Argentina's Credit Risk Indicator within a portfolio model for optimal credit risk capital requirements

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1. Introduction

Argentina's prudential regulations on bank solvency include a credit risk capital requirement for banks' "banking book", which departs somewhat from Basle recommendations. According to these, the requirement should be a minimum of 8% of risk weighted assets, where the weights can vary between 0 and 100% according to certain broad categories of assets and with some discretion for each country's regulatory authority.

The general basic requirement in Argentina is 11,5% but the Basel style requirement is multiplied by a bank specific "CAMEL factor" that varies between 0,97 and 1,125 according to the bank's CAMEL rating assigned by the Superintendence of Financial Institutions in such a way that riskier banks command a higher requirement. Additionally, the Basel style risk weight of each loan must be multiplied by a loan specific "Credit risk indicator (CRI)" (or factor) that is increasing with the spread between the loan rate and a certain basic rate and the loan rate.[2] This paper addresses the logic behind the CRI by placing it within a CAPM portfolio model for credit risk (Rochet (1992) that specifically accounts for the regulator's objective of placing a cap on bank's insolvency risk.

Basel style risk weights have often been criticized for ignoring the benefits of portfolio diversification and for not distinguishing between loans granted to debtors with different creditworthiness (say AAA firms versus A firms). Most

[1] The opinions expressed in this paper are the author's and do not necessarily reflect those of the B.C.R.A. The author gratefully acknowledges comments by participants in the B.C.R.A. seminar where the first version of the paper was discussed, as well as comments by Juan Pablo Nicolini to the version presented in the Cuartas Jornadas de Economía e Internacional de la Universidad de La Plata (May 6, 1999). Special thanks are due to María Elena Grubisic for her very careful revision of the final version.

[2] Until the end of 1999, the CRI started with a value of 0.8 for loans with rates not higher than 2 percentage points above Argentina's "prime rate" granted to clients with external ratings at least as high as Argentine government securities. Apart from this case, the factor varied between 1 (for loans with rates up to 18% p.a.) and 7 according to the level of the loan rate. For example, loans with rates between 18% and 21% had a factor of 1.1, etc. and at the other extreme loans with rates higher than 78% p.a. had a factor of 7. As of January 2000 this schedule has been modified to reflect more adequately the difference in administrative and loan loss costs of two groups of loans. Personal, credit card and accorded overdrafts, which have higher costs, have a schedule which starts with a factor of 1 for loans with rates up to 26% p.a. and then gradually increases to 7 for rates higher than 86% p.a. The schedule for other classes of loans starts with a factor of 1 for loans with rates up to 16% p.a. and increases to 7 for rates higher than 76% p.a. The special 0.8 factor remains in both cases as before.

loans in the Basel framework tend to have unitary risk weights unless they are backed by significant collateral. Within this framework there is no way of discriminating between banks with different levels of prudence in their allocation of loans. The recent consultative paper issued by the Basel Committee on Banking Supervision explicitly recognizes that “the current risk weighting of assets results, at best, in a crude measure of economic risk, primarily because degrees of credit risk exposure are not sufficiently calibrated as to adequately differentiate between borrowers’ differing default risks”.[3]

One way of addressing these deficiencies starts by recognizing that the loan interest rate reflects, among other things, the loan’s risk. Hence, using the loan rate to construct the regulatory risk weight can be *prima facie* appealing. However, the loan rate also reflects various other expected costs as well as the bank’s market power, so it is not very clear how to use the risk information that the rate contains. However, since entry in Argentina’s banking system is quite unrestricted and fierce competition prevails, I choose to ignore imperfect competition and assume that the banking system is (perfectly) competitive.

This paper delves into the microeconomic foundations for a CRI such as is used in Argentina by using portfolio theory. Perfectly competitive banks maximize von-Neumann-Morgenstern utility subject to capital requirements in a single period under multivariate normally distributed loan losses. Following Rochet (1992) it is shown that in order to place a cap on the bank’s failure probability the optimal risk weights should be proportional to the spread between the loan rate and expected costs. This is precisely what the “credit risk indicator” can approximate, if adequately constructed. In particular, the schedule should discriminate between loans (or categories of loans) that have different expected administrative or loan loss costs. If not, as in the basic Basel style risk weights, the regulatory incentive given to the bank may be precisely the opposite of the intended incentive: banks may engage in regulatory arbitrage by allocating very risky loans that have relatively low “risk weights” and thus end up having a higher probability of failure than if there were no minimum capital requirements.

[3] Basel Committee on Banking Supervision (1999), page 9.

2. The model

Bank managers act as unlimited liability risk averse portfolio managers that invest capital and deposits in risky loans and risk-less reserves under competitive conditions. Capital and deposits are exogenous and banks may raise funding in the risk-less inter-bank market instead of investing there.[4] There is a single period, after which deposits are returned to depositors with interest. To simplify, deposits earn the risk-less rate.

The bank earns a random return on its loans due to the existence of non-performing loans. It maximizes a Von Neumann-Morgenstern utility function that is increasing with end of period capital. For analytic tractability, I make the unrealistic assumption that loan losses net of recovery follow a multivariate normal distribution.

As in the "capital asset pricing model" (CAPM), when unrestricted by regulations banks determine the structure of their investments as a combination of risk-less reserves and the market portfolio of risky loans. Homogeneity of banks' perceptions of the return-risk characteristics of loans is assumed. These characteristics are given by the variance-covariance matrix of net loan losses and by the vector of loan interest rate spreads (above the sum of the risk-less rate, expected net loan losses and the administrative costs of loans).

All interest rates are market determined and non-stochastic. There are n types of loans, classified by their risk-return characteristics, that command loan rates $r_1 .... r_n$, Banks may also hold reserves in or fund themselves at the risk-less rate r_o . There are administrative and operating costs that banks homogeneously allocate to loan types according to $g_1 .... g_n$ and other costs given by a fraction g_o of initial capital (K). Default costs net of recovery are stochastic and given by $d_1^{\sim} .... d_n^{\sim}$ [5], which are multivariate normally distributed with expected values $d_1 .... d_n$, standard deviations $\sigma_1 .... \sigma_n$, and covariances σ_{ij}. Therefore, the return on loan i is $x_i^{\sim} = r_i - g_i - d_i^{\sim}$ and has expected value $x_i = r_i - g_i - d_i$ and standard deviation σ_i , while its covariance with loan j is σ_{ij}. Also, the spread between the return and the risk-less rate is

[4] Minimum reserve requirements are not explicitly addressed in this paper.

[5] A tilde (˜) in front of a variable means that the variable is stochastic. The same symbol without the tilde denotes the mean value of the variable.

$_i^{\sim} \equiv x_i^{\sim} - r_o = r_i - g_i - d_i^{\sim} - r_o$.

Therefore, the variances $V(s_i^{\sim})$, $V(x_i^{\sim})$, $V(d_i^{\sim})$ are all equal and σ_{ij} is also the covariance between $s_i^{\sim}$ and $s_j^{\sim}$. By assumption, $d^{\sim}$, and therefore $s^{\sim}$, are normally distributed:
$d^{\sim} \sim N(d, M)$, $s^{\sim} \sim N(s, M)$

where M is the variance-covariance matrix.

