,j} (b_{1,j} + b_2 + b_3)} \quad (14)$$

where $\tilde{\pi}_t$ and \tilde{h}_t are the components of inflation and output gap that do not depend on current and future interest rates.

From (14) one can see that, in order to calculate the optimal interest rate, it is necessary to have estimates of $E_t \tilde{\pi}_{t+i}$ and $E_t \tilde{h}_{t+i}$, $i \leq T$. The appendix shows that these variables depend on the realizations of π and h at time t , on the interest rate path from t

$$\min_{i_t} \ell = \frac{\lambda}{2} \sum_{j=1}^n \rho^j (E_t \pi_{t+j} - \pi_t^*)^2 + \frac{(1-\lambda)}{2} \sum_{j=1}^n \rho^j (E_t h_{t+j})^2$$

⁷ The derivation of equation (14) is shown in the appendix.

to $t+i-1$ and on expected exchange rate variation from t to $t+i$. At time t , only π_t and h_t are known. The interest rate path is the outcome of the optimization procedure. Therefore, it is still necessary to define the exchange rate variations path. This path was constructed assuming exchange rate follows a random walk. As it was explained before, a drawback of this hypothesis is the lack of response of exchange rate to interest rate decisions. However, an adequate way to incorporate such responses is beyond the scope of this paper and other simpler responses, like the widely used uncovered interest rate parity condition, may yield a worse fit than random walk.

Given the interest rate rule, a stochastic simulation was made in order to build an efficient frontier, showing the output gap and inflation variance for different values of λ and different time horizons. The result is shown in Chart 1 and will be commented below. The first step is to generate a series of simulated inflation and output. At period 1, both inflation and output gap are known and the Central Bank is assumed to choose an interest rate according to the optimization problem. At period 2, shocks on inflation, output gap and exchange rate hit the economy and a new interest rate is set. The errors are assumed to be normally distributed with mean zero and diagonal covariance matrix. The variances of inflation and output gap are the ones obtained in the regressions. Concerning the shock on exchange rate, it was imputed a standard deviation of 5%. This imputation was necessary because there were only two observations in the sample with a floating exchange rate regime. This procedure was repeated for 200 periods and the efficient frontier corresponded to the variance of inflation and output gap obtained in this simulation from period 50 on.

Chart 1 presents the trade-off between inflation and GDP output that is behind the selection of a specific λ and optimization periods. Along each line, the optimization period is held constant and λ varies from 1 (all weight given to inflation variance) to 0 (all weight given to output gap variance). For low values of λ , optimization taking into account only 2 periods ahead is the most efficient while for higher values of λ (ie, higher weight to inflation variance), the efficient frontier refers to optimization procedures with horizons of 3 and 4 quarters.

This result was to some extent surprising. It was already expected that for very short time horizons and high weight on inflation, output variance should high, given the

bigger sensitivity of output gap to interest rate. But we did not expect that output gap variance would reduce so sharply when the time horizon of the loss function increased only from 2 to 3 periods ahead. Table 1 shows that the bigger reduction in relative variances occurs exactly when the time horizon increases from 2 to 3 periods.

Chart 2 illustrates this point more clearly. Along each line in Chart 2, λ is held constant and the optimization horizon is varying from t+2 to t+8 periods. When $\lambda=1$, there is a slight increase in inflation variance is compensated by a substantial reduction in the variance of output gap, as the optimization horizon moves from t+2 to t+4 periods. In all cases, though, using more than 6 periods in the loss function is inefficient.

In order to compare the robustness of our model with other rules, we proceeded a test similar with the one done in Levin, Wieland and Williams (1998). The alternative rules follow the equation:

$$r_t = \rho r_{t-1} + \alpha(\pi_t - \pi^*) + \beta y_t \quad (15)$$

Table 2 shows different values for the coefficients above for alternative rules. Rule A is the traditional Taylor rule. All other rules preserve the stability condition that the coefficient α should be greater than 1⁸.

Table 2: Coefficients used in the Taylor-type rules

Coefficient	Rule		
	A	B	C
a	1.5	5	5
b	0.5	1.5	3

Chart 1a shows how these simple rules perform relative to the optimal ones. Actually, the traditional Taylor rule (A) presents a poor performance. As we increased the weights on both inflation and output gap variance, the performance improved. Rule

⁸ We conducted some simulations using different values for ρ , but the results were inferior to the ones presented in this paper.