At the initial instant the bank chooses the composition of its portfolio of reserves and loans R, L_1.... L_n, given its liabilities D+K, its costs $g_0, g_1 \ldots g_n$, and the probability distribution of returns $x_1^{\sim}$.... $x_n^{\sim}$. At the end of the period the bank is liquidated, deposits are returned with interest to depositors and owners receive the difference between assets and liabilities.

Due to the balance sheet identity:

(1) $$R = D + K - \Sigma_i L_i.$$

Whereas reserves may be positive or negative, it is assumed that banks cannot have short positions in loans ($L_i \geq 0$ for all i).

End of period capital is:

$$K_1^{\sim} = \Sigma_i L_i (1+ x_i^{\sim}) + (D + K - \Sigma_i L_i)(1+ r_o) - D(1+ r_o) - Kg_o =$$

$$= K(1+ r_o - g_o) + \Sigma_i L_i (x_i^{\sim} - r_o).$$

Defining "adjusted" initial capital as

$$K' = K(1 + r_o - g_o),$$

end of period capital may be expressed as:

(2) $$K_1^{\sim} = K' + \Sigma_i L_i s_i^{\sim}.$$

Therefore, the expected value and variance of $K_1^{\sim}$ are:

(3) $$\mu \equiv E(K_1^{\sim}) = K' + \Sigma_i L_i s_i$$

(4) $$\sigma^2 \equiv V(K_1^{\sim}) = \Sigma_i L_i^2\sigma_i^2 + \Sigma_{i\neq j} L_iL_j\sigma_{ij}.$$

Using vector notation, the last three expressions become

(2') $$K_1^{\sim} = K' + L's^{\sim}.$$

(3') $$\mu = K' + L's$$

(4') $$\sigma^2 = L'ML,$$

where vectors are column vectors and the apostrophe denotes transposition, L is the vector of loans, s is the vector of spreads and M is the (positive definite) variance-covariance matrix. By definition of x and s,

$$s \equiv x - r_o u \equiv r - g - d - r_o u,$$

where u is the vector of ones and x, r, d, g are the vectors of returns, loan interest rates, expected net loan losses and other (allocated) loan costs, respectively. Because $K_1^{\sim}$ is a linear function of normal variables, it is also normally distributed and completely characterized by its mean (3') and variance (4').

3. The unregulated bank's decision problem

The unregulated bank maximizes expected utility

(5) $$E(u(K_1^{\sim}))$$

where u(.) is a strictly quasi-concave and increasing Von Neumann-Morgenstern utility function. Under these assumptions the expected utility (5) and the probability of failure ($Prob(K_1^{\sim} < 0)$) are functions of μ and σ. Expected utility is:

(6) $$E(u(K_1^{\sim})) = \int_{-\infty}^{-\infty} u(\mu + \sigma y^{\sim})\phi(y)dy^{\sim} \equiv U(\mu,\sigma),$$

where $\phi(y)$ is the standardized normal density function.[6] The resulting function $U(\mu,\sigma)$ is increasing in μ and decreasing in σ as well as concave. Notice that under unlimited liability the bank maximizes expected utility over the whole set of states, including those that imply bank failure.

Due to normality, the probability of failure is:

$$(7) \qquad Prob(K_1^{\sim} < 0) = Prob((K_1^{\sim} - \mu)/\sigma < - \mu/\sigma) = $$
$$= Prob(y^{\sim} < - \mu/\sigma) = \Phi(- \mu/\sigma)$$

where $\Phi(.)$ is the cumulative standardized normal. Thus, the probability of failure is decreasing in μ and increasing in σ.

The bank's decision problem is to find a vector of loans L that maximizes utility subject to (3') and (4'):

$$(8) \qquad \max_{L} \; U(\mu(L), \sigma(L)) = U(K' + L's, (L'ML)^{1/2}).$$

The first order condition is[7]

$$(9) \qquad U_{\mu}s + U_{\sigma} (ML)/\sigma = 0$$

which gives the optimal loan vector:

$$(10) \qquad L^* = (1/\theta)M^{-1}s$$

where $\theta \equiv -U_{\sigma}/\sigma U_{\mu}$ is the bank's Arrow-Pratt coefficient of (absolute) risk aversion.[8] Therefore, the bank's optimal reserves are given by:

$$(11) \qquad R^* = D + K - u'L^* = D + K - (1/\theta)uM^{-1}s.$$

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[6] As $K_1^{\sim}$ is normally distributed with mean μ and standard deviation σ, expected utility is $\int u(K_1^{\sim})N(K_1^{\sim};\mu,\sigma)dK_1^{\sim}$, where N(.) is the density function of the normal distribution. With the change of variable $y^{\sim} = (K_1^{\sim} -\mu)/\sigma$, (6) is obtained.

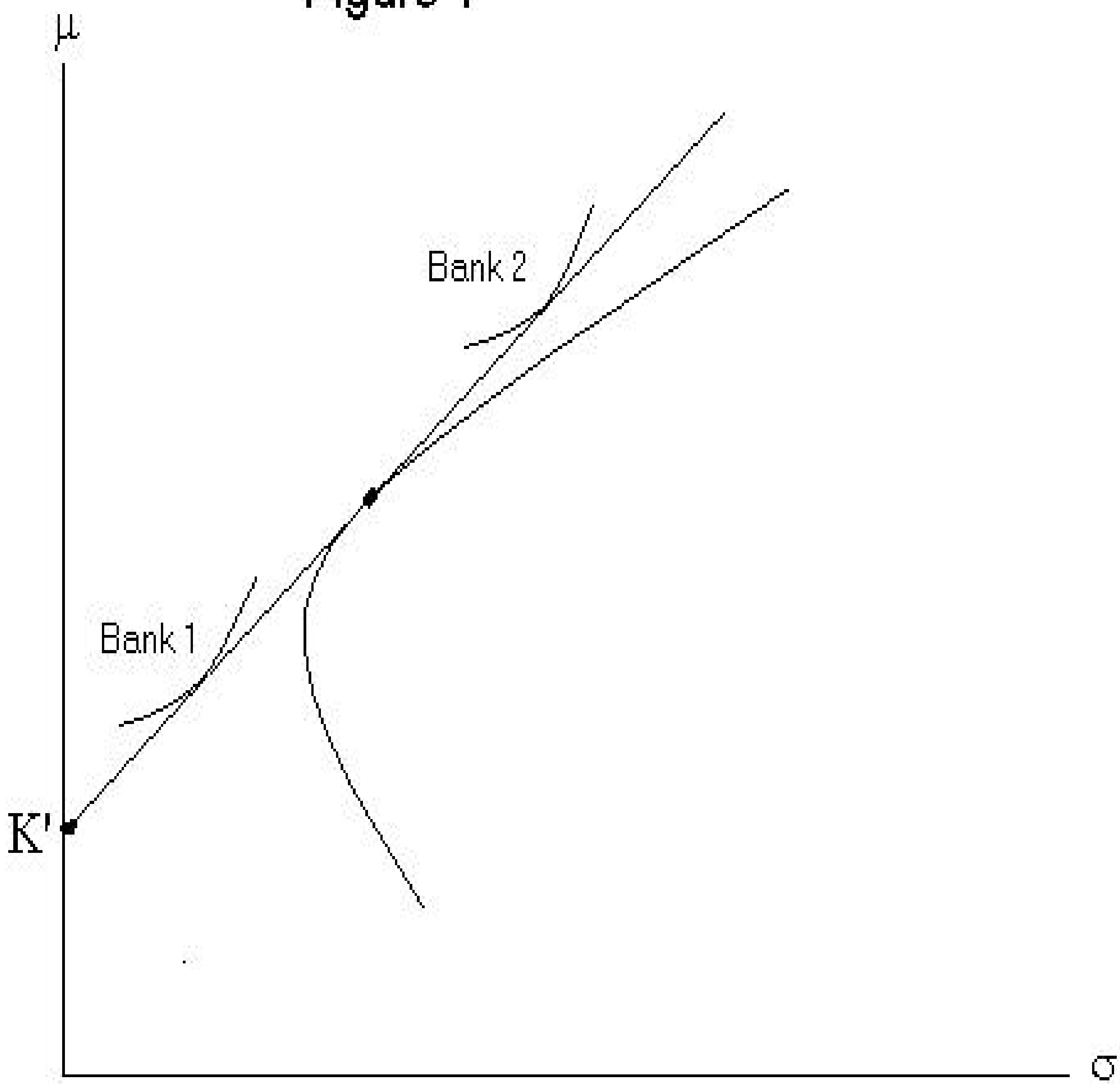
[7] The second order conditions are satisfied since M is positive definite (Cfr. Goldberger (1964), proposition (3.55) for the proof that a covariance matrix is non-negative definite and positive definite if and only if the $d_i^{\sim}$ are linearly independent which is here assumed).

[8] Assuming u(.) is of the constant absolute risk aversion (CARA) family (e.g. the negative exponential utility function $u(z) = A – Be^{-\theta z}$), θ is constant.

Given that all banks face the same loan interest rates and costs (and no transaction costs), they differ in their optimal loan vector only up to a scalar multiple that depends on their risk aversion coefficient.

Inserting (10) in (3) and (4), the optimal mean and standard deviation of $K_1^{\sim}$ for the unregulated bank are:

Figure 1



(12) $\mu^* = K' + (1/\theta)b$

(13) $\sigma^* = (1/\theta)\sqrt{b}$

where

(14) $b \equiv s'M^{-1}s.$

Figure 1 shows the optimal μ^* and σ^* for Banks 1 and 2, the first of which is more risk averse.

From (12) and (13) it is readily seen that for any bank the following linear relation between μ^* and σ^* holds:

(15) $\mu^* - K' = \sqrt{b}\, \sigma^*$

which means that the optimal risk-return combination of every bank lies on the efficiency frontier, which is the half-line that rises to the right from K' on the vertical axis. The bank's risk aversion coefficient determines its location on the efficiency frontier. The parabola represents the efficiency frontier when there is no risk-free asset.[9]

Also, inserting (12) and (13) in (7) gives the probability of failure as

(16) $\text{Prob}(K_1^{\sim} < 0) = \Phi(- \sqrt{b}\, [(\theta K'/b) + 1]).$

The probability of failure thus varies inversely with initial capital (contained in K') and the degree of risk aversion. Therefore, the greater probability of failure of less risk averse banks could in principle be corrected by having them constrained by a capital requirement. Figure 2 shows that an increase in K with an equivalent reduction in D shifts the market line upwards so that no matter what the bank's risk aversion is, the ratio μ^*/σ^* increases (and thus the probability of failure decreases).[10]

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[9] The Appendix derives the equations that define these frontiers.

[10] The change in K has the additional effect of moving the minimum variance frontier upwards, not shown in the graph for simplicity. The proof of this can be found in section 3 of the Appendix. As mentioned, if the utility function belongs to the CARA family θ is constant. Also, it is easy to show that the slope of any indifference curve is $-U_\sigma/U_\mu = \theta\sigma$, which is independent of μ. Therefore, if the efficiency frontier shifts upwards the new tangency point is directly above the old one.

4. The bank's decision problem under minimum capital requirements

Assume now that the regulatory authority imposes a minimum capital requirement defined as a fraction k of risk-weighted assets:

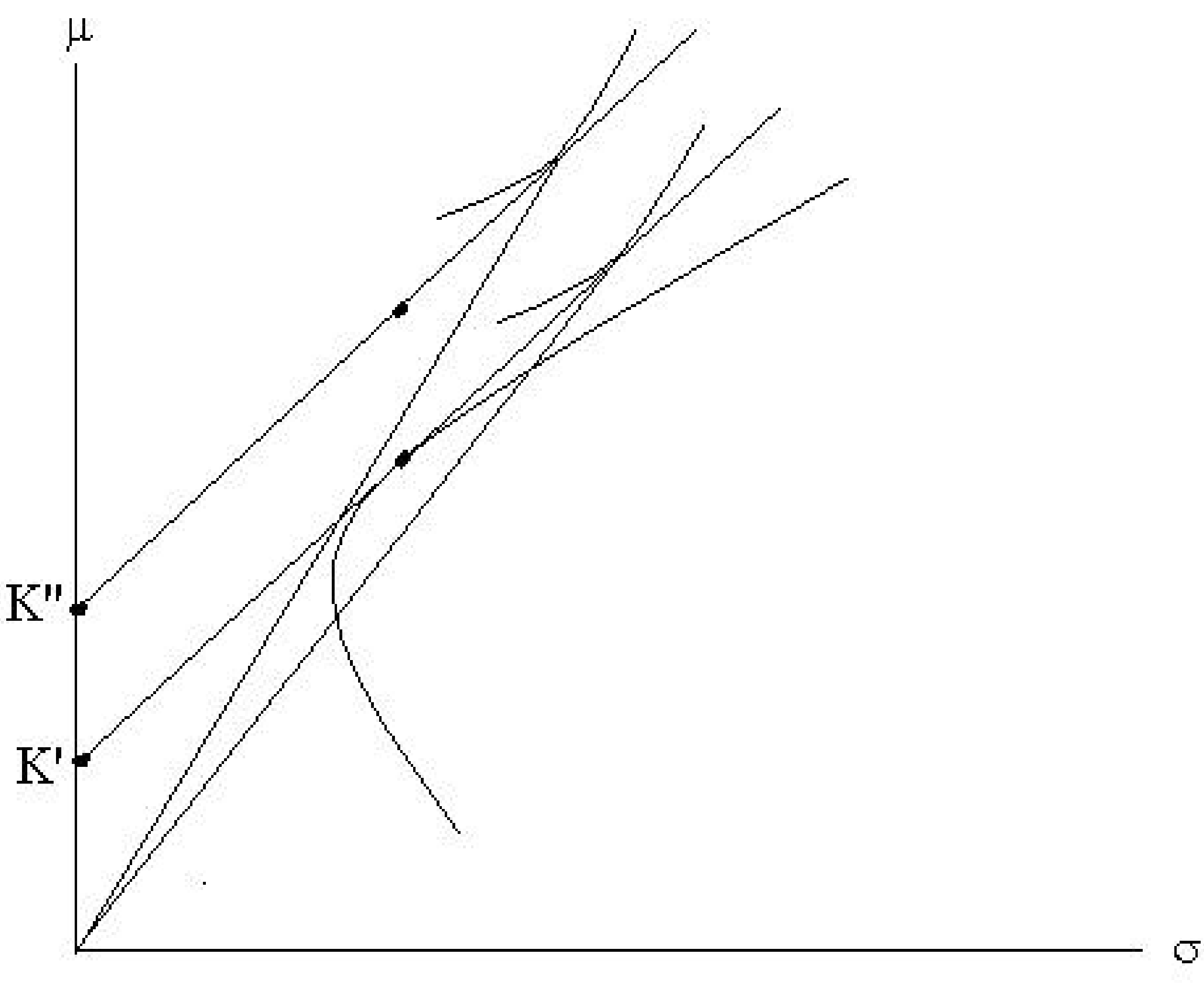
(17) $$K \geq kL'w$$

where $w' = (w_1 .... w_n)$ is the given vector of regulatory risk weights.