C can be even considered a reasonable alternative (to the optimal) rule if the Central Bank has a strong bias against inflation variance.

5. Concluding remarks

The estimation of the IS and Phillips equation was a positive surprise, especially the last one, due to the small sample. In the IS equation, all the estimated coefficients presented the expected sign and the only which was not significant was the first difference of the real exchange rate. From the Phillips equation we see that the pass-through from nominal exchange rate depreciation to inflation (measure by IGP-DI) is around 15%. The monetary transmission mechanism from interest rate for inflation implied each percentage point increase in real interest rate would decrease inflation by 0.12 percentage point after two quarters and by 0.6 pp in the long run.

Regarding the optimal rules, the stochastic simulation showed that output gap variance is more sensitive to the weights given in the loss function, compared to the variance in inflation and that this sensitivity drops sharply when the horizon embodied in the loss function increases from 2 to 3 periods. In general, the optimal horizons ranged from 2 to 4 periods and optimization procedures involving more than 6 quarters were inefficient.

Finally, sub-optimal but simple rules, like Taylor type rules can perform reasonably well if the Central Bank has a strong bias against inflation variance and if it reacts more fiercely to the output gap and to deviations of inflation than the traditional Taylor rule suggests.

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Chart 1: Efficient Frontiers

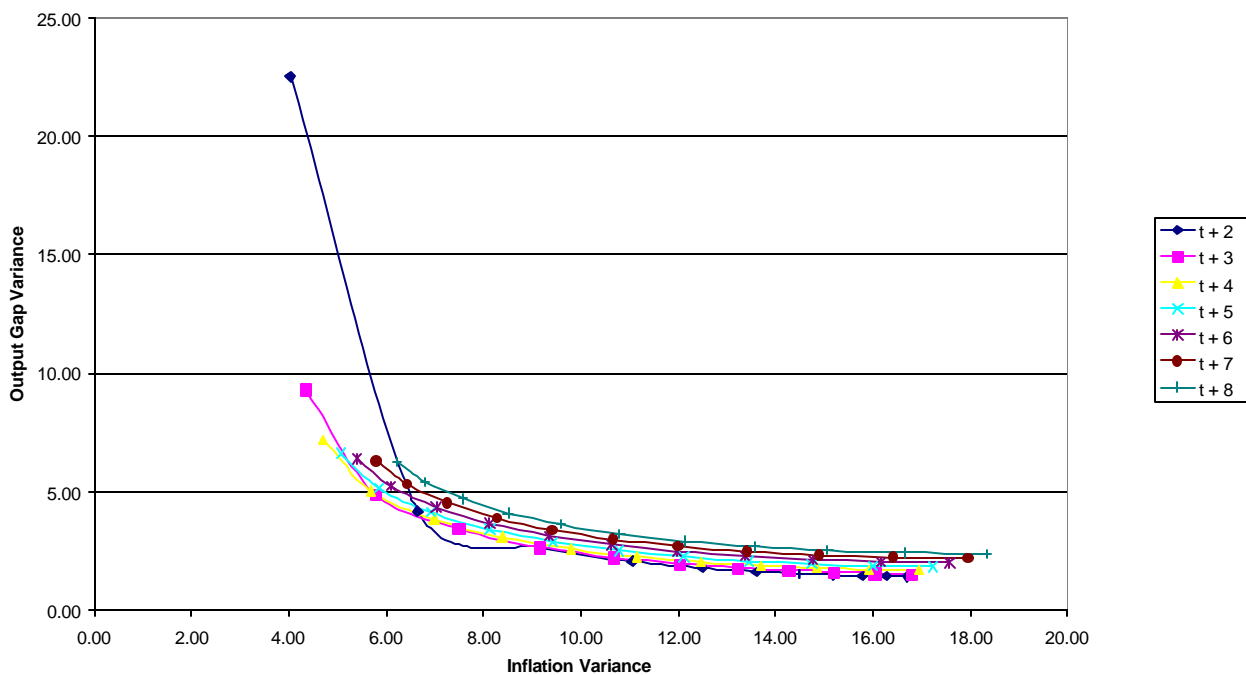


Chart 1a: Efficient Frontier with Alternative Taylor-Type Rules

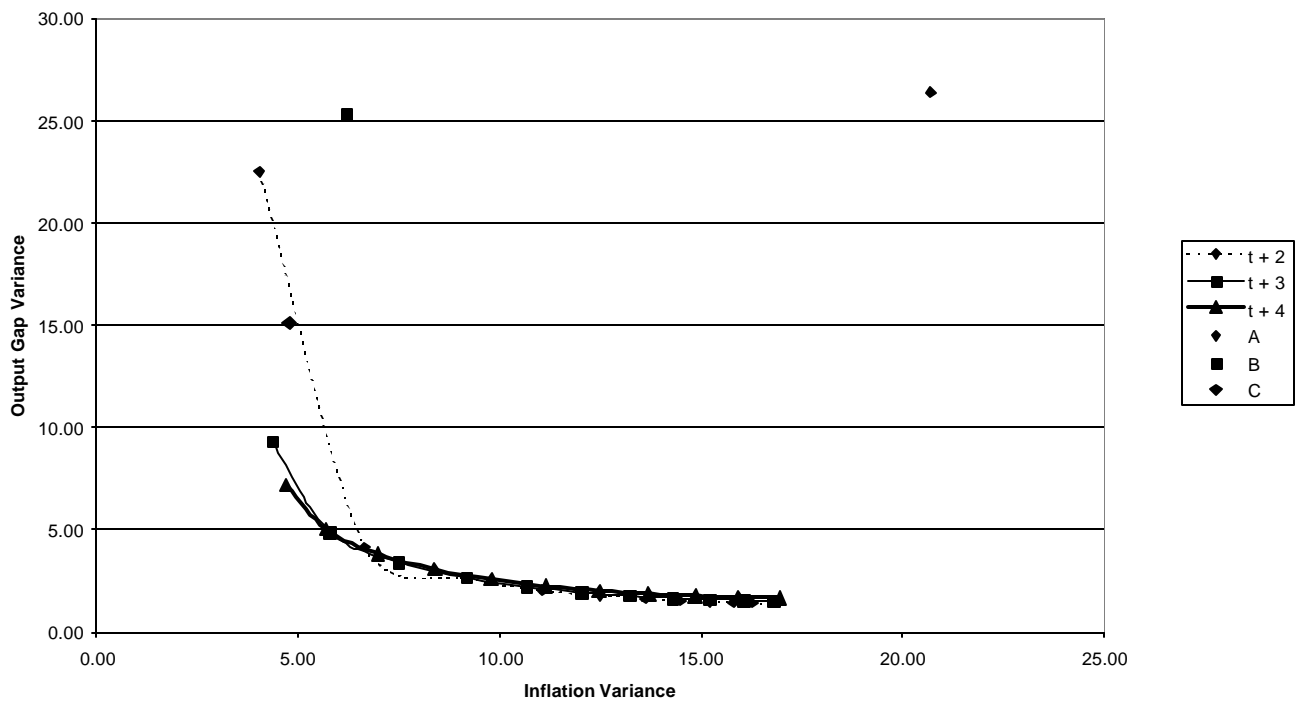


Chart 2: Inflation and Output Gap Variances by Lambda and Time Horizon

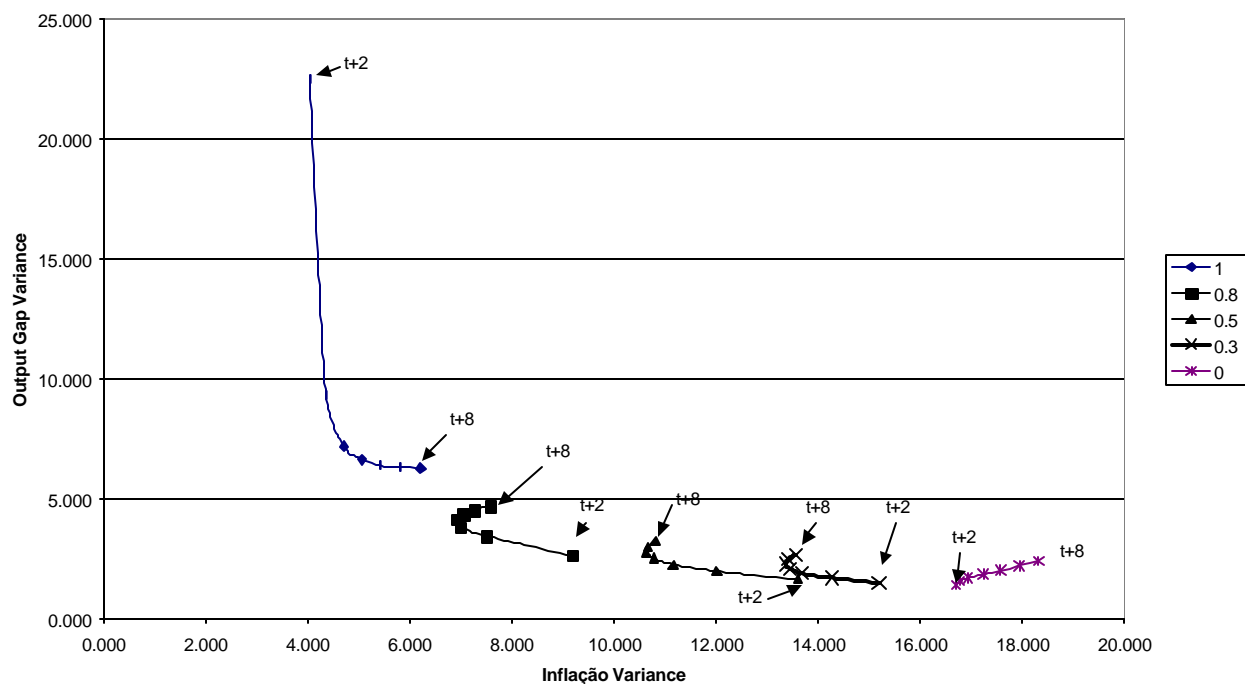


Table 1: Variance Ratios by Lambda and Time Horizon

Horizon	Variance	Lambda										
		1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0
t + 2	Inflation	1.0	1.6	2.3	2.7	3.1	3.4	3.6	3.8	3.9	4.0	4.1
	Output	15.9	2.9	1.9	1.5	1.3	1.2	1.1	1.1	1.0	1.0	1.0
t + 3	Inflation	1.0	1.3	1.7	2.1	2.5	2.8	3.0	3.3	3.5	3.7	3.9
	Output	6.0	3.2	2.2	1.7	1.4	1.3	1.2	1.1	1.0	1.0	1.0
t + 4	Inflation	1.0	1.2	1.5	1.8	2.1	2.4	2.7	2.9	3.2	3.4	3.6
	Output	4.3	3.0	2.3	1.8	1.5	1.3	1.2	1.1	1.1	1.0	1.0