The bank's problem is now (8) subject to the constraint (17). The Lagrangian is:

(18) $$U(K' + L's, (L'ML)^{1/2}) - \nu(kL'w - K)$$

Figure 2



where $\nu \geq 0$ is the Lagrange multiplier. The first order conditions are now:

$$(19) \qquad U_\mu s + U_\sigma ML/\sigma - \nu kw = 0,$$

$$(20) \qquad \nu(kL'w - K) = 0.$$

The first one implies that the optimal loan vector for the regulated bank is:

$$(21) \qquad L^\bullet = (1/\theta)M^{-1}(s - \nu kw/U_\mu).$$

If the inequality constraint is not binding, ν must be equal to zero, in which case (21) reduces to (10) and the bank is unaffected by the regulation. On the other hand, if the equality holds in (17), ν is positive and thus (21) shows that any loan with a positive risk weight will be strictly smaller than in the unconstrained case for. As Rochet (1992) stresses, except when the vector of risk weights w is proportional to the vector of spreads s, the optimal loan vector will have a different structure than in the unregulated case. It can be proved that when w is not proportional to s the loan vector does not minimize portfolio variance. Thus, the regulatory authority will have imposed non-optimal risk weights.[11]

Assume now that w is proportional to s, with a strictly positive proportionality factor γ:

$$(22) \qquad w = \gamma s$$

Then the optimal loan supply vector becomes

$$(23) \qquad L^\bullet = (1/\theta)\,(1-\nu k\gamma/U_\mu)M^{-1}s$$

Which is proportional to the unregulated optimal loan supply vector (10). Also, $L_i^\bullet < L_i^*$ whenever $L^\bullet_i > 0$. Furthermore, introducing (22) and (23) in (17) (with equality) gives:

$$(24) \qquad \nu k\gamma/U_\mu = 1 - \theta K/(k\gamma b)$$

[11] Section 5 of the Appendix shows that if w is chosen such that it minimizes portfolio variance for a bank constrained by a minimum capital requirement the optimal w must be proportional to s.

by which $L^{\bullet}$ becomes:

(25) $$L^{\bullet} = [K/(\gamma kb)]M^{-1}s.$$

Thus, inserting (25) in (2') and (3') one can verify that the constrained bank has optimal mean and standard deviation of end of period capital given by:

(26) $$\mu^{\bullet} = K' + K/\gamma k$$

(27) $$\sigma^{\bullet} = K/(\gamma k\sqrt{b}).$$

Furthermore, by (7) and the definition of K', the probability of failure of the constrained bank is:

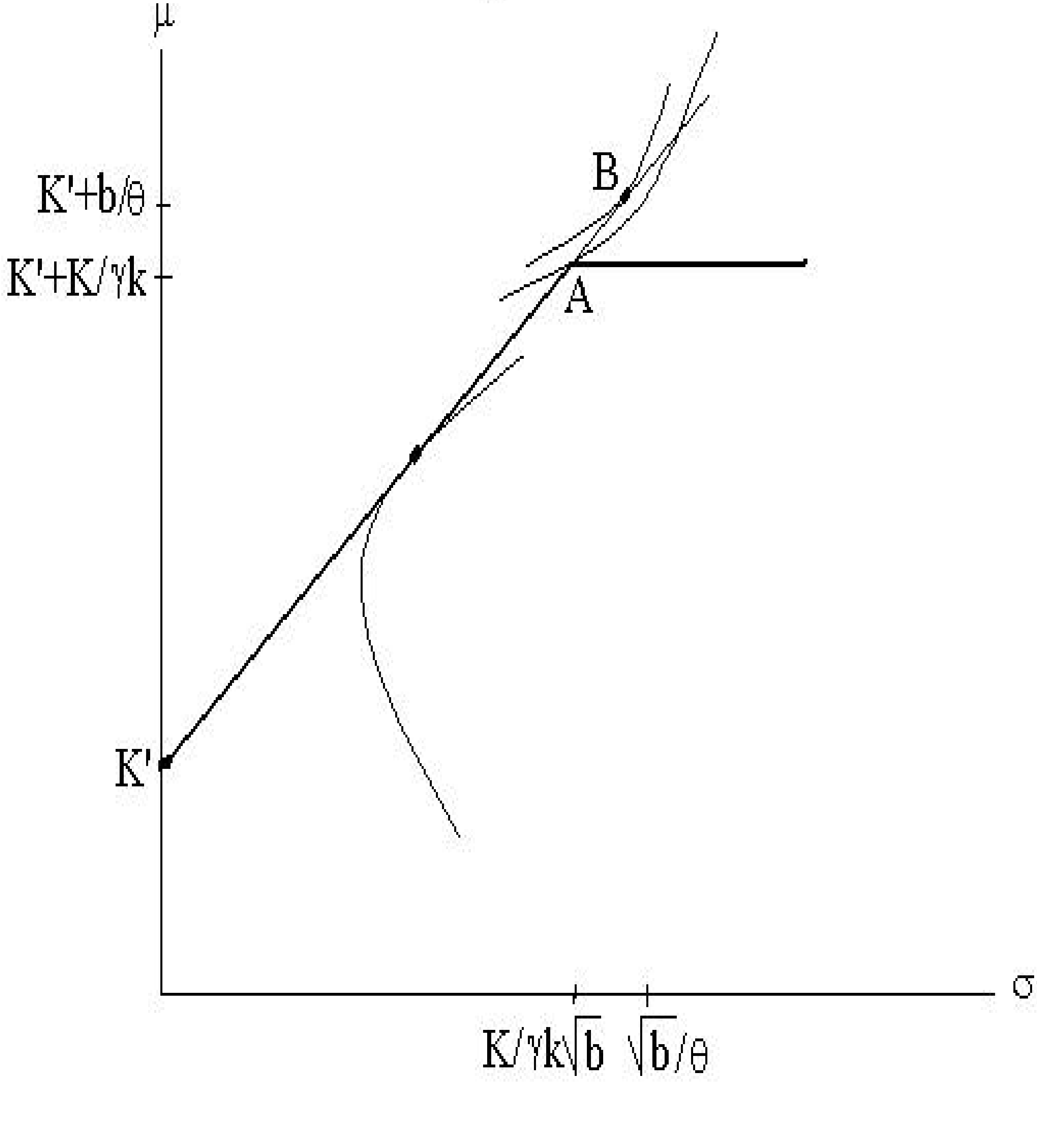
(30) $$\mathrm{Prob}(K_1^{\sim} < 0) = \Phi(- \sqrt{b}\, [k\gamma(1+ r_o - g) + 1]).$$

This shows that the higher the regulatory parameters k and γ are, the lower the probability of failure of a bank that is constrained by the capital requirement.

Graph 3 shows that the effect of the regulation for a constrained bank is to create a corner solution at A. The bank would like to have a higher return and be exposed to a greater risk (at B) but is restricted to the corner solution. A more risk averse bank, however, would not be affected.[12]

[12] Notice that $\mu^{\bullet} < \mu^{*}$ and $\sigma^{\bullet} < \sigma^{*}$ hold if and only if $\theta < \gamma kb/K$, that is, if risk aversion is sufficiently low. Furthermore, pre-multiplying (10) by s' it is seen that $b = \theta s'L^*$. Hence, the previous inequality holds if and only if $K < k(L^{*\prime}\gamma s)$, that is, initial capital is insufficient for the bank to attain its unconstrained optimum.

Figure 3



5. Market equilibrium and banking CAPM

Consider now a finite number B of banks indexed by b=1,...,B. By the previous results, when there is no minimum capital regulation, a bank's optimal loans and reserves are given by:

$$L_b = (1/\theta_b)M^{-1}s \tag{31}$$

$$R_b = D_b + K_b - (1/\theta_b)u'M^{-1}s \tag{32}$$

Therefore, adding over the market supply of loans (and interbank loans) gives:

$$L_M = (1/\theta_M)M^{-1}s \tag{33}$$

$$R_M = D_M + K_M - (1/\theta_M)u'M^{-1}s \tag{34}$$

where u is a vector of ones, the subscript M indicates addition over all banks and the market risk aversion coefficient θ_M is defined by

$$1/\theta_M \equiv \Sigma_b (1/\theta_b). \tag{35}$$

Assume that loan demands are given exogenously by the (vector) function

$$L^D(r, r_o), \tag{36}$$

where each component is a function of the vector of loan interest rates and the interbank interest rate. Equating supply (33) to demand (36) and taking into account the definition of s gives:

$$d + g + r_o u + \theta_M M L^D(r,r_o) = r. \tag{37}$$

This conveniently decomposes the loan interest rate into the sum of expected costs (expected loan losses, administrative costs and financing costs) and a risk premium. The latter is the product of the market risk aversion coefficient and the covariance between the return on a loan and the return on the market loan portfolio.[13]

In the interbank market, the sum of all (positive or negative) supplies must equal zero. Thus, from (34)

$u'M^{-1}s = \theta_M (D_M+K_M)$.

Hence, from the definition of s:

$u'M^{-1}(r - d - g - r_o u) = \theta_M (D_M+K_M)$.

and rearranging:

(38) $$[u'M^{-1}(r-d-g) - \theta_M (D_M+K_M)]/(u'M^{-1}u) = r_o.$$

Notice that (37) y (38) jointly show that (r, r_o) is a fixed point of the transformation defined by the left-hand sides of the two expressions. As seen below, this is a banking CAPM model. Instead of an equilibrium asset price model, however, it is a loan interest rate equilibrium model.

Adding (3') and (4') over all banks and taking into account the equality between aggregate supply and demand:

(39) $$\mu_M = K_M' + L^D(r,r_o)'s$$

(40) $$\sigma_M^2 = L^D(r,r_o)'M\, L^D(r,r_o).$$

Also, (37) is equivalent to

(41) $$\theta_M\, M\, L^D(r,r_o) = s.$$

Pre-multiplying both sides of this equality by L^D, and using (39) and (40) gives:

(42) $$\theta_M = (\mu_M - K_M')/\sigma_M^2.$$

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[13] The covariance between two portfolios (vectors) L_A and L_B is in general $L_A'ML_B$. If $L_A = e_i$ where e_i is the unit vector (with zeros everywhere except the i-th entry) and $L_B = L^D$, that is, the market portfolio, the covariance is $e_iML^D = M_i L^D$ where M_i is the i-th row of M.

Therefore, from (37) the following expression for the banking CAPM is obtained:

(43) $$r - d - g - r_o u = (\mu_M - K_M')\beta$$

where β is the vector of betas:

(44) $$\beta \equiv ML^D(r,r_o)/\sigma_M^2.$$

The elements of this vector are

(45) $$\beta_i \equiv M_i L^D(r,r_o)/\sigma_M^2 = \Sigma_j \sigma_{ij} L_j^D(r,r_o)/\sigma_M^2$$

and essentially reflect the covariance of the return on loan i with the return on the market portfolio. This covariance, divided by the variance of the return on the market portfolio is an adequate measure of the amount of risk that loan i represents. When a loan class has a beta greater than unity, it represents a higher risk than the market loan portfolio (which has a beta equal to one as can be checked by pre-multiplying (44) by $L^D(r,r_o)$) and by (43) commands a higher spread over expected costs than the market portfolio.

It has been said in previous sections (and is proved in the Appendix) that from the regulators point of view the optimal vector of risk weights is proportional to the vector of loan interest spreads over costs. (43) shows that the vector of spreads is also proportional to the vector of betas. This is basically the reason for the optimality of such risk weights: they are proportional to the loan betas, that is, to the non-diversifiable risk of these loans. With such optimal capital requirements (and the assumption of unlimited liability) the regulatory authority can limit banks' risk taking and avoid introducing distortions that lead to regulatory arbitrage by making banks more prone to reshuffle their portfolio of loans.

The preceding argument has assumed that banks are not constrained by minimum capital requirements. But as is shown below, even if some (or all) of the banks are constrained, if the risk weights are optimally designed the preceding argument still holds, although the aggregate loan portfolio will be smaller than in the unconstrained case and the equilibrium interest rate vector will be different.

Let there be N of the B banks that are not constrained by the minimum capital requirement while the other B-N are. Re-index the banks so that the first N are the unconstrained ones. Then the supplies of loans (and interbank loans) are:

$$L_b = (1/\theta_b)M^{-1}s, \qquad b=1,...,N$$

$$L_b = [K/(\gamma kb)]M^{-1}s, \qquad b=N+1,...,n.$$

$$R_b = D_b + K_b - (1/\theta_b)u'M^{-1}s, \qquad b=1,...,N$$

$$R_b = D_b + K_b - [K/(\gamma kb)]u'M^{-1}s \qquad b=N+1,...,n.$$

Adding over all n banks:

(46) $$L_M = (1/\xi)M^{-1}s$$

(47) $$R_M = D_M + K_M - (1/\xi)u'M^{-1}s,$$

where

(49) $$1/\xi \equiv 1/\theta_N + (B-N)K/(\gamma kb)$$

and $1/\theta_N$ is the sum of $1/\theta_b$ over the N unconstrained banks.