t + 5	Inflation	1.0	1.2	1.4	1.6	1.9	2.1	2.4	2.7	2.9	3.2	3.4
	Output	3.6	2.8	2.2	1.9	1.6	1.4	1.2	1.1	1.1	1.0	1.0
t + 6	Inflation	1.0	1.1	1.3	1.5	1.7	2.0	2.2	2.5	2.7	3.0	3.3
	Output	3.2	2.6	2.2	1.8	1.6	1.4	1.2	1.1	1.1	1.0	1.0
t + 7	Inflation	1.0	1.1	1.3	1.4	1.6	1.8	2.1	2.3	2.6	2.8	3.1
	Output	2.9	2.4	2.1	1.8	1.5	1.4	1.2	1.1	1.1	1.0	1.0
t + 8	Inflation	1.0	1.1	1.2	1.4	1.5	1.7	2.0	2.2	2.4	2.7	3.0
	Output	2.6	2.3	2.0	1.7	1.5	1.3	1.2	1.1	1.1	1.0	1.0

Appendix

1) Derivation of the optimal rule

The Phillips and IS equations (equations x and y) can be written in matrix notation as:

$$(I) \quad \begin{bmatrix} \pi_t \\ h_t \end{bmatrix} = \begin{bmatrix} \alpha_0 & \alpha_3 & \alpha_4 \\ \beta_0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \Delta e_t \\ DS_t \end{bmatrix} + \begin{bmatrix} \alpha_1 & \alpha_2 \\ -\beta_1 & \alpha_3 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ h_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \beta_1 \end{bmatrix} i_{t-1} + \begin{bmatrix} \varepsilon_\tau \\ n_\tau \end{bmatrix}$$

or:

$$X_t = A_0 K_t + A_1 X_{t-1} + A_2 i_{t-1} + E_t$$

where X is the vector of inflation and output gap; A_i are the coefficient matrices; and E is the error vector.