Equating aggregate loan supply (46) to aggregate loan demand gives:

(50) $$\xi M\, L^D(r,r_o) + d + g + r_o u = r.$$

and equating the aggregate of all (positive or negative) reserve demands to zero gives:

(51) $$[u'M^{-1}(r-d-g) - \xi(D_M + K_M)]/(u'M^{-1}u) = r_o.$$

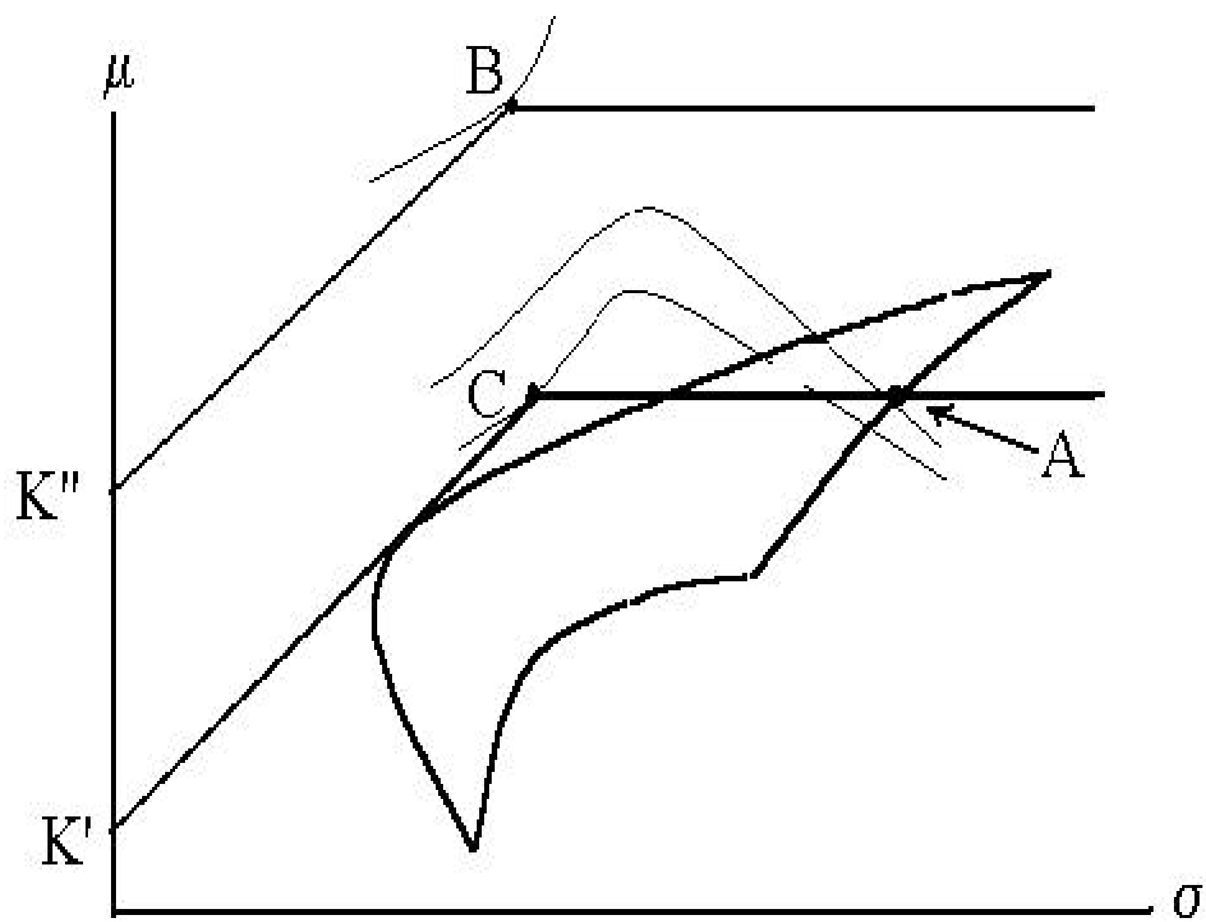
The equilibrium interest rate vector (r, r_o) is now a fixed point of the transformation defined by the left hand sides of (50) and (51). The steps necessary to prove the banking CAPM when there are constrained banks are exactly the same as in the case with no constrained banks. One need only replace θ by ξ and obtain (43) again. Observe, however, that neither the interest rate vector nor the aggregate loan vector will be the same as when there are no constrained banks.

6. The case of limited liability

When there is limited liability the bank receives nothing in states that imply its own failure, and may have a bankruptcy cost C≥0. As in Rochet (1992), expected utility now has two terms, associated to the two possibilities, "non-failure" and "failure", respectively:

(52) $$E(u(\tilde{K}_1)) = \int_{-\mu/\sigma}^{\infty} u(\mu + \sigma y)\phi(y)dy - C\Phi(-\mu/\sigma) \equiv U(\mu,\sigma).$$

Figure 4



The first term is similar to the unlimited liability case except that the integral runs over the y's that imply non-failure ($K_1^{\sim} \geq 0$, which is equivalent to $y^{\sim} = (K_1^{\sim} - \mu)/\sigma > -\mu/\sigma$). The second term reflects the fact that if the bank fails (which has probability $\Phi(-\mu/\sigma)$) there is an exogenous cost C.

The expected utility of end of period capital still depends exclusively on the two parameters μ and σ. However, the function U is no longer necessarily concave or decreasing with σ. Rochet (1992) proves that if μ (and thus K) is sufficiently low, the bank becomes risk loving for σ sufficiently high (that is, U becomes increasing in σ). There is a moral hazard problem derived from the protection that limited liability implies for the bank. In principle, this could invalidate the virtues of optimally designed risk weights along the previous lines. Figure 4 shows that the indifference curves can be decreasing so that there may be cases in which instead of choosing C the bank will prefere A, where the probability of failure is not only greater than in C but also greater than in B, the unregulated bank's optimal choice.

Rochet suggests that a solution might be to impose an additional absolute minimum capital requirement that is independent of any risk weights. This would shift the feasible set upwards to the area where the indifference curves have a more "normal" shape since banks have a large amount at stake. In the case of Argentina, this absolute minimum exists, and currently stands at $15 million, so that presumably the risk loving behavior that banks could have due to limited liability is considerably diminished by their having much to lose if loans turn sour.

7. <u>Conclusions</u>

Argentina's Credit Risk Indicator modifies the standard Basel style risk weights by allowing for a greater differentiation of weights and by making their relative magnitudes a closer approximation to the relative betas of the loans, that is their non-diversifiable risk. Under unlimited liability, optimal risk weights should be proportional to the spread between the loan rate and loan expected costs (financing, administrative, and loan loss). This is due to the fact that these spreads are proportional to the betas of the loans under the assumptions of perfect competition, homogeneous expectations with respect to the distribution of net loan losses, and normality. The betas of the loans reflect the covariance between the return on loans and the return on the market portfolio, and thus reflect the non-diversifiable (or systematic) risk of the loans. This paper shows this and by doing so heuristically suggests the ways in which future reforms of the CRI may help in adjusting the risk weights so that they are closer to the betas of the loans. Since the optimal risk weights should be proportional to the spread between the loan rate and loan expected costs (financing, administrative, and loan loss) this would imply making the Basel style risk weights more homogeneous (i.e. closer to 100%) and using the loan rates as well as whatever information on expected loan costs is available to produce the desired differentiation between the risk weights. The fact that most banks have limited liability introduces considerable complications in the analysis and is not pursued in this paper. However, the fact that Argentina additionally has a high minimum absolute capital requirement should dissuade banks from engaging in extreme risk loving behavior.

Appendix

1 The efficiency frontier when there is a risk free asset

When there is a risk free asset the efficiency frontier can be obtained by minimizing the variance of final capital (4'), given a level for the expected value of final capital:

$$\min_{L} \ (1/2)\, L'ML \quad \text{subject to} \quad \mu_o = K' + L's$$

The first order conditions are:

(A1) $ML - \rho s = 0$

(A2) $\mu_o = K' + L's$

where ρ is the Lagrange multiplier. From (A1):

(A3) $L = \rho\, M^{-1}s.$

Transposing and multiplying by s:

$L's = \rho\, s'M^{-1}s = \rho b$,

where $b \equiv s'M^{-1}s$. Using (A2) the value for ρ at a minimum is:

(A4) $\rho = (\mu_o - K')/b$

so that using (A4) in (A3) gives the loan vector that minimizes variance, given μ_o:

$L = [(\mu_o - K')/b]/ M^{-1}s$.

Also, from (A3) and (A4), the variance of final capital at a minimum is:

(A5) $\sigma^2 = L'ML = \rho^2 b = (\mu_o - K')^2/b$,

so that

$\sigma = |\mu_o - K'|/\sqrt{b}$.

Equivalently,

$\mu_o = K' + \sqrt{b}\sigma$ if $\mu_o \geq K'$
$\mu_o = K' - \sqrt{b}\sigma$ if $\mu_o < K'$.

The efficiency frontier is only the first of these half-lines since only there is expected final capital maximized for a given level of risk.

2 The efficiency frontier when there is no risk free asset

When there is no risk free asset there is the additional restriction that $R \equiv D+K-L'u = 0$. Hence, to find the efficiency frontier one must[14]:

Min (1/2) $L'ML$ subject to $\mu_o = K' + L's$, $D+K = L'u$

[14] Cfr. Merton (1972).

L

where u is the vector of ones. The first order conditions are now:

(A6) $ML - \rho s - \xi u = 0$

(A7) $\mu_o = K' + L's$

(A8) $D+K = L'u.$

From (A6), the optimal loan vector is:

(A9) $L = M^{-1}(\rho s + \xi u)$

so that replacing in (A7) and (A8) and rearranging:

(A10) $\mu_o - K' = \rho b + \xi a.$

(A11) $D + K = \rho a + \xi c$

where

$a \equiv u'M^{-1}s = s'M^{-1}u, \quad b \equiv s'M^{-1}s, \quad c \equiv u'M^{-1}u.$

Solving (A10) and (A11) for ρ and ξ gives:

(A12) $\rho = [c(\mu_o - K') - a(D+K)]/e$

(A13) $\xi = [b(D+K) - a(\mu_o - K')]/e$

where

$e \equiv bc - a^2.$

Pre-multiplying (A6) by L' and using (A7), (A8), (A12) and (A13) gives:

(A14) $\sigma^2 = L'ML = \rho L's + \xi L'u = \rho(\mu_o - K') + \xi(D+K)$

$= (1/e)[c(\mu_o - K')^2 - 2a(D+K)(\mu_o - K') + b(d+K)^2].$

Notice that since M^{-1} is positive definite[15],

$0 < (as - bu)'M^{-1}(as - bu) =$

$= a^2 s'M^{-1}s + b^2 u'M^{-1}u - ab\, s'M^{-1}u - ab\, u'M^{-1}s =$

$= a^2b + b^2c - 2a^2b = b(bc - a^2) = be.$

Since b is positive, so is e. This proves that in the μ_o - σ plane (A14) is the kind of parabola shown in the graphs.

3. Tangency between the two efficiency frontiers

By equating the right hand sides of (A5) and (A14) and remembering the definition of e, it is seen that the point of tangency between the line and the parabola is at the point defined by

(A15) $\mu_o = K' + (b/a)(D+K)$

(A16) $\sigma_o = (\sqrt{b}/a)(D+K).$

Hence, the loan portfolio that corresponds to the tangency point is:

(A17) $L_o = [(D+K)/a]\, M^{-1}s.$

Observe that this portfolio satisfies (A7) and (A8). The bank will choose this point only if (by chance) its risk aversion coefficient is $\theta = a/(D+K)$.

By means of (A15) and (A16) it is easy to see what happens in Graph 1 when K or D changes. If D rises without a change in K, the curve moves to the northeast with no change in the line. There is an additional amount of resources that can be used. If the utility function is of the CARA (constant absolute risk aversion) family[16], θ is constant so that the loan supply L* =

[15] The inverse of a positive definite matrix is also positive definite. Cfr. Goldberger (1964).

[16] A von Neumann-Morgenstern utility function of the CARA family (normalized so that u(0)=0) is:

$$u(K_1^{\sim}) = (1/\theta)[1 - \exp(-\theta K_1^{\sim})].$$

$(1/\theta)M^{-1}s$ does not change (as long as interest rates are constant) and the additional resources are invested in the interbank market.

If K rises with no change in D+K (that is, with a compensating reduction in K) both the line and the curve move upwards by $\Delta K'$. Again, with a CARA utility function the loan supply does not change and the additional resources are invested in reserves. Also, a change in K with no change in D can be obtained by a combination of the preceding two exercises.

4. The effect of minimum capital requirements on the efficiency frontier.

When the regulator determines a vector of risk weights w it is in fact transforming banks' efficiency frontier. The latter is then determined by the following program:

$$\min_{L} (1/2)\, L'ML \quad \text{subject to } \mu_o = K' + L's, \quad K \geq kL'w$$

The lagrangean is:

$$(1/2)\, L'ML + \rho(\mu_o - K' - L's) + \xi(K/k - L'w).$$

where ρ and ξ are the Lagrange multipliers and $\xi \geq 0$. Instead of (A1) and (A2), the first order conditions are now:

(A18) $\quad ML - \rho s - \xi w = 0$

(A19) $\quad \mu_o = K' + L's$

(A20) $\quad \xi(K/k - L'w) = 0.$

If $\xi > 0$, the second constraint must apply with equality and hence the bank is constrained by the capital requirement:

(A21) $\quad K/k - L'w = 0$

From (A18) one obtains

(A22) $L = M^{-1}(\rho s + \xi w)$,

which used in (A19) and (A21) gives:

(A23) $\mu_o - K' = \rho b + \xi a'$.

(A24) $D + K/k = \rho a' + \xi c'$

where

$a' \equiv w'M^{-1}s = s'M^{-1}w$, $b \equiv s'M^{-1}s$, $c' \equiv w'M^{-1}w$.

Thus, solving (A23) and (A24) for ρ and ξ gives:

(A25) $\rho = [c'(\mu_o - K') - a'K/k)]/e'$

(A26) $\xi = [bK/k - a'(\mu_o - K')]/e$

where

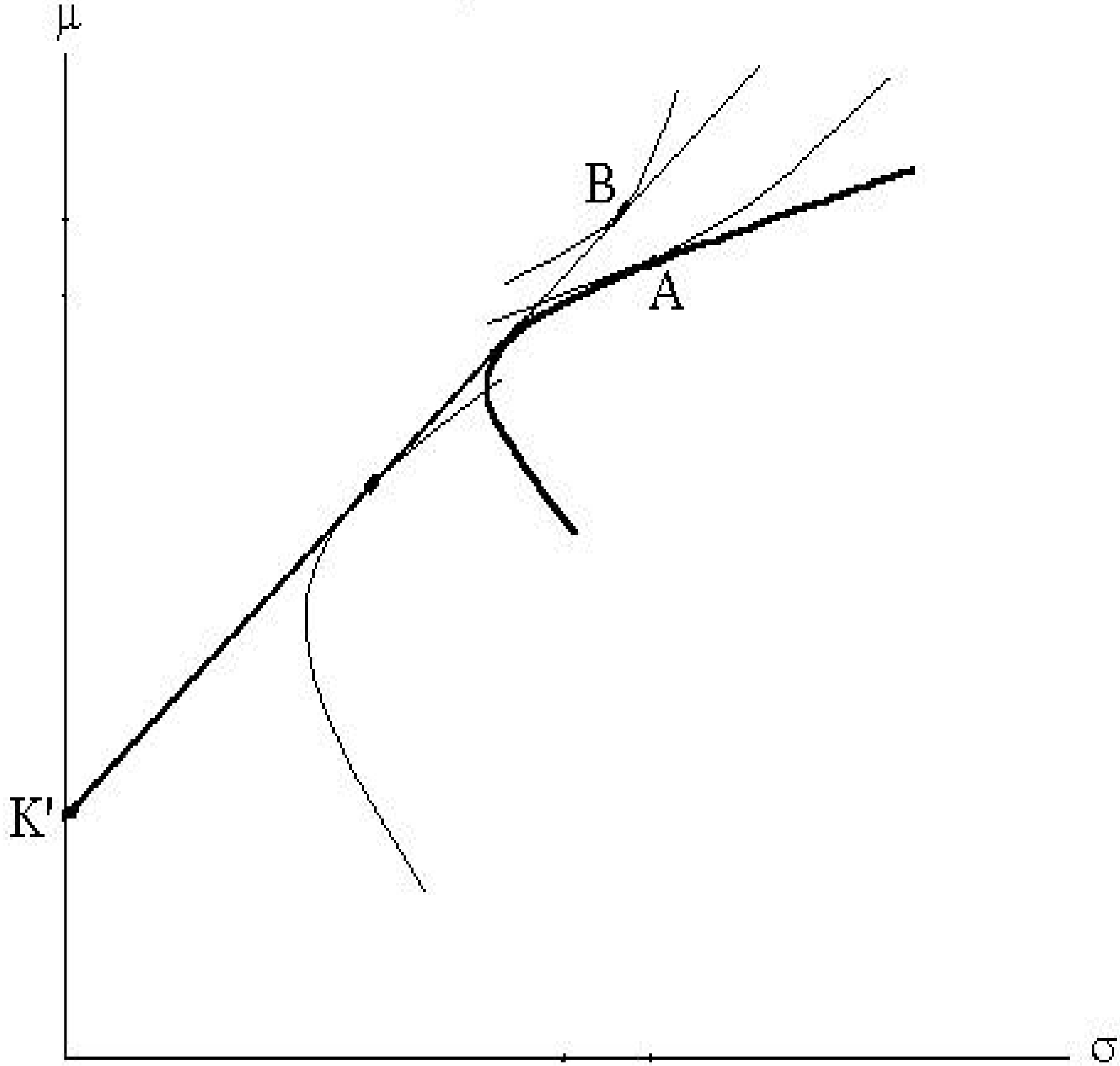
$e' \equiv bc' - (a')^2$.

Using (A25) and (A26) in (A22):

(A27) $L = \{[\, c'(\mu_o - K') - a'K/k]\, M^{-1}s + [bK/k - a'(\mu_o - K')]\, M^{-1}\, w\}/e'$

Therefore, portfolio variance is:

Figure A



(A28) $$\sigma^2 = L'ML = (1/e')[c'(\mu_o - K')^2 - 2a'(K/k)(\mu_o - K') + b(K/k)^2].$$

Analogously to parts 2 and 3 of this Appendix, this formula represents a parabola in the μ, σ plane that is tangent to the efficiency frontier in the case in which there is a risk free asset. But now the bank's efficiency frontier when there is a risk free asset is formed by the half-line until it meets the point of tangency with this new parabola. From that point on, the efficiency frontier continues on the new parabola, as in Figure A.

The bank would prefer to choose point B but must conform itself with point A. However, it is clear that at this point the probability of failure is greater than at point B. Thus, the regulation is a complete failure. The lesson is that the regulator must be careful in designing the risk weight structure. As will be proved below, it can determine the structure of risk weights by minimizing portfolio risk taking into account the way in which the bank determines its optimal portfolio when it is constrained by a minimum capital requirement.

5. Determination of the optimal risk weights.

In section 4 of the text we saw that a bank subject to minimum capital requirements maximizes

(A29) $U(K' + L's, (L'ML)^{1/2})$ subject to $K \geq kL'w$

obtaining the vector of optimal loans

(A30) $L^{\bullet} = (1/\theta)M^{-1}(s - \nu kw/U_{\mu})$.

where $\nu \geq 0$ is the Lagrange multiplier. Furthermore, if ν is positive the bank is constrained by the regulation: $kL'w = K$. Replacing (A30) in this equality gives:

(A31) $\nu k/U_{\mu} = (a' - \theta K/k)/c'$

where a' and c' are the same as in the previous section. Replacing this in (A30) gives

(A32) $L^{\bullet} = (1/\theta)M^{-1}\{s - [(a' - \theta K/k)/c']w\}$.

Therefore, portfolio risk is:

(A33) $\sigma^2 = L^{\bullet\prime}ML^{\bullet} = (1/\theta^2)\{b + [(a' - \theta K/k)^2 - 2a'(a' - \theta K/k)]/c'\}$

Notice that in this formula a' and c' depend on w.

To determine the structure (and not the level) of w we can restrict this vector to a certain hyper-plane that defines its level. It is convenient to define:[17]

(A34) $W \equiv \{w \geq 0 \,/\, a' = \gamma b\} \equiv \{ w \geq 0 \,/\, s'M^{-1}w = \gamma b \}$.

Notice that the vector w = γs that was used in the text is an element of this set. Using (A34) to eliminate a' from (A33) and introducing the definition of c' gives:

(A35) $\sigma^2 = L^{\bullet\prime}ML^{\bullet} = (1/\theta^2)\{b + [(\theta K/k)^2 - (\gamma b)^2]/(w'M^{-1}w)\}$.

Notice that the term in brackets is negative, since so is the left hand side of (A31)[18]. Therefore, if the regulator wants to choose w so as to minimize portfolio variance it is sufficient to minimize $w'M^{-1}w$ subject to $w \in W$, that is:

(A36) $\min_{w} (1/2)w'M^{-1}w$ subject to $s'M^{-1}w = \gamma b$.

The lagrangean is:

(A37) $(1/2)w'M^{-1}w - \lambda(\gamma b - s'M^{-1}w)$

and the first order condition is

(A38) $w'M^{-1} + \lambda s'M^{-1} = 0$.

Post-multiplying by s gives λ = -a'/b, and using this in (A38) gives w = (a'/b)s = γs. To make sure that it is a minimum, differentiate (A39) with respect to w to obtain M^{-1}. Since this matrix is positive definite, the second order condition for a minimum obtains.[19]

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[17] Notice that $W = \{w \geq 0 \,/\, k L^{\bullet\prime}w = K\}$ when $L^{\bullet}$ is defined according to (25) in the text. That is, W is the set of risk weights that makes the bank comply with the capital requirement with equality when the loan vector is determined according to risk weights that are proportional to margins (w=γs).

[18] In particular, c' is positive since w is different from zero and the inverse of a positive definite matrix is also positive definite.

[19] To verify that this is in accordance with the text, insert w = γs in (36) and simplify to obtain $\sigma = K/(\gamma k\sqrt{b})$ as in (27).

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