Through recursive substitution and taking expectations at time t , we find:

$$(II) \quad E_t X_{t+n} = \sum_{j=1}^n A_1^{n-j} A_0 E_t K_{t+j} + A_1^n X_t + A_1^{n-1} A_2 i_t + A_1 A_2 i_{t+n-2|n \geq 3} + A_2 i_{t+n-1|n \geq 2}$$

Now, define:

$$E_t \tilde{\pi}_{t+n} = [1 \quad 0] \left[\sum_{j=1}^n A_1^{n-j} A_0 E_t K_{t+j} + A_1^n X_t \right] \text{ as the expectation taken at time } t \text{ of the}$$

component of inflation at $(t+n)$ that does not depend on the interest rate decision at time t .

Also, define:

$$a_{1,n} = [1 \quad 0] A_1^{n-1} A_2, \text{ as the coefficient of } i_t \text{ on expected inflation at } (t+n);$$

$$a_2 = \begin{cases} [1 \quad 0] A_1 A_2 & \text{if } n \geq 3 \\ 0 & \text{otherwise} \end{cases}, \text{ as the coefficient of } i_{t+n-2} \text{ on expected inflation at}$$

$(t+n)$

$$a_3 = [1 \quad 0] A_2 = 0, \text{ as the coefficient of } i_{t+n-1} \text{ on expected inflation at } (t+n)$$

Similarly, define:

$$E_t \tilde{h}_{t+n} = [0 \quad 1] \left[\sum_{j=1}^n A_1^{n-j} A_0 E_t K_{t+j} + A_1^n X_t \right] \text{ as the expectation taken at time } t \text{ of the}$$

component of output gap at time $(t+n)$ that does not depend on the interest rate decision at time t .

$b_{1,n} = [0 \ 1]A_1^{n-1}A_2$, as the coefficient of i_t on expected output gap at $(t+n)$;

$$b_2 = \begin{cases} [0 \ 1]A_1 A_2 & \text{if } n \geq 3 \\ 0 & \text{otherwise} \end{cases}, \text{ as the coefficient of } i_{t+n-2} \text{ on expected output gap at}$$

$(t+n)$

$$b_3 = \begin{cases} [0 \ 1]A_2 & \text{if } n \geq 2 \\ 0 & \text{otherwise} \end{cases}, \text{ as the coefficient of } i_{t+n-1} \text{ on expected output gap at}$$

$(t+n)$

Therefore:

$$(IV) E_t \pi_{t+n} = E_t \tilde{\pi}_{t+n} + a_{1,n} i_t + a_{2,n} i_{t+n-1}, \text{ and}$$

$$(V) E_t h_{t+n} = E_t \tilde{h}_{t+n} + b_{1,n} i_t + b_{2,n} i_{t+n-2} + b_{3,n} i_{t+n-1}$$

The optimization problem is defined as:

$$\min_i \ell = \frac{\lambda}{2} \sum_{j=1}^n \rho^j E_t (\pi_{t+j} - \pi_t^*)^2 + \frac{(1-\lambda)}{2} \sum_{j=1}^n \rho^j E_t (h_{t+j})^2 \quad ^9$$

where ρ is the discount factor and λ is the weight given to inflation variance in the loss function.

Needs to assume $i_t = i_{t+n-1} = i_{t+n-2}$

Substituting (IV) and (V) into the loss function and solving for the optimal interest rate, i_t^* , we find:

$$i_t^* = \frac{- \left\{ \lambda \sum_{j=1}^n \rho^j (E_t \tilde{\pi}_{t+j} - \pi_t^*) a_{i,j} + (1-\lambda) \sum_{j=1}^n \rho^j E_t h_{t+j} b_{1,j} \right\}}{\lambda \sum_{j=1}^n \rho^j a_{i,j} (a_{i,j} + a_2) + (1-\lambda) \sum_{j=1}^n \rho^j b_{1,j} (b_{1,j} + b_2 + b_3)}$$

⁹ Estrella and Mishkin (1999) shows that if model parameters are uncorrelated with interest rate, the optimization problem above is the same as:

$$\min_i \ell = \frac{\lambda}{2} \sum_{j=1}^n \rho^j (E_t \pi_{t+j} - \pi_t^*)^2 + \frac{(1-\lambda)}{2} \sum_{j=1}^n \rho^j (E_t h_{t+j})^2$$